CORRECTIONS TO SOLUTIONS MANUAL

In the new edition, some chapter problems have been reordered and equations and figure references have changed. The solutions manual is based on the preview edition and therefore must be corrected to apply to the new edition. Below is a list reflecting those changes.

The “NEW” column contains the problem numbers in the new edition. If that problem was originally under another number in the preview edition, that number will be listed in the “PREVIEW” column on the same line. In addition, if a reference used in that problem has changed, that change will be noted under the problem number in quotes. Chapters and problems not listed are unchanged.

For example:

NEW	PREVIEW
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4.18	4.5
“Fig. 4.38”	“Fig. 4.35”
“Fig. 4.39”	“Fig. 4.36”

The above means that problem 4.18 in the new edition was problem 4.5 in the preview edition. To find its solution, look up problem 4.5 in the solutions manual. Also, the problem 4.5 solution referred to “Fig. 4.35” and “Fig. 4.36” and should now be “Fig. 4.38” and “Fig. 4.39,” respectively.

CHAPTER 3

NEW	PREVIEW
-----	---------
3.1	3.8
3.2	3.9
3.3	3.11
3.4	3.12
3.5	3.13
3.6	3.14
3.7	3.15
“From 3.6”	“From 3.14”
3.8	3.16
3.9	3.17
3.10	3.18
3.11	3.19
3.12	3.20
3.13	3.21
3.14	3.22
3.15	3.1

3.16	3.2
3.17	3.2’
3.18	3.3
3.19	3.4
3.20	3.5
3.21	3.6
3.22	3.7
3.23	3.10
3.24	3.23
3.25	3.24
3.26	3.25
3.27	3.26
3.28	3.27
3.29	3.28

CHAPTER 4

NEW	PREVIEW
-----	---------
4.1	4.12
4.2	4.13
4.3	4.14
4.4	4.15
4.5	4.16
4.6	4.17
4.7	4.18
“p. 4.6”	“p. 4.17”
4.8	4.19
4.9	4.20
4.10	4.21
4.11	4.22
4.12	4.23
4.13	4.24
“p. 4.9”	“p. 4.20”
4.14	4.1
“(4.52)”	“(4.51)”
“(4.53)”	“(4.52)”
4.15	4.2
4.16	4.3
4.17	4.4
4.18	4.5
“Fig. 4.38”	“Fig. 4.35”
“Fig. 4.39”	“Fig. 4.36”
4.19	4.6
“Fig 4.39(c)”	“Fig 4.36(c)”

4.20	4.7
4.21	4.8
4.22	4.9
4.23	4.10
4.24	4.11
4.25	4.25
4.26	4.26
“p. 4.9”	“p. 4.20”

CHAPTER 5

NEW	PREVIEW
-----	---------
5.1	5.16
5.2	5.17
5.3	5.18
5.4	5.19
5.5	5.20
5.6	5.21
5.7	5.22
5.8	5.23
5.9	5.1
5.10	5.2
5.11	5.3
5.12	5.4
5.13	5.5
5.14	5.6
5.15	5.7
5.16	5.8
5.17	5.9
5.18	5.10
“Similar to 5.18(a)”	“Similar to 5.10(a)”
5.19	5.11
5.20	5.12
5.21	5.13
5.22	5.14
5.23	5.15

CHAPTER 6

NEW	PREVIEW
-----	---------
6.1	6.7
6.2	6.8

6.3	6.9
“from eq(6.23)”	“from eq(6.20)”
6.4	6.10
6.5	6.11
“eq (6.52)”	“eq (6.49)”
6.6	6.1
6.7	6.2
6.8	6.3
6.9	6.4
6.10	6.5
6.11	6.6
6.13	6.13
“eq (6.56)”	“eq (6.53)”
“problem 3”	“problem 9”
6.16	6.16
“to (6.23) & (6.80)”	“to (6.20) & (6.76)”
6.17	6.17
“equation (6.23)”	“equation (6.20)”

CHAPTER 7

NEW	PREVIEW
-----	---------
7.2	7.2
“eqn. (7.59)”	“eqn. (7.57)”
7.17	7.17
“eqn. (7.59)”	“eqn. (7.57)
7.19	7.19
“eqns 7.66 and 7.67”	“eqns 7.60 and 7.61”
7.21	7.21
“eqn. 7.66”	“eqn. 7.60”
7.22	7.22
“eqns 7.70 and 7.71”	“eqns. 7.64 and 7.65”
7.23	7.23
“eqn. 7.71”	“eqn. 7.65”
7.24	7.24
“eqn 7.79”	“eqn 7.73”

CHAPTER 8

NEW	PREVIEW
-----	---------
8.1	8.5
8.2	8.6

8.3	8.7
8.4	8.8
8.5	8.9
8.6	8.10
8.7	8.11
8.8	8.1
8.9	8.2
8.10	8.3
8.11	8.4
8.13	8.13
“problem 8.5”	“problem 8.9”

CHAPTER 13

NEW	PREVIEW
3.17	3.17
“Eq. (3.123)”	“Eq. (3.119)”

CHAPTER 14 - New Chapter, “Oscillators”

CHAPTER 15 - New Chapter, “Phase-Locked Loops”

CHAPTER 16 - Was Chapter 14 in Preview Ed.

Change all chapter references in solutions manual from 14 to 16.

CHAPTER 17 - Was Chapter 15 in Preview Ed.

Change all chapter references in solutions manual from 15 to 17.

CHAPTER 18 - Was Chapter 16 in Preview Ed.

NEW	PREVIEW
18.3	16.3
“Fig. 18.12(c)”	“Fig. 16.13(c)”
18.8	16.8
“Fig. 18.33(a,b,c,d)”	“Fig. 16.34(a,b,c,d)”

Also, change all chapter references from 16 to 18.

Chapter 14
Oscillators

14.1 Open-Loop Transfer Function:

$$H(s) = \frac{-(g_m R_D)^2}{(1+\frac{s}{\omega_0})^2}, \quad \omega_0 = \frac{1}{R_D C_L}$$

The gain drops to unity at $\frac{g_m R_D}{(1+\frac{\omega_u^2}{\omega_0^2})^{1/2}} = 1$, which for $g_m R_D >> 1$, yields, $\omega_u >> \omega_0$ and $\omega_u \cong \omega_0 \cdot g_m R_D = \frac{g_m}{C_L}$. The phase changes from $-180°$ at $\omega \approx 0$ to $-2\tan^{-1}\frac{\omega_u}{\omega_0} - 180°$ at ω_u; i.e., the phase change at ω_u is $-2\tan^{-1}(g_m R_D)$ and the phase margin is equal to $180° - 2\tan^{-1}(g_m R_D)$.

14.2 (a) $g_m R_D \geq 2 \Rightarrow R_D \geq 400\,\Omega$.

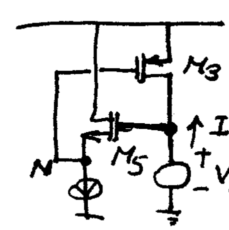
(b) $\begin{cases} \omega_{osc} = \sqrt{3}\,\omega_0 = \sqrt{3}/(R_D C_L) \\ \text{Total Gain} = (g_m R_D)^3 = 16 \Rightarrow R_D = 504\,\Omega \end{cases} \Rightarrow C_L = 0.547\,pF$

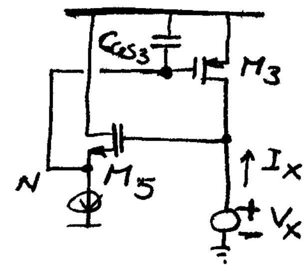
14.3 Each stage must provide a small-signal gain of 2. That is, $g_{m1} R_1 = 2$. With small swings, each transistor carries half of the tail current. For square-law devices, therefore, we have

$$g_{m1} R_1 = 2 = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}}\, R_1 = 2 \Rightarrow$$

$$I_{SS} \geq \frac{4}{\mu_n C_{ox} \frac{W}{L} R_1^2}$$

14.4 Neglecting body effect of M_5, we have $V_N \approx V_X$. Thus, the gate and drain of M_3 experience equal voltage variations. That is, M_3 operates as a diode-connected device, providing an impedance of $1/g_{m3}$.





14.5 $$\frac{V_N}{V_X} = \frac{\frac{1}{C_{GS3}s}}{\frac{1}{C_{GS3}s} + \frac{1}{g_{m5}}} \quad (\gamma = \lambda = 0)$$

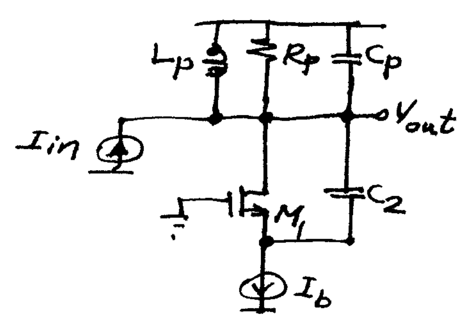
$$= \frac{g_{m5}}{g_{m5} + C_{GS3}s} \Rightarrow \frac{I_X}{V_X} = \frac{g_{m3}g_{m5}}{g_{m5} + C_{GS3}s}$$

$$\Rightarrow \frac{V_X}{I_X} = \frac{1}{g_{m3}} + \frac{C_{GS3}}{g_{m3}g_{m5}}s$$ $\Rightarrow$ The impedance is always inductive.

14.6 To avoid latchup, $g_m R_S < 1 \Rightarrow R_S < \frac{1}{g_m}$.

14.7 The drain currents saturate near I_{SS} and 0 for a short while, creating a "squarish" waveform. The output voltages are the result of injecting the currents into the tanks. Since the tanks provide suppression at higher harmonics, V_X and V_Y are filtered versions of I_{D1} and I_{D2}.

14.8 For the circuit to oscillate, the loop gain must exceed unity: $g_m R_P > 1 \Rightarrow g_m > 1/R_P$. For square-law devices, $\sqrt{\mu_n C_{ox}\frac{W}{L} I_{SS}} > \frac{1}{R_P}$. Thus, $I_{SS} > \frac{1}{\mu_n C_{ox}\frac{W}{L} R_P^2}$.
For M_1 and M_2 not to enter the triode region, the maximum value of V_X and the minimum value of V_Y must differ by no more than V_{TH}. That is, the peak-to-peak swing at X or Y must be less than V_{TH}. Since the peak-to-peak swing is $\approx I_{SS}R_P$, we must have $I_{SS}R_P < V_{TH}$.



14.9 Since the total current flowing thru M_1 and C_2 is equal to I_b, a constant value.
Thus, $\frac{V_{out}}{I_{in}} = (L_P s) \parallel R_P \parallel \frac{1}{C_P s}$.

14.10 Replace R_p with $R_p \| \frac{1}{C_p s} = \frac{R_p}{R_p C_p s + 1}$ in Eq. (14.40). The denominator then reduces to:

$$R_p C_1 C_2 L_p s^3 + (C_1 + C_2) L_p R_p C_p s^3 + (C_1+C_2) L_p s^2 + [g_m L_p R_p C_p s + g_m L_p + R_p(C_1+C_2)]s + g_m R_p$$

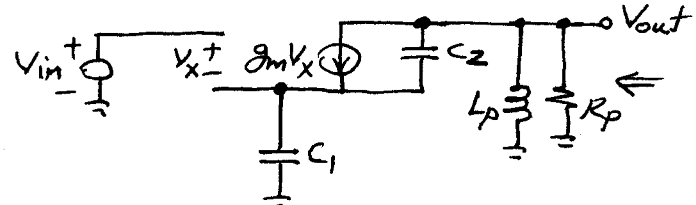
Grouping the imaginary terms and equating their sum to zero, we have

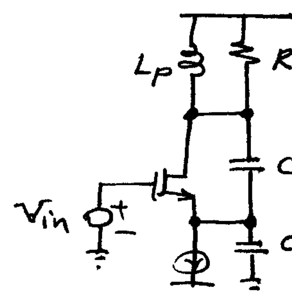
$$-R_p L_p \omega^3 [C_1 C_2 + (C_1+C_2)C_p] + [g_m L_p + R_p(C_1+C_2)]\omega = 0$$

Assuming $g_m L_p \ll R_p(C_1+C_2)$, we obtain

$$\omega^2 = \frac{1}{L_p\left(\frac{C_1 C_2}{C_1+C_2} + C_p\right)}.$$

14.11





The current thru $R_p \| (L_p s)$ is equal to $V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)$. The negative of this current flows thru C_1, generating a voltage $-V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)\frac{1}{C_1 s}$ across it. Thus, $V_X = V_{in} + V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)\frac{1}{C_1 s}$. Also, the current thru C_2 is equal to $\left[V_{out} + V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)\frac{1}{C_1 s}\right] C_2 s$.

Adding $g_m V_X$ and the current thru C_2 and equating the result to $-V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)$, we have

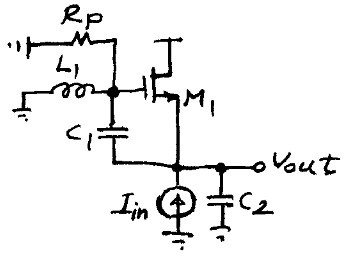
$$\left[V_{in} + V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)\frac{1}{C_1 s}\right] g_{m1} + \left[V_{out} + V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right)\frac{1}{C_1 s}\right] C_2 s = -V_{out}\left(\frac{1}{R_p} + \frac{1}{L_p s}\right).$$

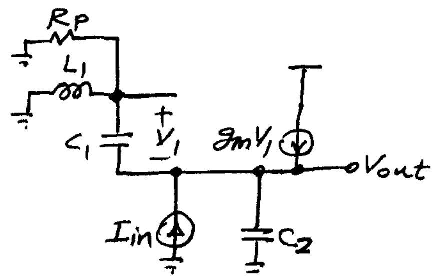
It follows that

$$\frac{V_{out}}{V_{in}} = \frac{-g_m L_p R_p C_1 s^2}{R_p L_p C_2 C_1 s^3 + L_p(C_1+C_2)s^2 + [g_{m1} L_p + R_p(C_1+C_2)]s + g_{m1} R_p}.$$

Note that the denomintor is the same as in Eq. (14.40).

14.12





$$V_1 = -(I_{in} - V_{out} C_2 s + g_m V_1)/C_1 s \Rightarrow V_1 (1 + g_m/C_1 s) = \frac{-I_{in} + V_{out} C_2 s}{C_1 s}$$

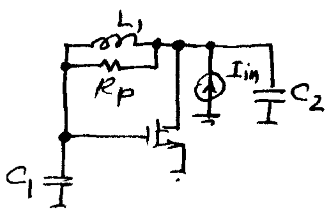
$$\Rightarrow V_1 = \frac{-I_{in} + V_{out} C_2 s}{g_m + C_1 s}$$

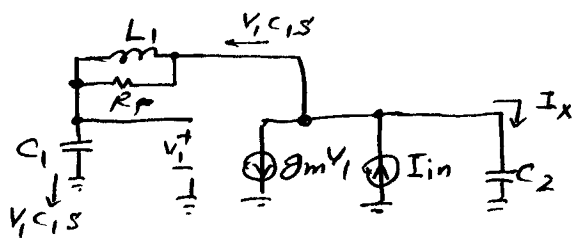
Writing a KVL, we have $\quad -V_1 C_1 s \dfrac{R_p L_1 s}{R_p + L_1 s} = V_1 + V_{out}$.

It follows that

$$V_{out} = -\frac{I_{in} + V_{out} C_2 s}{g_m + C_1 s}\left[1 + \frac{C_1 s R_p L_1 s}{R_p + L_1 s}\right].$$

Simplifying and calculating the denominator of V_{out}/I_{in}, we have $R_p L_1 C_1 C_2 s^3 + L_1(C_1 + C_2)s^2 + [R_p(C_1 + C_2) + g_m L_1]s + g_m R_p$, which is the same as Eq. (14.40). Thus, the oscillation conditions are the same as those of Colpitts oscillator.





We can consider V_1 as the output because for oscillation to begin the gain from I_{in} to V_1 must be infinite as well. First, assume $R_p = \infty$:

$$I_X = +V_1 C_1 s \left(L_1 s + \frac{1}{C_1 s}\right) C_2 s = -g_m V_1 + I_{in} - V_1 C_1 s$$

$$\Rightarrow V_1\left[C_1 C_2 s^2 \left(L_1 s + \frac{1}{C_1 s}\right) + g_m + C_1 s\right] = I_{in}$$

Now, include R_p: $\quad V_1\left[C_1 C_2 s^2\left(\dfrac{R_p L_1 s}{R_p + L_1 s} + \dfrac{1}{C_1 s}\right) + g_m + C_1 s\right] = I_{in}$

$$\Rightarrow V_1\left[\frac{C_1 C_2 s^2 (R_p C_1 L_1 s^2 + R_p + L_1 s) + (g_m + C_1 s)(C_1 s)(R_p + L_1 s)}{C_1 s (R_p + L_1 s)}\right] = I_{in}$$

$\Rightarrow$ denominator of V_1/I_{in} is $\qquad$ ($C_1 s$ is factored from numerator & denominator.)

$$R_p C_1 C_2 L_1 s^3 + R_p C_2 s + L_1 C_2 s^2 + g_m R_p + g_m L_1 s + C_1 R_p s + C_1 L_1 s^2$$

$$= R_p C_1 C_2 L_1 s^3 + L_1(C_1 + C_2)s^2 + [R_p(C_1 + C_2) + g_m L_1]s + g_m R_p,$$

the same as that in Eq. (14.40).

14.13 $\quad I_T = 1\,mA,\ \left(\frac{W}{L}\right)_{1,2} = 50/0.5$

(a) For a three-stage ring, the minimum gain per stage at low freqs must be 2. Thus, $g_{m1,2} R_{1,2} = 2$ (when no current flows thru M_3 and M_4). $\Rightarrow R_{1,2} = 2/g_{m1,2}$. $\left(g_{m1,2} = \sqrt{\mu_n C_{ox} \left(\frac{W}{L}\right)_{1,2} I_T}\,.\right)$

(b) $\quad g_{m3,4} R = 0.5$ with $I_{D3,4} = 0.5\,mA$.

$$g_{m3,4} = \sqrt{\mu_n C_{ox} \left(\frac{W}{L}\right)_{3,4} I_T} = \underbrace{g_{m1,2}}_{=\frac{2}{R}} \sqrt{\frac{(W/L)_{3,4}}{(W/L)_{1,2}}}$$

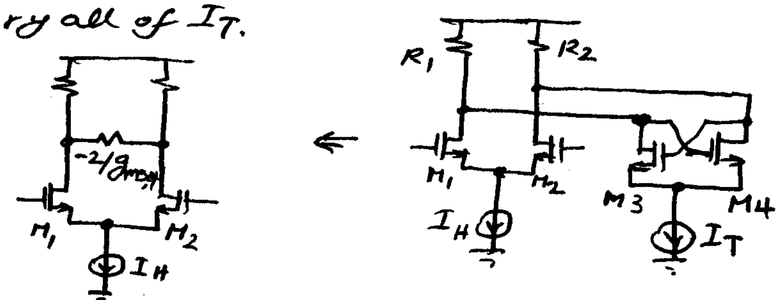
$$\Rightarrow \frac{2}{R} \sqrt{\frac{(W/L)_{3,4}}{(W/L)_{1,2}}}\, R = 0.5$$

$$\Rightarrow (W/L)_{3,4} = 0.25^2\,(W/L)_{1,2}.$$

(c) The voltage gain must be equal to 2 with a diff pair tail current of I_H while M_3 and M_4 carry all of I_T.

$$|A_V| = g_{m1,2}\left(R_{1,2} \,\|\, \frac{-1}{g_{m3,4}}\right)$$

$$= g_{m1,2} \frac{R_{1,2}}{1 - g_{m3,4} R_{1,2}}$$



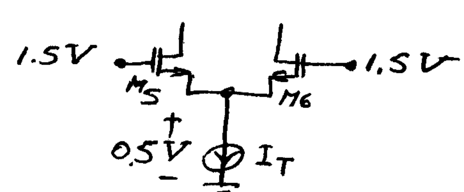
If $g_{m3,4} R_{1,2} < 1$ (to avoid latch-up), then

$$g_{m1,2} R_{1,2} > 2(1 - g_{m3,4} R_{1,2})$$

$$\Rightarrow \sqrt{2 \frac{I_H}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_{1,2}}\, R_{1,2} > 2\left(1 - \sqrt{2 \frac{I_T}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_{3,4}}\, R_{1,2}\right)$$

Thus, I_H can be determined.

(d) Neglecting body effect for simplicity, we have



$$\frac{I_T}{2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_{5,6} (V_{GS5,6} - V_{TH5,6})^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_{5,6} = \frac{I_T}{\mu_n C_{ox} (V_{GS5} - V_{TH5,6})^2} \text{ and } V_{GS5,6} + 0.5\,V = 1.5\,V.$$

14.14 If each inductor contributes a cap of C_1, then

$$f_{osc,min} = \frac{1}{2\pi\sqrt{L(C_0+C_1)}}, \quad f_{osc,max} = \frac{1}{2\pi\sqrt{L(0.62C_0+C_1)}}$$

Thus, the tuning range is given by $\frac{f_{osc,max}}{f_{osc,min}} = \sqrt{\frac{C_0+C_1}{0.62C_0+C_1}}$,

which is less than 27%. For example, if $C_1 = 0.2C_0$, then,

$f_{osc,max}/f_{osc,min} \cong 1.21$.

14.15 (a) $L_P = 5\ nH$, $C_X = 0.5\ pF$ $\quad f_{osc} = 1\ GHz = \frac{1}{2\pi\sqrt{5nH \times (C_X + C_D)}}$

$\Rightarrow C_D = 4.566$ pF.

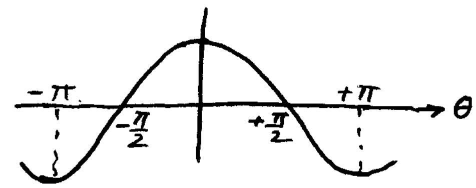
(b) $Q = \frac{L\omega}{R_P} = 4 \Rightarrow R_P = 125.7\ \Omega \Rightarrow$

With a 1-mA tail current, the peak-to-peak swing on each side is approximately equal to 126 mV.

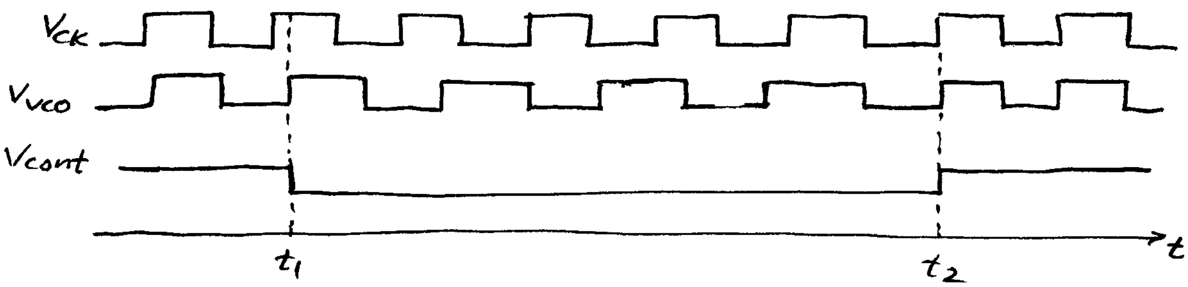
Chapter 15
Phase-Locked Loops

15.1 With two signals $V_1 \cos\omega t$ and $V_2 \cos(\omega t+\theta)$, the product is $V_{out} = \frac{1}{2}V_1V_2\left[\cos(2\omega t+\theta) + \cos\theta\right]$. If the high-freq. component is filtered out, $\overline{V_{out}} \propto \cos\theta$.

· The phase detector is linear only for a small neighborhood around $\theta = \pm\frac{\pi}{2}$.



15.2



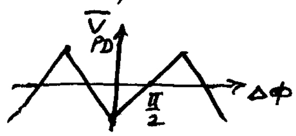
The difference between the two frequencies is integrated between t_1 and t_2 to accumulate a difference of ϕ_0:

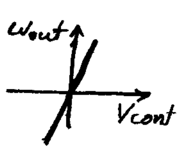
$$(f_H - f_L)(t_2 - t_1) = \frac{\phi_0}{2\pi}$$

$$\Rightarrow \quad t_2 - t_1 = \frac{\phi_0}{2\pi(f_H - f_L)}$$

15.3 The VCO still requires a dc voltage that defines the frequency of operation. A high-pass filter would not provide the dc component.

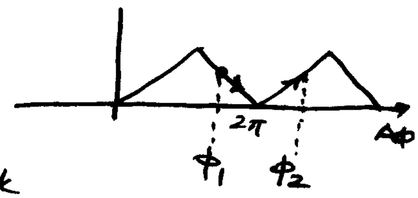
15.4 The loop must lock such that the phase difference is away from zero because the PD gain drops to zero at $\Delta\phi = 0$. With a large loop gain, the PD output settles around half of its full scale. This point can be better seen in a fully-differential implementation:



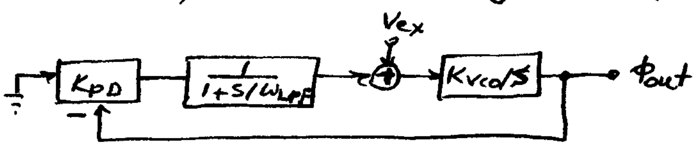


$\Rightarrow$ V_{cont} is small $\Rightarrow$ $\Delta\phi \simeq 90°$.

15.5 Suppose the loop begins with $\Delta\phi = \phi_1$.
If the feedback is positive, the loop accumulates so much phase to drive the PD toward ϕ_2, where the feedback is negative and the loop can settle.



15.6 Note: ϕ_{ex} should be changed to V_{ex}.

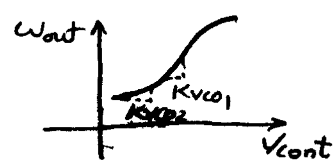


$$\left(-\phi_{out}\cdot K_{PD}\cdot\frac{1}{1+\frac{s}{\omega_{LPF}}}+V_{ex}\right)\frac{K_{VCO}}{s}=\phi_{out}$$

$$\Rightarrow \phi_{out}\left(1+\frac{K_{PD}K_{VCO}}{s(1+\frac{s}{\omega_{LPF}})}\right)=V_{ex}\frac{K_{VCO}}{s}\Rightarrow$$

$$\frac{\phi_{out}}{V_{ex}}=\frac{K_{VCO}}{s+\frac{K_{PD}K_{VCO}}{1+\frac{s}{\omega_{LPF}}}}=\frac{K_{VCO}(1+\frac{s}{\omega_{LPF}})}{\frac{s^2}{\omega_{LPF}}+s+K_{PD}K_{VCO}}$$

15.7 $$\zeta=\frac{1}{2}\sqrt{\frac{\omega_{LPF}}{K_{PD}K_{VCO}}}\qquad\sqrt{\frac{K_{VCO1}}{K_{VCO2}}}=1.5$$

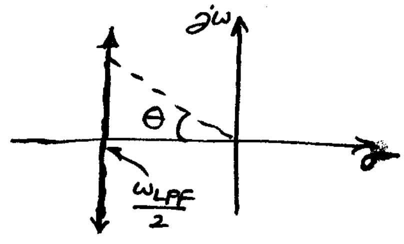


$$\Rightarrow\frac{K_{VCO1}}{K_{VCO2}}=2.25$$

The slope can vary by a factor of 2.25.

15.8 $$\tan\varphi=\frac{Im(pole)}{-Re(pole)}=\frac{\sqrt{1-\zeta^2}}{\zeta}$$

This is indeed as if $\zeta=\cos\varphi$ and $\sqrt{1-\zeta^2}=\sin\varphi$.



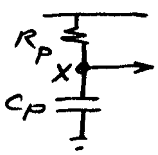
15.9 $K_{VCO}=100\ MHz/V$, $K_{PD}=1\ V/rad$, $\omega_{LPF}=2\pi(1\ MHz)$

$$\Rightarrow\zeta=\frac{1}{2}\sqrt{\frac{1\ MHz}{(1\ V/rad)\ (100\ MHz/V)}}=0.05\qquad\frac{\omega_n}{2\pi}=\sqrt{(1MHz)(1V/rad)(100MHz/V)}=10\ MHz.$$

The loop is heavily underdamped. $\tau=318\ ns$

Step response $\simeq[1-e^{-t/318\,ns}\sin(2\pi\times10\,MHz\times t+\theta)]\,u(t)$, $\theta\cong90°$.

15.10

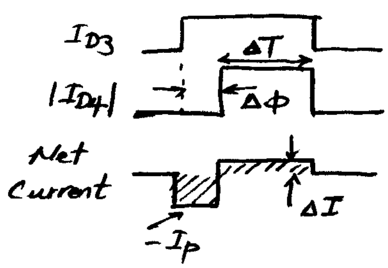


If the control voltage is sensed at node X, then R_P appears in series with the current sources in the charge pump, failing to provide a zero.

15.11 From (15.40), $\frac{I_{out}}{\Delta\phi}(s) = \frac{I_P}{2\pi}$. Since I_{out} is multiplied by the series combination of R_P and C_P:

$$\frac{V_{out}}{\Delta\phi}(s) = \frac{I_P}{2\pi}\left(R_P + \frac{1}{C_P s}\right).$$

15.12



$\Delta\phi$ must be such that the net current is zero. If the current mismatch equals ΔI and the width of $|I_{D4}|$ pulses is ΔT, then

$$\left(\frac{\Delta\phi}{2\pi}\cdot T_P\right) I_P = \Delta T \cdot \Delta I, \text{ where } T_P \text{ is the period.}$$

$$\Rightarrow \Delta\phi = 2\pi \frac{\Delta T}{T_P}\cdot\frac{\Delta I}{I_P}$$

15.13 $\omega_{out} = \omega_0 + K_{VCO} V_{cont}$, $V_{cont} = V_m \cos\omega_m t$. The VCO output is

$$V_{out} = V_0 \cos\left[\int \omega_{out}\, dt\right] = V_0 \cos\left[\omega_0 t + K_{VCO} V_m \int \cos\omega_m t\, dt\right]$$

$$= V_0 \cos\omega_0 t \cos\left(K_{VCO}\frac{V_m}{\omega_m}\sin\omega_m t\right) - V_0 \sin\omega_0 t \sin\left(K_{VCO}\frac{V_m}{\omega_m}\sin\omega_m t\right).$$

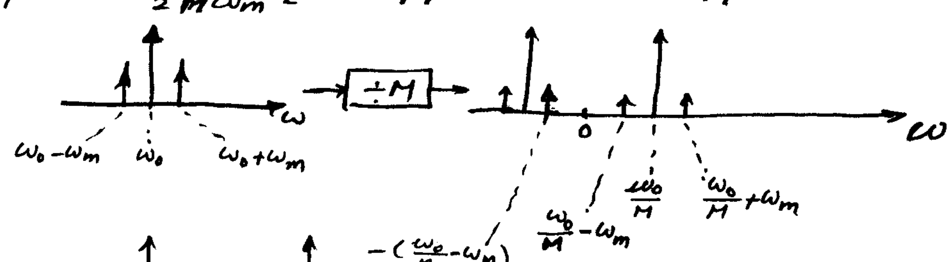
For small V_m, $V_{out}(t) \approx V_0\cos\omega_0 t - \frac{K_{VCO}V_m V_0}{2\omega_m}\left[\cos(\omega_0-\omega_m)t - \cos(\omega_0+\omega_m)t\right]$.

The divider output is expressed as

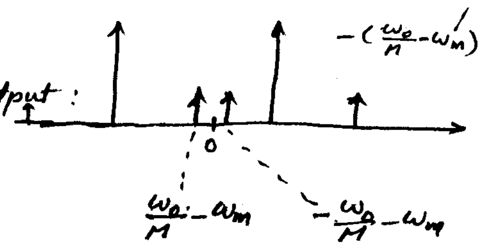
$$V_{out,M} = V_0 \cos\left[\frac{\omega_0 t}{M} + \frac{K_{VCO}V_m}{M}\int\cos\omega_m t\, dt\right]$$

$$\approx V_0 \cos\frac{\omega_0}{M}t - \frac{K_{VCO}V_m V_0}{2M\omega_m}\left[\cos\left(\frac{\omega_0}{M}-\omega_m\right)t - \cos\left(\frac{\omega_0}{M}+\omega_m\right)t\right].$$

If $\frac{\omega_0}{M} > \omega_m$,



If $\frac{\omega_0}{M} > \omega_m$, output:



(aliasing)

15.14 $s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$ $\xi \propto \sqrt{I_pK_{vco}}$

$\omega_n \propto \sqrt{I_pK_{vco}}$

As I_pK_{vco} starts from small values, $s_{1,2}$ are complex:

$Re\{s_{1,2}\} = -\xi\omega_n$ $Im\{s_{1,2}\} = \pm\omega_n\sqrt{1-\xi^2}$.

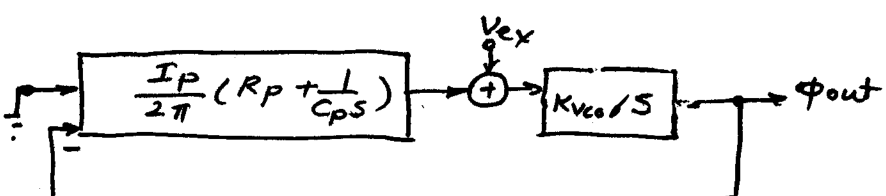
Noting that $\omega_n = \frac{2\xi}{R_pC_p}$, we can write $\omega_n^2 - \frac{2\xi\omega_n}{R_pC_p} = 0$

Adding $(\frac{1}{R_pC_p})^2$ to both sides and subtracting and adding $-\xi^2\omega_n^2$, we obtain $(-\xi\omega_n + \frac{1}{R_pC_p})^2 + \omega_n^2(1-\xi^2) = (\frac{1}{R_pC_p})^2$, which is a circle centered at $-\frac{1}{R_pC_p}$ with a radius equal to $\frac{1}{R_pC_p}$.

For $\xi \geq 1$, the poles become real and move away from each other: $-\xi\omega_n + \omega_n\sqrt{\xi^2-1}$ and $-\xi\omega_n - \omega_n\sqrt{\xi^2-1}$. If $\xi \to \infty$, then $-\xi\omega_n + \omega_n\sqrt{\xi^2-1} = \omega_n(-\xi + \sqrt{\xi^2-1}) = \omega_n\xi(-1+\sqrt{1-\frac{1}{\xi^2}})$

$\approx \omega_n\xi(-1+(1-\frac{1}{2\xi^2})) \approx \frac{-\omega_n}{2\xi} = \frac{-1}{R_pC_p}$.

15.15 Note: ϕ_{ex} should be changed to V_{ex}.



$$\left[-\phi_{out}\cdot\frac{I_p}{2\pi}\left(\frac{R_pC_ps+1}{C_ps}\right) + V_{ex}\right]\frac{K_{vco}}{s} = \phi_{out}$$

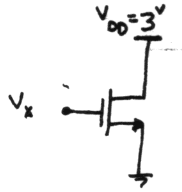
$$\Rightarrow \phi_{out}\left[1 + \frac{I_pK_{vco}(R_pC_ps+1)}{2\pi C_ps^2}\right] = V_{ex}\frac{K_{vco}}{s} \Rightarrow$$

$$\frac{\phi_{out}}{V_{ex}} = \frac{K_{vco}(2\pi C_ps^2)}{2\pi C_ps^2 + I_pK_{vco}R_pC_ps + 1}$$

15.16 When the VCO frequency is far from the input frequency, the PFD operates as a frequency detector, comparing the VCO and input frequencies. Thus, the VCO transfer function must relate the output frequency to the control voltage:
$\Delta\omega_{out} = K_{VCO}\,\Delta V_{cont}$ $\Rightarrow$ the order of the system falls by one (compared to when the VCO phase is of interest: K_{VCO}/s.)

Chapter 2

2.1) a) NMOS :

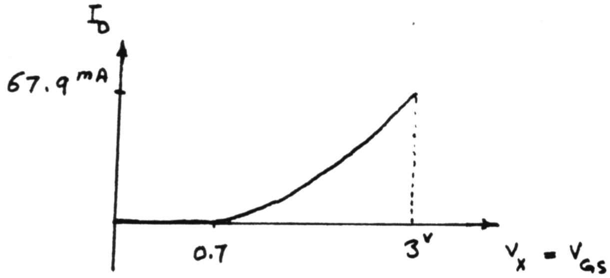


for $V_x < V_{TH} (=0.7)$ device is off , $I_D \simeq 0$

for $V_x \geq 0.7$

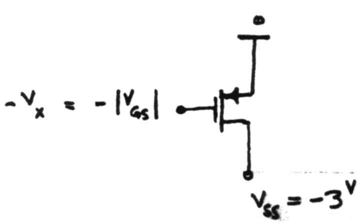
$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L_{eff}} (V_x - 0.7)^2 (1+\lambda \cdot 3^V) \quad (L_{eff} = 0.5^{\mu} - 2L_D)$$

$$I_D = 12.8 \left(\frac{mA}{V^2}\right) \cdot (V_x - 0.7)^2$$



b) PMOS :

Solution is the same

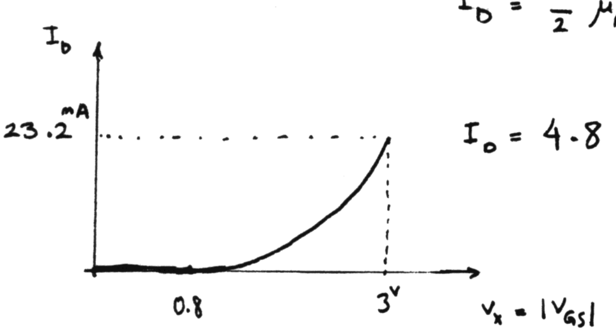


for $|V_{GS}| < V_{TH} (=0.8)$ $I_D \simeq 0$

for $|V_{GS}| \geq 0.8$

$$I_D = \frac{1}{2}\mu_p C_{ox} \frac{W}{L_{eff}} (V_x - 0.8)^2 (1+\lambda \cdot 3^V)$$

$$I_D = 4.8 \left(\frac{mA}{V^2}\right) \cdot (V_x - 0.8)^2$$



2.2) a) N mos

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 3.66 \ \frac{mA}{V} \qquad (\text{Neglecting } L_D)$$

$$r_o = \frac{1}{\lambda I_D} = 20^{K\Omega}$$

$$\text{Intrinsic gain} = g_m r_o = 7\ 33 \ \frac{V}{V}$$

b) PMOS

$$g_m = \sqrt{2\mu_p C_{ox} \frac{W}{L} I_D} = 1.96 \ \frac{mA}{V}$$

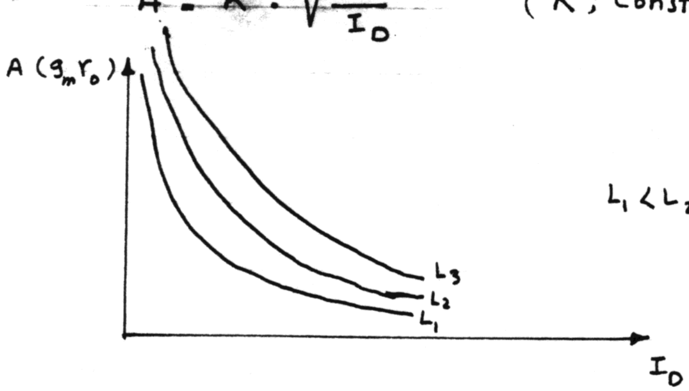
$$r_o = \frac{1}{\lambda I} = \frac{1}{0.2 \cdot 0.5^{mA}} = 10^{K\Omega}$$

$$g_m r_o = 19.6 \ \frac{V}{V}$$

2.3) $g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_D}$ $\qquad r_o = \frac{1}{\lambda I_D}$ $\qquad$ Assume $\lambda = \frac{\alpha}{L}$

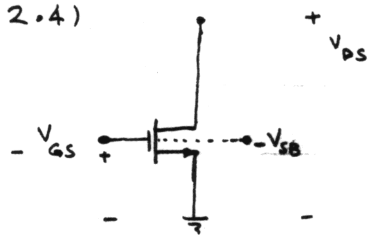
$$A = g_m r_o = \sqrt{2\mu C_{ox} \frac{W}{L} I_D} \cdot \frac{L}{\alpha I_D}$$

$$A = K \cdot \sqrt{\frac{WL}{I_D}} \qquad (K ; \text{Constant})$$



$L_1 < L_2 < L_3$

2.4)



I_D versus V_{GS} : (for NMOS)

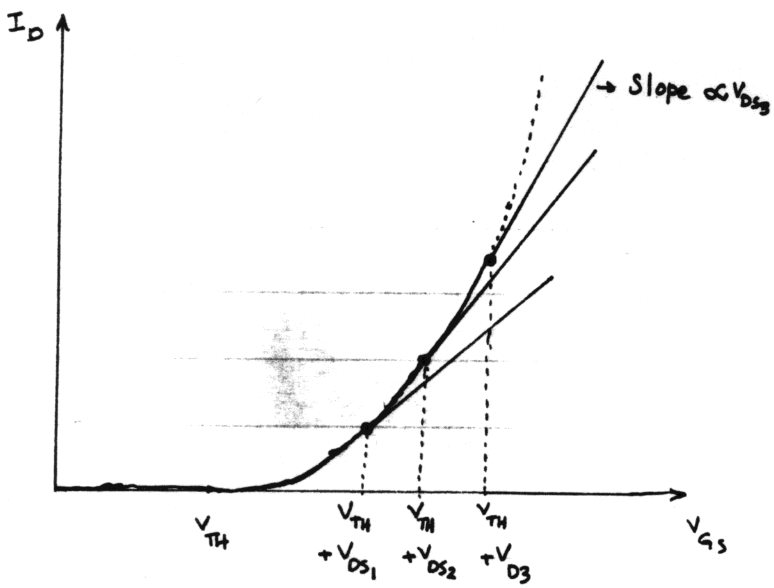
I) for $V_{GS} < V_{TH}$, $I_D \approx 0$

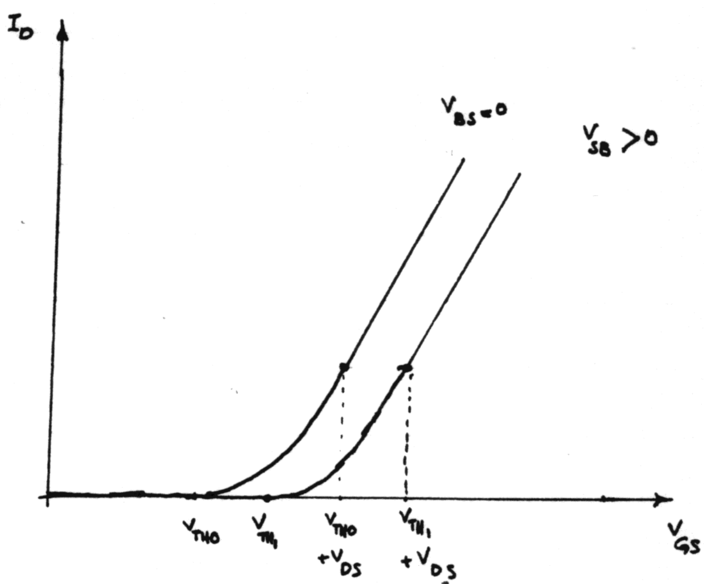
II) for $V_{TH} < V_{GS} < V_{TH} + V_{DS}$ $\Rightarrow$ Device is in the Saturation region

$$I_D \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$$

III) for $V_{GS} > V_{TH} + V_{DS}$ $\Rightarrow$ Device Operates in the triode region

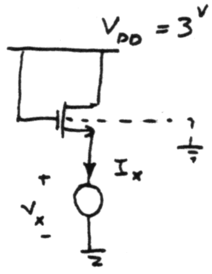
$$I_D \approx \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$





Changing V_{SB} just shifts the curve to the right for $V_{SB} > 0$ or to the left for $V_{SB} < 0$

2.5) a)



$\lambda = 0.1$, $\gamma = 0.45$, $2\phi_F = 0.9$, $V_{TH0} = 0.7$

$V_{GS} = 3 - V_x$, $V_{DS} = 3 - V_x$, $V_{SB} = V_x$

$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)$

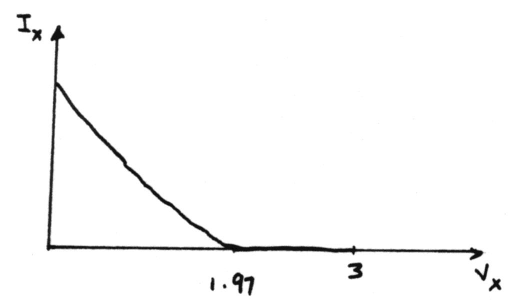
So, $I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(3 - V_x - 0.7 - 0.45\left(\sqrt{0.9+V_x} - \sqrt{0.9}\right)\right)^2\left(1 + \lambda(3 - V_x)\right)$

The above equation is valid for

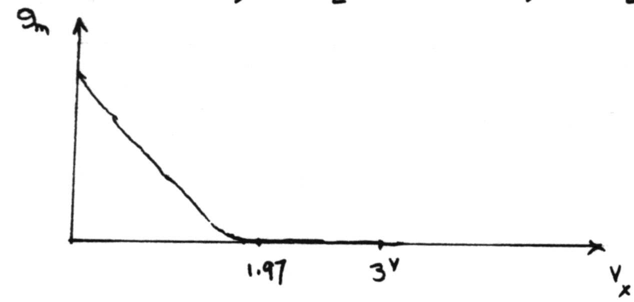
$3 - V_x - 0.7 - 0.45\left(\sqrt{0.9+V_x} - \sqrt{0.9}\right) > 0$, i.e. $V_x < 1.97^V$

So, $I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2.727 - V_x - 0.45\sqrt{0.9+V_x}\right)^2(1.3 - 0.1V_x)$

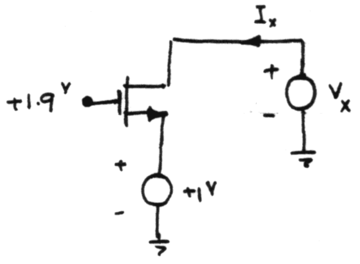
and $I_x = 0$ for $1.97 < V_x$



$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D} = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_x}$



2.5) b)



$\lambda = \gamma = 0 \qquad V_{TH} = 0.7$

for $0 < V_x < 1$, S and D exchang their roles.

$$V_{GS} = 1.9 - V_x \qquad V_{DS} = 1 - V_x \quad , V_{OD} = 1.2 - V_x$$

$$I_X = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(1.2 - V_X)\times 2\times(1 - V_X) - (1 - V_X)^2\right]$$

$$I_X = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(1 - V_X)(1.4 - V_X).$$

$$g_m = \mu_n C_{ox}\frac{W}{L}V_{DS} = \mu_n C_{ox}\frac{W}{L}(1 - V_X) \quad (\text{absolute Value})$$

The above equations are valid for $V_x < 1$

Then the direction of current is reversed.

$$V_{GS} = 1.9 - 1 = 0.9 \qquad V_{DS} = V_X - 1 \quad , V_{OD} = 0.9 - 0.7 = 0.2$$

for $V_X < 1.2$, device operates in the triode region.

$$I_X = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\times 0.2\times(V_X - 1) - (V_X - 1)^2\right]$$

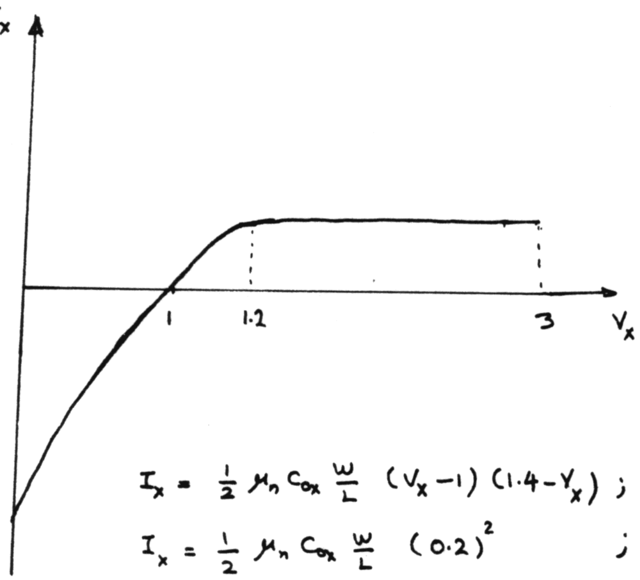
$$g_m = \mu_n C_{ox}\frac{W}{L}(V_X - 1)$$

for $V_x > 1.2$, Device goes into saturation region

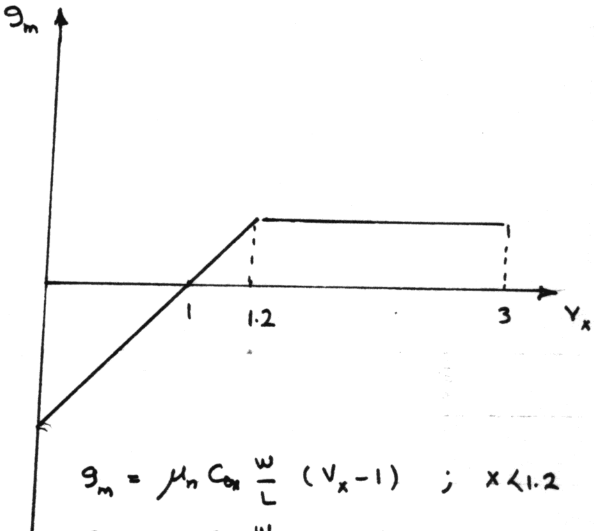
2.5) b Cont

$$\text{So,} \quad I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.2)^2 \,,$$

$$g_m = \mu_n C_{ox}\frac{W}{L}(0.2)$$

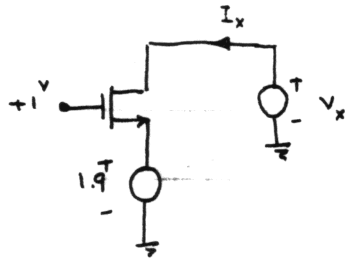


$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_x-1)(1.4-V_x) \; ; \; x<1.2$$
$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.2)^2 \; ; \; x>1.2$$



$$g_m = \mu_n C_{ox}\frac{W}{L}(V_x-1) \; ; \; x<1.2$$
$$g_m = \mu_n C_{ox}\frac{W}{L}(0.2) \; ; \; x>1.2$$

2.5) c



$\lambda = \gamma = 0$ $\qquad$ $V_{TH} = 0.7$

S and D exchange their roles.

$V_{GS} = 1 - V_x \qquad V_{DS} = 1.9 - V_x \qquad V_{OD} = V_{GS} - V_{TH} = 0.3 - V_x$

Device is in Saturation region, So, $I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.3 - V_x)^2$

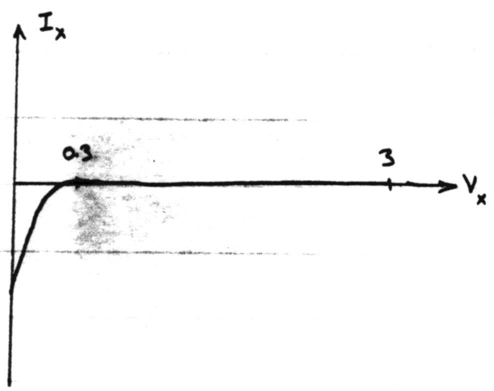
Device turns off when $V_x = 0.3$ and never turns on again.

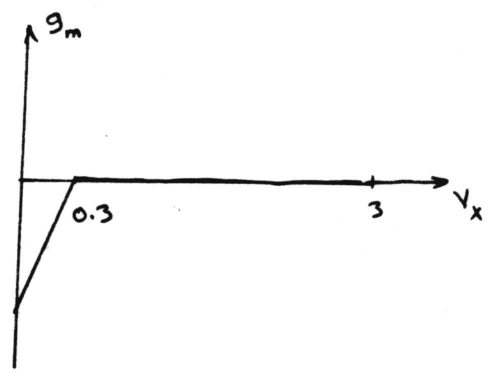
So, $\quad I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.3 - V_x)^2 \quad ; x < 0.3$

$I_x = 0 \quad$; otherwise

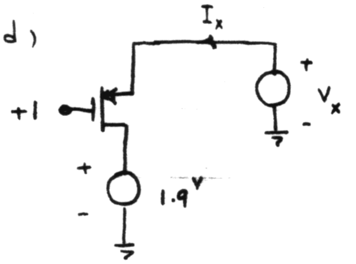
Then $\quad g_m = -\mu_n C_{ox}\frac{W}{L}(0.3 - V_x) \quad ; x < 0.3$

$g_m = 0 \quad$; Otherwise





2.5) d)



$V_{TH} = -0.8$ $\quad \gamma = 0$

D and S exchang their roles.

$V_{GS} = -0.9$ $\quad V_{DS} = V_x - 1.9$

for $V_x < 1.8$:

$$I_x = -\frac{1}{2}\mu_p C_{ox}\frac{W}{L}(0.1)^2$$

$$g_m = -\mu_p C_{ox}\frac{W}{L}(0.1)$$

Device remains in the saturation region until

$V_x = 1.9 - 0.1 = 1.8$, then device goes into the triode

region.

for $1.8 < V_x < 1.9$:

$$I_x = -\mu_p C_{ox}\frac{W}{L}\left[(-0.1)(V_x - 1.9) - \frac{1}{2}(V_x - 1.9)^2\right]$$

$$g_m = +\mu_p C_{ox}\frac{W}{L}(V_x - 1.9)$$

for $V_x > 1.9$:

S and D exchange their roles again, when $V_x = 1.9$

for $V_x > 1.9$, Device operates in the triode region.

$V_{GS} = 1 - V_x$, $\quad V_{DS} = 1.9 - V_x$

$$I_x = +\mu_p C_{ox}\frac{W}{L}\left[(1.8 - V_x)(1.9 - V_x) - \frac{1}{2}(1.9 - V_x)^2\right]$$

$$g_m = -\mu_p C_{ox}\frac{W}{L}(1.9 - V_x)$$

2.5) d So, $0 < V_x < 1.8$

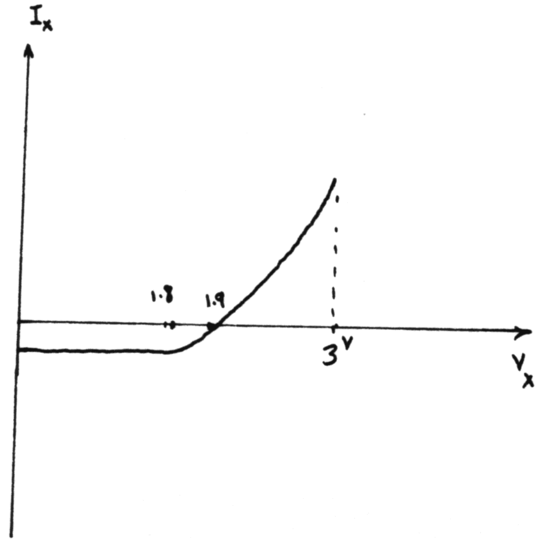
$$I_x = -\frac{1}{2}\mu_p C_{ox} \frac{W}{L} (0.1)^2$$

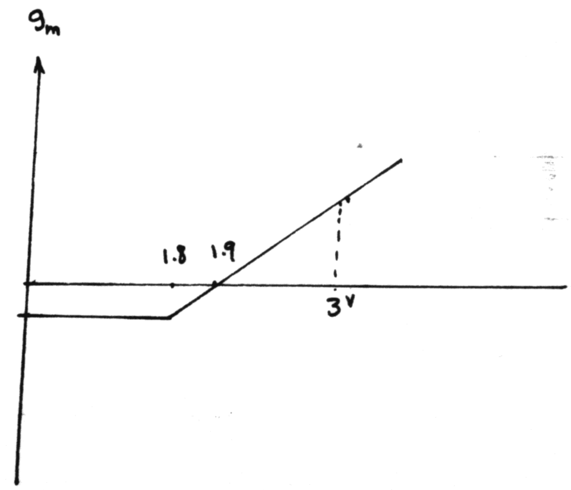
$$g_m = -\mu_p C_{ox} \frac{W}{L} (0.1)$$

$1.8 < V_x < 3$

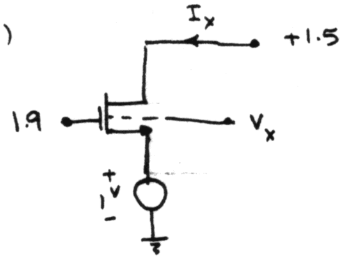
$$I_x = +\mu_p C_{ox} \frac{W}{L} \times \frac{1}{2} (V_x - 1.9)(V_x - 1.7)$$

$$g_m = \mu_p C_{ox} \frac{W}{L} (V_x - 1.9)$$





2.5) e)



$V_{TH0} = 0.7$ $\quad \gamma = 0.45$ $\quad 2\phi_F = 0.9$ $\quad , \lambda = 0$

$V_{SB} = 1 - V_x$

$V_{TH} = 0.7 + 0.45\left(\sqrt{0.9 + 1 - V_x} - \sqrt{0.9}\right)$

$V_{GS} = 0.9$ $\quad V_{DS} = 0.5$

for $V_x = 0$, $V_{TH} = 0.893$ So device is in Saturation region.

So $\quad I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(0.2 - 0.45\left(\sqrt{1.9 - V_x} - \sqrt{0.9}\right)\right)^2$

$$g_m = \mu_n C_{ox}\frac{W}{L}\left(0.2 - 0.45\left(\sqrt{1.9 - V_x} - \sqrt{0.9}\right)\right)$$

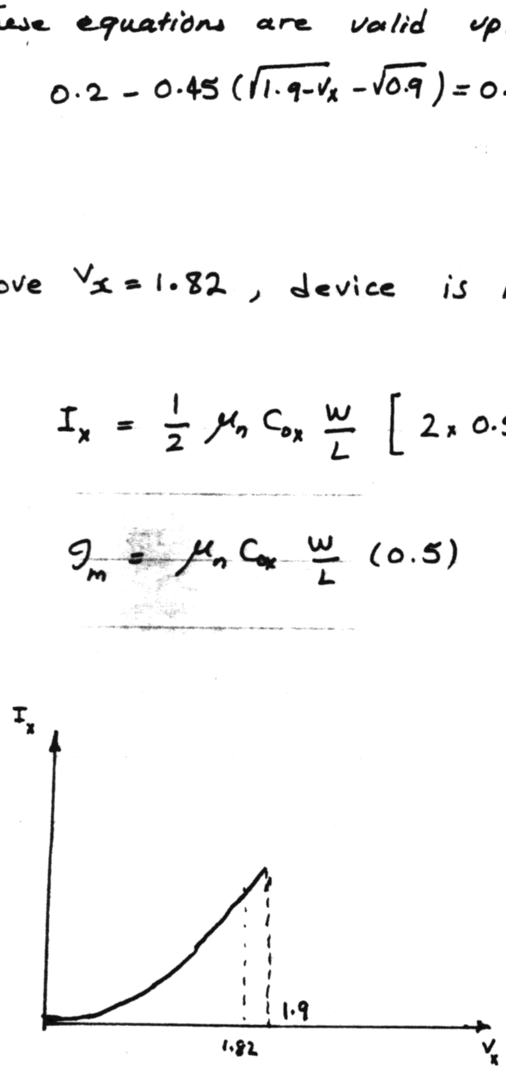
These equations are valid upto the edge of triode region, i.e.

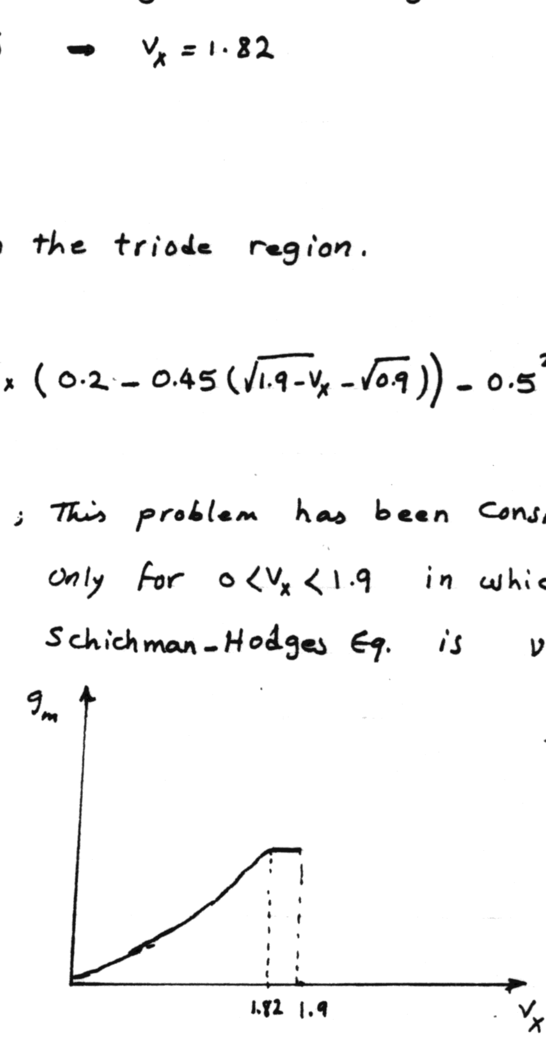
$0.2 - 0.45\left(\sqrt{1.9 - V_x} - \sqrt{0.9}\right) = 0.5 \quad \rightarrow \quad V_x = 1.82$

Above $V_x = 1.82$, device is in the triode region.

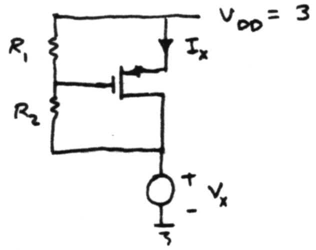
$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2 \times 0.5 \times \left(0.2 - 0.45\left(\sqrt{1.9 - V_x} - \sqrt{0.9}\right)\right) - 0.5^2\right]$$

$g_m = \mu_n C_{ox}\frac{W}{L}(0.5)$; This problem has been considered only for $0 < V_x < 1.9$ in which Schichman-Hodges Eq. is valid for V_{TH}.





2.6) a)



$\gamma = 0$

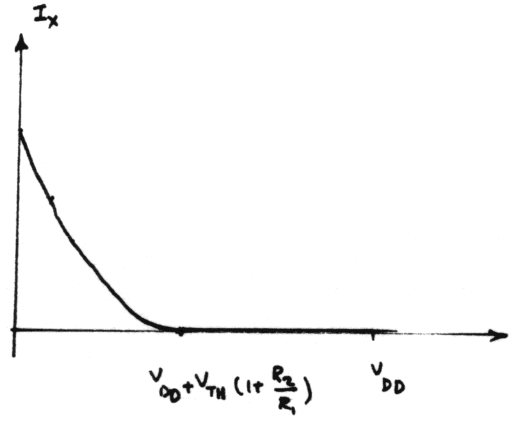
$$V_{SG} = (V_{DD} - V_X)\frac{R_1}{R_1 + R_2} \qquad V_{SD} = V_{DD} - V_X$$

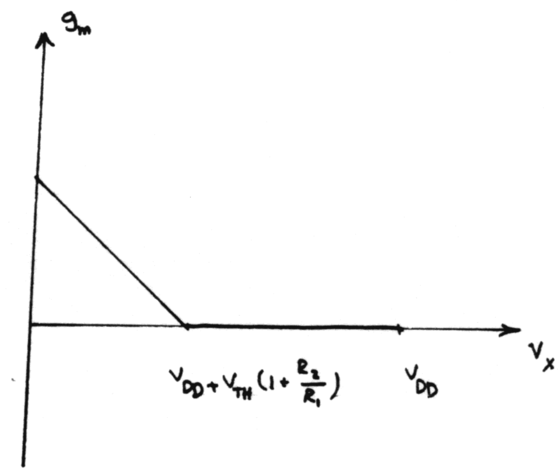
for $V_{SG} > |V_{TH}|$ Device is in the Saturation region (Device is off; otherwise)

$$(V_{DD} - V_X)\frac{R_1}{R_1+R_2} > -V_{TH}$$

$$V_X < V_{DD} + V_{TH}\left(1 + \frac{R_2}{R_1}\right) \Rightarrow I_X = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[(V_{DD}-V_X)\frac{R_1}{R_1+R_2} + V_{TH}\right]^2$$

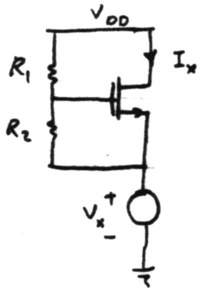
$$g_m = \mu_p C_{ox}\frac{W}{L}\left[(V_{DD}-V_X)\frac{R_1}{R_1+R_2} + V_{TH}\right]$$





If $V_{DD} + V_{TH}\left(1+\frac{R_2}{R_1}\right) < 0$ (e.g. for small value of R_1), device never turns on!

2.6) b)



$\gamma = 0$

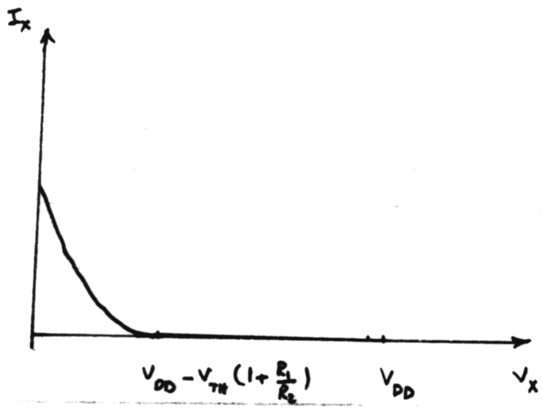
$$V_{GS} = (V_{DD} - V_x)\frac{R_2}{R_1 + R_2} \qquad V_{DS} = V_{DD} - V_x$$

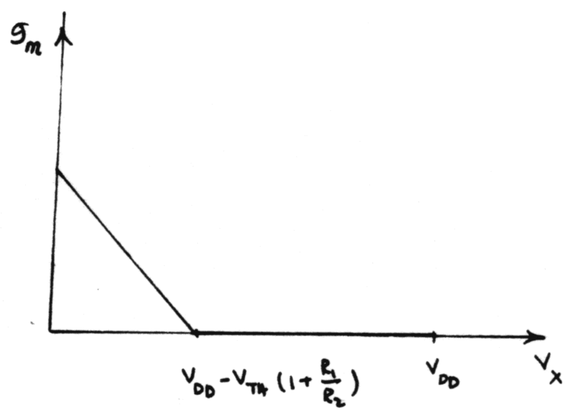
for $V_{GS} > V_{TH}$, Device is in the saturation region and

$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(V_{DD} - V_x)\frac{R_2}{R_1+R_2} - V_{TH}\right]^2$$

$$g_m = \mu_n C_{ox}\frac{W}{L}\left[(V_{DD} - V_x)\frac{R_2}{R_1+R_2} - V_{TH}\right]$$

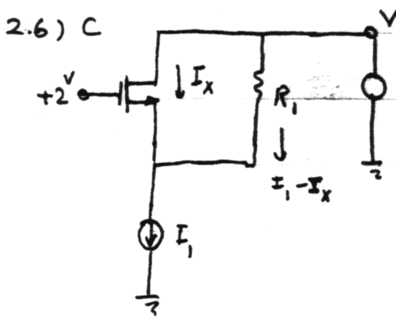
for $V_x < V_{DD} - V_{TH}\left(1 + \frac{R_1}{R_2}\right)$ (i.e. $V_{GS} > V_{TH}$)





If $V_{DD} - V_{TH}\left(1 + \frac{R_2}{R_1}\right) < 0$ device doesn't turn on.

2.6) C



I_x and $I_R = I_1 - I_x$ have the same polarity

So, $0 \leqslant I_x \leqslant I_1$

for $0 < V_x < 2 - V_{TH}$ (1.3) Device is in the triode.

$$V_{GS} = 2 - V_x + R_1 (I_1 - I_x) \quad , \quad V_{DS} = R_1 (I_1 - I_x)$$

$$I_x = I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH}) - V_{DS} \right] V_{DS}$$

$$\Rightarrow (*) \; I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[R_1 (I_1 - I_x) + 2(2 - V_{TH} - V_x) \right] \left\{ R_1 (I_1 - I_x) \right\}$$

The above equation peresents $I_x - V_x$ characteristics in this region.

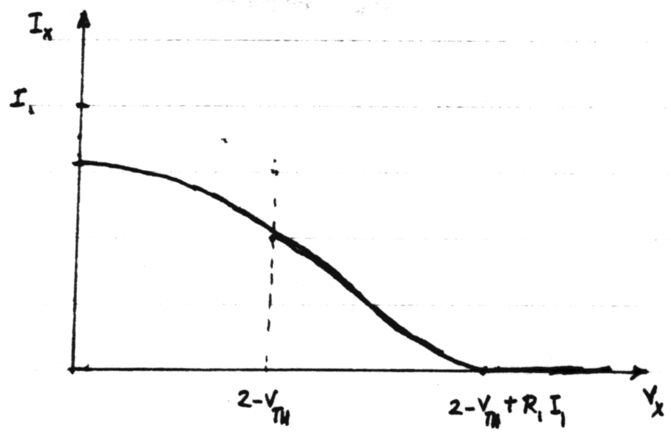
In this region $g_m = \mu_n C_{ox} V_{DS} = \mu_n C_{ox} R_1 (I_1 - I_x)$

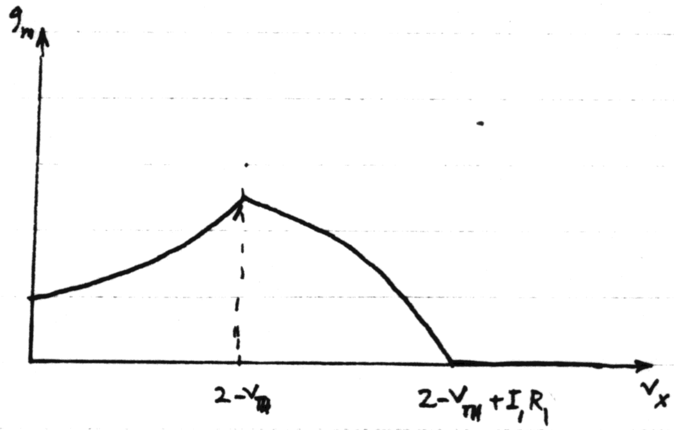
Then device enters the Saturation region; $V_{GS} = 2 - V_x + R_1 (I_1 - I_x)$

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 - V_x + R_1 (I_1 - I_x) - V_{TH} \right]^2$$

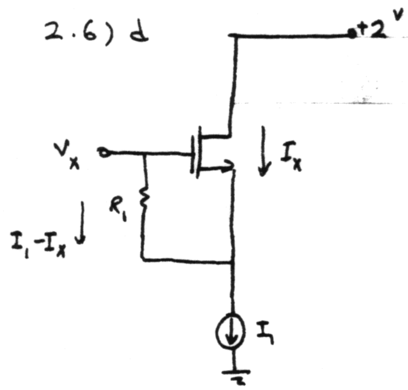
$$g_m = \mu_n C_{ox} \frac{W}{L} \left[2 - V_x + R_1 (I_1 - I_x) - V_{TH} \right]$$

Then device turns off when $V_x = 2 - V_{TH} + R_1 I_1$





2.6) d



Assumption : $R_1 I_1 > V_{TH}$

for $0 < V_x < 2 + V_{TH}$: Device is in the saturation region

$$V_{GS} = R_1 (I_1 - I_x)$$

$$I_D = I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[R_1 (I_1 - I_x) - V_{TH} \right]^2$$

I_x is a constant that can be derived by solving the above equation.

Then device enters the triode region for $V_x > 2 + V_{TH}$

In this case $V_{GS} = R_1 (I_1 - I_x)$ $\quad V_{DS} = 2 - \left[V_x - R_1 (I_1 - I_x) \right] = 2 - V_x + R_1 (I_1 - I_x)$

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 (V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \right] = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 \left[R (I_1 - I_x) - V_{TH} \right] - 2 + V_x - R_1 (I_1 - I_x) \right] \times \left(2 - V_x + R_1 (I_1 - I_x) \right)$$

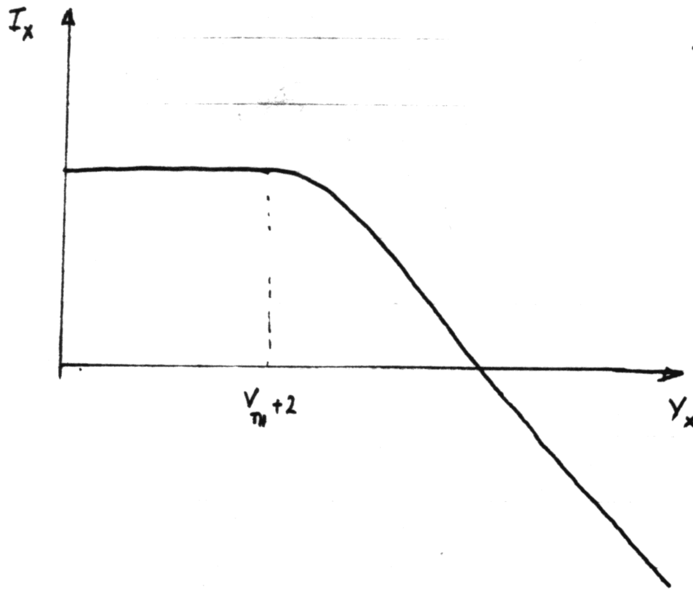
$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[\left(R_1 (I_1 - I_x) - V_{TH} \right) + \left(V_x - 2 - V_{TH} \right) \right] \left[\left(R_1 (I_1 - I_x) - V_{TH} \right) - \left(V_x - 2 - V_{TH} \right) \right]$$

(*) $$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[\left(R_1 (I_1 - I_x) - V_{TH} \right)^2 - \left(V_x - 2 - V_{TH} \right)^2 \right]$$

The second term shows that I_x decreases when we increase V_x

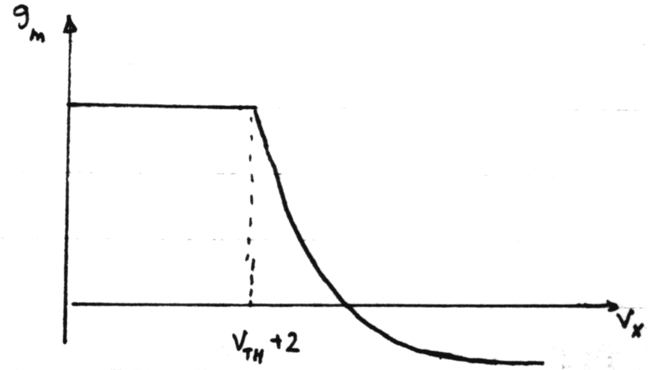
The polarity of I_x changes for higher V_x (Device still is in triode)

(*) presents $I_x - V_x$ relationship in this region.

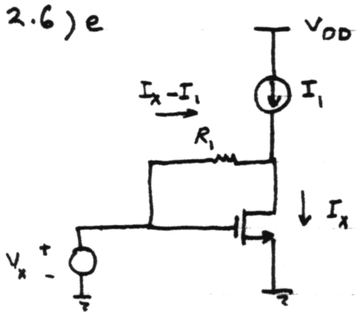


$$g_m = \mu_n C_{ox}\frac{W}{L}\left[R_1(I_1-I_X)-V_{TH}\right] \quad ; V_X < 2+V_{TH}$$

$$g_m = \mu_n C_{ox}\frac{W}{L}V_{DS} = \mu_n C_{ox}\frac{W}{L}\left[R_1(I_1-I_X)+2-V_X\right] \quad V_X > 2+V_{TH}$$



2.6) e



for $0 < V_X < V_{TH}$ Device is off $I_X = 0$ $g_m = 0$

Then device turns on (in the saturation region)

$$I_X = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_X - V_{TH})^2$$

Transistor is in the saturation until $V_{GD} = R_1(I_X - I_1) = V_{TH}$, Then device enters the triode region. (when $I_X = I_1 + \frac{V_{TH}}{R_1}$, i.e. $V_X = V_{TH} + \sqrt{\frac{2I_1 + 2V_{TH}/R_1}{\mu_n C_{ox}\frac{W}{L}}}$)

So, $V_{TH} < V_X < V_{TH} + \sqrt{\frac{2I_1 + 2V_{TH}/R_1}{\mu_n C_{ox}\frac{W}{L}}}$

$$I_X = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_X - V_{TH})^2$$

$$g_m = \mu_n C_{ox}\frac{W}{L}(V_X - V_{TH})$$

2.6) e Cont.

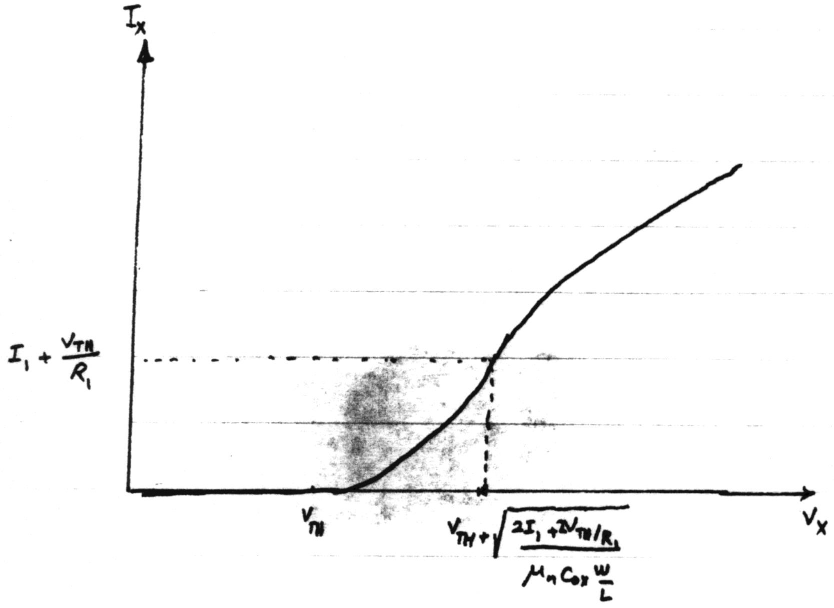
Then device enters the triode region. $V_{GS} = V_X$ $\quad V_{DS} = V_X - R_1(I_X - I_1)$

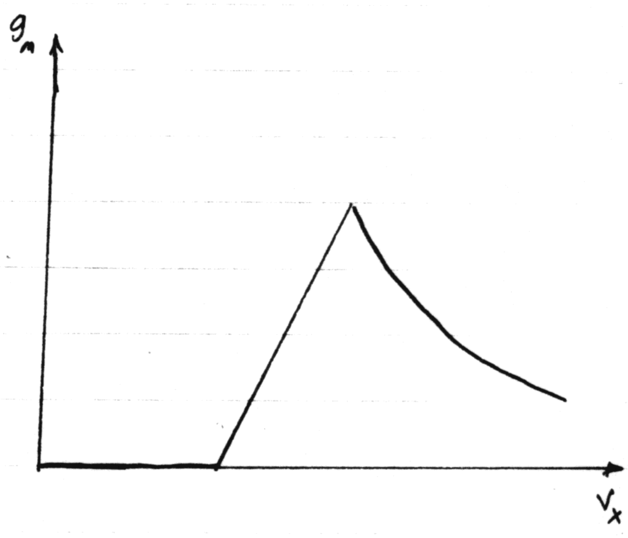
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})-V_{DS}\right]V_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_X-V_{TH})-V_X+R_1(I_X-I_1)\right]\times$$
$$(V_X - R_1(I_X-I_1))$$

$$(*)\quad I_X = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_X+R_1(I_X-I_1)-2V_{TH}\right)\left(V_X-R_1(I_X-I_1)\right)$$

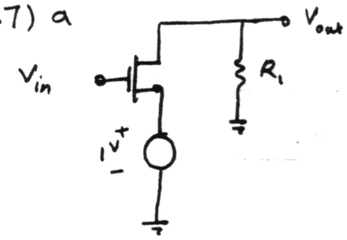
The above equation presents $I_X - V_X$ relationship in triode region.

In this region, $g_m = \mu_n C_{ox}\frac{W}{L}V_{DS} = \mu_n C_{ox}\frac{W}{L}(V_X - R_1(I_X-I_1))$





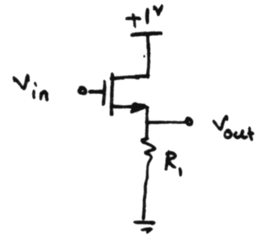
2.7) a



$\lambda = \gamma = 0$ $\qquad$ $V_{TH} = 0.7$

Drain and source exchange their roles.

$\Rightarrow$



for $0 < V_{in} < 0.7$ $\qquad$ device is off $\qquad$ $V_{out} = 0$

for $0.7 < V_{in} < 1.7$ $\qquad$ device is in the saturation region

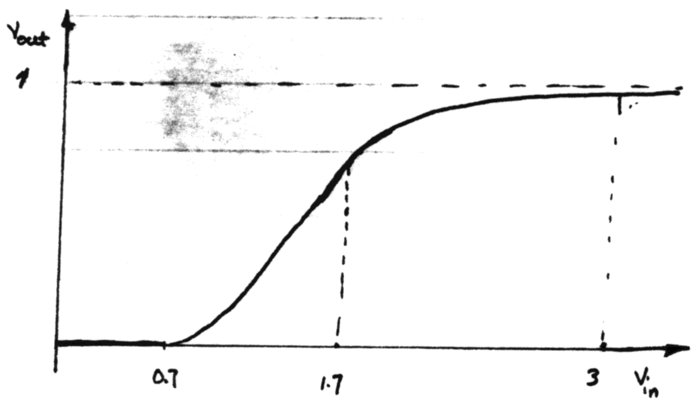
$$(*) \quad I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{in} - V_{out} - 0.7\right)^2 \Rightarrow \text{Input-Output relationship}$$

for $1.7 < V_{in} < 3$ $\qquad$ device is in the triode region

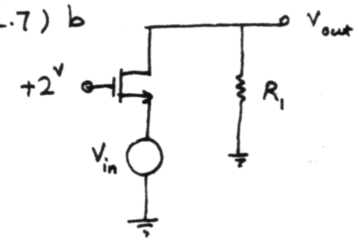
$V_{GS} = V_{in} - V_{out}$ $\qquad$ $V_{DS} = 1 - V_{out}$

$$(*) \quad I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\left(V_{in} - V_{out} - 0.7\right)\left(1 - V_{out}\right) - \left(1 - V_{out}\right)^2\right]$$

$\Rightarrow$ Input-output relationship



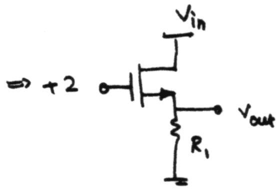
2.7) b



$\gamma = \lambda = 0 \qquad V_{TH} = 0.7$

Drain and source exchange their roles!

$\Rightarrow$



for $0 < V_{in} < 1.3$ device is in triode

$V_{GS} = 2 - V_{out} \qquad V_{DS} = V_{in} - V_{out}$

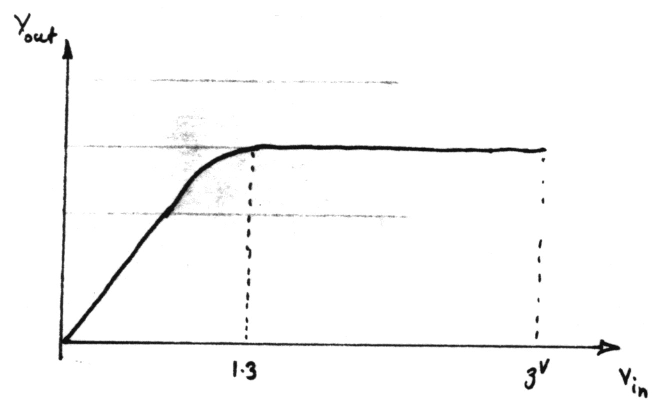
$$(*) \quad I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(2 - V_{out} - 0.7)(V_{in} - V_{out}) - (V_{in} - V_{out})^2\right]$$

Input output relationship is presented by the above equation.

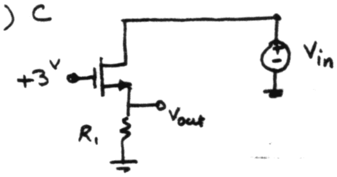
for $1.3 < V_{in} < 3$ device is in the saturation region

$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2 - V_{out} - 0.7)^2$$

V_{out} doesn't depend on V_{in} and it is constant for $V_{in} > 1.3$



2.7) c



$\gamma = \lambda = 0 \qquad V_{TH} = 0.7$

for $0 < V_{in} < 2.3$ device is in triode

$V_{GS} = 3 - V_{out} \qquad V_{DS} = V_{in} - V_{out}$

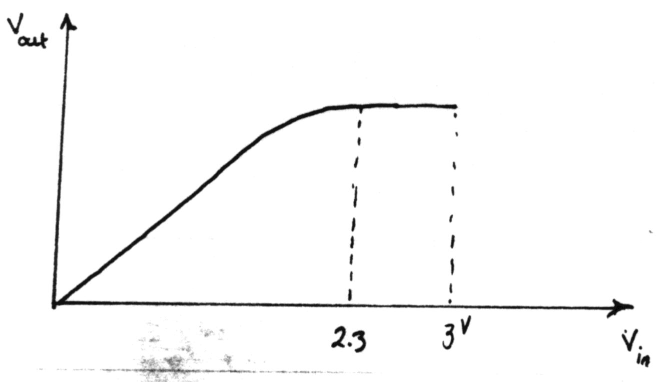
$$(*) \quad I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(3 - V_{out} - 0.7)(V_{in} - V_{out}) - (V_{in} - V_{out})^2\right]$$

Input - output relationship is presented by the above equation.

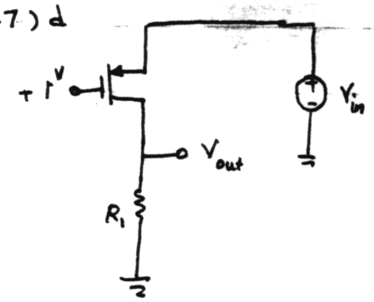
for $2.3 < V_{in} < 3$ device is in the saturation region

$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(3 - V_{out} - 0.7)^2$$

V_{out} is constant for $V_{in} > 2.3$ (It doesn't depend on V_{in})



2.7) d



$|V_{TH}| = 0.8 \qquad \gamma = \lambda = 0$

for $0 < V_{in} < 1.8$ device is off $\Rightarrow V_{out} = 0$

Then device tunns on (in sat.) and V_{out} goes up

Until $V_{out} = 1.8$, then device enters the triode region for $V_{in} > 1.8$ and $V_{out} < 1.8$

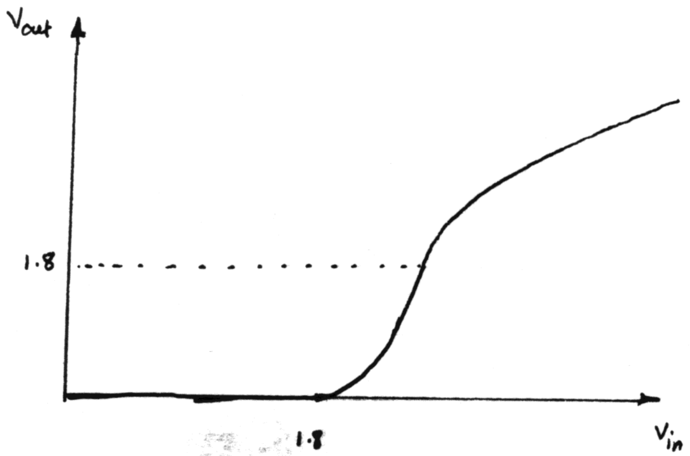
$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}(V_{in} - 1.8)^2 \Rightarrow V_{out} = \frac{1}{2}\mu_p C_{ox} R_1 \frac{W}{L}(V_{in} - 1.8)^2$$

This is good for $1.8 < V_{in} < 1.8 + \sqrt{\dfrac{2 \times 1.8^V}{\mu_p C_{ox}\frac{W}{L}R_1}}$

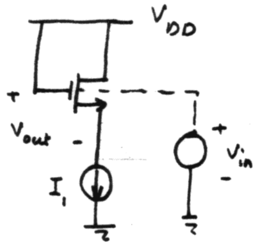
for $V_{in} > 1.8 + \sqrt{\dfrac{2 \times 1.8}{\mu_p C_{ox}\frac{W}{L}R_1}}$

$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[2(V_{in} - 1.8)(V_{in} - V_{out}) - (V_{in} - V_{out})^2\right].$$

Input - output relationship is presented by the above equation.



2.8) a



$$V_S = V_{DD} - V_{out} \qquad V_B = V_{in} \quad , V_{SB} = V_{DD} - V_{out} - V_{in}$$

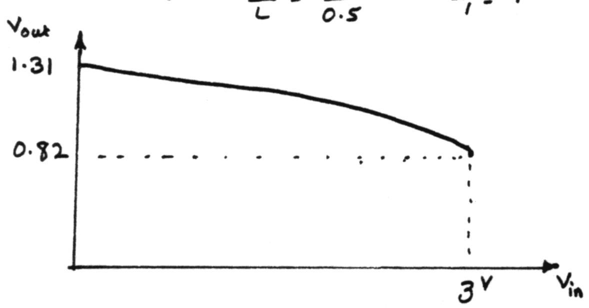
$$I_D = I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{out} - V_{TH} \right)^2$$

$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F} \right)$$

$$\Rightarrow I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{out} - V_{TH0} - \gamma \left(\sqrt{2\phi_F + V_{DD} - V_{out} - V_{in}} - \sqrt{2\phi_F} \right) \right)^2$$

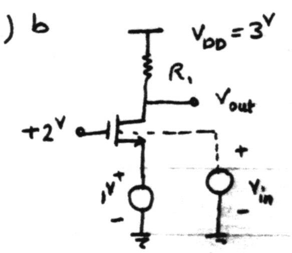
for each V_{in}, the above equation should be solved to obtain V_{out}

for $\frac{W}{L} = \frac{50}{0.5}$ $\quad I_1 = 1^{mA}$



Assumption: $2\phi_F + V_{DD} - V_{out} - V_{in} > 0$

2.8) b



$$V_{SB} = 1 - V_{in} \qquad V_{GS} = 1$$

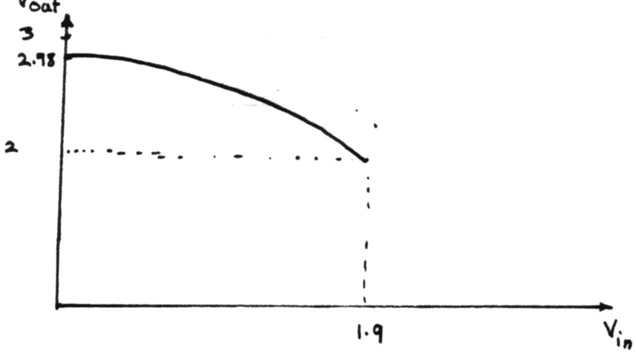
$$V_{TH} = V_{TH0} + \gamma \left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F} \right)$$

$$V_{TH} = 0.7 + 0.45 \left(\sqrt{1.9 - V_{in}} - \sqrt{0.9} \right)$$

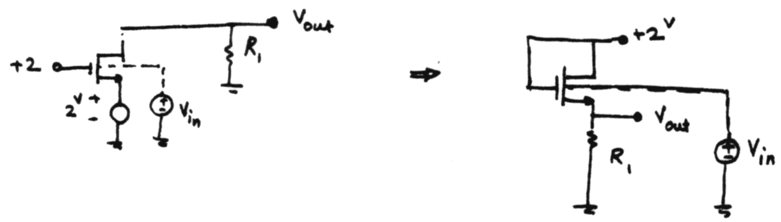
Assumption: V_{in} varies from 0 to 1.9 and R_1 is small enough to guarantee that the device remains in the saturation region.

$$V_{out} = 3 - R_1 \cdot \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(0.3 - 0.45 \left(\sqrt{1.9 - V_{in}} - \sqrt{0.9} \right) \right)^2$$

for $\frac{W}{L} = \frac{50\mu}{0.5\mu}$, $R = 0.2\,K$



2.8) c Drain and Source exchange their roles , ∴ $V_{THO} = 0.7$ $\gamma = 0.45$ $2\varphi_F = 0.9$



Assumption : $V_{SB} > -2\varphi_F$ $(V_{out} - V_{in} > -2\varphi_F)$ ⇒ Device is in the Saturation

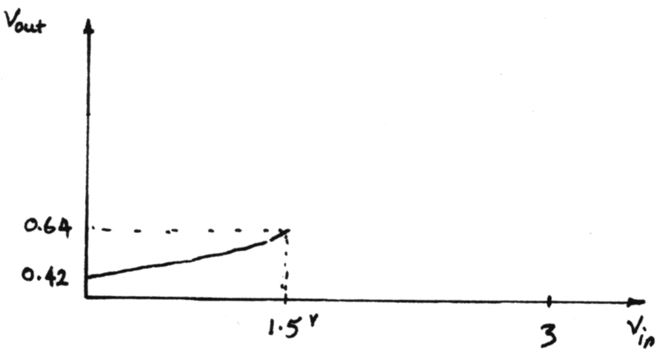
$$V_{TH} = 0.7 + 0.45\left(\sqrt{0.9 + V_{out} - V_{in}} - \sqrt{0.9}\right) \qquad V_{GS} = 2 - V_{out}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2 - V_{out} - 0.7 - 0.45\left(\sqrt{0.9 + V_{out} - V_{in}} - \sqrt{0.9}\right)\right)^2$$

$$I_D = \frac{V_{out}}{R_1}$$

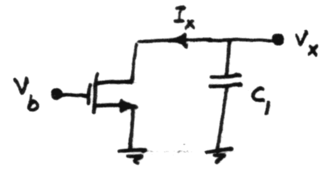
$$(*) \quad \frac{V_{out}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2 - V_{out} - 0.7 - 0.45\left(\sqrt{0.9 + V_{out} - V_{in}} - \sqrt{0.9}\right)\right)^2$$

Input – Output relationship is presented by the above equation.



$\frac{W}{L} = \frac{50}{0.5}$ $R = 100\,\Omega$

2.9) a



$\gamma = \lambda = 0$ $\qquad V_{TH} = 0.7$

for $V_b - 0.7 < V_x < 3$ device is in saturation

Assume $V_b > V_{TH}$

$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{TH})^2$$

$$V_x = -\frac{1}{C_1}\int I_x\,dt + 3^V = 3 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{TH})^2\,t$$

Then device goes into triode, for $0 < V_x < V_b - 0.7$

$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_b - 0.7)V_x - V_x^2\right] = -\frac{dV_x}{dt}\times C_1$$

$$\Rightarrow \quad -dt\;\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\times\frac{1}{C_1}}_{\alpha} = \frac{dV_x}{V_x\left[2(V_b - 0.7) - V_x\right]}$$

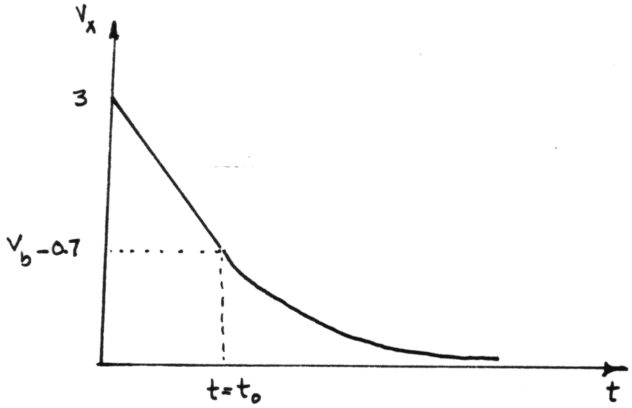
$$-\alpha\,dt = \left[\frac{1}{V_x} + \frac{1}{2(V_b - 0.7) - V_x}\right]\times\frac{1}{2(V_b - 0.7)}$$

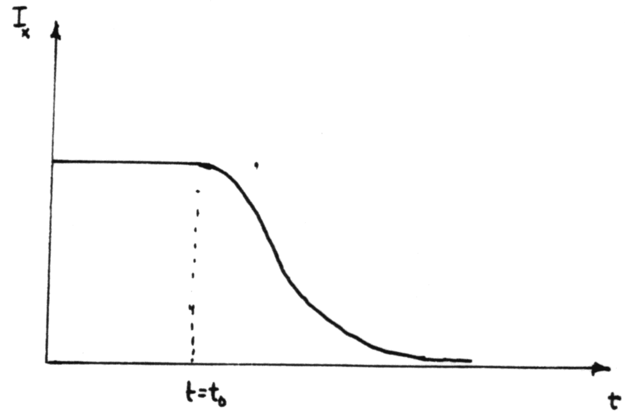
$$\Rightarrow \quad -\alpha(t - t_0) = \left[\ln\frac{V_x}{2(V_b - 0.7) - V_x}\right]\frac{1}{2(V_b - 0.7)} \qquad @\,t = t_0,\; V_x = V_b - 0.7$$

$$\Rightarrow \quad \frac{2(V_b - 0.7) - V_x}{V_x} = e^{2\alpha(V_b - 0.7)(t - t_0)}$$

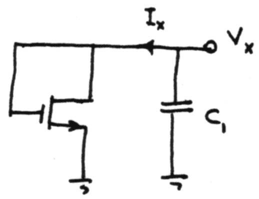
$$\Rightarrow \quad V_x = \frac{2(V_b - 0.7)}{1 + e^{2\alpha(V_b - 0.7)(t - t_0)}}$$

$$I_x = -C_1\frac{dV_x}{dt} = \frac{4\alpha C_1(V_b - 0.7)^2\,e^{2\alpha(V_b - 0.7)(t - t_0)}}{\left(1 + e^{2\alpha(V_b - 0.7)(t - t_0)}\right)^2}$$





2.9) b



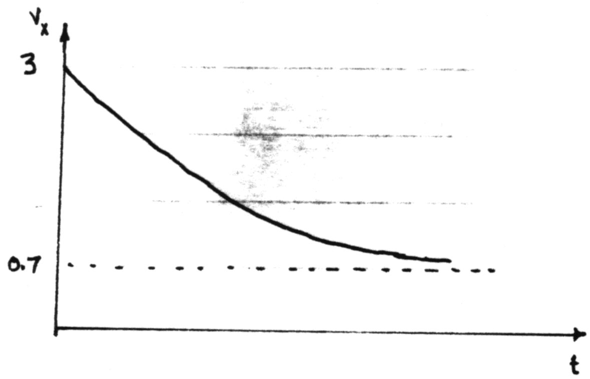
Device is always in the saturation region.

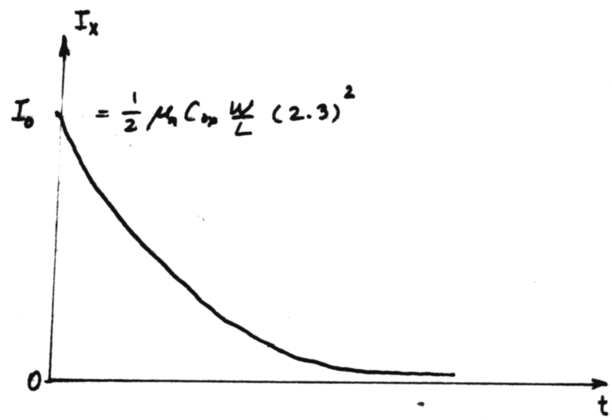
$$I_x = -C_1 \frac{dV_x}{dt} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_x - 0.7)^2$$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox} \frac{W}{L} \frac{1}{C_1}}_{\alpha} dt = -\frac{dV_x}{(V_x - 0.7)^2} \qquad \Rightarrow \alpha t = \frac{1}{V_x - 0.7} + K$$

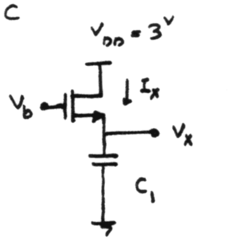
$$@\ t = 0,\ V_x = 3 \quad \Rightarrow \quad \alpha t = \frac{1}{V_x - 0.7} - \frac{1}{2.3} \quad \Rightarrow \quad V_x = 0.7 + \frac{1}{\alpha t + 1/2.3}$$

$$I_x = -C_1 \frac{dV_x}{dt} = \frac{\alpha C_1}{\left(\alpha t + \frac{1}{2.3}\right)^2}$$



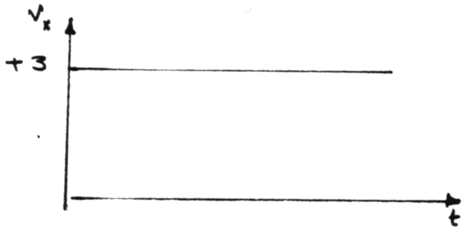


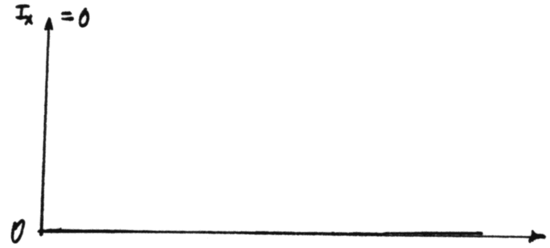
2.9) c



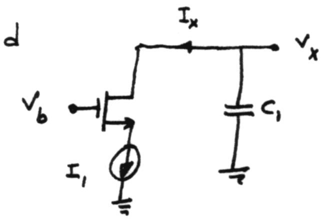
@ $t=0$ $\qquad V_x = 3 \;, \; V_{DD} = 3^V \Rightarrow V_{DS} = 0 \Rightarrow I_x = 0$

And the Circuit remains in this state





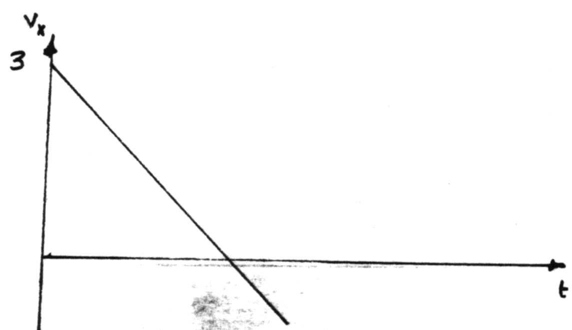
2.9) d

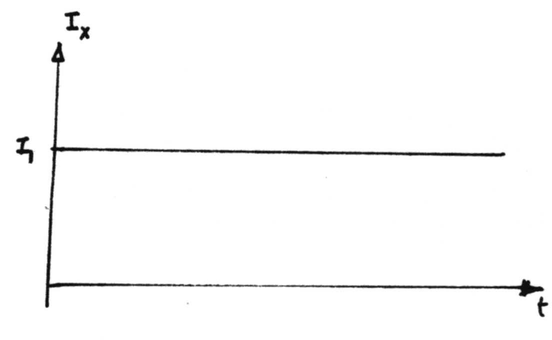


$$I_x = I_1$$

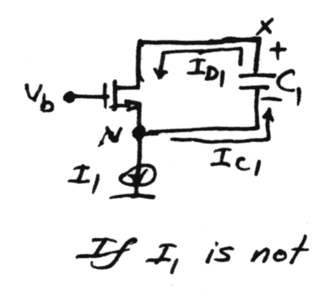
$$-C_1 \frac{dV_x}{dt} = I_1 \Rightarrow V_x = 3 - \frac{I_1}{C} t$$

Infact these Equations are valid until I_1 is no longer an ideal current Source.





2.9)e Initially, the current thru $M_1 = I_1 \Rightarrow$ certain V_{GS} is developed and $V_X = V_b - V_{GS1} + 3V$ and $I_X = I_1$. However, at $t = 0^+$, the drain current of M_1 flows from C_1: $I_{D1} - I_{C1} = I_1$. But,

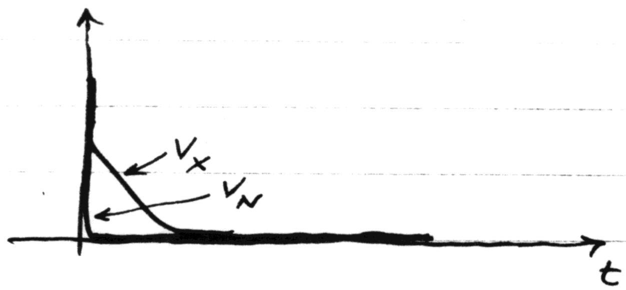


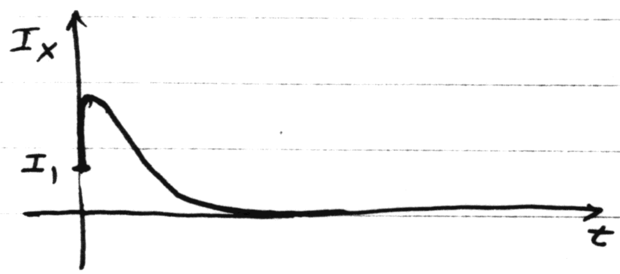
$I_{D1} = I_{C1} \Rightarrow I_1 = 0$. If the current source is ideal, V_N jumps to $-\infty$ (actually about 0.6 V below 0, where the S-B diode turns on.)

If I_1 is not ideal, V_N jumps to zero and C_1 discharges

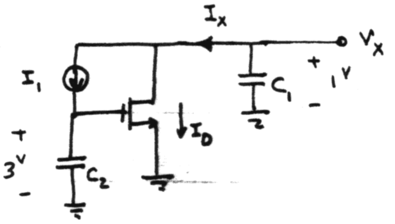
2.9) e (cnt'd)

through M_1:





2.10) a



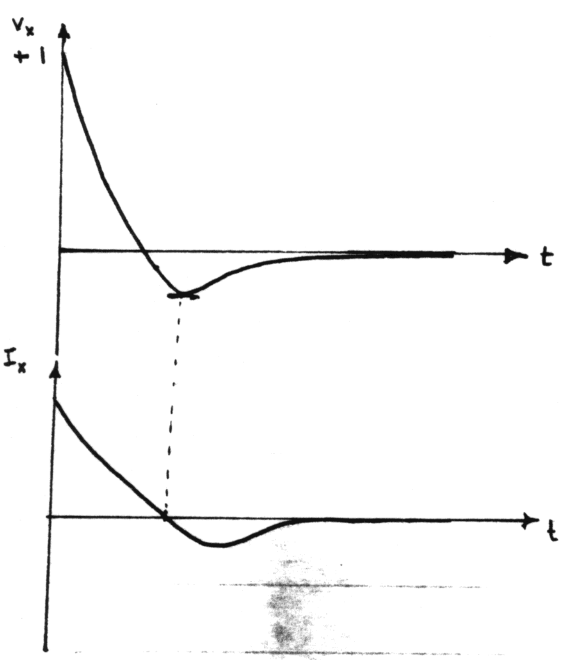
$V_G = 3 + \frac{I_1 t}{C}$

This circuit settles at $t=\infty$, when $V_G = \infty$

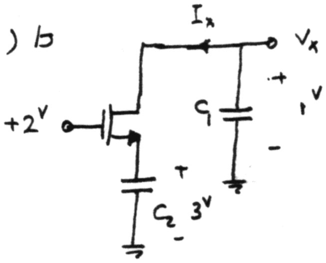
$I_D = -I_1$, $V_{DS} = 0$ (Actually, Drain and Source exchange their roles after a specific time at which $I_x = I_1$ and afterward V_x becomes negative) However, transistor always operates in the triode region.

$$I_x = I_1 + \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\left(3 + \frac{I_1}{C_2}t - 0.7\right)V_x - V_x^2\right] = -C_1\frac{dV_x}{dt}$$

The values of V_x can be obtained by numerical methods

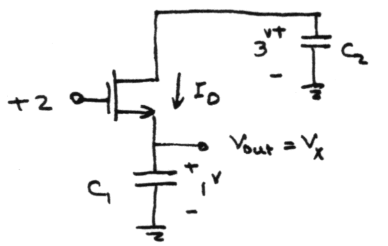


2.10) b



Drain and source exchange their roles.

$(\gamma=\lambda=0) \qquad V_{TH}=0.7$



$$\int I_D\,dt = q \qquad V_x = 1+\frac{q}{C_1}\ ,\ V_D = V_{C_2} = 3-\frac{q}{C_2}$$

V_x goes up until transistor turns off when $V_x = 1.3$

Assumption: Transistor is in saturation.

This assumption is correct if: $V_D = 3-\frac{q}{C_2} > 1.3 \quad (2-0.7)$

$$V_x(\infty) = 1+\frac{q(\infty)}{C_1} = 1.3 \qquad V_D(\infty) = 3-\frac{q(\infty)}{C_2} = 3-0.3\frac{C_1}{C_2} > 1.3$$

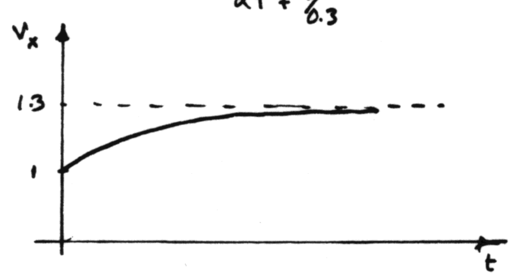
$$0.3\frac{C_1}{C_2} < 1.7 \qquad C_1 < 5.67\,C_2$$

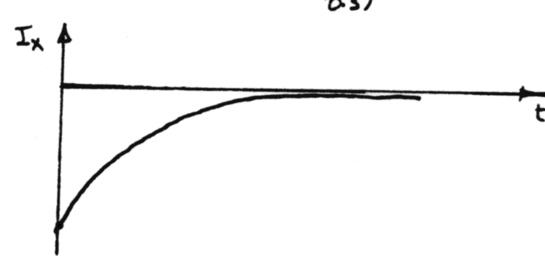
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2-1-\frac{q}{C_1}-0.7\right)^2 = \frac{dq}{dt}$$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{1}{C_1}}_{\alpha}\,dt = \frac{dq/C_1}{(0.3-q/C_1)^2} \qquad \Rightarrow \alpha t = \frac{1}{0.3-q/C_1}+K \quad (t=0, q=0)$$

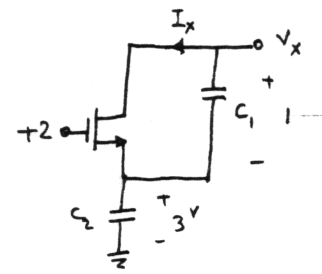
$$\Rightarrow \alpha t = \frac{1}{0.3-q/C_1}-\frac{1}{0.3} \qquad \Rightarrow \frac{q}{C_1} = 0.3-\frac{1}{\alpha t+\frac{1}{0.3}} \qquad V_x = 1+\frac{q}{C_1}$$

$$\Rightarrow V_x = 1.3-\frac{1}{\alpha t+1/0.3} \qquad I_x = -C_1\frac{dV_x}{dt} = \frac{-\alpha C_1}{(\alpha t+1/0.3)^2}$$





2.10) c

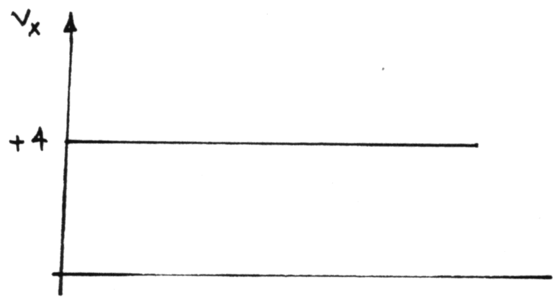


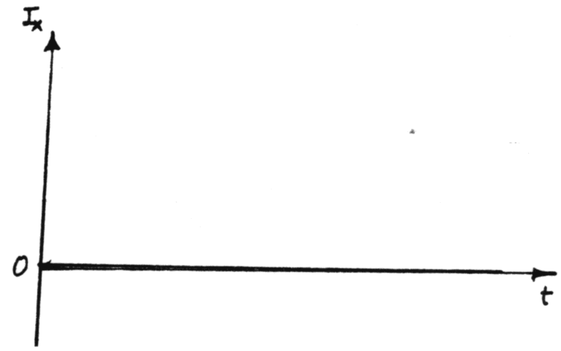
At $t=0$ $V_G = 2$ $V_S = 3$ $V_D = 4$

Device is off and doesn't turn on.

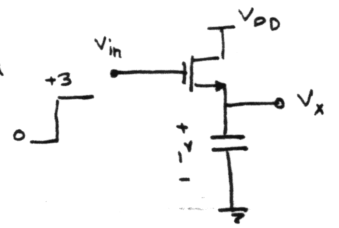
The Circuit remains in this state.

So, $V_x = 4$ $I_x = 0$





2.11) a



$\gamma = \lambda = 0$ $V_{TH} = 0.7$

At $t = 0^+$, device turns on (in Sat) and starts charging the capacitor, until device turns off when; $V_x = V_{in} - V_{TH} = 3 - 0.7 = 2.3$

$$I_c = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2.3 - V_x)^2 \quad ; \; V_{GS} = 3 - V_x - 0.7$$

$$I_c = C_1 \frac{dV_x}{dt}$$

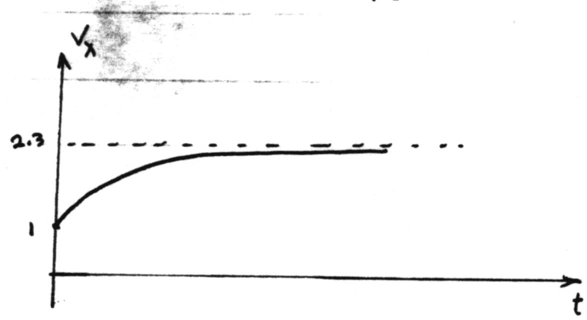
$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L} \times \frac{1}{C_1}}_{\alpha}(2.3 - V_x)^2 = \frac{dV_x}{dt}$$

$$\Rightarrow \alpha\, dt = \frac{dV_x}{(2.3 - V_x)^2} \quad \Rightarrow \alpha t + K_0 = \frac{1}{2.3 - V_x}$$

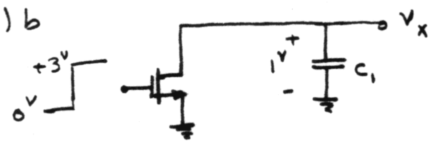
$(t = 0, V_x = 1)$ $\alpha \times 0 + K_0 = \dfrac{1}{2.3 - 1}$ $\Rightarrow K_0 = 1/1.3$

$$\Rightarrow \frac{1}{1.3} + \alpha t = \frac{1}{2.3 - V_x} \quad \Rightarrow 2.3 - V_x = \frac{1}{\alpha t + 1/1.3}$$

$$\Rightarrow V_x = 2.3 - \frac{1}{\alpha t + 1/1.3}$$



2.11) b



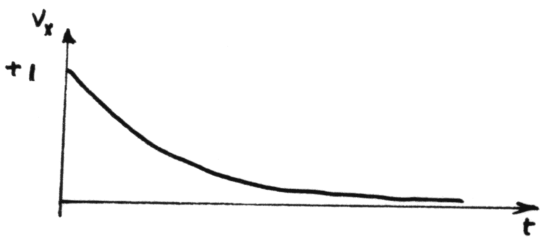
Transistor turns on at $t=0$, and discharges C_1 until $V_X = 0$, (device always operats in triode)

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(3-0.7)V_X - V_X^2\right] = -C_1\frac{dV_X}{dt}$$

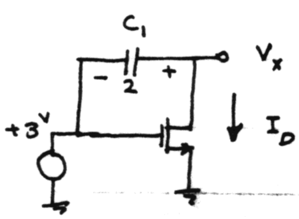
$$\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\times\frac{1}{C_1}}_{\alpha}\left[4.6\,V_X - V_X^2\right] = -\frac{dV_X}{dt} \Rightarrow -\alpha\, dt = \frac{dV_X}{V_X(4.6-V_X)}$$

$$\Rightarrow -\alpha t = \left(\frac{1}{V_X} + \frac{1}{4.6-V_X}\right)\frac{1}{4.6} + K \quad , \quad @\, t=0 \;, V_X = 1$$

$$\frac{1}{3.6}e^{-\alpha t} = \frac{V_X}{4.6-V_X} \longrightarrow V_X = \frac{4.6}{1+3.6\,e^{4.6\alpha t}}$$



2.11) c



At $t=0^+$, $V_X = 5$, device is in Saturation region

$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(3-0.7)^2$, V_X decreases until

$V_X = 2.3$ at $t = t_0$, then device enters triode region

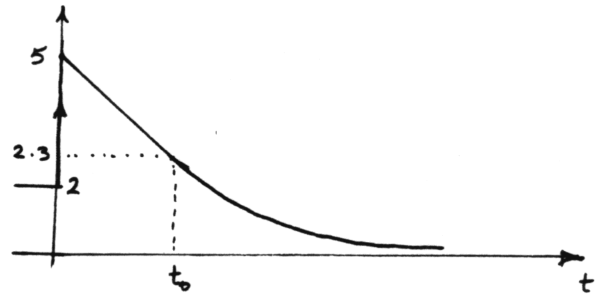
for $t < t_0$ $(V_X > 2.3)$ $\quad V_X = 5 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2.3)^2\, t/C_1$

for $t > t_0$ $\quad I_D = -C_1\frac{dV_X}{dt} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(3-0.7)V_X - V_X^2\right]$

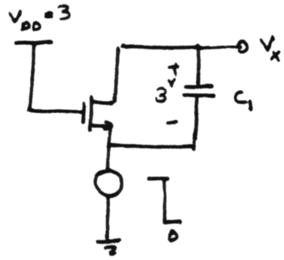
$$\Rightarrow \frac{dV_X}{V_X(4.6-V_X)} = -\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot\frac{1}{C_1}}_{\alpha}\, dt$$

2.11) c, Cont. $\quad -\alpha(t-t_0) = \left[\ln \dfrac{V_x}{4.6-V_x}\right] \cdot \dfrac{1}{4.6} \qquad t=t_0,\ V_x = 2.3$

$$\Rightarrow V_x = \frac{4.6}{1+e^{4.6\alpha(t-t_0)}}$$



2.11) d



At $t=0^+$, $V_x = 3$ device is in saturation

$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(3-0.7)^2$, V_x decreases until

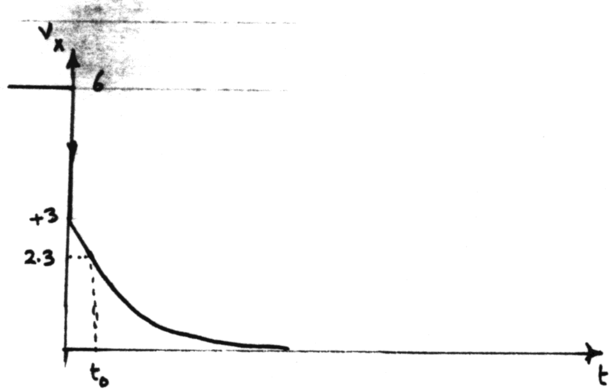
$V_x = 2.3$ at $t=t_0$, then device enters triode region.

for $t<t_0$ $\qquad V_x = 3 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(2.3)^2\frac{t}{C_1}$; $\quad 2.3 < V_x < 3$

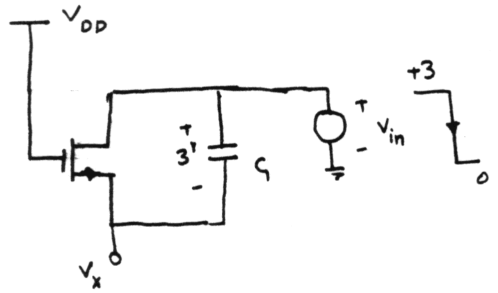
for $t>t_0$ $\qquad I_D = -C_1\frac{dV_x}{dt} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(3-0.7)V_x - V_x^2\right]$

$$\frac{dV_x}{V_x(4.6-V_x)} = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\frac{1}{C_1}dt \qquad ,(t=t_0,\ V_x=2.3)$$

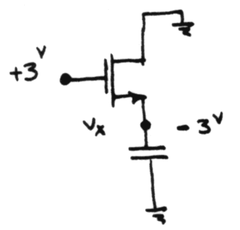
$$-\alpha(t-t_0) = \left[\ln\frac{V_x}{4.6-V_x}\right]\frac{1}{4.6} \Rightarrow V_x = \frac{4.6}{1+e^{4.6\alpha(t-t_0)}}$$



2.12) a)



$t \geqslant 0^+$



Device is in the triode region.

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\,(2.3 - V_x)(-V_x) - V_x^2\right]$$

$$\begin{cases} V_{GS} = 3 - V_x \\ V_{DS} = -V_x \end{cases}$$

$$I_D = C_1 \frac{dV_x}{dt}$$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\times\frac{1}{C_1}}_{\alpha}\left[V_x^2 - 4.6\,V_x\right] = \frac{dV_x}{dt}$$

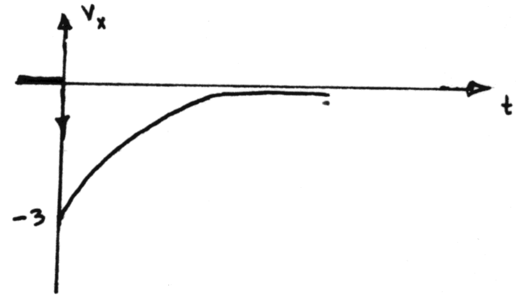
$$\Rightarrow \alpha\, dt = \frac{dV_x}{V_x^2 - 4.6\,V_x} = dV_x\left(\frac{1}{V_x - 4.6} + \frac{-1}{V_x}\right)\times\frac{1}{4.6}$$

$$\Rightarrow 4.6\,\alpha t + K_0 = \ln\left(\frac{V_x - 4.6}{V_x}\right) \quad ; \quad V_x(0^+) = -3$$

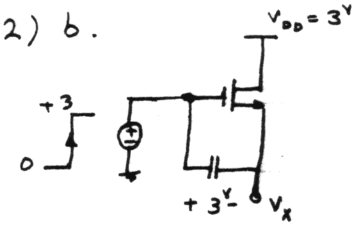
$$\Rightarrow K_0 = \ln\frac{7.6}{3} \qquad \Rightarrow \frac{V_x - 4.6}{V_x} = \frac{7.6}{3}\,e^{4.6\alpha t}$$

$$\Rightarrow \frac{4.6}{V_x} = 1 - \frac{7.6}{3}\,e^{4.6\alpha t}$$

$$\Rightarrow V_x = \frac{-4.6}{\frac{7.6}{3}\,e^{4.6\alpha t} - 1}$$

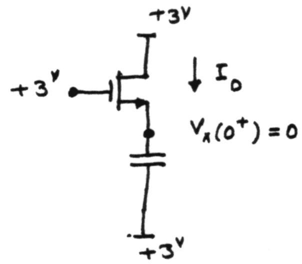


2.12) b.



Device is in saturation region

$t = 0^+$

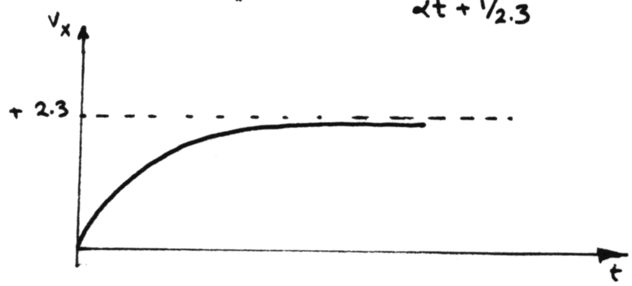


$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\,(3 - V_x - 0.7)^2 = C_1\frac{dV_x}{dt}$$

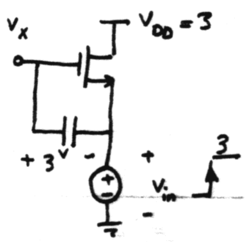
$$\frac{dV_x}{(2.3 - V_x)^2} = \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot\frac{1}{C_1}}_{\alpha}\,dt$$

$$\Rightarrow \frac{1}{2.3 - V_x} = \alpha t + K \qquad (t=0,\ V_x=0) \quad \Rightarrow \frac{1}{2.3 - V_x} - \frac{1}{2.3} = \alpha t$$

$$\Rightarrow V_x = 2.3 - \frac{1}{\alpha t + 1/2.3}$$

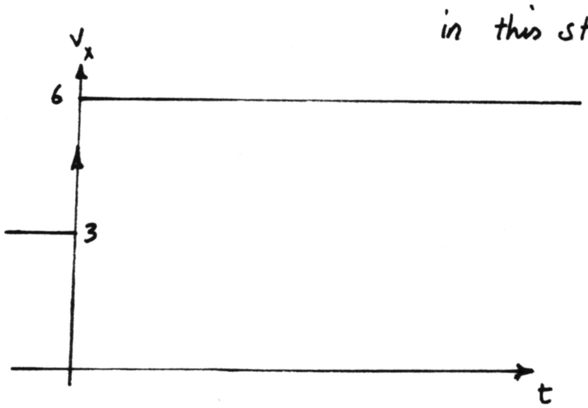


2.12) c

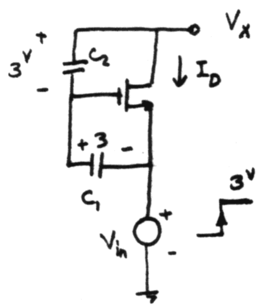


At $t = 0^+$ $\quad V_D = 3 \quad V_S = 3 \quad V_G = 6$

So, $V_{DS} = 0$ and $I_C = I_D = 0$ And circuit remains in this state. $\quad V_x(0^-) = 3$, $V_x(t) = 6$



2.12) d



Assume that the device remains in the saturation region until it turns off when $V_{gs} = 0.7$

$$V_{C_1} = V_{gs} = 3 - \frac{1}{C_1}\int I_D\,dt \qquad V_{C_2} = V_{dg} = 3 - \frac{1}{C_2}\int I_D\,dt$$

This assumption is correct if $V_{dg} > -0.7$ when $V_{gs} = 0.7$

$$\int I_D\,dt = q(t) \qquad V_{gs} = 3 - \frac{q}{C_1} = 0.7 \Rightarrow \frac{q}{C_1} = 2.3 \qquad V_{dg} = 3 - \frac{q}{C_2} > -0.7$$

$$\Rightarrow \frac{q}{C_2} < 3.7 \qquad 2.3\frac{C_1}{C_2} < 3.7 \Rightarrow C_1 < 1.61\,C_2$$

With this assumption,

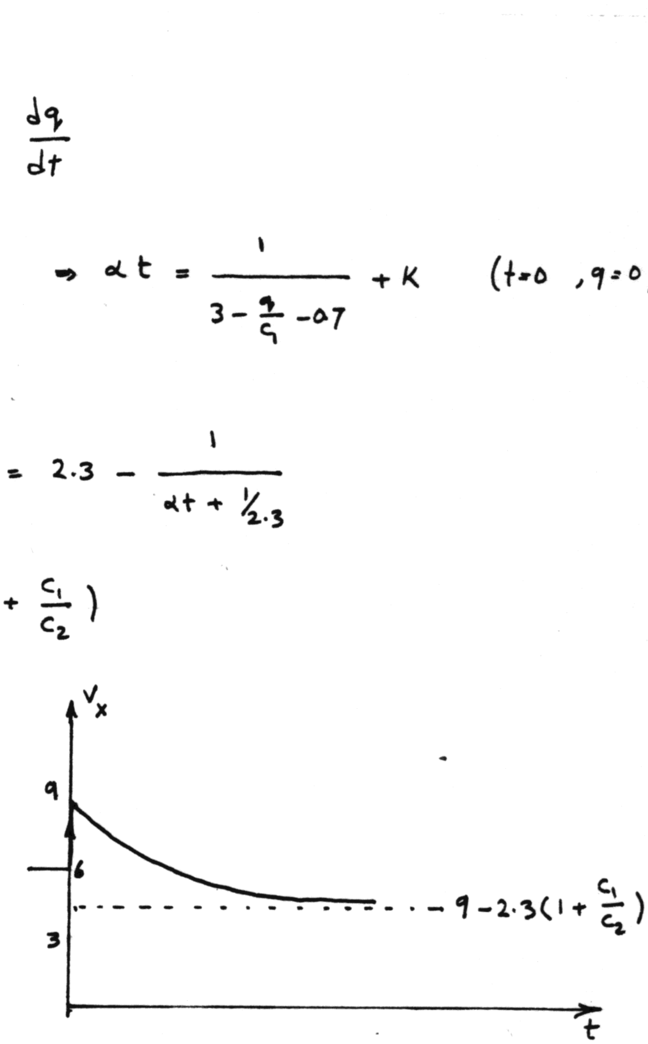
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(3 - \frac{q}{C_1} - 0.7\right)^2 = \frac{dq}{dt}$$

$$\Rightarrow \underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot\frac{1}{C_1}}_{\alpha}\,dt = \frac{dq/C_1}{\left(3 - \frac{q}{C_1} - 0.7\right)^2} \Rightarrow \alpha t = \frac{1}{3 - \frac{q}{C_1} - 0.7} + K \qquad (t=0,\ q=0)$$

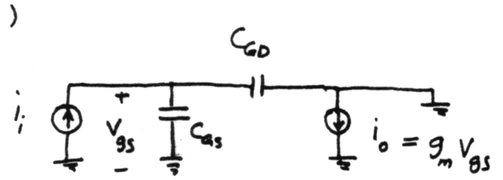
$$\Rightarrow \alpha t = \frac{1}{2.3 - \frac{q}{C_1}} - \frac{1}{2.3} \Rightarrow \frac{q}{C_1} = 2.3 - \frac{1}{\alpha t + 1/2.3}$$

$$V_x = 3 + 3 - \frac{q}{C_1} + 3 - \frac{q}{C_2} = 9 - \frac{q}{C_1}\left(1 + \frac{C_1}{C_2}\right)$$

$$V_x(t) = 9 - \left(1 + \frac{C_1}{C_2}\right)\frac{2.3\,\alpha t}{\alpha t + 1/2.3}$$



2.13) a)



$$i_i = (C_{GS} + C_{GD}) s V_{gs}$$

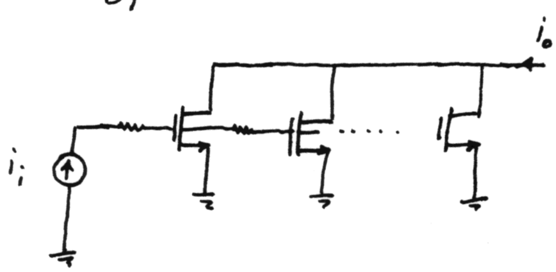
$$i_o = g_m V_{gs}$$

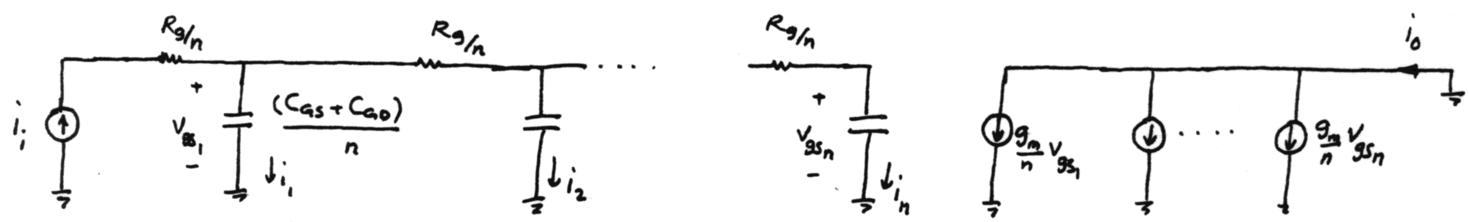
$$\beta = \frac{i_o}{i_i} = \frac{g_m}{(C_{GS} + C_{GD})s} \quad ; \quad |\beta| = 1 \Rightarrow \frac{g_m}{(C_{GS} + C_{GD})\omega_T} = 1$$

$$\Rightarrow \omega_T = \frac{g_m}{(C_{GS} + C_{GD})} \rightarrow f_T = \frac{\omega_T}{2\pi} = \frac{g_m}{2\pi(C_{GS} + C_{GD})}$$

Approximation : $g_m v_{gs}$ is the output current.

b)





$$i_K = \frac{1}{n} (C_{GS} + C_{GD}) s V_{gs_K} \qquad K = 1 \cdots n$$

$$(*) \quad i_i = i_1 + i_2 + \cdots + i_n = \frac{1}{n} (C_{GS} + C_{GD}) s (V_{gs_1} + V_{gs_2} + \cdots + V_{gs_n})$$

$$(**) \quad i_o = \frac{g_m}{n} V_{gs_1} + \cdots + \frac{g_m}{n} V_{gs_n} = \frac{g_m}{n} (V_{gs_1} + V_{gs_2} + \cdots + V_{gs_n})$$

$$(*), (**) \Rightarrow \beta = \frac{i_o}{i_i} = \frac{g_m}{(C_{GD} + C_{GS})s} \quad ; \quad |\beta| = 1 \Rightarrow f_T = \frac{\omega_T}{2\pi} = \frac{g_m}{2\pi(C_{GS} + C_{GD})}$$

c) $$f_T = \frac{g_m}{2\pi (C_{GS} + C_{GD})}$$

$$g_m = \mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$C_{GS} + C_{GD} \simeq C_{ox} WL$$

$$\Longrightarrow \quad f_T = \frac{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}{2\pi C_{ox} WL} \simeq \frac{\mu}{2\pi} \frac{(V_{GS} - V_{TH})}{L^2}$$

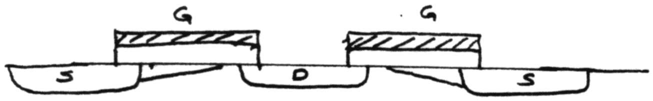
2.14)

$$f_T = \frac{g_m}{2\pi (C_{GS} + C_{GD})} \quad ; \quad g_m = \frac{I_D}{\xi V_T}$$

In the subthreshold $C_{GS} = C_{GD} = W C_{ov}$ (Fig 2.?.33)

So, $$f_T = \frac{I_D / \xi V_T}{4\pi W C_{ov}} = \frac{I_D}{4\pi \xi V_T W L_D C_{ox}}$$

2.15)



$C_{j0} = 0.56e{-3}\ F/m^2$ $\quad C_{jsw0} = 0.35e{-11}\ F/m$

$m_j = 0.6$ $\quad m_{jsw} = 0.2$

$$C_{DB} = \frac{W}{2} E C_j + 2\left(\frac{W}{2} + E\right) C_{jsw}$$

$$C_j = \frac{C_{j0}}{\left(1 + \frac{V_R}{2\phi_F}\right)^m}$$

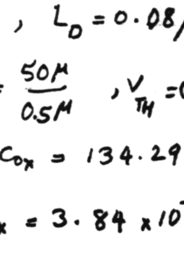
$$C_{SB} = 2\left[\frac{W}{2} E C_j' + 2\left(\frac{W}{2} + E\right) C_{jsw}'\right]$$

$$C_{GD} = 2\left(\frac{W}{2} C_{ov}\right) \qquad C_{ov} = L_D C_{ox}$$

$$C_{GS} = \frac{2WLC_{ox}}{3} + W C_{ov}$$

$$C_{GB} = (WLC_{ox}) C_d / (WLC_{ox} + C_d) \; ; \; C_d = WL\sqrt{q\epsilon_{si} N_{sub} / 2\phi_F}$$

$W = 50\mu \qquad L = 0.5\mu \qquad E = 1.5^{\mu}$



$$I = \frac{1}{2}\mu_n C_{ox} \frac{W}{L - 2L_D}(V_{GS} - V_{TH})^2 \; , \; 1^{mA} = \frac{1}{2} \times 0.13429 \times \frac{50}{0.5 - 0.16}(V_{GS} - 0.7)^2$$

$$V_{GS} = 1.0182 \qquad g_m = \frac{2I_D}{V_{GS} - V_{TH}} = 6.285\ ^{mA}/_{V} \; , \; V_{DB} = 1.0182$$

$\lambda = 0 \; , \; L_D = 0.08\,\mu m$

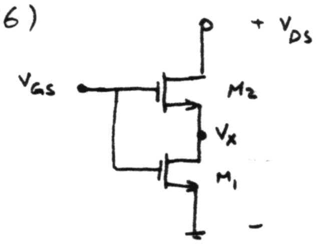
$\frac{W}{L} = \frac{50\mu}{0.5\mu} \; , V_{TH} = 0.7$ $\qquad C_{GD} = 15.4\ fF \qquad C_{GS} = 79.36\ fF$

$\mu_n C_{ox} = 134.29\ \mu A/V^2$

$C_{ox} = 3.84 \times 10^{-3}\ F/m^2$ $\qquad C_{SB} = 42.4\ fF \qquad C_{DB} = 13.5\ fF$

$$f_T = \frac{g_m}{2\pi(C_{GD} + C_{GS})} = 10.6\ GHz$$

2.16)



CASE I , M_1 : Triode M_2 : Triode

$$V_{OD1} = V_{GS} - V_{TH} \qquad V_{OD2} = V_{GS} - V_X - V_{TH}$$

$$V_{DS1} = V_X \qquad V_{DS2} = V_{DS} - V_X$$

$$I_{D1} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH})V_X - V_X^2 \right] \qquad (*)$$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH} - V_X)(V_{DS} - V_X) - (V_{DS} - V_X)^2 \right]$$

$$I_{D1} = I_{D2} \Rightarrow 2(V_{GS} - V_{TH})V_X - V_X^2 = 2(V_{GS} - V_{TH})V_{DS} + 2V_X^2 - 2V_X(V_{GS} - V_{TH})$$
$$- \cancel{2V_X V_{DS}} - V_{DS}^2 - V_X^2 + \cancel{2V_X V_{DS}}$$

$$\Rightarrow 2\left[2(V_{GS} - V_{TH})V_X - V_X^2 \right] = 2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2 \qquad (**)$$

$$(*),(**) \Rightarrow I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \times \frac{1}{2}\left[2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2 \right] \left(\frac{W}{2L} \text{ in Triode}\right)$$

CASE II , M_1 : Triode , M_2 : Sat

$$I_{D1} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH})V_X - V_X^2 \right] \qquad (*)$$

$$I_{D2} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_X - V_{TH})^2$$

$$I_{D1} = I_{D2} \Rightarrow V_X^2 - 2V_X(V_{GS} - V_{TH}) + (V_{GS} - V_{TH})^2 = 2(V_{GS} - V_{TH})V_X - V_X^2$$

$$\Rightarrow (V_{GS} - V_{TH})^2 = 2\left[2(V_{GS} - V_{TH})V_X - V_X^2 \right] \qquad (**)$$

2.16) Cont. $(*), (**) \Rightarrow I_{D_1} = I_{D_2} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \times \frac{1}{2} (V_{GS} - V_{TH})^2 \quad (\frac{W}{2L} \text{ in Sat})$

Note That M_1 is always in triode, because V_{OD2} is always positive

i.e. $V_{GS2} - V_{TH} > 0 \Rightarrow V_{GS} - V_X - V_{TH} > 0 \Rightarrow V_{GS} - V_{TH} > V_X$

$\Rightarrow V_{GS_1} - V_{TH} > V_{DS_1} \Rightarrow M_1$ is in the triode region.

Saturation – triode transition edge of M2:

We show that the transition point the saturation and triode region of the equivalent transitor is the same as that of M_2.

$V_{OD2} = V_{GS} - V_X - V_{TH} \qquad V_{DS2} = V_{DS} - V_X$

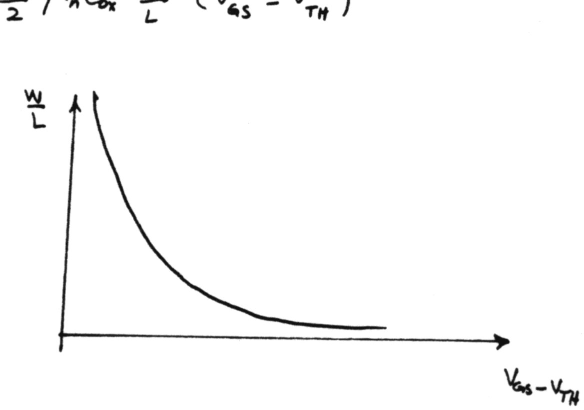
for $V_{OD2} > V_{DS2}$; M_2 is in the triode region, i.e. $V_{GS} - V_{TH} > V_{DS}$

It means that when M_2 is in the saturation, then the equivalent transistor is in the saturation, and vice versa.

2.17)

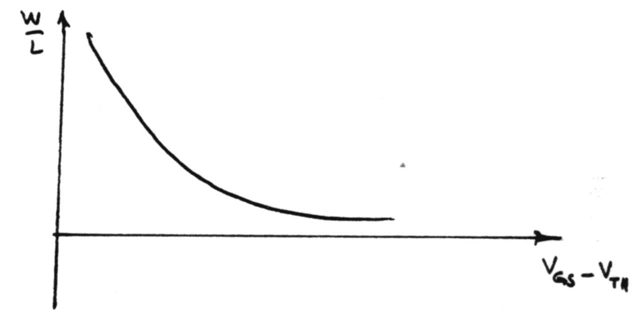
In Saturation region, $I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$

$$\Rightarrow \frac{W}{L} = \frac{2 I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^2}$$

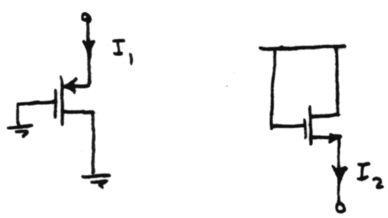


$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$\Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox} (V_{GS} - V_{TH})}$$



2.18)

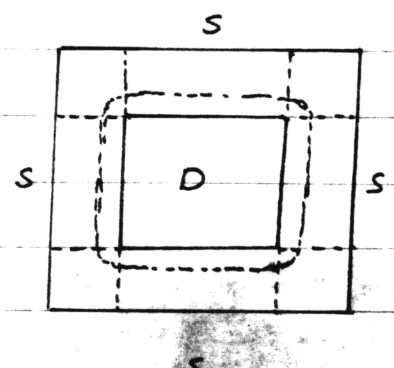


These structures cannot operate as current sources, because their currents strongly depend on source voltages, but an ideal current source should provide a constant current, independent of its voltage.

2.19) From Eq. (2.1) we know that $V_{TH} = \varphi_{MS} + 2\varphi_F + \frac{Q_{dep}}{C_{ox}}$, where φ_{MS} and φ_F are constant values, so any changes in V_{TH} come from the third term, in fact $\Delta V_{TH} = \frac{\Delta Q_{dep}}{C_{ox}}$ and

From Eq (2.22), we have $\Delta V_{TH} = \gamma(\sqrt{2\varphi_F + V_{SB}} - \sqrt{2\varphi_F})$ (in fact, this is definition of γ). from pn junction theory we know that Q_{dep} is proportional to $\sqrt{N_{sub}}$, so γ is directly proportional to $\sqrt{N_{sub}}$ and inversely proportional to C_{ox}.

2.20)



This structure operates as a traditional device does, in fact if we neglect edges we have four Mosfets in parallel, where the aspect ratio of each is $\frac{W}{L}$.
So the overal aspect ratio is almost $\frac{4W}{L}$

Drain junction capacitance: $C_{DB} = W^2 C_J + 4W C_{JSW}$

Drain junction capacitance of devices shown in fig 2.32 a, b for the aspect ratio of $\frac{4W}{L}$

$$C_{DB(a)} = 4WE\,C_J + (8W + 2E)\,C_{jsw}$$

$$C_{DB(b)} = 2WE\,C_J + (4W + 2E)\,C_{jsw}$$

The value of side wall capacitance in the ring structure is less than that in folded and traditional structures, but the bottom capacitance of ring structure is higher than that of the other two structures. (for $W > 4E$)

2.21) We first check the terminals of the device with a multimeter in order to find BS or BD junctions. There are 12 experiments in total of which two lead to conduction and remaining ones show no conduction. If we find one of those two conductions then we are done. Finding B and S (or D), we need to do one other experiment between B (Cathode of junction) and one of the two remaining terminals; In case of no Connection, the terminal under test is G, otherwise it is D (or S). In worst case with a maximum of 8 experiments, each terminal can be specified. It is as follows:

Assume, the two selected terminals do not conduct in both directions and this is the case for the other two terminals. Upto this point, four experiments have been done while not yet encountering any conduction. It is clear that one group consists of

G and B and the other Comprises from D and S , Because at least One Conduction should be observed if B were in the same group with one of the source or Drain. In the next step, we pick up one terminal from each group to undergo the conductivity test. Assume, no Conduction happens in either direction (Worst Case). It means that we had chosen G from (G B) group. Thusfor, we have done six experiments. we change both of terminals and now we have chosen B for sure. and in worst case, we will find a connection in 8th experiment. Now, we know B and S (D), Bulk's groupmate is Gate and Source's (Drain's) groupmate is Drain (Source).

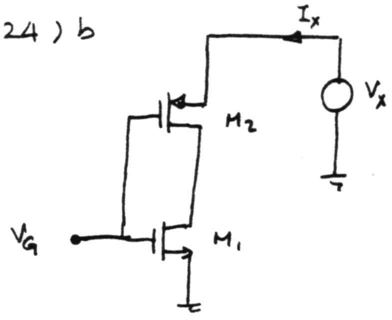
2.22) If we don't know the type of device, In eight experiment we cannot distinguish between B and S (D) and we should perform another experiment, which is exchanging one of

2.22) Cont. terminals with its groupmate. If we still had the Conduction then the exchanged terminal and its groupmate are Source and Drain, otherwise the exchanged terminal is Bulk.

2.23) a) NO, Because in DC model equations of MOSFET, we always have the product of $\mu_n C_{ox}$ and $\frac{W}{L}$.

b) NO, Because we cannot obtain as many independent equations as the unknown quantities. But if the difference between the aspect ratios is known, then $\mu_n C_{ox}$ and both $\frac{W}{L}$, are attainable.

2.24) b



CASE I : $V_G < V_{THN} \Rightarrow M_1$: off $I_x = 0$

$g_m = 0$

CASE II : $V_G > V_{THN}$

for $0 < V_X < V_G + |V_{THP}| \Rightarrow I_x = 0$ (M_2 : off) $g_m = \frac{\partial I_x}{\partial V_G} = 0$

Then M_2 turns on (in sat) , M_1 still is in triode region

$$I_x = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_X - V_G - |V_{THP}|)^2$$

This is correct until M_1 , goes into saturation , when

$$\frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_X - V_G - |V_{THP}|)^2 = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_N (V_G - V_{THN})^2$$

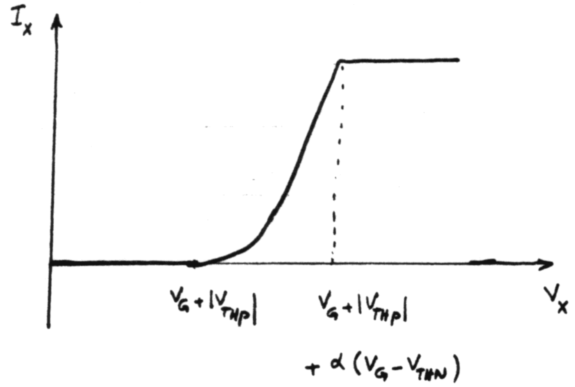
i.e. $$V_X = V_G + |V_{THP}| + \underbrace{\sqrt{\frac{\mu_n}{\mu_p} \frac{(W/L)_n}{(W/L)_p}}}_{\alpha} (V_G - V_{THN})$$

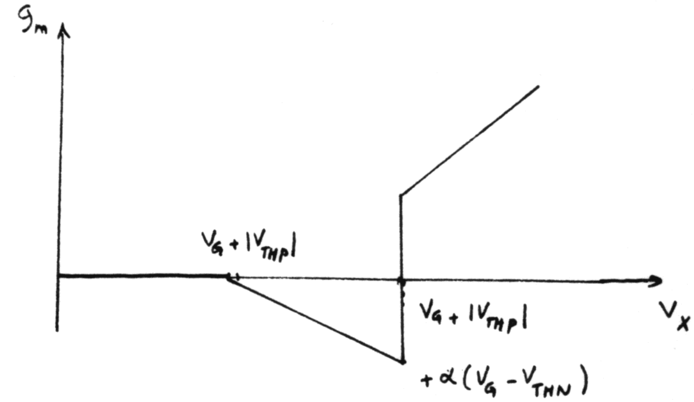
And afterward , M_2 goes into triode region and $I_x = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_N (V_G - V_{THN})^2$

So, $0 < V_X < V_G + |V_{THP}| \Rightarrow I_x = 0$ $g_m = \frac{\partial I_x}{\partial V_G} = 0$

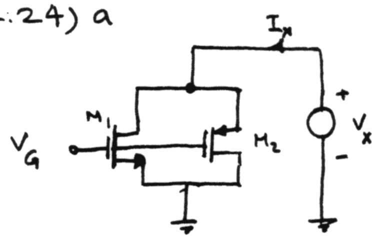
$V_G + |V_{THP}| < V_X < V_G + |V_{THP}| + \alpha (V_G - V_{THN})$ $I_x = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_X - V_G - |V_{THP}|)^2$ $g_m = \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_G + |V_{THP}| - V_X,$

$V_G + |V_{THP}| + \alpha (V_G - V_{THN}) < V_X$ $I_x = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_N (V_G - V_{THN})^2$ $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_N (V_G - V_{THN})$





2.24) a

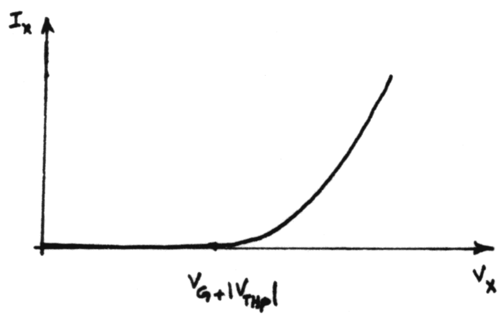


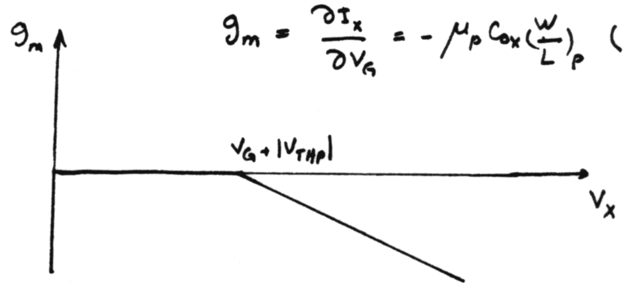
CASE I : $V_G < V_{THN}$ M_1 : off

for $0 < V_x < V_G + |V_{THP}|$ $I_x = 0$, $g_m = \frac{\partial I_x}{\partial V_G} = 0$

for $V_G + |V_{THP}| < V_x$ $\Rightarrow I_x = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p \left(V_x - V_G - |V_{THP}|\right)^2$

$$g_m = \frac{\partial I_x}{\partial V_G} = -\mu_p C_{ox}\left(\frac{W}{L}\right)_p \left(V_x - V_G - |V_{THP}|\right)$$





CASE II : $V_G > V_{THN}$

for $0 < V_x < V_G - V_{THN}$ (M_2 : off M_1 : triode)

$I_x = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n \left[2\left(V_G - V_{THN}\right)V_x - V_x^2\right]$ $g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)_n V_x$

for $V_G - V_{THN} < V_x < V_G + |V_{THP}|$ (M_2 : off M_1 : Sat)

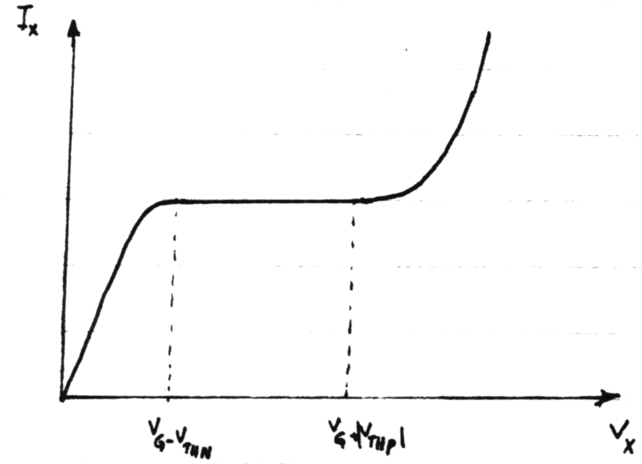
$I_x = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n \left(V_G - V_{THN}\right)^2$ $g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)_n \left(V_G - V_{THN}\right)$

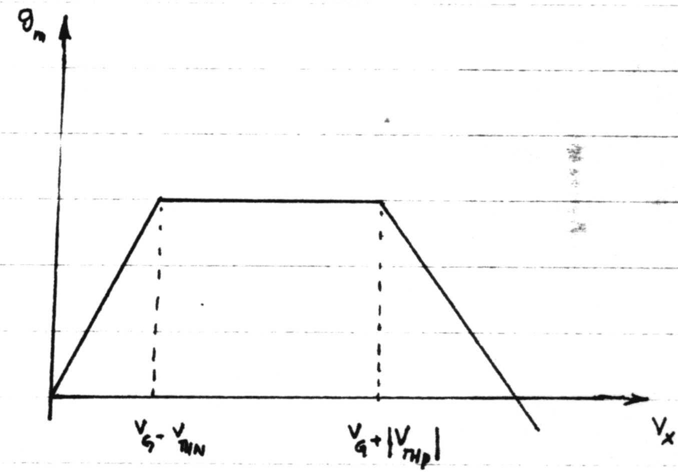
for $V_G + |V_{THP}| < V_x$ (M_2 : Sat M_1 : Sat)

2.24) a Cont.

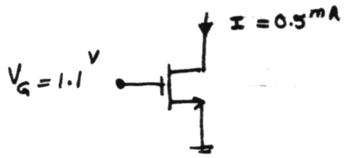
$$I_x = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_G - V_{THN})^2 + \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)^2$$

$$g_m = \frac{\partial I_x}{\partial V_G} = \mu_n C_{ox}\left(\frac{W}{L}\right)_n (V_G - V_{THN}) - \mu_p C_{ox}\left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)$$





2.25) $V_{TH} = 0.7$ $\quad \lambda = 0.1$ $\quad$ (for $L = 0.5^{\mu}$)



for $L = 0.5^{\mu}$ $\quad \lambda = 0.1 \rightarrow r_o = \dfrac{1}{\lambda I_D} = 20^{K\Omega}$

$$V_{OD} = V_{GS} - V_{TH} = 0.4 \Rightarrow V_{GS} = 1.1^V$$

Calculating W, $\quad I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2$

$$0.5^{mA} = \frac{1}{2} \times 0.1343 \frac{mA}{V^2} \times \frac{W}{0.5^{\mu} - 0.16^{\mu}} \times (0.4)^2$$

$\frac{W}{L_{eff}} \simeq 47 \Rightarrow \quad W = 15.82\ \mu M$

$$C_{gs} = \frac{2}{3} WL\, C_{ox} + W C_{ov} \simeq 25\, fF$$

$$C_{GD} = W C_{ov} = 4.85\, fF$$

$$C_{DB} = \frac{W}{2} E\, C_j + 2\left(\frac{W}{2} + E\right) C_{jsw} \quad (@\ V_D = 0.4) = 10.7\ fF$$

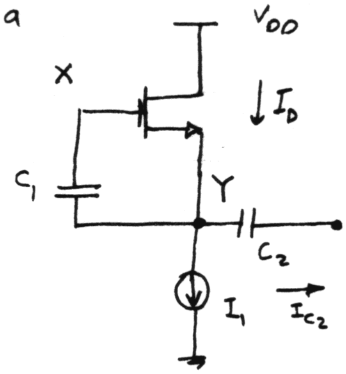
(for folded structure)

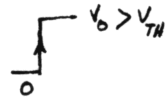
$$\left(C_j = \frac{C_{jo}}{\left(1 + \frac{V_{DB}}{2\varphi_F}\right)^{m_j}} = 0.449 \times 10^{-3} \frac{F}{M^2} \quad , \quad C_{jsw} = \frac{C_{jswo}}{\left(1 + \frac{V_{DB}}{2\varphi_F}\right)^{m_{jsw}}} = 0.325 \times 10^{-11} \, F/M \right)$$

$C_{ox} = 3.84 \times 10^{-3}\ F/m$ $\qquad C_{jo} = 0.56 \times 10^{-3}$ $\qquad m_j = 0.6$

$C_{jsw_o} = 0.35 \times 10^{-11}$ $\qquad m_{jsw_o} = 0.2$

2.26) a





Before applying the pulse

$$X(0^-) = V_{DD}$$

$$Y(0^-) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}}$$

After Applying the Pulse

$$X(0^+) = V_{DD} + V_0$$

$$Y(0^+) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} + V_0$$

for $t>0$

$$\begin{cases} X(t) = V_{DD} + \alpha(t) \\ Y(t) = V_{DD} - V_{TH} - \sqrt{\dfrac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} + \alpha(t) \end{cases}$$

$\alpha(0^+) = V_0$, Device is in triode

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2\right] = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} - \left(V_{TH} + \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} - \alpha(t)\right)\right]$$

$$\left(V_{TH} + \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} - \alpha(t)\right)$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}} - (\alpha(t) - V_{TH})^2\right] = I_1 - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(\alpha(t) - V_{TH})^2$$

$$I_{C_2} = I_D - I_1 = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(\alpha(t) - V_{TH})^2 = C_2\frac{dV_{C_2}}{dt} = C_2\frac{d\alpha(t)}{dt}$$

$$\underbrace{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\cdot\frac{1}{C_2}}_{K}\,dt = \frac{-d\alpha}{(\alpha - V_{TH})^2} \Rightarrow Kt = \frac{1}{\alpha - V_{TH}} - \frac{1}{V_0 - V_{TH}}$$

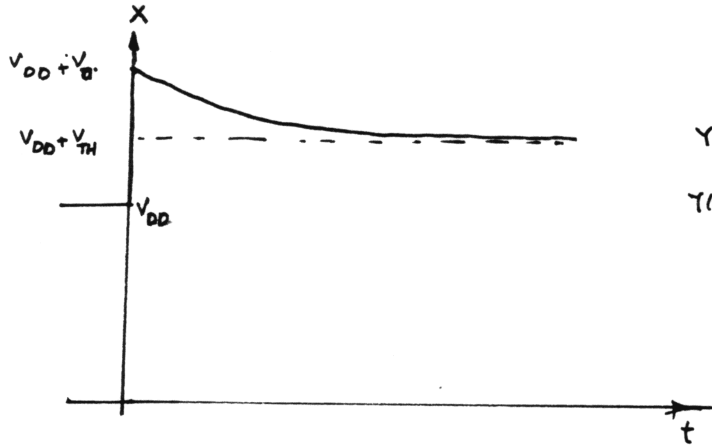
$$\Rightarrow \alpha(t) = V_{TH} + \frac{1}{Kt + \frac{1}{V_0 - V_{TH}}}$$

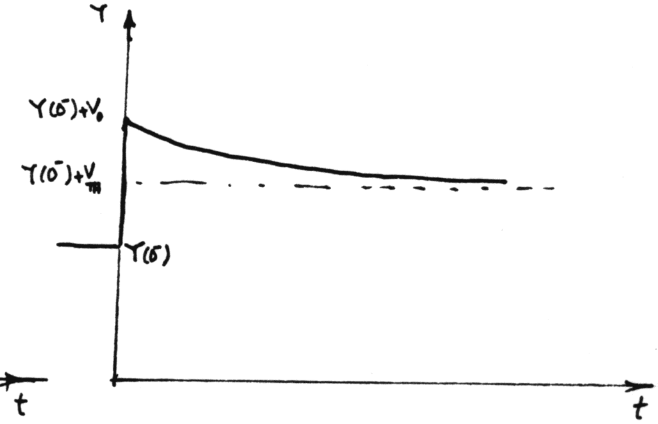
$$\alpha(\infty) = V_{TH}$$

2.26) a Cont.

$$X(\infty) = V_{DD} + V_{TH}$$

$$Y(\infty) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} + V_{TH} = V_{DD} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}}$$





2.26) b

Before applying the pulse.

$$X(0^-) = V_{DD}$$

$$Y(0^-) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}}$$

After applying the pulse

$$X(0^+) = V_{DD} - V_{TH}$$

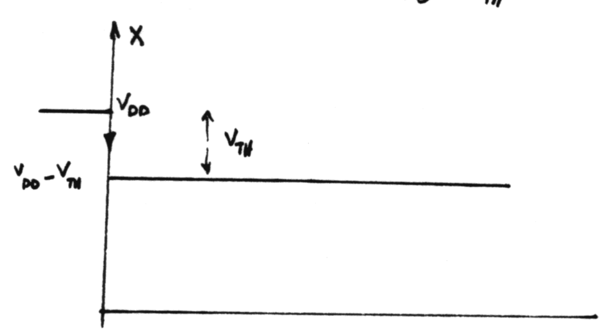
$$Y(0^+) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}} - V_{TH}$$

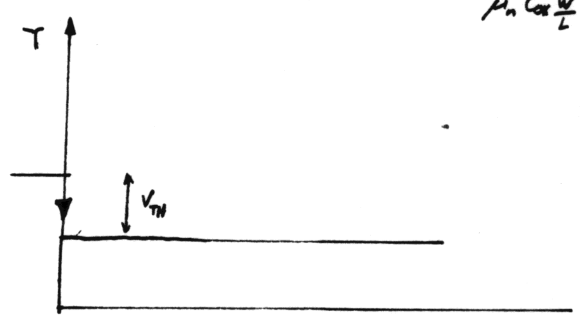
After applying the pulse, device remains in the saturation region, and its current doesn't chang, So, $I_{C_1} = I_{C_2} = 0$

Therefore, the circuit keeps its state.

$$X(t) = X(0^+) = V_{DD} - V_{TH}$$

$$Y(t) = Y(0^+) = V_{DD} - 2V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox}\frac{W}{L}}}$$





2.27)

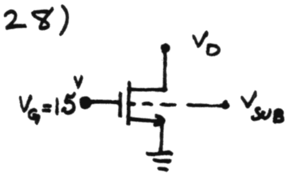
$$I_D = I_o \exp \frac{V_{GS}}{\xi V_T}$$

$$\frac{I_{D2}}{I_{D1}} = \exp \frac{V_{GS2} - V_{GS1}}{\xi V_T} \qquad \frac{I_{D2}}{I_1} = 10 \quad \Rightarrow \quad \Delta V_{GS} = \xi V_T \ln 10$$

$$\Delta V_{GS} = 1.5 \times \ln 10 \times 26^{mV} = 89.8 \, mV$$

$$g_m = \frac{I_D}{\xi V_T} = \frac{10^{\mu A}}{1.5 \times 26^{mV}} = 0.26 \, {}^{mA}/_{V}$$

2.28)



a) If we decrease V_D below zero, Source and drain exchange their roles and device operates in the triode region.

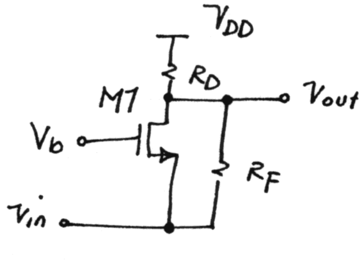
b) If we increase V_B, V_{TH} decreases, because

$$\Delta V_{TH} = \gamma \left(\sqrt{2\varphi_F - V_B} - \sqrt{2\varphi_F} \right) \text{ is negative.}$$

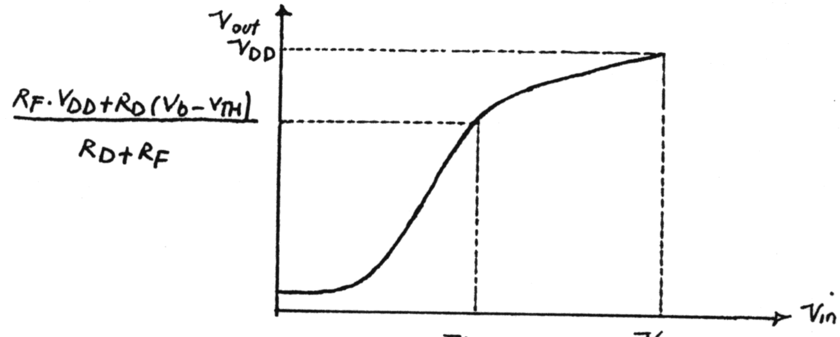
Therefore, I_D increases.

Chapter 3

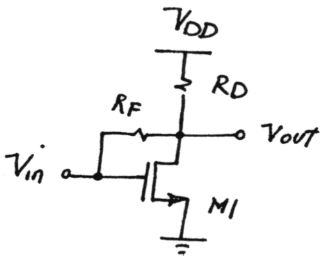
3.1.



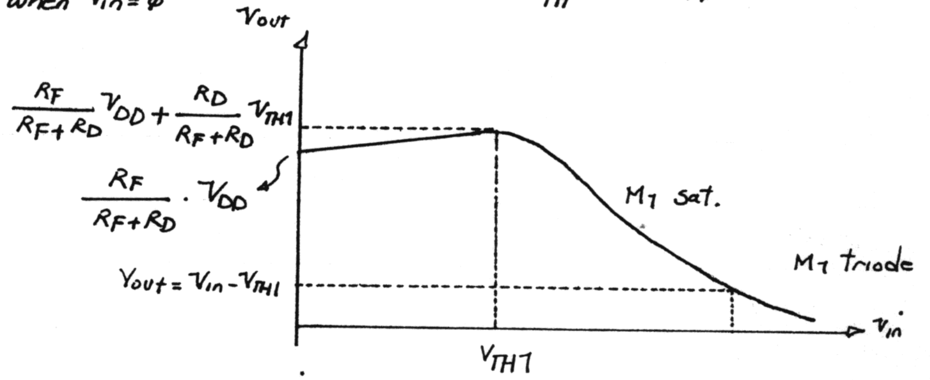
(a)



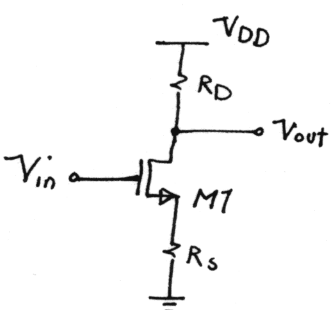
We assume that M_1 is saturated when $V_{in} = \phi$



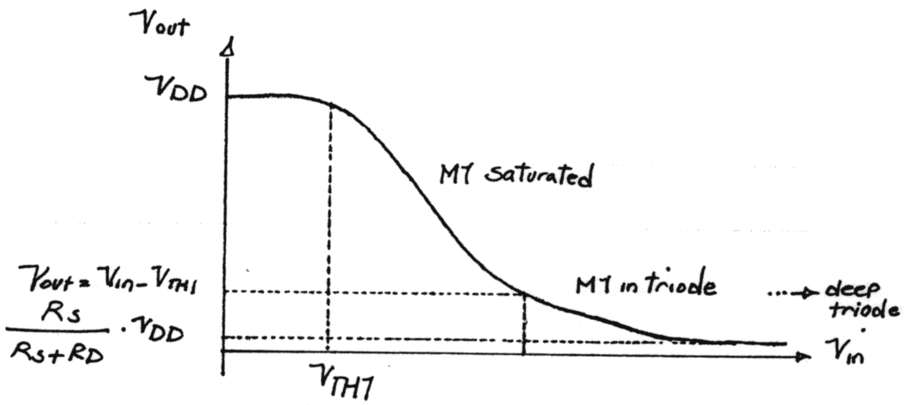
(b)

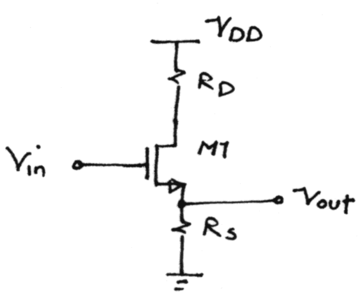


if $V_{in} < V_{TH1} \rightarrow V_{out} = \frac{R_F}{R_F+R_D} V_{DD} + \frac{R_D}{R_F+R_D} V_{in}$

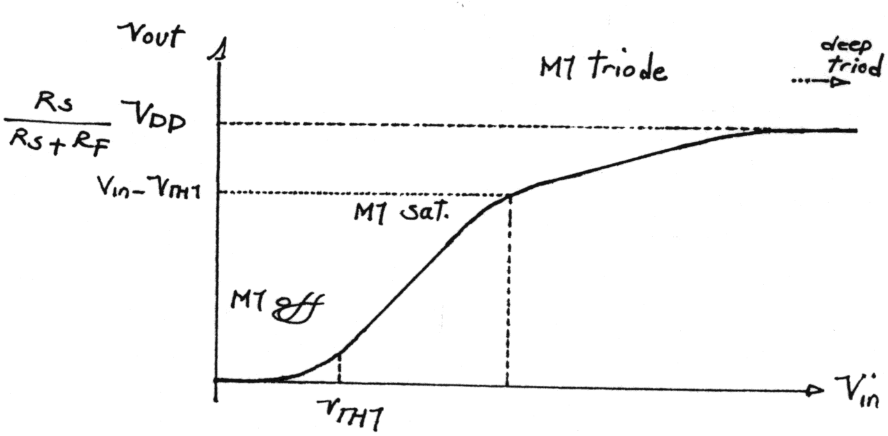


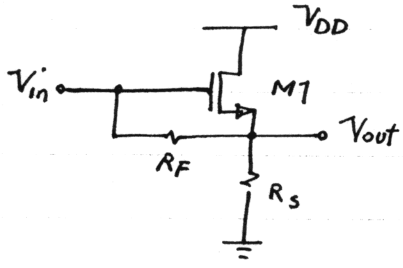
(c)



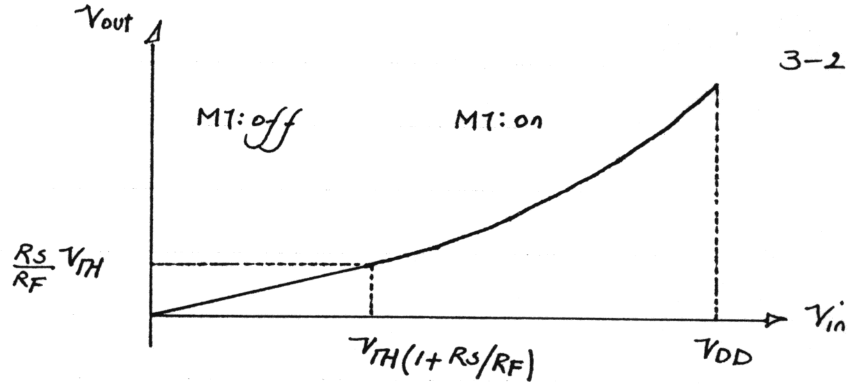


(d)



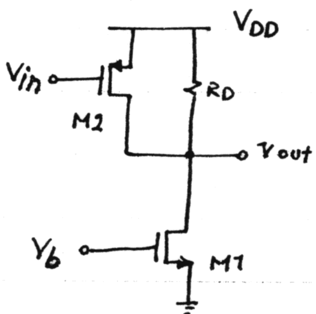


(e)

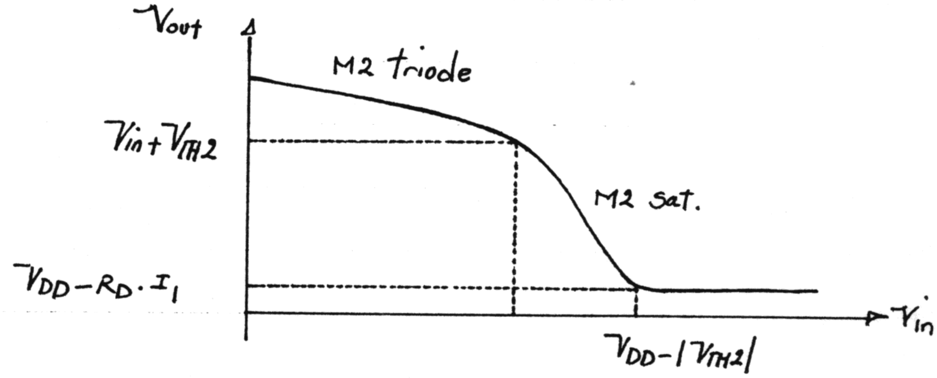


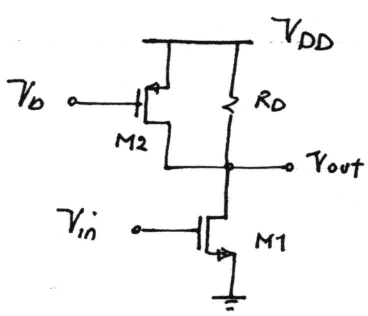
$$\text{if } V_{in} < (1+R_S/R_F)V_{TH} \rightarrow V_o = \frac{R_S}{R_S+R_F}V_{in}$$

3.2.

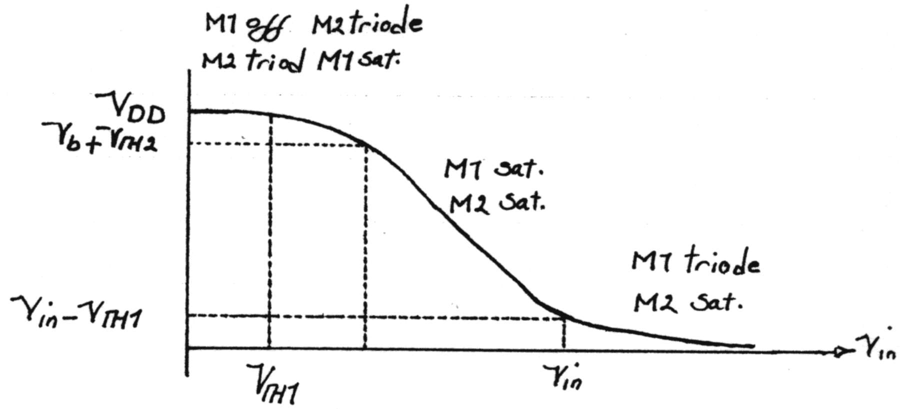


(b)

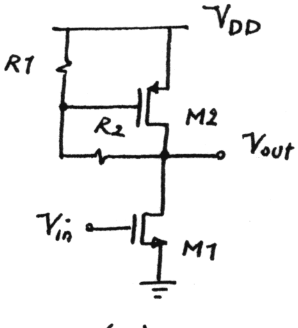




(a)

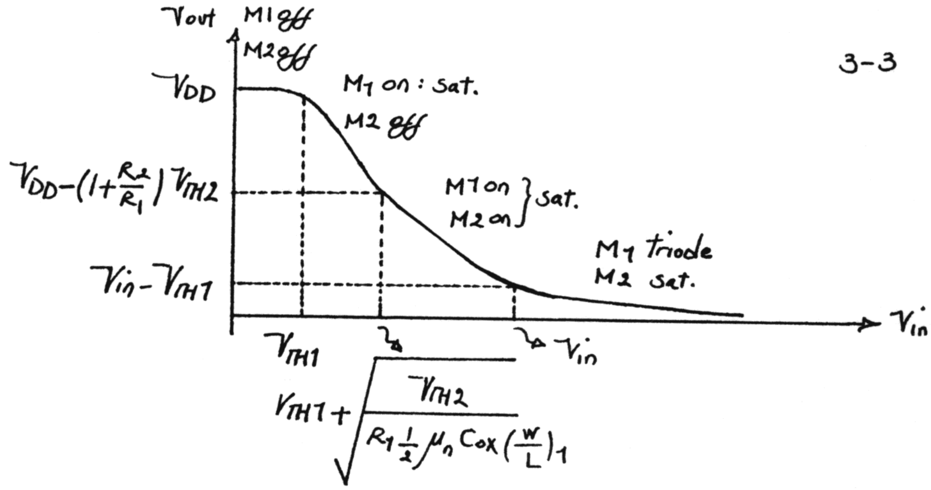


3-3

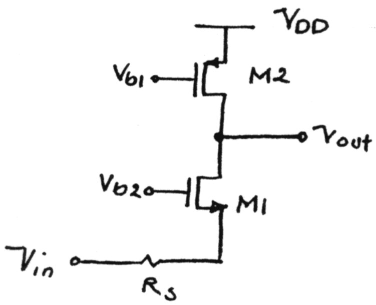


VDD
R1
R2
M2
Vout
Vin
M1

(c)

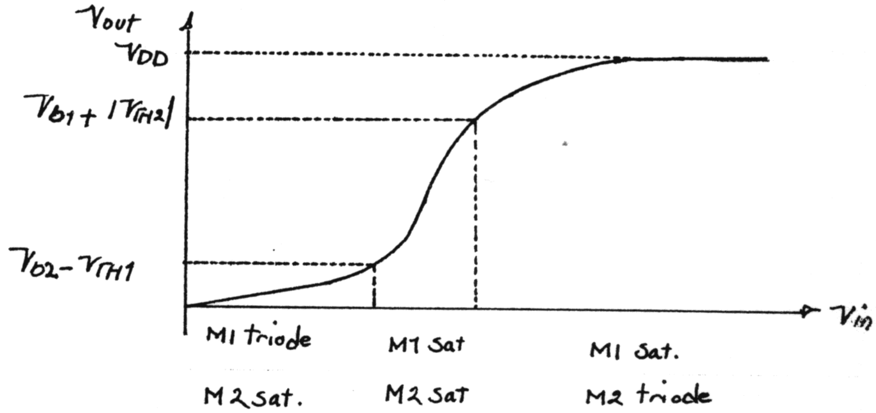


Vout
M1 off
M2 off
VDD
M1 on : sat.
M2 off
VDD-(1+R2/R1) VTH2
M1 on
M2 on
sat.
M1 triode
M2 sat.
Vin-VTH1
Vin
VTH1
Vin
VTH1 + √(VTH2 / (R1 ½ μn Cox (W/L)1))



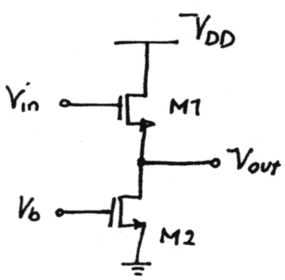
VDD
Vb1
M2
Vout
Vb2
M1
Vin
Rs

(d)



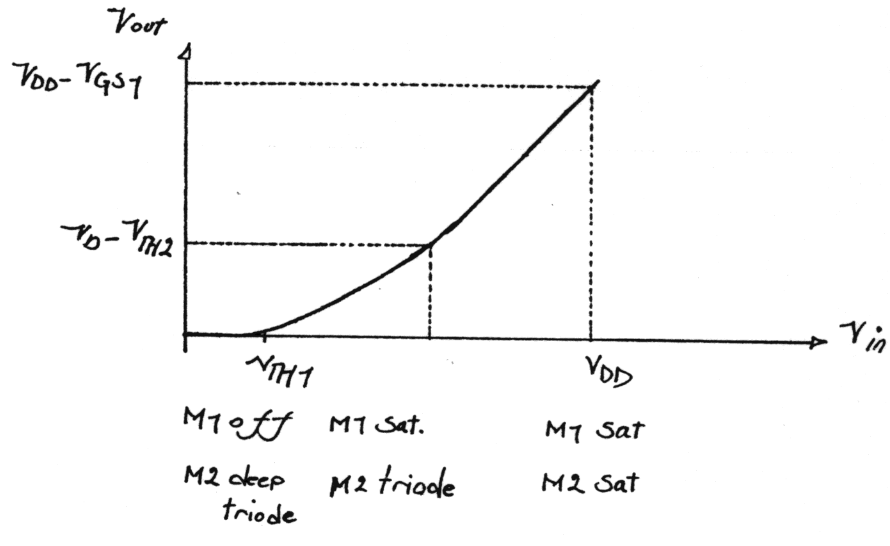
Vout
VDD
Vb1 + |VTH2|
Vb2- VTH1
Vin
M1 triode
M2 sat.
M1 Sat
M2 Sat
M1 Sat.
M2 triode

3.2:

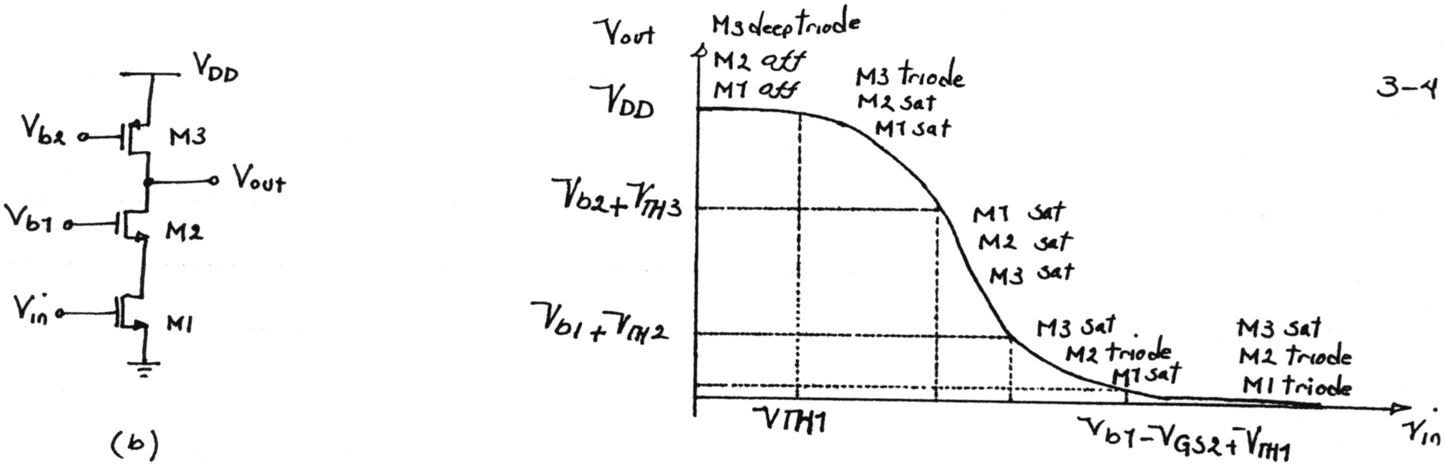


VDD
Vin
M1
Vout
Vb
M2

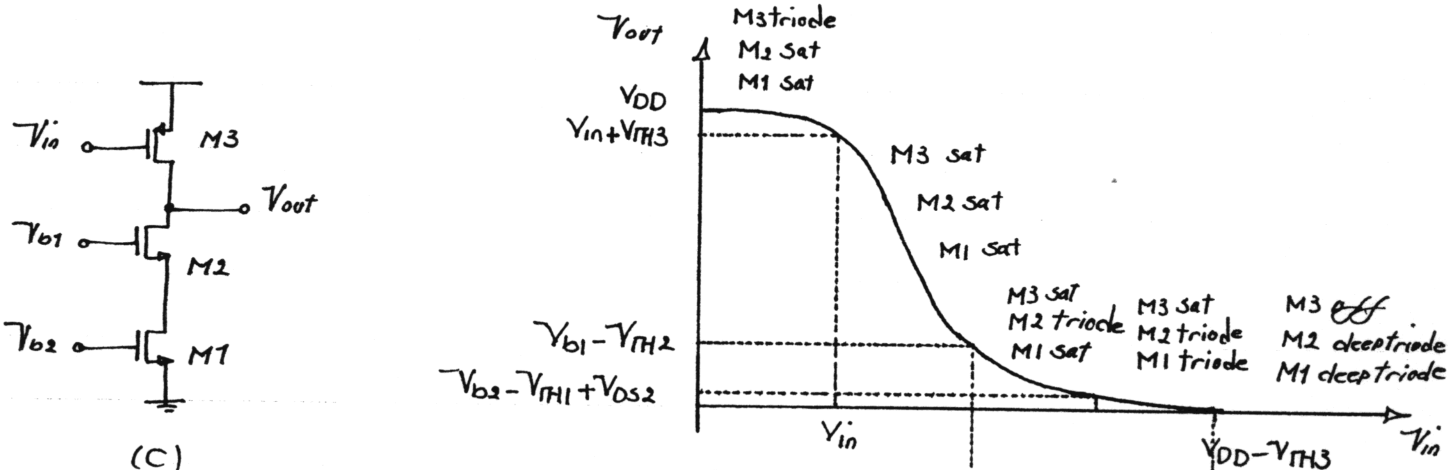
(a)



Vout
VDD- VGS1
Vb- VTH2
Vin
VTH1
VDD
M1 off
M2 deep triode
M1 Sat.
M2 triode
M1 Sat
M2 Sat

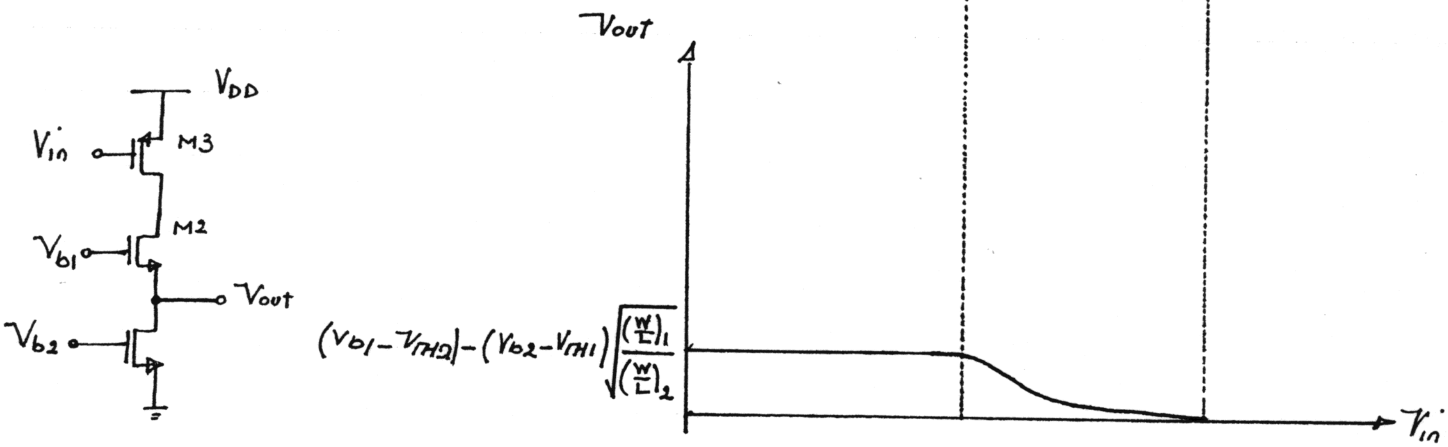


(b)



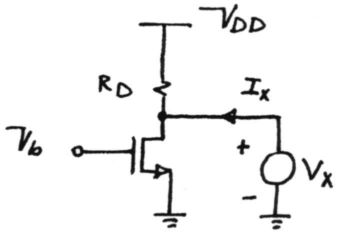
(c)

V_{DS2} is obtained from $\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b2}-V_{TH1})^2 = \mu_n C_{ox}\left(\frac{W}{L}\right)_2\left[(V_{b1}-V_{b2}+V_{TH1}-V_{TH2})V_{DS2}-\frac{V_{DS2}^2}{2}\right]$

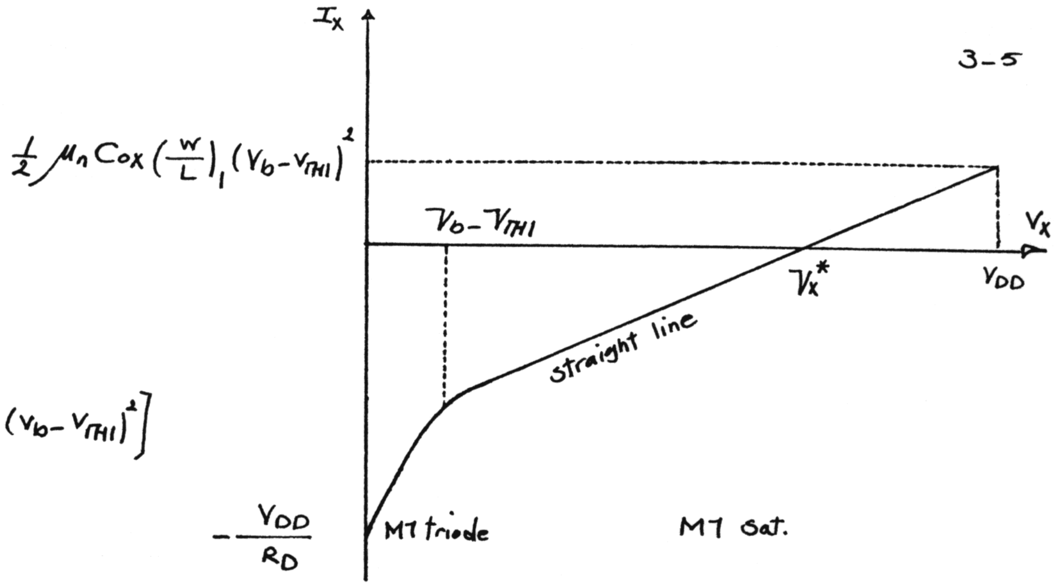


(d)

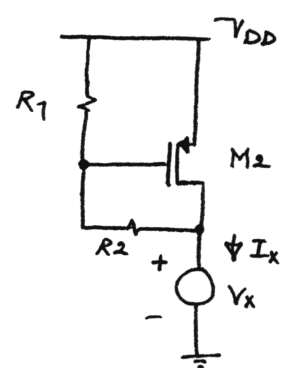
3.3.



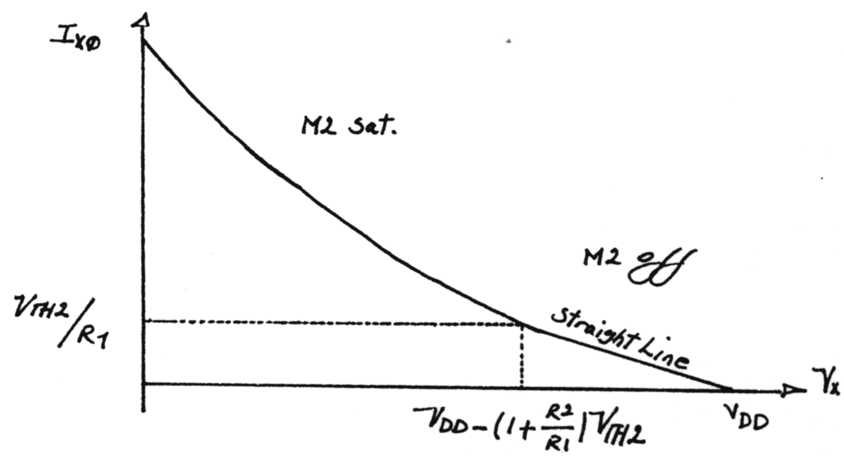
(a)



$$V_X^* = V_{DD} - R_D\left[\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_b - V_{TH1})^2\right]$$

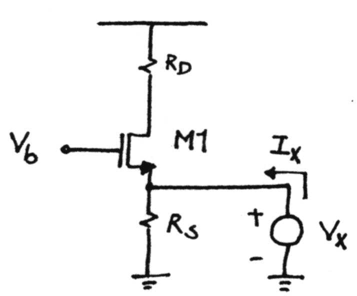


(b)

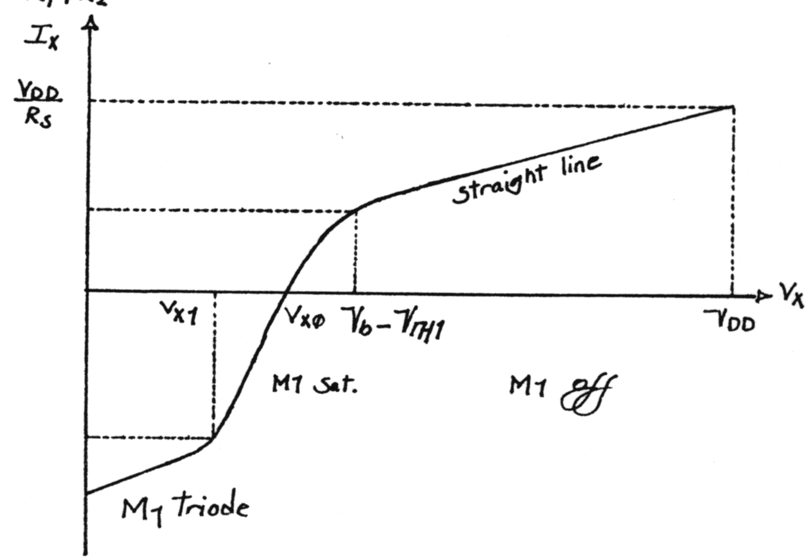


If $V_X < V_{DD} - \left(1+\frac{R_2}{R_1}\right)V_{TH2}$ $\quad I_X = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2\left[\frac{(V_{DD}-V_X)}{R_1+R_2}\cdot R_1 - V_{TH2}\right]^2 + \frac{V_{DD}-V_X}{R_1+R_2}$

If $V_X > V_{DD} - \left(1+\frac{R_2}{R_1}\right)V_{TH2}$ $\quad I_X = \frac{V_{DD}-V_X}{R_1+R_2}$



(c)

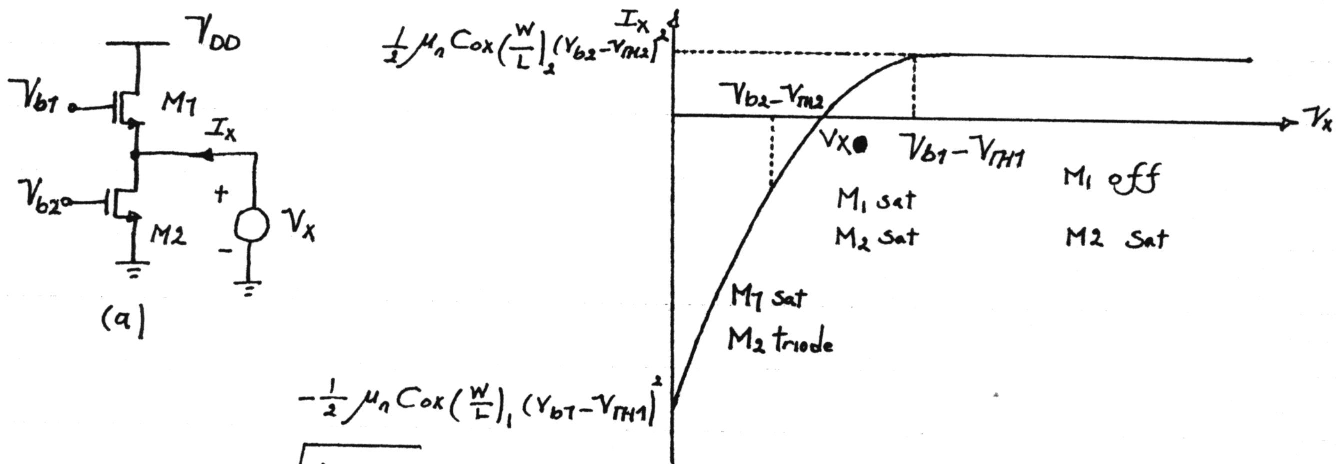


3-6

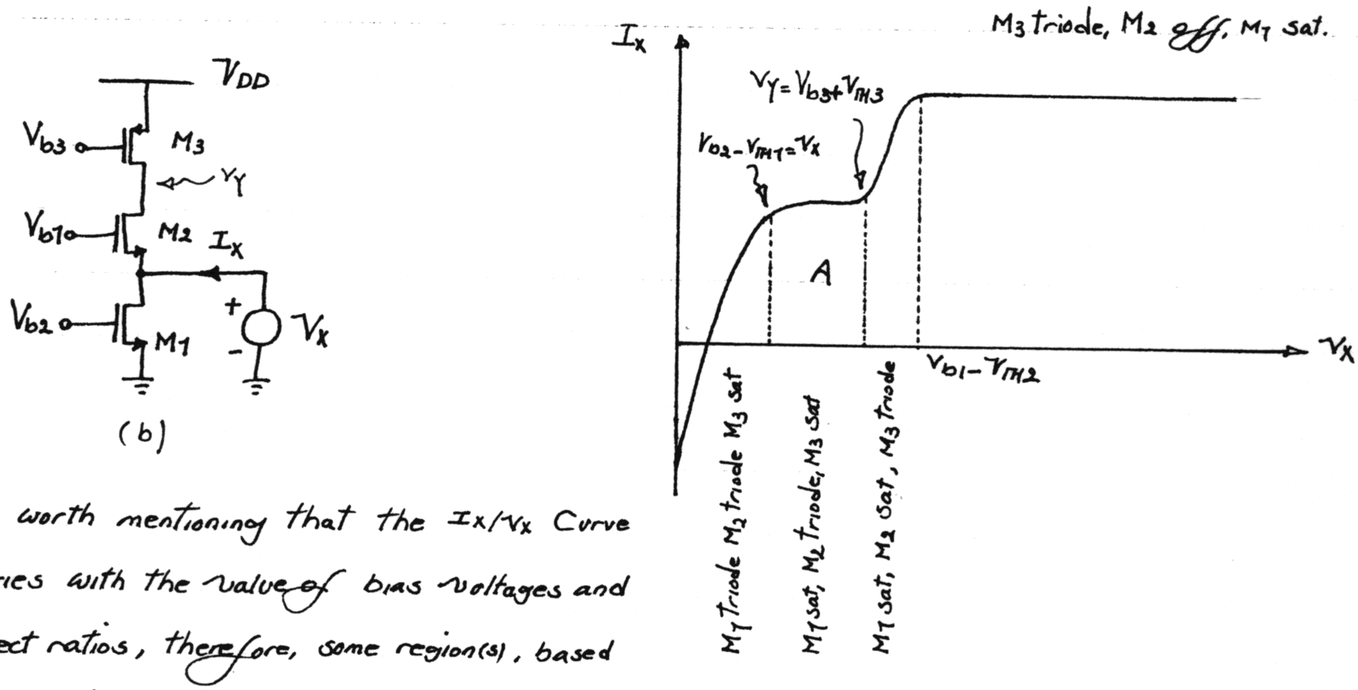
$$V_{X0} = V_{DD} - \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_b - V_{TH1})^2 \cdot R_D$$

$$V_{X1} = V_b - V_{TH1} - \left(\frac{2(V_{DD} - V_b + V_{TH1})}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot R_D}\right)^{1/2}$$

3.4.

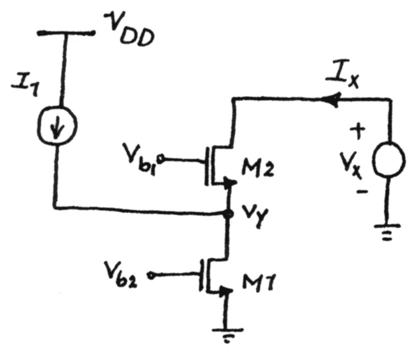


$$V_{X0} = V_{b1} - V_{TH1} - \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}\,(V_{b2} - V_{TH2})$$



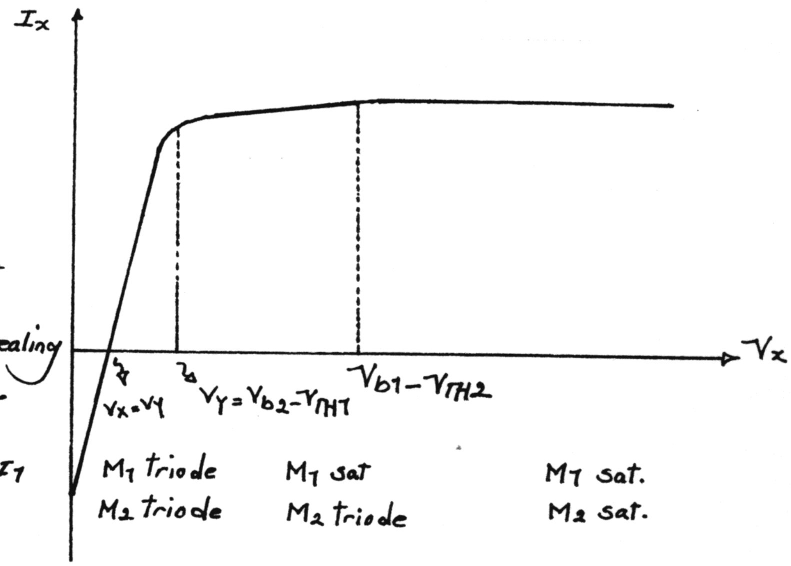
It's worth mentioning that the I_X/V_X Curve varies with the value of bias voltages and aspect ratios, therefore, some region(s), based on the aforementioned parameters, gets wider or narrower, especially the region Called "A" in the above figure.

3-7

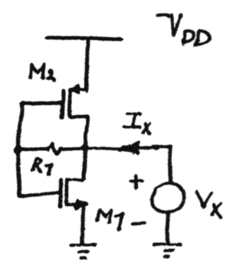


(c)

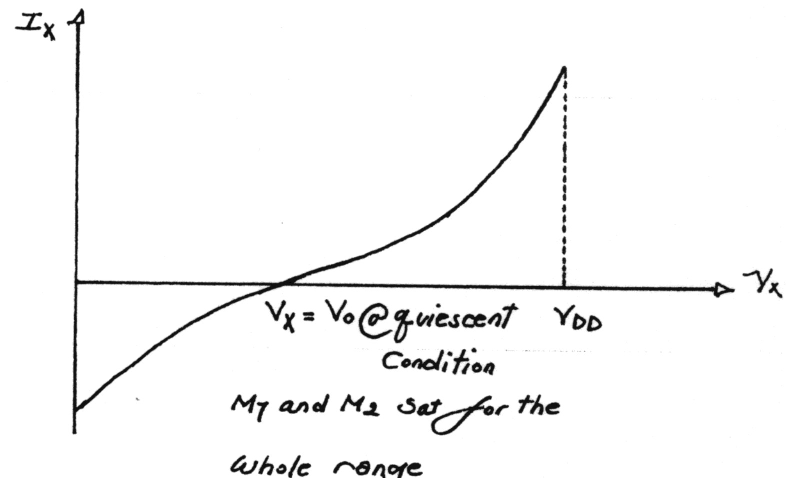
we assume $V_{b1} > V_{b2}$ and both M_1 and M_2 operate in Saturation region if $V_X = V_{DD}$

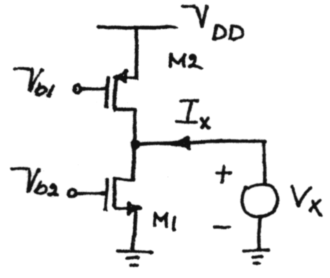


Below V_X, for which $V_X = V_Y$, drain current of M_2 flows in opposit direction, revealing the fact the drain and source terminals of M_2 are reversed. As expected, most of I_1 flow through M_2 when $V_X = 0$, because we assume that $V_{b1} > V_{b2}$.

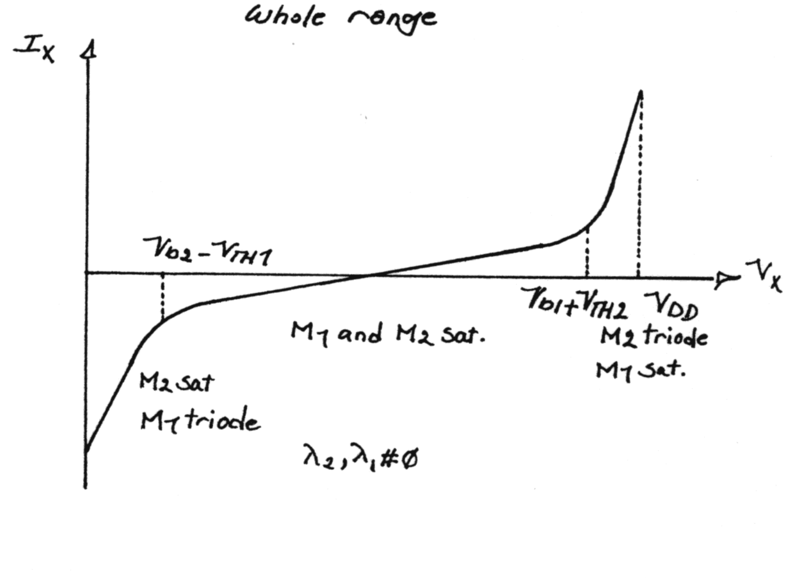


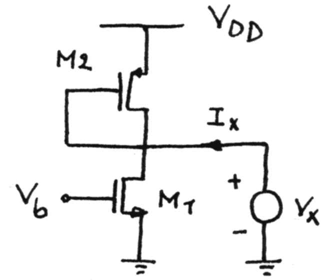
(d)



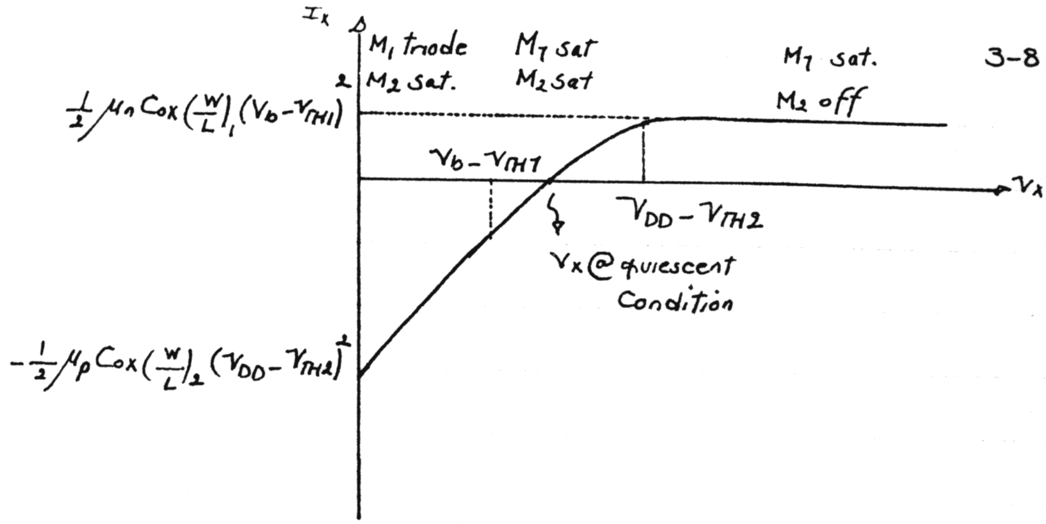


(e)



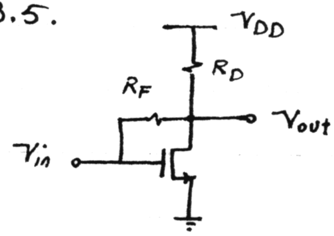


(f)



3-8

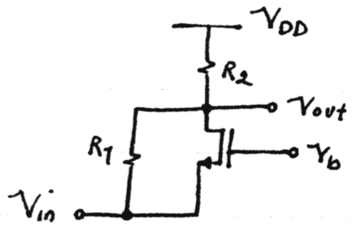
3.5.



(a)

$$\frac{V_o - V_{in}}{R_F} + g_{m1} V_{in} + \frac{V_o}{r_{o1}} + \frac{V_o}{R_D} = 0$$

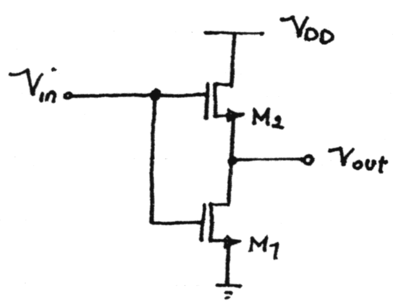
$$A_V = \frac{V_o}{V_{in}} = -\frac{g_{m1} - 1/R_F}{\frac{1}{R_F} + \frac{1}{r_{o1}} + \frac{1}{R_D}}$$



(b)

$$\frac{V_o}{R_2} + (V_o - V_{in})\left(\frac{1}{R_1} + \frac{1}{r_{o1}}\right) - g_{m1} V_{in} = 0$$

$$\frac{V_o}{V_{in}} = \frac{g_{m1} + \frac{1}{R_1} + \frac{1}{r_{o1}}}{\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{r_{o1}}}$$

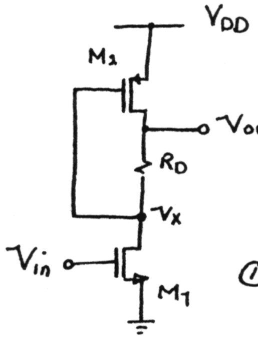


(C)

$$g_{m2}(V_{in} - V_{out}) + \frac{-V_{out}}{r_{o2}} = g_{m1} V_{in} + \frac{V_{out}}{r_{o1}}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1} - g_{m2}}{g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}}}$$

3-9



(d)

$$(1)\ \left(g_{m1}V_{in}+\frac{V_X}{r_{o1}}\right)R_D+V_X=V_{out},\quad (2)\ -\left(g_{m2}V_X+\frac{V_{out}}{r_{o2}}\right)=g_{m1}V_{in}+\frac{V_X}{r_{o1}},$$

$$(2)\rightarrow V_X\left(-g_{m2}-\frac{1}{r_{o1}}\right)=g_{m1}V_{in}+\frac{V_{out}}{r_{o2}}\quad (2)\rightarrow V_X=-\frac{g_{m1}\cdot V_{in}+V_{out}/r_{o2}}{g_{m2}+\frac{1}{r_{o1}}}\quad (3)$$

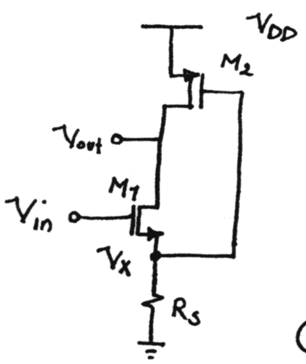
$$(1)\rightarrow g_{m1}R_D V_{in}+\left(1+\frac{R_D}{r_{o1}}\right)V_X=V_o\quad (4)$$

$$(3),(4)\rightarrow g_m R_D V_{in}-\frac{\left(1+\frac{R_D}{r_{o1}}\right)\left(g_{m1}V_{in}+\frac{V_{out}}{r_{o2}}\right)}{g_{m2}+\frac{1}{r_{o1}}}=V_{out}$$

$$\left[g_{m1}\cdot R_D-\frac{g_{m1}(1+R_D/r_{o1})}{g_{m2}+\frac{1}{r_{o1}}}\right]V_{in}=\left[1+\frac{\frac{1}{r_{o2}}(1+R_D/r_{o1})}{g_{m2}+\frac{1}{r_{o1}}}\right]V_{out}$$

$$\left[g_{m1}\cdot R_D\left(g_{m2}+\frac{1}{r_{o1}}\right)-g_{m1}\left(1+\frac{R_D}{r_{o1}}\right)\right]V_{in}=\left[g_{m2}+\frac{1}{r_{o1}}+\frac{1}{r_{o2}}\left(1+\frac{R_D}{r_{o1}}\right)\right]V_{out}$$

$$\frac{V_{out}}{V_{in}}=\frac{g_{m1}(g_{m2}R_D-1)}{g_{m2}+\frac{1}{r_{o1}}+\frac{1}{r_{o2}}\left(1+\frac{R_D}{r_{o1}}\right)}$$



(e)

$$-\left(\frac{V_{out}}{r_{o2}}+g_{m2}V_X\right)\overset{(1)}{=}\frac{V_{out}-V_X}{r_{o1}}+g_{m1}(V_{in}-V_X)\overset{(2)}{=}\overset{(3)}{\frac{V_X}{R_S}}$$

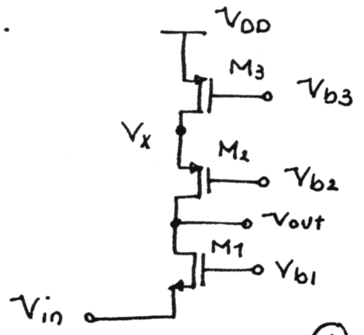
$$(1),(3)\rightarrow V_X=-\frac{V_{out}}{r_{o2}\left(g_{m2}+\frac{1}{R_S}\right)}$$

$$(2),(3)\quad \frac{V_{out}}{r_{o1}}+g_{m1}V_{in}=-\frac{V_{out}}{r_{o2}\left(g_{m2}+\frac{1}{R_S}\right)}\left(\frac{1}{R_S}+g_{m1}+\frac{1}{r_{o1}}\right)$$

$$\frac{V_{out}}{r_{o1}}\cdot r_{o2}\left(g_{m2}+\frac{1}{R_S}\right)+g_{m1}\cdot V_{in}\cdot r_{o2}\left(g_{m2}+\frac{1}{R_S}\right)=-V_{out}\left(\frac{1}{R_S}+g_{m1}+\frac{1}{r_{o1}}\right)$$

$$\frac{V_{out}}{V_{in}}=-\frac{g_{m1}(g_{m2}+1/R_S)r_{o2}}{g_{m1}+\frac{1}{R_S}+\frac{1}{r_{o1}}\left[1+r_{o2}\left(g_{m2}+\frac{1}{R_S}\right)\right]}$$

3.6.



(a)

(1) (2) (3)

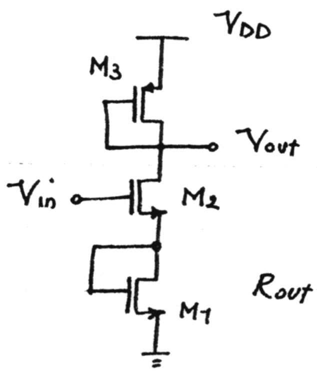
$$-\frac{V_X}{r_{o3}} = g_{m2}V_X + \frac{V_X - V_{out}}{r_{o2}} = \frac{V_{out} - V_{in}}{r_{o1}} - g_{m1}V_{in}$$

$$(1),(2) \rightarrow V_X = \frac{V_{out}}{1 + r_{o2}\left(g_{m2} + \frac{1}{r_{o3}}\right)}$$

$$(1),(3) \rightarrow \frac{V_{out} - V_{in}}{r_{o1}} - g_{m1}V_{in} = -\frac{V_{out}}{r_{o3} + r_{o2}(1 + g_{m2}\cdot r_{o3})}$$

$$V_{out}\left[\frac{1}{r_{o1}} + \frac{1}{r_{o3} + r_{o2}(1 + g_{m2}\cdot r_{o3})}\right] = \left(g_{m1} + \frac{1}{r_{o1}}\right)V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{(1 + g_{m1}\cdot r_{o1})\left[r_{o3} + r_{o2}(1 + g_{m2}\cdot r_{o3})\right]}{r_{o1} + r_{o3} + r_{o2}(1 + g_{m2}r_{o3})}$$

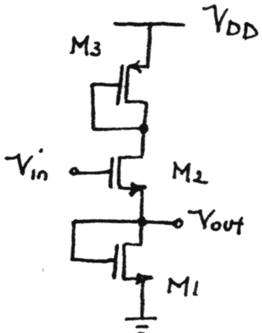


$$G_m = \frac{g_{m2}\cdot r_{o2}}{\left(\frac{1}{g_{m1}} \| r_{o1}\right) + \left[1 + g_{m2}\left(\frac{1}{g_{m1}} \| r_{o1}\right)\right] r_{o2}}$$

$$R_{out} = \left(\frac{1}{g_{m3}} \| r_{o3}\right) \| \left\{\left[1 + g_{m2}\left(\frac{1}{g_{m1}} \| r_{o1}\right)\right] r_{o2} + \left(\frac{1}{g_{m1}} \| r_{o1}\right)\right\}$$

(b)

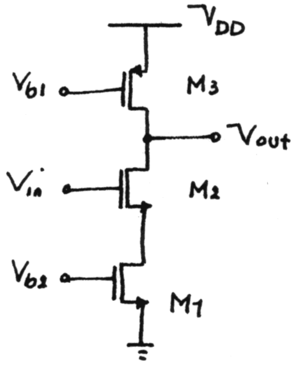
$$A_V = -G_m\cdot R_{out} = -\frac{g_{m2}r_{o2}\left(\frac{1}{g_{m3}} \| r_{o3}\right)}{\left(\frac{1}{g_{m3}} \| r_{o3}\right) + \left\{\left[1 + g_{m2}\left(\frac{1}{g_{m1}} \| r_{o1}\right)\right] r_{o2} + \left(\frac{1}{g_{m1}} \| r_{o1}\right)\right\}}$$



(C)

resistance seen looking up at the source of M_2, $\dfrac{\left(\frac{1}{g_{m3}} \| r_{o3}\right) + r_{o2}}{1 + g_{m2}\cdot r_{o2}}$

$$\frac{V_{out}}{V_{in}} = \frac{\left(\frac{1}{g_{m1}} \| r_{o1}\right)}{\left(\frac{1}{g_{m1}} \| r_{o1}\right) + \dfrac{\left(\frac{1}{g_{m3}} \| r_{o3}\right) + r_{o2}}{1 + g_{m2}\cdot r_{o2}}}$$

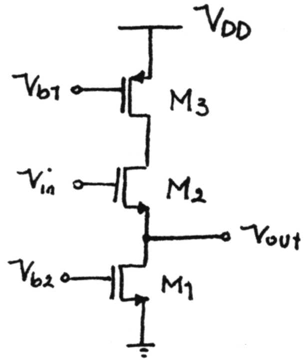


(d)

$$G_m = \frac{g_{m2} \cdot r_{o2}}{r_{o1} + \left[1 + g_{m2} \cdot r_{o1}\right] r_{o2}}$$

$$R_{out} = r_{o3} \parallel \left[(1 + g_{m2} \cdot r_{o1}) r_{o2} + r_{o1}\right]$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m2} \cdot r_{o2} \cdot r_{o3}}{r_{o3} + (1 + g_{m2} \cdot r_{o1}) r_{o2} + r_{o1}}$$

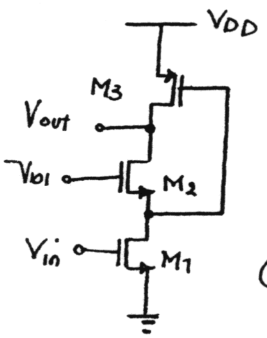


(e)

resistance seen looking up at the source of M_2

$$R_{in} = \frac{r_{o3} + r_{o2}}{1 + g_{m2} \cdot r_{o2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{r_{o1}}{r_{o1} + \dfrac{r_{o3} + r_{o2}}{1 + g_{m2} r_{o2}}} = \frac{r_{o1}(1 + g_{m2} \cdot r_{o2})}{r_{o1}(1 + g_{m2} \cdot r_{o2}) + r_{o2} + r_{o3}}$$



$$\overset{(1)}{-\left(\frac{V_{out}}{r_{o3}} + g_{m3} V_X\right)} = \overset{(2)}{\left(\frac{V_{out} - V_X}{r_{o2}} - g_{m2} V_X\right)} = \overset{(3)}{\frac{V_X}{r_{o1}} + g_{m1} V_{in}}$$

(1), (2) $\rightarrow$ $\dfrac{V_X}{r_{o2}} + g_{m2} V_X - g_{m3} V_X = \dfrac{V_{out}}{r_{o2}} + \dfrac{V_{out}}{r_{o3}} \rightarrow V_X = \dfrac{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}}{\frac{1}{r_{o2}} + g_{m2} - g_{m3}} V_{out}$

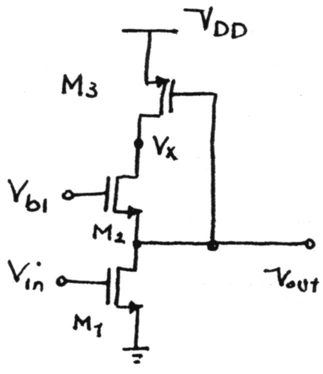
(1), (3) $\quad -\dfrac{V_{out}}{r_{o3}} - g_{m3} V_X = \dfrac{V_X}{r_{o1}} + g_{m1} V_{in}$

$$-\frac{V_{out}}{r_{o3}} - \left(g_{m3} + \frac{1}{r_{o1}}\right) \frac{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}}{\frac{1}{r_{o2}} + g_{m2} - g_{m3}} V_{out} = g_{m1} \cdot V_{in}$$

$$-V_{out}\left[\frac{1}{r_{o3}} + \frac{\left(g_{m3} + \frac{1}{r_{o1}}\right)\left(\frac{1}{r_{o2}} + \frac{1}{r_{o3}}\right)}{\frac{1}{r_{o2}} + g_{m2} - g_{m3}}\right] = g_{m1} \cdot V_{in}$$

$$-V_{out}\left[\frac{1}{r_{o3}}+\frac{(1+g_{m3}r_{o1})(r_{o3}+r_{o2})}{r_{o1}r_{o3}\left[1+(g_{m2}-g_{m3})r_{o2}\right]}\right]=g_{m1}\cdot V_{in}$$ 3-12

$$\frac{V_{out}}{V_{in}}=-\frac{g_{m1}r_{o1}r_{o3}\left[1+(g_{m2}-g_{m3})r_{o2}\right]}{r_{o1}\left[1+(g_{m2}-g_{m3})r_{o2}\right]+(1+g_{m3}\cdot r_{o1})(r_{o3}+r_{o2})}$$



(g)

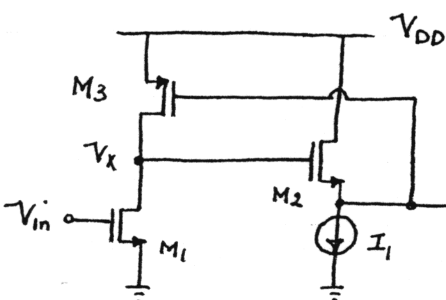
$$V_X=\frac{\frac{1}{r_{o2}}+g_{m2}-g_{m3}}{\frac{1}{r_{o2}}+\frac{1}{r_{o3}}}\cdot V_{out}$$

$$-\frac{V_X}{r_{o3}}-g_{m3}V_{out}=\frac{V_{out}}{r_{o1}}+g_{m1}\cdot V_{in}$$

$$-\frac{\frac{1}{r_{o2}}+g_{m2}-g_{m3}}{\frac{r_{o3}}{r_{o2}}+1}V_{out}-g_{m3}V_{out}=\frac{V_{out}}{r_{o1}}+g_{m1}V_{in}$$

$$-V_{out}\left[\frac{1+(g_{m2}-g_{m3})r_{o2}}{r_{o3}+r_{o2}}+g_{m3}+\frac{1}{r_{o1}}\right]=g_{m1}V_{in}$$

$$\frac{V_{out}}{V_{in}}=-\frac{g_{m1}r_{o1}(r_{o2}+r_{o3})}{r_{o1}\left[1+(g_{m2}-g_{m3})r_{o2}\right]+(r_{o2}+r_{o3})(1+g_{m3}\cdot r_{o1})}$$



(h)

$$-\left(\overset{①}{\frac{V_X}{r_{o3}}}+g_{m3}V_{out}\right)=g_{m1}V_{in}+\overset{②}{\frac{V_X}{r_{o1}}}$$

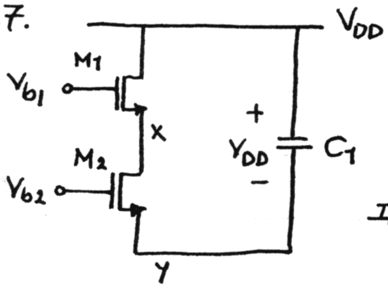
$$\overset{③}{-\frac{V_{out}}{r_{o2}}+g_{m2}(V_X-V_{out})=0}$$ @ output node

$$③\rightarrow\quad V_X=\frac{\frac{1}{r_{o2}}+g_{m2}}{g_{m2}}\cdot V_{out}=\frac{1+g_{m2}r_{o2}}{g_{m2}r_{o2}}V_{out}$$

$$①,②\rightarrow\quad -\left[\left(\frac{1}{r_{o3}}+\frac{1}{r_{o1}}\right)\frac{1+g_{m2}\cdot r_{o2}}{g_{m2}\cdot r_{o2}}+g_{m3}\right]V_{out}=g_{m1}\cdot V_{in}$$

$$\frac{V_{out}}{V_{in}}=-\frac{g_{m1}\cdot g_{m2}r_{o1}r_{o2}r_{o3}}{(r_{o1}+r_{o3})(1+g_{m2}\cdot r_{o2})+g_{m2}g_{m3}r_{o1}r_{o2}r_{o3}}$$

3.7.



(a)

3-13

$$V_Y(t=0) = -V_{C1} + V_{DD} = -V_{DD} + V_{DD} = 0$$

$$I_{D1} = I_{D2} \rightarrow \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[V_{b1} - V_X(t=0) - V_{TH1}\right]^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2$$

$$V_X(t=0) = V_{b1} - V_{TH1} - \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}\,(V_{b2} - V_{TH2})$$

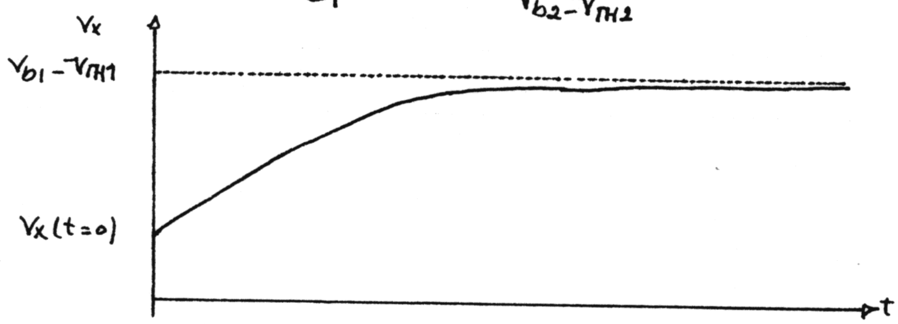
We assume that $V_X(t=0) > V_{b2} - V_{TH2}$, therefore, M_2 is always saturated.

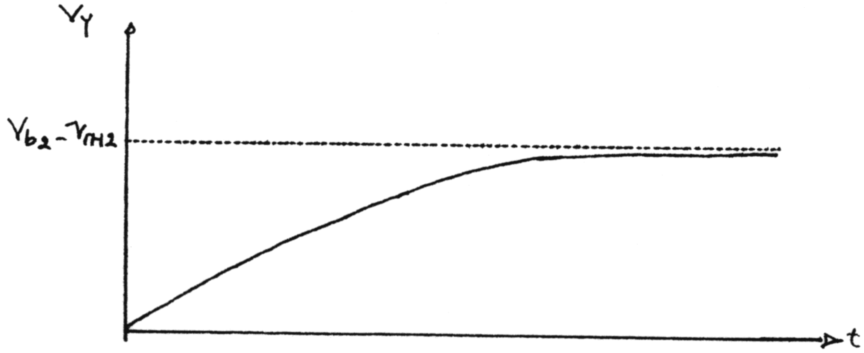
$$\underbrace{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_X - V_{TH1})^2}_{1} = \underbrace{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_Y - V_{TH2})^2}_{2} = \underbrace{C_1\frac{dV_Y}{dt}}_{3}$$

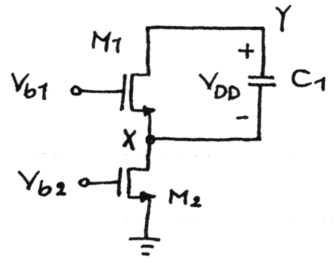
$$2, 3 \rightarrow \frac{dV_Y}{(V_{b2} - V_Y - V_{TH2})^2} = \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 \cdot dt$$

$$\frac{1}{V_{b2} - V_Y - V_{TH2}} = \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 t + K, \quad K = \frac{1}{V_{b2} - V_{TH2}} \quad \text{because } V_Y(t=0) = 0$$

$$V_Y = V_{b2} - V_{TH2} - \frac{1}{\frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 t + \frac{1}{V_{b2} - V_{TH2}}}, \quad V_X = V_{b1} - V_{TH1} - (V_{b2} - V_Y - V_{TH2})\sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}} \leftarrow 2, 1$$







(b)

The drain current of M_2 is zero, therefore, M_2 operates in deep triode region, pulling down V_X to zero potential.

$V_X = 0$ for $0 < t < \infty$

$V_Y(t=0) = V_{DD} \rightarrow M_1$ starts in saturation.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2 = -C_1\frac{dV_{C1}}{dt} = -C_1\frac{dV_Y}{dt}$$

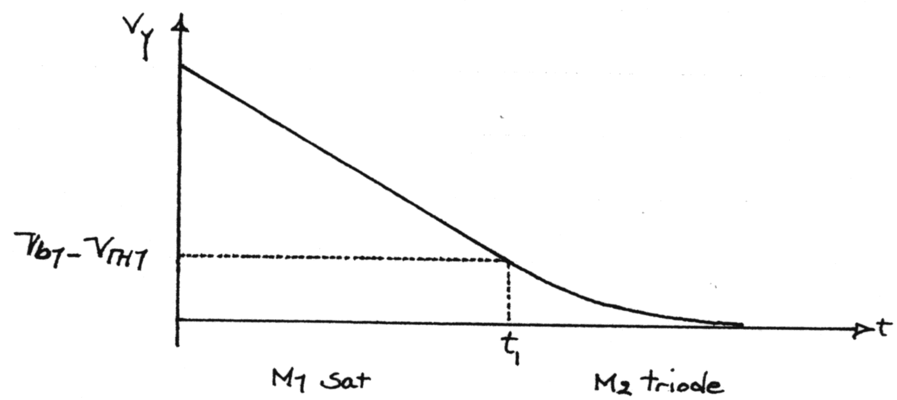
$$① \quad V_Y = V_{C1} = V_{DD} - \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2 t$$

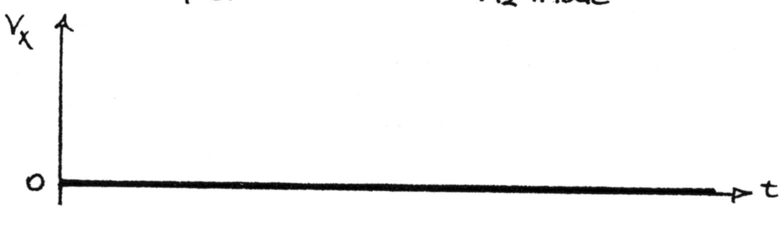
When $V_Y = V_{b1} - V_{TH1}$ ②, M_1 enters triode region.

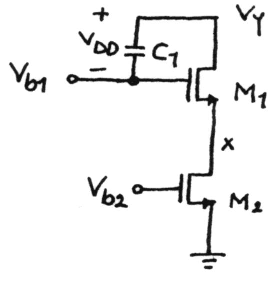
Substituting ② in ①, we calculate the time when M_1 is at the edge of triode region.

$$t_1 = \frac{V_{DD} - V_{b1} + V_{TH1}}{\frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2}$$

for $t > t_1$: $\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{b1} - V_{TH1})V_Y - \frac{V_Y^2}{2}\right] = -C_1\frac{dV_Y}{dt} \rightarrow V_Y = \ldots$







(c)

$V_Y(t=0) = V_{DD} + V_{b1}$, both transistors are saturated.

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_X - V_{TH1})^2$$

$$V_X = V_{b1} - V_{TH1} - (V_{b2} - V_{TH2})\sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}$$

$$C_1 \frac{dV_{C1}}{dt} = -\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 \rightarrow V_{C1} = V_{DD} - \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 t$$

$$V_Y = V_{C1} + V_{b1} = V_{DD} + V_{b1} - \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 t$$

@ $t = t_1$, we have $V_Y = V_{b1} - V_{TH1}$, polarity of voltage across C_1 has already changed.

$$V_{DD} + V_{b1} - \frac{1}{2}\mu_n \frac{C_{ox}}{C_1}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 t_1 = V_{b1} - V_{TH1}$$

$$t_1 = \frac{2(V_{DD} + V_{TH1})C_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2}$$

for $t > t_1$, M_1 enters triode region. We assume that still M_2 is saturated.

$V_Y = V_{DD} + V_{b1} - \frac{1}{C_1} I_{D2} \cdot t$ where $I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2$

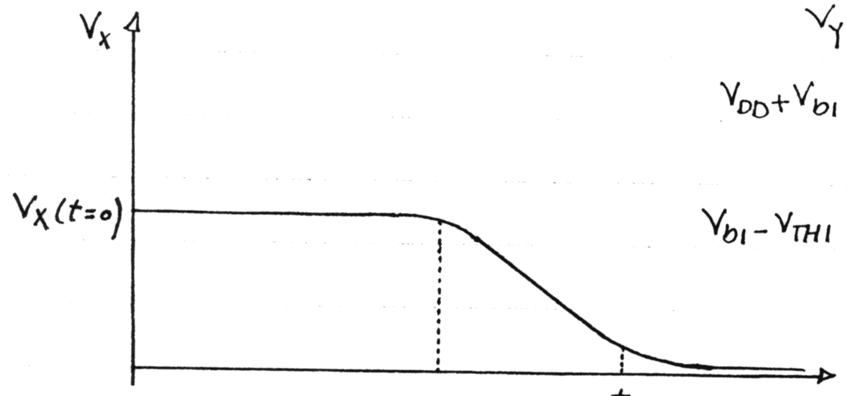
and $I_{D2} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{b1} - V_X)\left(V_{DD} + V_{b1} - \frac{1}{C_1} I_{D2}\cdot t - V_X\right) - \frac{\left(V_{DD} + V_{b1} - \frac{1}{C_1} I_{D2}\cdot t - V_X\right)^2}{2}\right]$

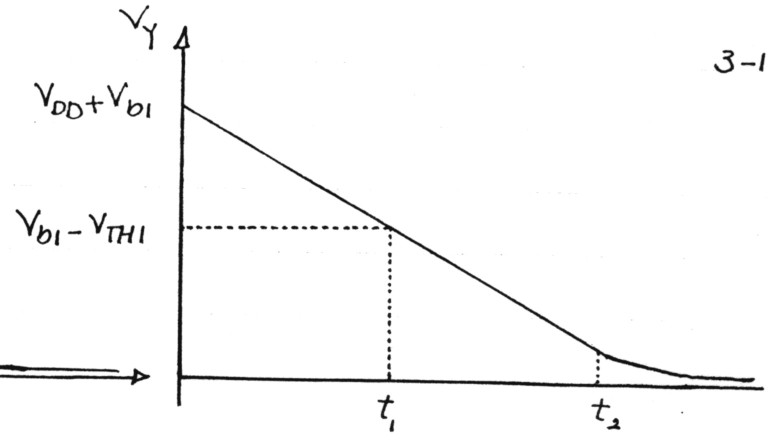
$\rightarrow V_X$ is obtained

When $V_X = V_{b2} - V_{TH2}$, M_2 enters the triode region, too.

$$\mu_n C_{ox}\left(\frac{W}{L}\right)_2 \left[(V_{b2} - V_{TH2})V_X - \frac{V_X^2}{2}\right] = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{b1} - V_X - V_{TH1})(V_Y - V_X) - \frac{(V_Y - V_X)^2}{2}\right] = -C_1\frac{dV_Y}{dt}$$

V_X and V_Y are obtained. This regime continues until V_X and V_Y drop to zero, and C_1 Charges up to $-V_{b1}$.



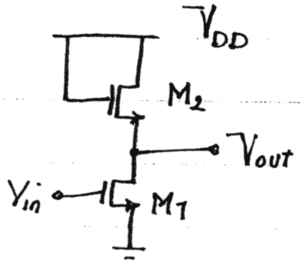


3-16

$$t_1 = \frac{2(V_{DD}+V_{TH1})C_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2}-V_{TH2})^2}$$

For $0<t<t_1$ M_1 Sat, M_2 Sat, for $t_1<t<t_2$ M_1 Triode, M_2 Sat.
for $t_2<t$ M_1 Triode, M_2 triode

3.8



$\left(\frac{W}{L}\right)_1 = \frac{50}{0.5}$, $\left(\frac{W}{L}\right)_2 = \frac{10}{0.5}$, $I_{D1} = I_{D2} = 0.5\,mA$

$$\mu_n C_{ox} = 350\,\frac{Cm^2}{V.S} \times \frac{8.85\times10^{-14}\times 3.9\ \text{Farad}/Cm}{9\times10^{-7}\,Cm} =$$

$$1.34225\times10^{-4}\ A/V^2$$

$$\mu_p C_{ox} = \frac{100\ Cm^2}{V.S} \times \frac{8.85\times10^{-14}\times3.9\ \text{Farad}/Cm}{9\times10^{-7}\,Cm} =$$

$$3.835\times10^{-5}\ A/V^2$$

$r_{o1} = r_{o2} = \frac{1}{\lambda_N I_1} = 20K$, $I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS2}-V_{TH2})^2(1+\lambda_N V_{DS2})$,

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\times20\left[3-V_o-0.7-0.45\left(\sqrt{0.9+V_o}-\sqrt{0.9}\right)\right]^2\left[1+0.1(3-V_o)\right]$$

$$2.3-0.45\left(\sqrt{0.9+V_o}-\sqrt{0.9}\right)-\sqrt{\frac{1}{2.6845\,(1.3-0.1V_o)}} = V_o \rightarrow V_o = 1.466\ V$$

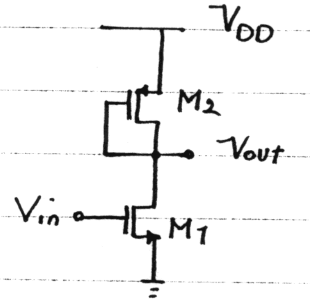
$$g_{m1} = \sqrt{2\times1.34225\times10^{-4}\times100\times0.5\times10^{-3}} = 3.66\times10^{-3}\ A/V$$

$$g_{m2} = \sqrt{2\times1.34225\times10^{-4}\times20\times0.5\times10^{-3}} = 1.63\times10^{-3}\ A/V$$

$$g_{mb2} = \frac{\gamma \cdot g_{m2}}{2\sqrt{2\phi_F+V_{SB}}} = \frac{0.45}{2\sqrt{0.9+1.466}}\times1.63\times10^{-3} = 2.3843\times10^{-4}\ A/V$$

$$R_{out} = \frac{1}{g_{m2} + g_{mb2} + r_{o2}^{-1}} \parallel r_{o1} = \frac{1}{1.63\times10^{-3} + 2.3843\times10^{-4} + (20\times10^{3})^{-1}} \parallel 20\times10^{3}$$ 3.17

$R_{out} = 508\ \Omega$ $\qquad A_V = -g_{m1}\cdot R_{out} = -3.66\times10^{-3}\times 508 = -1.85$



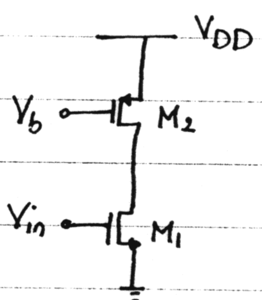
$$g_{m2} = \sqrt{2\times 3.835\times10^{-5}\times 20\times 0.5\times10^{-3}} = 8.7578\times10^{-4}$$

$$r_{o2} = \frac{1}{\lambda_p I} = \frac{1}{0.2\times0.5\times10^{-3}} = 10K$$

$$R_{out} = \frac{1}{g_{m2} + r_{o2}^{-1}} \parallel r_{o1} = 974.8628\ \Omega$$

$$A_V = -g_{m1}\cdot R_{out} = -0.8537$$

3.9.



$\left(\frac{W}{L}\right)_1 = 50/0.5, \quad \left(\frac{W}{L}\right)_2 = 50/2, \quad I_{D1} = I_{D2} = 0.5\,mA$

$r_{o1} = \frac{1}{\lambda_N I} = \frac{1}{0.1\times0.5\times10^{-3}} = 20K, \quad r_{o2} = \frac{1}{\lambda_p I} = \frac{1}{0.2\times0.5\times10^{-3}} = 40K$

$$g_{m1} = \sqrt{2\times 1.34225\times10^{-4}\times 100\times 0.5\times10^{-3}} = 3.6636\times10^{-3}$$

$$A_V = -g_{m1}(r_{o1}\parallel r_{o2}) = -48.84$$

If we assume that M_1 is in the edge of the triode region, then, we have:

$V_{GS} - V_{TH1} = V_{DS1} = V_{out}$, $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2 (1 + \lambda_N V_{DS})$

$0.5\times10^{-3} = \frac{1}{2}\times 1.34225\times10^{-4}\times 100\, V_{DS}^2(1+0.1V_{DS}) \rightarrow \sqrt{\frac{1}{13.4225(1+0.1V_{DS})}} = V_{DS}$

$V_{DSmin} = V_{o\,min} = 0.2693$

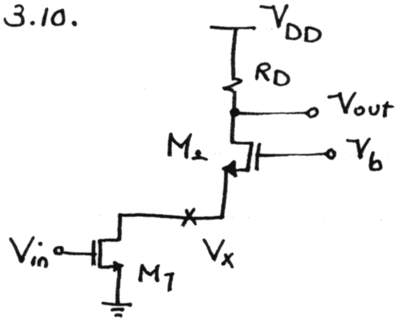
If we assume that M_2 is in the edge of the triode region, then, we have:

$I_D = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{SG} - |V_{TH2}|)^2 (1 + \lambda_p V_{SD})$, $0.5\times10^{-3} = \frac{1}{2}\times 3.835\times10^{-5}\times 25\, V_{SD}^2(1+\lambda_p V_{SD})$

$\sqrt{\frac{1}{0.95875(1+0.05V_{SD})}} = V_{SD} \rightarrow V_{SDmin} = 0.9967\,V$, $V_{o\,max} = V_{DD} - V_{SDmin}$,

$V_{o\,max} = 2V$

3.10.



$\left(\frac{W}{L}\right)_1 = 50/0.5,\ \left(\frac{W}{L}\right)_2 = 10/0.5\quad I_{D1} = I_{D2} = 0.5\,mA$ 3–18

$R_D = 1K\Omega$

$$V_{DS,sat1} = V_{GS1} - V_{TH1} = \left(\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\right)^{1/2} = \left(\frac{2\times0.5\times10^{-3}}{1.34225\times10^{-4}\times100}\right)^{1/2}$$

$$V_{DS,sat1} = 0.2729\,V$$

$$V_{X,Bias} = 0.2729 + 50\times10^{-3} = 0.3229\,V$$

$$V_{TH2} = V_{TH0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right) = 0.7 + 0.45\left(\sqrt{0.9+0.3229} - \sqrt{0.9}\right)$$

$$V_{TH2} = 0.77073\,V$$

$$V_{GS2} = V_{TH2} + \left(\frac{2I_{D2}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}\right)^{1/2} = 0.77073 + \left(\frac{2\times0.5\times10^{-3}}{1.34225\times10^{-4}\times20}\right)^{1/2} = 1.38107\,V,$$

$$V_b = V_{GS2} + V_X$$

$$V_b = 1.38107 + 0.3229 = 1.7\,V,\quad g_{m1} = \sqrt{2\times1.34225\times10^{-4}\times100\times0.5\times10^{-3}} = 3.6636\times10^{-3}\,A/V$$

$$g_{m2} = \sqrt{2\times1.34225\times10^{-4}\times20\times0.5\times10^{-3}} = 1.6384\times10^{-3}\,A/V$$

$$g_{mb2} = \frac{0.45}{2\sqrt{0.9+0.3229}}\,1.6384\times10^{-3} = 3.3336\times10^{-4},\quad r_{o1} = r_{o2} = \frac{1}{\lambda_N I} = \frac{1}{0.1\times0.5\times10^{-3}} = 20K$$

$$R_{out} = R_D \,\|\, \left\{\left[1 + (g_{m2}+g_{mb2})r_{o2}\right]r_{o1} + r_{o2}\right\} = 10^3 \,\|\, \left\{\left[1 + (1.6384\times10^{-3} + 3.3336\times10^{-4})\,20\times10^3\right]20\times10^3 + 20\times10^3\right\}$$

$$R_{out} = 998.7947\,\Omega,\quad G_m = \frac{g_{m1}\cdot r_{o1}\left[r_{o2}(g_{m2}+g_{mb2})+1\right]}{r_{o1}\cdot r_{o2}(g_{m2}+g_{mb2}) + r_{o1} + r_{o2}}$$

$$G_m = \frac{3.6636\times10^{-3}\times20\times10^3\left[20\times10^3(1.6384\times10^{-3} + 3.3336\times10^{-4})+1\right]}{(20\times10^3)^2(1.6384\times10^{-3} + 3.3336\times10^{-4}) + 2\times20\times10^3} = 3.5751\times10^{-3}\,A/V$$

$$A_v = -G_m R_{out} = -3.57$$

We obtain the small signal voltage gain from input to node x.

$$R_{out@X} = r_{o1} \,\|\, \frac{R_D + r_{o2}}{1+(g_{m2}+g_{mb2})r_{o2}} = 20\times10^3 \,\|\, \frac{10^3 + 20\times10^3}{1+(1.6384\times10^{-3} + 3.3336\times10^{-4})\,20\times10^3}$$

$$R_{out@X} = 506.2$$

$$A_{VX} = -g_{m1}\cdot R_{out@X} = -1.8545$$

If $V_X = V_{X\,min} = V_{DS,sat1},\ \Delta V_X = -50\,mV \rightarrow \Delta V_{in} = \dfrac{-50\times10^{-3}}{-1.8545} = 26.96\times10^{-3}$

$\Delta V_{out} = 26.96 \times 10^{-3} \times (-3.57) = -96.25 \times 10^{-3}$ 3-19

$V_{out\,min} = V_{DD} - R_D I_D + \Delta V_o = 3 - 1 \times 0.5 - 96.25 \times 10^{-3} = 2.4 V$

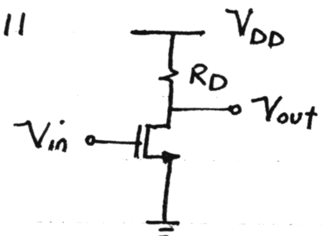
$V_{out,max} = 3 V, \quad \Delta V_o = 3 - 2.5 = 0.5 V, \quad \Delta V_{in} = \frac{0.5}{-3.57} = -0.14 V$

$\Delta V_X = -1.8545 (-0.14) = 0.2597$

$V_{X,max} = V_{X,Bias} + 0.2597 = 0.3229 + 0.2597 = 0.5826 V$

If we take $V_{out,min} = V_b - V_{TH2} = 1.7 - 0.77073 = 0.92924 V$, $\Delta V_o = -1.57$ which translates into a huge negative swing at x that makes the final voltage at node x negative. Therefore, M_1 limits the negative going output swing because the voltage gain from input to node x is quite large.

3.11



$\left(\frac{W}{L}\right)_1 = 50/0.5, \quad R_D = 2K\Omega, \quad \lambda = \phi$

$r_{o1} = \frac{1}{\lambda_N I_D} = \frac{1}{0.1 \times 10^{-3}} = 10K$

$R_{out} = r_{o1} \| R_D = 10K \| 2K = \frac{5000}{3} \Omega$

$g_{m1} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 10^{-3}} = 5.1812 \times 10^{-3}$

$A_V = -g_{m1} \cdot R_{out} = -5.1812 \times 10^{-3} \times \frac{5000}{3} = -8.6353$

At the edge of the triode region: $V_{out} = V_{GS} - V_{TH} = V_{GS} - 0.7$, $I_{D1} = \frac{V_{DD} - V_{out}}{R_D} =$

$\frac{3 - V_{GS} + 0.7}{2 \times 10^3} = \frac{3.7 - V_{GS}}{2 \times 10^3}, \quad I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2$

$\frac{3.7 - V_{GS}}{2 \times 10^3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \times 100 (V_{GS} - 0.7)^2$

$13.4225 V_{GS}^2 - 17.7915 V_{GS} - 10.277025 = 0 \rightarrow V_{GS} = 1.137 V$

I_D @ the edge of the triode $= \frac{1}{2} \times 1.34225 \times 10^{-4} \times 100 (1.137 - 0.7)^2 = 1.28151 \times 10^{-3}$

g_{m1} @ the edge of the triode $= \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 1.28151 \times 10^{-3}} = 5.8653 \times 10^{-3}$

$r_{o1} = \frac{1}{0.1 \times 1.28151\ 10^{-3}} = 7.8 \times 10^3$

A_V @ the edge of the triode $= -g_{m1} (r_{o1} \| R_D) = -5.8653 \times 10^{-3} (7.8 \times 10^3 \| 2 \times 10^3)$

$A_V = -9.3374$

V_o @ the edge of the triode $= V_{DD} - R_D \times I_D = 3 - 2 \times 1.2815 \times 10^{-3} = 0.4369\,V$ 3-20

$V_{DS} = V_{DS,sat1} - 50\times10^{-3} = 0.4369 - 50\times10^{-3} = 0.3869\,V$

$I_D = \dfrac{V_{DD}-V_{DS}}{R_D} = \dfrac{3-0.3869}{2\times10^{3}} = 1.3065\times10^{-3}$

$I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[(V_{GS}-V_{TH1})V_{DS} - \frac{V_{DS}^2}{2}\right]$

$1.3065\times10^{-3} = 1.34225\times10^{-4}\times100\left[(V_{GS}-0.7)\,0.3869 - \frac{(0.3869)^2}{2}\right] \Rightarrow V_{GS} = 1.145$

$g_{m1} = \dfrac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot V_{DS}$

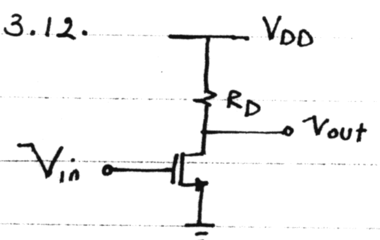
g_{m1} @ the point where 50mV into the triode $= 1.34225\times10^{-4}\times100\times0.3869 = 5.1942\times10^{-3}$

$R_o^{-1} = \dfrac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS}-V_{TH1}-V_{DS}) \rightarrow R_o = \dfrac{1}{1.34225\times10^{-4}\times100(1.145-0.7-0.38}$

$R_o = 1.2835\times10^{3}\,\Omega$

A_V @ 50 mV into the triode region $= -5.1942\times10^{-3}(1.2835\times10^{3} \,\|\, 2\times10^{3}) = -4$

3.12.



$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad R_D = 2K, \quad \lambda = 0$

I_{D1} @ $V_{out}=1V$ $= \dfrac{V_{DD}-V_o}{R_D} = \dfrac{3-1}{2\times10^{3}} = 10^{-3}\,A$

$V_{in} = V_{TH1} + \left(\dfrac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\right)^{1/2} = 0.7 + \left(\dfrac{2\times10^{-3}}{1.34225\times10^{-4}\times100}\right)^{1/2} \rightarrow V_{in}$ @ $V_{out}=1V$ $= 1.086\,V$

I_{D1} @ $V_{out}=2.5V$ $= \dfrac{3-2.5}{2\times10^{3}} = 2.5\times10^{-4}$, V_{in} @ $V_{out}=2.5V$ $= 0.7 + \left(\dfrac{2\times2.5\times10^{-4}}{1.34225\times10^{-4}\times100}\right)^{1/2} = 0.893\,V$

g_{m1} @ $V_{out}=1V$ $= \sqrt{2\times1.34225\times10^{-4}\times100\times10^{-3}} = 5.1812\times10^{-3}$

g_{m1} @ $V_{out}=2.5V$ $= \sqrt{2\times1.34225\times10^{-4}\times100\times2.5\times10^{-4}} = 2.59\times10^{-3}$

$$r_{o1}@V_{out}=1V = \frac{1}{0.1\times10^{-3}} = 10K, \quad R_{out} = r_{o1} \parallel R_D = 10000 \parallel 2000 = \frac{5000}{3}\,\Omega$$ 3-21

$$A_V@V_{out}=1V = -g_{m1}\cdot R_{out} = -5.1812\times10^{-3} \times \frac{5000}{3} = -8.6353$$

$$r_{o1}@V_{out}=2.5V = \frac{1}{0.1\times2.5\times10^{-4}} = 40K, \quad R_{out} = r_{o1} \parallel R_D = 40000 \parallel 2000 = 1.9\times10^{3}$$

$$A_V@V_{out}=2.5V = -g_{m1}\cdot R_{out} = -2.59\times10^{-3} \times 1.9\times10^{3} = -4.9221$$

3.13. $\left(\frac{W}{L}\right) = 50/0.5 \quad |I_D| = 0.5\,mA$
& $100/1$

For NMOS device with $\left(\frac{W}{L}\right) = 50/0.5$, $r_o = \frac{1}{\lambda_N I_D} = \frac{1}{0.1\times0.5\times10^{-3}} = 20K$

$g_m = \sqrt{2\times1.34225\times10^{-4}\times100\times0.5\times10^{-3}} = 3.6636\times10^{-3}$

$g_m r_o = 73.27$

For PMOS device with $\left(\frac{W}{L}\right) = 50/0.5$, $r_o = \frac{1}{\lambda_P I_D} = \frac{1}{0.2\times0.5\times10^{-3}} = 10K$

$g_m = \sqrt{2\times3.835\times10^{-5}\times100\times0.5\times10^{-3}} = 1.9583\times10^{-3}$

$g_m r_o = 19.5831$

For NMOS device with $\left(\frac{W}{L}\right) = 100/1$, $r_o = \frac{1}{\frac{0.1}{2}\times0.5\times10^{-3}} = 40K$

$g_m = 3.6636\times10^{-3}$, $g_m r_o = 146.5469$

For PMOS device with $\left(\frac{W}{L}\right) = 100/1$, $r_o = \frac{1}{\frac{0.2}{2}\times0.5\times10^{-3}} = 20K$

$g_m = 1.9583\times10^{-3}$, $g_m r_o = 39.1663$

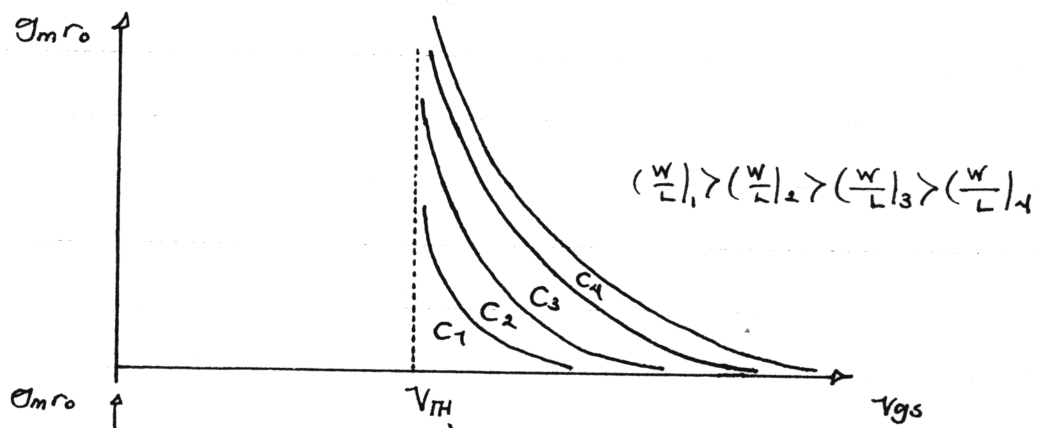
3.14. $I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$ (1)

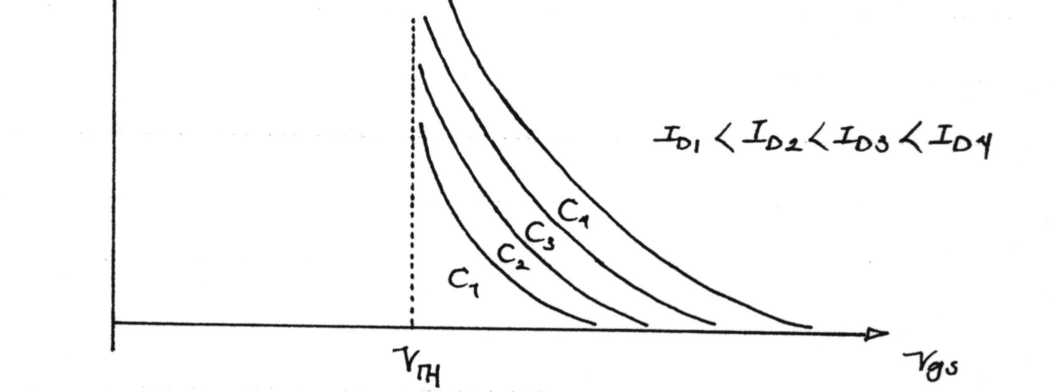
$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})(1+\lambda V_{DS})$ (2)

Substituting $(1+\lambda V_{DS})$ from ① in ②, we have,

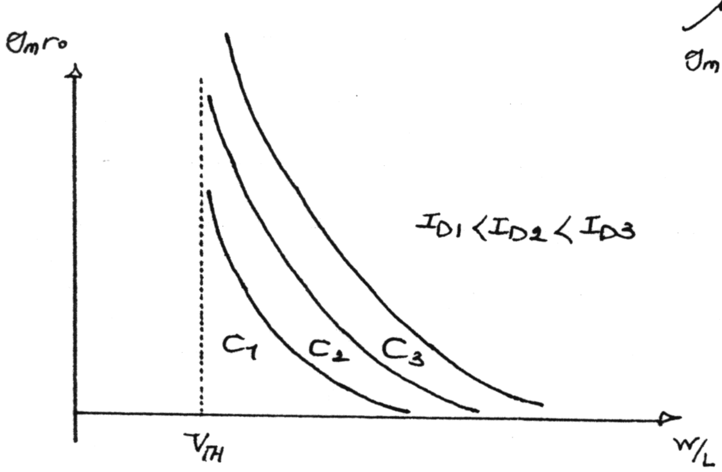
$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})\frac{I_D}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})^2} = \frac{2I_D}{V_{GS}-V_{TH}}$$

$$g_m r_o = \frac{2I_D}{V_{GS}-V_{TH}}\,\frac{1+\lambda V_{DS}}{\lambda I_D} = \frac{2(1+\lambda V_{DS})}{\lambda(V_{GS}-V_{TH})} = \frac{4I_D}{\mu_n C_{ox}(V_{GS}-V_{TH})^3\lambda\left(\frac{W}{L}\right)}$$

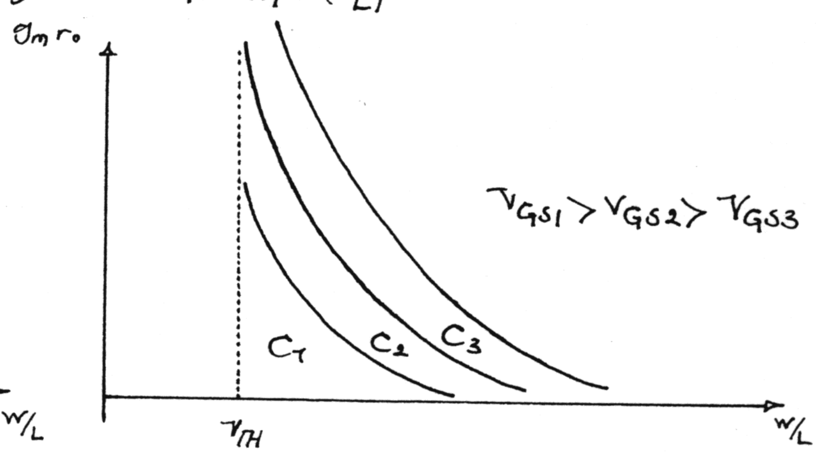




3.15. From 3.14. we have: $g_m r_o = \dfrac{4I_D}{\mu_n C_{ox}(V_{GS}-V_{TH})^3\lambda\left(\frac{W}{L}\right)}$



(a)



(b)

3.16. $\frac{W}{L} = 50/0.6 \quad V_G = +1.2V \quad V_S = 0 \quad 0 < V_D < 3 \quad V_{bulk} = 0$

$V_{Dsat} = V_{GS} - V_{TH} = 1.2 - 0.7 = 0.5V$, for a saturated device $g_m r_o = \frac{2(1+\lambda V_{DS})}{\lambda(V_{GS} - V_{TH})}$

@ the edge of the triode region $g_m r_o = \frac{2(1+0.5\times 0.1)}{0.1(1.2-0.7)} = 42$

We cannot neglect the channel-length modulation in the triode region, because it would lead to a discontinuity at the transition point between the saturation and the triode region.

@ triode region

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\left(\frac{W}{L}\right) V_{DS}(1+\lambda V_{DS})$$

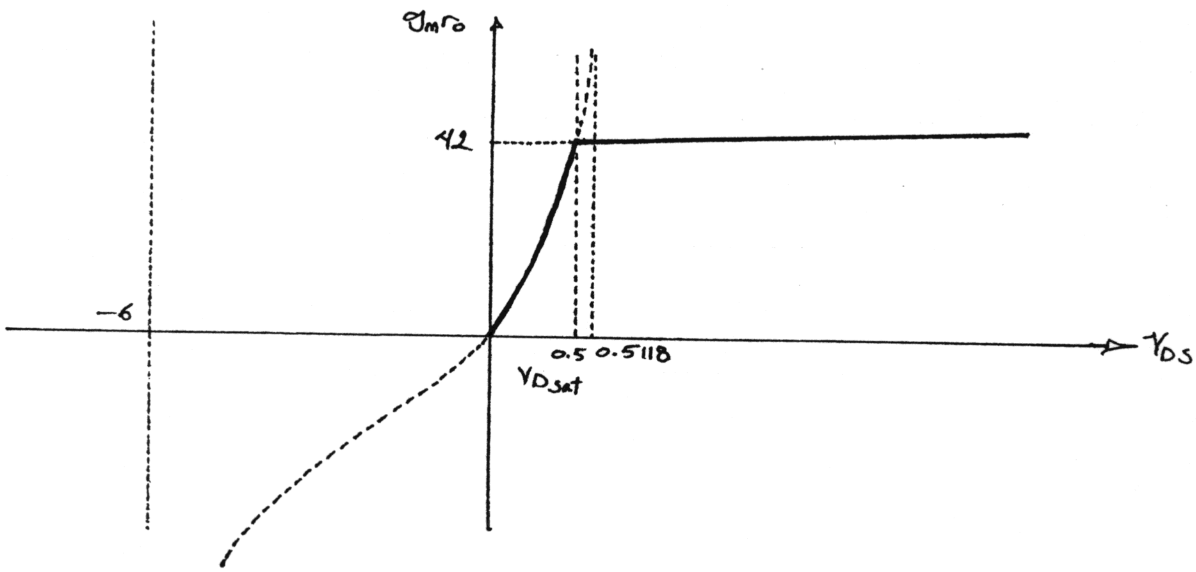
$$g_o = \frac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox}\left(\frac{W}{L}\right)\left\{(V_{GS}-V_{TH}-V_{DS})(1+\lambda V_{DS}) + \lambda\left[(V_{GS}-V_{TH})V_{DS} - \frac{V_{DS}^2}{2}\right]\right\}$$

in the triode region $g_m r_o = \dfrac{(1+\lambda V_{DS})V_{DS}}{(V_{GS}-V_{TH}-V_{DS})(1+\lambda V_{DS}) + \lambda\left[(V_{GS}-V_{TH})V_{DS} - \frac{V_{DS}^2}{2}\right]}$

In Saturation $\quad g_m r_o = \dfrac{2(1+0.1V_{DS})}{0.1(1.2-0.7)} = 40 + 4V_{DS} \qquad V_{DS} > 0.5V$

In triode $\quad g_m r_o = \dfrac{(1+0.1V_{DS})V_{DS}}{(0.5-V_{DS})(1+0.1V_{DS}) + 0.1\times 0.5 V_{DS}(1-V_{DS})}$

$$g_m r_o = \frac{0.1V_{DS}^2 + V_{DS}}{-0.15V_{DS}^2 - 0.9V_{DS} + 0.5}$$



3-24

$V_{bulk} = -1V$, $V_{SB} = +1V$

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right) = 0.7 + 0.45\left(\sqrt{0.9+1} - \sqrt{0.9}\right) = 0.8933\ V$$

In Saturation $$g_m r_o = \frac{2(1+0.1V_{DS})}{0.1(1.2-0.8933)} = 65.2262 + 6.5226\,V_{DS}$$

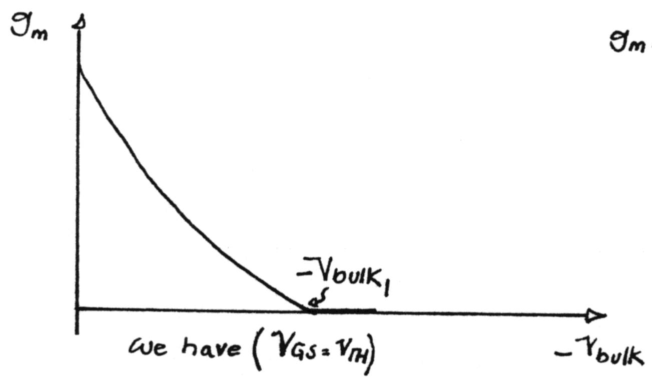
$V_{DSsat} = V_{GS} - V_{TH} = 1.2 - 0.8933 = 0.3066\ V$, @the edge of the Triode $g_m r_o = 67.2262$

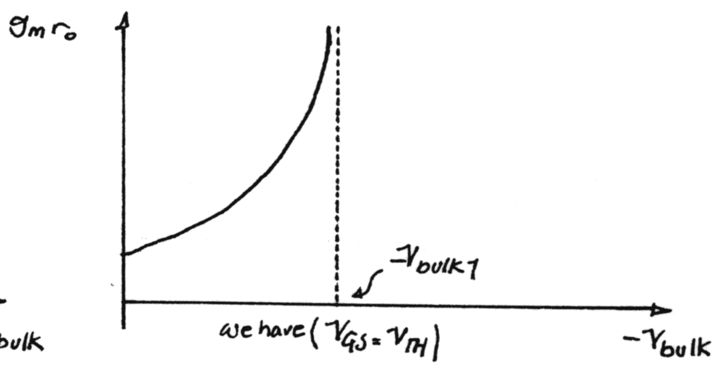
$$g_m r_o = \frac{(1+0.1V_{DS})V_{DS}}{(1.2-0.8933-V_{DS})(1+0.1V_{DS}) + 0.1\left[(1.2-0.8933)V_{DS} - 0.5V_{DS}^2\right]}$$

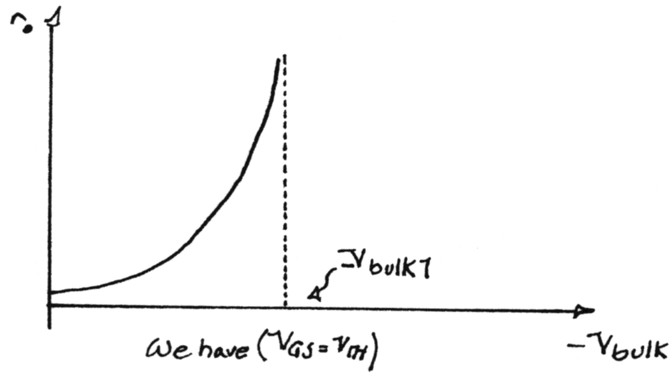
$$g_m r_o = \frac{(1+0.1V_{DS})V_{DS}}{-0.15V_{DS}^2 - 0.9386V_{DS} + 0.3066}$$

3.17.
$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[V_{GS} - V_{TH0} - \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right)\right](1+\lambda V_{DS})$$

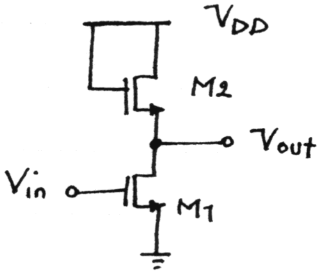
$$r_o = \frac{1}{\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})^2\lambda}\ ,\quad g_m r_o = \frac{2(1+\lambda V_{DS})}{\lambda(V_{GS}-V_{TH})}$$







3.18.



$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad \left(\frac{W}{L}\right)_2 = 10/0.5\ , \quad \lambda = \gamma = 0$

M_1 at the edge of the triode region, $\rightarrow V_{out} = V_{in} - V_{TH1}$

$$I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} + V_{TH1} - V_{TH2})^2$$

$$\left(\frac{W}{L}\right)_1^{1/2} (V_{in} - V_{TH1}) = \left(\frac{W}{L}\right)_2^{1/2} (V_{DD} - V_{in}) \rightarrow (V_{in} - V_{TH1}) = \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}\,(V_{DD} - V_{in})$$

$$V_{in} = \left(\sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}\,V_{DD} + V_{TH1}\right) \Big/ \left(1 + \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}\right) = \left[\left(\frac{10}{50}\right)^{1/2} \times 3 + 0.7\right] \Big/ \left[1 + \left(\frac{10}{50}\right)^{1/2}\right] = 1.41V$$

$$A_V = -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}} = -\sqrt{\frac{50}{10}} = -2.236\ .$$

At the edge of the triode region $V_{out} = 1.41 - 0.7 = 0.71\ V$

50 mV into the triode region $V_{out} = 0.71 - 50\times10^{-3} = 0.66V$

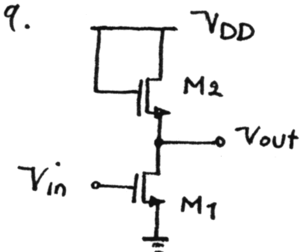
$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})^2 = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{in} - V_{TH1})V_{out} - \frac{V_{out}^2}{2}\right]$$

$$V_{in} = \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}\,\frac{(V_{DD} - V_{out} - V_{TH2})^2}{V_{out}} + \frac{V_{out}}{2} + V_{TH1} = \frac{10}{50}\,\frac{(3 - 0.66 - 0.7)^2}{0.66} + \frac{0.66}{2} + 0.7$$

$V_{in} = 1.8437\ ,\quad I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)_1\left[(V_{in} - V_{TH1})V_{out} - \frac{V_{out}^2}{2}\right],\quad \frac{\partial I_D}{\partial V_{in}} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot V_{out}$

$$A_V = -\frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot V_{out}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})} = -\frac{\frac{50}{0.5}\times 0.66}{\frac{10}{0.5}\times(3 - 0.66 - 0.7)} = -2.015$$

3.19.



$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad \left(\frac{W}{L}\right)_2 = 10/0.5 \quad \lambda = 0$

$V_{out} = V_{in} - V_{TH1}, \quad V_{TH2} = V_{TH2,0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right)$

$$I_{D1} = I_{D2} \Rightarrow \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 \left(V_{DD} - V_{in} + V_{TH1} - V_{TH2,0} - 0.45\left(\sqrt{0.9 + V_{out}} - \sqrt{0.9}\right)\right)^2$$

$$\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \left(\frac{W}{L}\right)_2 \left[V_{DD} - V_{in} - 0.45\left(\sqrt{0.9 + V_{in} - 0.7} - \sqrt{0.9}\right)\right]^2$$

$$V_{in} = \sqrt{\frac{1}{5}}\left[3 - V_{in} - 0.45\left(\sqrt{0.2 + V_{in}} - \sqrt{0.9}\right)\right] + 0.7 \rightarrow \text{After enough iterations} \rightarrow$$

$$V_{in} = 1.3685, \quad V_{out} = 0.6685, \quad \eta_2 = \frac{\gamma}{2(2|\phi_F| + V_{SB})^{1/2}} = \frac{0.45}{2(0.9 + 0.6685)^{1/2}} = 0.1796$$

$$A_V = -\frac{g_{m1}}{g_{m2}(1+\eta_2)} = -\sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}}\frac{1}{1+\eta_2} = -\sqrt{\frac{50}{10}}\frac{1}{1+0.1796} = -1.8955$$

$$V_{out} = 0.6685 - 50\times10^{-3} = 0.6185$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})^2 = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[(V_{in} - V_{TH1})V_{out} - \frac{V_{out}^2}{2}\right]$$

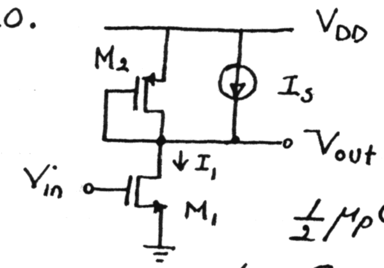
$$V_{TH2} = V_{TH2,0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right) = 0.7 + 0.45\left(\sqrt{0.9 + 0.6185} - \sqrt{0.9}\right) = 0.8276$$

$$24.1453 = \frac{50}{0.5}\left[(V_{in} - 0.7)\,0.6185 - \frac{0.6185^2}{2}\right] \rightarrow V_{in} = 1.3996$$

$$\eta_2 = \frac{0.45}{2(0.9 + 0.6185)^{1/2}} = 0.1825 \qquad A_V = -\frac{\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \cdot V_{out}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TH2})(1+\eta_2)} =$$

$$\frac{\left(\frac{W}{L}\right)_1 V_{out}}{\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})(1+\eta_2)} = \frac{50\times0.6185}{10(3 - 0.6185 - 0.8276)(1 + 0.1825)} = -1.6829$$

3.20.



$\left(\frac{W}{L}\right)_1 = 20/0.5,\ I_1 = 1mA,\ I_S = 0.75\,mA,\ \lambda = 0$ 3.27

M_1 at the edge of the triode region $V_{out} = V_{in} - V_{TH1}$

$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - |V_{TH2}|)^2 + I_S = \frac{1}{2}\mu_n C_{ox}(V_{in} - V_{TH1})^2 = 10^{-3}$$

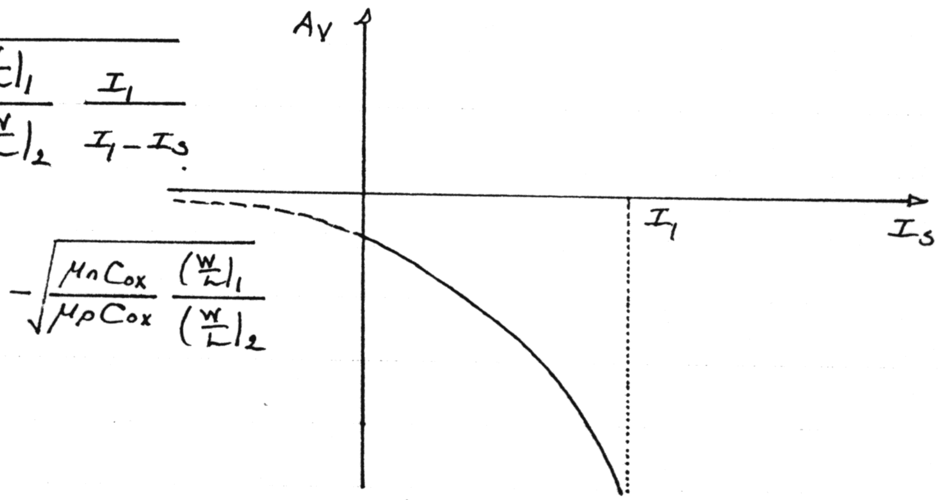
$$\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} + V_{TH1} - |V_{TH2}|)^2 + I_S = \frac{1}{2}\mu_n C_{ox}(V_{in} - V_{TH1})^2 = 10^{-3}$$

$$(V_{in} - V_{TH1})^2 = \frac{2I_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1} \rightarrow V_{in} = \sqrt{\frac{2I_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} + V_{TH1} = 0.7 + \sqrt{\frac{2\times10^{-3}}{1.34225\times10^{-4}\times 20/0.5}} = 1.31$$

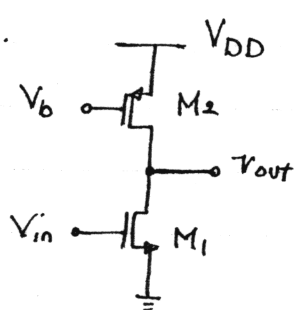
$$\frac{1}{2}\times 3.835\times10^{-5}\left(\frac{W}{L}\right)_2 (3 - 1.31 + 0.7 - 0.8)^2 + 0.75\times10^{-3} = 10^{-3},\quad \left(\frac{W}{L}\right)_2 = 5.159$$

$$A_V = -\frac{g_{m1}}{g_{m2}} = -\sqrt{\frac{\mu_n C_{ox}}{\mu_p C_{ox}}\times\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}\times\frac{I_1}{I_2}} = -\sqrt{\frac{1.34225\times10^{-4}}{3.835\times10^{-5}}\times\frac{20/0.5}{5.159}\times\frac{10^{-3}}{2.5\times10^{-4}}} = -10.418$$

3.21. $$A_V = -\sqrt{\frac{\mu_n C_{ox}}{\mu_p C_{ox}}\ \frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}\ \frac{I_1}{I_1 - I_S}}$$



3.22.



output voltage Swing = 2.2

$I_{D1} = I_{D2} = 1\ mA$

$A_V = 100$

$$V_{out,min} = \left(\frac{2I_{D1}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\right)^{1/2},\quad V_{out,max} = V_{DD} - \left(\frac{2I_{D2}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}\right)^{1/2}$$

3_28

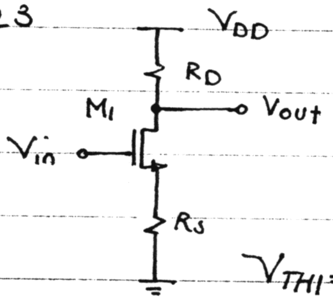
$$V_{DD} - \left(\frac{2I_D}{\mu_p C_{ox}\left(\frac{W}{L}\right)_2}\right)^{1/2} - \left(\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}\right)^{1/2} = 2.2,\quad r_{o1} = \frac{1}{\lambda_1 I_D} = \frac{1}{0.1\times10^{-3}} = 10K$$

$$r_{o2} = \frac{1}{\lambda_2 I_D} = \frac{1}{0.2\times10^{-3}} = 5K,\quad r_{o1}\| r_{o2} = \frac{10^4}{3},\quad g_{m1}(r_{o1}\| r_{o2}) = 100 \rightarrow g_{m1} = \frac{100\times3}{10^4} = 0.03$$

$$2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \times10^{-3} = 9\times10^{-4} \rightarrow \left(\frac{W}{L}\right)_1 = \frac{9\times10^{-4}}{2\times1.34225\times10^{-4}\times10^{-3}} = 3352.5796$$

$$3 - \left(\frac{2\times10^{-3}}{1.34225\times10^{-4}\times3352.5796}\right)^{1/2} - 2.2 = \left(\frac{2\times10^{-3}}{3.835\times10^{-5}\left(\frac{W}{L}\right)_2}\right)^{1/2} \rightarrow \left(\frac{W}{L}\right)_2 = 96.97$$

3.23



$$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad R_D = 2K \quad R_S = 200\Omega$$

$$r_{o1} = \frac{1}{\lambda I_D} = \frac{1}{0.1\times0.5\times10^{-3}} = 20K,\quad V_S = R_S I_D = 200\times0.5\times10^{-3} = 0.1$$

$$V_{TH1} = V_{TH1,0} + \gamma\left(\sqrt{2|\phi_F| + V_{SB}} - \sqrt{2|\phi_F|}\right) = 0.7 + 0.45\left(\sqrt{0.9+0.1} - \sqrt{0.9}\right.$$

$$V_{TH1} = 0.723,\quad V_{out} = V_{DD} - R_D\cdot I_D = 3 - 2\times10^3\times0.5\times10^{-3} = 2$$

$$V_{DS} = 2 - 0.1 = 1.9$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS}-V_{TH})^2(1+\lambda V_{DS})$$

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS}-V_{TH})(1+\lambda V_{DS}) = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1(1+\lambda V_{DS})I_D}$$

$$g_{m1} = \sqrt{2\times1.34225\times10^{-4}\times\frac{50}{0.5}(1+0.1\times0.9)\times0.5\times10^{-3}} = 3.8249\times10^{-3}$$

$$\eta_1 = \frac{0.45}{2(0.1+0.9)^{1/2}} = 0.225 \quad G_m = \frac{g_{m1}\cdot r_{o1}}{R_S + \left[1+(1+\eta_1)g_{m1}\cdot R_S\right]r_{o1}} =$$

$$G_m = \frac{3.8249\times10^{-3}\times20\times10^3}{200 + \left[1+(1+0.225)\,3.8249\times10^{-3}\times200\right]20\times10^3} = 1.9644\times10^{-3}$$

$$R_{out} = \left[1 + (g_m + g_{mb})r_o\right]R_S + r_o$$

Seen looking down at the drain of M1

$R_{out} = \left[1 + (1+0.225)\, 3.8249\times10^{-3}\right] 200 + 20\times10^{3} = 20.2\times10^{3}$ 3-29

$R_{out,total} = R_{out} \| R_D = 1819.8274$, $A_V = -G_m \cdot R_{out,total} = -1.96\times10^{-3}\times1819.8$

$A_V = -3.57$

$V_{out} = V_{in} - V_{TH1}$ @ the edge of the triode region

$V_{in} = V_{GS1} + R_S I_D$

$V_{DD} - R_D I_D = V_{out}$, $V_{DD} - R_D I_D = V_{GS1} + R_S I_D - V_{TH1}$, $V_{DD} - (R_S + R_D) I_D = V_{GS1} - V_{TH1}$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[V_{DD} - (R_S + R_D) I_D\right]^2 =$$
$$\frac{1}{2}\times 1.34225\times10^{-4}\times\frac{50}{0.5}\left[3 - (2000+200) I_D\right]^2$$

$I_D = 6.71125\times10^{-3}(3 - 2200 I_D)^2 \rightarrow 32482.45 I_D^2 - 89.5885 I_D + 60.40125\times10^{-3} = 0$

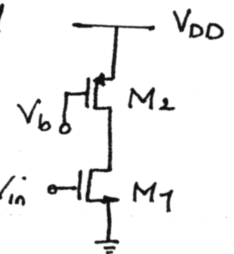
$I_{D1} = 1.5844\times10^{-3}$ (not acceptable), $I_{D2} = 1.17355\times10^{-3}$ (acceptable!)

$V_{in} = V_{DD} - R_D I_D + V_{TH1} = 3 - 2000\times1.17355\times10^{-3} + 0.7 = 1.35288\,V$

$g_{m1} = \sqrt{2\times1.34225\times10^{-4}\times\frac{50}{0.5}\times1.17355\times10^{-3}} = 5.6128\times10^{-3}$

$G_m = \frac{g_{m1}}{1 + g_{m1}\cdot R_S} = 2.6443\times10^{-3}$ $A_V = -G_m R_D = -2.6443\times10^{-3}\times2000 = -5.2887$

3.24



$A_V = -5$, $\left(\frac{W}{L}\right)_1 = 20/0.5$, $I_{D1} = 0.5\,mA$, $V_b = 0$

$g_{m1} = 2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (1 + \lambda V_{DS}) I_D$

$g_{m1} = \sqrt{2\times1.34225\times10^{-4}\times\frac{20}{0.5}(1 + 0.1 V_{DS})\times 0.5\times10^{-3}}$

$r_{o1} = \frac{1}{\lambda_N I} = \frac{1}{0.1\times0.5\times10^{-3}} = 20K$, $r_{o2} = \frac{1}{\lambda_P I} = \frac{1}{0.2\times0.5\times10^{-3}} = 10K$

The key point here is that the channel length modulation effect in M_1 cannot be neglected because its drain-source voltage is quite large. We take this effect into account with a few iterations.

First we let $V_{DS1}=0$, then we have, $g_{m1}=2.31711\times10^{-3}$ (as $A_v=-5$) 3-31

$R_{out,total}=2157.86\,\Omega$

$$r_{o2}=\frac{1}{\mu_p C_{ox}(\frac{W}{L})_2(V_{SG}-|V_{TH2}|-V_{SD})}=2418.8356$$

$0.5\times10^{-3}=\mu_p C_{ox}(\frac{W}{L})_2\left[(V_{SG}-|V_{TH2}|)V_{SD}-\frac{V_{SD}^2}{2}\right]$, by dividing these two relations together.

$$1.2094=\frac{(3-0.8)V_{SD}-0.5V_{SD}^2}{3-0.8-V_{SD}}=\frac{4.4V_{SD}-V_{SD}^2}{4.4-2V_{SD}},\quad V_{SD}^2-6.8188V_{SD}+5.3214=0$$

$V_{SD}=0.8989$, now second iteration starts, with the aid of the value we obtain for V_{SD} (or V_{DS}) from the first iteration, we have:

$g_{m1}=2.5489\times10^{-3}$, $R_{out}=1961.6020\,\Omega$

$$r_{o2}=2174.9182,\quad 1.087459=\frac{4.4V_{SD}-V_{SD}^2}{4.4-2V_{SD}},\quad V_{SD}^2-6.5749V_{SD}+4.7848=0$$

$V_{SD}=0.8336V$

Third iteration starts now:

By substituting the value of V_{SD} from the second iteration in the relation for g_{m1}, we get:

$g_{m1}=2.5558\times10^{-3}$, $R_{out}=1956.3119$, $r_{o2}=2168.4169\,\Omega$

$$1.0842=\frac{4.4V_{SD}-V_{SD}^2}{4.4-2V_{SD}},\quad V_{SD}^2-6.5684V_{SD}+4.77051=0$$

$V_{SD}=0.8315$

By doing the forth iteration:

$g_{m1}=2.5560\times10^{-3}$

$R_{out}=1956.1662$, $r_{o2}=2168.2379$, $1.0841189=\frac{4.4V_{SD}-V_{SD}^2}{4.4-2V_{SD}}$

$V_{SD}^2-6.5682V_{SD}+4.77012=0$

$$\boxed{V_{SD}=0.8315}$$

3.31

$$I=\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[(V_{SG}-|V_{TH2}|)V_{SD}-\frac{V_{SD}^2}{2}\right],\ \left(\frac{W}{L}\right)_2=\frac{0.5\times10^{-3}}{3.835\times10^{-5}\left[(3-0.8)0.8315-\frac{0.8315^2}{2}\right]}$$

$$\left(\frac{W}{L}\right)_2=8.7878$$

If M_1 is at the edge of the triode region: $V_{out}=V_{in}-V_{TH1}=V_{in}-0.7$

$$I_{D1}=\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{GS}-V_{TH1})=I_{D2}=\mu_p C_{ox}\left(\frac{W}{L}\right)_2\left[(V_{DD}-|V_{TH2}|)(V_{DD}-V_o)-\frac{(V_{DD}-V_o)^2}{2}\right]\times\left[1+0.2(V_{DD}-V_o)\right]$$

$$V_{out}=\sqrt{\frac{2\times3.835\times10^{-5}}{1.34225\times10^{-4}}\ \frac{8.7878}{40}\left[2.2(3-V_o)-\frac{(3-V_o)^2}{2}\right](1.6-0.2V_o)}$$

$V_o=0.6663$, $V_{in}=1.3663$, $g_{m1}=\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{GS}-V_{TH1})=\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{out}=$
$1.34225\times10^{-4}\times\frac{20}{0.5}\times0.6663=3.5773\times10^{-3}$

However, M_2 is no longer in triode region because $V_o=0.66<V_b+|V_{TH2}|=0.8$
Therefore, we should recalculate V_o with the assumption that M_2 is saturated

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{out}^2=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2(V_{DD}-V_b-|V_{TH2}|)^2\left[1+\lambda_p(V_{DD}-V_o)\right]$$

$536.9V_o^2+32.623V_o-260.9845=0$, $V_{out}=0.6674$, $V_{in}=1.3674$

$g_{m1}=\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{GS}-V_{TH})=3.5837\times10^{-3}$, $I_{D1}=\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{out}^2=1.196\times10^{-3}$

$$r_{out}=r_{o1}\,\|\,r_{o2}=\frac{1}{(\lambda_P+\lambda_N)I}=2786.962\,\Omega$$

$$A_V=-g_{m1}\cdot r_{out}=-9.9877$$

$V_{out}=0.8$, $\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{GS}-V_{TH1})^2=\frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2(V_{DD}-|V_{TH2}|)^2\left[1+\lambda_p(V_{DD}-V_o)\right]$

$$1.34225\times10^{-4}\times40\times(V_{GS}-0.7)^2=3.835\times10^{-5}\times8.7878(3-0.8)^2\left[1+0.2(3-0.8)\right]$$

$V_{in}=1.3614$

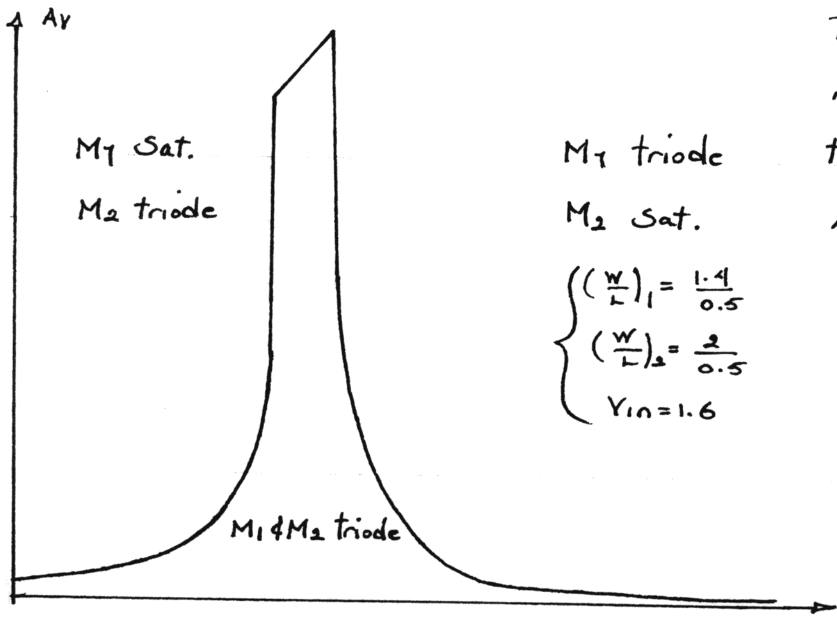
$$g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1}) = 3.5512 \times 10^{-3}$$

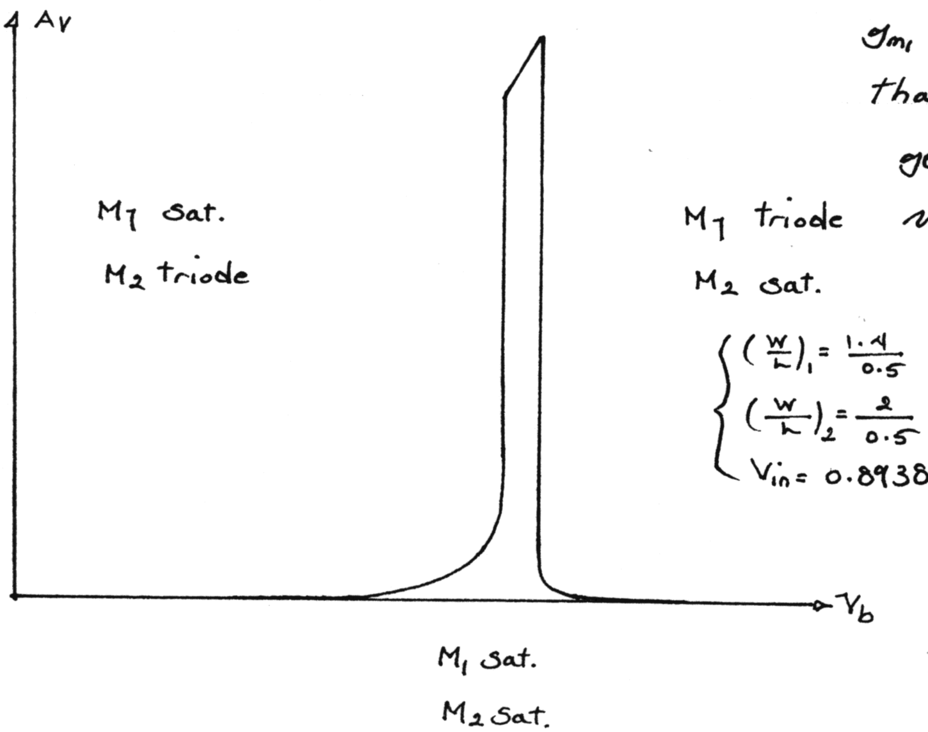
$$I = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2 = 1.1744 \times 10^{-3}$$

$$r_{out} = \frac{1}{(\lambda_P + \lambda_N) I} = 2838.2553$$

$$A_V = -g_{m1} \cdot r_{out} = -10.08$$

3.25

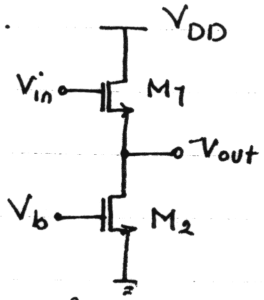




For M_1 to enter the triode region before M_2 is saturated, the overdrive voltage of M_1 must be increased.

Comparing the two curves, we observe that at $V_b = 0$ small signal voltage gain in (a) is higher than that in (b). That is because g_{m1} in (a) is higher than that in (b). However, generally, small signal voltage gain in (a) is less than that in (b), because when V_b sweeps all the way from 0 to V_{DD}, nowhere are both devices simultaneously in the saturation region.

3.26.



3.39

$V_{in} - V_{out} = 1V, \; I_{D1} = I_{D2} = 0.5\,mA, \; V_{GS2} - V_{GS1} = 0.5$

$\lambda = \gamma = 0$

$I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{out} - V_{TH1})^2 =$

$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_b - V_{TH2})^2$

$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \left(\frac{W}{L}\right)_1 (1 - 0.7)^2$

$\left(\frac{W}{L}\right)_1 = 82.77 \qquad V_{GS2} = 0.5 + 1 = 1.5$

$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \left(\frac{W}{L}\right)_2 (1.5 - 0.7)^2 \rightarrow \left(\frac{W}{L}\right)_2 = 11.64$

$\gamma = 0.45\,V^{-1}, \; V_{in} = 2.5V, \; V_{in} - V_{out} = 1, \; I_{D1} = I_{D2} = 0.5\,mA, \; V_{GS2} - V_{GS1} = 0.5$

$V_{out} = V_{in} - 1 = 2.5 - 1 = 1.5, \; V_{GS2} = 0.5 + (2.5 - 1.5) = 1.5V$

$V_{TH1} = V_{TH0} + \gamma\left(\sqrt{2|\phi_f| + V_0} - \sqrt{2|\phi_f|}\right) = 0.7 + 0.45\left(\sqrt{0.9 + 1.5} - \sqrt{0.9}\right) = 0.97022$

$I_{D1} = I_{D2} = 0.5 \times 10^{-3} = \frac{1}{2}\mu_n C_{ox} S_1 (V_{GS1} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox} S_2 (V_{GS2} - V_{TH2})^2$

$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} S_1 (1 - 0.97)^2 = \frac{1}{2} \times 1.34225 \times 10^{-4} S_2 (1.5 - 0.7)^2$

$S_1 = \left(\frac{W}{L}\right)_1 = 8278$

$S_2 = \left(\frac{W}{L}\right)_2 = 11.64$

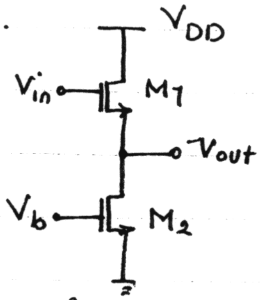
$V_{out} = V_b - V_{TH2} = 1.5 - 0.7 = 0.8$

$V_{TH1} = 0.7 + 0.45\left(\sqrt{0.9 + 0.8} - \sqrt{0.9}\right) = 0.8598$

$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \times 8278 (V_{in} - 0.8 - 0.8598)^2$

$V_{in} = 1.6897$

3.26.



$V_{in} - V_{out} = 1V,\ I_{D1} = I_{D2} = 0.5\,mA,\ V_{GS2} - V_{GS1} = 0.5$

$\lambda = \gamma = 0$

$$I_{D1} = I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{out} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_b - V_{TH2})^2$$

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\left(\frac{W}{L}\right)_1 (1-0.7)^2$$

$\left(\frac{W}{L}\right)_1 = 82.77$ $\qquad V_{GS2} = 0.5 + 1 = 1.5$

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\left(\frac{W}{L}\right)_2 (1.5-0.7)^2 \rightarrow \left(\frac{W}{L}\right)_2 = 11.64$$

$\gamma = 0.45\,V^{-1},\ V_{in} = 2.5V,\ V_{in} - V_{out} = 1,\ I_{D1} = I_{D2} = 0.5\,mA,\ V_{GS2} - V_{GS1} = 0.5$

$V_{out} = V_{in} - 1 = 2.5 - 1 = 1.5,\ V_{GS2} = 0.5 + (2.5 - 1.5) = 1.5V$

$$V_{TH1} = V_{TH0} + \gamma\left(\sqrt{2|\phi_F| + V_0} - \sqrt{2|\phi_F|}\right) = 0.7 + 0.45\left(\sqrt{0.9+1.5} - \sqrt{0.9}\right) = 0.97022$$

$$I_{D1} = I_{D2} = 0.5\times10^{-3} = \frac{1}{2}\mu_n C_{ox} S_1 (V_{GS1} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox} S_2 (V_{GS2} - V_{TH2})^2$$

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4} S_1 (1-0.97)^2 = \frac{1}{2}\times1.34225\times10^{-4} S_2 (1.5-0.7)^2$$

$S_1 = \left(\frac{W}{L}\right)_1 = 8278$

$S_2 = \left(\frac{W}{L}\right)_2 = 11.64$

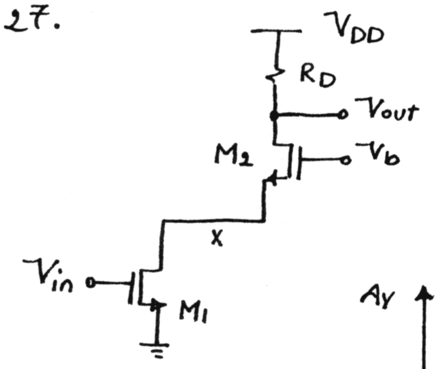
$V_{out} = V_b - V_{TH2} = 1.5 - 0.7 = 0.8$

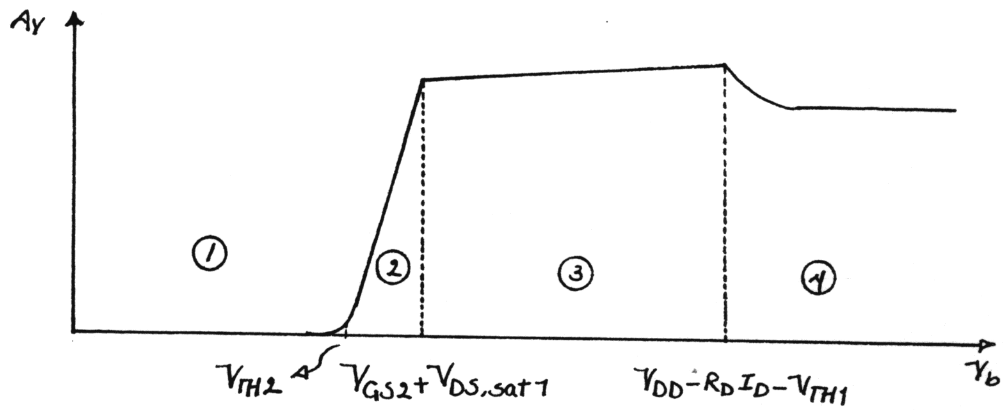
$V_{TH1} = 0.7 + 0.45\left(\sqrt{0.9+0.8} - \sqrt{0.9}\right) = 0.8598$

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\times8278\,(V_{in} - 0.8 - 0.8598)^2$$

$V_{in} = 1.6897$

3.27.





① In this region V_b is less than V_{TH2}, so M_1 and M_2 are off. It is worth mentioning that M_2 is saturated-off and M_1 is off in triode region.

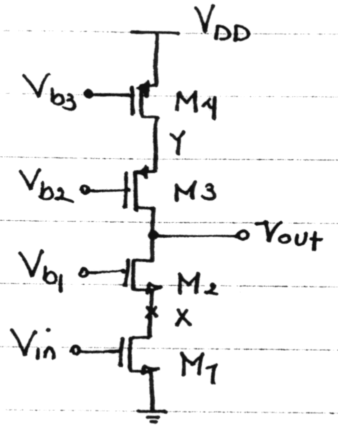
② V_b is increasing above V_{TH2}, as a result, a current establishes in circuit. M_1 operates in triode region and M_2 does in saturation. The higher V_b, the higher the drain-source voltage of M_1, increasing the output impedance of M_1 which, in turn, causes the small signal voltage gain of the circuit increases.

③ Both devices are in saturation region and the maximum gain is attainable in this region. The slight increase in A_v is because of increasing the transconductance of M_1 with increasing V_X (or V_b).

④ M_2 enters the triode region, as a result, the total output impedance decreases down to the limit of $r_{o1} \| R_D$. Consequently, the small signal voltage gain experiences a similar change.

3.28

3.35



output swing $= 1.9\,V$

$I_{bias} = 0.5\,mA$

$\gamma = 0 \quad \left(\frac{W}{L}\right)_{1-4} = \left(\frac{W}{L}\right)$

$$V_{b1} - V_{TH1} < V_{out} < V_{b2} + V_{TH3}$$

$$V_{b2} + V_{TH3} - (V_{b1} - V_{TH1}) = 1.9$$

$$V_{b2} + 0.8 - V_{b1} + 0.7 = 1.9\,, \quad V_{b2} - V_{b1} = 0.4$$

$$0.5\times10^{-3} = \frac{1}{2}\mu_n C_{ox} S (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox} S (V_{b1} - V_X - V_{TH2})^2 =$$
$$\frac{1}{2}\mu_p C_{ox} S (V_Y - V_{b2} - |V_{TH3}|)^2 = \frac{1}{2}\mu_p C_{ox} S (V_{DD} - V_{b3} - |V_{TH4}|)^2$$

$$V_{DD} - V_{SD\,min,4} - V_{SD\,min,3} - V_{DS\,min,1} - V_{DS\,min,2} = 1.9$$

$$1.1 = \left(\frac{2I_D}{\mu_p C_{ox} S}\right)^{1/2} + \left(\frac{2I_D}{\mu_p C_{ox} S}\right)^{1/2} + \left(\frac{2I_D}{\mu_n C_{ox} S}\right)^{1/2} + \left(\frac{2I_D}{\mu_n C_{ox} S}\right)^{1/2}$$

$$1.1 = 2\sqrt{2I_D}\left(\frac{1}{\sqrt{\mu C_{ox}}} + \frac{1}{\sqrt{\mu_n C_{ox}}}\right)\frac{1}{\sqrt{S}} \rightarrow S = \frac{8I_D\left(\frac{1}{\sqrt{\mu_p C_{ox}}} + \frac{1}{\sqrt{\mu_n C_{ox}}}\right)^2}{1.1^2}$$

$$S = \frac{8\times0.5\times10^{-3}\left(\frac{1}{\sqrt{1.34225\times10^{-4}}} + \frac{1}{\sqrt{3.835\times10^{-5}}}\right)^2}{1.1^2} = 202.98 \rightarrow S = 203$$

$$V_{DS\,min,1} = \left(\frac{2I_D}{\mu_n C_{ox} S}\right)^{1/2} = \left(\frac{2\times0.5\times10^{-3}}{1.34225\times10^{-4}\times203}\right)^{1/2} = 0.1915$$

$$V_{SD\,min,4} = \left(\frac{2\times0.5\times10^{-3}}{3.835\times10^{-5}\times203}\right)^{1/2} = 0.3584$$

$$0.5\times10^{-3} = \frac{1}{2}\times1.34225\times10^{-4}\times203\,(V_{b1} - V_X - 0.7)^2$$

$$V_{b1} - V_X = 0.8915$$

$$0.5\times10^{-3} = \frac{1}{2}\times3.835\times10^{-5}\times203\,(V_Y - V_{b2} - 0.8)^2$$

$V_Y - V_{b2} = 1.1584$

$V_{b2} - V_{b1} = 0.4$

If $V_X = 0.1915 \rightarrow V_{b1} = 1.083,\ V_{b2} = 1.483,\ V_Y = 2.6414$

$V_{SD4} = V_{DD} - V_Y = 0.3586$, as a result, M_7 and M_2 are at the edg of the triode region.

$$g_{m1} = \sqrt{2\mu_n C_{ox} S(1+\lambda V_{DS}) I_D} = \sqrt{2\times 1.34225\times 10^{-4} \times 203\times 0.5\times 10^{-3}}$$

$$g_{m1} = g_{m2} = 5.2199\times 10^{-3}$$

$$r_{o1} = r_{o2} = \frac{1}{0.1\times 0.5\times 10^{-3}} = 20K \qquad r_{o3} = r_{o4} = \frac{1}{0.2\times 0.5\times 10^{-3}} = 10K$$

$$G_m = \frac{g_{m1}\cdot r_{o1}\cdot(1+g_{m2}\cdot r_{o2})}{r_{o1}\cdot r_{o2}\, g_{m2} + r_{o1} + r_{o2}} = \frac{5.2199\times 10^{-3}\times 20\times 10^{3}\,(20\times 10^{3}\times 5.2199\times 10^{-3}+1)}{(20\times 10^{3})^2\times 5.2199\times 10^{-3} + 2\times 20\times 10^{3}}$$

$G_m = 5.17\times 10^{-3}$, neglecting the body effect.

$$R_{out} = \left[(1+g_{m2} r_{o2}) r_{o1} + r_{o2}\right] \| \left[(1+g_{m3} r_{o3}) r_{o1} + r_{o3}\right]$$

$$R_{out} = \left[(1+5.2199\times 10^{-3}\times 20\times 10^{3})\, 20\times 10^{3} + 20\times 10^{3}\right] \| \left[(1+2.79\times 10^{-3}\times 10\times 10^{3})\, 10\times 10^{3} + 10\times 10^{3}\right]$$

$R_{out} = 262.1766\times 10^{3}$, $A_V = -G_m R_{out} = -5.17\times 10^{-3}\times 262.1766\times 10^{3}$

$A_V = -1355.45$ $\qquad g_{m3} = g_{m4} = \sqrt{2\times 3.835\times 10^{-5}\times 203\times 0.5\times 10^{-3}} = 2.79\times 10^{-3}$

Chapter 4: Differential Amplifiers

4.1

(a) $A_v \cong -\dfrac{g_{mN}}{g_{mP}} = -\sqrt{\dfrac{\mu_n (W/L)_N}{\mu_p (W/L)_P}}$ (4.52)

$$A_v = -\sqrt{\frac{350}{100} \times \frac{50/0.5}{50/1}} = -\sqrt{7} = \underline{-2.65}$$

(b) $A_v = -g_{mN}(r_{ON} \| r_{OP})$ (4.53)

$I_D = \dfrac{I_{SS}}{2} = 0.5^{mA}$ $\mu_n C_{ox} = 350 \times \dfrac{8.85 \times 10^{-14} \times 3.9}{9 \times 10^{-7}} = 0.134\ ^{mA}/_{V^2}$

$$g_{m_N} = \sqrt{2 I_D \mu_n C_{ox} \frac{W}{L}} = \sqrt{2 \times 0.5^m \times 0.134^m \times 100} = 3.66\ m\Omega^{-1}$$

$L_N = 0.5^\mu \Rightarrow \lambda_n = 0.1 \Rightarrow r_{ON} = \dfrac{1}{\lambda_n I_D} = \dfrac{1}{0.1 \times 0.5^m} = 20^{K\Omega}$

$L_p = 1^\mu$; $\lambda_p = 0.2$ for $L = 0.5^\mu$; $\lambda \propto \dfrac{1}{L} \Rightarrow \lambda_p = 0.1$

$$r_{OP} = \frac{1}{\lambda_p I_D} = \frac{1}{0.1 \times 0.5^m} = 20^{K\Omega}$$

$$A_v = -g_{mN}(r_{ON} \| r_{OP}) = -3.66^m (20^K \| 20^K) = \underline{-36.6}$$

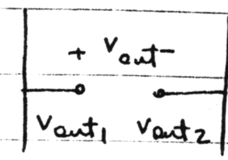
$(V_{in,cm})_{min} = 0.4 + V_{GS1}$ for both circuits

$$V_{GS1} = V_{TH} + \sqrt{\frac{2 I_D}{\mu_n C_{ox} (W/L)_N}} = 0.7 + \sqrt{\frac{2 \times 0.5^m}{0.134^m \times 100}} = 0.7 + 0.27 = 0.97^V$$

$\rightarrow (V_{in,CM})_{min} = 0.4 + 0.97 = 1.37^V$

max output voltage swing:

(a) $(V_{out1,2})_{max} = V_{DD} - |V_{TH,P}| = 3 - 0.8 = 2.2^V$



There are two constraints for $(V_{out1,2})_{min}$:

1) M_1 enters triode: $(V_{out1,2})_{min} = 0.4 + V_{GS1} - V_{TH,n}$

$= 0.4 + 0.97 - 0.7 = 0.67^V$

2) all of I_{SS} goes through $M3$:

$$(V_{out1,2})_{min} = V_{DD} - \left| V_{GS3}\Big|_{I_D = I_{SS}} \right| = V_{DD} - |V_{TH,P}| + \sqrt{\frac{2\, I_{SS}}{\mu_p C_{ox} (\frac{W}{L})_3}}$$

$$= 3 - 0.8 - \sqrt{\frac{2 \times 1^m}{38.3^\mu \times 50}} = 3 - 0.8 - 1.02 = 1.18^V$$

$$\mu_p C_{ox} = 100 \times \frac{8.85 \times 10^{-14} \times 3.9}{9 \times 10^{-7}} = 38.3^{\mu A/V^2} \qquad \Rightarrow (V_{out1,2})_{min} = 1.18^V$$

Max swing of $V_{out1,2} = 2.2 - 1.18 = 1.02^V$

Max swing of $V_{out} = 2 \times 1.02 = 2.04^V$

(b) $(V_{out1,2})_{max} = V_{DD} - |V_{GS3} - V_{TH,P}| = 3 - 0.72 = 2.28^V$

$(V_{out1,2})_{min} = 0.4 + V_{GS1} - V_{TH,n} = 0.67^V$

Max swing of $V_{out} = 2(2.28 - 0.67) = 3.22^V$

4.2 $I_{SS} = 1\ ^{mA}$

(a)

$$A_v = -g_{m1}\left(\frac{1}{g_{m3}} \| r_{o1} \| r_{o3} \| r_{o5}\right) \approx -\frac{g_{m1}}{g_{m3}} = \sqrt{\frac{\mu_n}{\mu_p} \times \frac{I_{D1}}{I_{D3}}}$$

$$= \sqrt{\frac{350}{100} \times \frac{\frac{1}{2} I_{SS}}{0.2\,\frac{I_{SS}}{2}}} = -4.18$$

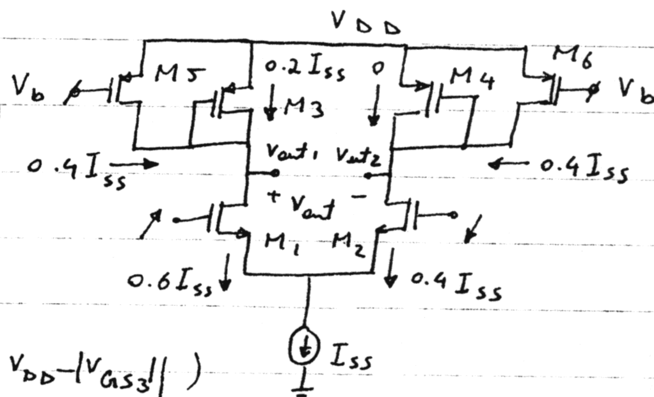
(b) $I_{D5} = I_{D6} = 0.8\left(\frac{I_{SS}}{2}\right) = 0.4\ ^{mA}$

$$|V_{GS5}| = V_{DD} - V_b \Rightarrow V_b = V_{DD} - |V_{GS5}| = V_{DD} - |V_{TH,P}| - \sqrt{\frac{2 I_{D5}}{\mu_p C_{ox} \frac{W}{L}}}$$

$$V_b = 3 - 0.8 - \sqrt{\frac{2 \times 0.4^m}{38.3^\mu \times 100}} = 1.74$$

(c)

$$(V_{out\,1,2})_{max} = \min(V_b + |V_{TH,p}|, V_{DD} - |V_{TH,p}|)$$



$$= (1.74 + 0.8,\ 3 - 0.8) = 2.2^V$$

$$(V_{out\,1,2})_{min} = \max\left(V_{I_{SS},min} + V_{GS1}\Big|_{I_D=0.6 I_{SS}} - V_{TH,n},\ V_{DD} - |V_{GS3}|\Big|_{I_D = 0.2 I_{SS}}\right)$$

$$V_{GS1}\Big|_{I_D=0.6 I_{SS}} = V_{TH,n} + \sqrt{\frac{2 \times 0.6 I_{SS}}{\mu_n C_{ox}\frac{W}{L}}} = V_{TH,n} + 0.299^V$$

$$|V_{GS3}|\Big|_{I_D=0.2 I_{SS}} = |V_{TH,P}| + \sqrt{\frac{2 \times 0.2 I_{SS}}{\mu_p C_{ox}\frac{W}{L}}} = 0.8 + 0.323^V = 1.12^V$$

$$(V_{out\,1,2})_{min} = \max(0.4 + 0.299,\ 3 - 1.12) = 1.88^V$$

Max swing of $V_{out} = 2(2.2 - 1.88) = 0.64^V$

4.2 $I_{SS} = 1\ mA$

(a)

$$A_v = -g_{m1}\left(\frac{1}{g_{m3}} \| r_{o1} \| r_{o3} \| r_{o5}\right) \approx -\frac{g_{m1}}{g_{m3}} = \sqrt{\frac{\mu_n}{\mu_p} \times \frac{I_{D1}}{I_{D3}}}$$

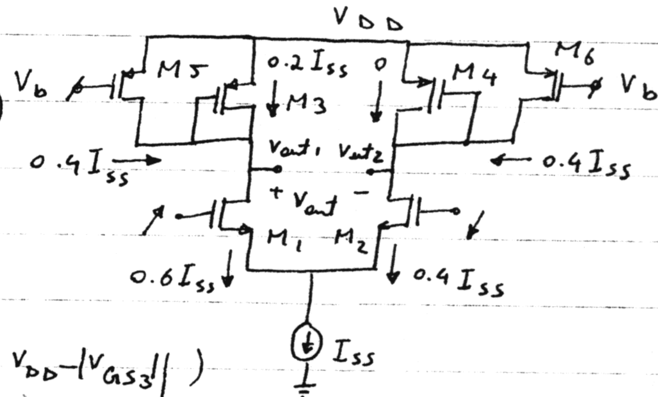
$$= \sqrt{\frac{350}{100} \times \frac{\frac{1}{2} I_{SS}}{0.2\,\frac{I_{SS}}{2}}} = -4.18$$

(b) $I_{D5} = I_{D6} = 0.8\left(\frac{I_{SS}}{2}\right) = 0.4\ mA$

$$|V_{GS5}| = V_{DD} - V_b \Rightarrow V_b = V_{DD} - |V_{GS5}| = V_{DD} - |V_{TH,P}| - \sqrt{\frac{2 I_{D5}}{\mu_p C_{ox} \frac{W}{L}}}$$

$$V_b = 3 - 0.8 - \sqrt{\frac{2 \times 0.4^{m}}{38.3^{\mu} \times 100}} = 1.74$$

(c)



$$(V_{out1,2})_{max} = \min(V_b + |V_{TH,p}|, V_{DD} - |V_{TH,p}|)$$

$$= (1.74 + 0.8, 3 - 0.8) = 2.2^{V}$$

$$(V_{out1,2})_{min} = \max\left(V_{I_{SS},min} + V_{GS1}\Big|_{I_D = 0.6 I_{SS}} - V_{TH,n},\ V_{DD} - |V_{GS3}|\Big|_{I_D = 0.2 I_{SS}}\right)$$

$$V_{GS1}\Big|_{I_D = 0.6 I_{SS}} = V_{TH,n} + \sqrt{\frac{2 \times 0.6\, I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = V_{TH,n} + 0.299^{V}$$

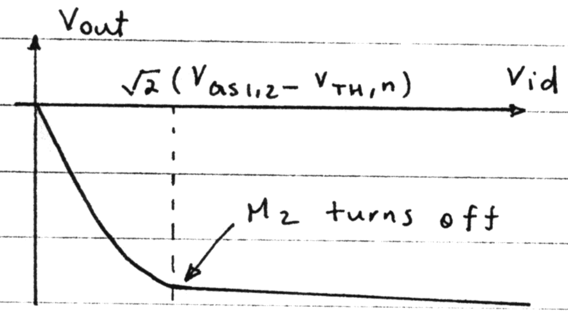
$$|V_{GS3}|\Big|_{I_D = 0.2 I_{SS}} = |V_{TH,P}| + \sqrt{\frac{2 \times 0.2\, I_{SS}}{\mu_p C_{ox} \frac{W}{L}}} = 0.8 + 0.323^{V} = 1.12^{V}$$

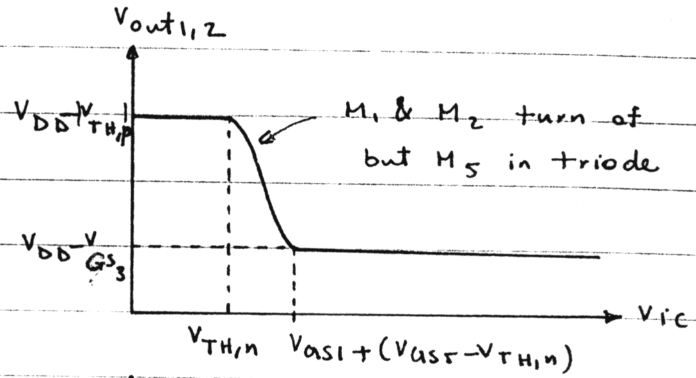
$$(V_{out1,2})_{min} = \max(0.4 + 0.299, 3 - 1.12) = 1.88^{V}$$

Max swing of $V_{out} = 2(2.2 - 1.88) = 0.64^{V}$

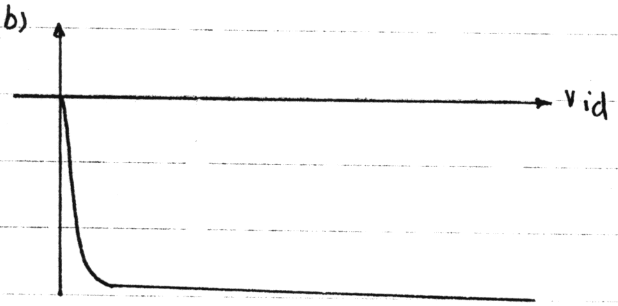
4.3

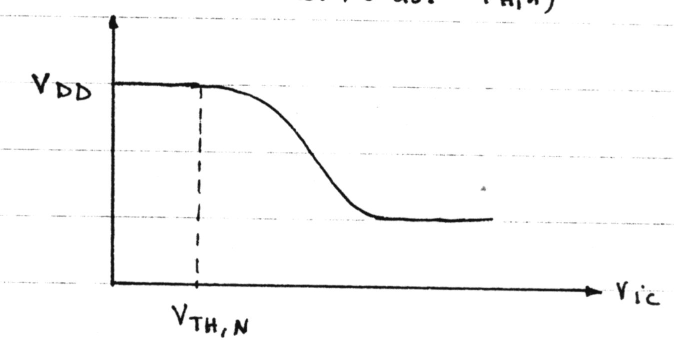
(a)


Vout
√2 (VGS1,2 − VTH,n)
Vid
M2 turns off

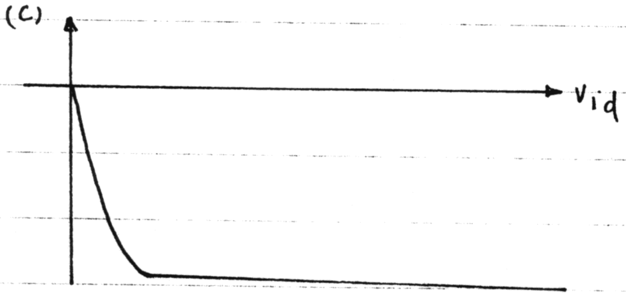

Vout1,2
VDD−VTH,p
M1 & M2 turn of
but M5 in triode
VDD−VGS3
Vic
VTH,n
VGS1 + (VGS5 − VTH,n)

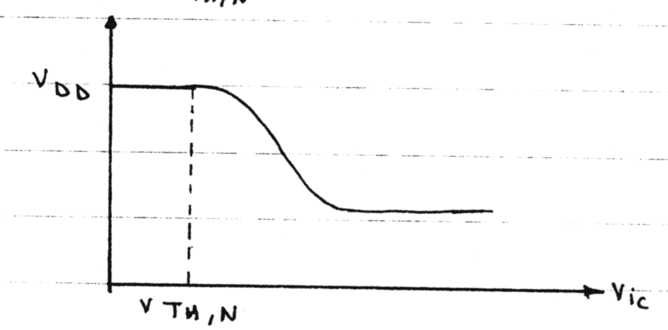
(b)


Vid

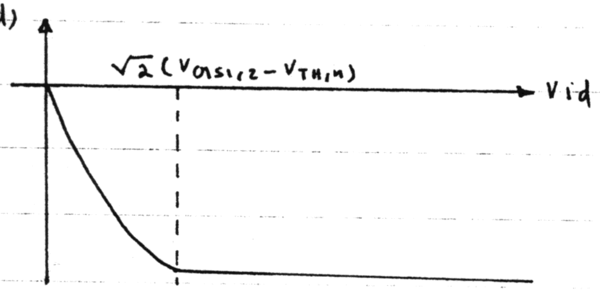

VDD
Vic
VTH,N

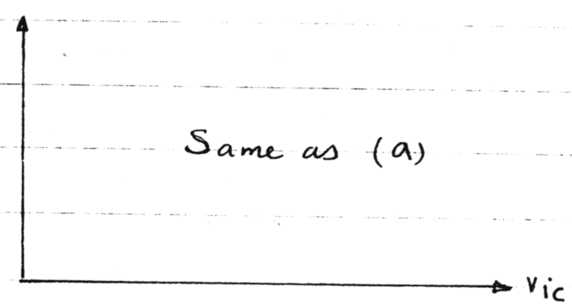
(c)


Vid

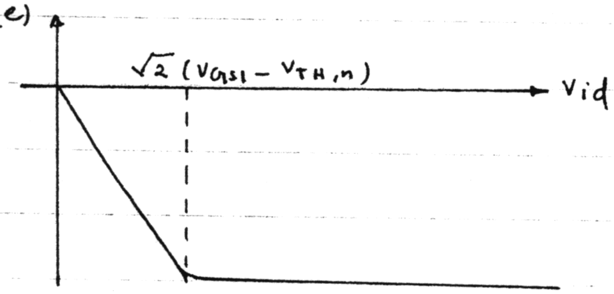

VDD
Vic
VTH,N

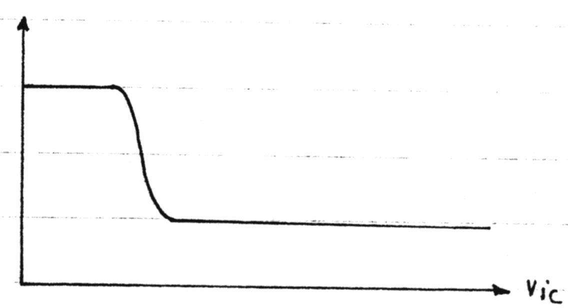
(d)


√2 (VGS1,2 − VTH,n)
Vid


Same as (a)
Vic

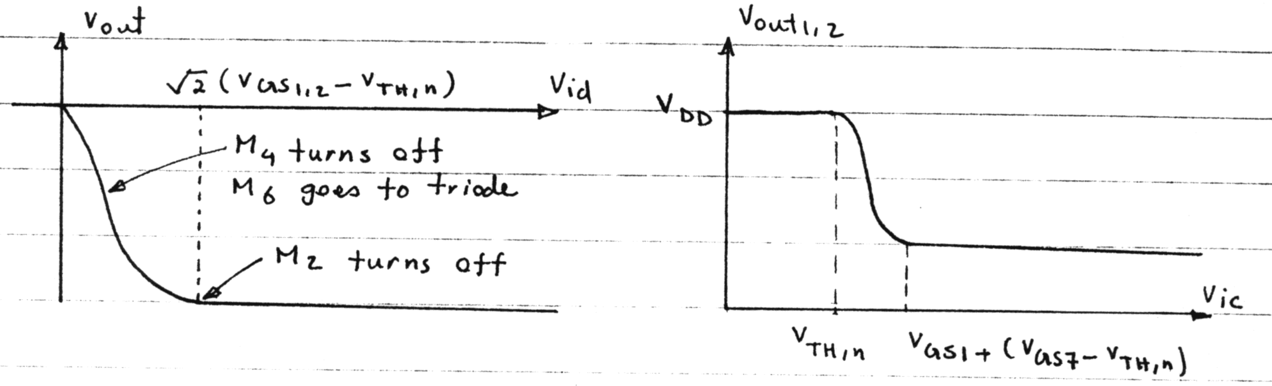
(e)


√2 (VGS1 − VTH,n)
Vid

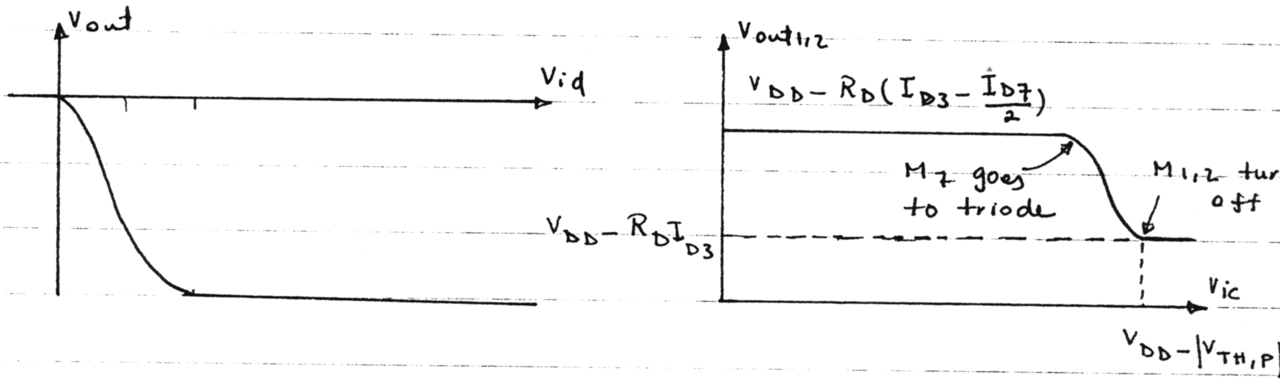

Vic

4.4

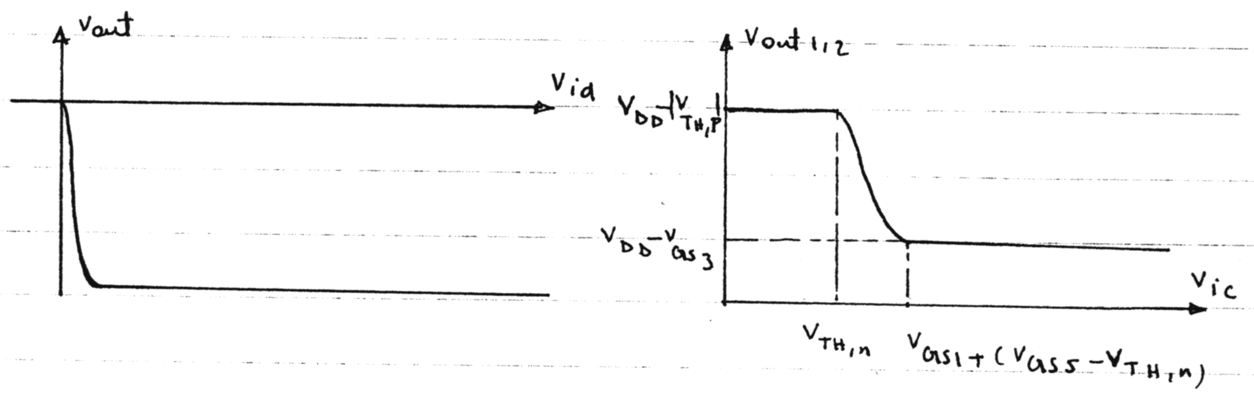
(a)


V_{out}
$\sqrt{2}\,(V_{GS1,2} - V_{TH,n})$
V_{id}
M_4 turns off
M_6 goes to triode
M_2 turns off
$V_{out1,2}$
V_{DD}
V_{ic}
$V_{TH,n}$
$V_{GS1} + (V_{GS7} - V_{TH,n})$

(b)


V_{out}
V_{id}
$V_{out1,2}$
$V_{DD} - R_D\left(I_{D3} - \frac{I_{D7}}{2}\right)$
M_7 goes to triode
$M_{1,2}$ turns off
$V_{DD} - R_D I_{D3}$
V_{ic}
$V_{DD} - |V_{TH,P}|$

(c)


V_{out}
V_{id}
$V_{out1,2}$
$V_{DD} - |V_{TH,P}|$
$V_{DD} - V_{GS3}$
V_{ic}
$V_{TH,n}$
$V_{GS1} + (V_{GS5} - V_{TH,n})$

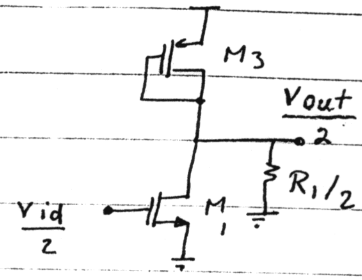
4.5

Fig. 4.35 using half circuit we have:

(a) we define $V_{id} = V_{in1} - V_{in2}$

$$A_v = \frac{V_{out}}{V_{id}} = -g_{m1}\left(\frac{1}{g_{m3}} \,\Big\|\, \frac{R_1}{2} \,\Big\|\, r_{o1} \,\|\, r_{o3}\right)$$

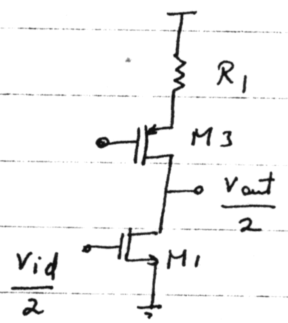
$$\approx -g_{m1}\left(\frac{1}{g_{m3}} \,\Big\|\, \frac{R_1}{2}\right) = -\frac{g_{m1}R_1}{2 + g_{m3}R_1}$$



(b)

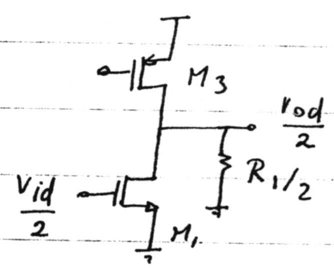
$$A_v = -g_{m1}\left[r_{o1} \,\|\, (R_1 g_{m3} r_{o3} + R_1 + r_{o3})\right]$$

$$\approx -g_{m1}\left(r_{o1} \,\|\, R_1 g_{m3} r_{o3}\right)$$



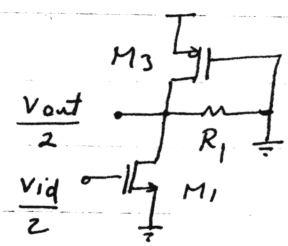
(c)

$$A_v = -g_{m1}\left(r_{o1} \,\|\, r_{o3} \,\Big\|\, \frac{R_1}{2}\right)$$

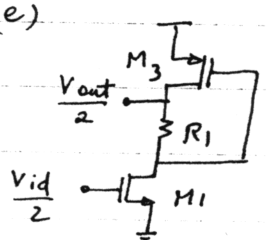


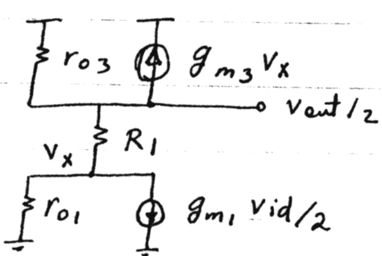
(d)

$$A_v = -g_{m1}\left(r_{o1} \,\|\, r_{o3} \,\|\, R_1\right)$$



(e)





KCL: $\frac{V_x}{r_{o1}} + \frac{V_x - V_{out}/2}{R_1} + g_{m1}\frac{V_{id}}{2} = 0$

$\frac{V_{out}/2}{r_{o3}} + \frac{V_{out}/2 - V_x}{R_1} + g_{m3} V_x = 0 \Rightarrow V_x = \frac{(r_{o3} + R_1)\, V_{out}}{2 r_{o3}(1 - R_1 g_{m3})}$

$\left(\frac{R_1 + r_{o1}}{R_1 r_{o1}} \times \frac{r_{o3} + R_1}{r_{o3}(1 - R_1 g_{m3})} - \frac{1}{R_1} \right) V_{out} + g_{m1} V_{id} = 0$

$\frac{(R_1 + r_{o1})(R_1 + r_{o3}) - r_{o1} r_{o3}(1 - R_1 g_{m3})}{R_1 r_{o1} r_{o3}(1 - R_1 g_{m3})} V_{out} + g_{m1} V_{id} = 0$

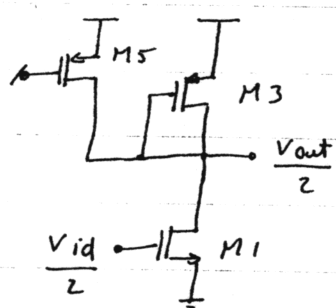
$\frac{R_1 + r_{o1} + r_{o2} + r_{o1} r_{o3} g_{m3}}{r_{o1} r_{o3}(1 - R_1 g_{m3})} V_{out} + g_{m1} V_{id} = 0$

$\Rightarrow A_v = \frac{V_{out}}{V_{id}} = -\frac{g_{m1} r_{o1} r_{o3}(1 - R_1 g_{m3})}{R_1 + r_{o1} + r_{o3} + r_{o1} r_{o3} g_{m3}} \approx -\frac{g_{m1}}{g_{m3}}(1 - R_1 g_{m3})$

$R_1 g_{m3} < 1$

Fig. 4.36

(a)

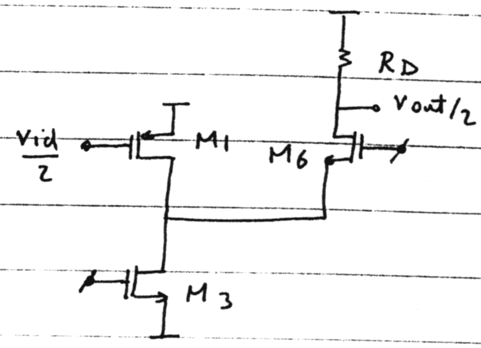


$A_v = -g_{m1}\left(\frac{1}{g_{m3}} \| r_{o1} \| r_{o3} \| r_{o5} \right)$

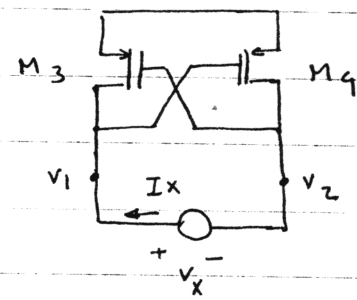
$\approx -\frac{g_{m1}}{g_{m3}}$

(b)

$$A_v = -g_{m1}\left(R_D \,||\, [r_{o6}\, g_{m6}\, (r_{o1}||r_{o3})]\right)$$



(c)



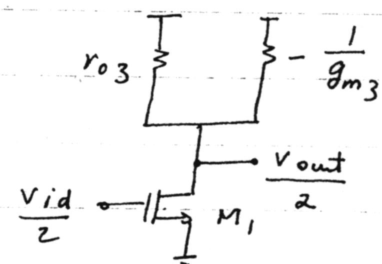
if we neglect r_{o3} & r_{o4} at the moment, we have:

$I_x = g_{m3} V_2$ $\qquad g_{m3} = g_{m4} = g_{m3,4}$

$I_x = -g_{m4} V_1$

$$2I_x = -g_{m3,4}(V_1 - V_2) = -g_{m3,4} V_x$$

$$\Rightarrow \frac{V_x}{I_x} = -\frac{2}{g_{m3,4}}$$



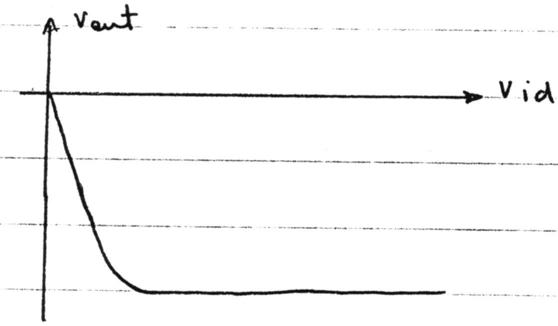
$$A_v = -g_{m1}\left(r_{o1} || r_{o3} || \frac{-1}{g_{m3}}\right)$$

$$A_v = -\frac{g_{m1}}{\frac{1}{r_{o1}} + \frac{1}{r_{o3}} - g_{m3}} \qquad \left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}} > g_{m3}\right)$$

if $g_{m3} \geq \frac{1}{r_{o1}} + \frac{1}{r_{o3}}$ then the circuit is not stable and small signal model is not valid.

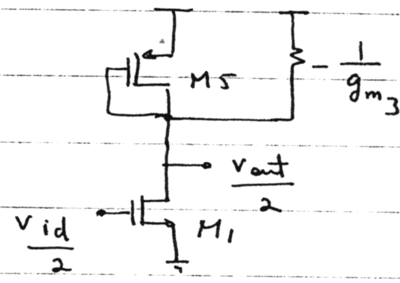
4.6

(a)



(b)

Similar to what we had in the previous problem (Fig 4.36 (c)), M_3 gives a negative resistance at the output.



$$A_V = -g_{m1}\left(\frac{1}{g_{m5}} \,\Big\|\, \frac{-1}{g_{m3}}\right) \qquad (\lambda = \infty)$$

$$A_V = -\frac{g_{m1}}{g_{m5} - g_{m3}} \qquad (g_{m3} \text{ must be less than } g_{m5})$$

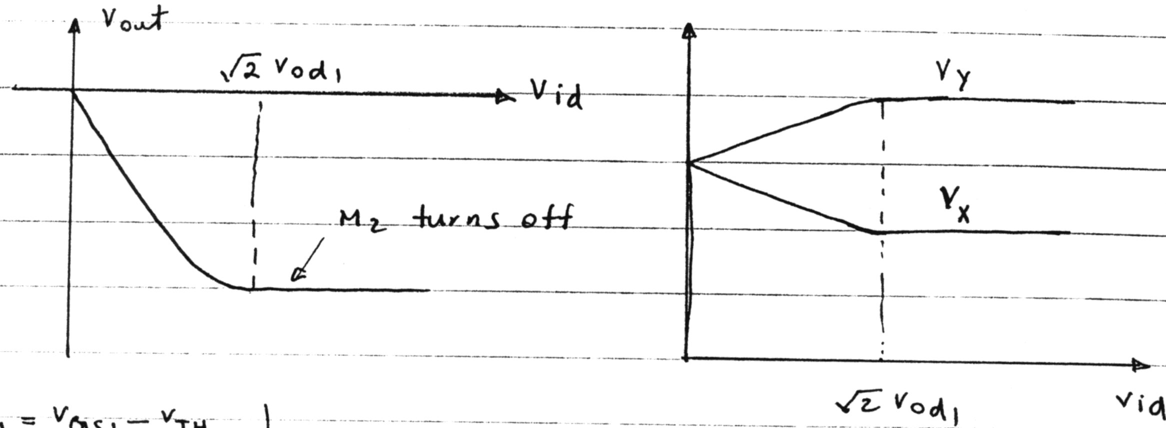
$$g_m = \mu_p C_{ox} \frac{W}{L}(V_{GS} - V_T) \qquad V_{GS\,3,4} = V_{GS\,5,6}$$

$$\Rightarrow \quad \frac{g_{m3,4}}{g_{m5,6}} = \frac{(W/L)_{3,4}}{(W/L)_{5,6}} = 0.8$$

$$\Rightarrow \quad A_V = -\frac{g_{m1}}{g_{m5} - 0.8\, g_{m5}} = -\frac{5\, g_{m1}}{g_{m5}}$$

4.7

(a)

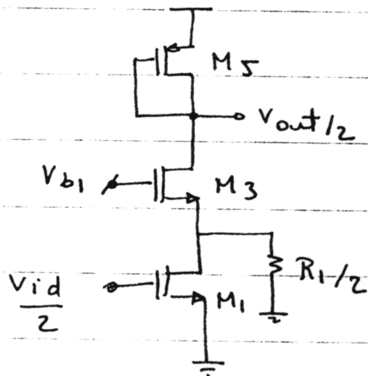


$V_{od1} = V_{GS1} - V_{TH,n} \Big|_{I_D = \frac{I_{SS}}{2}}$

(b)

$$A_v = \frac{V_{out}}{V_{id}} = -g_{m1}\left(\frac{R_1}{2} \Big\| \frac{1}{g_{m3}}\right) g_{m3}\, g_{m5}^{-1}$$

$$= -\frac{g_{m1}}{g_{m5}\left(1 + \frac{2}{R_1 g_{m3}}\right)}$$

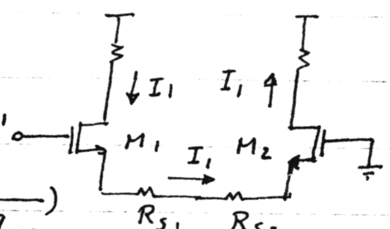


4.8

By using superposition, we have:

$$G_{m1} = \frac{I_1}{V_{in1}}\Bigg|_{V_{in2}=0} = \frac{g_{m1}}{1 + (g_{m1} + g_{mb1})\underbrace{\left(R_{S1} + R_{S2} + \frac{1}{g_{m2} + g_{mb2}}\right)}_{R_{eq1}}}$$

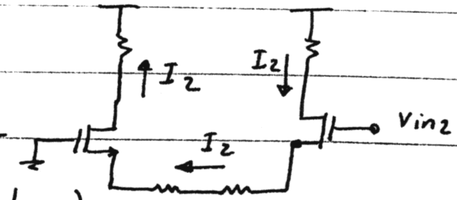
(Transconductance of CS)



$R_{eq1} = R_{S1} + R_{S2} + \frac{1}{g_{m2} + g_{mb2}}$

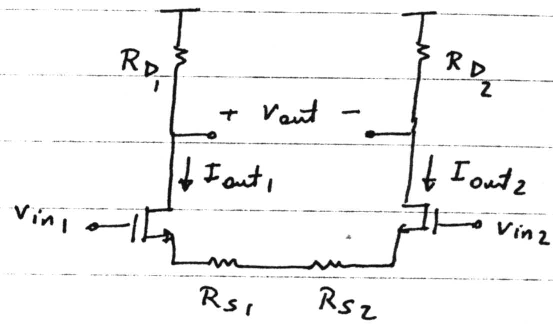
Similarly:

$$G_{m2} = \frac{I_2}{V_{in2}}\Bigg|_{V_{in1}=0} = \frac{g_{m2}}{1+(g_{m2}+g_{mb2})\left(R_{S1}+R_{S2}+\frac{1}{g_{m1}+g_{mb1}}\right)}$$



$$I_{out1} = I_1 - I_2$$

$$I_{out2} = I_2 - I_1$$



$$V_{out} = V_{out1} - V_{out2} = -R_{D1} I_{out1} + R_{D2} I_{out2}$$

$$V_{out} = -R_{D1}(I_1 - I_2) - R_{D2}(I_1 - I_2)$$

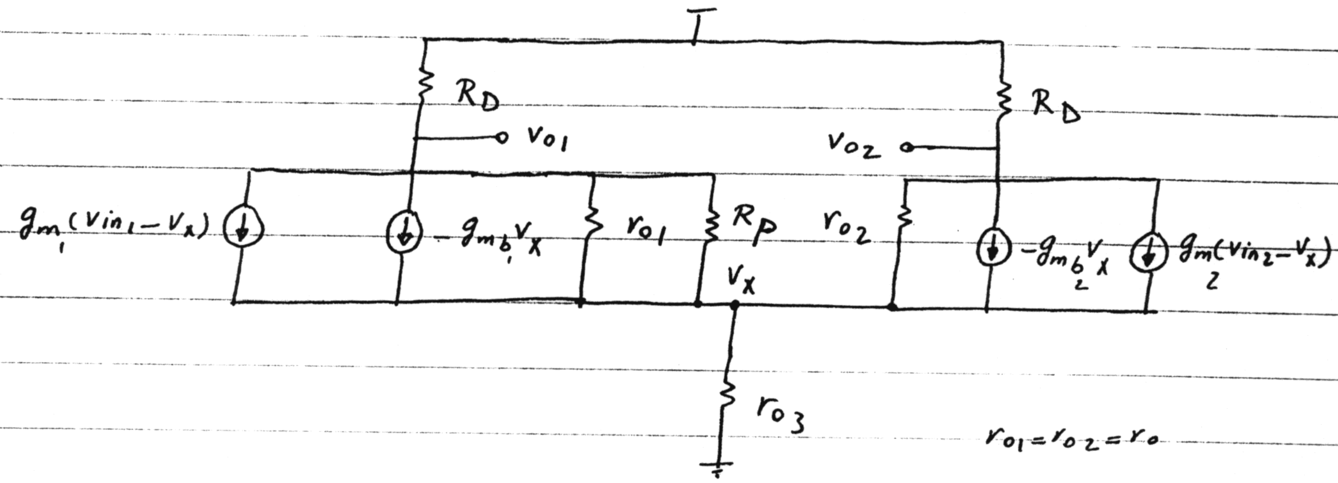
$$V_{out} = -(R_{D1} + R_{D2})(I_1 - I_2)$$

$$V_{out} = -(R_{D1} + R_{D2})(G_{m1} V_{in1} - G_{m2} V_{in2})$$

or equivalantly:

$$V_{out} = -(R_{D1} + R_{D2})\left[(G_{m1} + G_{m2})\frac{V_{in1} - V_{in2}}{2} + (G_{m1} - G_{m2})\frac{V_{in1} + V_{in2}}{2}\right]$$

4.9



$r_{o1} = r_{o2} = r_o$

$g_{m_1} = g_{m_2} = g_m$

$g_{mb_1} = g_{mb_2} = g_{mb}$

KCL:

$$
\begin{cases}
① \quad \dfrac{v_{o1}}{R_D} + \dfrac{v_{o1}-v_x}{r_o} + g_m(v_{in_1}-v_x) - g_{mb}v_x + \dfrac{v_{o1}-v_x}{R_P} = 0 \\[2ex]
② \quad \dfrac{v_{o2}}{R_D} + \dfrac{v_{o2}-v_x}{r_o} + g_m(v_{in_2}-v_x) - g_{mb}v_x = 0 \\[2ex]
\dfrac{v_{o1}}{R_D} + \dfrac{v_{o2}}{R_D} + \dfrac{v_x}{r_{o3}} = 0 \implies v_x = -r_{o3}\dfrac{v_{o1}+v_{o2}}{R_D}
\end{cases}
$$

we define:

$$
\begin{cases}
v_{od} = v_{o1} - v_{o2} \\
v_{oc} = \dfrac{v_{o1}+v_{o2}}{2}
\end{cases}
\implies
\begin{cases}
v_{o1} = v_{oc} + \dfrac{v_{od}}{2} \\
v_{o2} = v_{oc} - \dfrac{v_{od}}{2}
\end{cases}
$$

Now by substituting v_{o1}, v_{o2} and v_x in ① and ② we have:

$$
\begin{cases}
\left(v_{oc} + \dfrac{v_{od}}{2}\right)\left(\dfrac{1}{R_D} + \dfrac{1}{R_P} + v_o\right) + g_m v_{in_1} - \left(\dfrac{1}{R_P} + g_m + g_{mb} + \dfrac{1}{r_o}\right)\left(-\dfrac{2r_{o3}}{R_D}v_{oc}\right) = 0 \\[2ex]
\left(v_{oc} - \dfrac{v_{od}}{2}\right)\left(\dfrac{1}{R_D} + \dfrac{1}{r_o}\right) + g_m v_{in_2} - \left(g_m + g_{mb} + \dfrac{1}{r_o}\right)\left(-\dfrac{2r_{o3}v_{oc}}{R_D}\right) = 0
\end{cases}
$$

or:

$$
\left[\frac{1}{R_D} + \frac{1}{R_P} + \frac{1}{r_o} + \left(\frac{1}{R_P} + \frac{1}{r_o} + g_m + g_{mb}\right)\left(\frac{2r_{o3}}{R_D}\right)\right]v_{oc}
$$

$$
+ \frac{1}{2}\left(\frac{1}{R_D} + \frac{1}{R_P} + \frac{1}{r_o}\right)v_{od} + g_m v_{in_1} = 0 \qquad ③
$$

$$\left[\frac{1}{R_D} + \frac{1}{r_o} + \left(g_m + g_{mb} + \frac{1}{r_o}\right)\left(\frac{2r_{o3}}{R_D}\right)\right]V_{oc} - \frac{1}{2}\left(\frac{1}{R_D} + \frac{1}{r_o}\right)V_{od}$$

$$+ g_m V_{in2} = 0 \qquad (4)$$

From equation (3) and (4) V_{od} and V_{oc} can be solved in terms of V_{in1} and V_{in2}.

Now if $\lambda = \gamma = 0$, we have:

(5)
$$\begin{cases} \dfrac{V_{o1}}{R_D} + g_m(V_{in1} - V_X) + \dfrac{V_{o1} - V_X}{R_P} = 0 \\[2ex] \dfrac{V_{o2}}{R_D} + g_m(V_{in2} - V_X) = 0 \Rightarrow V_X = V_{in2} + \dfrac{V_{o2}}{g_m R_D} \\[2ex] V_{o1} + V_{o2} = 0 \quad , \quad V_{out} = V_{o1} - V_{o2} \Rightarrow \quad V_{o1} = \dfrac{V_{out}}{2} \quad V_{o2} = -\dfrac{V_{out}}{2} \end{cases}$$

(5) $\Rightarrow$
$$\frac{V_{out}}{2R_D} + g_m V_{in1} - g_m V_{in2} + \frac{V_{out}}{2R_D} + \frac{V_{out}}{2R_P} - \frac{V_{in2}}{R_P} + \frac{V_{out}}{2R_P g_m R_D} = 0$$

$$V_{out}\left(\frac{1}{R_D} + \frac{1}{2R_P} + \frac{1}{2g_m R_P R_D}\right) = -g_m(V_{in1} - V_{in2}) + \frac{V_{in2}}{R_P}$$

$$V_{in2} = \frac{V_{in1} + V_{in2}}{2} - \frac{V_{in1} - V_{in2}}{2}$$

$$\Rightarrow V_{out}\left(\frac{1}{R_D} + \frac{1}{2R_P} + \frac{1}{2g_m R_D R_P}\right) = -\left(g_m + \frac{1}{2R_P}\right)(V_{in1} - V_{in2}) + \frac{(V_{in1} + V_{in2})}{2R_P}$$

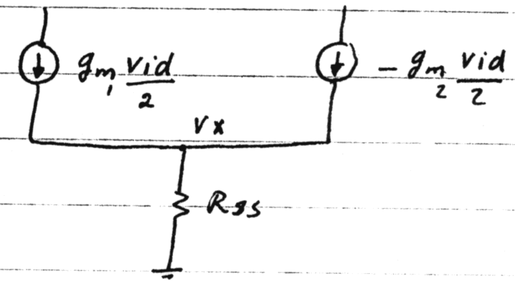
$$V_{out} = \left[-\left(g_m + \frac{1}{2R_P}\right)(V_{in1} - V_{in2}) + \frac{V_{in1} + V_{in2}}{2R_P}\right]\left(R_D \| 2R_P \| 2g_m R_D R_P\right)$$

$$CMRR = \frac{g_m + \frac{1}{2R_P}}{\frac{1}{R_P}} = \frac{2R_P g_m + 1}{2}$$

$$A_{dm-dm} = -\left(g_m + \frac{1}{2R_P}\right)\left(R_D \| 2R_P \| 2g_m R_D R_P\right) \qquad A_{cm-dm} = -\frac{R_D \| 2R_P \| 2g_m R_D R_P}{R_P}$$

4.10

$\lambda = 0$, so for a differential input, symmetry in the input is enough to for the tail node to be ground. in other words:

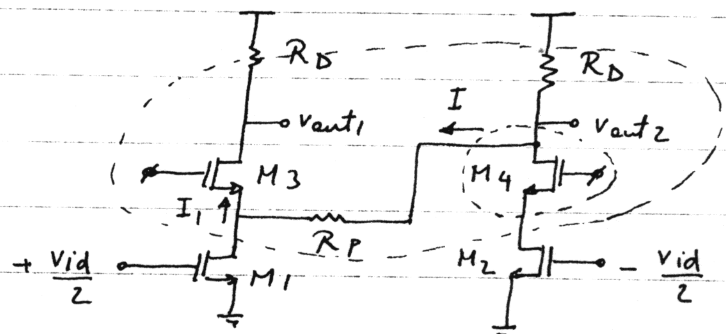


$$g_{m_1} \frac{V_{id}}{2} - g_{m_2} \frac{V_{id}}{2} = R_{SS} V_x$$

$$g_{m1} = g_{m2} \Rightarrow V_X = 0$$

($V_{GS1} = V_{GS2} \Rightarrow g_{m1} = g_{m2}$

but $g_{m3} \neq g_{m4}$)



For the cutsets shown:

$$\frac{V_{out1}}{R_D} + \frac{V_{out2}}{R_D} + \frac{V_{id}}{2} g_{m_1} - \frac{V_{id}}{2} g_{m_2} = 0 \quad \Rightarrow \quad V_{out1} + V_{out2} = 0 \quad (1)$$

KCL: $$\frac{V_{out2}}{R_D} + I - \frac{V_{id}}{2} g_{m_2} = 0 \quad \overset{g_{m_1} = g_{m_2}}{\Longrightarrow} \quad I = \frac{V_{id}\, g_{m_1}}{2} - \frac{V_{out2}}{R_D} \quad (2)$$

KVL: $$V_{out2} = I R_P + \frac{I_1}{g_{m3}} \quad , \quad \text{but} \quad I_1 = \frac{V_{out1}}{R_D}$$

$$\Rightarrow \quad V_{out2} = I R_P + \frac{V_{out1}}{R_D\, g_{m3}} \quad (3)$$

(2), (3) $$\Rightarrow \quad V_{out2} = \left(\frac{V_{id}\, g_{m1}}{2} - \frac{V_{out2}}{R_D} \right) R_P + \frac{V_{out1}}{R_D\, g_{m3}}$$

$$V_{out1} = -V_{out2} \quad \Rightarrow \quad -V_{out1} \left(1 + \frac{R_P}{R_D} + \frac{1}{R_D\, g_{m3}} \right) = \frac{g_{m_1} R_P}{2} V_{id}$$

$$A_{vd} = \frac{V_{out1} - V_{out2}}{V_{id}} = 2\,\frac{V_{out1}}{V_{id}} = -\frac{g_{m1} R_P}{1 + \frac{R_P}{R_D} + \frac{1}{R_D g_{m3}}}$$

$$= -\frac{g_{m1} R_D}{1 + \frac{R_D}{R_P} + \frac{1}{R_P g_{m3}}} = -\frac{g_{m1} R_D}{1 + \frac{1}{R_P}\left(R_D + \frac{1}{g_{m3}}\right)}$$

Since $\lambda = 0 \Rightarrow r_{o5} = \infty \Rightarrow A_{cm} = 0 \Rightarrow CMRR = \infty$

4.11

$$I_D = \frac{1}{2}\mu_p C_{ox}\frac{W}{L}\left[2(V_{GS}-V_{TH,P})V_{DS} - V_{DS}^2\right] \cong \mu_p C_{ox}\frac{W}{L}(V_{GS}-V_{TH,P})V_{DS}$$

$$R_{on} = \frac{|V_{DS}|}{I_D} = \frac{1}{\mu_p C_{ox}|V_{GS_3}-V_{TH,P}|}$$

$$\text{if } R_{on} = 2^{K\Omega} \Rightarrow |V_{GS_3} - V_{TH,P}| = \frac{1}{2^K \times 38.3^\mu \times 100} = 0.131^V$$

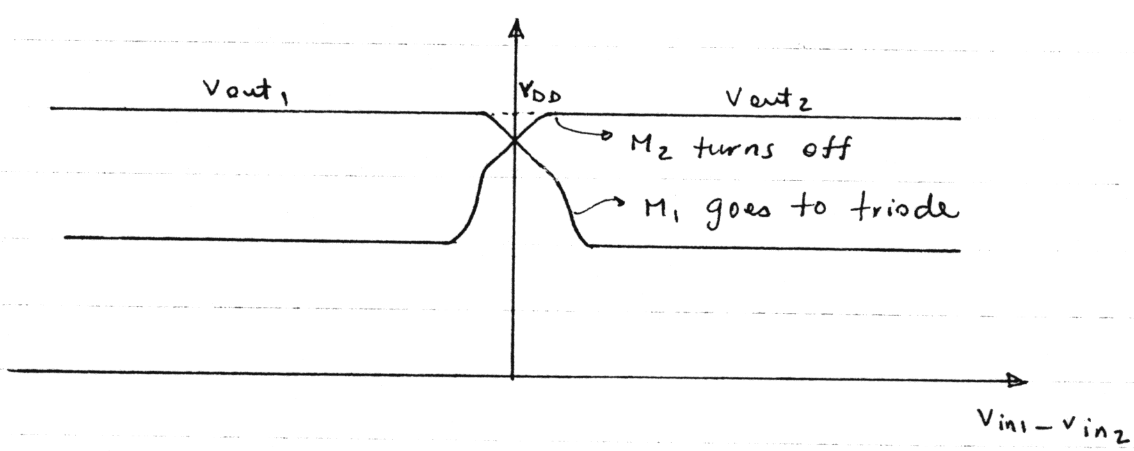
$$I_{SS} = 20^{\mu A} \Rightarrow V_{GS1} = V_{TH,N} + \sqrt{\frac{2 I_D}{\mu_n C_{ox}\frac{W}{L}}} = 0.7 + \sqrt{\frac{2\times 10^\mu}{0.134^m \times 100}} = 0.739^V$$

$$V_{DD} = |V_{GS3}| - V_{GS1} + V_{in,cm}$$

$$\Rightarrow V_{in,cm} = 3 - (0.131 + 0.8) + 0.739 = 2.81^V$$

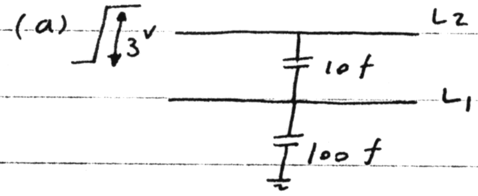
$|V_{DS3}| = R\frac{I_{SS}}{2} = 2^K \times 10^\mu = 20^{mV} \Rightarrow |V_{DS3}| < |V_{GS3} - V_{TH,P}| \Rightarrow M_3$ & M_4 are in triode

$V_{D1} - V_{G1} = (3 - 20^m) - 2.81 = 0.17^V > -V_{TH,n} \Rightarrow M_1$ & M_2 are in Sat.



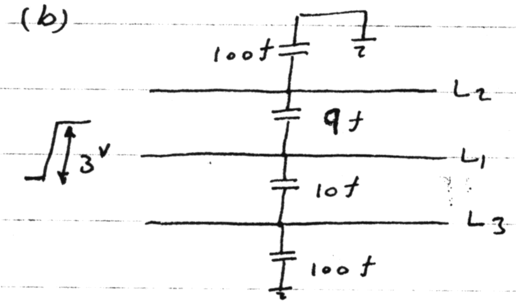
4.12

(a)



$$\Delta V_{L_1} = \frac{10}{10+100} 3^V = \frac{3}{11} = 0.273^V$$

(b)



$$\Delta V_2 = \frac{9}{9+100} \times 3^V = 0.248^V$$

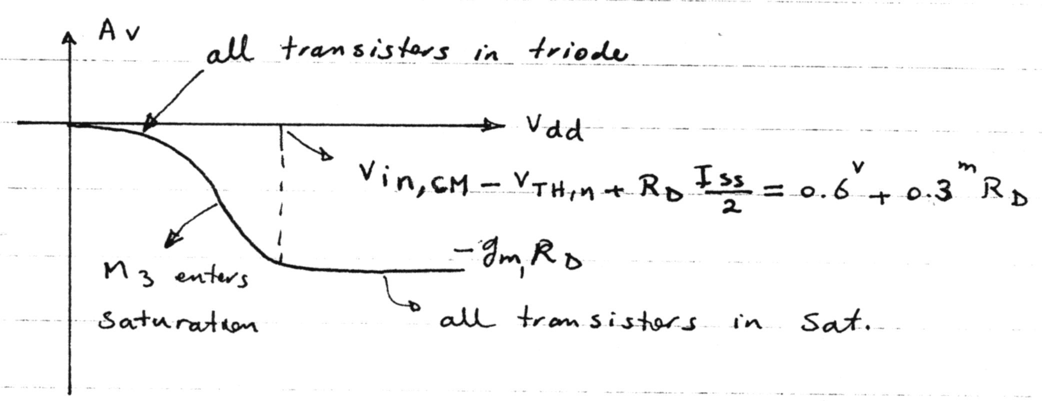
$$\Delta V_3 = \frac{10}{10+100} \times 3^V = 0.273^V$$

$$\Rightarrow \Delta V_{23} = 0.273 - 0.248 = 25^{mV}$$

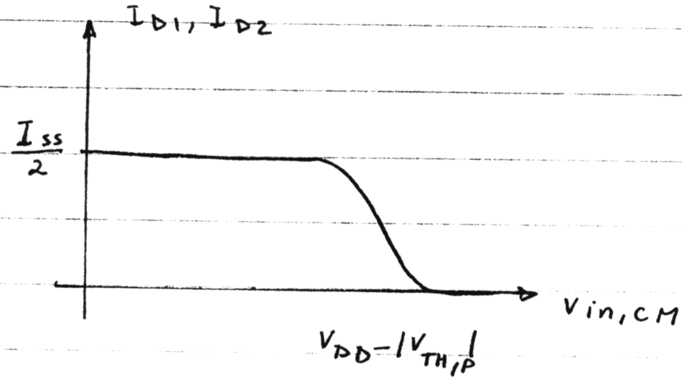
4.13

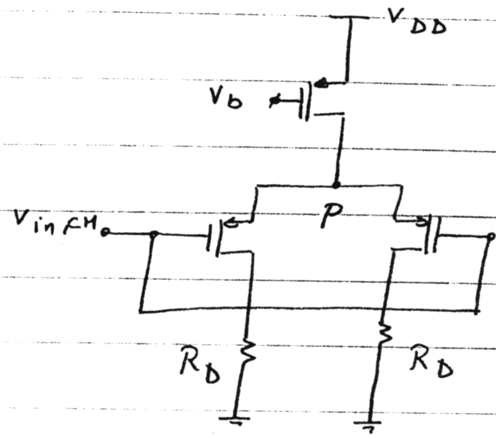
Fig. 4.8 (a) $\quad I_{SS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_3 (V_{GS3} - V_{TH,n})^2$

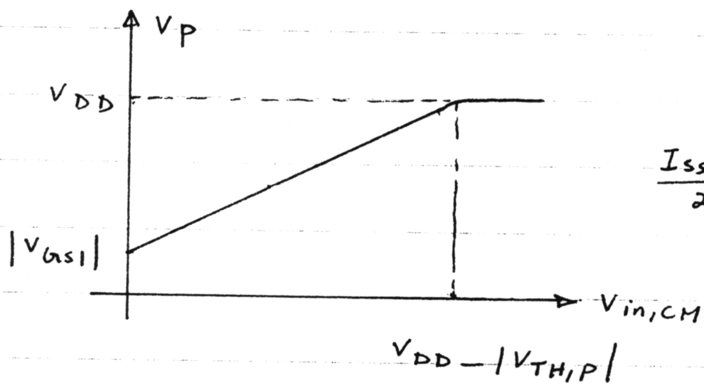
$$V_{GS3} = V_b = 1^V \Rightarrow I_{SS} = 0.5 \times 0.134^m \times 100(1-0.7)^2 = 0.603^{mA}$$

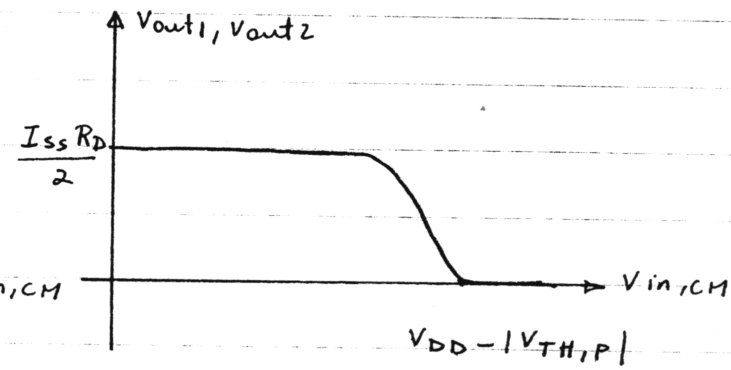


4.14









$$I_{D1,2} = \frac{V_{out1,2}}{R_D}$$

4.15 (a) $(V_{out\,1,2})_{max} = V_{DD} = 3^V$

$$(V_{out\,1,2})_{min} = V_{in,CM} - V_{TH,N} = 1.2 - 0.7 = 0.5^V$$

Max swing at $V_{out} = 2(3 - 0.5) = 5^V$

(b) $A_v = -g_m R_D$ $\quad g_m = \sqrt{2 \mu_n C_{ox} (\frac{W}{L}) I_D} = \sqrt{2 \times 0.134^m \times 100 \times \frac{0.5^m}{2}} = 2.59^m$

to get max swing: $R_D \frac{I_{ss}}{2} = \frac{(V_{out1,2})_{max} - (V_{out1,2})_{min}}{2} = 1.25$

$\Rightarrow R_D = 5^{K\Omega}$ $\quad \Rightarrow A_v = -2.59^m \times 5^K = -13$

4.16

(a) $V_{GS} - V_{TH} = \sqrt{\dfrac{2 I_D}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\dfrac{2 \times 0.5^m}{0.134^m \times 100}} = 0.273^V$

(b) $\Delta I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\dfrac{4 I_{ss}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$ (4.9)

$$\Delta I_D = \frac{1}{2} \times 0.134^m \times \frac{50}{0.5} \times 50^m \sqrt{\frac{4 \times 1^m}{0.134^m \times \frac{50}{0.5}} - (50^m)^2}$$

$$\Delta I_D = 182 \ \mu A \Rightarrow \begin{cases} I_{D1} = 0.5^m + \dfrac{0.182^m}{2} = 0.591^m \\ I_{D2} = 0.5^m - \dfrac{0.182^m}{2} = 0.409^m \end{cases}$$

(c)

$$G_m = \frac{\partial \Delta I_D}{\partial \Delta V_{in}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{\dfrac{4 I_{ss}}{\mu_n C_{ox} \frac{W}{L}} - 2 \Delta V_{in}^2}{\sqrt{\dfrac{4 I_{ss}}{\mu_n C_{ox} \frac{W}{L}} - \Delta V_{in}^2}} \quad (4.10)$$

$\Delta V_{in} = 50^{mV} \Rightarrow G_m = 3.61 \ m\Omega^{-1}$

(d) $\Delta V_{in} = 0 \Rightarrow G_{m_0} \doteq 3.66 \ m\Omega^{-1}$.

$$G_m = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{\dfrac{4 I_{ss}}{\mu_n C_{ox} W/L} - 2 \Delta V_{in}^2}{\sqrt{\dfrac{4 I_{ss}}{\mu_n C_{ox} \frac{W}{L}} - \Delta V_{in}^2}}$$

if we define $A = \dfrac{4 I_{ss}}{\mu_n C_{ox} \frac{W}{L}}$, $B = \left(\dfrac{G_m}{\frac{1}{2} \mu_n C_{ox} \frac{W}{L}} \right)^2$

$$\Rightarrow B = \frac{(A - 2 \Delta V_{in}^2)^2}{A - \Delta V_{in}^2} \qquad 4 \Delta V_{in}^4 - (4A - B) \Delta V_{in}^2 + A^2 - AB = 0$$

$$\Delta V_{in}^2 = \frac{4A - B \pm \sqrt{(4A-B)^2 - 16(A^2 - AB)}}{8}$$

taking the smaller value: $\Delta V_{in}^2 = \dfrac{4A - B - \sqrt{8AB + B^2}}{8}$

So for different value of G_m, B can be calculated and then ΔV_{in} is found.

$G_{m_{10\%}} = 0.9 \times 3.66^m = 3.294 \; m\Omega^{-1} \Rightarrow |\Delta V_{in}| = 139^{mV}$ 10% drop

$G_{m_{90\%}} = 0.1 \times 3.66^m = 0.366 \; m\Omega^{-1} \Rightarrow |\Delta V_{in}| = 372^{mV}$ 90% drop

4.17

(a) $V_{od} = V_{GS} - V_{TH} = 0.386^V$

(b) $\Delta I_D = 0.129^{mA} \Rightarrow I_{D1} = 0.565^{mA}$

$I_{D2} = 0.435^{mA}$

(c) $G_m = 2.57 \; m\Omega^{-1}$

(d) $\Delta V_{in} = 0 \Rightarrow G_{m_0} = 2.59 \; m\Omega^{-1}$

$G_{m_{10\%}} = 0.9 \times G_{m_0} = 0.9 \times 2.59^m \Rightarrow \Delta V_{in} = 197^{mV}$

$G_{m_{90\%}} = 0.1 \times G_{m_0} = 0.1 \times 2.59^m \Rightarrow \Delta V_{in} = 526^{mV}$

For a given current, by reducing $\frac{W}{L}$, overdrive voltage increases while G_m decreases. In this case for a fixed ΔV_{in}, I_D and G_m change less so the circuit has a wider linear range.

4.18

(a) $V_{od} = 0.386^{\,V}$

(b) $\Delta I_D = 0.258^{\,mA}$ $\begin{cases} I_{D1} = 1.13^{\,mA} \\ I_{D2} = 0.87^{\,mA} \end{cases}$

(c) $G_m = 5.14 \; m\Omega^{-1}$

(d) $\Delta V_{in} = 0 \rightarrow G_{m_0} = 5.18 \; m\Omega^{-1}$

$G_{m_{10\%}} = 0.9 \times 5.18^{\,m} \Rightarrow \Delta V_{in} = 197^{\,mV}$

$G_{m_{90\%}} = 0.1 \times 5.18^{\,m} \Rightarrow \Delta V_{in} = 526^{\,mV}$

In this case V_{od} and G_m have increased but the linearity range of G_m is same as (p. 4.17)

4.19 $I_{D1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH,n})^2$

$I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{2W}{L} (V_{in2} - V_{TH,n})^2$

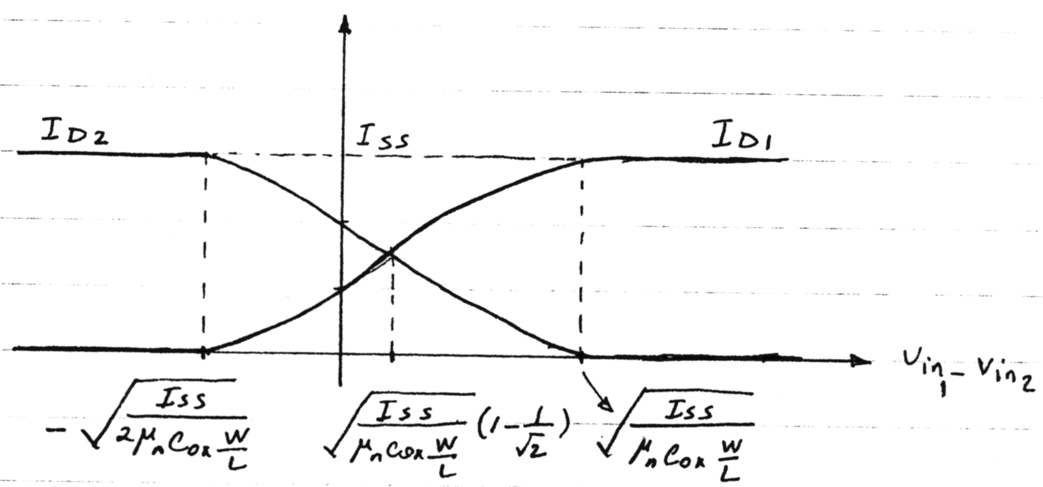
$I_{D1} + I_{D2} = I_{SS}$

if $I_{D1} = I_{D2}$ then:

$$I_{D1} = \frac{I_{SS}}{2} \Rightarrow V_{in1} = V_{TH,n} + \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

$$I_{D2} = \frac{I_{SS}}{2} \Rightarrow V_{in2} = V_{TH,n} + \sqrt{\frac{I_{SS}}{2\mu_n C_{ox} \frac{W}{L}}}$$

$$V_{in1} - V_{in2} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \left(1 - \frac{1}{\sqrt{2}}\right)$$



if $V_{in1} = V_{in2} \Rightarrow I_{D1} = \frac{I_{SS}}{3}$, $I_{D2} = \frac{2I_{SS}}{3}$

4.20

$$A_{dm-dm} = -g_m R_D$$

$$A_{cm-dm} = \frac{g_m}{1 + 2g_m R_{SS}} R_D - \frac{g_m}{1 + 2g_m R_{SS}} (R_D + \Delta R_D)$$

$$A_{cm-dm} = -\frac{g_m}{1+2g_m R_{SS}} \Delta R_D$$

$$SNR = \frac{(A_{dm-dm} \cdot V_{in,dM})^2}{(A_{cm-dm} \cdot V_{in,cm})^2} = \left(\frac{g_m R_D \times 10^m}{\frac{g_m}{1+2g_m R_{SS}} \Delta R_D \times 100^m}\right)^2$$

$$SNR = \frac{(1+2g_m R_{SS})^2}{\left(\frac{\Delta R}{R}\right)^2} \times \left(\frac{1}{10}\right)^2$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 3.66^{m\Omega^{-1}} \quad (I_D = 0.5^m)$$

$$L_{SS} = 0.5\ \mu m \Rightarrow \lambda = 0.1\ V^{-1} \Rightarrow R_{SS} = \frac{1}{\lambda I_D} = \frac{1}{0.1 \times 1^m} = 10^{K\Omega}$$

$$\Rightarrow SNR = \left(\frac{1+2\times 3.66^m \times 10^K}{0.05}\right)^2 \left(\frac{1}{10}\right)^2 = 22000$$

or $SNR = 10 \log 22000 = 43.4\ dB$

$$CMRR = \left|\frac{A_{dm-dm}}{A_{cm-dm}}\right| = \frac{g_m R_D}{\frac{g_m \Delta R_D}{1+2g_m R_{SS}}} = \frac{1+2g_m R_{SS}}{\Delta R_D / R_D}$$

$CMRR = 1484$ or $CMRR = 20 \log 1484 = 63.4\ dB$

4.21 $$A_{cm-dm} = -\frac{\Delta g_m R_D}{(g_{m1}+g_{m2})R_{SS}+1} \qquad g_{m1}+g_{m2} = 2g_m$$

$$SNR = \left(\frac{A_{dm-dm} \times V_{in-dm}}{A_{cm-dm} \times V_{in-cm}}\right)^2 = \left(\frac{g_m R_D}{\frac{\Delta g_m R_D}{(g_{m1}+g_{m2})R_{SS}+1}}\right)^2 \left(\frac{10^m}{100^m}\right)^2$$

$$SNR = \left(\frac{2 g_m R_{SS} + 1}{\frac{\Delta g_m}{g_m}} \right)^2 \times \left(\frac{1}{10}\right)^2$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \Rightarrow \Delta g_m = -\mu_n C_{ox} \frac{W}{L} \Delta V_{TH}$$

$$|\Delta g_m| = 0.134^m \times 100 \times 1^{mV} = 13.4\ \mu\Omega^{-1}$$

$$\Rightarrow SNR = \frac{(2 \times 3.66^m \times 10^K + 1)^2}{\left(\frac{13.4^\mu}{3.66^m}\right)^2} \times \left(\frac{1}{10}\right)^2 = 4.1 \times 10^6$$

or $SNR = 10 \log 4.1 \times 10^6 = 66.1\ dB$

$$CMRR = \left| \frac{A_{dm-dm}}{A_{cm-dm}} \right| = \frac{1 + 2 g_m R_{SS}}{\Delta g_m / g_m} = 20300$$

or $CMRR = 20 \log 20300 = 86.1\ dB$

4.22 (a)

$$\left(\frac{W}{L}\right)_{SS} = \frac{50}{0.5},\ I_{SS} = 0.5^{mA}$$

$$V_{od_{SS}} = V_{GS3} - V_{TH} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_{SS}}} = 0.273^V$$

$$I_{D1} = \frac{I_{SS}}{2} = 0.25^{mA} \Rightarrow V_{od_1} = \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} = 0.193^V$$

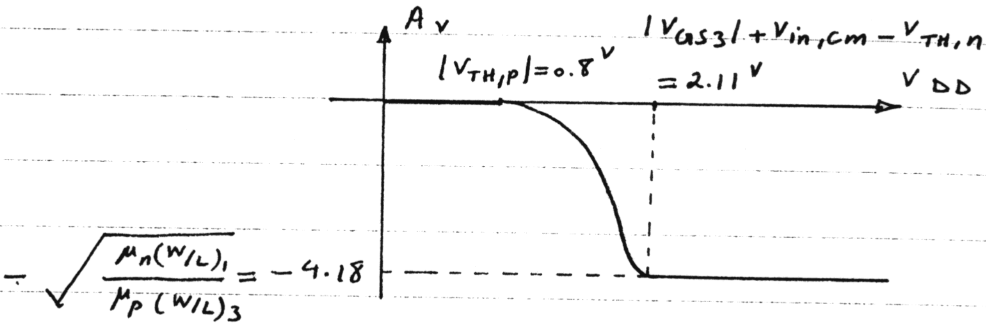
$$(V_{in,cm})_{min} = V_{GS1} + V_{od_{SS}} = 0.7 + 0.193 + 0.273 = 1.17^V$$

$$(V_{in,cm})_{max} = V_{DD} - |V_{GS3}| + V_{TH,N}$$

$$|V_{GS3}| = |V_{TH,p}| + \sqrt{\frac{2 I_{D3}}{\mu_p C_{ox}(\frac{W}{L})_3}} = 1.61^V$$

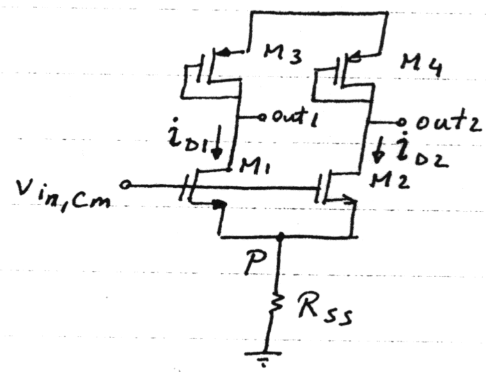
$$(V_{in,cm})_{max} = 3 - 1.61 + 0.7 = 2.09^V$$

(b) $V_{in,cm} = 1.2^V$



4.23

This mismatch in V_{TH} of M_1 and M_2 makes I_{D1} and I_D unequal. Therefore $g_{m1} \neq g_{m2}$, $g_{m3} \neq g_{m4}$. Using the equivalant circuit below to calculate A_{cm-dm}, we have:



$$i_{D1} = g_{m1}(V_{in,cm} - V_P)$$

$$i_{D2} = g_{m2}(V_{in,cm} - V_P)$$

$$V_{out_1} = -\frac{i_{D1}}{g_{m3}} = -\frac{g_{m1}}{g_{m3}}(V_{in,cm} - V_P)$$

$$V_{out_2} = -\frac{i_{D2}}{g_{m4}} = -\frac{g_{m2}}{g_{m4}}(V_{in,cm} - V_P)$$

$$\frac{g_{m1}}{g_{m3}} = \frac{\sqrt{2 I_{D1}\mu_n C_{ox}(\frac{W}{L})_{1,2}}}{\sqrt{2 I_{D3}\mu_p C_{ox}(\frac{W}{L})_{3,4}}} = \sqrt{\frac{\mu_n(\frac{W}{L})_{1,2}}{\mu_p(\frac{W}{L})_{3,4}}}, \quad \text{Similarly} \quad \frac{g_{m2}}{g_{m4}} = \sqrt{\frac{\mu_n(\frac{W}{L})_{1,2}}{\mu_p(\frac{W}{L})_{3,4}}}$$

$$\Rightarrow V_{out_1} = V_{out_2} \Rightarrow A_{cm-dm} = 0, \quad CMRR = \infty$$

4.24

P 4.20 $\quad CMRR = \dfrac{1 + 2g_{m_1}R_{SS}}{\Delta R_D / R_D}$

$$R_{D1} = \frac{1}{g_{m3}}, \quad R_{D2} = \frac{1}{g_{m4}}$$

$$\frac{\Delta R_D}{R_D} = \frac{R_{D1} - R_{D2}}{R_{D_1}} = 1 - \frac{R_{D2}}{R_{D1}} = 1 - \frac{g_{m3}}{g_{m4}} = 1 - \frac{\sqrt{2\mu_p C_{ox} (\frac{W}{L})_3 I_D}}{\sqrt{2\mu_p C_{ox} (\frac{W}{L})_4 I_D}}$$

$$\frac{\Delta R_D}{R_D} = 1 - \sqrt{\frac{10}{11}} = 0.0465$$

$\Rightarrow$ $CMRR = 2248$ or $CMRR = 20 \log 2247 = 67$ dB

4.25

(a) $\quad A_V = -g_{m_1} (r_{o1} \| r_{o3})$

$$g_{m_1} = 3.66^{\,m\Omega^{-1}} \qquad r_{o1} = \frac{1}{\lambda I_D} = \frac{1}{0.1 \times 0.5^m} = 20^{K\Omega}$$

$$r_{o3} = \frac{1}{\lambda I_D} = \frac{1}{0.2 \times 0.5^m} = 10^{K\Omega} \Rightarrow A_V = -3.66^m (10^k \| 20^k) = -24.4$$

(b) $\quad (V_{out1,2})_{min} = 1.5 - V_{TH,n} = 1.5 - 0.7 = 0.8^V$

$$(V_{out1,2})_{max} = V_{DD} - (|V_{GS3}| - |V_{TH,P}|)$$

$$= V_{DD} - \sqrt{\frac{2 I_D}{\mu_p C_{ox} (\frac{W}{L})_3}} = 3 - \sqrt{\frac{2 \times 0.5^m}{38.3^{\mu} \times 100}} = 2.49^V$$

Max swing of $V_{out} = 2(2.49 - 0.8) = 3.38^V$

4.26

P 4.20 : $CMRR = \dfrac{1 + 2g_{m1} R_{SS}}{\Delta R_D / R_D}$

$$R_D = \frac{1}{g_{m3}} \| r_{o1} \| r_{o3} \| r_{o5} \approx \frac{1}{g_{m3}}$$

$$I_{D3} + I_{D5} = I_{D4} + I_{D6} = \frac{I_{SS}}{2} \Rightarrow \Delta I_{D3} = -\Delta I_{D5} \qquad (1)$$

$$I_{D5} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{GS5} - V_{TH,P})^2 \Rightarrow \frac{\partial I_{D5}}{\partial V_{TH,P}} = -\mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{GS5} - V_{TH,P})$$

$$\Delta I_{D5} \approx \mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{DD} - V_b - |V_{TH,P}|) \Delta V_{TH,P} \qquad (2)$$

$$g_{m3} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right)_3 I_{D3}} \qquad \frac{\partial g_{m3}}{\partial I_{D3}} = \sqrt{\frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}{2 I_{D3}}}$$

$$\Rightarrow \Delta g_{m3} \approx \sqrt{\frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}{2 I_{D3}}} \Delta I_{D3} \qquad (3)$$

$$R_D = \frac{1}{g_{m3}} \qquad \frac{\partial R_D}{\partial g_{m3}} = -\frac{1}{g_{m3}^2} \Rightarrow \frac{\Delta R_D}{R_D} \approx -\frac{\Delta g_{m3}}{g_{m3}}$$

$$(1),(2),(3) \Rightarrow \frac{\Delta R_D}{R_D} = -\frac{\Delta g_{m3}}{g_{m3}} = -\sqrt{\frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}{2 I_{D3}}} \times \frac{\Delta I_{D3}}{\sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right)_3 I_{D3}}} = -\frac{\Delta I_{D3}}{2 I_{D3}}$$

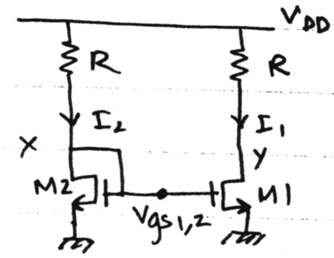
$$= \frac{\Delta I_{D5}}{2 I_{D3}} = \frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{DD} - V_b - |V_{TH,P}|) \Delta V_{TH,P}}{2 I_{D3}} = \frac{I_{D5}}{I_{D3}} \cdot \frac{\Delta V_{TH,P}}{V_{DD} - V_b - |V_{TH,P}|}$$

$$\text{or} \quad \frac{\Delta R}{R_D} = \frac{I_{D5}}{I_{D3}} \cdot \frac{V_{TH,P}}{V_{DD} - V_b - |V_{TH,P}|} \times \frac{\Delta V_{TH,P}}{V_{TH,P}}$$

$$\text{but} \quad \frac{I_{D5}}{I_{D3}} = 4$$

$$\Rightarrow \frac{\Delta R_D}{R_D} = 4 \cdot \frac{V_{TH,P}}{V_{DD} - V_b - |V_{TH,P}|} \cdot \frac{\Delta V_{TH,P}}{V_{TH,P}}$$

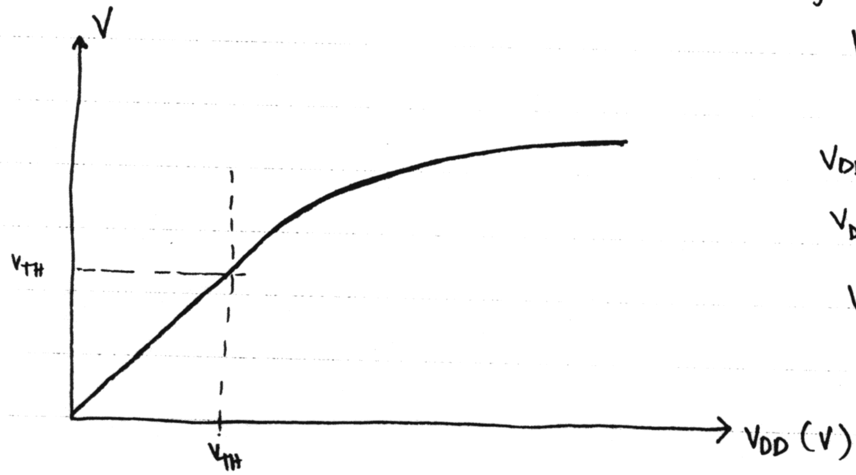
5.1 (a) M1 and M2 are off when $V_{DD} < V_{TH1,2}$.
At this time, $V_{x,y} = V_{DD}$ because $I_{1,2} = 0$.
Once $V_{DD} \geq V_{TH1,2}$, M1 and M2 turn on.
Since M1 and M2 are symmetric and the
V_{gs} for both are the same $I_1 = I_2$ and
$V_x = V_y = V_{gs2,1}$. V_{gs} is a quadratic solution as a function off V_{DD}
and follows the below relationship.



$$V_{gs2} = V_{DD} - I_2 R$$

$$= V_{DD} - \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs2} - V_{th})^2 \cdot R$$

$$K = \frac{1}{2}\mu C_{ox}\frac{W}{L}$$



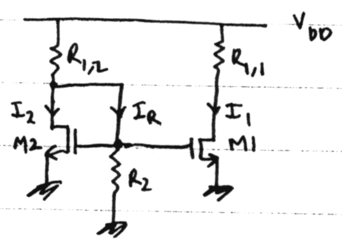
$V_{DD} < V_{TH}$: $V_x = V_y = V_{DD}$.

$V_{DD} \geq V_{TH}$:

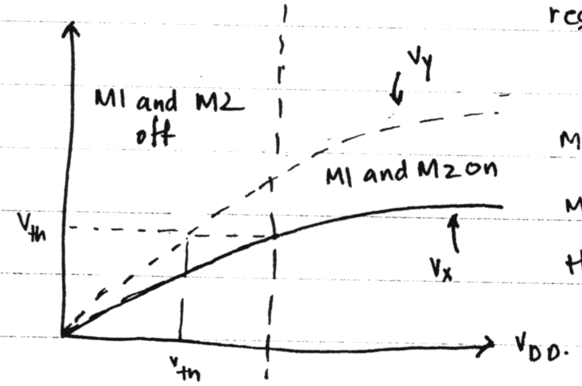
$$V_{gs} = \frac{2V_{th} - \frac{1}{KR} + \sqrt{\left(2V_{th} - \frac{1}{KR}\right)^2 - 4\left(V_{th}^2 - \frac{V_{DD}}{KR}\right)}}{2}$$

(b) We have the same solution as 5.1(a) because if
M1, M2 are symmetric and $V_{gs1} = V_{gs2}$ then $V_x = V_y$
and $I_1 = I_2$, In this case, no current ever flows
through R_2.

5.1 (c)



When $V_{DD} < V_{Th1,2}$ no current flows through M1 and M2. the only current that flows is I_R through $R_{1,2}$ and R_2. V_X is set by the resistor divider $V_X = V_{DD} \cdot \frac{R_2}{R_{1,2}+R_2}$ and $V_Y = V_{DD}$

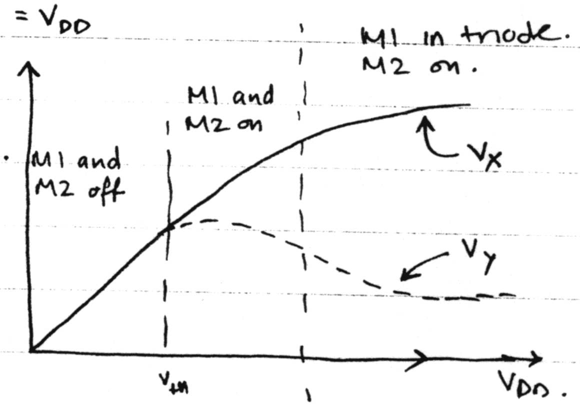


M1 and M2 turn on only when $V_X \geq V_{th1,2}$. Once M1 and M2 are on, I_1 and I_2 are equal because they have the same V_{gs}. But $V_Y > V_X$ because the current through $R_{1,2}$ consist of I_2 plus I_R which is greater than the current through $R_{1,1}$.

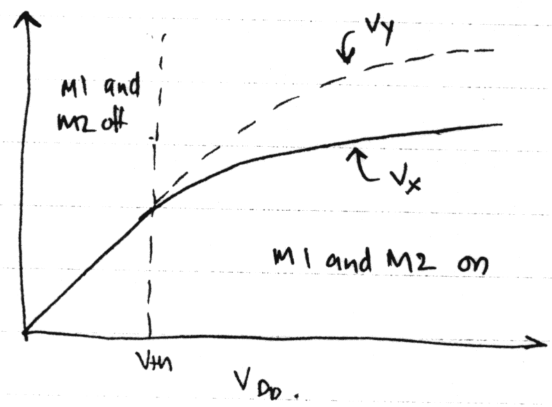
(d) When $V_{DD} < V_{th1,2}$, $I_1 = I_2 = 0$ and $V_X = V_Y = V_{DD}$

Once $V_{DD} \geq V_{th1,2}$, M1 and M2 turn on.

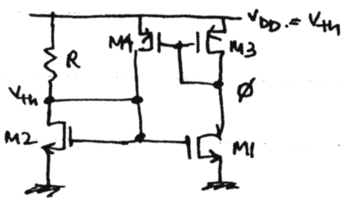
the R_2 at the source of M2 causes $V_{gs2} < V_{gs1}$. Thus $I_2 < I_1$ and $V_X > V_Y$. At some point, V_{gs1} becomes so large that M1 goes into triode as seen in the graph.

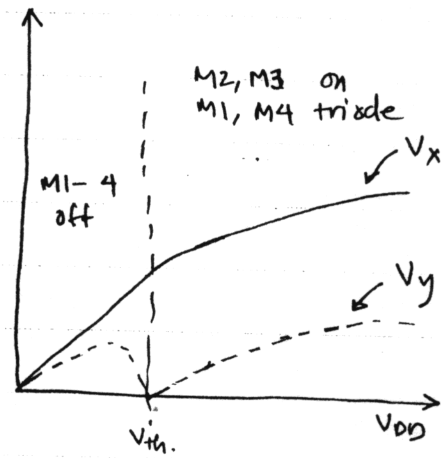


(e) Here when $V_{DD} > V_{th1,2}$, $V_{gs2} > V_{gs1}$ and $I_1 < I_2$. Thus $V_X < V_Y$, but because M2 is diode connected, M2 never goes into triode.



5.2 (a) When $V_{DD} < V_{th1,2}$, all transistors are off. $V_X = V_{DD}$ and V_Y is floating between GND and V_{DD} since it is isolated from either node.





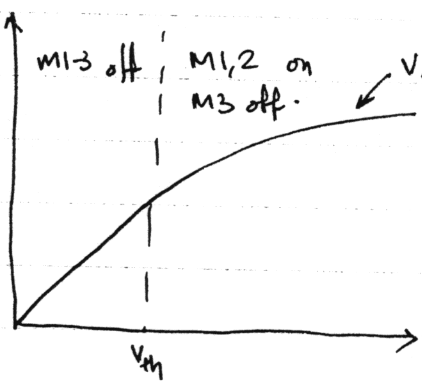
Once $V_{DD} = V_{th1,2}$ all transistors turn on and the voltages at nodes are as follows: $V_X = V_{DD} = V_{th2}$ and $V_Y = V_{DD} - V_{th3} \cong \varnothing$ if we assume $V_{th2} = V_{th3}$. M1 and M4 are in triode and stay in triode always as $V_{DD} > V_{th2}$.

As $V_{DD} > V_{th2}$, $V_X = V_{gs2}$ and $V_Y = V_{DD} - V_{gs3} \approx V_{DD} - V_{gs2}$

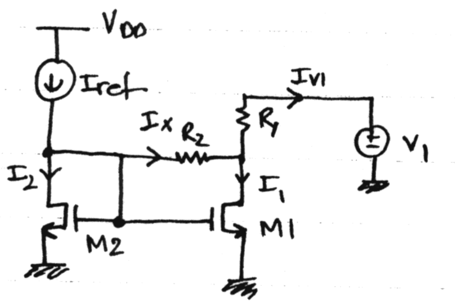
$V_X > V_Y$.

(b) when $V_{DD} < V_{th1,2}$ $I_1, I_2 = \varnothing$ and $V_X = V_Y = V_{DD}$

when $V_{DD} \geq V_{th1,2}$ $I_1 = I_2$ because $V_{gs1} = V_{gs2}$ and therefore $V_X = V_Y$. Since $V_X \cong V_Y$, $V_{gs3} = 0$ and M3 is always off.



5.3 (a)



for V_1: $0 \le V_1 \le V_{DD}$ M1 and M2 are always on: $I_1 = I_2$

V_y and V_x are related as a function of R_1, R_2 and V_1.

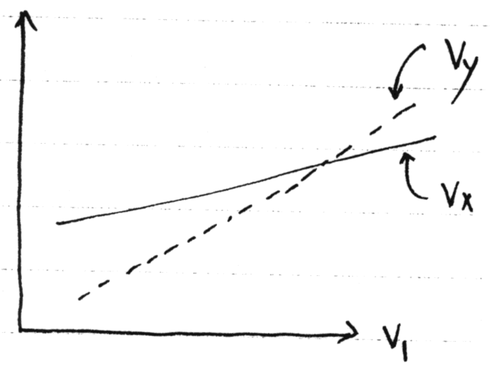
(1) $2I_1 = I_{ref} - I_{V1} = I_{ref} - \dfrac{V_y - V_1}{R_1}$

(2) $I_1 = \dfrac{V_x - V_y}{R_2} - \dfrac{V_y - V_1}{R_1}$

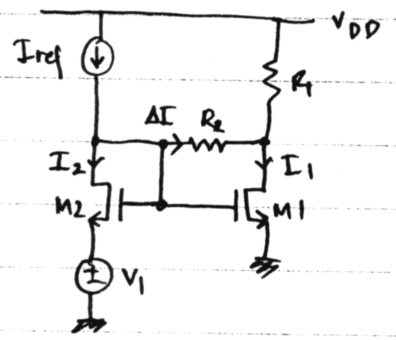
Solving these 2 equations for V_y, we get

$$V_y = \frac{R_2 V_1 + 2R_2 V_x - I_{Ref}}{R_1 + R_2}$$

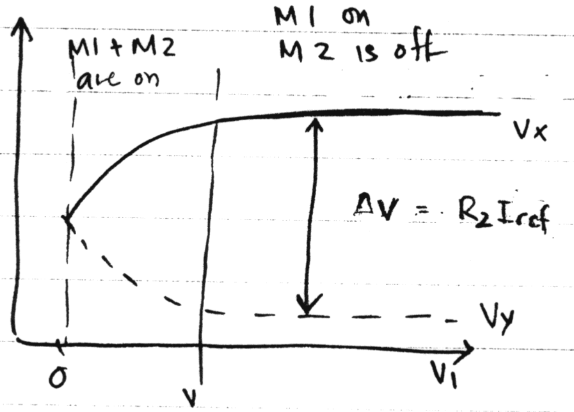
as V_1 increase, $I_{1,2}$ increase and therefore V_x and V_y increase. the slope of V_y is greater because it is a linear combination of $V_x + V_1$, but V_y starts off less than V_x because of the constant subtraction of $I_{ref}/R_1 + R_2$.

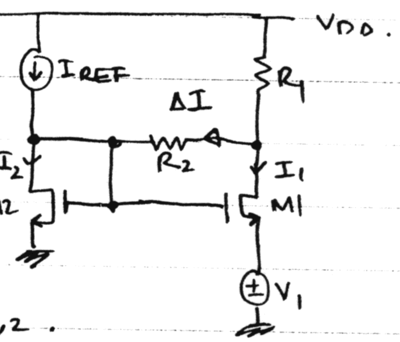


5.3 (b) When $V_1 = 0$, $I_1 = I_2 \doteq I_{ref}$ and $V_X \cong V_Y$, $\Delta I = 0$



as V_1 increases, I_2 gradually decreases and part of Iref flows throw R_2. V_X increases and $V_Y = V_X - R_2 \Delta I$, decreases. Finally when V_1 is large enough such that M2 turns off $V_Y = V_X - R_2 I_{ref}$ and both V_X and V_Y are set at a constant voltage



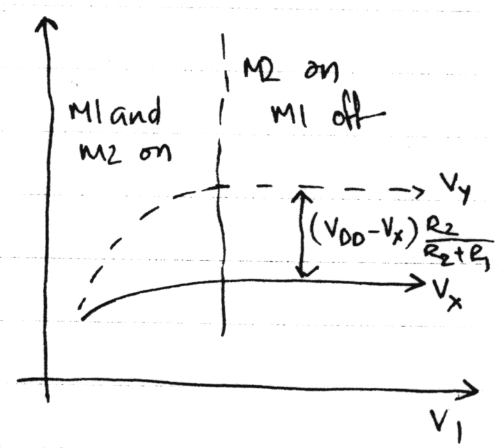


(c) When $V_1 = 0$, $I_1 = I_2$, $V_X \sim V_Y$. there maybe small variations if $V_{DD} - R_1 I_1 \neq V_{gs1,2}$.

as V_1 increases, I_1 decreases and the extra current flows through M2. I_2 increases.

$V_Y = V_X + \Delta I \, R_2$.

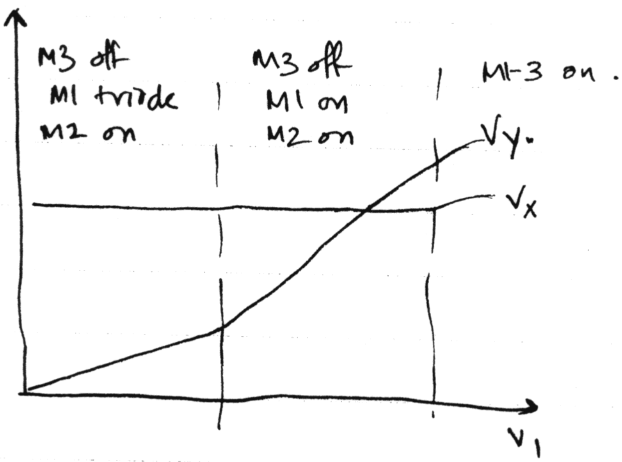
Once V_1 gets large enough, M1 shuts off. $V_Y = \dfrac{(V_{DD} - V_X) R_2}{R_1 + R_1} + V_X$



5.4 (a) V_X is constant until V_1 gets high enough that $V_Y - V_X$ is greater than V_{th3}.

Initially M1 is in triode with $V_Y = V_1 \frac{1}{1+g_{m1}R_2}$ until M1 is in saturation.

When M1 in sat., $V_Y = V_1 - I_{REF} R_1$

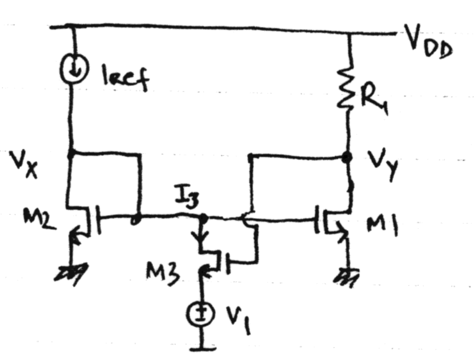


(b). When $V_1 = 0$, M1 and M2 are off and M3 is on, but in a flipped position, Source and drain switch as show below.

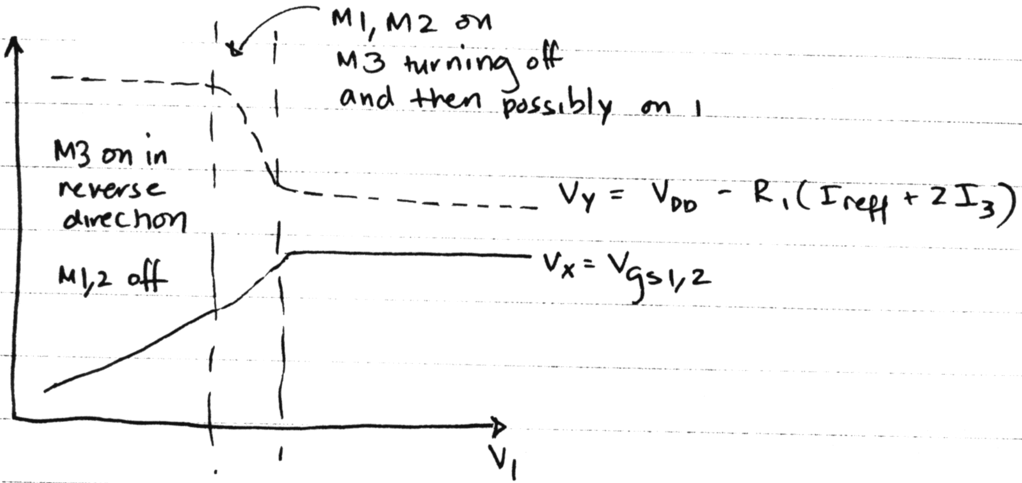
$V_Y = V_{DD}$ and $V_X = V_1 + V_{DS3}$ where $V_{DS3} = I_{REF}/g_{m3}$

As V_1 increase, V_X increases in the same amount until $V_X = V_{th1,2}$.

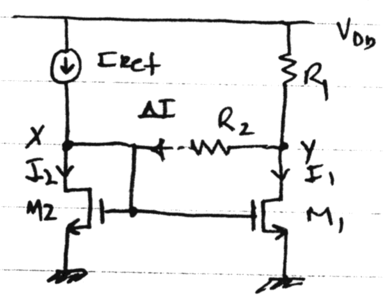
Now that M1 and M2 turn on, V_Y drops down due to I_1R_1 until M3 turns off. Once M3 is off, $V_X = V_{gs1,2}$ and $V_Y = V_{DD} - R_1 I_{Ref}$.



If at this point, $V_{th3} < V_Y - V_X$, M3 will turn on to increase the current through M2 and hence M1 so $V_Y = V_{DD} - R_1(I_{Ref} + 2I_3)$



5,5



When Iref = 0, current I_1 and I_2 are supplied by V_{DD} through R_1

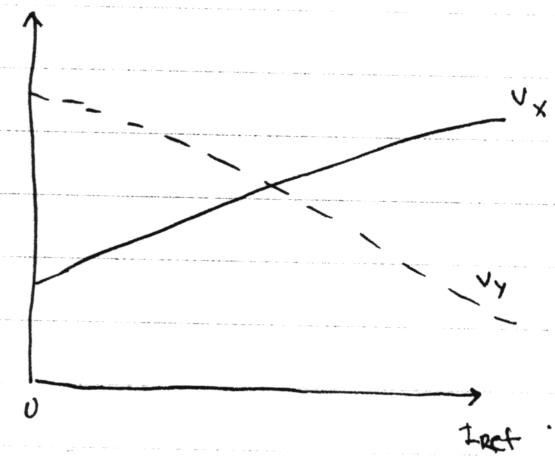
The initial points can be solved with

$$I_{1,2} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs1,2} - V_{th})^2 \quad (1)$$

$$V_{gs2,1} = V_{DD} - (2R_1 + R_2) I_{1,2} \quad (2)$$

where $V_x = V_{gs2,1}$

$$V_y = V_x + \frac{V_{DD} - V_x}{2R_1 + R_2} R_2 .$$



As Iref increases, I_2 increases and hence $V_{gs2} = V_{gs1} = V_x$ increases. Following KCL, we can find V_y as a function of V_x, Iref

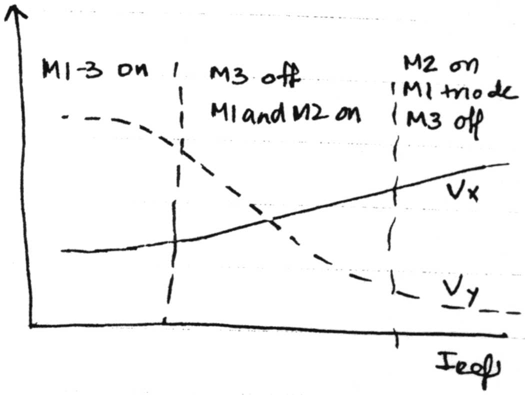
$$V_y = V_{DD}\frac{R_2}{2R_1 + R_2} - \frac{I_{ref} R_1 R_2}{2R_1 + R_2} + \frac{2R_1 V_x}{2R_1 + R_2}$$

5.5 (b) Initially when $I_{ref} = 0$, this ckt is on with the follow condition $I_1 = I_2 = I_3$ since all transistors are assumed equal and all on.

$$I_{1\text{-}3} = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs1\text{-}3} - V_{th})^2 \quad (1)$$

$$2V_{gs1\text{-}3} = V_{DD} - R_1 I_{1\text{-}3} \quad (2)$$

where $V_X = V_{gs1\text{-}3}$ and $V_Y = 2V_{gs1\text{-}3}$.

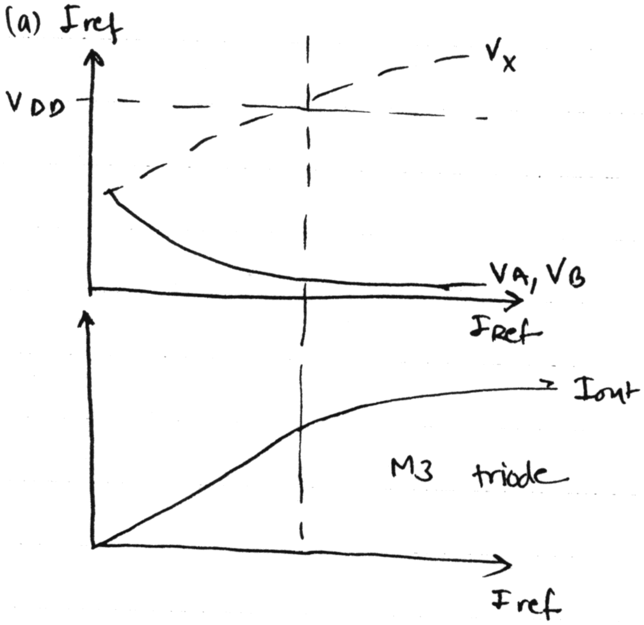


Once Iref increase, $V_{gs1,2}$ goes up and V_Y drops down, decreasing V_{gs3} until M3 turns off. Then M1 and M2 act as a typical current mirror for Iref. Then when Iref get so large, the $I_{REF}R_1$ drop increases to a point when M1 goes into triode and can't sustain the Iref current.

(c) At Iref = 0, all transistors are in saturation mode with $V_X = V_Y$. Once Iref turns on, M1 goes into triode and then M4 goes into triode also

$V_X \approx V_{gs2}$ and $V_Y = V_{DD} - V_{gs3} \approx V_{DD} - V_{gs2}$

5.6 (a)



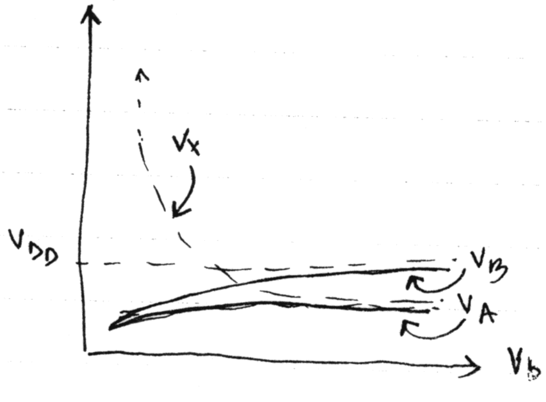
Iout follows Iref for all Iref until $V_X > V_{DD}$. Even if M1 and M2 go into triode, they still generate similar currents since V_A and V_B match.
As Iref increase, $V_{A,B}$ decrease since $V_{gs\,4,3}$ increases and V_X increases since $V_{gs1,2}$ also increase.

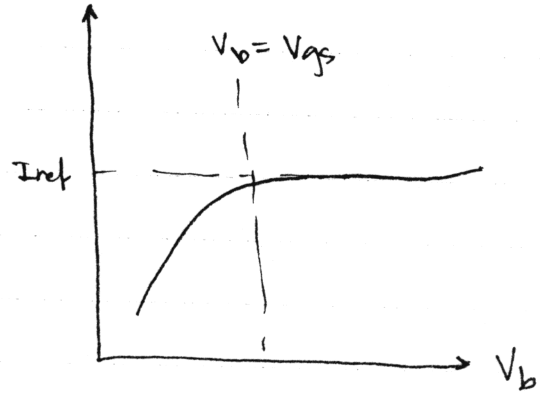
Once $V_X > V_{DD}$, M_3 goes into triod and reduces Iout w.r.t. Iref.

(b) If V_b is less than 1 V_{gs} for I_{REF} current, V_X goes up to a very large voltage to allow for I_{REF} to flow through M_2 and M_4 if we assume channel resistance.

$$I = \frac{1}{2}\mu C_{ox}\frac{W}{L}(V_{gs} - V_{th})(1 + \lambda V_{DS})$$

Once $V_b = 1V_{gs}$, V_A and V_B are ØV and Iout = Iref.
As V_b increases, V_A and V_B increase to turn M1 and M2 on. V_X is then equal to $1V_{gs}$. As V_B increase further, M3 and M4 go into triode while M1 and M2 are still in sat.





5.7 (a) $\gamma = 0$

$$K_o = \frac{1}{2}\mu C_{ox}$$

$$I_{REF} = K_o \frac{W_o}{L_o'} (V_{gs1} - V_{th})^2 (1 + \lambda V_{DS1})$$

$$I_{out} = K_o \frac{W_o}{L_o'} (V_{gs1} - V_{th})^2 (1 + \lambda V_{DS2})$$

where $V_{DS1} = V_{gs1}$ and $V_{DS2} = 2V_{gs1} - V_{gs4} - V_{gs3}$

if we assume $I_{out} \sim I_{REF}$, $V_{gs3} \approx V_{gs1}$,

so $V_{DS2} = V_{gs1} - V_{gs4}$

$$V_{gs1} = V_{th} + \sqrt{\frac{I_{REF}}{K_o} \frac{L_o'}{W_o}} \qquad V_{gs4} = V_{th} + \sqrt{\frac{I_1}{K_o} \frac{L_4'}{W_4}} \qquad L' = L - 2L_D$$

$$\frac{I_{out}}{I_{REF}} = \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}} = \frac{1 + \lambda \left(\sqrt{\frac{I_{REF} L_o'}{K_o W_o}} - \sqrt{\frac{I_1 L_4'}{K_o W_4}} \right)}{1 + \lambda \left(V_{th} + \sqrt{\frac{I_{REF}}{K_o} \frac{L_o'}{W_o}} \right)}$$

(b) $\gamma \neq 0$

$$V_{th} = V_{tho} + \gamma \left(\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right)$$

$\phi_f \simeq .45\,v$ (work func.)

$V_{SB} \equiv$ Source - substrate volt

Find V_{gs1}, V_{gso}, V_{gs4} and V_{gs3}

$$V_{gs1} = V_{tho} + \sqrt{\frac{I_{REF}}{K_o} \frac{L_o'}{W_o}}$$

$$V_{gso} = V_{tho} + \sqrt{\frac{I_{REF}}{K_o} \frac{L_o'}{W_o}} + \gamma \left(\sqrt{2\phi_f + V_{GS1}} - \sqrt{2\phi_f} \right)$$

$$V_{gs3} = V_{tho} + \sqrt{\frac{I_{out}}{K_o} \frac{L_o'}{W_o}} + \gamma \left(\sqrt{2\phi_f + V_{DS2}} - \sqrt{2\phi_f} \right)$$

if we assume $I_{out} \sim I_{REF}$ and $V_{DS2} \sim \sqrt{\frac{I_{REF}}{K_o} \frac{L_o}{W_o}} - \sqrt{\frac{I_1}{K_o} \frac{L_4}{W_4}}$

we can estimate V_{gs3}

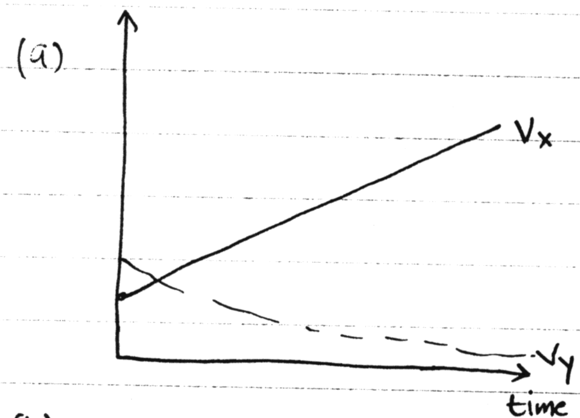
$$V_{gs3} \cong V_{tho} + \sqrt{\frac{I_{REF}}{K_o} \frac{L_o'}{W_o}} + \gamma \left(\sqrt{2\phi_f + \sqrt{\frac{I_{REF} L_o'}{K_o W_o}} - \sqrt{\frac{I_1 L_4'}{K_o W_4}}} - \sqrt{2\phi_f} \right)$$

5.7 $V_{gs4} = V_{tho} + \sqrt{\frac{I_1}{k_o}\frac{L_4'}{W_4}} + \gamma\left(\sqrt{2\phi_f + V_{gs3} + V_{DS2}} - \sqrt{2\phi_f}\right)$

Now we can plug every thing in to the final solution

$$\frac{I_{out}}{I_{ref}} \approx \frac{1 + \lambda(V_{gs1} + V_{gs0} - V_{gs4} - V_{gs3})}{1 + \lambda(V_{gs1})}$$

5.8 (a)

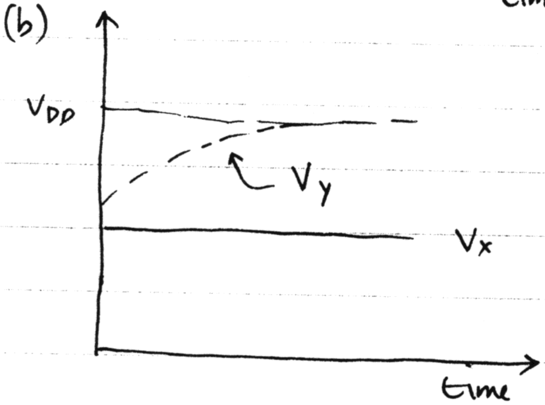


C_1 is continuously charged w/ I_{REF} so V_c and V_x increase with time $V_c = \int_0^t \frac{I_{REF}}{C_1} dt$

$V_x = V_c + V_{gs2}$

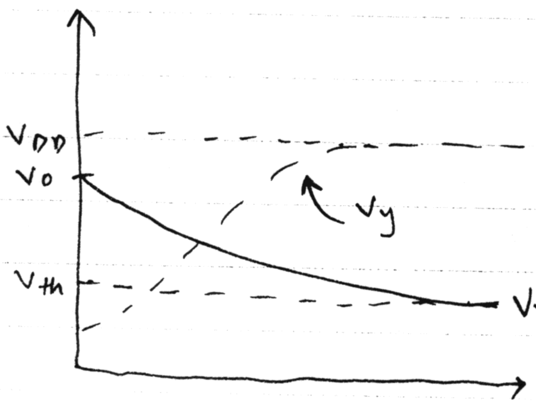
$V_{gs} = V_x$ and M_1 goes into triode

(b)



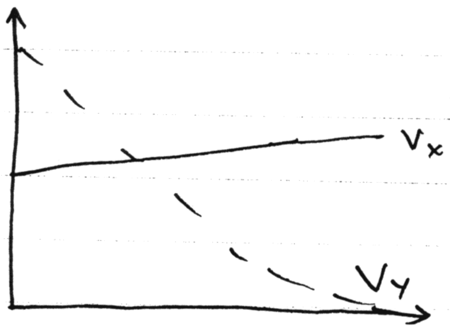
M2 is on with fixed $V_x = V_{gs2}$
C_1 is charged with current I_1 until M1 turns off.
$V_y = V_{DD} - I_1 R_1$ where
I_1 goes from I_{ref} to Ø A.

(c)



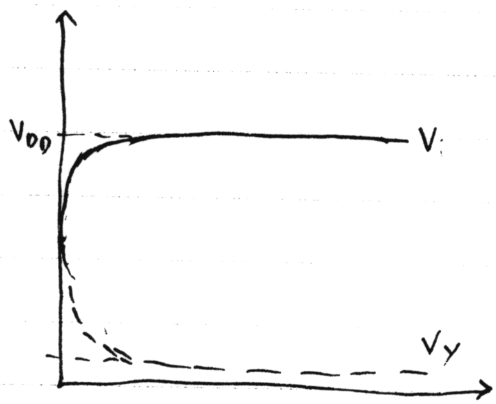
With C_1 initially charged w/ V_o, M2 is on and discharges C_1 until $V_x = V_{th}$ and $I_2 = 0$.
$V_y = V_{DD} - R_1 I_2$ and as I_2 goes to Ø, V_y goes to V_{DD}.

5.8 (d)



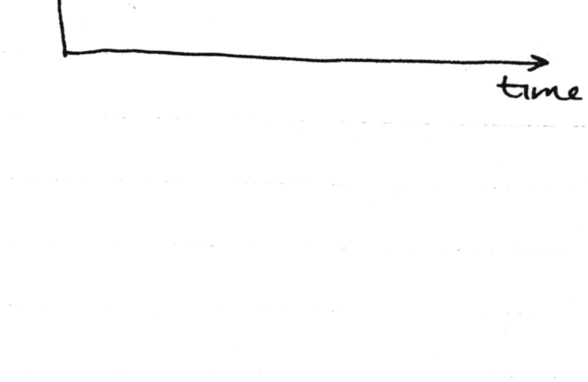
M2 and M1 are initially both on. M1 discharges all the charge in C_1 such that $V_y \to 0$ and M1 turns off. Since current through M1 reduces, current through M2 increases and V_x increases slightly

(e)

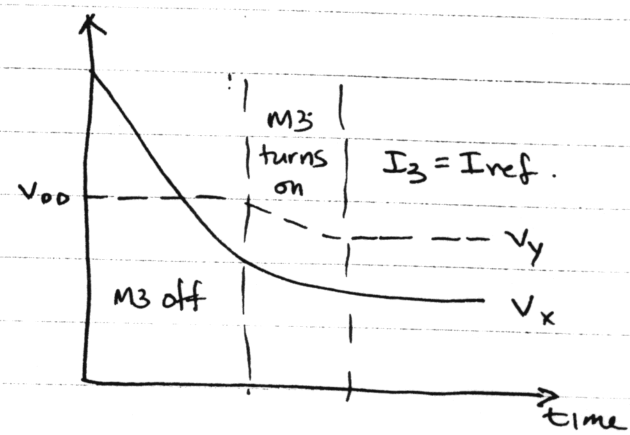


Since M2 can sustain no current, V_x goes up to V_{DD}. this pushes M1 into triode and Drops V_y to very small voltage. The Voltage accross C_1 drops to V_{th} to sustain $I_2 = 0$.

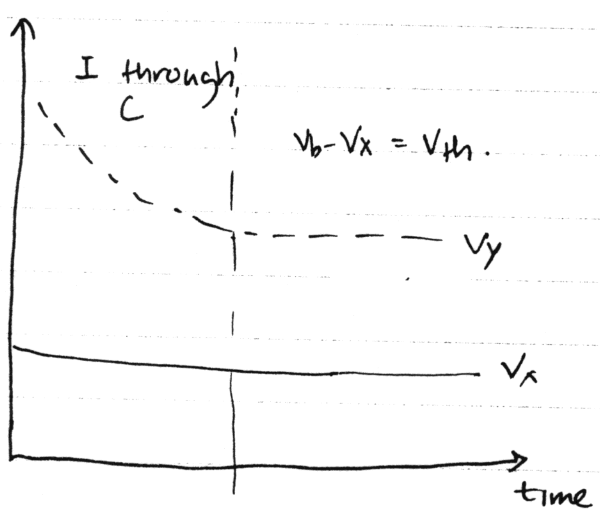
5.9 (a) At $t=0$, when C_1 is charged to 0V, $V_x = V_B - V_{gs3}$ where the $I_3 = I_{ref}$ and $V_y = V_{DD} - R_1 I_{ref}$. As the current flows into C_1, the cap charges up and shuts off M1. V_x is charged up to $V_b - V_{th}$ to sustain $I_{M} = 0$ and $V_y = V_{DD}$



5.9 (b) At $t=0$, V_x is so high that M3 is off. Current for M1 and M2 is generated by Iref and as I_1 flows through M1, C_1 discharges enough to allow current flow throw M3. Once $I_3 = I_{REF}$, C_1 no longer discharges and has a constant set voltage.

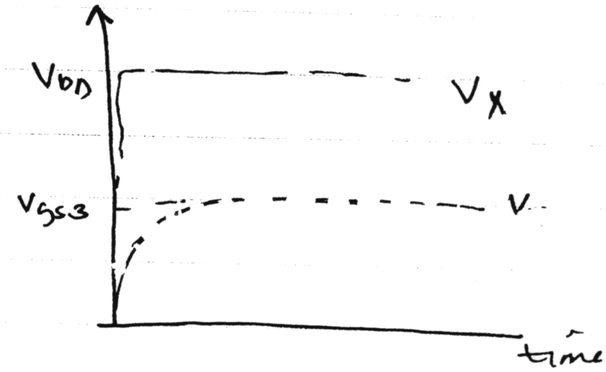


(c) At $t=0$, all transistors are on. $V_y = V_{gs2} + V_{DD}$ and V_x is such that $I_3 = k_0 \frac{W_3}{L_3}(V_b - V_x - V_{th})^2 (1+\lambda V_{DS3})$. as C_1 discharges, V_{DS3} decreases and V_{gs3} increases, lowering V_x. At the point where $V_b - V_x = V_{th}$ and $I_3 = I = 0$, V_y stops decresing.



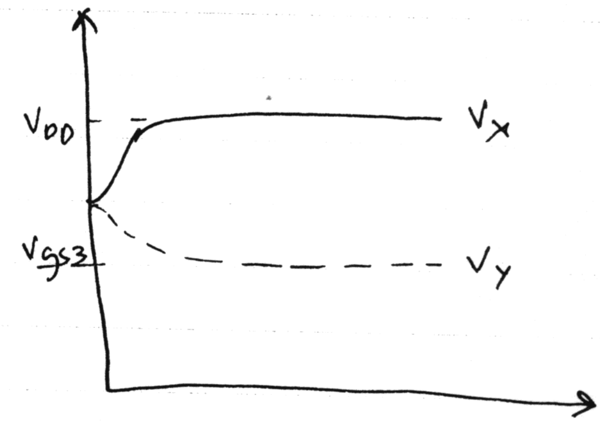
5.10 (a) at $t=0$, $V_{gs2} = 3V$ and forces current through M2. Since the source of M2 is attached to the gate of M1, no current can flow and $V_X = V_{DD}$

C_1 charges up such that $I_3 = I_1$ and $V_y = V_{DD} - V_{gs3}$

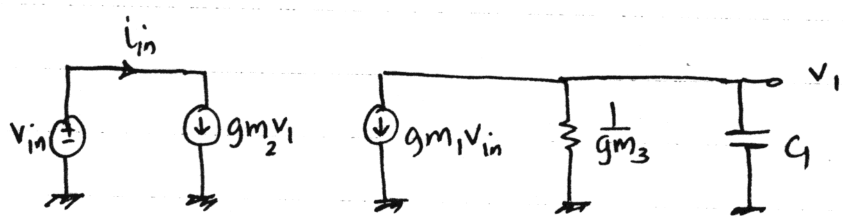


(b) Similar to 5.10(a), but since current can flow through C_1 to charge Capacitor, V_X doesn't instantaneously reach V_{DD}, but slowly charges to V_{DD}.

$V_y = V_{DD} - V_{gs3}$ also.



5.11 small signal model.



$$Z_{in} = \frac{V_{in}}{i_{in}} = \frac{V_{in}}{g_{m2} V_1} \qquad V_1 = -g_{m1}\left(\frac{1}{C_1 s + g_{m3}}\right) V_{in}$$

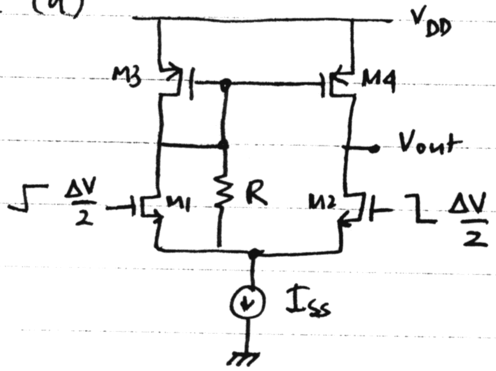
$$= -\frac{C_1 s + g_{m3}}{g_{m2} g_{m1}}$$

if all transistors are equal $g_{m1} = g_{m2} = g_{m3}$.

$$Z_{in} = -\frac{C_1 s}{g_m^2} - \frac{1}{g_m}$$

negative $C = -\frac{C_1}{g_m^2}$ and negative $R = -\frac{1}{g_m}$.

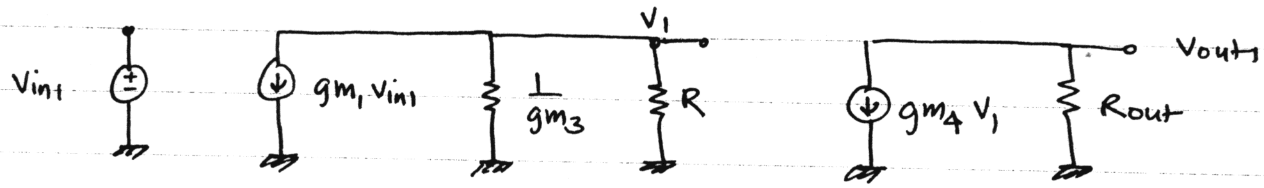
5.12 (a)



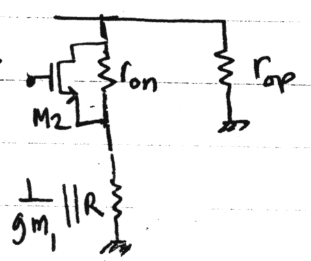
Solve for gain by super position

$$V_{in} = \Delta V = \frac{\Delta V}{2} - \left(-\frac{\Delta V}{2}\right) = V_{in1} + V_{in2}$$

1. for $+\frac{\Delta V}{2}$, ground M2 gate and find gain.



$R_{out1} \Rightarrow$



$$R_{out} = r_{op} \,\|\, \left(\frac{1}{g_{m1}} + r_{on} + \frac{g_{m2}}{g_{m1}} r_{on}\right)$$

$$\simeq r_{op} \,\|\, 2r_{on} \simeq \frac{2}{3} r_o = \frac{2}{3}\frac{1}{\lambda I_o} = \frac{4}{3}\frac{1}{\lambda I_{SS}}$$

$$\frac{V_{out1}}{V_{in1}} = +g_{m1}\left(\frac{1}{g_{m3}} \,\|\, R_o\right) g_{m4} R_{out}$$

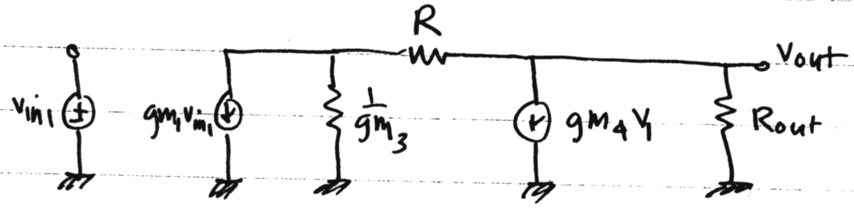
2. for $-\frac{\Delta V}{2}$

$V_{out} = -g_{m2} V_{in2} R_{out}$.

assume $g_{m1} = g_{m2}$ and $g_{m3} = g_{m4}$

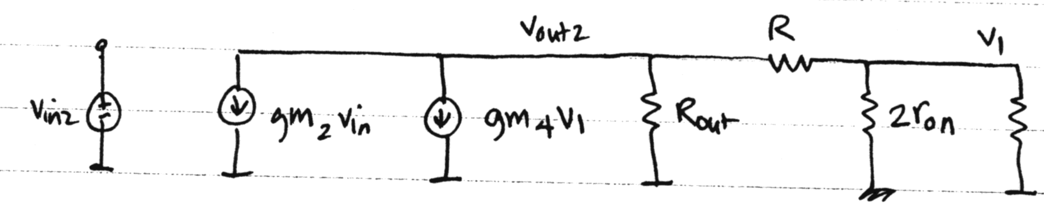
$$\text{Gain} = \frac{V_{out1} + V_{out2}}{\Delta V} = \frac{g_{m1}}{2} R_{out}\left[1 + \frac{R g_{m3}}{1 + R g_{m3}}\right]$$

(b) for $v_{in_1} = +\frac{\Delta V}{2}$



$$\frac{V_{out1}}{V_{in1}} = g_{m1} R_{out} \left[\frac{R g_{m3} - 1}{R g_{m3} + 2 R_{out} + 1} \right]$$

for $V_{in2} = -\frac{\Delta V}{2}$



$$V_1 = V_{out2} \cdot \frac{2r_{on} \| \frac{1}{g_{m3}}}{R + 2r_{on} \| \frac{1}{g_{m3}}} = V_{out2} \cdot \frac{2r_{on}}{R + 2r_{on} + 2g_{m3} r_{on} R}$$

$$V_{out2} = \left(g_{m2} V_{in2} - g_{m4} V_{out2} \frac{2r_{on}}{R + 2r_{on} + 2g_{m3} r_{on} R} \right) \left[R_{out} \| \left(R + 2r_{on} \| \frac{1}{g_{m3}} \right) \right]$$

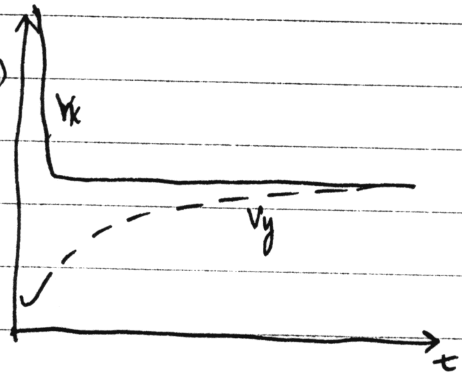
$$\frac{V_{out2}}{V_{in2}} = \frac{-g_{m2} \left[R_{out} \| \left(R + 2r_{on} \| \frac{1}{g_{m3}} \right) \right]}{1 + \frac{g_{m4}\, 2r_{on}}{R + r_{on} + 2g_{m3} r_{on} R} \cdot \left[R_{out} \| \left(R + 2r_{on} \| \frac{1}{g_{m3}} \right) \right]}$$

$$\text{Gain} = \frac{V_{out1} + V_{out2}}{\Delta V}$$

5.13 $V_{min} = V_p + V_{Dsat1,2}$ $\quad V_p = V_{CM1,2} - V_{gs1,2}$

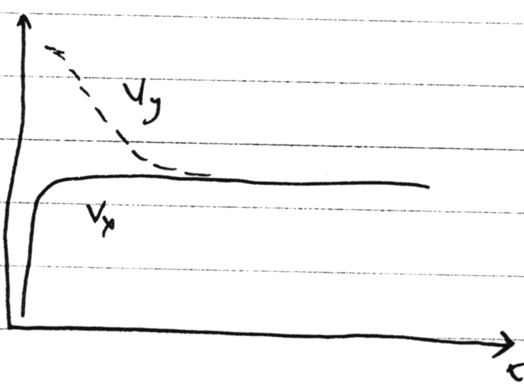
$= V_{CM1,2} - V_{gs1,2} + V_{DS1,2}$

5.14 (a)



M3 and M4 are initially off. M1 is on and M2 is in triode. the current through M1 initially comes from the C_1 charge until Vx drops in voltage and M3 and M4 turn on. M2 is still in triode until the current from M4 charges up Vy to the same voltage as Vx.

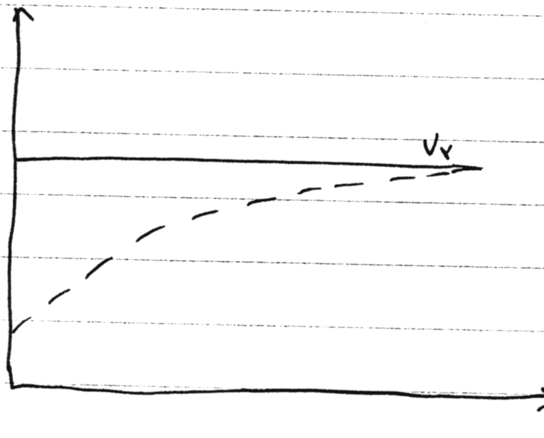
(b)



Vx starts at $1.5V - V_{gs1,2}$ where M1 is in triode and M3 is on strong. M4 and M2 can't sustain that high current w/ tail current $= I_{ss}$ so Vy goes up enough to put M4 in triode and reduce current. Current through M3 charge C_1 and Vy reduced its voltage w/ discharge in parasitics

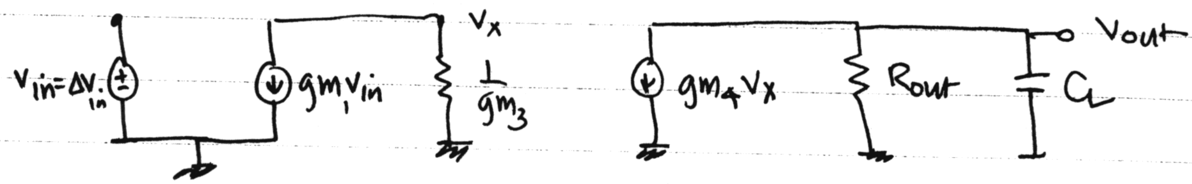
M3

(c)



Initial short between source + drain of M2 puts M2 in triode w/ minimal current flow. Current from M4 is used to charge up C_1. As C_1 charges up, some current starts flowing through M2 until V_{DS2} is high enough that M2 is in sat and all current is diverted to M2.

5.15 initial value ⇒ $V_{in} = V_1$ $V_{out} = V_X$

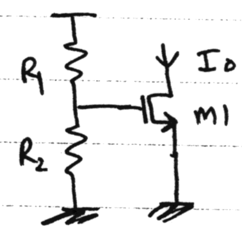


$$\Delta V_{out} = g_{m_1} \Delta V_{in} \frac{1}{g_{m_3}} g_{m_4} R_{out} \quad \text{(final)}$$

final value ⇒ $V_{in} = V_1 - \Delta V_{in}$ $V_{out} = V_X - \Delta V_{out}$

$\tau_c = R_{out} C_L$

5.16



$\frac{W}{L} = \frac{50\mu}{15\mu}$; $\lambda = 0$; $I = .5mA$; $K_p = \mu C_{ox} = 137\times10^{-4}$ $L_D = 90\,nm$

a) $R2/R1$

$$V_{gs1} = V_{DD}\frac{R_2}{R_1+R_2} = \sqrt{\frac{2I}{K_p \frac{W}{L'}}} + V_{th} \qquad L' = L - 2L_D$$

Let $RX = R_2/R_1$

$$R_X = \frac{\sqrt{2I/K_p \frac{W}{L'}} + V_{th}}{V_{DD} - \left(\sqrt{2I/K_p \frac{W}{L'}} + V_{th}\right)} = 0.4386$$

b) $I_o = \frac{1}{2}\mu C_{ox}\frac{W}{L'}\left(V_{DD}\frac{R_X}{1+R_X} - V_{th}\right)^2$

$$\frac{\left(\frac{\partial I_o}{\partial V_{DD}}\right)}{I_o} = \frac{\mu C_{ox}\frac{W}{L}\left(V_{DD}\frac{R_X}{1+R_X} - V_{th}\right)\frac{R_X}{1+R_X}}{\frac{1}{2}\mu C_{ox}\frac{W}{L}\left(V_{DD}\frac{R_X}{1+R_X} - V_{th}\right)^2}$$

$$= \frac{2}{V_{DD} - V_{th}\left(1+\frac{1}{R_X}\right)} = 2.84$$

5.16 (c) $\frac{\partial I_o}{\partial V_{th}} = -\mu C_{ox}\frac{W}{L}\left(V_{DD}\frac{R_x}{1+R_x} - V_{th}\right)$

$\Delta I_o \simeq -\mu C_{ox}\frac{W}{L}\left(V_{DD}\frac{R_x}{1+R_x} - V_{th}\right)\Delta V_{th} = -233\mu A$

$\Delta I_o = I_o(V_{th} = .75) - I_o(V_{th} = .7) = -205\mu A$

(d) $\frac{\partial I_o}{\partial T} = -\frac{3}{2}\left(\frac{T}{T_o}\right)^{-3/2} \cdot \frac{1}{T} \cdot I_o$ * $\quad T = T_o + \Delta T$

$\Delta I_o \simeq -\frac{3}{2}\left(\frac{T}{T_o}\right)^{-3/2}\frac{1}{T} \cdot I_o\,\Delta T = -103\,\mu A$ *

$\Delta I_o = I_o(T=370K) - I_o(T=300K) = -135\,\mu A$ *

(e) $\Delta I_{worst-case} = I_{worstcase} - I_o$

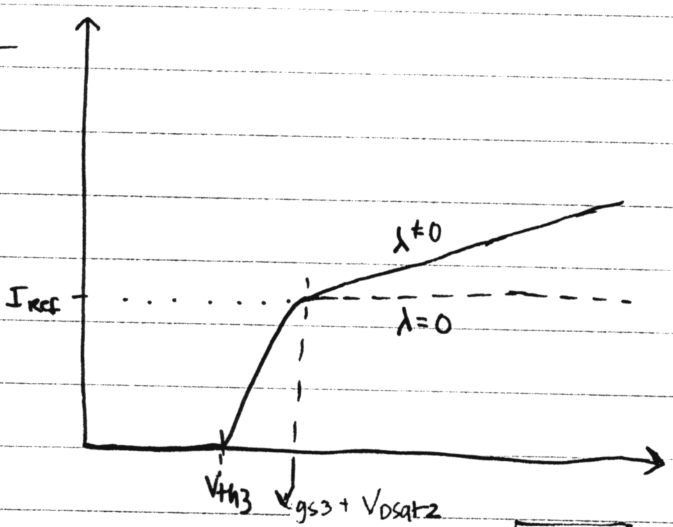
$I_{worst\,case} = \frac{1}{2}\mu_o\left(\frac{T_o + \Delta T}{T_o}\right)^{-3/2}\left(\left(V_{dd} - \Delta V_{dd}\right)\frac{R_x}{1+R_x} - (V_{th} + \Delta V_{th}\right)$

$= 43\,\mu A$

$\Delta I_{worst\,case} = -457\,\mu A$

* Note as temperature changes, so does V_{th}. In this calculation, we do not include the temperature effects on threshold voltage.

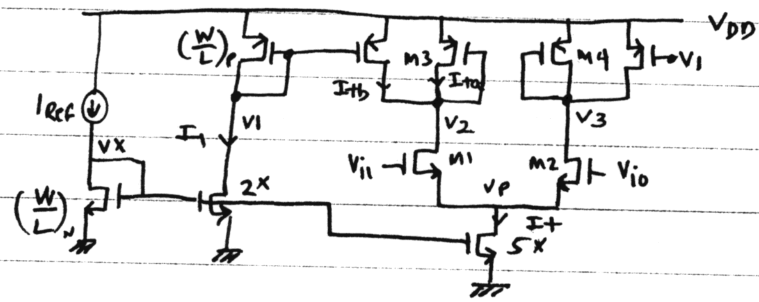
5.17



$$V_{gs3} = V_{th} + \sqrt{\frac{2 I_{ref}}{K_P \, W/L}}$$

$I_{out} = 0$ until $V_{DD} = V_{th3,4}$ when M3 and M4 are off, M2 in triode and M1 on. Once $V_{DD} > V_{th3,4}$, there is current flow but very little until M2 is in saturation at $V_{DD} = V_{gs3} + V_{Dsat2}$, $I_{out} \approx I_{ref}$. At $V_{DD} = V_{gs1} + V_{gB}$, $I_{out} = I_{ref}$ exactly and grows as V_{DS2} increases.

5.18



$(W/L)_N = \frac{10}{.5}$ $\quad (W/L)_{1,2}$ arbitrary

$(W/L)_P = \frac{10}{.5}$

$I_{REF} = 100\,\mu A$

$K_P = 38 \frac{\mu A}{V^2}$ $\quad K_n = 139 \frac{\mu A}{V^2}$

(a) $\lambda = 0$

$$I = \frac{1}{2} K_P \frac{W}{L - 2L_D} (V_{gs} - V_t)^2$$

$$V_1 = V_{DD} - V_{gs(P)} = V_{DD} - V_{tP} - \sqrt{\frac{2 \cdot 2 \cdot I_{Ref}}{K_P \frac{W}{L-2L_D}}} = 1.619\text{ V}$$

$$V_2 = V_3 = V_{DD} - V_{gs(3)} = V_{DD} - V_{tP} - \sqrt{\frac{I_{ref}}{K_P \frac{W}{L-2L_D}}} = 1.909\text{ V}$$

$$V_P = V_{cm} - V_{gs(1)} = 1.3 - V_{tn} - \sqrt{\frac{2.5 \cdot I_{ref}}{K_n \; W}} = .3747\text{ V} \quad (\gamma = 0)$$

V_P for $\gamma = .45$ is found iteratively by finding V_{th} iteratively also.

$$V_{tn}(V_P = .3747) = V_{TO} + \gamma\sqrt{2\phi + V_{SB}} - \gamma\sqrt{2\phi} \qquad \gamma = .45,\ \phi = .45$$

$$= .78$$

$V_P(V_{tn} = .78) = .29$ until $V_{tn}(\text{final}) = .767$

$V_P(\text{final}) = .307$

5.18 (b) $\lambda = .2$

$$V_{gs(0)} = V_{tn} + \sqrt{\frac{2 I_{ref}}{K_n \cdot \frac{W}{L-2L_D}}} = 0.9253$$

for I_t, initially assume $V_p = .307$ from part [a)

iterate with

$$\left\{ \begin{array}{l} I_t = I_{ref} \cdot 5 \cdot \dfrac{1+\lambda V_p}{1+\lambda V_{gs(0)}} \\ \text{and} \\ V_p = V_{cm\,1,2} - V_{gs(1,2)} \quad \text{with} \quad V_{tn\,1,2} = V_{to} + \gamma\sqrt{2\phi + V_p} - \gamma\sqrt{2\phi} \end{array} \right.$$

$I_t = 448\,\mu A$ and $V_p = 0.317$

for V_1, iterate also

$$\left\{ \begin{array}{l} I_1 = I_{ref} \cdot 2 \cdot \dfrac{1+\lambda V_1}{1+\lambda V_{gs(0)}} \\ V_1 = V_{DD} - |V_{gsp}| = V_{DD} - V_{tp} - \sqrt{\dfrac{2 \cdot I_1}{K_p \frac{W}{L-2L_D}}} \end{array} \right.$$

final $I_1 = 222\,\mu A$, $V_1 = 1.59\,V$

for V_2, V_3, iterate also.

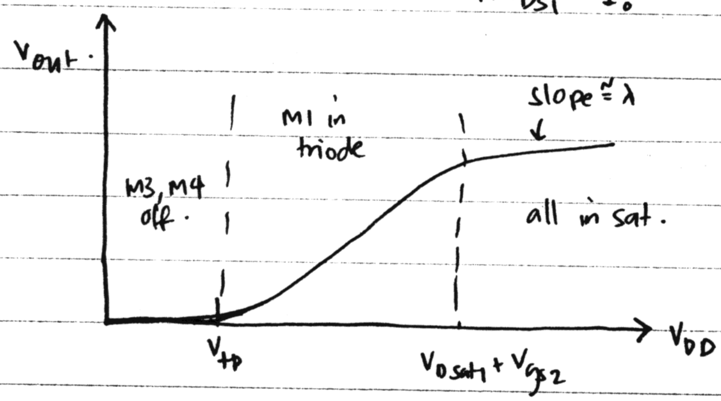
$$\left\{ \begin{array}{l} I_{ta} = I_t/2 - I_{tb} \\ V_{2,3} = V_{DD} - |V_{gs3,6}| = V_{DD} - V_{tp} - \sqrt{\dfrac{2 \cdot I_{ta}}{K_p \frac{W}{L-2L_D}}} \\ I_{tb} = I_1 \cdot \dfrac{1+\lambda |V_{gs3,6}|}{1+\lambda |V_{gsp}|} \end{array} \right.$$

$I_{ta} = 17.3\,\mu A$; $I_{tb} = 207\,\mu A$; $V_{2,3} = 2.029\,V$

5.19 $V_{DD} < V_{tp}$: M2 and M3 off, $I_3 = 0$, $V_{out} = 0$

$V_{tp} \le V_{DD} < V_{Dsat1} + V_{gs2}$: M1 in triode, linearly approaching saturation. M2 and M3 are on with V_{out} increasing linearly.

$V_{DD} > V_{Dsat1} + V_{gs2}$: All transistors are on and as V_{DD} increases, V_{DS1} increases so current increases as $\lambda V_{DS1} \cdot I_o$



5.20 $\gamma = 0$ $\left(\frac{W}{L}\right)_{1-3} = \frac{40}{.5}$ $I_{ref} = .3\,mA$ $K_n = 138 \frac{\mu A}{V^2}$ $L_D = 80\,nm$

(a) $V_b = 2V_{gs1}$ $V_{gs1} = V_{tn} + \sqrt{\frac{2I_{ref}}{k_n \frac{W}{L-2L_D}}} = .892\,V$

$V_b = 1.78\,V$

(b) $I_{out} = I_{ref} \cdot \frac{(1 + \lambda(V_{gs1} + \Delta V_b))}{1 + \lambda V_{gs1}} = 295\,\mu A$ $\Delta V_b = -.100\,V$

$\Delta I_{out} = \frac{I_{ref} \cdot \lambda \Delta V_b}{1 + \lambda V_{gs1}}$

5.20 (c) V_P increases by 1V

V_Y increases by ΔX

solve for 2 unknowns and 2 equations

$$\begin{cases} I_{out} = I_{ref}\left(\dfrac{1+\lambda(V_{gs1}-\Delta X)}{1+\lambda V_{gs1}}\right) \quad (M2) \quad \text{eq 1.} \\[2ex] I_{out} = \tfrac{1}{2}K_n\dfrac{W}{L-2L_D}(V_{gs1}-\Delta X)^2\left(\dfrac{1+\lambda(V_{gs1}+1V-\Delta X)}{1+\lambda V_{gs1}}\right) \quad (M3) \quad \text{eq 2} \end{cases}$$

$\Delta X = 13\,mV$

$V_y = V_{gs1} + \Delta X = .905\,V$

5.21 (a) $V_x = V_{gs1} = V_{tn} + V_{Dsat1} = .863$

$$V_{Dsat1} = \sqrt{\frac{2I_{ref}}{K_n\frac{W}{L-2L_D}}} = .163 \qquad V_{tn} = .7V$$

$V_b \geq V_{Dsat1} + V_{gs2}$

$$V_{gs2} = V_{tno} + \gamma\sqrt{2\phi + V_{Dsat1}} - \gamma\sqrt{2\phi} + \sqrt{\frac{2I_{REF}}{K_n\frac{W}{L-2L_D}}}$$

$$= .7 + .037 + .163 = .900v$$

as V_b increases, M2 and M4 go into triode and $V_A \sim V_X$ and $V_B \sim V_{M4,drain}$. As lond as $V_{M4,drain}$ does not drop Below V_{Dsat1}, I_{out} will reasonably follow I_{ref}.

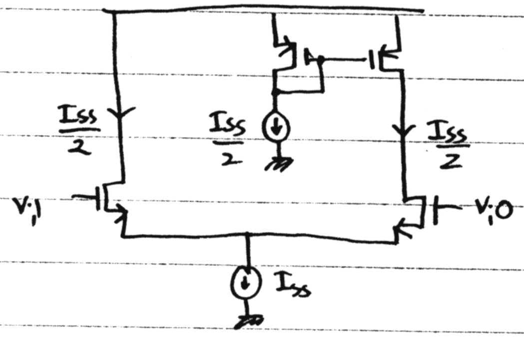
b) $V_{M4,drain} \uparrow 1V$, $V_B \uparrow \Delta X$

use eq 1.and 2. from 5.20 to solve for change in current.

$I_{out} = 301.1\,\mu A \quad \Delta X = 21\,mV$

5.22 Assume $\lambda = 0$ for bias purpose and $\lambda = .2$ for small signal analysis.

(a) DC Bias



$$gm_{1,2} = \frac{I_{ss}}{V_{Dsat\,1,2}} = 3.18\ mS$$

$$V_{dsat1,2} = \sqrt{\frac{I_{ss}}{K_n \frac{W}{L-2L_D}}} = .157$$

$$r_{on} = r_{op} = \frac{1}{\lambda \frac{I_{ss}}{2}} = 20K$$

small signal: $\text{Gain} = \frac{v_{out}}{v_{i1} - v_{i0}} = \frac{1}{2} gm_{1,2} R_{out} = 21.2$

$$R_{out} = r_{op} \parallel \left(2r_{on} + \frac{1}{gm_1}\right) \approx \frac{2}{3} r_o = 13.33\,K$$

(b) Maximum output voltage swing.

$$V_{out}(min) = V_{cm} - V_{gs1,2}\left(I = \frac{I_{ss}}{2}\right) + V_{dsat\,1,2}$$

$$= V_{cm} - V_{tn\,1,2}$$

$$= 1.3 - .778 = .522\ v$$

$V_{out}(max) = V_{dd}$ (M4 in triode)

$$V_{out}(max) - V_{out}(min) = V_{DD} - V_{cm} + V_{tn1,2}$$

$$= 3.V - 1.3V + .778 = 2.48V$$

$$V_{tn} = V_{to} + \gamma\sqrt{2\phi + .36} - \gamma\sqrt{2\phi} = .778$$

5.23. Assume $I_3 = I_4$ though $V_{th3} \neq V_{th4}$

(a) $I_3 = \frac{1}{2} K_p \frac{W}{L-2L_D} (V_{gs3} - V_{th3})^2 (1 + \lambda |V_{gs3}|)$

$I_4 = \frac{1}{2} K_p \frac{W}{L-2L_D} (V_{gs3} - V_{th4})^2 (1 + \lambda (|V_{gs3}| - \Delta x))$

$K' = \frac{1}{2} K_p \frac{W}{L-2L_D}$

$K' (V_{Dsat3})^2 (1 + \lambda |V_{gs3}|) = K' (|V_{dsat3}| - 1mV)^2 (1 + \lambda |V_{gs3}| + \lambda \Delta x)$

$$\Delta x \cong \frac{1mV \cdot (1 + \lambda |V_{gs3}|)}{\lambda |V_{dsat3}|}$$

$V_F = V_{gs3} - \Delta x$.

(b) $CMRR \triangleq \left| \frac{A_{dm}}{A_{cm}} \right|$

$A_{dm} = gm_{1,2} (r_{o3,4} \| r_{o1,2})$

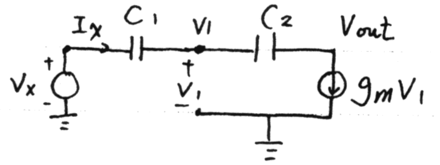
$= gm_{1,2} (\frac{1}{2} r_o)$

$$A_{cm} \cong \frac{-1}{1 + 2 gm_{1,2} r_o} \cdot \frac{gm_{1,2}}{gm_{3,4}}$$

$CMRR = (1 + 2 gm_{1,2} r_o) \, gm_{3,4} \, (r_{o1,2} \| r_{o3,4})$

Chapter 6

6.1 (a)



g_m : transconductance of M_1

$$I_X = SC_1(V_X - V_1) = SC_2(V_1 - V_{out}) = g_m V_1$$

$$\therefore \quad SC_1 V_X = (g_m + SC_1)V_1 \quad \Rightarrow V_1 = \left[\frac{SC_1}{g_m + SC_1}\right] V_X$$

$$\Rightarrow I_X = g_m V_1 = \left[{g_m SC_1}/{g_m + SC_1} \right] V_X$$

$$\Rightarrow Z_{in} = \frac{V_X}{I_X} = \frac{g_m + SC_1}{g_m SC_1}$$

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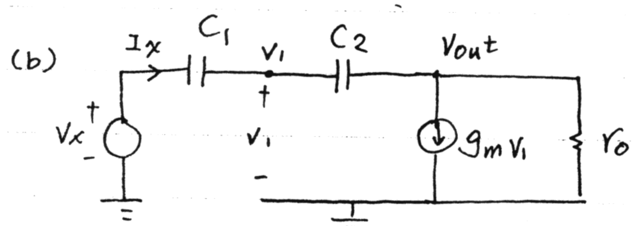
(b)



$g_m = g_{m1} + g_{m2}$

$r_o = r_{o1} // r_{o2}$

g_{m1}, g_{m2} : transconductance for M_1, M_2

r_{o1}, r_{o2} : output resistance for M_1, M_2

$$\therefore I_X = SC_1(V_X - V_1) = \overbrace{\underbrace{SC_2(V_1 - V_{out}) = g_m V_1 + \frac{V_{out}}{r_o}}_{①}}^{②}$$

from ① :

$$\frac{V_{out}}{V_1} = \frac{SC_2 - g_m}{SC_2 + \frac{1}{r_o}}$$

from ② :

$$(SC_1 + SC_2)V_1 = SC_1 V_X + SC_2 V_{out}$$

$$= SC_1 V_X + \frac{S^2 C_2^2 - g_m S C_2}{SC_2 + \frac{1}{r_o}} V_1$$

$$\Rightarrow \left[SC_1 + SC_2 - \frac{S^2 C_2^2 - g_m SC_2}{SC_2 + \frac{1}{r_o}} \right] V_1 = SC_1 V_X$$

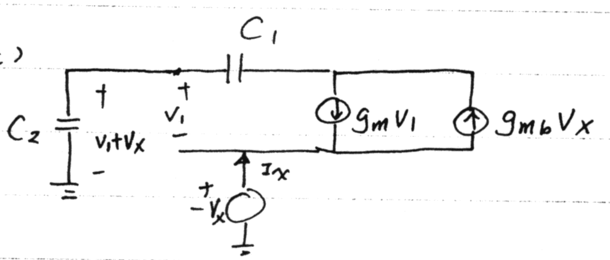
$$\Rightarrow \left[\frac{s^2C_1C_2 + \frac{sC_1}{r_o} + \frac{sC_2}{r_o} + g_m s C_2}{sC_2 + \frac{1}{r_o}} \right] V_1 = sC_1 V_x$$

$$\therefore V_1 = \left[\frac{sC_1C_2 + \frac{C_1}{r_o}}{sC_1C_2 + \frac{C_1}{r_o} + \frac{C_2}{r_o} + g_m C_2} \right] V_x$$

$$I_x = sC_1(V_x - V_1) = sC_1 \cdot \left[\frac{\frac{C_2}{r_o} + g_m C_2}{sC_1C_2 + \frac{C_1}{r_o} + \frac{C_2}{r_o} + g_m C_2} \right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{sC_1C_2 + \frac{C_1}{r_o} + \frac{C_2}{r_o} + g_m C_2}{sC_1C_2\left(\frac{1}{r_o} + g_m\right)} \quad \#$$

(c)



$$sC_2(V_1 + V_x) + g_m V_1 = g_{mb} V_x$$

$$\Rightarrow (sC_2 + g_m)V_1 = (g_{mb} - sC_2)V_x$$

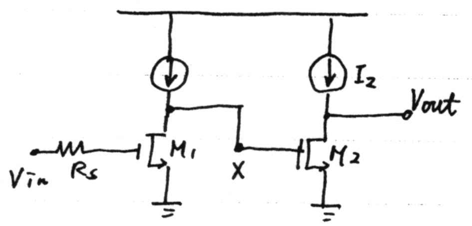
$$\frac{V_x}{V_1} = \frac{sC_2 + g_m}{g_{mb} - sC_2}$$

$$I_x = -g_m V_1 + g_{mb} V_x$$

$$= \left[-g_m \cdot \frac{g_{mb} - sC_2}{sC_2 + g_m} + g_{mb} \right] V_x = \left[\frac{(g_m + g_{mb})sC_2}{sC_2 + g_m} \right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{sC_2 + g_m}{sC_2(g_m + g_{mb})} \quad \#$$

6.2 (a)



There are three poles associated with this circuit.

The first pole @ Vout

$$\omega_{p,out} = \frac{1}{r_{o2} \cdot (C_{gd2} + C_{DB2})}$$

The pole @ the input

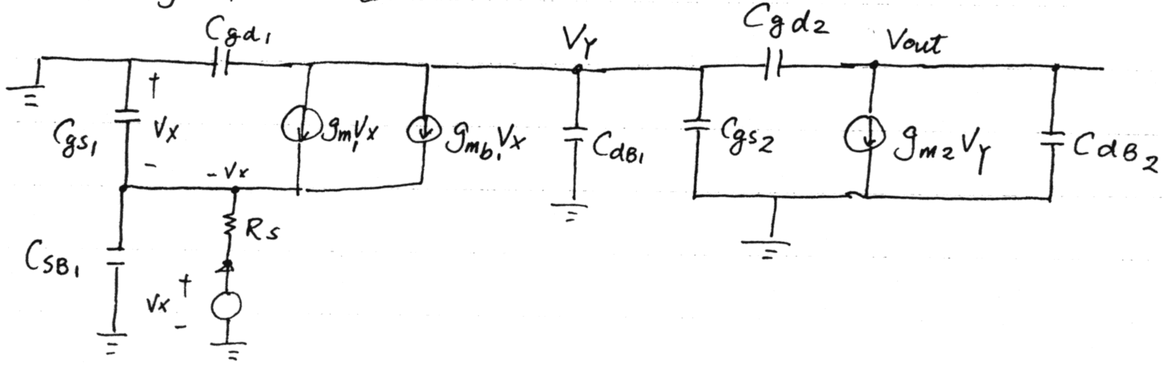
$$\omega_{p,in} = \frac{1}{R_S \cdot [(1+g_{m1} r_{o1}) C_{gd1} + C_{gs1}]}$$

The pole @ node X

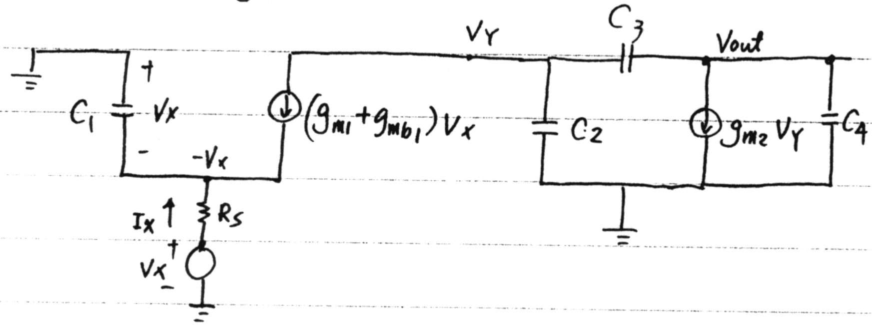
$$\omega_{p,X} = \frac{1}{r_{o1} \cdot [(C_{gd1} + C_{DB1} + C_{gs2}) + (1+g_{m2} r_{o2}) \cdot C_{gd2}]}$$

Please note that the above approximation is based on Miller effect. In order to get more accuracy approximation, transfer function has to be derived.

(b) Small signal model



Redraw small signal model



where,

$C_1 = C_{gs1} + C_{SB1}$

$C_2 = C_{gs2} + C_{dB1} + C_{gd1}$

$C_3 = C_{gd2}$

$C_4 = C_{dB2}$

KCL @ V_{out} : $sC_3(V_Y - V_{out}) = g_{m2}V_Y + sC_4 V_{out}$

$$\Rightarrow \frac{V_{out}}{V_Y} = \frac{-g_{m2} + sC_3}{s(C_3+C_4)}$$

KCL @ V_Y : $(g_{m1}+g_{mb1})V_X + sC_2V_Y + sC_3(V_Y - V_{out}) = 0$

$$(g_{m1}+g_{mb1})V_X = -V_Y\left(sC_2 + \frac{s^2C_3C_4 + sC_3\cdot g_{m2}}{s(C_3+C_4)}\right)$$

$$\frac{V_Y}{V_X} = -\frac{g_{m1}+g_{mb1}}{\left[s(C_2C_3+C_2C_4+C_3C_4) + C_3g_{m2}\right]/(C_3+C_4)}$$

KCL @ V_X : $\frac{V_{in}+V_X}{R_S} + sC_1V_X + (g_{m1}+g_{mb1})V_X = 0$

$$\frac{V_X}{V_{in}} = -\frac{1}{sC_1R_S + (1+(g_{m1}+g_{mb1})\cdot R_S)}$$

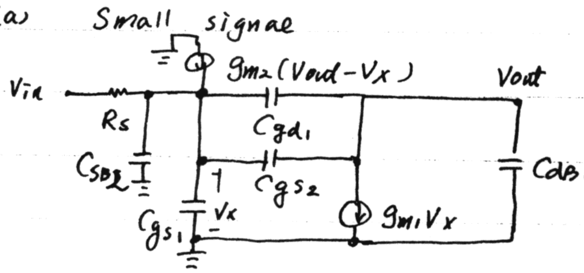
Thus. there are three poles

$\omega_{p0} = 0$

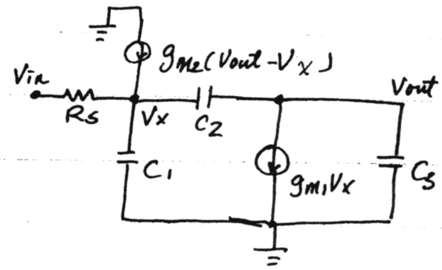
$$\omega_{p1} = \frac{-C_3g_{m2}}{C_2C_3+C_2C_4+C_3C_4} \quad ☆$$

$$\omega_{p2} = \frac{-(1+(g_{m1}+g_{mb1})\cdot R_S)}{C_1R_S} \quad ☆$$

6.3 (a) Small signal



$\Rightarrow$



$C_1 = C_{gs1} + C_{SB2}$

$C_2 = C_{gd1} + C_{gs2}$

$C_3 = C_{dB1}$

KCL @ Vout :

$$sC_2(V_x - V_{out}) = g_{m1}V_x + sC_3 V_{out}$$

$$\therefore \frac{V_{out}}{V_x} = \frac{sC_2 - g_{m1}}{s(C_2+C_3)} \quad \text{—} ①$$

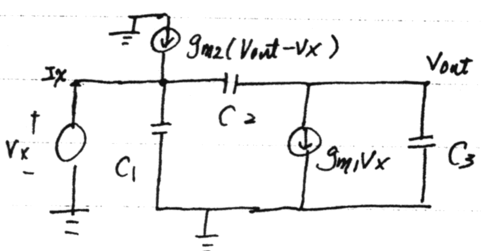
KCL @ Vx :

$$\frac{V_{in}-V_x}{R_s} + g_{m2}(V_{out}-V_x) = sC_1V_x + sC_2(V_x - V_{out})$$

$$\Rightarrow \frac{V_{in}}{R_s} = V_x\left(\frac{1}{R_s} + g_{m2} + sC_1 + sC_2\right) - (g_{m2}+sC_2)\cdot\left[\frac{sC_2 - g_{m1}}{s(C_2+C_3)}\right]V_x$$

$$= V_x \cdot \frac{s^2(C_1C_2+C_2C_3+C_1C_3)+s\left(\frac{1}{R_s}(C_2+C_3)+g_{m1}C_2+g_{m2}C_3\right)+g_{m1}g_{m2}}{s(C_2+C_3)}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_x}\cdot\frac{V_x}{V_{in}} = \frac{\frac{1}{R_s}(sC_2 - g_{m1})}{s^2(C_1C_2+C_2C_3+C_1C_3)+s\left[\frac{1}{R_s}(C_2+C_3)+g_{m1}C_2+g_{m2}C_3\right]+g_{m1}g_{m2}} \quad ☆$$



$$I_x = sC_1V_x + sC_2(V_x - V_{out}) + g_{m2}(V_x - V_{out})$$

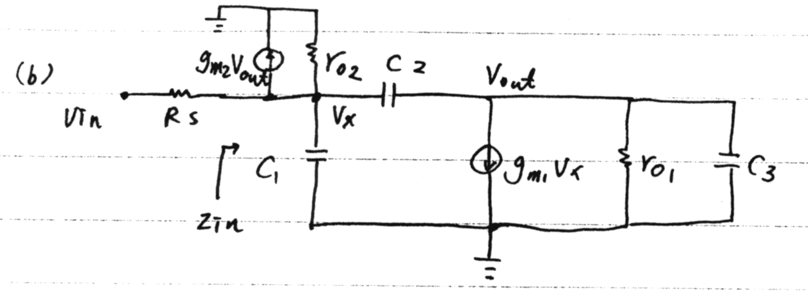
from ①: $$V_x - V_{out} = \left(\frac{sC_3 + g_{m1}}{s(C_2+C_3)}\right)V_x$$

$$\therefore I_x = \left[sC_1 + \frac{s^2C_2C_3 + g_{m1}sC_2 + g_{m2}sC_3 + g_{m1}g_{m2}}{s(C_2+C_3)}\right]V_x$$

$$= \left[\frac{s^2(C_1C_2+C_2C_3+C_1C_3) + s(g_{m1}C_2+g_{m2}C_3)+g_{m1}g_{m2}}{s(C_2+C_3)}\right]V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s(C_2+C_3)}{s^2(C_1C_2+C_2C_3+C_1C_3)+s(g_{m1}C_2+g_{m2}C_3)+g_{m1}g_{m2}} \quad ☆$$

(b)



$C_1 = C_{gs1} + C_{dB2}$

$C_2 = C_{gd1} + C_{gd2}$

$C_3 = C_{dB1} + C_{gs2}$

KCL @ Vout : $sC_2(V_x - V_{out}) = g_{m1}V_x + V_{out}\left(\frac{1}{r_{o1}} + sC_3\right)$

$$\frac{V_{out}}{V_x} = \frac{sC_2 - g_{m1}}{s(C_2+C_3) + \frac{1}{r_{o1}}}$$

KCL @ Vx : $\frac{V_{in}-V_x}{R_S} = g_{m2}V_{out} + V_x\left(\frac{1}{r_{o2}} + sC_1\right) + sC_2(V_x - V_{out})$

$$\frac{V_{in}}{R_S} = \left(sC_1 + sC_2 + \frac{1}{r_{o2}} + \frac{1}{R_S}\right)V_x - \frac{(-g_{m2}+sC_2)(sC_2-g_{m1})}{s(C_2+C_3)+\frac{1}{r_{o1}}}\cdot V_x$$

$$= \frac{s^2(C_1C_2+C_1C_3+C_2C_3) + s\left[\left(\frac{1}{r_{o2}}+\frac{1}{R_S}\right)(C_2+C_3) + \frac{1}{r_{o1}}(C_1+C_2) + g_{m1}C_2 + g_{m2}C_2\right] - g_{m1}g_{m2} + \frac{1}{r_{o1}}\left(\frac{1}{r_{o2}}+\frac{1}{R_S}\right)}{s(C_2+C_3)+\frac{1}{r_{o1}}}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{\frac{1}{R_S}(sC_2 - g_{m1})}{s^2(C_1C_2+C_1C_3+C_2C_3) + s\left[\left(\frac{1}{r_{o2}}+\frac{1}{R_S}\right)(C_2+C_3) + \frac{1}{r_{o1}}(C_1+C_2) + g_{m1}C_2 + g_{m2}C_2\right] - g_{m1}g_{m2} + \frac{1}{r_{o1}}\left(\frac{1}{r_{o2}}+\frac{1}{R_S}\right)}$$ #

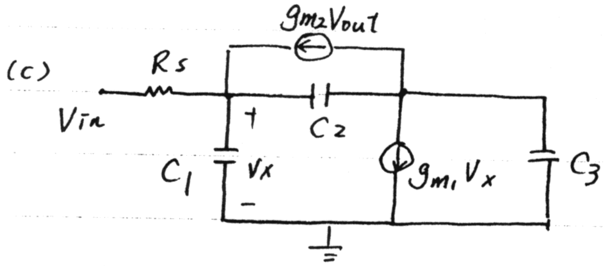
For Z_{in}

$$I_x = g_{m2}V_{out} + V_x\left(\frac{1}{r_{o2}} + sC_1\right) + sC_2(V_x - V_{out})$$

$$= \left[\frac{s^2(C_1C_2+C_1C_3+C_2C_3) + s\left[\frac{1}{r_{o2}}(C_2+C_3) + \frac{1}{r_{o1}}(C_1+C_2) + g_{m1}C_2 + g_{m2}C_2\right] + \left(-g_{m1}g_{m2} + \frac{1}{r_{o1}}\frac{1}{r_{o2}}\right)}{s(C_2+C_3)+\frac{1}{r_{o1}}}\right]V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s(C_2+C_3)+\frac{1}{r_{o1}}}{s^2(C_1C_2+C_1C_3+C_2C_3) + s\left[\frac{1}{r_{o2}}(C_2+C_3) + \frac{1}{r_{o1}}(C_1+C_2) + g_{m1}C_2 - g_{m2}C_2\right] + \left(-g_{m1}g_{m2} + \frac{1}{r_{o1}}\frac{1}{r_{o2}}\right)}$$

(c)



$C_1 = C_{gs1} + C_{dB2} + C_{gd2}$

$C_2 = C_{gd1}$

$C_3 = C_{dB1} + C_{SB2} + C_{gs2}$

KCL@ Vout : $sC_2(V_X - V_{out}) = g_{m1}V_X + sC_3V_{out} + g_{m2}V_{out}$

$$\Rightarrow \frac{V_{out}}{V_X} = \frac{sC_2 - g_{m1}}{s(C_2+C_3)+g_{m2}}$$

KCL@Vx : $\frac{V_{in}-V_X}{R_S} + g_{m2}V_{out} = sC_1V_X + sC_2(V_X - V_{out})$

$$\frac{V_{in}}{R_S} = \left(\frac{1}{R_S} + sC_1 + sC_2\right)V_X - (g_{m2}+sC_2)V_{out} = \left(\frac{1}{R_S}+sC_1+sC_2\right)V_X - \frac{(g_{m2}+sC_2)(sC_2-g_{m1})}{s(C_2+C_3)+g_{m2}}$$

$$\frac{V_i}{V} = \frac{s^2(C_1C_2+C_2C_3+C_1C_3)+s\left[\frac{1}{R_S}(C_2+C_3)+g_{m2}C_1+g_{m1}C_2\right]+\left(\frac{g_{m2}}{R_S}+g_{m1}g_{m2}\right)}{s(C_2+C_3)+g_{m2}}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_X}\cdot\frac{V_X}{V_{in}}$$

$$= \frac{\frac{1}{R_S}[sC_2 - g_{m1}]}{s^2(C_1C_2+C_2C_3+C_1C_3)+s\left[\frac{1}{R_S}(C_2+C_3)+g_{m2}C_1+g_{m1}C_2\right]+\left(\frac{g_{m2}}{R_S}+g_{m1}g_{m2}\right)}$$ #

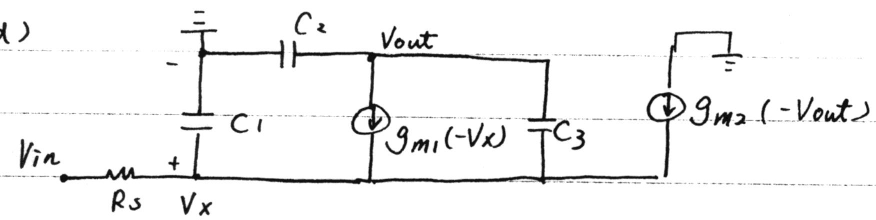
For Zin

$I_X = sC_1V_X - g_{m2}V_{out} + sC_2(V_X - V_{out})$

$$= \left[\frac{s^2(C_1C_2+C_2C_3+C_1C_3)+s(g_{m2}C_1+g_{m1}C_2)+g_{m1}g_{m2}}{s(C_2+C_3)+g_{m2}}\right]V_X$$

$$Z_{in} = \frac{s(C_2+C_3)+g_{m2}}{s^2(C_1C_2+C_2C_3+C_1C_3)+s(g_{m2}C_1+g_{m1}C_2)+g_{m1}g_{m2}}$$ #

(d)



$C_1 = C_{gs1} + C_{SB1} + C_{DB2}$

$C_2 = C_{gd1} + C_{gs2} + C_{dB1}$

$C_3 = C_{gd2}$

KCL @ Vout : $-sC_2 V_{out} = -g_{m1}V_x + sC_3(V_{out} - V_x)$

$$\frac{V_{out}}{V_x} = \frac{sC_3 + g_{m1}}{s(C_2 + C_3)}$$

KCL @ Vx : $\frac{V_{in} - V_x}{R_s} = sC_1V_x + g_{m1}V_x + sC_3(V_x - V_{out}) + g_{m2}V_{out}$

$$\frac{V_{in}}{R_s} = \left[\frac{1}{R_s} + s(C_1 + C_3) + g_{m1}\right]V_x + \frac{(sC_3 + g_{m1})(g_{m2} - sC_3)}{s(C_2 + C_3)}V_x$$

$$= V_x\left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[\frac{C_2}{R_s} + \frac{C_3}{R_s} + g_{m1}C_2 + g_{m2}C_3\right] + g_{m1}g_{m2}}{s(C_2 + C_3)}\right]$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{sC_3 + g_{m1}}{s^2R_s(C_1C_2 + C_2C_3 + C_1C_3) + s[C_2 + C_3 + R_s(g_{m1}C_2 + g_{m2}C_3)] + g_{m1}g_{m2}R_s}$$ #

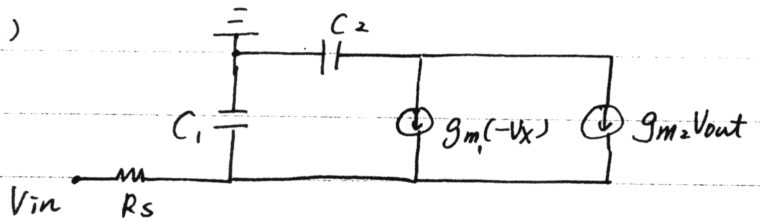
For Zin

$$I_X = sC_1V_x + g_{m1}V_x + sC_3(V_x - V_{out}) - g_{m2}V_{out}$$

$$= \left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m1}C_2 + g_{m2}C_3) + g_{m1}g_{m2}}{s(C_2 + C_3)}\right]V_x$$

$$\therefore Z_{in} = \frac{s(C_2 + C_3)}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m1}C_2 + g_{m2}C_3) + g_{m1}g_{m2}}$$ #

(e)



$C_1 = C_{gs1} + C_{SB1} + C_{dB2} + C_{gd2}$

$C_2 = C_{gd1} + C_{SB2} + C_{gs2} + C_{dB1}$

KCL @ Vout $\Rightarrow -sC_2 V_{out} = -g_{m1} V_x + g_{m2} V_{out}$

$$\Rightarrow \frac{V_{out}}{V_x} = \frac{g_{m1}}{sC_2 + g_{m2}}$$

KCL @ Vx $\Rightarrow \dfrac{V_{in} - V_x}{R_s} = sC_1 V_x + g_{m1} V_x - g_{m2} V_{out}$

$$\Rightarrow \frac{V_{in}}{R_s} = \left[\frac{1}{R_s} + sC_1 + g_{m1}\right] V_x - \frac{g_{m1} g_{m2}}{(sC_2 + g_{m2})} V_x$$

$$= \frac{s^2 C_1 C_2 + s\left[\left(\frac{1}{R_s} + g_{m1}\right) C_2 + g_{m2} C_1\right] + \frac{g_{m2}}{R_s}}{sC_2 + g_{m2}}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{g_{m1}}{s^2 R_s C_1 C_2 + s\left[(1 + g_{m1} R_s) C_2 + g_{m2} R_s C_1\right] + g_{m2}}$$ ※

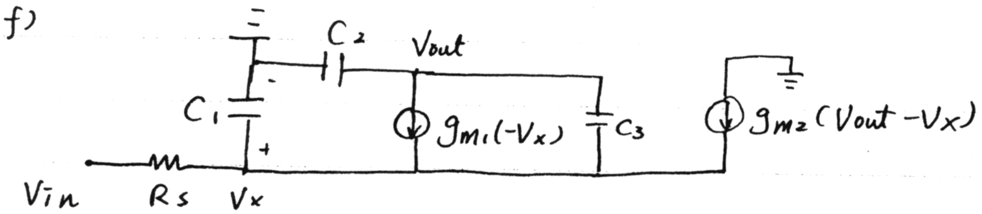
For Zin

$$I_x = sC_1 V_x + g_{m1} V_x - g_{m2} V_{out}$$

$$= \left[\frac{s^2 C_1 C_2 + s\left[g_{m1} C_2 + g_{m2} C_1\right]}{sC_2 + g_{m2}}\right]$$

$$\therefore Z_{in} = \frac{sC_2 + g_{m2}}{s^2 C_1 C_2 + s(g_{m1} C_2 + g_{m2} C_1)}$$ ※

(f)



$C_1 = C_{SB1} + C_{SB2} + C_{gs1}$
$C_2 = C_{gd1} + C_{gd2} + C_{dB1}$
$C_3 = C_{gs2}$

KCL @ Vout : $sC_2(-V_{out}) = g_{m1}(-V_x) + sC_3(V_{out} - V_x)$

$$\Rightarrow \frac{V_{out}}{V_x} = \frac{g_{m1} + sC_3}{s(C_2 + C_3)}$$

KCL @ Vx: $\frac{V_{in} - V_x}{R_s} + g_{m1}(-V_x) + g_{m2}(V_{out} - V_x) = sC_1 V_x + sC_3(V_x - V_{out})$

$$\frac{V_{in}}{R_s} = V_x\left(\frac{1}{R_s} + g_{m1} + g_{m2} + sC_1 + sC_3\right) - \frac{(g_{m2} + sC_3)(g_{m1} + sC_3)}{s(C_2 + C_3)} \cdot V_x$$

$$= \frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[\frac{C_2}{R_s} + \frac{C_3}{R_s} + g_{m1}C_2 + g_{m2}C_2\right] - g_{m1}g_{m2}}{s(C_2 + C_3)}$$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1} + sC_3}{s^2R_s(C_1C_2 + C_2C_3 + C_1C_3) + s[C_2 + C_3 + R_s(g_{m1}C_2 + g_{m2}C_2)] - g_{m1}g_{m2}R_s}$$

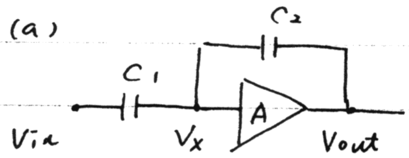
For Zin

$I_x = g_{m1}V_x + g_{m2}(V_x - V_{out}) + sC_1V_x + sC_3(V_x - V_{out})$

$$= \left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s[g_{m1}C_2 + g_{m2}C_2] - g_{m1}g_{m2}}{s(C_2 + C_3)}\right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s(C_2 + C_3)}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m1}C_2 + g_{m2}C_2) - g_{m1}g_{m2}}$$

6.4 (a)



$A = -\infty$

(i) At low frequency, Vx is like virtual ground

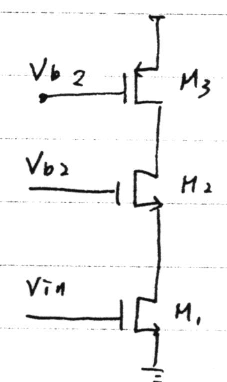
$SC_1 V_{in} = -SC_2 V_{out}$

$$\frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2}$$

(ii) At high frequency, C_1, C_2 is like a short circuit

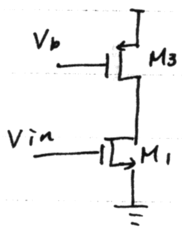
$$\frac{V_{out}}{V_{in}} = 1$$

(b) At low frequency, the equivalent circuit is shown as



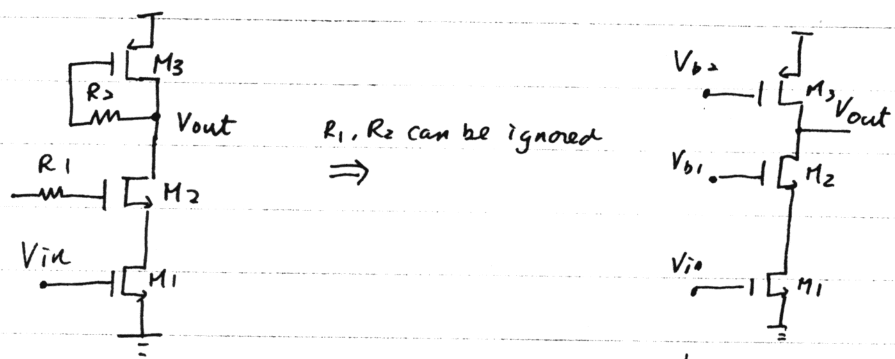
$A_v \cong -g_{m1} r_{o3} \to \infty$, if $\lambda = 0$

(ii) At high frequency, the equivalent circuit



$$A_V = -g_{m1}(r_{o1} // r_{o3}) \rightarrow \infty \text{ if } \lambda = 0$$

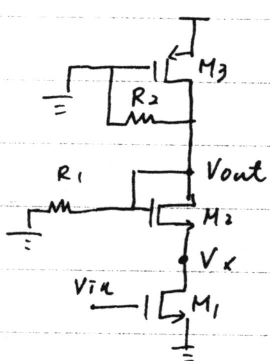
(c) (i) At low frequency, the equivalent circuit



The impedance @ Vout $= \frac{1}{g_{m3}}$

$$A_V \cong -g_{m1} \cdot 1/g_{m3} = -\frac{g_{m1}}{g_{m3}}$$

(ii) At high frequency.



$$\frac{V_X}{V_{in}} = -g_{m1} \cdot \underbrace{\frac{1}{g_{m2}}}$$

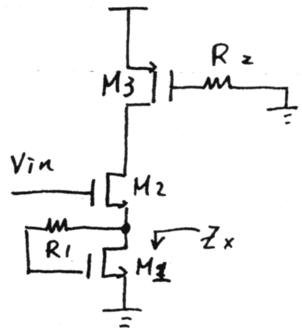
the impedance looking into V_x

The impedance @ Vout $= R_1 // R_2$

$$\therefore A_V = \left(-g_{m1} \cdot \frac{1}{g_{m2}}\right) \cdot g_{m2} \cdot (R_1 // R_2) = -g_{m1}(R_1 // R_2)$$

#

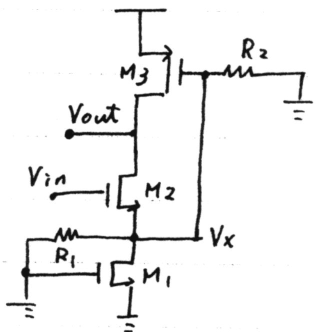
(d) (i) At low frequency, the equivalent circuit is



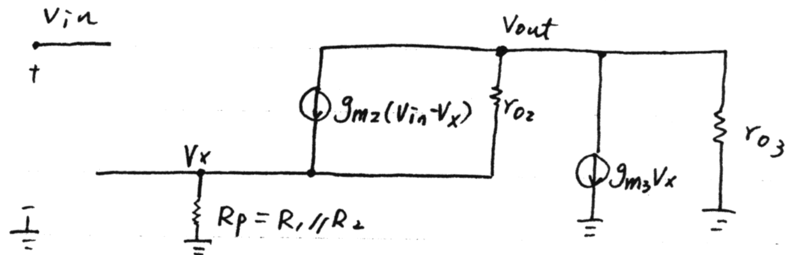
$$\frac{V_{out}}{V_{in}} = \frac{g_{m2}\left(r_{o3} // (1+g_{m2}r_{o2})Z_x\right]}{1+g_{m2}Z_x} \cong \frac{g_{m2}(r_{o3}//r_{o2})}{1+\frac{g_{m2}}{g_{m1}}} \to \infty \text{ if } \lambda=0$$

$$Z_x = \frac{1}{g_m}$$

(ii) At high frequency



Small-signal model



KCL @ V_x, V_{out} : $\frac{V_x}{R_p} = g_{m2}(V_{in}-V_x) + \frac{V_{out}-V_x}{r_{o2}} = -\left(g_{m3}V_x + \frac{V_{out}}{r_{o3}}\right)$

$$\frac{V_x}{R_p} = -\left(g_{m3}V_x + \frac{V_{out}}{r_{o3}}\right) \Rightarrow \frac{V_{out}}{V_x} = -r_{o3}\left(g_{m3}+\frac{1}{R_p}\right)$$

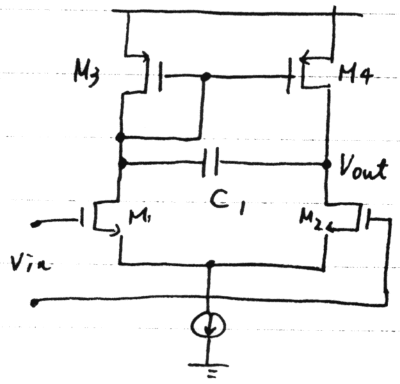
$$g_{m2}(V_{in}-V_x) + \frac{V_{out}-V_x}{r_{o2}} = \frac{V_x}{R_p} \Rightarrow g_{m2}V_{in} = \left(\frac{1}{R_p}+\frac{1}{r_{o2}}+g_{m2}\right)V_x + \frac{V_{out}}{r_{o2}}$$

$$\Rightarrow g_{m2}V_{in} = \left[-\left(\frac{1}{R_p}+\frac{1}{r_{o2}}+g_{m2}\right)\frac{1}{r_{o3}\left(g_{m3}+\frac{1}{R_p}\right)} + \frac{1}{r_{o2}}\right]V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{-g_{m2}r_{o3}\left(g_{m3}+\frac{1}{R_p}\right)}{\left(\frac{1}{R_p}+\frac{1}{r_{o2}}+g_{m2}\right) - \frac{r_{o3}}{r_{o2}}\left(g_{m3}+\frac{1}{R_p}\right)} \to \infty \text{ if } \lambda=0$$

#

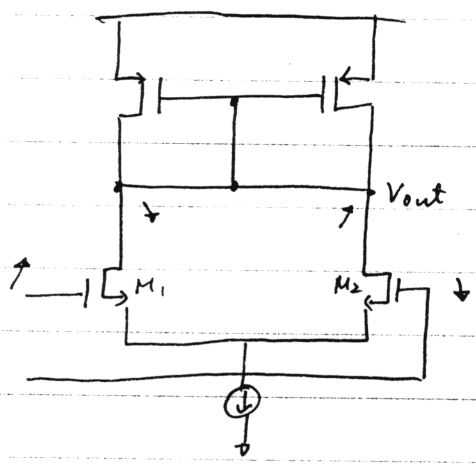
6.5 (a) (i) At low frequency



C_1 is like an open circuit @ very low frequency

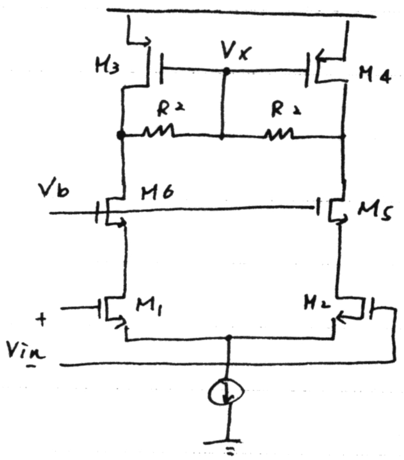
$$\Rightarrow \frac{V_{out}}{V_{in}} = -g_{m1}(r_{o2} \parallel r_{o4}) \rightarrow \infty \text{ if } \lambda = 0$$

(ii) At very high frequency, C_1 is like a short circuit

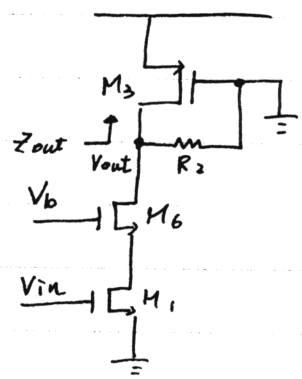


Gain = 0

(b) (i) At low frequency, the equivalent circuit is



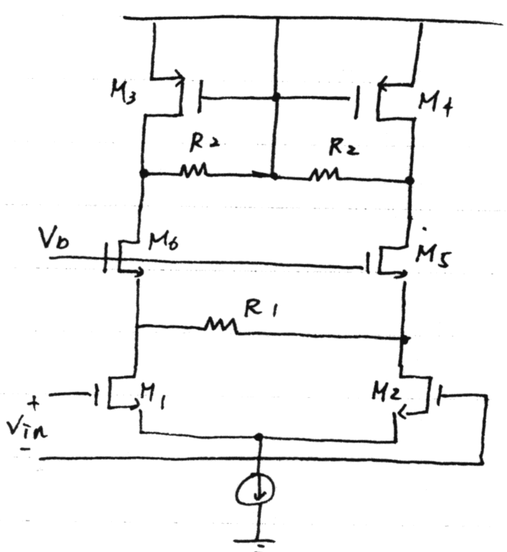
Vx is virtual ground

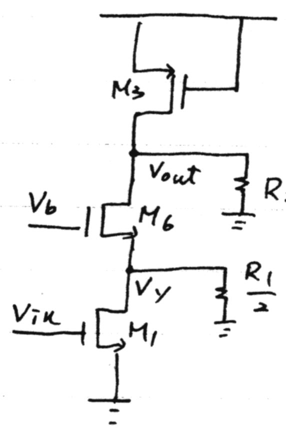


$Z_{out} \cong r_{o3} // R_2 \cong R_2$

$A_v = -g_{m1} \cdot (R_2 // r_{o3}) \cong -g_{m1} R_2$ ※

(ii) At high frequency

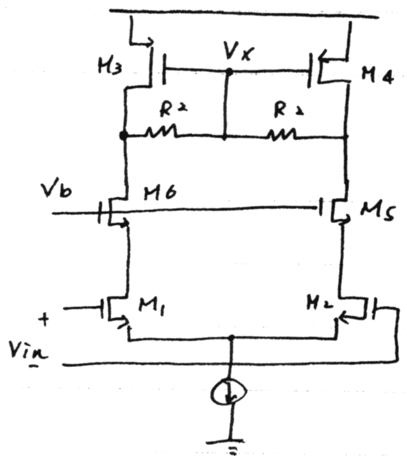




$\frac{V_y}{V_{in}} = -g_{m1} \left(\frac{1}{g_{m6}} // \frac{R_1}{2} \right)$, $\frac{V_{out}}{V_y} \cong +g_{m6} \cdot R_2$

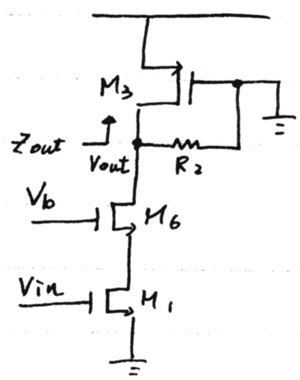
$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1} g_{m6} R_1 R_2}{(2 + g_{m6} \cdot R_1)}$$ ※

(b) (i) At low frequency, the equivalent circuit is



half circuit →

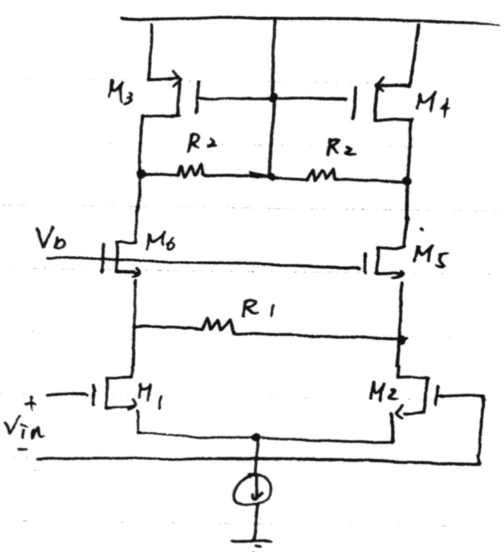
Vx is virtual ground



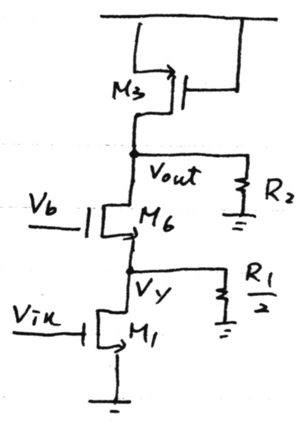
$$Z_{out} \cong r_{o3} // R_2 \cong R_2$$

$$A_V = -g_{m1} \cdot (R_2 // r_{o3}) \cong -g_{m1} R_2$$ *

(ii) At high frequency



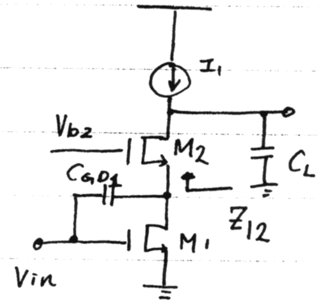
half circuit →



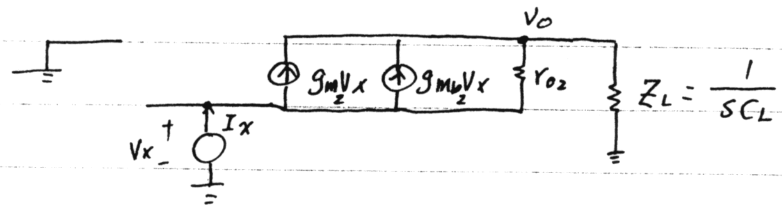
$$\frac{V_y}{V_{in}} = -g_{m1}\left(\frac{1}{g_{m6}} // \frac{R_1}{2}\right), \quad \frac{V_{out}}{V_y} \cong +g_{m6} \cdot R_2$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1} g_{m6} R_1 R_2}{(2 + g_{m6} \cdot R_1)}$$ *

6.6



The impedance Z_{12} can be derived from the following small signal model



KCL @ V_o : $\frac{V_o}{Z_L} + \frac{V_o - V_x}{r_{o_2}} = (g_{m_2} + g_{mb_2})V_x \Rightarrow \left(\frac{1}{Z_L} + \frac{1}{r_{o_2}}\right)V_o = \left(g_{m_2} + g_{mb_2} + \frac{1}{r_{o_2}}\right)V_x$

$$\Rightarrow V_o = \left(\frac{g_{m_2} + g_{mb2} + \frac{1}{r_{o2}}}{1 + \frac{Z_L}{r_{o2}}}\right)V_x$$

$$\Rightarrow I_x = \frac{V_o}{Z_L} = \left[\frac{g_{m_2} + g_{mb_2} + \frac{1}{r_{o2}}}{1 + \frac{Z_L}{r_{o2}}}\right]V_x \Rightarrow \frac{V_x}{I_x} = Z_{12} = \frac{1 + \frac{Z_L}{r_{o2}}}{g_{m_2} + g_{mb_2} + \frac{1}{r_{o2}}}$$

$$\Rightarrow Z_{12} = \frac{r_{o2} + Z_L}{1 + (g_{m_2} + g_{mb2})r_{o2}}$$

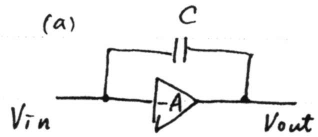
The miller multiplication for C_{GD1} = $1 + g_{m1} Z_{12}$

$$= 1 + \frac{g_{m1}(r_{o2} + Z_L)}{1 + (g_{m2} + g_{mb2})r_{o2}} \quad — (1)$$

If C_L is relatively large $\Rightarrow \left|\frac{1}{sC_L}\right| \ll r_{o2}$

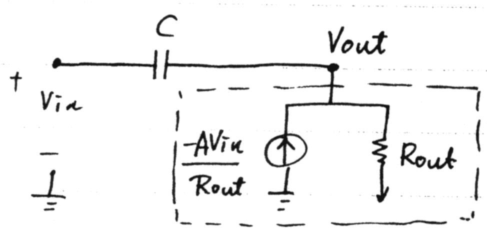
eq (1) can be approximated as $\cong 1 + \frac{g_{m1} r_{o2}}{1 + (g_{m2} + g_{mb2})r_{o2}} \cong 1 + \frac{g_{m1}}{g_{m2} + g_{mb2}}$

6.7 (a)



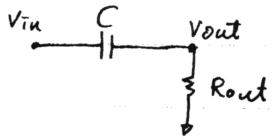
Assume the amplifier output resistance Rout

The small signal model is as follows



← The Norton equivalent circuit for amplifier

As we can see, the above circuit forms a high pass network



Thus, when there is a step ΔV at the input, output will follow Input, a step ΔV, first. Then, it will settle down to $-AV_{in}$ as the steady state

(b) KCL @ Vout :

$$-\frac{AV_{in}}{R_{out}} = \frac{V_{out}}{R_{out}} + sC(V_{out} - V_{in})$$

$$\Rightarrow \left(sC - \frac{A}{R_{out}}\right)V_{in} = \left(\frac{1}{R_{out}} + sC\right)V_{out} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{sCR_{out} - A}{1 + sCR_{out}}$$

✱

for the step response, $x(t) = u(t)$, $t \geq 0 \rightarrow X(s) = \frac{1}{s}$

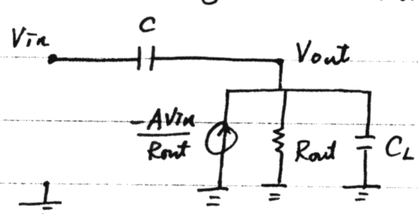
$$Y(s) = \frac{1}{s} \cdot \frac{V_{out}}{V_{in}}(s) = \frac{sCR_{out} - A}{s(1 + sCR_{out})} = \frac{-A}{s} + \frac{(A+1) \cdot R_{out}C}{1 + sCR_{out}}$$

$$\Rightarrow y(t) = -A\,u(t) + (A+1)\,e^{-\frac{t}{R_{out}C}}, \quad t \geq 0$$

For a ΔV input step, output $= -A \cdot \Delta V + (A+1) \cdot \Delta V \cdot e^{-\frac{t}{R_{out}C}}$

✱

6.8 (a) Small-signal circuit model



when input has ΔV jump, Vout will follow and the output jump $= \left(\frac{C}{C_L + C}\right) \Delta V$ ✱

(b) The transfer function $H(s)$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} \quad : \quad KCL \text{ @ } V_{out}$$

$$\Rightarrow \quad -\frac{A V_{in}}{R_{out}} = \frac{V_{out}}{R_{out}} + sC(V_{out} - V_{in}) + sC_L V_{out}$$

$$\Rightarrow \quad V_{in}\left(sC - \frac{A}{R_{out}}\right) = V_{out}\left(\frac{1}{R_{out}} + sC + sC_L\right)$$

$$\Rightarrow \quad \frac{V_{out}}{V_{in}} = \frac{sCR_{out} - A}{1 + sR_{out}(C + C_L)} \quad ✱$$

step response

$$Y(s) = \frac{1}{s} \cdot \frac{sCR_{out} - A}{[1 + sR_{out}(C + C_L)]} = \frac{-A}{s} + \frac{[(A+1)C + AC_L]R_{out}}{1 + sR_{out}(C + C_L)} = \frac{-A}{s} + \frac{\frac{(A+1)C + AC_L}{C + C_L}}{s + \frac{1}{R_{out}(C + C_L)}}$$

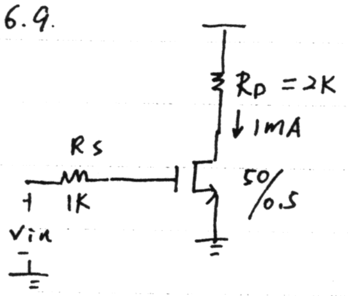
$$y(t) = -A\,u(t) + \frac{(A+1)C + AC_L}{C + C_L} \cdot e^{-\frac{t}{R_{out}(C + C_L)}}\, u(t)$$

For a step ΔV @ the input

$$\text{output} = -A\,\Delta V + \left[\frac{(A+1)C + A \cdot C_L}{C + C_L}\right] \cdot \Delta V \cdot e^{-\frac{t}{R_{out}(C + C_L)}}$$

✱

6.9.



$$\lambda = 0.1$$

$$C_{ox} = \frac{\epsilon_{SiO_2}}{t_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14}}{9 \times 10^{-7}} = 3.835 \times 10^{-7}$$

$$\mu_n = 350$$

$$I_D = 10^{-3} = \frac{1}{2} \times 350 \times 3.835 \times 10^{-7} \times \left(\frac{50}{0.5 - 2 \times 0.08}\right)(V_{gs} - V_T)^2 (1 + 0.1 \times V_{ds})$$

$$= 67.113 \times 10^{-6} \times \frac{50}{0.34} \times 1.1 \times (V_{in} - 0.7)^2$$

$$\Rightarrow V_{in} = 1.0035$$

$$g_m = \frac{2I_D}{(V_{gs} - V_t)} = 6.59 \times 10^{-3}$$

$$C_{gs} = \frac{2}{3} C_{ox} W L + C_{ov} \cdot W$$

$$= \frac{2}{3} \times 3.835 \times 10^{-7} \times 50 \times (0.5 - 0.08 \times 2) \times 10^{-8} + 3.835 \times 10^{-7} \times 0.08 \times 10^{-4} \times 50 \times 10^{-4}$$

$$= 53.7 \times 10^{-15}$$

$$C_{gd} = C_{gdo} \cdot W = 0.4 \times 10^{-11} \times 50 \times 10^{-6} = 2 \times 10^{-16}$$

$$C_{dB} = \frac{0.56 \times 10^{-3} \times 75 \times 10^{-12}}{\left(1 + \frac{1}{0.9}\right)^{0.6}} + \frac{0.35 \times 10^{-11} \times 103 \times 10^{-6}}{\left(1 + \frac{1}{0.9}\right)^{0.2}} = 2.714 \times 10^{-14}$$

According to eq (20)

$$\text{zero} = \frac{g_m}{C_{gd}} = \frac{6.59 \times 10^{-3}}{2 \times 10^{-6}} = 3.3 \times 10^{13}\ \text{rad/sec}$$ ☆

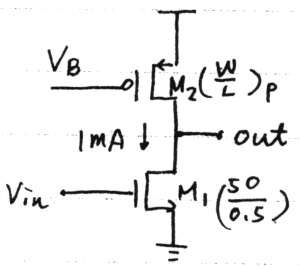
pole is the root of $R_S R_D (C_{gs} C_{gd} + C_{gs} C_{dB} + C_{gd} C_{dB}) s^2$
$+ [R_S (1 + g_m R_D) C_{gd} + R_S C_{gs} + R_D (C_{gd} + C_{dB})] s + 1$ ⋯ from eq (6.20)

$$\Rightarrow 2.95 \times 10^{-21} s^2 + 1.112 \times 10^{-10} s + 1 = 0$$

$$\omega_{p1} = -14.82 \times 10^9\ \text{rad/sec}$$ ☆

$$\omega_{p2}\ -22.88 \times 10^9\ \text{rad/sec}$$ ☆

6.10



(a) The maximum output level = 2.6 V

$\rightarrow$ V_B can be as low as $2.6 - |V_{THP}| = 2.6 - 0.8 = 1.8V$

Let's choose output DC bias @ 1.5V, such that M_1, M_2 are both in saturation region

Thus, $I_{D1} = 10^{-3} = \frac{1}{2} \times 350 \times 3.835 \times 10^{-7} \times \left(\frac{50}{0.5 - 2\times 0.08}\right)(V_{in} - 0.7)^2 (1 + 0.1 \times 1.5)$

$\Rightarrow V_{in} = 0.997 \cong 1V$

Also, $I_{D2} = 10^{-3} = \frac{1}{2} \times 100 \times 3.835 \times 10^{-7} \times \left(\frac{W_P}{0.5 - 2\times 0.09}\right)(3 - 1.8 - 0.8)^2 (1 + 0.2 \times 1.5)$

$W_P \cong 80.5 \mu m = 81 \mu m$

Therefore, we can choose $\left(\frac{W}{L}\right)_P = \left(\frac{81 \mu m}{0.5 \mu m}\right)$, with gate bias 1.8V so that M_1, M_2 are both in saturation region and $V_{out} \cong 1.5V$

$V_{out, low} = V_{in} - |V_{THN}| = 0.3V$

Thus, the maximum output peak-to-peak swing $= 2.6 - 0.3 = 1.3V$ ✱

(b) This problem is similar to problem 6.9 except

$R_D \rightarrow (r_{o1} \parallel r_{o2})$

$C_{DB} = C_{DB1} + C_{DB2} + C_{gd2}$

$\therefore g_{m1} = \frac{2 I_D}{V_{GS} - V_t} = 2\times 10^{-3} / 0.297 = 6.73 \times 10^{-3}$

$r_{oP} = 1 \Big/ \left(\frac{\lambda_P I_D}{1 + \lambda_P V_{DS}}\right) \cong 6.5K$

$r_{ON} = 1 \Big/ \left(\frac{\lambda_N I_D}{1 + \lambda_N V_{DS}}\right) = 11.5K$

$\therefore R_D = r_{oP} \parallel r_{ON} = 4.1K$

$R_S = 1K$

$C_{gs1} = \frac{2}{3} C_{ox} W_1 L_1 + C_{ox} W \cdot \Delta L = 58.8 \times 10^{-15} F$

$C_{gd1} = 2 \times 10^{-16}$

$$C_{dB_1} = \frac{0.56\times10^{-3}\times 75\times10^{-12}}{\left(1+\frac{1.5}{0.9}\right)^{0.6}} + \frac{0.35\times10^{-11}\times 103\times10^{-6}}{\left(1+\frac{1.5}{0.9}\right)^{0.2}} = 23.38\times10^{-15}F$$

$$C_{dB2} = \frac{0.94\times10^{-3}\times 121.5\times10^{-12}}{\left(1+\frac{1.5}{0.9}\right)^{0.5}} + \frac{0.32\times10^{-11}\times 165\times10^{-6}}{\left(1+\frac{1.5}{0.9}\right)^{0.3}} = 70.33\times10^{-15}F$$

$$C_{gd2} = 81\times 0.3\times10^{-11}\times10^{-6} = 0.243\times10^{-15}F$$

$$\omega_z = -\frac{g_{m1}}{C_{gd1}} = -\frac{6.59\times10^{-3}}{2\times10^{-16}} = -3.3\times10^{13}\ rad/sec$$

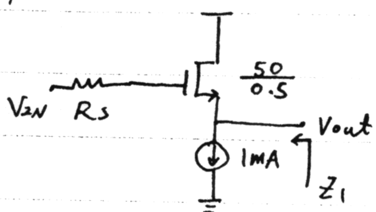
ω_{p1}, ω_{p2} is the root of the equation

$$R_S R_D(C_{gs1}C_{gd1} + C_{gs1}C_{dB} + C_{gd1}C_{dB}) + [R_S(1+g_{m1}R_D)C_{gd1} + R_S C_{gs1} + R_D(C_{gd1}+C_{dB})] + 1$$

$$\Rightarrow \omega_{p1} = -2.2\times10^{9}\ rad/sec$$

$$\omega_{p2} = -17.36\times10^{9}\ rad/sec$$

6.11



Assume $\gamma = 0$

$$g_m = 2I/(V_{gs}-V_t)$$

$$= \frac{2\times10^{-3}}{0.3} = 6.67\times10^{-3}$$

From eq (6.49) $Z_1 = \dfrac{R_s C_{gs}\cdot s + 1}{g_m + C_{gs} s}$

Since $\frac{1}{g_m} < R_s$

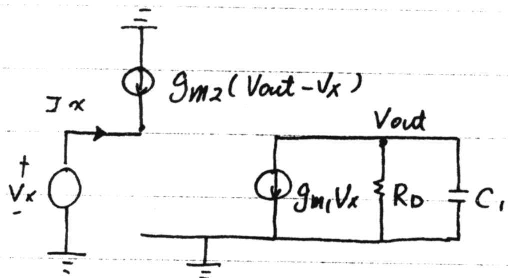
, thus Z_1 is inductive and the equivalent inductance is

$$= \frac{C_{gs}}{g_m}\left(R_s - \frac{1}{g_m}\right)$$

$C_{gs} = 58\times10^{-15}$ F

$\therefore L = 8.56\times10^{-8}$ H

6.12 (a)



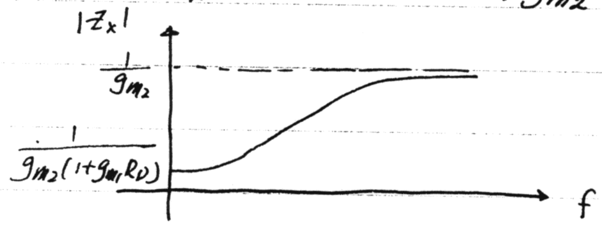
$$V_{out} = -g_{m1}V_X \left(R_D \| \frac{1}{sC_1}\right)$$

$$I_X = -g_{m2}(V_{out}-V_X) = -g_{m2}V_{out} + g_{m2}V_X = \left[g_{m2}g_{m1}\left(R_D\|\frac{1}{sC_1}\right) + g_{m2}\right]V_X$$

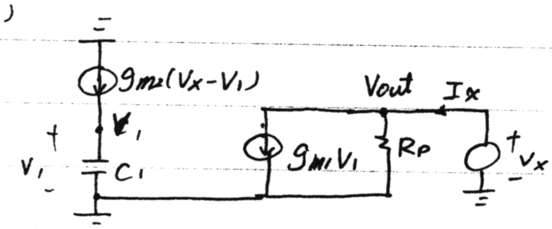
$$Z_X = \frac{V_X}{I_X} = \frac{1}{g_{m2}g_{m1}\frac{R_D/sC_1}{R_D+1/sC_1}+g_{m2}} = \frac{1}{g_{m2}\left[\left(\frac{g_{m1}R_D}{1+sR_DC_1}\right)+1\right]}$$

Thus, $Z_X(s\to 0) = \dfrac{1}{g_{m2}(1+g_{m1}R_D)}$

$Z_X(s\to\infty) = 1/g_{m2}$



(b)



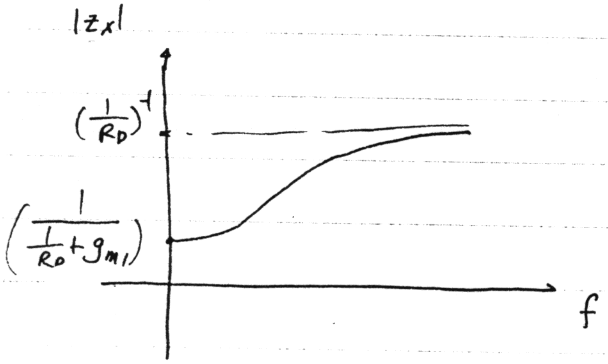
KCL @ V_1: $g_{m2}(V_X - V_1) = sC_1V_1$

$$\Rightarrow V_1 = \left(\frac{g_{m2}}{sC_1+g_{m2}}\right)V_X$$

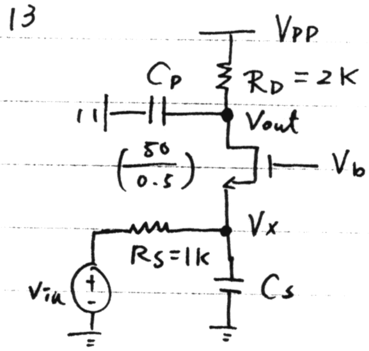
KCL @ V_{out}: $I_X = \frac{V_X}{R_D} + g_{m1}V_1$

$$= \frac{V_X}{R_D} + \frac{g_{m1}g_{m2}}{sC_1+g_{m2}}V_X$$

$$\Rightarrow Z_X = \frac{1}{\frac{1}{R_D}+\frac{g_{m1}g_{m2}}{sC_1+g_{m2}}}$$



6.13



from eq (6.53)

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb}) R_D}{1 + (g_m + g_{mb}) R_S} \frac{1}{\left(1 + \frac{C_S}{g_m + g_{mb} + R_S^{-1}} \cdot s\right)(1 + R_D C_D s)}$$

Assume V_b is chosen appropriately such that $V_x \sim 0$ (no body effect)

$g_m \sim 6.59 \times 10^{-3}$

$g_{mb} = \left[\gamma / 2\sqrt{2\phi_f} \right] g_m = 1.563 \times 10^{-3}$

$C_S = C_{SB} + C_{gs} = 42.4 \times 10^{-15} + 58.8 \times 10^{-15}$

$C_D = C_{DB} = 27.14 \times 10^{-15} F$ } from problem 9

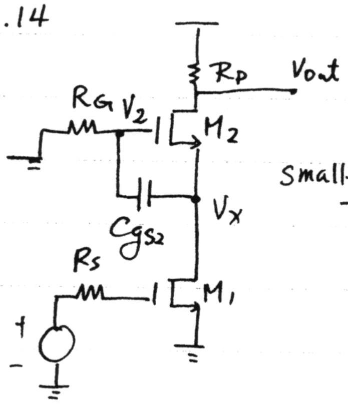
$$A_V(\text{low frequency}) = \frac{(g_m + g_{mb}) R_D}{1 + (g_m + g_{mb}) R_S} = 1.44$$

$$\omega_{P_1} = -\frac{g_m + g_{mb} + R_S^{-1}}{C_S} = -\frac{6.59 \times 10^{-3} + 1.563 \times 10^{-3} + 10^{-3}}{42.4 \times 10^{-15} + 58.8 \times 10^{-15}} = -9.044 \times 10^{10} \text{ rad/s}$$

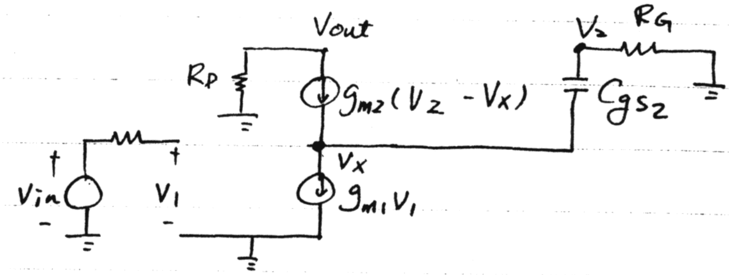
$$\omega_{P_2} = -\frac{1}{R_D C_D} = -\frac{1}{2 \times 10^3 \times 2.714 \times 10^{-14}} = -1.84 \times 10^{10} \text{ rad/s}$$

Compared with the pole locations in problem 9, the poles for Common-gate configuration are much larger because there is no Miller-effect for C_{gd} in this case

6.14



small-signal model



KCL @ V_2 : $\frac{V_2}{R_G} = sC_{gs2}(V_x - V_2)$

$$\Rightarrow \frac{V_2}{V_x} = \frac{sC_{gs2}}{\frac{1}{R_G} + sC_{gs2}} = \frac{sR_GC_{gs2}}{1+sR_GC_{gs2}} \quad \text{—} ①$$

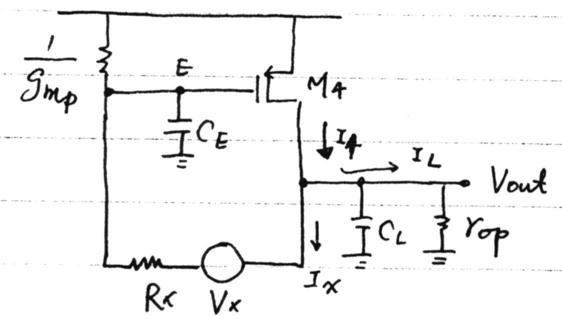
KCL @ V_{out} : $V_{out} = -g_{m2}(V_2 - V_x)R_D = \left[\frac{g_{m2}R_D}{1+sR_GC_{gs2}}\right]V_x \quad \text{—} ②$

KCL @ V_x : $g_{m1}V_1 = g_{m1}V_{in} = (g_{m2} + sC_{gs2})(V_2 - V_x)$

$$= \frac{-(g_{m2}+sC_{gs2})}{1+sR_GC_{gs2}}V_x \quad \text{—} ③$$

from ②, ③, $\frac{V_{out}}{V_{in}} = \frac{-g_{m1}g_{m2}R_D}{g_{m2}+sC_{gs2}}$ #

6.15



For zero frequency, $I_L = 0$ & $I_4 = I_x$

$$I_4 = -g_{mp} V_E$$

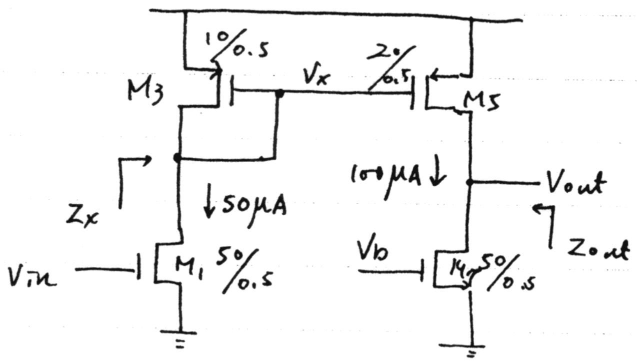
$$I_x = V_E (g_{mp} + sC_E)$$

$$\therefore \quad I_4 = I_x \Rightarrow \quad -g_{mp} = g_{mp} + sC_E$$

$$S_z = \frac{-2g_{mp}}{C_E}$$

*

6.18 Half circuit can be drawn as follows



Since $R_S = 0$, According to (6.20) & (6.76)

$$\frac{V_{out}}{V_{in}}(s) = -\frac{(C_{gd1}\cdot s - g_{m1})\cdot \frac{1}{g_{m3}}}{\frac{1}{g_{m3}}(C_{gd1}+C_X)s+1}\cdot g_{m5}\cdot(r_{o5}//r_{o7})\cdot\frac{1}{1+(r_{o5}//r_{o7})\cdot C_L\cdot s}$$

where $C_L = C_{dB7} + C_{gd7} + C_{dB5}$

$C_X = C_{gs3} + C_{dB1} + C_{gs5} + C_{dB3} + C_{gd5}(1+g_{m5}(r_{o5}//r_{o7}))$

$Z_{out} = (r_{o5}//r_{o7})$

$Z_X = \frac{1}{g_{m3}}$

First of all, let's calculate Vx operating point

$$I_{d3} = 50\times10^{-6} = \frac{1}{2}\times100\times3.835\times10^{-7}\times\frac{10}{0.5-0.09\times2}(3-V_X-0.8)^2(1+0.2(3-V_X))$$

$$V_X \doteqdot 1.94\,V$$

For Vin operating point

$$I_{d1} = 50\times10^{-6} = \frac{1}{2}\times350\times3.835\times10^{-7}\times\frac{50}{0.5-0.08\times2}(V_{gs1}-0.7)^2(1+0.1\cdot1.94)$$

$$V_{gs1} \doteqdot 0.765\,V$$

$$\Rightarrow g_{m1} = \frac{2I_{d1}}{(V_{gs1}-V_t)} \cong 1.54\times10^{-3}$$

$$g_{m3} = \frac{2I_{d1}}{(3-1.94-0.8)} \cong 3.73\times10^{-4} \Rightarrow g_{m5} = 2\cdot g_{m3} = 7.46\times10^{-4}$$

$g_{ds5} = 2\times 50\times 10^{-6}\cdot \lambda \Big/ (1+\lambda\cdot 1.06) = 10^{-4}\cdot 0.2 \Big/ 1.212 \cong 1.649\times 10^{-5}$

$g_{ds7} = 10^{-4}\times 0.1 \Big/ (1+0.196) \cong 8.36\times 10^{-6}$

$r_{o5} // r_{o7} = 40290$

$$C_L = C_{DB5} + C_{DB7} + C_{gd7} = \left[\frac{0.94\times 10^{-3}\times 30\times 10^{-12}}{\left(1+\frac{1.06}{0.9}\right)^{0.5}} + \frac{0.32\times 10^{-11}\times 43\times 10^{-6}}{\left(1+\frac{1.06}{0.9}\right)^{0.3}}\right]$$

$$+\left[\frac{0.56\times 10^{-3}\times 75\times 10^{-12}}{\left(1+\frac{1.94}{0.9}\right)^{0.6}} + \frac{0.35\times 10^{-11}\times 103\times 10^{-6}}{\left(1+\frac{1.94}{0.9}\right)^{0.2}}\right] + 50\times 0.4\times 10^{-17}$$

$$= 19.22\times 10^{-15} + 21.36\times 10^{-15} + 2\times 10^{-16} = 40.78\times 10^{-15}$$

$C_{gs3} = \frac{2}{3}\times 3.835\times 10^{-7}\times 10\times 0.32 + 3.835\times 10^{-7}\times 10\times 0.09 = 11.633\times 10^{-15}$

$C_{dB1} \cong C_{dB7} \cong 21.36\times 10^{-15}$

$C_{gs5} = 2\cdot C_{gs3} = 23.266\times 10^{-15}$

$C_{dB3} = \frac{1}{2}\cdot C_{DB5} = 9.61\times 10^{-15}$

$C_{gd5} = 6.3\times 10^{-11}\times 20\times 10^{-6} = 6\times 10^{-17}$

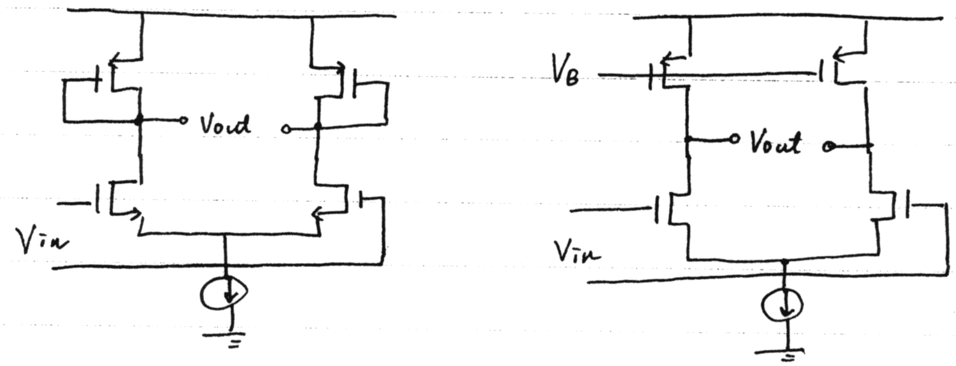
$C_X = \left[11.633 + 21.36 + 23.27 + 9.61 + 0.06\left(1+g_{m5}\cdot(r_{o5}//r_{o7})\right)\right]\times 10^{-15} = 67.732\times 10^{-15}$

$$\therefore\ \omega_z = \frac{g_{m1}}{C_{gd1}} = 7.7\times 10^{12}\ \text{rad/sec}$$

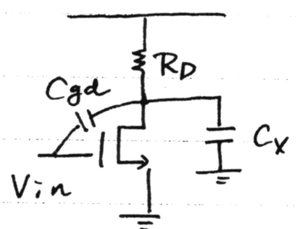
$$\omega_{p1} = -\frac{1}{C_L\cdot(r_{o5}//r_{o7})} = -\frac{1}{40290\times 40.78\times 10^{-15}} = 6.08\times 10^{8}\ \text{rad/sec}$$

$$\omega_{p2} = -\frac{g_{m3}}{(C_{gd1}+C_X)} = \frac{-3.73\times 10^{-4}}{2\times 10^{-16} + 67.732\times 10^{-15}} = 5.5\times 10^{9}\ \text{rad/sec}$$

6.17 (a)



Both of these two differential pair can be simplified as common-source amplifier with different load resistance and capacitance



Since $R_S = 0$, equation (6.20) can be simplified as

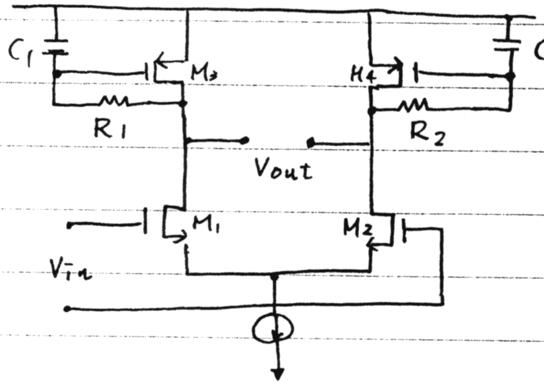
$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{gd}s - g_m)R_D}{s[R_D(C_{gd}+C_x)]+1}$$

where $\begin{cases} R_D = \frac{1}{g_{mp}} & \text{for diode connected load} \\ C_X = C_{dBN} + C_{dBp} + C_{gsp} \end{cases}$

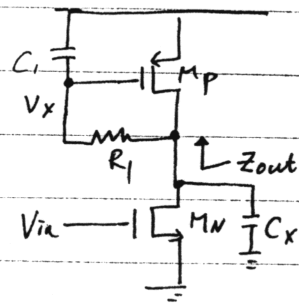
$\begin{cases} R_D \cong (r_{oN} \parallel r_{op}) & \text{for current mirror load} \\ C_X = C_{dBN} + C_{dBp} + C_{gdp} \end{cases}$

Although there is right-half-plane zero, however this zero is much larger than the dominant pole.
Therefore, the maximum phase shift it can achieve is $\sim 90°$ before the gain is down to unity.

(b)



half-circuit →



(i) At low frequency, C_1 is open circuit,

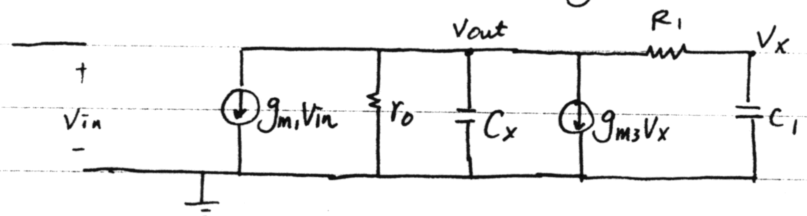
M_p is like a diode-connected device $\rightarrow Z_{out} \sim \frac{1}{g_{mp}}$

(ii) At high frequency, C_1 is short circuit.

M_p is like a current source device $\rightarrow Z_{out} \sim (r_{oN} // r_{op})$

Since $(r_{oN} // r_{op}) >> \frac{1}{g_{mp}}$, Z_{out} exhibits an inductive behavior

For transfer function, small-signal model



, $r_o = (r_{o1} // r_{o3})$

KCL @ V_x : $\dfrac{V_{out} - V_x}{R_1} = sC_1V_x \Rightarrow V_{out} = (1 + sR_1C_1)V_x$

$$\Rightarrow V_x = \frac{V_{out}}{1 + sR_1C_1}$$

KCL @ V_{out} : $-g_{m1}V_{in} = V_{out}\left(\frac{1}{r_o} + sC_x\right) + \frac{1}{R_1}(V_{out} - V_x) + g_{m3}V_x$

$$= V_{out}\left(\frac{1}{r_o} + sC_x + \frac{1}{R_1}\right) + \left(g_{m3} - \frac{1}{R_1}\right) \cdot \frac{V_{out}}{1 + sR_1C_1}$$

$$-g_{m1}V_{in} = V_{out}\left(\frac{\frac{1}{r_o}+\frac{1}{R_1}+sC_x+\left(\frac{R_1}{r_o}+1\right)sC_1+s^2R_1C_1C_x+g_{m3}-\frac{1}{R_1}}{1+sR_1C_1}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_{m1}(1+sR_1C_1)}{s^2R_1C_1C_x+s\left(C_1+\frac{R_1C_1}{r_o}+C_x\right)+\left(g_{m3}+\frac{1}{r_o}\right)}$$

From the above transfer function,

$$\omega_z = -\frac{1}{R_1C_1}$$

the sum of two poles $= -\frac{1}{R_1C_1C_x}\left(C_1+\frac{R_1C_1}{r_o}+C_x\right) = -\frac{1}{R_1C_1}\left(1+\frac{C_1}{C_x}\left(1+\frac{R_1}{r_o}\right)\right)$

usually $C_1 > C_x$, C_1 at least $= C_{gs3}$

Thus, the sum of two poles $> -\frac{2}{R_1C_1}$, which means that at least one of the poles are larger than zero $\Rightarrow$ It's quite impossible to produce 135° phase shift

Thus, this circuit still can't produce 135° phase shift.

However, it's more likely for it to generate 90° phase shift @ unity-gain frequency.

CHAPTER 7: NOISE

(7.1)

$$|A_v| = g_m R_D$$

$$\overline{V_{n,out}^2} = \left(4KT \tfrac{2}{3} g_m + \tfrac{4KT}{R_D}\right) R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{A_v^2} = 4KT\left(\frac{2}{3}\frac{1}{g_m} + \frac{1}{g_m^2 R_D}\right)$$

$$\overline{V_{n,in}}_{TOT} = \sqrt{\overline{V_{n,in}^2} \cdot BW}$$

$$g_m = \sqrt{2 I_D \mu_n C_{ox} (W/L_{eff})} = \sqrt{2(1mA)(134.28 \tfrac{\mu A}{V^2})(\tfrac{50\mu m}{0.34\mu m})} \cong 6.28 \tfrac{mA}{V}$$

$$4KT = 1.656 \times 10^{-20} V\text{-}C, \quad R_D = 2k\Omega, \quad BW = 100\,MHz$$

$$\therefore \overline{V_{n,in}}_{TOT} = \sqrt{(1.966 \times 10^{-18} \tfrac{V^2}{Hz})(100\,MHz)} \cong 14\,\mu V \text{ rms}$$ //

(7.2) using eqn. (7.57)

$$\overline{V_{n,in}^2} = 4KT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{2}{3}\frac{g_{m2}}{g_{m1}^2}\right)$$

$$\overline{V_{n,in}} = \sqrt{4KT \tfrac{2}{3}}\sqrt{\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}}$$

$$\frac{g_{m2}}{g_{m1}^2} = \left(\frac{1}{5}\right)^2 \frac{1}{g_{m1}} \Rightarrow g_{m2} = \left(\frac{1}{5}\right)^2 g_{m1}$$

$$g_m = \frac{2I_D}{V_{gs} - V_T} \Rightarrow V_{gs} - V_T = \frac{2I_D}{g_m}$$

$$\therefore \text{Output Swing} = V_{DD} - (V_{gs1} - V_{T1}) - |(V_{gs2} - V_{T2})|$$

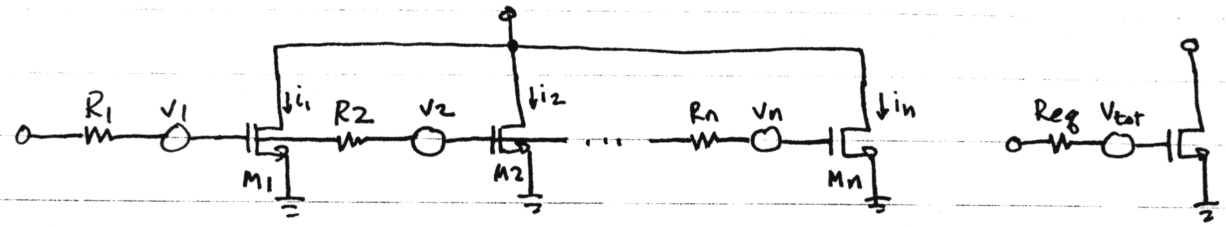
$$= V_{DD} - 2I_D\left(\frac{1}{g_{m1}} + \frac{1}{g_{m2}}\right)$$

$$= V_{DD} - 2I_D\left(\frac{1}{g_{m1}}\right)(1 + 5^2)$$

$$g_{m1} = \sqrt{2 I_D \mu_n C_{ox} (W/L_{eff})} = \sqrt{2(1mA)(134.28 \tfrac{\mu A}{V^2})(\tfrac{50\mu m}{0.34\mu m})} \approx 1.986 \tfrac{mA}{V}$$

$$\therefore \text{Output Swing} = 3V - 2(1mA)\left(\frac{1}{1.986\,mA/V}\right)(26) \approx 0.38\,V$$ //

(7.3)



The drain noise current of M_1 resulting from the gate resistance is

$$i_1 = g_{m1} v_1 \qquad \text{where } v_1 \text{ is the noise voltage of } R_1.$$

Similarly, $\quad i_2 = g_{m2}(v_1 + v_2)$

Thus, for transistor M_j, $\quad i_j = g_{mj}(v_1 + v_2 + \cdots + v_j)$

The total drain noise current is,

$$i_{tot} = i_1 + i_2 + \cdots + i_n$$
$$= g_{m1} v_1 + g_{m2}(v_1 + v_2) + \cdots + g_{mn}(v_1 + v_2 + \cdots + v_n)$$

If $g_{m1} = g_{m2} = \cdots = g_{mn} = \frac{g_m}{n}$ then

$$i_{tot} = \frac{g_m}{n}\left[n v_1 + (n-1) v_2 + \cdots + v_n\right]$$

Assuming $v_1, \ldots, v_n$ are uncorrelated,

$$\overline{i_{tot}^2} = \frac{g_m^2}{n^2}\left[n^2 \overline{v_1^2} + (n-1)^2 \overline{v_2^2} + \cdots + \overline{v_n^2}\right]$$

If $R_1 = R_2 = \cdots = R_n = \frac{R_g}{n}$ then $\overline{v_1^2} = \overline{v_2^2} = \cdots = \overline{v_n^2} = 4kTB\frac{R_g}{n}$

$$\overline{i_{tot}^2} = \frac{g_m^2}{n^2}\,\frac{4kTBR_g}{n}\left[n^2 + (n-1)^2 + \cdots + 1\right]$$

$$= g_m^2 (4kTB) R_g \frac{n(n+1)(2n+1)}{6n^3}$$

As $n \to \infty$

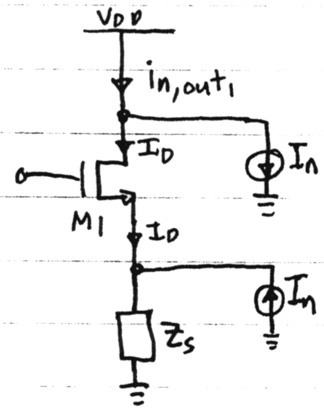
$$\overline{i_{tot}^2} = g_m^2 \left(4kTB\frac{R_g}{3}\right)$$

which can be referred to the input as

$$\overline{v_{tot}^2} = \frac{\overline{i_{tot}^2}}{g_m^2} = 4kTB\left(\frac{R_g}{3}\right)$$

$\Rightarrow$ lumped resistance $= \frac{R_g}{3}$ //

(7.4)



$I_{n,out_1} = I_D + I_n$, KCL @ drain

$I_D = \dfrac{-Z_s}{(\frac{1}{g_m} // r_o) + Z_s} I_n$, current divider @ source

$\therefore I_{n,out_1} = \left(\dfrac{-Z_s}{(1/g_m // r_o) + Z_s} + 1 \right) I_n$

$\therefore I_{n,out_1} = \dfrac{I_n}{Z_s(g_m + \frac{1}{r_o}) + 1}$ //

(7.5) $|A_v| = (g_{m_1} + g_{m_2})(r_{o_1} // r_{o_2})$

$\overline{V_{n,out}^2} = 4kT \frac{2}{3} (g_{m_1} + g_{m_2}) (r_{o_1} // r_{o_2})^2$

$\overline{V_{n,in}^2} = \dfrac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \frac{2}{3} \left(\dfrac{1}{g_{m_1} + g_{m_2}} \right)$

eqn(7.57) $\overline{V_{n,in}^2} = 4kT \frac{2}{3} \left(\frac{1}{g_{m_1}} + \frac{g_{m_2}}{g_{m_1}^2} \right)$

increasing g_{m_2} increases $\overline{V_{n,in}^2}$ in eqn. (7.57)
but reduces $\overline{V_{n,in}^2}$ for amplifier in figure 7.49.

(7.6)(a) $|A_v| = \dfrac{g_m R_D}{1 + g_m R_S}$

$\overline{V_{n,out}^2} = 4kT R_D + 4kT \frac{2}{3} \frac{1}{g_m} \left(\frac{g_m R_D}{1 + g_m R_S} \right)^2 + 4kT \frac{1}{R_S} \left(\frac{R_S}{\frac{1}{g_m} + R_S} \right)^2 R_D^2$

$= 4kT R_D + 4kT \frac{2}{3} \frac{1}{g_m} \left(\frac{g_m R_D}{1 + g_m R_S} \right)^2 + 4kT R_S \left(\frac{g_m R_D}{1 + g_m R_S} \right)^2$

$\overline{V_{n,in}^2} = \dfrac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \frac{2}{3} \frac{1}{g_m} + 4kT R_S + 4kT R_D \left(\frac{1 + g_m R_S}{g_m R_D} \right)^2$ //

(7.6)(b) $|A_v| = g_m \left(\frac{1}{g_m} // R_S \right)$

$\overline{V_{n,out}^2} = \left(4kT \frac{2}{3} g_m + 4kT \frac{1}{R_S} \right) \left(\frac{1}{g_m} // R_S \right)^2$

$\overline{V_{n,in}^2} = \dfrac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \frac{2}{3} \frac{1}{g_m} + 4kT \frac{1}{g_m^2 R_S}$ //

(7.6)(c) $|A_V| = \frac{g_m}{1+(g_m+\frac{1}{R_F})R_S} \cdot R_{out}$

$R_{out} = R_S + (1+g_m R_S)R_F$

$$\overline{V_{n,out}^2} = \left(\frac{4kT\frac{2}{3}g_m}{(1+(g_m+\frac{1}{R_F})R_S)^2} + \frac{4kT\frac{1}{R_F}}{(1+(g_m+\frac{1}{R_F})R_S)^2} + \frac{4kT\frac{1}{R_S}\cdot R_S^2}{(R_S+\frac{1}{g_m}//R_F)^2} \right) R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_V|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_m} + \frac{1}{g_m^2 R_F} + R_S\left(1+\frac{1}{g_m R_F}\right)^2\right)$$ //

(7.6)(d) $|A_V| = \frac{g_{m1}}{1+g_{m1}R_S} \cdot R_{out}$

$R_{out} = \frac{1}{g_{m2}}$

$$\overline{V_{n,out}^2} = 4kT\frac{2}{3}g_{m2}R_{out}^2 + 4kT\frac{2}{3}\frac{1}{g_{m1}}|A_V|^2 + 4kT\frac{1}{R_S}\left(\frac{R_S}{\frac{1}{g_{m1}}+R_S}\right)^2 R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_V|^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + R_S + \frac{2}{3}g_{m2}\left(\frac{1+g_{m1}R_S}{g_{m1}}\right)^2\right]$$ //

(7.6)(e) $|A_V| = g_{m1}R_D$

$$\overline{V_{n,out}^2} = \left(4kT\frac{2}{3}g_{m1} + 4kT\frac{1}{R_D}\right)R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_V|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right)$$ //

M2 + R_F do not contribute noise because $r_{o1} = \infty$

(7.6)(f) $|A_V| = g_{m1}\left(\frac{g_{m2}R_S}{1+g_{m2}R_S}\right)R_D$

$$\overline{V_{n,out}^2} = \left[4kT\frac{1}{R_D} + 4kT\frac{2}{3}\frac{1}{g_{m2}}\left(\frac{g_{m2}}{1+g_{m2}R_S}\right)^2 + 4kT\frac{2}{3}\frac{1}{g_{m1}}\left(\frac{g_{m1}R_S}{\frac{1}{g_{m2}}+R_S}\right)^2 + 4kT\frac{1}{R_S}\left(\frac{R_S}{\frac{1}{g_{m2}}+R_S}\right)^2\right]\cdot R_D^2$$

note: $\frac{R_S}{\frac{1}{g_{m2}}+R_S} = \frac{g_{m2}R_S}{1+g_{m2}R_S}$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_V|^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + \frac{2}{3}\frac{1}{g_{m2}}\frac{1}{(g_{m1}R_S)^2} + \frac{1}{g_{m1}^2 R_S} + \frac{1}{g_{m1}^2 R_D}\left(\frac{1+g_{m2}R_S}{g_{m2}R_S}\right)^2\right]$$ //

(7.7)(a) $|A_v| = \dfrac{g_{m1} R_{out}}{1+(g_{m1}+\frac{1}{R_F})R_S}$

$R_{out} = R_S + (1+g_{m1}R_S)R_F$

$$\overline{V_{n,out}^2} = \left[\frac{4KT\frac{2}{3}g_{m1}}{(1+(g_{m1}+\frac{1}{R_F})R_S)^2} + \frac{4KT\frac{1}{R_F}}{(1+(g_{m1}+\frac{1}{R_F})R_S)^2} + \frac{4KT\frac{1}{R_S}\cdot R_S^2}{(R_S+\frac{1}{g_{m1}}//R_F)^2} + 4KT\frac{2}{3}g_{m2}\right]R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4KT\left[\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_F} + R_S\left(1+\frac{1}{g_{m1}R_F}\right)^2 + \frac{2}{3}g_{m2}\left(\frac{1+(g_{m1}+\frac{1}{R_F})R_S}{g_{m1}}\right)^2\right]$$ //

(7.7)(b) $|A_v| = \left(g_{m2} + \dfrac{g_{m1}}{1+(g_{m1}+\frac{1}{R_F})R_S}\right)\cdot R_{out}$

$R_{out} = R_S + (1+g_{m1}R_S)R_F$

$$\overline{V_{n,out}^2} = \left[\frac{4KT\frac{2}{3}g_{m1}}{(1+(g_{m1}+\frac{1}{R_F})R_S)^2} + \frac{4KT\frac{1}{R_F}}{(1+(g_{m1}+\frac{1}{R_F})R_S)^2} + \frac{4KT\frac{1}{R_S}\cdot R_S^2}{(R_S+\frac{1}{g_{m1}}//R_F)^2} + 4KT\frac{2}{3}g_{m2}\right]R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4KT\left(\frac{1}{g_{m1}+g_{m2}(1+(g_{m1}+\frac{1}{R_F})R_S)}\right)^2\left[\frac{2}{3}g_{m1} + \frac{1}{R_F} + R_S\left(g_{m1}+\frac{1}{R_F}\right)^2 + \frac{2}{3}g_{m2}\left(1+(g_{m1}+\frac{1}{R_F})R_S\right)^2\right]$$ //

(7.7)(c) $|A_v| = \left(\dfrac{g_{m1}}{1+g_{m1}R_S}\right)(1+g_{m2}R_S)(R_D)$

$$\overline{V_{n,out}^2} = 4KTR_D + 4KT\frac{2}{3}g_{m2}R_D^2 + 4KT\frac{2}{3}\frac{1}{g_{m1}}|A_v|^2 + 4KT\frac{1}{R_S}\left[\frac{R_S}{\frac{1}{g_{m1}}+R_S} - \frac{\frac{1}{g_{m1}}R_S}{\frac{1}{g_{m1}}+R_S}g_{m2}\right]^2 R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4KT\left[\frac{2}{3}\frac{1}{g_{m1}} + \left(\frac{1}{R_D}+\frac{2}{3}g_{m2}\right)\left(\frac{1+g_{m1}R_S}{g_{m1}(1+g_{m2}R_S)}\right)^2 + R_S(g_{m1}-g_{m2})^2\left(\frac{1}{g_{m1}(1+g_{m2}R_S)}\right)^2\right]$$ //

(7.7)(d) $|A_v| = g_{m1}R_D$

$$\overline{V_{n,out}^2} = \left[4KT\frac{1}{R_D} + 4KT\frac{2}{3}g_{m3} + 4KT\frac{2}{3}g_{m1}\right]R_D^2$$

M2 does not contribute any noise because r_{o1} and $r_{o3} = \infty$.

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4KT\left[\frac{2}{3}\frac{1}{g_{m1}} + \frac{2}{3}\frac{g_{m3}}{g_{m1}^2} + \frac{1}{g_{m1}^2 R_D}\right]$$ //

(7.8)(a) $|A_v| = g_{m1} R_D$

$$\overline{V_{n,out}^2} = \left[4kT\frac{1}{R_D} + 4kT\frac{2}{3}g_{m1} + 4kT R_G g_{m1}^2\right] R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + R_G + \frac{1}{g_{m1}^2 R_D}\right) //$$

(7.8)(b) $|A_v| = (g_{m1} + \frac{1}{R_1})(R_1 // R_D)$

$$\overline{V_{n,out}^2} = \left[4kT\frac{2}{3}g_{m1} + 4kT\frac{1}{R_1} + 4kT\frac{1}{R_D}\right](R_1//R_D)^2$$

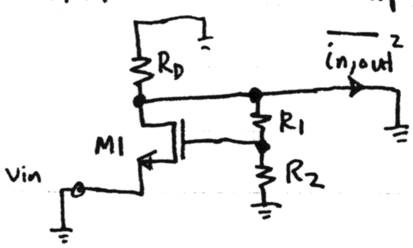
$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left(\frac{1}{g_{m1} + \frac{1}{R_1}}\right)^2\left[\frac{2}{3}g_{m1} + \frac{1}{R_1} + \frac{1}{R_D}\right] //$$

(7.8)(c) $A_v = (-g_{m1} + \frac{1}{R_F})(R_F // R_D)$

$$\overline{V_{n,out}^2} = 4kT\left(\frac{1}{R_F} + \frac{1}{R_D} + \frac{2}{3}g_{m1}\right)(R_F//R_D)^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left(\frac{1}{-g_{m1} + \frac{1}{R_F}}\right)^2\left[\frac{2}{3}g_{m1} + \frac{1}{R_F} + \frac{1}{R_D}\right] //$$

(7.8)(d) Find short circuit output noise current $\overline{i_{n,out}^2}$

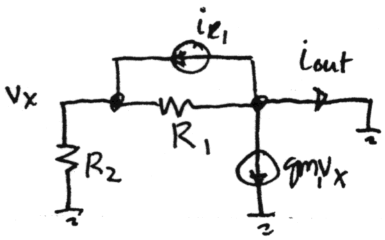


$$\overline{i_{n,out}^2} = 4kT\frac{1}{R_D} + 4kT\frac{2}{3}g_{m1} + \overline{i_{noise,R_1}^2} + \overline{i_{noise,R_2}^2}$$

$$\overline{i_{noise,R_1}^2} = 4kT\frac{1}{R_1}|A_{I,R_1}|^2$$

$$\overline{i_{noise,R_2}^2} = 4kT\frac{1}{R_2}|A_{I,R_2}|^2$$

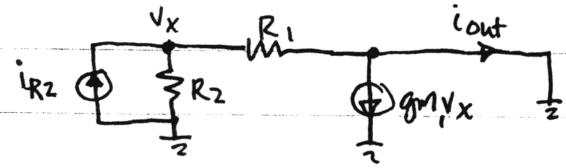
- small signal model used to find $A_{I,R1}$



$$A_{I,R_1} = \frac{i_{out}}{i_{R_1}} = -\left(1 + (R_1 // R_2)(g_{m1} - \frac{1}{R_1})\right)$$

(7.8) (d) cont.

- small signal model used to find $A_{I,R2}$



$$A_{I,R_2} = \frac{i_{out}}{i_{R2}} = \left[\frac{R_2}{R_1+R_2} - g_{m1}(R_1 \| R_2)\right]$$

$$\therefore\ \overline{i_{n,out}^2} = 4kT\frac{1}{R_D} + 4kT\frac{2}{3}g_{m1} + 4kT\frac{1}{R_1}\left(1+(R_1\|R_2)\left(g_{m1}-\frac{1}{R_1}\right)\right)^2 + 4kT\frac{1}{R_2}\left[\frac{R_2}{R_1+R_2} - g_{m1}(R_1\|R_2)\right]^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{i_{n,out}^2}}{g_{m1}^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} + \frac{1}{g_{m1}^2 R_1}\left(1+(R_1\|R_2)\left(g_{m1}-\frac{1}{R_1}\right)\right)^2 + \frac{1}{g_{m1}^2 R_2}\left(\frac{R_2}{R_1+R_2} - g_{m1}(R_1\|R_2)\right)^2\right] //$$

(7.9)(a) $|A_V| = g_{m1}R_D$

$$\overline{V_{n,out}^2} = 4kT\left(\frac{1}{R_D} + \frac{2}{3}g_{m1}\right)R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_V|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right) //$$

(7.9)(b) $|A_V| = g_{m1}\left(R_D \| \frac{1}{g_{m2}}\right)$

$$\overline{V_{n,out}^2} = 4kT\left(\frac{1}{R_D} + \frac{2}{3}g_{m1} + \frac{2}{3}g_{m2}\right)\left(R_D \| \frac{1}{g_{m2}}\right)^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_V|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} + \frac{2}{3}\frac{g_{m2}}{g_{m1}^2}\right) //$$

(7.9)(c) $|A_V| = g_{m1}R_D$

$$\overline{V_{n,out}^2} = 4kT\left(\frac{2}{3}g_{m1} + \frac{1}{R_D}\right)R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_V|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right) //$$

(7.9)(d) $|A_V| = \frac{g_{m1}}{g_{m1}+g_{m2}}$

$$\overline{V_{n,out}^2} = \left(4kT\frac{2}{3}g_{m1} + 4kT\frac{2}{3}g_{m2}\right)\left(\frac{1}{g_{m1}+g_{m2}}\right)^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_V|^2} = 4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{2}{3}\frac{g_{m2}}{g_{m1}^2}\right) //$$

(7.9)(e) $|A_V| = 1$, $\overline{V_{n,in}^2} = \overline{V_{n,out}^2} = 4kT\frac{2}{3}\frac{1}{g_{m1}}$ //

(7.10) • With the input shorted to ground,

$$\overline{V_{n,out}^2} = \frac{1}{C_{ox}f}\left[\frac{g_{m1}^2 k_n}{(WL)_1} + \frac{g_{m3}^2 k_p}{(WL)_3} + \frac{g_{m3}^2 k_p}{(WL)_4}\right](r_{o1} \parallel r_{o3})^2$$

$$|A_v| = (g_{m1}+g_{mb1})(r_{o1} \parallel r_{o3})$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = \frac{1}{C_{ox}f}\left[\frac{g_{m1}^2 k_n}{(WL)_1} + \frac{g_{m3}^2 k_p}{(WL)_3} + \frac{g_{m3}^2 k_p}{(WL)_4}\right]\frac{1}{(g_{m1}+g_{mb1})^2} \quad //$$

• With the input open,

$$\overline{V_{n,out}^2} = \frac{1}{C_{ox}f}\left[g_{m2}^2 k_n\left(\frac{1}{(WL)_0}+\frac{1}{(WL)_2}\right) + g_{m3}^2 k_p\left(\frac{1}{(WL)_3}+\frac{1}{(WL)_4}\right)\right] R_{out}^2$$

$$R_{out} \cong r_{o3} \parallel (g_{m1} r_{o1} r_{o2})$$

$$\overline{I_{n,in}^2} = \frac{1}{C_{ox}f}\left[g_{m2}^2 k_n\left(\frac{1}{(WL)_0}+\frac{1}{(WL)_2}\right) + g_{m3}^2 k_p\left(\frac{1}{(WL)_3}+\frac{1}{(WL)_4}\right)\right] \quad //$$

(7.11)

$$\overline{V_{n,out}^2} = \frac{K}{C_{ox}(WL)_1}\cdot\frac{1}{f}(g_{m1}R_{out})^2 + \frac{K}{C_{ox}(WL)_2}\cdot\frac{1}{f}(g_{m2}R_{out})^2$$

$$R_{out} = \left(\frac{1}{g_{m1}} \parallel \frac{1}{g_{mb1}} \parallel r_{o1} \parallel r_{o2}\right)$$

$$|A_v| = g_{m1}R_{out}$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = \frac{K}{C_{ox}f}\left[\frac{1}{(WL)_1} + \frac{g_{m2}^2}{(WL)_2\, g_{m1}^2}\right] \quad //$$

(7.12)(a)

$$\overline{V_{n,out}^2} = 4KT\frac{2}{3}(g_{m1}+g_{m2}+g_{m3}+g_{m4}+g_{m5}+g_{m6})R_{out}^2$$

$$|A_v| = g_{m1}R_{out}$$

$$g_{m1} = g_{m2}, \quad g_{m3,4} = 0.5\, g_{m5,6}$$

$$\overline{V_{n,in}^2} = 4KT\frac{2}{3}\left[\frac{2}{g_{m1}} + \frac{3g_{m5}}{g_{m1}^2}\right] \quad //$$

(7.12)(b)

$$|A_v| = g_{m1}(r_{o2} \parallel r_{o4})$$

$$\overline{V_{n,out}^2} = 4KT\frac{2}{3}[g_{m1}+g_{m2}+g_{m3}+g_{m4}]$$

$$g_{m1} = g_{m2}, \quad g_{m3} = g_{m4}$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4KT\left(\frac{2}{3}\right)\left[\frac{2}{g_{m1}} + \frac{2g_{m3}}{g_{m1}^2}\right] \quad //$$

(7.13)(a) $|A_v| = \dfrac{g_{m1} R_D}{1+g_{m1}R_S}$

$$\overline{V_{n,out}^2} = 4KTR_D + 4KT\frac{2}{3}\frac{1}{g_{m1}}|A_v|^2 + 4KT\frac{1}{R_S}\left(\frac{R_S}{R_S+\frac{1}{g_{m1}}}\right)^2 R_D^2$$

$$\overline{V_{n,in}^2} = 4KT\left[\frac{2}{3}\frac{1}{g_m} + R_S + \frac{1}{R_D}\left(\frac{1+g_{m1}R_S}{g_{m1}}\right)^2\right]$$

(7.13) (b)

$$IR_S = V_{GS}-V_T \Rightarrow R_S = \frac{V_{GS}-V_T}{I}$$

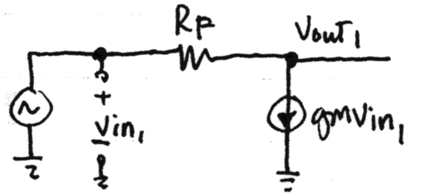
$$g_{m1} = \frac{2I}{V_{GS}-V_T} = \frac{2}{R_S}$$

$\therefore 4KT\left[\frac{2}{3}g_m\right] = 4KT\frac{R_S}{3}$ ← Thermal noise of M1

$4KTR_S$ ← Thermal noise of R_S

$\therefore$ R_S contributes 3x more noise power ($\overline{V_{n,in}^2}$) than M1 when $IR_S = V_{GS}-V_T$.

(7.14) Consider the following ckt with noise only due to resistor R_F:

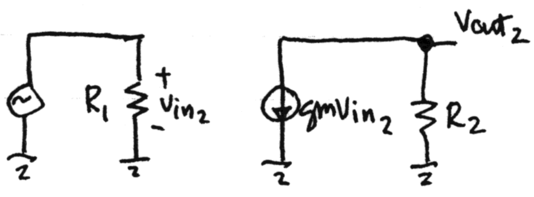


$$A_{v1} = \frac{V_{out1}}{V_{in1}} = -g_mR_F + 1$$

$$\overline{V_{n,out1}^2} = 4KTR_F$$

$$\overline{V_{n,in1}^2} = \frac{\overline{V_{n,out1}^2}}{|A_{v1}|^2} = 4KTR_F\left(\frac{1}{-g_mR_F+1}\right)^2$$

using the Miller effect, we have the following ckt:



$$A_{v2} = \frac{V_{out2}}{V_{in2}} = -g_mR_2 = \frac{-g_mR_F A_{v1}}{A_{v1}-1} = (-g_mR_F+1) = A_{v1}$$

$$\overline{V_{n,out2}^2} = 4KTR_2$$

$$\overline{V_{n,in2}^2} = \frac{\overline{V_{n,out2}^2}}{|A_{v2}|^2} = 4KT\frac{1}{g_m^2R_2} = 4KT\frac{1}{g_{m1}}\left(\frac{-1}{-g_{m1}R_F+1}\right)$$

$$R_1 = \frac{R_F}{1-A_{v1}} \qquad R_2 = \frac{R_F}{1-\frac{1}{A_{v1}}}$$

notice: $\overline{V_{n,in1}^2} \neq \overline{V_{n,in2}^2} \Rightarrow$ cannot use Miller's Theorem //

(7.15) Using equation (7.26)

$$\overline{V_{n,out}^2} = 4kT\left(\tfrac{2}{3}g_m\right)r_o^2$$
$$= (1.656\times10^{-20}\,V\cdot C)\left(\tfrac{2}{3}\right)\left(4.44\,\tfrac{mA}{V}\right)(20k\Omega)^2$$
$$= 19.6\times10^{-15}\,\tfrac{V^2}{Hz}$$
$$\overline{V_{n,out_{TOT}}} = \sqrt{\left(19.6\times10^{-15}\tfrac{V^2}{Hz}\right)(50\,MHz)} \cong 990\,\mu V\,rms \;//$$

(7.16) $|A_V|^2 = \dfrac{[g_m(R_D\|r_o)]^2}{1+(2\pi(R_D\|r_o)C_Lf)^2}$

$$\overline{V_{n,out}^2} = 4kT\frac{(R_D\|r_o)^2}{R_D}\frac{1}{1+(2\pi(R_D\|r_o)C_Lf)^2} + 4kT\frac{2}{3}g_m(R_D\|r_o)^2\frac{1}{1+(2\pi(R_D\|r_o)C_Lf)^2}$$
$$+\frac{k}{C_{ox}WL}g_m^2(R_D\|r_o)^2\frac{1}{1+(2\pi(R_D\|r_o)C_Lf)^2}$$

$$\overline{V_{n,out_{TOT}}^2} = 4kT(R_D\|r_o)\left[\frac{(R_D\|r_o)}{R_D}+\frac{2}{3}g_m(R_D\|r_o)\right]\int_{f_L}^{f_H}\frac{df}{1+(2\pi(R_D\|r_o)C_Lf)^2}$$
$$+\frac{k\,g_m^2(R_D\|r_o)^2}{C_{ox}WL}\int_{f_L}^{f_H}\frac{df}{f(1+(2\pi(R_D\|r_o)C_Lf)^2)}$$

$$\overline{V_{n,out_{TOT}}^2} = \frac{2kT}{\pi C_L}\left[\frac{(R_D\|r_o)}{R_D}+\frac{2}{3}g_m(R_D\|r_o)\right]\left[\tan^{-1}(2\pi(R_D\|r_o)C_Lf_H)-\tan^{-1}(2\pi(R_D\|r_o)C_Lf_L)\right]$$
$$+\frac{k\,g_m^2(R_D\|r_o)^2}{C_{ox}WL}\int_{f_L}^{f_H}\frac{df}{f[1+(2\pi(R_D\|r_o)C_Lf)^2]} \;//$$

(7.17) Using equation number (7.57)

$$\overline{V_{n,in}^2} = 4kT\left(\tfrac{2}{3}\right)\left(\frac{1}{g_{m1}}+\frac{g_{m2}}{g_{m1}^2}\right)$$
$$g_{m1} = \sqrt{2(0.5mA)\left(134.29\tfrac{\mu A}{V^2}\right)\left(\tfrac{50}{.34}\right)} = 4.44\,\tfrac{mA}{V}$$
$$g_{m2} = \sqrt{2(0.5mA)\left(38.37\tfrac{\mu A}{V^2}\right)\left(\tfrac{50}{.34}\right)} = 2.36\,\tfrac{mA}{V}$$
$$\therefore\ \overline{V_{n,in}^2} = (1.656\times10^{-20}\,V\cdot C)\left(\tfrac{2}{3}\right)(225.23\,\Omega+119.71\,\Omega)$$
$$\cong 3.81\times10^{-18}\,\tfrac{V^2}{Hz}$$
$$\overline{V_{n,in}} \cong 1.95\,\tfrac{nV}{\sqrt{Hz}}$$

//

(7.18) (a) $|A_v| = g_{m1} R_{out}$

$$R_{out} = r_{o1} \,//\, (R_S + (1+g_{m2}R_S) r_{o2})$$

$$\overline{V_{n,out}^2} = \left[4kT \frac{1}{R_S}\left(\frac{R_S}{R_S+\frac{1}{g_{m2}}}\right)^2 + 4kT\frac{2}{3}\frac{1}{g_{m2}}\left(\frac{g_{m2}}{1+g_{m2}R_S}\right)^2 + 4kT\frac{2}{3}g_{m1}\right] R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + R_S\left(\frac{g_{m2}}{1+g_{m2}R_S}\right)^2\frac{1}{g_{m1}^2} + \frac{2}{3}\frac{g_{m2}}{g_{m1}^2}\left(\frac{1}{1+g_{m2}R_S}\right)^2\right]$$

(b) R_S large

(7.19) Neglecting body effect and using eqns 7.60 and 7.61

$$\overline{V_{n,in}^2} = 4kT\left(\frac{2}{3}\frac{1}{g_m^2} + \frac{1}{g_m^2 R_D}\right)$$

$$\overline{I_{n,in}^2} = \frac{4kT}{R_D}$$

$$g_m = \sqrt{2(1\text{mA})(134.29\tfrac{\mu A}{V^2})(\tfrac{50}{.34})} = 6.28\ \tfrac{mA}{V}$$

$$\therefore\ \overline{V_{n,in}^2} = (1.656\times10^{-20}\,V\cdot C)(\tfrac{2}{3}\,159.24\Omega + 25.36\Omega) \cong 2.18\times10^{-18}\ \tfrac{V^2}{Hz}$$ //

$$\overline{I_{n,in}^2} = \frac{(1.656\times10^{-20}\,V\cdot C)}{1k\Omega} = 16.56\times10^{-24}\ \tfrac{A^2}{Hz}$$ //

(7.20) (a) $\overline{I_{n,in}^2} = 4kT\frac{1}{R_D} + 4kT\frac{2}{3}g_{m2}$

$$\overline{I_{n,in}} = \sqrt{4kT\left(\tfrac{1}{R_D} + \tfrac{2}{3}g_{m2}\right)}$$

$$\therefore\ \tfrac{2}{3}g_{m2} = \left(\tfrac{1}{5}\right)^2\left(\tfrac{1}{R_D}\right)$$

$$\Rightarrow g_{m2} = \left(\tfrac{1}{25}\right)\left(\tfrac{1}{1000}\right)\left(\tfrac{3}{2}\right) = 60\ \tfrac{\mu A}{V}$$

$$\left(\tfrac{W}{L}\right)_2 = \frac{g_{m2}^2}{2I_D\,\mu_n C_{ox}} = \frac{(60\tfrac{\mu A}{V})^2}{2(.05mA)(134.29\tfrac{\mu A}{V^2})} \cong 0.268$$ //

(b) $g_{m2} = \dfrac{2I_D}{(V_{gs}-V_T)_2} \Rightarrow (V_{gs}-V_T)_2 = \dfrac{2I_D}{g_m} = \dfrac{2(.05mA)}{60\frac{\mu A}{V}} \cong 1.67\ V$

$$(V_{gs}-V_T)_1 = \sqrt{\frac{2I_D}{\mu_n C_{ox}(\frac{W}{L})_1}} = \sqrt{\frac{2(.05mA)}{134.29\frac{\mu A}{V^2}(\frac{50}{.34})}} \cong 71.2\,mV$$

neglecting body effect

$V_b = (V_{gs}-V_t)_2 + V_{GS1} = 1.67 + 0.0712 + 0.7 = 2.4412V$ //

Output Swing $= V_{CC} - (V_{gs}-V_t)_1 - (V_{gs}-V_t)_2 = 3 - 1.67 - 0.0712 = 1.2588\,V$ //

Note: output swing is not symmetric.

(7.21) Neglecting body effect and using the result of eqn. 7.60

$$\overline{V_{n,in}} = \sqrt{4kT\left(\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D}\right)} = 3\,\frac{nV}{\sqrt{Hz}}$$

$$\frac{2}{3}\frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} = 543.4\,\Omega$$

note:

$$\frac{1}{g_{m1}} = \frac{(V_{gs}-V_T)}{2I_D} \triangleq \frac{\Delta V}{2I_D} \qquad \text{also define } R_N = 543.4\,\Omega$$

$$\Rightarrow \Delta V^2 + \frac{4}{3} I_D R_D \Delta V - 4 I_D^2 R_D R_N = 0 \quad ; \quad I_D = 0.5\,mA$$

One possible answer assumming $\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2$ and a 3V supply

$$\Delta V_1 = \Delta V_2 = 562\,mV \quad ; \quad R_D = 1875\,\Omega$$

$$\Rightarrow \text{Output swing} = 2 \cdot I_D R_D = 1.875\,V \;//$$

$$g_{m1} = g_{m2} = \frac{2I_D}{\Delta V} = 1.78\,\frac{mA}{V}$$

$$\left(\frac{W}{L_{eff}}\right) = \frac{g_m^2}{2 I_D \mu_n C_{ox}} = \frac{(1.78\,mA/V)^2}{2(0.5mA)(134.29\frac{\mu A}{V^2})} \cong 23.6 \;//$$

$$V_b = \Delta V_2 + V_{gs2} \cong 0.562\,V + 0.562\,V + 0.7\,V = 1.824\,V \;//$$

(7.22) Neglecting body effect and using eqns 7.64 + 7.65

$$\overline{V_{n,in}^2} = 4kT\frac{2}{3}\left(\frac{1}{g_{m1}} + \frac{g_{m3}}{g_{m1}^2}\right)$$

$$\overline{I_{n,in}^2} = 4kT\frac{2}{3}(g_{m2} + g_{m3})$$

$$g_{m1} = g_{m2} = \sqrt{2(0.5mA)(134.29\frac{\mu A}{V^2})(\frac{50}{.34})} \cong 4.44\,\frac{mA}{V}$$

$$g_{m3} = \sqrt{2(0.5mA)(38.37\frac{\mu A}{V^2})(\frac{50}{.34})} \cong 2.38\,\frac{mA}{V}$$

$$\therefore \quad \overline{V_{n,in}^2} = (1.656\times10^{-20}\,V\cdot C)\left(\frac{2}{3}\right)(225.23\,\Omega + 120.73\,\Omega) \cong 3.82\times10^{-18}\,\frac{V^2}{Hz} \;//$$

$$\overline{I_{n,in}^2} = (1.656\times10^{-20}\,V\cdot C)\left(\frac{2}{3}\right)\left(4.44\,\frac{mA}{V} + 2.38\,\frac{mA}{V}\right) \cong 75.3\times10^{-24}\,\frac{A^2}{Hz} \;//$$

(7.23) Neglect body effect and use eqn. 7.65

$$\overline{I_{n,in}^2} = 4kT\frac{2}{3}(g_{m2}+g_{m3})$$

$$g_{m1} = \sqrt{2(0.5mA)(134.29\tfrac{\mu A}{V^2})(\tfrac{50}{.134})} = 4.44\ \tfrac{mA}{V}$$

$$(V_{gs}-V_t)_1 = \frac{2I_D}{g_{m1}} = \frac{2(0.5mA)}{4.44mA/V} \cong 225.23\text{ mV}$$

define $\Delta V_x = |(V_{gs}-V_t)|_x$

$\therefore$ Output Swing $= V_{CC} - (\Delta V_1 + \Delta V_2 + \Delta V_3) = 2V$; $V_{CC} = 3V$

$$\Rightarrow \Delta V_2 + \Delta V_3 = 774.77\text{ mV}$$

note: $g_m = \dfrac{2I_D}{\Delta V}$

$$\therefore \overline{I_{n,in}^2} = 4kT\left(\frac{2}{3}\right)2I_D\left(\frac{\Delta V_2+\Delta V_3}{\Delta V_2\Delta V_3}\right)$$

to minimize $\overline{I_{n,in}^2}$ let $\Delta V_2 = \Delta V_3$ $\Rightarrow g_{m2} = g_{m3} = \dfrac{2(0.5mA)}{0.387V} = 2.58\ \dfrac{mA}{V}$

$$\left(\frac{W}{L_{eff}}\right)_2 = \frac{(g_{m2})^2}{2I_D\mu_nC_{ox}} = \frac{(2.58\ \frac{mA}{V})^2}{2(0.5mA)(134.29\frac{\mu A}{V^2})} \cong 49.7 \;//$$

$$\left(\frac{W}{L_{eff}}\right)_3 = \frac{(g_{m3})^2}{2I_D\mu_pC_{ox}} = \frac{(2.58\ \frac{mA}{V})^2}{2(0.5mA)(38.37\frac{\mu A}{V^2})} \cong 174 \;//$$

(7.24) (a) Neglecting body effect and r_{o1}, r_{o2}

$$R_{out} = \frac{1}{g_{m1}} = \frac{1}{\sqrt{2 I_D \mu_n C_{ox} \left(\frac{W}{L}\right)_1}} = 100\Omega$$

$$\left(\frac{W}{L_{eff}}\right)_1 = \frac{1}{2 I_D \mu_n C_{ox} R_{out}^2} = \frac{1}{2(1mA)(134.29\frac{\mu A}{V^2})(100\Omega)^2} \cong 3723$$

(b) Using the result from eqn 7.73

$$\overline{V_{n,in}} = \sqrt{4KT\left(\tfrac{2}{3}\right)\left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right)}$$

$$\frac{g_{m2}}{g_{m1}^2} = \left(\frac{1}{5}\right)^2 \frac{1}{g_{m1}} \Rightarrow g_{m2} = \left(\frac{1}{5}\right)^2 g_{m1} = \frac{1}{2500\Omega}$$

$$(V_{gs}-V_T)_2 = \frac{2I_D}{g_{m2}} = 0.5V \qquad \left(\frac{W}{L_{eff}}\right)_2 = \frac{(g_{m2})^2}{2I_D \mu_n C_{ox}} = \frac{(400\mu A/V)^2}{2(1mA)(134.29\frac{\mu A}{V^2})} \cong 5.96$$

$$(V_{gs}-V_T)_1 = \sqrt{\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)}} = \sqrt{\frac{2(0.1mA)}{(134.29\frac{\mu A}{V^2})(3723)}} \cong 20mV$$

neglecting body effect

$$V_{GS1} \approx 0.7 + 0.02 = 0.72$$

$\therefore$ Output swing $\approx V_{CC} - V_{GS1} - (V_{GS}-V_T)_2 = 3 - 0.72 - 0.5 = 1.78V$

(7.25) $|A_v|^2 = g_{m1}^2 \left(\frac{g_{m2}}{\omega C_X}\right)^2 \left(\frac{1}{1+\left(\frac{g_{m2}}{\omega C_X}\right)^2}\right) R_D^2$ $\qquad \omega = 2\pi f$

$$\overline{V_{n,out}^2} = \left[4kT\frac{1}{R_D} + 4kT\frac{2}{3}\frac{1}{g_{m2}}\frac{g_{m2}^2}{1+\left(\frac{g_{m2}}{\omega C_X}\right)^2} + 4kT\frac{2}{3}g_{m1}\left(\frac{g_{m2}}{\omega C_X}\right)^2\left(\frac{1}{1+\left(\frac{g_{m2}}{\omega C_X}\right)^2}\right)\right]R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT\left[\frac{2}{3}\frac{1}{g_{m1}} + \frac{2}{3}\frac{1}{g_{m2}}\left(\frac{\omega C_X}{g_{m1}}\right)^2 + \frac{1}{R_D}\left(\frac{g_{m2}^2 + (\omega C_X)^2}{g_{m1}^2 g_{m2}^2}\right)\right] \quad //$$

(7.26) (a) $M_1 = M_2$, M_3 does not contribute differential noise

$$|A_v| = \frac{g_{m1}R_L}{1+g_{m1}R_S}$$

$$\overline{V_{n,out_a}^2} = \left[2(4kT)\frac{1}{R_D} + 2\left(4kT\frac{2}{3}\right)\frac{1}{g_{m1}}\left(\frac{g_{m1}}{1+g_{m1}R_S}\right)^2 + 2\left(4kT\frac{1}{R_S}\right)\left(\frac{R_S}{\frac{1}{g_{m1}}+R_S}\right)^2\right]R_D^2$$

$$\overline{V_{n,in_a}^2} = \frac{\overline{V_{n,out_a}^2}}{|A_v|^2} = 2(4kT)\left[\frac{2}{3}\frac{1}{g_{m1}} + R_S + \frac{1}{R_D}\left(\frac{1+g_{m1}R_S}{g_{m1}}\right)^2\right] \quad //$$

(b) $M1 = M2$, $M_3 = M_4$

$$\overline{V_{n,out_b}^2} = \overline{V_{n,out_a}^2} + 2(4kT)\left(\frac{2}{3}\right)g_{m3}\left(\frac{R_S}{\frac{1}{g_{m1}}+R_S}\right)^2 R_D^2$$

$$\overline{V_{n,in_b}^2} = \frac{\overline{V_{n,out_b}^2}}{|A_v|^2} = 2(4kT)\left[\frac{2}{3}\frac{1}{g_{m1}} + R_S + \frac{1}{R_D}\left(\frac{1+g_{m1}R_S}{g_{m1}}\right)^2 + \frac{2}{3}g_{m3}R_S^2\right] \quad //$$

$$\therefore \overline{V_{n,in_b}^2} = \overline{V_{n,in_a}^2} + 2(4kT)\left(\frac{2}{3}g_{m3}R_S^2\right)$$

M3 + M4 contribute differential noise in (b)

8.1

$$V_{in} = I_{in}(Z_{in} + G_{22}) + G_{21} V_{out}$$

$$\frac{V_{out} - A_o I_{in} Z_{in}}{Z_{out}} = -G_{11} V_{out} - G_{12} I_{in}$$

$$\Rightarrow \quad I_{in}\left(\frac{A_o Z_{in}}{Z_{out}} - G_{12}\right) = V_{out}\left(\frac{1}{Z_{out}} + G_{11}\right)$$

$$V_{in} = V_{out}\left[G_{21} + \frac{(Z_{in} + G_{22})\left(\frac{1}{Z_{out}} + G_{11}\right)}{\frac{A_o Z_{in}}{Z_{out}} - G_{12}}\right]$$

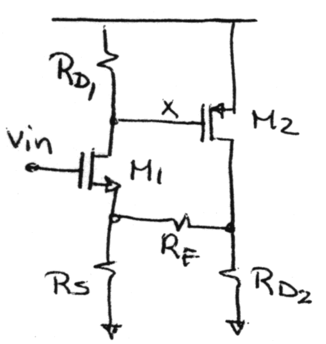
$$\frac{V_{out}}{V_{in}} = \frac{\frac{A_o Z_{in} - G_{12} Z_{out}}{Z_{out}}}{G_{21}\left(\frac{A_o Z_{in}}{Z_{out}} - G_{12}\right) + (Z_{in} + G_{22})\left(\frac{1}{Z_{out}} + G_{11}\right)}$$

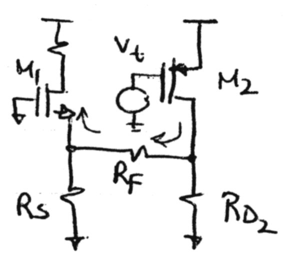
$$A_{v_{\text{open loop}}} = \frac{1}{Z_{out}}(A_o Z_{in} - G_{12} Z_{out}) \frac{1}{Z_{in} + G_{22}} \cdot \frac{1}{\frac{1}{Z_{out}} + G_{11}}$$

$$= \frac{G_{11}^{-1}}{G_{11}^{-1} + Z_{out}} \cdot \frac{1}{Z_{in} + G_{22}} (A_o Z_{in} - G_{12} Z_{out})$$

if $G_{12} \ll A_o Z_{in}/Z_{out}$ then the second term can be neglected

8.2





The current through R_S:

$$-\frac{g_{m2} \cdot V_t R_{D2}}{R_{D2} + R_F + (R_S \parallel \frac{1}{g_{m2}})}$$

The current through M_1:

$$-\frac{g_{m2} V_t R_{D2}}{R_{D2} + R_F + (R_S \parallel \frac{1}{g_{m2}})} \cdot \frac{R_S}{R_S + \frac{1}{g_{m2}}}$$

This current is multiplied by R_{D1} to produce V_F

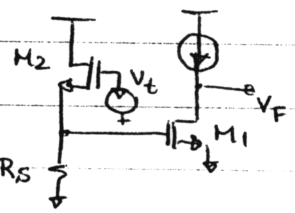
8.2

loop gain: $\dfrac{g_{m2} R_{D2} R_S R_{D1}}{(R_{D2}+R_F)(R_S+\frac{1}{g_{m2}}) + R_S \cdot \frac{1}{g_{m2}}}$

This result is accurate, whereas $G_{21} Av_{open}$ is approximate because it neglects the signal propagating thru the feedback network from the input to the output.

8.3

voltage-current



loop gain: $\dfrac{R_S}{R_S + \frac{1}{g_{m2}}} \cdot g_{m1} r_{o1}$

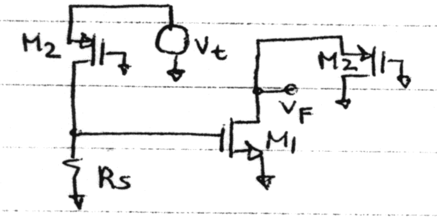
$\dfrac{V_{in}}{R_S} \times (R_S \| \frac{1}{g_{m2}})\, g_{m1} r_{o1} = V$

$\Rightarrow \quad \left.\dfrac{V_{out}}{\frac{V_{in}}{R_S}}\right|_{open} = g_{m1} r_{o1} (R_S \| \frac{1}{g_{m2}})$ $\qquad Zin_{open} = \dfrac{1}{g_{m2}}$

$Zout_{open} = r_{o1}$

$\left.\dfrac{V_{out}}{\frac{V_{in}}{R_S}}\right|_{closed} \underset{r_{o1}\to\infty}{=} \dfrac{1}{g_{m2}} \Rightarrow Av_{closed} = \dfrac{1}{g_{m2} R_S}$

$Zin_{closed} \underset{r_{o1}\to\infty}{=} 0 \qquad Zout_{closed} = \dfrac{R_S + \frac{1}{g_{m2}}}{g_{m1} R_S} = \dfrac{1}{g_{m1}} + \dfrac{1}{g_{m1} g_{m2} R_S}$

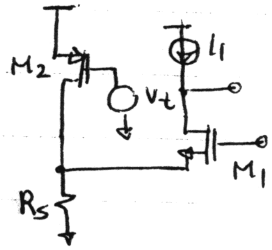


$\dfrac{V_F}{V_t} = g_{m2} R_S \dfrac{g_{m1}}{g_{m2}} = g_{m1} R_S$

$Zin_{open} = r_{o2} \qquad Zout_{open} = \dfrac{1}{g_{m2}}$

$\left.\dfrac{V_{out}}{\frac{V_{in}}{R_S}}\right|_{open} = R_S \cdot \dfrac{g_{m1}}{g_{m2}}$

$Av_{closed} = \dfrac{g_{m1}/g_{m2}}{1 + g_{m1} R_S} \qquad Zin_{closed} = \infty \qquad Zout_{closed} = \dfrac{1}{g_{m2}(1+g_{m1} R_S)}$



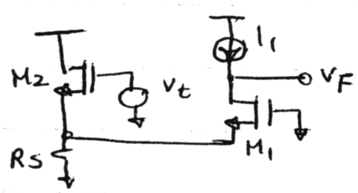
$$R_S\, g_{m2}\, V_t\, \frac{r_{o1}}{R_S + \frac{1}{g_{m1}}} = V_F \Rightarrow \text{loop gain: } \frac{R_S\, g_{m2}\, r_{o1}}{R_S + \frac{1}{g_{m1}}}$$

$$R_{out_{open}} = g_{m1} r_{o1} R_S + r_{o1} + R_S \approx g_{m1} r_{o1} R_S$$

$$R_{in_{open}} = \frac{1}{g_{m1}}$$

$$\left.\frac{V_{out}}{V_{in}}\right|_{open} = \frac{r_{o1}}{R_S + \frac{1}{g_{m1}}}$$

$r_{o1} \to \infty$ $\quad A_v = \frac{1}{R_S\, g_{m2}} \quad R_{in} = 0 \quad R_{out} = \frac{g_{m1}\left(R_S + \frac{1}{g_{m1}}\right)}{g_{m2}}$



$$\text{loop gain: } V_t \cdot \frac{R_S \| \frac{1}{g_{m1}}}{\frac{1}{g_{m2}} + R_S \| \frac{1}{g_{m1}}} \cdot g_{m1} r_{o1} = V_F$$

$$R_{in\,open} = \frac{1}{g_{m1} + g_{m2}} \qquad R_{out_{open}} = g_{m1} r_{o1} \left(R_S \| \frac{1}{g_{m2}}\right)$$

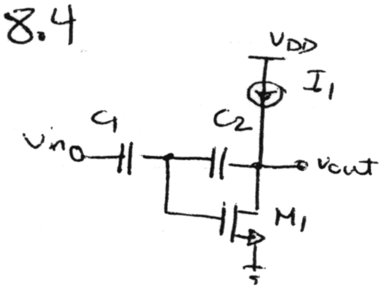
$$A_{v_{open}} \Rightarrow \frac{V_{in}}{R_S}\left(R_S \| \frac{1}{g_{m2}}\right) \times g_{m1} r_{o1} = V_{out} \Rightarrow A_{v_{open}} = \frac{g_{m1} r_{o1}\left(R_S \| \frac{1}{g_{m2}}\right)}{R_S}$$

$r_o \to \infty$

$$A_{v\,closed} = \frac{\left(R_S \| \frac{1}{g_{m2}}\right)\left(\frac{1}{g_{m2}} + R_S \| \frac{1}{g_{m1}}\right)}{R_S\left(R_S \| \frac{1}{g_{m1}}\right)}$$

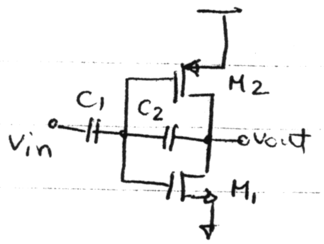
$$R_{in_{closed}} = 0 \qquad R_{out_{closed}} = \frac{\left(R_S \| \frac{1}{g_{m2}}\right)\left(\frac{1}{g_{m2}} + R_S \| \frac{1}{g_{m1}}\right)}{R_S \| \frac{1}{g_{m1}}}$$

8.4



$$R_{in} = \frac{1}{C_1 s} + \frac{1}{g_m}$$

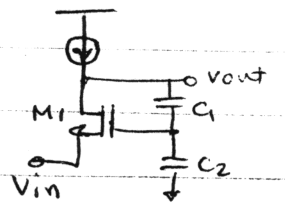
$$R_{out} = \frac{r_o}{1 + \frac{C_2}{C_1 + C_2} g_{m1} r_o} = \frac{C_1 + C_2}{g_{m1} C_2}$$



using the results in part (a)

$$R_{in} = \frac{1}{C_1 s} + \frac{1}{g_{m1} + g_{m2}}$$

$$R_{out} = \frac{C_1 + C_2}{(g_{m1} + g_{m2}) C_2}$$



$$\text{loop gain} = g_{m1} r_o \frac{C_1}{C_1 + C_2}$$

$$R_{in\,closed} = \frac{1}{g_{m1}} + \frac{1}{C_2 s}$$

$$R_{out\,closed} = \frac{r_o}{1 + g_{m1} r_o \frac{C_1}{C_1 + C_2}} = \frac{C_1 + C_2}{g_{m1} C_1}$$

8.5

$$-\frac{1}{\left(1+\frac{1}{g_{m1}r_{o1}}\right)\frac{C_2}{C_1}+\frac{1}{g_{m1}r_{o1}}} = -0.95\,\frac{C_1}{C_2} \xRightarrow{g_{m1}r_{o1}=50} \frac{C_1}{C_2}=1.63$$

Open loop output impedance : r_o

loop gain : $\frac{C_2}{C_1+C_2}\,g_m r_o$

closed loop $R_{out} = \frac{r_o}{1+\frac{C_2}{C_1+C_2}g_m r_o} = 0.49\,r_o$

8.6

$$R_{in\,closed} = \frac{1}{g_{m1}}\cdot\frac{1}{1+g_{m2}R_D\frac{C_1}{C_1+C_2}} \qquad I_1 = I_2 \Rightarrow g_{m2}=\sqrt{2}\,g_{m1}$$

$$= \frac{1}{g_{m1}}\cdot\frac{1}{1+1000\sqrt{2}\,g_{m1}} = 50 \Rightarrow g_{m1} = 3.42\ m\mho$$

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} \Rightarrow I_D = \frac{(3.42\times10^{-3})^2}{2\times1.342\times10^{-4}\times100} = 435\,\mu A$$

8.7

$$\frac{V_x}{I_x} = \frac{R_D}{1+\frac{g_{m2}R_S(g_{m1}+g_{mb1})R_D}{(g_{m1}+g_{mb1})R_S+1}\cdot\frac{C_1}{C_1+C_2}} \overset{R_D\to\infty}{=} \frac{(g_{m1}+g_{mb1})R_S+1}{g_{m2}R_S(g_{m1}+g_{mb1})}\cdot\frac{C_1+C_2}{C_1}$$

if $(g_{m1}+g_{mb1})R_S \gg 1 \Rightarrow \frac{V_x}{I_x} = \frac{1}{g_{m2}}\cdot\frac{C_1+C_2}{C_1}$

8.8

If f_{-3dB} of each stage is ω_o

$$\left|\frac{1}{(1+\frac{s}{\omega_o})^n}\right| = \frac{1}{\sqrt{2}} \Rightarrow \left(1+\left(\frac{\omega}{\omega_o}\right)^2\right)^n = 2$$

if we indicate the Gain x f_{-3dB} as $K = \text{const}$

$$\Rightarrow \left(\frac{K}{\omega_o}\right)^n = 500$$

$$\Rightarrow \frac{\ln 2}{\ln\left(1+\left(\frac{\omega}{\omega_o}\right)^2\right)} = \frac{\ln 500}{\ln\left(\frac{K}{\omega_o}\right)} \Rightarrow 1+\left(\frac{\omega}{\omega_o}\right)^2 = \left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}}$$

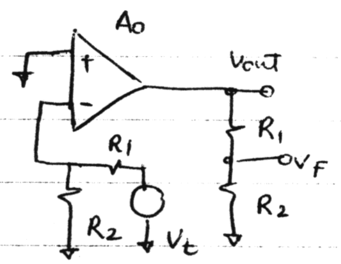
$$\Rightarrow \omega = \omega_o \sqrt{\left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}} - 1}$$

$$\frac{d\omega}{d\omega_o} = 0 \Rightarrow \sqrt{\left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}} - 1} + \frac{\omega_o}{2} \cdot \frac{1}{\sqrt{\left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}} - 1}} \cdot \left(-\frac{\ln 2}{\ln 500} \cdot \frac{1}{\omega_o} \cdot \left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}}\right)$$

$$\Rightarrow \left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}} - 1 - \frac{1}{2}\frac{\ln 2}{\ln 500}\left(\frac{K}{\omega_o}\right)^{\frac{\ln 2}{\ln 500}} = 0 \Rightarrow \frac{K}{\omega_o} = 1.67$$

$\Rightarrow$ Gain per stage $= 1.67$ Stage BW $= 598$ MHz

8.9



$$Av_{open} = A_o \frac{R_2}{R_1+R_2+R_o}$$

$$Rout_{open} = R_o \| (R_1+R_2)$$

Loop gain $=$

$$\left(\frac{R_2}{R_1+R_2}\right) A_o \left(\frac{R_2}{R_o+R_1+R_2}\right)$$

$$Av_{closed} = \frac{A_o \frac{R_2}{R_o+R_1+R_2}}{1+\left(\frac{R_2}{R_1+R_2}\right) A_o \left(\frac{R_2}{R_o+R_1+R_2}\right)}$$

$$Rout_{closed} = \frac{R_o \| (R_1+R_2)}{1+\left(\frac{R_2}{R_1+R_2}\right) A_o \left(\frac{R_2}{R_o+R_1+R_2}\right)}$$

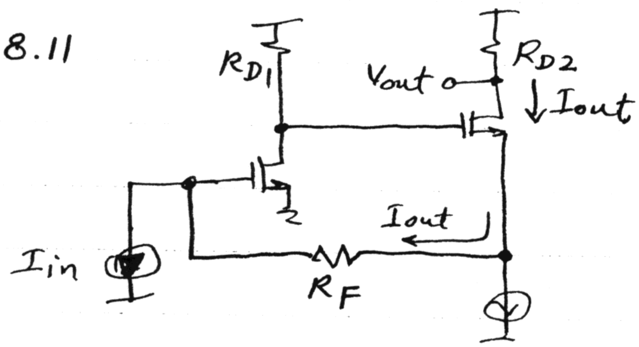
8.10
$$\frac{1+\frac{C_2}{C_1}}{\left(1+\frac{C_2}{C_1}\right)\frac{1}{g_{m1}(r_{o2}\|r_{o4})}+1} = 0.95\left(1+\frac{C_2}{C_1}\right)$$

↑ 5% gain error

$g_{m1}(r_{o2}\|r_{o4}) \approx 24.4$

$\Rightarrow \quad 1+\frac{C_2}{C_1} \le 1.28$

8.11

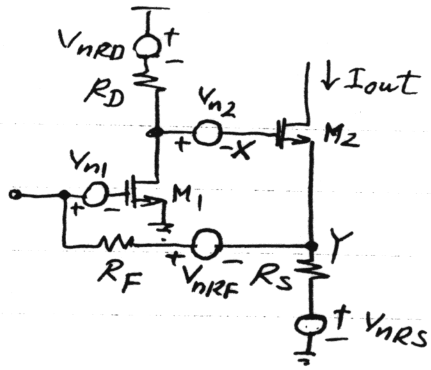


I_{out} fully flows through R_F $\therefore \Rightarrow I_{out} = I_{in}$ and $V_{out} = I_{out} \cdot R_{D2}$.

Thus, the transimpedance is equal to R_{D2}.

(continued on next page)

8.11 (cn'td)



$$\underbrace{-(I_{out} R_S + V_{nRS}}_{V_Y} + V_{nRF} + V_{n1}) g_{m1} R_D + V_{nRD} + V_{n2} = V_X$$

$$(V_X - V_Y) g_{m2} = I_{out}$$

$$\Rightarrow g_{m2}\left[(I_{out} R_S + V_{nRS} + V_{nRF} + V_{n1})(-g_{m1} R_D) + V_{nRD} + V_{n2} - (I_{out} R_S + V_{nRS})\right] = I_{out}$$

$$\Rightarrow I_{out}\left[1 + g_{m2} R_S (g_{m1} R_D + 1)\right] = g_{m2}\left[(-g_{m1} R_D - 1) V_{nRS} - g_{m1} R_D V_{nRF} - g_{m1} R_D V_{n1} + V_{nRD} + V_{n2}\right]$$

$$\Rightarrow I_{out} = \frac{g_{m2}\left[-(1 + g_{m1} R_D) V_{nRS} - g_{m1} R_D V_{nRF} - g_{m1} R_D V_{n1} + V_{nRD} + V_{n2}\right]}{1 + g_{m2} R_S (1 + g_{m1} R_D)}$$

If we apply a current of I_{in} to the input, the resulting output current is obtained as:

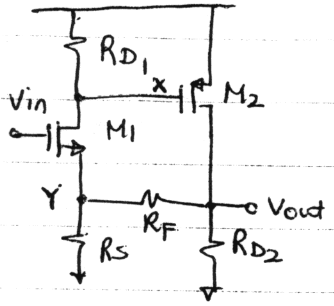
$$\Big\{\underbrace{\left[(I_{in} + I_{out}) R_S + I_{in} R_F\right](-g_{m1} R_D)}_{V_X} - \underbrace{(I_{in} + I_{out}) R_S}_{V_Y}\Big\} g_{m2} = I_{out}$$

$$\frac{I_{out}}{I_{in}} = \frac{\left[-g_{m1} R_D (R_S + R_F) - R_S\right] g_{m2}}{1 + g_{m2} R_S (1 + g_{m1} R_D)}$$

Dividing the output noise current by the gain yields the input-referred noise current:

$$I_{n,in} = \frac{-(1 + g_{m1} R_D) V_{nRS} - g_{m1} R_D V_{nRF} - g_{m1} R_D V_{n1} + V_{nRD} + V_{n2}}{-g_{m1} R_D (R_S + R_F) - R_S}$$

8.12



$I_{D_2} = 1\,mA \Rightarrow \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_X - |V_{TP}|)^2 = 1\,mA$

$\Rightarrow V_X = 1.478$

$I_{D_1} = 761\,\mu A$

$$\begin{cases} \dfrac{V_Y}{2K} + \dfrac{V_Y - V_{out}}{2K} = 761\,\mu A & \Rightarrow V_{out} = 1.841 \\ \dfrac{V_{out}}{2K} + \dfrac{V_{out} - V_Y}{2K} = 1\,mA & \Rightarrow V_Y = 1.681 \end{cases}$$

$I_{D_1} = 0.761\,mA \Rightarrow \frac{1}{2} \times 1.342\times10^{-4} \times 100 (V_{in} - V_Y - V_{Tn})^2 = 0.761\,mA$

$V_{in} - V_Y = 1.037 \Rightarrow V_{in} = 2.717\,V$

$G_{21} = 0.5$

$$\begin{cases} g_{m_1} = \dfrac{2 \times 0.761m}{1.037 - 0.7} = 4.52\,m\mho \\ g_{m_2} = \dfrac{2 \times 1\,mA}{1.522 - 0.8} = 2.77\,m\mho \end{cases}$$

$A_{v_{open}} = \dfrac{-2K}{1K + \dfrac{1}{4.52\,m\mho}} \left[-2.77\,m\mho \left[2K \| 4K\right]\right] = 6.048$

$A_{v_{closed}} = \dfrac{6.048}{1 + 3.024} = 1.503$

8.13

problem 8.9

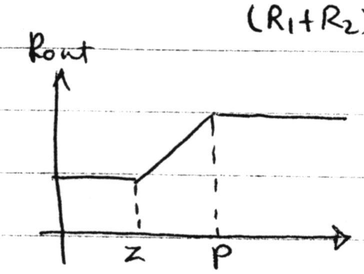
$$R_{out} = \frac{R_o(R_1+R_2)}{R_o + R_1 + R_2 + \dfrac{R_2^{\,2}}{R_1+R_2} \cdot \dfrac{A_o}{1 + \frac{s}{\omega_o}}}$$

Zero: ω_o

pole: $\omega_o + \dfrac{A_o R_2^2}{(R_1+R_2)(R_o+R_1+R_2)}\omega_o$

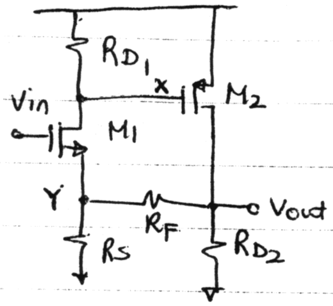
DC value: $\dfrac{R_o(R_1+R_2)}{R_o + R_1 + R_2 + A_o R_2^2/R_1+R_2}$

final value: $R_o \| (R_1+R_2)$



The output impedance is less reduced, as the loop gain gets smaller.

8.12



$I_{D2} = 1\,mA \Rightarrow \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_X - |V_{TP}|)^2 = 1\,mA$

$\Rightarrow V_X = 1.478$

$I_{D1} = 761\,\mu A$

$$\begin{cases} \frac{V_Y}{2K} + \frac{V_Y - V_{out}}{2K} = 761\,\mu A & \Rightarrow V_{out} = 1.841 \\ \frac{V_{out}}{2K} + \frac{V_{out} - V_Y}{2K} = 1\,mA & \Rightarrow V_Y = 1.681 \end{cases}$$

$I_{D1} = 0.761\,mA \Rightarrow \frac{1}{2} \times 1.342 \times 10^{-4} \times 100 (V_{in} - V_Y - V_{Tn})^2 = 0.761\,mA$

$V_{in} - V_Y = 1.037 \Rightarrow V_{in} = 2.717\,V$

$G_{21} = 0.5$

$$\begin{cases} g_{m1} = \frac{2 \times 0.761m}{1.037 - 0.7} = 4.52\,m\mho \\ g_{m2} = \frac{2 \times 1\,mA}{1.522 - 0.8} = 2.77\,m\mho \end{cases}$$

$A_{V_{open}} = \frac{-2K}{1K + \frac{1}{4.52\,m\mho}} \left[-2.77^{m\mho}\left[2^K \| 4^K\right]\right] = 6.048$

$A_{V_{closed}} = \frac{6.048}{1 + 3.024} = 1.503$

8.13

problem 8.9

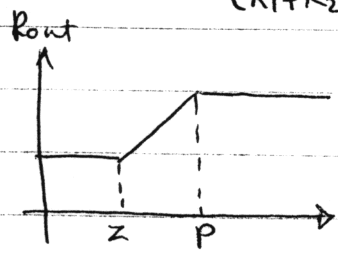
$$R_{out} = \frac{R_o(R_1 + R_2)}{R_o + R_1 + R_2 + \frac{R_2^2}{R_1 + R_2} \cdot \frac{A_o}{1 + \frac{s}{\omega_o}}}$$

Zero: ω_o

pole: $\omega_o + \frac{A_o R_2^2}{(R_1 + R_2)(R_o + R_1 + R_2)}\,\omega_o$

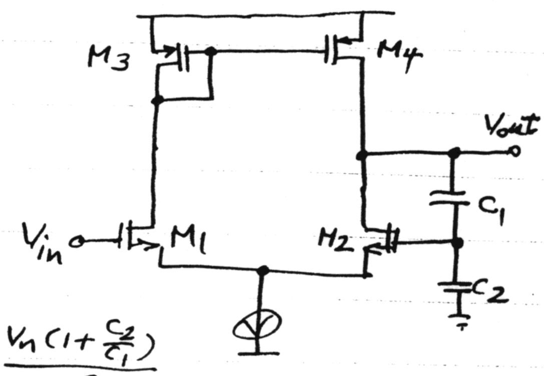
DC value: $\frac{R_o(R_1 + R_2)}{R_o + R_1 + R_2 + A_o\, R_2^2/R_1 + R_2}$

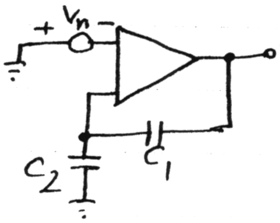
final value: $R_o \| (R_1 + R_2)$



The output impedance is less reduced, as the loop gain gets smaller.

8.14 The input-referred noise voltage of the circuit is the same as that of the open-loop circuit:





$V_{out} = V_n\left(1+\frac{C_2}{C_1}\right)$

$\Rightarrow$ input-referred noise $= \dfrac{V_n\left(1+\frac{C_2}{C_1}\right)}{1+\frac{C_2}{C_1}}$

$= V_n$

The noise produced by $M_1 - M_4$ referred to the input is:

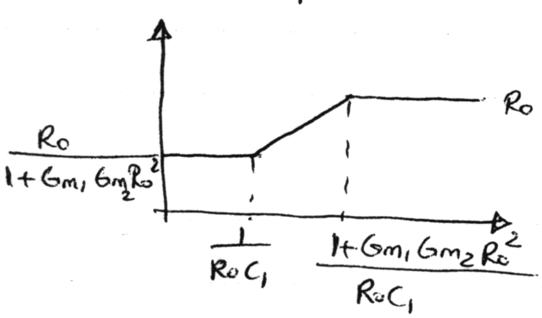
$$\overline{V_n^2} = 4kT\left(\frac{2}{3g_{m1,2}} + 2\,\frac{2\,g_{m3,4}}{3\,g_{m1,2}^2}\right) + 2\,\frac{K_N}{(WL)_{1,2}C_{ox}f} + 2\,\frac{K_P}{(WL)_{3,4}C_{ox}f} \times \frac{g_{m3,4}^2}{g_{m1,2}^2}$$

8.15

a) $Z_{in\ open} = R_o$

$$Z_{in\ closed} = \frac{R_o}{1 + G_{m1}G_{m2}R_o\,\dfrac{R_o}{1+R_oC_1s}} = \frac{R_o(1+R_oC_1s)}{R_oC_1s + 1 + G_{m1}G_{m2}R_o^2}$$

Zero: $\dfrac{1}{R_oC_1}$ Pole: $\dfrac{1+G_{m1}G_{m2}R_o^2}{R_oC_1}$ DC: $\dfrac{R_o}{1+G_{m1}G_{m2}R_o^2}$ final: R_o



b) Heavy feedback at lower frequency. As frequency increases, feedback weakens since the output impedance of the feedforward amplifier reduces

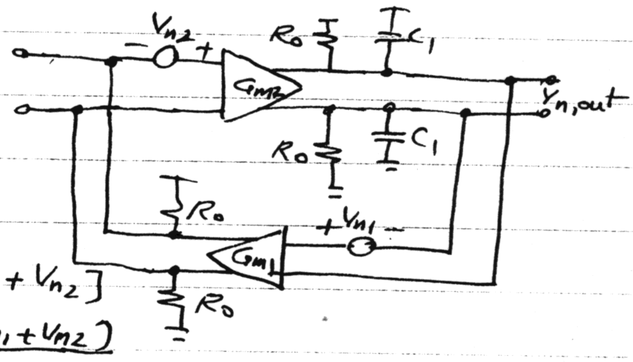
8.15 (c) For input-referred noise voltage, we short the input, hence $\overline{V_{n,out}^2} = 4kT \times 2\left(\frac{2}{3}g_m + \frac{1}{R_o}\right)\left(\left|R_o \| \frac{1}{C_1 s}\right|\right)$.

Dividing this by the voltage gain, $g_m^2\left(\left|R_o \| \frac{1}{C_1 s}\right|\right)^2$, we have

$$\overline{V_{n,in}^2} = 8kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_o}\right).$$

For the input noise current, we leave the input open.

Here, V_{n1} and V_{n2} represent the input noise of each differential pair (including the noise of resistors).



$$-V_{n,out} = G_{m2}\left(R_o \| \frac{1}{C_1 s}\right)\left[(V_{n,out} + V_{n1})G_{m1}R_o + V_{n2}\right]$$

$$\Rightarrow V_{n,out} = -\frac{G_{m2}\left(R_o \| \frac{1}{C_1 s}\right)(G_{m1}R_o V_{n1} + V_{n2})}{1 + G_{m2}\left(R_o \| \frac{1}{C_1 s}\right)G_{m1}R_o}$$

If we apply current between the two input terminals with value I_{in}, the output voltage is obtained as:

$$-V_{out} = (V_{out}G_{m1} + I_{in})R_o \cdot G_{m2}\left(R_o \| \frac{1}{C_1 s}\right)$$

$$\Rightarrow \frac{V_{out}}{I_{in}} = -\frac{G_{m2}R_o\left(R_o \| \frac{1}{C_1 s}\right)}{1 + G_{m1}G_{m2}R_o\left(R_o \| \frac{1}{C_1 s}\right)}$$

Dividning $V_{n,out}$ by this gain gives the input-referred noise current: $I_{n,in} = \frac{G_{m1}R_o V_{n1} + V_{n2}}{R_o} \Rightarrow \overline{I_{n,in}^2} = G_{m1}^2 V_{n1}^2 + \frac{V_{n2}^2}{R_o^2}$

8.16

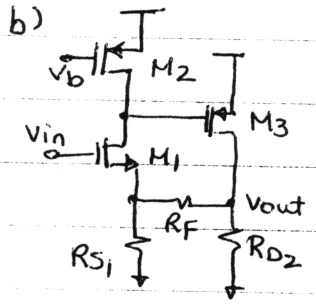
a) Due to symmetry of the π network, no current flows through R_F.

$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - R_{S1} \cdot I_{D1} - V_{Tn})^2 = 0.5\ mA$

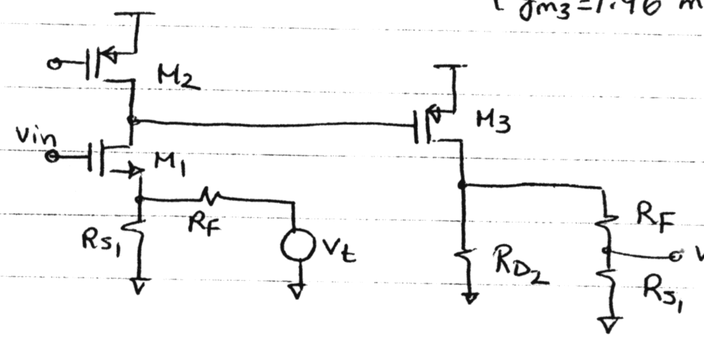
$(V_{in} - 2.2)^2 = \dfrac{2 \times 0.5 \times 10^{-3}}{1.342 \times 10^{-4} \times 100} \Rightarrow V_{in} = 2.473\ V$

$\left\{ \begin{array}{l} g_{m1} = 3.66\ m℧ \\ g_{m2} = 1.96\ m℧ \\ g_{m3} = 1.96\ m℧ \end{array} \right\} \begin{array}{l} r_{o1} = 20K \\ r_{o2} = 10K \\ r_{o3} = 10K \end{array}$

b)



$\Rightarrow$



$$A_{V_{open}} = \frac{r_{o2}}{R_{S1} \| R_F + \frac{1}{g_{m1}}} \times g_{m3}\left[r_{o3} \| R_{D2} \| (R_F + R_{S1}) \right] = 18.42$$

$$R_{out_{open}} = r_{o3} \| R_{D2} \| (R_F + R_{S1}) = 1.667\ K\Omega$$

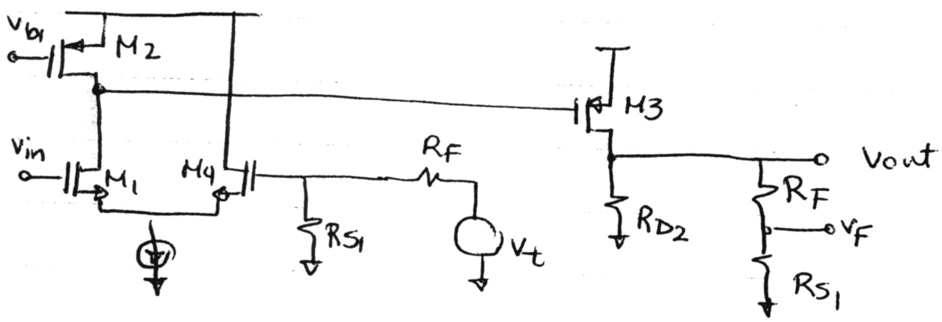
Loop gain: $\dfrac{V_t}{R_F} \times (R_F \| R_{S1}) \times \dfrac{r_{o2}}{R_{S1} \| R_F + \frac{1}{g_{m1}}} \times g_{m3}\left[r_{o3} \| R_{D2} \| (R_F + R_{S1}) \right] \times \dfrac{R_{S1}}{R_F + R_{S1}}$

$= V_F \Rightarrow$ loop gain $= 4.605$

$$A_V = \frac{18.42}{1 + 4.605} = 3.286$$

$$R_{out} = \frac{1.667^k}{1 + 4.605} = 297\ \Omega$$

8.17



$r_{o1} = r_{o4} = 20K$

$r_{o3} = 10K \quad r_{o2} = 10^K$

$g_{m1} = g_{m4} = 3.66\ m℧$

$g_{m2} = g_{m3} = 1.96\ m℧$

$$Av_{open} = \frac{1}{2} g_{m1} (r_{o1} \| r_{o2})\ g_{m3} \left(r_{o3} \| R_{D2} \| (R_F + R_{S1}) \right) = 39.85$$

$$R_{out} = r_{o3} \| R_{D2} \| (R_F + R_{S1}) = 1.667\ k$$

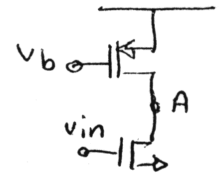
$$V_t \quad \frac{R_{S1}}{R_{S1} + R_F} \times \frac{g_{m1}}{2} \times (r_{o1} \| r_{o2})\ g_{m3} \left(r_{o3} \| R_{D2} \| (R_F + R_{S1}) \right) \frac{R_{S1}}{R_{S1} + R_F} = V_F$$

loop gain = 9.96

$$A_v = \frac{39.85}{1 + 9.96} = 3.63 \qquad R_{out} = \frac{1.667^K}{1 + 9.96} = 153\,\Omega$$

Smaller output impedance compared to 8.16.

8.18



a) $I_{D_n} = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} (V_{GS_n} - V_{TH_n})^2 \Rightarrow V_{GS_n} = 0.973\,V$

$V_{GS_p} = 1.311 \Rightarrow 3 - V_b = 1.311 \Rightarrow V_b = 1.69$

$V_{in} = R_1 \times I + V_{GS_n}$

M_3: saturation $\Rightarrow -V_A + V_b > |V_{THp}| \Rightarrow V_A < 1.689 - 0.8 = 0.889$

M_4: saturation $\Rightarrow -V_{out} + V_b > V_{THP} \Rightarrow V_{out} < 0.889 \Rightarrow R_1 I < 0.889 \Rightarrow R_1 \leq 1777$

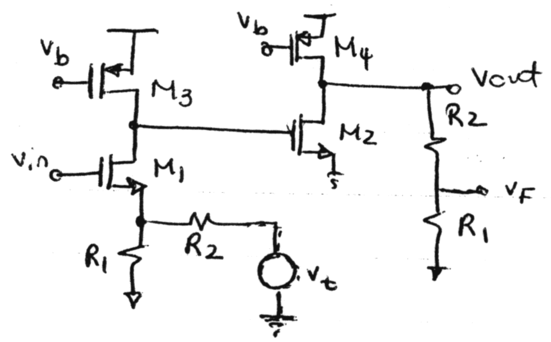
M_1: saturation $\Rightarrow V_A > R_1 I + (V_{GS_1} - V_{TH_n}) \Rightarrow 0.273 + R \times 0.5m < 0.889$

$\Rightarrow R_1 < 1232$

M_2: saturation: $V_{out} = R_1 I > V_A - V_{tn} \Rightarrow R_1 > \frac{0.889 - 0.7}{0.5 \times 10^{-3}} = 378\,\Omega$

$378 \leq R_1 \leq 1232 \Rightarrow 1.162 \leq V_{in} \leq 1.589$

b)



$R_1 = 805\,\Omega$ $\qquad r_{o1} = r_{o2} = 20K$

$g_{m1} = 3.66\,m℧$ $\qquad r_{o3} = r_{o4} = 10K$

$g_{m2} = 3.66\,m℧$

open loop gain $= \dfrac{r_{o3}}{R_1 \| R_2 + \frac{1}{g_{m1}}} \times g_{m2}\left(r_{o4} \| (R_1 + R_2) \| r_{o2}\right) = 97.6$

Output impedance $= r_{o4} \| (R_1 + R_2) \| r_{o2} = 2422\,\Omega$

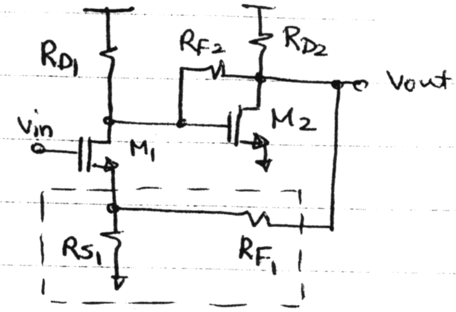
loop gain: $\dfrac{V_t}{R_2} \times (R_1 \| R_2) \times \dfrac{r_{o3}}{R_1 \| R_2 + \frac{1}{g_{m1}}} \times g_{m2}\left(r_{o4} \| (R_1 + R_2) \| r_{o2}\right) \dfrac{R_1}{R_1 + R_2} = V_F$

$\Rightarrow$ loop gain $= \dfrac{1}{3000} \times 635 \times 97.6 \times \dfrac{805}{3805} = 4.37$

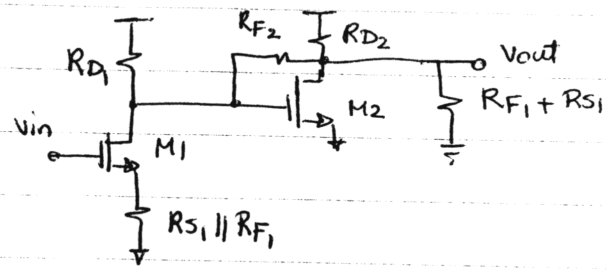
$$A_v = \frac{97.6}{1 + 4.37} = 18.17$$

$$R_{out} = \frac{2422}{1 + 4.37} = 451\,\Omega$$

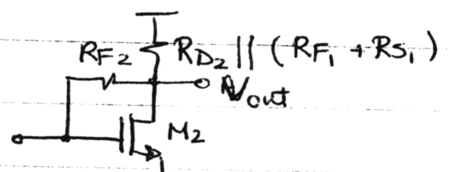
8.19



voltage - voltage



next we consider



$$R_{in_2} = \frac{R_{F_2}}{1 + g_{m_2}[R_{D_2} \| (R_{F_1} + R_{S_1})]} = 261\,\Omega$$

$$R_{out_2} = \frac{R_{D_2} \| (R_{F_1} + R_{S_1})}{1 + g_{m_2}[R_{D_2} \| (R_{F_1} + R_{S_1})]} = 174\,\Omega$$

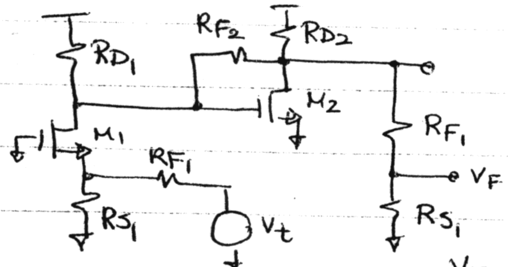
$$A_{v_2} = \frac{R_D}{R_D + R_{F_2}}(-g_{m_2} R_{F_2} + 1) = -3.6$$

$$R_D = R_{D_2} \| (R_{F_1} + R_{S_1}) = 1333$$

$$\text{Open loop gain} = -\frac{R_{D_1} \| R_{in_2}}{R_{S_1} \| R_{F_1} + \frac{1}{g_{m_1}}} \cdot A_{v_2} = \frac{231}{1000 + 200} \times 3.6 = 0.69$$

Open loop output impedance $= R_{out_2} = 174\,\Omega$

loop gain:



$$\frac{V_t}{R_{F_1}} \times (R_{S_1} \| R_{F_1}) \times \frac{R_{D_1}}{R_{S_1} \| R_{F_1} + \frac{1}{g_{m_1}}} \times A_{v_2} \times \frac{R_{S_1}}{R_{S_1} + R_{F_1}} = V_F$$

$$\frac{V_F}{V_t} = \frac{1}{2000} \times 1000 \times \frac{1000}{1200} \times 3.6 \times \frac{1}{2} = 0.75$$

$$A_{v_{closed}} = \frac{0.69}{1 + 0.75} = 0.394 \qquad R_{out_{closed}} = \frac{174}{1 + 0.75} = 99.5\,\Omega$$

8.20 $\quad I_{D1} = I_{D2} \Rightarrow V_{in} = 1.2538 \Rightarrow I_D = 2.316\ mA$

$$g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{THn}) = 1.342\times10^{-4} \times 100 \times (1.2538 - 0.7) = 7.432\ m\mho$$

$$g_{m2} = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{THp}) = 3.835\times10^{-5} \times 100\,(3 - 1.2538 - 0.8) = 3.628\ m\mho$$

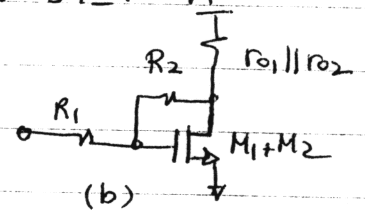
$$r_{o1} = \frac{1}{\lambda_1 I_1} = \frac{1}{0.1\times2.316\times10^{-3}} = 4.317\ k$$

$$r_{o2} = \frac{1}{\lambda_2 I_2} = \frac{1}{0.2\times2.316\times10^{-3}} = 2.159\ k$$

(a)

a) $A_V = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2}) = -(7.432 + 3.628)\,1.439 = -15.91$

$R_{out} = r_{o1} \| r_{o2} = 1439\ \Omega$



b) $A_V = \frac{1}{R_1} \cdot \frac{-(R_1 \| R_2)(g_{m1} + g_{m2})(R_2 \| r_{o1} \| r_{o2})}{1 + (g_{m1}+g_{m2})(R_2 \| r_{o1} \| r_{o2}) \frac{R_1}{R_1+R_2}}$

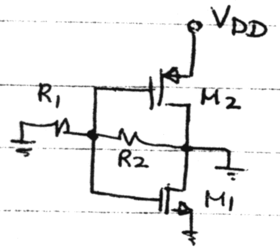
(eq. 8.70)

$$= \frac{1}{1} \times \frac{0.909\,(7.432 + 3.628)\,1.258}{1 + (7.432+3.628)\,1.258 \times \frac{1}{11}} = 5.58$$

$$R_{out} = \frac{R_2 \| r_{o1} \| r_{o2}}{1 + (g_{m1} + g_{m2})(R_2 \| r_{o1} \| r_{o2}) \frac{R_1}{R_1+R_2}}$$

$R_{out} = 1.258^k / 2.26 = 556\Omega$

(b) We figure out sensitivity for (b), (a) is a special case where $R_1 = 0$ $R_2 = \infty$



$G_m = g_{m2}$

R_{out} = same as before

$$A_V = \frac{g_{m2} (R_2 \| r_{o1} \| r_{o2})}{1 + (g_{m1} + g_{m2})(R_2 \| r_{o1} \| r_{o2}) \frac{R_1}{R_1+R_2}} = \frac{3.628 \times 1.258}{1 + 13.913 \times \frac{1.258}{11}} = 1.76$$

if $R_1 = 0$ and $R_2 = \infty$

$$A_V = g_{m2}(r_{o1} \| r_{o2}) = 3.628 \times 1.439 = 5.217$$

8-21

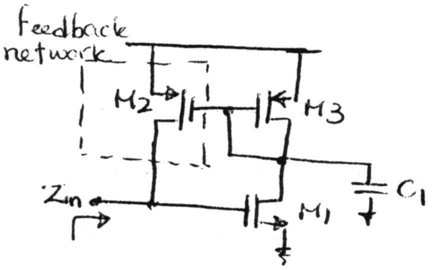
a) $\overline{v_{n\,out}^2} = 4KT\frac{2}{3}(g_{m1}+g_{m2})(r_{o1}||r_{o2})^2$

$\overline{v_{n\,in}^2} = \dfrac{4KT\frac{2}{3}}{g_{m1}+g_{m2}} =$

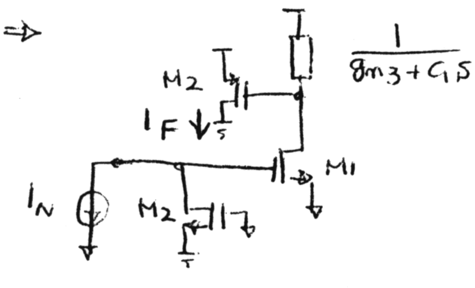
b) $\overline{v_{n\,in}^2} = 4KT\,R_1 + \dfrac{\left[\frac{4KT}{R_2} + 4KT\frac{2}{3}(g_{m1}+g_{m2})\right]R_o^2}{A_v^2} = 4KT\,R_1 + \dfrac{\frac{4KT}{R_2} + 4KT\frac{2}{3}(g_{m1}+g_{m}}{\left(\frac{R_2}{R_1+R_2}\right)^2(g_{m1}+g_{m2})}$

$= 4KTR_1 + \dfrac{4KT\frac{2}{3}}{g_{m1}+g_{m2}} + \dfrac{4KT/R_2}{\left(\frac{R_2}{R_1+R_2}\right)^2(g_{m1}+g_{m2})^2}$

8.22



Voltage - Current



$$\left|\frac{I_F}{I_N}\right| = r_{o2} \cdot g_{m1} \cdot \frac{1}{g_{m3}+C_1 s} \cdot g_{m2}$$

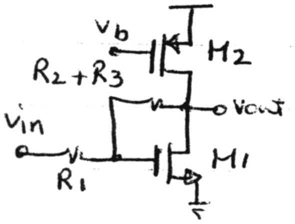
$r_{o2} \to \infty$

$$Z_{in_{open}} = r_{o2} \qquad Z_{in_{closed}} = \frac{r_{o2}}{1 - r_{o2}\, g_{m1} g_{m2} \frac{1}{g_{m3}+C_1 s}} = -\frac{g_{m3}+C_1 s}{g_{m1} g_{m2}}$$

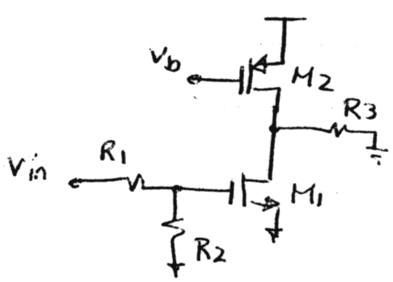
$$= -\frac{g_{m3}}{g_{m1} g_{m2}} - \underbrace{\frac{C_1}{g_{m1} g_{m2}}}\cdot s$$

8.23

very low freq.



very high freq.



eq. (8.70)

$$A_v = \frac{1}{R_1} \cdot \frac{-(R_1 \| (R_2+R_3))\, g_{m1} (R_2+R_3)}{1 + g_{m1}(R_2+R_3)\frac{R_1}{R_1+R_2+R_3}}$$

$$= \frac{-1}{2^k} \frac{(2^k \| 4^k)\, \frac{1}{200} \times 4^k}{1 + \frac{1}{200} \times 4^k \times \frac{1}{3}}$$

$$= -1.739$$

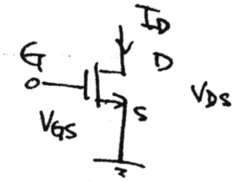
$$A_v = \frac{R_2}{R_1+R_2}(-g_{m1} R_3)$$

$$= \frac{1}{2}\left(-\frac{1}{200} \times 2000\right) = -5$$

Chapter 9

Problem 9.1

(a) For a MOSFET in triode region,



$$I_D = \mu_n C_{ox} \left(\frac{W}{L}\right)\left((V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right) \quad \text{— } (*)$$

Transconductance, $g_m = \frac{\partial I_D}{\partial V_{GS}}$ (from definition)

$$\frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L} V_{DS}$$

$$\boxed{g_m = \mu_n C_{ox} \frac{W}{L} V_{DS}}$$

Output resistance, $r_o = \frac{\partial V_{DS}}{\partial I_D}$

Take derivative of $(*)$ on both sides

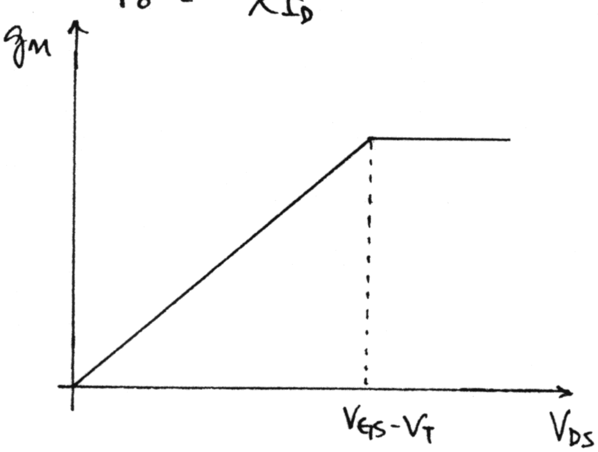
$$1 = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T - V_{DS}) \frac{\partial V_{DS}}{\partial I_D}$$

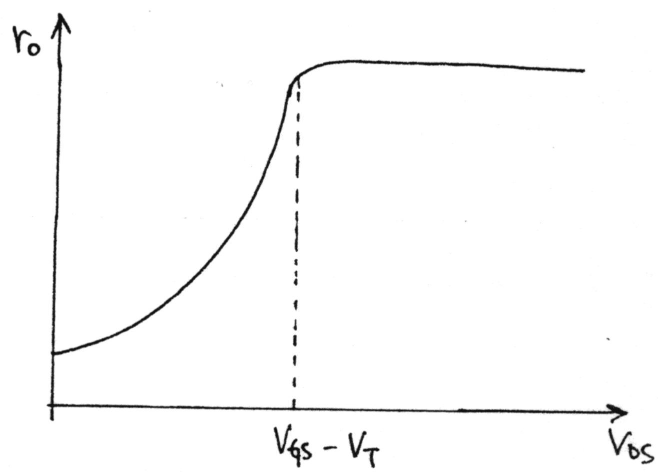
$$\boxed{r_o = \frac{\partial V_{DS}}{\partial I_D} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_T - V_{DS})}}$$

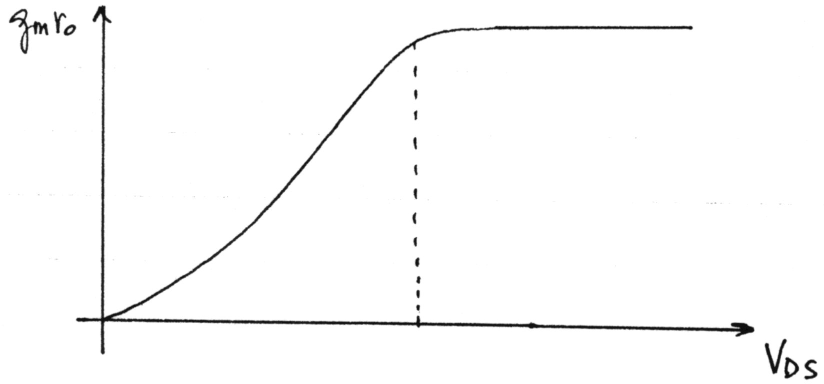
we know in saturation,

$g_m = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_T)$ &

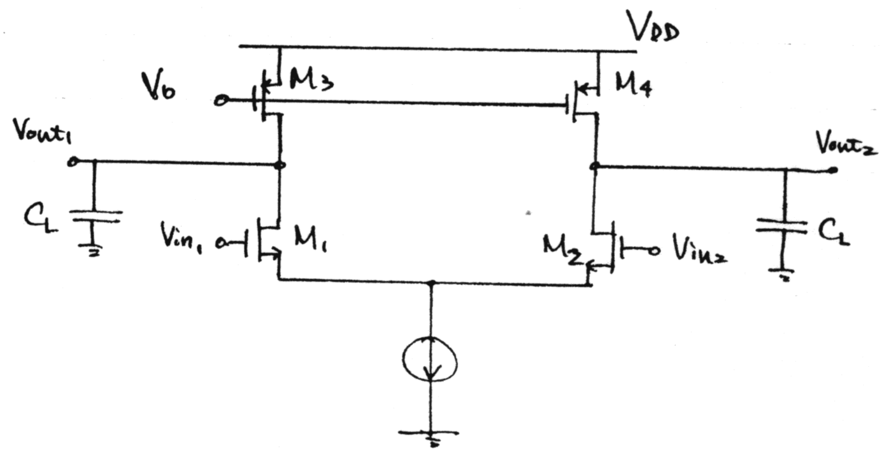
$r_o = \frac{1}{\lambda I_D}$



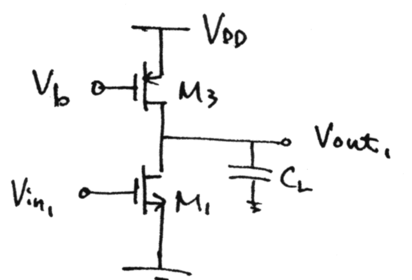




9.1 (b)



Use half-circuit concept:



$A_V = g_{m1}(r_{o1} \,//\, r_{o2})$

$g_{m1} = \sqrt{2\mu_n C_{ox} \frac{W}{L_{eff}} I_D}$ where $I_D = \frac{I_{SS}}{2} = 0.5mA$

$C_{ox}(@\, t_{ox} = 400\,\text{Å}) = 0.863 fF/\mu m^2 = 86.3 \times 10^{-9} F/cm^2$

$C_{ox}(@\, t_{ox} = 9 \times 10^{-9} m) = \frac{86.3 \times 10^{-9}}{9 \times 10^{-9}} \times 400 \times 10^{-10} = 383.56\ nF/cm^2$

$g_{m1} = \left[2 \times (350\ cm^2/V\cdot sec)(383.56 \times 10^{-9} F/cm^2)\left(\frac{50}{0.5 - 0.08 \times 2}\right)(0.5 \times 10^{-3} A)\right]^{\frac{1}{2}}$

$= 4.443\ m\Omega^{-1}$

$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.1)(1/V)(0.5\,mA)}$

$= 20k\Omega.$

$r_{o3} = \frac{1}{\lambda_3 I_{D3}} = \frac{1}{(0.2(1/V))(0.5\,mA)}$

$= 10k\Omega.$

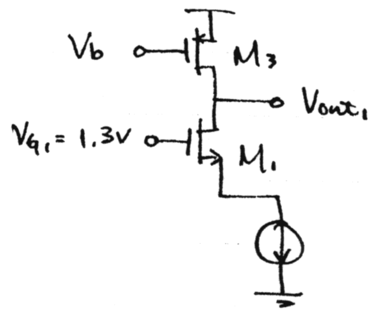
$A_V = g_{m1}(r_{o1} \,//\, r_{o3})$

$= (4.443 \times 10^{-3} \Omega^{-1})\left[\frac{20k \cdot 10k}{20k + 10k}\right](\Omega)$

$= \boxed{29.6 = A_V}$

To find maximum output swing

If we require both transistors are in saturation,



$$V_{DS1} \geq V_{GS1} - V_T$$

$$V_{D1} - V_{S1} \geq V_{G1} - V_{S1} - V_T$$

$$V_{D1} \geq V_{G1} - V_T = 1.3 - 0.7 = 0.6\text{V}.$$

$$\Rightarrow V_{out1\,min} = 0.6\text{V}$$

For M3: $\quad I_{D3} = \frac{1}{2}\mu_p C_{ox} \frac{W}{L_3} (V_{GS3} - V_{Tp})^2 (1 + \lambda V_{DS})$

For simplicity, assume channel length modulation is negligible, $\lambda \to 0$.

$$k_p' = \mu_p C_{ox}$$
$$= (100\ \text{cm}^2/\text{V.sec})(3.836 \times 10^{-7}\ \text{F/cm}^2)$$
$$= 3.836 \times 10^{-5}\ \text{A/V}^2$$

$$I_{D3} = 0.5\text{mA} = \frac{1}{2} k_p' \left(\frac{W}{L_{eff}}\right)(V_{GS3} - V_{Tp})^2$$

$$V_{GS3} - V_{Tp} = \left[\frac{(0.5\text{mA})(2)}{(3.836 \times 10^{-5}\ \text{A/V}^2)\left(\frac{50}{0.5 - 0.09 \times 2}\right)}\right]^{\frac{1}{2}} = 0.408$$

$$V_{DS3} \geq V_{GS3} - V_{Tp} = 0.408\text{V}.$$

$$V_{out1\,max} = V_{DD} - V_{DS3} = 3\text{V} - 0.408\text{V}$$
$$= 2.59\text{V}.$$

$$0.6\text{V} \leq V_{out1} \leq 2.59\text{V}$$

$\therefore$ One sided output swing $= 2.59 - 0.6 = 1.99\text{V}$.

$$\boxed{\text{Differential output swing} = (1.99 \times 2)\text{V} = 3.98\text{V}}$$

9.1 (c). From part (b), M_3 will enter the triode region when $V_{DS3} < V_{GS3} - V_{TP} = 0.408\,V$.

At the peak of the output swing $V_{DS_3} = 0.408 - 50mV$

$= 0.358\,V$

$$r_{o3} = \frac{1}{\mu_p C_{ox} \frac{W}{L_{eff}} (V_{GS_3} - V_{T_p} - V_{DS})} \quad \text{(from part a).}$$

$$= \left[(3.836 \times 10^{-5} A/V^2)\left(\frac{50}{0.32}\right)(0.408 - 0.358)\right]^{-1}$$

$$= 3.337\,k\Omega.$$

$$A_v = g_{m1} (r_{o1} \,//\, r_{o3})$$

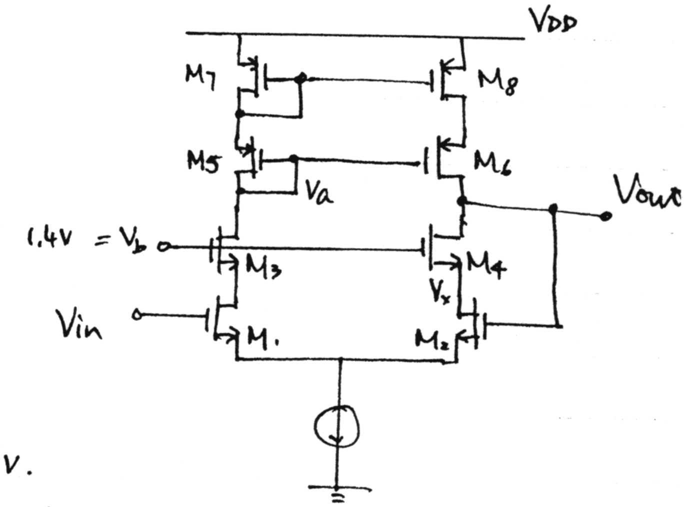
$$= (4.443 \times 10^{-3}\,\Omega^{-1})\left(\frac{20 \times 3.337}{20 + 3.337} k\Omega\right)$$

$$\boxed{A_v = 12.7}$$

9 P.5

Problem 9.2

(A) $V_b = 1.4V \quad I_{SS} = 1mA \quad \left(\frac{W}{L}\right)_{1-4} = \frac{100}{0.5}$



To keep M_3 in saturation,

$V_a > V_b - V_{THn} = 1.4V - 0.7V$

$= 0.7V$.

Assume $M_5 - M_8$ are identical,

so $V_{GS5} = V_{GS7}$

$$V_{GS5} = \frac{V_{DD} - V_a}{2} = \frac{3 - 0.7}{2} = 1.15V.$$

$$I_{D5} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_5 (V_{GS5} - V_{THp})^2 (1 + \lambda V_{DS5})$$

$$\left(\frac{W}{L_{eff}}\right)_5 = \frac{2I_{D5}}{\mu_p C_{ox} (V_{GS5} - V_{THp})^2 (1 + \lambda V_{DS})}$$

$$= \frac{2(0.5mA)}{(100)(3.836\times10^{-7})(1.15 - 0.8)^2(1 + 0.2(1.15))}$$

$$= 173.$$

$W_5 = 173 \times L_{eff} = 173 \times (0.5 - 0.09 \times 2)$.

$W_5 \cong 56\,\mu m$

$$\boxed{W_{5-8} = 56\,\mu m}$$

b. Max. output swing $= V_{TH4} - (V_{GS4} - V_{TH2}) = V_{TH4} + V_{TH2} - V_{GS4}$

$I_{D4} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_4 (V_{GS4} - V_{TH4})^2 (1 + \lambda V_{DS4})$ assume $\lambda \to 0$. for simplicity

$$V_{GS4} - V_{TH4} \cong \left[\frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_4}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2(0.5mA)}{(350)(3.836\times10^{-7})\left(\frac{100}{0.34}\right)}\right]^{\frac{1}{2}}$$

$$= 0.159V$$

$V_{GS4} = V_{TH4} + 0.159 = 0.859V$

Max. Output swing $= 0.7 + 0.7 - 0.859$

$= 0.541V$

9.2 (c).

$$A_{v\,\text{open loop}} = g_{m1}\,(g_{m4} r_{o4} r_{o2} \,//\, g_{m6} r_{o6} r_{o8})$$

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\tfrac{W}{L_{eff}}\right) I_D}$$

$$= \sqrt{2(350)(383.6\times10^{-9})\left(\tfrac{100}{0.34}\right)(0.5\,mA)}$$

$$= 6.28\,m\Omega^{-1}$$

$$g_{m4} = g_{m1} = 6.28\,m\Omega^{-1}$$

$$g_{m6} = \sqrt{2\mu_n C_{ox} \left(\tfrac{W}{L_{eff}}\right) I_D}$$

$$= \sqrt{2(100)(383.6\times10^{-9})\left(\tfrac{56}{0.5-0.09\times2}\right)(0.5\,mA)}$$

$$= 2.59\,m\Omega^{-1}$$

$$r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.5\,mA)} = 20\,k\Omega.$$

$$r_{o4} = r_{o2} = 20\,k\Omega.$$

$$r_{o6} = r_{o8} = \frac{1}{(0.2)(0.5\,mA)} = 10\,k\Omega.$$

$$A_v = (6.28m)\left[(6.28m \times 20k \times 20k) \,//\, (2.59m \times 10k \times 10k)\right]$$

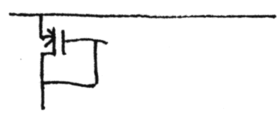
$$\boxed{A_v = 1474}$$

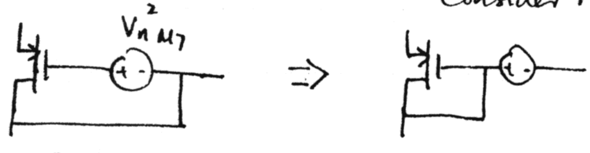
(d) Since this is a cascode configuration, the noise due to $M_{3,4,5,6}$ can be neglected.

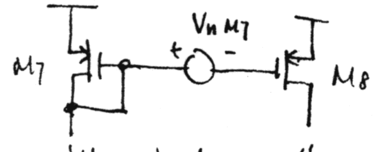
$\overline{V_n^2}_{\text{input}}$ due to $M_{1,2}$ $= 4kT\gamma \frac{1}{g_{m1,2}}$

$\overline{V_n^2}_{\text{input}}$ due to M_8 $= 4kT\gamma \frac{g_{m8}}{g_{m1,2}^2}$ same as in cascode.

$\overline{V_n^2}_{\text{input}}$ due to M_7

Consider M_7: M_7 



∴ 

M_7 will induce the same noise as M_8.

∴ input-referred noise voltage

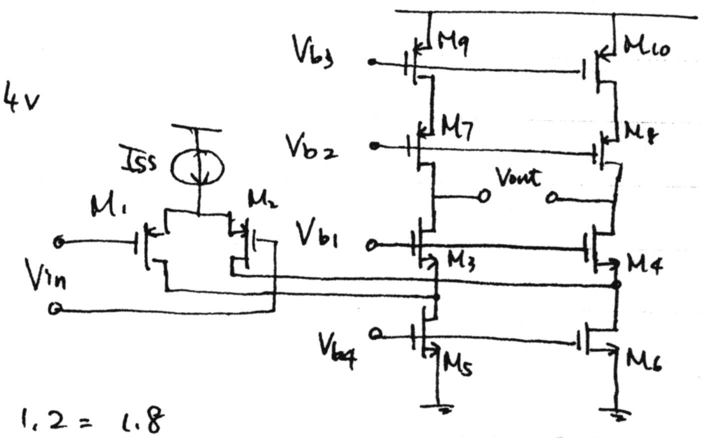
$$\overline{V_n^2} = \left[4kT\gamma \frac{1}{g_{m1,2}} + 4kT\gamma \frac{g_{m7,8}}{g_{m1,2}^2}\right] \times 2 \quad \text{where } \gamma = \frac{2}{3}$$

9.2 (d) cont.

$$\overline{V_n^2} = 4 \times 1.38 \times 10^{-23} \times 300 \times \frac{2}{3} \times 2 \left[\frac{1}{6.28m} + \frac{2.59m}{(6.28m)^2} \right]$$

$$\boxed{\begin{aligned} V_n^2 &= 4.966 \times 10^{-18} \ V^2/Hz \\ \text{or} &= 2.23 \times 10^{-9} \ V/\sqrt{Hz} \end{aligned}}$$

Problem 9.3



Requirements: Max. diff. swing 2.4V
$P_{total} = 6mW$

Max diff swing =
$= 2\left[V_{DD} - \left(V_{OD3} + V_{OD5} + |V_{OD7}| + |V_{OD9}|\right)\right]$
$= 2.4V$

$V_{OD3} + V_{OD5} + |V_{OD7}| + |V_{OD9}| = V_{DD} - 1.2 = 1.8$

In general, assign $|V_{OD7}|$ & $|V_{OD9}|$ to be large than V_{OD3} as PMOS has a smaller μ_p. Also M5 need large V_{OD} as I_{D5} is larger.

Let the followings:

$V_{OD3} = 0.3V$, $V_{OD5} = 0.44V$ $|V_{OD7}| = |V_{OD9}| = 0.53V$ $|V_{OD,1}| = 0.53V$

$P_{total} = 6mW = V_{DD} \times (I_{D5} + I_{D6})$.

$I_{D5} = I_{D6} = \frac{6mW}{(3)(2)} = 1mA$.

$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$

$\left(\frac{W}{L}\right) = \frac{2I_D}{\mu C_{ox}(V_{GS} - V_{TH})^2(1+\lambda V_{DS})}$

$\left(\frac{W}{L_{eff}}\right)_{5,6} = \frac{2(1mA)}{(350)(383.6\times10^{-9})(0.44)^2(1+(0.1)(0.44))}$ (Assume $V_{DS} \simeq V_{GS} - V_{TH}$,

$= 74$ Since λ is small, the result will not be affected too much.)

$W_{5,6} = (74)(0.34\mu m) = 25.16\mu m$.

Let $I_{D1,2} = 0.5mA$., $I_{D3,4} = 0.5mA$

$\left(\frac{W}{L_{eff}}\right)_{1,2} = \frac{2(0.5mA)}{(100)(383.6\times10^{-9})(0.53)^2(1+(0.2)(0.53))} = 84$

$W_{1,2} = 84(0.32\mu m) = 26.88\mu m$

$\left(\frac{W}{L_{eff}}\right)_{3,4} = \frac{2(0.5mA)}{(350)(383.6\times10^{-9})(0.3)^2(1+(0.1)(0.3))} = 80.$

$W_{3,4} = 80 \times 0.34\mu m = 27.2\mu m$.

$\left(\frac{W}{L_{eff}}\right)_{7,9} = \frac{2(0.5mA)}{(100)(383.6\times10^{-9})(0.53)^2(1+(0.2)(0.53))} = 84$

9.3 cont.

$W_{7,9} = 84 \times 0.32\mu m = 26.88\mu m.$

$V_{b4} = V_{GS5} = V_{OD5} + V_{TH5} = 0.44 + 0.7$

$V_{b4} = 1.14\,V$

$V_{b1} = V_{OD5} + V_{GS3} = V_{OD5} + V_{OD3} + V_{TH3} = V_{OD5} + V_{OD3} + V_{TH0} + \gamma\left(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|}\right)$

$= 0.44 + 0.3 + 0.7 + 0.45\left(\sqrt{0.9 + 0.44} - \sqrt{0.9}\right)$

$= 1.53\,V$

$V_{b3} = V_{DD} - |V_{GS9}| = V_{DD} - \left[|V_{OD9}| + |V_{TH9}|\right] = 3 - 0.53 - 0.8$

$V_{b3} = 1.67\,V$

$V_{b2} = V_{DD} - |V_{OD9}| - |V_{GS7}| = V_{DD} - |V_{OD9}| - |V_{OD7}| - |V_{TH7}|.$

$= 3 - 0.53 - 0.53 - \left[0.8 + 0.4\left(\sqrt{0.8 + 0.53} - \sqrt{0.8}\right)\right]$

$V_{b2} = 1.04\,V$

$V_{in,\text{common mode}} \leq V_{DD} - V_{I_{SS}} - V_{GS1} = 3 - 0.3 - 0.8 - 0.53 = 1.37\,V$ } Vin common mode can be zero ($V_{in_{CM}} = 0$)

$\geq V_{OD5} - V_{TH1} = 0.44 - 0.8 = -0.36\,V.$

$A_V = g_{m1}\left[(g_{m7} r_{o7} r_{o9}) // (g_{m3} r_{o3} (r_{o1} // r_{o5}))\right]$

$g_{m1} = \dfrac{2 I_{D1}}{V_{GS} - V_{TH}} = \dfrac{2(0.5mA)}{0.53} = 1.89\,m\Omega^{-1}$

$g_{m7} = g_{m1} = 1.89\,m\Omega^{-1}$

$g_{m3} = \dfrac{2 I_{D1}}{V_{GS} - V_{TH}} = \dfrac{2(0.5mA)}{0.3} = 3.33\,m\Omega^{-1}$

$r_{o7} = r_{o9} = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.2)(0.5mA)} = 10k\Omega = r_{o1}$

$r_{o3} = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.1)(0.5mA)} = 20k\Omega.$

$r_{o5} = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.1)(1mA)} = 10k\Omega$

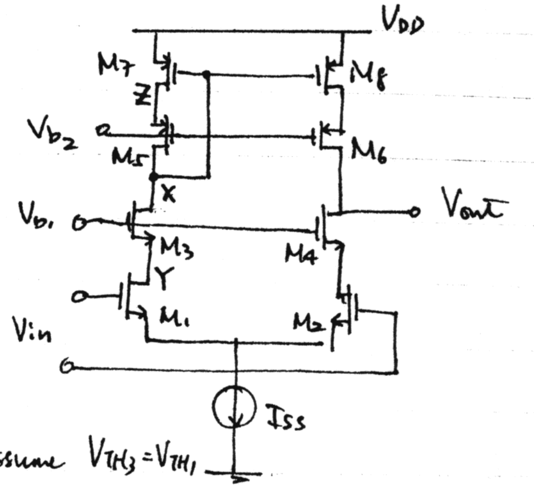
$A_v = (1.89m)\left[(1.89m\,(10k)^2) // (3.33m\,(20k)(10k/2))\right]$

$= 228$

$W_{1,2} = 26.88\mu m$	$V_{b1} = 1.53v$	$A_v = 228.$
$W_{3,4} = 27.2\mu m$	$V_{b2} = 1.04v$	
$W_{5,6} = 25.16\mu m$	$V_{b3} = 1.67v$	
$W_{7,8,9,10} = 26.88\mu m$	$V_{b4} = 1.14v$	

$-0.36 \leq V_{in,CM} \leq 1.37v$

Problem 9.4



$\left(\frac{W}{L}\right)_{1-8} = \frac{100}{0.5}$, $I_{SS} = 1mA$, $V_{b1} = 1.7V$, $\gamma = 0$

(a) $V_{in,CM,min} = V_{I_{SS}} + V_{GS1}$

$= V_{I_{SS}} + V_{THn} + V_{OD1}$

where $V_{I_{SS}}$ is the voltage across I_{SS}.

$V_{in,CM,max} = V_Y + V_{TH1}$;

$V_Y = V_{b1} - V_{GS3} = V_{b1} - V_{TH3} - V_{OD3}$

$V_{in,CM,max} = V_{b1} - V_{TH3} - V_{OD3} + V_{TH1}$ Assume $V_{TH3} = V_{TH1}$

$\boxed{V_{in,CM,max} = V_{b1} - V_{OD3}}$

To calculate V_{OD3}, $I_{D3} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_3 (V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$ Assume $\lambda \to 0$

$V_{OD3} = V_{GS3} - V_{TH} = \left[\frac{2I_{D3}}{\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right)}\right]^{\frac{1}{2}}$

$= \left[\frac{2(0.5mA)}{350(383.6\times10^{-9})\left(\frac{100}{0.34}\right)}\right]^{\frac{1}{2}} = 0.159V.$

$\therefore \boxed{V_{in,CM,max} = 1.7 - 0.159 = 1.541V}$

(b) $V_X = ?$ To find V_X, we can find V_{GS7}

$V_{GS7} - V_{THp} = \left[\frac{2I_{D7}}{\mu_p C_{ox}\left(\frac{W}{L_{eff}}\right)_7}\right]^{\frac{1}{2}}$

$= \left[\frac{2(0.5mA)}{(100)(383.6\times10^{-9})\left(\frac{100}{0.32}\right)}\right]^{\frac{1}{2}} = 0.289$

$V_{GS7} = 0.289 + V_{THp} = 1.089V$

$V_X = V_{DD} - V_{GS7} = 3 - 1.089V$

$\boxed{V_X = 1.911V}$

(c) For details, please see page 284 (chapter 9).

Max. output swing $= V_{TH4} - (V_{GS4} - V_{TH2})$

$V_{GS3} = V_{GS4}$ by symmetry, $V_{GS3} - V_{THn} = V_{GS4} - V_{THn} = 0.159V$

$V_{GS4} = 0.7 + 0.159 = 0.859V$

Max output swing $= 0.7 - (0.859 - 0.7) =$

$\boxed{\text{Max output swing} = 0.541V.}$

9.4 cont.

(d). We know $V_X = 1.911V$, $V_{GS5} = V_{GS7} = 1.089V$

To keep M_7 in saturation,

$V_Z < V_X + V_{THP}$; $V_{b2} = V_Z - |V_{GS5}|$

$V_{b2} < V_X + V_{THP} - |V_{GS5}| = 1.911V + 0.8 - 1.089$

$V_{b2} < 1.622V$

$V_{b2} > V_X - V_{TH5} = 1.911 - 0.8 \Rightarrow V_{b2} > 1.111V.$

$\therefore \boxed{1.11V < V_{b2} < 1.62V}$

e. As this is a cascode configuration, M_3, M_4, M_5, M_6 have negligible.

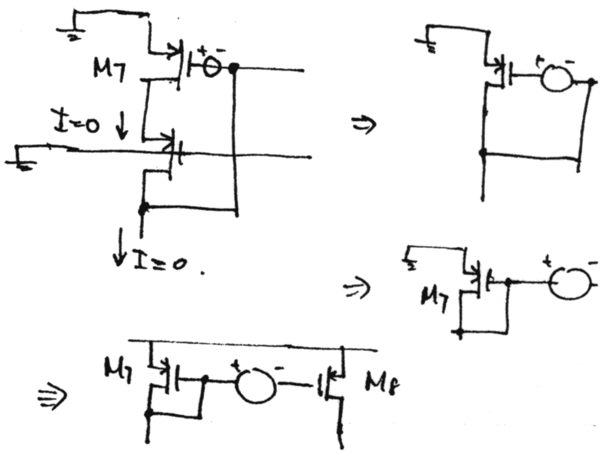
Input referred noise voltage due to M_1, M_2

$\overline{V_n^2}_{input} = 4kT\gamma \frac{1}{g_{m1,2}}$

$\overline{V_n^2}_{input\ M_8} = 4kT\gamma \frac{g_{m8}}{g_{m1,2}^2}$

$\overline{V_n^2}_{input\ M_7} = \overline{V_n^2}_{input\ due\ to\ M_8}$

$= 4kT\gamma \frac{g_{m7}}{g_{m1,2}^2}$



$\therefore$ Input referred noise voltage

$= \left[4kT\gamma\left(\frac{1}{g_{m1,2}} + \frac{g_{m7,8}}{g_{m1,2}^2}\right)\right] \times 2$

$g_{m1,2} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right) I_D}$

$= \left[2(350)(383.6\times10^{-9})\left(\frac{100}{0.34}\right)(0.5mA)\right]^{\frac{1}{2}}$

$= 6.28\times10^{-3}\ \Omega^{-1}$

$g_{m7,8} = \left[2(100)(383.6\times10^{-9})\left(\frac{100}{0.32}\right)(0.5mA)\right]^{\frac{1}{2}}$

$= 3.46m\Omega^{-1}$

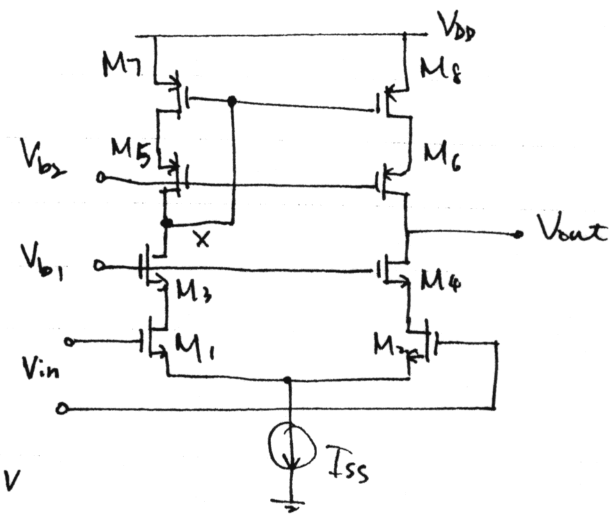
Input refered noise voltage $= \left[4\times1.38\times10^{-23}\times300\times\frac{2}{3}\times\left(\frac{1}{6.28m} + \frac{3.46m}{6.28m^2}\right)\right]\times2$

$\boxed{\begin{aligned} &= 5.45\times10^{-18}\ V^2/Hz \\ \text{or } &= 2.34\times10^{-9}\ V/\sqrt{Hz} \end{aligned}}$

Problem 9.5.

Requirement : Max. diff. swing = 2.4 V

Power$_{max}$ = 6mW.



$V_{DD} \cdot I_{SS} = \text{Power}_{max} = 6mW.$

$I_{SS} = \frac{6mW}{3V} = 2mA$

$|V_{GS7}| - |V_{THP}| = \left[\frac{2 I_{D7}}{\mu_p C_{ox} \left(\frac{W}{L_{eff}}\right)_7}\right]^{\frac{1}{2}}$ assume $W_{7,8} = 100\mu m$

$= \left[\frac{2(1mA)}{(100)(383.6\times10^{-9})\left(\frac{100}{0.32}\right)}\right]^{\frac{1}{2}} = 0.408\,V$

$|V_{GS7}| = 0.408 + |V_{THP}| = 1.208\,V$

$V_X = V_{DD} - |V_{GS7}| = 3 - 1.208 = 1.79\,V.$

$V_X - V_{TH5} < V_{b2} < V_X + V_{THp} - |V_{GS5}|$ assume $W_{5,6} = 100\mu m$

$0.99\,V < V_{b2} < 1.382\,V$

For larger output swing, choose $V_{b2} = 1.3V$.

$V_{out\,max} = V_{b2} + V_{THp} = 1.3 + 0.8 = 2.1V.$

We need one sided output swing $= 1.2V$, so $V_{out\,min} = 0.9V$.

$|V_{OD1}| = |V_{OD3}| = 0.3 = V_{ISS}$

$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L_{eff}}\right)(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$ Assume $V_{DS} \simeq V_{GS} - V_{TH}$

$\left(\frac{W}{L_{eff}}\right) = \frac{2 I_D}{\mu_n C_{ox}(V_{GS} - V_{TH})^2(1+\lambda V_{DS})} = \frac{2(1mA)}{(350)(383.6\times10^{-9})(0.3)^2(1+(0.1)(0.3))}$

$\left(\frac{W}{L_{eff}}\right) = 160 \Rightarrow W_{1,2,3,4} = (160)(L_{eff}) = (160)(0.34\mu m)$ let $L = 0.5\mu m$

$W_{1-4} = 55\mu m.$

$V_{in,CM} = V_{ISS} + V_{GS1,2} = V_{ISS} + V_{THn} + V_{OD1} = 0.3 + 0.7 + 0.3V$

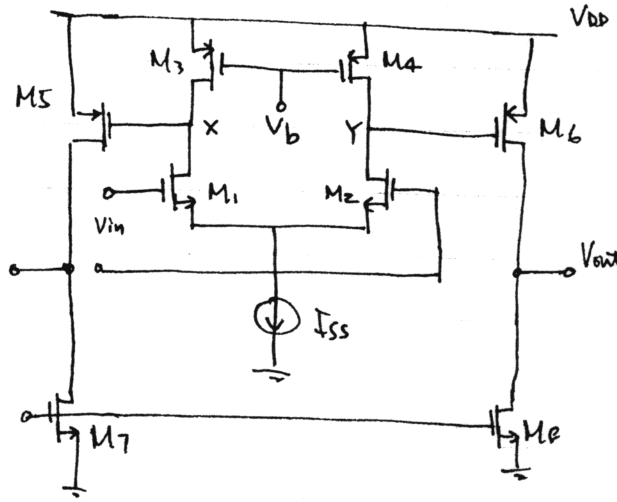
$= 1.3V$

$V_{b1} = V_{in,CM} + V_{OD1} = 1.3 + 0.3V = 1.6V.$

Summary

$\left(\frac{W}{L}\right)_{1-4} = \frac{55}{0.5}$	$V_{in\,CM} = 1.3V$	$I_{SS} = 2mA.$
$\left(\frac{W}{L}\right)_{5-8} = \frac{100}{0.5}$	$V_{b1} = 1.6V$	
	$V_{b2} = 1.3V$	

Problem 9.6



(a) Given: $\left(\frac{W}{L}\right)_{1-8} = \frac{100}{0.5}$ $\quad I_{SS} = 1mA$

$$I_{D5,6} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{5,6}(V_{GS5,6} - V_{THp})^2(1+\lambda V_{DS5,6})$$

$$V_{GS5,6} - V_{THp} = \left[\frac{2I_{D,5,6}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_{5,6}}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2(1mA)}{(100)(383.6\times10^{-9})\left(\frac{100}{0.32}\right)}\right]^{\frac{1}{2}} = 0.408v$$

$$V_{GS5,6} = 0.408 + 0.8 = 1.208v$$

$$V_{X,Y} = V_{DD} - V_{GS5,6} = 3 - 1.208v$$

$$\boxed{V_{X,Y} = 1.792v}$$

In order to keep M_1, M_2 in saturation,

$$V_{in,CM} < V_{X,Y} + V_{TH} = 1.792 + 0.7 = 2.492v$$

$$\boxed{\therefore\ V_{in,CM,max} = 2.49v}$$

(b) A_v of 1st stage $= g_{m1}(r_{o1} \| r_{o3})$

A_v of 2nd stage $= g_{m5}(r_{o5} \| r_{o7})$

$A_{vtot} = g_{m1}(r_{o1} \| r_{o3})\, g_{m5}(r_{o5} \| r_{o7})$.

$$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right)_1 (I_{D1})} = \left[2(350)(383.6\times10^{-9})\left(\frac{100}{0.34}\right)(0.5mA)\right]^{\frac{1}{2}}$$

$$= 6.28\ m\Omega^{-1}$$

$$g_{m5} = \frac{2I_D}{V_{GS} - V_{TH}} = \frac{2(1mA)}{0.408}$$

$$g_{m5} = 4.90\ m\Omega^{-1}$$

$$r_{o1} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.5mA)} = 20k\Omega$$

$$r_{o3} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(.5mA)} = 10k\Omega$$

$$r_{o5} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(1mA)} = 5k\Omega$$

$$r_{o7} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(1mA)} = 10k\Omega$$

$$A_v = (6.28m)[20k \| 10k](4.90m)[5k \| 10k]$$

$$\boxed{A_v = 684}$$

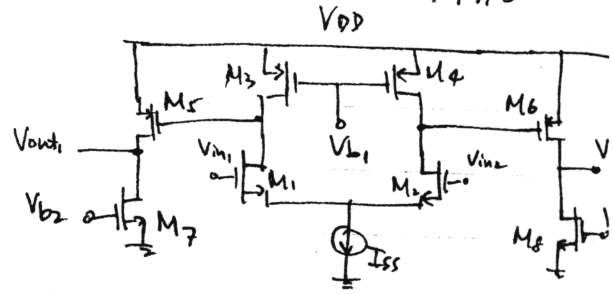
Max output swing $= 2(V_{DD} - |V_{OD5}| - V_{OD7})$

$$|V_{OD5}| = |V_{GS5}| - |V_{THp}| = 0.408V$$

$$V_{OD7} = V_{GS7} - V_{THn} = \left[\frac{2 I_D}{(\mu_n) C_{ox} \left(\frac{W}{L}\right)_7}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2(1mA)}{(350)(383.6\times10^{-9})\left(\frac{100}{0.34}\right)}\right]^{\frac{1}{2}} = 0.225$$

$$\boxed{\text{Max output swing} = 2(3 - 0.408 - 0.225) = 4.734V.}$$



Problem 9.7

Design the op amp of fig. 9.21

Max diff. swing = 4V

total Power = 6mW $\quad I_{SS} = 0.5mA$

Total current driven by $V_{DD} = \frac{6mW}{3V} = 2mA$

$I_{D5} + I_{D6} = 2mA - I_{SS} = 1.5mA \Rightarrow I_{D5} = I_{D6} = 0.75mA$

Max diff swing $= 2[V_{DD} - |V_{OD5}| - V_{OD7}] = 4V$

$\Rightarrow |V_{OD5}| + |V_{OD7}| = 1$ $\quad$ choose $V_{OD5} = 0.6V$, $V_{OD7} = 0.4V$

$I_D = \frac{1}{2}\mu C_{ox}(\frac{W}{L_{eff}})(V_{GS} - V_{TH})^2(1+\lambda V_{DS})$

$$(\frac{W}{L_{eff}})_5 = \frac{2I_D}{\mu_p C_{ox}(V_{GS} - V_{TH})^2(1+\lambda V_{DS})} = \frac{2(0.75mA)}{(100)(383.6\times10^{-9})(0.6)^2(1+0.2\times0.6)}$$

$(\frac{W}{L_{eff}})_5 = 97 \Rightarrow W_{5,6} = 97\times0.32\mu = 31\mu m$

$$(\frac{W}{L_{eff}})_7 = \frac{2(0.75m)}{(350)(383.6\times10^{-9})(0.4)^2(1+0.1(0.4))}$$

$= 67 \Rightarrow W_{7,8} = 67\times0.34\mu m = 23\mu m$

We are generally not worried about the swing of 1st stage,

assume $|V_{OD3}| = 1V$, $V_{OD1} = 1V$.

$$(\frac{W}{L_{eff}})_3 = \frac{2(0.25m)}{(100)(383.6\times10^{-9})(1)^2(1+0.2(1))} = 10.86$$

$W_{3,4} = 3.5\mu m$

$$(\frac{W}{L_{eff}})_1 = \frac{2(0.25m)}{(350)(383.6\times10^{-9})(1)^2(1+0.1)} = 3.4$$

$W_{1,2} = 1.2\mu m$

$V_{b1} = V_{DD} - |V_{OD3}| - V_{TH3} = 3 - 1 - 0.8 = 1.2V.$

$V_{in,CM} = V_{ISS} + V_{TH1} + V_{OD1} = 0.3 + 0.7 + \cancel{0.}\ 1.0 = 2V.$

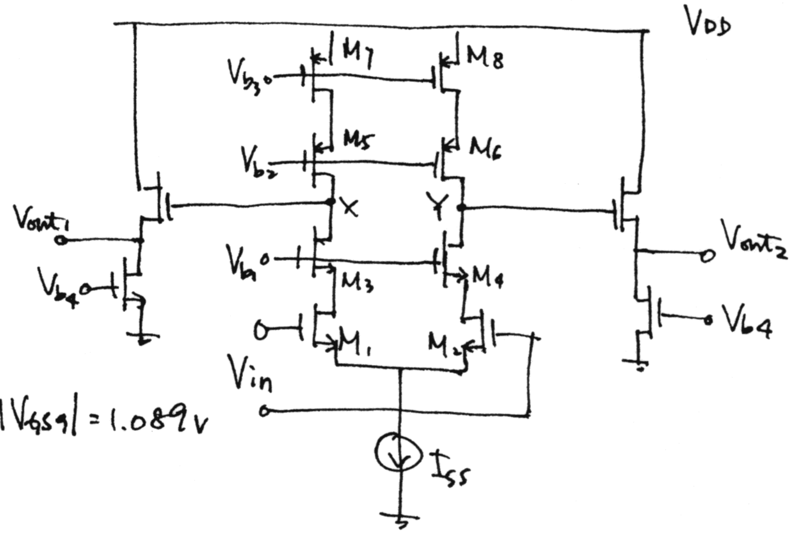
$V_{b2} = V_{TH7} + V_{OD7} = 0.7 + 0.4 = 1.1V.$

Summary $\quad L = 0.5\mu m$	
$W_1 = W_2 = 1.2\mu m$	$V_{b1} = 1.2V$
$W_{3,4} = 3.5\mu m$	$V_{b2} = 1.1V$
$W_{5,6} = 31\mu m$	$V_{in,CM} = 2V$
$W_{7,8} = 23\mu m$	

Problem 9.8

Given $I_{SS} = 1mA$, $I_{D9} - I_{D12} = 0.5mA$

$(\frac{W}{L})_{9-12} = \frac{100}{0.5}$



(a) $V_{X,Y,CM} = ?$

$$|V_{GS9}| - |V_{THp}| = \left[\frac{2I_{D9}}{\mu_p C_{ox}(\frac{W}{L})_9}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2(0.5mA)}{(100)(383.6n)(\frac{100}{0.32})}\right]^{\frac{1}{2}}$$

$$= 0.289V, \Rightarrow \quad |V_{GS9}| = 1.089V$$

$$\boxed{V_{XY,CM} = V_{DD} - |V_{GS9}| = 1.911V}$$

(b) V_X swing $= 0.2V$, $V_{X,CM} = 1.911V$

$V_{X\,max} = 2.011V$

$V_{X\,min} = 1.811V$

$$V_{OD7} = V_{OD5} = \frac{V_{DD} - V_{X\,max}}{2} = \frac{3 - 2.011}{2} = 0.495$$

$$V_{OD1} = V_{OD3} = \frac{V_{X\,min} - V_{I_{SS}}}{2} = \frac{1.811 - 0.4}{2} = 0.7055$$

$$(\frac{W}{L_{eff}})_{5-8} = \frac{2I_D}{\mu_p C_{ox}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})}$$

$$= \frac{2(0.5mA)}{(100)(383.6n)(0.495)(1 + 0.2)(0.495)}$$

$$= 97.02$$

$W_{5-8} = 97.02 \times L_{eff} = 31.05\mu m. \simeq 31.1\mu m$

$$(\frac{W}{L_{eff}})_{1-4} = \frac{2I_D}{\mu_n C_{ox}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})}$$

$$= \frac{2(0.5mA)}{(350)(383.6n)(0.7055)(1 + 0.1 \times 0.7055)}$$

$$= 14$$

$W_{1-4} \cong 4.8\mu m$

9 P.17

1c) $A_v = g_{m1}(g_{m3} r_{o3} r_{o1} \,||\, g_{m5} r_{o5} r_{o7}) g_{m9} (r_{o9} \,||\, r_{o11})$

$g_{m1} = \dfrac{2 I_D}{V_{GS1} - V_{TH}} = \dfrac{2(0.5mA)}{0.7055} = 1.417 m\Omega^{-1}$

$g_{m3} = g_{m1} = 1.417 m\Omega^{-1}$

$g_{m5} = \dfrac{2 I_D}{V_{GS5} - V_{THp}} = \dfrac{2(0.5mA)}{0.495} = 2.022 m\Omega^{-1}$

$g_{m9} = \dfrac{2(0.5mA)}{0.289} = 3.46 m\Omega^{-1}$

$r_{oN} = r_{o1} = r_{o3} = r_{o11} = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.1)(0.5mA)}$

$= 20 k\Omega$

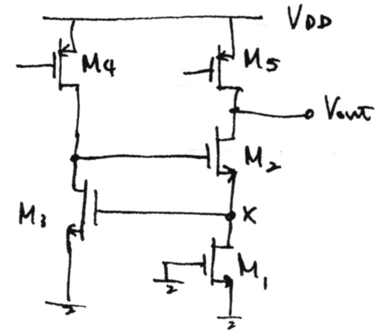
$r_{op} = r_{o5} = r_{o7} = r_{o9} = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.2)(0.5mA)}$

$= 10 k\Omega$.

$A_v = (1.417m)[1.417m \times 20k \times 20k \,||\, 2.022m \times 10k \times 10k] \times 3.46m \times (10k \,||\, 20k)$

$\boxed{A_v = 4871}$

Problem 9.9



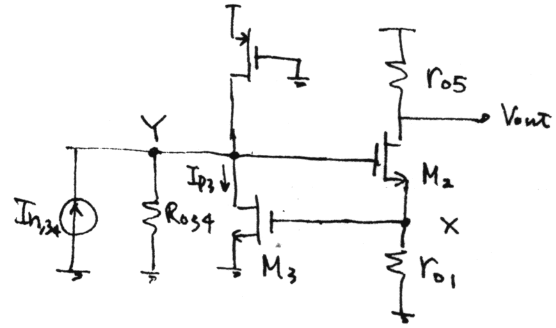
$\overline{V_n^2}, \text{input}|_{M_1} = 4kT\gamma \frac{1}{g_{m1}}$

$\overline{V_n^2}, \text{input}|_{M_2} \simeq 0$

$\overline{V_n^2}, \text{out}|_{M_5} = 4kT\gamma g_{m5} R_{out}^2$

$\overline{V_n^2}, \text{out}|_{M_5} = \frac{4kT\gamma g_{m5} R_{out}^2}{(g_{m1} R_{out})^2} = 4kT\gamma \frac{g_{m5}}{g_{m1}^2}$

Noise due to M3, M4 :



$I_{n,34} = 4kT\gamma(g_{m3} + g_{m4})$

$R_{o34} = r_{o3} // r_{o4}$

$V_X = r_{o1}\left(-\frac{V_{out}}{r_{o5}}\right)$

$I_{D3} = g_{m3} V_X = -\frac{g_{m3} r_{o1} V_{out}}{r_{o5}}$

$V_Y = R_{o34}\left(I_{n,34} + \frac{g_{m3} r_{o1}}{r_{o5}} V_{out}\right)$

neglect the r_{o2} to approximate the result,

$V_{out} = \frac{-r_{o5}}{\frac{1}{g_{m2}} + r_{o1}} V_Y = \frac{-r_{o5}}{\frac{1}{g_{m2}} + r_{o1}} R_{o34}\left(I_{n,34} + \frac{g_{m3} r_{o1}}{r_{o5}} V_{out}\right).$

$V_{out}\left(\frac{\frac{1}{g_{m2}} + r_{o2}}{+ r_{o5} R_{o34}} + \frac{g_{m3} r_{o1}}{r_{o5}}\right) = -I_n$

$V_{out} = \frac{-I_n r_{o5} R_{o34}}{\frac{1}{g_{m2}} + r_{o2} + g_{m3} r_{o1} R_{o34}} \simeq \frac{-I_n r_{o5} R_{o34}}{g_{m3} r_{o1} R_{o34}} = -\frac{r_{o5}}{r_{o1} g_{m3}} I_n$

$\overline{V_n^2}, \text{input}|_{M_3,M_4} = \frac{\left(\frac{r_{o5}}{r_{o1} g_{m3}}\right)^2 I_n}{g_{m1}^2 (r_{o5} // [g_{m2} r_{o2} r_{o1} g_{m3}(r_{o3}//r_{o4})])^2} \simeq \frac{I_n \left(\frac{\cancel{r_{o5}}}{g_{m3} r_{o1}}\right)^2}{g_{m1}^2 \cancel{r_{o5}^2}}$

$= 4kT\gamma(g_{m3} + g_{m4})\left[\frac{1}{g_{m1}^2 g_{m3}^2 r_{o1}^2}\right]$

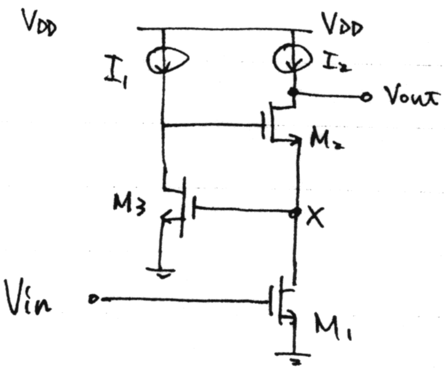
This is negligible compared with the noise due to M1, M5

$$\boxed{\therefore \overline{V_n^2}, \text{in total} = 4kT\gamma\left[\frac{1}{g_{m1}} + \frac{g_{m5}}{g_{m1}^2}\right]}$$

Problem 9.10

(a) $I_1 = 100\mu A$, $I_2 = 0.5mA$, $(\frac{W}{L})_{1-3} = \frac{100}{0.5}$

$(\frac{W}{L})_p = \frac{50}{0.5}$

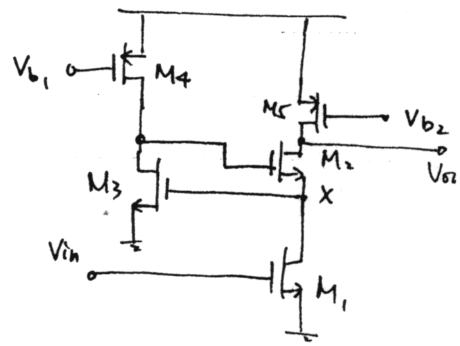


$I_{D3} = I_1 = \frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_3 (V_{GS} - V_{TH})^2(1+\lambda V_{DS})$

$V_{GS3} - V_{TH} = \left[\frac{2I_1}{\mu_n C_{ox}(\frac{W}{L_{eff}})_3}\right]^{\frac{1}{2}}$

$= \left[\frac{2(100\mu A)}{350 \times (383.6n)(\frac{100}{0.34})}\right]^{\frac{1}{2}} = 0.0712$

$\boxed{V_{GS3} = 0.7712V = V_{G3} = V_x}$



$V_{GS2} - V_{THn} = \left[\frac{2I_2}{\mu_n C_{ox}(\frac{W}{L_{eff}})_2}\right]^{\frac{1}{2}}$

$= \left[\frac{2(0.5mA)}{(350)(383.6n)(\frac{100}{0.34})}\right]^{\frac{1}{2}} = 0.159V$

$V_{GS2} = 0.859V$

$\boxed{V_{G2} = V_{GS2} + V_x = 1.630V.}$

(b) Max output swing:

$V_{out\,max} = V_{DD} - |V_{OD5}|$

$V_{out\,min} = V_x + V_{OD2} = V_{GS3} + V_{OD2}$

Max output swing $= V_{DD} - |V_{OD5}| - V_{OD2} - V_{GS3}$

$|V_{GS5}| - |V_{TH}| = \left[\frac{2I_D}{\mu_p C_{ox}(\frac{W}{L_{eff}})_5}\right]^{\frac{1}{2}} = \left[\frac{2(0.5mA)}{(100)(383.6n)(\frac{50}{0.32})}\right]^{\frac{1}{2}}$

$= 0.408V = |V_{OD5}|$

$\boxed{\text{Max output swing} = 3 - 0.408 - 0.159 - 0.7712 = 1.6618V}$

(c) $A_v = g_{m1}[r_{o5} \,||\, (g_{m2} r_{o2} r_{o1} g_{m3}(r_{o3} \,||\, r_{o4}))]$ Note: r_{o5} is limiting the gain.

$\simeq g_{m1} r_{o5}$

$g_{m1} = \sqrt{2\mu_n C_{ox}(\frac{W}{L})_1 I_{D1}} = \sqrt{2(350)(383.6n)(\frac{100}{0.34})(0.5mA)}$

$= 6.28 m\Omega^{-1}$

$r_{o5} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(0.5mA)} = 10k\Omega$

$\boxed{A_v = 62.8}$

9.10 cont.

(c) $\overline{V^2_{n,in}} = 4kT\gamma\left[\frac{1}{g_{m1}} + \frac{g_{m5}}{g_{m1}^2}\right] + 4kT\gamma\left(\frac{1}{g_{m3}} + \frac{g_{m4}}{g_{m3}^2}\right)\left[\frac{g_{m2} r_{o2} g_{m3} (r_{o3} // r_{o4})}{g_{m1}(r_{o1} + r_{o2} + r_{o5})}\right]^2$ (see

$g_{m1} = g_{m2} = 6.28 m\Omega^{-1}$

$g_{m5} = \left[2(100)(383.6n)\left(\frac{50}{0.32}\right)(0.5m)\right]^{\frac{1}{2}} = 2.45 m\Omega^{-1}$

$g_{m3} = \left[2(350)(383.6n)\left(\frac{100}{.34}\right)(100\mu)\right]^{\frac{1}{2}} = 2.81 m\Omega^{-1}$

$g_{m4} = \left[2(100)(383.6n)\left(\frac{50}{.32}\right)(100\mu)\right]^{\frac{1}{2}} = 1.09 m\Omega^{-1}$

$r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.5m)} = 20k\Omega$

$r_{o3} = \frac{1}{(0.1)(100\mu)} = 100k\Omega$

$r_{o4} = \frac{1}{(0.2)(100\mu)} = 50k\Omega$

$r_{o5} = \frac{1}{(0.2)(0.5m)} = 10k\Omega.$

$\overline{V^2_{n,in}} = 4(1.38\times10^{-23})(300)\left(\frac{2}{3}\right)\left[\frac{1}{6.28m} + \frac{2.45m}{6.28m^2}\right]$

$= 2.444\times10^{-18}\ V^2/Hz$

$\overline{V_{n,in}} = 1.56\times10^{-9}\ V/\sqrt{Hz}$

Problem 9.11

$V_P = 100mV$

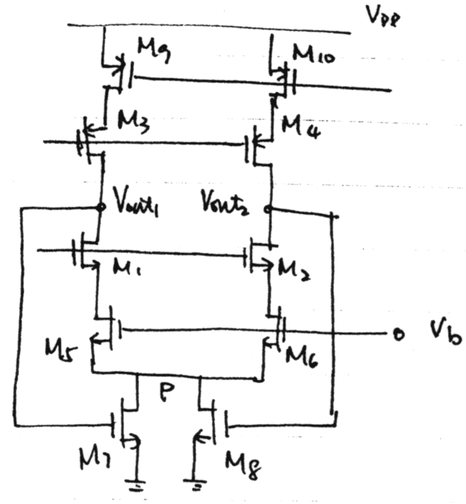
$V_{out,CM} = 1.5V$, $I_{D7,8} = 0.5mA$

$I_D = \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_7 \left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$

$\left(\frac{W}{L_{eff}}\right)_7 = \frac{2I_D}{\mu_n C_{ox}\left[(V_{GS} - V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]}$

$= \frac{2(0.5mA)}{(350)(383.6n)\left[(1.5-0.7)(0.1) - \frac{0.1^2}{2}\right]}$

$= 99.3$

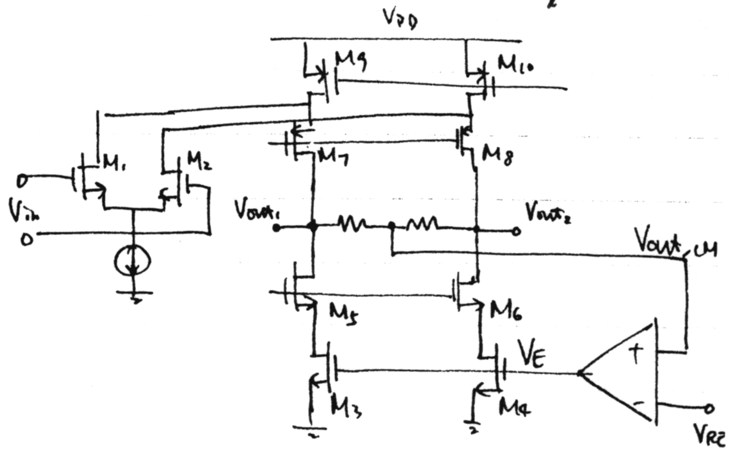


$W_{7,8} = 99.3 \times 0.34\mu m = 33.762\mu m \simeq 34\mu m$

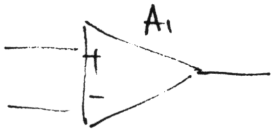
$\boxed{\left(\frac{W}{L}\right)_{7,8} = \frac{34}{0.5}}$

Problem 9.12

(a) PMOS devices should be used. Since $V_{out,CM}$ is in the middle voltage range, (around 1.5V), and $V_{GS3,4}$ are in low voltage range, (around 0.7 - 0.8V), we should use PMOS to bring down the voltage.



(b)



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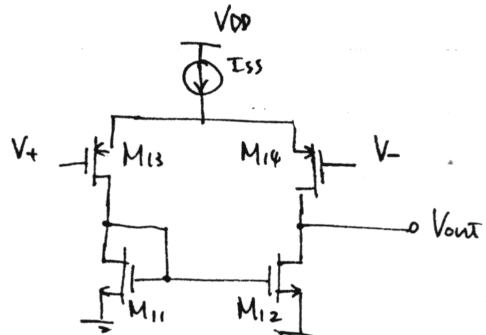


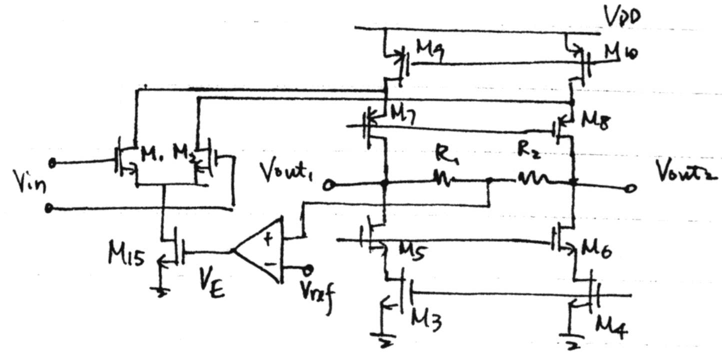
$A_1 = g_{m13}(r_{o12} \| r_{o14})$

$$\frac{V_{out,CM}}{V_E} = -g_{m3,4}\left(g_{m5} r_{o5} r_{o3} \| g_{m7} r_{o7} (r_{o1} \| r_{o9})\right)$$

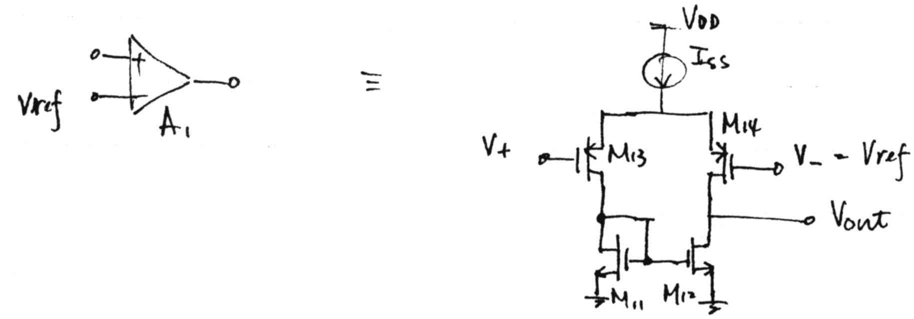
$$\boxed{\text{Loop gain} = -g_{m3,4}\left[(g_{m5} r_{o5} r_{o3}) \| (g_{m7} r_{o7} (r_{o1} \| r_{o9}))\right] g_{m13}(r_{o12} \| r_{o14})}$$

Problem 9.13

(a) Since we need to bring down $V_{out,CM}$ to fit the bias voltage of NMOS, which is relatively low, we should use PMOS for the input pair of amplifier.



(b)



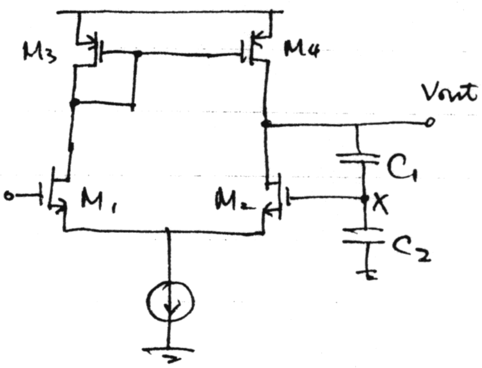
$A_1 = g_{m13}(r_{o12} \parallel r_{o14})$

$\frac{V_{out,CM}}{V_E} = -g_{m15}\left[(g_{m5} r_{o5} r_{o3}) \parallel (g_{m7} r_{o7}(r_{o9} \parallel g_{m1} r_{o1} r_{o15}))\right]$

loop gain $= -g_{m15}\left[(g_{m5} r_{o5} r_{o3}) \parallel (g_{m7} r_{o7}(r_{o9} \parallel g_{m1} r_{o1} r_{o15}))\right] g_{m13}(r_{o12} \parallel r_{o14})$

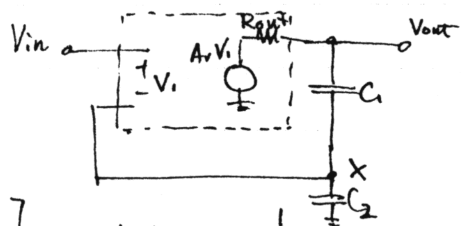
Problem 9.14

(a) $\left(\frac{W}{L}\right)_{1-4} = \frac{100}{0.5}$, $C_1 = C_2 = 0.5pF$, $I_{SS} = 1mA$



$A_v = g_{m1}(r_{o2} // r_{o4})$

$R_{out} = r_{o2} // r_{o4}$



$V_{in} = V_1 + V_x$

$V_x = V_{out} \frac{C_1}{C_1 + C_2}$

$$V_{out} = A_v V_1 \left[\frac{\frac{1}{C_1 // C_2 s}}{R_{out} + \frac{1}{C_1 // C_2 s}} \right] = A_v V_1 \frac{1}{(C_1 // C_2) R_{out} s + 1}$$

$$= \frac{A_v}{1 + (C_1 // C_2) R_{out} s} \left(V_{in} - V_{out} \frac{C_1}{C_1 + C_2}\right)$$

$$\left(1 + (C_1 // C_2) R_{out} s + A_v \frac{C_1}{C_1 + C_2}\right) V_{out} = A_v V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{A_v}{1 + A_v \frac{C_1}{C_1 + C_2} + (C_1 // C_2) R_{out} s} = \frac{\frac{A_v}{1 + A_v \frac{C_1}{C_1 + C_2}}}{1 + \frac{(C_1 // C_2) R_{out}}{1 + A_v \frac{C_1}{C_1 + C_2}} s}$$

$$\boxed{\tau = \frac{\frac{C_1 C_2}{C_1 + C_2} R_{out}}{1 + A_v \frac{C_1}{C_1 + C_2}}}$$

(b) $I_{D2} = 0.1 I_{SS}$,

Since I_{D2} is still small, we can solve this problem by assuming the current through C_1 & C_2 roughly equal to I_{SS}

$$V_X(t) - V_X(0) = \frac{I_{SS}}{C_2} t$$

At $t = 0^-$ $\quad I_{D1} = I_{D2} = 0.5mA$

$$V_{GS1,2} - V_{TH} = \left[\frac{2 I_D}{\mu_n C_{ox} \left(\frac{W}{L_{eff}}\right)} \right]^{\frac{1}{2}} = \left[\frac{2(0.5mA)}{(350)(383.6n)\left(\frac{100}{0.34}\right)} \right]^{\frac{1}{2}}$$

$$= 0.159V$$

At time $= t$, when $I_{D2} = 0.1 I_{SS} \Rightarrow I_{D1} = 0.9 I_{SS}$

$$I_D = \frac{1}{2} k_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_T)^2$$

$$\frac{I_{D1}}{I_{D2}} = \frac{0.9 I_{SS}}{0.1 I_{SS}} = \frac{(V_{GS1} - V_T)^2}{(V_{GS2} - V_T)^2} \Rightarrow \frac{(V_{GS1} - V_T)}{(V_{GS2} - V_T)} = 3$$

$$V_{GS1}(t) - V_T = 3(V_{GS2} - V_T)$$

$$(V_{GS1}(t) - V_T) = (V_{GS1}(0) - V_T) + 1V = 0.159 + 1V = 1.159V = 3(V_{GS2} - V_T)$$

$$V_{GS2}(t) - V_T = 0.386V$$

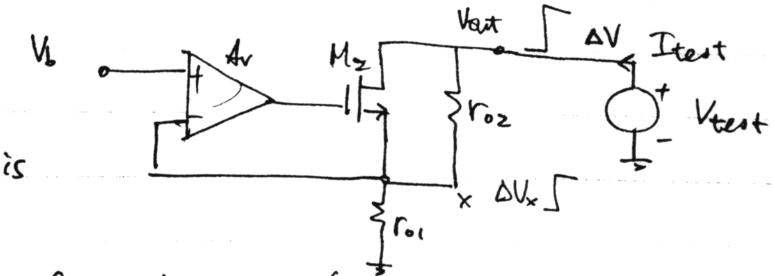
9.14 cont.

$$[V_{GS_2}(t) - V_T] - [V_{GS_2}(0) - V_T] = V_X(t) - V_X(0) = \frac{I_{SS}}{C_2} t$$

$$0.386 - 0.159 = 0.227 = \frac{1\,mA}{0.5\,pF}(t)$$

$$\boxed{t = 113.5\,ps.}$$

Problem 9.15



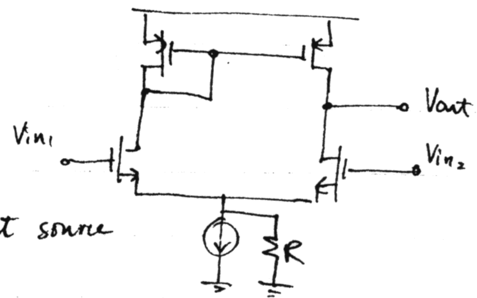
The mistake is made when we say the current from V_{test} is equal to $\Delta V / r_{o2}$.

We can see it when we start from the amplifier.

If we assume current from V_- is very small or negligible, the current through r_{o1} is equal to I_{test}, the current driven from V_{test}. The current through r_{o1} is $\frac{\Delta V_x}{r_{o1}}$, which is a much smaller value than $\Delta V / r_{o2}$.

The mistake is made because the current through r_{o2} is actually equal to $\Delta V / r_{o2}$ or $\simeq \frac{\Delta V - \Delta V_x}{r_{o2}}$. This current is larger ~~since~~ than I_{test} since some extra current from M_2 makes the current through r_{o2} larger. As a result, ΔV_{ro2} (Δ voltage across r_{o2}) increases by about ΔV, but the current from V_{test} ~~only~~ increases only by $\frac{\Delta V_x}{r_{o1}}$.

Problem 9.16

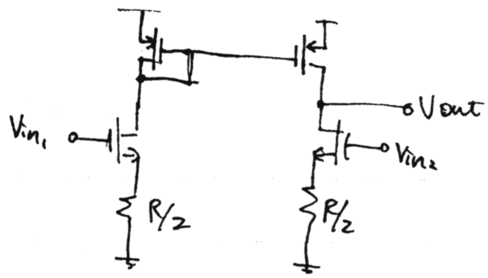


$$CMRR = \frac{\text{diff. gain}}{\text{CM. gain}}$$

$$\text{diff. gain} = g_{m1}(r_{o2} \| r_{o4})$$

CM gain: let R is the resistance of current source

$\Delta V_{in1} = \Delta V_{in2} = V_{CM}$



$$\frac{V_{out}}{\Delta V_{in1}} = \frac{-\frac{1}{g_{m3}}}{\frac{R}{2} + \frac{1}{g_{m1}}} g_{m4}(R_{out})$$

$$\frac{V_{out}}{\Delta V_{in2}} = \frac{-R_{out}}{\frac{R}{2} + \frac{1}{g_{m2}}}$$

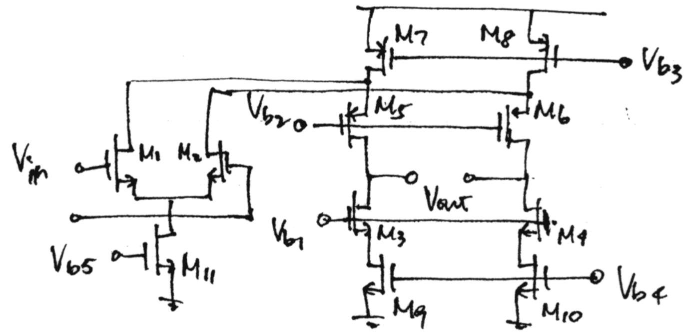
$$\left| \frac{V_{out}}{V_{CM}} \right| = \frac{2R_{out}}{\frac{R}{2} + \frac{1}{g_{m1}}} \approx \frac{4R_{out}}{R}$$

where $R_{out} = r_{o4} \| r_{o2}$

$$\text{CM gain} = \frac{4(r_{o4} \| r_{o2})}{R}$$

$$CMRR = \frac{g_{m1}(r_{o2} \| r_{o4})}{\frac{4(r_{o2} \| r_{o4})}{R}} = \boxed{\frac{g_{m1}R}{4} = CMRR}$$

Problem 9.17



Neglect the noise due to $M_{11}, M_3, M_4, M_5, M_6$.

Input-referred flicker noise due to $M_{7,8} = 2\left[\dfrac{\overline{V_{n,7,8}^2} \cdot g_{m7,8}^2 R_{out}^2}{A_v}\right]$

where $A_v = g_{m1}(R_{out})$, $\overline{V_{n,7,8}^2} = \dfrac{K_P}{C_{ox}(WL)_{7,8}} \cdot \dfrac{1}{f}$

$$\overline{V_{n,input}^2}\Big|_{M9,10} = \frac{\overline{V_{n,9,10}^2} \cdot g_{m9,10}^2 R_{out}^2}{g_{m1}^2 R_{out}^2} = 2\left[\overline{V_{n\,9,10}^2}\ \frac{g_{m9,10}^2}{g_{m1,2}^2}\right]$$

Total Input-referred flicker noise

$$\boxed{= \frac{2K_N}{C_{ox}f}\left[\frac{1}{(WL)_{1,2}} + \frac{1}{(WL)_{9,10}}\frac{g_{m9,10}^2}{g_{m1,2}^2}\right] + \frac{2K_P}{C_{ox}f}\frac{1}{(WL)_{7,8}}\frac{g_{m7,8}^2}{g_{m1,2}^2}}$$

Problem 9.18

$P = 6mW$, output swing $= 2.5V$

$L_{eff} = 0.5\mu m$

(a) $I_{D5,6} = 1mA$. $V_{OD5} \simeq V_{OD6} = \frac{V_{DD} - \text{Output Swing}}{2} = \frac{3-2.5}{2} = 0.25V$

$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2(1+\lambda V_{DS})$

$\left(\frac{W}{L}\right)_{eff} = \frac{2I_D}{\mu C_{ox}(V_{GS}-V_{TH})^2(1+\lambda V_{DS})}$

$\left(\frac{W}{L}\right)_5 = \frac{2(1mA)}{(350)(383.6n)(0.25)^2(1+(0.1)(0.25))}$ $\qquad I_{D5} = I_{D6} = 1mA$

$= 233$

$\left(\frac{W}{L}\right)_6 = \frac{2(1mA)}{(100)(383.6n)(0.25)^2(1+0.2(0.25))}$

$= 795$

$$\boxed{\left(\frac{W}{L}\right)_5 = 233 \qquad \left(\frac{W}{L}\right)_6 = 795}$$

b. A_v of 1st stage $= g_{m1}(r_{o2} \parallel r_{o4})$

A_v of 2nd stage $= g_{m5}(r_{o5} \parallel r_{o6})$

$g_{m5} = \frac{2I_D}{V_{GS}-V_{TH}} = \frac{2(1mA)}{0.25} = 8m\Omega^{-1}$

$r_{o5} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(1mA)} = 10k\Omega$

$r_{o6} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(1mA)} = 5k\Omega$

$$\boxed{A_v \text{ of output stage} = (8m)(10k \parallel 5k) = 26.67}$$

(c) $I_{D7} = 1mA \Rightarrow I_{D3} = I_{D4} = 0.5mA$

$V_{GS5} - V_{TH} = 0.25V \Rightarrow V_{GS5} = 0.25 + V_{TH} = 0.95V$

$V_{GS3} - V_{TH} = 0.25$

$\left(\frac{W}{L}\right)_{3,4} = \frac{2(0.5mA)}{(350)(383.6n)(0.25)^2(1+0.1 \times 0.25)}$

$$\boxed{\left(\frac{W}{L}\right)_{3,4} = 116}$$

9.18

(a)

$$A_{v\,tot} = g_{m1}(r_{o2} \,//\, r_{o4})\, g_{m5}(r_{o5} \,//\, r_{o6})$$

$$r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(0.5m)} = 10k\Omega.$$

$$r_{o4} = \frac{1}{(0.1)(0.5m)} = 20k\Omega.$$

$$r_{o2} \,//\, r_{o4} = 6.67k\Omega$$

$$A_{v\,tot} = g_{m1}(6.67k)(26.7) = 500$$

$$g_{m1} = 2.81\, m\Omega^{-1}$$

$$g_{m1} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right) I_D} \Rightarrow \left(\frac{W}{L}\right) = \frac{g_{m1}^2}{2\mu_p C_{ox} I_D}$$

$$\left(\frac{W}{L}\right)_{1,2} = \frac{(2.81\,m)^2}{2(100)(383.6n)(0.5m)}$$

$$\boxed{\left(\frac{W}{L}\right)_{1,2} = 206}$$

Problem 9.19

Av of 2nd stage = 20 $I_{D5,6}$ = 1mA

(a) $V_{GD5} = V_{GD6}$

Av of 2nd stage = $g_{m5}(r_{o5} \parallel r_{o6})$

$$= \frac{2I_D}{V_{GS5} - V_{TH}} \cdot \left[\frac{1}{\lambda_n I_{D5}} \parallel \frac{1}{\lambda_p I_{D6}} \right] = 20.$$

$r_{o5} = \frac{1}{(0.1)(1mA)} = 10k\Omega$ $r_{o6} = \frac{1}{(0.2)(1mA)} = 5k\Omega$. $r_{o5} \parallel r_{o6} = 3.33k\Omega$

$V_{GS5} - V_{TH} = \frac{2I_D(r_{o5} \parallel r_{o6})}{Av} = \frac{2(1mA)(3.33k\Omega)}{20} = 0.333V$

$$\left(\frac{W}{L}\right)_5 = \frac{2(1mA)}{(350)(383.6n)(0.33)^2(1+0.1\times0.33)} = \boxed{132. = (W/L)_5}$$

$$\left(\frac{W}{L}\right)_6 = \frac{2(1mA)}{(100)(383.6n)(0.33)^2(1+0.2\times0.33)} = \boxed{449 = (W/L)_6}$$

(b) $r_{o6} = \frac{1}{\mu_p C_{ox}(\frac{W}{L})(V_{GS6} - V_{THp} - V_{DS})}$ $V_{GS6} - V_{THp} - V_{DS} = 50mV$

$$= \frac{1}{(100)(383.6n)(449)(50m)} = 1.16k\Omega$$

Av of 2nd stage = $\left[\frac{2(1mA)}{0.333}\right]\left[10k \parallel 1.16k\right]$

$$= \boxed{6.24 = Av}$$

Problem 9.20

(a) $|V_{GS7} - V_{TH7}| = 0.4V = V_{OD7}$

$V_{in\,max} = V_{DD} - |V_{OD7}| - |V_{OD1}| - |V_{TH1}|$

In part (d), Prob 9.18. $g_{m1} = 2.81 m\Omega^{-1} = \frac{2I_D}{V_{GS} - V_{TH}} = \frac{2(0.5m)}{V_{OD1}}$

$|V_{OD1}| = 0.356V$

$V_{in\,max} = 3 - 0.4V - 0.356V - 0.8 = 1.444V$

$V_{in,min} = |V_{OD3}| = 0.25$ from Prob 9.18 (c)

Allowable input voltage range : $\boxed{0.25 \leq V_{in} \leq 1.44V}$

(b) At $V_{in} = V_{out}$, $V_{in1} = V_{in2}$ since V_{in2} is connect to V_{out}

Since $V_{in1} = V_{in2}$, $I_{D1} = I_{D2} \Rightarrow V_X = V_Y$.

$\Rightarrow I_{D5} = 1mA \Rightarrow I_{D1} = I_{D2} = 0.5mA$.

$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2(1+\lambda V_{DS})$

$V_{GS3} - V_{TH} = 0.25 \Rightarrow V_{GS3} = 0.95$

$\Rightarrow V_{DS} \cong V_{DD} - V_{DS7} - V_{GS3} \cong 3 - 0.7 - 0.95 \simeq 1.3V$

$V_{GS1} - V_{TH} = \sqrt{\frac{2I_D}{\mu C_{ox}\left(\frac{W}{L}\right)(1+\lambda V_{DS})}}$

$= \left[\frac{2(0.5m)}{(100)(383.6n)(206)(1+0.2\times1.3)}\right]^{\frac{1}{2}}$

$= 0.317V$

$V_{GS1} = 0.317 + 0.7 = 1.017V$

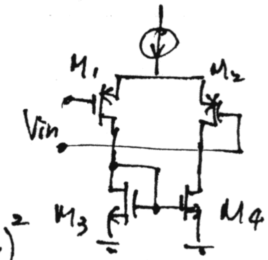
$V_{in} = V_{GS3} + V_{GS1} = 0.95 + 1.017V$

$V_{in} = 1.97V$

Problem 9.21

Noise due to M7 is negligible since induce common mode gain, which is very small.

Consider 1st stage:



$$\overline{V_n^2}_{,input}\Big|_{1st\ stage} = \left[4kT\gamma\left(\frac{1}{g_{m1,2}} + \frac{g_{m3,4}}{g_{m1,2}^2}\right)\right]\times 2$$

Consider 2nd stage

$$\overline{V_n^2}_{\,output}\Big|_{2nd\ stage} = \left[4kT\gamma(g_{m5} + g_{m6})\right](r_{o5}\,//\,r_{o6})^2$$

Overall:

$$\overline{V_n^2}_{,input} = \left[4kT\gamma\left(\frac{1}{g_{m1,2}} + \frac{g_{m3,4}}{g_{m1,2}^2}\right)\right]\times 2 + \frac{\left[4kT\gamma\left(\frac{1}{g_{m5}} + \frac{g_{m6}}{g_{m5}^2}\right)\right]}{\left[g_{m1}(r_{o2}\,//\,r_{o4})\right]^2}$$

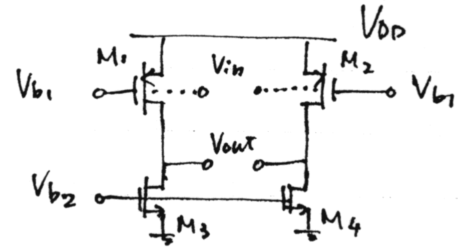
From Prob. 9.18

$g_{m1,2} = 2.81\,m\Omega^{-1}$, $g_{m3,4} = \dfrac{2I_D}{V_{GS3}-V_{TH}} = \dfrac{2(0.5m)}{0.25} = 4m\Omega^{-1}$

$g_{m5} = 8m\Omega^{-1}$ $g_{m6} = 8m\Omega^{-1}$, $r_{o2} = 10k$ $r_{o4} = 20k$ $r_{o2}//r_{o4} = 6.67k\Omega$

$$\overline{V_n^2}_{,input} = \left[4\times1.38\times10^{-23}\times300\times\frac{2}{3}\times\left(\frac{1}{2.81m} + \frac{4m}{2.81m^2}\right)\right]\times 2 + \frac{4\times1.38\times10^{-23}\times300\times\frac{2}{3}\times\left(\frac{1}{8m}+\frac{1}{8m}\right)}{\left[2.81m(6.67k)\right]^2}$$

$$\boxed{\begin{aligned} &= 1.905\times10^{-17}\ V^2/Hz \\ \overline{V_n^2}_{,input} &= 4.36\times10^{-9}\ V/\sqrt{Hz}\end{aligned}}$$



9.22.

(a) $A_v = g_{mb1,2}\,(r_{o1} \parallel r_{o3})$

(b) $V_{in} > V_{DD} - V_{D1}$ where V_{D1} is the diode junction voltage of the diode between source and body.

(c) $$g_{mb} = g_m \frac{\gamma}{2\sqrt{|2\phi_F| + |V_{SB}|}}$$

As $V_{in,CM}$ decreases, $|V_{SB}| \uparrow$, g_{mb} decreases.

More accurately, $g_{mb} \propto \dfrac{1}{\sqrt{|2\phi_F| + |V_{SB}|}}$

As a result, A_v decreases.

(d)

$$\overline{V_{n,out}^2} = \left[4kT\gamma\,(g_{m1} + g_{m3})\,R_{out}^2\right] \times 2$$

$$\overline{V_{n,in}^2} = \frac{4kT\gamma\,(g_{m1} + g_{m3})\,R_{out}^2 \times 2}{\left[g_{mb1,2}\,(R_{out})\right]^2}$$

$$= \left[4kT\gamma\,\frac{g_{m1} + g_{m3}}{(g_{mb1,2})^2}\right] \times 2$$

Problem 9.23

a. A_v of 1st stage $= g_{m1,2}(r_{o1} // r_{o3})$

A_v of 2nd stage $= [g_{m5,9}(r_{o7} // r_{o5})] \times 2$

$A_{v\text{-}tot} = g_{m1,2}(r_{o1} // r_{o3})\, g_{m5,9}(r_{o5} // r_{o7}) \times 2$

b. 1st major pole:

$$\omega_1 = \frac{1}{(r_{o9} // r_{o11})[C_{DG9} + C_{DB9} + C_{GS11} + C_{DB11} + C_{GS7} + C_{GD7}(1 + g_{m7}(r_{o5} // r_{o7}))]}$$

2nd major pole: node X, Y

$$\omega_X = \frac{1}{(r_{o1} // r_{o3})\left[\begin{array}{l} C_{DG1} + C_{DB1} + C_{DG3} + C_{DB3} + C_{GS10} + C_{GD10}\left(1 + \frac{g_{m10}}{g_{m12}}\right) + C_{GS5} \\ + C_{GD5}(1 + g_{m5}(r_{o5} // r_{o7}) \end{array}\right]}$$

3rd major pole: node output

$$\omega_{out} = \frac{1}{(r_{o5} // r_{o7})(C_{GD5} + C_{DB5} + C_{GD7} + C_{DB7})}$$

Prob. 9.24.

A_v of fast path: $g_{m1}'(r_{o5} // r_{o7})$

A_v of slow path: $g_{m1}(r_{o1} // r_{o3})\, g_{m5}(r_{o5} // r_{o7})$.

Overall gain $A_{v\,tot} = \left[\dfrac{g_{m1}' + g_{m1} g_{m5}(r_{o1} // r_{o3})}{2}\right](r_{o5} // r_{o7})$

The output swing is usually limited by M_{5-8}, i.e. $V_{DD} - |V_{OD7}| - V_{OD5}$.

Problem 9.25

Noise due to $M_{1,2}'$

$$\overline{V_n^2}, \text{input} \big| M_{1,2}' = 4kT\gamma\left(\frac{1}{g_{m1,2}'}\right) \times 2$$

$$\overline{V_n^2}, \text{input} \big| M_{1,2} = 4kT\gamma\left(\frac{1}{g_{m1,2}}\right) \times 2$$

$$\overline{V_n^2}\ \text{input} \big| M_{3,4} = 4kT\gamma\left(\frac{g_{m3,4}}{g_{m1,2}^2}\right) \times 2$$

$$\overline{V_n^2}\ \text{output} \big| M_{5,6} = 4kT\gamma\, g_{m5,6} R_{out} \times 2$$

$$\overline{V_n^2}\ \text{output} \big| M_{7,8} = 4kT\gamma\, g_{m7,8} R_{out} \times 2$$

$$\overline{V_n^2}, \text{input} \big| M_{5\text{-}8} = \frac{4kT\gamma\,(g_{m5,6} + g_{m7,8}) \times 2}{\left(\dfrac{g_{m1}' + g_{m1} g_{m5} (r_{o1} \| r_{o3})}{2}\right)^2}$$

$$\overline{V_n^2}\ \text{input} \big|_{tot} = 2\left[4kT\gamma\left(\frac{1}{g_{m1,2}'} + \frac{1}{g_{m1,2}} + \frac{g_{m3,4}}{g_{m1,2}^2} + \frac{4(g_{m5,6} + g_{m7,8})}{(g_{m1}' + g_{m1} g_{m5} (r_{o1} \| r_{o3}))^2}\right)\right]$$

$\boxed{10.1}$ Two poles $\omega_{p1} = 10\,MHz$ $\omega_{p2} = 500\,MHz$

First find ω_1 ($= GX$) that gives phase $= -120°$ (P.M. of 60°)

$-120° = -\tan^{-1}\frac{\omega_1}{\omega_{p1}} - \tan^{-1}\frac{\omega_1}{\omega_{p2}} \longrightarrow \omega_1 \cong 311\,MHz$

$A_o = (\log\frac{\omega_1}{\omega_{p1}})(20\,dB/dec) = (\log\frac{311}{10})(20) = \underline{\underline{29.9\,dB}}$

$\boxed{10.2}$ $\omega_{p1} = \omega_{p2} = \omega_p$

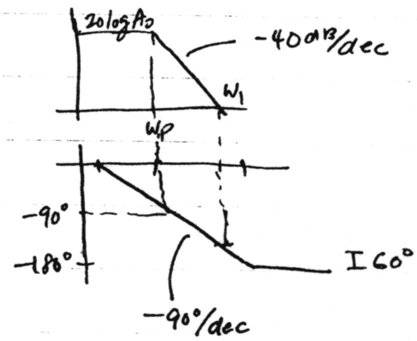
a) $60° \cdot \frac{1}{90°/dec} = 0.67$ decade

$\log\frac{10\omega_p}{\omega_1} = 0.67$ dec $\quad$ (ω_1 is GX)

$\Rightarrow \omega_1 = 2.14\,\omega_p$

$A_o = (\log\frac{2.14\omega_p}{\omega_p})(40\,dB/dec) = \underline{\underline{13.2\,dB}}$

($\beta = 1$)



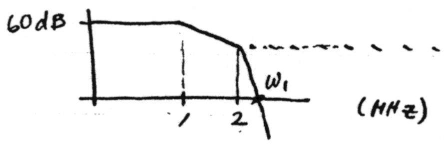
b) For closed-loop gain $= 4 \Longrightarrow \beta \approx \frac{1}{4}$

Thus A_o can increase by a factor of 4 to maintain 60° P.M.

$\Rightarrow A_o' = 13.2\,dB + 20\log 4 = \underline{\underline{25.2\,dB}}$

$\boxed{10.3}$ $A_o = 1000$ $\quad \omega_{p1} = 1\,MHz$

a)



$= 60\,dB - (\log\frac{2\,MHz}{1\,MHz})(20\frac{dB}{dec}) = \underline{54\,dB}$

$\log\frac{\omega_1}{2\,MHz} = 54\,dB \frac{1}{40\,dB/dec} = 1.35$ dec

$\underline{\omega_1 = 44.8\,MHz}$

$\angle H(j\omega_1) = -\tan^{-1}\frac{\omega_1}{1\,MHz} - \tan^{-1}\frac{\omega_1}{2\,MHz} = -176.2° \Longrightarrow P.M. = 180° - 176.2° = \underline{\underline{3.8°}}$

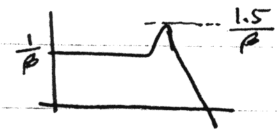
b) $\omega_{p2}' = 4\,MHz$

$\log\frac{\omega_1'}{4\,MHz} = [60\,dB - (\log\frac{4\,MHz}{1\,MHz})(20\,dB/dec)]\frac{1}{40\,dB/dec} = 1.199$ dec

$\Rightarrow \omega_1' = 63.2\,MHz$

$\angle H(j\omega_1') = -175.5° \Longrightarrow \underline{\underline{P.M. = 4.5°}}$

$\boxed{10.4}$



$\beta = 1$

At GX, $H(j\omega_1) = 1 \cdot e^{j\theta_1}$

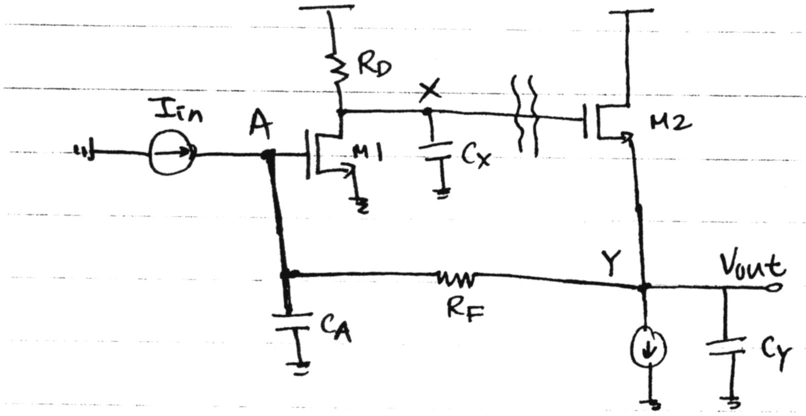
Closed loop: $\left|\frac{Y}{X}(j\omega_1)\right| = \left|\frac{H(j\omega_1)}{1+H(j\omega_1)}\right| = 1.5$

$$\left|\frac{1}{1+e^{j\theta_1}}\right| = 1.5$$

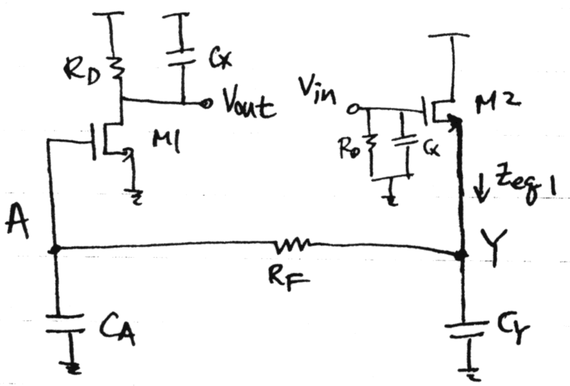
$$\rightarrow \frac{1}{\sqrt{1+2\cos\theta_1+1}} \rightarrow \frac{1}{2+2\cos\theta_1} = 1.5^2 \rightarrow \theta_1 = -141.1^\circ$$

$$\underline{\underline{P.M. = 38.9^\circ}}$$

$\boxed{10.5}$



Breaking the loop at node X as shown by {{ and replacing each end by the impedance each sees, we get the next circuit:



Next, calculate the loop gain

$$\frac{V_{out}}{V_{in}}(s) = \frac{V_Y}{V_{in}} \cdot \frac{V_A}{V_Y} \cdot \frac{V_{out}}{V_A}$$

$$= A_{v1} \cdot A_{v2} \cdot A_{v3}$$

$Z_{eq1} \cong \frac{1}{sC_Y}$ since $R_F = 10\,k\Omega >> \frac{1}{g_{m2}}$

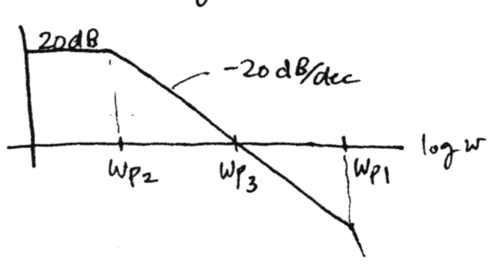
$$A_{v1} = \frac{g_{m2} Z_{eq1}}{1 + g_{m2} Z_{eq1}} \cong \frac{g_{m2}\frac{1}{sC_Y}}{1 + g_{m2}\frac{1}{sC_Y}} = \frac{1}{s\left(\frac{C_Y}{g_{m2}}\right) + 1}$$

$$A_{v2} = \frac{\frac{1}{sC_A}}{R_F + \frac{1}{sC_A}} = \frac{1}{sC_AR_F + 1}$$

$$A_{v3} = -g_{m1}\left(R_o \| \frac{1}{sC_X}\right) = \frac{-g_{m1}R_D}{1 + sC_XR_D}$$

Hence $\omega_{P1} = \frac{g_{m2}}{C_Y} = 1\times10^{11}\ \text{rad/s}$, $\omega_{P2} = \frac{1}{C_AR_F} = 1\times10^{9}\ \frac{\text{rad}}{s}$, $\omega_{P3} = \frac{1}{C_XR_D} = 1\times10^{10}\ \frac{\text{rad}}{s}$

and $g_{m1}R_D = 10 \rightarrow 20\ dB$



$\Rightarrow$ P.M. = 45°

$\boxed{10.6}$ $R_D' = 2\,k\Omega$

$\rightarrow g_{m1} R_D' = 20 \Rightarrow \underline{26.0\,dB}$, $\omega_{P3}' = \frac{1}{C_X R_D'} = \underline{5\times10^9\ rad/s}$.

$\omega_1' \Rightarrow 26\,dB - \left(\log \frac{\omega_{P3}'}{\omega_{P2}}\right)(20\ dB/dec) - \left(\log \frac{\omega_1}{\omega_{P3}'}\right)(40\ \frac{dB}{dec}) = 0\ dB$

$\underline{\omega_1' = 9.99\times10^9\ rad/s}$

$\phi = -\tan^{-1}\frac{\omega_1'}{1\times10^9} - \tan^{-1}\frac{\omega_1'}{5\times10^9} - \tan^{-1}\frac{\omega_1'}{1\times10^{11}} = -153.4°$

$\underline{\underline{P.M. = 26.6°}}$

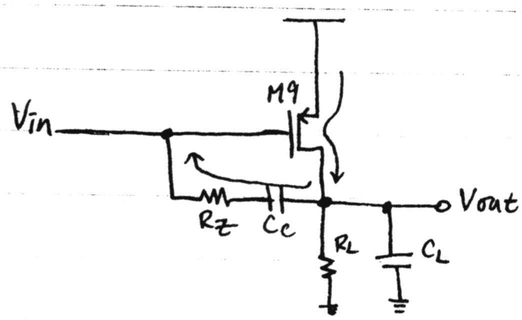
$\boxed{10.7}$ From 10.5 $\omega_{P1} = \frac{g_{m2}}{C_Y}$ $\omega_{P2} = \frac{1}{C_A R_F}$ $\omega_{P3} = \frac{1}{C_X R_D}$

a) Increasing C_Y causes ω_{P1} to move towards ω_{P3} and will be less than 1 decade from ω_{P3}. This will reduce the already 45°-phase margin. Hence $\underline{\underline{C_{Y\,max} = 100\,fF}}$.

b) Increasing C_A will increase phase margin.
Hence $\underline{\underline{C_{A\,max} = 100\,fF}}$.

c) $\underline{\underline{C_{X\,max} = 100\,fF}}$ since increasing C_X will reduce phase margin.

$\boxed{10.8}$ The approximation can be derived from the ideal case in which the circuit looks like the following:



At the zero, $V_{out} = 0$. and

$$\frac{-V_{in}}{R_z + \frac{1}{s_z C_c}} = -g_{m9} V_{in}$$

$$\left(\frac{1}{g_{m9}} - R_z\right)^{-1} = s_z C_c$$

$$\therefore\ s_z = \frac{1}{C_c\left(g_{m9}^{-1} - R_z\right)}$$

$\boxed{10.9}$ $\left(\frac{W}{L}\right)_{1-4} = \frac{50}{0.5}$ $I_{SS} = I_1 = 0.5mA$ $C_X = C_Y = 0.5pF$

a)

$$\omega_{PX} \cong \frac{1}{C_X(r_{op3} \| r_{op2})} \qquad \omega_{PY} \cong \frac{1}{C_Y \cdot (g_{m4}^{-1})}$$

In saturation :

$$r_o = \frac{1}{\lambda I_D} \qquad \lambda_p = 0.2 \quad \lambda_n = 0.1 \text{ from Table 2.1}$$

$$r_{op3} = \frac{1}{0.2(0.25mA)} = \underline{20K\Omega}$$

$$r_{on2} = 2\, r_{op3} = \underline{40K\Omega}$$

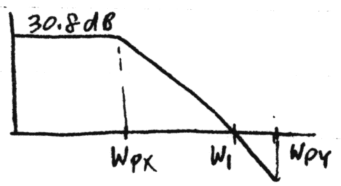
$$g_{m4} = \sqrt{2 I_D \mu_n C_{ox} \frac{W}{L}} = \sqrt{2(0.5mA)(1.34\times10^{-4})\left(\frac{50}{0.5}\right)} = \frac{1}{273} \ A/V$$

$$g_{m4}^{-1} = 273.0\ \Omega$$

$\Rightarrow$ $\omega_{PX} = 150\times10^6$ rad/s , $\omega_{PY} = 7.33\times10^9$ rad/s //

$|\text{Low frequency gain}| \cong g_{m2}(r_{on2} \| r_{op3}) \cdot (1)$ $\left(g_{m2} = \frac{g_{m4}}{\sqrt{2}}\right)$

$\cong 34.5$ V/V $\Longrightarrow$ 30.8 dB //



$$30.8\,dB - \left[\log\left(\frac{7.33\times10^9}{150\times10^6}\right)\right](20\,dB/dec) = -3.78\,dB$$

$$\left(\log \frac{\omega_1}{150\times10^6}\right)(20\,dB/dec) = 30.8\,dB$$

$\Rightarrow \omega_1 = 5.20\times10^9$ rad/s //

$$\phi = -\tan^{-1}\frac{\omega_1}{\omega_{PX}} - \tan^{-1}\frac{\omega_1}{\omega_{PY}} = -123.7°$$

$\Rightarrow$ $\underline{\underline{P.M. = 56.3°}}$

b)

$$\phi = -\tan^{-1}\frac{\omega_1}{150\times10^6} - \tan^{-1}\frac{\omega_1}{\omega_{PY}} = -120°$$

If ω_1 is same as (a), then $\omega_{PY} = 8.43\times10^9$ rad/s

$$= \frac{1}{C_{Ymax}\, g_{m4}^{-1}}$$

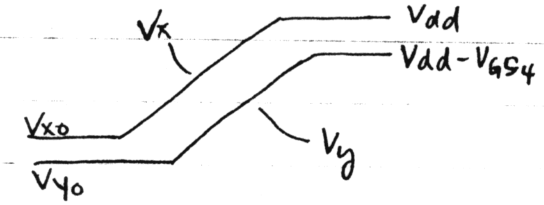
$\underline{\underline{C_{Ymax} = 434\,fF}}$.

$\boxed{10.10}$

For large positive step in V_{in}:

M_2 turns off. M_3 charges C_X and M_4 charges C_Y.

(M_4 also provides I_1.)



The slew rates of V_x and V_y (or V_{out}) must be exactly the same — regardless of C_X or C_Y.

Hence slew rate due to positive step input $\cong \frac{I_{D3}}{C_X}$ for both parts (a) and (b) of 10.9.

$$\text{slew rate} \cong \frac{I_{D3}}{C_X} \cong \frac{0.25\,mA}{0.5\,pF} = 5.00 \times 10^8\ V/s \quad //$$

For large negative step in V_{in}:

Again, V_{out} tracks V_x — as V_x drops, V_{out} drops at the same rate.

$$\text{Slew rate} \cong \frac{-I_{CX}}{C_X}, \quad I_{D3} \cong 0.25\,mA,\ I_{CX} \cong 0.5\,mA - I_{CX} = 0.25\,mA$$

For both C_Y's, slew rate $\cong -\frac{0.25\,mA}{0.5\,pF} = -5.00 \times 10^8\ V/s \quad //$

$\boxed{10.11}$ $\left(\frac{W}{L}\right)_{5,6} = \frac{60}{0.5}$ $I_{SS} = 0.25mA.$

a) CM level $V_X = V_Y = V_{DD} - V_{GS5} = V_{DD} - V_{GS6}$

$I = 1mA = \frac{1}{2}\mu_p C_{ox}\left(\frac{60}{0.5}\right)(|V_{gs6}| - |V_{tp}|)^2$

$\underline{V_{gs6} = 1.46\ V} \longrightarrow \underline{V_X = V_Y = 3 - 1.46 = 1.54V.}$

b) Max. output swing:

$V_{out,max} = V_{dd} - V_{overdrive6} = 3 - (1.46 - 0.8) = \underline{\underline{2.34\ V}}$

$V_{out,min} = V_{overdrive8} = V_b - V_{tn} = \underline{\underline{0.39V}}$

(since $V_b = 1.09V$ from $1mA = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - 0.7)^2$).

Total max. swing $= 2.34 - 0.39 = \underline{\underline{1.95V}}$

c) $r_{on2} = \frac{1}{(0.1)(0.125mA)} = \underline{80\ k\Omega}$

$r_{op4} = \underline{40\ k\Omega}$

$\rightarrow r_{on2} \| r_{op4} = \underline{26.67\ k\Omega}$

$r_{on8} = 10\ k\Omega,\ r_{op6} = 5k\Omega \rightarrow r_{on8} \| r_{op6} = \underline{3.33\ k\Omega}$

$g_{m2} = \sqrt{2(0.125mA)(1.34\times10^{-4})\left(\frac{50}{0.5}\right)} = \underline{1.83\times10^{-3}\ A/V}$

$g_{m6} = \mu_p C_{ox}\frac{W}{L}(V_{GS6} - V_{th}) = (3.83\times10^{-5})\left(\frac{60}{0.5}\right)(1.46 - 0.8) = \underline{3.03\times10^{-3}\ A/V}$

$A_{v2} = g_{m6}(3.33\ k\Omega) = \underline{-10.09\ V/V}$

$A_{v1} = g_{m2}(26.67\ k\Omega) = \underline{-48.8\ V/V}$

$\rightarrow A_{v1}A_{v2} = 492.4 \rightarrow \underline{\underline{53.8\ dB}}$

$C_{out} = C_L + \left[C_{db6} + \left(1 + \frac{1}{|A_{v2}|}\right)C_{gd6}\right] + (C_{db8} + C_{gd8})$

$\cong 1pF + 52.1 + \left(1 + \frac{1}{10.09}\right)0.18 + 23.4 + 0.2 = \underline{\underline{1.076pF}}$

$C_Y = C_{gd4} + C_{db4} + C_{gd2} + C_{db2} + C_{gs6} + C_{gd6}(1 + |A_{v2}|)$

$\cong 0.15 + 43.8 + 0.2 + 23.4 + 76.6 + 0.18(11.09) = \underline{\underline{146.1\ fF}}$

Using:

$C_{overlap} = CGDO \cdot W,\ C_{db} = \frac{(CJ)(W \cdot 1.5\mu m)}{\left[1 - \frac{V_D}{PB}\right]^{MJ}} + \frac{CJSW(2W + 3\mu m)}{\left[1 - \frac{V_D}{PB}\right]^{MJSW}}$

(V_D = reverse bias junction voltage.)

10.11 c) cont. $C_{gs} = \frac{2}{3} C_{ox} W \cdot L$, Values from Table 2.1.

Before Compensation:

Dominant Pole: $W_y = \frac{1}{C_y R_y} = \frac{1}{(146.1\,fF)(26.67\,k\Omega)} = \underline{2.57 \times 10^8\ rad/s}$

2nd Pole: $W_{out} = \frac{1}{C_{out} R_{out}} = \frac{1}{(1.076\,pF)(3.33\,k\Omega)} = \underline{2.79 \times 10^8\ rad/s}$

☆ After compensation:

2nd Pole: $W_{out}' \cong \frac{g_{m6}}{C_y + C_{out}} = \frac{3.03 \times 10^{-3}}{146.1\,fF + 1.076\,pF} = \underline{\underline{2.48 \times 10^9\ rad/s}}$

Dominant Pole: $W_y' = \frac{1}{[C_y + (1+|A_{v2}|)C_c]\, R_y}$

(For 60° P.M.) $90° + \tan^{-1} \frac{W_1'}{W_{out}'} = 120° \rightarrow W_1' = W_{out}' \tan 30° = \underline{1.43 \times 10^9\ rad/s}$

$\log \frac{W_1'}{W_y'} = \frac{53.8\,dB}{20\,dB/dec} \longrightarrow W_y' = \frac{1}{10^{53.8/20}} \cdot W_1' = \underline{\underline{2.91 \times 10^6\ rad/s}}$

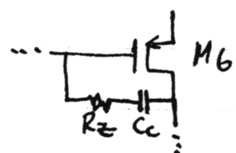
$C_c = \frac{[(2.91 \times 10^6)(26.67\,k\Omega)]^{-1} - 146.1\,fF}{1 + 10.08} = \underline{\underline{1.15\,pF}}$

(So $C_c >> C_y$)

Zero: $W_z' = \frac{g_{m6}}{C_c + C_{gd6}} = \frac{3.03 \times 10^{-3}}{1.15\,pF + 0.18\,fF} = \underline{\underline{2.63 \times 10^9\ rad/s}}$ ($> W_y'$, W_{out}')

d) $W_z = \frac{1}{C_c(\frac{1}{g_{m6}} - R_z)} = -|W_{out}'|$

$\hookrightarrow R_z = \frac{1}{g_{m6}} + \frac{1}{|W_{out}'| \cdot C_c} = \underline{\underline{680.7\,\Omega}}$



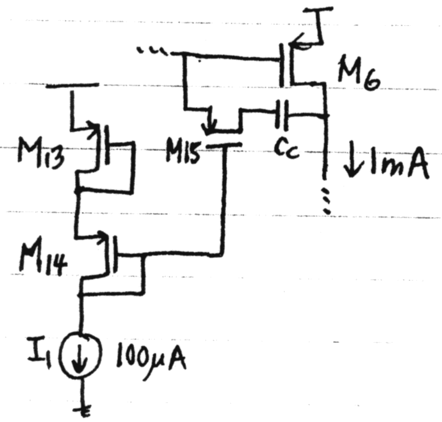
e) Slew rate: Symmetrical for large positive Vin or large negative Vin.

Large + Vin:

Slew rate of $V_{out2} \cong -\frac{I_{D4}}{C_c} \cong \frac{-0.125\,mA}{1.15\,pF} = \underline{\underline{-1.09 \times 10^8\ V/s}}$

Slew rate of V_{out1} = −(slew rate of V_{out2}) = $\underline{\underline{1.09 \times 10^8\ V/s}}$

$\boxed{10.12}$



Want $|V_{GS15}| = |V_{GS6}| = 1.46V$ (from 10.11a)

$$100\mu A = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{13}(|V_{GS6}| - |V_{tp}|)^2$$

$$\hookrightarrow \left(\frac{W}{L}\right)_{13} = \frac{2(100\mu A)}{(3.83\times10^{-5})(1.46-0.8)^2} = \underline{\underline{12.0}} \quad \text{e.g. } \left(\frac{W}{L}\right)_{13} = \frac{6}{0.5}$$

Allowing 0.5V across I_1 and maximizing $V_{gs14} = V_{gs15}$,
we get $V_{gs14} = V_{gs15} = V_{xy} - 0.5 = 1.54 - 0.5 = \underline{\underline{1.04\,V}}$

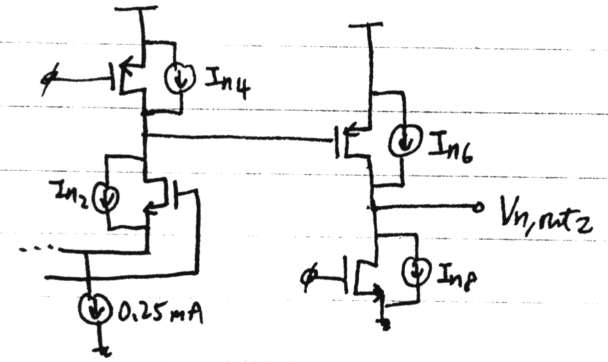
$$R_{on15} = \frac{1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_{15}(1.04-0.8)} = 680.7\,\Omega$$

$$\hookrightarrow \left(\frac{W}{L}\right)_{15} = \underline{\underline{384}} \quad \text{e.g. } \frac{192}{0.5}$$

$$I_{D14} = 100\mu A = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{14}(1.04-0.8)^2$$

$$\hookrightarrow \left(\frac{W}{L}\right)_{14} = \underline{\underline{90.7}} \quad \text{e.g. } \frac{45.5}{0.5}$$

$\boxed{10.13}$



$\overline{I_n^2} = 4KT\frac{2}{3} g_m$

$$V_{n,out_2} = (I_{n2} + I_{n4})(r_{o2} \| r_{o4}) \cdot A_{v2} + (I_{n6} + I_{n8})(r_{o6} \| r_{o8})$$

$$\overline{V_{n,out_2}^2} = (\overline{I_{n2}^2} + \overline{I_{n4}^2})\left[(r_{o2} \| r_{o4}) A_{v2}\right]^2 + (\overline{I_{n6}^2} + \overline{I_{n8}^2})(r_{o6} \| r_{o8})^2$$

$$\overline{V_{n,out_1}^2} = (\overline{I_{n1}^2} + \overline{I_{n3}^2})\left[(r_{o2} \| r_{o4}) A_{v2}\right]^2 + (\overline{I_{n5}^2} + \overline{I_{n7}^2})(r_{o6} \| r_{o8})^2$$

$$\overline{V_{n,out}^2} = \overline{V_{n,out_2}^2} + \overline{V_{n,out_1}^2}$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{(A_{v1} A_{v2})^2}$$

$$= \left(\frac{1}{A_{v1} A_{v2}}\right)^2 \left\{ \left[(r_{o2} \| r_{o4}) A_{v2}\right]^2 \left[\overline{I_{n1}^2} + \overline{I_{n2}^2} + \overline{I_{n3}^2} + \overline{I_{n4}^2}\right] + (r_{o6} \| r_{o8})^2 (\overline{I_{n5}^2} + \overline{I_{n6}^2} + \overline{I_{n7}^2} + \overline{I_{n8}^2}) \right\}$$

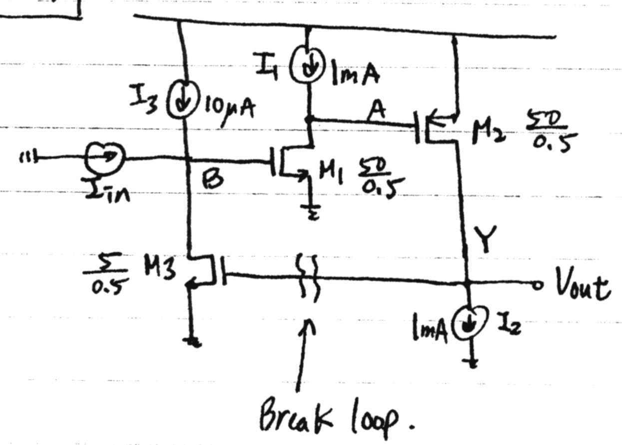
Aside
$$\left\{ \begin{array}{l} g_{mn_1} = g_{m2} = 1.83 \times 10^{-3}\ A/V \\ g_{mp_1} = g_{m4} = \sqrt{2 I_p \mu_p C_{ox} \frac{W}{L}} = \sqrt{2(0.125mA)(3.83 \times 10^{-5})(\frac{50}{0.5})} = 9.79 \times 10^{-4}\ A/V \\ g_{mn_2} = g_{m8} = 5.18 \times 10^{-3}\ A/V \ ; \qquad A_{v1} = 48.8 \ , \ A_{v2} = 10.09 \\ g_{mp_2} = g_{m6} = 3.03 \times 10^{-3}\ A/V \ ; \qquad r_{o2} \| r_{o4} = 26.67K\Omega, \ r_{o6} \| r_{o8} = 3.33K\Omega \end{array} \right.$$

$$\overline{V_{n,in}^2} = 4KT\frac{2}{3}\left[1678.1 + 0.7511\right] = 4KT\frac{2}{3}\left[1678.85\right] \qquad (4KT = 1.658 \times 10^{-20}$$

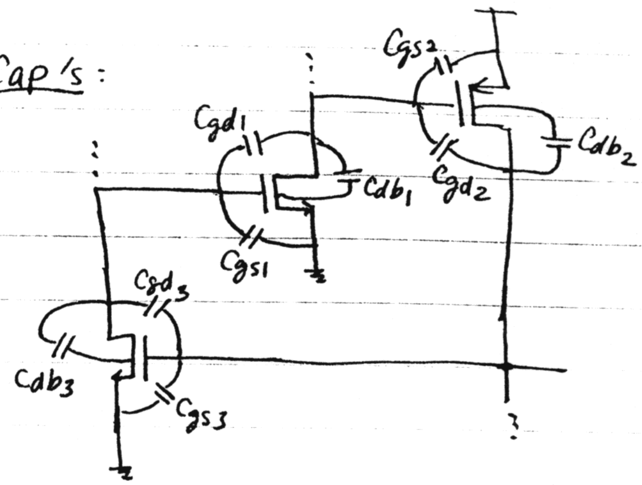
$$= \underline{\underline{1.86 \times 10^{-17}\ V^2/Hz}}$$

10.12

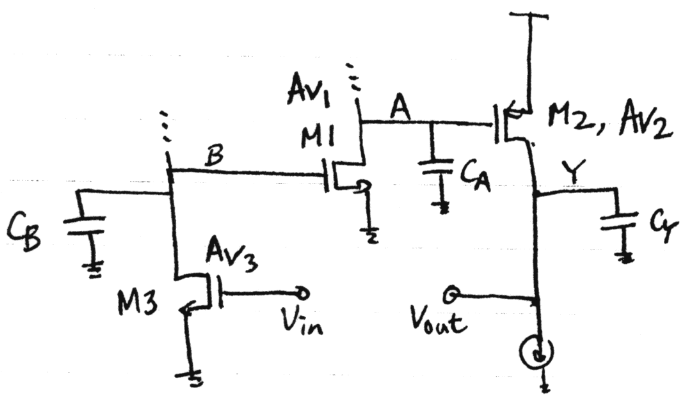
$\boxed{10.14}$



Cap's:



a)



$$C_B = C_{db3} + \left(1+\frac{1}{|A_{v3}|}\right)C_{gd3} + C_{gs1} + (1+|A_{v1}|)\,C_{gd1}$$

$$C_A = \left(1+\frac{1}{|A_{v1}|}\right)C_{gd1} + C_{gs2} + (1+|A_{v2}|)\,C_{gd2} + C_{db1}$$

$$C_Y = C_{db2} + \left(1+\frac{1}{|A_{v2}|}\right)C_{gd2} + C_{gs3} + (1+|A_{v3}|)\,C_{gd3}$$

$$r_{o1} = \frac{1}{(0.1)(1mA)} = 10\,k\Omega$$

$$r_{o2} = 5\,k\Omega$$

$$r_{o3} = 1\,M\Omega$$

Using capacitance formulas of (10.11), we get:

$C_A = 80.4\ fF$ $\qquad V_A = 1.48V$

$C_B = 77.1\ fF$ $\qquad V_B = 1.09V$

$C_Y = 105.6\ fF$ $\qquad V_Y = 0.822V$

Also,

$$g_{m1} = \sqrt{2(1mA)(1.34\times10^{-4})(50/0.5)} = 5.18\times10^{-3}\ A/V$$

$$g_{m2} = 2.77\times10^{-3}\ A/V$$

$$g_{m3} = 1.64\times10^{-4}\ A/V$$

$$\left.\begin{aligned} A_{v1} &= g_{m1} r_{o1} = -51.8\ V/V \\ A_{v2} &= g_{m2} r_{o2} = -13.85\ V/V \\ A_{v3} &= g_{m3} r_{o3} = -164.0\ V/V \end{aligned}\right\} \rightarrow |A_{v1} A_{v2} A_{v3}| = 1.18\times10^{5} \rightarrow \underline{\underline{101.4\,dB}}$$

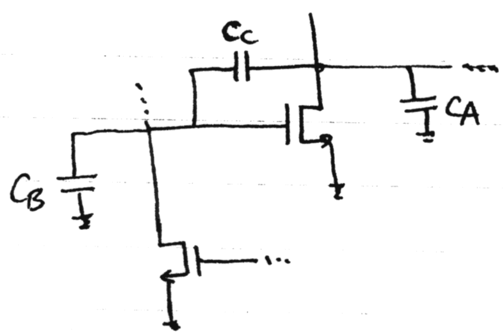
10.14 a) cont.

$$\boxed{\begin{aligned} W_A &= \frac{1}{C_A r_{o1}} = 1.24\times10^{9}\ \text{rad/s} \\ W_B &= \frac{1}{C_B r_{o3}} = 1.30\times10^{7}\ \text{rad/s} \\ W_Y &= \frac{1}{C_Y r_{o2}} = 1.89\times10^{9}\ \text{rad/s} \end{aligned}}$$

← 2nd
← Dominant
← 3rd

(Phase Margin is $-78.3°$ → unstable system)

b) Compensate by adding C_c across G and D of M_1.



$$\rightarrow W_A' \cong \frac{g_{m1}}{C_B + C_A} = \frac{5.18\times10^{-3}}{77.1\,fF + 80.4\,fF} = \underline{\underline{3.3\times10^{10}\ \text{rad/s}}}$$

$$\rightarrow W_y \text{ unchanged} = \underline{\underline{1.89\times10^{9}\ \text{rad/s}}}$$

$$90° + \tan^{-1}\frac{W_1}{W_y} = 120° \quad (\text{For } 60° \text{ P.M.})$$

$$\tan^{-1}\frac{W_1}{W_y} = 30°$$

$$\underline{W_1 = 1.09\times10^{9}\ \text{rad/s}}$$

$$\left(\log\frac{W_1}{W_B'}\right)\frac{20\,dB}{dec} = 101.4\,dB$$

$$\rightarrow \underline{\underline{W_B' = 9.28\times10^{3}\ \text{rad/s}}}$$

Dominant: W_B', 2nd: W_y, 3rd: W_A'

$$W_B' = \frac{1}{[C_B + (1+|A_{v1}|)C_c]\,r_{o3}} \longrightarrow C_c = \frac{\frac{1}{W_B'(1\times10^{6})} - 77.1\,fF}{52.8} = \underline{\underline{2.04\,pF}}$$

$$W_z' \cong \frac{g_{m1}}{C_c} = \frac{5.18\times10^{-3}}{2.04\,pF} = \underline{2.54\times10^{9}\ \text{rad/s}} \quad (> W_y.)$$

10.14 c)

$$W_z = \frac{1}{C_c(g_{m_1}^{-1} - R_z)} = -|W_y| = -1.89 \times 10^9$$

$$\hookrightarrow R_z = \frac{1}{g_{m_1}} + \frac{1}{|W_y|\, C_c} = \underline{\underline{452.4\ \Omega}}$$

$\boxed{10.15}$ a) Before compensation:

$C_A = 80.4\ fF$

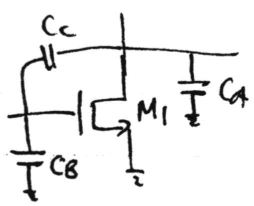
$C_B = 77.1\ fF$

$C_Y = 105.6\ fF + 0.5\ pF = 605.6\ fF$

$$\boxed{\begin{array}{ll} \omega_A = 1.24\times 10^9\ rad/s & \leftarrow \text{3rd} \\ \omega_B = 1.30\times 10^7\ rad/s & \leftarrow \text{Dominant} \\ \omega_Y = \frac{1}{(605.6\,fF)(5\times 10^3)} = 3.30\times 10^8\ rad/s & \leftarrow \text{2nd} \end{array}}$$

b) Two choices: We can put C_c across G & D of M_1 OR just add a C from G of M_1 to ground. Cannot take advantage of splitting 1st and 2nd pole here since 1st pole is at B and 2nd pole is at Y and the gain between those two nodes is >0.

Choose to put C_c across M_1: Splits 1st and 3rd poles, 2nd pole unchanged.



3rd: $\underline{\omega_A'} \cong \frac{g_{m1}}{C_A + C_B} = \underline{3.30\times 10^{10}\ rad/s}$

2nd: $\underline{\omega_Y = 3.30\times 10^8\ rad/s}$ (unchanged).

$\underline{\omega_1'} = \omega_Y \cdot \tan 30^\circ = \underline{1.91\times 10^8\ rad/s}$

Dominant: $\underline{\omega_B'} = \frac{1}{10^{(101.4/20)}} \cdot \omega_1' = \underline{1.63\times 10^3\ rad/s}$

$$C_c = \frac{\frac{1}{(1.63\times 10^3)(1\times 10^6)} - 77.1\,fF}{52.8} = \underline{\underline{11.6\ pF}}$$

10.15 c)

$$W_z = \frac{1}{C_c(g_{m1}^{-1} - R_z)} = -|W_g| = -3.30\times10^8 \text{ rad/s}$$

$$\hookrightarrow R_z = \frac{1}{g_{m1}} + \frac{1}{|W_g|\cdot C_c}$$

$$R_z = \frac{1}{5.18\times10^{-3}} + \frac{1}{(3.3\times10^8)(11.6pF)}$$

$$\underline{\underline{R_z = 454.3\,\Omega}}$$

10.16 If M_1 turns off momentarily, I_1 causes a positive jump in voltage at node A. This causes M_2 to shut off momentarily so the slew rate is determined by I_2 and C_y.

i) $$\text{Slew rate} = -\frac{I_2}{C_y} = -\frac{1\,mA}{105.6\,fF} = \underline{\underline{-9.47 \times 10^9\ V/s}}$$ (If unloaded)

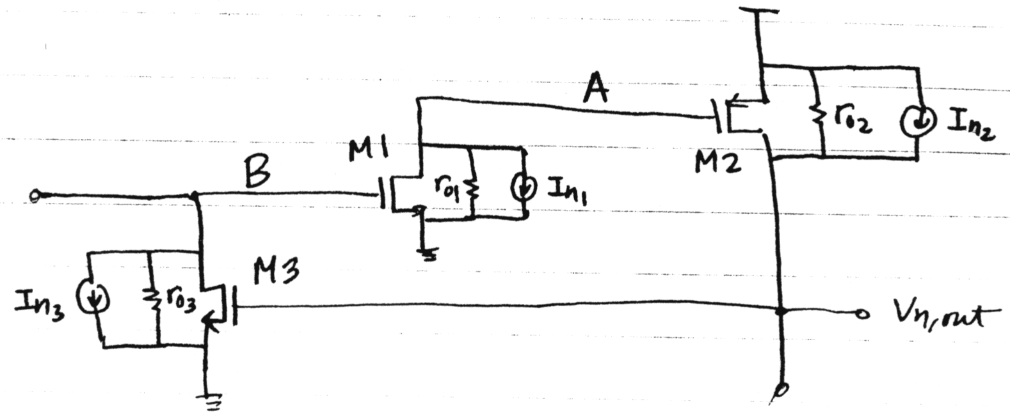
ii) $$\text{Slew rate} = -\frac{I_2}{C_y + C_L} = -\frac{1\,mA}{105.6\,fF + 0.5\,pF} = \underline{\underline{-1.65 \times 10^9\ V/s}}$$ (loaded)

10.17 For problem 10.14, Cc should not be placed "across" M_2 or M_3 because of the location of the poles. Since the dominant pole was at node B, the 2nd pole at node A, and the 3rd pole at node Y, we need to split the 1st two poles by placing Cc across M_1.

Putting Cc "across" M2 only splits the 2nd and 3rd pole keeping the dominant pole unchanged. It moves the 2nd pole toward the dominant pole and the 3rd pole away. That cannot give a 60° phase margin.

Putting Cc "across" M3 only affects the dominant pole and we cannot take advantage of pole-splitting to widen the bandwidth.

$\boxed{10.18}$



$$V_B = -r_{o3}(g_{m3} V_{n,out} + I_{n3})$$

$$V_A = -r_{o1}(g_{m1} V_B + I_{n1}) = -r_{o1}(-g_{m1} g_{m3} r_{o3} V_{n,out} - g_{m1} r_{o3} I_{n3} + I_{n1})$$

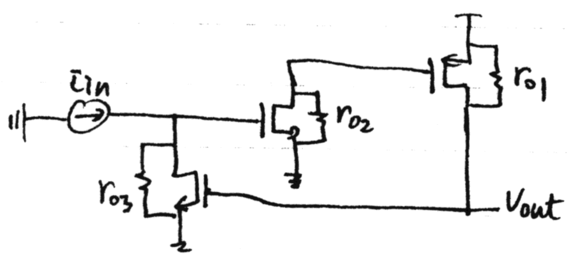
$$V_{n,out} = -r_{o2}(g_{m2} V_A + I_{n2})$$

$$= -r_{o2}[I_{n2} + g_{m1} g_{m2} g_{m3} r_{o1} r_{o3} V_{n,out} + g_{m1} g_{m2} r_{o1} r_{o3} I_{n3} - g_{m2} r_{o1} I_{n1}]$$

$$\overline{V_{n,out}^2} = \frac{\overline{I_{n2}^2} + (g_{m1} g_{m2} r_{o1} r_{o3})^2 \overline{I_{n3}^2} + (g_{m2} r_{o1})^2 \overline{I_{n1}^2}}{\left(\frac{1}{r_{o2}} + g_{m1} g_{m2} g_{m3} r_{o1} r_{o3}\right)^2}$$

//

Transresistance of the circuit is :



$$\frac{V_{out}}{i_{in}} = \frac{1}{g_{m3} + [g_{m1} g_{m2} r_{o1} r_{o2} r_{o3}]^{-1}}$$

//

☆ Input referred noise current: $\overline{i_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{\left(\frac{V_{out}}{i_{in}}\right)^2} = \underline{\underline{7.25 \times 10^{-24}\ A^2/Hz}}$

Aside {
Since from (10.14) : $g_{m1} = 5.18 \times 10^{-3}$ A/V, $g_{m2} = 2.77 \times 10^{-3}$ A/V, $g_{m3} = 1.64 \times 10^{-4}$ A/V.
$r_{o1} = 10k\Omega$, $r_{o2} = 5k\Omega$, $r_{o3} = 1M\Omega$.
$\overline{I_{n2}^2} = 4kT\frac{2}{3}g_{m2} = 3.062 \times 10^{-23}$ A²/Hz, $\overline{I_{n3}^2} = 1.813 \times 10^{-24}$ A²/Hz,
$\overline{I_{n1}^2} = 5.726 \times 10^{-23}$ A²/Hz.

$\boxed{10.19}$ $H_{open}(s) = \dfrac{A_o\left(1+\frac{s}{w_z}\right)}{\left(1+\frac{s}{w_{p1}}\right)\left(1+\frac{s}{w_{p2}}\right)}$ $\quad w_z \cong w_{p2}$

a)

$$H_{closed}(s) = \frac{A}{1+A\beta} = \frac{A_o\left(1+\frac{s}{w_z}\right)}{\left(1+\frac{s}{w_{p1}}\right)\left(1+\frac{s}{w_{p2}}\right)+A_o\left(1+\frac{s}{w_z}\right)}$$

$$= \frac{A_o\left(1+\frac{s}{w_z}\right)}{\frac{s^2}{w_{p1}w_{p2}} + s\left(\frac{1}{w_{p1}}+\frac{1}{w_{p2}}+\frac{A_o}{w_z}\right)+A_o+1} \qquad \text{Q.E.D.}$$

b) $D(s) = \left(1+\frac{s}{w_{pA}}\right)\left(1+\frac{s}{w_{pB}}\right) \cong 1+\frac{s}{w_{pB}}+\frac{s^2}{w_{pA}w_{pB}} \quad (w_{pB} << w_{pA})$

$$w_{pB} = \frac{A_o+1}{\frac{1}{w_{p1}}+\frac{1}{w_{p2}}+\frac{A_o}{w_z}} \quad //$$

$$w_{pA} = (1+A_o)w_{p1}w_{p2}\cdot\frac{1}{w_{pB}} = w_{p2}+w_{p1}+\frac{A_o}{w_z}w_{p1}w_{p2} \quad //$$

c) Using $w_z \cong w_{p2}$, $w_{p2} << (1+A_o)w_{p1}$ or $\frac{1}{w_{p1}} << \frac{A_o+1}{w_{p2}}$

$$w_{pB} = \frac{A_o+1}{\frac{1}{w_{p1}}+\frac{1}{w_{p2}}+\frac{A_o}{w_z}} \cong \frac{A_o+1}{\frac{1}{w_{p1}}+\frac{A_o+1}{w_{p2}}} \cong \underline{\underline{w_{p2}}}$$

$$w_{pA} = w_{p2}+w_{p1}+\frac{A_o}{w_z}w_{p1}w_{p2} \cong w_{p2}+(A_o+1)w_{p1} \cong \underline{\underline{(A_o+1)w_{p1}}}$$

$$H_{closed}(s) \cong \frac{\frac{A_o}{A_o+1}\left(1+\frac{s}{w_z}\right)}{\left(1+\frac{s}{(A_o+1)w_{p1}}\right)\left(1+\frac{s}{w_{p2}}\right)} \quad //$$

10.19 d) Step response: $Y(s)$

$$Y(s) = \frac{1}{s}\,\frac{A\left(1+\frac{s}{w_z}\right)}{\left(1+\frac{s}{w_{PA}}\right)\left(1+\frac{s}{w_{PB}}\right)}$$

$A = \frac{A_0}{1+A_0}$, $w_{PA} = (1+A_0)w_{P1}$, $w_{PB} = w_{P2}$.

$$= \frac{K_1}{s} + \frac{K_2}{\left(1+\frac{s}{w_{PA}}\right)} + \frac{K_3}{\left(1+\frac{s}{w_{PB}}\right)}$$

$$= \frac{s^2\left(\frac{K_1}{w_{PA}w_{PB}} + \frac{K_2}{w_{PB}} + \frac{K_3}{w_{PA}}\right) + s\left(\frac{K_1}{w_{PA}} + \frac{K_1}{w_{PB}} + K_2 + K_3\right) + K_1}{s\left(1+\frac{s}{w_{PA}}\right)\left(1+\frac{s}{w_{PB}}\right)}$$

$$\boxed{K_1 = A}$$

$$\frac{A}{w_{PA}} + \frac{A}{w_{PB}} + K_2 + K_3 = \frac{A}{w_z} \longrightarrow K_2 + K_3 = A\left(\frac{1}{w_z} - \frac{1}{w_{PA}} - \frac{1}{w_{PB}}\right) \quad \ldots (1)$$

$$\frac{A}{w_{PA}w_{PB}} + \frac{K_2}{w_{PB}} + \frac{K_3}{w_{PA}} = 0 \longrightarrow w_{PA}K_2 + w_{PB}K_3 = -A \quad \ldots (2)$$

$$-(1)\times w_{PA} + (2) \longrightarrow -\left[w_{PA}K_2 + w_{PA}K_3 = A\left(\frac{w_{PA}}{w_z} - 1 - \frac{w_{PA}}{w_{PB}}\right)\right]$$

$$+ \quad w_{PA}K_2 + w_{PB}K_3 = -A$$

$$K_3(w_{PB} - w_{PA}) = A\left(\frac{w_{PA}}{w_{PB}} - \frac{w_{PA}}{w_{PZ}}\right)$$

$$K_3 = \frac{A\left(\frac{w_{PA}}{w_B} - \frac{w_{PA}}{w_z}\right)}{w_{PB} - w_{PA}}$$

Plug back into (2) to get K_2:

$$w_{PA}K_2 + w_{PB}\left[\frac{A\,w_{PA}\left(\frac{1}{w_{PB}} - \frac{1}{w_z}\right)}{w_{PB} - w_{PA}}\right] = -A$$

$$w_{PA}K_2 = -A\left[1 + \frac{w_{PA} - \frac{w_{PA}w_{PB}}{w_z}}{w_{PB} - w_{PA}}\right] = -A\cdot\frac{w_{PB} - \frac{w_{PA}w_{PB}}{w_z}}{w_{PB} - w_{PA}}$$

$$K_2 = -A\,\frac{w_{PB}\left(1 - \frac{w_{PA}}{w_z}\right)}{w_{PA}(w_{PB} - w_{PA})}$$

10.19 d) (cont)

can simplify:

$$K_3 = \frac{A\,\omega_{PA}\left(\frac{1}{\omega_{PB}} - \frac{1}{\omega_z}\right)}{\omega_{PB} - \omega_{PA}} \cong \frac{\frac{A_o}{A_o+1}(A_o+1)\omega_{P1}\left(\frac{1}{\omega_{P2}} - \frac{1}{\omega_z}\right)}{\omega_{P2} - (A_o+1)\omega_{P1}}$$

$$\cong -\frac{A_o}{A_o+1}\left(\frac{1}{\omega_{P2}} - \frac{1}{\omega_{P2}}\right) = K_3$$

$$K_2 = -A\,\frac{\omega_{P2}\left(1 - \frac{(A_o+1)\omega_{P1}}{\omega_z}\right)}{(A_o+1)\omega_{P1}\left(\omega_{P2} - (A_o+1)\omega_{P1}\right)} \cong \frac{-A_o}{A_o+1}\;\frac{-(A_o+1)\omega_{P1}}{-(A_o+1)^2\omega_{P1}^2}$$

$$\cong \frac{-A_o}{(A_o+1)^2\omega_{P1}} = K_2$$

$$Y(s) = \frac{A}{s} + \frac{\frac{-A_o}{(A_o+1)^2\omega_{P1}}\cdot\omega_{PA}}{\omega_{PA}+s} + \frac{-\frac{A_o}{A_o+1}\left(\frac{1}{\omega_{P2}} - \frac{1}{\omega_z}\right)\cdot\omega_{PB}}{s+\omega_{PB}}$$

$$\cong \frac{A}{s} + \frac{-\frac{A_o}{A_o+1}}{s+(A_o+1)\omega_{P1}} + \frac{-\frac{A_o}{A_o+1}\left(1 - \frac{\omega_{P2}}{\omega_z}\right)}{s+\omega_{P2}}$$

★ $$y(t) = \frac{A_o}{1+A_o}\left[1 - \left(1 - \frac{\omega_{P2}}{\omega_z}\right)e^{-\omega_{P2}t} - e^{-(A_o+1)\omega_{P1}t}\right]u(t)$$

= small signal step response.

★ $$y(t) \cong \frac{A_o}{1+A_o}\left[1 - \left(1 - \frac{\omega_{P2}}{\omega_z}\right)e^{-\omega_{P2}t}\right]u(t) \text{ since } (1+A_o)\omega_{P1} \gg \omega_{P2}.$$

Hence if ω_z and ω_{P2} do not exactly cancel, there is an exponential term $\left(1 - \frac{\omega_{P2}}{\omega_z}\right)e^{-\omega_{P2}t}$ with a time constant $\frac{1}{\omega_{P2}} \approx \frac{1}{\omega_z}$. Q.E.D.

10.22

$\boxed{10.20}$

a) Perfect pole-zero cancellation.

Then

$$y(t) \cong \frac{A_o}{1+A_o}\left[1 - 0 - e^{-\omega_{p1}(A_o+1)t}\right]u(t)$$

$$\cong \frac{A_o}{1+A_o}\, u(t)$$

$\Rightarrow$ Step.

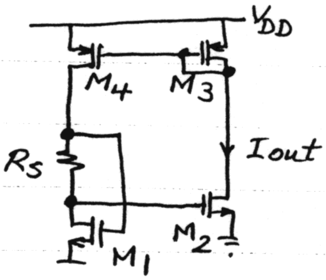
b) 10% mismatch.

$$y(t) \cong \frac{A_o}{1+A_o}\left[1 - 0.9\, e^{-\omega_{p2} t}\right]u(t)$$

$\Rightarrow$

$\tau = \frac{1}{\omega_{p2}}.$

Chapter 11

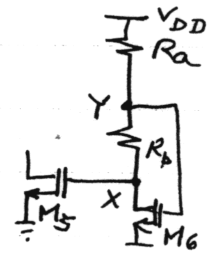


11.1 Assuming all transistors are in saturation, we have

$$I_{out} R_S + \sqrt{\frac{2 I_{out}}{\mu_n C_{ox} (\frac{W}{L})_2}} + V_{TH2} = \sqrt{\frac{2 I_{out}}{\mu_n C_{ox} (\frac{W}{L})_2}} + V_{TH1},$$

where we have assumed $(\frac{W}{L})_4 = (\frac{W}{L})_3$ and $\lambda = 0$.

Thus, $$I_{out} = \frac{1}{\mu_n C_{ox} R_S^2} \left(\sqrt{(\tfrac{L}{W})_1} - \sqrt{(\tfrac{L}{W})_2} \right)^2$$



11.2 When the circuit turns on, initially both M_5 and M_6 are off and V_X and V_Y rise together, i.e., $V_X = V_Y$. When V_Y reaches V_{TH6}, V_X is also near V_{TH5}. Thus, M_6 and M_5 turn on almost simultaneously. The surge in the drain current of M_5 turns the rest of the circuit on. As V_Y increases further, V_X begins to drop if M_6 is turned on sufficiently because the voltage gain of M_6 and R_b exceeds unity. For high values of V_Y, V_X can be lower than V_{TH5}.

Since $(V_{DD} - I_{D6} \cdot R_a - V_{TH})^2 \cdot \mu_n C_{ox} (\frac{W}{L})_6 = I_{D6}$, we solve the quadratic equation:

$$R_a^2 I_{D6}^2 - I_{D6} \left(2 R_a (V_{DD} - V_{TH}) + \frac{1}{\mu_n C_{ox} (\frac{W}{L})_6} \right) + (V_{DD} - V_{TH})^2 = 0$$

$$\Rightarrow I_{D6} = \frac{2 R_a (V_{DD} - V_{TH}) + \frac{1}{\mu_n C_{ox} (\frac{W}{L})_6} + \sqrt{\left[2 R_a (V_{DD} - V_{TH}) + \frac{1}{\mu_n C_{ox} (\frac{W}{L})_6} \right]^2 - 4 R_a^2 (V_{DD} - V_{TH})^2}}{2 R_a^2}$$

This value is substituted in the other condition:

$$V_{DD} - I_{D6} (R_a + R_b) \leq V_{TH5}$$

to give the condition for turning off M_5.

11.3 (a) Since the output voltage is near 2.5V whereas $V_X \approx 2V_{BE}$,

we write $\dfrac{I_{D1}}{I_{D2}} \approx \dfrac{1+\lambda(V_{DD}-2V_{BE})}{1+\lambda(V_{DD}-2.5V)}$

$$\approx 1+\lambda(2.5V-2V_{BE})$$

$$\Rightarrow V_{BE2}-V_{BE4} = V_T \ln n + V_T \ln \frac{I_{D1}}{I_D} \qquad \ln(1+\varepsilon)\approx\varepsilon$$

$$= V_T \ln n + V_T \lambda (2.5V - 2V_{BE})$$

The error $V_T\lambda(2.5V-2V_{BE})$ directly appears in V_{out}.

This error is also divided by R_1 and multiplied by R_2, giving another error component at the output. So the overall error is equal to $(1+\frac{R_2}{R_1})V_T\lambda(2.5V-2V_{BE})$.

(b) $$\frac{I_{D3}}{I_{D4}} \approx \frac{1+\lambda(V_{DD}-V_{BE1})}{1+\lambda(V_{DD}-V_{BE1}+V_T\ln n)}$$

$$\approx 1-\lambda V_T \ln n$$

The output error is then equal to $V_T \ln(1-\lambda V_T \ln n)$

$$\approx -V_T^2\lambda \ln n.$$

(c) $V_{TH1}=V_{TH}$, $V_{TH2}=V_{TH}+\Delta V_{TH}$

For small V_{TH}, we have $I_{D2}=I_{D1}+g_m\Delta V_{TH}$, where g_m is the mean transconductance of M_1 and M_2. Thus,

$\dfrac{I_{D1}}{I_{D2}} = 1-\dfrac{g_m\Delta V_{TH}}{I_{D2}} = 1-\dfrac{2\Delta V_{TH}}{|V_{GS}-V_{TH}|_2}$. Using the method of part (a), we have: output error $=(1+\frac{R_2}{R_1})(-V_T)\dfrac{2\Delta V_{TH}}{|V_{GS}-V_{TH}|_2}$.

(d) $\dfrac{I_{D3}}{I_{D4}} = 1-\dfrac{2\Delta V_{TH}}{|V_{GS}-V_{TH}|_4} \Rightarrow$ output error $= -V_T\cdot\dfrac{2\Delta V_{TH}}{|V_{GS}-V_{TH}|_4}$

11.4 $$-V_{XY}\cdot A_1 = V_{DD} - |V_{GS2}| \qquad |V_{GS2}| = \sqrt{\frac{2(V_T \ln n)/R_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} + |V_{TH2}|$$

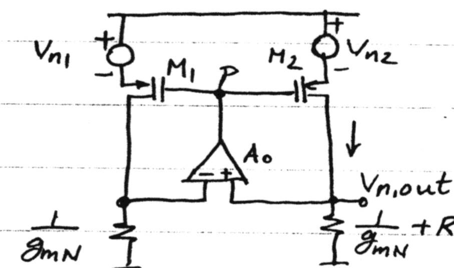
$$A_1 \geq \left[V_{DD} - \sqrt{\frac{2(V_T \ln n)/R_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} - |V_{TH2}|\right] / (-V_e)$$

11.5 The collector current of Q_4 is less than its emitter current. Thus, the current thru R_1 and R_2 is given by $\frac{(V_T \ln n)}{R_1} \times \frac{\beta+1}{\beta}$, and hence the output has an error equal to $\frac{1}{\beta}\frac{V_T \ln n}{R_1}R_2$.

Another source of error is the flow of base currents of Q_2 and Q_4 from M_3 and M_4, respectively. That is, $|V_{BE1}|$ and $|V_{BE3}|$ are slightly less than the predicted value.

$$\text{error} = V_T \ln \frac{\beta}{\beta+1}.$$

11.6



For the noise due to M_1:

$$\frac{V_{n,out}}{R_1 + g_{mN}^{-1}}\cdot\frac{1}{g_{mp}} = V_P, \quad \left(+\frac{V_P}{A_0} + V_{n,out}\right)g_{mN} = |I_{D1}|$$

$\Rightarrow$

$$\underbrace{\left(\frac{V_{n,out}}{R_1 + g_{mN}^{-1}}\cdot\frac{1}{g_{mp}}\cdot\frac{1}{A_0} + V_{n,out}\right)g_{mN}}_{I_{D1}}\left(\frac{1}{g_{mp}}\right) = \frac{V_{n,out}}{R_1 + g_{mN}^{-1}}\cdot\frac{1}{g_{mp}} + V_{n1}$$

$$\Rightarrow V_{n,out} = V_{n1}\frac{1}{\left(\frac{1}{R_1 + g_{mN}^{-1}}\cdot\frac{1}{g_{mp}}\cdot\frac{1}{A_0} + 1\right)\frac{g_{mN}}{g_{mp}} - \frac{1}{(R_1 + g_{mN}^{-1})}\cdot\frac{1}{g_{mp}}},$$

where $$\overline{V_{n1}^2} = 4kT\left(\frac{2}{3g_{mp}}\right) + \frac{K_{Fp}}{WLC_{ox}}\cdot\frac{1}{f}$$

11.4

For the noise due to M_2:
$$\begin{cases} \dfrac{V_{n,out}}{R_1+g_{mN}^{-1}} \cdot \dfrac{1}{g_{mP}} + V_{n2} = V_P \\[2ex] \left(\dfrac{V_P}{A_0} + V_{n,out}\right) g_{mN} = |I_{D1}| \end{cases}$$

$$\Rightarrow \left[\left(\frac{V_{n,out}}{R_1+g_{mN}^{-1}} \cdot \frac{1}{g_{mP}} + V_{n2}\right)\frac{1}{A_0} + V_{n,out}\right] g_{mN} \left(\frac{1}{g_{mP}}\right) = \frac{V_{n,out}}{R_1+g_{mN}^{-1}} \cdot \frac{1}{g_{mP}} + V_{n2}$$

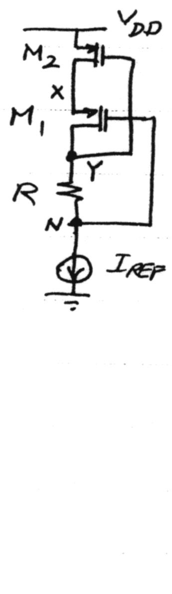
$$\Rightarrow V_{n,out}\left\{\left[\frac{1}{(R_1+g_{mN}^{-1})g_{mP}A_0} + 1\right] \times \frac{g_{mN}}{g_{mP}} - \frac{1}{R_1+g_{mN}^{-1}} \cdot \frac{1}{g_{mP}}\right\} = V_{n2}\left[1 - \frac{g_{mN}}{A_0 g_{mP}}\right]$$

$$\Rightarrow V_{n,out} = V_{n2} \frac{1 - \dfrac{g_{mN}}{g_{mP}}}{\dfrac{g_{mN}/g_{mP}}{(R_1+g_{mN}^{-1})g_{mP}A_0} + \dfrac{g_{mN}}{g_{mP}} - \dfrac{1}{R_1+g_{mN}^{-1}} \cdot \dfrac{1}{g_{mP}}},$$

where $\overline{V_{n2}^2} = 4kT\left(\dfrac{2}{3g_{mP}}\right) + \dfrac{K_{F,P}}{WLC_{ox}} \cdot \dfrac{1}{f}$

The overall noise is obtained by adding the noise powers.

11.7 For M_1 to be in saturation, $RI_{REF} \leq |V_{TH1}|$.



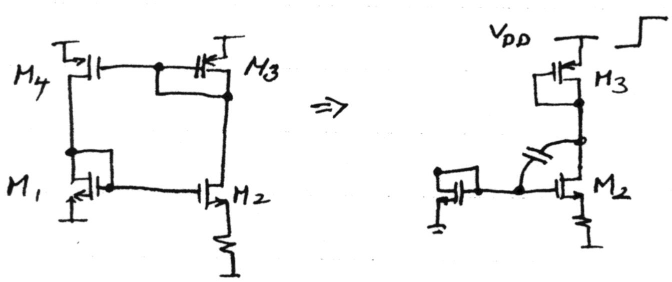
For M_2 to be in saturation,

$$V_N + |V_{GS1}| \leq \underbrace{V_{DD} - |V_{GS2}|}_{V_Y = V_N + RI_{REF}} + |V_{TH2}|$$

$$\Rightarrow |V_{GS1}| \leq RI_{REF} + |V_{TH2}|$$

$$\Rightarrow |V_{GS1}| - |V_{TH2}| \leq RI_{REF} \leq |V_{TH1}|$$

11.8



When V_{DD} rises, M_3 turns on because the gate-drain overlap capacitance of M_2 must charge. The current flowing thru this capacitance may increase the gate voltage of M_2 sufficiently, turning this transistor on as well. When M_3 turns on, M_4 also turns on.

11.9 $$\frac{\partial V_{BE}}{\partial T} = \frac{V_{BE} - E_g/q}{T} - (4+m)\frac{k}{q}$$

As T increased, V_{BE} drops. Thus, the TC becomes more negative.

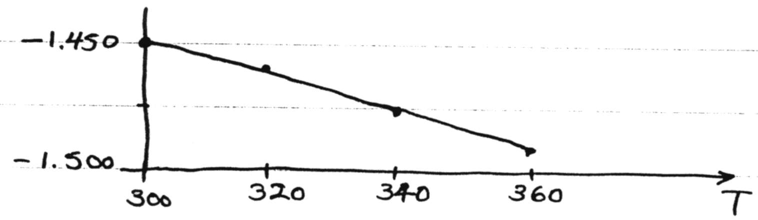
We can sketch the behavior by a piecewise linear approximation.

$T = 300°K$, $V_{BE} \approx 750\ mV \Rightarrow TC = -1.45\ mV/°K$

$T = 320°K$, $V_{BE} \approx 750 - 20(1.45) = 721\ mV \Rightarrow TC = -1.46\ mV/°K$

$T = 340°K$, $V_{BE} \approx 721 - 20(1.46) = 692\ mV \Rightarrow TC = -1.476\ mV/°K$

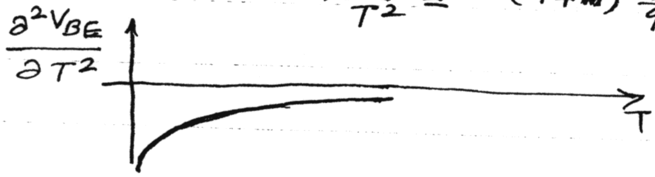
$T = 360°K$, $V_{BE} \approx 692 - 20(1.476) = 662\ mV \Rightarrow TC = -1.489\ mV/°K$



11.10 $$\frac{\partial^2 V_{BE}}{\partial T^2} = \frac{(\partial V_{BE}/\partial T)T - (V_{BE} - \frac{E_g}{q})}{T^2} = \frac{1}{T}\frac{\partial V_{BE}}{\partial T} - \frac{1}{T^2}(V_{BE} - \frac{E_g}{q})$$

$$= \frac{V_{BE} - (4+m)V_T - E_g/q}{T^2} - \frac{1}{T^2}V_{BE} + \frac{1}{T^2}\frac{E_g}{q}$$

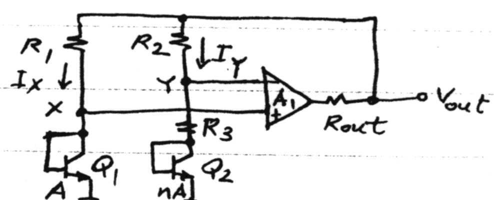
$$= -(4+m)\frac{V_T}{T^2} = -(4+m)\frac{k}{q}\cdot\frac{1}{T}$$



11.11 $V_Y - V_X = R_3 I_Y - V_T \ln n$

$\Rightarrow -A_1(R_3 I_Y - V_T \ln n) - R_{out}(2 I_Y) = V_{out}$

Assumed $I_X \approx I_Y$ here.



We also note that: $I_Y = \dfrac{V_{out} - V_{BE2}}{R_2 + R_3}$.

Thus, $$V_{out} = \frac{(R_2+R_3)(V_T \ln n)A_1 + (A_1R_3 + 2R_{out})V_{BE2}}{R_2 + 2R_{out} + A_1R_3 + R_3}$$

Dividing the numerator and denominator by $A_1 R_3$ and assuming $\frac{R_3+R_2+2R_{out}}{A_1 R_3} \ll 1$, we have:

$$V_{out} \cong \left[\left(1+\frac{R_2}{R_3}\right)V_T \ln n + V_{BE} + \left(\frac{2R_{out}}{A_1 R_3}V_{BE}\right)\right]\left(1-\frac{R_3+R_2+2R_{out}}{A_1 R_3}\right)$$

The error is then equal to:

$$\frac{2R_{out}}{A_1 R_3}V_{BE} - \frac{R_3+R_2+2R_{out}}{A_1 R_3}\left[\left(1+\frac{R_2}{R_3}\right)V_T \ln n + V_{BE}\right]$$

11.12 $R_3 = 1\,k\Omega$ $I_{R3} = 50\,\mu A$ $R_1 = R_2$

$V_{out} = V_{BE2} + (V_T \ln n)\left(1+\frac{R_2}{R_3}\right) = 1.25\,V$, $V_{BE2} \approx 750\,mV$

$$I_{R3} = \frac{V_{out}-V_{BE2}}{R_2+R_3} = \frac{(V_T \ln n)\left(1+\frac{R_2}{R_3}\right)}{R_2+R_3} = 50\,\mu A$$

$$(\ln n)\left(1+\frac{R_2}{R_3}\right) \cong 17.2$$

$\Rightarrow R_2 = 7.944\,k\Omega$

$\Rightarrow n \approx 6.84.$

Some iteration is usually necessary to arrive at an integer n. (Of course, the current thru R_3 will be slightly different from $50\,\mu A$.)

11.13 $I_{C1} = I_{C2} = 100\,\mu A$ $I_{C3} = I_{C4} = 50\,\mu A$ $R_1 = 1\,k\Omega$.

V_{DD} must be equal to 3 V.

Since $V_{out} \cong 2.5\,V$, M_2 and hence M_1 must be sized such that they remain in saturation.

$$\begin{cases} V_{BE3} + V_{BE4} + \left(1+\frac{R_2}{R_1}\right)(2V_T)\ln(\underbrace{m}_{I_{C2}/I_{C3}}\, n) \cong 2.5\,V \\ V_{out} - (V_{BE4}+V_{BE3}) = (50\,\mu A)(R_1+R_2) \end{cases}$$

The two unknowns here are R_2 and n. Since Q_3 and Q_4 are biased at a relatively low current, we assume $V_{BE3} = V_{BE} \approx 700\,mV \Rightarrow \left(1+\frac{R_2}{R_1}\right)(2V_T)\ln(mn) \approx 1.8\,V$

From the second equation, $R_1 + R_2 \approx 36\,k\Omega$, $\Rightarrow R_2 = 35\,k\Omega$.

Dividing the numerator and denominator by $A_1 R_3$ and assuming $\frac{R_3+R_2+2R_{out}}{A_1R_3} \ll 1$, we have:

$$V_{out} \cong \left[\left(1+\frac{R_2}{R_3}\right)V_T \ln n + V_{BE} + \left(\frac{2R_{out}}{A_1R_3}V_{BE}\right)\right]\left(1-\frac{R_3+R_2+2R_{out}}{A_1R_3}\right)$$

The error is then equal to:

$$\frac{2R_{out}}{A_1R_3}V_{BE} - \frac{R_3+R_2+2R_{out}}{A_1R_3}\left[\left(1+\frac{R_2}{R_3}\right)V_T \ln n + V_{BE}\right]$$

11.12 $\quad R_3 = 1\,k\Omega \quad I_{R3} = 50\,\mu A \quad R_1 = R_2$

$$V_{out} = V_{BE2} + (V_T \ln n)\left(1+\frac{R_2}{R_3}\right) = 1.25\,V, \qquad V_{BE2} \approx 750\,mV$$

$$I_{R3} = \frac{V_{out}-V_{BE2}}{R_2+R_3} = \frac{(V_T \ln n)\left(1+\frac{R_2}{R_3}\right)}{R_2+R_3} = 50\,\mu A$$

$$(\ln n)\left(1+\frac{R_2}{R_3}\right) \cong 17.2 \qquad \Rightarrow R_2 = 7.944\,k\Omega \quad \Rightarrow n \approx 6.84.$$

Some iteration is usually necessary to arrive at an integer n. (Of course, the current thru R_3 will be slightly different from 50 μA.)

11.13 $\quad I_{C1} = I_{C2} = 100\,\mu A \quad I_{C3} = I_{C4} = 50\,\mu A \quad R_1 = 1\,k\Omega$.

V_{DD} must be equal to 3 V.

Since $V_{out} \cong 2.5\,V$, M_2 and hence M_1 must be sized such that they remain in saturation.

$$\begin{cases} V_{BE3} + V_{BE4} + \left(1+\frac{R_2}{R_1}\right)(2V_T)\ln(\overbrace{m}^{I_{C2}/I_{C3}}\, n) \cong 2.5\,V \\ V_{out} - (V_{BE4} + V_{BE3}) = (50\,\mu A)(R_1+R_2) \end{cases}$$

The two unknowns here are R_2 and n. Since Q_3 and Q_4 are biased at a relatively low current, we assume

$V_{BE3} = V_{BE} \approx 700\,mV \Rightarrow \left(1+\frac{R_2}{R_1}\right)(2V_T)\ln(mn) \approx 1.8\,V$

From the second equation, $R_1 + R_2 \approx 36\,k\Omega$, $\Rightarrow R_2 = 35\,k\Omega$.

From the first equation, $n \cong 1.31$.

Since $|V_{DS2}| \approx 0.5\,V$ with a 3-V supply, with $|I_{DS}| = 50\,\mu A$, we have $(W/L)_2 \geq 10.4$. With $I_{D1} = 2\,I_{D2}$, $(W/L)_1 = 2(W/L)_2$. Similarly, $(W/L)_3 = 2(W/L)_2$ and $(W/L)_4 = (W/L)_2$.

11.14 When we set (11.34) to zero, we obtain a relationship that is valid at only one temperature. Thus, (11.35) is only valid at one temperature and so is (11.36). In other words, the V_{BE} in (11.35) is at a single temperature T_0 whereas the V_{BE} in (11.33) is at a general temperature T. When we say $V_{REF} \to \frac{E_g}{q}$ if $T \to 0$, we really mean if V_{REF} is extrapolated, it reaches E_g/q.

11.15 $\frac{\partial}{\partial T}(g_m R_D) = 0 \qquad g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D} \qquad \mu_n \propto T^{-3/2}$

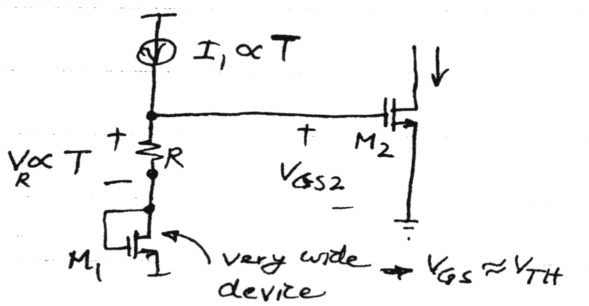
Thus, $I_D \propto T^{3/2} \approx \alpha T + \beta T^2$.

$$\Rightarrow \left.\begin{matrix} T^{1/2} \approx \alpha + \beta T \\ \text{derivative} \rightarrow \frac{3}{2}T^{1/2} \approx \alpha + 2\beta T \end{matrix}\right\} \text{using } T = 300^\circ K\text{, we have}$$

$\alpha \approx 8.66$ and $\beta = 0.0289$.

$$\Rightarrow I_D \propto \underbrace{8.66\,T}_{PTAT} + \underbrace{0.0289\,T^2}_{PTAT^2}$$

(Note that the coefficient of T^2 is quite small.)



$V_{GS2} \cong V_R + V_{TH1} \Rightarrow$

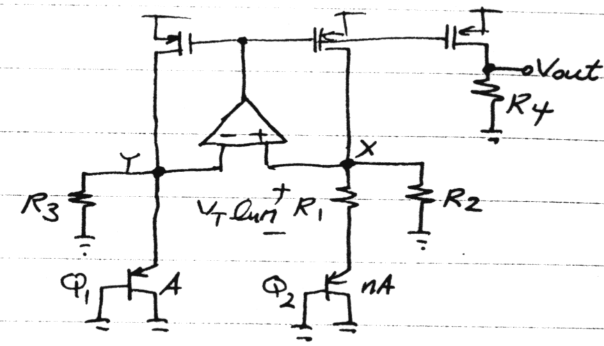
$I_{D2} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_R)^2$

$\propto T^2$

This current and a PTAT current are simply added with proper weighting to produce $8.66\,T + 0.0289\,T^2$.

11.16 $g_m \propto T^{-3/4}$. Thus, $\frac{\partial R}{\partial T} = T^{3/4}$.

11.17 The current thru R_1 is PTAT and $V_X = V_Y = V_{BE1}/R_3$.



The current thru each PMOS device is $\frac{V_T \ln n}{R_1} + \frac{V_{BE1}}{R_3}$ and hence

$$V_{out} = R_4\left(\frac{V_T \ln n}{R_1} + \frac{V_{BE1}}{R_3}\right)$$

$$= \frac{R_4}{R_3} V_{BE1} + \frac{R_4}{R_1} V_T \ln n.$$

Since V_{BE1} is multiplied by R_4/R_3, the output voltage can be arbitrarily scaled.

11.18 $V_X = V_Y - V_{OS} \Rightarrow V_{REF} = V_{BE1}\frac{R_4}{R_3} + \frac{R_4}{R_1} V_T \ln n$

$$- \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) V_{OS}.$$

11.19 (a) When S_1 is on and S_2 is off, $V_{out} \approx V_T \ln \frac{I_1}{I_{S1}}$.

(b) when S_1 turns off and S_2 turns on,

$V_X = V_T \ln \frac{I_1 + I_2}{I_{S1}}$. This change is amplified by $\frac{C_2}{C_1}+1$ and added to the original voltage across C_2:

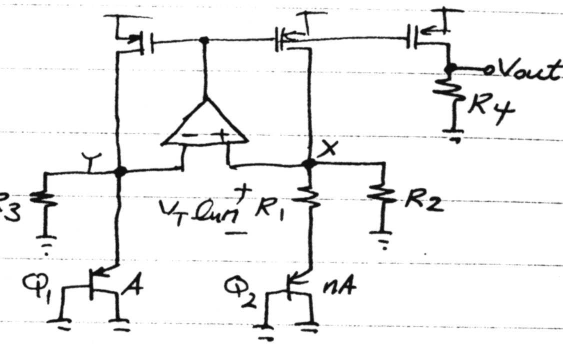
$$V_{out} = \left(1+\frac{C_2}{C_1}\right)\left(V_T \ln \frac{I_1+I_2}{I_{S1}} - V_T \ln \frac{I_1}{I_{S1}}\right) + \underbrace{V_T \ln \frac{I_1}{I_{S1}}}_{V_{BE}}$$

$$= \left(1+\frac{C_2}{C_1}\right) V_T \ln \left(\overbrace{1 + \frac{I_2}{I_1}}^{m}\right) + V_{BE}$$

$$= \left(1+\frac{C_2}{C_1}\right) V_T \ln m + V_{BE}$$

11.16 $\quad g_m \propto T^{-3/4}$. Thus, $\frac{\partial R}{\partial T} = T^{3/4}$.

11.17 The current thru R_1 is PTAT and $V_X = V_Y = V_{BE1}/R_3$.



The current thru each PMOS device is $\frac{V_T \ln n}{R_1} + \frac{V_{BE1}}{R_3}$ and hence

$$V_{out} = R_4\left(\frac{V_T \ln n}{R_1} + \frac{V_{BE1}}{R_3}\right)$$
$$= \frac{R_4}{R_3} V_{BE1} + \frac{R_4}{R_1} V_T \ln n.$$

Since V_{BE1} is multiplied by R_4/R_3, the output voltage can be arbitrarily scaled.

11.18 $\quad V_X = V_Y - V_{OS} \Rightarrow V_{REF} = V_{BE1}\frac{R_4}{R_3} + \frac{R_4}{R_1} V_T \ln n$
$$- \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) V_{OS}.$$

11.19 (a) When S_1 is on and S_2 is off, $V_{out} \approx V_T \ln \frac{I_1}{I_{S1}}$.

(b) when S_1 turns off and S_2 turns on,

$V_X = V_T \ln \frac{I_1 + I_2}{I_{S1}}$. This change is amplified by $\frac{C_2}{C_1}+1$ and added to the original voltage across C_2:

$$V_{out} = \left(1 + \frac{C_2}{C_1}\right)\left(V_T \ln \frac{I_1 + I_2}{I_{S1}} - V_T \ln \frac{I_1}{I_{S1}}\right) + \underbrace{V_T \ln \frac{I_1}{I_{S1}}}_{V_{BE}}$$

$$= \left(1 + \frac{C_2}{C_1}\right) V_T \ln \overbrace{\left(1 + \frac{I_2}{I_1}\right)}^{m} + V_{BE}$$

$$= \left(1 + \frac{C_2}{C_1}\right) V_T \ln m + V_{BE}$$

(c) for zero TC : $\quad (1+\frac{C_2}{C_1})\ln(1+\frac{I_2}{I_1}) \approx 17.2.$

11.20 $\quad V_{out} = (1+\frac{C_2}{C_1})V_T \ln(1+\frac{I_2}{I_1}) + V_{BE}$

If $\frac{I_2}{I_1} = N+\varepsilon \Rightarrow V_{out} = (1+\frac{C_2}{C_1})V_T\ln(1+N+\varepsilon) + V_{BE}$

$$= (1+\frac{C_2}{C_1})V_T\left[\ln(1+N) + \ln(1+\frac{\varepsilon}{1+N})\right] + V_{BE}$$

$$\cong (1+\frac{C_2}{C_1})V_T\left[\ln m + \frac{\varepsilon}{1+N}\right] + V_{BE}$$

The error is thus equal to $(1+\frac{C_2}{C_1})V_T\frac{\varepsilon}{1+N}$.

11.21 $\quad R_1 = 1\,K\Omega,\ R_2 = 2\,k\Omega$

(a) $\quad V_{out} = \frac{V_T \ln n}{R_1}\cdot R_2 + V_{BE3} \quad \Rightarrow \ln n \approx \frac{17.2}{2} = 8.6$

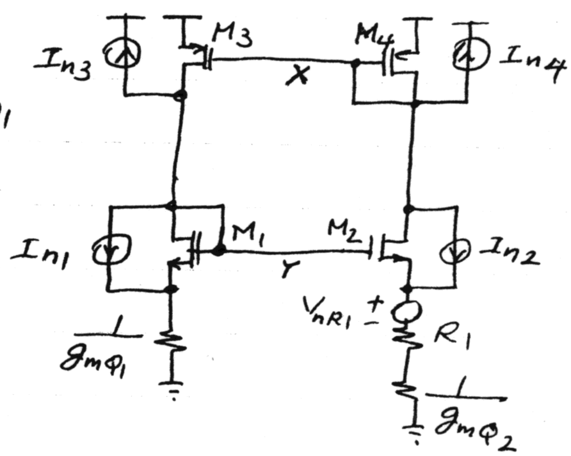
$\Rightarrow n = 5432$ (!)

Alternatively, we can scale $(W/L)_5$ up by a factor α such that: $V_{out} = \frac{V_T \ln n}{R_1}\alpha\cdot R_2 + V_{BE3}$.

For example, for $\alpha = 4$, $n = 8.58$.

(b) V_Y is given by:

$$\underbrace{(\underbrace{-g_{m3}V_X - I_{n3}}_{I_{D3}} - I_{n1})}_{\text{current thru } M_1}\frac{1}{g_{m1}} + \underbrace{(-g_{m3}V_X - I_{n3})}_{\text{current thru } \frac{1}{g_{mQ}}}\frac{1}{g_{mQ_1}}$$



Also:

$$(-g_{m4}V_X - I_{n4})(R_1 + \frac{1}{g_{mQ_2}}) + V_{nR_1} + \underbrace{(-g_{m4}V_X - I_{n4} - I_{n2})}_{I_{D2}}\frac{1}{g_{m2}} = V_Y$$

Equating these, we have

$$V_X = \frac{1}{g_{m4}R_1}\left[I_{n3}(\frac{1}{g_{m1}} + \frac{1}{g_{mQ_1}}) + \frac{I_{n1}}{g_{m1}} + I_{n4}(R_1 + \frac{1}{g_{mQ_2}} + \frac{1}{g_{m2}}) + \frac{I_{n2}}{g_{m2}} + V_{nR_1}\right]$$

This noise is amplified by $g_{m5}(R_2 + \frac{1}{g_{mQ3}})$ when it appears at the output.

$$\overline{V_{n,out,tot}^2} = \frac{g_{m5}^2 \left(R_2 + \frac{1}{g_{mQ3}}\right)^2}{(g_{m4} R_1)^2} \left[2\, I_{n3}^2 \left(\frac{1}{g_{m1}} + \frac{1}{g_{mQ1}}\right)^2 + \frac{2\, I_{n1}^2}{g_{m1}^2} + I_{n4}^2 R_1^2 + V_{nR_1}^2 \right]$$

$$+ I_{n5}^2 \left(R_2 + \frac{1}{g_{mQ3}}\right)^2 + V_{nR_2}^2$$

11.22 $f_{CK} = 50\,MHz$ power budget $= 1\,mW$. $g_{m1} = \frac{1}{500\Omega}$

$g_{m1} = \frac{2}{R_S}\left(1 - \frac{1}{\sqrt{K}}\right)$ $R_S = \frac{1}{f_{CK} C_S}$ $I_{D1} = I_{D2} = \frac{0.5\,mW}{3V}$
$= 167\,\mu A$

$$I_{out} = \frac{2}{\mu_n C_{ox} (W/L)_N} \cdot \frac{1}{R^2}\left(1 - \frac{1}{\sqrt{K}}\right)^2 = \frac{2}{\mu_n C_{ox} (W/L)_N} \left(\frac{g_{m1}}{2}\right)^2$$

$\Rightarrow \left(\frac{W}{L}\right)_N = 89.4$

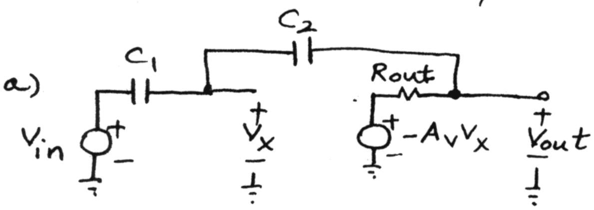
We assume $K = 4$. $\Rightarrow$ $\frac{1}{500\Omega} = \frac{2}{R_S}\left(1 - \frac{1}{2}\right) = \frac{1}{R_S}$

$\Rightarrow R_S = 500\,\Omega \Rightarrow C_S = 40\,pF$.

$\left(\frac{W}{L}\right)_2 = 4 \times 89.4$ For M_3 and M_4, there is some freedom so long as the transistors remain saturated. For example
$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 50$

Chapter 12

12.1 (a)



$$V_X = (V_{out} - V_{in})\frac{C_2}{C_1+C_2} + V_{in}$$

$$\frac{V_{out} - (-A_V V_X)}{R_{out}} = -\frac{V_{out} - V_X}{\frac{1}{C_2 s}} \Rightarrow \quad = \frac{C_2}{C_1+C_2} V_{out} - \frac{C_1}{C_1+C_2} V_{in}$$

$$\Rightarrow V_{out}\left[1 + \frac{A_V C_2}{C_1+C_2} + \frac{C_2}{C_1+C_2} R_{out} C_2 s\right] = V_{in}\left[A_V \frac{C_1}{C_1+C_2} + \frac{C_1}{C_1+C_2} R_{out} C_2 s\right]$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = A_V \frac{C_1}{C_2} \frac{1 + R_{out} C_2 s}{1 + \frac{C_1}{C_2} + A_V + R_{out} C_2 s}$$

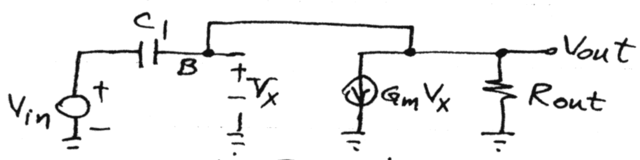
(b) $$A_V(s) = -\frac{R_F \| \frac{1}{C_2 s}}{\frac{1}{C_1 s}} = -\frac{R_F C_1 s}{R_F C_2 s + 1}$$

Nominal Gain $= 4$ $$\frac{R_F C_1 \omega}{\sqrt{1 + R_F^2 C_2^2 \omega^2}} = 3.96 \qquad \omega = 2\pi(1\,MHz)$$

$$(3.96^2 R_F^2 C_2^2 - R_F^2 C_1^2)\omega^2 + 3.96^2 = 0 \Rightarrow R_F^2 = \frac{3.96^2}{\omega^2 (C_1^2 - 3.96^2 C_2^2)}$$

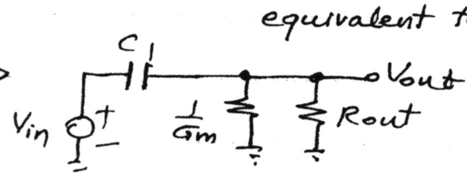
$$\Rightarrow R_F = 2.23\ M\Omega$$

12.2 (a)



equivalent to a resistor of $\frac{1}{G_m}$

$\Rightarrow$

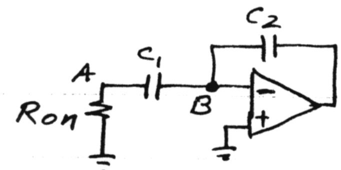


$$\frac{V_{out}}{V_{in}} = \frac{(\frac{1}{G_m} \| R_{out}) C_1 s}{(\frac{1}{G_m} \| R_{out}) C_1 s + 1} \approx \frac{(1/G_m) C_1 s}{\frac{C_1}{G_m} s + 1}$$

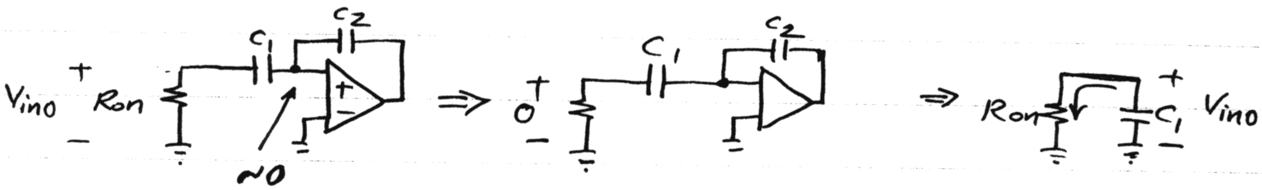
(b) $\omega = 2\pi\,(100\,MHz)$, $C_1 = 1\,pF$, $\frac{1}{G_m} = 100\,\Omega \Rightarrow \frac{C_1}{G_m}\omega = 0.0628$

$\Rightarrow \frac{V_{out}}{V_{in}} \approx 0.0628$, with a phase shift of nearly $90°$.

12.3



Since node B is at virtual ground, $\tau \approx R_{on} C_1$.



$\Rightarrow$ Total energy is that stored on $C_1 = \frac{1}{2} C_1 V_{ino}^2$

12.4 (a) $\left(\frac{W}{L}\right)_1 = \frac{20}{0.5}$, $C_H = 1\ pF$ $\quad I_{D,sat} = 20.8\ mA$

$\Rightarrow t_1 = 146\ ps$

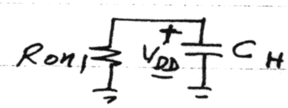
$$+1\ mV = \frac{2(2.3V)\exp\left[-(2.3V)\frac{\mu_n C_{ox} W/L}{C_H}(t-t_1)\right]}{1+\exp\left[-(2.3V)\frac{\mu_n C_{ox} W/L}{C_H}(t-t_1)\right]}$$

$$\Rightarrow \exp\left[-(2.3V)\frac{\mu_n C_{ox} W/L}{C_H}(t-t_1)\right] \approx \frac{+1\ mV}{2(2.3V)}$$

$\Rightarrow t - t_1 = 465\ ps \Rightarrow$ total time $= 611\ ps$

(b) $R_{on_1} = 55\Omega \Rightarrow \tau = 55\ ps$

$V_{out} = V_{DD} \exp\frac{-t}{\tau} \Rightarrow t = 440\ ps$

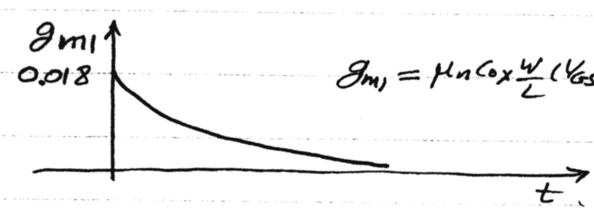


It underestimates the required time.

12.5 (a) $$2.1 = 2.3 - \frac{1}{\frac{1}{2}\frac{\mu_n C_{ox}}{C_H}\frac{W}{L}t + \frac{1}{2.3}} \qquad (\gamma = 0)$$

$$\Rightarrow \frac{1}{2}\frac{\mu_n C_{ox}}{C_H}\frac{W}{L}t + \frac{1}{2.3} = 5 \Rightarrow t \approx 1.16\ ns$$

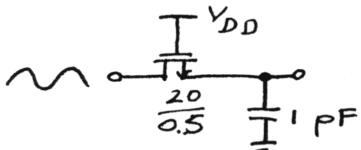
(b)



$g_{m1} = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})$

$\Rightarrow g_{m1}(t=0) = 0.018\ \mho$

12.6



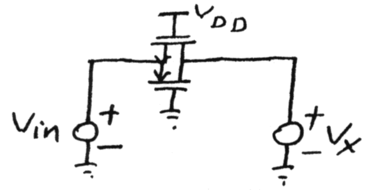
(a) $R_{on1} = 55\,\Omega$

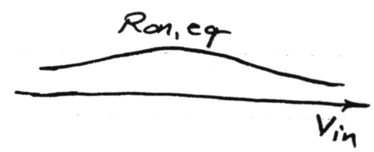
$|\theta| = \tan^{-1}(RC\omega)$

$= 1.98^\circ$

(b) $R_{on1} = 97.6\,\Omega \Rightarrow |\theta| \approx 3.96^\circ$

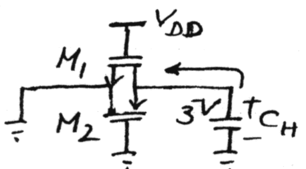
12.7





V_X is a voltage-dependent voltage source that follows Vin with a, say, 20-mV difference. We can then monitor the current drawn by either source, invert it, and normalize it to 20 mV in a dc sweep that varies Vin across the range of interest.

12.8



$R_{on1} = 55\,\Omega \quad R_{on2} = 64.3\,\Omega, C_H = 1\,pF$

$R_{on1} \| R_{on2} = 29.6\,\Omega \Rightarrow \tau = 29.6\,ps$

$\Rightarrow +1\,mV = +3V \exp\frac{-t}{\tau}$

$\Rightarrow t \approx 237\,ps.$

12.9 $V_{GS} - V_{TH} = 2.3V \Rightarrow V_{error} = \dfrac{WLC_{ox}(V_{GS} - V_{TH})}{C_H}$ eff.

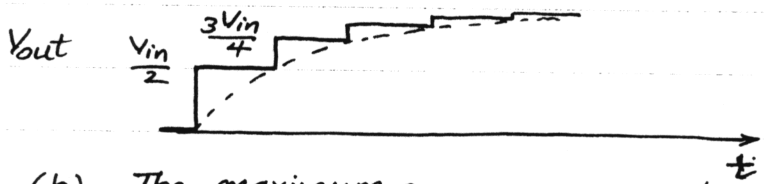
$= 60\,mV$

For clock feedthrough: $C_{ov} = (0.4 \times 10^{-11}\,F/m) \times 20\,\mu m = 0.08\,fF$

$V_{error} \cong \dfrac{C_{ov}}{C_H} V_{CK} = 0.24\,mV$

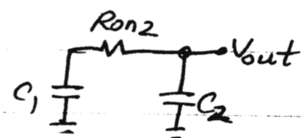
The overlap capacitance in Table 2.1 should actually be 0.4e-9 for NMOS. Thus, the error due to clock feedthrough will be about 24 mV, somewhat less than that due to worst-case charge injection.

12.10 (a) C_1 ~~together~~ with M_1 and M_2 can be viewed as a resistor. Thus, C_2 charges to $2V$ with an envelope given by $1 - \exp\frac{-t}{\tau}$, where $\tau = \frac{1}{f_{CK} C_1} \cdot C_2$.



(b) The maximum error occurs when $V_{GS} - V_{TH}$ is maximum. If all of M_1 channel charge is injected onto C_1, then after V_{C_1} has reached V_{in} and M_1 turns off, V_{C_1} incurs an error equal to $(V_{GS} - V_{in} - V_{TH}) WL C_{ox} / C_1$. When M_2 turns on, it absorbs some charge into its channel and when it turns off, it injects the charge back onto C_1 and C_2. Thus, only the charge due to M_1 need be considered. This error is divided equally between C_1 and C_2, yielding an overall output error of $\frac{WL C_{ox}}{2C_1}(V_{GS} - V_{in} - V_{TH})$.

(c) When M_1 turns off, a voltage equal to $\sqrt{\frac{kT}{C_1}}$ is stored across C_1. When M_2 is on, this voltage is distributed between C_1 and C_2. Moreover, M_2 itself produces thermal noise:

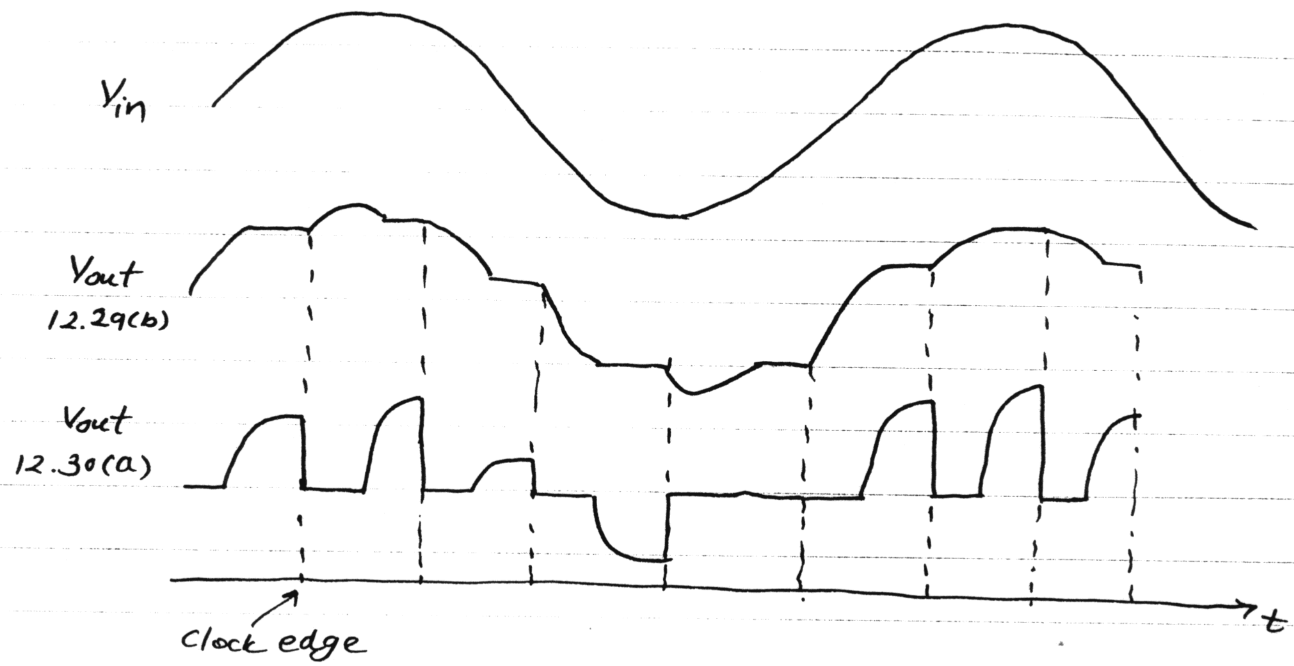


$$\Rightarrow V_{n,out} = \sqrt{\frac{kT}{2C_1}}$$

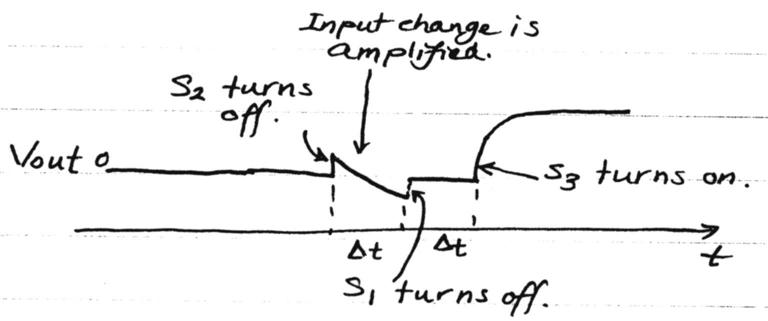
$$\Rightarrow V_{n,out,tot}^2 = \frac{1}{4}\frac{kT}{C_1} + \frac{kT}{2C_1} = \frac{3kT}{4C_1}$$

$$\Rightarrow V_{n,out,tot} = \sqrt{\frac{3kT}{4C_1}}$$

12.5



12.12



12.13 Gain error $\approx (C_2+C_1+C_{in})/(C_2 A_{v_1}) = 0.01$

$$\Rightarrow 1+\frac{C_1}{C_2}+\frac{C_{in}}{C_2} = 10 \Rightarrow 9C_2 = C_1 + C_{in} = 2.2\,pF$$

$$\Rightarrow C_2 = \frac{2.2\,pF}{9} \Rightarrow$$

$$\frac{C_1}{C_2} = 8.2 \rightarrow 8.0$$

12.14 $G_m = 100^{-1}\,\mho$ $\quad \tau_{amp} = \dfrac{C_L C_{eq} + C_L C_2 + C_{eq} C_2}{G_m C_2}$ $\quad C_{eq} = C_1 + C_{in}$

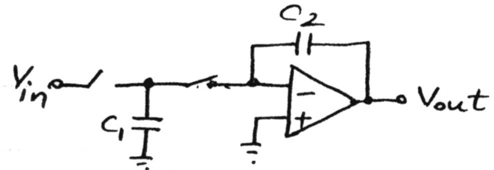
$$= \frac{C_{eq}}{G_m} \quad \text{because } C_L = 0$$

$$= 2\,ns \Rightarrow C_{eq} = 20\,pF$$

Since $C_{in} = 0.2\,pF$, $C_1 = 19.8\,pF$. Also, $9C_2 = 20\,pF \Rightarrow$

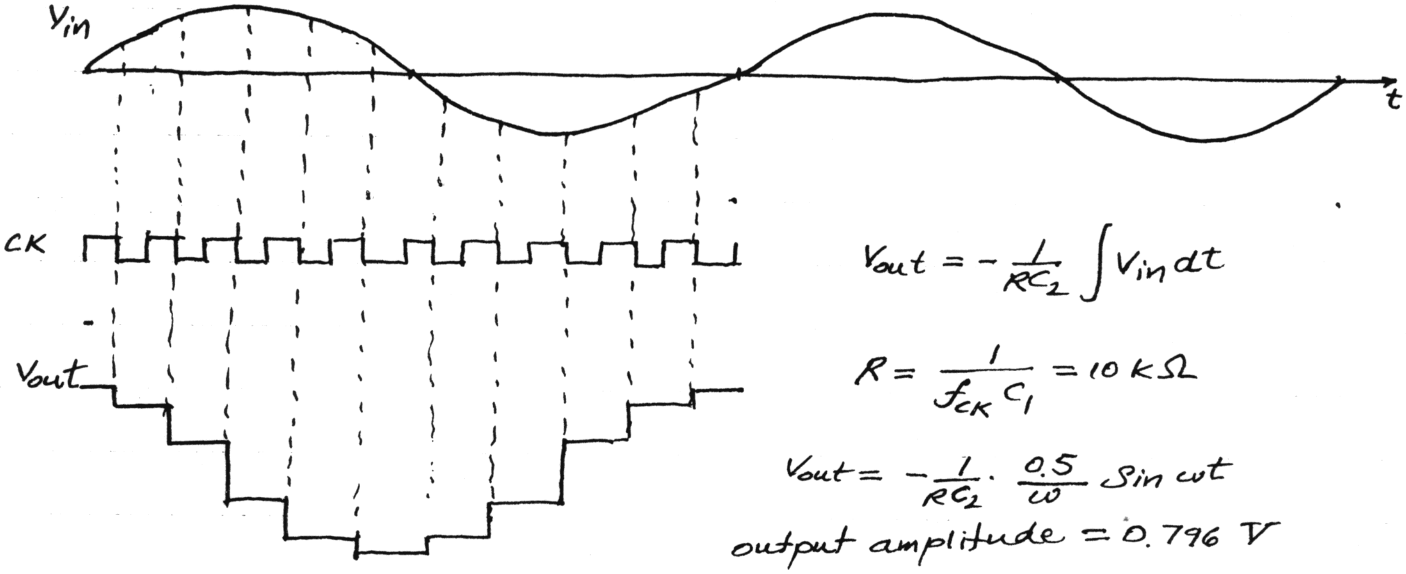
$\frac{C_1}{C_2} = 44$

12.15



$C_1 = C_2 = 1\ pF$ $f_{CK} = 100\ MHz$

$V_{in} = 0.5 \cos[2\pi (10\ MHz) t]$



$$V_{out} = -\frac{1}{RC_2} \int V_{in}\, dt$$

$$R = \frac{1}{f_{CK} C_1} = 10\ k\Omega$$

$$V_{out} = -\frac{1}{RC_2} \cdot \frac{0.5}{\omega} \sin \omega t$$

output amplitude $= 0.796\ V$

12.16 (a) The minimum level $= 1.5V - V_{TH1,2} \approx 0.8V$.

The maximum level places M_3 or M_4 at the edge of the triode region. $|V_{GS} - V_{TH}|_{3,4} = 0.421\ V \Rightarrow$ max. level $= 2.58V$.

$\Rightarrow$ Max. Swing $= 1.78\ V$.

(b) $A_{v,open} \approx g_{m1,2}(r_{o1} \| r_{o2}) = 27.3$

Gain Error $= \dfrac{C_1 + C_3}{C_3 A_v} = 18.3\%$

(c) $\tau_{amp} \approx \dfrac{C_1}{G_m} = 0.488\ ns$

12.17 (a) same.

(b) The gate-source cap is equal to $\frac{2}{3} W L_{eff} C_{ox} + W C_{ov}$
$\approx 44\ fF$

(The overlap cap in Table 2.1 must actually be $0.4e-9$, in which case $C_{in} \cong 64\ fF$.)

The gate-drain overlap capacitance (of $M_{1,2}$) changes the gain equation because it appears in parallel with the feedback capacitor:

$$\frac{V_{out}}{V_{in}} \approx -\frac{C_1}{C_1 + WC_{ov}}\left(1 - \frac{C_3 + WC_{ov} + C_1 + C_{in}}{C_3 + WC_{ov}} \cdot \frac{1}{A_v}\right)$$

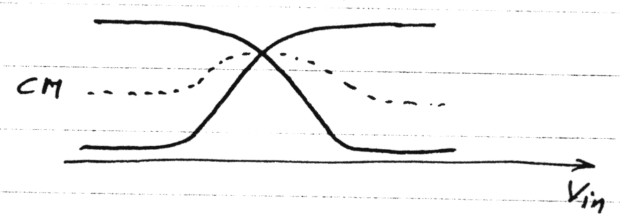
$$\approx -\frac{C_1}{C_3}\left(1 - \frac{WC_{ov}}{C_3}\right)\left(1 - \frac{C_3 + WC_{ov} + C_1 + C_{in}}{C_3 + WC_{ov}} \cdot \frac{1}{A_v}\right)$$

Thus, the gain error rises to $\frac{WC_{ov}}{C_3} + \frac{C_3 + C_1 + WC_{ov} + C_{in}}{C_3 + WC_{ov}} \cdot \frac{1}{A_v}$.

Assuming $C_{ov} = 0.4e{-9}$, we obtain a gain error of 22.2%.

(c) Neglecting the drain junction caps at the output, we have $\tau_{amp} \approx \frac{(C_1 + C_{in})}{G_m} \cong 0.503$ ns

12.18 Plotting the CM level, we see that it changes with the differential output.



This usually means that the CM feedback network, in particular the devices sensing the CM level, are quite nonlinear.

12.19 Since $I_{D5} = 1$ mA, $(V_{GS} - V_{TH})_5 = 319$ mV $\Rightarrow$ Minimum input level $= V_{GS1,2} + 319$ mV $\cong 1.245$ V (neglecting body effect.)

Since $I_{D6} = 50\,\mu A$, $(V_{GS} - V_{TH})_6 = 71.3$ mV $\Rightarrow$

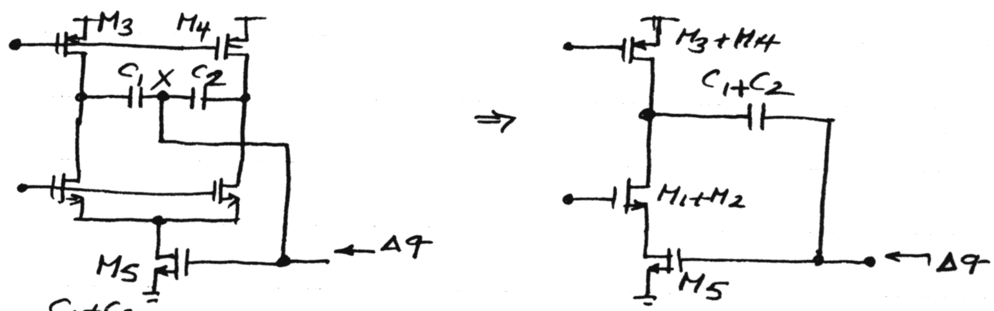
$V_{out,CM} = 71.3\text{mV} + V_{TH6} + V_{GS5} \cong 1.79$ (neglecting body effect.)

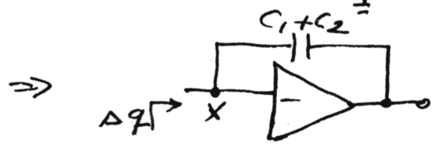
$\Rightarrow V_{in,max} = 1.79 + V_{TH1,2} \approx 2.49$ V

· 12.20 $V_{in,min}$ remains the same.

$$V_{in,max} = V_{GS6} + V_{GS5} + V_{TH1,2}$$
$$= 0.859 + 1.019 + 0.7 = 2.578\ V \quad \text{(neglecting body effect)}$$

12.21

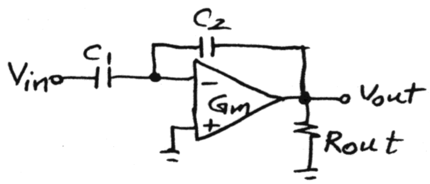




The charge resides on C_1+C_2 ⇒ output CM level changes by $\sim \frac{\Delta q}{C_1+C_2}$.

The voltage at node X changes by $\frac{\Delta q}{C_1+C_2}\cdot\frac{1}{A_v}$,
where $A_v = g_{m5}\left[r_{O3+4} \,\|\, (g_{m1+2}\, r_{O1+2}\, r_{O5})\right]$.

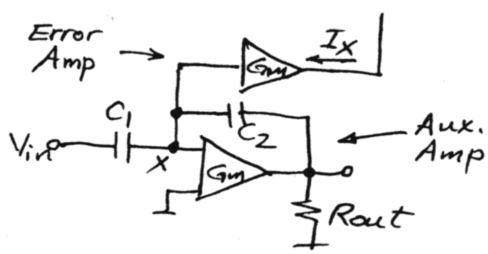
12.22 For a simple stage :

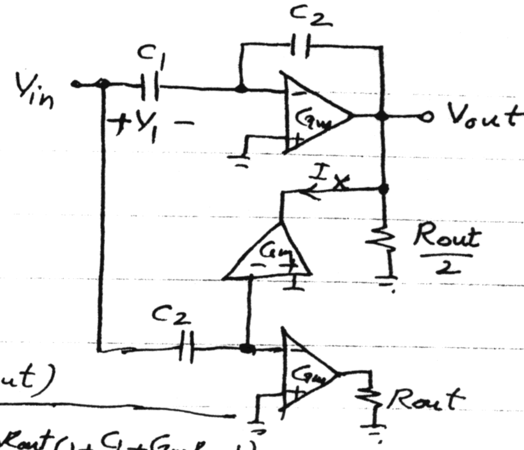


$$\frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2}\,\frac{1}{1+\left(1+\frac{C_1}{C_2}\right)\frac{1}{G_m R_{out}}}$$

Thus, the voltage at node X and hence the current drawn by the error amplifier can be easily calculated.

$$I_X = \frac{C_1}{C_2}\,\frac{G_m}{1+\frac{C_1}{C_2}+G_m R_{out}}\cdot V_{in}$$





$$\left[-G_m(V_{in}-V_1)+\frac{G_m\frac{C_1}{C_2}V_{in}}{1+\frac{C_1}{C_2}+G_mR_{out}}\right]\frac{R_{out}}{2}=V_{out}$$

$$V_1=\frac{V_{in}-V_{out}}{1+\frac{C_1}{C_2}}$$

$$\Rightarrow\frac{V_{out}}{V_{in}}=-\frac{C_1}{C_2}\frac{\frac{G_mR_{out}}{2}\left(2+2\frac{C_1}{C_2}+G_mR_{out}\right)}{\left(1+\frac{C_1}{C_2}\right)\left(1+\frac{C_1}{C_2}+G_mR_{out}\right)+\frac{G_mR_{out}}{2}\left(1+\frac{C_1}{C_2}+G_mR_{out}\right)}$$

$$=-\frac{C_1}{C_2}\frac{1+2\frac{1+\frac{C_1}{C_2}}{G_mR_{out}}}{1+3\frac{1+C_1/C_2}{G_mR_{out}}+2\left(\frac{1+C_1/C_2}{G_mR_{out}}\right)^2}$$

$$=-\frac{C_1}{C_2}\frac{1+2X}{(1+X)(1+2X)}\qquad X=\frac{1+C_1/C_2}{G_mR_{out}}$$

$$=-\frac{C_1}{C_2}\frac{1}{1+\frac{1+C_1/C_2}{G_mR_{out}}}$$

Interestingly, the gain error is the same. But if the Gm stage in the error amplifier has a very high output impedance, then the load resistor of the main amplifier is Rout rather than Rout/2 and

$$\frac{V_{out}}{V_{in}}=-\frac{C_1}{C_2}\frac{1+2\frac{1+C_1/C_2}{G_mR_{out}}}{1+2\frac{1+C_1/C_2}{G_mR_{out}}+\left(\frac{1+C_1/C_2}{G_mR_{out}}\right)^2}$$

$$=-\frac{C_1}{C_2}\frac{1}{1+\underbrace{\frac{(1+C_1/C_2)^2/(G_mR_{out})^2}{1+2\frac{1+C_1/C_2}{G_mR_{out}}}}_{\approx 1}}$$

$$\cong-\frac{C_1}{C_2}\frac{1}{1+\left(\frac{1+C_1/C_2}{G_mR_{out}}\right)^2},$$ as if the open-loop gain of the amplifier is squared.

Chapter 13

13.1 $y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) \quad x = [0 \;\; x_{max}]$

(a) Straight line passing through the end points:

$$y_1 = \frac{\alpha_1 x_{max} + \alpha_2 x_{max}^2}{x_{max}} \cdot x = (\alpha_1 + \alpha_2 x_{max})\, x$$

$$y(t) - y_1 = -\alpha_2 x_{max} \cdot x + \alpha_2 x^2$$

$\Rightarrow$ Error is maximum at $x = \frac{x_{max}}{2}$ and equal to $-\frac{\alpha_2 x_{max}^2}{4}$. This value is usually normalized to the maximum output level.

(b)
$$y(t) = \alpha_1 \frac{x_{max}\cos\omega t}{2} + \alpha_1 \frac{x_{max}}{2} + \frac{\alpha_2}{4} x_{max}^2 \underbrace{\cos^2\omega t}_{\frac{1+\cos 2\omega t}{2}} + \frac{\alpha_2}{4} 2 x_{max}^2 \cos\omega t + \frac{\alpha_2}{4} x_{max}^2$$

$\Rightarrow$ Fundamental: $\left(\frac{\alpha_1 x_{max}}{2} + \frac{\alpha_2}{2} x_{max}^2\right)\cos\omega t$

Second Harmonic: $\frac{\alpha_2}{8} x_{max}^2 \cos 2\omega t$

$$\Rightarrow \quad THD = \frac{\alpha_2^2 x_{max}^4 / 64}{\left(\frac{\alpha_1 x_{max}}{2} + \frac{\alpha_2}{2} x_{max}^2\right)^2}$$

$$= \frac{\alpha_2^2 x_{max}^2}{16(\alpha_1 + \alpha_2 x_{max})^2}$$

13.2 For Fig. 13.6 (a) : $\frac{A_{HD2}}{A_F} = \frac{V_m}{4(V_{GS} - V_{TH})}$ $\quad V_{GS} - V_{TH} = 356\,mV$

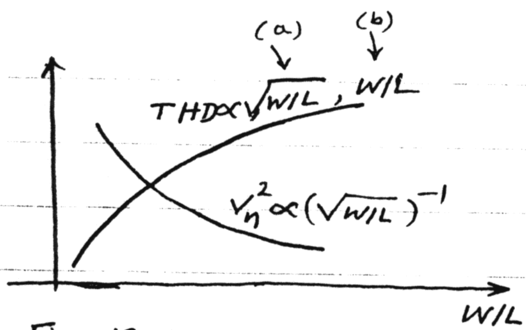
$\Rightarrow \frac{A_{H2}}{A_F} = 7\% \quad (-23\ dB)$

For Fig. 13.6 (b) : $\frac{A_{HD3}}{A_F} \approx \frac{V_m^2}{32(V_{GS} - V_{TH})^2}$

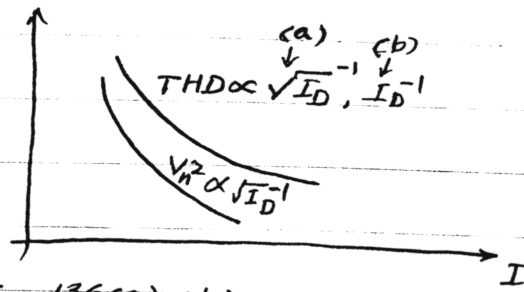
$= 0.25\% \quad (-52\ dB)$

- If we double W/L, $V_{GS} - V_{TH}$ is divided by $\sqrt{2}$.
  $\Rightarrow$ For (a), distortion goes up by $\sqrt{2}$ (3 dB) and for (b), by 2 (6 dB).
- If we double I, $V_{GS} - V_{TH}$ is multiplied by $\sqrt{2}$.
  $\Rightarrow$ Distortion goes down by $\sqrt{2}$ and 2 for (a) & (b), respectively.

13.3



Figs. 13.6 (a), (b)



Figs. 13.6(a), (b)

(Note that here THD is the ratio of voltages. If we take the ratio of powers, the relations must be squared.)

Increasing I and hence power dissipation decreases both the noise and the nonlinearity, whereas increasing W/L degrades the linearity while reducing the noise.

13.4 (1) As (W/L) is increased to increase the voltage gain, the linearity degrades (with a constant I).

(2) As I is increased to linearize the circuit, the load resistance must be decreased to maintain the same voltage headroom $\Rightarrow$ gain $\downarrow$.

13.5

$$\frac{b}{a} = \frac{\alpha_2}{2} V_m \frac{1}{\alpha_1} \frac{1}{(1+\beta\alpha_1)^2}$$

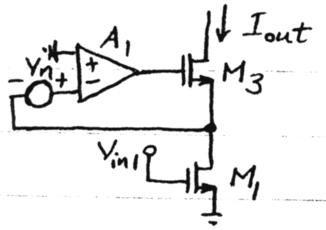
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS0}+V_m\cos\omega t - V_{TH})^2 \qquad V_{GS0}-V_{TH} = \text{overdrive with } I_D = 1\,mA$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[V_m^2\cos^2\omega t + 2V_m\cos\omega t\cdot(V_{GS0}-V_{TH}) + (V_{GS0}-V_{TH})^2\right]$$

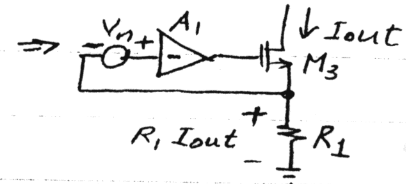
$$\Rightarrow \begin{cases} \left|\dfrac{\alpha_2}{\alpha_1}\right| = \dfrac{1}{2\cdot(V_{GS0}-V_{TH})} = 1.57\,V^{-1} \\ \alpha_1 = \mu_n C_{ox}\dfrac{W}{L}(V_{GS0}-V_{TH})R_D = 6286\,\mu A/V \times 2\,k\Omega = 12.57 \end{cases}$$

$$\Rightarrow \frac{b}{a} = 6.36\times 10^{-4}$$

13.6



$\Rightarrow$



$$R_1^{-1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{GS}-V_{TH}) - 2V_{DS}\right]$$

$$\left[(R_1 I_{out} + V_n)(-A_1) - R_1 I_{out}\right] g_{m3} = I_{out}$$

$$\Rightarrow I_{out} = \frac{-g_{m3} A_1}{1+g_{m3}R_1A_1+g_{m3}R_1} V_n \approx \frac{-A_1}{R_1(A_1+1)} V_n$$

$$\approx \frac{-1}{R_1} V_n$$

$$\Rightarrow |V_{n,in}| = \frac{\frac{1}{R_1} V_n}{g_{m1}}, \quad g_{m1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (2V_{DS})$$

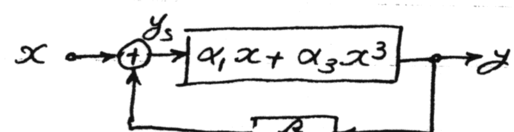
13.7 while increasing W/L raises the open-loop gain, it also makes the circuit more nonlinear (if I remains constant.) Since $\frac{W}{L}$ is multiplied by a factor of 4 $\Rightarrow \left|\frac{\alpha_2}{\alpha_1}\right| \uparrow$ by 2X, and $\alpha_1 \uparrow$ by 2X $\Rightarrow$

$$\frac{b}{a} = 4.32 \times 10^{-4}$$

13.8 $\quad \beta\alpha_1 = 5.03 \quad \Rightarrow \quad \frac{b}{a} \approx \frac{\alpha_2}{\alpha_1} \cdot \frac{V_m}{2} \cdot \frac{1}{\beta^2\alpha_1^2}$

$$= 3.1 \times 10^{-4}$$

13.9



Assume $y = a\cos\omega t + b\cos(3\omega t)$ and $x = V_m \cos\omega t$.

$$y_s = V_m \cos\omega t - \beta(a\cos\omega t + b\cos 3\omega t)$$

$$\Rightarrow y(t) = \alpha_1(V_m - \beta a)\cos\omega t - \alpha_1\beta b\cos 3\omega t + \alpha_3(V_m-\beta a)^3\cos^3\omega t$$
$$-\alpha_3\beta^3 b^3\cos^3 3\omega t - 3\alpha_3(V_m-\beta a)^2\cos^2\omega t \cdot \beta b\cos 3\omega t$$
$$+ 3\alpha_3(V_m-\beta a)\cos\omega t \cdot \beta^2 b^2\cos^2 3\omega t.$$

Neglecting higher order terms: $\quad a \approx \frac{\alpha_1}{1+\beta\alpha_1} V_m, \quad V_m - \beta a \approx \frac{a}{\alpha_1}$

$$b \approx -\alpha_1\beta b + \frac{\alpha_3}{4}(V_m - \beta a)^3$$

$$\Rightarrow \frac{b}{a} \approx \frac{1}{4} \frac{\alpha_3}{\alpha_1} \frac{V_m^2}{(1+\beta\alpha_1)^3}$$

13.10 $\quad I_D = I_0 \exp \frac{V_{GS}}{\xi V_T} \qquad V_{GS} = V_{GS0} + V_m \cos\omega t$

$$\Rightarrow I_D = \left(I_0 \exp \frac{V_{GS0}}{\xi V_T}\right)\left[1 + \frac{V_m \cos\omega t}{\xi V_T} + \frac{1}{2}\left(\frac{V_m \cos\omega t}{\xi V_T}\right)^2 + \cdots\right]$$

If $V_m \ll \xi V_T$, only second harmonic is significant: $\frac{1}{4}\left(\frac{V_m}{\xi V_T}\right)^2 \cos 2\omega t$.

For the differential pair, $I_{D1} + I_{D2} = I_{SS}$, and

$V_{in} - V_{GS1} + V_{GS2} = 0 \Rightarrow \quad V_{in} = \xi V_T \ln \frac{I_{D1}}{I_0} - \xi V_T \ln \frac{I_{D2}}{I_0}$

$$= \xi V_T \ln \frac{I_{D1}}{I_{D2}}$$

It follows that: $\quad I_{D1} = \frac{I_{SS} \exp[V_{in}/(\xi V_T)]}{1 + \exp[V_{in}/(\xi V_T)]}$

$$I_{D2} = \frac{I_{SS}}{1 + \exp[V_{in}/(\xi V_T)]}$$

Thus, $\quad I_{D1} - I_{D2} = I_{SS} \tanh \frac{V_{in}}{2\xi V_T} \qquad \tanh \varepsilon \approx \varepsilon - \frac{\varepsilon^3}{3}$

$$\approx I_{SS}\left[\frac{V_{in}}{2\xi V_T} - \left(\frac{V_{in}}{2\xi V_T}\right)^3\right]$$

If $V_{in} = V_{in0} + V_m \cos\omega t$, the third harmonic is given by $\quad -I_{SS} \frac{1}{(2\xi V_T)^3} V_m^3 \frac{1}{4} \cos 3\omega t$.

13.11 $\quad I_D = \frac{1}{2} \frac{\mu_0 C_{ox}}{1 + \theta(V_{GS} - V_{TH})} \frac{W}{L} (V_{GS} - V_{TH})^2$

$$\approx \frac{1}{2} \mu_0 C_{ox} \frac{W}{L} [1 - \theta(V_{GS} - V_{TH})] (V_{GS} - V_{TH})^2$$

If $V_{GS} = V_{GS0} + V_m \cos\omega t$, then the third harmonic is given by $\quad \frac{1}{2} \mu_0 C_{ox} \frac{W}{L} (-\theta) \frac{V_m^3}{4} \cos 3\omega t$.

13.12 (a) $\Delta V_{TH} = \dfrac{0.1\, t_{ox}}{\sqrt{WL}}$ $t_{ox} = 90\,\text{Å}$

$\Rightarrow W = 6.5\ \mu m$

(b) $THD = \dfrac{V_m^2}{32(V_{GS}-V_{TH})^2}$ $I_D = 1\,mA$ $\frac{W}{L} = \frac{6.5}{0.5}$

$\Rightarrow V_{GS}-V_{TH} = 1.07\ V$

$\Rightarrow V_{m,max} = 0.61\ V$

13.13 (a) $W = 6.5\ \mu m \times \left(\dfrac{5\ mV}{2\ mV}\right)^2 = 41\ \mu m.$

(b) $V_{GS}-V_{TH} = 1.07\,V \times \sqrt{\dfrac{6.5}{41}} = 0.426\ V$

$\Rightarrow V_{m,max} = 0.61 \times \sqrt{\dfrac{6.5}{41}} = 0.243\ V$

We see a trade-off between input offset and nonlinearity (if the channel length remains constant.)

13.14 $\left|\dfrac{\Delta I_D}{I_D}\right| = \dfrac{2\,\Delta V_{TH}}{V_{GS}-V_{TH}} = 0.02 \Rightarrow \Delta V_{TH} = 5\ mV = \dfrac{0.1 \times 90\text{Å}}{\sqrt{WL}}$

$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})^2 \Rightarrow \frac{W}{L} = 29.9$

$\Rightarrow \begin{cases} L = 0.033\ \mu m \\ W = 0.984 \end{cases}$ But if $L_{min} \approx 0.5\ \mu m \Rightarrow \dfrac{W}{L} = \dfrac{15\ \mu m}{0.5\ \mu m}$.

13.15 $I_D R_S + \sqrt{\dfrac{2 I_D}{\mu_n C_{ox} W/L}} + V_{TH} = V_b$

Take the total differential of both sides and substitute $g_m = \dfrac{2 I_D}{V_{GS}-V_{TH}}$. Then, the result is obtained.

13.16 $y_1(t) = \alpha_1 A \cos\omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t$

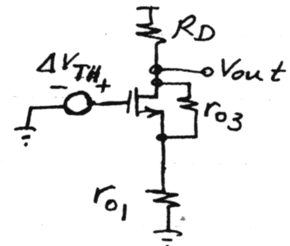
$y_2(t) = \alpha_1 A \cos(\omega t + \theta) + \alpha_2 A^2 \cos^2(\omega t + \theta) + \alpha_3 A^3 \cos^3(\omega t + \theta)$

The second harmonic arises from $\alpha_2 A^2 [\cos^2 \omega t - \cos^2(\omega t + \theta)]$

$$= \alpha_2 A^2 \frac{\cos(2\omega t) - \cos(2\omega t + 2\theta)}{2}$$

$$= \frac{\alpha_2 A^2}{4} \sin\theta \sin(2\omega t)$$

13.17 We calculate the output offset first. Viewing offset as noise, we have the following circuit:



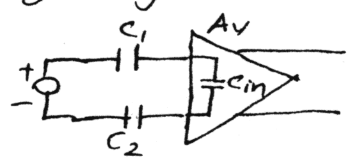
$$\Rightarrow V_{out} = \Delta V_{TH} \frac{-g_{m3} r_{O3} R_D}{R_D + r_{O1} + r_{O3} + g_{m3} r_{O3} r_{O1}}$$

This must be divided by the voltage gain, which for moderate R_D is given by $g_{m1} R_D \Rightarrow |V_{os,in}| \approx \dfrac{\frac{g_{m3}}{g_{m1}} r_{O3}}{R_D + g_{m3} r_{O3} r_{O1}}$.

If $R_D \to \infty$, $|V_{out}| \to \Delta V_{TH} \cdot g_{m3} r_{O3}$. The voltage gain is obtained from Eq. (3.119) as $\sim g_{m1} r_{O1} g_{m3} r_{O3} \Rightarrow$

$|V_{os,in}| \approx \dfrac{\Delta V_{TH}}{g_{m1} r_{O1}}$. This is why we usually neglect the offset contributed by cascode devices.

13.18 With a finite input capacitance, the gain of the circuit is no longer A_V.



$$A_{V,tot} = \frac{C_1}{C_1 + 2C_{in}} \cdot A_V$$

$$\Rightarrow V_{os,in} = \frac{V_{os}}{\frac{C_1}{C_1 + 2C_{in}} \cdot A_V}.$$

13.19 If W is doubled, the gain increases by approximately a factor of $\sqrt{2}$. Also, the input devices exhibit a smaller mismatch. For example, the threshold voltage mismatch decreases by a factor of $\sqrt{2}$. Thus, if the input devices dominate the offset, the overall input offset drops by a factor of 2.

13.20 To minimize the input offset, we maximize the overdrive of M_3 and M_4. But this limits the high level of the output swings.

Chapter 14

14.1

$$I_{D,scaled} = \frac{1}{2}\mu_n C_{ox}\frac{W/\alpha}{L/\alpha}\left(\frac{V_{GS}}{\alpha} - \frac{V_{TH}}{\alpha}\right)^2$$
$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\frac{1}{\alpha^2}$$

$$C_{ch,scaled} = \frac{W}{\alpha}\cdot\frac{L}{\alpha}C_{ox}$$
$$= \frac{1}{\alpha^2}WLC_{ox}$$

If the junction capacitances of S/D are neglected, then,

$$T_{d,scaled} = \frac{C/\alpha^2}{I/\alpha^2}\cdot\frac{V_{DD}}{\alpha}$$
$$= \left(\frac{C}{I}V_{DD}\right)\frac{1}{\alpha}.$$

Same as ideal scaling. But,

$$g_{m,scaled} = \mu C_{ox}\frac{W/\alpha}{L/\alpha}\frac{V_{GS}-V_{TH}}{\alpha}$$
$$= \mu C_{ox}\frac{W}{L}(V_{GS}-V_{TH})\cdot\frac{1}{\alpha}$$

14.2

$$W_{d,scaled} \approx \sqrt{\frac{2\varepsilon_{si}}{q}\Big(\underbrace{\frac{1}{N_A}}_{S/D} + \underbrace{\frac{1}{\alpha N_D}}_{Sub.}\Big)\frac{V_R}{\alpha}} \qquad N_A \gg \alpha N_D$$

$$\approx \frac{1}{\sqrt{\alpha}}\sqrt{\frac{2\varepsilon_{si}}{q}\frac{1}{N_A}V_R}$$

The depletion region capacitance per unit area therefore increases by $\sqrt{\alpha}$ rather than α. The series resistance increases.

DIBL arises primarily from the depletion region in the substrate rather than in the drain. Thus, DIBL remains relatively constant.

14.3 (a) Since $V_{n,rms} = \sqrt{\frac{kT}{C}}$, the capacitors must increase by a factor of 4.

(b) G_m should increase by a factor of 4.

(c) For square-law devices, W/L and bias current must increase by a factor of 4. $\Rightarrow$ Power increases by a factor of 2.

(d) $SR = I/C \Rightarrow$ Bias current must increase by a factor of 4.

14.4

$$I_D = \mu C_d \frac{W}{L} V_T^2 \left(\exp \frac{V_{GS}-V_{TH}}{\xi V_T}\right)\left(1-\exp \frac{-V_{DS}}{V_T}\right)$$

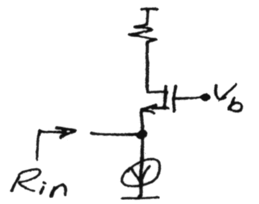
$C_d = \sqrt{\varepsilon_{si} q N_{sub}/(4\Phi_B)}$ $\quad N_{sub} \to \alpha N_{sub}$, $\Phi_B \sim$ constant.

$V_{GS}-V_{TH} \to \dfrac{V_{GS}-V_{TH}}{\alpha}$ $\qquad \xi = 1+\dfrac{C_d}{C_{ox}} \to 1+\dfrac{\sqrt{\alpha}}{\alpha}\dfrac{C_d}{C_{ox}}$

$V_{DS} \to \dfrac{V_{DS}}{\alpha}$

$S_{scaled} = 2.3 V_T \left(1+\dfrac{\sqrt{\alpha}\, C_d}{\alpha C_{ox}}\right) \Rightarrow S\downarrow$, i.e., subthreshold behavior improves.

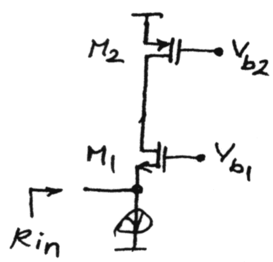
14.5



$$R_{in} = \frac{1}{\sqrt{\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})}} = 50\,\Omega$$

$$R_{in,scaled} = \frac{1}{\sqrt{\alpha \frac{\alpha}{\alpha}\frac{1}{\alpha}}} \times 50\,\Omega = 50\,\Omega$$

14.6



$$R_{in} = \frac{r_{o2}+r_{o1}}{1+(g_{m1}+g_{mb1})r_{o1}} \approx \frac{r_{o2}+r_{o1}}{(g_{m1}+g_{mb1})r_{o1}}$$

$$R_{in,scaled} \cong \frac{r_{o2}+r_{o1}}{(g_{m1}+g_{mb1,scaled})r_{o1}}$$

$$g_{mb1,scaled} = \frac{\gamma_{scaled}}{2\sqrt{2\Phi_F+\frac{V_{sub}}{\alpha}}}\, g_{m1} \qquad \gamma_{scaled} = \frac{\sqrt{2q\varepsilon\alpha N_{sub}}}{\alpha C_{ox}}$$

If $\dfrac{V_{sub}}{\alpha} \gg 2\Phi_F \Rightarrow g_{mb1,scaled} = g_{mb1}$.

14.7

$$\left.\frac{g_m}{I_D}\right|_{sat.} = \frac{\sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}}{I_D} = \sqrt{\frac{2\mu_n C_{ox}\frac{W}{L}}{I_D}}$$

$$\left.\frac{g_m}{I_D}\right|_{sub.} \cong \frac{\frac{I_D}{\xi V_T}}{I_D} = \frac{1}{\xi V_T}$$

The two are equal at $I_D = \dfrac{2\mu_n C_{ox}\frac{W}{L}}{(\xi V_T)^2}$.

14.4 $$I_D = \mu C_d \frac{W}{L} V_T^2 \left(\exp \frac{V_{GS}-V_{TH}}{\xi V_T}\right)\left(1-\exp \frac{-V_{DS}}{V_T}\right)$$

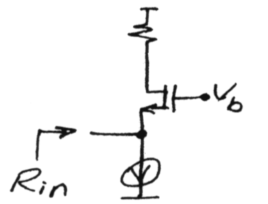
$C_d = \sqrt{\varepsilon_{si} q N_{sub}/(4\Phi_B)}$ $\quad N_{sub} \to \alpha N_{sub}$, $\Phi_B \sim$ constant.

$V_{GS}-V_{TH} \to \dfrac{V_{GS}-V_{TH}}{\alpha}$ $\qquad \xi = 1 + \dfrac{C_d}{C_{ox}} \to 1 + \dfrac{\sqrt{\alpha}}{\alpha}\dfrac{C_d}{C_{ox}}$

$V_{DS} \to \dfrac{V_{DS}}{\alpha}$

$S_{scaled} = 2.3 V_T \left(1+\dfrac{\sqrt{\alpha}\, C_d}{\alpha C_{ox}}\right) \Rightarrow S\downarrow$, i.e., subthreshold behavior improves.

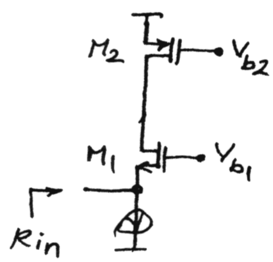
14.5



$$R_{in} = \frac{1}{\sqrt{\mu_n C_{ox} \frac{W}{L}(V_{GS}-V_{TH})}} = 50\,\Omega$$

$$R_{in,scaled} = \frac{1}{\sqrt{\alpha \frac{\alpha}{\alpha}\frac{1}{\alpha}}} \times 50\,\Omega = 50\,\Omega$$

14.6



$$R_{in} = \frac{r_{o2}+r_{o1}}{1+(g_{m1}+g_{mb1})r_{o1}} \approx \frac{r_{o2}+r_{o1}}{(g_{m1}+g_{mb1})r_{o1}}$$

$$R_{in,scaled} \cong \frac{r_{o2}+r_{o1}}{(g_{m1}+g_{mb1\,scaled})r_{o1}}$$

$$g_{mb1,scaled} = \frac{\gamma_{scaled}}{2\sqrt{2\Phi_F + \frac{V_{sub}}{\alpha}}}\, g_{m1} \qquad \gamma_{scaled} = \frac{\sqrt{2q\varepsilon\,\alpha N_{sub}}}{\alpha C_{ox}}$$

If $\dfrac{V_{sub}}{\alpha} \gg 2\Phi_F \Rightarrow g_{mb1,scaled} = g_{mb1}$.

14.7

$$\left.\frac{g_m}{I_D}\right|_{sat.} = \frac{\sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}}{I_D} = \sqrt{\frac{2\mu_n C_{ox}\frac{W}{L}}{I_D}}$$

$$\left.\frac{g_m}{I_D}\right|_{sub.} \cong \frac{\frac{I_D}{\xi V_T}}{I_D} = \frac{1}{\xi V_T}$$

The two are equal at $I_D = \dfrac{2\mu_n C_{ox}\frac{W}{L}}{(\xi V_T)^2}$.

14.8 Since $I = v \cdot Q$, if Q drops to zero, $v \to \infty$. But the velocity is limited to v_{sat}. Thus, at the pinch-off point, the charge density is not zero. Carriers reach their saturated velocity and shoot through the depletion region surrounding the drain.

14.9
$$I_D = \frac{1}{2} \frac{\mu_0 C_{ox}}{1+\theta(V_{GS}-V_{TH})} \frac{W}{L} (V_{GS}-V_{TH})^2$$

$$\frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2}\mu_0 C_{ox}\frac{W}{L}\left[\frac{-\theta(V_{GS}-V_{TH})^2}{[1+\theta(V_{GS}-V_{TH})]^2} + \frac{2(V_{GS}-V_{TH})}{1+\theta(V_{GS}-V_{TH})}\right]$$

$$= \mu_0 C_{ox}\frac{W}{L}\frac{V_{GS}-V_{TH}}{1+\theta(V_{GS}-V_{TH})}\left[1-\frac{\frac{\theta}{2}(V_{GS}-V_{TH})}{1+\theta(V_{GS}-V_{TH})}\right]$$

$$= \mu_0 C_{ox}\frac{W}{L}\frac{V_{GS}-V_{TH}}{1+\theta(V_{GS}-V_{TH})}\frac{1+\frac{\theta}{2}(V_{GS}-V_{TH})}{1+\theta(V_{GS}-V_{TH})}$$

$$= \frac{2I_D}{V_{GS}-V_{TH}} \cdot \frac{1+\frac{\theta}{2}(V_{GS}-V_{TH})}{1+\theta(V_{GS}-V_{TH})}$$

For small overdrives, $g_m \to \frac{2I_D}{V_{GS}-V_{TH}}$. For large overdrives, $g_m \to \frac{I_D}{V_{GS}-V_{TH}}$.

14.10 Using the results of Prob. 14.9 and replacing θ with $\frac{\mu_0}{2v_{sat}L}+\theta$, we have:

$$g_m = \frac{I_D}{V_{GS}-V_{TH}} \frac{2+\left(\frac{\mu_0}{2v_{sat}L}+\theta\right)(V_{GS}-V_{TH})}{1+\left(\frac{\mu_0}{2v_{sat}L}+\theta\right)(V_{GS}-V_{TH})}$$

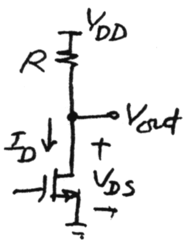
$$= \frac{I_D}{V_{GS}-V_{TH}}\left[1+\frac{1}{1+\left(\frac{\mu_0}{2v_{sat}L}+\theta\right)(V_{GS}-V_{TH})}\right]$$

14.11 $$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\left(1+\frac{\lambda}{1+kV_{DS}}V_{DS}\right)$$

$$r_o^{-1} = \frac{\partial I_D}{\partial V_{DS}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\frac{\lambda V_{DS}}{(1+kV_{DS})^2}$$

$$\approx I_D\frac{\lambda V_{DS}}{(1+kV_{DS})^2}$$

$$\Rightarrow r_o = \frac{1}{\lambda\frac{I_D V_{DS}}{(1+kV_{DS})^2}}$$



$$I_D \approx \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\left[1+\lambda V_{DS}(1-kV_{DS})\right] \quad \text{if } kV_{DS} \ll 1$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{DS}-\lambda k V_{DS}^2)$$

We note that the voltage across $R_D = V_{DD} - I_D R_D$ $= V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{DS}-\lambda k V_{DS}^2)R_D$. Thus, even if V_{GS} changed by a very small value, the nonlinear dependence on V_{DS} results in nonlinearity in the voltage across R_D.

14.12 (a) $g_m = v_{sat} W C_{ox} \Rightarrow |A_v| \approx v_{sat} W C_{ox}$

(b) $$|A_v| \approx \frac{g_{m1}}{g_{m3}} = \frac{v_{sat} W_1 C_{ox}}{v_{sat} W_3 C_{ox}} = \frac{W_1}{W_3}.$$

14.13 $$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \mu C_{ox}\frac{W}{L}\left(-\frac{2}{3}\gamma\right)\left(\frac{-\frac{3}{2}}{\sqrt{V_{DS}-V_{BS}+2\phi_F}} - \frac{-3/2}{\sqrt{-V_{BS}+2\phi_F}}\right)$$

$$= \mu C_{ox}\frac{W}{L}\,\gamma\,\frac{\sqrt{-V_{BS}+2\phi_F}-\sqrt{V_{DS}-V_{BS}+2\phi_F}}{\sqrt{V_{DS}-V_{BS}+2\phi_F}\sqrt{-V_{BS}+2\phi_F}}$$

14.14

$$\frac{\partial E_g}{\partial T} = -7.02\times10^{-4}\,\frac{2T(T+1108)-T^2}{(T+1108)^2}$$

$$= -7.02\times10^{-4}\,\frac{T^2+2216T}{(T+1108)^2}$$

For example, at $T=300\,^{\circ}K$, $\frac{\partial E_g}{\partial T} = -0.267\ \text{meV}/^{\circ}K$

For bandgap references, Eq. (11.10) must be modified:

$$\frac{\partial I_S}{\partial T} = b(4+m)T^{3+m}\exp\frac{-E_g}{kT} + bT^{4+m}\left(\exp\frac{E_g}{kT}\right)\left(\frac{E_g}{kT^2}-\frac{1}{kT}\frac{\partial E_g}{\partial T}\right)$$

$$\Rightarrow \frac{V_T}{I_S}\cdot\frac{\partial I_S}{\partial T} = (4+m)\frac{V_T}{T} + \left(\frac{E_g}{kT^2}-\frac{1}{kT}\frac{\partial E_g}{\partial T}\right)V_T$$

$$\Rightarrow \frac{\partial V_{BE}}{\partial T} = \frac{V_{BE}-(4+m)V_T}{T} - \left(\frac{E_g}{qT}-\frac{1}{q}\frac{\partial E_g}{\partial T}\right)$$

Thus, the TC of V_{BE} is slightly more positive.

14.15 (a) $|A_V| = \frac{g_{m1}}{g_{m2}} = \sqrt{\frac{\mu_n C_{ox}(\frac{W}{L})_1}{\mu_p C_{ox}(\frac{W}{L})_2}}$ $\Rightarrow |A_V|$ is highest for fast N, slow P, etc.

Input thermal noise voltage:

$$\overline{v_n^2} = 4kT\frac{2}{3g_{m1}} + 4kT\frac{2}{3}\frac{g_{m2}}{g_{m1}^2}$$ $\Rightarrow v_n$ is lowest for fast N, slow P, etc.

(b) $|A_V| = g_{m1}(r_{o1}\|r_{o2})$ $\Rightarrow |A_V|$ highest for fast N.

input noise: same as above.

14.16 (a) If V_{GS1} and V_{GS2} are constant $\Rightarrow g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})$

$$\Rightarrow |A_V| = \frac{\mu_n C_{ox}(\frac{W}{L})_1(V_{GS}-V_{TH})_1}{\mu_p C_{ox}(\frac{W}{L})_2(V_{GS}-V_{TH})_2}$$ $\rightarrow |A_V|$ highest for fast N, slow P, etc.

same result for thermal noise.

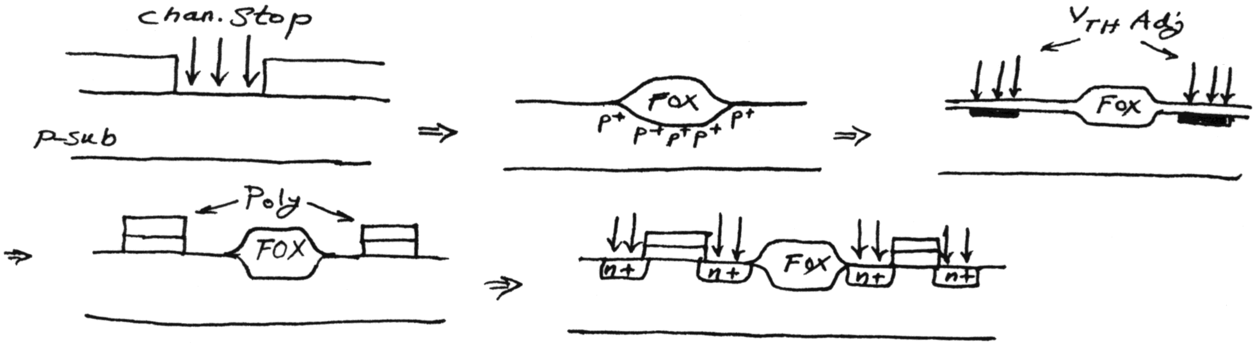
(b) $$|A_V| = \mu_n C_{ox}\left(\frac{W}{L}\right)_1(V_{GS}-V_{TH})_1\left[\frac{1}{\lambda_1\frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_1(V_{GS}-V_{TH})_1^2}\,\Big\|\,\frac{1}{\frac{\lambda_2}{2}\mu_p C_{ox}(\frac{W}{L})_2(V_{GS}-V_{TH})_2^2}\right]$$

$\Rightarrow |A|$ highest for fast N and slow P, etc.

Noise is as before.

Chapter 15

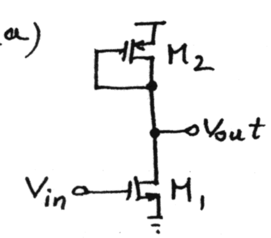
15.1 Simplifying the flow shown in Fig. 15.8, we note that n-well is not necessary.



The back-end processing is similar to that shown in Figs. 15.10 and 15.11. Thus, the process requires one fewer mask.

15.2 Since the dopants are not concentrated near the surface, their effect is less than expected. For example, if the implant aims to increase the threshold of an NFET from zero to 0.5 V, the actual value will be less than 0.5 V.

15.3 (a)



For M_1 in saturation:

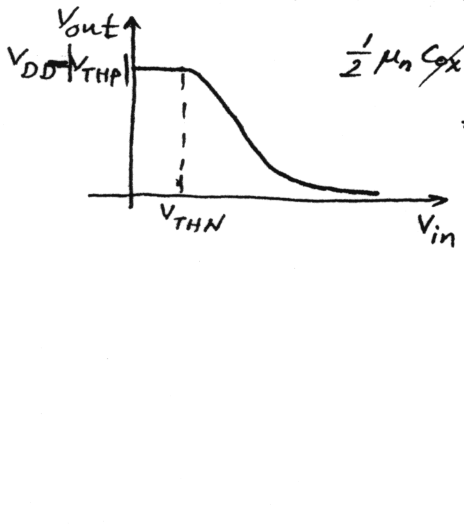
$$\frac{1}{2}\mu_n \cancel{C_{ox}}\left(\frac{W}{L}\right)_1 (V_{in}-V_{THN})^2 = \frac{1}{2}\mu_p \cancel{C_{ox}}\left(\frac{W}{L}\right)_2 (V_{out}-|V_{THP}|)^2$$

$\Rightarrow$ result independent of C_{ox}.

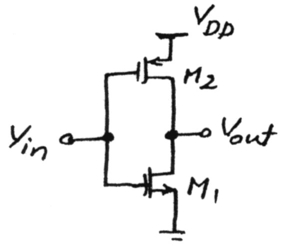
when M_1 enters the triode region:

$$\frac{1}{2}\mu_n \cancel{C_{ox}}\left(\frac{W}{L}\right)_1 \left[2(V_{in}-V_{THN})V_{out}-V_{out}^2\right] = \frac{1}{2}\mu_p \cancel{C_{ox}}\left(\frac{W}{L}\right)_2 (V_{out}-|V_{THP}|)^2$$

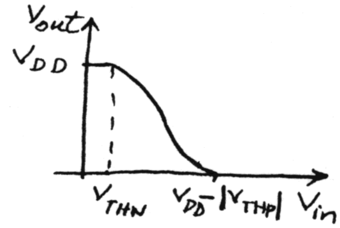
$\Rightarrow$ same.



(b)

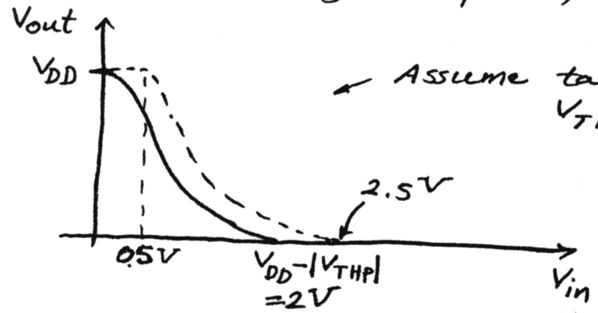


Using similar arguments, one can show that the result does not depend on C_{ox}.



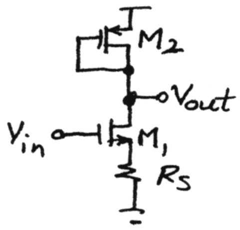
15.4 Without a threshold-adjust implant, $V_{THN} \approx 0$ and $V_{THP} \approx -1V$.

$V_{DD} = 3V$



Assume target values are: $V_{THN} = +0.5\ V$, $V_{THP} = -0.5\ V$

15.5

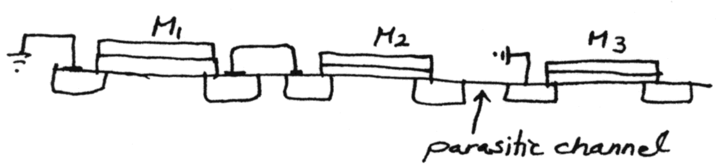


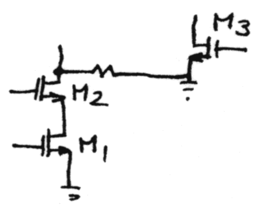
(a) Source of M_1 is spiked to the substrate, shorting R_S out.

(b) Drain of M_2 is spiked to its n-well.

15.6 (a) Channeling during S/D implant leads to deep junctions, intensifying DIBL. But the effect is not significant as far as the output impedance is concerned. (Just slightly lower.)

(b) With no channel-stop implant, it is possible that an unrelated high-voltage line passing over the field oxide between the transistors creates a channel between them.:

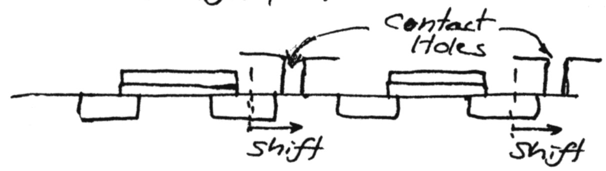




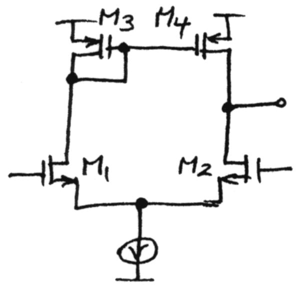
(c) Insufficient gate oxide growth typically does not degrade the output impedance.

15.7

The zero output current is probably caused by a contact misalignment.



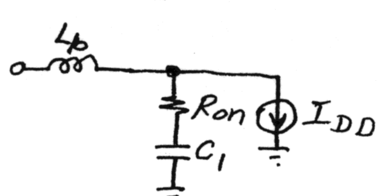
15.8



Long gate oxidation cycle is probably the reason: $A_v = g_{m1,2}(r_{O2} \| r_{O4})$, $g_{m1,2}$ is lower than expected. The output resistance remains constant or decreases as $t_{ox}\uparrow$.

15.9 (b) If the bottom plate of C_1 is heavily doped, then the oxide grows faster in C_1, leading to a smaller value for the capacitor. From Chapter 12, we note that if the input capacitance of the op amp is taken into account, then a lower value of C_1 yields a higher gain error.

15.10 (a) $R_{on} = \left[\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS}-V_{TH})\right]^{-1} = 11\ \Omega$



$C_1 = 100 \times 0.34 \times 3.84 + 100 \times 2 \times 0.4$

$= 210.6\ fF$

For critically-damped response: $R_{on} = 2\sqrt{\frac{L_b}{C_1}} \Rightarrow$
$L_b \leq 6.37\ pH$.

15.11 $g_m r \dfrac{N(N+1)}{2} = 0.01$

$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) = 1/(254\,\Omega)$

$\Rightarrow \quad r = 4.8\ m\Omega.$

15.12 $t = 1\ \mu m$, $h = 3\ \mu m$ Parallel Plate $\propto \frac{W}{h}$, the remaining terms determine the fringe capacitance:

$\frac{W}{3} = 0.77 + 1.06 (\frac{W}{3})^{0.25} + 1.06 (\frac{1}{3})^{0.5}$

$\Rightarrow W \approx 8.25\ \mu m$

If $h = 5\ \mu m$, then:

$\frac{W}{8} = 0.77 + 1.06 (\frac{W}{8})^{0.25} + 1.06 (\frac{1}{8})^{0.5}$

$\Rightarrow W \approx 19.7\ \mu m$

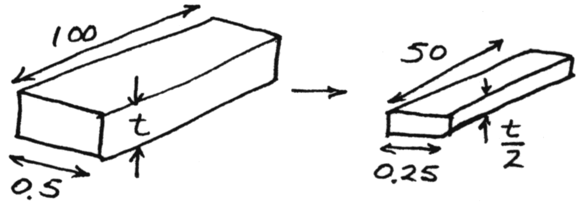
Chapter 16

16.1 $R_{\square,poly} = 30\ \Omega/\square \quad R_{\square,M1} = 80\ m\Omega/\square$

$$R_\square = \frac{\rho}{t} \Rightarrow \frac{\rho_{poly}}{\rho_{M1}} = \frac{R_{\square,poly} \times t_{poly}}{R_{\square,M_1} \times t_{M_1}} = \frac{30 \times 0.2}{0.08 \times 1.0}$$

$$= 75$$

16.2 $\frac{W}{L} = \frac{100}{0.5} \rightarrow \frac{50}{0.25}$



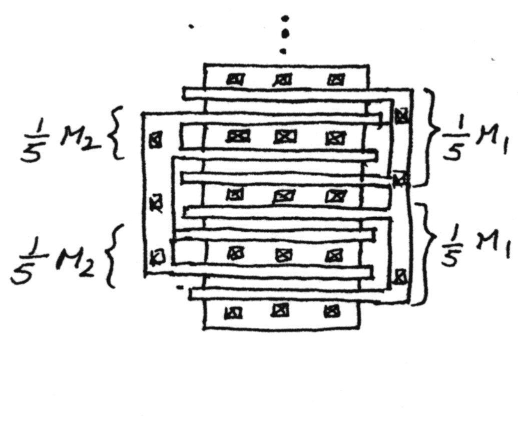
The sheet resistivity increases by a factor of 2. Since the number of squares is constant, the total gate resistance also increases by a factor of 2.

16.3 For a total gate resistance of $10\ \Omega$, suppose each device consists of N fingers each $\frac{100\ \mu m}{N}$ wide. The total gate resistance is then equal to $R_G = \left(\frac{200}{N}\right) \cdot \frac{1}{N} \cdot (5\ \Omega/\square)$

$$= \frac{1000}{N^2}\ \Omega$$

$$\Rightarrow N = 10$$

From Fig. 16.13(c), a possible solution is:



16.4. A_1: a finite resistance may appear between the drains, degrading the voltage gain.

A_2: a large resistance may appear in series with the sources, introducing unwanted degeneration or, more importantly, input-referred offset.

A_3: Gate of NMOS current source on the bottom may be shorted to its source.

A_4: Part of contact hole may fall on FOX, increasing the contact resistance: source degeneration or offsets.

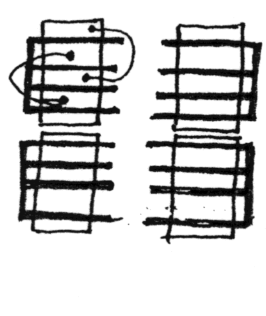
A_5: If the poly contact area is too close to the active area, the active area may be damaged during the etching of poly $\Rightarrow$ offsets, even poor transistor operation.

A_6: Latch-up may occur.

A_7: Latch-up may occur.

A_8: A finite resistance may appear between the gates of the input transistors.

16.5. In principle, only two layers of interconnect are sufficient for any routing. However, for reasonable symmetry, interconnect resistance, and area, approximately four layers are needed here.



16.6 In Fig. 6.22, temp. gradients introduce threshold and mobility mismatch between M_{REF} and each of $M_1 - M_n$. Thus, the output currents suffer from additional mismatches.

In Fig. 6.23, temp. gradients have much less effect because M_{REF1} and M_{REF2} are quite close to their mirrors.

16.7 $R = 500\ \Omega \Rightarrow$ poly must be $\frac{500}{60}$ squares long and

n-well must be $\frac{500}{2000}$ square long.

Poly width $= 3\ \mu m \Rightarrow$ Poly length $= 25\ \mu m$

n-well length $= 6\ \mu m \Rightarrow$ n-well width $= 24\ \mu m$.

$\Rightarrow$ Poly Cap $= 3 \times 25 \times 100\ aF/\mu m^2 = 7.5\ fF$

n-well Cap $= 6 \times 24 \times 1000\ aF/\mu m^2 = 144\ fF$.

Thus, the poly structure is preferable.

16.8 Assuming $C_1 = C_2 = C_3 = 40\ aF/\mu m^2$ and $C_4 = 60\ aF/\mu m^2$ in Fig. 16.34(d), we have (neglecting fringe cap.):

Fig. 16.34 (a): $C_1 = 40\ aF/\mu m^2$, $C_p = 9\ aF/\mu m^2$

(b): $C_1 + C_2 = 80\ aF/\mu m^2$, $C_p = 15\ aF/\mu m^2$

(c): $C_1 + C_2 + C_3 = 120\ aF/\mu m^2$, $C_p = 30\ aF/\mu m^2$

(d): $C_1 + \cdots + C_4 = 180\ aF/\mu m^2$, $C_p = 90\ aF/\mu m^2$

Thus, the lowest C_p/C occurs for (b).

16.9 Wire Propagation Delay $\approx \dfrac{R_{tot} C_{tot}}{2} = \dfrac{40 \times 37\ fF}{2}$

$= 0.74\ ps$

Lumped Delay $\approx 500\ \Omega \times 37\ fF$

$= 18.5\ ps$

Thus, the propagation delay thru the wire is negligible.

16.10 Wire Delay $\approx \dfrac{20\,\Omega \times 44\,aF}{2}$

$= 0.44\ ps$

Lumped Delay $\approx 22\ ps$

16.11 Metal 1 : $C_{tot} = (1000\,\mu m \times 0.35\,\mu m \times 30\,aF/\mu m^2) + 1000\,\mu m \times 80\,aF/\mu m$

$= 90.5\ fF$

Metal 2 : $C_{tot} = (1000\,\mu m \times 0.45\,\mu m \times 15\,aF/\mu m^2) + 1000\,\mu m \times 50\,aF/\mu m$

$= 56.75\ fF$

Metal 3 : $C_{tot} = (1000\,\mu m \times 0.5\,\mu m \times 9\,aF/\mu m^2) + 1000\,\mu m \times 40\,aF/\mu m$

$= 44.5\ fF$

Metal 4 : $C_{tot} = (1000\,\mu m \times 0.6\,\mu m \times 7\,aF/\mu m^2) + 1000\,\mu m \times 30\,aF/\mu m$

$= 34.2\ fF$

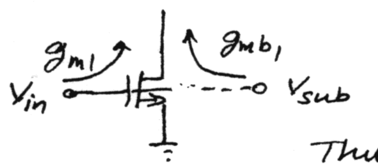
Thus, metal 4 provides the smallest delay.

16.12 The results do not change because the capacitance of metal 4 is still largest.

16.13 $(W/L)_1 = 100/0.5$, $I_{D1} = 1mA \Rightarrow g_{m1} = \sqrt{2 \times 1mA \times \dfrac{100}{0.34} \times 134\,\mu A/V^2}$

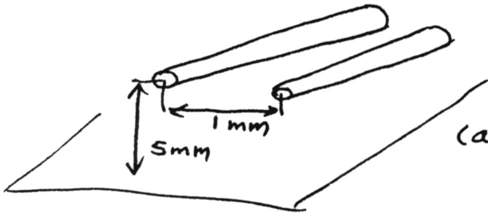
$= 8.88\ m℧$

$$g_{mb1} = \frac{\gamma\, g_{m1}}{2\sqrt{\underbrace{V_{SB}}_{0} + |2\phi_F|}} = \frac{0.45}{2\sqrt{0.9}} \times 8.88\,m℧ = 2.11\ m℧$$



V_{sub} generates a drain current of $g_{mb1}V_{sub}$. Thus, referred to the gate, the effect is equal to $\dfrac{g_{mb1}}{g_{m1}} = \dfrac{\gamma}{2\sqrt{|2\phi_F|}} = 4.21^{-1} \Rightarrow$ input-referred noise $= 11.9\ mV_{pp}$.

16.14.



(a) $L_m = 0.1 \ln\left[1 + \left(\frac{10}{1}\right)^2\right] \times 4\,mm$

$= 1.85\ nH.$

(b) $V = L_m \frac{di}{dt}$

$= 1.85\,nH \times 2\pi \times 10^8 \times 1\,mA$

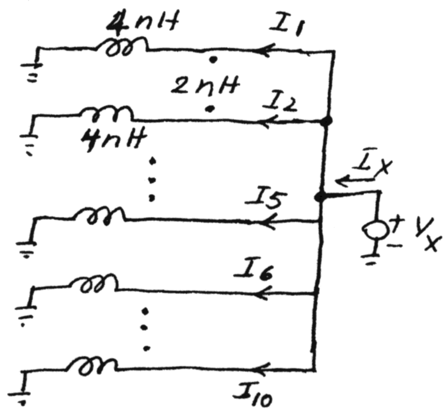
$= 1.16\ mV_p.$

16.15 L_m must decrease by a factor of 4. $\Rightarrow$

$0.1 \ln\left[1 + \left(\frac{2h}{d}\right)^2\right] \times 4\,mm = \frac{1.85}{4}$

$\Rightarrow \frac{2h}{d} = 1.476 \Rightarrow d = 6.78\ mm.$

16.16 (a)



By symmetry:

$I_1 = I_{10},\ I_2 = I_9, \cdots, I_5 = I_6$

we then construct equations for

$I_1 - I_5$

$$\begin{cases} (4\,nH)sI_1 + (2\,nH)sI_2 = V_X \\ (4\,nH)sI_2 + (2\,nH)sI_1 + (2\,nH)sI_3 = V_X \\ \vdots \end{cases}$$

$\Rightarrow I_1 = \frac{5}{22}\frac{V_X}{s},\ I_2 = \frac{1}{22}\frac{V_X}{s},\ I_3 = \frac{4}{22}\frac{V_X}{s},\ I_4 = \frac{2}{22}\frac{V_X}{s},\ I_5 = \frac{3}{22}\frac{V_X}{s}$

$I_X = 2(I_1 + \cdots + I_5) \Rightarrow L_{eq} = \frac{22}{30}\,nH$ for each of ground and V_{DD} lines.

16.17 (a) $L_a = 0.2 \ln \frac{2h}{25\mu m}$ nH $\quad C_a = 100^2 C_0$

(b) $L_b = 0.2 \ln \frac{2h}{12.5\mu m}$ nH $\quad C_b = 50^2 C_0$

$$\frac{L_a C_a}{L_b C_b} = \frac{\ln \frac{2h}{25}}{\ln \frac{2h}{12.5}} \cdot \frac{4}{1}$$

Is the first fraction greater or less than 1/4?

$$\frac{\ln \frac{2h}{25}}{\ln \frac{2h}{12.5}} = \frac{1}{4} \Rightarrow h \approx 15.7\ \mu m$$

Thus, for $h > 15.7\ \mu m$ (which is quite realistic), case b is certainly preferable. For $h \ll 15.7\ \mu m$, case (a) may be preferable.

Design of Analog CMOS Integrated Circuits

Behzad Razavi

Errata in Problem Sets

Chapter 2

- In Eq. (2.44), μ_n must be in the numerator.

Chapter 3

- Call the third problem 3.2'.
- In Problem 3.2, Fig. 3.68(d), change the gate voltage of M_2 to V_{b2}.
- In Problem 3.4, Fig. 3.71(a), change the gate voltage of $M_=1$ to V_{b1}.
- In Fig. 3.72(e), V_{b1} must be changed to V_{in}.
- In Fig. 3.73(h), the output is at the source of M_2.
- In Problem 3.10(c), the question must be phrased as: Which device enters the triode region first as V_{out} falls?
- In Problem 3.13, first sentence should read: ... with $W/L = 50/0.5$ ...
- In Problem 3.16(a), do not neglect channel-length modulation in the triode region.

Chapter 4

- In Problem 4.2, assume $I_{SS} = 1$ mA and change part (a) to: Determine the voltage gain.
- In Problem 4.6, assume $\lambda = 0$.
- In Problem 4.9, assume $\lambda = \gamma = 0$.
- In Problem 4.11, assume $I_{D5} = 20\ \mu$A.
- In Problem 4.13, change the figure number to 4.8(a).

Chapter 5

- In Problem 5.16(d), assume V_{TH} does not vary with temperature.

Chapter 6

- In Problem 6.4(b) and (d), assume $\lambda \neq 0$.

Chapter 7

- The second sentence of Problem 7.2 should read: Assume $(W/L)_1 = 50/0.5$, $I_{D1} = I_{D2} = 0.1$ mA ...
- In Problem 7.20, change I_{D1} and I_{D2} to 0.05 mA.
- In Problem 7.24, change the bias current to 0.1 mA.

Chapter 8

- In Problem 8.10, change the tolerable gain error to 5%.
- In Problem 8.15, Fig. 8.55(b), call label the top G_m block G_{m2}. The output is at the output nodes of G_{m2}.

Chapter 10

- In Problem 10.11, change I_{SS} to 0.25 mA and $(W/L)_{5,6}$ to 60/0.5.
- In Problem 10.12, add: Maximize $V_{GS14} = V_{GS15}$ while leaving at least 0.5 V across I_1. Also, in part (b), change M_2 to M_1.
- Problem 10.17 should read: ... between the gate and the drain of M_2 or M_3.
- In Fig. 10.42, change the gate voltage of $M_{3,4}$ to V_{b1}.
- In Problem 10.19(c), change A_0 in the numerator to A.

Chapter 11

- In Problem 11.13, ... such that the circuit operates with $V_{DD} = 3$ V.
- In Problems 11.17 and 11.18, the top terminal of R_2 should be connected to the top terminal of R_1.
- In Problem 11.22, assume $K = 4$.

Chapter 12

- In Problem 12.8, assume $C_H = 1$ pF.
- In Problem 12.12, assume all switches are NMOS devices.
- In Problem 12.14, assume $C_{in} = 0.2$ pF and calculate C_1 and C_2.
- In Problem 12.16, the output is sensed at the drains of M_1 and M_2.

Chapter 13

- In Problem 13.5, change the figure number to 13.6(a).