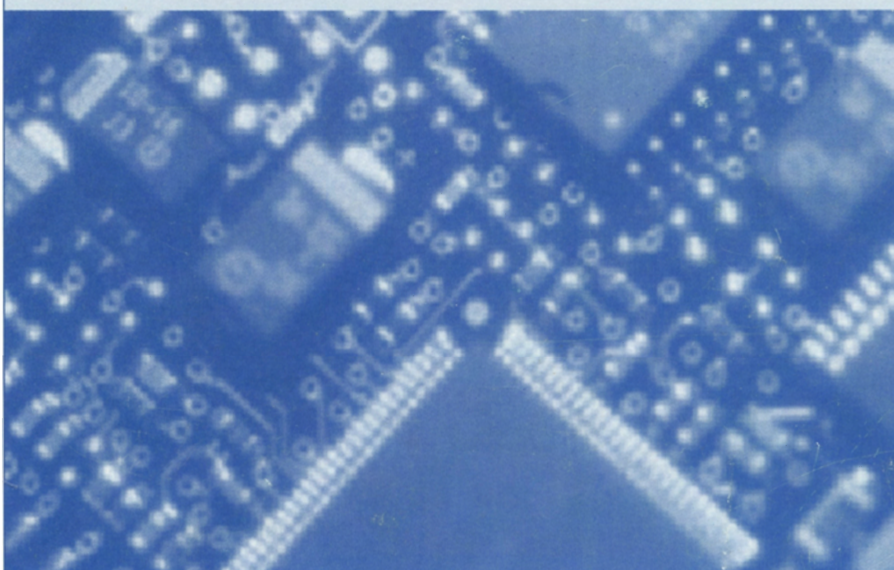
Solutions Manual

to accompany

Electronic Circuit Analysis and Design

Second Edition



Donald A. Neamen

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Donald A. Neamen
University of New Mexico

McGraw-Hill

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ELECTRONIC CIRCUIT ANALYSIS AND DESIGN
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Exercise Solutions

E1.1

$$n_i = BT^{3/2}\exp\left(-\frac{E_g}{2kT}\right)$$

GaAs :

$$n_i = (2.1\times10^{14})(300)^{3/2}\exp\left(\frac{-1.4}{2(86\times10^{-6})(300)}\right)$$

$\underline{n_i = 1.8\times10^{6}\ \text{cm}^{-3}}$

Ge :

$$n_i = (1.66\times10^{15})(300)^{3/2}\exp\left(\frac{-0.66}{2(86\times10^{-6})(300)}\right)$$

$\underline{n_i = 2.40\times10^{13}\ \text{cm}^{-3}}$

E1.2

Si :

$$n_i = (5.23\times10^{15})(400)^{3/2}\exp\left(\frac{-1.1}{2(86\times10^{-6})(400)}\right)$$

$\underline{n_i = 4.76\times10^{12}\ \text{cm}^{-3}}$

GaAs :

$$n_i = (2.1\times10^{14})(400)^{3/2}\exp\left(\frac{-1.4}{2(86\times10^{-6})(400)}\right)$$

$\underline{n_i = 2.44\times10^{9}\ \text{cm}^{-3}}$

Ge :

$$n_i = (1.66\times10^{15})(400)^{3/2}\exp\left(\frac{-0.66}{2(86\times10^{-6})(400)}\right)$$

$\underline{n_i = 9.06\times10^{14}\ \text{cm}^{-3}}$

E1.3

a. majority carrier: $\underline{p_0 = 10^{17}\ \text{cm}^{-3}}$
minority carrier:

$$n_0 = \frac{n_i^2}{N_a} = \frac{(1.5\times10^{10})^2}{10^{17}} = 2.25\times10^{3}\ \text{cm}^{-3}$$

b. $n_0 = N_d - N_a = 5\times10^{15}$

majority carrier: $\underline{n_0 = 5\times10^{15}\ \text{cm}^{-3}}$

minority carrier:

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5\times10^{10})^2}{5\times10^{15}} = 4.5\times10^{4}\ \text{cm}^{-3}$$

E1.4

(a) $n_o = 5x10^{16}\ cm^{-3},\quad p_o << n_o$

$\sigma \cong e\mu_n n_o = (1.6x10^{-19})(1350)(5x10^{16}) \Rightarrow$

$\underline{\sigma = 10.8\,(\Omega-cm)^{-1}}$

(b) $p_o = 5x10^{16}\ cm^{-3},\quad n_o << p_o$

$\sigma \cong e\mu_p p_o = (1.6x10^{-19})(480)(5x10^{16}) \Rightarrow$

$\underline{\sigma = 3.84\,(\Omega-cm)^{-1}}$

E1.5

a. $n_0 = N_d = 8\times10^{15}\ \text{cm}^{-3}$

$$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5\times10^{10})^2}{8\times10^{15}} \Rightarrow$$

$\underline{p_0 = 2.81\times10^{4}\ \text{cm}^{-3}}$

b. $n = n_0 + \delta n = 8\times10^{15} + 0.1\times10^{15} \Rightarrow$

$\underline{n = 8.1\times10^{15}\ \text{cm}^{-3}}$

$p = p_0 + \delta p \Rightarrow$

$\underline{p \approx 10^{14}\ \text{cm}^{-3}}$

E1.6

$J = \sigma E = (10)(15) \Rightarrow$

$\underline{J = 150\text{A}/\text{cm}^2}$

E1.7

a. $V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$V_{bi} = (0.026)\ln\left[\frac{(10^{15})(10^{17})}{(1.5\times10^{10})^2}\right]$$

$\underline{V_{bi} = 0.697\ \text{V}}$

b. $$V_{bi} = (0.026)\ln\left[\frac{(10^{17})(10^{17})}{(1.5\times10^{10})^2}\right]$$

$\underline{V_{bi} = 0.817\ \text{V}}$

E1.8

$$V_{bi} = V_T \ln\left[\frac{N_a N_d}{n_i^2}\right] = (0.026)\ln\left[\frac{(10^{6})(10^{17})}{(1.8\times 10^{6})^2}\right]$$

$\underline{V_{bi} = 1.23\ \text{V}}$

E1.9

$$C_j = C_{j0}\left(1+\frac{V_R}{V_{bi}}\right)^{-1/2}$$

$$V_{bi} = V_T \ln\left[\frac{N_a N_d}{n_i^2}\right]$$

$$= (0.026)\ln\left[\frac{(10^{17})(10^{16})}{(1.5\times 10^{10})^2}\right] = 0.757\ \text{V}$$

$$0.8 = C_{j0}\left(1+\frac{5}{0.757}\right)^{-1/2} = C_{j0}(7.61)^{-1/2}$$

$$= C_{j0}(0.362)$$

$\underline{C_{j0} = 2.21\ \text{pF}}$

E1.10

a. $I = I_S\left[\exp\left(\frac{V_D}{V_T}\right)-1\right]$

For $V_D = 0.5$: $I \cong 10^{-14}\exp\left(\frac{0.5}{0.026}\right)$

For $V_D = 0.6$: $I = 10^{-14}\exp\left(\frac{0.6}{0.026}\right)$

For $V_D = 0.7$: $I = 10^{-14}\exp\left(\frac{0.7}{0.026}\right)$

So we have

V_D	I
0.5	2.25 μA
0.6	0.105 mA
0.7	4.93 mA

b. 10^{-14} A both cases

E1.11

$$I = I_S\left[\exp\left(\frac{V_D}{V_T}\right)-1\right]$$

$$10^{-3} = (10^{-13})\left[\exp\left(\frac{V_D}{0.026}\right)-1\right]$$

$$(0.026)\ln(10^{10}) \cong V_D$$

$\underline{V_D = 0.599\ \text{V}}$

E1.12

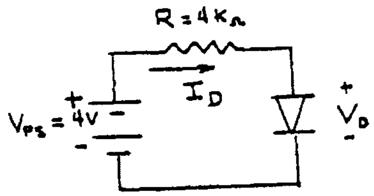
$\Delta T = 100°\text{C}$

$\Delta V_D \approx 2\times 100 = 200\ \text{mV}$

$\Rightarrow V_D = 0.650 - 0.2$

$\Rightarrow \underline{V_D = 0.450\ \text{V}}$

E1.13



$I_S = 10^{-12}$ A

$V_{PS} = I_D R + V_D$ and $I_D \cong I_S \exp\left(\frac{V_D}{V_T}\right)$

So

$4 = I_D(4) + V_D \Rightarrow I_D = (4 - V_D)/4$ mA

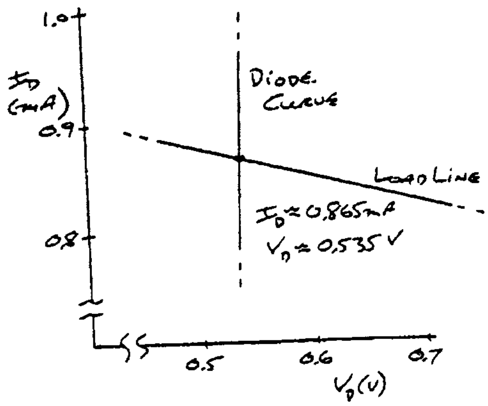
and

$I_D \cong I_S \exp\left(\frac{V_D}{V_T}\right) \Rightarrow I_D = 10^{-9}\exp\left(\frac{V_D}{0.026}\right)$ mA

By trial and error:

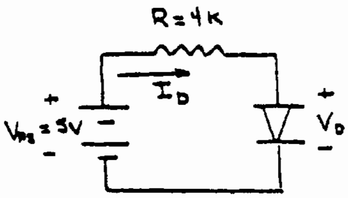
$I_D = 0.864$ mA and $V_D = 0.535$ V

E1.14



E1.15

a.

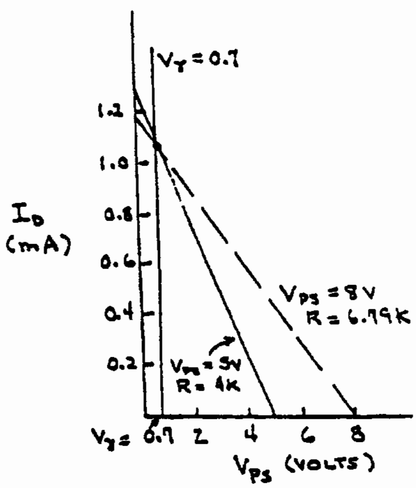


$$I_D = \frac{V_{PS} - V_\gamma}{R} = \frac{5 - 0.7}{4} \Rightarrow \underline{I_D = 1.08 \text{ mA}}$$

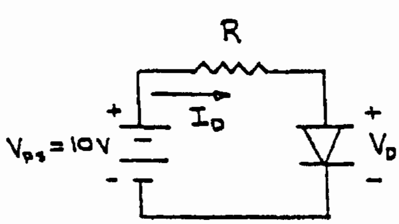
b. $$I_D = \frac{V_{PS} - V_\gamma}{R} \Rightarrow R = \frac{V_{PS} - V_\gamma}{I_D}$$

$$R = \frac{8 - 0.7}{1.075} \Rightarrow \underline{R = 6.79 \text{ k}\Omega}$$

c.



E1.16



Power dissipation in diode = $I_D V_D$

$1.05 \text{ mW} = I_D(0.7) \Rightarrow \underline{I_D = 1.5 \text{ mA}}$

$$R = \frac{V_{PS} - V_\gamma}{I_D} = \frac{10 - 0.7}{1.5} \Rightarrow \underline{R = 6.2 \text{ k}\Omega}$$

E1.17

$$g_d = \frac{I_D}{V_T} = \frac{0.8}{0.026} = \underline{30.8 \text{ mS}}$$

E1.18

$$r_d = \frac{V_T}{I_D} \Rightarrow 50 = \frac{0.026}{I_D} \Rightarrow I_D = \frac{0.026}{50}$$

$\underline{I_D = 0.52 \text{ mA}}$

E1.19

$$I_D \cong I_S \exp\left(\frac{V_D}{V_T}\right)$$

$$V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$$

pn junction: $V_D = (0.026) \ln\left(\frac{10^{-3}}{10^{-12}}\right)$

$$\underline{V_D = 0.539 \text{ V}}$$

Schottky Diode: $V_D = (0.026) \ln\left(\frac{10^{-3}}{10^{-8}}\right)$

$$\underline{V_D = 0.299 \text{ V}}$$

E1.20

For the *pn* junction diode

$$V_D = V_T \ln\left(\frac{I_D}{I_S}\right) = (0.026) \ln\left(\frac{1.2 \times 10^{-3}}{4 \times 10^{-15}}\right)$$

$V_D = 0.6871 \text{ V}$

Schottky diode voltage will be smaller

$\Rightarrow V_D = 0.6871 - 0.265 = 0.4221 \text{ V}$

$$I_D = I_S \exp\left(\frac{V_D}{V_T}\right)$$

$$I_S = \frac{1.2 \times 10^{-3}}{\exp\left(\frac{0.4221}{0.026}\right)} \Rightarrow \underline{I_S = 1.07 \times 10^{-10} \text{ A}}$$

E1.21

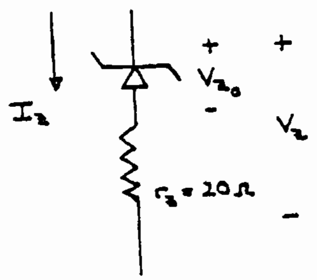
Power $= I \cdot V_Z$

$10 = I(5.6) \Rightarrow I = 1.79$ mA

$I = \dfrac{10 - 5.6}{R} = 1.79$

$R = \dfrac{10 - 5.6}{1.79} \Rightarrow \underline{R = 2.46\ \text{k}\Omega}$

E1.22



$V_Z = V_{Z0} + I_Z r_Z$

So $V_{Z0} = V_Z - I_Z r_Z$

$V_{Z0} = 5.20 - (10^{-3})(20) = 5.20 - 0.02 = 5.18$

Then

$V_Z = 5.18 + (10 \times 10^{-3})(20) \Rightarrow \underline{V_Z = 5.38\ \text{V}}$

Chapter 1

Problem Solutions

1.1

(a) $n_i = BT^{3/2}e^{-E_g/2kT}$

(i) Silicon, T=275K

$n_i = (5.23x10^{15})(275)^{3/2} e^{-1.1/2(86x10^{-6})(275)}$

$n_i = 1.90x10^9\ cm^{-3}$

(ii) T=325K

$n_i = (5.23x10^{15})(325)^{3/2} e^{-1.1/2(86x10^{-6})(325)}$

$n_i = 8.71x10^{10}\ cm^{-3}$

(b) GaAs

(i) T=275K

$n_i = (2.1x10^{14})(275)^{3/2} e^{-1.4/2(86x10^{-6})(275)}$

$n_i = 1.34x10^5\ cm^{-3}$

(ii) T=325K

$n_i = (2.10x10^{14})(325)^{3/2} e^{-1.4/2(86x10^{-6})(325)}$

$n_i = 1.63x10^7\ cm^{-3}$

1.2

a. $n_i = BT^{3/2}\exp\left(\frac{-Eg}{2kT}\right)$

$10^{12} = 5.23 \times 10^{15} T^{3/2}\exp\left(\frac{-1.1}{2(86\times10^{-6})(T)}\right)$

$1.91 \times 10^{-4} = T^{3/2}\exp\left(-\frac{6.40\times10^3}{T}\right)$

By trial and error, $T \approx 367°$ K

b. $n_i = 10^9\ cm^{-3}$

$10^9 = 5.23 \times 10^{15} T^{3/2}\exp\left(\frac{-1.1}{2(86\times10^{-6})(T)}\right)$

$1.91 \times 10^{-7} = T^{3/2}\exp\left(-\frac{6.40\times10^3}{T}\right)$

By trial and error, $T \approx 268°$ K

1.3

a. $N_d = 5 \times 10^{15}\ cm^{-3} \Rightarrow \underline{n-type}$

$\underline{n_0 = N_d = 5 \times 10^{15}\ cm^{-3}}$

$p_0 = \frac{n_i^2}{n_0} = \frac{(1.5\times10^{10})^2}{5\times10^{15}} \Rightarrow \underline{p_0 = 4.5 \times 10^4\ cm^{-3}}$

b. $N_d = 5 \times 10^{15}\ cm^{-3} \Rightarrow \underline{n-type}$

$\underline{n_0 = N_d = 5 \times 10^{15}\ cm^{-3}}$

$n_i = (2.10\times10^{14})(300)^{3/2}\exp\left(\frac{-1.4}{2(86\times10^{-6})(300)}\right)$

$= (2.10\times10^{14})(300)^{3/2}(1.65\times10^{-12})$

$= 1.80 \times 10^6\ cm^{-3}$

$p_0 = \frac{n_i^2}{n_0} = \frac{(1.8\times10^6)^2}{5\times10^{15}} \Rightarrow \underline{p_0 = 6.48 \times 10^{-4}\ cm^{-3}}$

1.4

a. $N_a = 10^{16}\ cm^{-3} \Rightarrow \underline{p-type}$

$\underline{p_0 = N_a = 10^{16}\ cm^{-3}}$

$n_0 = \frac{n_i^2}{p_0} = \frac{(1.5\times10^{10})^2}{10^{16}} \Rightarrow \underline{n_0 = 2.25 \times 10^4\ cm^{-3}}$

b. Germanium

$N_a = 10^{16}\ cm^{-3} \Rightarrow \underline{p-type}$

$\underline{p_0 = N_a = 10^{16}\ cm^{-3}}$

$n_i = (1.66\times10^{15})(300)^{3/2}\exp\left(\frac{-0.66}{2(86\times10^{-6})(300)}\right)$

$= (1.66\times10^{15})(300)^{3/2}(2.79\times10^{-6})$

$= 2.4 \times 10^{13}\ cm^{-3}$

$n_0 = \frac{n_i^2}{p_0} = \frac{(2.4\times10^{13})^2}{10^{16}} \Rightarrow \underline{n_0 = 5.76 \times 10^{10}\ cm^{-3}}$

1.5

(a) $n_o = 5x10^{15}\ cm^{-3}$

$p_o = \frac{n_i^2}{n_o} = \frac{(1.5x10^{10})^2}{5x10^{15}} \Rightarrow p_o = 4.5x10^4\ cm^{-3}$

(b) $n_o >> p_o \Rightarrow$ n-type

(c) $n_o \cong N_d = 5x10^{15}\ cm^{-3}$

1.6

a. <u>Add Donors</u>

$\underline{N_d = 7 \times 10^{15}\ cm^{-3}}$

b. Want $p_0 = 10^6\ cm^{-3} = n_i^2/N_d$

So $n_i^2 = (10^6)(7\times10^{15}) = 7 \times 10^{21}$

$= B^2T^3\exp\left(\frac{-Eg}{kT}\right)$

$7 \times 10^{21} = (5.23\times10^{15})^2 T^3\exp\left(\frac{-1.1}{(86\times10^{-6})(T)}\right)$

By trial and error, $T \approx 324°$ K

1.7

$I = J \cdot A = \sigma E A$

$I = (2.2)(15)(10^{-4}) \Rightarrow I = 3.3\ mA$

1.8

$J = \sigma E \Rightarrow \sigma = \dfrac{J}{E} = \dfrac{85}{12}$

$\underline{\sigma = 7.08\ (\text{ohm} - \text{cm})^{-1}}$

1.9

(a) For n-type,

$\sigma \cong e\mu_n N_d = (1.6x10^{-19})(8500)N_d$

For $10^{15} \le N_d \le 10^{19}\ cm^{-3} \Rightarrow$

$1.36 \le \sigma \le 1.36x10^4\ (\Omega - cm)^{-1}$

(b) $J = \sigma E = \sigma(0.1) \Rightarrow$

$0.136 \le J \le 1.36x10^3\ A/cm^2$

1.10

a. $N_a = 10^{17}\ \text{cm}^{-3} \Rightarrow \underline{p_0 = 10^{17}\ \text{cm}^{-3}}$

$n_0 = \dfrac{n_i^2}{p_0} = \dfrac{(1.8 \times 10^6)^2}{10^{17}} \Rightarrow \underline{n_0 = 3.24 \times 10^{-5}\ \text{cm}^{-3}}$

b. $n = n_0 + \delta n = 3.24 \times 10^{-5} + 10^{15} \Rightarrow \underline{n = 10^{15}\ \text{cm}^{-3}}$
$p = p_0 + \delta p = 10^{17} + 10^{15} \Rightarrow \underline{p = 1.01 \times 10^{17}\ \text{cm}^{-3}}$

1.11

$V_{bi} = V_T \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

a. $V_{bi} = (0.026)\ln\left[\dfrac{(10^{15})(10^{15})}{(1.5 \times 10^{10})^2}\right] \Rightarrow V_{bi} = 0.578\ \text{V}$

b. $V_{bi} = (0.026)\ln\left[\dfrac{(10^{15})(10^{18})}{(1.5 \times 10^{10})^2}\right] \Rightarrow V_{bi} = 0.757\ \text{V}$

c. $V_{bi} = (0.026)\ln\left[\dfrac{(10^{18})(10^{18})}{(1.5 \times 10^{10})^2}\right] \Rightarrow V_{bi} = 0.937\ \text{V}$

1.12

$V_{bi} = V_T \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

a. $V_{bi} = (0.026)\ln\left[\dfrac{(10^{15})(10^{15})}{(1.8 \times 10^6)^2}\right] \Rightarrow V_{bi} = 1.05\ \text{V}$

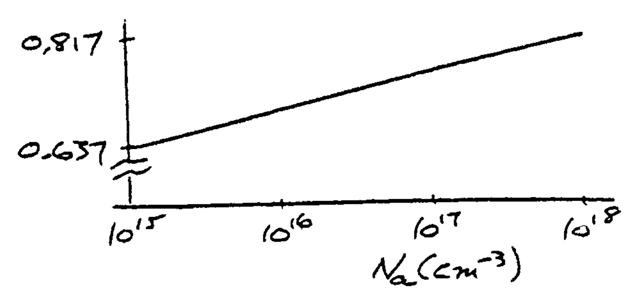
b. $V_{bi} = (0.026)\ln\left[\dfrac{(10^{15})(10^{18})}{(1.8 \times 10^6)^2}\right] \Rightarrow V_{bi} = 1.23\ \text{V}$

c. $V_{bi} = (0.026)\ln\left[\dfrac{(10^{18})(10^{18})}{(1.8 \times 10^6)^2}\right] \Rightarrow V_{bi} = 1.41\ \text{V}$

1.13

$V_{bi} = V_T \ln\left(\dfrac{N_a N_d}{n_i^2}\right) = (0.026)\ln\left[\dfrac{N_a(10^{16})}{(1.5x10^{10})^2}\right]$

For $N_a = 10^{15}\ cm^{-3}$, $V_{bi} = 0.637\ V$
For $N_a = 10^{18}\ cm^{-3}$, $V_{bi} = 0.817\ V$



1.14

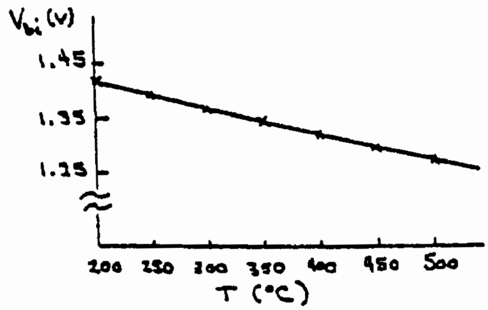
$kT = (0.026)\left(\dfrac{T}{300}\right)$

T	kT	$(T)^{3/2}$
200	0.01733	2828.4
250	0.02167	3952.8
300	0.026	5196.2
350	0.03033	6547.9
400	0.03467	8000.0
450	0.0390	9545.9
500	0.04333	11,180.3

$n_i = (2.1 \times 10^{14})(T^{3/2})\exp\left(\dfrac{-1.4}{2(86 \times 10^{-6})(T)}\right)$

$V_{bi} = V_T \ln\left(\dfrac{N_a N_d}{n_i^2}\right)$

T	n_i	V_{bi}
200	1.256	1.405
250	6.02×10^3	1.389
300	1.80×10^6	1.370
350	1.09×10^8	1.349
400	2.44×10^9	1.327
450	2.80×10^{10}	1.302
500	2.00×10^{11}	1.277



1.15

$$C_j = C_{j0}\left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2}$$

$$V_{bi} = (0.026)\ln\left[\frac{(2 \times 10^{16})(2 \times 10^{15})}{(1.5 \times 10^{10})^2}\right] = 0.673 \text{ V}$$

a. $V_R = 1$ V

$$C_j = (1)\left(1 + \frac{1}{0.673}\right)^{-1/2} \Rightarrow \underline{C_j = 0.634 \text{ pF}}$$

b. $V_R = 5$ V

$$C_j = (1)\left(1 + \frac{5}{0.673}\right)^{-1/2} \Rightarrow \underline{C_j = 0.344 \text{ pF}}$$

1.16

(a) $C_j = C_{jo}\left(1 + \frac{V_R}{V_{bi}}\right)^{-1/2}$

For $V_R = 5V$,

$$C_j = (0.02)\left(1 + \frac{5}{0.8}\right)^{-1/2} = 0.00743\ pF$$

For $V_R = 1.5V$,

$$C_j = (0.02)\left(1 + \frac{1.5}{0.8}\right)^{-1/2} = 0.0118\ pF$$

$$C_j(avg) = \frac{0.00743 + 0.0118}{2} = 0.00962\ pF$$

$$v_C(t) = v_C(final) + \left(v_C(initial) - v_C(final)\right)e^{-t/\tau}$$

where

$$\tau = RC = RC_j(avg) = (47x10^3)(0.00962x10^{-12})$$

or

$$\tau = 4.52x10^{-10}\ s$$

Then

$$v_C(t) = 1.5 = 0 + (5 - 0)e^{-t_1/\tau}$$

$$\frac{5}{1.5} = e^{+t_1/\tau} \Rightarrow t_1 = \tau \ln\left(\frac{5}{1.5}\right)$$

$$\underline{t_1 = 5.44x10^{-10}\ s}$$

(b) For $V_R = 0\,V$,

$$C_j = C_{jo} = 0.02\ pF$$

For $V_R = 3.5\,V$,

$$C_j = (0.02)\left(1 + \frac{3.5}{0.8}\right)^{-1/2} = 0.00863\ pF$$

$$C_j(avg) = \frac{0.02 + 0.00863}{2} = 0.0143\ pF$$

$$\tau = RC_j(avg) = 6.72x10^{-10}\ s$$

$$v_C(t) = v_C(final) + \left(v_C(initial) - v_C(final)\right)e^{-t/\tau}$$

$$3.5 = 5 + (0 - 5)e^{-t_2/\tau} = 5\left(1 - e^{-t_2/\tau}\right)$$

so that $\underline{t_2 = 8.09x10^{-10}\ s}$

1.17

$$V_{bi} = (0.026)\ln\left[\frac{(10^{18})(10^{15})}{(1.5 \times 10^{10})^2}\right] = 0.757 \text{ V}$$

a. $V_R = 1$ V

$$C_j = (0.25)\left(1 + \frac{1}{0.757}\right)^{-1/2} = 0.164 \text{ pF}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(2.2 \times 10^{-3})(0.164 \times 10^{-12})}}$$

$\underline{f_0 = 8.38 \text{ MHz}}$

b. $V_R = 10$ V

$$C_j = (0.25)\left(1+\frac{10}{0.757}\right)^{-1/2} = 0.0663 \text{ pF}$$

$$f_0 = \frac{1}{2\pi\sqrt{(2.2\times10^{-3})(0.0663\times10^{-12})}}$$

$f_0 = 13.2$ MHz

1.18

a. $I = I_S\left[\exp\left(\frac{V_D}{V_T}\right) - 1\right]$

$$-0.90 = \exp\left(\frac{V_D}{V_T}\right) - 1$$

$$\exp\left(\frac{V_D}{V_T}\right) = 1 - 0.90 = 0.10$$

$V_D = V_T \ln(0.10) \Rightarrow V_D = -0.0599$ V

b.

$$\left|\frac{I_F}{I_R}\right| = \frac{I_S}{I_S}\cdot\frac{\left[\exp\left(\frac{V_F}{V_T}\right)-1\right]}{\left[\exp\left(\frac{V_R}{V_T}\right)-1\right]} = \left|\frac{\exp\left(\frac{0.2}{0.026}\right)-1}{\exp\left(\frac{-0.2}{0.026}\right)-1}\right|$$

$$= \left|\frac{2190}{-1}\right|$$

$$\frac{I_F}{I_R} = 2190$$

1.19

a.

$I \cong (10^{-11})\exp\left(\frac{0.5}{0.026}\right) \Rightarrow I = 2.25$ mA

$I = (10^{-11})\exp\left(\frac{0.6}{0.026}\right) \Rightarrow I = 0.105$ A

$I = (10^{-11})\exp\left(\frac{0.7}{0.026}\right) \Rightarrow I = 4.93$ A

b.

$I \cong (10^{-13})\exp\left(\frac{0.5}{0.026}\right) \Rightarrow I = 22.5\ \mu$A

$I = (10^{-13})\exp\left(\frac{0.6}{0.026}\right) \Rightarrow I = 1.05$ mA

$I = (10^{-13})\exp\left(\frac{0.7}{0.026}\right) \Rightarrow I = 49.3$ mA

1.20

(a) $I = I_S\left(e^{V_D/V_T} - 1\right)$

$$150x10^{-6} = 10^{-11}\left(e^{V_D/V_T} - 1\right) \cong 10^{-11}e^{V_D/V_T}$$

Then

$$V_D = V_T \ln\left(\frac{150x10^{-6}}{10^{-11}}\right) = (0.026)\ln\left(\frac{150x10^{-6}}{10^{-11}}\right)$$

Or

$V_D = 0.430\,V$

(b)

$$V_D = V_T \ln\left(\frac{150x10^{-6}}{10^{-13}}\right)$$

Or

$V_D = 0.549\,V$

1.21

a. $I_D \cong I_S \exp\left(\frac{V_D}{nV_T}\right)$

$10^{-3} = I_S \exp\left(\frac{0.7}{2(0.026)}\right) \Rightarrow I_S = 1.42\times10^{-9}$ A

b. $I_D = (1.42\times10^{-9})\exp\left(\frac{0.8}{2(0.026)}\right)$

$I_D = 6.82$ mA

c. $10^{-3} = I_S \exp\left(\frac{0.7}{0.026}\right)$

$I_S = 2.03\times10^{-15}$ A

$I_D = (2.03\times10^{-15})\exp\left(\frac{0.8}{0.026}\right)$

$I_D = 46.8$ mA

1.22

I_S doubles for every 5C increase in temperature.

$I_S = 10^{-12}\,A$ at T = 300K

For $I_S = 0.5x10^{-12}\,A \Rightarrow$ T = 295K

For $I_S = 50x10^{-12}\,A$, $(2)^n = 50 \Rightarrow n = 5.64$

Where n equals number of 5C increases.
Then

$\Delta T = (5.64)(5) = 28.2\,K$

So

$295 \le T \le 328.2\,K$

1.23

$$\frac{I_S(T)}{I_S(-55)} = 2^{\Delta T/5}, \quad \Delta T = 155^\circ\text{C}$$

$$\frac{I_S(100)}{I_S(-55)} = 2^{155/5} = 2.147 \times 10^9$$

$V_T@100^\circ\text{C} \Rightarrow 373^\circ\text{K} \Rightarrow V_T = 0.03220$

$V_T@-55^\circ\text{C} \Rightarrow 216^\circ\text{K} \Rightarrow V_T = 0.01865$

$$\frac{I_D(100)}{I_D(-55)} = (2.147 \times 10^9) \times \frac{\exp\left(\frac{0.6}{0.0322}\right)}{\exp\left(\frac{0.6}{0.01865}\right)}$$

$$= \frac{(2.147 \times 10^9)(1.237 \times 10^8)}{(9.374 \times 10^{13})}$$

$$\underline{\frac{I_D(100)}{I_D(-55)} = 2.83 \times 10^3}$$

1.24

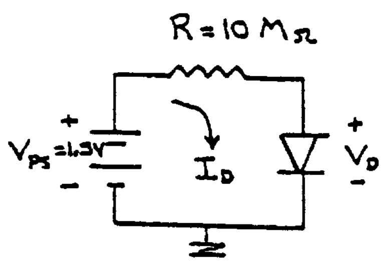
a. $\frac{I_{D2}}{I_{D1}} = 10 = \exp\left(\frac{V_{D2} - V_{D1}}{V_T}\right)$

$\Delta V_D = V_T \ln(10) \Rightarrow \underline{\Delta V_D = 59.9 \text{ mV} \approx 60 \text{ mV}}$

b. $\Delta V_D = V_T \ln(100) \Rightarrow \underline{\Delta V_D = 119.7 \text{ mV} \approx 120 \text{ mV}}$

1.25

a.



$1.5 = I_D(10 \times 10^6) + V_D$ and

$I_D = I_S\left[\exp\left(\frac{V_D}{V_T}\right) - 1\right]$

$1.5 = (10 \times 10^6)(30 \times 10^{-9})\left[\exp\left(\frac{V_D}{V_T}\right) - 1\right] + V_D$

$= 0.3\left[\exp\left(\frac{V_D}{V_T}\right) - 1\right] + V_D$

By trial and error, $\underline{V_D = 0.046 \text{ V}}$

Then $I_D = \frac{1.5 - 0.046}{10} \Rightarrow \underline{I_D = 0.145\ \mu\text{A}}$

b. Reverse-Bias

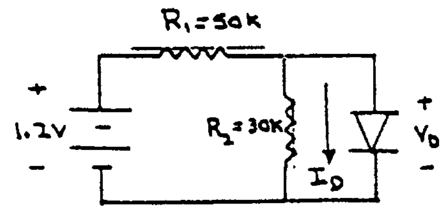
$\underline{I = I_S = 30 \text{ nA}}$

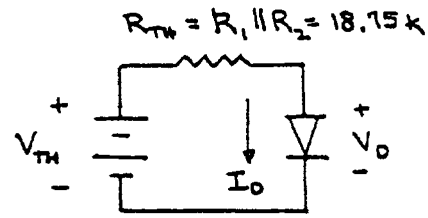
$V_R = (30 \times 10^{-9})(10 \times 10^6) = 0.30 \text{ V}$

$V_D = -1.5 + 0.3 \Rightarrow \underline{V_D = -1.2 \text{ V}}$

1.26

$I_S = 5 \times 10^{-13}$ A





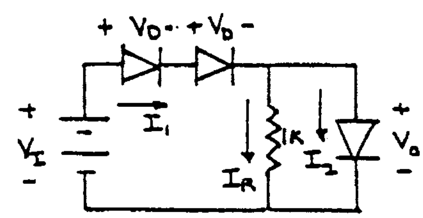
$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(1.2) = \left(\frac{30}{80}\right)(1.2) = 0.45 \text{ V}$$

$0.45 = I_D R_{TH} + V_D, \quad V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$

By trial and error:

$\underline{I_D = 2.6\ \mu\text{A}}, \quad \underline{V_D = 0.402 \text{ V}}$

1.27



$I_S = 2 \times 10^{-13} A$

$V_0 = 0.60$ V

$I_2 = I_S \exp\left(\frac{V_0}{V_T}\right) = (2 \times 10^{-13}) \exp\left(\frac{0.60}{0.026}\right)$

$= 2.105$ mA

$I_R = \frac{0.6}{1\text{ K}} = 0.60$ mA

$I_1 = I_2 + I_R = 2.705$ mA

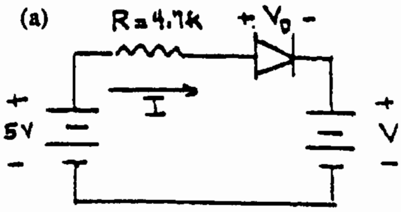
$V_D = V_T \ln\left(\frac{I_1}{I_S}\right) = (0.026)\ln\left(\frac{2.705 \times 10^{-3}}{2 \times 10^{-13}}\right)$

$= 0.6065$

$V_I = 2V_D + V_0 \Rightarrow \underline{V_I = 1.81 \text{ V}}$

1.28

(a)



$I_S = 5 \times 10^{-12}$ A

$I = 0.50$ mA

$V_D = (0.026) \ln\left(\frac{0.5 \times 10^{-3}}{5 \times 10^{-12}}\right)$

$\underline{V_D = 0.479 \text{ V}}$

$5 = IR + V_D + V$

$= (0.5 \times 10^{-3})(4.7 \times 10^{3}) + 0.479 + V$

$\underline{V = 2.17 \text{ V}}$

(b) $P = I_D V_D = (0.5)(0.479)$

or

$\underline{P = 0.24\, mW}$

1.29

(a) Assume diode is conducting.

Then, $V_D = V_\gamma = 0.7\, V$

So that $I_{R2} = \frac{0.7}{30} \Rightarrow 23.3\, \mu A$

$I_{R1} = \frac{1.2 - 0.7}{10} \Rightarrow 50\, \mu A$

Then $I_D = I_{R1} - I_{R2} = 50 - 23.3$

Or

$\underline{I_D = 26.7\, \mu A}$

(b) Let $R_1 = 50\, k\Omega$ Diode is cutoff.

$V_D = \frac{30}{30+50} \cdot (1.2) = 0.45\, V$

Since $V_D < V_\gamma$, $I_D = 0$

1.30

(a) Diode is conducting

$5 = I_D(10) + V_\gamma - 5$

or

$I_D = \frac{10 - 0.6}{10} \Rightarrow \underline{I_D = 0.94\, mA}$

$V_O = V_\gamma - 5 = 0.6 - 5 \Rightarrow \underline{V_O = -4.4\, V}$

(b) Diode is conducting

$5 = V_\gamma + I_D(10) - 5$

or

$I_D = \frac{10 - 0.6}{10} \Rightarrow \underline{I_D = 0.94\, mA}$

$V_O = I_D R - 5 = (0.94)(10) - 5 \Rightarrow \underline{V_O = 4.4\, V}$

(c) Diode is reverse biased

$\underline{I_D = 0}$ ~~$V_D = -10V$~~

$V_O = +5$ VDC

1.31

Minimum diode current for $V_{PS}(\min)$

$I_D(\min) = 2\, mA$, $V_D = 0.7\, V$

$I_2 = \frac{0.7}{R_2}$, $I_1 = \frac{5 - 0.7}{R_1} = \frac{4.3}{R_1}$

We have

$I_1 = I_2 + I_D$

so

(1) $\frac{4.3}{R_1} = \frac{0.7}{R_2} + 2$

Maximum diode current for $V_{PS}(\max)$

$P = I_D V_D$ $10 = I_D(0.7) \Rightarrow I_D = 14.3\, mA$

$I_1 = I_2 + I_D$

or

(2) $\frac{9.3}{R_1} = \frac{0.7}{R_2} + 14.3$

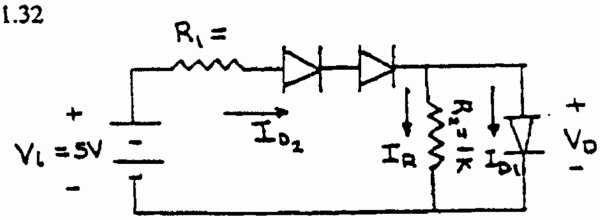
Using Eq. (1),

$\frac{9.3}{R_1} = \frac{4.3}{R_1} - 2 + 14.3 \Rightarrow \underline{R_1 = 0.41\, k\Omega}$

Then

$\underline{R_2 = 82.5\, \Omega}$

1.32

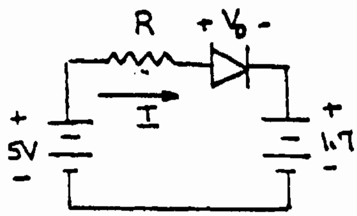


$I_{D1} = \frac{1}{2} I_{D2} \Rightarrow I_{D1} = I_R = \frac{0.65}{1} = 0.65$ mA

$I_{D2} = 1.3 \text{ mA} = \frac{5 - 3(0.65)}{R_1}$

$\underline{R_1 = 2.35\text{ k}\Omega}$, $\underline{I_{D1} = 0.65\text{ mA}}$, $\underline{I_{D2} = 1.30\text{ mA}}$

1.33



$V_\gamma = 0.65$ V

Power $= I \cdot V_\gamma = 0.2$ mW $= I(0.65)$

$\Rightarrow I = 0.308$ mA

$$I = \frac{5 - 0.65 - 1.7}{R} = 0.308$$

$$\Rightarrow R = \frac{2.65}{0.308} \Rightarrow \underline{R = 8.60\ \text{k}\Omega}$$

1.34

For forward bias

$$I_D = \frac{15 - 0.7}{10\ M\Omega} = 0.08\ \mu A$$

For reverse bias

$I_D = 0,\ \ V_D = -15V$

1.35

a. $r_d = \dfrac{V_T}{I_{DQ}} = \dfrac{(0.026)}{1} = 0.026\ \text{k}\Omega = 26\Omega$

$i_d = 0.05 I_{DQ} = 50\ \mu$A peak-to-peak

$v_d = i_d r_d = (26)(50)\ \mu$A

$\Rightarrow \underline{v_d = 1.30\ \text{mV}\ \text{peak-to-peak}}$

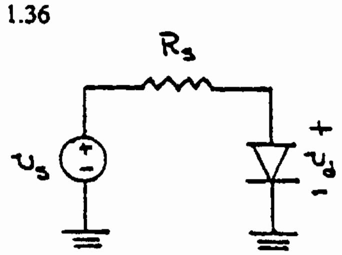
b. For $I_{DQ} = 0.1$ mA $\Rightarrow r_d = \dfrac{(0.026)}{0.1} = 260\Omega$

$i_d = 0.05 I_{DQ} = 5\ \mu$A peak-to-peak

$v_d = i_d r_d = (260)(5)\ \mu$V

$\Rightarrow \underline{v_d = 1.30\ \text{mV}\ \text{peak-to-peak}}$

1.36



ac equivalent circuit

a. diode resistance $r_d = V_T/I$

$$v_d = \left(\frac{r_d}{r_d + R_S}\right) v_S = \left(\frac{V_T/I}{\frac{V_T}{I} + R_S}\right) v_S$$

$$v_d = \left(\frac{V_T}{V_T + I R_S}\right) v_S = v_0$$

b. $R_S = 260\Omega$

$$I = 1\ \text{mA},\ \frac{v_0}{v_S} = \left(\frac{V_T}{V_T + IR_S}\right) = \frac{0.026}{0.026 + (1)(0.26)}$$

$$\Rightarrow \underline{\frac{v_0}{v_S} = 0.0909}$$

$$I = 0.1\ \text{mA},\ \frac{v_0}{v_S} = \frac{0.026}{0.026 + (0.1)(0.26)}$$

$$\Rightarrow \underline{\frac{v_0}{v_S} = 0.50}$$

$$I = 0.01\ \text{mA},\ \frac{v_0}{v_S} = \frac{0.026}{0.026 + (0.01)(0.26)}$$

$$\Rightarrow \underline{\frac{v_0}{v_S} = 0.909}$$

1.37

$$I \cong I_S \exp\left(\frac{V_a}{V_T}\right),\quad V_a = V_T \ln\left(\frac{I}{I_S}\right)$$

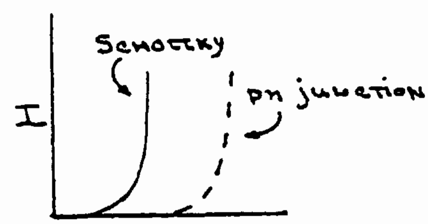
pn junction, $V_a = (0.026)\ln\left(\dfrac{100 \times 10^{-6}}{10^{-14}}\right)$

$\underline{V_a = 0.599\ \text{V}}$

Schottky diode, $V_a = (0.026)\ln\left(\dfrac{100 \times 10^{-6}}{10^{-9}}\right)$

$\underline{V_a = 0.299\ \text{V}}$

1.38



Schottky: $I \cong I_S \exp\left(\dfrac{V_a}{V_T}\right)$

$$V_a = V_T \ln\left(\frac{I}{I_S}\right) = (0.026)\ln\left(\frac{0.5 \times 10^{-3}}{5 \times 10^{-7}}\right)$$

$$= 0.1796\ \text{V}$$

Then

V_a of pn junction $= 0.1796 + 0.30$

$= 0.4796$

$$I_S = \frac{I}{\exp\left(\frac{V_a}{V_T}\right)} = \frac{0.5 \times 10^{-3}}{\exp\left(\frac{0.4796}{0.026}\right)}$$

$\underline{I_S = 4.87 \times 10^{-12}\ \text{A}}$

1.39

pn junction $I_D = 0.5\,mA$

$I_D \cong I_S e^{V_D/V_T} \Rightarrow V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$

Then

$V_D = (0.026)\ln\left(\frac{0.5x10^{-3}}{10^{-12}}\right) = 0.521\,V$

Schottky diode

$V_D = (0.026)\ln\left(\frac{0.5x10^{-3}}{10^{-8}}\right) = 0.281\,V$

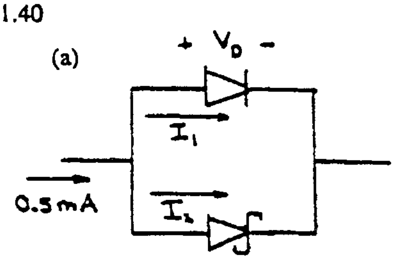
Then

$R = \frac{V_D(pn) - V_D(S)}{0.5} = \frac{0.521 - 0.281}{0.5} \Rightarrow$

$\underline{R = 480\,\Omega}$

1.40

(a)



$I_1 + I_2 = 0.5 \times 10^{-3}$

$5 \times 10^{-8} \exp\left(\frac{V_D}{V_T}\right) + 10^{-12} \exp\left(\frac{V_D}{V_T}\right) = 0.5 \times 10^{-3}$

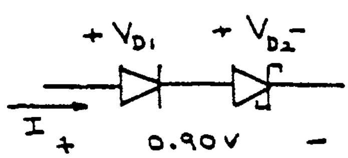
$5.001 \times 10^{-8} \exp\left(\frac{V_D}{V_T}\right) = 0.5 \times 10^{-3}$

$V_D = (0.026)\ln\left(\frac{0.5 \times 10^{-3}}{5.001 \times 10^{-8}}\right) \Rightarrow \underline{V_D = 0.2395}$

Schottky diode, $\underline{I_2 = 0.49999\text{ mA}}$

pn junction, $\underline{I_1 = 0.00001\text{ mA}}$

(b)



$I = 10^{-12} \exp\left(\frac{V_{D1}}{V_T}\right) = 5 \times 10^{-8} \exp\left(\frac{V_{D2}}{V_T}\right)$

$V_{D1} + V_{D2} = 0.9$

$10^{-12} \exp\left(\frac{V_{D1}}{V_T}\right) = 5 \times 10^{-8} \exp\left(\frac{0.9 - V_{D1}}{V_T}\right)$

$= 5 \times 10^{-8} \exp\left(\frac{0.9}{V_T}\right) \cdot \exp\left(\frac{-V_{D1}}{V_T}\right)$

$\exp\left(\frac{2V_{D1}}{V_T}\right) = \left(\frac{5 \times 10^{-8}}{10^{-12}}\right) \exp\left(\frac{0.9}{0.026}\right)$

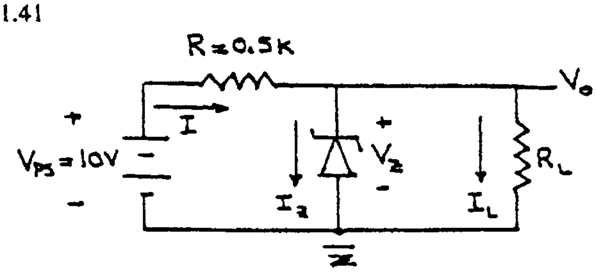
$2V_{D1} = V_T \ln\left(\frac{5 \times 10^{-8}}{10^{-12}}\right) + 0.9 = 1.1813$

$\underline{V_{D1} = 0.591}$ pn junction

$\underline{V_{D2} = 0.309}$ Schottky diode

$I = 10^{-12} \exp\left(\frac{0.5907}{0.026}\right) \Rightarrow \underline{I = 7.36\text{ mA}}$

1.41



$V_Z = V_{Z0} = 5.6$ V at $I_Z = 0.1$ mA

$r_Z = 10\Omega$

$I_Z r_Z = (0.1)(10) = 1$ mV

$V_{Z0} = 5.599$

a. $R_L \to \infty \Rightarrow$

$I_Z = \frac{10 - 5.599}{R + r_Z} = \frac{4.401}{0.50 + 0.01} = 8.63$ mA

$V_Z = V_{Z0} + I_Z r_Z = 5.599 + (0.00863)(10)$

$\underline{V_Z = V_0 = 5.685\text{ V}}$

b. $V_{PS} = 11\text{ V} \Rightarrow I_Z = \frac{11 - 5.599}{0.51} = 10.59$ mA

$V_Z = V_0 = 5.599 + (0.01059)(10) = 5.705$ V

$V_{PS} = 9\text{ V} \Rightarrow I_Z = \frac{9 - 5.599}{0.51} = 6.669$ mA

$V_Z = V_0 = 5.599 + (0.006669)(10) = 5.666$ V

$\Delta V_0 = 5.705 - 5.666 \Rightarrow \underline{\Delta V_0 = 0.039\text{ V}}$

c. $I = I_Z + I_L$

$I_L = \frac{V_0}{R_L}$, $I = \frac{V_{PS} - V_0}{R}$, $I_Z = \frac{V_0 - V_{Z0}}{r_Z}$

$$\frac{10 - V_0}{0.50} = \frac{V_0 - 5.599}{0.010} + \frac{V_0}{2}$$

$$\frac{10}{0.50} + \frac{5.599}{0.010} = V_0\left[\frac{1}{0.50} + \frac{1}{0.010} + \frac{1}{2}\right]$$

$$20.0 + 559.9 = V_0(102.5)$$

$\underline{V_0 = 5.658 \text{ V}}$

1.42

a. $I_Z = \frac{9 - 6.8}{0.2} \Rightarrow \underline{I_Z = 11 \text{ mA}}$

$P_Z = (11)(6.8) \Rightarrow \underline{P_Z = 74.8 \text{ mW}}$

b. $I_Z = \frac{12 - 6.8}{0.2} \Rightarrow \underline{I_Z = 26 \text{ mA}}$

$\% = \frac{26 - 11}{11} \times 100 \Rightarrow \underline{136\%}$

$P_Z = (26)(6.8) = 176.8 \text{ mW}$

$\% = \frac{176.8 - 74.8}{74.8} \times 100 = \underline{136\%}$

1.43

$I_Z r_Z = (0.1)(20) = 2 \text{ mV}$

$V_{Z0} = 6.8 - 0.002 = 6.798 \text{ V}$

a. $R_L = \infty$

$I_Z = \frac{10 - 6.798}{0.5 + 0.02} \Rightarrow I_Z = 6.158 \text{ mA}$

$V_0 = V_Z = V_{Z0} + I_Z r_Z = 6.798 + (0.006158)(20)$

$\underline{V_0 = 6.921 \text{ V}}$

b. $I = I_Z + I_L$

$$\frac{10 - V_0}{0.50} = \frac{V_0 - 6.798}{0.020} + \frac{V_0}{1}$$

$$\frac{10}{0.50} + \frac{6.798}{0.020} = V_0\left[\frac{1}{0.50} + \frac{1}{0.020} + \frac{1}{1}\right]$$

$359.9 = V_0(53)$

$V_0 = 6.791 \text{ V}$

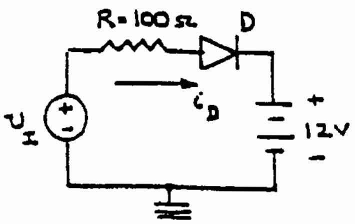
$\Delta V_0 = 6.791 - 6.921$

$\underline{\Delta V_0 = -0.13 \text{ V}}$

Chapter 2

Exercise Solutions

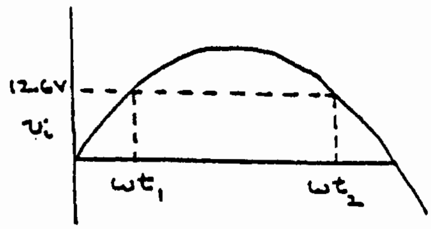
E2.1



a. $i_D(\text{peak}) = \dfrac{24 - 12 - 0.6}{0.10} = 114 \text{ mA}$

b. $V_R(\max) = 24 + 12 = 36 \text{ V}$

c.



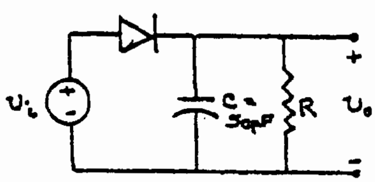
$v_I = 24 \sin \omega t = 12.6$

$\omega t_1 = \sin^{-1}\left(\dfrac{12.6}{24}\right) = 31.7°$

By symmetry, $\omega t_2 = 180 - 31.7 = 148.3°$

$\% = \left(\dfrac{148.3 - 31.7}{360}\right) \times 100\% \Rightarrow \underline{32.4\%}$

E2.2



$v_s = 75 \sin(2\pi 60 t)$

$V_r = \dfrac{V_m}{fRC}$

or $R = \dfrac{V_m}{fCV_r} = \dfrac{75}{(60)(50 \times 10^{-6})(4)}$

$\underline{R = 6.25 \text{ k}\Omega}$

E2.3

$v_s = 120 \sin(2\pi 60 t)$, $V_\gamma = 0.7$, $R = 2.5 \text{ k}\Omega$

Full-wave rectifier

Turns ratio $1:2 \Rightarrow v_S = v_s$

$V_M = 120 - 0.7 = 119.3 \text{ V}$

$V_r = 119.3 - 100 = 19.3 \text{ V}$

So $C = \dfrac{V_m}{2fRV_r} = \dfrac{119.3}{2(60)(2.5 \times 10^3)(19.3)}$

$C = 2.06 \times 10^{-5} = 20.6 \times 10^{-6} \Rightarrow C = 20.6 \ \mu\text{F}$

E2.4

$v_s = 50 \sin(2\pi 60 t)$, $V_\gamma = 0.7$, $R = 10 \text{ k}\Omega$

Full-wave rectifier

$C = \dfrac{V_m}{2fRV_r} = \dfrac{(50 - 1.4)}{2(60)(10 \times 10^3)(2)}$

$C = 2.025 \times 10^{-5} = 20.25 \times 10^{-6} \Rightarrow C = 20.3 \ \mu\text{F}$

E2.5

Using Eq. (2-10)

a. $\omega \Delta t = \sqrt{\dfrac{2V_r}{V_M}} = \sqrt{\dfrac{2(4)}{75}} = 0.327$

$\% = \left(\dfrac{0.327}{2\pi}\right) \times 100\% = \underline{5.2\%}$

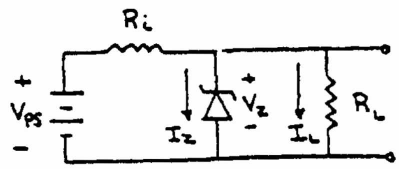
b. $\omega \Delta t = \sqrt{\dfrac{2V_r}{V_M}} = \sqrt{\dfrac{2(19.3)}{119.3}} = 0.569$

$\% = \left(\dfrac{0.569}{\pi}\right) \times 100\% = \underline{18.1\%}$

c. $\omega \Delta t = \sqrt{\dfrac{2V_r}{V_M}} = \sqrt{\dfrac{2(2)}{48.6}} = 0.287$

$\% = \left(\dfrac{0.287}{\pi}\right) \times 100\% = \underline{9.14\%}$

E2.6



$10 \le V_{PS} \le 14 \text{ V}$, $V_Z = 5.6$

$20 \le R_L \le 100$

$I_L(\max) = \dfrac{5.6}{20} = 0.28 \text{ A}$, $I_L(\min) = \dfrac{5.6}{100} = 0.056 \text{ A}$

$I_Z(\max)$

$$= \frac{[V_{PS}(\max) - V_Z]I_L(\max)}{V_{PS}(\min) - 0.9V_Z - 0.1V_{PS}(\max)} - \frac{[V_{PS}(\min) - V_Z]I_L(\min)}{V_{PS}(\min) - 0.9V_Z - 0.1V_{PS}(\max)}$$

$$= \frac{[14 - 5.6](280) - [10 - 5.6](56)}{10 - (0.9)(5.6) - (0.1)(14)}$$

$$= \frac{2352 - 246.4}{3.56}$$

$I_Z(\max) = 591.5$ mA

$\text{Power}(\min) = I_Z(\max) \cdot V_Z = (0.5915)(5.6)$

<u>Power = 3.31 W</u>

$$R_i = \frac{V_{PS}(\max) - V_Z}{I_Z(\max) + I_L(\min)} = \frac{14 - 5.6}{0.5915 + 0.056}$$

$$= \frac{8.4}{0.6475}$$

<u>$R_i \approx 13\Omega$</u>

E2.7

$$I_Z = \frac{V_{PS} - V_Z}{R_i} - I_L$$

For $V_{PS}(\min)$ and $I_L(\max)$, then

$$I_Z(\min) = \frac{11-9}{20} - 0.1 = 0$$

(Minimum Zener current is zero.)

For $V_{PS}(\max)$ and $I_L(\min)$, then

$$I_Z(\max) = \frac{13.6-9}{20} - 0 \Rightarrow 230\ mA$$

The characteristic of the minimum Zener current being one-tenth of the maximum value is violated. The proper circuit operation is questionable.

E2.8

$$I_Z(\min) = \frac{V_{PS}(\min) - V_Z}{R_i} - I_L(\max)$$

so

$$30 = \frac{10-9}{0.0153} - I_L(\max)$$

Or

<u>$I_L(\max) = 35.4\ mA$</u>

E2.9

$$\%\ \text{Regulation} = \frac{V_L(\max) - V_L(\min)}{V_L(\text{nominal})}$$

$V_L(\text{nominal}) = 5.6$

$V_L(\max) = V_L(\text{nominal}) + I_Z(\max)r_Z$

$$= 5.6 + (0.5915)(1.5) = 6.487$$

$V_L(\min) = V_L(\text{nominal}) + I_Z(\min)r_Z$

$$= 5.6 + (I_Z(\min))(1.5)$$

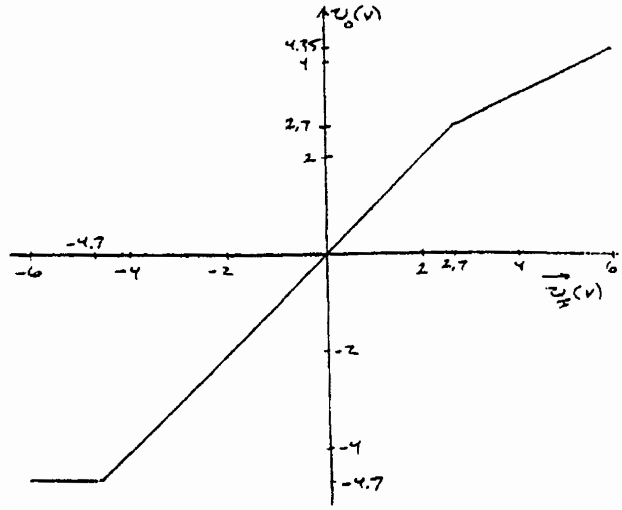
$$I_Z(\min) = \frac{10 - 5.6}{13} - 0.280$$

$$= 0.0585 \text{ A}$$

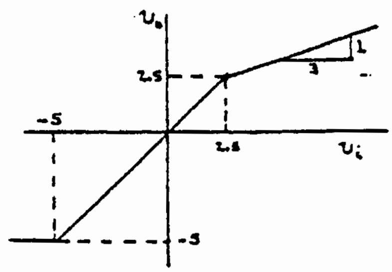
$V_L(\min) = 5.6 + (0.0585)(1.5) = 5.688$

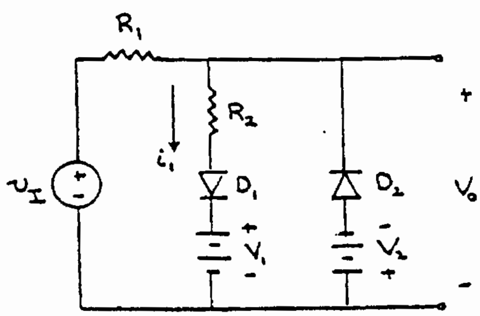
$$\%\ \text{Reg} = \frac{6.487 - 5.688}{5.6} = 0.143 \Rightarrow \underline{14.3\%}$$

E2.10



E2.11





$V_\gamma = 0.7$ V

For $v_I < 5$, D_2 on $\Rightarrow V_0 = -5$ V $\Rightarrow \underline{V_2 = 4.3\ \text{V}}$

D_1 turns on when $v_I = 2.5 \Rightarrow \underline{V_1 = 1.8\ \text{V}}$

For $v_I > 2.5$, $\dfrac{\Delta v_0}{\Delta v_I} = \dfrac{1}{3} \Rightarrow \dfrac{R_2}{R_1 + R_2} = \dfrac{1}{3}$

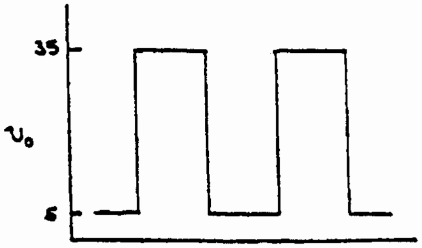
$$\Rightarrow R_1 = 2R_2$$

E2.12

For $V_\gamma = 0$, $v_O(\max) = -2\,V$
Now, $\Delta v_O = 8\,V$, so that
$v_O(\min) = -10\,V$

E2.13

As v_S goes negative, D turns on and $v_0 = +5$ V.
As v_S goes positive, D turns off.



Output, a square wave oscillating between +5 and +35 volts.

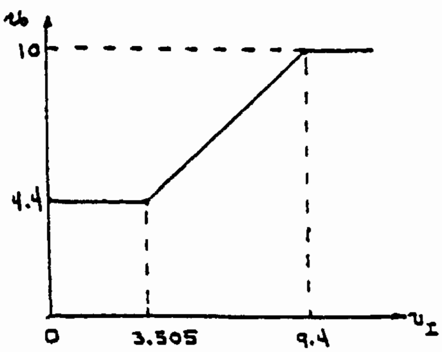
E2.14

$v_0 = 4.4$, $I = \dfrac{10 - 4.4}{9.5} = 0.5895$ mA

Set $I = I_{D_1}$

$v_I = 4.4 - 0.6 - (0.5895)(0.5)$

$v_I = 3.305$



Summary: $0 \le v_I \le 3.5$, $v_0 = 4.4$

For $v_I > 3.5$. $D2$ turns off and when $v_I \ge 9.4$.

$v_0 = 10$

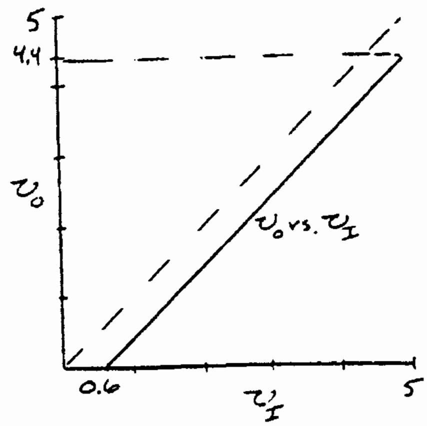
E2.15

$V_0 = -0.6$ V. $I_{D1} = 0$.

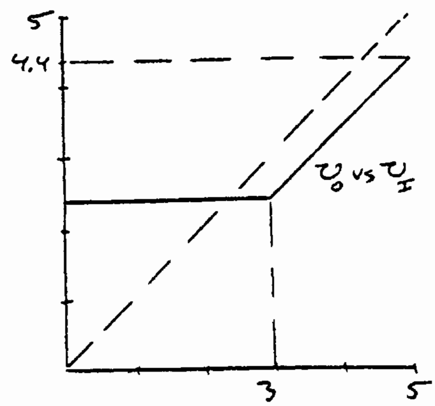
$I_{D2} = I = \dfrac{-0.6 - (-10)}{2.2} \Rightarrow I_{D2} = I = 4.27$ mA

E2.16

(a)

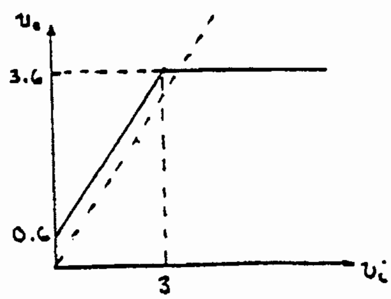


(b)



E2.17

a. $V_0 = 0.6$ V for all V_I
b.



E2.18

a. $I_{Ph} = \eta e \Phi$ A

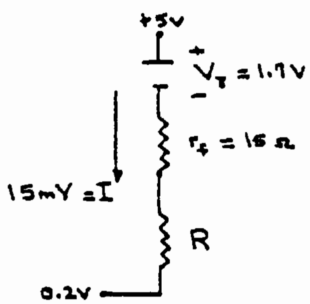
$$I_{Ph} = (0.8)(1.6 \times 10^{-19})\left[\frac{6.4 \times 10^{-2}}{(2)(1.6 \times 10^{-19})}\right](0.5)$$

$\underline{I_{Ph} = 12.8 \text{ mA}}$

b. We have $v_0 = (12.8)(1) = 12.8$ volts.
The diode must be reverse biased so that
$V_{PS} > 12.8$ volts.

E2.19

The equivalent circuit is



$$I = \frac{5 - 1.7 - 0.2}{r_f + R} = 15 \text{ mA}$$

$$r_f + R = \frac{5 - 1.7 - 0.2}{15} = \frac{3.1}{15} = 0.207 \text{ k}\Omega$$

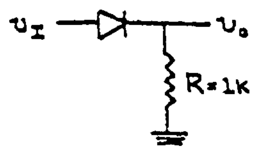
$$= 207\Omega$$

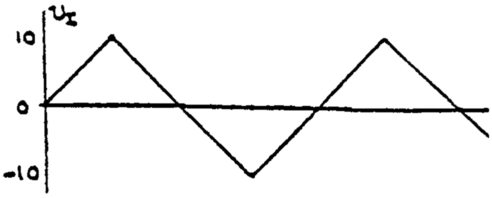
$R = 207 - 15 \Rightarrow \underline{R = 192\Omega}$

Chapter 2

Problem Solutions

2.1



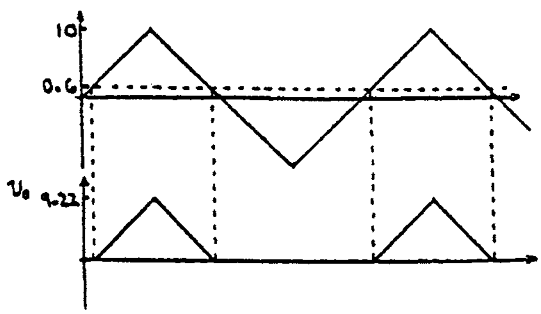


$V_\gamma = 0.6$ V, $r_f = 20\ \Omega$

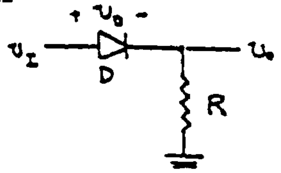
For $v_I = 10$ V, $v_O = \left(\dfrac{R}{R + r_f}\right)(10 - 0.6)$

$= \left(\dfrac{1}{1 + 0.02}\right)(9.4)$

$v_O = 9.22$



2.2



$v_O = v_I - v_D$

$v_D = V_T \ln\left(\dfrac{i_D}{I_S}\right)$ and $i_D = \dfrac{v_O}{R}$

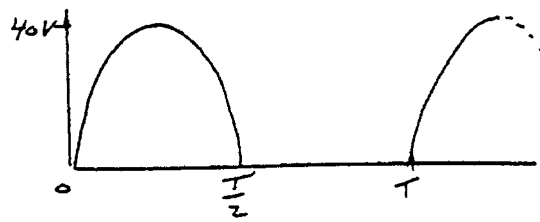
$v_O = v_I - V_T \ln\left(\dfrac{v_O}{I_S R}\right)$

2.3

(a) $v_s(\max) = \dfrac{160}{4} = 40\,V$

(b) $PIV = |v_s(\max)| = 40\,V$

(c)



$$v_O(avg) = \frac{1}{T_O}\int_{T_0} v_O(t)dt = \frac{1}{2\pi}\int_0^{\pi} 40\sin x dx$$

$$= \frac{40}{2\pi}[-\cos x]_0^{\pi} = \frac{40}{2\pi}[-(-1-1)] = \frac{40}{\pi}$$

or

$v_O(avg) = 12.7\,V$

(d) 50%

2.4

$v_O = v_S - 2V_\gamma \Rightarrow v_S(\max) = v_O(\max) + 2V_\gamma$

a. For $v_O(\max) = 25$ V $\Rightarrow v_S(\max) = 25 + 2(0.7)$
$= 26.4$ V

$\dfrac{N_1}{N_2} = \dfrac{160}{26.4} \Rightarrow \dfrac{N_1}{N_2} = 6.06$

b. For $v_O(\max) = 100$ V $\Rightarrow v_S(\max) = 101.4$ V

$\dfrac{N_1}{N_2} = \dfrac{160}{101.4} \Rightarrow \dfrac{N_1}{N_2} = 1.58$

Full Bridge

From part (a)
$PIV = ~~2v_s(\max) - V_\gamma = 2(26.4) - 0.7~~$
or
$PIV = 52.1\,V$ $\quad V_s(\max) - V_\gamma = PIV$

26.5

or, from part (b)
$PIV = ~~2~~(101.4) - 0.7$

or

$PIV = ~~202.1\,V~~$

101.5

2.5

a. $v_0(\max) = 24\text{ V} \Rightarrow v_S(\max) = 24 + 2(0.7)$

$v_S(\max) = 25.4\text{ V}$

$v_S(\text{rms}) = \dfrac{25.4}{\sqrt{2}} \Rightarrow \underline{v_S(\text{rms}) = 17.96\text{ V}}$

b. $V_r = \dfrac{V_M}{2fRC} \Rightarrow C = \dfrac{V_M}{2fV_rR}$

$C = \dfrac{24}{2(60)(0.5)(150)} \Rightarrow \underline{C = 2667\ \mu\text{F}}$

c. $i_{D,\max} = \dfrac{V_m}{R}\left(1 + 2\pi\sqrt{\dfrac{V_M}{2V_r}}\right)$

$i_{D,\max} = \dfrac{24}{150}\left(1 + 2\pi\sqrt{\dfrac{24}{2(0.5)}}\right)$

$\underline{i_{D,\max} = 5.08\text{ A}}$

2.6

(a) $v_s(\max) = 24 + 0.7 = 24.7\ V$

$v_s(rms) = \dfrac{v_S(\max)}{\sqrt{2}} \Rightarrow v_s(rms) = 17.5\ V$

(b) $V_r = \dfrac{V_M}{fRC} \Rightarrow C = \dfrac{V_M}{fRV_r} = \dfrac{24}{(60)(150)(0.5)}$

or

$\underline{C = 5333\ \mu F}$

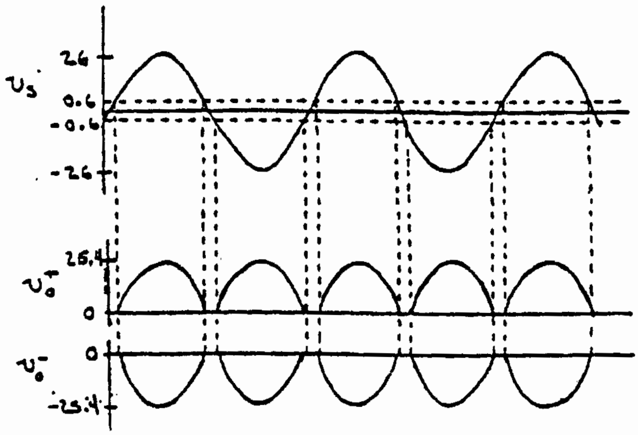
(c) For the half-wave rectifier

$i_{D,\max} = \dfrac{V_M}{R}\left(1 + 4\pi\sqrt{\dfrac{V_M}{2V_r}}\right) = \dfrac{24}{150}\left(1 + 4\pi\sqrt{\dfrac{24}{2(0.5)}}\right)$

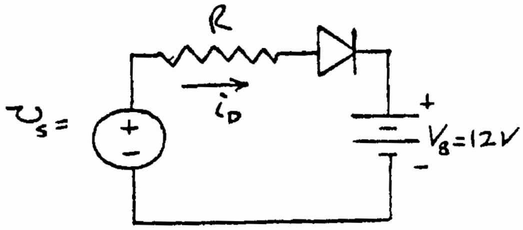
or

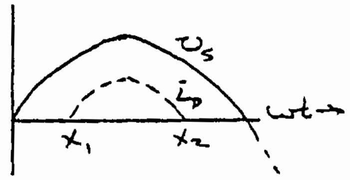
$\underline{i_{D,\max} = 10.0\ A}$

2.7



2.8





$v_s(t) = 24\sin\omega t$

Now

$i_D(avg) = \dfrac{1}{T}\int_0^T i_D(t)dt$

We have for $x_1 \le \omega t \le x_2$

$$i_D = \frac{24\sin x - 12.7}{R}$$

To find x_1 and x_2,

$$24\sin x_1 = 12.7$$

$x_1 = 0.558\ rad$

$x_2 = \pi - 0.558 = 2.584\ rad$

Then

$i_D(avg) = 2 = \dfrac{1}{2\pi}\int_{x_1}^{x_2}\left[\dfrac{24\sin x - 12.7}{R}\right]dx$

$= \dfrac{1}{2\pi}\left(\dfrac{24}{R}\right)(-\cos x)_{x_1}^{x_2} - \dfrac{1}{2\pi}\left(\dfrac{12.7}{R}\right)\cdot x_{x_1}^{x_2}$

or

$2 = \dfrac{6.482}{R} - \dfrac{4.095}{R} \Rightarrow \underline{R = 1.19\ \Omega}$

Fraction of time diode is conducting

$= \dfrac{x_2 - x_1}{2\pi} \times 100\% = \dfrac{2.584 - 0.558}{2\pi} \times 100\%$

or

$\underline{\text{Fraction} = 32.2\%}$

Power rating

$P_{avg} = R \cdot i_{rms}^2 = \dfrac{R}{T}\int_0^T i_D^2 dt = \dfrac{R}{2\pi}\int_{x_1}^{x_2}\left[\dfrac{24\sin x - 12.7}{R}\right]^2 dx$

$= \dfrac{1}{2\pi R}\int_{x_1}^{x_2}\left[(24)^2\sin^2 x - 2(12.7)(24)\sin x + (12.7)^2\right]dx$

$= \dfrac{1}{2\pi R}\left[(24)^2\left[\dfrac{x}{2} - \dfrac{\sin 2x}{4}\right]_{x_1}^{x_2} - 2(12.7)(24)(-\cos x)_{x_1}^{x_2} + (12.7)^2 x\Big|_{x_1}^{x_2}\right]$

For $R = 1.19\ \Omega$, then

$\underline{P_{avg} = 17.9\ W}$

2.9

$R = \frac{15}{0.1} = 150\,\Omega$

$v_S(\max) = v_O(\max) + V_\gamma = 15 + 0.7$

or

$v_S(\max) = 15.7\,V$

Then

$v_S(rms) = \frac{15.7}{\sqrt{2}} = 11.1\,V$

Now

$\frac{N_1}{N_2} = \frac{120}{11.1} \Rightarrow \frac{N_1}{N_2} = 10.8$

$V_r = \frac{V_M}{2fRC} \Rightarrow C = \frac{V_M}{2fRV_r} = \frac{15}{2(60)(150)(0.4)}$

or

$C = 2083\,\mu F$

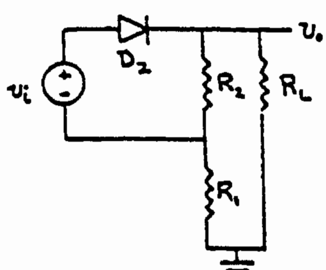
$PIV = 2v_S(\max) - V_\gamma = 2(15.7) - 0.7$

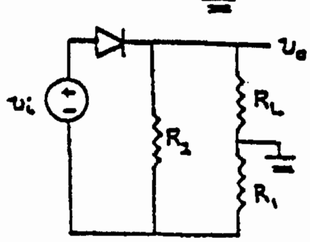
or

$PIV = 30.7\,V$

2.10

For $v_i > 0$

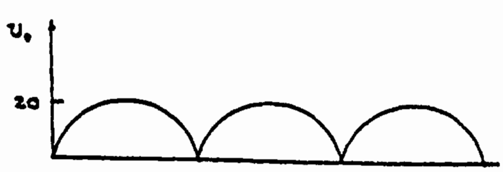




$V_\gamma = 0$

Voltage across $R_L + R_1 = v_i$

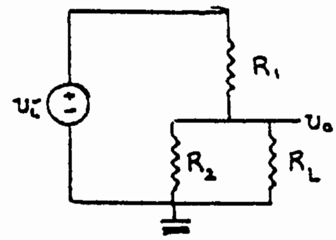
Voltage Divider $\Rightarrow v_0 = \left(\frac{R_L}{R_L + R_1}\right) v_i = \frac{1}{2} v_i$



2.11

For $v_i > 0$, $(V_\gamma = 0)$

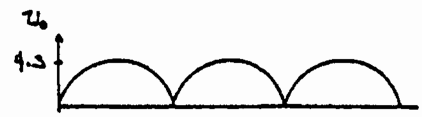
a.



$v_0 = \left(\frac{R_2 \| R_L}{R_2 \| R_L + R_1}\right) v_i$

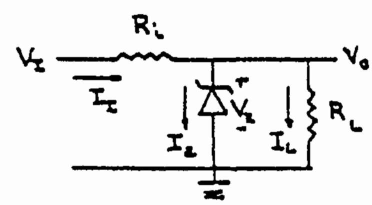
$R_2 \| R_L = 2.2 \| 6.8 = 1.66\ k\Omega$

$v_0 = \left(\frac{1.66}{1.66 + 2.2}\right) v_i = 0.43 v_i$



b. $v_0(rms) = \frac{v_0(\max)}{\sqrt{2}} \Rightarrow v_0(rms) = 3.04\ V$

2.12



$V_I = 6.3\ V$, $R_i = 12\Omega$, $V_Z = 4.8$

a. $I_I = \frac{6.3 - 4.8}{12} \Rightarrow 125\ mA$

$I_L = I_I - I_Z = 125 - I_Z$

$25 \le I_L \le 120\ mA \Rightarrow 40 \le R_L \le 192\Omega$

b. $P_Z = I_Z V_Z = (100)(4.8) \Rightarrow P_Z = 480\ mW$

$P_L = I_L V_0 = (120)(4.8) = P_L = 576\ mW$

2.13

a. $I_I = \frac{20 - 10}{222} \Rightarrow I_I = 45.0\ mA$

$I_L = \frac{10}{380} \Rightarrow I_L = 26.3\ mA$

$I_Z = I_I - I_L \Rightarrow I_Z = 18.7\ mA$

b. $P_Z(\max) = 400 \text{ mW} \Rightarrow I_Z(\max) = \frac{400}{10} = 40 \text{ mA}$

$\Rightarrow I_L(\min) = I_I - I_Z(\max) = 45 - 40$

$\Rightarrow I_L(\min) = 5 \text{ mA} = \frac{10}{R_L}$

$\Rightarrow \underline{R_L = 2 \text{ k}\Omega}$

For $R_i = 175\Omega$

$\underline{I_I = 57.1 \text{ mA}}$ $\underline{I_L = 26.3 \text{ mA}}$ $\underline{I_Z = 30.8 \text{ mA}}$

$I_Z(\max) = 40 \text{ mA} \Rightarrow I_L(\min) = 57.1 - 40 = 17.1 \text{ mA}$

$R_L = \frac{10}{17.1} \Rightarrow \underline{R_L = 585\Omega}$

2.14

a. From Eq. (2-23)

$$I_Z(\max) = \frac{500[20-10] - 50[15-10]}{15 - (0.9)(10) - (0.1)(20)}$$
$$= \frac{5000 - 250}{4}$$

$I_Z(\max) = 1.1875 \text{ A}$

$I_Z(\min) = 0.11875 \text{ A}$

From Eq. (2-21(b))

$$R_i = \frac{20-10}{1187.5 + 50} \Rightarrow \underline{R_i = 8.08\Omega}$$

b. $P_Z = (1.1875)(10) \Rightarrow \underline{P_Z = 11.9 \text{ W}}$
$P_L = I_L(\max)V_0 = (0.5)(10) \Rightarrow \underline{P_L = 5 \text{ W}}$

2.15

(a) As approximation, assume $I_Z(\max)$ and $I_Z(\min)$ are the same as in problem 2-14.

$V_0(\max) = V_0(\text{nom}) + I_Z(\max)r_Z$
$= 20 + (0.453)(2) = 20.906$

$V_0(\min) = V_0(\text{nom}) + I_Z(\min)r_Z$
$= 20 + (0.0453)(2) = 20.0906$

b. $\% \text{ Reg} = \frac{20.906 - 20.0906}{20} \times 100\%$

$\Rightarrow \underline{\% \text{ Reg} = 4.08\%}$

2.16

$$\%\text{Reg} = \frac{V_L(\max) - V_L(\min)}{V_L(nom)} \times 100\%$$
$$= \frac{V_L(nom) + I_Z(\max)r_z - (V_L(nom) + I_Z(\min)r_z)}{V_L(nom)}$$
$$= \frac{[I_Z(\max) - I_Z(\min)](3)}{6} = 0.05$$

So

$I_Z(\max) - I_Z(\min) = 0.1 \ A$

Now

$I_L(\max) = \frac{6}{500} = 0.012 \ A, \quad I_L(\min) = \frac{6}{1000} = 0.006 \ A$

Now

$$R_i = \frac{V_{PS}(\min) - V_Z}{I_Z(\min) + I_L(\max)}$$

or

$$280 = \frac{15-6}{I_Z(\min) + 0.012} \Rightarrow I_Z(\min) = 0.020 \ A$$

Then

$I_Z(\max) = 0.1 + 0.02 = 0.12 \ A$

and

$$R_i = \frac{V_{PS}(\max) - V_Z}{I_Z(\max) + I_L(\min)}$$

or

$$280 = \frac{V_{PS}(\max) - 6}{0.12 + 0.006} \Rightarrow \underline{V_{PS}(\max) = 41.3 V}$$

2.17

Using Figure 2.17

a. $V_{PS} = 20 \pm 25\% \Rightarrow 15 \le V_{PS} \le 25 \text{ V}$

For $V_{PS}(\min)$:

$I_I = I_Z(\min) + I_L(\max) = 5 + 20 = 25 \text{ mA}$

$R_i = \frac{V_{PS}(\min) - V_Z}{I_I} = \frac{15-10}{25} \Rightarrow \underline{R_i = 200\Omega}$

b. For $V_{PS}(\max)$

$\Rightarrow I_I(\max) = \frac{25-10}{R_i} \Rightarrow I_I(\max) = 75 \text{ mA}$

For $I_L(\min) = 0 \Rightarrow I_Z(\max) = 75 \text{ mA}$

$V_{Z0} = V_Z - I_Z r_Z = 10 - (0.025)(5) = 9.875 \text{ V}$

$V_0(\max) = 9.875 + (0.075)(5) = 10.25$

$V_0(\min) = 9.875 + (0.005)(5) = 9.90$

$\underline{\Delta V_0 = 0.35 \text{ V}}$

c. $\% \text{ Reg} = \frac{\Delta V_0}{V_0(\text{nom})} \times 100\% \Rightarrow \underline{\% \text{ Reg} = 3.5\%}$

2.18

From Equation (2.21(a))

$$R_i = \frac{V_{PS}(\min) - V_Z}{I_Z(\min) + I_L(\max)} = \frac{24-16}{40+400}$$

or

$\underline{R_i = 18.2 \ \Omega}$

Also

$$V_r = \frac{V_M}{2fRC} \Rightarrow C = \frac{V_M}{2fRV_r}$$

$R \cong R_i + r_z = 18.2 + 2 = 20.2 \ \Omega$

Then

$$C = \frac{24}{2(60)(1)(20.2)} \Rightarrow \underline{C = 9901 \ \mu F}$$

2.19

$V_Z = V_{Z0} + I_Z r_Z \quad V_Z(\text{nom}) = 8\ \text{V}$

$8 = V_{Z0} + (0.1)(0.5) \Rightarrow V_{Z0} = 7.95\ \text{V}$

$I_i = \dfrac{V_S(\max) - V_Z(\text{nom})}{R_i} = \dfrac{12-8}{3} = 1.333\ \text{A}$

For $I_L = 0.2\ \text{A} \Rightarrow I_Z = 1.133\ \text{A}$

For $I_L = 1\ \text{A} \Rightarrow I_Z = 0.333\ \text{A}$

$V_L(\max) = V_{Z0} + I_Z(\max) r_Z$

$= 7.95 + (1.133)(0.5) = 8.5165$

$V_L(\min) = V_{Z0} + I_Z(\min) r_Z$

$= 7.95 + (0.333)(0.5) = 8.1165$

$\Delta V_L = 0.4\ \text{V}$

$\% \text{Reg} = \dfrac{\Delta V_L}{V_0(\text{nom})} = \dfrac{0.4}{8} \Rightarrow \underline{\% \text{Reg} = 5.0\%}$

$V_r = \dfrac{V_M}{2fRC} \Rightarrow C = \dfrac{V_M}{2fRV_r}$

$R = R_i + r_z = 3 + 0.5 = 3.5\,\Omega$

Then

$C = \dfrac{8}{2(60)(3.5)(0.8)} \Rightarrow \underline{C = 0.0238\ F}$

2.20

(a) For $-10 \le v_I \le 0$, both diodes are conducting $\Rightarrow v_O = 0$

For $0 \le v_I \le 3$, Zener not in breakdown, so

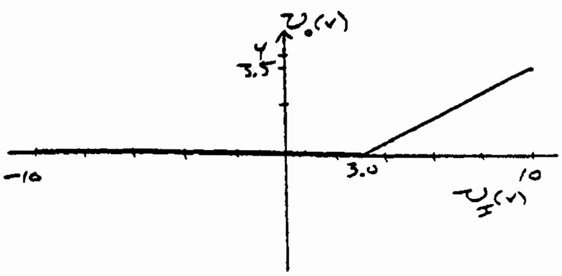
$\underline{i_1 = 0, \quad v_O = 0}$

For $v_I > 3$

$i_1 = \dfrac{v_I - 3}{20}\ mA$

$v_O = \left(\dfrac{v_I - 3}{20}\right)(10) = \dfrac{1}{2}v_I - 1.5$

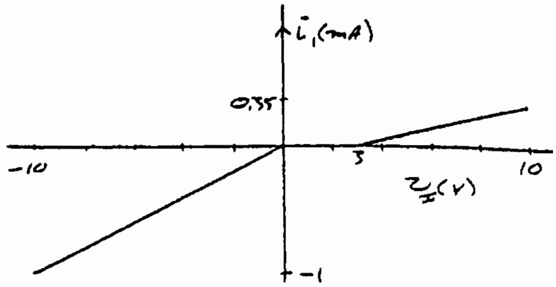
At $v_I = 10\,V$, $\quad v_O = 3.5\,V$, $\quad i_1 = 0.35\,mA$



(b) For $v_I < 0$, both diodes forward biased

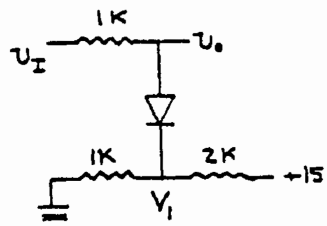
$-i_1 = \dfrac{0 - v_I}{10}$. At $v_I = -10\,V, i_1 = -1\,mA$

For $v_I > 3$, $i_1 = \dfrac{v_I - 3}{20}$. At $v_I = 10\,V, i_1 = 0.35\,mA$



2.21

(a)



$V_1 = \dfrac{1}{3} \times 15 = 5\ \text{V} \Rightarrow$ for $v_I \le 5.7$, $v_0 = v_I$

$\dfrac{v_I - (V_1 + 0.7)}{1} + \dfrac{15 - V_1}{2} = \dfrac{V_1}{1}$, $v_0 = V_1 + 0.7$

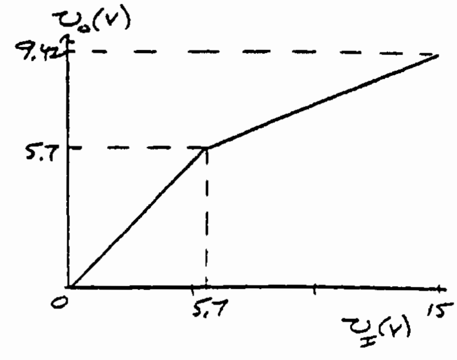
$\dfrac{v_I - v_0}{1} + \dfrac{15 - (v_0 - 0.7)}{2} = \dfrac{v_0 - 0.7}{1}$

$\dfrac{v_I}{1} + \dfrac{15.7}{2} + \dfrac{0.7}{1} = v_0\left(\dfrac{1}{1} + \dfrac{1}{2} + \dfrac{1}{1}\right) = v_0(2.5)$

$v_I + 8.55 = v_0(2.5) \Rightarrow v_0 = \dfrac{1}{2.5}v_I + 3.42$

$v_I = 5.7 \Rightarrow v_0 = 5.7$

$v_I = 15 \Rightarrow v_0 = 9.42$



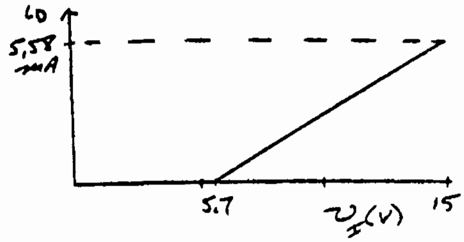
(b) $i_D = 0$ for $0 \le v_I \le 5.7$

Then

$i_D = \dfrac{v_I - v_O}{1} = \dfrac{v_I - \left(\dfrac{v_I}{2.5} + 3.42\right)}{1}$

or

$i_D = \dfrac{0.6v_I - 3.42}{1} \qquad$ For $v_I = 15$, $\; i_D = 5.58\,mA$



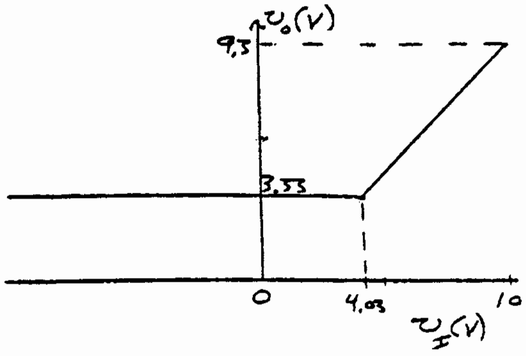
2.22

(a) For D off, $v_O = \left(\frac{20}{30}\right)(20) - 10 = 3.33\,V$

Then for $v_I \le 3.33 + 0.7 = 4.03\,V \Rightarrow v_O = 3.33\,V$

For $v_I > 4.03$, $v_O = v_I - 0.7$;

For $v_I = 10$, $v_O = 9.3$

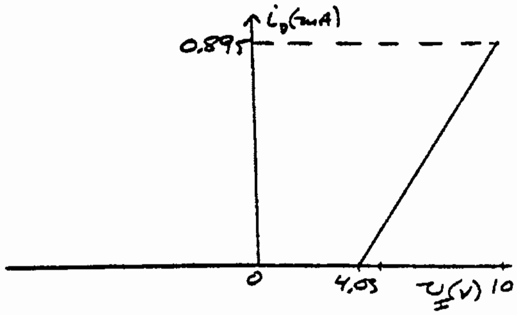


(b) For $v_I \le 4.03\,V$, $i_D = 0$

For $v_I > 4.03$, $i_D + \frac{10 - v_O}{10} = \frac{v_O - (-10)}{20}$

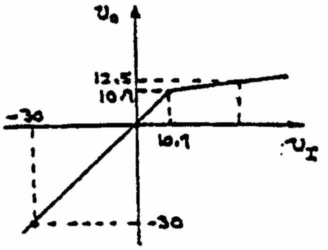
Which yields $i_D = \frac{3}{20} v_I - 0.605$

For $v_I = 10$, $i_D = 0.895\,mA$



2.23

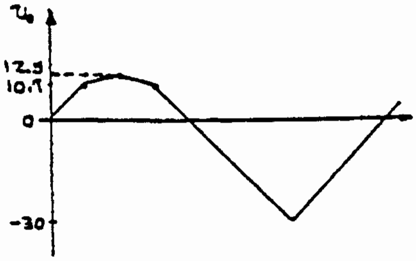
a.



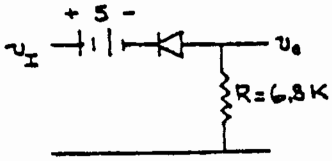
For $v_I = 30$ V, $i = \frac{30 - 10.7}{100 + 10} = 0.175$ A

$v_0 = i(10) + 10.7 = 12.5$ V

b.

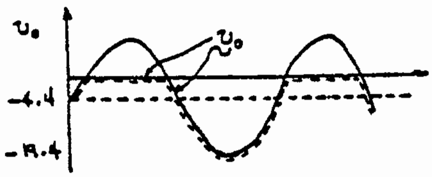


2.24



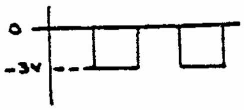
$V_\gamma = 0.6$ V

$v_I = 15 \sin \omega t$

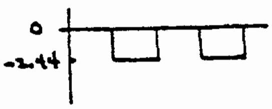


2.25

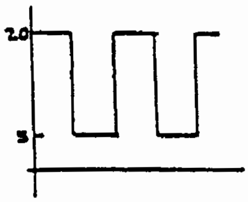
a. $V_\gamma = 0$



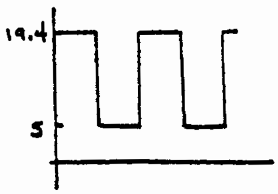
$V_\gamma = 0.6$



b. $V_\gamma = 0$

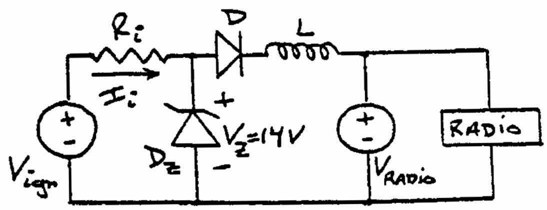


$V_\gamma = 0.6$



2.26

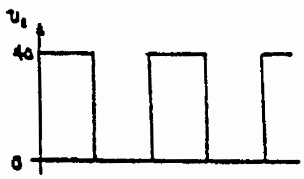
One possible example is shown.



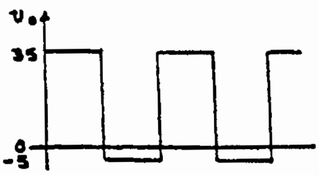
L will tend to block the transient signals
D_Z will limit the voltage to +14 V and –0.7 V.
Power ratings depends on number of pulses per second and duration of pulse.

2.27
$V_\gamma = 0$

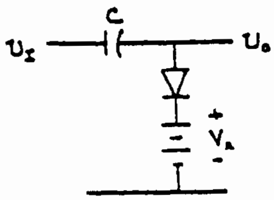
a.



b.

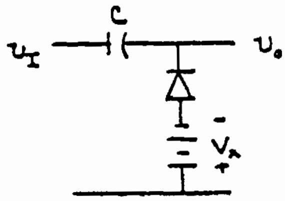


2.28



a. For $V_\gamma = 0 \Rightarrow V_B = 2.7$ V
b. For $V_\gamma = 0.7$ V $\Rightarrow V_B = 2.0$ V

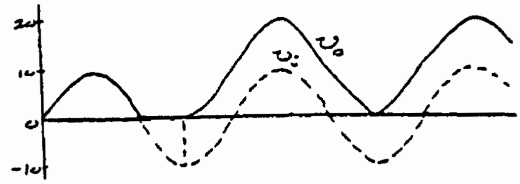
2.29



For $V_\gamma = 0$; $V_B = 10$ V

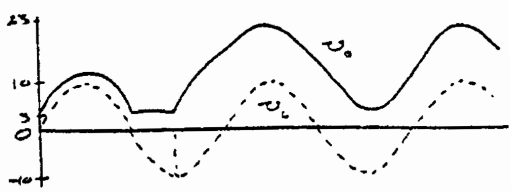
2.30

For circuit in Figure P2.27(a)



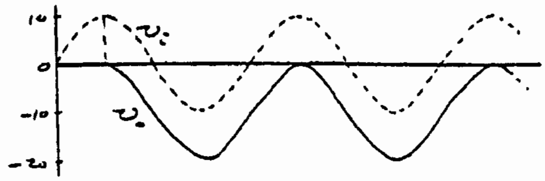
For circuit in Figure P2.27(b)
(i) For $V_B = +3V$

(ii) For $V_B = -3V$



2.31

For Figure P2.27(a)



2.32

a. $I_{D1} = \frac{10-0.6}{9.5+0.5} \Rightarrow \underline{I_{D1} = 0.94 \text{ mA}}$ $\underline{I_{D2} = 0}$

$V_0 = I_{D1}(9.5) \Rightarrow \underline{V_0 = 8.93 \text{ V}}$

b. $I_{D1} = \frac{5-0.6}{9.5+0.5} \Rightarrow \underline{I_{D1} = 0.44 \text{ mA}}$ $\underline{I_{D2} = 0}$

$V_0 = I_{D1}(9.5) \Rightarrow \underline{V_0 = 4.18 \text{ V}}$

c. Same as (a)

d. $10 = \frac{(I)}{2}(0.5) + 0.6 + I(9.5) \Rightarrow I = 0.964 \text{ mA}$

$V_0 = I(9.5) \Rightarrow \underline{V_0 = 9.16 \text{ V}}$

$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.482 \text{ mA}}$

2.33

a. $\underline{I = I_{D1} = I_{D2} = 0}$ $\underline{V_0 = 10}$

b. $10 = I(9.5) + 0.6 + I(0.5) \Rightarrow$

$\underline{I = I_{D2} = 0.94 \text{ mA}}$ $\underline{I_{D1} = 0}$

$V_0 = 10 - I(9.5) \Rightarrow \underline{V_0 = 1.07 \text{ V}}$

c. $10 = I(9.5) + 0.6 + I(0.5) + 5 \Rightarrow$

$\underline{I = I_{D2} = 0.44 \text{ mA}}$ $\underline{I_{D1} = 0}$

$V_0 = 10 - I(9.5) \Rightarrow \underline{V_0 = 5.82 \text{ V}}$

d. $10 = I(9.5) + 0.6 + \frac{I}{2}(0.5) \Rightarrow \underline{I = 0.964 \text{ mA}}$

$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.482 \text{ mA}}$

$V_0 = 10 - I(9.5) \Rightarrow \underline{V_0 = 0.842 \text{ V}}$

2.34

a. $V_1 = V_2 = 0 \Rightarrow D_1, D_2, D_3$, on $\underline{V_0 = 4.4 \text{ V}}$

$I = \frac{10-4.4}{9.5} \Rightarrow$ $\underline{I = 0.589 \text{ mA}}$

$I_{D1} = I_{D2} = \frac{4.4-0.6}{0.5} \Rightarrow \underline{I_{D1} = I_{D2} = 7.6 \text{ mA}}$

$I_{D3} = I_{D1} + I_{D2} - I = 2(7.6) - 0.589 \Rightarrow$

$\underline{I_{D3} = 14.6 \text{ mA}}$

b. $V_1 = V_2 = 5 \text{ V}$ D_1 and D_2 on, D_3 off

$10 = I(9.5) + 0.6 + \frac{I}{2}(0.5) + 5 \Rightarrow \underline{I = 0.451 \text{ mA}}$

$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.226 \text{ mA}}$

$\underline{I_{D3} = 0}$

$V_0 = 10 - I(9.5) = 10 - (0.451)(9.5) \Rightarrow$

$\underline{V_0 = 5.72 \text{ V}}$

c. $V_1 = 5 \text{ V}, V_2 = 0$ D_1 off, D_2, D_3 on

$\underline{V_0 = 4.4 \text{ V}}$

$I = \frac{10-4.4}{9.5} \Rightarrow$ $\underline{I = 0.589 \text{ mA}}$

$I_{D2} = \frac{4.4-0.6}{0.5} \Rightarrow$ $\underline{I_{D2} = 7.6 \text{ mA}}$

$\underline{I_{D1} = 0}$

$I_{D3} = I_{D2} - I = 7.6 - 0.589 \Rightarrow \underline{I_{D3} = 7.01 \text{ mA}}$

d. $V_1 = 5 \text{ V}, V_2 = 2 \text{ V}$ D_1 off, D_2, D_3 on

$\underline{V_0 = 4.4 \text{ V}}$

$I = \frac{10-4.4}{9.5} \Rightarrow$ $\underline{I = 0.589 \text{ mA}}$

$I_{D2} = \frac{4.4-0.6-2}{0.5} \Rightarrow$ $\underline{I_{D2} = 3.6 \text{ mA}}$

$\underline{I_{D1} = 0}$

$I_{D3} = I_{D2} - I = 3.6 - 0.589 \Rightarrow \underline{I_{D3} = 3.01 \text{ mA}}$

2.35

(a) D_1 *on*, D_2 *off*, D_3 *on*

So $\underline{I_{D2} = 0}$

Now $\underline{V_2 = -0.6V}$, $I_{D1} = \frac{10-0.6-(-0.6)}{R_1+R_2} = \frac{10}{2+6} \Rightarrow$

$\underline{I_{D1} = 1.25\,mA}$

$V_1 = 10 - 0.6 - (1.25)(2) \Rightarrow \underline{V_1 = 6.9\,V}$

$I_{R3} = \frac{-0.6-(-5)}{2} = 2.2\,mA$

$I_{D3} = I_{R3} - I_{D1} = 2.2 - 1.25 \Rightarrow \underline{I_{D3} = 0.95\,mA}$

(b) D_1 *on*, D_2 *on*, D_3 *off*

So $\underline{I_{D3} = 0}$

$\underline{V_1 = 4.4\,V}$, $I_{D1} = \frac{10-0.6-4.4}{R_1} = \frac{5}{6}$

or

$\underline{I_{D1} = 0.833\,mA}$

$I_{R2} = \frac{4.4-(-5)}{R_2+R_3} = \frac{9.4}{10} = 0.94\,mA$

$I_{D2} = I_{R2} - I_{D1} = 0.94 - 0.833 \Rightarrow \underline{I_{D2} = 0.107\,mA}$

$V_2 = I_{R2}R_3 - 5 = (0.94)(5) - 5 \Rightarrow \underline{V_2 = -0.3\,V}$

(c) All diodes are on

$V_1 = 4.4\,V, \quad V_2 = -0.6\,V$

$$I_{D_1} = 0.5\,mA = \frac{10-0.6-4.4}{R_1} \Rightarrow R_1 = 10\,k\Omega$$

$$I_{R2} = 0.5+0.5 = 1\,mA = \frac{4.4-(-0.6)}{R_2} \Rightarrow$$

$R_2 = 5\,k\Omega$

$$I_{R3} = 1.5\,mA = \frac{-0.6-(-5)}{R_3} \Rightarrow R_3 = 2.93\,k\Omega$$

2.36

For v_I small, both diodes off

$$v_O = \left(\frac{0.5}{0.5+5}\right)v_I = 0.0909v_I$$

When $v_I - v_O = 0.6$, D_1 turns on. So we have

$v_I - 0.0909v_I = 0.6 \Rightarrow v_I = 0.66, \quad v_O = 0.06$

For D_1 on

$$\frac{v_I - 0.6 - v_O}{5} + \frac{v_I - v_O}{5} = \frac{v_O}{0.5} \text{ which yields}$$

$$v_O = \frac{2v_I - 0.6}{12}$$

When $v_O = 0.6$, D_2 turns on. Then

$$0.6 = \frac{2v_I - 0.6}{12} \Rightarrow v_I = 3.9\,V$$

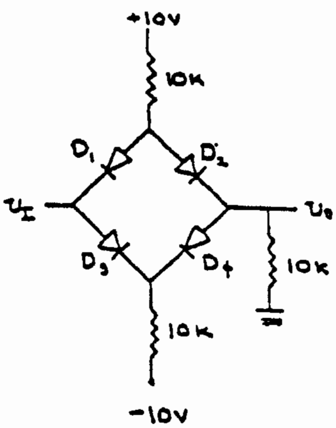
Now for $v_I > 3.9$

$$\frac{v_I - 0.6 - v_O}{5} + \frac{v_I - v_O}{5} = \frac{v_O}{0.5} + \frac{v_O - 0.6}{0.5}$$

Which yields

$$v_O = \frac{2v_I + 5.4}{22};\ For\ v_I = 10 \Rightarrow v_O = 1.15\,V$$

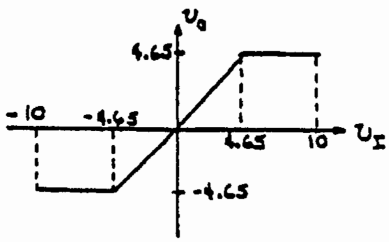
2.37



For $v_I > 0$, when D_1 turns off

$$I = \frac{10-0.7}{20} = 0.465\text{ mA}$$

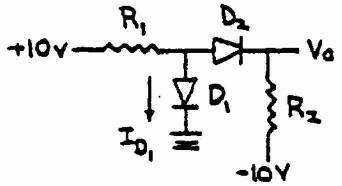
$v_O = I(10\text{ k}\Omega) = 4.65\text{ V}$



$v_O = v_I$ for $-4.65 \le v_I \le 4.65$

2.38

a.



$R_1 = 5\text{ k}\Omega,\ R_2 = 10\text{ k}\Omega$

D_1 and D_2 on $\Rightarrow$ $V_O = 0$

$$I_{D1} = \frac{10-0.7}{5} - \frac{0-(-10)}{10} = 1.86 - 1.0$$

$I_{D1} = 0.86\text{ mA}$

b. $R_1 = 10\text{ k}\Omega,\ R_2 = 5\text{ k}\Omega,\ D_1$ off, D_2 on

$I_{D1} = 0$

$$I = \frac{10-0.7-(-10)}{15} = 1.287$$

$V_O = IR_2 - 10 \Rightarrow V_O = -3.57\text{ V}$

2.39

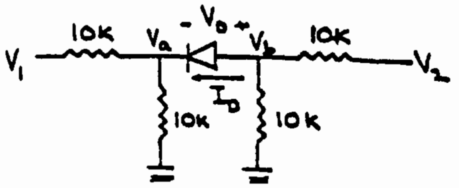
$$\frac{15-(V_O+0.7)}{10} = \frac{V_O+0.7}{20} + \frac{V_O}{20}$$

$$\frac{15}{10} - \frac{0.7}{10} - \frac{0.7}{20} = V_O\left(\frac{1}{10} + \frac{1}{20} + \frac{1}{20}\right) = V_O\left(\frac{4.0}{20}\right)$$

$V_O = 6.975\text{ V}$

$$I_D = \frac{V_O}{20} \Rightarrow I_D = 0.349\text{ mA}$$

2.40



a. $V_1 = 15$ V, $V_2 = 10$ V Diode off

$V_a = 7.5$ V, $V_b = 5$ V $\Rightarrow$ $\underline{V_D = -2.5\text{ V}}$

$\underline{I_D = 0}$

b. $V_1 = 10$ V, $V_2 = 15$ V Diode on

$$\frac{V_2 - V_b}{10} = \frac{V_b}{10} + \frac{V_a}{10} + \frac{V_a - V_1}{10} \Rightarrow V_a = V_b - 0.6$$

$$\frac{15}{10} + \frac{10}{10} = V_b\left(\frac{1}{10} + \frac{1}{10}\right) + V_b\left(\frac{1}{10} + \frac{1}{10}\right)$$

$$- 0.6\left(\frac{1}{10} + \frac{1}{10}\right)$$

$$2.62 = V_b\left(\frac{4}{10}\right) \Rightarrow V_b = 6.55\text{ V}$$

$$I_D = \frac{15 - 6.55}{10} - \frac{6.55}{10} \Rightarrow \underline{I_D = 0.19\text{ mA}}$$

$\underline{V_D = 0.6\text{ V}}$

2.41

$v_I = 0$, D_1 off, D_2 on

$$I = \frac{10 - 2.5}{15} = 0.5\text{ mA}$$

$v_0 = 10 - (0.5)(5) \Rightarrow \underline{v_0 = 7.5\text{ V for } 0 < v_I < 7.5}$

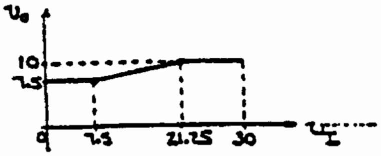
For $v_I = 30$ V, D_2 off, $v_0 = 10$ V

Determine v_I when $V_a = 10$

$$I = \frac{v_I - 2.5}{25}$$

$V_a = 10 = I(10) + 2.5 \Rightarrow I = 0.75$ mA

$v_I = (0.75)(25) + 2.5 = 21.25$



2.42

a. $\underline{V_{01} = V_{02} = 0}$

b. $\underline{V_{01} = 4.4\text{ V}}$, $\underline{V_{02} = 3.8\text{ V}}$

c. $\underline{V_{01} = 4.4\text{ V}}$, $\underline{V_{02} = 3.8\text{ V}}$

Logic "1" level degrades as it goes through additional logic gates.

2.43

a. $\underline{V_{01} = V_{02} = 5\text{ V}}$

b. $\underline{V_{01} = 0.6\text{ V}}$, $\underline{V_{02} = 1.2\text{ V}}$

c. $\underline{V_{01} = 0.6\text{ V}}$, $\underline{V_{02} = 1.2\text{ V}}$

Logic "0" signal degrades as it goes through additional logic gates.

2.44

$(V_1\ AND\ V_2)\ OR\ (V_3\ AND\ V_4)$

2.45

$$I = \frac{10 - 1.5 - 0.2}{R + 10} = 12\text{ mA} = 0.012$$

$$R + 10 = \frac{8.3}{0.012} = 691.7\Omega$$

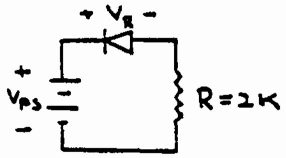
$\underline{R = 681.7\Omega}$

2.46

$$I = \frac{10 - 1.7 - V_I}{0.75} = 8$$

$V_I = 10 - 1.7 - 8(0.75) \Rightarrow \underline{V_I = 2.3\text{ V}}$

2.47



$V_R = 1$ V, $I = 0.8$ mA

$V_{PS} = 1 + (0.8)(2)$

$\underline{V_{PS} = 2.6\text{ V}}$

2.48

$I_{Ph} = \eta e \Phi A$

$0.6 \times 10^{-3} = (1)(1.6 \times 10^{-19})(10^{17})A$

$\underline{A = 3.75 \times 10^{-2}\text{ cm}^2}$

Chapter 3

Exercise Solutions

E3.1

$\beta = \frac{\alpha}{1-\alpha}$

For $\alpha = 0.980$, $\beta = \frac{0.980}{1-0.980} = 49$

For $\alpha = 0.995$, $\beta = \frac{0.995}{1-0.995} = 199$

$\underline{49 \le \beta \le 199}$

E3.2

$\alpha = \frac{\beta}{1+\beta} = \frac{75}{76} = \underline{0.9868}$

$\alpha = \frac{125}{126} = \underline{0.9921}$

E3.3

$I_E = (1+\beta)I_B$

So $(1+\beta) = \frac{I_E}{I_B} = \frac{0.780}{0.00960} = 81.25 \Rightarrow \underline{\beta = 80.3}$

$\alpha = \frac{\beta}{1+\beta} = \frac{80.3}{81.3} \Rightarrow \underline{\alpha = 0.9877}$

$I_C = \beta I_B = (80.3)(9.60\ \mu A) \Rightarrow \underline{I_C = 0.771\ mA}$

E3.4

$\beta = \frac{\alpha}{1-\alpha} = \frac{0.990}{1-0.990} \Rightarrow \underline{\beta = 99}$

$I_B = \frac{I_E}{(1+\beta)} = \frac{2.150}{100} \Rightarrow \underline{I_B = 21.50\ \mu A}$

$I_C = \alpha I_E = (0.990)(2.150) \Rightarrow \underline{I_C = 2.13\ mA}$

E3.5

$r_0 = \frac{V_A}{I_C} = \frac{150}{I_C}$

$I_C = 0.1\ mA \Rightarrow r_0 = 1.5\ M\Omega$

$I_C = 1.0\ mA \Rightarrow r_0 = 150\ k\Omega$

$I_C = 10\ mA \Rightarrow r_0 = 15\ k\Omega$

E3.6

$I_C = I_0\left(1 + \frac{V_{CE}}{V_A}\right)$

At $V_{CE} = 1$, $I_C = 1$

a. $V_A = 75$

$I_C = 1 = I_0\left(1 + \frac{1}{75}\right) \Rightarrow I_0 = 0.9868\ mA$

At $V_{CE} = 10$, $I_C = (0.9868)\left(1 + \frac{10}{75}\right) \Rightarrow$

$\underline{I_C = 1.12\ mA}$

b. $V_A = 150$

$I_C = 1 = I_0\left(1 + \frac{1}{150}\right) \Rightarrow I_0 = 0.9934\ mA$

At $V_{CE} = 10$, $I_C = (0.9934)\left(1 + \frac{10}{150}\right) \Rightarrow$

$\underline{I_C = 1.06\ mA}$

E3.7

$BV_{CEo} = \frac{BV_{CBo}}{\sqrt[n]{\beta}} = \frac{200}{\sqrt[3]{120}} = 40.5$ volts

E3.8

$BV_{CEo} = \frac{BV_{CBo}}{\sqrt[3]{\beta}}$

$BV_{CBo} = (\sqrt[3]{100})(30) = 139\ V$

E3.9

a. $V_I = 0.2 < V_{BE}(on) \Rightarrow \underline{I_B = I_C = 0}$, $\underline{V_0 = 5\ V}$
$P = 0$

b. $V_I = 3.6$ Transistor is driven into saturation.

$I_B = \frac{3.6 - 0.7}{0.64} \Rightarrow \underline{I_B = 4.53\ mA}$

$I_C = \frac{5 - V_{CE}(sat)}{R_C} = \frac{5 - 0.2}{0.44} \Rightarrow \underline{I_C = 10.9\ mA}$

Note that $\frac{I_C}{I_B} = \frac{10.9}{4.53} = 2.41 < \beta$ which shows that the transistor is indeed in saturation.

$P = I_C V_{CE} + I_B V_{BE} = (10.9)(0.2) + (4.53)(0.7)$

$= 2.18 + 3.17$

$P = 5.35\ mW$

E3.10

For $V_{BC} = 0 \Rightarrow V_0 = 0.7\ V$

Then $I_C = \frac{5 - 0.7}{0.44} \Rightarrow I_C = 9.77\ mA$

and $I_B = \frac{I_C}{\beta} = \frac{9.77}{50} = 0.195\ mA$

$V_I = I_B R_B + V_{BE}(on) = (0.195)(0.64) + 0.7$

$\Rightarrow V_I = 0.825\ V$

Power $= I_C V_{CE} + I_B V_{BE}$

$= (9.77)(0.7) + (0.195)(0.7)$

$\underline{\text{Power} = 6.98\ mW}$

E3.11

For $V_C = 4$ V and $I_{CQ} = 1.5$ mA

$R_C = \frac{10-4}{1.5} \Rightarrow \underline{R_C = 4\text{ k}\Omega}$

$I_E = \frac{-V_{BE}(\text{on}) - (-10)}{R_E}$

$I_E = \left(\frac{101}{100}\right) I_C = 1.515$ mA

$R_E = \frac{10-0.70}{1.515} \Rightarrow \underline{R_E = 6.14\text{ k}\Omega}$

E3.12

$I_C = \frac{10 - V_C}{R_C} = \frac{10-6.34}{4} \Rightarrow I_C = 0.915$ mA

$I_E = \frac{-V_{BE}(\text{on}) - (-10)}{R_E} = \frac{10-0.7}{10} \Rightarrow$

$\underline{I_E = 0.930\text{ mA}}$

$I_C = \alpha I_E \Rightarrow \alpha = \frac{I_C}{I_E} = \frac{0.915}{0.930} \Rightarrow \underline{\alpha = 0.9839}$

$I_B = I_E - I_C = 0.930 - 0.915 \Rightarrow \underline{I_B = 0.0150\text{ mA}}$

$\beta = \frac{I_C}{I_B} = \frac{0.915}{0.015} \Rightarrow \underline{\beta = 61}$

$V_{CE} = V_C - V_E = 6.34 - (-0.70) \Rightarrow$

$\underline{V_{CE} = 7.04\text{ V}}$

E3.13

$I_E = \frac{10 - V_{EB}(\text{on})}{R_E} = \frac{10-0.7}{8} \Rightarrow \underline{I_E = 1.16\text{ mA}}$

$I_B = \frac{I_E}{(1+\beta)} = \frac{1.16}{51} \Rightarrow \underline{I_B = 2.27\ \mu\text{A}}$

$I_C = \frac{\beta}{1+\beta} I_E = \frac{50}{51}(1.16) \Rightarrow \underline{I_C = 1.14\text{ mA}}$

$V_C = I_C R_C - 10 = (1.14)(4) - 10 = -5.44$

$V_{EC} = 0.7 - (-5.44) \Rightarrow \underline{V_{EC} = 6.14\text{ V}}$

E3.14

$I_E = \frac{V_{BB} - V_{EB}(\text{on})}{R_E}$

$\Rightarrow R_E = \frac{4-0.7}{1.0} \Rightarrow \underline{R_E = 3.3\text{ k}\Omega}$

$I_C = \alpha I_E = (0.9920)(1.0) \Rightarrow \underline{I_C = 0.992\text{ mA}}$

$I_B = I_E - I_C = 1.0 - 0.9920 \Rightarrow \underline{I_B = 0.0080\text{ mA}}$

$V_{CB} = -V_{BC} = I_C R_C - V_{CC} = (0.992)(1) - 5$

$\Rightarrow \underline{V_{BC} = 4.01\text{ V}}$

E3.15

$V_{BB} = I_B R_B + V_{BE}(\text{on}) + I_E R_E$

$= I_B R_B + V_{BE}(\text{on}) + (1+\beta) I_B R_E$

$I_B = \frac{V_{BB} - V_{BE}}{R_B + (1+\beta) R_E} = \frac{2-0.7}{10 + (76)(1)}$

$\Rightarrow \underline{I_B = 15.1\ \mu\text{A}}$

$I_C = \beta I_B = (75)(15.1\ \mu\text{A}) \Rightarrow \underline{I_C = 1.13\text{ mA}}$

$I_E = (1+\beta) I_B = (76)(15.1\ \mu\text{A}) \Rightarrow \underline{I_E = 1.15\text{ mA}}$

$V_{CE} = V_{CC} + V_{BB} - I_C R_C - I_E R_E$

$= 8 + 2 - (1.13)(2.5) - (1.15)(1)$

$\underline{V_{CE} = 6.03\text{ V}}$

E3.16

$V_{CE} = 2.5 \Rightarrow V_E = 2.5\text{ V} = I_E R_E$

$V_{BB} = I_B R_B + V_{BE}(\text{on}) + V_E$

$I_B = \frac{V_{BB} - V_{BE}(\text{on}) - V_E}{R_B} = \frac{5 - 0.7 - 2.5}{10}$

$I_B = 0.18\text{ mA} \Rightarrow I_E = (101)(0.18)$

$\Rightarrow I_E = 18.18$ mA

So $R_E = \frac{2.5}{18.18} \Rightarrow \underline{R_E = 0.138\text{ k}\Omega = 138\ \Omega}$

E3.17

$V_{BB} = I_E R_E + V_{EB}(\text{on}) + I_B R_B$

$I_E = 2.2\text{ mA} \Rightarrow I_B = \frac{2.2}{51} = 0.0431$ mA

$I_C = \left(\frac{\beta}{1+\beta}\right) I_E = \left(\frac{50}{51}\right)(2.2) \Rightarrow \underline{I_C = 2.16\text{ mA}}$

$V_{BB} = (2.2)(1) + 0.7 + (0.0431)(50)$

$\Rightarrow \underline{V_{BB} = 5.06\text{ V}}$

$V_{EC} = 5 - I_E R_E = 5 - (2.2)(1)$

$\Rightarrow \underline{V_{EC} = 2.8\text{ V}}$

E3.18

(1) $6 = I_B R_B + V_{BE}(\text{on}) + I_E R_E$

(2) $5 = I_C R_C + V_{CE}(\text{sat}) + I_E R_E$

$I_E = I_B + I_C$

(1) $6 = 10 I_B + 0.7 + (I_B + I_C)(1)$

(2) $5 = 4 I_C + 0.2 + (I_B + I_C)(1)$

(1) $[5.3 = I_C + 11 I_B] \times 5 \rightarrow \quad 26.5 = 5 I_C + 55 I_B$

(2) $4.8 = 5 I_C + I_B \qquad\qquad \underline{4.8 = 5 I_C + I_B}$

$21.7 = 54 I_B$

$\Rightarrow \underline{I_B = 0.402\text{ mA}}$

From (1), $I_C = 5.3 - 11 I_B \Rightarrow \underline{I_C = 0.880\text{ mA}}$

$\underline{I_E = 1.28\text{ mA}}$, $\underline{V_{CE} = V_{CE}(\text{sat}) = 0.2\text{ V}}$

E3.19

$\underline{V_{EC} = V_{EC}(\text{sat}) = 0.2\text{ V}}$

$I_C = \frac{-0.2 - (-5)}{R_C} = \frac{5-0.2}{10} \Rightarrow \underline{I_C = 0.48\text{ mA}}$

$I_B = \frac{I_C}{2} = \frac{0.48}{2} = \underline{0.24 = I_B}$

$V_I + I_B R_B + V_{EB}(\text{on}) = 0$

$\Rightarrow V_I = -(0.24)(20) - 0.7 \Rightarrow \underline{V_I = -5.5\text{ V}}$

E3.20

a. $V_I = -4.5\text{ V} \Rightarrow V_{BE} < V_{BE}(\text{on}) \Rightarrow$ Transistor is cutoff. $\underline{I_B = I_C = I_E = 0}$, $\underline{V_{CE} = 10\text{ V}}$

b. $V_I = -3.5$ V Transistor is active.

$V_I = I_B R_B + V_{BE}(\text{on}) + I_E R_E - 5$

$5 - 3.5 = I_B(10) + 0.7 + (76)I_B(4)$

$$I_B = \frac{5 - 3.5 - 0.7}{10 + (76)(4)} = 0.00255\text{ mA}$$

$\Rightarrow \underline{I_B = 2.55\ \mu\text{A}}$

$I_C = \beta I_B = (75)(2.55\ \mu\text{A}) \Rightarrow \underline{I_C = 0.191\text{ mA}}$

$I_E = (1 + \beta)I_B = (76)(2.55\ \mu\text{A})$

$\Rightarrow \underline{I_E = 0.194\text{ mA}}$

$V_{CE} = 10 - I_C R_C - I_E R_E$

$= 10 - (0.191)(2) - (0.194)(4)$

$\underline{V_{CE} = 8.84\text{ V}}$

c. $V_I = +3.5$ V Transistor is in saturation.

(1) $3.5 = I_B R_B + V_{BE}(\text{on}) + I_E R_E - 5$

(2) $5 = I_C R_C + V_{CE}(\text{sat}) + I_E R_E - 5$

(3) $I_E = I_B + I_C$

(1) $3.5 + 5 - 0.7 = 10 I_B + 4(I_B + I_C)$

(2) $5 + 5 - 0.2 = 2I_C + 4(I_B + I_C)$

(1) $7.8 = 14 I_B + 4 I_C$

(2) $9.8 = 4 I_B + 6 I_C$

$3 \times (1) \Rightarrow 23.4 = 42 I_B + 12 I_C$

$2 \times (2) \Rightarrow \underline{19.6 = 8 I_B + 12 I_C}$

$3.8 = 34 I_B$

$\Rightarrow \underline{I_B = 0.112\text{ mA}}$

$7.8 = 14(0.112) + 4I_C = 1.568 + 4I_C$

$\Rightarrow \underline{I_C = 1.56\text{ mA}}$

$\frac{I_C}{I_B} = 13.9 < \beta \Rightarrow$ In saturation.

$I_E = I_B + I_C \Rightarrow \underline{I_E = 1.67\text{ mA}}$

$\underline{V_{CE} = V_{CE}(\text{sat}) = 0.2}$

E3.21

$$I_C(sat) = \frac{5 - 1.5 - 0.2}{R} = 15 \Rightarrow \underline{R = 0.220\ k\Omega}$$

$$I_B = \frac{I_C}{20} = \frac{15}{20} = 0.75\ mA = \frac{5 - 0.8}{R_B}$$

or

$\underline{R_B = 5.6\ k\Omega}$

E3.22

a. $V_1 = V_2 = 0$, $I_{B1} = I_{B2} = I_{C1} = I_{C2} = I_R = 0$
$V_0 = 5$ V

b. $V_1 = 5$ V, $V_2 = 0$, $I_{B2} = I_{C2} = 0$

$$I_{B1} = \frac{5 - 0.7}{0.95} \Rightarrow I_{B1} = 4.53\text{ mA}$$

$$I_{C1} = \frac{5 - 0.2}{0.6} \Rightarrow I_{C1} = I_R = 8\text{ mA}$$

$V_0 = 0.2$ V

c. $V_1 = V_2 = 5$ V. $I_{B1} = I_{B2} = 4.53$ mA

$I_R = 8$ mA. $I_{C1} = I_{C2} = 4$ mA, $V_0 = 0.2$ V

E3.23

$v_O = 5 - i_C R_C = 5 - \beta i_B R_C$

and

$$i_B = \frac{V_{BB} + \Delta v_I - V_{BE}(on)}{R_B}$$

Then

$$\Delta v_O = \frac{-\beta R_C \Delta v_I}{R_B}$$

or

$$\frac{\Delta v_O}{\Delta v_I} = \frac{-\beta R_C}{R_B}$$

Let $\beta = 100$, $R_C = 5\ k\Omega$, $R_B = 100\ k\Omega$

Then

$$\frac{\Delta v_O}{\Delta v_I} = \frac{-(100)(5)}{100} = -5$$

Want Q-point to be

$v_O(Q-pt) = 2.5 = 5 - (100)I_{BQ}(5)$

Then

$$\underline{I_{BQ} = 0.005\ mA}\ ,\ I_{BQ} = 0.005 = \frac{V_{BB} - 0.7}{100}$$

or

$\underline{V_{BB} = 1.2\ V}$

Also $I_{CQ} = \beta I_{BQ} = (100)(0.005)$

Or

$\underline{I_{CQ} = 0.5\ mA}$

E3.24

a. For $V_{CEQ} = 2.5\text{ V} \Rightarrow I_{CQ} = \frac{5 - 2.5}{2}$

$\Rightarrow \underline{I_{CQ} = 1.25\text{ mA}}$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{1.25}{100} \Rightarrow \underline{I_{BQ} = 12.5\ \mu\text{A}}$$

Then $R_B = \frac{5 - 0.7}{0.0125} \Rightarrow \underline{R_B = 344\text{ k}\Omega}$

b. I_{BQ} is independent of β.

For $V_{CEQ} = 1$ V. $I_C = \frac{5 - 1}{2} = 2$ mA

$$\beta = \frac{I_C}{I_B} = \frac{2}{0.0125} \Rightarrow \beta = 160$$

For $V_{CEQ} = 4$ V. $I_C = \frac{5-4}{2} = 0.5$ mA

$$\beta = \frac{I_C}{I_B} = \frac{0.5}{0.0125} \Rightarrow \beta = 40$$

So $\underline{40 \le \beta \le 160}$

E3.25

$$I_{BQ} = \frac{5-0.7}{800} \Rightarrow I_{BQ} = 0.005375 \text{ mA}$$

$\beta = 75 \Rightarrow I_{CQ} = (75)(0.005375) = 0.403$ mA

$\beta = 150 \Rightarrow I_{CQ} = (150)(0.005375) = 0.806$ mA

Largest $I_{CQ} \Rightarrow$ Smallest V_{CEQ}

$$\beta = 150 \Rightarrow R_C = \frac{5-1}{0.806} = 4.96 \text{ k}\Omega$$

$$\beta = 75 \Rightarrow R_C = \frac{5-4}{0.403} = 2.48 \text{ k}\Omega$$

For a nominal $I_C = 0.604$ mA and $V_{CEQ} = 2.5$

$$R_C = \frac{5-2.5}{0.604} = \underline{4.14 = R_C}$$

For $I_{CQ} = 0.403$,

$V_{CEQ} = 5 - (0.403)(4.14) = 3.33$ V

For $I_{CQ} = 0.806$,

$V_{CEQ} = 5 - (0.806)(4.14) = 1.66$ V

So for $R_C = 4.14$, $\underline{1.66 \text{ V} \le V_{CEQ} \le 3.33 \text{ V}}$

E3.26

a. $R_{TH} = R_1 \| R_2 = 9 \| 2.25 \Rightarrow \underline{R_{TH} = 1.8 \text{ k}\Omega}$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{2.25}{9 + 2.25}\right)(5)$$

$\Rightarrow \underline{V_{TH} = 1.0 \text{ V}}$

b. $$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{1-0.7}{1.8 + (151)(0.2)}$$

$\Rightarrow \underline{I_{BQ} = 9.375\ \mu\text{A}}$

$I_{CQ} = \beta I_{BQ} = (150)(9.375\ \mu\text{A})$

$\Rightarrow \underline{I_{CQ} = 1.41 \text{ mA}}$

$I_{EQ} = (1+\beta)I_{BQ} \Rightarrow \underline{I_{EQ} = 1.42 \text{ mA}}$

$$V_{CEQ} = 5 - I_{CQ}R_C - I_{EQ}R_E$$

$$= 5 - (1.41)(1) - (1.42)(0.2)$$

$\underline{V_{CEQ} = 3.31 \text{ V}}$

c. For $\beta = 75$

$$I_{BQ} = \frac{1-0.7}{1.8 + (76)(0.2)} \Rightarrow \underline{I_{BQ} = 17.6\ \mu\text{A}}$$

$I_{CQ} = \beta I_{BQ} = (75)(17.6\ \mu\text{A})$

$\Rightarrow \underline{I_{CQ} = 1.32 \text{ mA}}$

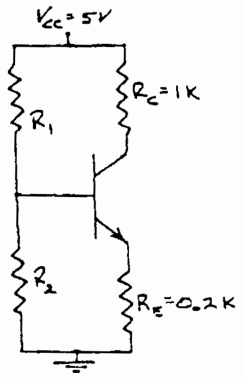
$I_{EQ} = (1+\beta)I_{BQ} = (76)(17.6\ \mu\text{A})$

$\Rightarrow \underline{I_{EQ} = 1.34 \text{ mA}}$

$V_{CEQ} = 5 - (1.32)(1) - (1.34)(0.2)$

$\Rightarrow \underline{V_{CEQ} = 3.41 \text{ V}}$

E3.27



$R_1 + R_2 = 11.25$ kΩ, $\beta = 150$

$I_C \approx I_E$. $V_{CEQ} = 2.5$ V

So $I_{CQ} \approx I_{EQ} = \frac{5-2.5}{1+0.2} = 2.081$ mA

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{2.08}{150} = 13.9\ \mu\text{A}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E}$$

$$0.0139 = \frac{\left(\frac{R_2}{R_1+R_2}\right)V_{CC} - V_{BE}(\text{on})}{\frac{R_1 R_2}{R_1 + R_2} + (1+\beta)R_E} = \frac{\left(\frac{R_2}{11.25}\right)(5) - 0.7}{\frac{R_1 R_2}{(11.25)} + (151)(0.2)}$$

$R_2 = 11.25 - R_1$, so

$$0.0139[R_1(11.25 - R_1) + (151)(0.2)(11.25)]$$

$$= 5R_2 - (0.7)(11.25)$$

$$= 5(11.25 - R_1) - (0.7)(11.25)$$

$$0.156R_1 - 0.0139R_1^2 + 4.72 = 56.25 - 5R_1 - 7.875$$

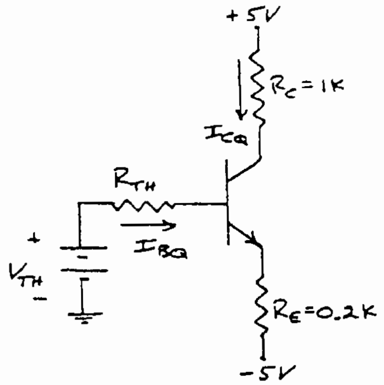
$$0.0139R_1^2 - 5.156R_1 + 43.66 = 0$$

$$R_1 = \frac{5.156 \pm \sqrt{(5.156)^2 - 4(0.0139)(43.66)}}{2(0.0139)}$$

$\Rightarrow R_1 = 8.67$ kΩ and $R_2 = 2.58$ kΩ

E3.28

dc equivalent circuit



$\beta = 150,\ R_{TH} = R_1\|R_2,$

$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(10) - 5$$

$$I_{CQ} = \frac{5-0}{1} = 5 \text{ mA}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{5}{150} = 0.0333 \text{ mA}$$

$$I_{BQ} = \frac{V_{TH}+5-0.7}{R_{TH}+(1+\beta)R_E}$$

Set $R_{TH} = (0.1)(1+\beta)R_E$

$$I_{BQ} = \frac{V_{TH}+4.3}{(1.1)(1+\beta)R_E}$$

$$\Rightarrow 0.0333 = \frac{\left(\frac{R_2}{R_1+R_2}\right)(10) - 5 + 4.3}{(1.1)(151)(0.2)}$$

$$(0.0333)(1.1)(151)(0.2) = \left(\frac{R_2}{R_1+R_2}\right)(10) - 0.7$$

So $\left(\frac{R_2}{R_1+R_2}\right) = 0.1806$

$$R_{TH} = \frac{R_1R_2}{R_1+R_2} = (0.1)(151)(0.2) = 3.02 \text{ k}\Omega$$

Then $R_1(0.1806) = 3.02 \Rightarrow \underline{R_1 = 16.7 \text{ k}\Omega}$

$R_2 = (0.1806)(16.7 + R_2) \Rightarrow 0.8194R_2 = 3.02$

$\Rightarrow \underline{R_2 = 3.68 \text{ k}\Omega}$

E3.29

$\beta = 120,\ V_{CEQ} = 5$ V

$R_{TH} = R_1\|R_2,\ V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(10) - 5$

$I_{CQ} \approx I_{EQ}$

So $I_{CQ} = \dfrac{10 - V_{CEQ}}{R_C + R_E}$

$$I_{CQ} = \frac{10-5}{1.2+0.3} \Rightarrow \underline{I_{CQ} = 3.33 \text{ mA}}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{3.33}{120} \Rightarrow I_{BQ} = 0.0278 \text{ mA}$$

$$I_{BQ} = \frac{V_{TH}+5-0.7}{R_{TH}+(1+\beta)R_E}$$

Set $R_{TH} = (0.1)(1+\beta)R_E$

$$I_{BQ} = 0.0278 = \frac{\left(\frac{R_2}{R_1+R_2}\right)(10) - 5 + 5 - 0.7}{(1.1)(121)(0.3)}$$

$$(0.0278)(1.1)(121)(0.3) = \left(\frac{R_2}{R_1+R_2}\right)(10) - 0.7$$

$$\left(\frac{R_2}{R_1+R_2}\right) = 0.181$$

$$R_{TH} = \frac{R_1R_2}{R_1+R_2} = R_1(0.181) = (0.1)(121)(0.3)$$

$\Rightarrow \underline{R_1 = 20.1 \text{ k}\Omega}$

$R_2 = (0.181)(20.1 + R_2) \Rightarrow 0.819R_2 = 3.63 \text{ k}\Omega$

$\Rightarrow \underline{R_2 = 4.44 \text{ k}\Omega}$

E3.30

a. $I_{CQ} = \left(\dfrac{\beta}{1+\beta}\right)I_{EQ} = \left(\dfrac{100}{101}\right)(1) = 0.99$ mA

$$I_{BQ} = \frac{1 \text{ mA}}{1+\beta} = \frac{1}{101} \Rightarrow 9.90\ \mu\text{A}$$

$V_B = -I_{BQ}R_B = -(0.0099)(50)$

$\Rightarrow \underline{V_B = -0.495 \text{ V}}$

$$V_{BE} = V_T \ln\left(\frac{I_{CQ}}{I_S}\right) = (0.026)\ln\left(\frac{0.99\times10^{-3}}{3\times10^{-14}}\right)$$

$\Rightarrow V_{BE} = 0.630$ V

$V_E = V_B - V_{BE} = -0.495 - 0.630$

$\Rightarrow V_E = -1.13$ V

$V_C = 10 - (0.99)(5) = 5.05$ V

$V_{CEQ} = V_C - V_E = 5.05 - (-1.13)$

$\Rightarrow \underline{V_{CEQ} = 6.18 \text{ V}}$

b. $I_{EQ} = 1$ mA. $I_B = \dfrac{1}{51} = 0.0196$ mA

$V_B = -(0.0196)(50) = \underline{-0.98 \text{ V} = V_B}$

$$I_{CQ} = \left(\frac{50}{51}\right)(1) = 0.98 \text{ mA}$$

$$V_{BE} = (0.026)\ln\left(\frac{0.98\times10^{-3}}{3\times10^{-14}}\right) = 0.629 \text{ V}$$

$V_E = -0.98 - 0.629 = -1.61$

$V_C = 10 - (0.98)(5) = 5.1$ V

$V_{CEQ} = 5.1 - (-1.61) \Rightarrow \underline{V_{CEQ} = 6.71 \text{ V}}$

E3.31

$$I_B = \frac{I_Q}{1+\beta} = \frac{I_Q}{121},\ V_B = \left(\frac{I_Q}{121}\right)(20) = I_Q(0.165)$$

$$V_E = I_Q(0.165) + 0.7$$

$$I_{CQ} = \left(\frac{\beta}{1+\beta}\right)I_{EQ} = \left(\frac{120}{121}\right)I_Q = (0.992)I_Q$$

$$V_C = I_{CQ}R_C - 5 = (0.992)I_Q(4) - 5$$

$$= 3.97I_Q - 5$$

$$V_{ECQ} = V_E - V_C$$

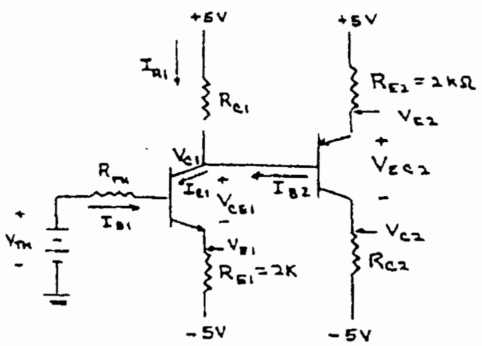
$$= [I_Q(0.165) + 0.7] - [3.97I_Q - 5]$$

$$= -3.805I_Q + 5.7$$

$$-3.805I_Q + 5.7 = 3$$

$$\Rightarrow \underline{I_Q = 0.710 \text{ mA}}$$

E3.32



$$R_{TH} = 50\|100 = 33.3 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{50}{150}\right)(10) - 5 = -1.67 \text{ V}$$

$$I_{B1} = \frac{5 - 1.67 - 0.7}{33.3 + (101)(2)} = \frac{2.63}{235} = 11.2\ \mu\text{A}$$

$$I_{C1} = 1.12 \text{ mA},\ I_{E1} = 1.13 \text{ mA}$$

$$V_{E1} = I_{E1}R_{E1} - 5 = (1.13)(2) - 5 = -2.74 \text{ V}$$

$$V_{CE1} = 3.25 \text{ V} \Rightarrow V_{C1} = 0.51 \text{ V}$$

$$\Rightarrow V_{E2} = 0.51 + 0.7 = 1.21 \text{ V}$$

$$I_{E2} = \frac{5 - 1.21}{2} = 1.90 \text{ mA} \Rightarrow I_{B2} = 18.8\ \mu\text{A}$$

$$I_{C2} = 1.88 \text{ mA}$$

$$I_{R1} = I_{C1} - I_{B2} = 1.12 - 0.0188 = 1.10 \text{ mA}$$

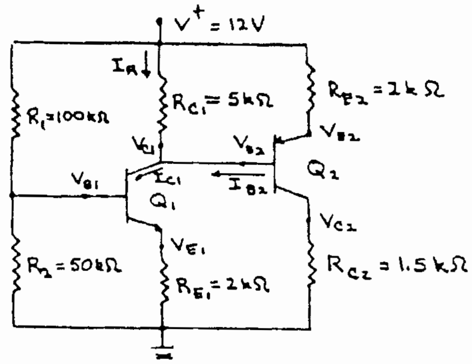
$$R_{C1} = \frac{5 - 0.51}{1.10} \Rightarrow \underline{R_{C1} = 4.08 \text{ k}\Omega}$$

$$V_{EC2} = 2.5 \Rightarrow V_{C2} = V_{E2} - V_{EC2}$$

$$= 1.21 - 2.5 = -1.29$$

$$R_{C2} = \frac{-1.29 - (-5)}{1.88} \Rightarrow \underline{R_{C2} = 1.97 \text{ k}\Omega}$$

E3.33



$$I_{B1} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_{E1}} = \frac{4 - 0.7}{33.3 + (101)(2)}$$

$$\underline{I_{B1} = 14\ \mu\text{A}},\ \underline{I_{C1} = 1.40 \text{ mA}},\ \underline{I_{E1} = 1.42 \text{ mA}},$$

$$V_{B1} = 4 - (0.014)(33.3)$$

$$\Rightarrow \underline{V_{B1} = 3.53 \text{ V}},\ \underline{V_{E1} = 2.83 \text{ V}}$$

$$I_R + I_{B2} = I_{C1}$$

$$\Rightarrow \frac{12 - V_{C1}}{5} + \frac{12 - (V_{C1} + 0.7)}{(101)(2)} = 1.40$$

$$\frac{12}{5} + \frac{(12 - 0.7)}{(101)(2)} - 1.40 = \frac{V_{C1}}{5} + \frac{V_{C1}}{(101)(2)}$$

$$2.4 + 0.0559 - 1.40 = V_{C1}(0.2 + 0.00495)$$

$$\underline{V_{C1} = V_{B2} = 5.15 \text{ V}},\ \underline{V_{E2} = 5.85}$$

$$I_{E2} = \frac{12 - 5.85}{2} \Rightarrow \underline{I_{E2} = 3.08 \text{ mA}},$$

$$\underline{I_{C2} = 3.04 \text{ mA}},\ \underline{I_{B2} = 30.4\ \mu\text{A}}$$

$$V_{C2} = (3.04)(1.5) \Rightarrow \underline{V_{C2} = 4.56 \text{ V}}$$

Chapter 3

Problem Solutions

3.1

(a) $\beta_F = \frac{i_C}{i_B} = \frac{510}{6} \Rightarrow \underline{\beta_F = 85}$

$\alpha_F = \frac{\beta_F}{1+\beta_F} = \frac{85}{86} \Rightarrow \underline{\alpha_F = 0.9884}$

$i_E = (1+\beta_F)i_B = (86)(6) \Rightarrow \underline{i_E = 516\ \mu A}$

(b) $\beta_F = \frac{2.65}{0.050} \Rightarrow \underline{\beta_F = 53}$

$\alpha_F = \frac{53}{54} \Rightarrow \underline{\alpha_F = 0.9815}$

$i_E = (1+\beta_F)i_B = (54)(0.050) \Rightarrow \underline{i_E = 2.70\ mA}$

3.2

(a)

For $\beta = 110$:

$\alpha = \frac{\beta}{1+\beta} = \frac{110}{111} = 0.99099$

For $\beta = 180$:

$\alpha = \frac{180}{181} = 0.99448$

$\underline{0.99099 \le \alpha \le 0.99448}$

(b) $I_C = \beta I_B = 110(50\ \mu A) \Rightarrow \quad I_C = 5.50\ mA$

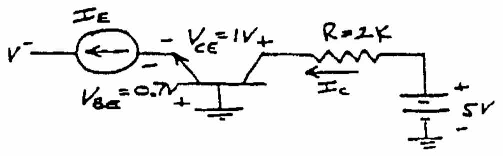
or

$I_C = 180(50\ \mu A) \Rightarrow \quad I_C = 9.00\ mA$

so

$\underline{5.50 \le I_C \le 9.0\ mA}$

3.3



$5 = I_C R + V_{CE} - V_{BE} = I_C(2) + 1 - 0.7$

$\Rightarrow I_C = 2.35$ mA

$I_E = \frac{I_C}{\alpha} = \frac{2.35}{0.982} \Rightarrow \underline{I_E = 2.39 \text{ mA}}$

3.4

Same Figure as Problem 3.3

$v_C = -0.7 + 2 = 1.3\,V$; $i_C = \frac{5-1.3}{2} \Rightarrow \underline{i_C = 1.85\ mA}$

For $\beta_F = 120$, $i_B = \frac{i_C}{\beta_F} = \frac{1.85}{120} \Rightarrow \underline{i_B = 15.4\ \mu A}$

$i_E = \left(\frac{1+\beta_F}{\beta_F}\right)i_C = \left(\frac{121}{120}\right)(1.85) \Rightarrow \underline{i_E = 1.865\ mA}$

3.5

$\alpha = \frac{\beta}{1+\beta} = \frac{60}{61} \Rightarrow \underline{\alpha = 0.9836}$

$I_E = \frac{I_C}{\alpha} = \frac{0.85}{0.9836} \Rightarrow \underline{I_E = 0.864 \text{ mA}}$

$I_B = \frac{I_C}{\beta} = \frac{0.85}{60} \Rightarrow \underline{I_B = 14.2\ \mu\text{A}}$

3.6

$i_E = I_S e^{v_{BE}/V_T} = (10^{-13})e^{0.685/0.026} \Rightarrow \underline{i_E = 27.7\ mA}$

$i_C = \left(\frac{90}{91}\right)(27.7) \Rightarrow \underline{i_C = 27.4\ mA}$

$i_B = \frac{i_E}{1+\beta} = \frac{27.7}{91} \Rightarrow \underline{i_B = 0.304\ mA}$

3.7

Device 1: $i_E = I_{S1}e^{v_{BE}/V_T} \Rightarrow 0.5x10^{-3} = I_{S1}e^{0.650/0.026}$

So that

$\underline{I_{S1} = 6.94x10^{-15}\ A}$

Device 2: $12.2x10^{-3} = I_{S2}e^{0.650/0.026}$

Or

$\underline{I_{S2} = 1.69x10^{-13}\ A}$

Ratio of areas $= \frac{I_{S2}}{I_{S1}} = \frac{1.69x10^{-13}}{6.94x10^{-15}} \Rightarrow$ Ratio = 24.4

3.8

(a) $r_o = \frac{V_A}{I_C} = \frac{250}{1} \Rightarrow \quad r_o = 250\ k\Omega$

(b) $r_o = \frac{V_A}{I_C} = \frac{250}{0.1} \Rightarrow \quad r_o = 2.50\ M\Omega$

3.9

$BV_{CEO} = \frac{BV_{CBO}}{\sqrt[3]{\beta}} = \frac{60}{\sqrt[3]{100}}$

$\underline{BV_{CEO} = 12.9 \text{ V}}$

3.10

$$BV_{CEO} = \frac{BV_{CBO}}{\sqrt[3]{\beta}}$$

$$56 = \frac{220}{\sqrt[3]{\beta}} \Rightarrow \sqrt[3]{\beta} = \frac{220}{56} = 3.93$$

$\underline{\beta = 60.6}$

3.11

$$BV_{CEO} = \frac{BV_{CBO}}{\sqrt[3]{\beta}}$$

$$BV_{CBO} = (BV_{CEO})\sqrt[3]{\beta} = (50)\sqrt[3]{50}$$

$\underline{BV_{CBO} = 184\text{ V}}$

3.12

(a) $I_E = \frac{12-0.7}{10} \Rightarrow \underline{I_E = 1.13\ mA}$

$$I_C = \left(\frac{75}{76}\right)(1.13) = 1.12\ mA$$

$$V_{CE} = 24 - (1.13)(10) - (1.12)R_C = 6$$

so that

$R_C = 5.98\ k\Omega$

(b) $I_B = \frac{1}{76} = 0.0132\ mA$

$V_B = -I_B R_B = -(0.0132)(50) \Rightarrow \underline{V_B = -0.658\ V}$

$$I_C = \left(\frac{75}{76}\right)(1) = 0.987\ mA$$

$R_C = \frac{5-2}{0.987} \Rightarrow \underline{R_C = 3.04\ k\Omega}$

c. $I_B = \frac{8 - 0.7 - (-2)}{10 + (76)(10)} \Rightarrow I_B = 12.1\ \mu\text{A}$

$\underline{I_C = 0.906\text{ mA}}$

$V_E = 0.7 + (0.0121)(10) - 2$

$V_E = -1.18\text{ V}$

$V_C = I_C R_C - 8 = (0.906)(3) - 8$

$\Rightarrow V_C = -5.28\text{ V}$

$V_{EC} = V_E - V_C = -1.18 - (-5.28)$

$\Rightarrow \underline{V_{EC} = 4.1\text{ V}}$

d. $5 = (1+\beta)I_B(10) + I_B(20) + 0.7 + (1+\beta)I_B(2)$

$5 = I_B[760 + 20 + 152] + 07 \Rightarrow \underline{I_B = 4.61\ \mu\text{A}}$

$I_C = \beta I_B = (75)(4.61) \Rightarrow I_C = 0.346\text{ mA}$

$V_C = 5 - (1+\beta)I_B R_C = 5 - (76)(0.00461)(10)$

$\Rightarrow \underline{V_C = 1.50}$

3.13

(a) Figure P3.12(c)

$8 = (76)I_B(10) + 0.7 + I_B(10) - 2$

$$I_B = \frac{10-0.7}{10+(76)(10)} = 0.01208\ mA$$

Then

$I_C = (75)I_B \Rightarrow \underline{I_C = 0.906\ mA}$

and $\underline{I_E = 0.918\ mA}$

$V_{EC} = 8 - I_E R_E - I_C R_C - (-8)$

$V_{EC} = 16 - (0.918)(10) - (0.906)(R_C)$

$V_{EC} = 6.82 - (0.906)R_C$

$R_C = 3\ k\Omega \pm 5\% \Rightarrow\ 2.85 \le R_C \le 3.15\ k\Omega$

Then

$\underline{3.97 \le V_{EC} \le 4.24\ V}$

(b) Figure P3.12(d)

$5 = (1+\beta)I_B R_C + I_B(20) + 0.7 + (1+\beta)I_B(2)$

$5 = (76)I_B R_C + I_B(20) + 0.7 + (76)I_B(2)$

Now $R_C = 10\ k\Omega \pm 5\% \Rightarrow\ 9.5 \le R_C \le 10.5\ k\Omega$

Then $0.00443 \le I_B \le 0.00481\ mA$

And $V_C = 5 - (1+\beta)I_B R_C$

So that $\underline{1.46 \le V_C \le 1.53\ V}$

3.14

$R_B = \frac{V_{BB} - V_{EB}}{I_B} = \frac{2.5 - 0.7}{0.015} \Rightarrow \underline{R_B = 120\ k\Omega}$

$I_{CQ} = (70)(15\ \mu A) \Rightarrow 1.05\ mA$

$R_C = \frac{V_{CC} - V_{ECQ}}{I_{CQ}} = \frac{5-2.5}{1.05} \Rightarrow \underline{R_C = 2.38\ k\Omega}$

3.15

a. $V_B = -I_B R_B \Rightarrow I_B = \frac{-V_B}{R_B} = \frac{-(-1)}{500}$

$I_B = 2.0\ \mu\text{A}$

$V_E = -1 - 0.7 = -1.7\text{ V}$

$I_E = \frac{V_E - (-3)}{R_E} = \frac{-1.7 + 3}{4.8} = 0.2708\text{ mA}$

$\frac{I_E}{I_B} = (1+\beta) = \frac{0.2708}{0.002} = 135.4 \Rightarrow \underline{\beta = 134.4}$

$\alpha = \frac{\beta}{1+\beta} \Rightarrow \underline{\alpha = 0.9926}$

$I_C = \beta I_B \Rightarrow \underline{I_C = 0.269\text{ mA}}$

$V_{CE} = 3 - V_E = 3 - (-1.7) \Rightarrow \underline{V_{CE} = 4.7\text{ V}}$

b. $I_E = \frac{5-4}{2} \Rightarrow I_E = 0.5\text{ mA}$

$4 = 0.7 + I_B R_B + (I_B + I_C)R_C - 5$

$I_B + I_C = I_E$

$4 = 0.7 + I_B(100) + (0.5)(8) - 5$

$I_B = 0.043 \Rightarrow \frac{I_E}{I_B} = (1+\beta) = \frac{0.5}{0.043} = 11.63$

$\underline{\beta = 10.63}, \quad \alpha = \frac{\beta}{1+\beta} \Rightarrow \underline{\alpha = 0.9140}$

3.16

a. $V_B = 0 \Rightarrow$ Cutoff $\Rightarrow \underline{I_E = 0}, \ \underline{V_C = 6\text{ V}}$

b. $V_B = 1\text{ V},\ I_E = \frac{1-0.7}{1} \Rightarrow \underline{I_E = 0.3\text{ mA}}$

$I_C \approx I_E \Rightarrow V_C = 6 - (0.3)(10) \Rightarrow \underline{V_C = 3\text{ V}}$

c. $V_B = 2\text{ V}$. Assume active-mode

$I_E = \frac{2-0.7}{1} = I_E = 1.3\text{ mA} \approx I_C$

$V_C = 6 - (1.3)(10) = -7\text{ V!}$

Transistor in saturation

$I_E = \frac{2-0.7}{1} \Rightarrow \underline{I_E = 1.3\text{ mA}}$

$V_E = 1.3\text{ V},\ V_{CE}(\text{sat}) = 0.2\text{ V}$

$V_C = V_E + V_{CE}(\text{sat}) = 1.3 + 0.2$

$\Rightarrow \underline{V_C = 1.5\text{ V}}$

3.17

a. $V_{BB} = 0$,

Cutoff $V_0 = \left(\frac{R_L}{R_C + R_L}\right)V_{CC} = \left(\frac{10}{10+5}\right)(5)$

$\underline{V_0 = 3.33\text{ V}}$

b. $V_{BB} = 1\text{ V}$

$I_B = \frac{1-0.7}{50} \Rightarrow 6\ \mu\text{A}$

$I_C = \beta I_B = (75)(6) \Rightarrow I_C = 0.45\text{ mA}$

$\frac{5 - V_0}{5} = I_C + \frac{V_0}{10}$

$1 - 0.45 = V_0\left(\frac{1}{5} + \frac{1}{10}\right) \Rightarrow \underline{V_0 = 1.83\text{ V}}$

c. Transistor in saturation

$\underline{V_0 = V_{CE}(\text{sat}) = 0.2\text{ V}}$

3.18

(a) $\beta_F = 100$

(i) $I_Q = 0.1\,mA \quad I_C = \left(\frac{100}{101}\right)(0.1) = 0.0990\,mA$

$V_O = 5 - (0.099)(5) \Rightarrow \underline{V_O = 4.505\,V}$

(ii) $I_Q = 0.5\,mA \quad I_C = \left(\frac{100}{101}\right)(0.5) = 0.495\,mA$

$V_O = 5 - (0.495)(5) \Rightarrow \underline{V_O = 2.525\,V}$

(iii) $I_Q = 2\,mA$ Transistor is in saturation

$V_O = -V_{BE}(sat) + V_{CE}(sat) = -0.8 + 0.2 \Rightarrow$

$\underline{V_O = -0.6\,V}$

(b) $\beta_F = 150$

(i) $I_Q = 0.1\,mA \quad I_C = \left(\frac{150}{151}\right)(0.1) = 0.09934\,mA$

$V_O = 5 - (0.09934)(5) \Rightarrow \underline{V_O = 4.503\,V}$

$\%\text{ change} = \frac{4.503 - 4.505}{4.503} x100\% = \underline{-0.044\%}$

(ii) $I_Q = 0.5\,mA \quad I_C = \left(\frac{150}{151}\right)(0.5) = 0.4967\,mA$

$V_O = 5 - (0.4967)(5) \Rightarrow \underline{V_O = 2.517\,V}$

$\%\text{ change} = \frac{2.517 - 2.525}{2.525} x100\% = \underline{-0.32\%}$

(iii) $I_Q = 2\,mA$ Transistor in saturation

$\underline{V_O = -0.6\,V}$ No change

3.19

$I_E = \frac{V_B - 0.7}{1}$

$I_C = \left(\frac{\beta}{1+\beta}\right)I_E = \left(\frac{50}{51}\right)(V_B - 0.7) = \frac{6 - V_C}{10}$

and $V_C = V_B$

$\left(\frac{50}{51}\right)(V_B - 0.7) = \frac{6 - V_B}{10}$

$9.80(V_B - 0.7) = 6 - V_B$

$10.80V_B = 6 + (0.7)(9.80) \Rightarrow \underline{V_B = 1.19\text{ V}}$

$I_E = \frac{1.19 - 0.7}{1} \Rightarrow \underline{I_E = 0.49\text{ mA}}$

3.20

$V_{CB} = 0.5\,V \Rightarrow V_O = 0.5\,V,\ I_C = \frac{5 - 0.5}{5} = 0.90\,mA$

$I_Q = \left(\frac{101}{100}\right)(0.90) \Rightarrow \underline{I_Q = 0.909\,mA}$

3.21

$$I_E = \frac{10 - V_E}{10} = \frac{10 - 2}{10} \Rightarrow \underline{I_E = 0.80 \text{ mA}}$$

$$V_B = V_E - 0.7 = 2 - 0.7 = 1.3 \text{ V}$$

$$I_B = \frac{V_B}{R_B} = \frac{1.3}{50} \Rightarrow \underline{I_B = 0.026 \text{ mA}}$$

$$I_C = I_E - I_B = 0.80 - 0.026 \Rightarrow \underline{I_C = 0.774 \text{ mA}}$$

$$\beta = \frac{I_C}{I_B} = \frac{0.774}{0.026} \Rightarrow \underline{\beta = 29.77}$$

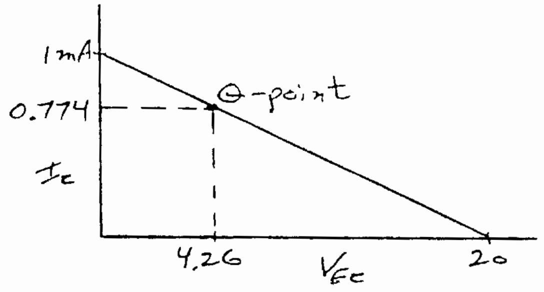
$$\alpha = \frac{\beta}{1+\beta} = \frac{29.77}{30.77} \Rightarrow \underline{\alpha = 0.9675}$$

$$V_{EC} = V_E - V_C = V_E - (I_C R_C - 10)$$

$$= 2 - [(0.774)(10) - 10]$$

$$\underline{V_{EC} = 4.26 \text{ V}}$$

Load line developed assuming the V_B voltage can change and the R_B resistor is removed.



3.22

$$I_C = \left(\frac{50}{51}\right)(1) = 0.98\, mA$$

$$V_C = I_C R_C - 9 = (0.98)(4.7) - 9$$

or $\underline{V_C = -4.39\, V}$

$$I_B = \frac{1}{51} = 0.0196\, mA$$

$$V_E = I_B R_B + V_{EB}(on) = (0.0196)(50) + 0.7$$

or $\underline{V_E = 1.68\, V}$

3.23

$$I_C = \left(\frac{50}{51}\right)(0.5) = 0.49\, mA, \quad I_B = \frac{0.5}{51} = 0.0098\, mA$$

$$V_E = I_B R_B + V_{EB}(on) = (0.0098)(50) + 0.7$$

or $V_E = 1.19\, V$

$$V_C = I_C R_C - 9 = (0.49)(4.7) - 9 = -6.70\, V$$

Then $V_{EC} = V_E - V_C = 1.19 - (-6.7) = 7.89\, V$

$$P_Q = I_C V_{EC} + I_B V_{EB} = (0.49)(7.89) + (0.0098)(0.7)$$

or $\underline{P_Q = 3.87\, mW}$

Power Supplied $= P_S = I_Q(9 - V_E) = (0.5)(9 - 1.19)$

Or $\underline{P_S = 3.91\, mW}$

3.24

For $I_Q = 0$, then $\underline{P_Q = 0}$

For $I_Q = 0.5\, mA$, $I_C = \left(\frac{50}{51}\right)(0.5) = 0.49\, mA$

$$I_B = \frac{0.5}{51} = 0.0098\, mA, \; V_B = 0.490\, V, \; V_E = 1.19\, V$$

$$V_C = (0.49)(4.7) - 9 = -6.70\, V \Rightarrow V_{EC} = 7.89\, V$$

$$P \cong I_C V_{EC} = (0.49)(7.89) \Rightarrow \underline{P = 3.87\, mW}$$

For $I_Q = 1.0\, mA$, Using the same calculations as above, we find $P = 5.95\, mW$

For $I_Q = 1.5\, mA$, $P = 6.26\, mW$
For $I_Q = 2\, mA$, $P = 4.80\, mW$
For $I_Q = 2.5\, mA$, $P = 1.57\, mW$

For $I_Q = 3\, mA$, Transistor is in saturation.

$$0.7 + I_B(50) = 0.2 + I_C(4.7) - 9$$

$$I_E = I_Q = I_B + I_C \Rightarrow I_B = 3 - I_C$$

Then, $0.7 + (3 - I_C)(50) = 0.2 + I_C(4.7) - 9$

Which yields $I_C = 2.916\, mA$ and $I_B = 0.084\, mA$

$$P = I_B V_{EB} + I_C V_{EC} = (0.084)(0.7) + (2.916)(0.2)$$

or $P = 0.642\, mW$

3.25

$$I_{E1} = I_{E2} = \frac{I}{2} \Rightarrow \underline{I_{E1} = I_{E2} = 0.5 \text{ mA}}$$

$$I_{C1} = I_{C2} \approx 0.5 \text{ mA}$$

$$V_{C1} = V_{C2} = 5 - (0.5)(4) \Rightarrow \underline{V_{C1} = V_{C2} = 3 \text{ V}}$$

3.26

a. $I_{BQ} = \dfrac{V_{CC} - V_{BE}(\text{on})}{R_B}$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{2}{60} = 0.0333 \text{ mA}$$

$$R_B = \frac{24 - 0.7}{0.0333} \Rightarrow \underline{R_B = 699 \text{ k}\Omega}$$

$$I_{CQ} = \frac{V_{CC} - V_{CEQ}}{R_C} \Rightarrow R_C = \frac{24 - 12}{2}$$

$$\Rightarrow \underline{R_C = 6 \text{ k}\Omega}$$

b. $I_{BQ} = \dfrac{V_{CC} - V_{BE}(\text{on})}{R_B} = \dfrac{24 - 0.7}{699}$

$= 0.0333$ mA (Unchanged)

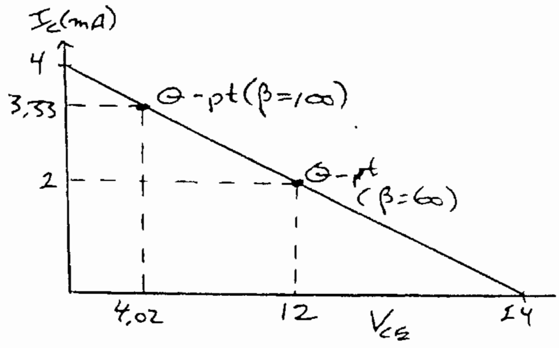
$$I_{CQ} = \beta I_{BQ} = (100)(0.0333)$$

$$\Rightarrow \underline{I_{CQ} = 3.33 \text{ mA}}$$

$$V_{CEQ} = V_{CC} - I_{CQ} R_C = 24 - (3.33)(6)$$

$$\Rightarrow \underline{V_{CEQ} = 4.02 \text{ V}}$$

(c) $V_{CE} = V_{CC} - I_C R_C = 24 - I_C(6)$



3.27

$$I_E = \frac{V_{EE} - V_{EB}(\text{on})}{R_E} = \frac{9 - 0.7}{4}$$

$\Rightarrow \underline{I_E = 2.075 \text{ mA}}$

$I_C = \alpha I_E = (0.9920)(2.075)$

$\Rightarrow \underline{I_C = 2.06 \text{ mA}}$

$V_{BC} + I_C R_C = V_{CC}$

$V_{BC} = 9 - (2.06)(2.2) \Rightarrow \underline{V_{BC} = 4.47 \text{ V}}$

3.28

$$I_{CQ} = \frac{V_{CC} - V_{CEQ}}{R_C} = \frac{12 - 6}{2.2} = 2.73 \text{ mA}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{2.73}{30} \Rightarrow I_{BQ} = 0.091 \text{ mA}$$

$$I_{R2} = \frac{0.7 - (-12)}{100} = 0.127 \text{ mA}$$

$I_{R1} = I_{R2} + I_{BQ} = 0.127 + 0.091 = 0.218 \text{ mA}$

$V_1 = I_{R1} R_1 + 0.7 = (0.218)(15) + 0.7$

$\Rightarrow \underline{V_1 = 3.97 \text{ V}}$

3.29

For $V_{CE} = 4.5$

$$I_{CQ} = \frac{5 - 4.5}{1} = 0.5 \text{ mA}$$

$$I_{BQ} = \frac{0.5}{25} = 0.02 \text{ mA}$$

$$I_{R2} = \frac{0.7 - (-5)}{100} = 0.057 \text{ mA}$$

$I_{R1} = I_{R2} + I_{BQ} = 0.057 + 0.02 = 0.077 \text{ mA}$

$V_1 = I_{R1} R_1 + V_{BE}(\text{on}) = (0.077)(15) + 0.7$

$= 1.86 \text{ V}$

For $V_{CE} = 1.0$

$$I_{CQ} = \frac{5 - 1}{1} = 4 \text{ mA}$$

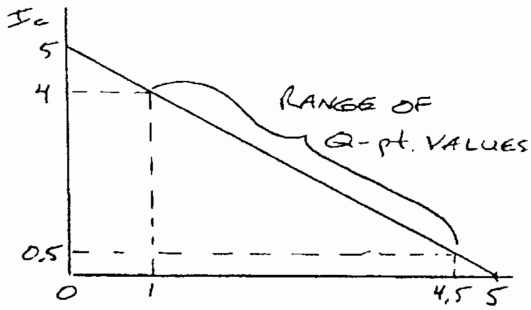
$$I_{BQ} = \frac{4}{25} = 0.16 \text{ mA}$$

$I_{R2} = 0.057 \text{ mA}$

$I_{R1} = I_{R2} + I_{BQ} = 0.057 + 0.16 = 0.217 \text{ mA}$

$V_1 = (0.217)(15) + 0.7 \Rightarrow 3.96 \text{ V}$

So $\underline{1.86 \le V_1 \le 3.96 \text{ V}}$



3.30

$R_{TH} = R_1 \| R_2 = 33 \| 10 = 7.67 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{10}{10 + 33}\right)(18)$$

$= 4.19 \text{ V}$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1 + \beta) R_E} = \frac{4.19 - 0.7}{7.67 + (51)(1)}$$

$I_{BQ} = 0.0595 \text{ mA}$

$I_{CQ} = \beta I_{BQ} \Rightarrow \underline{I_{CQ} = 2.97 \text{ mA}}$

$I_{EQ} = 3.03 \text{ mA}$

$V_{CEQ} = V_{CC} - I_{CQ} R_C - I_{EQ} R_E$

$= 18 - (2.97)(2.2) - (3.03)(1)$

$\Rightarrow \underline{V_{CEQ} = 8.44 \text{ V}}$

3.31

$I_{CQ} = 1.2\, mA, \quad V_{CEQ} = 9\, V, \quad R_{TH} = 50\, k\Omega$

Also $I_B = \frac{1.2}{80} = 0.015\, mA$

$V_{TH} = I_{BQ} R_{TH} + V_{BE}(on) + (1 + \beta) I_{BQ} R_E$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{CC}) = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(50)(18)$$

Then

$$\frac{1}{R_1}(50)(18) = (0.015)(50) + 0.7 + (81)(0.015)(1)$$

or $\underline{R_1 = 338\, k\Omega}$. Then $\frac{338 R_2}{338 + R_2} = 50 \Rightarrow$

$\underline{R_2 = 58.7\, k\Omega}$ $\quad I_{EQ} = \left(\frac{81}{80}\right)(1.2) = 1.215\, mA$

$18 = I_{CQ} R_C + V_{CEQ} + I_{EQ} R_E$

$18 = (1.2) R_C + 9 + (1.215)(1) \Rightarrow \underline{R_C = 6.49\, k\Omega}$

3.32

$R_{TH} = R_1 \| R_2 = 20 \| 15 = 8.57\ k\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{CC}) = \left(\frac{15}{15+20}\right)(10) = 4.29\ V$$

$$V_{CC} = I_{EQ}R_E + V_{EB}(on) + \frac{I_{EQ}}{1+\beta} \cdot R_{TH} + V_{TH}$$

$$10 = I_{EQ}(1) + 0.7 + I_{EQ}\left(\frac{8.57}{101}\right) + 4.29$$

Then

$$I_{EQ} = \frac{10 - 0.7 - 4.29}{1 + \frac{8.57}{101}} = \frac{5.01}{1.085} \Rightarrow \underline{I_{EQ} = 4.62\ mA}$$

$$V_B = \frac{I_{EQ}}{1+\beta} \cdot R_{TH} + V_{TH} = \left(\frac{4.62}{101}\right)(8.57) + 4.29$$

or

$\underline{V_B = 4.68\ V}$

3.33

a. $R_{TH} = R_1 \| R_2 = 58 \| 42 = 24.36\ k\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\frac{42}{42+58}\right)(24) = 10.1\ V$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1+\beta)R_E} = \frac{10.1 - 0.7}{24.36 + (126)(10)}$$

$I_{BQ} = 0.00732$ mA

$I_{CQ} = \beta I_{BQ} = (125)(0.00732)$

$\Rightarrow \underline{I_{CQ} = 0.915\ mA}$

$I_{EQ} = 0.922 \Rightarrow V_{CEQ} = 24 - (0.922)(10)$

$\Rightarrow \underline{V_{CEQ} = 14.8\ V}$

b. Let

$R_2 = 42\ k\Omega + 5\% = 44.1\ k\Omega$

$R_1 = 58\ k\Omega - 5\% = 55.1\ k\Omega$

$R_E = 10\ k\Omega - 5\% = 9.5\ k\Omega$

$R_{TH} = R_1 \| R_2 = 24.5\ k\Omega$

$$V_{TH} = \left(\frac{44.1}{44.1 + 55.1}\right)(24) = 10.7\ V$$

$$I_{BQ} = \frac{10.7 - 0.7}{24.5 + (126)(9.5)} = 0.00819$$

$\underline{I_{CQ} = 1.02\ mA}$, $I_{EQ} = 1.03$ mA

$V_{CEQ} = 24 - (1.03)(9.5) \Rightarrow \underline{V_{CEQ} = 14.2\ V}$

Let

$R_2 = 42\ k\Omega - 5\% = 39.9\ k\Omega$

$R_1 = 58\ k\Omega + 5\% = 60.9\ k\Omega$

$R_E = 10\ k\Omega + 5\% = 10.5\ k\Omega$

$R_{TH} = R_1 \| R_2 = 24.1\ k\Omega$

$$V_{TH} = \left(\frac{39.9}{39.9 + 60.9}\right)(24) = 9.5\ V$$

$$I_{BQ} = \frac{9.5 - 0.7}{24.1 + (126)(10.5)} = 0.00653\ mA$$

$\underline{I_{CQ} = 0.817\ mA}$, $I_{EQ} = 0.823$ mA

$V_{CEQ} = 24 - (0.823)(10.5) \Rightarrow \underline{V_{CEQ} = 15.4\ V}$

So $0.817 \le I_{CQ} \le 1.02$ mA and $14.2 \le V_{CEQ} \le 15.4$ V.

3.34

a. $R_{TH} = R_1 \| R_2 = 25 \| 8 = 6.06\ k\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\frac{8}{8+25}\right)(24) = 5.82\ V$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1+\beta)R_E} = \frac{5.82 - 0.7}{6.06 + (76)(1)}$$

$\underline{I_{BQ} = 0.0624\ mA}$, $\underline{I_{CQ} = 4.68\ mA}$

$I_{EQ} = 4.74$

$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$

$= 24 - (4.68)(3) - (4.74)(1)$

$\underline{V_{CEQ} = 5.22\ V}$

b. $$I_{BQ} = \frac{5.82 - 0.7}{6.06 + (151)(1)} \Rightarrow \underline{I_{BQ} = 0.0326\ mA}$$

$\underline{I_{CQ} = 4.89\ mA}$

$I_{EQ} = 4.92$

$V_{CEQ} = 24 - (4.89)(3) - (4.92)(1)$

$\underline{V_{CEQ} = 4.41\ V}$

3.35

(a) $I_{CQ} \cong I_{EQ} = 0.4\ mA$

$$R_C = \frac{3}{0.4} \Rightarrow R_C = 7.5\ k\Omega;\quad R_E = \frac{3}{0.4} \Rightarrow R_E = 7.5\ k\Omega$$

$$R_1 + R_2 \cong \frac{9}{(0.2)(0.4)} = 112.5\ k\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{CC}) = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$$

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(112.5 - R_2)R_2}{112.5},$$

$$I_{BQ} = \frac{0.4}{100} = 0.004\ mA$$

$$R_2\left(\frac{9}{112.5}\right) = (0.004)\left[\frac{(112.5 - R_2)R_2}{112.5}\right] + 0.7 + (101)(0.004)(7.5)$$

We obtain

$R_2(0.08) = 0.004R_2 - 3.56x10^{-5}R_2^2 + 3.73$

From this quadratic, we find

$\underline{R_2 = 48\ k\Omega \Rightarrow R_1 = 64.5\ k\Omega}$

(b) Standard resistor values:
Set $R_E = R_C = 7.5\ k\Omega$ and
$R_1 = 62\ k\Omega,\quad R_2 = 47\ k\Omega$
Now $R_{TH} = R_1 \| R_2 = 62\|47 = 26.7\ k\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{CC}) = \left(\frac{47}{47+62}\right)(9) = 3.88\ V$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$$

So

$$I_{BQ} = \frac{3.88 - 0.7}{26.7 + (101)(7.5)} = 0.00406\ mA$$

Then
$I_{CQ} = 0.406\ mA$
$V_{RC} = V_{RE} = (0.406)(7.5) = 3.05\ V$

3.36

a. $R_{TH} = R_1 \| R_2 = 12\|2 = \underline{1.71\text{ k}\Omega = R_{TH}}$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5 = \left(\frac{2}{12+2}\right)(10) - 5$$
$$= \underline{-3.57\text{ V}} = V_{TH}$$

b.

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on}) - (-5)}{R_{TH} + (1+\beta)R_E}$$
$$= \frac{-3.57 - 0.7 + 5}{1.71 + (101)(0.5)} = \frac{0.73}{52.2}$$

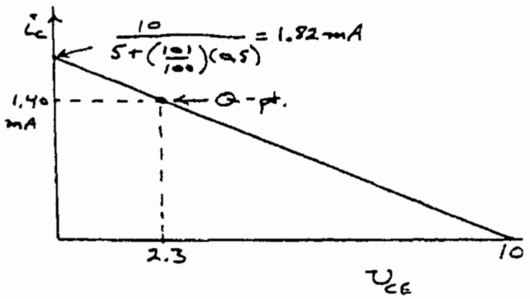
$\Rightarrow I_{BQ} = 0.0140$ mA

$\underline{I_{CQ} = 1.40\text{ mA}}$, $I_{EQ} = 1.41$ mA

$$V_{CEQ} = 10 - I_{CQ}R_C - I_{EQ}R_E$$
$$= 10 - (1.40)(5) - (1.41)(0.5)$$

$\underline{V_{CEQ} = 2.30\text{ V}}$

c.



d. For

$R_2 = 2\text{ k}\Omega + 5\% = 2.1\text{ k}\Omega$

$R_1 = 12\text{ k}\Omega - 5\% = 11.4\text{ k}\Omega$

$R_E = 0.5\text{ k}\Omega - 5\% = 0.475\text{ k}\Omega$

$R_{TH} = R_1 \| R_2 = 1.77\text{ k}\Omega$

$$V_{TH} = \left(\frac{2.1}{2.1 + 1.4}\right)(10) - 5 = -3.44\text{ V}$$

$$I_{BQ} = \frac{-3.44 - 0.7 + 5}{1.77 + (101)(0.475)} = \frac{0.86}{49.7} = 0.0173\text{ mA}$$

$\underline{I_{CQ} = 1.73\text{ mA}}$, $I_{EQ} = 1.75$ mA

For $R_C = 5\text{ k}\Omega + 5\% = 5.25\text{ k}\Omega$

$V_{CEQ} = 10 - (1.73)(5.25) - (1.75)(0.475)$

$\Rightarrow V_{CEQ} = 0.0863$ V (Saturation)

For

$R_2 = 2\text{ k}\Omega - 5\% = 1.9\text{ k}\Omega$

$R_1 = 12\text{ k}\Omega + 5\% = 12.6\text{ k}\Omega$

$R_E = 0.5\text{ k}\Omega + 5\% = 0.525\text{ k}\Omega$

$R_{TH} = R_1 \| R_2 = 1.65\text{ k}\Omega$

$$V_{TH} = \left(\frac{1.9}{12.6 + 1.9}\right)(10) - 5 = -3.69\text{ V}$$

$$I_{BQ} = \frac{-3.69 - 0.7 + 5}{1.65 + (101)(0.525)} = \frac{0.61}{54.7} = 0.0112\text{ mA}$$

$\underline{I_{CQ} = 1.12\text{ mA}}$, $I_{EQ} = 1.13$ mA

For $R_C = 5\text{ k}\Omega - 5\% = 4.75\text{ k}\Omega$

$V_{CEQ} = 10 - (1.12)(4.75) - (1.13)(0.525)$

$\Rightarrow V_{CEQ} = 4.09$ V

So $1.12 \le I_{CQ} \le 1.73$ mA and
$\underbrace{0.0863}_{\text{Saturation}} \le V_{CEQ} \le 4.09$ V.

3.37

$R_{TH} = R_1 \| R_2 = 9\|1 = 0.90\text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(-12) = \left(\frac{1}{1+9}\right)(-12)$$
$$= -1.2\text{ V}$$

$$I_{EQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH} = 0$$

$$I_{BQ} = \frac{-V_{TH} - V_{EB}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{1.2 - 0.7}{0.90 + (76)(0.1)}$$

$I_{BQ} = 0.0588$, $\underline{I_{CQ} = 4.41\text{ mA}}$

$I_{EQ} = 4.47$ mA

Center of load line $\Rightarrow \underline{V_{ECQ} = 6\text{ V}}$

$I_{EQ}R_E + V_{ECQ} + I_{CQ}R_C - 12 = 0$

$(4.47)(0.1) + 6 + (4.41)R_C = 12$

$\Rightarrow \underline{R_C = 1.26\text{ k}\Omega}$

3.38

(a) $R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(0.5) = 5.05\ k\Omega$

$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$

$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.8}{100} = 0.008\ mA$

Then

$\frac{1}{R_1}(5.05)(10) = (0.008)(5.05) + 0.7 + (101)(0.008)(0.5)$

or

$R_1 = 44.1\ k\Omega$, $\frac{44.1R_2}{44.1+R_2} = 5.05 \Rightarrow$ $R_2 = 5.70\ k\Omega$

Now $I_{EQ} = \left(\frac{101}{100}\right)(0.8) = 0.808\ mA$

$V_{CC} = I_{CQ}R_C + V_{CEQ} + I_{EQ}R_E$

$10 = (0.8)R_C + 5 + (0.808)(0.5)$

$R_C = 5.75\ k\Omega$

(b) For $75 \le \beta \le 150$

$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(V_{CC}) = \left(\frac{5.7}{5.7+44.1}\right)(10) = 1.145\ V$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$

For $\beta = 75$, $I_{BQ} = \frac{1.145-0.7}{5.05+(76)(0.5)} = 0.0103\ mA$

Then $I_{CQ} = (75)(0.0103) = 0.775\ mA$

For $\beta = 150$, $I_{BQ} = \frac{1.145-0.7}{5.05+(151)(0.5)} = 0.00552\ mA$

Then $I_{CQ} = 0.829\ mA$

% Change $= \frac{\Delta I_{CQ}}{I_{CQ}} = \frac{0.829-0.775}{0.80} x100\% \Rightarrow$

% Change = 6.75%

(c) For $R_E = 1\ k\Omega$

$R_{TH} = (0.1)(101)(1) = 10.1\ k\Omega$

$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(10.1)(10) =$

$(0.008)(10.1) + 0.7 + (101)(0.008)(1)$

which yields $R_1 = 63.6\ k\Omega$

And $\frac{63.6R_2}{63.6+R_2} = 10.1 \Rightarrow$ $R_2 = 12.0\ k\Omega$

Now $V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(V_{CC}) = \left(\frac{12}{12+63.6}\right)(10) = 1.587\ V$

For $\beta = 75$, $I_{BQ} = \frac{1.587-0.7}{10.1+(76)(1)} = 0.0103\ mA$

So $I_{CQ} = 0.773\ mA$

For $\beta = 150$, $I_{BQ} = \frac{1.587-0.7}{10.1+(151)(1)} = 0.00551\ mA$

Then $I_{CQ} = 0.826\ mA$

% Change $= \frac{\Delta I_{CQ}}{I_{CQ}} = \frac{0.826-0.773}{0.8} x100\% \Rightarrow$

% Change = 6.63%

3.39

$V_{CC} \cong I_{CQ}(R_C + R_E) + V_{CEQ}$

$10 = (0.8)(R_C + R_E) + 5 \Rightarrow R_C + R_E = 6.25\ k\Omega$

Let $R_E = 1\ k\Omega$

Then, for bias stable $R_{TH} = (0.1)(121)(1) = 12.1\ k\Omega$

$I_{BQ} = \frac{0.8}{120} = 0.00667\ mA$

$\frac{1}{R_1}(12.1)(10) = (0.00667)(12.1) + 0.7$

$+(121)(0.00667)(1)$

So $R_1 = 76.2\ k\Omega$ and $\frac{76.2R_2}{76.2+R_2} = 12.1 \Rightarrow$

$R_2 = 14.4\ k\Omega$

Then $I_R \cong \frac{10}{76.2+14.4} = 0.110\ mA$

This is close to the design specification.

3.40

$I_{CQ} \approx I_{EQ} \Rightarrow V_{CEQ} = V_{CC} - I_{CQ}(R_C + R_E)$

$6 = 12 - I_{CQ}(2 + 0.2)$

$I_{CQ} = 2.73$ mA, $I_{BQ} = 0.0218$ mA

$V_{CEQ} = 6$ V

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E - 6$

$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(12) - 6$, $R_{TH} = R_1 \| R_2$

Bias stable $\Rightarrow$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(126)(0.2) = 2.52\ k\Omega$

$V_{TH} = \left(\frac{1}{R_1}\right)(R_{TH})(12) - 6$

$\frac{1}{R_1}(2.52)(12) - 6 = (0.0218)(2.52) + 0.7$

$+ (126)(0.0218)(0.2) - 6$

$\frac{1}{R_1}(30.24) = 0.7549 + 0.5494$

$R_1 = 23.2\ k\Omega$, $\frac{23.2R_2}{23.2+R_2} = 2.52$

$R_2 = 2.83\ k\Omega$

3.41

a. $I_{CQ} = 1$ mA. $I_{EQ} = \left(\frac{80}{81}\right)(1) = 1.01$ mA

$V_{CEQ} = 12 - (1)(2) - (1.01)(0.2) \Rightarrow \underline{V_{CEQ} = 9.80 \text{ V}}$

$I_{BQ} = \frac{1}{80} = 0.0125$ mA

$R_{TH} = +(0.1)(1+\beta)R_E = (0.1)(81)(0.2) = 1.62$ kΩ

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(12) - 6 = \frac{1}{R_1}(R_{TH})(12) - 6$$

$$= \frac{1}{R_1}(19.44) - 6$$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E - 6$

$$\frac{1}{R_1}(19.44) - 6 = (0.0125)(1.62) + 0.7$$
$$+ (81)(0.0125)(0.2) - 6$$

$\frac{1}{R_1}(19.44) = 0.923$

$\underline{R_1 = 21.1 \text{ k}\Omega}$, $\frac{21.1R_2}{21.1 + R_2} = 1.62$

$\underline{R_2 = 1.75 \text{ k}\Omega}$

b.

$R_1 = 22.2$ kΩ or $R_1 = 20.0$ kΩ

$R_2 = 1.84$ kΩ or $R_2 = 1.66$ kΩ

$R_E = 0.21$ kΩ or $R_E = 0.19$ kΩ

$R_C = 2.1$ kΩ or $R_C = 1.9$ kΩ

$R_2(\max)$, $R_1(\min)$, $R_E(\min)$

$R_{TH} = (1.84)\|(20.0) = 1.685$ kΩ

$$V_{TH} = \left(\frac{1.84}{1.84 + 20.0}\right)(12) - 6 = -4.99 \text{ V}$$

$$I_{BQ} = \frac{-4.99 - 0.7 - (-6)}{1.685 + (81)(0.19)} = \frac{0.31}{17.08} = 0.0182 \text{ mA}$$

$\underline{I_{CQ} = 1.45 \text{ mA}}$

For max, $R_C \Rightarrow$

$V_{CE} = 12 - (1.45)(2.1) - (1.47)(0.19)$

$V_{CE} = 8.68$ V

$R_2(\min)$, $R_1(\max)$, $R_E(\max)$

$R_{TH} = (1.66)\|(22.2) = 1.547$ kΩ

$$V_{TH} = \left(\frac{1.66}{1.66 + 22.2}\right)(12) - 6 = -5.165 \text{ V}$$

$$I_{BQ} = \frac{-5.165 - 0.7 + 6}{1.547 + (81)(0.21)} = \frac{0.135}{18.56} = 0.00727 \text{ mA}$$

For min, $R_C \Rightarrow \underline{I_{CQ} = 0.582 \text{ mA}}$, $I_E = 0.589$

$V_{CEQ} = 12 - (0.582)(1.9) - (0.589)(0.21)$

$V_{CEQ} = 10.77$ V

So $0.582 \le I_C \le 1.45$ mA

$8.68 \le V_{CEQ} \le 10.77$ V

3.42

$V_{CEQ} \cong V_{CC} - I_{CQ}(R_C + R_E)$

$5 = 12 - 3(R_C + R_E) \Rightarrow R_C + R_E = 2.33$ kΩ

Let $\underline{R_E = 0.33 \text{ k}\Omega}$ and $\underline{R_C = 2 \text{ k}\Omega}$

Nominal value of $\beta = 100$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(0.33) = 3.33$ kΩ

$I_{BQ} = \frac{3}{100} = 0.03$ mA

$$V_{TH} = \frac{1}{R_1}\cdot R_{TH}\cdot(12) - 6 = \frac{1}{R_1}(3.33)(12) - 6$$

Then

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E - 6$

$$\frac{1}{R_1}(3.33)(12) - 6 = (0.03)(3.33) + 0.7 + (101)(0.03)(0.33) - 6$$

which yields $\underline{R_1 = 22.2 \text{ k}\Omega}$ and $\underline{R_2 = 3.92 \text{ k}\Omega}$

Now

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(12) - 6 = \left(\frac{3.92}{3.92 + 22.2}\right)(12) - 6$$

or

$V_{TH} = -4.20$ V

For $\beta = 75$,

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E - 6$

$$I_{BQ} = \frac{V_{TH} + 6 - 0.7}{R_{TH} + (1+\beta)R_E} = \frac{-4.2 + 6 - 0.7}{3.33 + (76)(0.33)}$$

$= 0.0387$ mA $\Rightarrow \underline{I_C = 2.90 \text{ mA}}$

For $\beta = 150$,

$$I_{BQ} = \frac{-4.2 + 6 - 0.7}{3.33 + (151)(0.33)} = 0.0207 \text{ mA}$$

Then

$\underline{I_C = 3.10 \text{ mA}}$

Specifications are met.

3.43

$R_{TH} = R_1\|R_2 = 3\|12 = 2.4$ kΩ

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\frac{12}{12+3}\right)(20) = 16 \text{ V}$$

(a) For $\beta = 75$

$20 = (1+\beta)I_{BQ}R_E + V_{EB}(on) + I_{BQ}R_{TH} + V_{TH}$

$20 - 0.7 - 16 = I_{BQ}[(76)(2) + 2.4]$

So

$I_{BQ} = 0.0214$ mA, $I_{CQ} = 1.60$ mA, $I_{EQ} = 1.62$ mA

$V_{ECQ} = 20 - (1.6)(1) - (1.62)(2)$

or

$V_{ECQ} = 15.16$ V

(b) For $\beta = 100$, we find

$I_{BQ} = 0.0161$ mA, $I_{CQ} = 1.61$ mA, $V_{ECQ} = 15.13$ V

3.44

$I_{CQ} = 4.8 \text{ mA} \rightarrow I_{EQ} = 4.84 \text{ mA}$

$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$

$6 = 18 - (4.8)(2) - (4.84)R_E \Rightarrow \underline{R_E = 0.496 \text{ k}\Omega}$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(121)(0.496) = 6.0 \text{ k}\Omega$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E$

$I_{BQ} = 0.040 \text{ mA}$

$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(6.0)(18)$

$\frac{1}{R_1}(6.0)(18) = (0.04)(6.0) + 0.70$

$+ (121)(0.04)(0.496)$

$\frac{1}{R_1}(108) = 3.34$

$\underline{R_1 = 32.3 \text{ k}\Omega}$. $\frac{32.3R_2}{32.3 + R_2} = 6.0$

$\underline{R_2 = 7.37 \text{ k}\Omega}$

3.45

For nominal $\beta = 70$

$I_{BQ} = \frac{2}{70} = 0.0286 \text{ mA} \rightarrow I_{EQ} = 2.03 \text{ mA}$

$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$

$10 = 20 - (2)(4) - (2.03)R_E \Rightarrow \underline{R_E = 0.985 \text{ K}}$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(71)(0.985) = 6.99 \text{ K}$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E$

$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E$

$\frac{1}{R_1}(6.99)(20) = (0.0286)(6.99) + 0.70$

$+ (2.03)(0.985)$

$\frac{1}{R_1}(139.8) = 2.90$

$\underline{R_1 = 48.2 \text{ K}}$. $\frac{48.2R_2}{48.2 + R_2} = 6.99$

$\underline{R_2 = 8.18 \text{ K}}$

Check: For $\beta = 50$

$V_{TH} = \left(\frac{8.18}{8.18 + 48.2}\right)(20) = 2.90$

$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{2.90 - 0.7}{6.99 + (51)(0.985)}$

$= 0.0384 \text{ mA}$

$\underline{I_{CQ} = 1.92 \text{ mA}}$

For $\beta = 90$

$I_{BQ} = \frac{2.90 - 0.7}{6.99 + (91)(0.985)} = 0.0228 \text{ mA}$

$\underline{I_{CQ} = 2.05 \text{ mA}}$

Design criterion is satisfied.

3.46

$I_{CQ} = 1 \text{ mA} \rightarrow I_{EQ} = 1.02 \text{ mA}$

$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$

$5 = 15 - (1)(5) - (1.02)R_E \Rightarrow \underline{R_E = 4.90 \text{ k}\Omega}$

Bias stable:

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(61)(4.9) = 29.9 \text{ k}\Omega$

$I_{BQ} = \frac{1}{60} = 0.0167 \text{ mA}$

$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}(\text{on})$

$+ I_{EQ}R_E$

$\frac{1}{R_1}(29.9)(15) = (0.0167)(29.9) + 0.70$

$+ (1.02)(4.90)$

$\frac{1}{R_1}(448.5) = 6.197$

$\underline{R_1 = 72.4 \text{ k}\Omega}$. $\frac{72.4R_2}{72.4 + R_2} = 29.9$

$\underline{R_2 = 50.9 \text{ k}\Omega}$

Check: For $\beta = 45$

$V_{TH} = \left(\frac{50.9}{50.9 + 72.4}\right)(15) = 6.19 \text{ V}$

$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{6.19 - 0.7}{29.9 + (46)(4.90)}$

$= 0.0215 \text{ mA}$

$\underline{I_{CQ} = 0.968 \text{ mA}}$, $\frac{\Delta I_C}{I_C} = 3.2\%$

Check: For $\beta = 75$

$I_{BQ} = \frac{6.19 - 0.7}{29.9 + (76)(4.90)} = 0.0136 \text{ mA}$

$\underline{I_{CQ} = 1.02 \text{ mA}}$, $\frac{\Delta I_C}{I_C} = 2.0\%$

Design criterion is satisfied.

3.47

(a) $V_{CC} \cong I_{CQ}(R_C + R_E) + V_{CEQ}$

$3 = (0.1)(5R_E + R_E) + 1.4 \Rightarrow \underline{R_E = 2.67 \, k\Omega}$

$\underline{R_C = 13.3 \, k\Omega}$, $I_{BQ} = \frac{100}{120} = 0.833 \, \mu A$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(121)(2.67) = 32.3 \, k\Omega$

$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(32.3)(3)$

$= I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$

$= (0.000833)(32.3) + 0.7 + (121)(0.000833)(2.67)$

which gives $\underline{R_1 = 97.3 \, k\Omega}$, and $\underline{R_2 = 48.4 \, k\Omega}$

(b) $I_R \cong \frac{3}{R_1+R_2} = \frac{3}{97.3+48.4} \Rightarrow 20.6\ \mu A$

$I_{CQ} = 100\ \mu A$

$P = (I_{CQ} + I_R)V_{CC} = (100+20.6)(3)$

or

$\underline{P = 362\ \mu W}$

3.48

$I_E = \frac{5-V_E}{R_E} = \frac{5}{3} = 1.67 \text{ mA}$

$R_{TH} = R_1 \| R_2 = (0.1)(1+\beta)R_E$

$= (0.1)(101)(3) = 30.3 \text{ k}\Omega$

$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(4) - 2 = \frac{1}{R_1} \cdot R_{TH} \cdot (4) - 2$

$I_{BQ} = \frac{I_{EQ}}{1+\beta} = 0.0165 \text{ mA}$

$5 = I_{EQ}R_E + V_{EB}(\text{on}) + I_B R_{TH} + V_{TH}$

$5 = (1.67)(3) + 0.7 + (0.0165)(30.3)$

$+ \frac{1}{R_1}(30.3)(4) - 2$

$0.80 = \frac{1}{R_1}(30.3)(4) \Rightarrow \underline{R_1 = 152 \text{ k}\Omega}$

$\frac{152R_2}{152+R_2} = 30.3 \Rightarrow \underline{R_2 = 37.8 \text{ k}\Omega}$

3.49

a. $R_{TH} = R_1 \| R_2 = 10\|20 \Rightarrow \underline{R_{TH} = 6.67 \text{ k}\Omega}$

$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(10) - 5 = \left(\frac{20}{20+10}\right)(10) - 5$

$\Rightarrow \underline{V_{TH} = 1.67 \text{ V}}$

b. $10 = (1+\beta)I_{BQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_{TH} + V_{TH}$

$I_{BQ} = \frac{10 - 0.7 - 1.67}{6.67 + (61)(2)} = \frac{7.63}{128.7}$

$\Rightarrow \underline{I_{BQ} = 0.0593 \text{ mA}}$

$\underline{I_{CQ} = 3.65 \text{ mA}}, \quad I_{EQ} = 3.62 \text{ mA}$

$V_E = 10 - I_{EQ}R_E = 10 - (3.62)(2)$

$\underline{V_E = 2.76 \text{ V}}$

$V_C = I_{CQ}R_C - 10 = (3.56)(2.2) - 10$

$\underline{V_C = -2.17 \text{ V}}$

3.50

$V^+ - V^- \cong I_{CQ}(R_C + R_E) + V_{ECQ}$

$20 = (0.5)(R_C + R_E) + 8 \Rightarrow (R_C + R_E) = 24\ k\Omega$

Let $\underline{R_E = 10\ k\Omega}$ then $\underline{R_C = 14\ k\Omega}$

Let $\beta = 60$ from previous problem.

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(61)(10)$

Or $R_{TH} = 61\ k\Omega$

$I_{BQ} = \frac{0.5}{60} = 0.00833\ mA$

$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(10) - 5 = \frac{1}{R_1} \cdot R_{TH} \cdot 10 - 5$

Now

$10 = (1+\beta)I_{BQ}R_E + V_{EB}(on) + I_{BQ}R_{TH} + V_{TH}$

$10 = (61)(0.00833)(10) + 0.7 + (0.00833)(61)$

$+ \frac{1}{R_1}(61)(10) - 5$

Then $\underline{R_1 = 70.0\ k\Omega}$ and $\underline{R_2 = 474\ k\Omega}$

$I_R \cong \frac{10}{R_1+R_2} = \frac{10}{70+474} \Rightarrow 18.4\ \mu A$

So the $40\ \mu A$ current limit is met.

3.51

a. $R_{TH} = R_1 \| R_2 = 35\|20 \Rightarrow \underline{R_{TH} = 12.7 \text{ k}\Omega}$

$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(7) - 5 = \left(\frac{20}{20+35}\right)(7) - 5$

$\Rightarrow \underline{V_{TH} = -2.45 \text{ V}}$

b.

$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on}) - (-10)}{R_{TH} + (1+\beta)R_E}$

$= \frac{-2.45 - 0.7 + 10}{12.7 + (76)(0.5)}$

$\Rightarrow \underline{I_{BQ} = 0.135 \text{ mA}}$

$\underline{I_{CQ} = 10.1 \text{ mA}}, \quad I_{EQ} = 10.3 \text{ mA}$

$V_{CEQ} = 20 - I_{CQ}R_C - I_{EQ}R_E$

$= 20 - (10.1)(0.8) - (10.3)(0.5)$

$\underline{V_{CEQ} = 6.77 \text{ V}}$

c.

$R_2 = 20 + 5\% = 21 \text{ k}\Omega$

$R_1 = 35 - 5\% = 33.25 \text{ k}\Omega$

$R_E = 0.5 - 5\% = 0.475 \text{ k}\Omega$

$R_{TH} = R_1 \| R_2 = 21\|33.25 = 12.9 \text{ k}\Omega$

$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(7) - 5$

$= \left(\frac{21}{21+33.25}\right)(7) - 5 = -2.29 \text{ V}$

$I_{BQ} = \frac{-2.29 - 0.7 - (-10)}{12.9 + (76)(0.475)} = 0.143 \text{ mA}$

$\underline{I_{CQ} = 10.7 \text{ mA}}, \quad I_{EQ} = 10.9 \text{ mA}$

For $R_C = 0.8 + 5\% = 0.84 \text{ k}\Omega$

$V_{CEQ} = 20 - (10.7)(0.84) - (10.9)(0.475)$

$\Rightarrow V_{CEQ} = 5.83 \text{ V}$

For $R_C = 0.8 - 5\% = 0.76$ kΩ

$V_{CEQ} = 20 - (10.7)(0.76) - (10.9)(0.475)$

$\Rightarrow V_{CEQ} = 6.69$ V

$R_2 = 20 - 5\% = 19$ kΩ

$R_1 = 35 + 5\% = 36.75$ kΩ

$R_E = 0.5 + 5\% = 0.525$ kΩ

$R_{TH} = R_1 \| R_2 = 19 \| 36.75 = 12.5$ kΩ

$$V_{TH} = \left(\frac{19}{19 + 36.75}\right)(7) - 5 = -2.61 \text{ V}$$

$$I_{BQ} = \frac{-2.61 - 0.7 - (-10)}{12.5 + (76)(0.525)} = 0.128 \text{ mA}$$

$\underline{I_{CQ} = 9.58 \text{ mA}}$, $I_{EQ} = 9.70$ mA

For $R_C = 0.84$ kΩ

$V_{CEQ} = 20 - (9.58)(0.84) - (9.70)(0.525)$

$\Rightarrow V_{CEQ} = 6.86$ V

For $R_C = 0.76$ kΩ

$V_{CEQ} = 20 - (9.58)(0.76) - (9.70)(0.525)$

$\Rightarrow V_{CEQ} = 7.63$ V

So $9.58 \le I_{CQ} \le 10.7$ mA

and

$5.83 \le V_{CEQ} \le 7.63$ V

3.52

a. $R_{TH} = 500 \text{ k}\Omega \| 500 \text{ k}\Omega \| 70 \text{ k}\Omega = 250 \text{ k}\Omega \| 70 \text{ k}\Omega$

$\Rightarrow \underline{R_{TH} = 54.7 \text{ k}\Omega}$

$$\frac{5 - V_{TH}}{500} + \frac{3 - V_{TH}}{500} = \frac{V_{TH} - (-5)}{70}$$

$$\frac{5}{500} + \frac{3}{500} - \frac{5}{70} = V_{TH}\left(\frac{1}{500} + \frac{1}{500} + \frac{1}{70}\right)$$

$-0.0554 = V_{TH}(0.0183)$

$\underline{V_{TH} = -3.03 \text{ V}}$

b.

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on}) - (-5)}{R_{TH} = (1+\beta)R_E}$$

$$= \frac{-3.03 - 0.7 + 5}{54.7 + (101)(5)}$$

$I_{BQ} = 0.00227$ mA

$\underline{I_{CQ} = 0.227 \text{ mA}}$, $I_{EQ} = 0.229$

$V_{CEQ} = 20 - (0.227)(50) - (0.229)(5)$

$\underline{V_{CEQ} = 7.51 \text{ V}}$

3.53

$R_{TH} = 30 \| 60 \| 20 \Rightarrow \underline{R_{TH} = 10 \text{ k}\Omega}$

$$\frac{5 - V_{TH}}{30} + \frac{5 - V_{TH}}{60} = \frac{V_{TH}}{20}$$

$$\left(\frac{5}{30} + \frac{5}{60}\right) = V_{TH}\left(\frac{1}{30} + \frac{1}{60} + \frac{1}{20}\right)$$

$\underline{V_{TH} = 2.5 \text{ V}}$

For $\beta = 100$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on}) - (-5)}{R_{TH} + (1+\beta)R_E}$$

$$= \frac{2.5 - 0.7 + 5}{10 + (101)(0.2)}$$

$I_{BQ} = 0.225$ mA

$\underline{I_{CQ} = 22.5 \text{ mA}}$, $I_{EQ} = 22.7$ mA

$V_{CEQ} = 15 - (22.5)(0.5) - (22.7)(0.2)$

$V_{CEQ} = -0.79$! In saturation

$\Rightarrow \underline{V_{CEQ} = 0.2 \text{ V}}$

$V_E = V_{TH} - I_{BQ}R_{TH} - V_{BE}(\text{on})$

$= 2.5 - (0.225)(10) - 0.7$

$V_E = -0.45 \text{ V} \Rightarrow V_C = -0.45 + 0.2 = -0.25$ V

$$I_{CQ} = \frac{10 - (-0.25)}{0.5} \Rightarrow \underline{I_{CQ} = 20.5 \text{ mA}}$$

3.54

$R_{TH} = R_1 \| R_2 = 100 \| 40 = 28.6$ kΩ

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(10) = \left(\frac{40}{40 + 100}\right)(10) = 2.86 \text{ V}$$

$$I_{B1} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_{E1}} = \frac{2.86 - 0.7}{28.6 + (121)(1)}$$

$\underline{I_{B1} = 0.0144 \text{ mA}}$

$\underline{I_{C1} = 1.73 \text{ mA}}$, $\underline{I_{E1} = 1.75 \text{ mA}}$

$$\frac{10 - V_{B2}}{3} = I_{C1} + I_{B2}$$

$$I_{E2} = \frac{V_{B2} - V_{BE}(\text{on}) - (-10)}{5}$$

$$\frac{10 - V_{B2}}{3} = I_{C1} + \frac{V_{B2} - 0.7 + 10}{(121)(5)}$$

$$\frac{10}{3} - 1.73 - \frac{9.3}{605} = V_{B2}\left(\frac{1}{3} + \frac{1}{(121)(5)}\right)$$

$1.59 = V_{B2}(0.335) \Rightarrow V_{B2} = 4.75$ V

$$I_{E2} = \frac{4.75 - 0.7 - (-10)}{5} \Rightarrow \underline{I_{E2} = 2.81 \text{ mA}}$$

$\underline{I_{B2} = 0.0232 \text{ mA}}$

$\underline{I_{C2} = 2.79 \text{ mA}}$

$V_{CEQ1} = 4.75 - (1.75)(1) \Rightarrow \underline{V_{CEQ1} = 3.0 \text{ V}}$

$V_{CEQ2} = 10 - (4.75 - 0.7) \Rightarrow \underline{V_{CEQ2} = 5.95 \text{ V}}$

3.55

$$V_{E1} = -0.7$$
$$I_{R1} = \frac{-0.7-(-5)}{20} = 0.215 \text{ mA}$$
$$V_{E2} = -0.7 - 0.7 = -1.4$$
$$I_{E2} = \frac{-1.4-(-5)}{1} \Rightarrow \underline{I_{E2} = 3.6 \text{ mA}}$$
$$\underline{I_{B2} = 0.0444 \text{ mA}}$$
$$\underline{I_{C2} = 3.56 \text{ mA}}$$

$$I_{E1} = I_{R1} + I_{B2} = 0.215 + 0.0444$$
$$\underline{I_{E1} = 0.259 \text{ mA}}$$
$$\underline{I_{B1} = 0.00320 \text{ mA}}$$
$$\underline{I_{C1} = 0.256 \text{ mA}}$$

3.56

Current through V^- source $= I_{E1} + I_{E2}$ and

$I_{E1} = I_{E2} = (1+\beta)I_{B1} = (51)(8.26)\ \mu\text{A}$

So total current $= 2(51)(8.26)\ \mu\text{A} = 843\ \mu\text{A}$

$P^- = I \cdot |V^-| = (0.843)(5) \Rightarrow \underline{P^- = 4.22 \text{ mW}}$

(From V^- source)

From Example 3.15, $I_Q = 0.413$ mA

So $I_{C0} = \left(\frac{50}{51}\right)(0.413) = 0.405$ mA

$P^+ = I \cdot V^+ = (0.405)(5) \Rightarrow \underline{P^+ = 2.03 \text{ mW}}$

(From V^+ source)

3.57

$$R_{TH} = R_1 \| R_2 = 50 \| 100 = 33.3 \text{ k}\Omega$$
$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(10) - 5$$
$$= \left(\frac{100}{100+50}\right)(10) - 5 = 1.67 \text{ V}$$
$$5 = I_{E1}R_{E1} + V_{EB}(\text{on}) + I_{B1}R_{TH} + V_{TH}$$

$$I_{E1} = \left(\frac{101}{100}\right)(0.8) = 0.808 \text{ mA}$$
$$I_{B1} = 0.008 \text{ mA}$$
$$5 = (0.808)R_{E1} + 0.7 + (0.008)(33.3) + 1.67$$
$$R_{E1} = 2.93 \text{ k}\Omega$$

$$V_{E1} = 5 - (0.808)(2.93) = 2.63 \text{ V}$$
$$V_{C1} = V_{E1} - V_{ECQ1} = 2.63 - 3.5 = -0.87 \text{ V}$$
$$V_{E2} = -0.87 - 0.70 = -1.57 \text{ V}$$
$$I_{E2} = \frac{-1.57-(-5)}{R_{E2}} = 0.808 \Rightarrow \underline{R_{E2} = 4.25 \text{ k}\Omega}$$

$$V_{CEQ2} = 4 \Rightarrow V_{C2} = -1.57 + 4 = 2.43 \text{ V}$$
$$R_{C2} = \frac{5-2.43}{0.8} \Rightarrow \underline{R_{C2} = 3.21 \text{ k}\Omega}$$
$$I_{RC1} = I_{C1} - I_{B2} = 0.8 - 0.008 = 0.792 \text{ mA}$$
$$R_{C1} = \frac{-0.87-(-5)}{0.792} \Rightarrow \underline{R_{C1} = 5.21 \text{ k}\Omega}$$

Chapter 4

Exercise Solutions

E4.1

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.25}{0.026} \Rightarrow \underline{g_m = 9.62 \text{ mA/V}}$$

$$r_\pi = \frac{V_T\beta}{I_{CQ}} = \frac{(0.026)(120)}{0.25} \Rightarrow \underline{r_\pi = 12.5 \text{ k}\Omega}$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{150}{0.25} \Rightarrow \underline{r_o = 600 \text{ k}\Omega}$$

E4.2

$$r_o = \frac{V_A}{I_{CQ}} \Rightarrow I_{CQ} = \frac{V_A}{r_o} = \frac{75}{200 \text{ k}\Omega}$$

$$\Rightarrow \underline{I_{CQ} = 0.375 \text{ mA}}$$

E4.3

$$I_{BQ} = \frac{V_{BB} - V_{BE}(\text{on})}{R_B} = \frac{0.92 - 0.7}{100}$$

$$\Rightarrow I_{BQ} = 0.0022 \text{ mA}$$

$$I_{CQ} = (150)(0.0022) = 0.33 \text{ mA}$$

a. $$g_m = \frac{I_{CQ}}{V_T} = \frac{0.33}{0.026} \Rightarrow \underline{g_m = 12.7 \text{ mA/V}}$$

$$r_\pi = \frac{V_T\beta}{I_{CQ}} = \frac{(0.0026)(150)}{0.33} \Rightarrow \underline{r_\pi = 11.8 \text{ k}\Omega}$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{200}{0.33} \Rightarrow \underline{r_o = 606 \text{ k}\Omega}$$

b. $$v_o = -g_m v_\pi (r_o \| R_C), \quad v_\pi = \left(\frac{r_\pi}{r_\pi + R_B}\right) v_S$$

$$A_v = \frac{v_o}{v_S} = -g_m\left(\frac{r_\pi}{r_\pi + R_B}\right)(r_o \| R_C)$$

$$= -(12.7)\left(\frac{11.8}{11.8 + 100}\right)(606 \| 15)$$

$$= -(12.7)(0.1055)(14.64)$$

$$\Rightarrow \underline{A_v = -19.6}$$

E4.4

$$g_m = \frac{I_{CQ}}{V_T}$$

a. $$I_{BQ} = \frac{V_{BB} - V_{EB}(\text{on})}{R_B} = \frac{1.145 - 0.70}{50}$$

$$\Rightarrow I_{BQ} = 0.0089 \text{ mA}$$

$$I_{CQ} = (90)(0.0089) \Rightarrow I_{CQ} = 0.801 \text{ mA}$$

$$g_m = \frac{0.801}{0.026} \Rightarrow \underline{g_m = 30.8 \text{ mA/V}}$$

$$r_\pi = \frac{V_T\beta}{I_{CQ}} = \frac{(0.0026)(90)}{0.801} \Rightarrow \underline{r_\pi = 2.92 \text{ k}\Omega}$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{120}{0.801} \Rightarrow \underline{r_o = 150 \text{ k}\Omega}$$

b. $$v_o = g_m v_\pi (r_o \| R_C). \quad v_\pi = -\left(\frac{r_\pi}{r_\pi + R_B}\right) v_S$$

$$A_v = \frac{v_o}{v_S} = -g_m\left(\frac{r_\pi}{r_\pi + R_B}\right)(r_o \| R_C)$$

$$= -(30.8)\left(\frac{2.92}{2.92 + 50}\right)(150 \| 2.5)$$

$$= -(30.8)(0.055)(2.46)$$

$$\Rightarrow \underline{A_v = -4.17}$$

E4.5

$$R_{TH} = 250 \| 75 = 57.7 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(5) = \left(\frac{75}{75 + 250}\right)(5) = 1.154$$

$$I_{BQ} = \frac{1.154 - 0.7}{57.7 + (121)(0.6)} = 3.48 \ \mu\text{A}$$

$$I_{CQ} = 0.418 \text{ mA}$$

a. $$r_\pi = \frac{(120)(0.026)}{0.418} = 7.46 \text{ k}\Omega$$

$$g_m = \frac{0.418}{0.026} = 16.08 \text{ mA/V}$$

$$V_0 = -g_m V_\pi R_C$$

$$R_{ib} = r_\pi + (1+\beta)R_E = 7.46 + (121)(0.6) = 80.1 \ k\Omega$$

$$R_1 \| R_2 = 250 \| 75 = 57.7 \ k\Omega$$

$$R_1 \| R_2 \| R_{ib} = 57.7 \| 80.1 = 33.54 \ k\Omega$$

$$V_S' = \left(\frac{R_1 \| R_2 \| R_{ib}}{R_1 \| R_2 \| R_{ib} + R_S}\right) \cdot V_S = \left(\frac{33.54}{33.54 + 0.5}\right) \cdot V_S$$

$$V_S' = (0.985)V_S$$

$$V_S' = V_\pi\left[1 + \left(\frac{1+\beta}{r_\pi}\right)R_E\right]$$

Then

$$A_v = (0.985)(-8.39) = -8.27$$

b. $$R_{ib} = r_\pi + (1+\beta)(R_E) = 7.46 + (121)(0.6)$$

$$\Rightarrow \underline{R_{ib} = 80.1 \text{ k}\Omega}$$

E4.6

As a first approximation,

$$A_v \cong -\frac{R_C}{R_E}$$

Resulting gain is always smaller than this value. The effect of R_S is very small.

Set $\frac{R_C}{R_E} = 10$

$5 \cong I_C(R_C + R_E) + V_{CEQ}$

$5 = 0.5(R_C + R_E) + 2.5$

So that $\underline{R_E = 0.454\ k\Omega}$ and $\underline{R_C = 4.54\ k\Omega}$

$I_{BQ} = \frac{0.5}{100} = 0.005\ mA$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(0.454) = 4.59\ k\Omega$

$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(4.59)(5)$

or $V_{TH} = \frac{23}{R_1}$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$

$\frac{23}{R_1} = (0.005)(4.59) + 0.7 + (101)(0.005)(0.454)$

which yields, $\underline{R_1 = 24.1\ k\Omega}$ and $\underline{R_2 = 5.67\ k\Omega}$

E4.7

dc analysis

$R_{TH} = R_1 \| R_2 = 15 \| 85 = 12.75$

$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{85}{85 + 15}\right)(12)$

$V_{TH} = 10.2$ V

$I_{BQ} = \frac{12 - 0.7 - V_{TH}}{R_{TH} + (1+\beta)R_E}$

$= \frac{12 - 0.7 - 10.2}{12.75 + (101)(0.5)} = \frac{1.1}{63.25} = 0.0174$

$I_{CQ} = 1.74$ mA

ac analysis

$V_0 = h_{fe} I_b (R_C \| R_L)$

$I_b = \frac{-V_S}{h_{ie} + (1 + h_{fe})R_E}$

$A_v = \frac{-h_{fe}(R_C \| R_L)}{h_{ie} + (1 + h_{fe})R_E}$

For $I_{CQ} = 1.7$ mA

$h_{fe}(\max) = 110 \qquad h_{fe}(\min) = 70$

$h_{ie}(\max) = 2\ k\Omega \qquad h_{ie}(\min) = 1.1\ k\Omega$

$A_v(\max) = \frac{-110(4\|2)}{2 + (111)(0.5)} = -2.54$

$A_v(\min) = \frac{-70(4\|2)}{1.1 + (71)(0.5)} = -2.54$

E4.8

First approximation, $A_v = -\frac{R_C}{R_E}$ which predicts a low value. Set $\frac{R_C}{R_E} = 9$. Now

$V_{CC} \cong I_{CQ}(R_C + R_E) + V_{ECQ}$

$7.5 = (0.6)(9R_E + R_E) + 3.75$

So $\underline{R_E = 0.625\ k\Omega}$ and $\underline{R_C = 5.62\ k\Omega}$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(0.625) = 6.31\ k\Omega$

$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(6.31)(7.5)$

$V_{CC} = (1+\beta)I_{BQ}R_E + V_{EB}(on) + I_{BQ}R_{TH} + V_{TH}$

$I_{BQ} = \frac{0.6}{100} = 0.006\ mA$

$7.5 = (101)(0.006)(0.625) + 0.7 + (0.006)(6.31) + \frac{1}{R_1}(6.31)(7.5)$

Then $\underline{R_1 = 7.40\ k\Omega}$ and $\underline{R_2 = 42.8\ k\Omega}$

E4.9

dc analysis

$I_{BQ} = \frac{10 - 0.7}{100 + (101)(20)} = 0.00439$ mA

$I_{CQ} = 0.439$ mA. $I_{EQ} = 0.443$ mA

$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(100)}{0.439} = 5.92\ k\Omega$

$g_m = \frac{I_{CQ}}{V_T} = \frac{0.439}{0.026} = 16.88$ mA/V

$r_0 = \frac{V_A}{I_{CQ}} = \frac{100}{0.439} = 228\ k\Omega$

(a) $V_o = -g_m V_\pi (r_o \| R_C)$

$V_\pi = V_s' = \left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_S}\right) \cdot V_s$

$R_B \| r_\pi = 100 \| 5.92 = 5.59\ k\Omega$

Then

$V_\pi = \left(\frac{5.59}{5.59 + 0.5}\right) \cdot V_s = 0.918 V_s$

Then

$A_v = -(161.7)(0.918) = -148$

b. $R_{in} = R_B \| r_\pi = (100) \| (5.92)$

$\Rightarrow \underline{R_{in} = 5.59\ k\Omega}$

$R_0 = R_C \| r_0 = 10 \| 228 \Rightarrow \underline{R_0 = 9.58\ k\Omega}$

E4.10

$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_E} = -(0.95)\left(\frac{R_C}{R_E}\right) = -(0.95)\left(\frac{2}{0.4}\right)$

or $A_v = -4.75$

Assume $r_\pi = 1.2\ k\Omega$ from Example 4.5. Then

$\frac{-\beta(2)}{1.2 + (1+\beta)(0.4)} = -4.75$

which yields $\underline{\beta = 76}$

E4.11

dc analysis: $V_{TH} = 0, R_{TH} = R_1 \| R_2 = 10\ k\Omega$

$$I_{BQ} = \frac{5 - 0.7}{10 + (126)(5)} = 0.00672$$

$$I_{CQ} = 0.84 \text{ mA}$$

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(125)}{0.84} = 3.87 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.84}{0.026} = 32.3 \text{ mA/V}$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{200}{0.84} = 238 \text{ k}\Omega$$

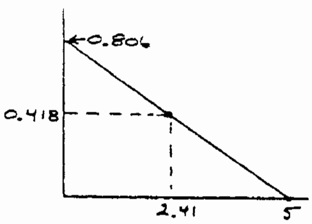
a. $v_0 = -g_m v_\pi (r_0 \| R_C \| R_L)$, $v_\pi = v_S$

$$A_v = -g_m (r_0 \| R_C \| R_L) = -(32.3)(238 \| 2.3 \| 5)$$

$$A_v = -(32.3)(1.56) \Rightarrow \underline{A_v = -50.4}$$

b. $R_0 = r_0 \| R_C \Rightarrow \underline{R_0 = 2.28 \text{ k}\Omega}$

E4.12



$$V_{CEQ} = 5 - (0.418)(5.6) - \left(\frac{121}{120}\right)(0.418)(0.6)$$

$$= 5 - 2.34 - 0.253$$

$$V_{CEQ} = 2.41$$

ΔV_{CE} variation $(2.41 - 0.5)2 = 3.82$ V peak-to-peak

E4.13

a. dc load line:

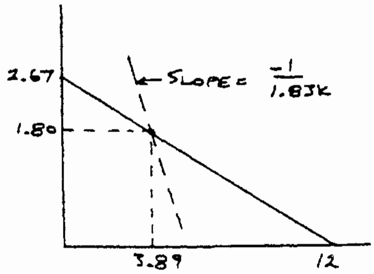
$$V_{EC} = V_{CC} - I_E R_E - I_C R_C$$

$$V_{EC} \cong 12 - I_C (R_E + R_C) = 12 - I_C (4.5)$$

ac load line:

$$v_{ec} \approx -i_C (R_E + R_C \| R_L)$$

$$v_{ec} = -i_C (0.5 + 4 \| 2) = -i_C (1.83)$$



$$I_{BQ} = \frac{12 - 0.7 - 10.2}{12.75 + (121)(0.5)} = \frac{1.1}{73.25}$$

$$I_{BQ} = 0.0150$$

$$I_{CQ} = 1.80,\ I_{EQ} = 1.82$$

$$\Rightarrow V_{ECQ} = 3.89$$

b. For $\Delta i_C = 1.8 \Rightarrow \Delta v_{EC} = (1.8)(1.83) = 3.29$

For $\Delta v_{EC} = -3.29 \Rightarrow \Delta v_{CE} = 3.89 - 3.29 = 0.6$

$\Rightarrow$ Max. symmetrical swing

$= 2 \times (3.29) = \underline{6.58 \text{ V}}$ peak-to-peak

E4.14

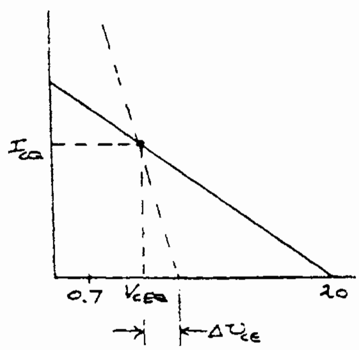
dc load line:

$$V_{CE} \approx (10 + 10) - I_C (R_C + R_E)$$

$$V_{CE} = 20 - I_C (10 + R_E)$$

$$v_{ce} = -i_C R_C = -i_C (10)$$

ac load line:



$$\Delta v_{ce} = V_{CEQ} - 0.7 = \Delta i_C (10) = I_{CQ} (10)$$

So $V_{CEQ} - 0.7 = I_{CQ}(10)$

We have

$$V_{CEQ} = 20 - I_{CQ}(10 + R_E)$$

$$I_{CQ}(10) + 0.7 = 20 - I_{CQ}(10 + R_E) \qquad (1)$$

$$I_{BQ} = \frac{10-0.7}{100+(101)R_E} \Rightarrow I_{CQ} = \frac{(100)(9.3)}{100+(101)R_E} \qquad (2)$$

From (1)

$I_{CQ}[10+10+R_E] = 20-0.7$

Substitute (2)

$\left[\frac{(100)(9.3)}{100+(101)R_E}\right](20+R_E) = 19.3$

$930(20+R_E) = 19.3[100+(101)R_E]$

$18,600 + 930R_E = 1930 + 1949.3R_E$

$16,670 = 1019.3R_E \Rightarrow \underline{R_E = 16.35\ \text{k}\Omega}$

So

$I_{CQ} = \frac{(100)(9.3)}{100+(101)(16.35)} \Rightarrow I_{CQ} = 0.531\ \text{mA}$

$V_{CEQ} = 20 - (0.531)(10+16.35) \Rightarrow V_{CEQ} = 6.0\ \text{V}$

$\Delta v_{CE} = V_{CEQ} - 0.7 = 6 - 0.7 = 5.3$

Max symmetrical swing

$2 \times (5.3) = \underline{10.6\ \text{V}}$ peak-to-peak

E4.15

a. $I_{BQ} = \frac{5-0.7}{10+(126)(5)} = 0.00672$

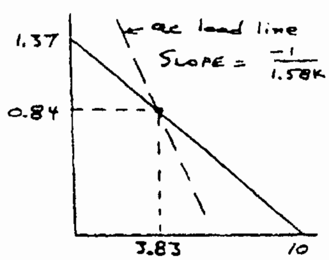
$I_{CQ} = 0.84\ \text{mA},\ I_{EQ} = 0.847\ \text{mA}$

$r_0 = \frac{V_A}{I_{CQ}} = \frac{200}{0.84} = 238\ \text{k}\Omega$

$V_{CEQ} = 10 - (0.84)(2.3) - (0.847)(5)$

$= 10 - 1.932 - 4.235$

$V_{CEQ} = 3.83\ \text{V}$



dc load line

$V_{CE} \approx 10 - I_C(7.3)$

ac load line

$v_{ce} = -i_c(R_C \| R_L) = -i_c(2.3\|5) = -i_c(1.58)$

(neglecting r_0)

b. $\Delta i_C = 0.84$

$\Rightarrow \Delta v_{CE} = (0.84)(1.58) = 1.33\ \text{V}$

$V_{CE}(\text{min}) = 3.83 - 1.33 = 2.5\ \text{V}$

$V_{CE}(\text{max}) = 3.83 + 1.33 = 5.16\ \text{V}$

So max symmetrical swing

$= 2 \times (1.33) = \underline{2.66\ \text{V}}$ peak-to-peak

E4.16

a. $V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(12)$. $V_{TH} = \frac{1}{R_1}(R_{TH})(12)$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(121)R_E$

$= 12.1R_E = 12.1\ \text{k}\Omega$

$I_{BQ} = \frac{12-0.7-V_{TH}}{R_{TH}+(1+\beta)R_E} = \frac{11.3 - \frac{1}{R_1}(12.1)(12)}{12.1+(121)(1)}$

$I_{CQ} = 1.6 \Rightarrow I_{BQ} = \frac{1.6}{120} = 0.01333\ \text{mA}$

$0.01333 = \frac{11.3 - \frac{1}{R_1}(145.2)}{133.1}$

$\frac{1}{R_1}(145.2) = 11.3 - (0.01333)(133.1)$

$\Rightarrow \underline{R_1 = 15.24\ \text{k}\Omega}$

$\frac{R_1R_2}{R_1+R_2} = 12.1 = \frac{15.24R_2}{15.24+R_2}$

$(12.1)(15.24) = (15.24-12.1)R_2$

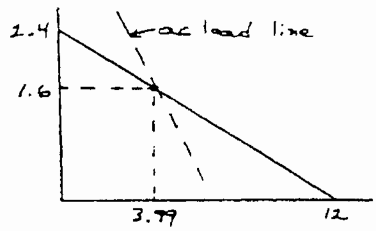
$\Rightarrow \underline{R_2 = 58.7\ \text{k}\Omega}$

$V_{ECQ} = 12 - (1.6)(4) - (1.61)(1) \Rightarrow \underline{V_{ECQ} = 3.99\ \text{V}}$

b. $v_0 = g_m v_\pi(R_C \| R_L) = -g_m(R_C \| R_L) = -v_{ec}$

$i_c = g_m v_\pi = -g_m v_{\pi}$

or $-v_{ec} = i_c(R_C \| R_L)$



Want $\Delta i_C = 1.6 - 0.1 = 1.5$

$\Delta v_{ec} = 3.99 - 0.5 = 3.49$

$\frac{\Delta v_{ec}}{\Delta i_C} = \frac{3.49}{1.5} = 2.327\ \text{k}\Omega = R_C \| R_L$

$\frac{R_C R_L}{R_C+R_L} = \frac{4R_L}{4+R_L} = 2.327$

$(4-2.327)R_L = (4)(2.327) \Rightarrow \underline{R_L = 5.56\ \text{k}\Omega}$

E4.17

$R_{TH} = R_1 \| R_2 = 25\|50 = 16.7\ \text{k}\Omega$

$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)V_{CC} = \left(\frac{50}{50+25}\right)(5)$

$V_{TH} = 3.33\ \text{V}$

$I_{BQ} = \frac{V_{BB}-V_{BE}(\text{on})}{R_{TH}+(1+\beta)R_E} = \frac{3.33-0.70}{16.7+(121)(1)}$

$= \frac{2.63}{137.7}$

$\Rightarrow I_{BQ} = 0.0191$

$I_{CQ} = 2.29\ \text{mA}$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.29}{0.026} = 88.1 \text{ mA/V}$$

$$r_\pi = \frac{V_T \beta}{I_{CQ}} = \frac{(0.026)(120)}{2.29} = 1.36 \text{ k}\Omega$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{100}{2.29} = 43.7 \text{ k}\Omega$$

a.

$$V_s' = \left(\frac{R_1 \| R_2 \| R_{ib}}{R_1 \| R_2 \| R_{ib} + R_S}\right) \cdot V_s$$

$$R_{ib} = r_\pi + (1+\beta)(R_E \| r_o) = (1.36) + (121)(1 \| 43.7) \Rightarrow$$

$$R_{ib} = 120 \text{ k}\Omega \text{ and } R_1 \| R_2 = 16.7 \text{ k}\Omega$$

Then $R_1 \| R_2 \| R_{ib} = 16.7 \| 120 = 14.7 \text{ k}\Omega$

Then

$$V_s' = \left(\frac{14.7}{14.7 + 0.5}\right) \cdot V_s = (0.967) V_s$$

Now

$$V_o = \left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right)(R_E \| r_o) = V_\pi \left(\frac{1+\beta}{r_\pi}\right) R_E \| r_o$$

$$V_\pi = \frac{V_s'}{1 + \left(\frac{1+\beta}{r_\pi}\right) R_E \| r_o} = \frac{(0.967) V_s}{1 + \left(\frac{1+\beta}{r_\pi}\right) R_E \| r_o}$$

So

$$\frac{V_o}{V_s} = \frac{(0.967)\left(\frac{1+\beta}{r_\pi}\right) R_E \| r_o}{1 + \left(\frac{1+\beta}{r_\pi}\right) R_E \| r_o} = \frac{(0.967)(1+\beta) R_E \| r_o}{r_\pi + (1+\beta) R_E \| r_o}$$

$$R_E \| r_o = 1 \| 43.7 = 0.978 \text{ k}\Omega$$

Then

$$A_v = \frac{(0.967)(121)(0.978)}{1.36 + (121)(0.978)} \Rightarrow \underline{A_v = 0.956}$$

b. $R_{ib} = r_\pi + (1+\beta) R_E \| r_0 = 1.36 + (121)(0.978)$

$\Rightarrow \underline{R_{ib} = 120 \text{ k}\Omega}$

(c)

$$R_o = R_E \| r_o \left\| \frac{r_\pi + R_1 \| R_2 \| R_S}{1+\beta} = 1 \right\| 43.7 \left\| \frac{1.36 + 16.7 \| 0.5}{121} \right.$$

which yields

$\underline{R_o = 15.1 \, \Omega}$

E4.18

$$V_{CEQ} = 5 \text{ V} \Rightarrow I_{EQ} = \frac{5}{2} = 2.5 \text{ mA}$$

$$I_{BQ} = \frac{2.5}{101} = 0.0248 \text{ mA}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(\text{on}) + (1+\beta) I_{BQ} R_E$$

$$R_{in} = R_{TH} \| [r_\pi + (1+\beta) R_E]$$

$$r_\pi = 1.05 \text{ k}\Omega$$

$$65 = R_{TH} \| 203 = \frac{R_{TH} \cdot 203}{R_{TH} + 203}$$

$$\Rightarrow R_{TH} = 95.6 \text{ k}\Omega$$

$$\frac{1}{R_1}(95.6)(10) = (0.0248)(95.6) + 0.7 + 2.5(2)$$

$$= 8.07$$

$\underline{R_1 = 118 \text{ k}\Omega}$, $\quad \frac{118 R_2}{118 + R_2} = 95.6$

$\underline{R_2 = 504 \text{ k}\Omega}$

$$R_{in} = 65 \text{ k}\Omega$$

$$V_s' = \left(\frac{R_{in}}{R_{in} + R_S}\right) \cdot V_s = \left(\frac{65}{65 + 0.5}\right) \cdot V_s = 0.992 V_s$$

Then

$$A_v = \frac{(0.992)(1+\beta) R_E}{r_\pi + (1+\beta) R_E} = \frac{(0.992)(101)(2)}{1.05 + (101)(2)} \Rightarrow$$

$\underline{A_v = 0.987}$

Neglecting R_S, $\underline{A_v = 0.995}$

$$R_o = R_E \left\| \frac{r_\pi + R_1 \| R_2 \| R_S}{1+\beta} = 2 \right\| \frac{1.05 + 95.6 \| 0.5}{101}$$

or

$$R_o = 15.2 \, \Omega$$

Neglecting R_S, $R_o = 2 \left\| \frac{1.05}{102} \right. \Rightarrow \underline{R_o = 10.3 \, \Omega}$

E4.19

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5$$

$\beta = 100$, $V_A = 125$ V, $V_{BE}(\text{on}) = 0.7$ V

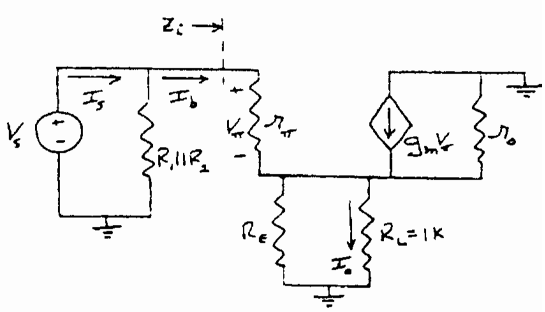
$I_{CQ} = 0.75$ mA

Then $r_0 = \frac{125}{0.75} = 167 \text{ k}\Omega$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.75} \Rightarrow r_\pi = 3.47 \text{ k}\Omega$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on}) - (-5)}{R_{TH} + (1+\beta) R_E}$$

$$I_{CQ} = \beta I_{BQ}$$



$$Z_i = r_\pi + (1+\beta)[R_E \| R_L \| r_o]$$

$$I_o = \left(\frac{R_E \| r_o}{R_E \| r_o + R_L}\right)(1+\beta)I_b$$

$$I_b = \left(\frac{R_1 \| R_2}{R_1 \| R_2 + Z_i}\right)I_S$$

$$A_I = \frac{I_o}{I_S}$$

$$= \left(\frac{R_E \| r_o}{R_E \| r_o + R_L}\right)(1+\beta)\left(\frac{R_1 \| R_2}{R_1 \| R_2 + Z_i}\right)$$

Assume that $R_E \| r_o \approx R_E$

$$R_1 \| R_2 = (0.1)(1+\beta)R_E$$

$$= (0.1)(101)R_E = 10.1R_E$$

Assume $R_L = 1\,k\Omega$

Then

$$A_I = 15 = \left(\frac{R_E}{R_E+1}\right)(101)\times \left(\frac{10.1R_E}{10.1R_E + 3.47 + (101)[R_E \| 1\text{ k}\Omega]}\right)$$

where $R_E \| R_L \| r_o \approx R_E \| R_L = R_E \| 1\text{ k}\Omega$

$$15 = \frac{(101)(10.1)R_E^2}{R_E+1} \times \frac{1}{\left[10.1R_E + 3.47 + \dfrac{101R_E}{1+R_E}\right]}$$

$$15 = \frac{(101)(10.1)R_E^2}{R_E+1} \times \frac{1}{\dfrac{(1+R_E)[10.1R_E + 3.47] + 101R_E}{1+R_E}}$$

$$15 = \frac{(101)(10.1)R_E^2}{10.1R_E + 3.47 + 10.1R_E^2 + 3.47R_E + 101R_E}$$

$$15 = \frac{(101)(10.1)R_E^2}{10.1R_E^2 + 114.57R_E + 3.47}$$

$$(101)(10.1)R_E^2 = 15[10.1R_E^2 + 114.57R_E + 3.47]$$

$$1020.1R_E^2 = 151.5R_E^2 + 1718.55R_E + 52.05$$

$$868.6R_E^2 - 1718.55R_E - 52.05 = 0$$

$$16.7R_E^2 - 33.0R_E - 1 = 0$$

$$R_E = \frac{33 \pm \sqrt{(33)^2 + 4(16.7)}}{2(16.7)}$$

Must use + sign $\Rightarrow \underline{R_E = 2.0\text{ k}\Omega}$

Then $R_1 \| R_2 = 10.1R_E = 20.2\text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(10) - 5$$

$$= \frac{1}{R_1}\left(\frac{R_1R_2}{R_1+R_2}\right)(10) - 5$$

$$= \frac{1}{R_1}(20.2)(10) - 5$$

$$I_{CQ} = 0.75$$

$$= (100)\left\{\frac{\frac{1}{R_1}(20.2)(10) - 5 - 0.7 + 5}{20.2 + (101)(2)}\right\}$$

$$1.67 = \frac{1}{R_1}(202) - 0.7 \Rightarrow \underline{R_1 = 85.2\text{ k}\Omega}$$

$$\frac{R_1R_2}{R_1+R_2} = 20.2 = \frac{85.2R_2}{85.2+R_2} \Rightarrow \underline{R_2 = 26.5\text{ k}\Omega}$$

E4.20

a. $\beta = 100$. $V_{BE}(\text{on}) = 0.7$, $I_{CQ} = 1.25$ mA

$$I_{EQ} = 1.26\text{ mA},\ I_{BQ} = 0.0125\text{ mA}$$

$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(10) - 5$$

$$V_{TH} = \frac{1}{R_1}(R_1 \| R_2)(10) - 5$$

$$I_{BQ} = \frac{V_{TH} - 0.7 - (-5)}{R_{TH} + (1+\beta)R_E}$$

$$V_{CEQ} = 10 - I_{EQ}R_E = 4$$

$$I_{EQ}R_E = 6 \Rightarrow I_{EQ} = \frac{6}{R_E} \Rightarrow R_E = \frac{6}{1.26} = 4.76\text{ k}\Omega$$

$$\Rightarrow \underline{R_E = 4.76\text{ k}\Omega}$$

$$R_{TH} = (0.1)(1+\beta)R_E = 10.1R_E$$

Then

$$I_{BQ} = \frac{I_{EQ}}{101} = \frac{\frac{1}{R_1}(101)R_E(10) - 5 - 0.7 + 5}{10.1R_E + (101)R_E}$$

$$0.0125 = \frac{\frac{1}{R_1}(101)(4.76) - 0.7}{(111.1)(4.76)} \Rightarrow \underline{R_1 = 65.8\text{ k}\Omega}$$

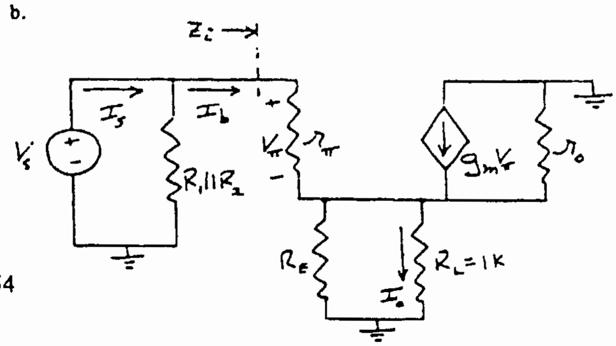
$$\frac{R_1R_2}{R_1+R_2} = (10.1)R_E = (10.1)(4.76) = 48.1\text{ k}\Omega$$

$$(65.8)R_2 = (48.1)(65.8) + (48.1)R_2$$

$$(65.8 - 48.1)R_2 = (48.1)(65.8)$$

$$\Rightarrow \underline{R_2 = 178.8\text{ k}\Omega}$$

b.



$$r_O = \frac{V_A}{I_{CQ}} = \frac{125}{1.25} = 100 \text{ k}\Omega$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.25} = 2.08 \text{ k}\Omega$$

$$g_m V_\pi = g_m(i_b r_\pi) = \beta I_b$$

$$Z_i = r_\pi + (1+\beta)[R_E \| R_L \| r_O]$$

$$= 2.08 + (101)[4.76\|1\|100]$$

$$= 2.08 + (101)[0.826\|100]$$

$$= 2.08 + (101)(0.819)$$

$$Z_i = 84.8 \text{ k}\Omega$$

Assume $R_L = 1\ k\Omega$

$$I_O = \left(\frac{R_E\|r_O}{R_E\|r_O + R_L}\right)(1+\beta)I_b$$

$$I_b = \left(\frac{R_1\|R_2}{R_1\|R_2 + Z_i}\right)I_S$$

$$A_I = \frac{I_O}{I_S} = \left(\frac{R_E\|r_O}{R_E\|r_O + R_L}\right)(1+\beta)\left(\frac{R_1\|R_2}{R_1\|R_2 + Z_i}\right)$$

$$R_E\|r_O = 4.76\|100 = 4.54$$

$$A_I = \left(\frac{4.54}{4.54+1}\right)(101)\left(\frac{48.1}{48.1+84.8}\right)$$

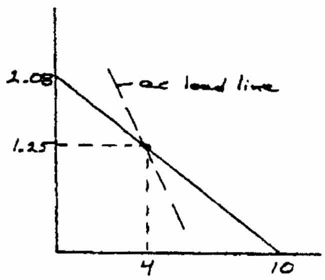
$$= \left(\frac{4.54}{5.54}\right)(101)\left(\frac{48.1}{132.9}\right)$$

$$\Rightarrow \underline{A_I = 30.0}$$

c. $R_O = \dfrac{r_\pi}{1+\beta} \,\Big\|\, R_E \,\Big\|\, r_O = \dfrac{2.08}{101} \,\Big\|\, 4.76 \,\Big\|\, 100$

$$\Rightarrow R_O = 20.4\ \Omega$$

d.



$$v_O = v_{ce} = (1+\beta)i_b(R_E\|R_L\|r_O)$$

$$i_b = \frac{i_C}{\beta}$$

$$v_{ce} = i_C\left(\frac{1+\beta}{\beta}\right)(R_E\|R_L\|r_O)$$

$$= i_C\left(\frac{101}{100}\right)(4.76\|1\|100)$$

$$= i_C(0.828)$$

If $\Delta i_C = 1.25$ mA, $\rightarrow \Delta v_{ce} = 1.035$ V

Maximum symmetrical swing in output voltage

is $= 2\Delta v_{ce} =$ **2.07 V peak-to-peak**

E4.21

For $\beta = 130$

$$I_{BQ} = \frac{10 - 0.7}{100 + (131)(10)} \Rightarrow 6.596\ \mu A$$

$$I_{CQ} = 0.857 \text{ mA}$$

From Figure 4.21

$3 < h_{ie} < 5$ kΩ Let $h_{re} = 0$

$98 < h_{fe} < 170$

$8 < h_{oe} < 16\ \mu$S

$$h_{ie} = 4 \text{ k}\Omega$$

$$h_{fe} = 134$$

$$h_{oe} = 12\ \mu S \Rightarrow \frac{1}{h_{oe}} = 83.3 \text{ k}\Omega$$

a. $R_S = R_L = 10$ kΩ

$$R_{ib} = h_{ie} + (1+h_{fe})\left(R_E \,\Big\|\, R_L \,\Big\|\, \frac{1}{h_{oe}}\right)$$

$$= 4 + (135)(10\|10\|83.3)$$

$$\Rightarrow \underline{R_{ib} = 641 \text{ k}\Omega}$$

$$A_v = \left(\frac{R_B\|R_{ib}}{R_S + R_B\|R_{ib}}\right) \times \left(\frac{(1+h_{fe})\left(R_E \,\Big\|\, R_L \,\Big\|\, \frac{1}{h_{oe}}\right)}{h_{ie} + (1+h_{fe})\left(R_E \,\Big\|\, R_L \,\Big\|\, \frac{1}{h_{oe}}\right)}\right)$$

$$A_v = \left(\frac{100\|641}{10 + 100\|641}\right)\left[\frac{(135)(10\|10\|83.3)}{4 + (135)(10\|10\|83.3)}\right]$$

$$= \left(\frac{86.5}{10+86.5}\right)\left[\frac{637}{641}\right]$$

$$\Rightarrow \underline{A_v = 0.891}$$

$$A_i = \left(\frac{R_E \,\Big\|\, \frac{1}{h_{oe}}}{R_E \,\Big\|\, \frac{1}{h_{oe}} + R_L}\right)(1+h_{fe})\left(\frac{R_B}{R_B + R_{ib}}\right)$$

$$= \left(\frac{10\|83.3}{10\|83.3 + 10}\right)(135)\left(\frac{100}{100+641}\right)$$

$$\Rightarrow \underline{A_i = 8.59}$$

$$R_O = R_E \,\Big\|\, \frac{1}{h_{oe}} \,\Big\|\, \frac{h_{ie} + R_S\|R_B}{1 + h_{fe}}$$

$$= 10 \,\Big\|\, 83.3 \,\Big\|\, \frac{4 + 10\|100}{135} = 8.93\|0.0970$$

$$\Rightarrow \underline{R_O = 96.0\ \Omega}$$

b. $R_S = 1$ kΩ, $\underline{R_{ib} = 641 \text{ k}\Omega}$, $\underline{A_i = 8.59}$

$$A_v = \left(\frac{86.5}{1+86.5}\right)\left(\frac{637}{641}\right) \Rightarrow \underline{A_v = 0.982}$$

$$R_O = 10 \,\Big\|\, 8.33 \,\Big\|\, \left[\frac{4 + 1\|100}{135}\right]$$

$$= 8.93\|0.03696 \Rightarrow \underline{R_O = 36.8\ \Omega}$$

E4.22

$V_{TH} = 2.5\,V,\ R_{TH} = 25\,k\Omega$

$I_{BQ} = \dfrac{5 - 0.7 - 2.5}{25 + (101)(2)} = \dfrac{1.8}{227} = 0.00793 \text{ mA}$

$I_{CQ} = 0.793 \text{ mA}$

$g_m = \dfrac{I_{CQ}}{V_T} = \dfrac{0.793}{0.026} = 30.5 \text{ mA/V}$

$r_\pi = \dfrac{V_T\beta}{I_{CQ}} = \dfrac{(0.0026)(100)}{0.793} = 3.28 \text{ k}\Omega$

$r_0 = \dfrac{V_A}{I_{CQ}} = \dfrac{125}{0.793} = 158 \text{ k}\Omega$

a. $R_E \| R_L \| r_0 = 2\|0.5\|158 = 0.4\|158 \approx 0.4$

$A_\nu = \dfrac{(1+\beta)(R_E\|R_L\|r_0)}{r_\pi + (1+\beta)(R_E\|R_L\|r_0)}$

$= \dfrac{(101)(0.4)}{3.28 + (101)(0.4)} \Rightarrow \underline{A_\nu = 0.925}$

b. $R_{ib} = r_\pi + (1+\beta)(R_E\|R_L\|r_0)$

$R_{ib} = 3.28 + (101)(0.4)$

$\Rightarrow \underline{R_{ib} = 43.7 \text{ k}\Omega}$

$R_0 = \dfrac{r_\pi}{1+\beta} \,\Big\|\, R_E \,\Big\|\, r_0 = \dfrac{3.28}{101} \,\Big\|\, 2 \,\Big\|\, 98.3$

$\Rightarrow \underline{R_0 = 32.0\ \Omega}$

c. $I_B(\text{max})$ —

$R_E(\text{min}) = 1.9 \text{ k}\Omega$

$R_2(\text{min}) = 47.5 \text{ k}\Omega$

$R_1(\text{max}) = 52.5 \text{ k}\Omega$

$R_{TH} = 24.9 \text{ k}\Omega$

$V_{TH} = \left(\dfrac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\dfrac{47.5}{100}\right)(5) = 2.375$

$I_{BQ} = \dfrac{5 - 0.7 - 2.375}{24.9 + (101)(1.9)} = \dfrac{1.925}{216.8}$

$I_{CQ} = 0.888 \text{ mA}$

$R_E(\text{max}) = 2.1 \text{ k}\Omega$

$R_2(\text{max}) = 52.5 \text{ k}\Omega$

$R_1(\text{min}) = 47.5 \text{ k}\Omega$

$R_{TH} = 24.9 \text{ k}\Omega$

$V_{TH} = \left(\dfrac{52.5}{100}\right)(5) = 2.625$

$I_{CQ} = (100)\left[\dfrac{5 - 0.7 - 2.625}{24.9 + (101)(2.1)}\right] = \dfrac{(100)(1.675)}{237}$

$I_{CQ} = 0.707 \text{ mA}$

$r_\pi(\text{max}) = \dfrac{(100)(0.026)}{0.707} = 3.68 \text{ k}\Omega$

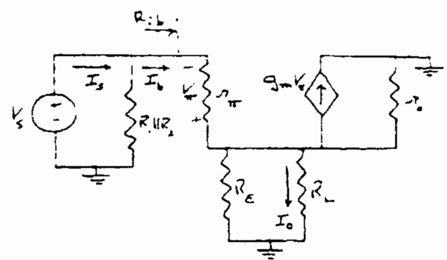
$R_0 = 1.96 \,\Big\|\, \dfrac{3.68}{101} = 1.96\|0.0364 = 35.7\ \Omega$

$r_\pi(\text{min}) = 2.93 \text{ k}\Omega$

$R_0 = 1.96 \,\Big\|\, \dfrac{2.93}{101} = 1.96\|0.0290 = 28.6\ \Omega$

$\Rightarrow \underline{28.6 \le R_0 \le 35.7\ \Omega}$

E4.23



For $V_{ECQ} = 2.5$ V,

$I_{EQ} = \dfrac{5 - 2.5}{R_E} = \dfrac{5 - 2.5}{0.5} = 5 \text{ mA}$

$I_{CQ} = \left(\dfrac{75}{76}\right)(5) = 4.93 \text{ mA} \Rightarrow I_{BQ} = 0.0658 \text{ mA}$

$V_\pi = -I_b r_\pi$

$r_\pi = \dfrac{\beta V_T}{I_{CQ}} = \dfrac{(75)(0.026)}{4.93} = 0.396 \text{ k}\Omega$

$r_0 = \dfrac{V_A}{I_{CQ}} = \dfrac{75}{4.93} = 15.2 \text{ k}\Omega$

$g_m V_\pi = g_m(-I_b r_\pi) = -\beta I_b$

$I_0 = \left(\dfrac{R_E\|r_0}{R_E\|r_0 + R_L}\right) \times (1+\beta) I_b$

$I_b = \left(\dfrac{R_1\|R_2}{R_1\|R_2 + Z_{ib}}\right) I_S$

$A_I = \dfrac{I_0}{I_S} = \left(\dfrac{R_E\|r_0}{R_E\|r_0 + R_L}\right)(1+\beta)\left(\dfrac{R_1\|R_2}{R_1\|R_2 + Z_{ib}}\right)$

a. $R_E = R_L = 0.5 \text{ k}\Omega$

$R_{ib} = r_\pi + (1+\beta)(R_E\|R_L\|r_0)$

$= 0.396 + (76)[0.5\|0.5\|15.2]$

$= 0.396 + (76)(0.246)$

$\Rightarrow R_{ib} = 19.1 \text{ k}\Omega$

$R_E\|r_0 = 0.5\|15.2 = 0.484 \text{ k}\Omega$

$A_I = 10 = \left(\dfrac{0.484}{0.484 + 0.5}\right)(76)\left(\dfrac{R_1\|R_2}{R_1\|R_2 + 19.1}\right)$

$10 = 37.38\left(\dfrac{R_1\|R_2}{R_1\|R_2 + 19.1}\right)$

$0.2675(R_1\|R_2 + 19.1) = R_1\|R_2$

$R_1\|R_2 = 6.975 \text{ k}\Omega$

$V_{TH} = \left(\dfrac{R_2}{R_1 + R_2}\right) V_{CC} = \dfrac{1}{R_1}(R_1\|R_2) V_{CC}$

$= \dfrac{1}{R_1}(6.975)(5)$

$$I_{BQ} = \frac{5 - V_{EB}(\text{on}) - V_{TH}}{R_{TH} + (1+\beta)R_E}$$

$$0.0658 = \frac{5 - 0.7 - V_{TH}}{6.975 + (76)(0.5)}$$

$$2.96 = 4.3 - V_{TH} \Rightarrow V_{TH} = 1.34 = \frac{1}{R_1}(6.975)(5)$$

$$\Rightarrow \underline{R_1 = 26.0\ \text{k}\Omega}$$

$$\frac{R_1 R_2}{R_1 + R_2} = 6.975 = \frac{26 R_2}{26 + R_2}$$

$$6.975(26 + R_2) = 26 R_2$$

$$\Rightarrow \underline{R_2 = 9.53\ \text{k}\Omega}$$

b. For $R_E = 4R_L = 4(0.5) \Rightarrow R_E = 2\ \text{k}\Omega$

$$I_{EQ} = \frac{5 - 2.5}{2} = 1.25\ \text{mA} \rightarrow I_{CQ} = 1.23\ \text{mA}$$

$$\rightarrow I_{BQ} = 0.0164\ \text{mA}$$

$$r_\pi = \frac{(75)(0.026)}{1.23} = 1.59\ \text{k}\Omega$$

$$r_o = \frac{75}{1.23} = 60.9\ \text{k}\Omega$$

$$Z_{ib} = r_\pi + (1+\beta)[R_E \| R_L \| r_o]$$

$$= 1.59 + (76)[2\|0.5\|60.9]$$

$$\Rightarrow Z_{ib} = 31.8\ \text{k}\Omega$$

$$R_E \| r_o = 2\|60.9 = 1.94\ \text{k}\Omega$$

$$A_I = \left(\frac{1.94}{1.94 + 0.5}\right)(76)\left(\frac{R_1\|R_2}{R_1\|R_2 + 31.8}\right) = 10$$

$$10 = 60.4\left(\frac{R_1\|R_2}{R_1\|R_2 + 31.8}\right)$$

$$0.166(R_1\|R_2 + 31.8) = R_1\|R_2$$

$$R_1\|R_2 = 6.33\ \text{k}\Omega$$

Then $I_{BQ} = 0.0164 = \dfrac{4.3 - V_{TH}}{6.33 + (76)(2)}$

$$V_{TH} = 1.70 = \frac{1}{R_1}(R_1\|R_2)V_{CC} = \frac{1}{R_1}(6.33)(5)$$

$$\Rightarrow \underline{R_1 = 18.6\ \text{k}\Omega}$$

$$\frac{R_1 R_2}{R_1 + R_2} = 6.33 = \frac{(18.6)R_2}{18.6 + R_2}$$

$$6.33(18.6 + R_2) = (18.6)R_2$$

$$\Rightarrow \underline{R_2 = 9.6\ \text{k}\Omega}$$

E4.24

a. $I_{EQ} = \dfrac{10 - 0.7}{10} = 0.93\ \text{mA}$

$$I_{CQ} = \left(\frac{\beta}{1+\beta}\right) I_{EQ} = \left(\frac{100}{101}\right)(0.93)$$

$$\Rightarrow \underline{I_{CQ} = 0.921\ \text{mA}}$$

$$V_{ECQ} = 10 + 10 - I_{CQ}R_C - I_{EQ}R_E$$

$$V_{ECQ} = 20 - (0.921)(5) - (0.93)(10)$$

$$\Rightarrow \underline{V_{ECQ} = 6.1\ \text{V}}$$

b. $r_\pi = \dfrac{\beta V_T}{I_{CQ}} = \dfrac{(100)(0.026)}{0.921} = 2.82\ \text{k}\Omega$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.921}{0.026} = 35.42\ \text{mA/V}$$

$i_0 = g_m V_\pi$ and $V_\pi = v_S$

$$i_i = \frac{v_S}{R_E\|r_\pi} + g_m V_\pi = v_S\left(\frac{1}{R_E\|r_\pi} + g_m\right)$$

$$A_I = \frac{i_0}{i_i} = \frac{g_m v_S}{v_S\left(\frac{1}{R_E\|r_\pi} + g_m\right)} = \frac{g_m(R_E\|r_\pi)}{1 + g_m(R_E\|r_\pi)}$$

$$= \frac{(35.42)(10\ \text{k}\Omega\|2.82\ \text{k}\Omega)}{1 + (35.42)(10\ \text{k}\Omega\|2.82\ \text{k}\Omega)}$$

$$\Rightarrow \underline{A_I = 0.987}$$

$A_v = \dfrac{v_0}{v_S}$ and $v_0 = g_m V_\pi R_C = g_m v_S R_C$

$$A_v = g_m R_C = (35.42)(5) \Rightarrow \underline{A_v = 177.1}$$

c. $V_{ECQ} = 6.1\ \text{V} \Rightarrow V_{ECQ} = V_E - V_C$

$$V_C = V_E - V_{ECQ} = 0.7 - 6.1 = -5.4\ \text{V}$$

$$v_C = V_C + i_0 R_C$$

For $v_{EC} = 0.5 \Rightarrow v_C = +0.2$

$$+0.2 = -5.4 + i_0 R_C$$

$$i_0 = \frac{0.2 + 5.4}{5} = 1.12\ \text{mA}$$

$\Rightarrow$ Current limited

$i_0(\text{max}) = 0.921$

$\Rightarrow v_0(\text{peak}) = (0.921)(5) = 4.61$

$\Rightarrow$ <u>9.21 V</u> peak-to-peak

E4.25

a. $I_{BQ} = \dfrac{V_{EE} - V_{BE}(\text{on})}{R_B + (1+\beta)R_E} = \dfrac{10 - 0.7}{100 + (101)(10)}$

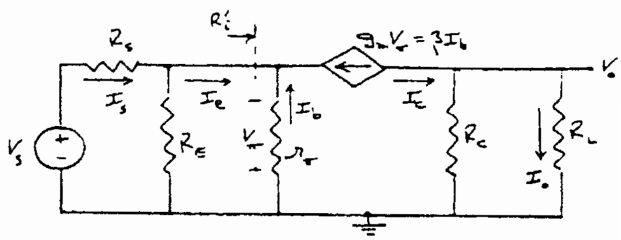
$\Rightarrow I_{BQ} = 8.38\ \mu\text{A},\ I_{CQ} = 0.838\ \text{mA}$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.838} \Rightarrow \underline{r_\pi = 3.10\ \text{k}\Omega}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.838}{0.026} \Rightarrow \underline{g_m = 32.23\ \text{mA/V}}$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{\infty}{0.838} \Rightarrow \underline{r_0 = \infty}$$

b.



$$g_m V_\pi + \frac{V_\pi}{r_\pi} = \left(\frac{-V_\pi}{R_E}\right) + \frac{(-V_\pi - V_S)}{R_S}$$

$$V_\pi\left[\left(\frac{1+\beta}{r_\pi}\right) + \frac{1}{R_E} + \frac{1}{R_S}\right] = -\frac{V_S}{R_S}$$

$$V_\pi = -\frac{V_S}{R_S}\left[\left(\frac{1+\beta}{r_\pi}\right) \| R_E \| R_S\right]$$

$$V_0 = -g_m V_\pi (R_C \| R_L)$$

$$A_v = \frac{V_0}{V_S} = g_m \frac{(R_C\|R_L)}{R_S}\left[\left(\frac{r_\pi}{1+\beta}\right) \| R_E \| R_S\right]$$

$$= \frac{(32.23)[10\|1]}{(1)}\left\{\frac{3.10}{101} \| 10 \| 1\right\}$$

$$= (32.23)(0.909)\{0.0297\} \Rightarrow \underline{A_v = 0.870}$$

$$R_i' = \frac{r_\pi}{1+\beta} = \frac{3.10}{101} = 0.0307 \text{ k}\Omega$$

$$I_e = \left(\frac{R_E}{R_E + R_i}\right) I_S \approx I_S$$

$$I_C = \left(\frac{\beta}{1+\beta}\right) I_e. \quad I_0 = \left(\frac{R_C}{R_C + R_L}\right) I_C$$

$$I_e = \left(\frac{R_C}{R_C + R_L}\right)\left(\frac{\beta}{1+\beta}\right) I_S$$

$$A_I = \frac{I_0}{I_S} = \left(\frac{10}{10+1}\right)\left(\frac{100}{101}\right) \Rightarrow \underline{A_I = 0.900}$$

c. $R_i = R_E \| R_i' = 10\|0.0307$

$$\Rightarrow \underline{R_i \approx 30.7\ \Omega}$$

$$\underline{R_0 = R_C = 10 \text{ k}\Omega}$$

E4.26

$$5 = I_B R_B + V_{BE}(\text{on}) + I_E R_E$$

$$I_B = \frac{5 - 0.7}{R_B + (101)R_E} = \frac{4.3}{R_B + (101)R_E}$$

$$I_C = \frac{(100)(4.3)}{R_B + (101)R_E}$$

$$5 = I_C R_C + V_{CE} + I_E R_E - 5$$

$$V_{CE} = 10 - I_C\left(R_C + \left(\frac{101}{100}\right) R_E\right)$$

ac analysis

$$V_0 = -g_m V_\pi (R_C \| R_L)$$

$$V_S = -V_\pi - \frac{V_\pi}{r_\pi} \cdot R_B = -V_\pi\left(1 + \frac{R_B}{r_\pi}\right)$$

$$\text{or } V_\pi = -\left(\frac{r_\pi}{r_\pi + R_B}\right) V_S$$

$$\frac{V_0}{V_S} = -g_m\left[-\left(\frac{r_\pi}{r_\pi + R_B}\right)\right](R_C \| R_L)$$

$$A_v = \frac{V_0}{V_S} = \frac{\beta}{r_\pi + R_B}(R_C \| R_L)$$

For $I_C = 1$ mA,

$$r_\pi = \frac{\beta V_T}{I_C} = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega$$

$$A_v = 20 = \frac{(100)(1)}{2.6 + R_B}$$

$$R_B = \frac{(100)(1)}{20} - 2.6 \Rightarrow \underline{R_B = 2.4 \text{ k}\Omega}$$

$$I_C = 1 = \frac{(100)(4.3)}{2.4 + (101)R_E}$$

$$R_E = \frac{\frac{(100)(4.3)}{1} - 2.4}{(101)} \Rightarrow R_E = 4.23 \text{ k}\Omega$$

E4.27

a. $R_{TH} = 70\|6 = 5.526$ kΩ

$$V_{TH} = \left(\frac{6}{6+70}\right)(10) - 5 = -4.21 \text{ V}$$

$$I_{B1} = \frac{5 - 4.21 - 0.70}{5.526 + (126)(0.2)} = \frac{0.090}{30.726}$$

$$\Rightarrow I_{B1} = 2.93\ \mu\text{A}. \quad \underline{I_{CQ1} = 0.366 \text{ mA}}$$

$$\frac{5 - V_{C1}}{5} = I_{C1} + \frac{(V_{C1} - 0.7) - (-5)}{(1+\beta)1.5}$$

$$\frac{5 - V_{C1}}{5} = 0.366 + \frac{V_{C1}}{(126)(1.5)} + \frac{4.3}{(126)(1.5)}$$

$$1 - 0.366 - \frac{4.3}{(126)(1.5)} = \frac{V_{C1}}{5} + \frac{V_{C1}}{(126)(1.5)}$$

$$0.6112 = V_{C1}(0.2053) \Rightarrow V_{C1} = 2.977 \text{ V}$$

$$I_{E1} = 0.369 \text{ mA}$$

$$V_{E1} = (0.369)(0.2) - 5 \Rightarrow V_{E1} = -4.926 \text{ V}$$

$$\Rightarrow V_{CE1} = V_{C1} - V_{E1} = 2.977 - (-4.926)$$

$$\Rightarrow \underline{V_{CE1} = 7.90 \text{ V}}$$

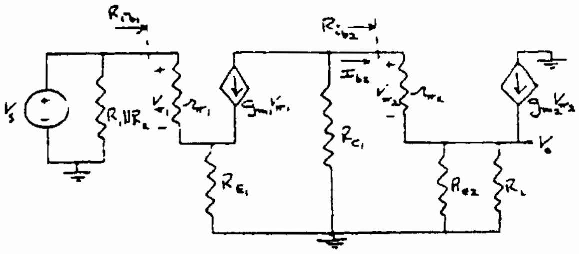
$$I_{EQ2} = \frac{(V_{C1} - 0.7) - (-5)}{1.5} = \frac{5 + 2.98 - 0.7}{1.5}$$

$$= 4.85 \text{ mA}$$

$$I_{CQ2} = \left(\frac{\beta}{1+\beta}\right) I_{EQ2} \Rightarrow \underline{I_{CQ2} = 4.81 \text{ mA}}$$

$$V_{E2} = V_{C1} - 0.7 = 2.98 - 0.7 = 2.28$$

$$V_{CEQ2} = 5 - V_{E2} = 5 - 2.28 \Rightarrow \underline{V_{CEQ2} = 2.72 \text{ V}}$$



$$r_{\pi1} = \frac{\beta V_T}{I_{CQ1}} = \frac{(125)(0.026)}{0.366} = 8.88 \text{ k}\Omega$$

$$r_{\pi2} = \frac{\beta V_T}{I_{CQ2}} = \frac{(125)(0.026)}{4.81} = 0.676 \text{ k}\Omega$$

$$g_{m1} = \frac{I_{CQ1}}{V_T} = \frac{0.366}{0.026} = 14.1 \text{ mA/V}$$

$$g_{m2} = \frac{I_{CQ2}}{V_T} = \frac{4.81}{0.026} = 185 \text{ mA/V}$$

$$R_{ib1} = r_{\pi 1} + (1+\beta)R_{E1} = 8.88 + (126)(0.2)$$
$$= 34.08 \text{ k}\Omega$$
$$R_{ib2} = r_{\pi 2} + (1+\beta)(R_{E2}\|R_L)$$
$$= 0.676 + (126)(1.5\|10) = 165 \text{ k}\Omega$$

$$V_0 = (1+\beta)I_{b2}(R_{E2}\|R_L)$$
$$I_{b2} = \left(\frac{R_{C1}}{R_{C1}+R_{ib2}}\right)(-g_{m1}V_{\pi 1})$$
$$V_{\pi 1} = \frac{V_S}{Z_1}\cdot r_{\pi 1}$$
$$A_v = (1+\beta)(R_{E2}\|R_L)\left(\frac{R_{C1}}{R_{C1}+R_{ib2}}\right)\left(\frac{-g_{m1}r_{\pi 1}}{R_{ib1}}\right)$$
$$A_v = \frac{V_0}{V_S}$$
$$= -(126)(125)(1.5\|10)\left(\frac{5}{5+165}\right)\left(\frac{1}{34.08}\right)$$
$$= -(126)(125)(1.30)\left(\frac{5}{170}\right)\left(\frac{1}{34.08}\right)$$
$$\underline{A_v = -17.7}$$

c. $R_i = R_1\|R_2\|R_{ib1} = (5.53)\|(34.1)$

$$\Rightarrow \underline{R_i = 4.76 \text{ k}\Omega}$$
$$R_0 = \left(\frac{r_{\pi 2}+R_{C1}}{1+\beta}\right)\|R_{E2} = \left(\frac{0.676+5}{126}\right)\|1.5$$
$$= 0.0450\|1.5$$
$$\Rightarrow \underline{R_0 = 43.7 \ \Omega}$$

E4.28

a. $I_{CQ2} = \left(\frac{100}{101}\right)(1 \text{ mA}) \Rightarrow I_{CQ2} = 0.990 \text{ mA}$

$$I_{EQ1} = \frac{I_{EQ2}}{1+\beta} = \frac{1}{101} \Rightarrow I_{EQ1} = 0.0099 \text{ mA}$$
$$I_{BQ1} = \frac{I_{EQ1}}{1+\beta} = \frac{0.0099}{101}$$
$$\Rightarrow I_{BQ1} = 0.000098 \text{ mA}. \quad I_{CQ1} = 0.0098 \text{ mA}$$
$$V_{B1} = -I_{BQ1}R_B = -(0.000098)(10)$$
$$= -0.00098 \text{ V} \approx 0$$
$$V_{E1} = -0.7 \text{ V}, \quad V_{E2} = -1.4 \text{ V}$$

$$I_1 = I_{CQ2} + I_{CQ1} = 0.990 + 0.0098$$
$$I_1 \cong 1 \text{ mA} \Rightarrow V_0 = 5 - (1)(4) = 1 \text{ V}$$
$$V_{CEQ2} = 1 - (-1.4) = 2.4$$
$$\Rightarrow \underline{V_{CEQ2} = 2.4 \text{ V}}, \quad I_{CQ2} = 0.990 \text{ mA}$$
$$V_{CEQ1} = 1 - (-0.7) = 1.7$$
$$\Rightarrow \underline{V_{CEQ1} = 1.7 \text{ V}}, \quad I_{CQ1} = 0.0098 \text{ mA}$$

b. $r_\pi = \frac{\beta V_T}{I_{CQ}}$ and $g_m = \frac{I_{CQ}}{V_T}$

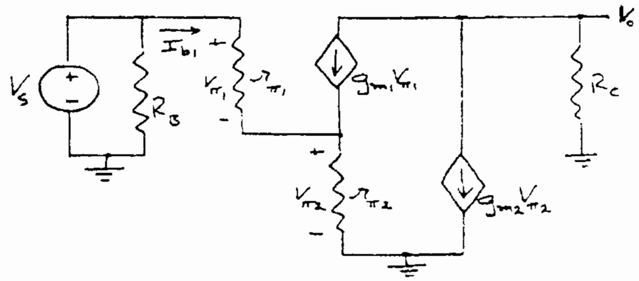
For Q_1 :

$$r_{\pi 1} = \frac{(100)(0.026)}{0.0098} \Rightarrow r_{\pi 1} = 265 \text{ k}\Omega$$
$$g_{m1} = \frac{0.0098}{0.026} \Rightarrow g_{m1} = 0.377 \text{ mA/V}$$

For Q_2 :

$$r_{\pi 2} = \frac{(100)(0.026)}{0.990} \Rightarrow r_{\pi 2} = 2.63 \text{ k}\Omega$$
$$g_{m2} = \frac{0.99}{0.026} \Rightarrow g_{m2} = 38.1 \text{ mA/V}$$
$$r_{01} = r_{02} = \infty$$

c.



$$V_0 = -(g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2})R_C$$
$$V_S = V_{\pi 1} + V_{\pi 2}$$
$$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1}\right)r_{\pi 2} = \frac{(1+\beta)}{r_{\pi 1}}V_{\pi 1}r_{\pi 2}$$

Then

$$V_0 = -\left[g_{m1}V_{\pi 1} + g_{m2}\left(\frac{(1+\beta)r_{\pi 2}}{r_{\pi 1}}\right)V_{\pi 1}\right]\cdot R_C$$

Also

$$V_S = V_{\pi 1} + \left(\frac{1+\beta}{r_{\pi 1}}\right)V_{\pi 1}r_{\pi 2}$$
$$= V_{\pi 1}\left[1 + (1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)\right]$$
$$V_{\pi 1} = \frac{V_S}{1 + (1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)}$$
$$V_0 = -\left[g_{m1} + g_{m2}(1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)\right]R_C$$
$$\cdot \frac{V_S}{1 + (1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)}$$

$$A_v = -\frac{\left[g_{m1} + g_{m2}(1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)\right]R_C}{1 + (1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)}$$

$$A_v = -\frac{\left[0.377 + (38.08)(101)\left(\frac{2.626}{265.3}\right)\right](4)}{1 + (101)\left(\frac{2.626}{265.3}\right)}$$

$$A_v = -\frac{153.8}{1.9997} \Rightarrow \underline{A_v = -76.9}$$

d. $R_i = r_{\pi 1} + (1+\beta) r_{\pi 2}$

$R_i = 265.3 + (101)(2.626)$

$\Rightarrow \underline{R_i = 531\ \text{k}\Omega}$

E4.29

a. $I_{E1} = \dfrac{V_{E1} - (-10)}{20} + \dfrac{(V_{E1} - 0.7) - (-10)}{(1+\beta)(10)}$

$-I_{B1}R_B - V_{BE}(\text{on}) = V_{E1}$

So

$$(1+\beta)I_{B1} = \frac{10 - I_{B1}R_B - 0.7}{20} + \frac{10 - 0.7 - I_{B1}R_B - 0.7}{(101)(10)}$$

$$(101)I_{B1} + I_{B1}\left(\frac{20}{20}\right) + I_{B1} \cdot \frac{20}{(101)(10)} = \frac{9.3}{20} + \frac{8.6}{(101)(10)}$$

$(102)I_{B1} = 0.465 + 0.00851$

$I_{B1} = 0.00464\ \text{mA} \Rightarrow V_{B1} = -0.09281$

$\Rightarrow V_{E1} = -0.793\ \text{V} \Rightarrow V_{E2} = -1.493\ \text{V}$

$I_{C1} = 0.464\ \text{mA},\ I_{E1} = 0.469\ \text{mA}$

$I_1 = \dfrac{10 - 0.793}{20} = 0.46035\ \text{mA}$

$\Rightarrow I_{B2} = I_{E1} - I_1$

$I_{B2} = 0.00865\ \text{mA} \Rightarrow I_{C2} = 0.865\ \text{mA}$

or $I_{E2} = \dfrac{10 - 1.493}{10} = 0.851 \Rightarrow I_{C2} = 0.842\ \text{mA}$

$I_C = I_{C1} + I_{C2} = 0.464 + 0.842$

$= 1.306\ \text{mA}$

$V_O = 10 - (1.306)(2) = 7.39\ \text{V}$

$V_{CEQ2} = 7.39 - (-1.493)$

$\Rightarrow \underline{V_{CEQ2} = 8.88\ \text{V}},\ \underline{I_{CQ2} = 0.842\ \text{mA}}$

$V_{CEQ1} = 7.39 - (-0.793)$

$\Rightarrow \underline{V_{CEQ1} = 8.18\ \text{V}},\ \underline{I_{CQ1} = 0.464\ \text{mA}}$

b. $r_\pi = \dfrac{\beta V_T}{I_{CQ}},\quad g_m = \dfrac{I_{CQ}}{V_T}$

For Q_1:

$r_{\pi 1} = \dfrac{(100)(0.026)}{0.464} \Rightarrow r_{\pi 1} = 5.60\ \text{k}\Omega$

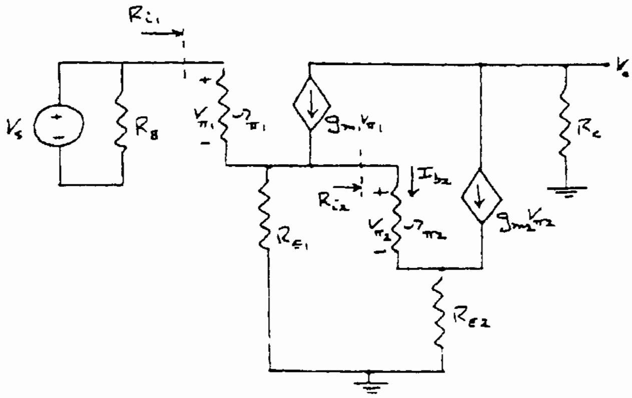
$g_{m1} = \dfrac{0.464}{0.026} \Rightarrow g_{m1} = 17.8\ \text{mA/V}$

$r_{\pi 2} = \dfrac{(100)(0.026)}{0.842} \Rightarrow r_{\pi 2} = 3.09\ \text{k}\Omega$

$g_{m2} = \dfrac{0.842}{0.026} \Rightarrow g_{m2} = 32.4\ \text{mA/V}$

$r_{O1} = r_{O2} = \infty$

c.



$R_{i2} = r_{\pi 2} + (1+\beta)R_{E2} = 3.09 + (101)(10)$

$= 1013.1\ \text{k}\Omega$

$R_{i1} = r_{\pi 1} + (1+\beta)[R_{E1} \| R_{i2}]$

$= 5.60 + (101)[20 \| 1013.1]$

d. $\underline{R_{i1} = 1.986\ \text{M}\Omega}$

Note that $g_{m1}V_{\pi 1} = \beta I_{b1}$ and $g_{m2}V_{\pi 2} = \beta I_{b2}$

So $V_O = -[\beta I_{b1} + \beta I_{b2}]R_C$

$I_{b1} = \dfrac{V_S}{R_{i1}}$ and $I_{b2} = \left(\dfrac{R_{E1}}{R_{E1} + R_{i2}}\right)(1+\beta)I_{b1}$

So

$$V_O = -[I_{b1} + I_{b2}]\beta R_C$$

$$= -\left\{I_{b1} + \left(\frac{R_{E1}}{R_{E1} + R_{i2}}\right)(1+\beta)I_{b1}\right\}\beta R_C$$

$$A_v = \frac{V_O}{V_S} = -\left[1 + \left(\frac{R_{E1}}{R_{E1} + R_{i2}}\right)(1+\beta)\right]\frac{\beta R_C}{R_{i1}}$$

$$= -\left[1 + \left(\frac{20}{20 + 1013.1}\right)(101)\right]\frac{(100)(2)}{1986}$$

$\underline{A_v = -0.298}$

E4.30

a. dc analysis

$\beta = 100,\ V_{BE}(\text{on}) = 0.7\ \text{V},\ V_A = \infty$

Want: $I_{CQ2} = 0.5\ \text{mA},\ V_{CE1} = V_{CE2} = 4\ \text{V}$

$R_1 + R_2 + R_3 = 100\ \text{k}\Omega$

Neglecting base currents:

$I_1 = \frac{12}{100} = 0.12 \text{ mA}$

$V_{E1} = I_{CQ2}R_E = (0.5)(0.5) = 0.25 \text{ V}$

$V_{C1} = V_{CEQ1} + V_{E1} = 4 + 0.25 = 4.25$

So

$V_{C2} = V_{C1} + V_{CEQ2} = 4.25 + 4 = 8.25 \text{ V}$

$R_C = \frac{V_{CC} - V_{C2}}{I_{CQ}} = \frac{12 - 8.25}{0.5} \Rightarrow \underline{R_C = 7.5 \text{ k}\Omega}$

$V_{B1} = V_{E1} + 0.7 = 0.25 + 0.7 = 0.95 \text{ V}$

$V_{B1} = \left(\frac{R_3}{R_1 + R_2 + R_3}\right)(12) \Rightarrow 0.95 = \frac{R_3}{100}(12)$

$\Rightarrow \underline{R_3 = 7.92 \text{ k}\Omega}$

$V_{B2} = V_{C1} + 0.7 = 4.25 + 0.7 = 4.95$

$V_{B2} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right)(12)$

$\Rightarrow 4.95 = \frac{R_2 + 7.92}{100}(12)$

$R_2 = \frac{(4.95)(100)}{12} - 7.92 \Rightarrow \underline{R_2 = 33.3 \text{ k}\Omega}$

$R_1 = 100 - R_2 - R_3 = 100 - 33.3 - 7.92$

$\Rightarrow \underline{R_1 = 58.8 \text{ k}\Omega}$

b. For both Q_1 and Q_2

$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.5}$

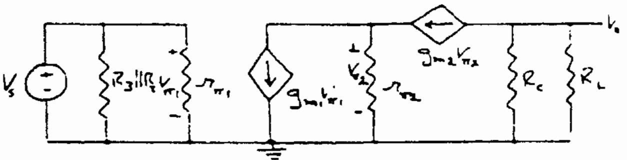
$\Rightarrow \underline{r_{\pi 1} = r_{\pi 2} = 5.2 \text{ k}\Omega}$

$g_m = \frac{I_{CQ}}{V_T} = \frac{0.5}{0.026}$

$\Rightarrow \underline{g_{m1} = g_{m2} = 19.23 \text{ mA/V}}$

$\underline{r_{01} = r_{02} = \infty}$

c.



$v_0 = -g_{m2}V_{\pi 2}(R_C \| R_L)$

$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} = g_{m1}V_{\pi 1}$

$V_{\pi 2}\left(\frac{1 + \beta}{r_{\pi 2}}\right) = g_{m1}V_{\pi 1}$

$V_{\pi 2} = g_{m1}V_{\pi 1}\left(\frac{r_{\pi 2}}{1 + \beta}\right)$ and $V_{\pi 1} = v_S$

So

$A_v = \frac{V_0}{V_S} = -g_{m2}(R_C \| R_L) \cdot g_{m1}\left(\frac{r_{\pi 2}}{1 + \beta}\right)$

$= -g_{m2}(R_C \| R_L)\left(\frac{\beta}{1 + \beta}\right)$

$A_v = -\left(\frac{100}{101}\right)(19.23)(7.5 \| 2) \Rightarrow \underline{A_v = -30.1}$

E4.31

a. dc analysis

$\beta = 80,\ V_{BE}(\text{on}) = 0.7,\ V_A = \infty$

$I_{BQ} = \frac{2.32 - 0.7}{24.2 + (81)(0.5)} = \frac{1.62}{64.7}$

$I_{BQ} = 0.0250 \text{ mA},\ I_{CQ} = 2.00 \text{ mA}$

Power dissipated in $R_C = I_{CQ}^2 R_C = (2.0)^2(2)$

$\Rightarrow \underline{P_C = 8.0 \text{ mW}}$

Power dissipated in $\underline{R_L = 0,\ P_L = 0}$

$V_{CE} = V_{CC} - I_C\left[R_C + \left(\frac{1 + \beta}{\beta}\right)R_E\right]$

$= 12 - 2\left[2 + \left(\frac{81}{80}\right)(0.5)\right]$

$V_{CE} = 6.99 \text{ V}$

$P_T = I_B V_{BE} + I_C V_{CE} = (0.0259)(0.7) + (2)(6.99)$

$\Rightarrow \underline{P_T = 14.0 \text{ mW}}$

b. $r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(80)(0.026)}{2.0} \Rightarrow r_\pi = 1.04 \text{ k}\Omega$

For $v_s = 18 \cos \omega t$ mV

From the text, power dissipation in the transistor

$P_T = V_{CEQ}I_{CQ} - \left(\frac{\beta}{r_\pi}\right)^2\left(\frac{V_P}{\sqrt{2}}\right)^2(R_C \| R_L)$

$= (6.99)(2 \times 10^{-3})$

$- \left(\frac{80}{1.04 \times 10^3}\right)^2\left(\frac{0.018}{\sqrt{2}}\right)^2(2 \times 10^3 \| 2 \times 10^3)$

$P_T = (14 - 0.96) \text{ m}\omega \Rightarrow \underline{P_T = 13.0 \text{ mW}}$

From notes

$|v_{ce}| = \frac{\beta}{r_\pi}(R_C \| R_L)V_P \cos \omega t$

Power dissipated in R_L

$P_L = \frac{|v_{ce}|^2}{R_L}\Big|_{rms} = \left[\frac{\beta}{r_\pi}(R_C \| R_L)\right]^2 \times \frac{1}{R_L} \times \frac{V_P^2}{2}$

$= \left[\frac{80}{1.04}(1.0)\right]^2 \times \frac{1}{2 \times 10^3} \times \left(\frac{0.018}{2}\right)^2$

$\Rightarrow \underline{P_L = 0.479 \text{ mW}}$

$R_C = 2 \text{ k}\Omega$ also so $P_C = 8.0 + 0.479$

$\Rightarrow \underline{P_C = 8.48 \text{ mW}}$

E4.32

$\beta = 100$, $V_{BE}(\text{on}) = 0.7$ V, $V_A = \infty$

a. $R_{TH} = R_1 \| R_2 = 10 \| 53.8 = 8.43$ kΩ

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(5) = \left(\frac{10}{10 + 53.8}\right)(5)$$

$V_{TH} = 0.7837$

$$I_{BQ} = \frac{0.7837 - 0.7}{8.43} = 0.00993 \text{ mA}$$

$I_{CQ} = 0.993$ mA

$V_{CEQ} = V_{CC} - I_{CQ}R_C$

$2.5 = 5 - (0.993)R_C \Rightarrow \underline{R_C = 2.52 \text{ k}\Omega}$

b. Power in $R_C = P_R = I_C^2 R_C = (0.993)^2(2.52)$

$\Rightarrow \underline{P_R = 2.48 \text{ mW}}$

Power in $Q \cong P_Q \approx I_{CQ}V_{CEQ} = (0.993)(2.5)$

$\Rightarrow \underline{P_Q = 2.48 \text{ mW}}$

c. $i_C = 0.993 \cos \omega t$

ac power $= \frac{1}{2} \times (0.993)^2 \times R_C = 1.24$ mW

in R_C

$$\frac{1.24}{2.48 + 2.48} = \underline{0.25}$$

Chapter 4

Problem Solutions

4.1

a. $g_m = \frac{I_{CQ}}{V_T} = \frac{2}{0.026} \Rightarrow \underline{g_m = 76.9 \text{ mA/V}}$

$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(180)(0.026)}{2} \Rightarrow \underline{r_\pi = 2.34 \text{ k}\Omega}$

$r_0 = \frac{V_A}{I_{CQ}} = \frac{150}{2} \Rightarrow \underline{r_0 = 75 \text{ k}\Omega}$

b. $g_m = \frac{0.5}{0.026} \Rightarrow \underline{g_m = 19.2 \text{ mA/V}}$

$r_\pi = \frac{(180)(0.026)}{0.5} \Rightarrow \underline{r_\pi = 9.36 \text{ k}\Omega}$

$r_0 = \frac{150}{0.5} \Rightarrow \underline{r_0 = 300 \text{ k}\Omega}$

4.2

$g_m = \frac{I_{CQ}}{V_T} \Rightarrow 200 = \frac{I_{CQ}}{0.026} \Rightarrow \underline{I_{CQ} = 5.2 \text{ mA}}$

$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(125)(0.026)}{5.2} \Rightarrow \underline{r_\pi = 0.625 \text{ k}\Omega}$

$r_0 = \frac{V_A}{I_{CQ}} = \frac{200}{5.2} \Rightarrow \underline{r_0 = 38.5 \text{ k}\Omega}$

4.3

(a) $I_{BQ} = \frac{2-0.7}{250} = 0.0052\, mA$

$I_C = (120)(0.0052) = 0.624\, mA$

$g_m = \frac{0.624}{0.026} \Rightarrow \quad \underline{g_m = 24\, mA/V}$

$r_\pi = \frac{(120)(0.026)}{0.624} \Rightarrow \quad \underline{r_\pi = 5\, k\Omega}$

$\underline{r_o = \infty}$

(b) $A_v = -g_m R_C \left(\frac{r_\pi}{r_\pi + R_B}\right) = -(24)(4)\left(\frac{5}{5+250}\right) \Rightarrow$

$\underline{A_v = -1.88}$

(c) $v_s = \frac{v_O}{A_v} = \frac{v_O}{-1.88} \Rightarrow$

$\underline{v_s = -0.426 \sin 100t \quad V}$

4.4

$g_m = \frac{I_{CQ}}{V_T}, \quad 1.08 \le I_{CQ} \le 1.32 \text{ mA}$

$\frac{1.08}{0.026} \le g_m \le \frac{1.32}{0.026} \Rightarrow \underline{41.5 \le g_m \le 50.8 \text{ mA/V}}$

$r_\pi = \frac{\beta V_T}{I_{CQ}}; \; r_\pi(\text{max}) = \frac{(120)(0.026)}{1.08} = 2.89 \text{ k}\Omega$

$r_\pi(\text{min}) = \frac{(80)(0.026)}{1.32} = 1.58 \text{ k}\Omega$

$\underline{1.58 \le r_\pi \le 2.89 \text{ k}\Omega}$

4.5

a. $r_\pi = 5.4 = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{I_{CQ}}$

$\Rightarrow I_{CQ} = 0.578 \text{ mA}$

$V_{CEQ} = \frac{1}{2}V_{CC} = \frac{1}{2}(5) = 2.5 \text{ V}$

$V_{CEQ} = V_{CC} - I_{CQ}R_C \Rightarrow 2.5 = 5.0 - (0.578)R_C$

$\Rightarrow \underline{R_C = 4.33 \text{ k}\Omega}$

$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.578}{120} = 0.00482 \text{ mA}$

$V_{BB} = I_{BQ}R_B + V_{BE}(\text{on})$

$= (0.00482)(25) + 0.70$

$\Rightarrow \underline{V_{BB} = 0.821}$

b. $r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.578} = 5.40 \text{ k}\Omega$

$g_m = \frac{I_{CQ}}{V_T} = \frac{0.578}{0.026} = 22.2 \text{ mA/V}$

$r_0 = \frac{V_A}{I_{CQ}} = \frac{100}{0.578} = 173 \text{ k}\Omega$

$V_0 = -g_m(r_0 \| R_C)V_\pi, \; V_\pi = \left(\frac{r_\pi}{r_\pi + R_B}\right)V_S$

$A_v = -g_m\left(\frac{r_\pi}{r_\pi + R_B}\right)(r_0 \| R_C) = -\frac{\beta(r_0 \| R_C)}{r_\pi + R_B}$

$A_v = -\frac{(120)[173 \| 4.33]}{5.40 + 25} = -\frac{(120)(4.22)}{30.4}$

$\Rightarrow \underline{A_v = -16.7}$

4.6

a. $V_{ECQ} = \frac{1}{2}V_{CC} = 5 \text{ V}$

$V_{ECQ} = 10 - I_{CQ}R_C \Rightarrow 5 = 10 - (0.5)R_C$

$\Rightarrow \underline{R_C = 10 \text{ k}\Omega}$

$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.5}{100} = 0.005$

$V_{EB}(\text{on}) + I_{BQ}R_B = V_{BB} = (0.70) + (0.005)(50)$

$\Rightarrow \underline{V_{BB} = 0.95 \text{ V}}$

b. $g_m = \frac{I_{CQ}}{V_T} = \frac{0.5}{0.026} \Rightarrow \underline{g_m = 19.2 \text{ mA/V}}$

$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.5} \Rightarrow \underline{r_\pi = 5.2 \text{ k}\Omega}$

$r_0 = \frac{V_A}{I_{CQ}} = \frac{\infty}{0.5} \Rightarrow \underline{r_0 = \infty}$

c. $A_v = -\frac{\beta R_C}{r_\pi + R_B} = -\frac{(100)(10)}{5.2 + 50} \Rightarrow \underline{A_v = -18.1}$

4.7

a. $I_E = 0.35 \text{ mA}, \ I_B = \frac{0.35}{101} = 0.00347 \text{ mA}$

$V_B = -I_B R_B = -(0.00347)(10)$

$\Rightarrow \underline{V_B = -0.0347 \text{ V}}$

$V_E = V_B - V_{BE}(\text{on}) \Rightarrow \underline{V_E = -0.735 \text{ V}}$

b. $V_C = V_{CEQ} + V_E = 3.5 - 0.735 = 2.77 \text{ V}$

$I_C = \left(\frac{\beta}{1+\beta}\right) I_E = \left(\frac{100}{101}\right)(0.35) = 0.347 \text{ mA}$

$R_C = \frac{V^+ - V_C}{I_C} = \frac{5 - 2.77}{0.347} \Rightarrow \underline{R_C = 6.43 \text{ k}\Omega}$

(c) $A_v = -g_m \left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_S}\right)(R_C \| r_o)$

$g_m = \frac{0.347}{0.026} = 13.3 \, mA/V \ , \ r_o = \frac{100}{0.347} = 288 \, k\Omega$

$r_\pi = \frac{(100)(0.026)}{0.347} = 7.49 \, k\Omega$

$R_B \| r_\pi = 10 \| 7.49 = 4.28 \, k\Omega$

$A_v = -(13.3)\left(\frac{4.28}{4.28 + 0.1}\right)(6.43 \| 288) \Rightarrow$

$A_v = -81.7$

d. $A_v = -g_m \left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_S}\right)(R_C \| r_0)$

$R_B \| r_\pi = 10 \| 7.49 = 4.28 \text{ k}\Omega$

$A_v = -(13.3)\left(\frac{4.28}{4.28 + 0.5}\right)(6.43 \| 288)$

$\Rightarrow \underline{A_v = -74.9}$

4.8

a. $R_{TH} = R_1 \| R_2 = 6 \| 1.5 = 1.2 \text{ k}\Omega$

$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V^+ = \left(\frac{1.5}{1.5 + 6}\right)(5) = 1.0 \text{ V}$

$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{1.0 - 0.7}{1.2 + (181)(0.1)}$

$= 0.0155 \text{ mA}$

$\underline{I_{CQ} = 2.80 \text{ mA}}, \ \underline{I_{EQ} = 2.81}$

$V_{CEQ} = V^+ - I_{CQ} R_C - I_{EQ} R_E$

$= 5 - (2.8)(1) - (2.81)(0.1)$

$\Rightarrow \underline{V_{CEQ} = 1.92 \text{ V}}$

b. $r_\pi = \frac{(180)(0.026)}{2.80} \Rightarrow \underline{r_\pi = 1.67 \text{ k}\Omega}$

$g_m = \frac{2.80}{0.026} \Rightarrow \underline{g_m = 108 \text{ mA/V}}, \ r_0 = \infty$

(c) $A_v = -g_m \left(\frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S}\right)(R_C \| R_L)$

$R_1 \| R_2 \| r_\pi = 6 \| 1.5 \| 1.67 = 0.698 \, k\Omega$

$A_v = -(108)\left(\frac{0.698}{0.698 + 0.2}\right)(1 \| 1.2) \Rightarrow$

$A_v = -45.8$

4.9

a. $I_{CQ} \approx I_{EQ}$

$V_{CEQ} = 5 = 10 - I_{CQ}(R_C + R_E)$

$= 10 - I_{CQ}(1.2 + 0.2)$

$I_{CQ} = 3.57 \text{ mA}$

$I_{BQ} = \frac{3.57}{150} = 0.0238 \text{ mA}$

$R_1 \| R_2 = R_{TH} = (0.1)(1+\beta)R_E$

$= (0.1)(151)(0.2) = 3.02 \text{ k}\Omega$

$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot (10) - 5$

$V_{TH} = I_{BQ} R_{TH} + V_{BE}(\text{on}) + (1+\beta) I_{BQ} R_E - 5$

$\frac{1}{R_1}(3.02)(10) - 5 = (0.0238)(3.02) + 0.7$

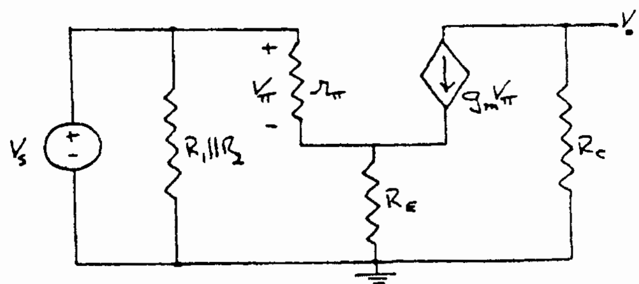
$+ (151)(0.0238)(0.2) - 5$

$\frac{1}{R_1}(30.2) = 1.49 \Rightarrow R_1 = 20.3 \, k\Omega$

$\frac{20.3 R_2}{20.3 + R_2} = 3.02 \Rightarrow \underline{R_2 = 3.55 \text{ k}\Omega}$

b. $r_\pi = \frac{(150)(0.026)}{3.57} = 1.09 \text{ k}\Omega$

$g_m = \frac{3.57}{0.026} = 137 \text{ mA/V}$



$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_E} = -\frac{(150)(1.2)}{1.09 + (151)(0.2)}$

$\Rightarrow \underline{A_v = -5.75}$

4.10

a. $V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{50}{50 + 10}\right)(12) = 10 \text{ V}$

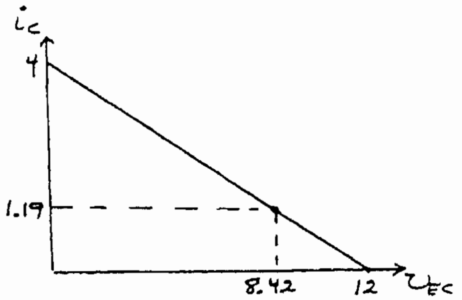
$R_{TH} = R_1 \| R_2 = 50 \| 10 = 8.33 \text{ k}\Omega$

$I_{BQ} = \frac{12 - 0.7 - 10}{8.33 + (101)(1)} = 0.0119 \text{ mA}$

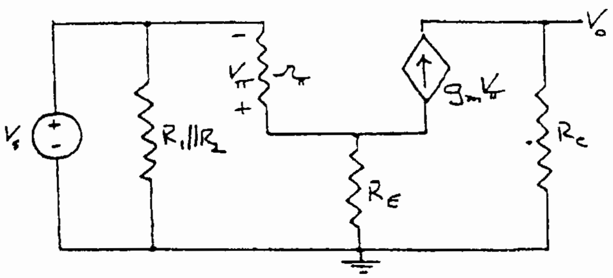
$\underline{I_{CQ} = 1.19 \text{ mA}}$, $I_{EQ} = 1.20 \text{ mA}$

$V_{ECQ} = 12 - (1.20)(1) - (1.19)(2)$

$\underline{V_{ECQ} = 8.42 \text{ V}}$



b.



$r_\pi = \frac{(100)(0.026)}{1.19} = 2.18 \text{ k}\Omega$

$V_0 = g_m V_\pi R_C$

$V_S = -V_\pi - \left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right) R_E$

$= -V_\pi \left[\frac{r_\pi + (1+\beta) R_E}{r_\pi}\right]$

$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta) R_E} = \frac{-(100)(2)}{2.18 + (101)(1)}$

$\Rightarrow \underline{A_v = -1.94}$

c. Approximation: Assume r_π does not vary significantly.

$R_C = 2 \text{ k}\Omega \pm 5\% = 2.1 \text{ k}\Omega$ or $1.9 \text{ k}\Omega$

$R_E = 1 \text{ k}\Omega \pm 5\% = 1.05 \text{ k}\Omega$ or $0.95 \text{ k}\Omega$

For $R_C(\text{max}) = 2.1 \text{ k}\Omega$ and $R_E(\text{min})$

$$A_v = \frac{-(100)(2.1)}{2.18 + (101)(0.95)} = -2.14$$

For $R_C(\text{min}) = 1.9 \text{ k}\Omega$ and $R_E(\text{max}) = 1.05 \text{ k}\Omega$

$$A_v = \frac{-(100)(1.9)}{2.18 + (101)(1.05)} = -1.76$$

So $\underline{1.76 \le |A_v| \le 2.14}$

4.11

(a) $V_{CC} = \left(\frac{1+\beta}{\beta}\right) I_{CQ} R_E + V_{ECQ} + I_{CQ} R_C$

$12 = \left(\frac{101}{100}\right) I_{CQ}(1) + 6 + I_{CQ}(2)$

so that $\underline{I_{CQ} = 1.99 \, mA}$

$I_{BQ} = \frac{1.99}{100} = 0.0199 \, mA$

$R_{TH} = (0.1)(1+\beta) R_E = (0.1)(101)(1) = 10.1 \, k\Omega$

$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(10.1)(12)$

$V_{CC} = (1+\beta) I_{BQ} R_E + V_{EB}(on) + I_{BQ} R_{TH} + V_{TH}$

$12 = (101)(0.0199)(1) + 0.7 + (0.0199)(10.1) + \frac{121.2}{R_1}$

which yields $\underline{R_1 = 13.3 \, k\Omega}$ and $\underline{R_2 = 42 \, k\Omega}$

(b) $A_v = \frac{-\beta R_C}{r_\pi + (1+\beta) R_E} = \frac{-(100)(2)}{1.31 + (101)(1)} \Rightarrow$

$\underline{A_v = -1.95}$

4.12

$I_{CQ} = 0.25 \text{ mA}$, $I_{EQ} = 0.2525 \text{ mA}$

$I_{BQ} = 0.0025 \text{ mA}$

$I_{BQ} R_B + V_{BE}(\text{on}) + I_{EQ}(R_S + R_E) - 5 = 0$

$(0.0025)(50) + 0.7 + (0.2525)(0.1 + R_E) = 5$

$\underline{R_E = 16.4 \text{ k}\Omega}$

$V_E = -(0.0025)(50) - 0.7 = -0.825 \text{ V}$

$V_C = V_{CEQ} + V_E = 3 - 0.825 = 2.175 \text{ V}$

$R_C = \frac{5 - 2.175}{0.25} \Rightarrow \underline{R_C = 11.3 \text{ k}\Omega}$

$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta) R_S}$

$r_\pi = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$

$A_v = \frac{-(100)(11.3)}{10.4 + (101)(0.1)} \Rightarrow \underline{A_v = -55.1}$

$R_i = R_B \| [r_\pi + (1+\beta) R_S]$

$= 50 \| [10.4 + (101)(0.1)]$

$R_i = 50 \| 20.5 \Rightarrow \underline{R_i = 14.5 \text{ k}\Omega}$

4.13

a. $9 = I_{EQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_S$

$I_{EQ} = 0.75 \text{ mA}, \quad I_{BQ} = \dfrac{0.75}{81} = 0.00926 \text{ mA}$

$I_{CQ} = 0.741 \text{ mA}$

$9 = (0.75)R_E + 0.7 + (0.00926)(2)$

$\Rightarrow \underline{R_E = 11.0 \text{ k}\Omega}$

b. $V_E = 9 - (0.75)(11) = 0.75 \text{ V}$

$V_C = V_E - V_{ECQ} = 0.75 - 7 = -6.25 \text{ V}$

$R_C = \dfrac{V_C - (-9)}{I_{CQ}} = \dfrac{9 - 6.25}{0.741} \Rightarrow \underline{R_C = 3.71 \text{ k}\Omega}$

c. $A_v = -g_m\left(\dfrac{r_\pi}{r_\pi + R_S}\right)(R_C \| R_L \| r_o)$

$r_\pi = \dfrac{(80)(0.026)}{0.741} = 2.81 \text{ k}\Omega$

$r_o = \dfrac{80}{0.741} = 108 \text{ k}\Omega$

$A_v = \dfrac{-80}{2.81 + 2}(3.71 \| 10 \| 108)$

$\underline{A_v = -43.9}$

d. $R_i = R_S + r_\pi = 2 + 2.81 \Rightarrow \underline{R_i = 4.81 \text{ k}\Omega}$

4.14

(a) $V_{CC} \cong I_{CQ}(R_C + R_E) + V_{CEQ}$

$9 = I_{CQ}(2.2 + 2) + 3.75$ So that

$\underline{I_{CQ} = 1.25\, mA}$

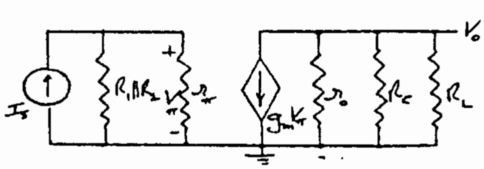
(b) $g_m = \dfrac{1.25}{0.026} = 48.1\, mA/V$

$r_\pi = \dfrac{(120)(0.026)}{1.25} = 2.50\, k\Omega$

$r_o = \dfrac{100}{1.25} = 80\, k\Omega$

Assume circuit is to be designed to be bias stable.

$R_{TH} = R_1 \| R_2 = (0.1)(1+\beta)R_E = (0.1)(121)(2) = 24.2\ \Omega$



$V_o = -g_m V_\pi (r_o \| R_C \| R_L)$

$V_\pi = I_S (R_1 \| R_2 \| r_\pi)$

Then

$R_m = \dfrac{V_o}{I_s} = -g_m (R_1 \| R_2 \| r_\pi)(r_o \| R_C \| R_L)$

$R_m = -48.1(24.2 \| 2.5)(80 \| 2.2 \| 1) = -48.1(2.27)(0.682)$

or

$\underline{R_m = \dfrac{V_o}{I_s} = -74.5\, k\Omega = -74.5\, V/mA}$

4.15

a. $I_{EQ} = 0.80 \text{ mA}, \quad I_{BQ} = \dfrac{0.80}{66} = 0.0121 \text{ mA}$

$I_{CQ} = 0.788 \text{ mA}$

$V_B = I_{BQ}R_B \Rightarrow R_B = \dfrac{0.3}{0.0121} \Rightarrow \underline{R_B = 24.8 \text{ k}\Omega}$

$R_C = \dfrac{V_C - (-5)}{I_{CQ}} = \dfrac{5 - 3}{0.788} \Rightarrow \underline{R_C = 2.54 \text{ k}\Omega}$

b. $g_m = \dfrac{0.788}{0.026} = 30.3 \text{ mA/V}$

$r_\pi = \dfrac{(65)(0.026)}{0.788} = 2.14 \text{ k}\Omega$

$r_o = \dfrac{75}{0.788} = 95.2 \text{ k}\Omega$

$i_o = \left(\dfrac{R_C \| r_o}{R_C \| r_o + R_L}\right) g_m V_\pi, \quad V_\pi = -v_S$

$G_f = \dfrac{i_o}{v_S} = -g_m\left(\dfrac{R_C \| r_o}{R_C \| r_o + R_L}\right)$

$= -(30.3)\left(\dfrac{2.54 \| 95.2}{2.54 \| 95.2 + 4}\right)$

$\underline{G_f = -11.6 \text{ mA/V}}$

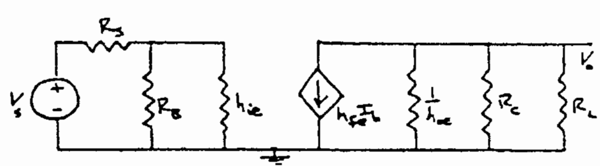
4.16

$I_{BQ} = \dfrac{4 - 0.7}{5 + (101)(5)} = 0.00647$

$I_{CQ} = 0.647 \text{ mA}$

a. $80 \le h_{fe} \le 120, \quad 10 \le h_{oe} \le 20\ \mu\text{S}$

$\underbrace{2.45 \text{ k}\Omega}_{\text{low gain}} \le h_{ie} \le \underbrace{3.7 \text{ k}\Omega}_{\text{high gain}}$



$V_0 = -h_{fe}I_b\left(\dfrac{1}{h_{oe}} \,\Big\|\, R_C \,\Big\|\, R_L\right)$

$I_b = \dfrac{\dfrac{R_B}{R_B + R_S} \cdot V_S}{R_{TH} + h_{ie}}$

$R_{TH} = R_B \| R_S = 5 \| 1 = 0.833 \text{ k}\Omega$

High-gain

$$I_b = \frac{\left(\frac{5}{5+1}\right)V_S}{0.833 + 3.7} = 0.1838 V_S$$

Low-gain

$$I_b = \frac{\left(\frac{5}{5+1}\right)V_S}{0.833 + 2.45} = 0.2538 V_S$$

For

$$h_{oe} = 10 \Rightarrow \frac{1}{h_{oe}} \parallel R_C \parallel R_L = \frac{1}{0.010} \parallel 4 \parallel 4$$

$$= 100\|2 = 1.96 \text{ k}\Omega$$

For

$$h_{oe} = 20 \Rightarrow \frac{1}{0.020} \parallel 4 \parallel 4 = 50\|2 = 1.92 \text{ k}\Omega$$

$|A_v|_{max} = (120)(0.1838)(1.96) = 43.2$

$|A_v|_{min} = (80)(0.2538)(1.92) = 39.0$

$\underline{39.0 \le |A_v| \le 43.2}$

b. $R_i = R_B \| h_{ie} = 5\|3.7 = 2.13 \text{ k}\Omega$

or $R_i = 5\|2.45 = 1.64 \text{ k}\Omega$

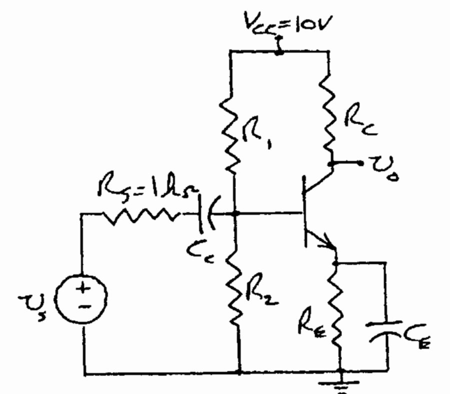
$\underline{1.64 \le R_i \le 2.13 \text{ k}\Omega}$

$$R_O = \frac{1}{h_{oe}} \parallel R_C = \frac{1}{0.010} \parallel 4 = 100\|4 = 3.85 \text{ k}\Omega$$

$$\text{or } R_O = \frac{1}{0.020} \parallel 4 = 50\|4 = 3.70 \text{ k}\Omega$$

$\underline{3.70 \le R_O \le 3.85 \text{ k}\Omega}$

4.17



Assume an npn transistor with $\beta = 100$ and $V_A = \infty$. Let $V_{CC} = 10\,V$.

$$|A_v| = \frac{0.5}{0.01} = 50$$

Bias at $I_{CQ} = 1\,mA$ and let $R_E = 1\,k\Omega$

For a bias stable circuit

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(1) = 10.1\,k\Omega$$

$$V_{TH} = \frac{1}{R_1}\cdot R_{TH}\cdot V_{CC} = \frac{1}{R_1}(10.1)(10) = \frac{101}{R_1}$$

$$I_{BQ} = \frac{1}{100} = 0.01\,mA$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$$

$$\frac{101}{R_1} = (0.01)(10.1) + 0.7 + (101)(0.01)(1)$$

which yields $\underline{R_1 = 55.8\,k\Omega}$ and $\underline{R_2 = 12.3\,k\Omega}$

Now

$$r_\pi = \frac{(100)(0.026)}{1} = 2.6\,k\Omega$$

$$g_m = \frac{1}{0.026} = 38.46\,mA/V$$

$$V_o = -g_m V_\pi R_C$$

where

$$V_\pi = \left(\frac{R_1\|R_2\|r_\pi}{R_1\|R_2\|r_\pi + R_S}\right)\cdot V_s = \left(\frac{10.1\|2.6}{10.1\|2.6+1}\right)\cdot V_s$$

or

$$V_\pi = 0.674 V_s$$

Then

$$A_v = \frac{V_o}{V_s} = -(0.674)g_m R_C = -(0.674)(38.46)R_C = -50$$

which yields $\underline{R_C = 1.93\,k\Omega}$

With this R_C, the dc bias is OK.

4.18

a. $I_{BQ} = \dfrac{6 - 0.7}{10 + (101)(3)} = 0.0169 \text{ mA}$

$\underline{I_{CQ} = 1.69 \text{ mA}}$, $\underline{I_{EQ} = 1.71 \text{ mA}}$

$V_{CEQ} = (16 + 6) - (1.69)(6.8) - (1.71)(3)$

$\underline{V_{CEQ} = 5.38 \text{ V}}$

b. $g_m = \dfrac{1.69}{0.026} \Rightarrow \underline{g_m = 65 \text{ mA/V}}$

$r_\pi = \dfrac{(100)(0.026)}{1.69} \Rightarrow \underline{r_\pi = 1.54 \text{ k}\Omega}$, $\underline{r_o = \infty}$

(c) $$A_v = \frac{-\beta(R_C\|R_L)}{r_\pi + (1+\beta)R_E}\cdot\frac{R_B\|R_{ib}}{R_B\|R_{ib} + R_S}$$

$$R_{ib} = r_\pi + (1+\beta)R_E = 1.54 + (101)(3) = 304.5\,k\Omega$$

$$R_B\|R_{ib} = 10\|304.5 = 9.68\,k\Omega$$

Then

$$A_v = \frac{-(100)(6.8\|6.8)}{1.54 + (101)(3)}\cdot\left(\frac{9.68}{9.68+0.5}\right) \Rightarrow$$

$\underline{A_v = -1.06}$

$$i_o = \left(\frac{R_C}{R_C + R_L}\right)(-\beta i_b)$$

$$i_b = \left(\frac{R_B}{R_B + r_\pi + (1+\beta)R_E}\right) i_S$$

$$A_i = -(\beta)\left(\frac{R_C}{R_C + R_L}\right)\left(\frac{R_B}{R_B + r_\pi + (1+\beta)R_E}\right)$$

$$= -(100)\left(\frac{6.8}{6.8 + 6.8}\right)\left(\frac{10}{10 + 1.54 + (101)(3)}\right)$$

$\Rightarrow \underline{A_i = -1.59}$

(d) $R_{is} = R_S + R_B \| R_{ib} = 0.5 + 10 \| 30.45 = 10.2\ k\Omega$

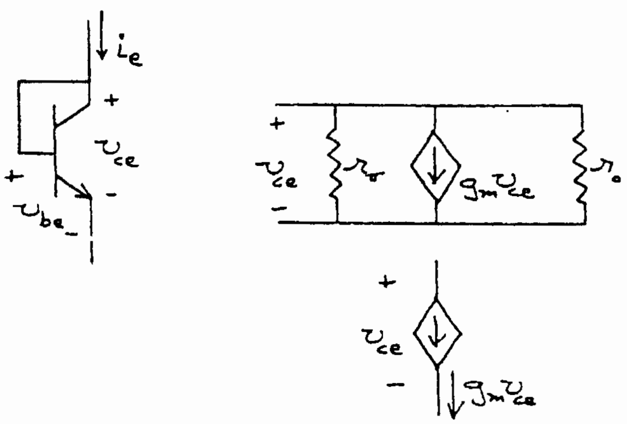
(e) $A_v = \dfrac{-\beta(R_C \| R_L)}{r_\pi + (1+\beta)R_E} \cdot \dfrac{R_B \| R_{ib}}{R_B \| R_{ib} + R_S}$

$A_v = \dfrac{-(100)(6.8 \| 6.8)}{1.54 + (101)(3)} \cdot \left(\dfrac{9.68}{9.68+1}\right) \Rightarrow$

$A_v = -1.01$

A_i = same as (c) $\Rightarrow \underline{A_i = -1.59}$

4.19

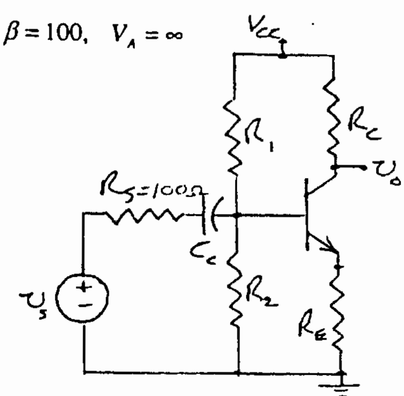


$r = \dfrac{v_{ce}}{g_m v_{ce}} = \dfrac{1}{g_m}$

So $\underline{r_e = r_\pi \left\| \left(\dfrac{1}{g_m}\right) \right\| r_o}$

4.20

Let $\beta = 100$, $V_A = \infty$



Let $V_{CC} = 2.5\,V$

$P = (I_R + I_C)V_{CC} \Rightarrow 0.12 = (I_R + I_C)(2.5) \Rightarrow$

$I_R + I_C = 48\,\mu A$, Let $I_R = 8\,\mu A$, $I_C = 40\,\mu A$

$R_1 + R_2 \cong \dfrac{V_{CC}}{I_R} = \dfrac{2.5}{8} \Rightarrow 312.5\ k\Omega$

$I_{BQ} = \dfrac{40}{100} = 0.4\,\mu A$

Let $R_E = 2\ k\Omega$. For a bias stable circuit

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(2) = 20.2\ k\Omega$

$V_{TH} = \dfrac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$

$\dfrac{1}{R_1}(20.2)(2.5) = (0.0004)(20.2) + 0.7$

$+(101)(0.0004)(2)$

which yields $R_1 = 64\ k\Omega$ and $R_2 = 29.5\ k\Omega$

$r_\pi = \dfrac{(100)(0.026)}{0.04} = 65\ k\Omega$ Neglect R_S

$A_v = \dfrac{V_o}{V_s} \cong \dfrac{-\beta R_C}{r_\pi + (1+\beta)R_E}$

$-10 = \dfrac{-100R_C}{65 + (101)(2)} \Rightarrow \underline{R_C = 26.7\ k\Omega}$

With this R_C, dc biasing is OK.

4.21

Need a voltage gain of $\dfrac{100}{5} = 20$.

Assume a sign inversion from a common-emitter is not important. Use the configuration for Figure 4.28. Need an input resistance of

$R_i = \dfrac{5x10^{-3}}{0.2x10^{-6}} = 25x10^3 = 25\ k\Omega$

$R_i = R_{TH} \| R_{ib}$. Let $R_{TH} = 50\ k\Omega$, $R_{ib} = 50\ k\Omega$

$R_{ib} = r_\pi + (1+\beta)R_E \cong (1+\beta)R_E$

For $\beta = 100$, $R_E = \dfrac{R_{ib}}{1+\beta} = \dfrac{50}{101} = 0.495\ k\Omega$

Let $\underline{R_E = 0.5\ k\Omega}$, $\underline{V_{CC} = 10\,V}$, $\underline{I_{CQ} = 0.2\ mA}$

Then $I_{BQ} = \dfrac{0.2}{100} = 0.002\ mA$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$

$\dfrac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \dfrac{1}{R_1}(50)(10) = (0.002)(50) + 0.7$

$+(101)(0.002)(0.5)$

which yields $R_1 = 555\ k\Omega$ and $R_2 = 55\ k\Omega$

Now

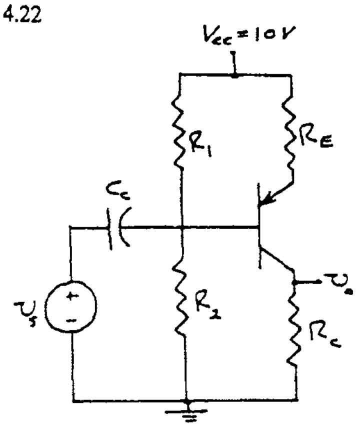
$A_v = \dfrac{-\beta R_C}{r_\pi + (1+\beta)R_E}$, $r_\pi = \dfrac{(100)(0.026)}{0.2} = 13\ k\Omega$

so

$-20 = \dfrac{-(100)R_C}{13 + (101)(0.5)} \Rightarrow \underline{R_C = 12.7\ k\Omega}$

[Note: $I_{CQ}R_C = (0.2)(12.7) = 2.54\,V$. So dc biasing is OK.]

4.22



$\beta = 80, \quad A_v = \dfrac{-\beta R_C}{r_\pi + (1+\beta)R_E}$

First approximation:

$(A_v) \approx \dfrac{R_C}{R_E} = 10 \Rightarrow R_C = 10R_E$

Set $R_C = 12R_E$

$V_{EC} \approx V_{CC} - I_C(R_C + R_E) = 10 - I_C(13R_E)$

For $V_{EC} = \frac{1}{2}V_{CC} = 5$

$5 = 10 - I_C(13R_E)$

For $I_C = 0.7$ mA

$I_E = 0.709, \quad I_B = 0.00875$ mA

$\Rightarrow R_E = 0.55\ \text{k}\Omega \rightarrow R_C = 6.6\ \text{k}\Omega$

Bias stable $\Rightarrow$

$R_1\|R_2 = R_{TH} = (0.1)(1+\beta)R_E$

$= (0.1)(81)(0.55) = 4.46\ \text{k}\Omega$

$10 = (0.709)(0.55) + 0.7 + (0.00875)(4.46)$

$+ \dfrac{1}{R_1}(4.46)(10)$

$8.87 = \dfrac{1}{R_1}(4.46) \Rightarrow \underline{R_1 = 5.03\ \text{k}\Omega}$

$\dfrac{5.03R_2}{5.03 + R_2} = 4.46 \Rightarrow \underline{R_2 = 39.4\ \text{k}\Omega}$

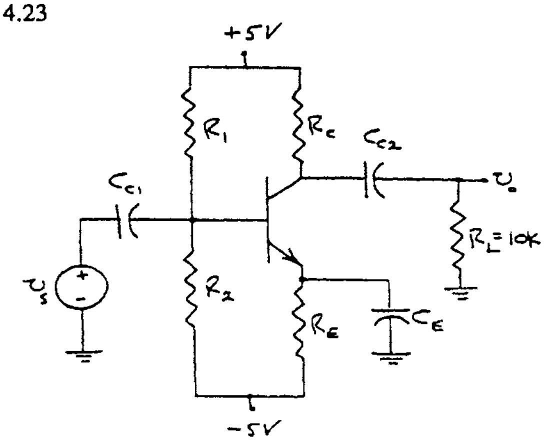
$\dfrac{10}{R_1 + R_2} = \dfrac{10}{5.03 + 39.4} = 0.225$ mA

$0.7 + 0.225 \cong 0.925$ mA from V_{CC} source.

Now $r_\pi = \dfrac{(80)(0.026)}{0.7} = 2.97\ \text{k}\Omega$

$|A_v| = \dfrac{(80)(6.6)}{2.97 + (81)(0.55)} = 11.1$

4.23



$\beta = 120$

Let $I_{CQ} = 0.35$ mA, $I_{EQ} = 0.353$ mA

$I_{BQ} = 0.00292$ mA

Let $R_E = 2\ \text{k}\Omega$. For $V_{CEQ} = 4\ \text{V} \Rightarrow$

$10 = 4 + (0.35)R_C + (0.353)(2)$

$\underline{R_C = 15.1\ \text{k}\Omega}, \quad r_\pi = \dfrac{(120)(0.026)}{0.35} = 8.91\ \text{k}\Omega$

$A_v = \dfrac{-\beta(R_C\|R_L)}{r_\pi} = -\dfrac{(120)(15.1\|10)}{8.91}$

$\underline{A_v = -81.0}$

For bias stable circuit:

$R_1\|R_2 = R_{TH} = (0.1)(1+\beta)R_E$

$= (0.1)(121)(2) = 24.2\ \text{k}\Omega$

$V_{TH} = \left(\dfrac{R_2}{R_1 + R_2}\right)(10) - 5 = \dfrac{1}{R_1}\cdot R_{TH}\cdot(10) - 5$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E - 5$

$\dfrac{1}{R_1}(24.2)(10) - 5 = (0.00292)(24.2) + 0.7$

$+ (121)(0.00292)(2) - 5$

$\dfrac{1}{R_1}(242) = 1.477, \quad \underline{R_1 = 164\ \text{k}\Omega}$

$\dfrac{164R_2}{164 + R_2} = 24.2 \Rightarrow \underline{R_2 = 28.4\ \text{k}\Omega}$

$\dfrac{10}{164 + 28.4} = 0.052, \quad 0.35 + 0.052 = \underline{0.402\ \text{mA}}$

4.24

From Prob. 4-10:

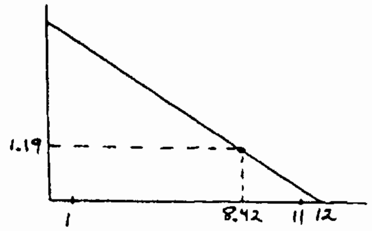
$R_{TH} = R_1\|R_2 = 10\|50 = 8.33\ \text{k}\Omega$

$V_{TH} = \left(\dfrac{R_2}{R_1 + R_2}\right)(12) = \left(\dfrac{50}{50 + 10}\right)(12) = 10\ \text{V}$

$I_{BQ} = \dfrac{12 - 0.7 - 10}{8.33 + (101)(1)} = 0.0119$ mA

$I_{CQ} = 1.19$ mA, $I_{EQ} = 1.20$ mA

$V_{ECQ} = 12 - (1.19)(2) - (120)(1) = 8.42$ V



For $1 \le v_{EC} \le 11$

$\Delta v_{EC} = 11 - 8.42 = 2.58$

$\Rightarrow$ Output voltage swing $= 5.16$ V (peak-to-peak)

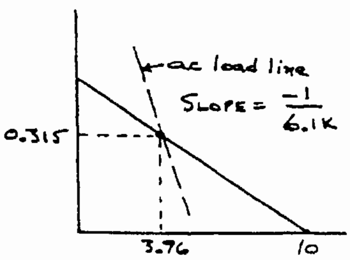
4.25

$$I_{BQ} = \frac{5 - 0.7}{50 + (101)(0.1 + 12.9)} = 0.00315 \text{ mA}$$

$I_{CQ} = 0.315$ mA, $I_{EQ} = 0.319$ mA

$V_{CEQ} = (5 + 5) - (0.315)(6) - (0.319)(13)$

$V_{CEQ} = 3.96$ V



$\Delta i_C = -\frac{1}{6.1}\Delta v_{eC}$

For $\Delta i_C = 0.315 - 0.05 = 0.265$

$\Rightarrow |\Delta v_{EC}| = 1.62$

$v_{EC}(\min) = 3.96 - 1.62 = 2.34$

Output signal swing determined by current:

Max. output swing = 3.24 V peak-to-peak

4.26

For $R_C = 6\,k\Omega$, $V_C = 5 - \left(\frac{100}{101}\right)(0.35)(6) = 2.92\,V$

$V_E = -I_{BQ}R_B - V_{BE}(on) = -\frac{0.35}{100}(10) - 0.7 = -0.735\,V$

Then $V_{CE} = V_C - V_E = 2.92 - (-0.735) = 3.66\,V$

$\Delta v_{CE} = \Delta i_C \cdot R_C \Rightarrow (4.5 - 3.66) = \Delta i_C(6)$

so that $\Delta i_C = 0.14\,mA$

(a) Total $\Delta v_{CE} = 2(4.5 - 3.66) = 1.68\,V$ peak-to-peak

(b) Total $\Delta i_C = 2(0.14) = 0.28\,mA$ peak-to-peak

4.27

$I_{EQ} = 0.80$ mA, $I_{CQ} = 0.792$ mA

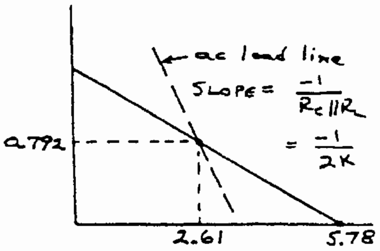
$I_{BQ} = 0.008$ mA

$V_E = 0.7 + (0.008)(10) = 0.78$ V

$V_C = I_{CQ}R_C - 5 = (0.792)(4) - 5 = -1.83$ V

$V_{ECQ} = 0.78 - (-1.83) = 2.61$ V

Load line: Assume V_E remains constant at ≈ 0.78 V



$\Delta i_C = \frac{-1}{2\text{ k}\Omega} \cdot v_{ec}$

Collector current swing $= 0.792 - 0.08 = 0.712$ mA

$|\Delta v_{ec}| = (0.712)(2) = 1.42$ V

Output swing determined by current.

Max. output swing = 2.84 V peak-to-peak

Swing in i_o current $= \frac{2.84}{4} = 0.71$ mA peak-to-peak

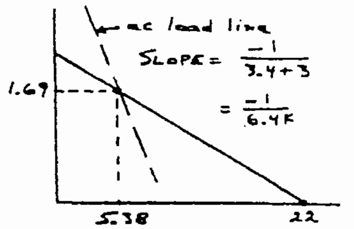
4.28

$$I_{BQ} = \frac{6 - 0.7}{10 + (101)(3)} = 0.0169 \text{ mA}$$

$I_{CQ} = 1.69$ mA, $I_{EQ} = 1.71$ mA

$V_{CEQ} = (16 + 6) - (1.69)(6.8) - (1.71)(3)$

$V_{CEQ} = 5.38$ V



$\Delta i_C = -\frac{1}{6.4}\Delta v_{ce}$

For $v_{ce}(\min) = 1$ V, $\Delta v_{ce} = 5.38 - 1 = 4.38$ V

$\Rightarrow |\Delta i_C| = \frac{4.38}{6.4} = 0.684$ mA

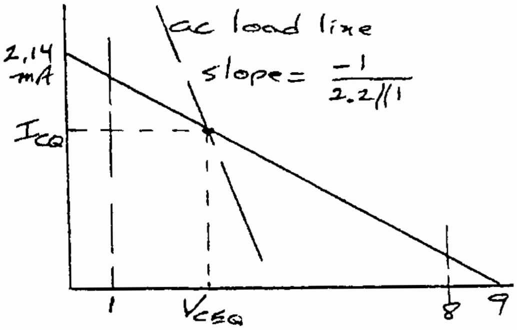
Output swing limited by voltage:

Δv_{ce} = Max. swing in output voltage

= 8.76 V peak-to-peak

$\Delta i_0 = \frac{1}{2}\Delta i_C \Rightarrow \Delta i_0 = 0.342$ mA (peak-to-peak)

4.29



$\Delta v_{CE}(\max) = V_{CEQ} - 1$, $\Delta i_C(\max) = I_{CQ}$

$\Delta v_{CE} = \Delta i_C(0.6875)$. So

$V_{CEQ} - 1 = I_{CQ}(0.6875)$ and

$V_{CEQ} = V_{CC} - I_{CQ}(R_C + R_E) \Rightarrow V_{CEQ} = 9 - I_{CQ}(4.2)$

Then

$9 - I_{CQ}(4.2) - 1 = I_{CQ}(0.6875)$

So $I_{CQ} = 1.64\ mA$ and $V_{CEQ} = 2.11\ V$

$V_{TH} = \frac{1}{R_1}\cdot R_{TH}\cdot V_{CC}$ and $R_{TH} = (0.1)(1+\beta)R_E$

$R_{TH} = (0.1)(151)(2) = 30.2\ k\Omega$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$

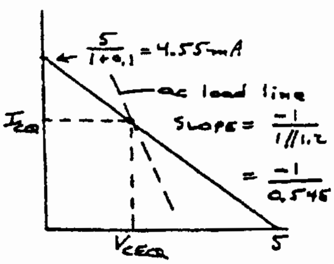
$I_{BQ} = \frac{1.64}{150} = 0.0109\ mA$

$\frac{1}{R_1}(30.2)(9) = (0.0109)(30.2) + 0.7 + (151)(0.0109)(2)$

which yields $R_1 = 62.9\ k\Omega$ and $R_2 = 58.1\ k\Omega$

4.30

dc load line



For maximum symmetrical swing

$\Delta i_C = I_{CQ} - 0.25$

$\Delta v_{CE} = V_{CEQ} - 0.5$

and $|\Delta i_C| = \frac{1}{0.545\ k\Omega}\cdot|\Delta v_{CE}|$

$I_{CQ} - 0.25 = \frac{V_{CEQ} - 0.5}{0.545}$

$V_{CEQ} = 5 - I_{CQ}(1.1)$

$0.545(I_{CQ} - 0.25) = [5 - I_{CQ}(1.1)] - 0.5$

$(0.545 + 1.1)I_{CQ} = 5 - 0.5 + 0.136$

$I_{CQ} = 2.82$ mA, $I_{BQ} = 0.0157$ mA

$R_{TH} = R_1\|R_2 = (0.1)(1+\beta)R_E$

$= (0.1)(181)(0.1) = 1.81$ kΩ

$V_{TH} = \frac{1}{R_1}\cdot R_{TH}\cdot V^+ = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E$

$\frac{1}{R_1}(1.81)(5) = (0.0157)(1.81) + 0.7 + (181)(0.0157)(0.1)$

$\frac{1}{R_1}(9.05) = 1.01 \Rightarrow R_1 = 8.96$ kΩ

$\frac{8.96R_2}{8.96 + R_2} = 1.81 \Rightarrow R_2 = 2.27$ kΩ

4.31

$I_{CQ} = 0.647\ mA$, $V_{CEQ} = 10 - (0.647)(9) = 4.18\ V$

$\Delta i_C = I_{CQ} = 0.647\ mA$

So $\Delta v_{CE} = \Delta i_C(4\|4) = (0.647)(2) = 1.294\ V$

Voltage swing is well within the voltage specification. Then

$\Delta v_{CE} = 2(1.294) = 2.59\ V$ peak-to-peak

4.32

a. $R_{TH} = R_1\|R_2 = 10\|10 = 5$ kΩ

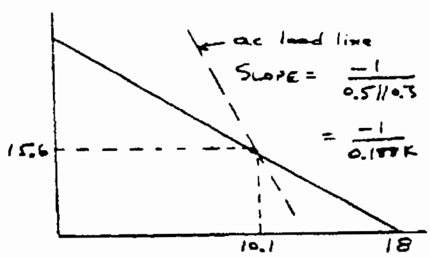
$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(18) - 9 = \left(\frac{10}{10+10}\right)(18) - 9 = 0$

$I_{BQ} = \frac{0 - 0.7 - (-9)}{5 + (181)(0.5)} = 0.0869$ mA

$I_{CQ} = 15.6$ mA, $I_{EQ} = 15.7$ mA

$V_{CEQ} = 18 - (15.7)(0.5) \Rightarrow V_{CEQ} = 10.1$ V

b.



c. $r_\pi = \dfrac{(180)(0.026)}{15.6} = 0.30\ \text{k}\Omega$

$$A_v = \frac{(1+\beta)(R_E\|R_L)}{r_\pi + (1+\beta)(R_E\|R_L)} \cdot \left(\frac{R_1\|R_2\|R_{ib}}{R_1\|R_2\|R_{ib} + R_S}\right)$$

$R_{ib} = r_\pi + (1+\beta)(R_E\|R_L) = 0.30 + (181)(0.5\|0.3)$

or $R_{ib} = 34.2\ k\Omega$

$R_1\|R_2\|R_{ib} = 5\|34.2 = 4.36\ k\Omega$

$$A_v = \frac{(181)(0.5\|0.3)}{0.3+(181)(0.5\|0.3)} \cdot \left(\frac{4.36}{4.36+1}\right) \Rightarrow \underline{A_v = 0.806}$$

d. $R_{ib} = r_\pi + (1+\beta)(R_E\|R_L)$

$R_{ib} = 0.30 + (181)(0.188) \Rightarrow \underline{R_{ib} = 34.3\ \text{k}\Omega}$

$$R_o = R_E \left\| \frac{r_\pi + R_1\|R_2\|R_S}{1+\beta} = 0.5 \right\| \frac{0.3+5\|1}{181} \Rightarrow$$

$\underline{R_o = 6.18\ \Omega}$

4.33

a. $R_{TH} = R_1\|R_2 = 10\|10 = 5\ \text{k}\Omega$

$V_{TH} = \left(\dfrac{R_2}{R_1+R_2}\right)(-10) = -5\ \text{V}$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E - 10$

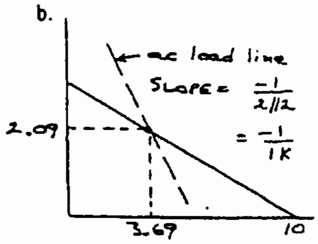
$I_{BQ} = \dfrac{-5-0.7-(-10)}{5+(121)(2)} = 0.0174\ \text{mA}$

$\underline{I_{CQ} = 2.09\ \text{mA}}$, $\underline{I_{EQ} = 2.11\ \text{mA}}$

$V_{CEQ} = 10 - (2.09)(1) - (2.11)(2)$

$\Rightarrow \underline{V_{CEQ} = 3.69\ \text{V}}$

b.



c. $r_\pi = \dfrac{(120)(0.026)}{2.09} = 1.49\ \text{k}\Omega$

$$A_v = \frac{(1+\beta)(R_E\|R_L)}{r_\pi + (1+\beta)(R_E\|R_L)} \cdot \left(\frac{R_1\|R_2\|R_{ib}}{R_1\|R_2\|R_{ib} + R_S}\right)$$

$R_{ib} = r_\pi + (1+\beta)(R_E\|R_L) = 1.49 + (121)(2\|2)$

$R_{ib} = 122.5\ k\Omega$, $R_1\|R_2\|R_{ib} = 5\|122.5 = 4.80\ k\Omega$

$$A_v = \frac{(121)(2\|2)}{1.49+(121)(2\|2)} \cdot \left(\frac{4.80}{4.80+5}\right) \Rightarrow$$

$\underline{A_v = 0.484}$

d. $R_{ib} = r_\pi + (1+\beta)(R_E\|R_L)$

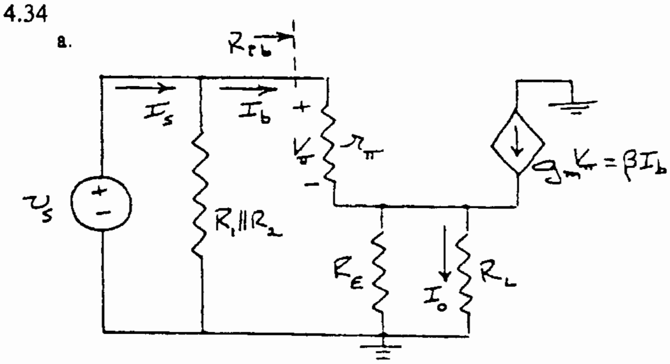
$R_{ib} = 1.49 + (121)(2\|2) \Rightarrow \underline{R_{ib} = 122\ \text{k}\Omega}$

$$R_o = R_E \left\| \frac{r_\pi + R_1\|R_2\|R_S}{1+\beta} = 2 \right\| \frac{1.49+5\|5}{121} \Rightarrow$$

$R_o = 32.5\ \Omega$

4.34

a.



$I_o = (1+\beta)I_b\left(\dfrac{R_E}{R_E+R_L}\right)$

$I_b = I_S\left(\dfrac{R_1\|R_2}{R_1\|R_2 + R_{ib}}\right)$

$R_{ib} = r_\pi + (1+\beta)(R_E\|R_L)$

$V_{CC} = 10\ \text{V}$, For $\underline{V_{CEQ} = 5\ \text{V}}$

$5 = 10 - \left(\dfrac{1+\beta}{\beta}\right)I_{CQ}R_E$

$\beta = 80$, For $R_E = 0.5\ \text{k}\Omega$

$I_{CQ} = 9.88\ \text{mA}$, $I_{EQ} = 10\ \text{mA}$, $I_{BQ} = 0.123\ \text{mA}$

$r_\pi = \dfrac{(80)(0.026)}{9.88} = 0.211\ \text{k}\Omega$

$R_{ib} = 0.211 + (81)(0.5\|0.5) \Rightarrow \underline{R_{ib} = 20.46\ \text{k}\Omega}$

$A_i = \dfrac{I_0}{I_S} = (1+\beta)\left(\dfrac{R_E}{R_E+R_L}\right)\left(\dfrac{R_1\|R_2}{R_1\|R_2 + R_{ib}}\right)$

$8 = (81)\left(\dfrac{1}{2}\right)\left(\dfrac{R_1\|R_2}{R_1\|R_2 + 20.46}\right)$

$0.1975[R_1\|R_2 + 20.46] = R_1\|R_2$

$R_1\|R_2 \Rightarrow 5.04\ \text{k}\Omega$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E$

$\dfrac{1}{R_1}(5.04)(10) = (0.123)(5.04) + 0.7 + (10)(0.5)$

$\Rightarrow \underline{R_1 = 7.97\ \text{k}\Omega}$

$\dfrac{7.97R_2}{7.97+R_2} = 5.04 \Rightarrow \underline{R_2 = 13.7\ \text{k}\Omega}$

(b) $R_{ib} = 0.211 + (81)(0.5\|2) = 32.6\ k\Omega$

$$A_i = (81)\left(\frac{0.5}{0.5+2}\right)\left(\frac{5.04}{5.04+32.6}\right) = (81)(0.2)(0.134)$$

$\underline{A_i = 2.17}$

4.35

$R_i = R_{TH}\|R_{ib}$ where $R_{ib} = r_\pi + (1+\beta)R_E$

$V_{CEQ} = 3.5,\quad I_{CQ} = \dfrac{5-3.5}{2} = 0.75\ mA$

$$r_\pi = \frac{(120)(0.026)}{0.75} = 4.16\ k\Omega$$

$R_{ib} = 4.16 + (121)(2) = 246\ k\Omega$

Then $R_i = 120 = R_{TH}\|245 \Rightarrow R_{TH} = 235\ k\Omega$

$$I_{BQ} = \frac{0.75}{120} = 0.00625\ mA$$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$

$$\frac{1}{R_1}\cdot R_{TH}\cdot V_{CC} = \frac{1}{R_1}(235)(5) = (0.00625)(235)$$
$$+0.7 + (121)(0.00625)(2)$$

which yields $\underline{R_1 = 319\ k\Omega}$ and $\underline{R_2 = 892\ k\Omega}$

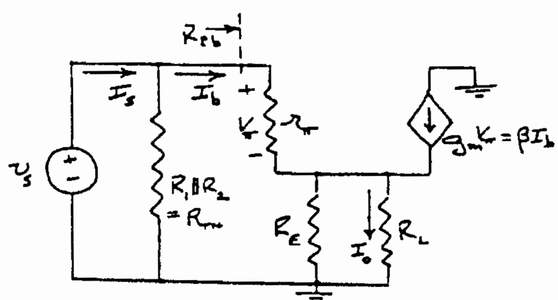
4.36

a. Let $R_E = 24\ \Omega$ and $V_{CEQ} = \frac{1}{2}V_{CC} = 12$ V

$$\Rightarrow I_{EQ} = \frac{12}{24} = 0.5 \text{ A}$$

$I_{CQ} = 0.493$ A, $I_{BQ} = 6.58$ mA

$$r_\pi = \frac{(75)(0.026)}{0.493} = 3.96\ \Omega$$



$$I_o = (1+\beta)I_b\left(\frac{R_E}{R_E+R_L}\right)$$

$$I_b = I_S\left(\frac{R_{TH}}{R_{TH}+R_{ib}}\right)$$

$R_{ib} = r_\pi + (1+\beta)(R_E\|R_L)$

$= 3.96 + (76)(24\|8) \Rightarrow R_{ib} = 460\ \Omega$

$$A_i = \frac{I_0}{I_S} = (1+\beta)\left(\frac{R_E}{R_E+R_L}\right)\left(\frac{R_{TH}}{R_{TH}+R_{ib}}\right)$$

$$8 = (76)\left(\frac{24}{24+8}\right)\left(\frac{R_{TH}}{R_{TH}+460}\right)$$

$$0.140 = \frac{R_{TH}}{R_{TH}+460}$$

$\Rightarrow \underline{R_{TH} = 74.9\ \Omega}$ (Minimum value)

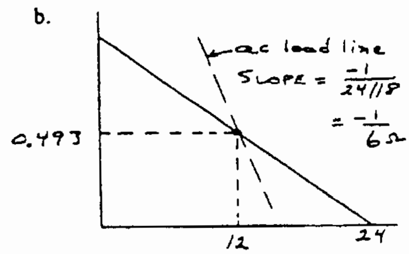
dc analysis:

$$V_{TH} = \frac{1}{R_1}\cdot R_{TH}\cdot V_{CC}$$
$$= I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E$$

$$\frac{1}{R_1}(74.9)(24) = (0.00658)(74.9) + 0.70$$
$$+ (0.5)(24)$$
$$= 12.75$$

$\underline{R_1 = 136\ \Omega}$, $\dfrac{136R_2}{136+R_2} = 74.9 \Rightarrow \underline{R_2 = 167\ \Omega}$

b.



$$\Delta i_C = -\frac{1}{6}\Delta v_{ce}$$

For $\Delta i_C = 0.493$

$\Rightarrow |\Delta v_{ce}| = (0.493)(6)$

$\Rightarrow$ Max. swing in output voltage for this design

$= \underline{5.92 \text{ V peak-to-peak}}$

c. $R_o = \dfrac{r_\pi}{1+\beta}\,\Big\|\, R_E = \dfrac{3.96}{76}\,\Big\|\, 24 = 0.0521\|24$

$\Rightarrow \underline{R_o = 52\ m\Omega}$

4.37

a. $R_{TH} = R_1\|R_2 = 60\|40 = 24\ k\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)V_{CC} = \left(\frac{40}{40+60}\right)(5) = 2 \text{ V}$$

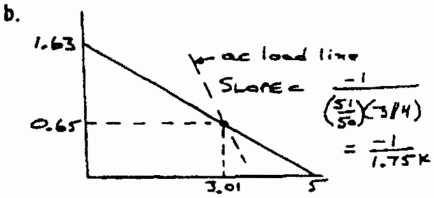
$$I_{BQ} = \frac{5-0.7-2}{24+(51)(3)} = 0.0130 \text{ mA}$$

$\underline{I_{CQ} = 0.650 \text{ mA}}$, $I_{EQ} = 0.663$ mA

$V_{ECQ} = 5 - I_{EQ}R_E = 5 - (0.663)(3)$

$\Rightarrow \underline{V_{CEQ} = 3.01 \text{ V}}$

b.



c. $r_\pi = \dfrac{(50)(0.026)}{0.650} = 2\ \text{k}\Omega,\ r_o = \dfrac{80}{0.65} = 123\ \text{k}\Omega$

Define $R_L' = R_E \| R_L \| r_o = 3\|4\|123 = 1.69\ \text{k}\Omega$

$$A_v = \frac{(1+\beta)R_L'}{r_\pi + (1+\beta)R_L'} = \frac{(51)(1.69)}{2 + (51)(1.69)}$$

$\Rightarrow \underline{A_v = 0.977}$

$$A_i = (1+\beta)I_b\left(\frac{R_E\|r_o}{R_E\|r_o + R_L}\right)$$

$$I_b = I_S\left(\frac{R_{TH}}{R_{TH} + R_{ib}}\right)$$

$R_{ib} = r_\pi + (1+\beta)R_L' = 2 + (51)(1.69) = 88.2$

$R_E\|r_o = 3\|r_o = 3\|123 = 2.93$

$$A_i = (51)\left(\frac{2.93}{2.93 + 4}\right)\left(\frac{24}{24 + 88.2}\right)$$

$\Rightarrow \underline{A_i = 4.61}$

d. $R_{ib} = r_\pi + (1+\beta)R_E\|R_L\|r_o = 2 + (51)(1.69)$

$\Rightarrow \underline{R_{ib} = 88.2\ \text{k}\Omega}$

$$R_o = \frac{r_\pi}{1+\beta} \,\Big\|\, R_E = \left(\frac{2}{51}\right) \Big\| \,3 = 0.0392\|3$$

$R_o = 38.7\ \Omega$

e. Assume variations in r_π and r_o have negligible effects

$R_1 = 60 \pm 5\%$ —	$R_1 = 63\ \text{k}\Omega$,	$R_1 = 57\ \text{k}\Omega$
$R_2 = 40 \pm 5\%$	$R_2 = 42\ \text{k}\Omega$,	$R_2 = 38\ \text{k}\Omega$
$R_E = 3 \pm 5\%$	$R_E = 3.15\ \text{k}\Omega$,	$R_E = 2.85\ \text{k}\Omega$
$R_L = 4 \pm 5\%$	$R_L = 4.2\ \text{k}\Omega$,	$R_L = 3.8\ \text{k}\Omega$

$$A_i = (1+\beta)\left(\frac{R_E\|r_o}{R_E\|r_o + R_L}\right)\left(\frac{R_{TH}}{R_{TH} + R_{ib}}\right)$$

$R_{ib} = r_\pi + (1+\beta)(R_E\|R_L\|r_o)$

$R_{TH}(\text{max}) = 25.2\ \text{k}\Omega,\ R_{TH}(\text{min}) = 22.8\ \text{k}\Omega$

$R_{ib}(\text{max}) = 92.5\ \text{k}\Omega,\ R_{ib}(\text{min}) = 84.0\ \text{k}\Omega$

$R_E(\text{max}),\ R_L(\text{min}),\ R_{ib} = 88.6\ \text{k}\Omega$

$R_E(\text{min}),\ R_L(\text{max}),\ R_{ib} = 87.4\ \text{k}\Omega$

$R_E(\text{max})\|r_o = 3.07\ \text{k}\Omega$

$R_E(\text{min})\|r_o = 2.79\ \text{k}\Omega$

For $R_E(\text{min}),\ R_L(\text{max}),\ R_{TH}(\text{min})$

$$A_i = (51)\left(\frac{2.79}{2.79 + 4.2}\right)\left(\frac{22.8}{22.8 + 87.4}\right)$$

$\Rightarrow \underline{A_i = 4.21}$

For $R_E(\text{max}),\ R_L(\text{min}),\ R_{TH}(\text{max})$

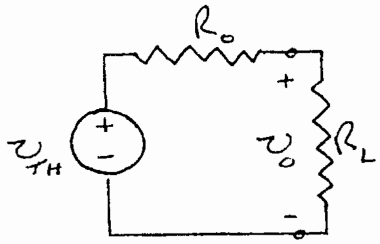
$$A_i = (51)\left(\frac{3.07}{3.07 + 3.8}\right)\left(\frac{25.2}{25.2 + 88.6}\right)$$

$\Rightarrow \underline{A_i = 5.05}$

4.38

The output of the emitter follower is

$$v_O = \left(\frac{R_L}{R_L + R_o}\right)\cdot v_{TH}$$



For v_O to be within 5% for a range of R_L, we have

$$\frac{R_L(\text{min})}{R_L(\text{min}) + R_o} = (0.95)\frac{R_L(\text{max})}{R_L(\text{max}) + R_o}$$

$\dfrac{4}{4 + R_o} = (0.95)\dfrac{10}{10 + R_o}$ which yields

$R_o = 0.364\ k\Omega$

We have $R_o = \left(\dfrac{r_\pi + R_1\|R_2\|R_S}{1+\beta}\right)\|R_E\|r_o$

The first term dominates

Let $R_1\|R_2\|R_S \cong R_S$, then

$$R_o \cong \frac{r_\pi + R_S}{1+\beta} \Rightarrow 0.364 = \frac{r_\pi + 4}{1+\beta}$$

or

$$0.364 = \frac{r_\pi}{1+\beta} + \frac{4}{1+\beta} = \frac{\beta V_T}{I_{CQ}(1+\beta)} + \frac{4}{1+\beta}$$

$$0.364 \cong \frac{V_T}{I_{CQ}} + \frac{4}{1+\beta}$$

The factor $\dfrac{4}{1+\beta}$ is in the range of $\dfrac{4}{91} = 0.044$ to $\dfrac{4}{131} = 0.0305$. We can set $R_o \cong 0.32 = \dfrac{V_T}{I_{CQ}}$

Or $I_{CQ} = 0.08125\ mA$. To take into account other factors, set $I_{CQ} = 0.15\ mA$,

$$I_{BQ} = \frac{0.15}{110} = 0.00136\ mA$$

For $V_{CEQ} \cong 5\,V$, set $R_E = \dfrac{5}{0.15} = 33.3\ k\Omega$

Design a bias stable circuit.

$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(10) - 5 = \frac{1}{R_1}(R_{TH})(10) - 5$$

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(111)(33.3) = 370\ k\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E - 5$$

So $\frac{1}{R_1}(370)(10) - 5 = (0.00136)(370) + 0.7$

$$+(111)(0.00136)(33.3) - 5$$

which yields $\underline{R_1 = 594\ k\Omega}$ and $\underline{R_2 = 981\ k\Omega}$

Now

$$A_v = \frac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \cdot \left(\frac{R_{TH} \| R_{ib}}{R_{TH} \| R_{ib} + R_S}\right)$$

$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L)$ and $r_\pi = \frac{\beta V_T}{I_{CQ}}$.

For $\beta = 90$, $R_L = 4\ k\Omega$,

$r_\pi = 15.6\ k\Omega$, $R_{ib} = 340.6\ k\Omega$

$$A_v = \frac{(91)(33.3\|4)}{15.6 + (91)(33.3\|4)} \cdot \frac{370\|340.6}{370\|340.6 + 4} \Rightarrow$$

$A_v = 0.9332$

For $\beta = 90$, $R_L = 10\ k\Omega$

$R_{ib} = 715.4\ k\Omega$

$$A_v = \frac{(91)(33.3\|10)}{15.6 + (91)(33.3\|10)} \cdot \frac{370\|715.4}{370\|715.4 + 4} \Rightarrow$$

$A_v = 0.9625$

For $\beta = 130$, $R_L = 4\ k\Omega$

$r_\pi = 22.5\ k\Omega$, $R_{ib} = 490\ k\Omega$

$$A_v = \frac{(131)(33.3\|4)}{22.5 + (131)(33.3\|4)} \cdot \frac{370\|490}{370\|490 + 4} \Rightarrow$$

$A_v = 0.9360$

For $\beta = 130$, $R_L = 10\ k\Omega$

$R_{ib} = 1030\ k\Omega$

$$A_v = \frac{(131)(33.3\|10)}{22.5 + (131)(33.3\|10)} \cdot \frac{370\|1030}{370\|1030 + 4} \Rightarrow$$

$A_v = 0.9645$

Now $v_O(\min) = |A_v(\min)| \cdot v_S = 3.73 \sin \omega t$

$v_O(\max) = |A_v(\max)| \cdot v_S = 3.86 \sin \omega t$

$\frac{\Delta v_O}{v_O} = 3.5\%$

4.39

a. $R_{TH} = R_1 \| R_2 = 40\|60 = 24\ k\Omega$

$$V_{TH} = \left(\frac{60}{60+40}\right)(10) = 6\ V$$

$\beta = 75$

$$I_{BQ} = \frac{6 - 0.7}{24 + (76)(5)} = 0.0131\ mA$$

$I_{CQ} = 0.984\ mA$

$\beta = 150$

$$I_{BQ} = \frac{6 - 0.7}{24 + (151)(5)} = 0.00680\ mA$$

$I_{CQ} = 1.02\ mA$

$\beta = 75$

$$r_\pi = \frac{(75)(0.026)}{0.984} = \underline{1.98\ k\Omega}$$

$\beta = 150$

$\underline{r_\pi = 3.82\ k\Omega}$

$\beta = 75$

$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L) = 65.3\ k\Omega$

$\beta = 150$

$R_{ib} = 130\ k\Omega$

$$A_v = \frac{(1+\beta)(R_E\|R_L)}{r_\pi + (1+\beta)(R_E\|R_L)} \cdot \frac{R_1\|R_2\|R_{ib}}{R_1\|R_2\|R_{ib} + R_S}$$

For $\beta = 75$, $R_1\|R_2\|R_{ib} = 40\|60\|65.3 = 17.5\ k\Omega$

$$A_v = \frac{(76)(0.833)}{1.98 + (76)(0.833)} \cdot \frac{17.5}{17.5+4} \Rightarrow$$

$\underline{A_v = 0.789}$

For $\beta = 150$, $R_1\|R_2\|R_{ib} = 40\|60\|130 = 20.3\ k\Omega$

$$A_v = \frac{(151)(0.833)}{3.82 + (151)(0.833)} \cdot \frac{20.3}{20.3+4} \Rightarrow$$

$\underline{A_v = 0.811}$

$$A_i = (1+\beta)\left(\frac{R_E}{R_E + R_L}\right)\left(\frac{R_{TH}}{R_{TH} + R_{ib}}\right)$$

$\beta = 75$

$$A_i = (76)\left(\frac{5}{5+1}\right)\left(\frac{24}{24 + 65.3}\right) \Rightarrow \underline{A_i = 17.0}$$

$\beta = 150$

$$A_i = (151)\left(\frac{5}{6}\right)\left(\frac{24}{24 + 130}\right) \Rightarrow \underline{A_i = 19.6}$$

$\underline{17.0 \le A_i \le 19.6}$

b. Current gain is the same as part (a)

(b)For $R_S = 5\ k\Omega$

$\beta = 75 \Rightarrow A_v = 0.754$

$\beta = 150 \Rightarrow A_v = 0.779$

4.40

(a) $I_{BQ} = \frac{0.5}{81} = 0.00617\ mA$

$V_B = I_{BQ}R_B = (0.00617)(10) \Rightarrow \underline{V_B = 0.0617\ V}$

$V_E = V_B + 0.7 \Rightarrow \underline{V_E = 0.7617\ V}$

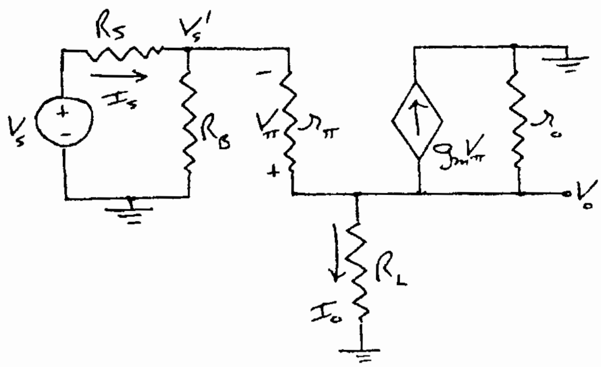
(b) $I_{CQ} = (0.5)\left(\frac{80}{81}\right) = 0.494\ mA$

$g_m = \frac{I_{CQ}}{V_T} = \frac{0.494}{0.026} \Rightarrow \underline{g_m = 19\ mA/V}$

$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(80)(0.026)}{0.494} \Rightarrow \underline{r_\pi = 4.21\ k\Omega}$

$r_o = \frac{V_A}{I_{CQ}} = \frac{150}{0.494} \Rightarrow \underline{r_o = 304\ k\Omega}$

(c)



For $R_S = 0$

$$V_o = -\left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right)(R_L \| r_o)$$

so that

$$V_\pi = \frac{-V_o}{\left(\frac{1+\beta}{r_\pi}\right)(R_L \| r_o)}$$

Now

$V_s + V_\pi = V_o$

or

$$V_s = V_o - V_\pi = V_o + \frac{V_o}{\left(\frac{1+\beta}{r_\pi}\right)(R_L \| r_o)}$$

We find

$$A_v = \frac{V_o}{V_s} = \frac{(1+\beta)(R_L \| r_o)}{r_\pi + (1+\beta)(R_L \| r_o)} = \frac{(81)(0.5 \| 304)}{4.21 + (81)(0.5 \| 304)}$$

$$\cong \frac{(81)(0.5)}{4.21 + (81)(0.5)} \Rightarrow \underline{A_v = 0.906}$$

$R_{ib} = r_\pi + (1+\beta)(R_L \| r_o) \cong 4.21 + (81)(0.5) = 44.7\ k\Omega$

$I_b = \left(\frac{R_B}{R_B + R_{ib}}\right) \cdot I_s$ and $I_o = \left(\frac{r_o}{r_o + R_L}\right)(1+\beta) I_b$

Then

$$A_i = \frac{I_o}{I_s} = (1+\beta)\left(\frac{R_B}{R_B + R_{ib}}\right)\left(\frac{r_o}{r_o + R_L}\right)$$

$$A_i \cong (81)\left(\frac{10}{10+44.7}\right)(1) \Rightarrow \underline{A_i = 14.8}$$

(d)

$$V_s' = \left(\frac{R_B \| R_{ib}}{R_B \| R_{ib} + R_S}\right) \cdot V_s = \left(\frac{10 \| 44.7}{10 \| 44.7 + 2}\right) \cdot V_s = (0.803) V_s$$

Then

$A_v = (0.803)(0.906) \Rightarrow \underline{A_v = 0.728}$

$\underline{A_i = 14.8}$ (Unchanged)

4.41

$V_o = (1+\beta) I_b R_L$

$$I_b = \frac{V_s}{r_\pi + (1+\beta) R_L}$$

so $A_v = \frac{(1+\beta) R_L}{r_\pi + (1+\beta) R_L}$

For $\beta = 100$, $R_L = 0.5\ k\Omega$

$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2\ k\Omega$

Then $A_v(\min) = \frac{(101)(0.5)}{5.2 + (101)(0.5)} = 0.9066$

For $\beta = 180$, $R_L = 500\ k\Omega$

$r_\pi = \frac{(180)(0.026)}{0.5} = 9.36\ k\Omega$

Then $A_v(\max) = \frac{(181)(500)}{9.36 + (181)(500)} = 0.9999$

4.42

a. $I_{EQ} = 1$ mA, $V_{CEQ} = V_{CC} - I_{EQ} R_E$

$5 = 10 - (1)(R_E) \Rightarrow \underline{R_E = 5\ k\Omega}$

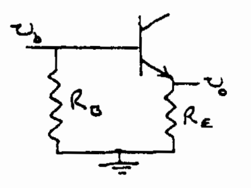
$I_{BQ} = \frac{1}{101} = 0.0099$

$10 = I_{BQ} R_B + V_{BE}(\text{on}) + I_{EQ} R_E$

$10 = (0.0099) R_B + 0.7 + (1)(5)$

$\Rightarrow \underline{R_B = 434\ k\Omega}$

b.

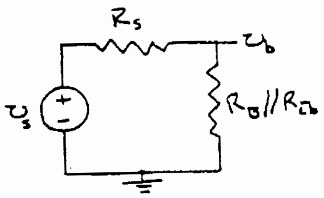


$r_\pi = \frac{(100)(0.026)}{0.99} = 2.63\ k\Omega$

$$\frac{v_o}{v_b} = \frac{(1+\beta) R_E}{r_\pi + (1+\beta) R_E} = \frac{(101)(5)}{2.63 + (101)(5)} = 0.995$$

$\Rightarrow v_b = \frac{v_o}{0.995} = \frac{4}{0.995}$

$\Rightarrow \underline{v_b = 4.02\ \text{V peak-to-peak at base}}$



$R_{ib} = r_\pi + (1+\beta)R_E = 508\ k\Omega$

$R_B \| R_{ib} = 434 \| 508 = 234\ k\Omega$

$$\nu_b = \frac{R_B \| R_{ib}}{R_B \| R_{ib} + R_S} \cdot \nu_S = \frac{234 \nu_S}{234 + 0.7} = \frac{234}{234.7} \nu_S$$

$\nu_b = 0.997 \nu_S$

$$\Rightarrow \nu_S = \frac{4.02}{0.997} \Rightarrow \nu_S = 4.03 \text{ V peak-to-peak}$$

c. $R_{ib} = r_\pi + (1+\beta)(R_E \| R_L)$

$R_{ib} = 2.63 + (101)(5\|1) = 86.8\ k\Omega$

$R_B \| R_{ib} = 434 \| 86.8 = 72.3\ k\Omega$

$$\nu_b = \left(\frac{72.3}{72.3+0.7}\right)\nu_S = 0.99\nu_S = (0.99)(4.03)$$

$\nu_b = 3.99$ V peak-to-peak

$$\nu_O = \frac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \cdot \nu_b$$

$$= \frac{(101)(0.833)}{2.63 + (101)(0.833)}(3.99)$$

$\nu_O = 3.87$ V peak-to-peak

4.43

$P_{AVG} = i_L^2(rms)R_L \Rightarrow 1 = i_L^2(rms)(12)$

so $i_L(rms) = 0.289\ A \Rightarrow i_L(peak) = \sqrt{2}(0.289)$

$i_L(peak) = 0.409\ A$

$v_L(peak) = i_L(peak) \cdot R_L = (0.409)(12) = 4.91\ V$

Need a gain of $\frac{4.91}{5} = 0.982$

With $R_S = 10\ k\Omega$, we will not be able to meet this voltage gain requirement. Need to insert a buffer or an op-amp voltage follower (see Chapter 9) between R_S and C_{C1}.

Set $I_{EQ} = 0.5\ A$, $V_{CEQ} = \frac{1}{3}(12-(-12)) = 8\ V$

$24 = I_{EQ}R_E + V_{CEQ} = (0.5)R_E + 8 \Rightarrow R_E = 32\ \Omega$

Let $\beta = 50$, $I_{CQ} = \frac{50}{51}(0.5) = 0.49\ A$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(50)(0.026)}{0.49} = 2.65\ \Omega$$

$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L) = 2.65 + (51)(32\|12)$

$R_{ib} = 448\ \Omega$

$$I_{BQ} = \frac{0.49}{50} = 0.0098\ A = 9.8\ mA$$

Let $I_R \cong \frac{24}{R_1 + R_2} \cong 10 I_B = 98\ mA$

So that $R_1 + R_2 = 245\ \Omega$

$$V_{TH} = \frac{R_2}{R_1+R_2}(24) - 12 = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E - 12$$

$$\left(\frac{R_2}{245}\right)(24) = \frac{(0.0098)R_1R_2}{245} + 0.7 + (0.5)(32)$$

Now $R_1 = 245 - R_2$

So we obtain

$4x10^{-5}R_2^2 + 0.0882R_2 - 16.7 = 0$

which yields $R_2 = 175\ \Omega$ and $R_1 = 70\ \Omega$

4.44

(a) $R_{TH} = R_1 \| R_2 = 25.6 \| 10.4 = 7.40\ k\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(V_{CC}) = \left(\frac{10.4}{10.4+25.6}\right)(18) = 5.2\ V$$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$

$$I_{BQ} = \frac{5.2 - 0.7}{7.40 + (126)(3)} = 0.0117\ mA$$

Then $I_{CQ} = 1.46\ mA$ and $I_{EQ} = 1.47\ mA$

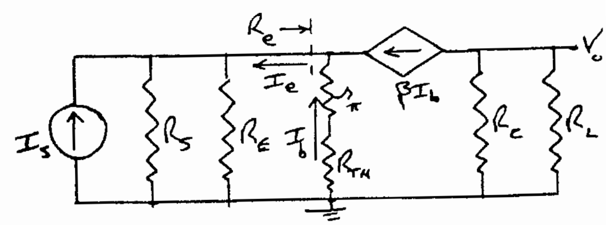
$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$

$V_{CEQ} = 18 - (1.46)(4) - (1.47)(3) \Rightarrow V_{CEQ} = 7.75\ V$

(b)

$$r_\pi = \frac{(125)(0.026)}{1.46} = 2.23\ k\Omega$$

$$g_m = \frac{1.46}{0.026} = 56.2\ mA/V$$



$$R_e = \frac{r_\pi + R_{TH}}{1+\beta} = \frac{2.23 + 7.40}{126} = 0.0764\ k\Omega$$

$$I_e = \frac{-(R_S \| R_E)}{(R_S \| R_E) + R_e} \cdot I_s = \frac{-(100\|3)}{(100\|3) + 0.0764} \cdot I_s$$

or $I_e = -(0.974)I_s$

$$V_o = -I_c(R_C \| R_L) = -\left(\frac{\beta}{1+\beta}\right)I_e(R_C \| R_L)$$

Then

$$\frac{V_o}{I_s} = -\left(\frac{\beta}{1+\beta}\right)(-0.974)(R_C \| R_L) = \left(\frac{125}{126}\right)(0.974)(4\|4)$$

Then

$$R_m = \frac{V_o}{I_s} = 1.93\ k\Omega = 1.93\ V/mA$$

(c)

$$V_s = I_s(R_S \| R_E \| R_e) = I_s(100\|3\|0.0764) = I_s(0.0744)$$

or $I_s = \dfrac{V_s}{0.0744}$

which yields $\dfrac{V_o}{I_s} = \dfrac{V_o}{V_s}(0.0744) = 1.93$

or $A_v = \dfrac{V_o}{V_s} = 25.9$

4.45

(a) $A_v = \dfrac{\beta(R_C \| R_L)}{r_\pi + R_1 \| R_2}$, $R_L = 12\ k\Omega$, $\beta = 100$

Let $R_1 \| R_2 = 50\ k\Omega$, $I_{CQ} = 0.5\ mA$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(on) + (1+\beta) I_{BQ} R_E$$

$$I_{BQ} = \frac{0.5}{100} = 0.005\ mA, \quad r_\pi = \frac{(100)(0.026)}{0.5} = 5.2\ k\Omega$$

$$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(50)(12) = (0.005)(50) + 0.7 + (101)(0.005)(0.5)$$

which yields $R_1 = 500\ k\Omega$ and $R_2 = 55.6\ k\Omega$

$A_v = \dfrac{(100)(12\|12)}{5.2+50} = 10.7$, Design criterion is met.

(b)

$I_{CQ} = 0.5\ mA$, $I_{EQ} = 0.505\ mA$

$V_{CEQ} = 12 - (0.5)(12) - (0.505)(0.5) \Rightarrow$

$V_{CEQ} = 5.75\ V$

$A_v = g_m(R_C \| R_L)$, $g_m = \dfrac{0.5}{0.026} = 19.23\ mA/V$

$A_v = (19.23)(12\|12) \Rightarrow A_v = 115$

4.46

$i_s(peak) = 2.5\ \mu A$, $v_o(peak) = 5\ mV$

So we need $R_m = \dfrac{v_o}{i_s} = \dfrac{5x10^{-3}}{2.5x10^{-6}} = 2x10^3 = 2\ k\Omega$

From Problem 4.44

$$\frac{V_o}{I_s} = \left(\frac{\beta}{1+\beta}\right)(R_C \| R_L)\left(\frac{R_S \| R_E}{R_S \| R_E + R_e}\right)$$

Let $R_C = 4\ k\Omega$, $R_L = 5\ k\Omega$, $R_E = 2\ k\Omega$

Now $\beta = 120$, so we have

$$2 = \left(\frac{120}{121}\right)(4\|5)\left(\frac{R_S \| R_E}{R_S \| R_E + R_e}\right) = 2.20\left(\frac{R_S \| R_E}{R_S \| R_E + R_e}\right)$$

Then $\dfrac{R_S \| R_E}{R_S \| R_E + R_e} = 0.909$

$R_S \| R_E = 50\|2 = 1.92\ k\Omega$, so that $R_e = 0.192\ k\Omega$

Assume $V_{CEQ} = 6\ V$

$$V_{CC} \cong I_{CQ}(R_C + R_E) + V_{CEQ}$$

$$12 = I_{CQ}(4+2) + 6 \Rightarrow I_{CQ} = 1\ mA$$

$$r_\pi = \frac{(120)(0.026)}{1} = 3.12\ k\Omega$$

$$R_e = \frac{r_\pi + R_{TH}}{1+\beta} \Rightarrow 0.192 = \frac{3.12 + R_{TH}}{121}$$

which yields $R_{TH} = 20.1\ k\Omega$

Now $V_{TH} = I_{BQ} R_{TH} + V_{BE}(on) + I_{EQ} R_E$

$$I_{BQ} = \frac{1}{120} = 0.00833\ mA, \quad I_{EQ} = \left(\frac{121}{120}\right)(1) = 1.008\ mA$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(20.1)(5) = (0.00833)(20.1) + 0.7 + (1.008)(2)$$

which yields $R_1 = 34.9\ k\Omega$ and $R_2 = 47.4\ k\Omega$

4.47

a. Emitter current

$$I_{EQ} = I_{CC} = 0.5\ \text{mA}$$

$$I_{BQ} = \frac{0.5}{101} = 0.00495\ \text{mA}$$

$$V_E = I_{EQ} R_E = (0.5)(1) \Rightarrow \underline{V_E = 0.5\ \text{V}}$$

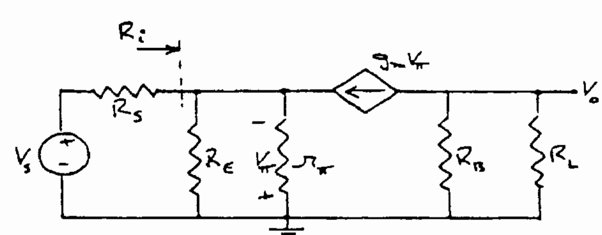
$$V_B = V_E + V_{BE}(\text{on}) = 0.5 + 0.7 \Rightarrow \underline{V_B = 1.20\ \text{V}}$$

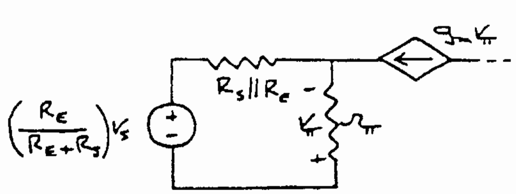
$$V_C = V_B + I_{BQ} R_B = 1.20 + (0.00495)(100)$$

$$\Rightarrow \underline{V_C = 1.7\ \text{V}}$$

b. $r_\pi = \dfrac{(100)(0.026)}{(100)(0.00495)} = 5.25\ k\Omega$

$g_m = \dfrac{(100)(0.00495)}{0.026} = 19.0\ \text{mA/V}$





$V_0 = -g_m V_\pi (R_B \| R_L)$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{\left(\frac{R_E}{R_E + R_S}\right) V_S - (-V_\pi)}{R_S \| R_E} = 0$$

$$V_\pi \left[g_m + \frac{1}{r_\pi} + \frac{1}{R_S \| R_E} \right] = \frac{-\left(\frac{R_E}{R_E + R_S}\right) V_S}{R_S \| R_E}$$

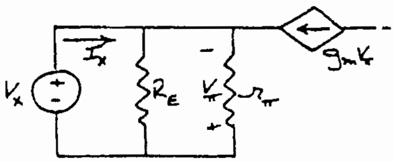
$$V_\pi \left[19.0 + \frac{1}{5.25} + \frac{1}{0.05 \| 1} \right] = \frac{-\left(\frac{1}{1 + 0.05}\right) V_S}{0.05 \| 1}$$

$V_\pi (40.19) = -20 V_S \Rightarrow V_\pi = -(0.4976) V_S$

$V_0 = (19)(0.4976) V_S (100 \| 1)$

$\underline{A_V = 9.36}$

c.



$$I_X = \frac{V_X}{R_E} + \frac{V_X}{r_\pi} - g_m V_\pi, \quad V_\pi = -V_X$$

$$\frac{I_X}{V_X} = \frac{1}{R_i} = \frac{1}{R_E} + \frac{1}{r_\pi} + g_m$$

or $R_i = R_E \left\| r_\pi \right\| \frac{1}{g_m} = 1 \left\| 5.25 \right\| \frac{1}{19}$

$R_i = 0.84 \| 0.0526$

$\Rightarrow \underline{R_i = 49.5\ \Omega}$

4.48

a. $I_{EQ} = \dfrac{20 - 0.7}{10} = 1.93$ mA

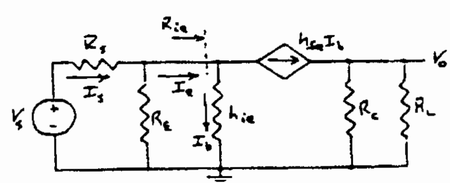
$\underline{I_{CQ} = 1.91 \text{ mA}}$

$V_{ECQ} = V_{CC} + V_{EB}(\text{on}) - I_C R_C$

$= 25 + 0.7 - (1.91)(6.5)$

$\Rightarrow \underline{V_{ECQ} = 13.3 \text{ V}}$

b.



Neglect effect of h_{oe}

From Problem 4-16, assume

$2.45 \le h_{ie} \le 3.7$ kΩ

$80 \le h_{fe} \le 120$

$V_0 = (h_{fe} I_b)(R_C \| R_L)$

$$R_{ie} = \frac{h_{ie}}{1 + h_{fe}}, \quad I_e = \left(\frac{R_E}{R_E + R_{ie}}\right) I_S$$

$$I_b = \left(\frac{I_e}{1 + h_{fe}}\right), \quad I_S = \frac{V_S}{R_S + R_E \| R_{ie}}$$

$$A_v = \left(\frac{h_{fe}}{1 + h_{fe}}\right)(R_C \| R_L)\left(\frac{R_E}{R_E + R_{ie}}\right) \times \left(\frac{1}{R_S + R_E \| R_{ie}}\right)$$

High gain device: $h_{ie} = 3.7$ kΩ, $h_{fe} = 120$

$R_{ie} = \dfrac{3.7}{121} = 0.0306$ kΩ

$R_E \| R_{ie} = 10 \| 0.0306 = 0.0305$

$$A_v = \left(\frac{120}{121}\right)(6.5 \| 5)\left(\frac{10}{10 + 0.0306}\right)\left(\frac{1}{1 + 0.0305}\right)$$

$\Rightarrow A_v = 2.711$

Low gain device: $h_{ie} = 2.45$ kΩ, $h_{fe} = 80$

$R_{ie} = \dfrac{2.45}{81} = 0.03025$ kΩ

$R_E \| R_{ie} = 10 \| 0.03025 = 0.0302$

$$A_v = \left(\frac{80}{81}\right)(6.5 \| 5)\left(\frac{10}{10 + 0.03025}\right)\left(\frac{1}{1 + 0.0302}\right)$$

$\Rightarrow A_v = 2.70$ So $A_v \approx$ constant

$\underline{2.70 \le A_v \le 2.71}$

c. $R_i = R_E \| R_{ie}$

We found $\underline{0.0302 \le R_i \le 0.0305 \text{ k}\Omega}$

Neglecting h_{oe}, $\underline{R_0 = R_C = 6.5 \text{ k}\Omega}$

4.49

a. Small-signal voltage gain

$A_v = g_m (R_C \| R_L) \Rightarrow 25 = g_m (R_C \| 1)$

For $V_{ECQ} = 3$ V

$\Rightarrow V_C = -V_{ECQ} + V_{EB}(\text{on}) = -3 + 0.7$

$\Rightarrow V_C = -2.3$

$V_{CC} - I_{CQ} R_C + V_C = 0$

$$\Rightarrow I_{CQ} = \frac{5 - 2.3}{R_C} = \frac{2.7}{R_C} = I_{CQ}$$

For $I_{CQ} = 1$ mA, $\underline{R_C = 2.7 \text{ k}\Omega}$

$g_m = \dfrac{1}{0.026} = 38.5$ mA/V

$A_v = (38.5)(2.7 \| 1) = \underline{28.1}$

Design criterion satisfied and V_{ECQ} satisfied.

$$I_E = \left(\frac{101}{100}\right)(1) = 1.01 \text{ mA}$$
$$V_{EE} = I_E R_E + V_{EB}(\text{on})$$
$$\Rightarrow R_E = \frac{5-0.7}{1.01} \Rightarrow \underline{R_E = 4.26 \text{ k}\Omega}$$

b. $$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1}$$
$$\Rightarrow \underline{r_\pi = 2.6 \text{ k}\Omega}, \quad \underline{g_m = 38.5 \text{ mA/V}}, \quad \underline{r_0 = \infty}$$

4.50

a. $$V_{TH1} = \left(\frac{R_2}{R_1+R_2}\right)V_{CC} = \left(\frac{20}{20+80}\right)(10)$$
$$\Rightarrow V_{TH1} = 2.0 \text{ V}$$
$$R_{TH1} = R_1 \| R_2 = 20\|80 = 16 \text{ k}\Omega$$
$$I_{B1} = \frac{2-0.7}{16+(101)(1)} = 0.0111 \text{ mA}$$
$$I_{C1} = 1.11 \text{ mA}$$
$$\Rightarrow g_{m1} = \frac{1.11}{0.026} \Rightarrow \underline{g_{m1} = 42.7 \text{ mA/V}}$$
$$r_{\pi 1} = \frac{(100)(0.026)}{1.11} \Rightarrow \underline{r_{\pi 1} = 2.34 \text{ k}\Omega}$$
$$r_{01} = \frac{\infty}{1.11} \Rightarrow \underline{r_{01} = \infty}$$

$$V_{TH2} = \left(\frac{R_4}{R_3+R_4}\right)V_{CC} = \left(\frac{15}{15+85}\right)(10) = 1.50 \text{ V}$$
$$R_{TH2} = R_3 \| R_4 = 15\|85 = 12.75 \text{ k}\Omega$$
$$I_{B2} = \frac{1.50-0.70}{12.75+(101)(0.5)} = 0.0126 \text{ mA}$$
$$I_{C2} = 1.26 \text{ mA}$$
$$\Rightarrow g_{m2} = \frac{1.26}{0.026} \Rightarrow \underline{g_{m2} = 48.5 \text{ mA/V}}$$
$$r_{\pi 2} = \frac{(100)(0.026)}{1.26} \Rightarrow \underline{r_{\pi 2} = 2.06 \text{ k}\Omega}$$
$$\underline{r_{02} = \infty}$$

b. $$A_{v1} = -g_{m1}R_{C1} = -(42.7)(2)$$
$$\Rightarrow \underline{A_{v1} = -85.4}$$
$$A_{v2} = -g_{m2}(R_{C2}\|R_L) = -(48.5)(4\|4)$$
$$\Rightarrow \underline{A_{v2} = -97}$$

c. Input resistance of 2nd stage
$$R_{i2} = R_3\|R_4\|r_{\pi 2} = 15\|85\|2.06$$
$$= 12.75\|2.06 \Rightarrow R_{i2} = 1.77 \text{ k}\Omega$$
$$A'_{v1} = -g_{m1}(R_{C1}\|R_{i2}) = -(42.7)(2\|1.77)$$
$$A'_{v1} = -40.1$$

Overall gain: $A_v = (-40.1)(-97) \Rightarrow \underline{A_v = 3890}$

If we had $A_{v1} \cdot A_{v2} = (-85.4)(-97) = \underline{8284}$

Loading effect reduces overall gain

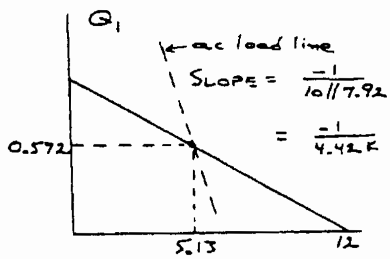
4.51

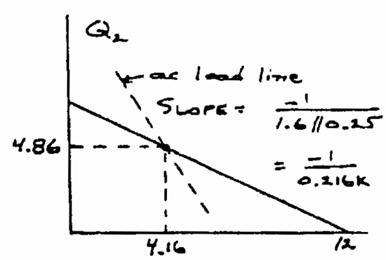
a. $$V_{TH1} = \left(\frac{R_2}{R_1+R_2}\right)V_{CC} = \left(\frac{12.7}{12.7+67.3}\right)(12)$$
$$\Rightarrow V_{TH1} = 1.905 \text{ V}$$
$$R_{TH1} = R_1\|R_2 = 12.7\|67.3 = 10.68 \text{ k}\Omega$$
$$I_{B1} = \frac{1.905-0.70}{10.68+(121)(2)} = 0.00477 \text{ mA}$$
$$I_{C1} = 0.572 \text{ mA}$$
$$g_{m1} = \frac{0.572}{0.026} \Rightarrow \underline{g_{m1} = 22 \text{ mA/V}}$$
$$r_{\pi 1} = \frac{(120)(0.026)}{0.572} \Rightarrow \underline{r_{\pi 1} = 5.45 \text{ k}\Omega}$$
$$r_{01} = \frac{\infty}{0.572} \Rightarrow \underline{r_{01} = \infty}$$

$$V_{TH2} = \left(\frac{R_4}{R_3+R_4}\right)V_{CC} = \left(\frac{45}{45+15}\right)(12)$$
$$\Rightarrow V_{TH2} = 9.0 \text{ V}$$
$$R_{TH2} = R_3\|R_4 = 15\|45 = 11.25 \text{ k}\Omega$$
$$I_{B2} = \frac{9.0-0.70}{11.25+(121)(1.6)} = 0.0405 \text{ mA}$$
$$I_{C2} = 4.86 \text{ mA}$$
$$g_{m2} = \frac{4.86}{0.026} \Rightarrow \underline{g_{m2} = 187 \text{ mA/V}}$$
$$r_{\pi 2} = \frac{(120)(0.026)}{4.86} \Rightarrow \underline{r_{\pi 2} = 0.642 \text{ k}\Omega}$$
$$\underline{r_{02} = \infty}$$

b. $$I_{E1} = 0.577 \text{ mA}$$

$$V_{CEQ1} = 12 - (0.572)(10) - (0.577)(2)$$
$$\Rightarrow \underline{V_{CEQ1} = 5.13 \text{ V}}$$
$$I_{E2} = 4.90$$
$$V_{CEQ2} = 12 - (4.90)(1.6) \Rightarrow \underline{V_{CEQ2} = 4.16 \text{ V}}$$





$R_{i2} = R_3 \| R_4 \| R_{ib}$

$R_{ib} = r_{\pi 2} + (1+\beta)(R_{E2} \| R_L)$

$= 0.642 + (121)(1.6 \| 0.25)$

$R_{ib} = 26.8$

$R_{i2} = 15 \| 45 \| 26.8$

$R_{i2} = 7.92 \text{ k}\Omega$

c. $A_{v1} = -g_{m1}(R_{C1} \| R_{i2}) = -(22)(10 \| 7.92)$

$\Rightarrow A_{v2} = -97.2$

$$A_{v2} = \frac{(1+\beta)(R_{E2} \| R_L)}{r_{\pi 2} + (1+\beta)(R_{E2} \| R_L)}$$

$$= \frac{(121)(0.216)}{0.642 + (121)(0.216)} = 0.976$$

Overall gain $= (-97.2)(0.976) = \underline{-94.9}$

d. $R_{iS} = R_1 \| R_2 \| r_{\pi 1} = 67.3 \| 12.7 \| 5.45$

$\Rightarrow \underline{R_{iS} = 3.61 \text{ k}\Omega}$

$R_0 = \dfrac{r_{\pi 2} + R_S}{1+\beta} \Big\| R_{E2}$ where

$R_S = R_3 \| R_4 \| R_{C1}$

$= 15 \| 45 \| 10 \Rightarrow R_S = 5.29 \text{ k}\Omega$

$R_0 = \dfrac{0.642 + 5.29}{121} \Big\| 1.6 \Rightarrow 0.049 \| 1.6$

$\Rightarrow \underline{R_0 = 47.5 \ \Omega}$

e. $\Delta i_C = \dfrac{-1}{0.216 \text{ k}\Omega} \cdot \Delta v_{ce}$. $\Delta i_C = 4.86$

$|\Delta v_{ce}| = (4.86)(0.216) = 1.05$ V

Max. output voltage swing

$= \underline{2.10 \text{ V peak-to-peak}}$

4.52

(a) $I_{R1} = \dfrac{5 - 2(0.7)}{0.050} = 72 \ mA$

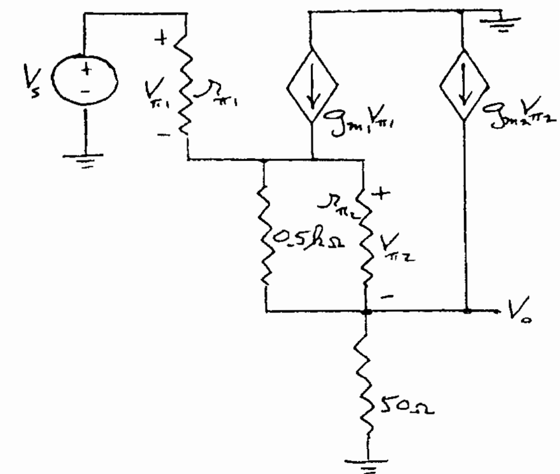
$I_{R2} = \dfrac{0.7}{0.5} = 1.4 \ mA$

$I_{C2} = \left(\dfrac{\beta}{1+\beta}\right)(72 - 1.4) \Rightarrow \underline{I_{C2} = 69.9 \ mA}$

$I_{B2} = \dfrac{69.9}{100} = 0.699 \ mA$

$I_{C1} = \left(\dfrac{\beta}{1+\beta}\right)(1.4 + 0.699) \Rightarrow \underline{I_{C1} = 2.08 \ mA}$

(b)



$V_s = V_{\pi 1} + V_{\pi 2} + V_o$

(1) $V_o = \left(\dfrac{V_{\pi 2}}{0.5} + \dfrac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2}\right)(0.05)$

$r_{\pi 2} = \dfrac{(100)(0.026)}{69.9} = 0.0372 \ k\Omega$

$g_{m2} = \dfrac{69.9}{0.026} = 2688 \ mA/V$

$V_o = V_{\pi 2}\left(\dfrac{1}{0.05} + \dfrac{1}{0.0372} + 2688\right)(0.05)$

so that (1) $V_{\pi 2} = \dfrac{V_o}{136.7}$

(2) $\dfrac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \dfrac{V_{\pi 2}}{0.5} + \dfrac{V_{\pi 2}}{r_{\pi 2}}$

$r_{\pi 1} = \dfrac{(100)(0.026)}{2.08} = 1.25 \ k\Omega$

$g_{m1} = \dfrac{2.08}{0.026} = 80 \ mA/V$

$V_{\pi 1}\left(\dfrac{1}{1.25} + 80\right) = V_{\pi 2}\left(\dfrac{1}{0.5} + \dfrac{1}{0.0372}\right)$

$V_{\pi 1}(80.8) = V_{\pi 2}(28.88) = \left(\dfrac{V_o}{136.7}\right)(28.88)$

or (2) $V_{\pi 1} = V_o(0.00261)$

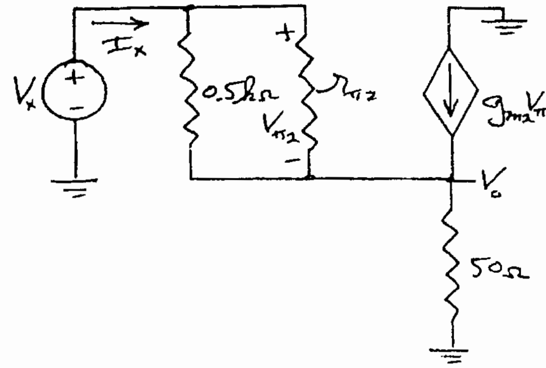
Then

$V_s = V_o(0.00261) + \dfrac{V_o}{136.7} + V_o = V_o(1.00993)$

or $\underline{A_v = \dfrac{V_o}{V_s} = 0.990}$

(c)

$R_{ib} = r_{\pi 1} + (1+\beta)[R_x]$



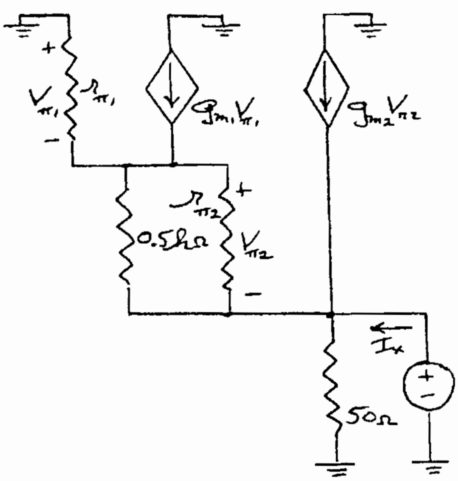
$$I_x = \frac{V_{\pi 2}}{0.5} + \frac{V_{\pi 2}}{r_{\pi 2}} = V_{\pi 2}\left(\frac{1}{0.5} + \frac{1}{r_{\pi 2}}\right)$$

$$\frac{V_o}{0.05} = \frac{V_x - V_{\pi 2}}{0.05} = I_x + g_{m2}V_{\pi 2}$$

$$\frac{V_x}{0.05} - I_x = V_{\pi 2}\left(\frac{1}{0.05} + g_{m2}\right) = \frac{I_x\left(\frac{1}{0.05} + g_{m2}\right)}{\left(\frac{1}{0.05} + \frac{1}{r_{\pi 2}}\right)}$$

We find $\frac{V_x}{I_x} = R_x = 2.89\ k\Omega$

Then $R_{ib} = 1.25 + (101)(2.89) \Rightarrow$ $\underline{R_{ib} = 293\ k\Omega}$



To find R_o:

(1) $I_x = \frac{V_x}{0.05} - g_{m2}V_{\pi 2} - \frac{V_{\pi 2}}{0.05 \| r_{\pi 2}}$

(2) $V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1}\right)(0.05 \| r_{\pi 2})$

$$= V_{\pi 1}\left(\frac{1}{1.25} + 80\right)(0.05 \| 0.0372)$$

or $V_{\pi 2} = (1.72)V_{\pi 1}$

(3) $V_{\pi 1} + V_{\pi 2} + V_x = 0 \Rightarrow V_{\pi 1} + (1.72)V_{\pi 1} + V_x = 0$

so that $V_{\pi 1} = -(0.368)V_x$

and $V_{\pi 2} = (1.72)[-(0.368)V_x] = -(0.633)V_x$

Now $I_x = \frac{V_x}{0.05} - V_{\pi 2}\left(g_{m2} + \frac{1}{0.05 \| r_{\pi 2}}\right)$

So that

$$I_x = \frac{V_x}{0.05} + (0.633)V_x\left[2688 + \frac{1}{0.05 \| 0.0372}\right]$$

which yields

$$R_o = \frac{V_x}{I_x} = 0.583\ \Omega$$

4.53

a. $R_{TH} = R_1 \| R_2 = 335 \| 125 = 91.0\ k\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC}$$

$$= \left(\frac{125}{125 + 335}\right)(10) = 2.717\ V$$

$V_{TH} = I_{B1}R_{TH} + V_{BE1} + V_{BE2} + I_{E2}R_{E2}$

$I_{E2} = (1 + \beta)I_{E1} = (1 + \beta)^2 I_{B1}$

$$I_{B1} = \frac{2.717 - 1.40}{91.0 + (101)^2(1)} \Rightarrow I_{B1} = 0.128\ \mu A$$

$\underline{I_{C1} = 12.8\ \mu A}$

$I_{C2} = \beta I_{E1} = \beta(1 + \beta)I_{B1} = (100)(101)(0.128\ \mu A)$

$\underline{I_{C2} = 1.29\ mA}$, $I_{E2} = 1.31\ mA$

$I_{RC} = I_{C2} + I_{C1} = 1.29 + 0.0128 = 1.31\ mA$

$V_C = 10 - I_{RC}R_C = 10 - (1.31)(2.2) = 7.12\ V$

$V_E = I_{E2}R_{E2} = (1.31)(1) = 1.31\ V$

$V_{CE2} = 7.12 - 1.31 = 5.81\ V$

$V_{CE1} = V_{CE2} - V_{BE2} = 5.81 - 0.7$

$V_{CE1} = 5.11\ V$

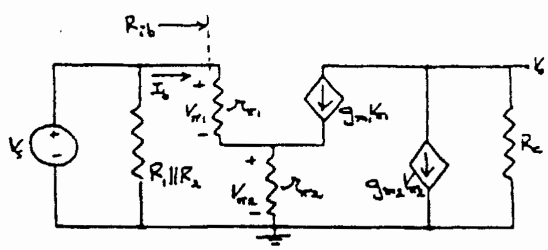
Summary:

$I_{C1} = 12.8\ \mu A$ $\quad I_{C2} = 1.29\ mA$

$V_{CE1} = 5.11\ V$ $\quad V_{CE2} = 5.81\ V$

b. $g_{m1} = \frac{0.0128}{0.026} = 0.492\ mA/V$

$g_{m2} = \frac{1.29}{0.026} = 49.6\ mA/V$



$$V_0 = -(g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2})R_C$$

$$V_S = V_{\pi 1} + V_{\pi 2}, \quad V_{\pi 1} = V_S - V_{\pi 2}$$

$$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1}\right) r_{\pi 2}$$

$$V_{\pi 2} = V_{\pi 1}\left(\frac{1+\beta}{r_{\pi 1}}\right) r_{\pi 2}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{0.0128} = 203 \text{ k}\Omega$$

$$r_{\pi 2} = \frac{(100)(0.026)}{1.29} = 2.02 \text{ k}\Omega$$

$$V_0 = -[g_{m1}(V_S - V_{\pi 2}) + g_{m2}V_{\pi 2}]R_C$$

$$V_0 = -[g_{m1}V_S + (g_{m2} - g_{m1})V_{\pi 2}]R_C$$

$$V_{\pi 2} = (V_S - V_{\pi 2})(1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)$$

$$V_{\pi 2}\left[1 + (1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)\right] = V_S(1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)$$

$$V_0 = -\left\{ g_{m1}V_S + (g_{m2} - g_{m1}) \cdot \frac{V_S(1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)}{1 + (1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)} \right\} R_C$$

$$A_v = \frac{V_0}{V_S}$$

$$= -\left\{ (0.492) + \frac{(49.6 - 0.492)(101)\left(\frac{2.02}{203}\right)}{1 + (101)\left(\frac{2.02}{203}\right)} \right\} 2.2$$

$$A_v = -55.2$$

c. $R_{is} = R_1 \| R_2 \| R_{ib}$

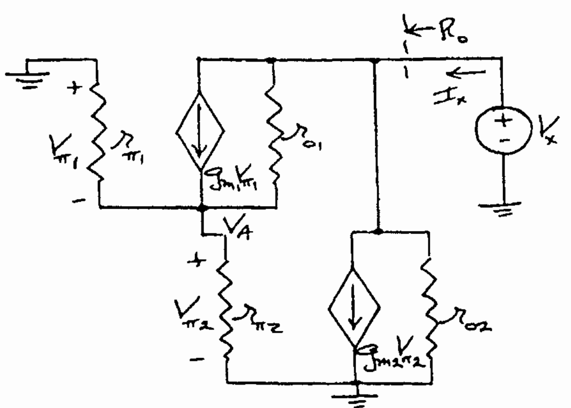
$$R_{ib} = r_{\pi 1} + (1+\beta) r_{\pi 2}$$

$$= 203 + (101)(2.02) = 407 \text{ k}\Omega$$

$R_{is} = 91\|407 = \underline{74.4 \text{ k}\Omega = R_{is}}$

$\underline{R_0 = R_C = 2.2 \text{ k}\Omega}$

4.54



(1) $I_x = g_{m2}V_{\pi 2} + \dfrac{V_x}{r_{o2}} + \dfrac{V_x - V_A}{r_{o1}} + g_{m1}V_{\pi 1}$

(2) $\dfrac{V_x - V_A}{r_{o1}} + g_{m1}V_{\pi 1} = \dfrac{V_A}{r_{\pi 1}\|r_{\pi 2}}$

(3) $V_{\pi 2} = V_A = -V_{\pi 1}$

Then from (2)

$$\frac{V_x}{r_{o1}} = V_A\left(\frac{1}{r_{o1}} + g_{m1} + \frac{1}{r_{\pi 1}\|r_{\pi 2}}\right)$$

(1) $I_x = g_{m2}V_A + \dfrac{V_x}{r_{o2}} + \dfrac{V_x}{r_{o1}} - \dfrac{V_A}{r_{o1}} - g_{m1}V_A$

or $I_x = V_x\left(\dfrac{1}{r_{o1}} + \dfrac{1}{r_{o2}}\right) + V_A\left(g_{m2} - \dfrac{1}{r_{o1}} - g_{m1}\right)$

Solving for V_A from Equation (2) and substituting into Equation (1), we find

$$R_o = \frac{V_x}{I_x} = \frac{\frac{1}{r_{o1}} + g_{m1} + \frac{1}{r_{\pi 1}\|r_{\pi 2}}}{\frac{1}{r_{o2}}\left(\frac{1}{r_{o1}} + g_{m1} + \frac{1}{r_{\pi 1}\|r_{\pi 2}}\right) + \frac{1}{r_{o1}}\left(\frac{1}{r_{\pi 1}\|r_{\pi 2}} + g_{m2}\right)}$$

For $\beta = 100, V_A = 100\,V, I_{C1} = I_{Bias} = 1\,mA$

$$r_{o1} = r_{o2} = \frac{100}{1} = 100\,k\Omega$$

$$r_{\pi 1} = r_{\pi 2} = \frac{(100)(0.026)}{1} = 2.6\,k\Omega$$

$$g_{m1} = g_{m2} = \frac{1}{0.026} = 38.46\,mA/V$$

Then

$$R_o = \frac{\frac{1}{100} + 38.46 + \frac{1}{2.6\|2.6}}{\frac{1}{100}\left(\frac{1}{100} + 38.46 + \frac{1}{2.6\|2.6}\right) + \frac{1}{100}\left(\frac{1}{2.6\|2.6} + 38.46\right)}$$

or

$\underline{R_o = 50.0\,k\Omega}$

Now

$I_{C2} = 1\,mA, I_{Bias} = 0$

Replace I_{Bias} by $\dfrac{I_{C2}}{\beta} \cdot \dfrac{\beta}{1+\beta} = \dfrac{I_{C2}}{1+\beta}$, $I_{C1} \cong 0.01\,mA$

$$r_{o2} = \frac{100}{1} = 100\,k\Omega, r_{o1} = \frac{100}{0.01} = 10{,}000\,k\Omega$$

$$g_{m_2} = \frac{1}{0.026} = 38.46\,mA/V, g_{m1} = 0.3846\,mA/V$$

$$r_{\pi 2} = \frac{(100)(0.026)}{1} = 2.6\,k\Omega, r_{\pi 1} = 260\,k\Omega$$

Then

$\underline{R_o = 66.4\,k\Omega}$

4.55

a. $R_{TH} = R_1 \| R_2 = 93.7 \| 6.3 = 5.90\ \text{k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{6.3}{6.3 + 93.7}\right)(12) = 0.756\ \text{V}$$

$$I_{BQ} = \frac{0.756 - 0.70}{5.90} = 0.00949\ \text{mA}$$

$I_{CQ} = 0.949$ mA

$V_{CEQ} = 12 - (0.949)(6) \Rightarrow V_{CEQ} = 6.31$ V

Transistor:

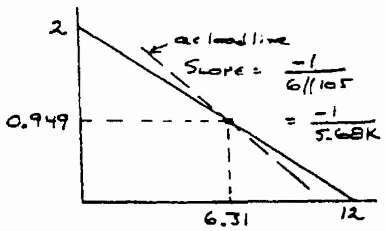
$P_Q \approx I_{CQ} V_{CEQ} = (0.949)(6.31)$

$\Rightarrow \underline{P_Q = 5.99\ \text{mW}}$

R_C: $P_R = I_{CQ}^2 R_C = (0.949)^2(6)$

$\Rightarrow \underline{P_R = 5.40\ \text{mW}}$

b.



$$r_0 = \frac{100}{0.949} = 105\ \text{k}\Omega$$

Peak signal current = 0.949 mA

$|V_0(\text{max})| = (5.68)(0.949) = 5.39$ V

$$P_{RC} = \frac{1}{2} \cdot \frac{V_0^2(\text{max})}{R_C} = \frac{1}{2}\left[\frac{(5.39)^2}{6}\right]$$

$\Rightarrow P_{RC} = 2.42$ mW

4.56

(a) $10 = I_{BQ} R_B + V_{BE}(on) + (1+\beta) I_{BQ} R_E$

$$I_{BQ} = \frac{10 - 0.7}{100 + (121)(20)} = 0.00369\ mA$$

$I_{CQ} = 0.443\ mA,\ \ I_{EQ} = 0.447\ mA$

For R_C: $P_{RC} = (0.443)^2(10) \Rightarrow \underline{P_{RC} = 1.96\ mW}$

For R_E: $P_{RE} = (0.447)^2(20) \Rightarrow \underline{P_{RE} = 4.0\ mW}$

(b)

$\Delta i_C = 0.443\ mA,\ \ \Delta v_{CE} = (0.443)(10) = 4.43\ V$

Then

$$\overline{P}_{RC} = \frac{1}{2}(\Delta i_C)^2 R_C = \frac{1}{2}(0.443)^2(10)$$

$\underline{\overline{P}_{RC} = 0.981\ mW}$

4.57

a. $$I_{BQ} = \frac{10 - 0.7}{50 + (151)(10)} = 0.00596\ \text{mA}$$

$I_{CQ} = 0.894$ mA, $I_{EQ} = 0.90$ mA

$V_{ECQ} = 20 - (0.894)(5) - (0.90)(10)$

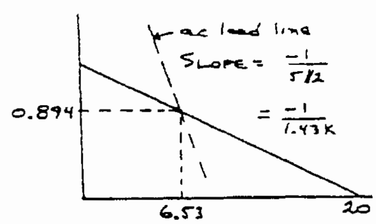
$\Rightarrow V_{ECQ} = 6.53$ V

$P_Q \cong I_{CQ} V_{ECQ} = (0.894)(6.53) \Rightarrow \underline{P_Q = 5.84\ \text{mW}}$

$P_{RC} \cong I_{CQ}^2 R_C = (0.894)^2(5) \Rightarrow \underline{P_{RC} = 4.0\ \text{mW}}$

$P_{RE} \cong I_{EQ}^2 R_C = (0.90)^2(10) \Rightarrow \underline{P_{RE} = 8.1\ \text{mW}}$

b.



$$\Delta i_C = \frac{-1}{1.43\ \text{k}\Omega} \cdot \Delta v_{ec}$$

$\Delta i_C = 0.894 \Rightarrow |\Delta v_{ec}| = (0.894)(1.43) = 1.28$ V

$$\Delta i_0 = \left(\frac{5}{5+2}\right)\Delta i_C = 0.639\ \text{mA}$$

$\overline{P_{RL}} = \frac{1}{2}(0.639)^2(2) \Rightarrow \underline{\overline{P_{RL}} = 0.408\ \text{mW}}$

$\overline{P_{RC}} = \frac{1}{2} \cdot (0.894 - 0.639)^2(5) \Rightarrow \underline{\overline{P_{RC}} = 0.163\ \text{mW}}$

$\underline{\overline{P_{RE}} = 0}$

$\overline{P_Q} = 5.84 - 0.408 - 0.163 \Rightarrow \underline{\overline{P_Q} = 5.27\ \text{mW}}$

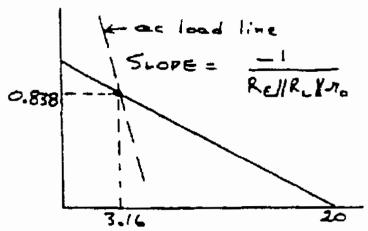
4.58

$I_{BQ} = \dfrac{10 - 0.70}{100 + (101)(10)} = 0.00838$ mA

$I_{CQ} = 0.838$ mA, $I_{EQ} = 0.846$ mA

$V_{CEQ} = 20 - (0.838)(10) - (0.846)(10)$

$\Rightarrow V_{CEQ} = 3.16$ V



$r_0 = \dfrac{100}{0.838} = 119$ kΩ

Neglecting base currents:

a. $R_L = 1$ kΩ

$\text{slope} = \dfrac{-1}{10\|1\|119} = \dfrac{-1}{0.902 \text{ k}\Omega}$

$\Delta i_C = \dfrac{-1}{0.902 \text{ k}\Omega} \cdot \Delta v_{ce}$

$\Delta i_C = 0.838 \Rightarrow |\Delta v_{ce}| = (0.902)(0.838)$

$= 0.756$ V

$\overline{P_{RL}} = \dfrac{1}{2}\dfrac{(0.756)^2}{1} \Rightarrow \overline{P_{RL}} = 0.286$ mW

b. $R_L = 10$ kΩ

$\text{slope} = \dfrac{-1}{10\|10\|119} = \dfrac{-1}{4.80}$

For

$\Delta i_C = 0.838 \Rightarrow |\Delta v_{ce}| = (0.838)(4.80) = 4.02$

Max. swing determined by voltage

$\overline{P_{RL}} = \dfrac{1}{2}\dfrac{(3.16)^2}{10} \Rightarrow \overline{P_{RL}} = 0.499$ mW

4.59

a. $I_{BQ} = \dfrac{10 - 0.7}{100 + (101)(10)} = 0.00838$ mA

$I_{CQ} = 0.838$ mA, $I_{EQ} = 0.846$ mA

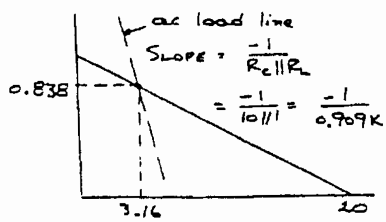
$V_{CEQ} = 20 - (0.838)(10) - (0.846)(10)$

$\Rightarrow V_{CEQ} = 3.16$ V

$P_Q \cong I_{CQ}V_{CEQ} = (0.838)(3.16) \Rightarrow P_Q = 2.65$ mW

$P_{RC} \cong I_{CQ}^2 R_C = (0.838)^2(10) \Rightarrow P_{RC} = 7.02$ mW

b.



$\Delta i_C = \dfrac{-1}{0.909 \text{ k}\Omega} \cdot \Delta v_{ce}$

For

$\Delta i_C = 0.838 \Rightarrow |\Delta v_{ce}| = (0.909)(0.838) = 0.762$ V

$\Delta i_0 = \left(\dfrac{R_C}{R_C + R_L}\right)\Delta i_C = \left(\dfrac{10}{10+1}\right)\Delta i_C = 0.762$ mA

$\overline{P_{RL}} = \dfrac{1}{2}(0.762)^2(1) \Rightarrow \overline{P_{RL}} = 0.290$ mW

$\overline{P_{RC}} = \dfrac{1}{2} \cdot (0.838 - 0.762)^2(10) \Rightarrow \overline{P_{RC}} = 0.0289$ mW

$\overline{P_Q} = 2.65 - 0.290 - 0.0289 \Rightarrow \overline{P_Q} = 2.33$ mW

Chapter 5

Exercise Solutions

E5.1

(a) $V_{TN} = 1.2\,V$, $V_{GS} = 2\,V$

$V_{DS}(sat) = V_{GS} - V_{TN} = 2 - 1.2 = 0.8\,V$

(i) $V_{DS} = 0.4 \Rightarrow$ Nonsaturation

(ii) $V_{DS} = 1 \Rightarrow$ Saturation

(iii) $V_{DS} = 5 \Rightarrow$ Saturation

(b) $V_{TN} = -1.2\,V$, $V_{GS} = 2\,V$

$V_{DS}(sat) = V_{GS} - V_{TN} = 2 - (-1.2) = 3.2\,V$

(i) $V_{DS} = 0.4 \Rightarrow$ Nonsaturation

(ii) $V_{DS} = 1 \Rightarrow$ Nonsaturation

(iii) $V_{DS} = 5 \Rightarrow$ Saturation

E5.2

(a) $K_n = \dfrac{W\mu_n C_{ox}}{2L}$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{450x10^{-8}} = 7.67x10^{-8}\ F/cm$$

$$K_n = \frac{(100)(500)(7.67x10^{-8})}{2(7)} \Rightarrow \underline{K_n = 0.274\ mA/V^2}$$

(b) $V_{TN} = 1.2\,V$, $V_{GS} = 2\,V$

(i) $V_{DS} = 0.4\,V \Rightarrow$ Nonsaturation

$I_D = (0.274)\left[2(2-1.2)(0.4) - (0.4)^2\right] \Rightarrow$

$\underline{I_D = 0.132\,mA}$

(ii) $V_{DS} = 1\,V \Rightarrow$ Saturation

$I_D = (0.274)(2-1.2)^2 \Rightarrow \underline{I_D = 0.175\,mA}$

(iii) $V_{DS} = 5\,V \Rightarrow$ Saturation

$I_D = (0.274)(2-1.2)^2 \Rightarrow \underline{I_D = 0.175\,mA}$

$V_{TN} = -1.2\,V$, $V_{GS} = 2\,V$

(i) $V_{DS} = 0.4\,V \Rightarrow$ Nonsaturation

$I_D = (0.274)\left[2(2+1.2)(0.4) - (0.4)^2\right] \Rightarrow$

$I_D = 0.658\,mA$

(ii) $V_{DS} = 1\,V \Rightarrow$ Nonsaturation

$I_D = (0.274)\left[2(2+1.2)(1) - (1)^2\right] \Rightarrow \underline{I_D = 1.48\,mA}$

(iii) $V_{DS} = 5\,V \Rightarrow$ Saturation

$I_D = (0.274)(2+1.2)^2 \Rightarrow \underline{I_D = 2.81\,mA}$

E5.3

$V_{TN} = 1\,V$, $V_{GS} = 3\,V$, $V_{DS} = 4.5\,V$

$V_{DS} = 4.5 > V_{DS}(sat) = V_{GS} - V_{TN} = 3 - 1 = 2\,V$

Transistor biased in the saturation region

$I_D = K_n(V_{GS} - V_{TN})^2 \Rightarrow 0.8 = K_n(3-1)^2 \Rightarrow$

$K_n = 0.2\,mA/V^2$

(a) $V_{GS} = 2\,V$, $V_{DS} = 4.5\,V$

Saturation region:

$I_D = (0.2)(2-1)^2 \Rightarrow \underline{I_D = 0.2\,mA}$

(b) $V_{GS} = 3\,V$, $V_{DS} = 1\,V$

Nonsaturation region:

$I_D = (0.2)\left[2(3-1)(1) - (1)^2\right] \Rightarrow \underline{I_D = 0.6\,mA}$

E5.4

(a) $V_{TP} = -2\,V$, $V_{SG} = 3\,V$

$V_{SD}(sat) = V_{SG} + V_{TP} = 3 - 2 = 1\,V$

(i) $V_{SD} = 0.5\,V \Rightarrow$ Nonsaturation

(ii) $V_{SD} = 2\,V \Rightarrow$ Saturation

(iii) $V_{SD} = 5\,V \Rightarrow$ Saturation

(b) $V_{TP} = 0.5\,V$, $V_{SG} = 3\,V$

$V_{SD}(sat) = V_{SG} + V_{TP} = 3 + 0.5 = 3.5\,V$

(i) $V_{SD} = 0.5\,V \Rightarrow$ Nonsaturation

(ii) $V_{SD} = 2\,V \Rightarrow$ Nonsaturation

(iii) $V_{SD} = 5\,V \Rightarrow$ Saturation

E5.5

(a) $\lambda = 0$, $V_{DS}(sat) = 2.5 - 0.8 = 1.7\,V$

For $V_{DS} = 2\,V$, $V_{DS} = 10\,V \Rightarrow$ Saturation Region

$I_D = (0.1)(2.5-0.8)^2 \Rightarrow \underline{I_D = 0.289\,mA}$

(b) $\lambda = 0.02\,V^{-1}$

$I_D = K_n(V_{GS} - V_{TN})^2(1 + \lambda V_{DS})$

For $V_{DS} = 2\,V$

$I_D = (0.1)(2.5-0.8)^2\left[1 + (0.02)(2)\right] \Rightarrow$

$\underline{I_D = 0.300\,mA}$

$V_{DS} = 10\,V$

$I_D = (0.1)\left[(2.5-0.8)^2(1 + (0.02)(10))\right] \Rightarrow$

$\underline{I_D = 0.347\,mA}$

(c) For part (a), $\lambda = 0 \Rightarrow \underline{r_o = \infty}$

For part (b), $\lambda = 0.02\,V^{-1}$,

$r_o = \left[\lambda K_n(V_{GS} - V_{TN})^2\right]^{-1} = \left[(0.02)(0.1)(2.5-0.8)^2\right]^{-1}$

or $\underline{r_o = 173\,k\Omega}$

E5.6

$$V_{TN} = V_{TNO} + \gamma\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right]$$

$2\phi_f = 0.70\,V$, $V_{TNO} = 1\,V$

(a) $V_{SB} = 0 \Rightarrow$, $\underline{V_{TN} = 1\,V}$

(b) $V_{SB} = 1\,V$,

$$V_{TN} = 1 + (0.35)\left[\sqrt{0.7+1} - \sqrt{0.7}\right] \Rightarrow \quad V_{TN} = 1.16\,V$$

(c) $V_{SB} = 4\,V$,

$$V_{TN} = 1 + (0.35)\left[\sqrt{0.7+4} - \sqrt{0.7}\right] \Rightarrow \quad V_{TN} = 1.47\,V$$

E5.7

$$V_G = \left(\frac{R_2}{R_1+R_2}\right)(10) - 5 = \left(\frac{40}{40+60}\right)(10) - 5$$

$V_G = -1\,V$

$V_S = I_D R_S - 5$

Then

$V_{GS} = V_G - V_S = -1 - (I_D R_S - 5) = 4 - I_D R_S$

Assume transistor is biased in saturation region

$$I_D = K_n(V_{GS} - V_{TN})^2 = \frac{4 - V_{GS}}{R_S}$$

$4 - V_{GS} = (0.5)(0.1)[V_{GS} - 1]^2 \Rightarrow$

$0.5V_{GS}^2 - 3.5 = 0 \Rightarrow \quad \underline{V_{GS} = 2.65\,V}$

$I_D = (0.5)(2.65-1)^2 \Rightarrow \quad \underline{I_D = 1.36\,mA}$

$V_{DS} = 10 - I_D(R_S + R_D) = 10 - (1.36)(1+2) \Rightarrow$

$\underline{V_{DS} = 5.92\,V}$

$V_{DS} > V_{DS}(sat)$, Yes

E5.8

$$V_G = \left(\frac{R_2}{R_1+R_2}\right)(10) - 5$$

$$= \left(\frac{200}{350}\right)(10) - 5 = 0.714\text{ V}$$

$V_S = 5 - I_D R_S = 5 - (1.2)I_D$

So

$V_{SG} = V_S - V_G = 5 - (1.2)I_D - 0.714$

$= 4.286 - (1.2)I_D$

$$I_D = \frac{4.286 - V_{SG}}{1.2}$$

$I_D = K_p(V_{SG} + V_{TP})^2$

$4.286 - V_{SG} = (1.2)(0.25)\times (V_{SG}^2 - 2V_{SG}(-1) + (-1)^2)$

$4.286 - V_{SG} = (0.3)V_{SG}^2 - 0.6V_{SG} + 0.3$

$0.3V_{SG}^2 + 0.4V_{SG} - 3.986 = 0$

$$V_{SG} = \frac{-0.4 \pm \sqrt{(0.4)^2 + 4(0.3)(3.986)}}{2(0.3)}$$

Must use + sign $\Rightarrow \underline{V_{SG} = 3.04\text{ V}}$

$I_D = (0.25)(3.04 - 1)^2 \Rightarrow \underline{I_D = 1.04\text{ mA}}$

$V_{SD} = 10 - I_D(R_S + R_D) = 10 - (1.04)(1.2 + 4)$

$\Rightarrow \underline{V_{SD} = 4.59\text{ V}}$

$V_{SD} > V_{SD}(sat)$, Yes

E5.9

$I_D = K_n(V_{GS} - V_{TN})^2$

$0.4 = 0.25(V_{GS} - 0.8)^2 \Rightarrow V_{GS} = 2.06\text{ V}$

$$V_{GS} = \left(\frac{R_2}{R_1+R_2}\right)V_{DD}$$

$$2.06 = \left(\frac{R_2}{250}\right)(7.5) \Rightarrow \underline{R_2 = 68.7\text{ k}\Omega}$$

$\underline{R_1 = 181.3\text{ k}\Omega}$

$V_{DS} = 4 = V_{DD} - I_D R_D$

$$R_D = \frac{7.5 - 4}{0.4} \Rightarrow \underline{R_D = 8.75\text{ k}\Omega}$$

$V_{DS} > V_{DS}(sat)$, Yes

E5.10

$$I_D = \frac{V_S - (-5)}{R_S} \text{ and } V_S = -V_{GS}$$

$$\text{So } R_S = \frac{5 - V_{GS}}{0.1}$$

$I_D = K_n(V_{GS} - V_{TN})^2$

$0.1 = (0.080)(V_{GS} - 1.2)^2 \Rightarrow V_{GS} = 2.32\text{ V}$

$$\text{So } R_S = \frac{5 - 2.32}{0.1} \Rightarrow \underline{R_S = 26.8\text{ k}\Omega}$$

$V_{DS} = V_D - V_S \Rightarrow V_D = V_{DS} + V_S = 4.5 - 2.32$

$V_D = 2.18$

$$R_D = \frac{5 - V_D}{I_D} = \frac{5 - 2.18}{0.1} \Rightarrow \underline{R_D = 28.2\text{ k}\Omega}$$

$V_{DS} > V_{DS}(sat)$, Yes

E5.11

$$I_D = \frac{10 - V_{SG}}{R_S} \text{ and } \quad I_D = K_p(V_{SG} + V_{TP})^2$$

$0.12 = (0.050)(V_{SG} - 0.8)^2$

$V_{SG} = 2.35\text{ V}$

$$R_S = \frac{10 - 2.35}{0.12} \Rightarrow \underline{R_S = 63.75\text{ k}\Omega}$$

$V_{SD} = 8 = 20 - I_D(R_S + R_D)$

$8 = 20 - (0.12)(63.75) - (0.12)R_D$

$$R_D = \frac{20 - (0.12)(63.75) - 8}{0.12}$$

$\Rightarrow R_D = 36.25\text{ k}\Omega$

E5.12

$I_D = \frac{V_{DD} - V_{GS}}{R_S}$, $I_D = K_n(V_{GS} - V_{TN})^2$

$10 - V_{GS} = (10)(0.2)(V_{GS}^2 - 2V_{GS}V_{TN} + V_{TN}^2)$

$10 - V_{GS} = 2V_{GS}^2 - 8V_{GS} + 8$

$2V_{GS}^2 - 7V_{GS} - 2 = 0$

$$V_{GS} = \frac{7 \pm \sqrt{(7)^2 + 4(2)2}}{2(2)}$$

Use + sign: $\underline{V_{GS} = V_{DS} = 3.77 \text{ V}}$

$I_D = \frac{10 - 3.77}{10} \Rightarrow \underline{I_D = 0.623 \text{ mA}}$

Power $= I_D V_{DS} = (0.623)(3.77)$

$\Rightarrow$ Power = 2.35 mW

E5.13

For $V_{DS} = 2.2\,V$

$I_D = \frac{5-2.2}{5} \Rightarrow \underline{I_D = 0.56\,mA}$

$I_D = K_n(V_{GS} - V_{TN})^2$

$0.56 = K_n(2.2 - 1)^2$

$K_n = 0.389\,mA/V = \frac{W}{L} \cdot \frac{\mu_n C_{ox}}{2}$

$\frac{W}{L} = \frac{(389)(2)}{(40)} \Rightarrow \underline{\frac{W}{L} = 19.5}$

E5.14

(a) The transition point is

$$V_{It} = \frac{V_{DD} - V_{TNL} + V_{TND}\left(1 + \sqrt{K_{nD}/K_{nL}}\right)}{1 + \sqrt{K_{nD}/K_{nL}}}$$

$$= \frac{5 - 1 + 1\left(1 + \sqrt{0.05/0.01}\right)}{1 + \sqrt{0.05/0.01}}$$

$$= \frac{7.236}{3.236} \Rightarrow \underline{V_{It} = 2.24 \text{ V}}$$

$V_{Ot} = V_{It} - V_{TND} = 2.24 - 1 \Rightarrow \underline{V_{Ot} = 1.24\,V}$

(b) We may write

$I_D = K_{nD}(V_{GSD} - V_{TND})^2 = (0.05)(2.24 - 1)^2$

$\Rightarrow \underline{I_D = 76.9\,\mu A}$

E5.15

$$V_{It} = \frac{V_{DD} - V_{TNL} + V_{TND}\left(1 + \sqrt{K_{nD}/K_{nL}}\right)}{1 + \sqrt{K_{nD}/K_{nL}}}$$

$$2.5 = \frac{5 - 1 + 1\left(1 + \sqrt{K_{nD}/K_{nL}}\right)}{1 + \sqrt{K_{nD}/K_{nL}}}$$

$2.5 + 2.5\sqrt{K_{nD}/K_{nL}} = 5 + \sqrt{K_{nD}/K_{nL}} \Rightarrow$

$\sqrt{K_{nD}/K_{nL}} = \frac{5 - 2.5}{1.5} = 1.67 \Rightarrow$

$\underline{K_{nD}/K_{nL} = 2.78}$

b. For $V_I = 5$, driver in nonsaturated region.

$I_{DD} = I_{DL}$

$K_{nD}\left[2(V_I - V_{TND})V_O - V_O^2\right] = K_{nL}(V_{GSL} - V_{TNL})^2$

$\frac{K_{nD}}{K_{nL}}\left[2(V_I - V_{TND})V_O - V_O^2\right] = \left[V_{DD} - V_O - V_{TNL}\right]^2$

$2.78[2(5 - 1)V_0 - V_0^2] = [5 - V_0 - 1]^2$

$22.24V_0 - 2.78V_0^2 = (4 - V_0)^2$

$= 16 - 8V_0 + V_0^2$

$3.78V_0^2 - 30.24V_0 + 16 = 0$

$$V_0 = \frac{30.24 \pm \sqrt{(30.24)^2 - 4(3.78)(16)}}{2(3.78)}$$

$\Rightarrow \underline{V_0 = 0.57 \text{ V}}$

E5.16

If the transistor is biased in the saturation region

$I_D = K_n(V_{GS} - V_{TN})^2 = K_n(-V_{TN})^2$

$I_D = (0.25)(2.5)^2 \Rightarrow \underline{I_D = 1.56 \text{ mA}}$

$V_{DS} = V_{DD} - I_D R_S = 10 - (1.56)(4)$

$\Rightarrow \underline{V_{DS} = 3.76}$

$V_{DS} > V_{GS} - V_{TN} = -V_{TN}$

$3.76 > -(-2.5)$

Yes — biased in the saturation region

Power $= I_D V_{DS} = (1.56)(3.76)$

$\Rightarrow$ Power = 5.87 mW

E5.17

We have $V_{DS} = 1.2\,V < V_{GS} - V_{TN} = -V_{TN} = 1.8\,V$

Transistor is biased in the nonsaturation region.

$I_D = K_n\left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2\right]$

and

$I_D = \frac{V_{DD} - V_{DS}}{R_S} = \frac{5 - 1.2}{8} \Rightarrow$

$I_D = 0.475\,mA$

$0.475 = K_n\left[2(0 - (-1.8))(1.2) - (1.2)^2\right]$

$0.475 = K_n(2.88) \Rightarrow \quad K_n = 0.165\,mA/V^2$

$K_n = \frac{W}{L} \cdot \frac{\mu_n C_{ox}}{2}$

$\frac{W}{L} = \frac{(165)(2)}{35} \Rightarrow \underline{\frac{W}{L} = 9.43}$

E5.18

(a) Transition point for the load transistor – Driver is in the saturation region.

$I_{DD} = I_{DL}$

$K_{nD}(V_{GSD} - V_{TND})^2 = K_{nL}(V_{GSL} - V_{TNL})^2$

$V_{DSL}(sat) = V_{GSL} - V_{TNL} = -V_{TNL}$

$\Rightarrow V_{DSL} = V_{DD} - V_{Ot} = 2\,V$

Then $V_{Ot} = 5 - 2 = 3\,V$, $\underline{V_{Ot} = 3\,V}$

$$\sqrt{\frac{K_{nD}}{K_{nL}}}(V_{It} - 1) = (-V_{TNL})$$

$$\sqrt{\frac{0.08}{0.01}}(V_{It} - 1) = 2 \Rightarrow \underline{V_{It} = 1.89\,V}$$

(b) For the driver:

$V_{Ot} = V_{It} - V_{TND}$

$\underline{V_{It} = 1.89\,V,\ V_{Ot} = 0.89\,V}$

E5.19

(a) For $V_I = 5\,V$, Load in saturation and driver in nonsaturation.

$I_{DD} = I_{DL}$

$K_{nD}\left[2(V_I - V_{TND})V_O - V_O^2\right] = K_{nL}(-V_{TNL})^2$

$$\frac{K_{nD}}{K_{nL}}\left[2(5-1)(0.25) - (0.25)^2\right] = 4 \Rightarrow$$

$$\frac{K_{nD}}{K_{nL}} = 2.06$$

(b) $I_{DL} = K_{nL}(-V_{TNL})^2 \Rightarrow 0.2 = K_{nL}[-(-2)]^2$

$\underline{K_{nL} = 50\,\mu A/V^2}$ and $\underline{K_{nD} = 103\,\mu A/V^2}$

E5.20

(a) $I_{REF} = K_{n3}(V_{GS3} - V_{TN})^2 = K_{n4}(V_{GS4} - V_{TN})^2$

$V_{GS3} = 2\,V \Rightarrow V_{GS4} = 3\,V$

$$(2-1)^2 = \frac{K_{n4}}{K_{n3}}(3-1)^2 \Rightarrow \underline{\frac{K_{n4}}{K_{n3}} = \frac{1}{4}}$$

(b) $I_Q = K_{n2}(V_{GS2} - V_{TN})^2$

But $V_{GS2} = V_{GS3} = 2\,V$

$0.1 = K_{n2}(2-1)^2 \Rightarrow \underline{K_{n2} = 0.1\,mA/V^2}$

(c) $0.2 = K_{n3}(2-1)^2 \Rightarrow \underline{K_{n3} = 0.2\,mA/V^2}$

$0.2 = K_{n4}(3-1)^2 \Rightarrow \underline{K_{n4} = 0.05\,mA/V^2}$

E5.21

For $R_D = 10\,k\Omega, V_{DD} = 5\,V$, and $V_o = 1\,V$

$$I_D = \frac{5-1}{10} = 0.4\,mA$$

$I_D = K_n\left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2\right]$

$I_D = 0.4 = K_n\left[2(5-1)(1) - (1)^2\right] \Rightarrow$

$K_n = 0.057\,mA/V^2$

$P = I_D \cdot V_{DS} = (0.4)(1) \Rightarrow \underline{P = 0.4\,mW}$

E5.22

$I_D = K_n\left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2\right]$

$= (0.050)[2(10 - 0.7)(0.35) - (0.35)^2]$

$I_D = 0.319$ mA

$$R_D = \frac{V_{DD} - V_O}{I_D} = \frac{10 - 0.35}{0.319}$$

$\Rightarrow \underline{R_D = 30.3\ k\Omega}$

E5.23

(a) Transistor biased in the nonsaturation region

$$I_D = \frac{5 - 1.5 - V_{DS}}{R} = 12$$

$I_D = K_n\left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2\right]$

$12 = 4\left[2(5-0.8)V_{DS} - V_{DS}^2\right]$

$4V_{DS}^2 - 33.6V_{DS} + 12 = 0 \Rightarrow \underline{V_{DS} = 0.374\,V}$

Then

$$R = \frac{5 - 1.5 - 0.374}{12} \Rightarrow \underline{R = 261\,\Omega}$$

E5.24

a. $V_1 = 5$ V, $V_2 = 0$, M_2 cutoff $\Rightarrow \underline{I_{D2} = 0}$

$$I_D = K_n\left[2(V_I - V_{TN})V_O - V_O^2\right] = \frac{5 - V_O}{R_D}$$

$(0.05)(30)[2(5-1)V_O - V_O^2] = 5 - V_O$

$1.5V_O^2 - 13V_O + 5 = 0$

$$V_O = \frac{13 \pm \sqrt{(13)^2 - 4(1.5)(5)}}{2(1.5)} \Rightarrow \underline{V_O = 0.40\ V}$$

$$I_R = I_{D1} = \frac{5 - 0.40}{30} \Rightarrow \underline{I_R = I_{D1} = 0.153\ \text{mA}}$$

b. $V_1 = V_2 = 5$ V

$$\frac{5 - V_O}{R_D} = 2\left\{K_n\left[2(V_I - V_{TN})V_O - V_O^2\right]\right\}$$

$5 - V_O = 2(0.05)(30)[2(5-1)V_O - V_O^2]$

$3V_O^2 - 25V_O + 5 = 0$

$$V_0 = \frac{25 \pm \sqrt{(25)^2 - 4(3)(5)}}{2(3)} \Rightarrow \underline{V_0 = 0.205 \text{ V}}$$

$$I_R = \frac{5 - 0.205}{30} \Rightarrow \underline{I_R = 0.160 \text{ mA}}$$

$$\underline{I_{D1} = I_{D2} = 0.080 \text{ mA}}$$

E5.25

(a) $$I_D = \frac{5 - V_O}{R_D} = K_n\left[2(V_2 - V_{TN})V_O - V_O^2\right]$$

$$\frac{5-(0.10)}{25} = K_n\left[2(5-1)(0.10) - (0.10)^2\right] \Rightarrow$$

$$\underline{K_n = 0.248 \; mA/V^2}$$

b. $$\frac{5 - V_0}{25} = 2(0.248)\left[2(5-1)V_0 - V_0^2\right]$$

$$5 - V_0 = 12.4\left[8V_0 - V_0^2\right]$$

$$12.4V_0^2 - 100.2V_0 + 5 = 0$$

$$V_0 = \frac{100.2 \pm \sqrt{(100.2)^2 - 4(12.4)(5)}}{2(12.4)}$$

$$\Rightarrow \underline{V_0 = 0.0502 \text{ V}}$$

E5.26

$$V_{DS}(\text{sat}) = V_{GS} - V_P = -1.2 - (-4.5)$$

$$\Rightarrow \underline{V_{DS}(\text{sat}) = 3.3 \text{ V}}$$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 12\left(1 - \frac{(-1.2)}{(-4.5)}\right)^2$$

$$\Rightarrow \underline{I_D = 6.45 \text{ mA}}$$

E5.27

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$1.2 = 2\left(1 - \frac{V_{GS}}{(-2.5)}\right)^2 \Rightarrow \underline{V_{GS} = -0.564 \text{ V}}$$

E5.28

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$3 = I_{DSS}\left(1 - \frac{0.8}{3.8}\right)^2 \Rightarrow \underline{I_{DSS} = 4.81 \text{ mA}}$$

$$V_{SD}(\text{sat}) = V_P - V_{GS} = 3.8 - 0.8$$

$$\underline{V_{SD}(\text{sat}) = 3.0 \text{ V}}$$

E5.29

$$I_D = K(V_{GS} - V_{TN})^2$$

a. $V_{GS} = 0.35 \Rightarrow I_D = 25(0.35 - 0.25)^2$
$\Rightarrow \underline{I_D = 0.25 \; \mu\text{A}}$

b. $V_{GS} = 0.50 \Rightarrow I_D = 25(0.50 - 0.25)^2$
$\Rightarrow \underline{I_D = 1.56 \; \mu\text{A}}$

E5.30

Assume the transistor is biased in the saturation region.

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$8 = 18\left(1 - \frac{V_{GS}}{(-3.5)}\right)^2 \Rightarrow \underline{V_{GS} = -1.17 \text{ V}}$$

$$\Rightarrow V_S = -V_{GS} = 1.17$$

$$V_D = 15 - (8)(0.8) = 8.6$$

$$V_{DS} = 8.6 - (1.17) = 7.43 \text{ V}$$

$$V_{DS} = 7.43 > V_{GS} - V_P = -1.17 - (-3.5) = 2.33$$

Yes, the transistor is biased in the saturation region.

E5.31

$$I_D = 2.5 \text{ mA}$$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2.5 = 6\left(1 - \frac{V_{GS}}{(-4)}\right)^2 \Rightarrow V_{GS} = -1.42 \text{ V}$$

$$V_S = I_D R_S - 5 = (2.5)(0.25) - 5$$

$$V_S = -4.375$$

$$V_{DS} = 6 \Rightarrow V_D = 6 - 4.375 = 1.625$$

$$R_D = \frac{5 - 1625}{2.5} \Rightarrow \underline{R_D = 1.35 \text{ k}\Omega}$$

$$\frac{(20)^2}{R_1 + R_2} = 2 \Rightarrow R_1 + R_2 = 200 \text{ k}\Omega$$

$$V_G = V_{GS} + V_S = -1.42 - 4.375 = -5.795$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(20) - 10$$

$$-5.795 = \left(\frac{R_2}{200}\right)(20) - 10 \Rightarrow$$

$$\underline{R_2 = 42.05 \text{ k}\Omega} - \underline{42 \text{ k}\Omega}$$

$$\underline{R_1 = 157.95 \text{ k}\Omega} - \underline{158 \text{ k}\Omega}$$

E5.32

$$V_S = -V_{GS},\quad I_D = \frac{0 - V_S}{R_S} = \frac{V_{GS}}{R_S}$$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$\frac{V_{GS}}{1} = 6\left(1 - \frac{V_{GS}}{4}\right)^2 = 6\left(1 - \frac{V_{GS}}{2} + \frac{V_{GS}^2}{16}\right)$$

$$0.375V_{GS}^2 - 4V_{GS} + 6 = 0$$

$$V_{GS} = \frac{4 \pm \sqrt{16 - 4(0.375)(6)}}{2(0.375)}$$

$\underbrace{V_{GS} = 8.86}_{\text{impossible}}$ or $\underline{V_{GS} = 1.81 \text{ V}}$

$$\underline{I_D = \frac{V_{GS}}{R_S} = 1.81 \text{ mA}}$$

$$V_D = I_D R_D - 5 = (1.81)(0.4) - 5 = -4.276$$

$$V_{SD} = V_S - V_0 = -1.81 - (-4.276)$$

$$\Rightarrow \underline{V_{SD} = 2.47 \text{ V}}$$

$$V_{SD}(\text{sat}) = V_P - V_{GS} = 4 - 1.81 = 2.19$$

So $V_{SD} > V_{SD}(\text{sat})$

E5.33

$$R_{in} = R_1 \| R_2 = \frac{R_1 R_2}{R_1 + R_2} = 100 \text{ k}\Omega$$

$$I_{DQ} = 5 \text{ mA},\quad V_S = -I_{DQ}R_S = -(5)(1.2) = -6 \text{ V}$$

$$V_{SDQ} = 12 \text{ V} \Rightarrow V_D = V_S - V_{SDQ}$$

$$= -6 - 12 = -18 \text{ V}$$

$$R_D = \frac{-18 - (-20)}{5} \Rightarrow \underline{R_D = 0.4 \text{ k}\Omega}$$

$$I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 \Rightarrow 5 = 8\left(1 - \frac{V_{GS}}{4}\right)^2$$

$$V_{GS} = 0.838 \text{ V}$$

$$V_G = V_{GS} + V_S = 0.838 - 6 = -5.162$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(-20)$$

$$-5.162 = \frac{1}{R_1}(100)(-20) \Rightarrow \underline{R_1 = 387 \text{ k}\Omega}$$

$$\frac{R_1 R_2}{R_1 + R_2} = 100 \Rightarrow (387)R_2 = 100(387) + 100R_2$$

$$(387 - 100)R_2 = (100)(387)$$

$$\Rightarrow \underline{R_2 = 135 \text{ k}\Omega}$$

E5.34

$$I_{DQ} = K(V_{GS} - V_{TN})^2 \Rightarrow 5 = 50(V_{GS} - 0.15)^2$$

$$\Rightarrow \underline{V_{GS} = 0.466 \text{ V}}$$

$$V_S = (0.005)(10) = 0.050 \text{ V}$$

$$\Rightarrow V_{GG} = V_{GS} + V_S = 0.466 + 0.050$$

$$\Rightarrow \underline{V_{GG} = 0.516 \text{ V}}$$

$$V_D = 5 - (0.005)(100) \Rightarrow V_D = 4.5 \text{ V}$$

$$V_{DS} = V_D - V_S = 4.5 - 0.050$$

$$\Rightarrow \underline{V_{DS} = 4.45 \text{ V}}$$

E5.35

$$I_D = K\left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2\right]$$

$$= 100\left[2(0.7 - 0.2)(0.1) - (0.1)^2\right]$$

$$I_D = 9 \ \mu\text{A}$$

$$R_D = \frac{2.5 - 0.1}{0.009} \Rightarrow \underline{R_D = 267 \text{ k}\Omega}$$

Chapter 5

Problem Solutions

5.1

(a) $V_{DS} = 6V > V_{GS} - V_{TN} = 5 - 1.5 = 3.5V$
Biased in the saturation region
$I_D = K_n(V_{GS} - V_{TN})^2 = (0.25)(5-1.5)^2 \Rightarrow$
$\underline{I_D = 3.06\, mA}$

(b) $V_{DS} = 2.5V < V_{DS}(sat) = 3.5V$
Biased in nonsaturation region
$I_D = K_n[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2]$
$I_D = (0.25)[2(5-1.5)(2.5) - (2.5)^2] \Rightarrow$
$\underline{I_D = 2.81\, mA}$

5.2

(a) $I_D = \frac{k_n'}{2} \cdot \frac{W}{L}(V_{GS} - V_{TN})^2$
$0.5 = \left(\frac{0.080}{2}\right)(5)(V_{GS} - 0.8)^2$
$\sqrt{\frac{0.5}{0.2}} + 0.8 = V_{GS} \Rightarrow \underline{V_{GS} = 2.38\, V}$
$V_{DS}(sat) = V_{GS} - V_{TN} = 2.38 - 0.8 \Rightarrow$
$\underline{V_{DS}(sat) = 1.58\, V}$

(b) $1.5 = \left(\frac{0.080}{2}\right)(5)(V_{GS} - 0.8)^2$
$\sqrt{\frac{1.5}{0.2}} + 0.8 = V_{GS} \Rightarrow \underline{V_{GS} = 3.54\, V}$
$V_{DS}(sat) = 3.54 - 0.8 \Rightarrow \underline{V_{DS}(sat) = 2.74\, V}$

5.3

a. $V_{GS} = 0$
$V_{DS}(sat) = V_{GS} - V_{TN} = 0 - (-2.5) = 2.5\, V$

i. $V_{DS} = 0.5$ V $\Rightarrow$ Biased in nonsaturation
$I_D = (1.1)[2(0 - (-2.5))(0.5) - (0.5)^2]$
$\Rightarrow \underline{I_D = 2.48 \text{ mA}}$

ii. $V_{DS} = 2.5$ V $\Rightarrow$ Biased in saturation
$I_D = (1.1)(0 - (-2.5))^2$
$\Rightarrow \underline{I_D = 6.88 \text{ mA}}$

iii. $V_{DS} = 5$ V Same as (ii) $\Rightarrow \underline{I_D = 6.88 \text{ mA}}$

b. $V_{GS} = 2$ V

$V_{DS}(\text{sat}) = 2 - (-2.5) = 4.5$ V

i. $V_{DS} = 0.5$ V $\Rightarrow$ Nonsaturation
$I_D = (1.1)[2(2 - (-2.5))(0.5) - (0.5)^2]$
$\Rightarrow \underline{I_D = 4.68 \text{ mA}}$

ii. $V_{DS} = 2.5$ V $\Rightarrow$ Nonsaturation
$I_D = (1.1)[2(2 - (-2.5))(2.5) - (2.5)^2]$
$\Rightarrow \underline{I_D = 17.9 \text{ mA}}$

iii. $V_{DS} = 5$ V $\Rightarrow$ Saturation
$I_D = (1.1)(2 - (-2.5))^2$
$\Rightarrow \underline{I_D = 22.3 \text{ mA}}$

5.4

$V_{DS} > V_{GS} - V_{TN} = 0 - (-2) = 2\, V$
Biased in the saturation region
$I_D = \frac{k_n'}{2} \cdot \frac{W}{L}(V_{GS} - V_{TN})^2$
$1.5 = \left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)[0 - (-2)]^2 \Rightarrow \underline{\frac{W}{L} = 9.375}$

5.5

a. $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{450 \times 10^{-8}}$
$\Rightarrow \frac{\epsilon_{ox}}{t_{ox}} = 7.67 \times 10^{-8}$ F/cm²
$K_n = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L}$
$= \frac{1}{2}(650)(7.67x10^{-8})\left(\frac{64}{4}\right)$
$\underline{K_n = 0.399\, mA/V^2}$

b. $V_{GS} = V_{DS} = 3$ V $\Rightarrow$ Saturation
$I_D = K_n(V_{GS} - V_{TN})^2 = (0.399)(3 - 0.8)^2$
$\Rightarrow \underline{I_D = 1.93\, mA}$

5.6

a. $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{600 \times 10^{-8}}$
$\Rightarrow \frac{\epsilon_{ox}}{t_{ox}} = 5.75 \times 10^{-8}$ F/cm²
$K_n = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L}$
$= \frac{1}{2}(500)(5.75x10^{-8})\left(\frac{100}{5}\right) \Rightarrow$
$\underline{K_n = 0.288\, mA/V^2}$

b. i. $V_{GS} = 0$, $V_{DS} = 5$ V

$V_{DS}(\text{sat}) = 0 - (-2) = 2$ V

Biased in saturation

$I_D = (0.288)(0 - (-2))^2 \Rightarrow \underline{I_D = 1.15 \text{ mA}}$

ii. $V_{GS} = 2$ V, $V_{DS} = 1$ V

$V_{DS}(\text{sat}) = 2 - (-2) = 4$ V

Nonsaturation

$I_D = (0.288)[2(2 - (-2))(1) - (1)^2]$

$\Rightarrow \underline{I_D = 2.02 \text{ mA}}$

5.7

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{400 \times 10^{-8}}$$

$$= 8.63 \times 10^{-8} \text{ F/cm}^2$$

$$K_n = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L}$$

$$= \frac{1}{2}(600)(8.63 x 10^{-8})\left(\frac{W}{2.5}\right)$$

$$\underline{K_n = (1.04 x 10^{-5}) W}$$

$$I_D = K_n (V_{GS} - V_{TN})^2$$

$1.2 \times 10^{-3} = (1.04 \times 10^{-5}) W (5 - 1)^2$

$\Rightarrow \underline{W = 7.21 \ \mu\text{m}}$

5.8

Biased in the saturation region in both cases.

$$I_D = \frac{k_p'}{2} \cdot \frac{W}{L} (V_{SG} + V_{TP})^2$$

(1) $0.225 = \left(\frac{0.040}{2}\right)\left(\frac{W}{L}\right)(3 + V_{TP})^2$

(2) $1.40 = \left(\frac{0.040}{2}\right)\left(\frac{W}{L}\right)(4 + V_{TP})^2$

Take ratio of (2) to (1):

$$\frac{1.40}{0.225} = 6.222 = \frac{(4 + V_{TP})^2}{(3 + V_{TP})^2}$$

$$\sqrt{6.222} = 2.49 = \frac{4 + V_{TP}}{3 + V_{TP}} \Rightarrow \underline{V_{TP} = -2.33 \, V}$$

Then

$$0.225 = \left(\frac{0.040}{2}\right)\left(\frac{W}{L}\right)(3 - 2.33)^2 \Rightarrow \underline{\frac{W}{L} = 25.1}$$

5.9

$V_S = 5$ V, $V_G = 0 \Rightarrow V_{SG} = 5$ V

$V_{TP} = -0.5 \, V$

$\Rightarrow V_{SD}(sat) = V_{SG} + V_{TP} = 5 - 0.5 = 4.5 \, V$

a. $V_D = 0 \Rightarrow V_{SD} = 5$ V

$\Rightarrow$ Biased in saturation

$I_D = 2(5 - 0.5)^2 \Rightarrow \underline{I_D = 40.5 \text{ mA}}$

b. $V_D = 2$ V $\Rightarrow V_{SD} = 3$ V

$\Rightarrow$ Nonsaturation

$I_D = 2[2(5 - 0.5)(3) - (3)^2]$

$\Rightarrow \underline{I_D = 36 \text{ mA}}$

c. $V_D = 4$ V $\Rightarrow V_{SD} = 1$ V

$\Rightarrow$ Nonsaturation

$I_D = 2[2(5 - 0.5)(1) - (1)^2]$

$\Rightarrow \underline{I_D = 16 \text{ mA}}$

d. $V_D = 5$ V $\Rightarrow V_{SD} = 0 \Rightarrow \underline{I_D = 0}$

5.10

$V_{SD}(sat) = V_{SG} + V_{TP}$

(a) $V_{SD}(sat) = -1 + 2 \Rightarrow V_{SD}(sat) = 1 \, V$

(b) $V_{SD}(sat) = 0 + 2 \Rightarrow V_{SD}(sat) = 2 \, V$

(c) $V_{SD}(sat) = 1 + 2 \Rightarrow V_{SD}(sat) = 3 \, V$

$$I_D = \frac{k_p'}{2} \cdot \frac{W}{L} (V_{SG} + V_{TP})^2 = \frac{k_p'}{2} \cdot \frac{W}{L} \cdot [V_{SD}(sat)]^2$$

(a) $I_D = \left(\frac{0.040}{2}\right)(6)(1)^2 \Rightarrow \underline{I_D = 0.12 \, mA}$

(b) $I_D = \left(\frac{0.040}{2}\right)(6)(2)^2 \Rightarrow \underline{I_D = 0.48 \, mA}$

(c) $I_D = \left(\frac{0.040}{2}\right)(6)(3)^2 \Rightarrow \underline{I_D = 1.08 \, mA}$

5.11

$V_{SD}(sat) = V_{SG} + V_{TP} = 0 + 2 = 2 \, V$

(a) $V_{SD} = 1 \, V$, Nonsaturation

$I_D = K_p[2(V_{SG} + V_{TP})V_{SD} - V_{SD}^2]$

$I_D = (0.5)[2(0 + 2)(1) - (1)^2] \Rightarrow \underline{I_D = 1.5 \, mA}$

(b) $V_{SD} = 2 \, V$, Saturation

$I_D = K_p (V_{SG} + V_{TP})^2 = (0.5)(0 + 2)^2 \Rightarrow$

$\underline{I_D = 2 \, mA}$

(c) $V_{SD} = 3 \, V$, Saturation

Same as (b), $\underline{I_D = 2 \, mA}$

5.12

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{(3.9)(8.85 \times 10^{-14})}{500 \times 10^{-8}}$$
$$= 6.90 \times 10^{-8} \text{ F/cm}^2$$

$$k_n' = (\mu_n C_{ox}) = (675)(6.90 \times 10^{-8})$$
$$\Rightarrow 46.6\ \mu\text{A/V}^2$$
$$k_p' = (\mu_p C_{ox}) = (375)(6.90 \times 10^{-8})$$
$$\Rightarrow 25.9\ \mu\text{A/V}^2$$

PMOS:

$$I_D = \frac{k_p'}{2}\left(\frac{W}{L}\right)_p (V_{SG} + V_{TP})^2$$
$$0.8 = \left(\frac{0.0259}{2}\right)\left(\frac{W}{L}\right)_p (5 - 0.6)^2$$
$$\Rightarrow \left(\frac{W}{L}\right)_p = 3.19$$
$$L = 4\ \mu\text{m} \Rightarrow \underline{W_p = 12.8\ \mu\text{m}}$$

$$K_p = \left(\frac{0.0259}{2}\right)(3.19) \Rightarrow \underline{K_p = 41.3\ \mu A/V^2 = K_n}$$

Want $K_n = K_p$

$$\frac{k_n'}{2}\left(\frac{W}{L}\right)_N = \frac{k_p'}{2}\left(\frac{W}{L}\right)_P = 41.3$$
$$\left(\frac{46.6}{2}\right)\left(\frac{W}{L}\right)_N = 41.3 \Rightarrow \left(\frac{W}{L}\right)_N = 1.77$$
$$L = 4\ \mu\text{m} \Rightarrow \underline{W_N = 7.08\ \mu\text{m}}$$

5.13

$V_{GS} = 2$ V, $I_D = (0.2)(2 - 1.2)^2 = 0.128$ mA

$$r_0 = \frac{1}{\lambda I_D} = \frac{1}{(0.01)(0.128)} \Rightarrow \underline{r_0 = 781\ \text{k}\Omega}$$

$V_{GS} = 4$ V, $I_D = (0.2)(4 - 1.2)^2 = 1.57$ mA

$$r_0 = \frac{1}{(0.01)(1.57)} \Rightarrow \underline{r_0 = 63.7\ \text{k}\Omega}$$
$$V_A = \frac{1}{\lambda} = \frac{1}{(0.01)} \Rightarrow \underline{V_A = 100\ \text{V}}$$

5.14

$$I_D = \left(\frac{0.080}{2}\right)(4)(3 - 0.8)^2 = (0.16)(3 - 0.8)^2 \Rightarrow$$
$$I_D = 0.774\ mA$$
$$r_o = \frac{1}{\lambda I_D} \Rightarrow \lambda = \frac{1}{r_o I_D} = \frac{1}{(200)(0.774)} \Rightarrow$$
$$\underline{\lambda(\max) = 0.00646\ V^{-1}}$$
$$V_A(\min) = \frac{1}{\lambda(\max)} = \frac{1}{0.00646} \Rightarrow \underline{V_A(\min) = 155\ V}$$

5.15

$$V_{TN} = V_{TNO} + \gamma\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right]$$
$$\Delta V_{TN} = 2 = (0.8)\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2(0.35)}\right]$$
$$2.5 + 0.837 = \sqrt{2(0.35) + V_{SB}} \Rightarrow \underline{V_{SB} = 10.4\ V}$$

5.16

$$\Delta V_{TN} = \gamma\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right]$$
$$1.2 = \gamma\left[\sqrt{2(0.37) + 10} - \sqrt{2(0.37)}\right] = \gamma(2.42)$$

Then $\gamma = 0.496\ V^{1/2}$

5.17

a. $V_G = \epsilon_{ox} t_{ox} = (6 \times 10^6)(275 \times 10^{-8})$

$\underline{V_G = 16.5\ \text{V}}$

b. $V_G = \frac{16.5}{3} \Rightarrow \underline{V_G = 5.5\ \text{V}}$

5.18

Want $V_G = (3)(24) = \epsilon_{ox} t_{ox} = (6 \times 10^6) t_{ox}$

$\underline{t_{ox} = 1.2 \times 10^{-5}\ \text{cm} = 1200\ \text{Angstroms}}$

5.19

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \left(\frac{18}{18 + 32}\right)(10) = 3.6\ \text{V}$$

Assume transistor biased in saturation region

$$I_D = \frac{V_S}{R_S} = \frac{V_G - V_{GS}}{R_S} = K_n (V_{GS} - V_{TN})^2$$
$$3.6 - V_{GS} = (0.5)(2)(V_{GS} - 0.8)^2$$
$$= V_{GS}^2 - 1.6 V_{GS} + 0.64$$

$$V_{GS}^2 - 0.6 V_{GS} - 2.96 = 0$$
$$V_{GS} = \frac{0.6 \pm \sqrt{(0.6)^2 + 4(2.96)}}{2}$$
$$\Rightarrow \underline{V_{GS} = 2.05\ \text{V}}$$

$$I_D = \frac{V_G - V_{GS}}{R_S} = \frac{3.6 - 2.05}{2} \Rightarrow \underline{I_D = 0.775\ \text{mA}}$$
$$V_{DS} = V_{DD} - I_D(R_D + R_S)$$
$$= 10 - (0.775)(4 + 2)$$
$$\Rightarrow \underline{V_{DS} = 5.35\ \text{V}}$$
$$\underline{V_{DS} > V_{DS}(\text{sat})}$$

5.20

$$V_G = \left(\frac{R_2}{R_1+R_2}\right)(20) - 10 = \left(\frac{6}{14+6}\right)(20) - 10 \Rightarrow$$

$V_G = -4\ V$

Assume transistor is biased in saturation region

$$I_D = \frac{V_S - (-10)}{R_S} = \frac{V_G - V_{GS} + 10}{R_S} = K_n(V_{GS} - V_{TN})^2$$

$$K_n = \left(\frac{0.060}{2}\right)(60) \Rightarrow 1.8\ mA/V^2$$

$$-4 - V_{GS} + 10 = (1.8)(0.5)(V_{GS} - 2)^2$$
$$= 0.9V_{GS}^2 - 3.6V_{GS} + 3.6$$

Then $0.9V_{GS}^2 - 2.6V_{GS} - 2.4 = 0$

$$V_{GS} = \frac{2.6 \pm \sqrt{(2.6)^2 + 4(0.9)(2.4)}}{2(0.9)} \Rightarrow \underline{V_{GS} = 3.62\ V}$$

$$I_D = \frac{V_G - V_{GS} + 10}{R_S} = \frac{-4 - 3.62 + 10}{0.5} \Rightarrow$$

$\underline{I_D = 4.76\ mA}$

$V_{DS} = 20 - I_D(R_D + R_S) = 20 - (4.76)(1.2 + 0.5) \Rightarrow$

$\underline{V_{DS} = 11.9\ V}$

$\underline{V_{DS} = 11.9\ V > V_{GS} - V_{TN} = 3.62 - 2 = 1.62\ V}$

5.21

$$I_D = \frac{10 - V_S}{R_S} = K_p(V_{SG} + V_{TP})^2$$

Assume transistor biased in saturation region

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(20) - 10$$
$$= \left(\frac{22}{8 + 22}\right)(20) - 10$$

$\Rightarrow V_G = 4.67\ \text{V}$

$V_S = V_G + V_{SG}$

$10 - (4.67 + V_{SG}) = (1)(0.5)(V_{SG} - 2)^2$

$5.33 - V_{SG} = 0.5(V_{SG}^2 - 4V_{SG} + 4)$

$0.5V_{SG}^2 - V_{SG} - 3.33 = 0$

$$V_{SG} = \frac{1 \pm \sqrt{(1)^2 + 4(0.5)(3.33)}}{2(0.5)}$$

$\Rightarrow \underline{V_{SG} = 3.77\ \text{V}}$

$$I_D = \frac{10 - (4.67 + 3.77)}{0.5} \Rightarrow \underline{I_D = 3.12\ \text{mA}}$$

$V_{SD} = 20 - I_D(R_S + R_D)$

$= 20 - (3.12)(0.5 + 2)$

$\Rightarrow \underline{V_{SD} = 12.2\ \text{V}}$

$\underline{V_{SD} > V_{SD}(\text{sat})}$

5.22

$V_G = 0.\ \ V_{SG} = V_S$

Assume saturation region

$I_D = 0.4 = K_p(V_{SG} + V_{TP})^2$

$0.4 = (0.2)(V_S - 0.8)^2$

$$V_S = \sqrt{\frac{0.4}{0.2}} + 0.8 \Rightarrow \underline{V_S = 2.21\ \text{V}}$$

$V_D = I_D R_D - 5 = (0.4)(5) - 5 = -3\ \text{V}$

$V_{SD} = V_S - V_D = 2.21 - (-3)$

$\Rightarrow \underline{V_{SD} = 5.21\ \text{V}}$

$\underline{V_{SD} > V_{SD}(\text{sat})}$

5.23

$V_{DD} = I_{DQ}R_D + V_{DSQ} + I_{DQ}R_S$

(1) $10 = I_{DQ}(5) + 5 + V_{GS}$ and

$$I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$$

or (2) $I_{DQ} = \left(\frac{0.060}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - 1.2)^2$

Let $\underline{V_{GS} = 2.5\ V}$

Then from (1), $10 = I_{DQ}(5) + 5 + 2.5 \Rightarrow I_D = 0.5\ mA$

Then from (2), $0.5 = \left(\frac{0.060}{2}\right)\left(\frac{W}{L}\right)(2.5 - 1.2)^2 \Rightarrow$

$\underline{\frac{W}{L} = 9.86}$

$I_{DQ}R_S = V_{GS} \Rightarrow R_S = \frac{V_{GS}}{I_{DQ}} = \frac{2.5}{0.5} \Rightarrow \underline{R_S = 5\ k\Omega}$

$$I_R = \frac{10}{R_1 + R_2} = (0.5)(0.05) = 0.025\ mA$$

Then $R_1 + R_2 = \frac{10}{0.025} = 400\ k\Omega$

$$\left(\frac{R_2}{R_1 + R_2}\right)(V_{DD}) = 2V_{GS} \Rightarrow \left(\frac{R_2}{400}\right)(10) = 2(2.5) \Rightarrow$$

$\underline{R_1 = R_2 = 200\ k\Omega}$

5.24

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5$$
$$= \left(\frac{5.5}{14.5 + 5.5}\right)(10) - 5 = -2.25\ \text{V}$$

$$I_D = \frac{V_G - V_{GS} - (-5)}{R_S} = K_n(V_{GS} - V_{TN})^2$$

Assume transistor biased in saturation region

$-2.25 - V_{GS} + 5 = (0.5)(0.6)(V_{GS} - (-1))^2$

$2.75 - V_{GS} = (0.3)(V_{GS}^2 + 2V_{GS} + 1)$

$0.3V_{GS}^2 + 1.6V_{GS} - 2.45 = 0$

$V_{GS} = \dfrac{-1.6 \pm \sqrt{(1.6)^2 + 4(0.3)(2.45)}}{2(0.3)}$

$\Rightarrow \underline{V_{GS} = 1.24\text{ V}}$

$I_D = \dfrac{-2.25 - 1.24 + 5}{0.6} \Rightarrow \underline{I_D = 2.52\text{ mA}}$

$V_{DS} = 10 - I_D(R_S + R_D)$

$= 10 - (2.52)(0.6 + 0.8)$

$\Rightarrow \underline{V_{DS} = 6.47\text{ V}}$

$\underline{V_{DS} > V_{DS}(\text{sat})}$

5.25

$20 = I_{DQ}R_S + V_{SDQ} + I_{DQ}R_D$

(1) $20 = V_{SG} + 10 + I_{DQ}R_D$

$I_{DQ} = \left(\dfrac{k_p'}{2}\right)\left(\dfrac{W}{L}\right)(V_{SG} + V_{TP})^2$

(2) $I_{DQ} = \left(\dfrac{0.040}{2}\right)\left(\dfrac{W}{L}\right)(V_{SG} - 2)^2$

For example, let $\underline{I_{DQ} = 0.8\ mA}$ and $\underline{V_{SG} = 4\ V}$

Then $0.8 = \left(\dfrac{0.040}{2}\right)\left(\dfrac{W}{L}\right)(4-2)^2 \Rightarrow \underline{\dfrac{W}{L} = 10}$

$I_{DQ}R_S = V_{SG} \Rightarrow (0.8)R_S = 4 \Rightarrow \underline{R_S = 5\ k\Omega}$

From (1) $20 = 4 + 10 + (0.8)R_D \Rightarrow \underline{R_D = 7.5\ k\Omega}$

$I_R = \dfrac{20}{R_1 + R_2} = (0.8)(0.1) \Rightarrow R_1 + R_2 = 250\ k\Omega$

$\left(\dfrac{R_1}{R_1 + R_2}\right)(20) = 2V_{SG} = (2)(4)$

$\dfrac{R_1}{250}(20) = 8 \Rightarrow \underline{R_1 = 100\ k\Omega,\quad R_2 = 150\ k\Omega}$

5.26

(a) (i) $I_Q = 50 = 500(V_{GS} - 1.2)^2 \Rightarrow \underline{V_{GS} = 1.516\ V}$

$V_{DS} = 5 - (-1.516) \Longrightarrow \underline{V_{DS} = 6.516\ V}$

(iv) $I_Q = 1 = (0.5)(V_{GS} - 1.2)^2 \Rightarrow \underline{V_{GS} = 2.61\ V}$

$V_{DS} = 5 - (-2.61) \Rightarrow \underline{V_{DS} = 7.61\ V}$

(b) (i) Same as (a) $\underline{V_{GS} = V_{DS} = 1.516\ V}$

(iv) $\underline{V_{GS} = V_{DS} = 2.61\ V}$

5.27

$I_D = K_n(V_{GS} - V_{TN})^2$

$0.25 = (0.2)(V_{GS} - 0.6)^2$

$V_{GS} = \sqrt{\dfrac{0.25}{0.2}} + 0.6 \Rightarrow V_{GS} = 1.72\text{ V}$

$\Rightarrow \underline{V_S = -1.72\text{ V}}$

$V_D = 9 - (0.25)(24) \Rightarrow \underline{V_D = 3\text{ V}}$

5.28

$I_D = \dfrac{5 - V_D}{R_D} \Rightarrow 0.8 = \dfrac{5-1}{R_D} \Rightarrow \underline{R_D = 5\text{ k}\Omega}$

$V_G = 0$

$I_D = K_n(V_{GS} - V_{TN})^2 \Rightarrow 0.8 = (0.4)(V_{GS} - 1.7)^2$

$V_{GS} = \sqrt{\dfrac{0.8}{0.4}} + 1.7 \Rightarrow V_{GS} = 3.11\text{ V}$

$\Rightarrow V_S = -3.11\text{ V}$

$I_D = 0.8 = \dfrac{-3.11 - (-5)}{R_S} \Rightarrow \underline{R_S = 2.36\text{ k}\Omega}$

5.29

$V_{DD} = V_{SD} + I_{DQ}R$

$9 = 2.5 + (0.1)R \Rightarrow \underline{R = 65\ k\Omega}$

$I_{DQ} = \left(\dfrac{k_p'}{2}\right)\left(\dfrac{W}{L}\right)(V_{SG} + V_{TP})^2$

$(0.1) = \left(\dfrac{0.025}{2}\right)\left(\dfrac{W}{L}\right)(2.5 - 1.5)^2 \Rightarrow \underline{\dfrac{W}{L} = 8}$

Then for $\underline{L = \mu m,\quad W = 32\ \mu m}$

5.30

$5 = I_{DQ}R_S + V_{SDQ} = I_{DQ}(2) + 2.5$

$\underline{I_{DQ} = 1.25\ mA}$

$I_R = \dfrac{10}{R_1 + R_2} = (1.25)(0.1) \Rightarrow R_1 + R_2 = 80\ k\Omega$

$I_{DQ} = K_p(V_{SG} + V_{TP})^2$

$1.25 = 0.5(V_{SG} + 1.5)^2 \Rightarrow \sqrt{\dfrac{1.25}{0.5}} - 1.5 = V_{SG}$

$\underline{V_{SG} = 0.0811\ V}$

$\underline{V_G = V_S - V_{SG} = 2.5 - 0.0811 = 2.42\ V}$

$V_G = \left(\dfrac{R_2}{R_1 + R_2}\right)(10) - 5$

$2.42 = \left(\dfrac{R_2}{80}\right)(10) - 5 \Rightarrow$

$R_2 = 59.4\ k\Omega,\quad R_1 = 20.6\ k\Omega$

5.31

$K_p = \left(\dfrac{0.030}{2}\right)(20) \Rightarrow 0.30\ mA/V^2$

$I_D = K_p(V_{SG} + V_{TP})^2$

$0.5 = 0.30(V_{SG} - 1.2)^2 \Rightarrow \underline{V_{SG} = 2.49\ V}$

$V_S = V_{SG} = 2.49\ V$

$I_D = \dfrac{5 - V_S}{R_S} \Rightarrow R_S = \dfrac{5 - 2.49}{0.5} \Rightarrow \underline{R_S = 5.02\ k\Omega}$

$R_D = \dfrac{V_D - (-5)}{I_D} = \dfrac{5-3}{0.5} \Rightarrow \underline{R_D = 4\ k\Omega}$

5.32

$$I_D = \frac{-V_{SD} - (-10)}{R_D} \Rightarrow 5 = \frac{-6 + 10}{R_D}$$

$$\Rightarrow \underline{R_D = 0.8 \text{ k}\Omega}$$

$$I_D = K_p(V_{SG} + V_{TP})^2 \Rightarrow 5 = 3(V_{SG} - 1.75)^2$$

$$V_{SG} = \sqrt{\frac{5}{3}} + 1.75 = 3.04 \text{ V} \Rightarrow V_G = -3.04$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5 = -3.04$$

$$R_{in} = R_1 \| R_2 = 80 \text{ k}\Omega$$

$$\frac{1}{R_1} \cdot (80)(10) = 5 - 3.04 \Rightarrow \underline{R_1 = 408 \text{ k}\Omega}$$

$$\frac{408 R_2}{408 + R_2} = 80 \Rightarrow \underline{R_2 = 99.5 \text{ k}\Omega}$$

5.33

(a) $K_{n1} = \left(\frac{60}{2}\right)(4) = 120\ \mu A/V^2$

$K_{n2} = \left(\frac{60}{2}\right)(1) = 30\ \mu A/V^2$

For $v_I = 1V$, M_1 Sat. region, M_2 Non-sat region.

$I_{D2} = I_{D1}$

$$30\left[2(-V_{TNL})(5 - v_O) - (5 - v_O)^2\right] = 120(1 - 0.8)^2$$

We find $v_O^2 - 6.4v_O + 7.16 = 0 \Rightarrow \underline{v_O = 4.955\,V}$

(b) For $v_I = 3V$, M_1 Non-sat region, M_2 Sat. region. $I_{D2} = I_{D1}$

$$30[-(-1.8)]^2 = 120[2(3 - 0.8)v_O - v_O^2]$$

We find $4v_O^2 - 17.6v_O + 3.24 = 0 \Rightarrow \underline{v_O = 0.193\,V}$

(c) For $v_I = 5V$, biasing same as (b)

$$30[-(-1.8)]^2 = 120[2(5 - 0.8)v_O - v_O^2]$$

We find $4v_O^2 - 33.6v_O + 3.24 = 0 \Rightarrow \underline{v_O = 0.0976\,V}$

5.34

For $v_I = 5V$, M_1 Non-sat region, M_2 Sat. region.

$I_{D1} = I_{D2}$

$$\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_1\left[2(V_{GS1} - V_{TN1})V_{DS1} - V_{DS1}^2\right] = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_2(V_{GS2} - V_{TN2})^2$$

$$\left(\frac{W}{L}\right)_1\left[2(5 - 0.8)(0.15) - (0.15)^2\right] = (1)[0 - (-2)]^2$$

which yields $\underline{\left(\frac{W}{L}\right)_1 = 3.23}$

5.35

a. M_1 and M_2 in saturation

$$K_{n1}(V_{GS1} - V_{TN1})^2 = K_{n2}(V_{GS2} - V_{TN2})^2$$

$$K_{n1} = K_{n2},\quad V_{TN1} = V_{TN2}$$

$$\Rightarrow \underline{V_{GS1} = V_{GS2} = 2.5 \text{ V}},\quad \underline{V_0 = 2.5 \text{ V}}$$

$$I_D = (15)(40)(2.5 - 0.8)^2 \Rightarrow \underline{I_D = 1.73 \text{ mA}}$$

b. $\left(\frac{W}{L}\right)_1 > \left(\frac{W}{L}\right)_2 \Rightarrow V_{GS1} < V_{GS2}$

$$40(V_{GS1} - 0.8)^2 = (15)(V_{GS2} - 0.8)^2$$

$$V_{GS2} = 5 - V_{GS1}$$

$$1.63(V_{GS1} - 0.8) = (5 - V_{GS1} - 0.8)$$

$$2.63V_{GS1} = 5.50 \Rightarrow \underline{V_{GS1} = 2.09 \text{ V}}$$

$$\underline{V_{GS2} = 2.91 \text{ V}},\quad \underline{V_0 = V_{GS1} = 2.09 \text{ V}}$$

$$I_D = (15)(15)(2.91 - 0.8)^2 \Rightarrow \underline{I_D = 1.0 \text{ mA}}$$

5.36

Each transistor biased in saturation.

M_3 : $V_1 = V_{GS3} = 2$ V

$$I_D = 0.5 = 0.018\left(\frac{W}{L}\right)_3(2 - 1)^2$$

$$\Rightarrow \underline{\left(\frac{W}{L}\right)_3 = 27.8}$$

M_2 : $V_{GS2} = V_2 - V_1 = 5 - 2 = 3$ V

$$I_D = 0.5 = 0.018\left(\frac{W}{L}\right)_2(3 - 1)^2$$

$$\Rightarrow \underline{\left(\frac{W}{L}\right)_2 = 6.94}$$

M_1 : $V_{GS1} = 10 - V_2 = 10 - 5 = 5$ V

$$I_D = 0.5 = 0.018\left(\frac{W}{L}\right)_1(5 - 1)^2$$

$$\Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 1.74}$$

5.37

M_L in saturation

M_D in nonsaturation

$$\left(\frac{W}{L}\right)_L(V_{GSL} - V_{TNL})^2 = \left(\frac{W}{L}\right)_D\left[2(V_{GSD} - V_{TND})V_{DSD} - V_{DSD}^2\right]$$

$$(1)(5 - 0.1 - 0.8)^2 = \left(\frac{W}{L}\right)_D[2(5 - 0.8)(0.1) - (0.1)^2]$$

$$16.81 = \left(\frac{W}{L}\right)_D[0.83]$$

$$\underline{\left(\frac{W}{L}\right)_D = 20.3}$$

5.38

M_L in saturation
M_D in nonsaturation

$$\left(\frac{W}{L}\right)_L (V_{GSL} - V_{TNL})^2 = \left(\frac{W}{L}\right)_D \left[2(V_{GSD} - V_{TND})V_{DSD} - V_{DSD}^2\right]$$

$$(1)(1.8)^2 = \left(\frac{W}{L}\right)_D \left[2(5-0.8)(0.05) - (0.05)^2\right]$$

$$3.24 = \left(\frac{W}{L}\right)_D [0.4175]$$

$$\underline{\left(\frac{W}{L}\right)_D = 7.76}$$

5.39

$$I_{REF} = K_{n4}(V_{GS4} - V_{TN})^2 = K_{n3}(V_{GS3} - V_{TN})^2$$

$$V_{GS4} = 5 - V_{GS3}$$

$$\sqrt{\frac{400}{200}}(5 - V_{GS3} - 1) = (V_{GS3} - 1)$$

$2.41V_{GS3} = 6.66 \Rightarrow \underline{V_{GS3} = 2.76\text{ V}}$

$\underline{V_{GS4} = 2.24\text{ V}}$, $\underline{V_{GS2} = V_{GS3} = 2.76\text{ V}}$

$$I_{REF} = K_{n3}(V_{GS3} - V_{TN})^2 = (0.2)(2.76-1)^2$$

$\underline{I_{REF} = 0.620\ mA}$

$$I_Q = K_{n2}(V_{GS2} - V_{TN})^2 = (0.1)(2.76-1)^2$$

$\underline{I_Q = 0.310\ mA}$

$$I_Q = K_{n1}(V_{GS1} - V_{TN})^2$$

$$\Rightarrow 0.310 = (0.08)(V_{GS1} - 1)^2$$

Then $V_{GS1} = \sqrt{\frac{0.310}{0.08}} + 1 \Rightarrow \underline{V_{GS1} = 2.97\ V}$

5.40

$$I_D = \frac{V_{DD} - V_0}{R_D} = \frac{5 - 0.1}{10} = 0.49\text{ mA}$$

Transistor biased in nonsaturation

$$I_D = 0.49$$

$$= (0.015)\left(\frac{W}{L}\right)\left[2(4.2-0.8)(0.1) - (0.1)^2\right]$$

$$0.49 = \left(\frac{W}{L}\right)[0.67] \Rightarrow \frac{W}{L} = 0.731$$

5.41

$$5 = I_D R_D + V_\gamma + V_{DS}$$

$5 = (12)R_D + 1.6 + 0.2 \Rightarrow \underline{R_D = 267\ \Omega}$

$$I_D = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$$

$12 = \left(\frac{0.040}{2}\right)\left(\frac{W}{L}\right)(5-0.8)^2 \Rightarrow \underline{\frac{W}{L} = 34}$

5.42

$$5 = V_{SD} + I_D R_D + V_\gamma$$

$5 = 0.15 + (15)R_D + 1.6 \Rightarrow \underline{R_D = 217\ \Omega}$

$$I_D = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)(V_{SG} + V_{TP})^2$$

$15 = \left(\frac{0.020}{2}\right)\left(\frac{W}{L}\right)(5-0.8)^2 \Rightarrow \underline{\frac{W}{L} = 85}$

5.43

$$V_{DS}(\text{sat}) = V_{GS} - V_P$$

So $\underline{V_{DS} > V_{DS}(\text{sat}) = -V_P}$, $\underline{I_D = I_{DSS}}$

5.44

$V_{DS}(\text{sat}) = V_{GS} - V_P = \underline{V_{GS} + 3 = V_{DS}(\text{sat})}$

a. $V_{GS} = 0 \Rightarrow \underline{I_D = I_{DSS} = 6\text{ mA}}$

b. $I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 6\left(1 - \frac{-1}{-3}\right)^2$

$\Rightarrow \underline{I_D = 2.67\text{ mA}}$

c. $I_D = 6\left(1 - \frac{-2}{-3}\right)^2 \Rightarrow \underline{I_D = 0.667\text{ mA}}$

d. $\underline{I_D = 0}$

5.45

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2.8 = I_{DSS}\left(1 - \frac{1}{V_P}\right)^2$$

$$0.30 = I_{DSS}\left(1 - \frac{3}{V_P}\right)^2$$

$$\frac{2.8}{0.30} = \frac{\left(1 - \frac{1}{V_P}\right)^2}{\left(1 - \frac{3}{V_P}\right)^2} = 9.33$$

$$\frac{\left(1 - \frac{1}{V_P}\right)}{\left(1 - \frac{3}{V_P}\right)} = 3.055$$

$$1 - \frac{1}{V_P} = 3.055 - \frac{9.165}{V_P}$$

$\frac{8.165}{V_P} = 2.055 \Rightarrow \underline{V_P = 3.97\text{ V}}$

$$2.8 = I_{DSS}\left(1 - \frac{1}{3.97}\right)^2 = I_{DSS}(0.560)$$

$\Rightarrow \underline{I_{DSS} = 5.0\text{ mA}}$

5.46

$V_S = -V_{GS},\ V_{SD} = V_S - V_{DD}$

Want $V_{SD} \geq V_{SD}(\text{sat}) = V_P - V_{GS}$

$V_S - V_{DD} \geq V_P - V_{GS}$

$-V_{GS} - V_{DD} \geq V_P - V_{GS} \Rightarrow \underline{V_{DD} \leq -V_P}$

So $\underline{V_{DD} \leq -2.5\text{ V}}$

$$I_D = 2 = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2 = 6\left(1 - \frac{V_{GS}}{2.5}\right)^2 \Rightarrow V_{GS} = 1.06\text{ V}$$

$\Rightarrow \underline{V_S = -1.06\text{ V}}$

5.47

$I_D = K_n(V_{GS} - V_{TN})^2$

$18.5 = K_n(0.35 - V_{TN})^2$

$86.2 = K_n(0.5 - V_{TN})^2$

Then

$$\frac{18.5}{86.2} = 0.2146 = \frac{(0.35 - V_{TN})^2}{(0.50 - V_{TN})^2} \Rightarrow \underline{V_{TN} = 0.221\,V}$$

$18.5 = K_n(0.35 - 0.221)^2 \Rightarrow \underline{K_n = 1.11\,mA/V^2}$

5.48

$I_D = K(V_{GS} - V_{TN})^2$

$250 = K(0.75 - 0.24)^2 \Rightarrow \underline{K = 0.961\,mA/V^2}$

5.49

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = \frac{V_S}{R_S} = -\frac{V_{GS}}{R_S}$$

$$10\left(1 - \frac{V_{GS}}{-5}\right)^2 = -\frac{V_{GS}}{0.2}$$

$$2\left(1 + \frac{2V_{GS}}{5} + \frac{V_{GS}^2}{25}\right) = -V_{GS}$$

$$\frac{2}{25}V_{GS}^2 + \frac{9}{5}V_{GS} + 2 = 0$$

$$2V_{GS}^2 + 45V_{GS} + 50 = 0$$

$$V_{GS} = \frac{-45 \pm \sqrt{(45)^2 - 4(2)(50)}}{2(2)}$$

$\Rightarrow \underline{V_{GS} = -1.17\text{ V}}$

$$I_D = -\frac{V_{GS}}{R_S} = \frac{1.17}{0.2} \Rightarrow \underline{I_D = 5.85\text{ mA}}$$

$V_D = 20 - (5.85)(2) = 8.3\text{ V}$

$V_{DS} = V_D - V_S = 8.3 - 1.17 \Rightarrow \underline{V_{DS} = 7.13\text{ V}}$

5.50

$V_{DS} = V_{DD} - V_S$

$8 = 10 - V_S \Rightarrow V_S = 2\text{ V} = I_D R_S = (5)R_S$

$\Rightarrow \underline{R_S = 0.4\text{ k}\Omega}$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$5 = I_{DSS}\left(1 - \frac{-1}{V_P}\right)^2 \quad \text{Let } \underline{I_{DSS} = 10\text{ mA}}$$

$$5 = 10\left(1 - \frac{-1}{V_P}\right)^2 \Rightarrow \underline{V_P = -3.41\text{ V}}$$

$V_G = V_{GS} + V_S = -1 + 2 = 1\text{ V}$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \frac{1}{R_1}\cdot R_{in}\cdot V_{DD}$$

$$1 = \frac{1}{R_1}(500)(10) \Rightarrow \underline{R_1 = 5\text{ M}\Omega}$$

$$\frac{5R_2}{5 + R_2} = 0.5 \Rightarrow \underline{R_2 = 0.556\text{ M}\Omega}$$

5.51

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$5 = 8\left(1 - \frac{V_{GS}}{4}\right)^2 \Rightarrow \underline{V_{GS} = 0.838\text{ V}}$$

$V_{SD} = V_{DD} - I_D(R_S + R_D)$

$= 20 - (5)(0.5 + 2) \Rightarrow \underline{V_{SD} = 7.5\text{ V}}$

$V_S = 20 - (5)(0.5) = 17.5\text{ V}$

$V_G = V_S + V_{GS} = 17.5 + 0.838 = 18.3\text{ V}$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \frac{1}{R_1}\cdot R_{in}\cdot V_{DD}$$

$$18.3 = \frac{1}{R_1}(100)(20) \Rightarrow \underline{R_1 = 109\text{ k}\Omega}$$

$$\frac{109R_2}{109 + R_2} = 100 \Rightarrow \underline{R_2 = 1.21\text{ M}\Omega}$$

5.52

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$5 = 7\left(1 - \frac{V_{GS}}{3}\right)^2 \Rightarrow \underline{V_{GS} = 0.465\text{ V}}$$

$V_{SD} = V_{DD} - I_D(R_S + R_D)$

$6 = 12 - (5)(0.3 + R_D) \Rightarrow \underline{R_D = 0.9\text{ k}\Omega}$

$V_S = 12 - (5)(0.3) = 10.5\text{ V}$

$V_G = V_S + V_{GS} = 10.5 + 0.465 = 10.965\text{ V}$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD}$$

$$10.965 = \left(\frac{R_2}{100}\right)(12) \Rightarrow \underline{R_2 = 91.4\text{ k}\Omega}$$

$\Rightarrow \underline{R_1 = 8.6\text{ k}\Omega}$

5.53

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \left(\frac{60}{140 + 60}\right)(20)$$

$\Rightarrow \underline{V_G = 6 \text{ V}}$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = \frac{V_S}{R_S} = \frac{V_G - V_{GS}}{R_S}$$

$$(8)(2)\left(1 - \frac{V_{GS}}{(-4)}\right)^2 = 6 - V_{GS}$$

$$16\left(1 + \frac{V_{GS}}{2} + \frac{V_{GS}^2}{16}\right) = 6 - V_{GS}$$

$$V_{GS}^2 + 9V_{GS} + 10 = 0$$

$$V_{GS} = \frac{-9 \pm \sqrt{(9)^2 - 4(10)}}{2} \Rightarrow \underline{V_{GS} = -1.30}$$

$$I_D = 8\left(1 - \frac{(-1.30)}{(-4)}\right)^2 \Rightarrow \underline{I_D = 3.65 \text{ mA}}$$

$$V_{DS} = V_{DD} - I_D(R_S + R_D)$$

$$= 20 - (3.65)(2 + 2.7)$$

$$V_{DS} = 2.85 \text{ V}$$

$$V_{DS} > V_{DS}(\text{sat}) = V_{GS} - V_P$$

$$= -1.30 - (-4)$$

$$= 2.7 \text{ V} \quad \text{(Yes)}$$

5.54

$$V_{DS} = V_{DD} - I_D(R_S + R_D)$$

$5 = 12 - I_D(0.5 + 1) \Rightarrow \underline{I_D = 4.67 \text{ mA}}$

$$V_S = I_D R_S = (4.67)(0.5) \Rightarrow V_S = 2.33 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \left(\frac{20}{450 + 20}\right)(12)$$

$\Rightarrow V_G = 0.511 \text{ V}$

$$V_{GS} = V_G - V_S = 0.511 - 2.33$$

$\Rightarrow \underline{V_{GS} = -1.82 \text{ V}}$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$4.67 = 10\left(1 - \frac{(-1.82)}{V_P}\right)^2 \Rightarrow \underline{V_P = -5.75 \text{ V}}$$

5.55

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2, \quad V_{GS} = 0$$

$\underline{I_D = I_{DSS} = 4 \text{ mA}}$

$$R_D = \frac{V_{DD} - V_{DS}}{I_D} = \frac{10 - 3}{4} \Rightarrow \underline{R_D = 1.75 \text{ k}\Omega}$$

5.56

$$V_{SD} = V_{DD} - I_D R_S$$

$10 = 20 - (1)R_S \Rightarrow \underline{R_S = 10 \text{ k}\Omega}$

$$R_1 + R_2 = \frac{V_{DD}}{I} = \frac{20}{0.1} = 200 \text{ k}\Omega$$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$1 = 2\left(1 - \frac{V_{GS}}{2}\right)^2 \Rightarrow \underline{V_{GS} = 0.586 \text{ V}}$$

$$V_G = V_S + V_{GS} = 10 + 0.586 = 10.586$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD}$$

$$10.586 = \left(\frac{R_2}{200}\right)(20) \Rightarrow \underline{R_2 = 106 \text{ k}\Omega}$$

$\underline{R_1 = 94 \text{ k}\Omega}$

5.57

$$V_{DS} = V_{DD} - I_D(R_S + R_D)$$

$2 = 3 - (0.040)(10 + R_D) \Rightarrow \underline{R_D = 15 \text{ k}\Omega}$

$$I_D = K(V_{GS} - V_{TN})^2$$

$40 = 250(V_{GS} - 0.20)^2 \Rightarrow \underline{V_{GS} = 0.60 \text{ V}}$

$$V_G = V_{GS} + V_S = 0.60 + (0.040)(10) = 1.0 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD}$$

$$1 = \left(\frac{R_2}{150}\right)(3) \Rightarrow \underline{R_2 = 50 \text{ k}\Omega}$$

$\underline{R_1 = 100 \text{ k}\Omega}$

5.58

For $V_O = 0.70 V \Rightarrow$

$V_{DS} = 0.70 > V_{DS}(sat) = V_{GS} - V_{TN}$

$0.75 - 0.15 = 0.6$

Biased in the saturation region

$$I_D = \frac{V_{DD} - V_{DS}}{R_D} = \frac{3 - 0.7}{50} \Rightarrow \underline{I_D = 46\ \mu A}$$

$$I_D = K(V_{GS} - V_{TN})^2 \Rightarrow 46 = K(0.75 - 0.15)^2 \Rightarrow$$

$\underline{K = 128\ \mu A / V^2}$

Chapter 6

Exercise Solutions

E6.1

$g_m = 2K_n(V_{GS} - V_{TN})$ and

$I_D = K_n(V_{GS} - V_{TN})^2$

$0.75 = 0.5(V_{GS} - 0.8)^2 \Rightarrow V_{GS} = 2.025\,V$

$g_m = 2(0.5)(2.025 - 0.8) \Rightarrow \underline{g_m = 1.22\,mA/V}$

$r_o = \left[\lambda K_n(V_{GS} - V_{TN})^2\right]^{-1}$

$= \left[(0.01)(0.5)(2.025 - 0.8)^2\right]^{-1} \Rightarrow$

$\underline{r_o = 133\,k\Omega}$

E6.2

$g_m = 2K_n(V_{GS} - V_{TN})$ and $I_D = K_n(V_{GS} - V_{TN})^2$

$\Rightarrow V_{GS} - V_{TN} = \sqrt{\dfrac{I_{DQ}}{K_n}}$ and

$g_m = 2K_n\sqrt{\dfrac{I_{DQ}}{K_n}} = 2\sqrt{K_n I_{DQ}}$

$K_n = \dfrac{g_m^2}{4I_{DQ}} = \dfrac{(3.4)^2}{4(2)} = 1.45\,mA/V$

$K_n = \dfrac{\mu_n C_{ox}}{2} \cdot \dfrac{W}{L}$

$1.45 = (0.018)\left(\dfrac{W}{L}\right) \Rightarrow \underline{\dfrac{W}{L} = 80.6}$

$r_o = \left[\lambda K_n(V_{GS} - V_{TN})^2\right]^{-1} = \left[\lambda I_{DQ}\right]^{-1}$

$r_o = \left[(0.015)(2)\right]^{-1} \Rightarrow \underline{r_o = 33.3\,k\Omega}$

E6.3

a. $I_{DQ} = K_n(V_{GS} - V_{TN})^2$

$0.4 = 0.5(V_{GS} - 2)^2 \Rightarrow \underline{V_{GS} = 2.89\text{ V}}$

$V_{DSQ} = V_{DD} - I_{DQ}R_D = 10 - (0.4)(10)$

$\Rightarrow \underline{V_{DSQ} = 6\text{ V}}$

b. $g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.5)(2.89 - 2)$

$\Rightarrow \underline{g_m = 0.89\text{ mA/V}}$

$r_o = [\lambda I_{DQ}]^{-1}$, $\lambda = 0 \Rightarrow \underline{r_o = \infty}$

$A_v = \dfrac{v_o}{v_i} = -g_m R_D = -(0.89)(10)$

$\Rightarrow \underline{A_v = -8.9}$

c. $v_i = 0.4\sin\omega t \Rightarrow v_{ds} = -(8.9)(0.4)\sin\omega t$

$v_{ds} = -3.56\sin\omega t$

At $V_{DS1} = 6 - 3.56 = 2.44$

$V_{GS1} = 2.89 + 0.4 = 3.29$

$V_{GS1} - V_{TN} = 3.29 - 2 = 1.29$

So $V_{DS1} > V_{GS1} - V_{TN} \Rightarrow$ Biased in saturation region

E6.4

a. $V_{SDQ} = V_{DD} - I_{DQ}R_D$

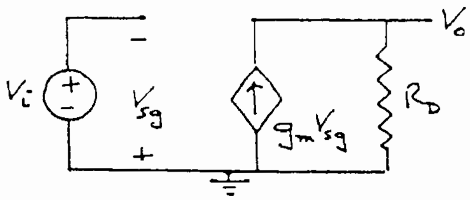
$7 = 12 - I_{DQ}(6) \Rightarrow I_{DQ} = 0.833\text{ mA}$

$I_{DQ} = K_p(V_{SG} - |V_{TP}|)^2$

$0.833 = 2(V_{SG} - 1)^2 \Rightarrow \underline{V_{SG} = 1.65\text{ V}}$

b. $g_m = 2K_p(V_{SG} - |V_{TP}|) = 2(2)(1.65 - 1)$

$\Rightarrow \underline{g_m = 2.6\text{ mA/V}}$, $\underline{r_o = \infty}$



$A_v = \dfrac{v_o}{v_i} = -g_m R_D = -(2.6)(6)$

$\Rightarrow \underline{A_v = -15.6}$

E6.5

$I_{DQ} = K_n(V_{GS} - V_{TN})^2 \Rightarrow V_{GS} - V_{TN} = \sqrt{\dfrac{I_{DQ}}{K_n}}$

$g_m = 2K_n(V_{GS} - V_{TN}) = 2K_n\sqrt{\dfrac{I_{DQ}}{K_n}}$

So $g_m = 2\sqrt{K_n I_{DQ}}$

E6.6

$\eta = \dfrac{\gamma}{2\sqrt{2\phi_f + v_{SB}}}$

(a) $\eta = \dfrac{0.40}{2\sqrt{2(0.35) + 1}} \Rightarrow \underline{\eta = 0.153}$

(b) $\eta = \dfrac{0.40}{2\sqrt{2(0.35)+3}} \Rightarrow \underline{\eta = 0.104}$

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.75)} = 1.22\ mA/V$

For (a), $g_{mb} = g_m\eta = (1.22)(0.153) \Rightarrow$

$\underline{g_{mb} = 0.187\ mA/V}$

For (b), $g_{mb} = (1.22)(0.104) \Rightarrow \underline{g_{mb} = 0.127\ mA/V}$

E6.7

a. $I_{DQ} = K_n(V_{GS} - V_{TN})^2 = (0.25)(2-0.8)^2$

$\Rightarrow \underline{I_{DQ} = 0.36\ \text{mA}}$

$V_{DSQ} = V_{DD} - I_{DQ}R_D = 5 - (0.36)(5)$

$\Rightarrow \underline{V_{DSQ} = 3.2\ \text{V}}$

b. $g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.25)(2-0.8)$

$\Rightarrow \underline{g_m = 0.6\ \text{mA/V}}, \quad \underline{r_0 = \infty}$

c. $A_v = \dfrac{v_0}{v_i} = -g_m R_D = -(0.6)(5)$

$\Rightarrow \underline{A_v = -3.0}$

E6.8

$v_i = v_{gs} = 0.1 \sin \omega t$

$i_d = g_m v_{gs} = (0.6)(0.1)\sin \omega t$

$i_d = 0.06 \sin \omega t\ \text{mA}$

$v_{ds} = (-3)(0.1)\sin \omega t = -0.3 \sin \omega t$

Then $i_D = I_{DQ} + i_d = 0.36 + 0.06 \sin \omega t$

$= i_D\ \text{mA}$

$v_{DS} = V_{DSQ} + v_{ds} = \underline{3.2 - 0.3 \sin \omega t = v_{DS}}$

E6.9

$V_{SDQ} = 3\ \text{V}$ and $I_{DQ} = 0.5\ \text{mA}$

$\Rightarrow R_D = \dfrac{5-3}{0.5} \Rightarrow \underline{R_D = 4\ \text{k}\Omega}$

$I_{DQ} = K_p(V_{SG} - |V_{TP}|)^2$

$0.5 = 1(V_{SG} - 1)^2 \Rightarrow V_{SG} = 1.71\ \text{V}$

$\Rightarrow V_{GG} = 5 - 1.71 \Rightarrow \underline{V_{GG} = 3.29\ \text{V}}$

$A_v = -g_m R_D$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(0.5)}$

$g_m = 1.41\ \text{mA/V}$

$A_v = -(1.41)(4) \Rightarrow \underline{A_v = -5.64}$

$A_v = \dfrac{v_0}{v_i} = \dfrac{-v_{sd}}{v_i} = -\dfrac{0.46 \sin \omega t}{v_i} = -5.64$

$\Rightarrow \underline{v_i = 0.0816 \sin \omega t}$

E6.10

a. $V_{SG} = 9 - I_{DQ}R_S, \quad I_{DQ} = K_p(V_{SG} - |V_{TP}|)^2$

$V_{SG} = 9 - (2)(1.2)(V_{SG} - 2)^2$

$= 9 - 2.4(V_{SG}^2 - 4V_{SG} + 4)$

$2.4V_{SG}^2 - 8.6V_{SG} + 0.6 = 0$

$V_{SG} = \dfrac{8.6 \pm \sqrt{(8.6)^2 - 4(2.4)(0.6)}}{2(2.4)}$

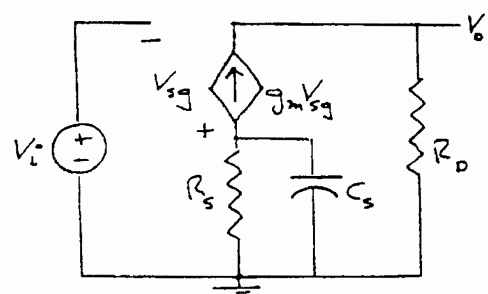
$V_{SG} = 3.51\ \text{V}, \quad I_{DQ} = 2(3.51 - 2)^2$

$\Rightarrow \underline{I_{DQ} = 4.56\ \text{mA}}$

$V_{SDQ} = 9 + 9 - I_{DQ}(1.2 + 1) = 18 - (4.56)(2.2)$

$\Rightarrow \underline{V_{SDQ} = 7.97\ \text{V}}$

b.



$V_o = g_m V_{SG} R_D$

$A_v = -g_m R_D = -(6.04)(1) \Rightarrow \underline{A_v = -6.04}$

E6.11

$I_{DQ} = I_Q = 0.5\ \text{mA}$

Let $\dfrac{W}{L} = 25$

$K_n = (20)(25) = 500\ \mu A/V^2$

$V_{GS} = \sqrt{\dfrac{0.5}{0.5}} + 1.5 = 2.5\ \text{V} \Rightarrow V_S = -2.5\ \text{V}$

$A_v = -g_m R_D$

$g_m = 2(0.5)(2.5 - 1.5) = 1\ \text{mA/V}$

For $A_v = -4.0 \Rightarrow R_D = 4\ \text{k}\Omega$

$V_D = 5 - (0.5)(4) = 3\ \text{V}$

$\Rightarrow V_{DSQ} = 3 - (-2.5) = 5.5\ \text{V}$

E6.12

a. With $R_G \Rightarrow V_{GS} = V_{DS} \Rightarrow$ transistor biased in sat. region

$I_D = K_n(V_{GS} - V_{TN})^2 = K_n(V_{DS} - V_{TN})^2$

$V_{DS} = V_{DD} - I_D R_D$

$= V_{DD} - K_n R_D (V_{DS} - V_{TN})^2$

$$V_{DS} = 15 - (0.15)(10)(V_{DS} - 1.8)^2$$
$$= 15 - 1.5(V_{DS}^2 - 3.6V_{DS} + 3.24)$$
$$1.5V_{DS}^2 - 4.4V_{DS} - 10.14 = 0$$
$$V_{DS} = \frac{4.4 \pm \sqrt{(4.4)^2 + (4)(1.5)(10.14)}}{2(1.5)}$$
$$\Rightarrow \underline{V_{DSQ} = 4.45 \text{ V}}$$
$$I_{DQ} = (0.15)(4.45 - 1.8)^2 \Rightarrow \underline{I_{DQ} = 1.05 \text{ mA}}$$

b. Neglecting effect of R_G :

$$A_v = -g_m(R_D \| R_2)$$
$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.15)(4.45 - 1.8)$$
$$\Rightarrow g_m = 0.795 \text{ mA/V}$$
$$A_v = -(0.795)(10\|5) \Rightarrow \underline{A_v = -2.65}$$

c. $R_G \Rightarrow$ establishes $V_{GS} = V_{DS} \Rightarrow$ essentially no effect on small-signal voltage gain.

E6.13

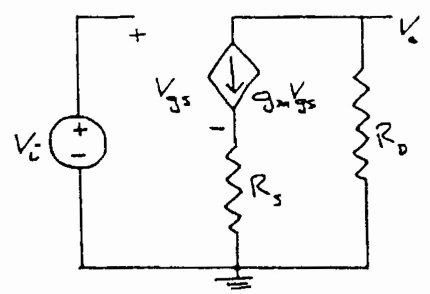
a. $I_{DQ} = K_n(V_{GS} - V_{TN})^2$

$$I_{DQ} = 0.8(2 - V_{SG})^2 = \frac{V_{SG}}{R_S} = \frac{V_{SG}}{4}$$
$$3.2(4 - 4V_{SG} + V_{SG}^2) = V_{SG}$$
$$3.2V_{SG}^2 - 13.8V_{SG} + 12.8 = 0$$
$$V_{SG} = \frac{13.8 \pm \sqrt{(13.8)^2 - 4(3.2)(12.8)}}{2(3.2)}$$
$$V_{SG} = 1.35 \text{ V} \Rightarrow I_{DQ} = 0.8(2 - 1.35)^2$$
$$\Rightarrow \underline{I_{DQ} = 0.338 \text{ mA}}$$

b. $V_{DSQ} = V_{DD} - I_{DQ}(R_D + R_S)$

$$6 = 10 - (0.338)(R_D + 4)$$
$$R_D = \frac{10 - (0.338)(4) - 6}{0.338} \Rightarrow \underline{R_D = 7.83 \text{ k}\Omega}$$

c.



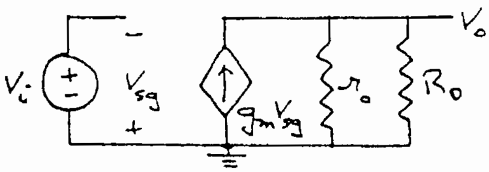
$$V_i = V_{gs} + g_m V_{gs} R_S \Rightarrow V_{gs} = \frac{V_i}{1 + g_m R_S}$$
$$V_0 = -g_m V_{gs} R_D$$
$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.8)(-1.35 + 2)$$
$$= 1.04 \text{ mA/V}$$
$$A_v = \frac{V_0}{V_i} = \frac{-g_m R_D}{1 + g_m R_S} = \frac{-(1.04)(7.83)}{1 + (1.04)(4)}$$
$$\Rightarrow \underline{A_v = -1.58}$$

E6.14

a. $5 = I_{DQ}R_S + V_{SG}$ and
$I_{DQ} = K_p(V_{SG} + V_{TP})^2$

$$0.8 = 0.5(V_{SG} + 0.8)^2 \Rightarrow V_{SG} = 0.465 \text{ V}$$
$$5 = (0.8)R_S + 0.465 \Rightarrow \underline{R_S = 5.67 \text{ k}\Omega}$$
$$V_{SDQ} = 10 - I_{DQ}(R_S + R_D)$$
$$3 = 10 - (0.8)(5.67 + R_D)$$
$$R_D = \frac{10 - (0.8)(5.67) - 3}{0.8} \Rightarrow \underline{R_D = 3.08 \text{ k}\Omega}$$

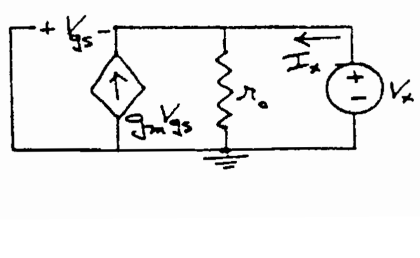
b.



$$V_0 = g_m V_{sg}(R_D \| r_0) = -g_m V_i(R_D \| r_0)$$
$$A_v = \frac{V_0}{V_i} = -g_m(R_D \| r_0)$$
$$g_m = 2K_p(V_{SG} + V_{TP}) = 2(0.5)(0.465 + 0.8)$$
$$= 1.27 \text{ mA/V}$$
$$r_0 = \frac{1}{\lambda I_0} = \frac{1}{(0.02)(0.8)} = 62.5 \text{ k}\Omega$$
$$A_v = -(1.27)(3.08\|62.5) \Rightarrow \underline{A_v = -3.73}$$

E6.15

$$V_0 = g_m V_{gs} r_0$$
$$V_i = V_{gs} + V_0 \Rightarrow V_{gs} = V_i - V_0$$
$$\text{So } V_0 = g_m r_0(V_i - V_0)$$
$$A_v = \frac{V_0}{V_i} = \frac{g_m r_0}{1 + g_m r_0} = \frac{(4)(50)}{1 + (4)(50)}$$
$$\Rightarrow \underline{A_v = 0.995}$$



$I_x + g_m V_{gs} = \frac{V_x}{r_0}$ and $V_{gs} = -V_x$

$I_x = g_m V_x + \frac{V_x}{r_0} \Rightarrow R_0 = r_0 \left\| \frac{1}{g_m} = 50 \right\| \frac{1}{4}$

$\Rightarrow \underline{R_0 \cong 0.25 \text{ k}\Omega}$

With $R_S = 4 \text{ k}\Omega \Rightarrow A_v = \frac{g_m(r_0\|R_S)}{1 + g_m(r_0\|R_S)}$

$r_0\|R_S = 50\|4 = 3.7 \text{ k}\Omega \Rightarrow A_v = \frac{(4)(3.7)}{1 + (4)(3.7)}$

$\Rightarrow \underline{A_v = 0.937}$

E6.16

$V_{DSQ} = V_{DD} - I_{DQ}R_S$

$5 = 10 - (1.5)R_S \Rightarrow \underline{R_S = 3.33 \text{ k}\Omega}$

$I_{DQ} = K_n(V_{GS} - V_{TN})^2 \Rightarrow 1.5 = (1)(V_{GS} - 0.8)^2$

$V_{GS} = 2.02 \text{ V} = V_G - V_S = V_G - 5$

$\Rightarrow V_G = 7.02 \text{ V} = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \frac{R_2}{400} \cdot 10$

So $\underline{R_2 = 280.8 \text{ k}\Omega}$, $\underline{R_1 = 119.2 \text{ k}\Omega}$

Neglecting R_{Si},

$A_v = \frac{g_m(R_S\|r_0)}{1 + g_m(R_S\|r_0)}$

$r_0 = [\lambda I_{DQ}]^{-1} = [(0.015)(1.5)]^{-1} = 44.4 \text{ k}\Omega$

$R_S\|r_0 = 3.33\|44.4 = 3.1 \text{ k}\Omega$

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(1.5)} = 2.45 \, mA/V$

$A_v = \frac{(2.45)(3.1)}{1 + (2.45)(3.1)} \Rightarrow \underline{A_v = 0.884}$

$R_0 = \frac{1}{g_m} \left\| R_S \right\| r_0 = \frac{1}{2.45} \left\| 3.33 \right\| 44.4$

$= 0.408\|3.1$

$\Rightarrow \underline{R_0 = 0.36 \text{ k}\Omega}$

E6.17

$V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \left(\frac{9.3}{70.7 + 9.3}\right)(5)$

$= 0.581 \text{ V}$

$I_{DQ} = K_p(V_{SG} - |V_{TP}|)^2 = K_p(V_S - V_G - |V_{TP}|)^2$

$= \frac{5 - V_S}{R_S}$

Then $(0.4)(5)(V_S - 0.581 - 0.8)^2 = 5 - V_S$

$2(V_S - 1.381)^2 = 5 - V_S$

$2(V_S^2 - 2.762V_S + 1.907) = 5 - V_S$

$2V_S^2 - 4.52V_S - 1.19 = 0$

$V_S = \frac{4.52 \pm \sqrt{(4.52)^2 + 4(2)(1.19)}}{2(2)}$

$V_S = 2.50 \text{ V} \Rightarrow I_{DQ} = \frac{5 - 2.5}{5} = 0.5 \text{ mA}$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.4)(0.5)} = 0.894 \, mA/V$

$A_v = \frac{g_m R_S}{1 + g_m R_S} \cdot \frac{R_1\|R_2}{R_1\|R_2 + R_{Si}}$

$= \frac{(0.894)(5)}{1 + (0.894)(5)} \cdot \frac{70.7\|9.3}{70.7\|9.3 + 0.5} \Rightarrow \underline{A_v = 0.770}$

Neglecting R_{Si}, $A_v = 0.817$

$R_0 = R_S \left\| \frac{1}{g_m} = 5 \right\| \frac{1}{0.894} = 5\|1.12$

$\Rightarrow \underline{R_0 = 0.915 \text{ k}\Omega}$

E6.18

(a) $g_m = 2\sqrt{K_n I_{DQ}} \Rightarrow 2 = 2\sqrt{K_n(0.8)} \Rightarrow$

$K_n = 1.25 \, mA/V^2$

$K_n = \frac{\mu_n C_{ox}}{2} \cdot \frac{W}{L} \Rightarrow 1.25 = (0.020)\left(\frac{W}{L}\right)$

So $\underline{\frac{W}{L} = 62.5}$

$I_{DQ} = K_n(V_{GS} - V_{TN})^2 \Rightarrow 0.8 = 1.25(V_{GS} - 2)^2$

$\Rightarrow \underline{V_{GS} = 2.8 \text{ V}}$

b. $r_0 = [\lambda I_{DQ}]^{-1} = [(0.01)(0.8)]^{-1} = 125 \text{ k}\Omega$

$A_v = \frac{g_m(r_0\|R_L)}{1 + g_m(r_0\|R_L)}$

$r_0\|R_L = 125\|4 = 3.88$

$A_v = \frac{(2)(3.88)}{1 + (2)(3.88)} \Rightarrow \underline{A_v = 0.886}$

$R_0 = \frac{1}{g_m} \left\| r_0 = \frac{1}{2} \right\| 125 \Rightarrow \underline{R_0 \approx 0.5 \text{ k}\Omega}$

E6.19

$I_{DQ} = K_p(V_{SG} - |V_{TP}|)^2$

$3 = 2(V_{SG} - 2)^2 \Rightarrow V_{SG} = 3.22 \text{ V}$

$I_{DQ} = \frac{5 - V_{SG}}{R_S} \Rightarrow 3 = \frac{5 - 3.22}{R_S}$

$\Rightarrow \underline{R_S = 0.593 \text{ k}\Omega}$

$r_0 = [\lambda I_{DQ}]^{-1} = [(0.02)(3)]^{-1} = 16.7 \text{ k}\Omega$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(3)} = 4.9 \, mA/V$

For $R_L = \infty$, $A_v = \frac{g_m(r_0\|R_S)}{1 + g_m(r_0\|R_S)}$

$r_0\|R_S = 16.7\|0.593 = 0.573 \text{ k}\Omega$

$A_v = \frac{(4.9)(0.573)}{1 + (4.9)(0.573)} \Rightarrow \underline{A_v = 0.737}$

If A_v is reduced by 10%

$\Rightarrow A_v = 0.737 - 0.0737 = 0.663$

$A_v = \frac{g_m(r_0\|R_S\|R_L)}{1 + g_m(r_0\|R_S\|R_L)}$

Let $r_o \| R_S \| R_L = x$

$$0.663 = \frac{(4.9)x}{1+(4.9)x} \Rightarrow 0.663 = 4.9x(1-0.663)$$

$$x = 0.402 = 0.573 \| R_L$$

$$\frac{0.573 R_L}{R_L + 0.573} = 0.402$$

$$\Rightarrow (0.573 - 0.402)R_L = (0.402)(0.573)$$

$$\Rightarrow \underline{R_L = 1.35\ \text{k}\Omega}$$

E6.20

$$R_{in} = \frac{1}{g_m} = 0.35\ \text{k}\Omega \Rightarrow \underline{g_m = 2.86\ \text{mA/V}}$$

$$\frac{V_0}{I_i} = R_D \| R_L = 2.4 = R_D \| 4 = \frac{4R_D}{4+R_D}$$

$$(4-2.4)R_D = (2.4)(4) \Rightarrow \underline{R_D = 6\ \text{k}\Omega}$$

$$g_m = 2\sqrt{K_n I_{DQ}}$$

$$2.86 = 2\sqrt{K_n(0.5)} \Rightarrow K_n = 4.09\ mA/V^2$$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$0.5 = 4.09(V_{GS} - 1)^2 \Rightarrow V_{GS} = 1.35\ \text{V}$$

$$\Rightarrow V_S = -1.35\ \text{V}, \quad V_D = 5 - (0.5)(6) = 2\ \text{V}$$

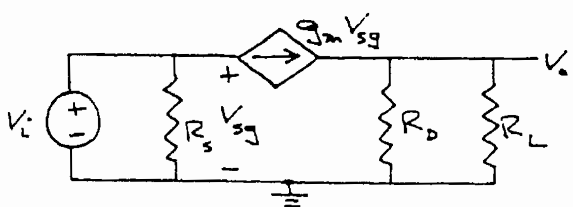
$$V_{DS} = V_D - V_S = 2 - (-1.35) = 3.35\ \text{V}$$

We have

$V_{DS} = 3.35 > V_{GS} - V_{TN} = 1.35 - 1 = 0.35\ V$

$\Rightarrow$ Biased in the saturation region

E6.21



$V_0 = g_m V_{sg}(R_D \| R_L)$ and $V_{sg} = V_i$

$$A_v = g_m(R_D \| R_L)$$

$$I_{DQ} = \frac{5 - V_{SG}}{R_S} = K_p(V_{SG} - |V_{TP}|)^2$$

$$5 - V_{SG} = (1)(4)(V_{SG} - 0.8)^2$$

$$5 - V_{SG} = 4(V_{SG}^2 - 1.6V_{SG} + 0.64)$$

$$4V_{SG}^2 - 5.4V_{SG} - 2.44 = 0$$

$$V_{SG} = \frac{5.4 \pm \sqrt{(5.4)^2 + (4)(4)(2.44)}}{2(4)}$$

$$V_{SG} = 1.71\ \text{V}$$

$$I_{DQ} = \frac{5 - 1.71}{4} = 0.822\ \text{mA}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(0.822)} = 1.81\ mA/V$$

$$A_v = (1.81)(2\|4) = (1.81)(1.33) \Rightarrow \underline{A_v = 2.41}$$

$$R_{in} = R_S \left\| \frac{1}{g_m} = 4 \right\| \frac{1}{1.81} = 4\|0.552$$

$$\Rightarrow \underline{R_{in} = 0.485\ \text{k}\Omega}$$

E6.22

$$K_{n1} = \frac{\mu_n C_{ox}}{2} \cdot \left(\frac{W}{L}\right)_1 = (0.020)(80) = 1.6\ mA/V^2$$

$$K_{n2} = \frac{\mu_n C_{ox}}{2} \cdot \left(\frac{W}{L}\right)_2 = (0.020)(1) = 0.020\ mA/V^2$$

$$A_v = -\sqrt{\frac{K_{n1}}{K_{n2}}} = -\sqrt{\frac{1.6}{0.020}} \Rightarrow \underline{A_v = -8.94}$$

The transition point is determined from

$$v_{GSt} - V_{TND} = V_{DD} - V_{TNL} - \sqrt{\frac{K_{n1}}{K_{n2}}}(v_{GSt} - V_{TND})$$

$$v_{GSt} - 0.8 = (5 - 0.8) - (8.94)(v_{GSt} - 0.8)$$

$$v_{GSt} = \frac{(5 - 0.8) + (8.94)(0.8) + 0.8}{1 + 8.94}$$

$$v_{GSt} = 1.22\ \text{V}$$

For Q-point in middle of saturation region

$$V_{GS} = \frac{1.22 - 0.8}{2} + 0.8 \Rightarrow \underline{V_{GS} = 1.01\ \text{V}}$$

E6.23

$$K_{n2} = \frac{\mu_n C_{ox}}{2} \cdot \left(\frac{W}{L}\right)_2 = (0.015)(2) = 0.030\ mA/V^2$$

$$A_v = -\sqrt{\frac{K_{n1}}{K_{n2}}} = -6 \Rightarrow \frac{K_{n1}}{K_{n2}} = 36$$

$$K_{n1} = (36)(0.030) = 1.08\ mA/V^2$$

$$1.08 = (0.015)\left(\frac{W}{L}\right)_1 \Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 72}$$

The transition point is found from

$$v_{GSt} - 1 = (10 - 1) - (6)(v_{GSt} - 1)$$

$$v_{GSt} = \frac{10 - 1 + 6 + 1}{1 + 6} = 2.29\ \text{V}$$

For Q-point in middle of saturation region

$$V_{GS} = \frac{2.29 - 1}{2} + 1 \Rightarrow \underline{V_{GS} = 1.645\ \text{V}}$$

E6.24

(a) Transition points:

For M_2: $v_{OB} = V_{DD} - |V_{TNL}| = 5 - 1.2 = 3.8\ V$

For M_1:

$$K_{n1}\left[(v_{OA})^2(1 + \lambda v_{OA})\right] = K_{n2}\left[(V_{TNL})^2(1 + \lambda_2[V_{DD} - v_{OA}])\right]$$

$$250\left[v_{OA}^2 + (0.01)v_{OA}^3\right] = 25\left[(1.2)^2(1 + (0.01)(5) - (0.01)v_{OA})\right]$$

$10\left[v_{OA}^2+(0.01)v_{OA}^3\right]=1.512-0.0144v_{OA}$

$(0.01)v_{OA}^3+v_{OA}^2+0.00144v_{OA}-0.512=0$

which yields $v_{OA}\cong 0.388\,V$

Then middle of saturation region

$$v_{OQ}=\frac{3.8-0.388}{2}+0.388 \Rightarrow V_{DSQ1}=2.094\text{ V}$$

$$K_{n1}\left[(V_{GS1}-V_{TND})^2(1+\lambda_1 v_O)\right]=K_{n2}\left[(V_{TNL})^2(1+\lambda_2[V_{DD}-v_O])\right]$$

$250[(V_{GS1}-0.8)^2(1+[0.01][2.094])]$

$=25[(1.2)^2(1+[0.01][5-2.094])]$

$10[(V_{GS1}-0.8)^2(1.0209)]=1.482$

$(V_{GS1}-0.8)^2=0.145 \Rightarrow \underline{V_{GS1}=1.18\text{ V}}$

b. $I_{DQ}=K_{n1}\left[(V_{GS1}-0.8)^2(1+(0.01)(2.094))\right]$

$I_{DQ}=(0.25)[(0.145)^2(1.02094)]$

$\Rightarrow \underline{I_{DQ}=37.0\ \mu\text{A}}$

c. $A_v=\dfrac{-g_{m1}}{I_{DQ}(\lambda_1+\lambda_2)}=-g_{m1}(r_{01}\|r_{02})$

$g_{m1}=2K_{n1}(V_{GS1}-V_{TND})$

$=2(0.25)(1.18-0.8)=0.19\text{ mA/V}$

$$A_v=\frac{-0.19}{(0.037)(0.01+0.01)} \Rightarrow \underline{A_v=-257}$$

E6.25

$R_0=R_{S2}\left\|\dfrac{1}{g_{m2}}\right.$

$g_{m2}=0.632\text{ mA/V},\quad R_{S2}=8\text{ k}\Omega$

$R_0=8\left\|\dfrac{1}{0.632}\right.=8\|1.58 \Rightarrow \underline{R_0=1.32\text{ k}\Omega}$

E6.26

a. $I_{DQ2}=2mA,\quad V_{SDQ2}=10\text{ V}$

$I_{DQ2}\cdot R_{S2}=10=2R_{S2} \Rightarrow \underline{R_{S2}=5\text{ k}\Omega}$

$I_{DQ2}=K_{n2}(V_{GS2}-V_{TN2})^2$

$2=1(V_{GS2}-2)^2 \Rightarrow V_{SG2}=3.41\text{ V}$

$\Rightarrow V_{G2}=3.41\text{ V}$

Then $R_{D1}=\dfrac{10-3.41}{2} \Rightarrow \underline{R_{D1}=3.3\text{ k}\Omega}$

For $V_{DSQ1}=10\text{ V} \Rightarrow V_{S1}=3.41-10=-6.59\text{ V}$

Then $R_{S1}=\dfrac{-6.59-(-10)}{2} \Rightarrow \underline{R_{S1}=1.71\text{ k}\Omega}$

$I_{D1}=K_{n1}(V_{GS1}-V_{TN1})^2$

$2=1(V_{GS1}-2)^2 \Rightarrow V_{GS1}=3.41\text{ V}$

$V_{GS1}=\left(\dfrac{R_2}{R_1+R_2}\right)(20)-I_{DQ1}R_{S1}$

$\dfrac{R_2}{R_1+R_2}=\dfrac{1}{R_1}\cdot R_{in}$

$3.41=\dfrac{1}{R_1}(200)(20)-(2)(1.71)$

$\Rightarrow \underline{R_1=586\text{ k}\Omega}$

$\dfrac{586R_2}{586+R_2}=200 \Rightarrow (586-200)R_2=(200)(586)$

$\Rightarrow \underline{R_2=304\text{ k}\Omega}$

b. $g_{m1}=2\sqrt{K_{n1}I_{DQ1}}=2\sqrt{(1)(2)}$

$\Rightarrow g_{m1}=g_{m2}=2.83\text{ mA/V}$

From Example 6-16

$$A_v=\frac{-g_{m1}g_{m2}R_{D1}(R_{S2}\|R_L)}{1+g_{m2}(R_{S2}\|R_L)}$$

$R_{S2}\|R_L=5\|4=2.22\text{ k}\Omega$

$$A_v=\frac{-(2.83)(2.83)(3.3)(2.22)}{1+(2.83)(2.22)}$$

$\Rightarrow \underline{A_v=-8.06}$

$R_0=\dfrac{1}{g_{m2}}\left\|R_{S2}=\dfrac{1}{2.83}\right\|5=0.353\|5$

$\Rightarrow \underline{R_0=0.330\text{ k}\Omega}$

E6.27

a. $I_{DQ1}=K_{n1}(V_{GS1}-V_{TN1})^2$

$1=1.2(V_{GS1}-2)^2 \Rightarrow V_{GS1}=V_{GS2}=2.91\text{ V}$

$R_S=10\text{ k}\Omega \Rightarrow V_{S1}=I_{DQ}R_S-10$

$=(1)(10)-10=0$

$V_{G1}=2.91=\left(\dfrac{R_3}{R_1+R_2+R_3}\right)(10)$

$=\left(\dfrac{R_3}{500}\right)(10)$

$\Rightarrow \underline{R_3=145.5\text{ k}\Omega}$

$V_{DSQ1}=3.5 \Rightarrow V_{S2}=3.5\text{ V} \Rightarrow 3.5+2.91$

$\Rightarrow V_{G2}=6.41$

$V_{G2}=\left(\dfrac{R_2+R_3}{R_1+R_2+R_3}\right)(10)=6.41$

$=\left(\dfrac{R_2+R_3}{500}\right)(10)$

$R_2+R_3=320.5=R_2+145.5 \Rightarrow \underline{R_2=175\text{ k}\Omega}$

Then $R_1+R_2+R_3=500=R_1+175+145.5$

$\Rightarrow \underline{R_1=179.5\text{ k}\Omega}$

Now $V_{S2}=3.5 \Rightarrow V_{D2}=V_{S2}+V_{SDQ2}$

$=3.5+3.5=7\text{ V}$

So $R_D = \frac{10-7}{1} \Rightarrow \underline{R_D = 3\ k\Omega}$

b. From Example 6-18:

$A_v = -g_{m1} R_D$

$g_{m1} = 2\sqrt{K_{n1} I_{DQ}} = 2\sqrt{(1.2)(1)} = 2.19\ mA/V$

$A_v = -(2.19)(3) \Rightarrow \underline{A_v = -6.57}$

E6.28

From Example 6-18:

$g_m = 2.98$ mA/V, $r_0 = 42.1$ kΩ

$R_1 \| R_2 = 420 \| 180 = 126$ kΩ

$$V_{gs} = \left(\frac{R_1\|R_2}{R_1\|R_2 + R_i}\right) V_i$$

$$= \left(\frac{126}{126+20}\right) V_i = 0.863 V_i$$

$$A_v = \frac{-g_m V_{gs} (r_0 \| R_D \| R_L)}{V_i}$$

$= -(2.98)(0.863)(42.1 \| 2.7 \| 4)$

$= -(2.57)(42.1 \| 1.61) = -(2.57)(1.55)$

$\Rightarrow \underline{A_v = -3.98}$

E6.29

$V_S = I_{DQ} R_S = (1.2)(2.7) = 3.24$ V

$V_D = V_S + V_{DSQ} = 3.24 + 12 = 15.24$

$R_D = \frac{20 - 15.24}{1.2} \Rightarrow \underline{R_D = 3.97\ k\Omega}$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$1.2 = 4\left(1 - \frac{V_{GS}}{V_P}\right)^2 \Rightarrow \frac{V_{GS}}{V_P} = 0.452$$

$V_{GS} = (0.452)(-3) = -1.356$

$V_G = V_S + V_{GS} = 3.24 - 1.356 = 1.88$ V

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(20) = \left(\frac{R_2}{500}\right)(20) = 1.88$$

$\Rightarrow \underline{R_2 = 47\ k\Omega}$. $\underline{R_1 = 453\ k\Omega}$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.005)(1.2)} = 167\ k\Omega$$

$$g_m = \frac{2I_{DSS}}{(-V_P)}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(4)}{3}\left(1 - \frac{1.356}{3}\right)$$

$= 1.46$ mA/V

$A_v = -g_m(r_0 \| R_D \| R_L) = -(146)(167 \| 3.97 \| 4)$

$\Rightarrow \underline{A_v = -2.87}$

E6.30

a. $V_{G1} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{DD})$

$$V_{G1} = \left(\frac{430}{430+70}\right)(20) = 17.2\ V$$

$$I_{DQ1} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 6\left(1 - \frac{V_{G1} - V_{S1}}{2}\right)^2$$

$$= 6\left(1 - \frac{17.2}{2} + \frac{V_{S1}}{2}\right)^2 = 6\left(\frac{V_{S1}}{2} - 7.6\right)^2$$

and $I_{DQ1} = \frac{20 - V_{S1}}{4}$

Then $\frac{20 - V_{S1}}{4} = 6\left(\frac{V_{S1}}{2} - 7.6\right)^2$

$$20 - V_{S1} = 24\left(\frac{V_{S1}^2}{4} - 7.6V_{S1} + 57.76\right)$$

$= 6V_{S1}^2 - 182.4V_{S1} + 1386.24$

$6V_{S1}^2 - 181.4V_{S1} + 1366.24 = 0$

$$V_{S1} = \frac{181.4 \pm \sqrt{(181.4)^2 - 4(6)(1366.24)}}{2(6)}$$

$V_{S1} = 14.2$ V $\Rightarrow V_{GS1} = 17.2 - 14.2$

$= 3$ V $> V_P$

So $V_{S1} = 16.0 \Rightarrow V_{GS1} = 17.2 - 16$

$= 1.2 < V_P - Q$

on $I_{DQ1} = \frac{20-16}{4} \Rightarrow \underline{I_{DQ1} = 1\ mA}$

$V_{SDQ1} = 20 - I_{DQ1}(R_{S1} + R_{D1})$

$= 20 - (1)(8)$

$\Rightarrow \underline{V_{SDQ1} = 12\ V}$

$V_{D1} = I_{DQ1} R_{D1} = (1)(4) = 4\ V = V_{G2}$

$$I_{DQ2} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 6\left(1 - \frac{V_{G2} - V_{S2}}{(-2)}\right)^2$$

$$= 6\left(1 + \frac{4}{2} - \frac{V_{S2}}{2}\right)^2 = 6\left(3 - \frac{V_{S2}}{2}\right)^2$$

and $I_{DQ2} = \frac{V_{S2}}{R_{S2}} = \frac{V_{S2}}{4}$

Then

$$\frac{V_{S2}}{4} = 6\left(3 - \frac{V_{S2}}{2}\right)^2$$

$$V_{S2} = 24\left(9 - 3V_{S2} + \frac{V_{S2}^2}{4}\right)$$

$6V_{S2}^2 - 73V_{S2} + 216 = 0$

$$V_{S2} = \frac{73 \pm \sqrt{(73)^2 - 4(6)(216)}}{2(6)}$$

$\Rightarrow V_{S2} = 7.09$ V or $= 5.08$ V

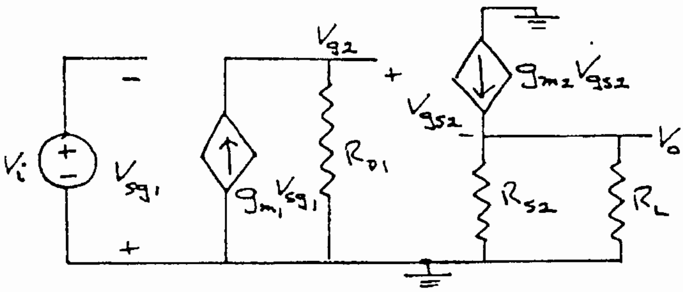
For $V_{S2} = 5.08$ V

$\Rightarrow V_{GS2} = 4 - 5.08 = -1.08$ transistor biased ON

$I_{DQ2} = \frac{5.08}{4} \Rightarrow \underline{I_{DQ2} = 1.27 \text{ mA}}$

$V_{DS2} = 20 - V_{S2} = 20 - 5.08 \Rightarrow \underline{V_{DS2} = 14.9 \text{ V}}$

b.



$V_{g2} = g_{m1}V_{sg1}R_{D1} = -g_{m1}V_iR_{D1}$

$V_o = g_{m2}V_{gs2}(R_{S2}\|R_L)$

$V_{g2} = V_{gs2} + V_o \Rightarrow V_{gs2} = \frac{V_{g2}}{1 + g_{m2}(R_{S2}\|R_L)}$

$A_v = \frac{V_o}{V_i} = \frac{-g_{m1}R_{D1}}{1 + g_{m2}(R_{S2}\|R_L)}$

$g_{m1} = \frac{2I_{DSS}}{|V_P|}\left(1 - \frac{V_{GS}}{V_P}\right)$

$= \frac{2(6)}{2}\left(1 - \frac{1.2}{2}\right) = 2.4 \text{ mA/V}$

$g_{m2} = \frac{2(6)}{2}\left(1 - \frac{1.08}{2}\right) = 2.76 \text{ mA/V}$

Then $A_v = \frac{-(2.4)(4)}{1 + (2.76)(4)(2)} = \underline{-2.05 = A_v}$

E6.31

a. $I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

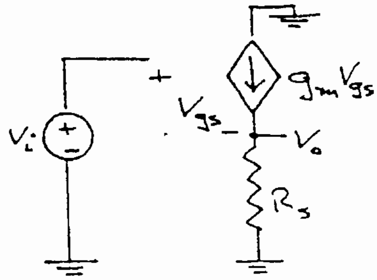
$2 = 8\left(1 - \frac{V_{GS}}{V_P}\right)^2 \Rightarrow \frac{V_{GS}}{V_P} = 0.5$

$V_{GS} = (0.5)(-3.5) \Rightarrow V_{GS} = -1.75$

Also $I_{DQ} = \frac{-V_{GS} - (-10)}{R_S}$

$2 = \frac{1.75 + 10}{R_S} \Rightarrow \underline{R_S = 5.88 \text{ k}\Omega}$

b.



$g_m = \frac{2I_{DSS}}{|V_P|}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(8)}{3.5}\left(1 - \frac{1.75}{3.5}\right)$

$= 2.29 \text{ mA/V}$

$r_o = \frac{1}{(0.01)(2)} = 50 \text{ k}\Omega$

$V_i = V_{gs} + g_mR_SV_{gs} \Rightarrow V_{gs} = \frac{V_i}{1 + g_mR_S}$

$A_v = \frac{V_o}{V_i} = \frac{g_mR_S\|r_o}{1 + g_mR_S\|r_o} = \frac{(2.29)[5.88\|50]}{1 + (2.29)[5.88\|50]}$

$\Rightarrow \underline{A_v = 0.923}$

c. $A_v = \frac{g_m(R_S\|R_L)}{1 + g_m(R_S\|R_L)} = 0.931 - 0.186 = 0.745$

$\frac{(2.29)(R_S\|R_L)}{1 + (2.29)(R_S\|R_L)} = 0.745$

$(2.29)(R_S\|R_L)(1 - 0.745) = 0.745$

$\Rightarrow R_S\|R_L = 1.28 \text{ k}\Omega$

$\frac{5.88R_L}{5.88 + R_L} = 1.28$

$\Rightarrow (5.88 - 1.28)R_L = (1.28)(5.88)$

$\Rightarrow \underline{R_L = 1.64 \text{ k}\Omega}$

Chapter 6

Problem Solutions

6.1

(a) $g_m = 2\sqrt{K_n I_D} = 2\sqrt{\left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right) I_D}$

$0.5 = 2\sqrt{(0.020)\left(\frac{W}{L}\right)(0.5)} \Rightarrow \underline{\frac{W}{L} = 12.5}$

(b) $I_D = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$

$0.5 = (0.02)(12.5)(V_{GS} - 0.8)^2$

$\Rightarrow \underline{V_{GS} = 2.21 \text{ V}}$

6.2

(a) $g_m = 2\sqrt{K_p I_D} = 2\sqrt{\left(\frac{\mu_p C_{ox}}{2}\right)\left(\frac{W}{L}\right) I_D}$

$\left(\frac{50}{2}\right)^2 = (10)\left(\frac{W}{L}\right)(100) \Rightarrow \underline{\frac{W}{L} = 0.625}$

(b) $I_D = \left(\frac{\mu_p C_{ox}}{2}\right)\left(\frac{W}{L}\right)(V_{SG} + V_{TP})^2$

$100 = (10)(0.625)(V_{SG} - 1.2)^2$

$\Rightarrow \underline{V_{SG} = 4.2 \text{ V}}$

6.3

$I_D = K_n(V_{GS} - V_{TN})^2(1 + \lambda V_{DS})$

$\frac{I_{D1}}{I_{D2}} = \frac{1 + \lambda V_{DS1}}{1 + \lambda V_{DS2}} \Rightarrow \frac{3.4}{3.0} = \frac{1 + \lambda(10)}{1 + \lambda(5)}$

$3.4[1 + 5\lambda] = 3.0[1 + 10\lambda]$

$3.4 - 3.0 = \lambda(3 \cdot 10 - (3.4) \cdot 5)$

$\Rightarrow \underline{\lambda = 0.0308}$

$r_o \approx \frac{1}{\lambda I_D} = \frac{1}{(0.0308)(3)} \Rightarrow \underline{r_o = 10.8 \text{ k}\Omega}$

6.4

$r_o = \frac{1}{\lambda I_D}$

$I_D = \frac{1}{\lambda r_o} = \frac{1}{(0.012)(100)} \Rightarrow \underline{I_D = 0.833 \text{ mA}}$

6.5

$A_v = -g_m(r_o \| R_D) = -(1)(50\|10)$

$\Rightarrow \underline{A_v = -8.33}$

6.6

a. $R_D = \frac{V_{DD} - V_{DSQ}}{I_{DQ}} = \frac{10 - 6}{0.5} \Rightarrow \underline{R_D = 8 \text{ k}\Omega}$

For $V_{GSQ} = 2$ V, for example,

$I_{DQ} = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)(V_{GSQ} - V_{TN})^2$

$0.5 = (0.030)\left(\frac{W}{L}\right)(2 - 0.8)^2$

$\Rightarrow \underline{\frac{W}{L} = 11.6}$

b. $g_m = 2\sqrt{K_n I_{DQ}} = 2K_n(V_{GSQ} - V_{TN})$

$g_m = 2(0.030)(11.6)(2 - 0.8)$

$\Rightarrow \underline{g_m = 0.835 \text{ mA/V}}$

$I_{DQ} = (0.030)(11.6)(2 - 0.8)^2 = 0.501$ mA

$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(0.501)} \Rightarrow \underline{r_o = 133 \text{ k}\Omega}$

c. $A_v = -g_m(r_o \| R_D) = -(0.835)(133\|8)$

$\Rightarrow \underline{A_v = -6.30}$

6.7

$K_n v_{gs}^2 = K_n[V_{gs} \sin \omega t]^2 = K_n V_{gs}^2 \sin^2 \omega t$

$\sin^2 \omega t = \frac{1}{2}[1 - \cos 2\omega t]$

So $K_n v_{gs}^2 = \frac{K_n V_{gs}^2}{2}[1 - \cos 2\omega t]$

Ratio of signal at 2ω to that at ω:

$$\frac{\frac{K_n V_{gs}^2}{2} \cdot \cos 2\omega t}{2K_n(V_{GSQ} - V_{TN})V_{gs} \cdot \sin \omega t}$$

The coefficient of this expression is then:

$$\underline{\frac{V_{gs}}{4(V_{GSQ} - V_{TN})}}$$

6.8

$0.01 = \frac{V_{gs}}{4(V_{GSQ} - V_{TN})}$

So $V_{gs} = (0.01)(4)(3 - 1)$

$\Rightarrow \underline{V_{gs} = 0.08 \text{ V}}$

6.9

$V_o = -g_m V_{gs}(r_o \| R_D)$

$V_{gs} = \frac{R_1 \| R_2}{R_1 \| R_2 + R_{Si}} \cdot V_i = \left(\frac{50}{50 + 2}\right) \cdot V_i = (0.962)V_i$

$A_v = -g_m(0.962)(r_o \| R_D) = -(1)(0.962)(50\|10) \Rightarrow$

$\underline{A_v = -8.02}$

6.10

$$A_v = -g_m(r_o \| R_D)$$
$$-10 = -g_m(100\|5)$$
$$\Rightarrow \underline{g_m = 2.1 \text{ mA/V}}$$

6.11

a. $V_G = \left(\frac{R_2}{R_1+R_2}\right)V_{DD}$

$$V_G = \left(\frac{200}{200+300}\right)(12) = 4.8 \text{ V}$$
$$I_D = \frac{V_G - V_{GS}}{R_S} = K_n(V_{GS} - V_{TN})^2$$
$$4.8 - V_{GS} = (1)(2)(V_{GS}^2 - 4V_{GS} + 4)$$
$$2V_{GS}^2 - 7V_{GS} + 3.2 = 0$$
$$V_{GS} = \frac{7 \pm \sqrt{(7)^2 - 4(2)(3.2)}}{2(2)} = 2.96 \text{ V}$$

$$I_D = (1)(2.96-2)^2 \Rightarrow \underline{I_D = 0.920 \text{ mA}}$$
$$V_{DS} = V_{DD} - I_D(R_D + R_S)$$
$$= 12 - (0.92)(3+2)$$
$$\Rightarrow \underline{V_{DS} = 7.4 \text{ V}}$$

(b) $V_o = \frac{-g_m V_{gs}(R_D \| R_L)}{1 + g_m R_S}$

where $V_{gs} = \frac{R_1\|R_2}{R_1\|R_2 + R_{Si}} \cdot V_i = \frac{300\|200}{300\|200 + 2} \cdot V_i$

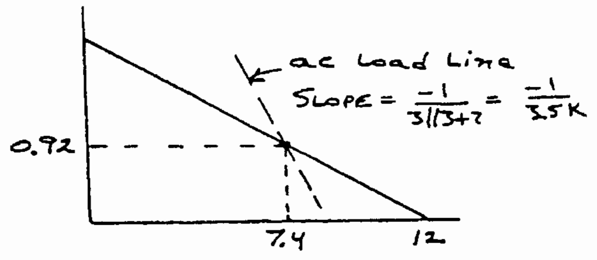
$$= \frac{120}{120+2} \cdot V_i = (0.984)V_i$$

Then

$$A_v = \frac{-g_m(0.984)(R_D\|R_L)}{1+g_m R_S}$$
$$g_m = 2(1)(2.96-2) = 1.92 \text{ mA/V}$$

So $A_v = \frac{-(1.92)(0.984)(3\|3)}{1+(1.92)(2)} \Rightarrow \underline{A_v = -0.586}$

c.



$$\Delta i_D = \frac{-1}{3.5 \text{ k}\Omega} \cdot \Delta v_{ds}$$
$$\Delta i_D = 0.92 \text{ mA}$$
$$\Rightarrow |\Delta v_{ds}| = (0.92)(3.5) = 3.22$$
$$\Rightarrow \underline{6.44 \text{ V peak-to-peak}}$$

6.12

a. $I_{DQ} = 3 \text{ mA} \Rightarrow V_S = I_{DQ}R_S = (3)(0.5) = 1.5 \text{ V}$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$
$$3 = (2)(V_{GS} - 2)^2 \Rightarrow V_{GS} = 3.22 \text{ V}$$
$$V_G = V_{GS} + V_S = 3.22 + 1.5 = 4.72 \text{ V}$$
$$V_G = \left(\frac{R_2}{R_1+R_2}\right)V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$
$$4.72 = \frac{1}{R_1}(200)(15) \Rightarrow \underline{R_1 = 636 \text{ k}\Omega}$$
$$\frac{636R_2}{636 + R_2} = 200 \Rightarrow \underline{R_2 = 292 \text{ k}\Omega}$$

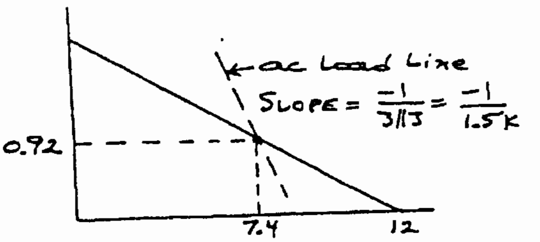
b. $A_v = \frac{-g_m(R_D\|R_L)}{1+g_m R_S}$

$$g_m = 2(2)(3.22-2) = 4.88 \text{ mA/V}$$
$$A_v = \frac{-(4.88)(2\|5)}{1+(4.88)(0.5)} \Rightarrow \underline{A_v = -2.03}$$

6.13

From Problem 6-11: $I_D = 0.920$ mA

$$V_{DS} = 7.4 \text{ V}$$
$$g_m = 1.92 \text{ mA/V}$$
$$A_v = -g_m(R_D\|R_L)\left(\frac{R_1\|R_2}{R_1\|R_2 + R_{Si}}\right)$$
$$= -(1.92)(3\|3)\left(\frac{200\|300}{200\|300+2}\right) = -(2.88)(0.984)$$
$$\underline{A_v = -2.83}$$



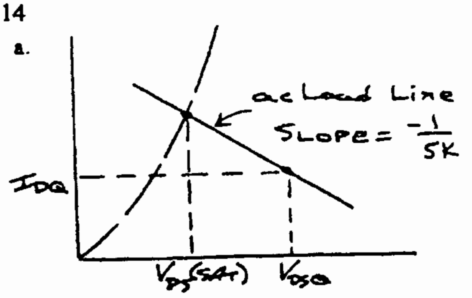
$\Delta i_D = \frac{-1}{1.5 \text{ k}\Omega} \cdot \Delta v_{DS}$, $\Delta i_D = 0.92$ mA

$$\Rightarrow |\Delta v_{DS}| = (0.92)(1.5) = 1.38$$

$\Rightarrow$ 2.76 V peak-to-peak output voltage swing

6.14

a.



$V_{DSQ} = V^+ - I_{DQ}R_D - (-V_{GSQ})$

Output Voltage Swing $= V_{DSQ} - V_{DS}(\text{sat})$

$= [V^+ - I_{DQ}R_D + V_{GSQ}] - (V_{GSQ} - V_{TN})$

$= V^+ - I_{DQ}R_D + V_{TN}$

So $|\Delta I_D| = I_{DQ} = \frac{1}{5\ k\Omega} \cdot |\Delta V_{DS}|$

$= \frac{1}{5\ k\Omega}[V^+ - I_{DQ}R_D + V_{TN}]$

$I_{DQ} = \frac{1}{5\ k\Omega}[5 - I_{DQ}(10) + 1]$

$= 1.2 - 2I_{DQ} = I_{DQ}$

$\Rightarrow \underline{I_{DQ} = 0.4\text{ mA} = I_Q}$

b. $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.4)} = 0.894\ mA/V$

$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(0.4)} = 250\ k\Omega$

$A_v = -g_m(R_D \| R_L \| r_0)$

$= -(0.894)(10\|10\|250)$

$\Rightarrow \underline{A_v = -4.38}$

6.15

a. $I_D = K_n(V_{GS} - V_{TN})^2$

$2 = 4(V_{GS} - (-1))^2$

$V_{GS} = -0.293\text{ V}$

$\Rightarrow V_S = 0.293\text{ V} = I_D R_S = (2)R_S$

$\Rightarrow \underline{R_S = 0.146\ k\Omega}$

$V_D = V_{DS} + V_S = 6 + 0.293 = 6.293$

$R_D = \frac{10 - 6.293}{2} \Rightarrow \underline{R_D = 1.85\ k\Omega}$

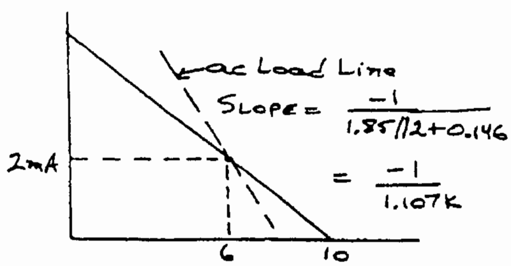
b. $A_v = \frac{-g_m(R_D \| R_L)}{1 + g_m R_S}$

$g_m = 2K_n(V_{GS} - V_{TN})$

$g_m = 2(4)(-0.293 + 1) = 5.66\text{ mA/V}$

$A_v = \frac{-(5.66)(1.85\|2)}{1 + (5.66)(0.146)} \Rightarrow \underline{A_v = -2.98}$

c.



$\Delta v_0 = (\Delta i_D)(1.85\|2) = (2)(1.85\|2)$

$= 1.92\text{ V}$

$\Delta v_i = \frac{1.92}{2.98} = 0.645 \Rightarrow \underline{V_i = 0.645\text{ V}}$

6.16

a. $V_{DSQ} = V_{DD} - I_{DQ}(R_D + R_S)$

$2.5 = 5 - I_{DQ}(2 + R_S)$

$I_{DQ} = \frac{2.5}{2 + R_S}$

$I_{DQ} = K_n(V_{GS} - V_{TN})^2 = \frac{-V_{GS}}{R_S}$

$\Rightarrow V_{GS} = -I_{DQ}R_S = \frac{-2.5R_S}{2 + R_S}$

$K_n\left[\frac{-2.5R_S}{2 + R_S} - V_{TN}\right]^2 = \frac{2.5}{2 + R_S}$

$4\left[1 - \frac{2.5R_S}{2 + R_S}\right]^2 = \frac{2.5}{2 + R_S}$

$4\left[\frac{2 + R_S - 2.5R_S}{2 + R_S}\right]^2 = \frac{2.5}{2 + R_S}$

$4\frac{(2 - 1.5R_S)^2}{2 + R_S} = 2.5$

$4(4 - 6R_S + 2.25R_S^2) = 2.5(2 + R_S)$

$9R_S^2 - 26.5R_S + 11 = 0$

$R_S = \frac{26.5 \pm \sqrt{(26.5)^2 - 4(9)(11)}}{2(9)}$

$R_S = 0.5\ k\Omega$ or $2.44\ k\Omega$

But $R_S = 2.44\ k\Omega \Rightarrow V_{GS} = -1.37$ Cut off.

$\Rightarrow \underline{R_S = 0.5\ k\Omega}, \quad \underline{I_{DQ} = 1.0\text{ mA}}$

b. $A_v = \frac{-g_m(R_D \| R_L)}{1 + g_m R_S}$

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(1)} = 4\ mA/V$

$A_v = \frac{-(4)(2\|2)}{1 + (4)(0.5)} \Rightarrow \underline{A_v = -1.33}$

6.17

a. $5 = I_{DQ}R_S + V_{SDQ} + I_{DQ}R_D - 5$

$5 = I_{DQ}R_S + 6 + I_{DQ}(10) - 5$

1. $\underline{I_{DQ} = \frac{4}{R_S + 10}}$

$V_S = V_{SDQ} + I_{DQ}R_D - 5 = V_{SGQ}$

2. $1 + I_{DQ}(10) = V_{SGQ}$

3. $I_{DQ} = K_p(V_{SGQ} - 2)^2$

Choose $\underline{R_S = 10\ k\Omega} \Rightarrow I_{DQ} = \frac{4}{20} = 0.20$ mA

$V_{SGQ} = 1 + (0.2)(10) = 3$ V

$0.20 = K_p(3-2)^2 \Rightarrow \underline{K_p = 0.20\ mA/V^2}$

b. $I_{DQ} = (0.20)(3-2)^2 = 0.20$ mA

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.2)(0.2)} = 0.4\ mA/V$

$A_v = -g_m(R_D \| R_L) = -(0.4)(10\|10)$

$\Rightarrow \underline{A_v = -2.0}$

c. Choose $\underline{R_S = 20\ k\Omega} \Rightarrow I_{DQ} = \frac{4}{30} = 0.133$ mA

$V_{SGQ} = 1 + (0.133)(10) = 2.33$ V

$0.133 = K_p(2.33-2)^2 \Rightarrow \underline{K_p = 1.22\ mA/V^2}$

$g_m = 2\sqrt{(1.22)(0.133)} = 0.806$ mA/V

$A_v = -(0.806)(10\|10) \Rightarrow \underline{A_v = -4.03}$

A larger gain can be achieved.

6.18

a. $I_{DQ} = 1 = K_p(V_{SGQ} + V_{TP})^2$

$1 = 5(V_{SGQ} - 1.5)^2 \Rightarrow V_{SGQ} = 1.95$ V

$R_S = \frac{5 - 1.95}{1} \Rightarrow \underline{R_S = 3.05\ k\Omega}$

$V_{SDQ} = 10 - I_{DQ}(R_S + R_D)$

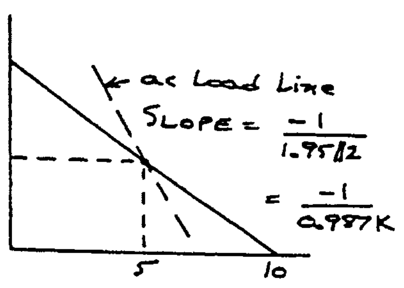
$5 = 10 - (1)(3.05 + R_D) \Rightarrow \underline{R_D = 1.95\ k\Omega}$

b. $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(5)(1)} = 4.47\ mA/V$

$A_v = -g_m(R_D \| R_L) = -(4.47)(1.95\|2)$

$\Rightarrow \underline{A_v = -4.41}$

c.



$\Delta i_D = -\frac{1}{0.987\ k\Omega} \cdot \Delta v_{DS}$

$\Rightarrow |\Delta v_{DS}| = (1)(0.987) = 0.987$ V

Swing in output voltage = 1.97 V peak-to-peak

6.19

$I_{DQ} = K_n(V_{GSQ} - V_{TN})^2$

$g_m = 2\sqrt{K_n I_{DQ}}$

$2.2 = 2\sqrt{K_n(6)} \Rightarrow \underline{K_n = 0.202\ mA/V^2}$

$6 = 0.202(2.8 - V_{TN})^2 \Rightarrow \underline{V_{TN} = -2.65\ V}$

$V_{DSQ} = 18 - I_{DQ}(R_S + R_D)$

$R_S + R_D = \frac{18 - 10}{6} = 1.33\ k\Omega \Rightarrow R_S = 1.33 - R_D$

$$A_v = -\frac{g_m(R_D \| R_L)}{1 + g_m R_S}$$

$$-1 = \frac{-2.2\left(\frac{R_D \cdot 1}{R_D + 1}\right)}{1 + (2.2)(1.33 - R_D)}$$

$1 + 2.93 - 2.2R_D = \frac{2.2R_D}{1 + R_D}$

$(3.93 - 2.2R_D)(1 + R_D) = 2.2R_D$

$3.93 + 1.73R_D - 2.2R_D^2 = 2.2R_D$

$2.2R_D^2 + 0.47R_D - 3.93 = 0$

$$R_D = \frac{-0.47 + \sqrt{(0.47)^2 + 4(2.2)(3.93)}}{2(2.2)}$$

$\Rightarrow \underline{R_D = 1.23\ k\Omega}, \quad \underline{R_S = 0.10\ k\Omega}$

$V_G = V_{GS} + V_S = 2.8 + (6)(0.1) = 3.4$ V

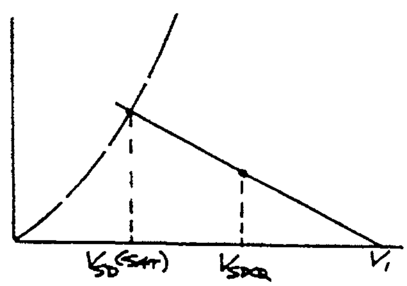
$V_G = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} = \frac{1}{R_1}(100)(18) = 3.4$

$\Rightarrow \underline{R_1 = 529\ k\Omega}$

$\frac{529R_2}{529 + R_2} = 100 \Rightarrow \underline{R_2 = 123\ k\Omega}$

6.20

a.



$V_1 = 9 + V_{SG}, \quad V_{SD}(sat) = V_{SG} + V_{TP}$

$$V_{SDQ} = \frac{V_1 - V_{SD}(sat)}{2} + V_{SD}(sat)$$

$$= \frac{(9 + V_{SG}) - (V_{SG} + V_{TP})}{2} + (V_{SG} + V_{TP})$$

$$= \frac{9 + 1.5}{2} + V_{SG} - 1.5$$

$V_{SDQ} = 3.75 + V_{SG} = 9 + V_{SG} - I_{DQ}R_D$

$I_{DQ} = K_p(V_{SG} + V_{TP})^2$

Set $R_D = 0.1R_L = 2\ \text{k}\Omega$

$3.75 = 9 - I_{DQ}(2) \Rightarrow I_{DQ} = 2.625\ \text{mA}$

b. $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(2.625)} = 4.58\ mA/V$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(2.625)} = 38.1\ \text{k}\Omega$$

Open circuit.

$A_v = -g_m(R_D \| r_0)$

$A_v = -4.58(2\|38.1) \Rightarrow \underline{A_v = -8.70}$

c. With R_L

$A_v = -4.58(2\|20\|38.1) \Rightarrow \underline{A_v = -7.95}$

$\Rightarrow$ Change = 8.62%

6.21

a. $I_{DQ} = K_p(V_{SG} + V_{TP})^2$

$2 = (0.5)(V_{SG} + 2)^2 \Rightarrow V_{SG} = 0\ \text{V}$

$R_S = \frac{10 - 0}{2} \Rightarrow \underline{R_S = 5\ \text{k}\Omega}$

$V_D = 0 - V_{SDQ} = 0 - 6 \Rightarrow R_D = \frac{-6 - (-10)}{2}$

$\Rightarrow \underline{R_D = 2\ \text{k}\Omega}$

$A_v = -g_m R_D$

b. $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.5)(2)} = 2\ mA/V$

$A_v = -(2)(2) \Rightarrow \underline{A_v = -4.0}$

6.22

$A_v = -g_m(R_D \| R_L)$

$V_{DSQ} = V_{DD} - I_{DQ}(R_S + R_D)$

$10 = 20 - (1)(R_S + R_D) \Rightarrow R_S + R_D = 10\ \text{k}\Omega$

Let $\underline{R_D = 8\ \text{k}\Omega}$. $\underline{R_S = 2\ \text{k}\Omega}$

$A_v = -10 = -g_m(8\|20)$

$g_m = 1.75\ mA/V = 2\sqrt{K_n I_{DQ}} = 2\sqrt{K_n(1)} \Rightarrow$

$\underline{K_n = 0.766\ mA/V^2}$

$V_S = I_{DQ}R_S = (1)(2) = 2\ \text{V}$

$I_{DQ} = K_n(V_{GS} - V_{TN})^2 \Rightarrow 1 = 0.766(V_{GS} - 2)^2$

$\Rightarrow V_{GS} = 3.31\ \text{V}$

$V_G = V_{GS} + V_S = 3.31 + 2 = 5.31$

$V_G = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$

$\Rightarrow \frac{1}{R_1}(200)(20) = 5.31 \Rightarrow \underline{R_1 = 753\ \text{k}\Omega}$

$\frac{753 R_2}{753 + R_2} = 200 \Rightarrow \underline{R_2 = 272\ \text{k}\Omega}$

6.23

$$A_v = \frac{g_m r_0}{1 + g_m r_0} = \frac{(4)(50)}{1 + (4)(50)} \Rightarrow \underline{A_v = 0.995}$$

$$R_0 = \frac{1}{g_m} \| r_0 = \frac{1}{4} \| 50 \Rightarrow \underline{R_0 = 0.249\ \text{k}\Omega}$$

$$A_v = \frac{g_m(r_0\|R_S)}{1 + g_m(r_0\|R_S)} = \frac{4(50\|2.5)}{1 + 4(50\|2.5)}$$

$$= \frac{4(2.38)}{1 + 4(2.38)} \Rightarrow \underline{A_v = 0.905}$$

$$R_0 = \frac{1}{g_m} \| r_0 \| R_S = \frac{1}{4} \| 50 \| 2.5$$

$\Rightarrow \underline{R_0 = 0.226\ \text{k}\Omega}$

6.24

a. $V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \left(\frac{396}{396 + 1240}\right)(10)$

$\Rightarrow V_G = 2.42\ \text{V}$

$$I_{DQ} = \frac{10 - (V_{SG} + 2.42)}{R_S} = K_p(V_{SG} + V_{TP})^2$$

$7.58 - V_{SG} = (2)(4)(V_{SG}^2 - 4V_{SG} + 4)$

$8V_{SG}^2 - 31V_{SG} + 24.4 = 0$

$$V_{SG} = \frac{31 \pm \sqrt{(31)^2 - 4(8)(24.4)}}{2(8)}$$

$\Rightarrow V_{SG} = 2.78\ \text{V}$

$I_{DQ} = (2)(2.78 - 2)^2 \Rightarrow \underline{I_{DQ} = 1.21\ \text{mA}}$

$V_{SDQ} = 10 - I_{DQ}R_S = 10 - (1.21)(4)$

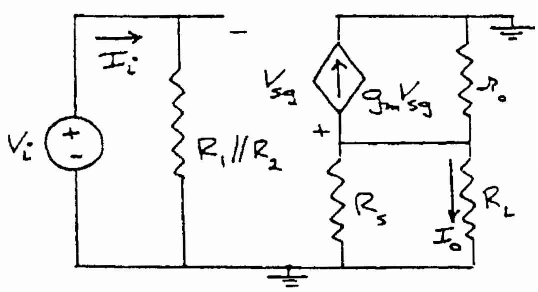
$\Rightarrow \underline{V_{SDQ} = 5.16\ \text{V}}$

b. $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(1.21)} = 3.11\ mA/V$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(1.21)} = 41.3\ \text{k}\Omega$$

$$A_v = \frac{g_m(R_S\|R_L\|r_0)}{1 + g_m(R_S\|R_L\|r_0)}$$

$$= \frac{3.11(4\|4\|41.3)}{1 + (3.11)(4\|4\|41.3)} \Rightarrow \underline{A_v = 0.886}$$



$$I_0 = -(g_m V_{sg})\left(\frac{R_S\|r_0}{R_S\|r_0 + R_L}\right)$$

$$V_{sg} = \frac{-V_i}{1 + g_m(R_S\|R_L\|r_0)}$$

$V_i = I_i(R_1\|R_2)$

$$A_i = \frac{I_o}{I_i} = \frac{g_m(R_1\|R_2)}{1+g_m(R_S\|R_L\|r_o)} \cdot \frac{R_S\|r_o}{R_S\|r_o + R_L}$$
$$= \frac{(3.11)(396\|1240)}{1+(3.11)(4\|4\|41.3)} \cdot \frac{4\|41.3}{4\|41.3+4}$$
$$= \frac{(3.11)(300)}{1+(3.11)(1.908)} \cdot \frac{3.647}{3.647+4}$$
$\Rightarrow \underline{A_v = 64.2}$

$$R_o = \frac{1}{g_m} \,\Big\|\, R_S \,\Big\|\, R_L \,\Big\|\, r_o = \frac{1}{3.11} \,\Big\|\, 4 \,\Big\|\, 4 \,\Big\|\, 41.3$$
$\Rightarrow \underline{R_o = 0.275\ \text{k}\Omega}$

6.25

$$g_m = 2\sqrt{K_n I_Q} = 2\sqrt{(1)(1)} = 2\ mA/V$$
$$r_o = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(1)} = 100\ \text{k}\Omega$$
$$A_v = \frac{g_m(r_o\|R_L)}{1+g_m(r_o\|R_L)} = \frac{2(100\|4)}{1+2(100\|4)}$$
$\Rightarrow \underline{A_v = 0.885}$

$$R_o = \frac{1}{g_m} \,\Big\|\, r_o = \frac{1}{2} \,\Big\|\, 100 \Rightarrow \underline{R_o = 0.490\ \text{k}\Omega}$$

6.26

a. $A_v = \dfrac{g_m R_L}{1+g_m R_L} \Rightarrow 0.95 = \dfrac{g_m(4)}{1+g_m(4)}$

$0.95 = 4(1-0.95)g_m \Rightarrow g_m = 4.75\ \text{mA/V}$

$$g_m = 2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)I_Q}$$
$$4.75 = 2\sqrt{(0.030)\left(\frac{W}{L}\right)(4)} \Rightarrow \underline{\frac{W}{L} = 47.0}$$

b. $g_m = 2\sqrt{\left(\dfrac{1}{2}\mu_n C_{ox}\right)\left(\dfrac{W}{L}\right)I_Q}$

$4.75 = 2\sqrt{(0.030)(60)I_Q} \Rightarrow \underline{I_Q = 3.13\ \text{mA}}$

6.27

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$
$5 = 5(V_{GS}+2)^2 \Rightarrow V_{GS} = -1\ \text{V}$

$\Rightarrow V_S = -V_{GS} = 1\ \text{V}$

$$I_{DQ} = \frac{V_S - (-5)}{R_S} \Rightarrow R_S = \frac{1+5}{5} \Rightarrow \underline{R_S = 1.2\ \text{k}\Omega}$$
$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(5)(5)} = 10\ mA/V$$
$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20\ \text{k}\Omega$$
$$A_v = \frac{g_m(r_o\|R_S\|R_L)}{1+g_m(r_o\|R_S\|R_L)}$$
$$= \frac{(10)(20\|1.2\|2)}{1+(10)(20\|1.2\|2)} \Rightarrow \underline{A_v = 0.878}$$
$$R_o = \frac{1}{g_m} \,\Big\|\, r_o \,\Big\|\, R_S = \frac{1}{10} \,\Big\|\, 20 \,\Big\|\, 1.2$$
$\Rightarrow \underline{R_o = 91.9\ \Omega}$

6.28

$$A_v = \frac{g_m R_S}{1+g_m R_S} \Rightarrow 0.90 = \frac{g_m(10)}{1+g_m(10)}$$
$0.90 = 10(1-0.90)g_m \Rightarrow \underline{g_m = 0.90\ \text{mA/V}}$

$$R_o = \frac{1}{g_m} \,\Big\|\, R_S = \frac{1}{0.90} \,\Big\|\, 10 \Rightarrow \underline{R_o = 1\ \text{k}\Omega}$$

With R_L :

$$A_v = \frac{g_m(R_S\|R_L)}{1+g_m(R_S\|R_L)} = \frac{(0.90)(10\|2)}{1+(0.90)(10\|2)}$$
$\Rightarrow \underline{A_v = 0.60}$

6.29

$$R_o = \frac{1}{g_m} \,\Big\|\, R_S$$

Output resistance determined primarily by g_m

Set $\dfrac{1}{g_m} = 0.2\ \text{k}\Omega \Rightarrow g_m = 5\ \text{mA/V}$

$$g_m = 2\sqrt{K_n I_{DQ}}$$
$\Rightarrow 5 = 2\sqrt{(4)I_{DQ}} \Rightarrow I_{DQ} = 1.56\ \text{mA}$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$
$1.56 = 4(V_{GS}+2)^2$

$V_{GS} = -1.38\ \text{V},\quad V_S = -V_{GS} = 1.38\ \text{V}$

$$R_S = \frac{1.38-(-5)}{1.56} \Rightarrow \underline{R_S = 4.09\ \text{k}\Omega}$$
$$A_v = \frac{g_m(R_S\|R_L)}{1+g_m(R_S\|R_L)} = \frac{5(4.09\|2)}{1+5(4.09\|2)}$$
$\Rightarrow \underline{A_v = 0.870}$

6.30

a. $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(5)(5)} = 10\ mA/V$

$$R_o = \frac{1}{g_m} = \frac{1}{10} \Rightarrow \underline{R_o = 100\ \Omega}$$

b. Open-circuit gain

$A_v = \dfrac{g_m r_o}{1+g_m r_o}$ But $r_o = \infty$ so $A_v = 1.0$

With R_L :

$$A_v = \frac{g_m R_L}{1+g_m R_L}$$
$$0.50 = \frac{10R_L}{1+10R_L} \Rightarrow 0.50 = 10(1-0.5)R_L$$
$\Rightarrow \underline{R_L = 0.1\ \text{k}\Omega}$

6.31

$$|\Delta i_D| = I_{DQ} = \frac{-1}{R_S\|R_L} \cdot \Delta v_0$$

$$\Delta v_0 = -I_{DQ} \cdot R_S\|R_L = -\frac{I_{DQ} \cdot R_S R_L}{R_S + R_L}$$

$$\underline{v_0(\min) = -\frac{I_{DQ}R_S}{1 + \frac{R_S}{R_L}}}$$

$$A_v = \frac{g_m(R_S\|R_L)}{1 + g_m(R_S\|R_L)} = \frac{v_0}{v_i}$$

$$v_i = \frac{-I_{DQ}(R_S\|R_L)[1 + g_m(R_S\|R_L)]}{g_m(R_S\|R_L)}$$

$$\underline{v_i(\min) = -\frac{I_{DQ}}{g_m}[1 + g_m(R_S\|R_L)]}$$

6.32

a. $V_{DSQ} = V_{DD} - I_{DQ}R_S$

$3 = 5 - (1.7)R_S \Rightarrow \underline{R_S = 1.18\ \text{k}\Omega}$

$I_{DQ} = K_n(V_{GS} - V_{TN})^2$

$1.7 = (1)(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.30\ \text{V}$

$V_S = V_{DD} - V_{DSQ} = 5 - 3 = 2\ \text{V}$

$V_G = V_S + V_{GS} = 2 + 2.30 = 4.30\ \text{V}$

$V_G = \frac{1}{R_1} \cdot R_i \cdot V_{DD} = \frac{1}{R_1}(300)(5) = 4.30$

$\Rightarrow \underline{R_1 = 349\ \text{k}\Omega}$

$\frac{349R_2}{349 + R_2} = 300 \Rightarrow \underline{R_2 = 2.14\ \text{M}\Omega}$

b. $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(1.7)} = 2.61\ mA/V$

$A_v = \frac{g_m R_S}{1 + g_m R_S} = \frac{2.61(1.18)}{1 + (2.61)(1.18)}$

$\Rightarrow \underline{A_v = 0.755}$

$R_0 = \frac{1}{g_m}\Big\| R_S = \frac{1}{2.61}\Big\| 1.18 = 0.383\|1.18$

$\Rightarrow \underline{R_0 = 0.289\ \text{k}\Omega}$

6.33

a. $V_{GS} + I_{DQ}R_S = 5$

$I_{DQ} = \frac{5 - V_{GS}}{R_S} = K_n(V_{GS} - V_{TN})^2$

$5 - V_{GS} = (10)(3)(V_{GS}^2 - 2V_{GS} + 1)$

$30V_{GS}^2 - 59V_{GS} + 25 = 0$

$V_{GS} = \frac{59 \pm \sqrt{(59)^2 - 4(30)(25)}}{2(30)} \Rightarrow \underline{V_{GS} = 1.35\ \text{V}}$

$I_{DQ} = (3)(1.35 - 1)^2 \Rightarrow \underline{I_{DQ} = 0.365\ \text{mA}}$

$V_{DSQ} = 10 - (0.365)(5 + 10) \Rightarrow \underline{V_{DSQ} = 4.53\ \text{V}}$

b. $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(3)(0.365)} \Rightarrow$

$\underline{g_m = 2.09\ mA/V}$

$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0)(0.365)} \Rightarrow \underline{r_0 = \infty}$

c. $A_v = g_m(R_D\|R_L) = (2.09)(5\|4)$

$\Rightarrow \underline{A_v = 4.64}$

6.34

a. $I_{DQ} = K_p(V_{SG} + V_{TP})^2$

$0.75 = (0.5)(V_{SG} - 1)^2 \Rightarrow V_{SG} = 2.22\ \text{V}$

$5 = I_{DQ}R_S + V_{SG} \Rightarrow R_S = \frac{5 - 2.22}{0.75}$

$\Rightarrow \underline{R_S = 3.71\ \text{k}\Omega}$

$V_{SDQ} = 10 - I_{DQ}(R_S + R_D)$

$6 = 10 - (0.75)(3.71 + R_D) \Rightarrow \underline{R_D = 1.62\ \text{k}\Omega}$

b. $R_i = \frac{1}{g_m}$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.5)(0.75)} = 1.22\ mA/V$

$R_i = \frac{1}{1.22} \Rightarrow \underline{R_i = 0.816\ \text{k}\Omega}$

$R_0 = R_D \Rightarrow \underline{R_0 = 1.62\ \text{k}\Omega}$

c. $i_0 = \left(\frac{R_D}{R_D + R_L}\right)\left(\frac{R_S}{R_S + [1/g_m]}\right) \cdot i_i$

$i_0 = \left(\frac{1.62}{1.62 + 2}\right)\left(\frac{3.71}{3.71 + 0.816}\right)i_i$

$i_0 = 0.367i_i \Rightarrow \underline{i_0 = 1.84 \sin \omega t\ (\mu\text{A})}$

$v_0 = i_0 R_L = (1.84)(2) \sin \omega t$

$\Rightarrow \underline{v_0 = 3.68 \sin \omega t\ (\text{mV})}$

6.35

a. $I_{DQ} = K_n(V_{GS} - V_{TN})^2$

$5 = 4(V_{GS} - 2)^2 \Rightarrow V_{GS} = 3.12\ \text{V}$

$V^+ = I_{DQ}R_D + V_{DSQ} - V_{GS}$

$10 = (5)R_D + 12 - 3.12 \Rightarrow \underline{R_D = 0.224\ \text{k}\Omega}$

b. $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(5)} \Rightarrow \underline{g_m = 8.94\ mA/V}$

$R_i = \frac{1}{g_m} = \frac{1}{8.94} \Rightarrow \underline{R_i = 0.112\ \text{k}\Omega}$

c. $A_v = g_m(R_D\|R_L) = (8.94)(0.224\|2)$

$\Rightarrow \underline{A_v = 1.80}$

6.36

a. $I_{DQ} = K_p(V_{SG} + V_{TP})^2$

$3 = 2(V_{SG} - 2)^2 \Rightarrow V_{SG} = 3.22$ V

$V^+ = I_{DQ}R_S + V_{SG}$

$R_S = \frac{10 - 3.22}{3} \Rightarrow \underline{R_S = 2.26 \text{ k}\Omega}$

$V_{SDQ} = 20 - I_{DQ}(R_S + R_D)$

$10 = 20 - (3)(2.26 + R_D) \Rightarrow \underline{R_D = 1.07 \text{ k}\Omega}$

b. $A_v = g_m(R_D \| R_L)$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(3)} = 4.90\ mA/V$

$A_v = (4.90)(1.07\|10) \Rightarrow \underline{A_v = 4.74}$

6.37

a. $|A_v| = \sqrt{\frac{(W/L)_D}{(W/L)_L}} = 5$

$\Rightarrow (W/L)_D = 25(0.5) \Rightarrow \underline{(W/L)_D = 12.5}$

$K_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_D = (30)(12.5) = 375\ \mu A/V^2$

$K_L = (30)(0.5) = 15\ \mu A/V^2$

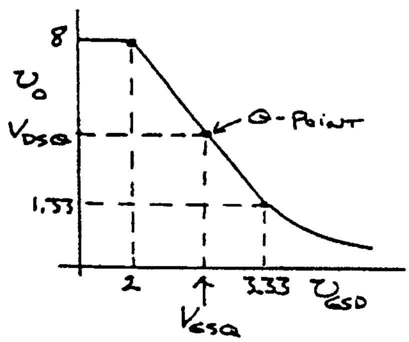
Transition point:

$v_{GSD} - V_{TND} = (V_{DD} - V_{TNL}) - \sqrt{\frac{K_D}{K_L}}(v_{GSD} - V_{TND})$

$v_{GSD} - 2 = (10 - 2) - \sqrt{\frac{375}{15}}(v_{GSD} - 2)$

$v_{GSD}(1 + 5) = (10 - 2) + 2 + 5(2)$

$v_{GSD} = 3.33$ V and $v_{DSD} = 1.33$ V



$V_{GSQ} = \frac{3.33 - 2}{2} + 2 \Rightarrow \underline{V_{GSQ} = 2.67 \text{ V}}$

b. $I_{DQ} = K_D(V_{GSQ} - V_{TND})^2$

$I_{DQ} = 0.375(2.67 - 2)^2 \Rightarrow \underline{I_{DQ} = 0.167 \text{ mA}}$

$V_{DSQ} = \frac{8 - 1.33}{2} + 1.33 \Rightarrow \underline{V_{DSQ} = 4.67 \text{ V}}$

6.38

(a) Transition point: Load:

$v_{OB} = V_{DD} - |V_{TNL}| = 10 - 2 = 8\ V$

Driver:

$K_D\left[(v_{OA})^2(1 + \lambda_D v_{OA})\right]$

$= K_L\left[(-V_{TNL})^2(1 + \lambda_L(V_{DD} - v_{OA}))\right]$

$0.5[v_{OtA}^2 + (0.02)v_{OtA}^3]$

$= 0.1[(4)(1 + 0.02(10 - v_{OtA}))]$

We have

$0.01v_{OtA}^3 + 0.5v_{OtA}^2 + 0.008v_{OtA} - 0.48 = 0$

Therefore $v_{OtA} = 0.963$ V

Now

$v_{OQ} = \frac{8 - 0.963}{2} + 0.963 = 4.48 \text{ V} = V_{DSQ}$

Then

$K_D\left[(V_{GSD} - V_{TND})^2(1 + \lambda_D v_{OQ})\right]$

$= K_L\left[(-V_{TNL})^2(1 + \lambda_L(V_{DD} - v_{OQ}))\right]$

$0.5[(V_{GSD} - 1.2)^2(1 + (0.02)(4.48))]$

$= 0.1[(4)(1 + 0.02(10 - 4.48))]$

which yields $\underline{V_{GSD} = 2.103 \text{ V}}$

b. $I_{DQ} = K_D\left[(V_{GSD} - V_{TND})^2(1 + \lambda_D v_{OQ})\right]$

$I_{DQ} = 0.5[(2.103 - 1.2)^2(1 + (0.02)(4.48))]$

So $\underline{I_{DQ} = 0.444 \text{ mA}}$

c. $r_{OD} = r_{OL} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.444)} = 113 \text{ k}\Omega$

$g_{mD} = 2K_D(V_{GSD} - V_{TND}) = 2(0.5)(2.103 - 1.2) \Rightarrow$

$g_{mD} = 0.903\ mA/V$

Then

$A_v = -g_{mD}(r_{OD}\|r_{OL}) = -(0.903)(113\|113)$

or $\underline{A_v = -51.0}$

6.39

$I_D = K_n(V_{GS} - V_{TN})^2, \quad V_{DS} = V_{GS}$

$I_D = 0$ when $V_{DS} = 1.5\,V \Rightarrow V_{TN} = 1.5\,V$

$0.8 = K_n(3 - 1.5)^2 \Rightarrow K_n = 0.356\,mA/V^2$

$$g_o = \frac{dI_D}{dV_{DS}} = \frac{1}{R_o} = 2K_n(V_{DS} - V_{TN})$$

$$= 2(0.356)(3 - 1.5)$$

$\Rightarrow \underline{R_o = 0.936\,k\Omega}$

6.40

a.

$I_{DQ} = K_{nD}(V_{GS} - V_{TND})^2 = (0.5)(0 - (-1))^2$

$\underline{I_{DQ} = 0.5\,mA}$

$I_{DQ} = K_{nL}(V_{GSL} - V_{TNL})^2 = K_{nL}(V_{DD} - V_O - V_{TNL})^2$

$0.5 = 0.030(10 - V_0 - 1)^2$

$\sqrt{\frac{0.5}{0.030}} = 9 - V_0 \Rightarrow \underline{V_0 = 4.92\text{ V}}$

b. $I_{DD} = I_{DL}$

$K_{nD}(V_i - V_{TND})^2 = K_{nL}(V_{DD} - V_o - V_{TNL})^2$

$\sqrt{\frac{K_{nD}}{K_{nL}}}(V_i - V_{TND}) = V_{DD} - V_o - V_{TNL}$

$V_o = V_{DD} - V_{TNL} - \sqrt{\frac{K_{nD}}{K_{nL}}}(V_i - V_{TND})$

$$A_v = \frac{dV_o}{dV_i} = -\sqrt{\frac{K_{nD}}{K_{nL}}} = -\sqrt{\frac{(W/L)_D}{(W/L)_L}}$$

$A_v = -\sqrt{\frac{500}{30}} \Rightarrow \underline{A_v = -4.08}$

6.41

(a) $I_{DQ} = K_L(V_{GSL} - V_{TNL})^2 = K_L(V_{DSL} - V_{TNL})^2$

$I_D = (0.1)(4 - 1)^2 = 0.9\,mA$

$I_{DQ} = K_D(V_{GSD} - V_{TND})^2$

$0.9 = (1)(V_{GSD} - 1)^2 \Rightarrow V_{GSD} = 1.95\text{ V}$

$V_{GG} = V_{GSD} + V_{DSL} = 1.95 + 4$

$\Rightarrow \underline{V_{GG} = 5.95\text{ V}}$

b. $I_{DD} = I_{DL}$

$K_D(V_{GSD} - V_{TND})^2 = K_L(V_{GSL} - V_{TNL})^2$

$\sqrt{\frac{K_D}{K_L}}(V_{GG} + V_i - V_o - V_{TND}) = V_o - V_{TNL}$

$V_o\left(1 + \sqrt{\frac{K_D}{K_L}}\right) = \sqrt{\frac{K_D}{K_L}}(V_{GG} + V_i - V_{TND}) + V_{TNL}$

$$A_v = \frac{dV_o}{dV_i} = \frac{\sqrt{K_D/K_L}}{1 + \sqrt{K_D/K_L}} \Rightarrow \underline{A_v = \frac{1}{1 + \sqrt{K_L/K_D}}}$$

(c) From Problem 6.39:

$$R_{LD} = \frac{1}{2K_L(V_{DSL} - V_{TNL})}$$

$$= \frac{1}{2(0.1)(4 - 1)} = 1.67\,k\Omega$$

$g_m = 2\sqrt{K_D I_{DQ}} = 2\sqrt{(1)(0.9)} = 1.90\,mA/V$

$$A_v = \frac{g_m(R_{LD}\|R_L)}{1 + g_m(R_{LD}\|R_L)} = \frac{(1.90)(1.67\|4)}{1 + (1.90)(1.67\|4)}$$

$\Rightarrow \underline{A_v = 0.691}$

6.42

a. From Problem 6-41:

$$A_v = \frac{g_m(R_{LD}\|R_L)}{1 + g_m(R_{LD}\|R_L)} = \frac{(1.90)(1.67\|10)}{1 + (1.90)(1.67\|10)}$$

$\underline{A_v = 0.731}$

b. $R_0 = \frac{1}{g_m} \| R_{LD} = \frac{1}{1.90} \| 1.67 = 0.526\|1.67$

$\underline{R_0 = 0.40\text{ k}\Omega}$

6.43

a. $I_{D1} = K_1(V_{GS1} - V_{TN1})^2$

$0.4 = 0.1(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 4\text{ V}$

$V_{S1} = I_{D1}R_{S1} = (0.4)(4) = 1.6\text{ V}$

$V_{G1} = V_{S1} + V_{GS1} = 1.6 + 4 = 5.6\text{ V}$

$V_{G1} = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$

$5.6 = \frac{1}{R_1}(200)(10) \Rightarrow \underline{R_1 = 357\text{ k}\Omega}$

$\frac{357R_2}{357 + R_2} = 200 \Rightarrow \underline{R_2 = 455\text{ k}\Omega}$

$V_{DS1} = V_{DD} - I_{D1}(R_{S1} + R_{D1})$

$4 = 10 - (0.4)(4 + R_{D1}) \Rightarrow \underline{R_{D1} = 11\text{ k}\Omega}$

$V_{D1} = 10 - (0.4)(11) = 5.6\text{ V}$

$I_{D2} = K_2(V_{SG2} + V_{TP2})^2$

$2 = 1(V_{SG2} - 2)^2 \Rightarrow V_{SG2} = 3.41\text{ V}$

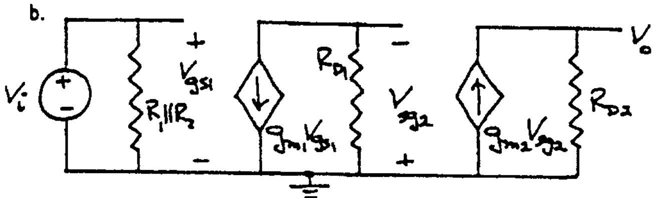
$V_{S2} = V_{D1} + V_{SG2} = 5.6 + 3.41 = 9.01$

$R_{S2} = \frac{10 - 9.01}{2} \Rightarrow \underline{R_{S2} = 0.495\text{ k}\Omega}$

$V_{SD2} = V_{DD} - I_{D2}(R_{S2} + R_{D2})$

$5 = 10 - (2)(0.495 + R_{D2}) \Rightarrow \underline{R_{D2} = 2.01\text{ k}\Omega}$

b.



$V_0 = g_{m2}V_{sg2}R_{D2} = (g_{m2}R_{D2})(g_{m1}V_{gs1}R_{D1})$

$V_{gs1} = V_i$

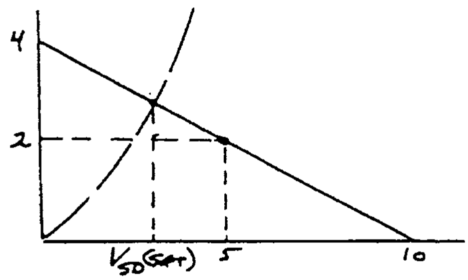
$A_v = \frac{V_0}{V_i} = g_{m1}g_{m2}R_{D1}R_{D2}$

$g_{m1} = 2\sqrt{K_1 I_{D1}} = 2\sqrt{(0.1)(0.4)} = 0.4\ mA/V$

$g_{m2} = 2\sqrt{K_2 I_{D2}} = 2\sqrt{(1)(2)} = 2.83\ mA/V$

$A_v = (0.4)(2.83)(11)(2.01) \Rightarrow \underline{A_v = 25.0}$

c.



$V_{SD}(\text{sat}) = V_{SG} + V_{Th}$

$= V_{DD} - I_{D2}(R_{D2} + R_{S2})$

$= V_{DD} - k_{p2}(R_{D2} + R_{S2})V_{SD}^2(\text{sat})$

So

$(1)(2.01 + 0.495)V_{SD}^2(\text{sat}) + V_{SD}(\text{sat}) - V_{DD} = 0$

$2.505V_{SD}^2(\text{sat}) + V_{SD}(\text{sat}) - 10 = 0$

$V_{SD}(\text{sat}) = \frac{-1 \pm \sqrt{1 + 4(2.505)(10)}}{2(2.505)}$

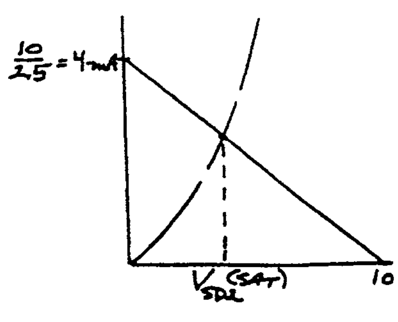
$V_{SD}(\text{sat}) = 1.81\ \text{V}$

$5 - 1.81 = 3.19$

$\Rightarrow$ Max. output swing = 6.38 V peak-to-peak

6.44

a.



$V_{SD2}(sat) = V_{DD} - I_{D2}(R_{D2} + R_{S2})$

$= V_{DD} - K_{p2}(R_{D2} + R_{S2})V_{SD2}^2(sat)$

$(1)(2 + 0.5)V_{SD2}^2(\text{sat}) + V_{SD2}(\text{sat}) - 10 = 0$

$2.5V_{SD2}^2(\text{sat}) + V_{SD2}(\text{sat}) - 10 = 0$

$V_{SD2}(\text{sat}) = \frac{-1 \pm \sqrt{1 + 4(2.5)(10)}}{2(2.5)} = 1.81\ \text{V}$

$V_{SDQ2} = \frac{10 - 1.81}{2} + 1.81 \Rightarrow V_{SDQ2} = 5.91\ \text{V}$

$V_{SDQ2} = V_{DD} - I_{DQ2}(R_{D2} + R_{S2})$

$5.91 = 10 - I_{DQ2}(2 + 0.5) \Rightarrow I_{DQ2} = 1.64\ \text{mA}$

$V_{S2} = 10 - (1.64)(0.5) = 9.18\ \text{V}$

$I_{DQ2} = K_{p2}(V_{SG2} + V_{TP2})^2$

$1.64 = (1)(V_{SG2} - 2)^2 \Rightarrow V_{SG2} = 3.28\ \text{V}$

$V_{D1} = V_{S2} - V_{SG2} = 9.18 - 3.28 = 5.90\ \text{V}$

$R_{D1} = \frac{10 - 5.90}{0.4} \Rightarrow \underline{R_{D1} = 10.25\ \text{k}\Omega}$

$I_{DQ1} = K_{n1}(V_{GS1} - V_{TN1})^2$

$0.4 = (0.1)(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 4\ \text{V}$

$V_{S1} = (0.4)(1) = 0.4\ \text{V}$

$V_{G1} = V_{S1} + V_{GS1} = 0.4 + 4 = 4.4\ \text{V}$

$V_{G1} = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$

$4.4 = \frac{1}{R_1} \cdot (200)(10) \Rightarrow \underline{R_1 = 455\ \text{k}\Omega}$

$\frac{455R_2}{455 + R_2} = 200 \Rightarrow \underline{R_2 = 357\ \text{k}\Omega}$

b. $\underline{I_{DQ2} = 1.64\ \text{mA}}$

$V_{SDQ2} = 10 - (1.64)(2 + 0.5) \Rightarrow \underline{V_{SDQ2} = 5.90\ \text{V}}$

$V_{DSQ1} = 10 - (0.4)(10.25 + 1) \Rightarrow \underline{V_{DSQ1} = 5.50\ \text{V}}$

(c) $g_{m1} = 2\sqrt{K_{n1}I_{DQ1}} = 2\sqrt{(0.1)(0.4)} = 0.4\ mA/V$

$g_{m2} = 2\sqrt{K_{p2}I_{DQ2}} = 2\sqrt{(1)(1.64)} = 2.56\ mA/V$

$A_v = g_{m1}g_{m2}R_{D1}R_{D2} = (0.4)(2.56)(10.25)(2)$

$\Rightarrow \underline{A_v = 21.0}$

6.45

a. $V_{DSQ2} = 7 = V_{DD} - I_{DQ2}R_{S2} = 10 - I_{DQ2}(6)$

$I_{DQ2} = 0.5\ \text{mA}$

$I_{DQ2} = K_2(V_{GS2} - V_{TN2})^2$

$0.5 = (0.2)(V_{GS2} - 0.8)^2 \Rightarrow V_{GS2} = 2.38\ \text{V}$

$V_{S1} = V_{S2} + V_{GS2} = 3 + 2.38 = 5.38$

$I_{DQ1} = \frac{V_{S1}}{R_{S1}} = \frac{5.38}{20} = 0.269\ \text{mA}$

$I_{DQ1} = K_1(V_{GS1} - V_{TN1})^2$

$0.269 = (0.2)(V_{GS1} - 0.8)^2 \Rightarrow V_{GS1} = 1.96\ \text{V}$

$V_{G1} = V_{S1} + V_{GS1} = 5.38 + 1.96 = 7.34\ \text{V}$

$V_{G1} = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$

$7.34 = \frac{1}{R_1}(400)(10) \Rightarrow \underline{R_1 = 545\ \text{k}\Omega}$

$\frac{545 R_2}{545 + R_2} = 400 \Rightarrow \underline{R_2 = 1.50\ \text{M}\Omega}$

b. $\underline{I_{DQ1} = 0.269\ \text{mA}}$, $\underline{I_{DQ2} = 0.5\ \text{mA}}$

$V_{DSQ1} = 10 - (0.269)(20) \Rightarrow \underline{V_{DSQ1} = 4.62\ \text{V}}$

c. $A_v = \frac{g_{m1} R_{S1}}{1 + g_{m1} R_{S1}} \cdot \frac{g_{m2} R_{S2}}{1 + g_{m2} R_{S2}}$

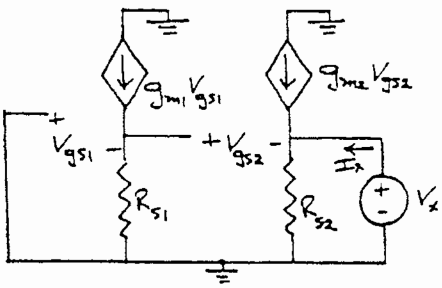
$g_{m1} = 2\sqrt{K_1 I_{DQ1}} = 2\sqrt{(0.2)(0.269)} = 0.464\ mA/V$

$g_{m2} = 2\sqrt{K_2 I_{DQ2}} = 2\sqrt{(0.2)(0.5)} = 0.632\ mA/V$

$A_v = \frac{(0.464)(20)}{1 + (0.464)(20)} \cdot \frac{(0.632)(6)}{1 + (0.632)(6)}$

$= (0.9027)(0.7913)$

$= \underline{A_v = 0.714}$



$R_0 = \frac{1}{g_{m2}} \| R_{S2} = \frac{1}{0.632} \| 6 = 1.582 \| 6$

$\Rightarrow \underline{R_0 = 1.25\ \text{k}\Omega}$

6.46

(a) $I_{DQ1} = \frac{10 - V_{GS1}}{R_{S2}} = K_{n1}(V_{GS1} - V_{TN1})^2$

$10 - V_{GS1} = (4)(10)(V_{GS1}^2 - 4V_{GS1} + 4)$

$40V_{GS1}^2 - 159V_{GS1} + 150 = 0$

$V_{GS1} = \frac{159 \pm \sqrt{(159)^2 - 4(40)(150)}}{2(40)}$

$\Rightarrow V_{GS1} = 2.435\ \text{V}$

$I_{DQ1} = (4)(2.435 - 2)^2 \Rightarrow \underline{I_{DQ1} = 0.757\ \text{mA}}$

$V_{DSQ1} = 20 - (0.757)(10) \Rightarrow \underline{V_{DSQ1} = 12.4\ \text{V}}$

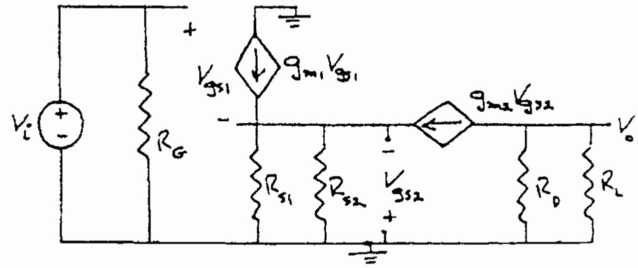
Also $\underline{I_{DQ2} = 0.757\ \text{mA}}$

$V_{DSQ2} = 20 - (0.757)(10 + 5) \Rightarrow \underline{V_{DSQ2} = 8.65\ \text{V}}$

(b) $g_{m1} = g_{m2} = 2\sqrt{KI_{DQ}} = 2\sqrt{(4)(0.757)} \Rightarrow$

$\underline{g_{m1} = g_{m2} = 3.48\ mA/V}$

c.



$V_0 = -(g_{m2} V_{gs2})(R_D \| R_L)$

$V_{gs2} = (-g_{m1} V_{gs1} - g_{m2} V_{gs2})(R_{S1} \| R_{S2})$

$V_i = V_{gs1} - V_{gs2} \Rightarrow V_{gs1} = V_i + V_{gs2}$

$V_{gs2} + g_{m2} V_{gs2}(R_{S1} \| R_{S2})$

$= -g_{m1}(V_i + V_{gs2})(R_{S1} \| R_{S2})$

$V_{gs2} + g_{m2} V_{gs2}(R_{S1} \| R_{S2}) + g_{m1} V_{gs2}(R_{S1} \| R_{S2})$

$= -g_{m1} V_i (R_{S1} \| R_{S2})$

$V_{gs2} = \frac{-g_{m1} V_i (R_{S1} \| R_{S2})}{1 + g_{m2}(R_{S1} \| R_{S2}) + g_{m1}(R_{S1} \| R_{S2})}$

$A_v = \frac{V_0}{V_i} = \frac{g_{m1} g_{m2} (R_{S1} \| R_{S2})(R_D \| R_L)}{1 + (g_{m1} + g_{m2})(R_{S1} \| R_{S2})}$

$A_v = \frac{(3.48)^2 (10 \| 10)(5 \| 2)}{1 + (3.48 + 3.48)(10 \| 10)}$

$\Rightarrow \underline{A_v = 2.42}$

6.47

a. $I_{DQ} = 3\ \text{mA}$

$V_{S1} = I_{DQ} R_S - 5 = (3)(1.2) - 5 = -1.4\ \text{V}$

$I_{DQ} = K_1 (V_{GS} - V_{TN})^2$

$3 = 2(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.22\ \text{V}$

$V_{G1} = V_{GS} + V_{S1} = 2.22 - 1.4 = 0.82\ \text{V}$

$V_{G1} = \left(\frac{R_3}{R_1 + R_2 + R_3}\right)(5) \Rightarrow 0.82 = \left(\frac{R_3}{500}\right)(5)$

$\Rightarrow \underline{R_3 = 82\ \text{k}\Omega}$

$V_{D1} = V_{S1} + V_{DSQ1} = -1.4 + 2.5 = 1.1\ \text{V}$

$V_{G2} = V_{D1} + V_{GS} = 1.1 + 2.22 = 3.32\ \text{V}$

$V_{G2} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right)(5)$

$\Rightarrow 3.32 = \left(\frac{R_2 + R_3}{500}\right)(5)$

$R_2 + R_3 = 332 \Rightarrow \underline{R_2 = 250\ \text{k}\Omega}$

$R_1 = 500 - 250 - 82 \Rightarrow \underline{R_1 = 168\ k\Omega}$

$V_{D2} = V_{D1} + V_{DSQ2} = 1.1 + 2.5 = 3.6\ V$

$R_D = \frac{5 - 3.6}{3} \Rightarrow \underline{R_D = 0.467\ k\Omega}$

b. $A_v = -g_{m1} R_D$

$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(3)} = 4.90\ mA/V$

$A_v = -(4.90)(0.467) \Rightarrow \underline{A_v = -2.29}$

6.48

a. $V_{S1} = I_{DQ} R_S - 10 = (5)(2) - 10 \Rightarrow V_{S1} = 0$

$I_{DQ} = K_1(V_{GS1} - V_{TN})^2$

$5 = 4(V_{GS1} - 1.5)^2 \Rightarrow V_{GS1} = 2.618\ V$

$V_{G1} = V_{GS1} + V_{S1} = 2.618\ V = IR_3 = (0.1)R_3$

$\Rightarrow \underline{R_3 = 26.2\ k\Omega}$

$V_{D1} = V_{S1} + V_{DSQ1} = 0 + 3.5 = 3.5\ V$

$V_{G2} = V_{D1} + V_{GS} = 3.5 + 2.62 = 6.12\ V$

$= (0.1)(R_2 + R_3)$

$R_2 + R_3 = 61.2\ k\Omega \Rightarrow \underline{R_2 = 35\ k\Omega}$

$V_{D2} = V_{D1} + V_{DSQ2} = 3.5 + 3.5 = 7.0\ V$

$R_D = \frac{10 - 7}{5} \Rightarrow \underline{R_D = 0.6\ k\Omega}$

$R_1 = \frac{10 - 6.12}{0.1} \Rightarrow \underline{R_1 = 38.8\ k\Omega}$

b. $A_v = -g_{m1} R_D$

$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(5)} = 8.94\ mA/V$

$A_v = -(8.94)(0.6) \Rightarrow \underline{A_v = -5.36}$

6.49

a. $I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$4 = 6\left(1 - \frac{V_{GS}}{(-3)}\right)^2$

$V_{GS} = (-3)\left[1 - \sqrt{\frac{4}{6}}\right] \Rightarrow \underline{V_{GS} = -0.551\ V}$

$V_{DSQ} = V_{DD} - I_{DQ} R_D$

$6 = 10 - (4)R_D \Rightarrow \underline{R_D = 1\ k\Omega}$

b. $g_m = \frac{2I_{DSS}}{(-V_P)}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(6)}{3}\left(1 - \frac{-0.551}{-3}\right)$

$\Rightarrow \underline{g_m = 3.27\ mA/V}$

$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(4)} \Rightarrow \underline{r_0 = 25\ k\Omega}$

c. $A_v = -g_m(r_0 \| R_D) = -(3.27)(25\|1)$

$\Rightarrow \underline{A_v = -3.14}$

6.50

$V_{GS} + I_{DQ}(R_{S1} + R_{S2}) = 0$

$I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$V_{GS} + I_{DSS}(R_{S1} + R_{S2})\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 0$

$V_{GS} + (2)(0.1 + 0.25)\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 0$

$V_{GS} + 0.7\left(1 - \frac{2V_{GS}}{(-2)} + \frac{V_{GS}^2}{(-2)^2}\right) = 0$

$0.175V_{GS}^2 + 1.7V_{GS} + 0.7 = 0$

$V_{GS} = \frac{-1.7 \pm \sqrt{(1.7)^2 - 4(0.175)(0.7)}}{2(0.175)}$

$\Rightarrow V_{GS} = -0.431\ V$

$g_m = \frac{2I_{DSS}}{-V_P}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(2)}{2}\left(1 - \frac{-0.431}{-2}\right)$

$\Rightarrow \underline{g_m = 1.57\ mA/V}$

$A_v = \frac{-g_m(R_D \| R_L)}{1 + g_m R_{S1}} = \frac{-(1.57)(8\|4)}{1 + (1.57)(0.1)}$

$\Rightarrow \underline{A_v = -3.62}$

$A_i = \frac{i_0}{i_i} = \frac{(v_0/R_L)}{(v_i/R_G)} = \frac{v_0}{v_i} \cdot \frac{R_G}{R_L} = (-3.62)\left(\frac{50}{4}\right)$

$\Rightarrow \underline{A_i = -45.3}$

6.51

$I_{DQ} = \frac{I_{DSS}}{2} = 4\ mA$

$V_{DSQ} = \frac{V_{DD}}{2} = 10\ V$

$V_{DSQ} = V_{DD} - I_{DQ}(R_S + R_D)$

$10 = 20 - (4)(R_S + R_D) \Rightarrow R_S + R_D = 2.5\ k\Omega$

$V_S = 2\ V = I_{DQ} R_S = 4R_S$

$\Rightarrow \underline{R_S = 0.5\ k\Omega}, \quad \underline{R_D = 2.0\ k\Omega}$

$I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$4 = 8\left(1 - \frac{V_{GS}}{(-4.2)}\right)^2 \Rightarrow V_{GS} = (-4.2)\left(1 - \sqrt{\frac{4}{8}}\right)$

$\Rightarrow V_{GS} = -1.23\ V$

$V_G = V_S + V_{GS} = 2 - 1.23$

$V_G = 0.77\ V = \left(\frac{R_2}{R_1 + R_2}\right)(20) = \left(\frac{R_2}{100}\right)(20)$

$\Rightarrow \underline{R_2 = 3.85\ k\Omega}, \quad \underline{R_1 = 96.2\ k\Omega}$

6.52

a. $I_{DQ} = \frac{I_{DSS}}{2} = 5 \text{ mA}$

$V_{DSQ} = \frac{V_{DD}}{2} = \frac{12}{2} = 6 \text{ V}$

$R_S = \frac{12-6}{5} \Rightarrow \underline{R_S = 1.2 \text{ k}\Omega}$

$I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$5 = 10\left(1 - \frac{V_{GS}}{(-5)}\right)^2 \Rightarrow V_{GS} = (-5)\left(1 - \sqrt{\frac{5}{10}}\right)$

$\Rightarrow V_{GS} = -1.46 \text{ V}$

$V_G = V_S + V_{GS} = 6 - 1.46 = 4.54 \text{ V}$

$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$

$4.54 = \frac{1}{R_1}(100)(12) \Rightarrow \underline{R_1 = 264 \text{ k}\Omega}$

$\frac{264 R_2}{264 + R_2} = 100 \Rightarrow \underline{R_2 = 161 \text{ k}\Omega}$

b. $g_m = \frac{2I_{DSS}}{(-V_P)}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(10)}{5}\left(1 - \frac{-1.46}{-5}\right)$

$\Rightarrow g_m = 2.83 \text{ mA/V}$

$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20 \text{ k}\Omega$

$A_v = \frac{g_m(r_0\|R_S\|R_L)}{1 + g_m(r_0\|R_S\|R_L)}$

$A_v = \frac{(2.83)(20\|1.2\|0.5)}{1 + (2.83)(20\|1.2\|0.5)}$

$\Rightarrow \underline{A_v = 0.495}$

$R_0 = \frac{1}{g_m} \,\Big\|\, R_S = \frac{1}{2.83} \,\Big\|\, 1.2 = 0.353\|1.2$

$\Rightarrow \underline{R_0 = 0.273 \text{ k}\Omega}$

6.53

a. $V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \left(\frac{110}{110 + 90}\right)(10) = 5.5 \text{ V}$

$I_{DQ} = \frac{10 - (V_G - V_{GS})}{R_S} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$10 - 5.5 + V_{GS} = (2)(5)\left(1 - \frac{V_{GS}}{1.75}\right)^2$

$4.5 + V_{GS} = 10(1 - 1.143V_{GS} + 0.3265V_{GS}^2)$

$3.265V_{GS}^2 - 12.43V_{GS} + 5.5 = 0$

$V_{GS} = \frac{12.43 \pm \sqrt{(12.43)^2 - 4(3.265)(5.5)}}{2(3.265)}$

$\Rightarrow V_{GS} = 0.511 \text{ V}$

$I_{DQ} = (2)\left(1 - \frac{0.511}{1.75}\right)^2 \Rightarrow \underline{I_{DQ} = 1.42 \text{ mA}}$

$V_{SDQ} = 10 - (1.42)(5) \Rightarrow \underline{V_{SDQ} = 2.9 \text{ V}}$

b. $g_m = \frac{2I_{DSS}}{V_P}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(2)}{1.75}\left(1 - \frac{0.511}{1.75}\right)$

$\Rightarrow g_m = 1.62 \text{ mA/V}$

$A_v = \frac{g_m(R_S\|R_L)}{1 + g_m(R_S\|R_L)} = \frac{(1.62)(5\|10)}{1 + (1.62)(5\|10)}$

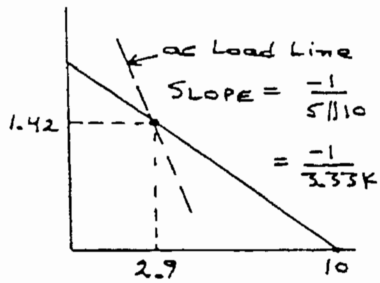
$\Rightarrow \underline{A_v = 0.844}$

$A_i = \frac{i_0}{i_i} = \frac{(v_0/R_L)}{(v_i/R_i)} = A_v \cdot \left(\frac{R_i}{R_L}\right)$

$R_i = R_1\|R_2 = 90\|110 = 49.5 \text{ k}\Omega$

$A_i = (0.844)\left(\frac{49.5}{10}\right) \Rightarrow \underline{A_i = 4.18}$

c.



Maximum swing in output voltage
= 5.8 V peak-to-peak

6.54

$I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$4 = 8\left(1 - \frac{V_{GS}}{4}\right)^2 \Rightarrow V_{GS} = 4\left(1 - \sqrt{\frac{4}{8}}\right)$

$\Rightarrow V_{GS} = 1.17 \text{ V}$

$V_{SDQ} = V_{DD} - I_{DQ}(R_S + R_D)$

$7.5 = 20 - 4(R_S + R_D) \Rightarrow R_S + R_D = 3.125 \text{ k}\Omega$

$g_m = \frac{2I_{DSS}}{V_P}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(8)}{4}\left(1 - \frac{1.17}{4}\right)$

$\Rightarrow g_m = 2.83 \text{ mA/V}$

$R_S = 3.125 - R_D$

$$A_v = \frac{-g_m R_D}{1 + g_m R_S}$$

$$-3(1 + g_m R_S) = -g_m R_D$$

$$3[1 + (2.83)(3.125 - R_D)] = (2.83) R_D$$

$$9.844 - 2.83 R_D = 0.7075 R_D \Rightarrow \underline{R_D = 2.78 \text{ k}\Omega}$$

$$\underline{R_S = 0.345 \text{ k}\Omega}$$

$$V_S = 20 - (4)(0.345) \Rightarrow V_S = 18.6 \text{ V}$$

$$V_G = V_S - V_{GS} = 18.6 - 1.17 = 17.4 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \left(\frac{R_2}{400}\right)(20)$$

$$\Rightarrow \underline{R_2 = 348 \text{ k}\Omega}, \quad \underline{R_1 = 52 \text{ k}\Omega}$$

Chapter 7

Exercise Solutions

E7.1

a. $R_S = R_P = 4\ \text{k}\Omega$

$$\omega = \frac{1}{\tau_S} = \frac{1}{(R_S + R_P)C_S}$$

$$C_S = \frac{1}{2\pi f(R_S + R_P)} = \frac{1}{2\pi(20)(4+4)\times 10^3}$$

$\underline{C_S = 0.995\ \mu\text{F}}$

b. $$|T(j\omega)| = \left(\frac{R_P}{R_S + R_P}\right)\frac{\omega\tau_S}{\sqrt{1+\omega^2\tau_S^2}}$$

$$\tau_S = (R_S + R_P)C_S = 7.96 \times 10^{-3}$$

$$\frac{R_P}{R_S + R_P} = \frac{4}{4+4} = 0.5$$

$f = 40$ Hz

$$|T(j\omega)| = \frac{(0.5)(2\pi)(40)(7.96\times 10^{-3})}{\sqrt{1+[2\pi(40)(7.96\times 10^{-3})]^2}}$$

$\underline{|T(j\omega)| = 0.447}$

$f = 80$ Hz

$$|T(j\omega)| = \frac{(0.5)(2\pi)(80)(7.96\times 10^{-3})}{\sqrt{1+[2\pi(80)(7.96\times 10^{-3})]^2}}$$

$\underline{|T(j\omega)| = 0.485}$

$f = 200$ Hz

$$|T(j\omega)| = \frac{(0.5)(2\pi)(200)(7.96\times 10^{-3})}{\sqrt{1+[2\pi(200)(7.96\times 10^{-3})]^2}}$$

$\underline{|T(j\omega)| = 0.498}$

E7.2

$$\omega = \frac{1}{\tau_P} = \frac{1}{(R_S \| R_P)C_P}$$

$$C_P = \frac{1}{2\pi f(R_S \| R_P)}$$

$$= \frac{1}{2\pi(500\times 10^3)(10\|10)\times 10^3}$$

$\underline{C_P = 63.7\ \text{pF}}$

E7.3

a. $V_0 = -(g_m V_\pi)R_L$

$$V_\pi = \frac{r_\pi}{r_\pi + \frac{1}{sC_C} + R_S} \times V_i$$

$$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{-g_m r_\pi R_L}{r_\pi + R_S + (1/sC_C)}$$

$$= \frac{-g_m r_\pi R_L(sC_C)}{1 + s(r_\pi + R_S)C_C}$$

$g_m r_\pi = \beta$

$$\underline{T(s) = \frac{-\beta R_L}{r_\pi + R_S} \times \left(\frac{s(r_\pi + R_S)C_C}{1 + s(r_\pi + R_S)C_C}\right)}$$

b. $$\underline{f_{3-dB} = \frac{1}{2\pi(r_\pi + R_S)C_C}}$$

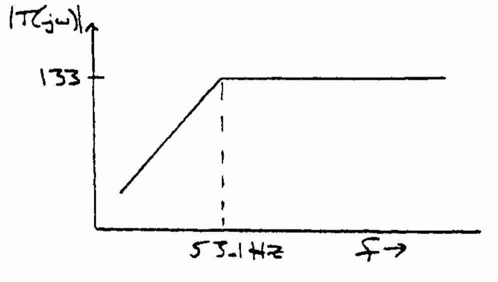
$$f_{3-dB} = \frac{1}{2\pi[2\times 10^3 + 1\times 10^3][10^{-6}]}$$

$\Rightarrow \underline{f_{3dB} = 53.1\ \text{Hz}}$

$$|T(j\omega)|_{max} = \frac{r_\pi g_m R_L}{r_\pi + R_S} = \frac{(2)(50)(4)}{2+1}$$

$\underline{|T(j\omega)|_{max} = 133}$

c.



E7.4

a. $$V_0 = -g_m V_\pi\left(R_L \| \frac{1}{sC_L}\right)$$

$$V_\pi = \left(\frac{r_\pi}{r_\pi + R_S}\right) \times V_i$$

$$T(s) = \frac{V_0(s)}{V_i(s)} = -g_m \frac{r_\pi}{r_\pi + R_S}\left(\frac{R_L \times \frac{1}{sC_L}}{R_L + \frac{1}{sC_L}}\right)$$

$$\underline{T(s) = \frac{-\beta R_L}{r_\pi + R_S} \times \left(\frac{1}{1 + sR_L C_L}\right)}$$

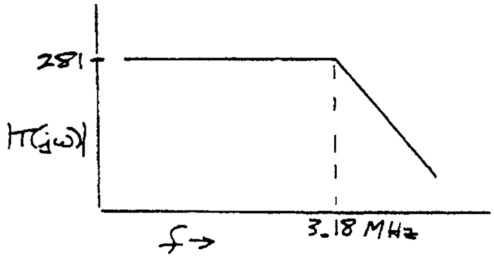
b. $$f_{3-dB} = \frac{1}{2\pi R_L C_L} = \frac{1}{2\pi(5\times 10^3)(10\times 10^{-12})}$$

$\Rightarrow \underline{f_{3dB} = 3.18\text{ MHz}}$

$|T(j\omega)| = \dfrac{g_m r_\pi R_L}{r_\pi + R_S} = \dfrac{(75)(1.5)(5)}{1.5 + 0.5}$

$\underline{|T(j\omega)|_{\max} = 281}$

c.



E7.5

a. $20\log_{10}\left(\dfrac{R_P}{R_P + R_S}\right) = -1$

$\Rightarrow \dfrac{R_P}{R_P + R_S} = 0.891 = \dfrac{R_P}{R_P + 1}$

$\Rightarrow (1 - 0.891)R_P = 0.891 \Rightarrow \underline{R_P = 8.17\text{ k}\Omega}$

$f_L = \dfrac{1}{2\pi(R_S + R_P)C_S}$

$\Rightarrow C_S = \dfrac{1}{2\pi(100)(1 + 8.17)\times 10^3}$

$\underline{C_S = 0.174\ \mu\text{F}}$

$f_H = \dfrac{1}{2\pi(R_S \| R_P)C_P}$

$\Rightarrow C_P = \dfrac{1}{2\pi(10^6)(1\|8.17)\times 10^3}$

$\underline{C_P = 179\text{ pF}}$

b. $\tau_S = (R_S + R_P)C_S$

$\tau_S = (1\times 10^3 + 8.17\times 10^3)(0.174\times 10^{-6})$

$\underline{\tau_S = 1.60\text{ ms}}$ open-circuit time-constant

$\tau_P = (R_S\|R_P)C_P$

$\tau_P = (1\|8.17)\times 10^3(179\times 10^{-12})$

$\underline{\tau_P = 0.160\ \mu\text{s}}$ short-circuit time-constant

E7.6

a. Open-circuit time constant ($C_L \to$ open)

$\tau_S = (R_S + r_\pi)C_C$

$= (0.25 + 2)\times 10^3(2\times 10^{-6}) = 4.5\text{ ms}$

Short-circuit time constant ($C_C \to$ short)

$\tau_P = R_L C_L = (4\times 10^3)(50\times 10^{-12})$

$\underline{\tau_P = 0.2\ \mu\text{s}}$

b. Midband gain

$V_0 = -g_m V_\pi R_L, \quad V_\pi = \left(\dfrac{r_\pi}{r_\pi + R_S}\right)V_i$

$A_v = \dfrac{V_0}{V_i} = \dfrac{-g_m r_\pi R_L}{r_\pi + R_S}$

$= \dfrac{-(65)(2)(4)}{2 + 0.25}$

$\underline{A_v = -231}$

c. $f_L = \dfrac{1}{2\pi\tau_S} = \dfrac{1}{2\pi(4.5\times 10^{-3})} \Rightarrow \underline{f_L = 35.4\text{ Hz}}$

$f_H = \dfrac{1}{2\pi\tau_P} = \dfrac{1}{2\pi(0.2\times 10^{-6})} \Rightarrow \underline{f_H = 0.796\text{ MHz}}$

E7.7

a. $\tau_S = (R_i + R_S)C_C$

b. $f = \dfrac{1}{2\pi\tau_S}$

$R_{TH} = R_1\|R_2 = 2.2\|20 = 1.98\text{ k}\Omega$

$V_{TH} = \left(\dfrac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\dfrac{2.2}{2.2 + 20}\right)(10)$

$= 0.991\text{ V}$

$I_{BQ} = \dfrac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \dfrac{0.991 - 0.7}{1.98 + (201)(0.1)}$

$= 0.0132\text{ mA}$

$I_{CQ} = 2.64\text{ mA}$

$r_\pi = \dfrac{\beta V_T}{I_{CQ}} = \dfrac{(200)(0.026)}{2.64} = 1.97\text{ k}\Omega$

$g_m = \dfrac{I_{CQ}}{V_T} = \dfrac{2.64}{0.026} = 102\text{ mA/V}$

$R_{ib} = r_\pi + (1+\beta)R_E = 1.97 + (201)(0.1)$

$= 22.1\text{ k}\Omega$

$R_B = R_1\|R_2 = 1.98\text{ k}\Omega$

$R_i = R_B\|R_{ib} = 1.98\|22.1 = 1.82\text{ k}\Omega$

$\tau_S = (R_i + R_S)C_C$

$= (1.82 + 0.1)(\times 10^3)(47\times 10^{-6})$

$= 90.24\text{ ms}$

$f = \dfrac{1}{2\pi(90.24\times 10^{-3})} \Rightarrow \underline{f = 1.76\text{ Hz}}$

Midband Gain

$A_v = \dfrac{-\beta R_C}{r_\pi + (1+\beta)R_E}\cdot\dfrac{R_i}{R_i + R_S}$

$= \dfrac{-(200)(2)}{1.97 + (201)(0.1)}\cdot\dfrac{1.82}{1.82 + 0.1}$

$\underline{A_v = -17.2}$

E7.8

$I_{DQ} = K_n(V_{GS} - V_{TN})^2$

a. $\sqrt{\frac{0.8}{0.5}} + 2 = V_{GS} \Rightarrow V_{GS} = 3.26 \text{ V}$

$V_S = -3.26 \Rightarrow I_{DQ} = \frac{V_S - (-5)}{R_S}$

$R_S = \frac{-3.26 + 5}{0.8} \Rightarrow \underline{R_S = 2.18 \text{ k}\Omega}$

$V_D = 0 \Rightarrow R_D = \frac{5}{0.8} \Rightarrow \underline{R_D = 6.25 \text{ k}\Omega}$

b. $\tau_S = (R_D + R_L)C_C = (10 + 6.25) \times 10^3 \times C_C$

$f = \frac{1}{2\pi\tau_S} \Rightarrow C_C = \frac{1}{2\pi f(16.25 \times 10^3)}$

$C_C = \frac{1}{2\pi(20)(16.25 \times 10^3)}$

$\Rightarrow \underline{C_C = 0.49 \ \mu\text{F}}$

E7.9

$\tau_S = (R_L + R_E \| R_0)C_{C2}$

$f = \frac{1}{2\pi\tau_S} \Rightarrow C_{C2} = \frac{1}{2\pi f(R_L + R_E \| R_0)}$

$R_0 = r_0 \,\|\, \left\{\frac{r_\pi + (R_S \| R_B)}{1 + \beta}\right\}$

From Example 7-5, $R_0 = 35.6 \ \Omega$

$R_0 \| R_E = 0.0356 \| 10 \approx 0.0356 \text{ k}\Omega$

$C_{C2} = \frac{1}{2\pi(10)[10 \times 10^3 + 35.6]}$

$\underline{C_{C2} = 1.59 \ \mu\text{F}}$

E7.10

a. $I_{DQ} = K_p(V_{SG} + V_{TP})^2$

$\sqrt{\frac{1}{0.5}} - (-2) = V_{SG} \Rightarrow V_{SG} = 3.41 \text{ V}$

$V_S = 3.41$

$R_S = \frac{5 - 3.41}{1} \Rightarrow \underline{R_S = 1.59 \text{ k}\Omega}$

For $V_{SDQ} = V_{SGQ} \Rightarrow V_D = 0$

$\Rightarrow R_D = \frac{5}{1} \Rightarrow \underline{R_D = 5 \text{ k}\Omega}$

b. $\tau_P = (R_D \| R_L)C_L$

$f = \frac{1}{2\pi\tau_P} \Rightarrow C_L = \frac{1}{2\pi f(R_D \| R_L)}$

$C_L = \frac{1}{2\pi(10^6)(5 \| 10) \times 10^3}$

$\Rightarrow \underline{C_L = 47.7 \text{ pF}}$

E7.11

a. $I_{BQ} = \frac{0 - 0.7 - (-10)}{0.5 + (101)(4)} = 0.0230 \text{ mA}$

$I_{CQ} = 2.30 \text{ mA}$

$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{2.30} = 1.13 \text{ k}\Omega$

$g_m = \frac{I_{CQ}}{V_T} = \frac{2.30}{0.026} = 88.5 \text{ mA/V}$

$\tau_B = \frac{R_E(R_S + r_\pi)C_E}{R_S + r_\pi + (1+\beta)R_E}$

$= \frac{(4 \times 10^3)(0.5 + 1.13)C_E}{0.5 + 1.13 + (101)(4)}$

$\tau_B = \frac{1}{2\pi f_B} = \frac{1}{2\pi(200)} = 0.796 \text{ ms}$

$\tau_B = 16.07 C_E \Rightarrow C_E = \frac{0.796 \times 10^{-3}}{16.07}$

$\Rightarrow \underline{C_E = 49.5 \ \mu\text{F}}$

b. $\tau_A = R_E C_E = (4 \times 10^3)(49.5 \times 10^{-6})$

$\Rightarrow \tau_A = 0.198 \text{ s}$

$f_A = \frac{1}{2\pi\tau_A} = \frac{1}{2\pi(0.198)} \Rightarrow \underline{f_A = 0.80 \text{ Hz}}$

E7.14

$r_\pi = \frac{\beta_0 V_T}{I_{CQ}} = \frac{(150)(0.026)}{0.5} = 7.8 \text{ k}\Omega$

$f_\beta = \frac{1}{2\pi r_\pi(C_\pi + C_\mu)}$

$= \frac{1}{2\pi(7.8 \times 10^3)(2 + 0.3) \times 10^{-12}}$

$\Rightarrow \underline{f_\beta = 8.87 \text{ MHz}}$

E7.15

$r_\pi = \frac{\beta_0 V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$

$f_\beta = \frac{1}{2\pi r_\pi(C_\pi + C_\mu)}$

$C_\pi + C_\mu = \frac{1}{2\pi f_\beta r_\pi} = \frac{1}{2\pi(11.5 \times 10^6)(10.4 \times 10^3)}$

$C_\pi + C_\mu = 1.33 \text{ pF}$

$C_\mu = 0.1 \text{ pF}$

$\Rightarrow \underline{C_\pi = 1.23 \text{ pF}}$

E7.16

$h_{fe} = \frac{\beta_0}{1 + j(f/f_\beta)}$

$f_\beta = 5 \text{ MHz}, \ \beta_0 = 100$

At $f = 50$ MHz

$|h_{fe}| = \frac{100}{\sqrt{1 + \left(\frac{50}{5}\right)^2}} \Rightarrow \underline{|h_{fe}| = 9.95}$

Phase $= -\tan^{-1}\left(\frac{50}{5}\right) \Rightarrow$ $\underline{\text{Phase} = -84.3^\circ}$

E7.17

$$f_\beta = \frac{f_T}{\beta_0} = \frac{500}{120} \Rightarrow \underline{f_\beta = 4.17 \text{ MHz}}$$

$$f_\beta = \frac{1}{2\pi r_\pi (C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = \frac{1}{2\pi f_\beta r_\pi} = \frac{1}{2\pi (4.17 \times 10^6)(5 \times 10^3)}$$

$$C_\pi + C_\mu = 7.63 \text{ pF}$$

$$C_\mu = 0.2 \text{ pF}$$

$$\Rightarrow \underline{C_\pi = 7.43 \text{ pF}}$$

E7.18

$$r_\pi = \frac{\beta_0 V_T}{I_{CQ}} = \frac{(150)(0.026)}{1} \Rightarrow r_\pi = 3.9 \text{ k}\Omega$$

$$f_\beta = \frac{1}{2\pi r_\pi (C_\pi + C_\mu)}$$

$$= \frac{1}{2\pi (3.9 \times 10^3)(4 + 0.5)(10^{-12})}$$

$$\Rightarrow \underline{f_\beta = 9.07 \text{ MHz}}$$

$$f_T = \beta_0 f_\beta = (150)(9.07)$$

$$\Rightarrow \underline{f_T = 1.36 \text{ GHz}}$$

E7.19

$$R_{TH} = R_1 \| R_2 = 200 \| 220 = 105 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{220}{200 + 220}\right)(5)$$

$$= 2.62 \text{ V}$$

$$I_{BQ} = \frac{2.62 - 0.7}{105 + (101)(1)} = 0.00932 \text{ mA}$$

$$I_{CQ} = 0.932 \text{ mA}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.932}{0.026} \Rightarrow g_m = 35.8 \text{ mA/V}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.932} \Rightarrow r_\pi = 2.79 \text{ k}\Omega$$

a. $C_M = C_\mu [1 + g_m (R_C \| R_L)]$

$$C_M = (2)[1 + (35.8)(2.2 \| 4.7)]$$

$$\Rightarrow \underline{C_M = 109 \text{ pF}}$$

b. $R_B = r_S \| R_1 \| R_2 = 100 \| 200 \| 220 = 51.2 \text{ k}\Omega$

$$f_{3dB} = \frac{1}{2\pi (R_B \| r_\pi)(C_\pi + C_\mu)}$$

$$= \frac{1}{2\pi [51.2 \| 2.79] \times 10^3 \times (10 + 109) \times 10^{-12}}$$

$$\Rightarrow \underline{f_{3dB} = 0.505 \text{ MHz}}$$

E7.20

(a) $g_m = 2K_n (V_{GS} - V_{TN}) = 2(0.4)(3 - 1)$

$$\Rightarrow g_m = 1.6 \text{ mA/V}$$

$$g'_m = 80\% \text{ of } g_m = 1.28 \text{ mA/V}$$

$$g'_m = \frac{g_m}{1 + g_m r_S}$$

$$1 + g_m r_S = \frac{g_m}{g'_m}$$

$$r_S = \frac{1}{g_m}\left(\frac{g_m}{g'_m} - 1\right) = \frac{1}{1.6}\left(\frac{1.6}{1.28} - 1\right)$$

$$r_S = 0.156 \text{ k}\Omega \Rightarrow \underline{r_S = 156 \text{ ohms}}$$

(b) $g_m = 2K_n (V_{GS} - V_{TN}) = 2(0.4)(5 - 1)$

$$\Rightarrow g_m = 3.2 \text{ mA/V}$$

$$g'_m = \frac{g_m}{1 + g_m r_S} = \frac{3.2}{1 + (3.2)(0.156)} = 2.13$$

$$\frac{\Delta g_m}{g_m} = \frac{3.2 - 2.13}{3.2} \Rightarrow \text{A 33.4\% reduction}$$

E7.21

$$f_T = \frac{g_m}{2\pi (C_{gs} + C_{gd})}$$

$$I_{DQ} = K_n (V_{GS} - V_{TN})^2$$

$$0.4 = 0.2(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.41 V$$

$$g_m = 2K_n (V_{GS} - V_{TN}) = 2(0.2)(2.41 - 1)$$

$$= 0.564 \, mA/V$$

$$f_T = \frac{0.564 x 10^{-3}}{2\pi (0.25 + 0.02) x 10^{-12}} \Rightarrow$$

$$\underline{f_T = 332 \, MHz}$$

E7.22

$$f_T = \frac{g_m}{2\pi (C_{gsT} + C_{gdT})}$$

$$= \frac{g_m}{2\pi (C_{gs} + C_{gsp} + C_{gdp})}$$

$$C_{gs} = \frac{g_m}{2\pi f_T} - C_{gsp} - C_{gdp}$$

$$= \frac{0.5 \times 10^{-3}}{2\pi (500 \times 10^6)} - (0.01 + 0.01) \times 10^{-12}$$

$$\Rightarrow \underline{C_{gs} = 0.139 \text{ pF}}$$

E7.23

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gsp} + C_{gdp})}$$

$$C_{gsp} = C_{gdp}$$

$$2C_{gsp} = \frac{g_m}{2\pi f_T} - C_{gs} = \frac{1 \times 10^{-3}}{2\pi(350 \times 10^6)} - 0.4 \times 10^{-12}$$

$$2C_{gsp} = 0.0547 \text{ pF}$$

$$\Rightarrow \underline{C_{gsp} = C_{gdp} \cong 0.0274 \text{ pF}}$$

E7.24

dc analysis

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \left(\frac{166}{166 + 234}\right)(10) = 4.15 \text{ V}$$

$$I_D = \frac{V_S}{R_S} \text{ and } V_S = V_G - V_{GS}$$

$$K_n(V_{GS} - V_{TN})^2 = \frac{V_G - V_{GS}}{R_S}$$

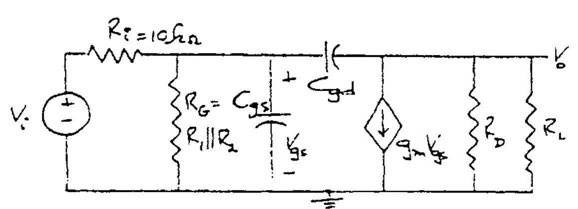
$$(0.5)(0.5)(V_{GS}^2 - 4V_{GS} + 4) = 4.15 - V_{GS}$$

$$0.25V_{GS}^2 - 3.15 = 0 \Rightarrow V_{GS} = 3.55 \text{ V}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.5)(2.55 - 2)$$

$$= 1.55 \text{ mA/V}$$

Small-signal equivalent circuit.



a. $C_M = C_{gd}\left(1 + g_m\left(R_D \| R_L\right)\right)$

$$C_M = (0.1)[1 + (1.55)(4\|20)]$$

$$\Rightarrow \underline{C_M = 0.617 \text{ pF}}$$

b. $f_H = \dfrac{1}{2\pi\tau_P}$

$$\tau_P = \left(R_G \| R_i\right)\left(C_{gs} + C_M\right)$$

$$R_G = R_1 \| R_2 = 234\|166 = 97.1 \text{ k}\Omega$$

$$R_G \| R_i = 97.1\|10 = 9.07 \text{ k}\Omega$$

$$\tau_P = (9.07 \times 10^3)(1 + 0.617) \times 10^{-12} = 14.7 \text{ ns}$$

$$f_H = \frac{1}{2\pi(14.7 \times 10^{-9})} \Rightarrow \underline{f_H = 10.9 \text{ MHz}}$$

E7.25

dc analysis

$$V_{TH} = 0, \quad R_{TH} = 10 \text{ k}\Omega$$

$$I_{BQ} = \frac{0 - 0.7 - (-5)}{10 + (126)(5)} = 0.00672 \text{ mA}$$

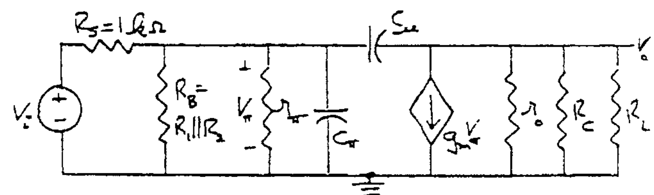
$$I_{CQ} = 0.840 \text{ mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(125)(0.026)}{0.840} = 3.87 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.840}{0.026} = 32.3 \text{ mA/V}$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{200}{0.84} = 238 \text{ k}\Omega$$

High-frequency equivalent circuit



a. Miller capacitance

$$C_M = C_\mu(1 + g_m R_L')$$

$$R_L' = r_0 \| R_C \| R_L$$

$$R_L' = 238\|2.3\|5 = 1.57 \text{ k}\Omega$$

$$C_M = (3)[1 + (32.3)(1.57)] \Rightarrow \underline{C_\mu = 155 \text{ pF}}$$

b. $R_{eq} = R_S \| R_B \| r_\pi = R_S \| R_1 \| R_2 \| r_\pi$

$$R_{eq} = 1\|20\|20\|3.87 = 0.736 \text{ k}\Omega$$

$$\tau_P = R_{eq}(C_\pi + C_M)$$

$$= (0.736 \times 10^3)(24 + 155) \times 10^{-12}$$

$$= 1.32 \times 10^{-7}$$

$$f_H = \frac{1}{2\pi(1.32 \times 10^{-7})} \Rightarrow \underline{f_H = 1.21 \text{ MHz}}$$

c. $(A_v)_M = -g_m R_L' \left[\dfrac{R_B \| r_\pi}{R_B \| r_\pi + R_S}\right]$

$$(A_v)_M = -(32.3)(1.57)\left[\frac{10\|3.87}{10\|3.87 + 1}\right]$$

$$\Rightarrow \underline{(A_v)_M = -37.3}$$

E7.26

dc analysis

$$V_G = \left(\frac{50}{50+150}\right)(10) - 5 = -2.5$$

$$V_S = V_G - V_{GS}. \quad I_D = \frac{V_S - (-5)}{R_S}$$

$$K_n(V_{GS} - V_{TN})^2 = \frac{V_G - V_{GS} + 5}{R_S}$$

$$(1)(2)[V_{GS}^2 - 1.6V_{GS} + 0.64] = -2.5 - V_{GS} + 5$$

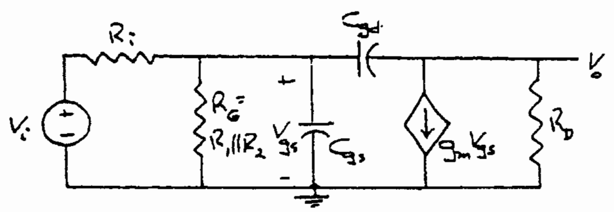
$$2V_{GS}^2 - 2.2V_{GS} - 1.22 = 0$$

$$V_{GS} = \frac{2.2 \pm \sqrt{(2.2)^2 + 4(2)(1.22)}}{2(2)}$$

$$\Rightarrow V_{GS} = 1.51 \text{ V}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(1)(1.51 - 0.8)$$
$$= 1.42 \text{ mA/V}$$

Equivalent circuit



(a) $C_M = C_{gd}(1 + g_m R_D) = (0.2)[1 + (1.42)(5)] \Rightarrow$
$\underline{C_M = 1.62 \text{ pF}}$

(b) $\tau_P = (R_S \| R_G)(C_{gs} + C_M)$

$$\tau_P = [20\|50\|150] \times 10^3 \times (2 + 1.62) \times 10^{-12}$$
$$= (13 \times 10^3)(3.62 \times 10^{-12}) = 4.71 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau_P} = \frac{1}{2\pi(4.7 \times 10^{-8})}$$

$$\Rightarrow \underline{f_H = 3.38 \text{ MHz}}$$

c. $(A_v)_M = -g_m R_D \left(\frac{R_G}{R_G + R_S}\right)$

$$(A_v)_M = -(1.42)(5)\left(\frac{37.5}{37.5 + 20}\right)$$

$$\Rightarrow \underline{(A_v)_M = -4.63}$$

E7.27

The dc analysis

$$I_{BQ} = \frac{10 - 0.7}{100 + (101)(10)} = 0.00838 \text{ mA}$$

$$I_{CQ} = 0.838 \text{ mA}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.838} = 3.10 \text{ k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = 32.2 \text{ mA/V}$$

For the input

$$\tau_{P\pi} = \left[\left(\frac{r_\pi}{1+\beta}\right) \Big\| R_E \Big\| R_S\right] C_\pi$$
$$= \left[\frac{3.10}{101} \Big\| 10 \Big\| 1\right] \times 10^3 \times 24 \times 10^{-12}$$
$$= 7.13 \times 10^{-10} \text{ s}$$

$$f_{H\pi} = \frac{1}{2\pi\tau_{P\pi}} = \frac{1}{2\pi(7.13 \times 10^{-10})}$$

$$\Rightarrow \underline{f_{H\pi} = 223 \text{ MHz}}$$

For the output

$$\tau_{P\mu} = [R_C \| R_L] C_\mu = (10\|1) \times 10^3 \times 3 \times 10^{-12}$$
$$= 2.73 \times 10^{-9}$$

$$f_{H\mu} = \frac{1}{2\pi\tau_{P\mu}} = \frac{1}{2\pi(2.73 \times 10^{-9})}$$

$$\Rightarrow \underline{f_{H\mu} = 58.3 \text{ MHz}}$$

$$(A_v)_M = g_m(R_C \| R_L)\left[\frac{R_E \Big\| \left(\frac{r_\pi}{1+\beta}\right)}{R_E \Big\| \left(\frac{r_\pi}{1+\beta}\right) + R_S}\right]$$

$$= (32.2)(10\|1)\left[\frac{10 \Big\| \left(\frac{3.1}{101}\right)}{10 \Big\| \left(\frac{3.1}{101}\right) + 1}\right]$$

$$\Rightarrow \underline{(A_v)_M = 0.869}$$

Chapter 7

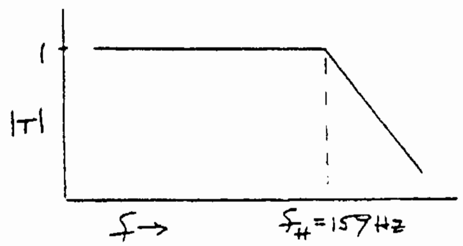
Problem Solutions

7.1

a. $T(s) = \dfrac{V_0(s)}{V_i(s)} = \dfrac{1/(sC_1)}{[1/(sC_1)] + R_1}$

$\underline{T(s) = \dfrac{1}{1 + sR_1C_1}}$

b.



$f_H = \dfrac{1}{2\pi R_1 C_1} = \dfrac{1}{2\pi(10^3)(10^{-6})}$

$\Rightarrow \underline{f_H = 159 \text{ Hz}}$

c. $V_0(s) = V_i(s) \cdot \dfrac{1}{1 + sR_1C_1}$

For a step function $V_i(s) = \dfrac{1}{s}$

$V_0(s) = \dfrac{1}{s} \cdot \dfrac{1}{1 + sR_1C_1} = \dfrac{K_1}{s} + \dfrac{K_2}{1 + sR_1C_1}$

$= \dfrac{K_1(1 + sR_1C_1) + K_2 s}{s(1 + sR_1C_1)}$

$= \dfrac{K_1 + s(K_1R_1C_1 + K_2)}{s(1 + sR_1C_1)}$

$K_2 = -K_1R_1C_1$ and $K_1 = 1$

$V_0(s) = \dfrac{1}{s} + \dfrac{-R_1C_1}{1 + sR_1C_1}$

$= \dfrac{1}{s} - \dfrac{1}{\dfrac{1}{R_1C_1} + s}$

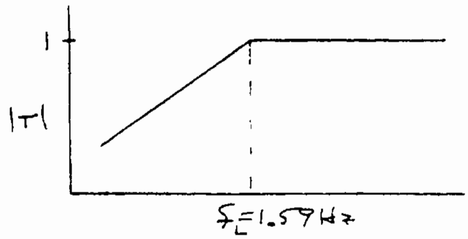
$\underline{v_0(t) = 1 - e^{-t/R_1C_1}}$

7.2

a. $T(s) = \dfrac{V_0(s)}{V_i(s)} = \dfrac{R_2}{R_2 + [1/(sC_2)]}$

$\underline{T(s) = \dfrac{sR_2C_2}{1 + sR_2C_2}}$

b.



$f_L = \dfrac{1}{2\pi R_2 C_2} = \dfrac{1}{2\pi(10^4)(10 \times 10^{-6})}$

$\Rightarrow \underline{f_L = 1.59 \text{ Hz}}$

c. $V_0(s) = V_i(s) \cdot \dfrac{sR_2C_2}{1 + sR_2C_2}$

$V_i(s) = \dfrac{1}{s}$

$V_0(s) = \dfrac{R_2C_2}{1 + sR_2C_2} = \dfrac{1}{s + \dfrac{1}{R_2C_2}}$

$\underline{v_0(t) = e^{-t/R_2C_2}}$

7.3

a. $T(s) = \dfrac{V_0(s)}{V_i(s)} = \dfrac{R_P \| \dfrac{1}{sC_P}}{R_P \| \dfrac{1}{sC_P} + \left(R_S + \dfrac{1}{sC_S}\right)}$

$R_P \| \dfrac{1}{sC_P} = \dfrac{R_P \cdot \dfrac{1}{sC_P}}{R_P + \dfrac{1}{sC_P}} = \dfrac{R_P}{1 + sR_PC_P}$

Then

$T(s) = \dfrac{R_P}{R_P + \left(R_S + \dfrac{1}{sC_S}\right)(1 + sR_PC_P)}$

$= \dfrac{R_P}{R_P + R_S + \dfrac{R_PC_P}{C_S} + \dfrac{1}{sC_S} + sR_SR_PC_P}$

$T(s)$

$= \left(\dfrac{R_P}{R_P + R_S}\right) \times \left(1\Big/\left[1 + \dfrac{R_P}{R_P + R_S} \cdot \dfrac{C_P}{C_S} + \dfrac{1}{s(R_S + R_P)C_S} + \dfrac{sR_PR_S}{R_S + R_P} \cdot C_P\right]\right)$

b.

$T(s)$

$$= \left(\frac{10}{10+10}\right) \times \left(1/\left[1 + \frac{10}{20}\cdot\frac{10^{-11}}{10^{-6}} + \frac{1}{s(2\times 10^{4})\cdot 10^{-6}} + s(5\times 10^{3})\cdot 10^{-11}\right]\right)$$

$$\cong \frac{1}{2}\cdot\frac{1}{1 + \frac{1}{s(0.02)} + s(5\times 10^{-8})}$$

$s = j\omega$

$$T(j\omega) = \frac{1}{2}\cdot\frac{1}{1 + j\left[\omega(5\times 10^{-8}) - \frac{1}{\omega(0.02)}\right]}$$

For $\omega_L = \frac{1}{(R_S + R_P)C_S} = \frac{1}{(2\times 10^{4})(10^{-6})} = 50$

$$T(j\omega) = \frac{1}{2}\cdot\frac{1}{1 + j\left[(50)(5\times 10^{-8}) - \frac{1}{(50)(0.02)}\right]}$$

$$\approx \frac{1}{2}\cdot\frac{1}{1-j} \Rightarrow \underline{|T(j\omega)| = \frac{1}{2}\cdot\frac{1}{\sqrt{2}}}$$

For

$$\omega_H = \frac{1}{(R_S \| R_P)C_P} = \frac{1}{(5\times 10^{3})(10^{-11})} = 2\times 10^{7}$$

$T(j\omega)$

$$= \frac{1}{2}\cdot\frac{1}{1 + j\left[(2\times 10^{7})(5\times 10^{-8}) - \frac{1}{(2\times 10^{7})(0.02)}\right]}$$

$$T(j\omega) \approx \frac{1}{2}\cdot\frac{1}{1+j} \Rightarrow \underline{|T(j\omega)| = \frac{1}{2}\cdot\frac{1}{\sqrt{2}}}$$

In each case, $|T(j\omega)| = \frac{1}{\sqrt{2}}\cdot\frac{R_P}{R_P + R_S}$

c. $R_S = R_P = 10\ \text{k}\Omega,\ C_S = C_P = 0.1\ \mu\text{F}$

$$T(s) = \frac{1}{2}\cdot\left(1/\left[1 + \frac{1}{2}\cdot\frac{1}{1} + \frac{1}{s(2\times 10^{4}(10^{-7}))} + s(5\times 10^{3})(10^{-7})\right]\right)$$

$s = j\omega$

$$T(j\omega) = \frac{1}{2}\cdot\frac{1}{1 + \frac{1}{2} + j\left[\omega(5\times 10^{-4}) - \frac{1}{\omega(2\times 10^{-3})}\right]}$$

For $\omega = \frac{1}{(2\times 10^{4})(10^{-7})} = 500$

$T(j\omega)$

$$= \frac{1}{2}\cdot\frac{1}{1.5 + j\left[(500)(5\times 10^{-4}) - \frac{1}{(500)(2\times 10^{3})}\right]}$$

$$= \frac{1}{2}\cdot\frac{1}{1.5 - j(0.75)} \Rightarrow \underline{|T(j\omega)| = 0.298}$$

For $\omega = \frac{1}{(5\times 10^{3})(10^{-7})} = 2\times 10^{3}$

$$T(j\omega) = \frac{1}{2}\cdot\left\{1/\left(1.5 + j\left[(2\times 10^{3})(5\times 10^{-4}) - \frac{1}{(2\times 10^{3})(2\times 10^{-3})}\right]\right)\right\}$$

$$= \frac{1}{2}\cdot\frac{1}{1.5 + j(0.75)} \Rightarrow \underline{|T(j\omega)| = 0.298}$$

In each case, $|T(j\omega)| < \frac{1}{\sqrt{2}}\cdot\frac{R_P}{R_P + R_S}$

7.4

Circuit (a):

$$T = \frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1 \left\| \frac{1}{sC_1}\right.} = \frac{R_2}{R_2 + \frac{R_1(1/sC_1)}{R_1 + (1/sC_1)}}$$

$$= \frac{R_2}{R_2 + \frac{R_1}{1 + sR_1C_1}} = \frac{R_2(1 + sR_1C_1)}{R_2 + sR_1R_2C_1 + R_1}$$

or

$$\frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2}\cdot\frac{(1 + sR_1C_1)}{1 + sR_1\|R_2C_1}$$

Low frequency:

$$\left|\frac{V_o}{V_i}\right| = \frac{R_2}{R_1 + R_2} = \frac{20}{10 + 20} = \frac{2}{3}$$

High frequency:

$$\left|\frac{V_o}{V_i}\right| = 1$$

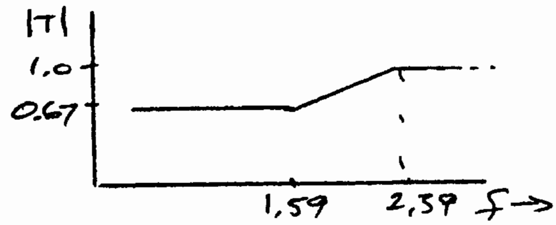
$$\tau_1 = R_1C_1 = (10^{4})(10x10^{-6}) = 0.10 \Rightarrow$$

$$f_1 = \frac{1}{2\pi\tau_1} = 1.59\ Hz$$

$$\tau_2 = (R_1\|R_2)C_1 = (10\|20)x10^{3}x(10x10^{-6}) \Rightarrow$$

$$\tau_2 = 0.0667 \Rightarrow$$

$$f_2 = \frac{1}{2\pi\tau_2} = 2.39\ Hz$$



Circuit (b):

$$T = \frac{V_o}{V_i} = \frac{R_2 \left\| \frac{1}{sC_2} \right.}{R_2 \left\| \frac{1}{sC_2} \right. + R_1} = \frac{\frac{R_2}{1+sR_2C_2}}{\frac{R_2}{1+sR_2C_2} + R_1}$$

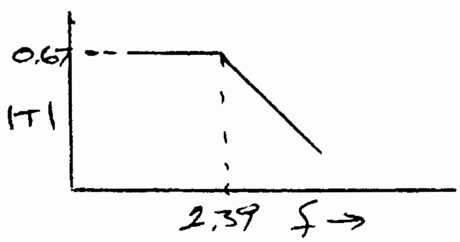
$$= \left(\frac{R_2}{R_1+R_2}\right)\left(\frac{1}{1+s(R_1 \| R_2)C_2}\right)$$

Low frequency:

$$\left|\frac{V_o}{V_i}\right| = \frac{R_2}{R_1+R_2} = \frac{20}{20+10} = \frac{2}{3}$$

$$\tau = (R_1 \| R_2)C_2 = (10\|20)x10^3x10x10^{-6} = 0.0667$$

$$f = \frac{1}{2\pi\tau} = 2.39\ Hz$$



7.5

a. $\tau_S = (R_i + R_P)C_S = [30 + 10] \times 10^3 \times 10 \times 10^{-6}$

$\Rightarrow$ $\underline{\tau_S = 0.40\ \text{s}}$

$\tau_P = (R_i \| R_P)C_P = [30\|10] \times 10^3 \times 50 \times 10^{-12}$

$\Rightarrow$ $\underline{\tau_P = 0.375\ \mu s}$

b. $f_L = \frac{1}{2\pi\tau_S} = \frac{1}{2\pi(0.4)} \Rightarrow \underline{f_L = 0.398\ \text{Hz}}$

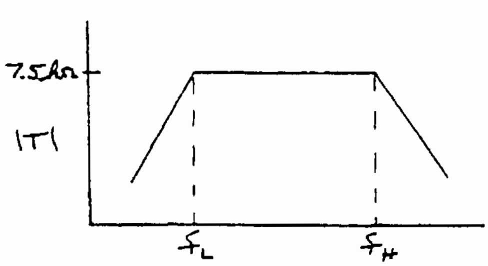
$f_H = \frac{1}{2\pi\tau_P} = \frac{1}{2\pi(0.375 \times 10^{-6})} \Rightarrow \underline{f_H = 424\ \text{kHz}}$

At midband, $C_S \rightarrow$ short, $C_P \rightarrow$ open

$V_0 = I_i(R_i \| R_P)$

$T(s) = R_i \| R_P = 30\|10 \Rightarrow \underline{T(s) = 7.5\ \text{k}\Omega}$

c.



7.6

(a) $T = \frac{1}{(1+j2\pi f\tau)^2} \Rightarrow$

$$|T| = \frac{1}{\left(\sqrt{1+(2\pi f\tau)^2}\right)^2} = \frac{1}{1+(2\pi f\tau)^2}$$

$|T|_{max} = 1$

At $f = \frac{1}{2\pi\tau} \Rightarrow |T| = \frac{1}{1+(1)^2} = \frac{1}{2}$

$|T|_{dB} = 20\log_{10}\left(\frac{1}{2}\right) \Rightarrow \underline{|T|_{dB} \cong -6\,dB}$

Phase $= 2\tan^{-1}(2\pi f\tau) = -2\tan^{-1}(1) = -2(45°) \Rightarrow$

$\underline{\text{Phase} = -90°}$

(b) Slope

$= -2(6\,dB/oct) =$

$\underline{-12\,dB/oct = -40\,dB/decade}$

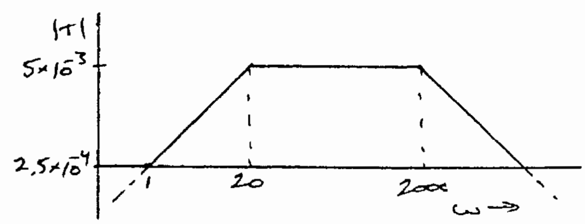
Phase $= -2(90°) \Rightarrow$ $\underline{\text{Phase} = -180°}$

7.7

(a) $$T(j\omega) = \frac{-10(j\omega)}{20\left(1+\frac{j\omega}{20}\right)(2000)\left(1+\frac{j\omega}{2000}\right)}$$

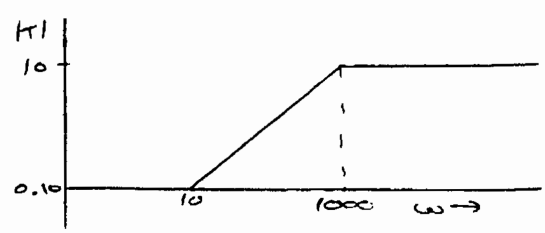
$$= \frac{2.5x10^{-4}(j\omega)}{\left(1+\frac{j\omega}{20}\right)\left(1+\frac{j\omega}{2000}\right)}$$

$$|T| = \frac{2.5x10^{-4}(\omega)}{\sqrt{1+\left(\frac{\omega}{20}\right)^2} \cdot \sqrt{1+\left(\frac{\omega}{2000}\right)^2}}$$



(b) $$T(j\omega) = \frac{(10)(10)\left(1+\frac{j\omega}{10}\right)}{1000\left(1+\frac{j\omega}{1000}\right)}$$

$$|T| = \frac{(0.10)\sqrt{1+\left(\frac{\omega}{10}\right)^2}}{\sqrt{1+\left(\frac{\omega}{1000}\right)^2}}$$



7.8

a. $V_0 = -g_m V_\pi R_L \quad V_\pi = \left(\frac{r_\pi}{r_\pi + R_S}\right)V_i$

$|T| = g_m R_L\left(\frac{r_\pi}{r_\pi + R_S}\right) = (29)(6)\left(\frac{5.2}{5.2+0.5}\right)$

$\underline{|T_{midband}| = 159}$

b. $\tau_S = (R_S + r_\pi)C_C$

$f_L = \frac{1}{2\pi\tau_S} \Rightarrow \tau_S = \frac{1}{2\pi f_L} = \frac{1}{2\pi(30)}$

$\Rightarrow \underline{\tau_S = 5.31 \text{ ms}}$ Open-circuit

$\tau_P = \frac{1}{2\pi f_H} = \frac{1}{2\pi(480\times 10^3)}$

$\Rightarrow \underline{\tau_P = 0.332\ \mu s}$ Short-circuit

c. $C_C = \frac{\tau_S}{(R_S + r_\pi)} = \frac{5.31\times 10^{-3}}{(0.5+5.2)\times 10^3}$

$\Rightarrow \underline{C_C = 0.932\ \mu F}$

$\tau_P = R_L C_L$

$C_L = \frac{\tau_P}{R_L} = \frac{0.332\times 10^{-6}}{6\times 10^3} \Rightarrow \underline{C_L = 55.3 \text{ pF}}$

7.11

a. $R_{TH} = R_1\|R_2 = 10\|1.5 = 1.30\ k\Omega$

$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)V_{CC} = \left(\frac{1.5}{1.5+10}\right)(12)$

$= 1.565$ V

$I_{BQ} = \frac{1.565 - 0.7}{1.30 + (101)(0.1)} = 0.0759$ mA

$I_{CQ} = 7.59$ mA

$r_\pi = \frac{(100)(0.026)}{7.59} = 0.343\ k\Omega$

$g_m = \frac{7.59}{0.026} = 292$ mA/V

$R_i = R_1\|R_2\|[r_\pi + (1+\beta)R_E]$

$= 10\|1.5\|[0.343 + (101)(0.1)]$

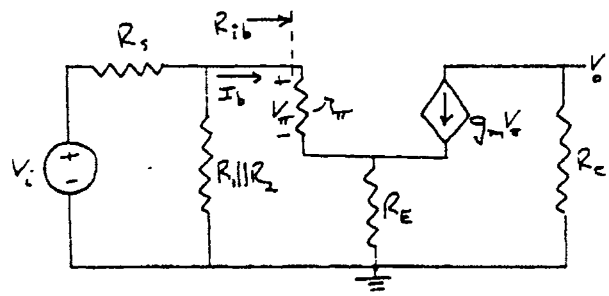
$= 1.30\|10.4 \Rightarrow R_i = 1.16\ k\Omega$

$\tau = (R_S + R_i)C_C = [0.5 + 1.16]\times 10^3 \times 0.1\times 10^{-6}$

$\tau = 1.66\times 10^{-4}$

$f_L = \frac{1}{2\pi\tau} = \frac{1}{2\pi(1.66\times 10^{-4})} \Rightarrow \underline{f_L = 959 \text{ Hz}}$

b.



$V_0 = -(\beta I_b)R_C$

$R_{ib} = r_\pi + (1+\beta)R_E$

$= 0.343 + (101)(0.1) = 10.4\ k\Omega$

$I_b = \left(\frac{R_1\|R_2}{R_1\|R_2 + R_{ib}}\right)I_i$

$= \left(\frac{1.30}{1.30 + 10.4}\right)I_i = (0.111)I_i$

$I_i = \frac{V_i}{R_S + R_1\|R_2\|R_{ib}}$

$= \frac{V_i}{0.5 + (1.3)\|(10.4)}$

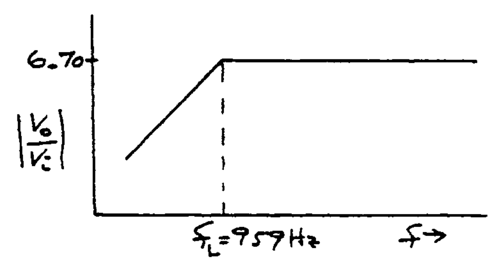
$I_i = \frac{V_i}{1.656}$

$\left|\frac{V_0}{V_i}\right| = \frac{\beta R_C(0.111)}{1.656}$

$\Rightarrow \left|\frac{V_0}{V_i}\right|_{midband} = \frac{(100)(1)(0.111)}{1.656}$

$\Rightarrow \underline{\left|\frac{V_0}{V_i}\right|_{midband} = 6.70}$

c.



7.12

$I_{DQ} = 0.5 \text{ mA} \Rightarrow V_S = (0.5)(0.5) = 0.25$ V

$I_{DQ} = K_n(V_{GS} - V_{TN})^2$

$\Rightarrow V_{GS} = \sqrt{\frac{0.5}{0.2}} + 1.5 = 3.08$ V

$V_G = V_{GS} + V_S = 3.08 + 0.25 \Rightarrow V_G = 3.33$ V

$V_G = \left(\frac{R_2}{R_1+R_2}\right)V_{DD} \Rightarrow 3.33 = \frac{1}{R_1}\cdot R_{in}\cdot V_{DD}$

$$3.33 = \frac{1}{R_1}(200)(9) \Rightarrow \underline{R_1 = 541\ \text{k}\Omega}$$

$$\frac{541 R_2}{541 + R_2} = 200 \Rightarrow \underline{R_2 = 317\ \text{k}\Omega}$$

$$V_D = V_{DSQ} + V_S = 4.5 + 0.25 = 4.75$$

$$R_D = \frac{9 - 4.75}{0.5} \Rightarrow \underline{R_D = 8.5\ \text{k}\Omega}$$

$$f_L = \frac{1}{2\pi\tau_L} \Rightarrow \tau_L = \frac{1}{2\pi f_L} = \frac{1}{2\pi(20)} = 7.96\ \text{ms}$$

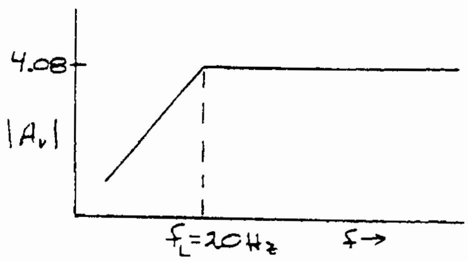
$$\tau_L = R_{in} \cdot C_C \Rightarrow C_C = \frac{\tau_L}{R_{in}} = \frac{7.96 \times 10^{-3}}{200 \times 10^3}$$

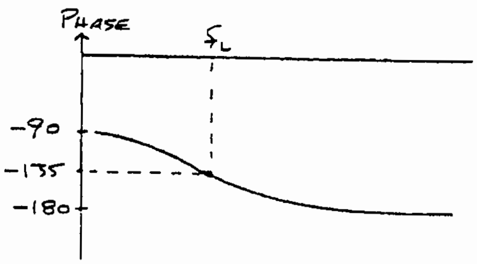
$$\Rightarrow \underline{C_C = 0.0398\ \mu\text{F}}$$

$$g_m = 2(0.2)(3.08 - 1.5) = 0.632\ \text{mA/V}$$

$$|A_v|_{\text{midband}} = \frac{g_m R_D}{1 + g_m R_S} = \frac{(0.632)(8.5)}{1 + (0.632)(0.5)}$$

$$\Rightarrow |A_v| = 4.08$$





7.13

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$\Rightarrow V_{GS} = \sqrt{\frac{I_{DQ}}{K_n}} + V_{TN} = \sqrt{\frac{1}{0.5}} + 1 = 2.41\,V$$

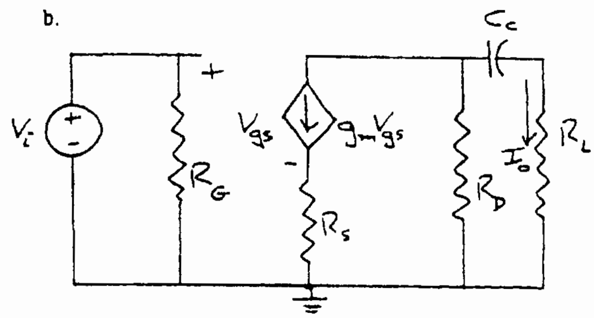
$$V_S = -2.41\ \text{V}$$

$$R_S = \frac{-2.41 - (-5)}{1} \Rightarrow \underline{R_S = 2.59\ \text{k}\Omega}$$

$$V_D = V_{DSQ} + V_S = 3 - 2.41 = 0.59\ \text{V}$$

$$R_D = \frac{5 - 0.59}{1} \Rightarrow \underline{R_D = 4.4\ \text{k}\Omega}$$

b.



$$I_0 = -(g_m V_{gs})\left(\frac{R_D}{R_D + R_L + \frac{1}{sC_C}}\right)$$

$$V_{gs} = \frac{V_i}{1 + g_m R_S}$$

$$\frac{I_0(s)}{V_i(s)} = \frac{-g_m}{1 + g_m R_S} \cdot R_D \left[\frac{sC_C}{1 + s(R_D + R_L)C_C}\right]$$

$$T(s) = \frac{I_0(s)}{V_i(s)}$$

$$= \frac{-g_m R_D}{1 + g_m R_S} \cdot \frac{1}{R_D + R_L} \cdot \frac{s(R_D + R_L)C_C}{1 + s(R_D + R_L)C_C}$$

c. $$f_L = \frac{1}{2\pi\tau_L} \rightarrow \tau_L = \frac{1}{2\pi f_L} = \frac{1}{2\pi(10)} = 15.9\ \text{ms}$$

$$\tau_L = (R_D + R_L)C_C$$

$$\Rightarrow C_C = \frac{\tau_L}{R_D + R_L} = \frac{15.9 \times 10^{-3}}{(4.41 + 4) \times 10^3}$$

$$\Rightarrow \underline{C_C = 1.89\ \mu\text{F}}$$

7.14

a. $$\frac{9 - V_{SG}}{R_S} = I_D = K_p(V_{SG} + V_{TP})^2$$

$$9 - V_{SG} = (0.5)(12)(V_{SG}^2 - 4V_{SG} + 4)$$

$$6V_{SG}^2 - 23V_{SG} + 15 = 0$$

$$V_{SG} = \frac{23 \pm \sqrt{(23)^2 - 4(6)(15)}}{2(6)} \Rightarrow V_{SG} = 3\ \text{V}$$

$$g_m = 2K_p(V_{SG} + V_{TP}) = 2(0.5)(3 - 2)$$

$$\Rightarrow g_m = 1\ \text{mA/V}$$

$$R_0 = \frac{1}{g_m} \Big\| R_S = 1 \| 12 \Rightarrow \underline{R_0 = 0.923\ \text{k}\Omega}$$

b. $$\underline{\tau = (R_0 + R_L)C_C}$$

a. $$f_L = \frac{1}{2\pi\tau} \Rightarrow \tau = \frac{1}{2\pi f_L} = \frac{1}{2\pi(20)} = 7.96\ \text{ms}$$

$$C_C = \frac{\tau}{R_0 + R_L} = \frac{7.96 \times 10^{-3}}{(0.923 + 10) \times 10^3}$$

$$\Rightarrow \underline{C_C = 0.729\ \mu\text{F}}$$

7.15

a. $I_{CQ} = 1\text{ mA},\ I_{BQ} = \frac{1}{120} = 0.00833\text{ mA}$

$R_1 \| R_2 = (0.1)(1+\beta)(R_E)$

$= (0.1)(121)(4) = 48.4\text{ k}\Omega$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E$

$\frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$

$= (0.00833)(48.4) + 0.7 + (121)(0.00833)(4)$

$\frac{1}{R_1}(48.4)(12) = 5.13$

$\underline{R_1 = 113\text{ k}\Omega}$

$\frac{113R_2}{113+R_2} = 48.4 \Rightarrow \underline{R_2 = 84.7\text{ k}\Omega}$

b. $R_o = \frac{r_\pi}{1+\beta} \| R_E \| r_o$

$r_\pi = \frac{(120)(0.026)}{1} = 3.12\text{ k}\Omega$

$r_o = \frac{80}{1} = 80\text{ k}\Omega$

$R_o = \frac{3.12}{121} \| 4 \| 80 = 0.0258 \| 4 \| 80$

$\Rightarrow \underline{R_o = 25.6\ \Omega}$

c. $\tau = (R_o + R_L)C_{C2}$

$\tau = (0.0256 + 4) \times 10^3 \times 2 \times 10^{-6} = 8.05 \times 10^{-3}\text{ s}$

$f = \frac{1}{2\pi\tau} = \frac{1}{2\pi(8.05 \times 10^{-3})} \Rightarrow \underline{f = 19.8\text{ Hz}}$

7.16

(a) $\frac{5 - V_{SG}}{R_1} = K_p(V_{SG} + V_{TP})^2$

$5 - V_{SG} = (1)(1.2)(V_{SG} - 1.5)^2 = (1.2)(V_{SG}^2 - 3V_{SG} + 2.25)$

$1.2V_{SG}^2 - 2.6V_{SG} - 2.3 = 0 \Rightarrow \underline{V_{SG} = 2.84\ V}$

$\underline{I_{DQ} = 1.8\ mA}$

$V_{SDQ} = 10 - (1.8)(1.2 + 1.2) \Rightarrow \underline{V_{SDQ} = 5.68\ V}$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(1.8)} = 2.68\ mA/V$

$r_o = \infty$

(b) $R_{is} = \frac{1}{g_m} = \frac{1}{2.68} = 0.373\ k\Omega$

$R_i = 1.2 \| 0.373 = 0.285\ k\Omega$

For C_{C1}, $\tau_{s1} = (285 + 200)(4.7x10^{-6}) = 2.28\ ms$

For C_{C2}, $\tau_{s2} = (1.2x10^3 + 50x10^3)(10^{-6}) = 51.2\ ms$

(c) C_{C2} dominates,

$f_{3-dB} = \frac{1}{2\pi\tau_{s2}} = \frac{1}{2\pi(51.2x10^{-3})} = 3.1\ Hz$

7.17

Assume $V_{TN} = 1V$, $k_n' = 80\ \mu A/V^2$, $\lambda = 0$

Neglecting $R_{Si} = 200\ \Omega$, Midband gain is:

$|A_v| = g_m R_D$

Let $I_{DQ} = 0.2\ mA$, $V_{DSQ} = 5V$

Then $R_D = \frac{9-5}{0.2} \Rightarrow \underline{R_D = 20\ k\Omega}$

We need

$g_m = \frac{|A_v|}{R_D} = \frac{10}{20} = 0.5\ mA/V^2$

and

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_{DQ}}$

or

$0.5 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.2)} \Rightarrow \underline{\frac{W}{L} = 7.81}$

Let $R_1 + R_2 = \frac{9}{(0.2)I_{DQ}} = \frac{9}{(0.2)(0.2)} = 225\ k\Omega$

$I_{DQ} = 0.2 = \left(\frac{0.080}{2}\right)(7.81)(V_{GS} - 1)^2$

$\Rightarrow V_{GS} = 1.80 = \left(\frac{R_2}{R_1 + R_2}\right)(9) = \left(\frac{R_2}{225}\right)(9) \Rightarrow$

$\underline{R_2 = 45\ k\Omega,\ R_1 = 180\ k\Omega}$

$R_{TH} = R_1 \| R_2 = 180 \| 45 = 36\ k\Omega$

$\tau_1 = \frac{1}{2\pi f_1} = \frac{1}{2\pi(200)} = 7.96x10^{-4}\ s = (R_{Si} + R_{TH})C_C$

or

$C_C = \frac{7.96x10^{-4}}{(200 + 36x10^3)} \Rightarrow \underline{C_C = 220\ \mu F}$

$\tau_2 = \frac{1}{2\pi f_2} = \frac{1}{2\pi(3x10^3)} = 5.31x10^{-5}\ s = R_D C_L$

or

$C_L = \frac{5.31x10^{-5}}{20x10^3} \Rightarrow \underline{C_L = 2.66\ nF}$

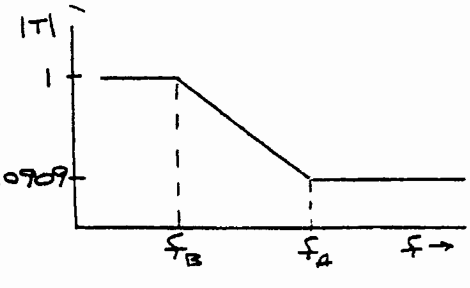
7.18

a. $T(s) = \frac{R_2 + (1/sC)}{R_2 + (1/sC) + R_1}$

$T(s) = \frac{1 + sR_2C}{1 + s(R_1 + R_2)C}$

$\tau_A = R_2C,\ \tau_B = (R_1 + R_2)C$

b.



c. $\tau_A = R_2 C = (10^3)(100 \times 10^{-12}) = \underline{10^{-7}\ s = \tau_A}$

$\tau_B = (R_1 + R_2)C = [10 + 1] \times 10^3 \times 100 \times 10^{-12}$

$= \underline{1.1 \times 10^{-6}\ s = \tau_B}$

$f_A \approx \dfrac{1}{2\pi\tau_A} = \dfrac{1}{2\pi(10^{-7})} \Rightarrow \underline{f_A = 1.59\ \text{MHz}}$

$f_B \approx \dfrac{1}{2\pi\tau_B} = \dfrac{1}{2\pi(1.1 \times 10^{-6})}$

$\Rightarrow \underline{f_B = 0.145\ \text{MHz}}$

7.19

$I_{BQ} = \dfrac{10-0.7}{430+(201)(2.5)} = 0.00997\ mA$

$I_{CQ} = (200)I_{BQ} = 1.99\ mA$

$r_\pi = \dfrac{(200)(0.026)}{1.99} = 2.61\ k\Omega$

$R_{ib} = 2.61+(201)(2.5) = 505\ k\Omega$

$\tau_s = \dfrac{1}{2\pi f_L} = \dfrac{1}{2\pi(15)} = 0.0106\ s$

$= R_{eq}C_C = (0.5+505\|430)x10^3 C_C = 232.7x10^3 C_C$

Or

$\underline{C_C = 4.56x10^{-8}\ F \Rightarrow 45.6\ nF}$

7.20

$R_{TH} = R_1\|R_2 = 1.2\|1.2 = 0.6\ k\Omega$

$V_{TH} = \left(\dfrac{R_2}{R_1+R_2}\right)(V_{CC}) = \left(\dfrac{1.2}{1.2+1.2}\right)(5) = 2.5V$

$I_{BQ} = \dfrac{2.5-0.7}{0.6+(101)(0.05)} = 0.319\ mA$

$I_{CQ} = 31.9\ mA$

$r_\pi = \dfrac{(100)(0.026)}{31.9} = 0.0815\ k\Omega$

$\tau_{C_{C1}} >> \tau_{C_{C2}}$ and $f = \dfrac{1}{2\pi\tau}$ so that

$f_{3\text{-}dB}(C_{C1}) << f_{3\text{-}dB}(C_{C2})$

Then, for $f_{3\text{-}dB}(C_{C1}) \Rightarrow C_{C2}$ acts as an open and for $f_{3\text{-}dB}(C_{C2}) \Rightarrow C_{C1}$ acts as a short circuit.

$f_{3\text{-}dB}(C_{C2}) = 25\ Hz = \dfrac{1}{2\pi\tau_2}$, so that

$\tau_2 = \dfrac{1}{2\pi(25)} = 0.00637\ s = R_{eq}C_{C2}$

where $R_{eq} = R_L + R_E \left\| \left(\dfrac{r_\pi + R_1\|R_2\|R_S}{1+\beta}\right)\right.$

$= 10+50\left\|\left(\dfrac{81.5+600\|300}{101}\right)\right. = 10+50\|2.79 \Rightarrow$

$R_{eq} = 12.6\,\Omega \Rightarrow C_{C2} = \dfrac{0.00637}{12.6} \Rightarrow \underline{C_{C2} = 506\ \mu F}$

$R_{ib} = r_\pi + (1+\beta)R_E$ Assume C_{C2} an open

$R_{ib} = 81.5+(101)(50) = 5132\,\Omega$

$\tau_1 = (100)\tau_2 = (100)(0.00637) = 0.637\ s = R_{eq1}C_{C1}$

$R_{eq1} = R_S + R_{TH}\|R_{ib} = 300+600\|5132 = 837\,\Omega$

So $C_{C1} = \dfrac{0.637}{837} \Rightarrow \underline{C_{C1} = 761\ \mu F}$

7.21

a. $I_D = K_n(V_{GS} - V_{TN})^2$

$V_{GS} = \sqrt{\dfrac{I_D}{K_n}} + V_{TN} = \sqrt{\dfrac{0.5}{0.5}} + 0.8 = 1.8\ V$

$R_S = \dfrac{-V_{GS} - (-5)}{0.5} = \dfrac{5 - 1.8}{0.5} \Rightarrow \underline{R_S = 6.4\ k\Omega}$

$V_D = V_{DSQ} + V_S = 4 - 1.8 = 2.2\ \text{V}$

$R_D = \dfrac{5 - 2.2}{0.5} \Rightarrow \underline{R_D = 5.6\ k\Omega}$

(b) $g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.5)(0.5)} = 1\ mA/V$

From Problem 7.20,

$\tau_A = R_S C_S = (6.4 \times 10^3)(5 \times 10^{-6})$

$= 3.2 \times 10^{-2}\ \text{s}$

$f_A = \dfrac{1}{2\pi\tau_A} = \dfrac{1}{2\pi(3.2 \times 10^{-2})} \Rightarrow \underline{f_A = 4.97\ \text{Hz}}$

$\tau_B = \left(\dfrac{R_S}{1+g_m R_S}\right)C_S = \left[\dfrac{6.4 \times 10^3}{1+(1)(6.4)}\right](5 \times 10^{-6})$

$= 4.32 \times 10^{-3}\ \text{s}$

$f_B = \dfrac{1}{2\pi\tau_B} = \dfrac{1}{2\pi(4.32 \times 10^{-3})} \Rightarrow \underline{f_B = 36.8\ \text{Hz}}$

c.

$$|A_v| = \frac{g_m R_D(1 + sR_S C_S)}{(1 + g_m R_S)\left[1 + s\left(\dfrac{R_S}{1+g_m R_S}\right)C_S\right]}$$

As R_S becomes large

$$|A_v| \to \frac{g_m R_D(sR_S C_S)}{(g_m R_S)\left[1 + s\left(\dfrac{R_S}{g_m R_S}\right)C_S\right]}$$

$$A_v = \frac{(g_m R_D)\left[s\left(\dfrac{1}{g_m}\right)C_S\right]}{1 + s\left(\dfrac{1}{g_m}\right)C_S}$$

The corner frequency $f_B = \dfrac{1}{2\pi(1/g_m)C_S}$ and the corresponding $\underline{f_A \to 0}$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.5)(0.5)} = 1\ mA/V$$

$$f_B = \frac{1}{2\pi\left(\dfrac{1}{10^{-3}}\right)(5\times10^{-6})} \Rightarrow \underline{f_B = 31.8 \text{ Hz}}$$

7.22

a. $f_B = \dfrac{1}{2\pi\tau_B}$

and $\tau_B = \dfrac{R_E(R_S + r_\pi)C_E}{R_S + r_\pi + (1+\beta)R_E}$

For $R_S = 0$ $\tau_B = \dfrac{R_E r_\pi C_E}{r_\pi + (1+\beta)R_E}$

$$I_{EQ} = \frac{-0.7-(-10)}{5} = 1.86 \text{ mA}$$

$\beta = 75 \Rightarrow I_{CQ} = 1.84$ mA

$\beta = 125 \Rightarrow I_{CQ} = 1.85$ mA

For $f_B \le 200$ Hz

$$\Rightarrow \tau_B \ge \frac{1}{2\pi(200)} = 0.796 \text{ ms}$$

$r_\pi \alpha \beta$ so smallest τ_B will occur for smallest β.

$$\beta = 75;\ r_\pi = \frac{(75)(0.026)}{1.84} = 1.06 \text{ k}\Omega$$

$$0.796\times10^{-3} = \frac{(5\times10^3)(1.06)C_E}{1.06 + (76)(5)}$$

$\Rightarrow \underline{C_E = 57.2\ \mu\text{F}}$

b. For $\beta = 125$; $r_\pi = \dfrac{(125)(0.026)}{1.85} = 1.76 \text{ k}\Omega$

$$\tau_B = \frac{(5\times10^3)(1.76)(57.2\times10^{-6})}{1.76 + (126)(5)} = 0.797 \text{ ms}$$

$$f_B = \frac{1}{2\pi\tau_B} = \frac{1}{2\pi(0.797\times10^{-3})}$$

$\Rightarrow \underline{f_B = 199.7 \text{ Hz}}$ Essentially independent of β.

$$\tau_A = R_E C_E = (5\times10^3)(57.2\times10^{-6}) = 0.286 \text{ sec}$$

$$f_A = \frac{1}{2\pi\tau_A} = \frac{1}{2\pi(0.286)}$$

$\Rightarrow \underline{f_A = 0.556 \text{ Hz}}$ Independent of β.

7.23

a. Expression for the voltage gain is the same as Equation (7.58) with $R_S = 0$.

b. $\tau_A = R_E C_E$

$$\tau_B = \frac{R_E r_\pi C_E}{r_\pi + (1+\beta)R_E}$$

7.24

$$\tau_H = (R_L \| R_C)C_L = (10\|5)\times10^3\times15\times10^{-12}$$

$$\tau_H = 5\times10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi(5\times10^{-8})} \Rightarrow \underline{f_H = 3.18 \text{ MHz}}$$

$$I_{EQ} = \frac{10-0.7}{10} = 0.93 \text{ mA},\ I_{CQ} = 0.921 \text{ mA}$$

$$g_m = \frac{0.921}{0.026} = 35.4 \text{ mA/V}$$

$$A_v = g_m(R_C\|R_L) = 35.4(5\|10) \Rightarrow \underline{A_v = 118}$$

7.25

$$V_G = \left(\frac{R_2}{R_1+R_2}\right)V_{DD} = \left(\frac{166}{166+234}\right)(10)$$

$$= 4.15 \text{ V}$$

$$I_D = \frac{V_G - V_{GS}}{R_S} = K_n(V_{GS}-V_{TN})^2$$

$$4.15 - V_{GS} = (0.5)(0.5)(V_{GS}^2 - 4V_{GS} + 4)$$

$$0.25V_{GS}^2 - 3.15 = 0 \Rightarrow V_{GS} = 3.55 \text{ V}$$

$$g_m = 2K_n(V_{GS}-V_{TN}) = 2(0.5)(3.55-2)$$

$$g_m = 1.55\ mA/V$$

$$R_0 = R_S \left\| \frac{1}{g_m} = 0.5 \right\| \frac{1}{1.55} = 0.5\|0.645$$

$$R_0 = 0.282 \text{ k}\Omega$$

$\tau = (R_0\|R_L)C_L$ and $f_H = \dfrac{1}{2\pi\tau}$

$B\omega \approx f_H = 5$ MHz

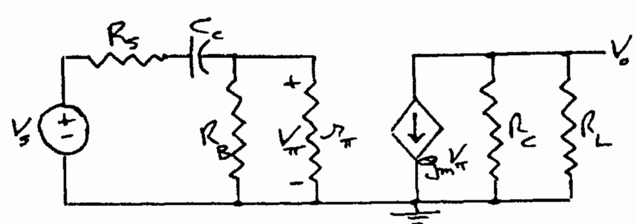
$$\Rightarrow \tau = \frac{1}{2\pi(5\times10^6)} = 3.18\times10^{-8} \text{ s}$$

$$C_L = \frac{\tau}{R_0\|R_L} = \frac{3.18\times10^{-8}}{(0.282\|4)\times10^3}$$

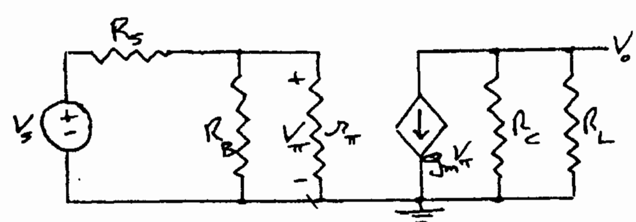
$\Rightarrow \underline{C_L = 121 \text{ pF}}$

7.26

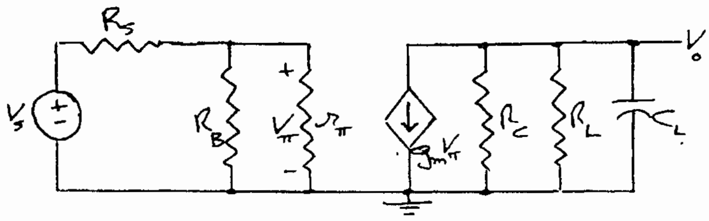
(a) Low-frequency



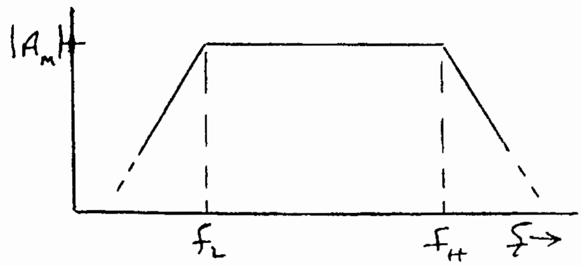
Mid-Band



High-frequency



(b)



(c) $I_{BQ} = \dfrac{12-0.7}{1\ M\Omega} = 11.3\ \mu A$

$I_{CQ} = 1.13\ mA$

$r_\pi = \dfrac{(100)(0.026)}{1.13} = 2.3\ k\Omega$

$g_m = \dfrac{1.13}{0.026} = 43.46\ mA/V$

$A_m = \dfrac{V_o}{V_s}(midband) = -g_m(R_C \| R_L)\left(\dfrac{R_B \| r_\pi}{R_B \| r_\pi + R_S}\right)$

$= -(43.46)(5.1\|500)\left(\dfrac{1000\|2.3}{1000\|2.3+1}\right)$

$= -(43.46)(5.05)\left(\dfrac{2.29}{2.29+1}\right) \Rightarrow \quad |A_m| = 153$

$|A_m|_{dB} = 43.7\ dB$

$f_L = \dfrac{1}{2\pi\tau_L}, \quad \tau_L = (R_S + R_B\|r_\pi)C_C$

or

$\tau_L = (1 + 1000\|2.3)x10^3(10x10^{-6}) \Rightarrow$

$\tau_L = 3.29x10^{-2}\ s \Rightarrow \quad f_L = 4.84\ Hz$

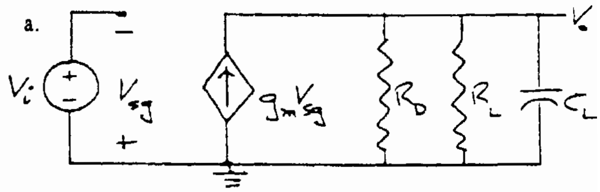
$f_H = \dfrac{1}{2\pi\tau_H}, \quad \tau_H = (R_C\|R_L)C_L \Rightarrow$

$\tau_L = (5.1\|500)x10^3(10x10^{-12}) = 5.05x10^{-8}\ s$

$\Rightarrow f_H = 3.15\ MHz$

7.27

a.



$V_0 = (g_m V_{sg})\left(R_0 \,\|\, R_L \,\|\, \dfrac{1}{sC_L}\right)$

$V_{sg} = -V_i$

$A_v(s) = \dfrac{V_0(s)}{V_i(s)} = -g_m\left(R_D \,\|\, R_L \,\|\, \dfrac{1}{sC_L}\right)$

$= -g_m\left[\dfrac{R_D\|R_L \cdot \dfrac{1}{sC_L}}{R_D\|R_L + \dfrac{1}{sC_L}}\right]$

$A_v(s) = -g_m(R_D\|R_L) \cdot \dfrac{1}{1 + s(R_D\|R_L)C_L}$

b. $\tau = (R_D\|R_L)C_L$

c. $\tau = (10\|20) \times 10^3 \times 10 \times 10^{-12}$

$\Rightarrow \tau = 6.67 \times 10^{-8}$ s

$f_H = \dfrac{1}{2\pi\tau} = \dfrac{1}{2\pi(6.67 \times 10^{-8})}$

$\Rightarrow f_H = 2.39$ MHz

From Example 7.6, $g_m = 0.705$ mA/V

$|A_v| = g_m(R_D\|R_L) = (0.705)(10\|20)$

$\Rightarrow |A_v| = 4.7$

7.31

$f_T = \dfrac{g_m}{2\pi(C_\pi + C_\mu)}$

$g_m = \dfrac{I_{CQ}}{V_T} = \dfrac{1}{0.026} = 38.5$ mA/V

$f_T = \dfrac{38.5 \times 10^{-3}}{2\pi(10+2) \times 10^{-12}}$

$f_T = 511$ MHz

$f_\beta = \dfrac{f_T}{\beta} = \dfrac{511}{120} \Rightarrow f_\beta = 4.26$ MHz

7.32

$$f_\beta = \frac{f_T}{\beta} = \frac{5000\text{ MHz}}{150} \Rightarrow \underline{f_\beta = 33.3\text{ MHz}}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$g_m = \frac{0.5}{0.026} = 19.2\text{ mA/V}$$

$$5 \times 10^9 = \frac{19.2 \times 10^3}{2\pi(C_\pi + 0.15) \times 10^{-12}}$$

$$C_\pi + 0.15 = \frac{19.2 \times 10^3}{2\pi(10^{-12})(5 \times 10^9)} = 0.611\text{ pF}$$

$$\underline{C_\pi = 0.461\text{ pF}}$$

7.33

a. $f_\beta = \dfrac{f_T}{\beta} = \dfrac{2000\text{ MHz}}{150} = \underline{13.3\text{ MHz} = f_\beta}$

b. $h_{fe} = \dfrac{150}{1 + j(f/f_\beta)}$

$$|h_{fe}| = \frac{150}{\sqrt{1 + (f/f_\beta)^2}} = 10$$

$$1 + \left(\frac{f}{f_\beta}\right)^2 = \left(\frac{150}{10}\right)^2 = 225$$

$$f = f_\beta \cdot \sqrt{224} = (13.3)\sqrt{224}$$

$$\Rightarrow \underline{f = 199\text{ MHz}}$$

7.34

(a) $V_o = -g_m V_\pi R_L$ where

$$V_\pi = \frac{r_\pi \left\| \frac{1}{sC_1} \right.}{r_\pi \left\| \frac{1}{sC_1} \right. + r_b} \cdot V_i = \frac{\frac{r_\pi}{1 + sr_\pi C_1}}{\frac{r_\pi}{1 + sr_\pi C_1} + r_b} \cdot V_i$$

$$= \frac{r_\pi}{r_\pi + r_b + sr_b r_\pi C_1} \cdot V_i = \left(\frac{r_\pi}{r_\pi + r_b}\right)\left(\frac{1}{1 + s(r_b \| r_\pi)C_1}\right) \cdot V_i$$

So

$$A_v(s) = \frac{V_o(s)}{V_i(s)} = -g_m R_L \left(\frac{r_\pi}{r_\pi + r_b}\right)\left(\frac{1}{1 + s(r_b \| r_\pi)C_1}\right)$$

(b) Midband gain:

$$r_\pi = \frac{(100)(0.026)}{1} = 2.6\,k\Omega\ ,$$

$$g_m = \frac{1}{0.026} = 38.46\,mA/V$$

(i) For $r_b = 100\,\Omega$

$$A_{v1} = -(38.46)(4)\left(\frac{2.6}{2.6 + 0.1}\right) \Rightarrow \underline{A_{v1} = -148.1}$$

(ii) For $r_b = 500\,\Omega$

$$A_{v2} = -(38.46)(4)\left(\frac{2.6}{2.6 + 0.5}\right) \Rightarrow \underline{A_{v2} = -129.0}$$

(c) $f_{3\text{-}dB} = \dfrac{1}{2\pi\tau}$, $\tau = (r_b \| r_\pi)C_1$

(i) For $r_b = 100\,\Omega$

$$\tau_1 = (0.1 \| 2.6)x10^3(2.2x10^{-12}) = 2.12x10^{-10}\ s$$

$$\underline{\Rightarrow f_{3\text{-}db} = 751\ MHz}$$

(ii) For $r_b = 500\,\Omega$

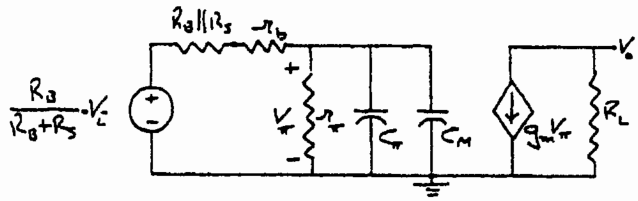
$$\tau_2 = (0.5 \| 2.3)x10^3(2.2x10^{-12}) = 9.04x10^{-10}\ s$$

$$\underline{f_{3\text{-}dB} = 176\ MHz}$$

7.35

a. $\underline{C_M = C_\mu(1 + g_m R_L)}$

b.



$V_0 = -g_m V_\pi R_L$　Let $C_\pi + C_M = C_i$

$$V_\pi = \frac{r_\pi \left\| \frac{1}{sC_i} \right.}{r_\pi \left\| \frac{1}{sC_i} \right. + R_B \| R_S + r_b} \cdot \left(\frac{R_B}{R_B + R_S}\right) V_i$$

$$A_v(s) = \frac{V_0(s)}{V_i(s)}$$

$$= -g_m R_L \left(\frac{R_B}{R_B + R_S}\right) \left[\frac{\dfrac{r_\pi \cdot \frac{1}{sC_i}}{r_\pi + \frac{1}{sC_i}}}{\dfrac{r_\pi \cdot \frac{1}{sC_i}}{r_\pi + \frac{1}{sC_i}} + R_B \| R_S + r_b} \right]$$

$$= -g_m R_L \left(\frac{R_B}{R_B + R_S}\right) \times \left[\frac{r_\pi}{r_\pi + (1 + sr_\pi C_i)(R_B \| R_S + r_b)}\right]$$

Let $R_{eq} = (R_B \| R_S + r_b)$

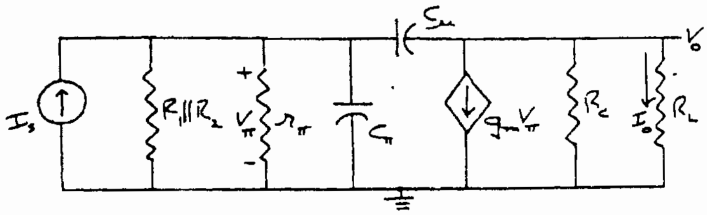
$$A_v(s) = -\beta R_L \left(\frac{R_B}{R_B + R_S}\right) \times \left[\frac{1}{(r_\pi + R_{eq})[1 + s(r_\pi \| R_{eq})C_i]}\right]$$

$$\underline{A_v(s) = \frac{-\beta R_L}{r_\pi + R_{eq}} \cdot \left(\frac{R_B}{R_B + R_S}\right) \cdot \frac{1}{1 + s(r_\pi \| R_{eq})C_i}}$$

c. $\underline{f_H = \dfrac{1/}{2\pi(r_\pi \| R_{eq})C_i}}$

7.36

High Freq. ⇒ C_{C1}, C_{C2}, C_E → short circuits



$$g_m = \frac{I_{CQ}}{V_T} = \frac{5}{0.026} = 192 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow 250 \times 10^6 = \frac{192 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = 122 \text{ pF} \Rightarrow C_\mu = 5 \text{ pF}, \; C_\pi = 117 \text{ pF}$$

$$C_M = C_\mu(1 + g_m(R_C \| R_L))$$

$$= 5[1 + (192)(1\|1)] \Rightarrow C_M = 485 \text{ pF}$$

$$C_i = C_\pi + C_M = 117 + 485 = 602 \text{ pF}$$

$$r_\pi = \frac{(200)(0.026)}{5} = 1.04 \text{ k}\Omega$$

$$R_{eq} = R_1 \| R_2 \| r_\pi = 5 \| 1.04 = 0.861 \text{ k}\Omega$$

$$\tau = R_{eq} \cdot C_i = (0.861 \times 10^3)(602 \times 10^{-12})$$

$$= 5.18 \times 10^{-7} \text{ s}$$

$$f = \frac{1}{2\pi\tau} = \frac{1}{2\pi(5.18 \times 10^{-7})} \Rightarrow \underline{f = 307 \text{ kHz}}$$

7.37

$$R_{TH} = R_1 \| R_2 = 60 \| 5.5 = 5.04 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{5.5}{5.5 + 60}\right)(15) = 1.26 \text{ V}$$

$$I_{BQ} = \frac{1.26 - 0.7}{5.04 + (101)(0.2)} = 0.0222 \text{ mA}$$

$$I_{CQ} = 2.22 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{2.22} = 1.17 \text{ k}\Omega$$

$$g_m = \frac{2.22}{0.026} = 85.4 \text{ mA/V}$$

Lower 3 – dB frequency:

$$\tau_L = R_{eq} \cdot C_{C1}$$

$$R_{eq} = R_S + R_1 \| R_2 \| r_\pi$$

$$= 2 + 60 \| 5.5 \| 1.17 = 2.95 \text{ k}\Omega$$

$$\tau_L = (2.95 \times 10^3)(0.1 \times 10^{-6}) = 2.95 \times 10^{-4} \text{ s}$$

$$f_L = \frac{1}{2\pi\tau_L} = \frac{1}{2\pi(2.95 \times 10^{-4})} \Rightarrow \underline{f_L = 540 \text{ Hz}}$$

Upper 3 – dB frequency:

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow 400 \times 10^6 = \frac{85.4 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

$$C_\pi + C_\mu = 34 \text{ pF}; \; C_\mu = 2 \text{ pF} \Rightarrow C_\pi = 32 \text{ pF}$$

$$C_M = C_\mu(1 + g_m R_C) = 2(1 + (85.4)(4))$$

$$\Rightarrow C_M = 685 \text{ pF}$$

$$C_i = C_\pi + C_M = 32 + 685 = 717 \text{ pF}$$

$$R_{eq} = R_S \| R_1 \| R_2 \| r_\pi = 2 \| 60 \| 5.5 \| 1.17$$

$$= 0.644 \text{ k}\Omega$$

$$\tau = R_{eq} \cdot C_i = (0.644 \times 10^3)(717 \times 10^{-12})$$

$$= 4.62 \times 10^{-7} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau} \Rightarrow \underline{f_H = 344 \text{ kHz}}$$

7.38

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})}$$

$$g_m = 2\sqrt{K_n I_D}, \quad K_n = (15)\left(\frac{40}{10}\right) = 60 \; \mu A/V^2$$

$$g_m = 2\sqrt{(60)(100)} = 155 \; \mu A/V^2$$

$$f_T = \frac{155x10^{-6}}{2\pi(0.5 + 0.05)x10^{-12}} \Rightarrow \underline{f_T = 44.9 \; MHz}$$

7.39

a. $C_M = C_{gd'} \; (1 + g_m(r_O \| R_D))$

$$C_M = 5[1 + (3)(15\|10)] \Rightarrow \underline{C_M = 95 \text{ pF}}$$

b. $\tau = (r_i)(C_{gs} \; + C_M)$

$$\tau = (10 \times 10^3)(50 + 95) \times 10^{-12} = 1.45 \times 10^{-6} \text{ s}$$

$$f = \frac{1}{2\pi\tau} = \frac{1}{2\pi(1.45 \times 10^{-6})}$$

$$\Rightarrow \underline{f = 110 \text{ kHz}}$$

7.40

$$f_T = \frac{g_m}{2\pi(C_{gsT} + C_{gdT})} \quad \text{(Eq. 7.90)}$$

Let $C_{gdT} = 0$ and $C_{gsT} = \left(\frac{2}{3}\right)(WLC_{ox})$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{\left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right) I_D}$$

$$\text{So } f_T = \frac{2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)I_D}}{2\pi\left(\frac{2}{3}\right)(WLC_{ox})}$$

$$= \frac{3}{2\pi L}\cdot\frac{\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)I_D}}{WC_{ox}}$$

$$\underline{f_T = \frac{3}{2\pi L}\cdot\sqrt{\frac{\mu_n I_D}{2WC_{ox}L}}}$$

7.41

(a) $g_m' = \dfrac{g_m}{1+g_m r_S}$

$g_m = 2K_n(V_{GS} - V_{TN})$

$K_n = \left(\dfrac{\mu_n C_{ox}}{2}\right)\left(\dfrac{W}{L}\right) = \left(\dfrac{(400)(7.25x10^{-8})}{2}\right)(10)$

$K_n = 1.45x10^{-4}\ mA/V^2$

For $V_{GS} = 5$ V

$g_m = 2(1.45\times10^{-4})(5-0.65) = 1.26\times10^{-3}$

$g_m' = (0.80)g_m = 1.01\times10^{-3}$

$1.01\times10^{-3} = \dfrac{1.26\times10^{-3}}{1+(1.26\times10^{-3})r_S}$

$1+(1.26\times10^{-3})r_S = 1.25$

$\Rightarrow \underline{r_S = 198\ \Omega}$

b. For $V_{GS} = 3$ V

$g_m = 2(1.45\times10^{-4})(3-0.65) = 0.6815\times10^{-3}$

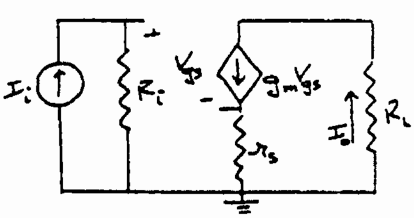
$g_m' = \dfrac{0.6815\times10^{-3}}{1+(0.6815\times10^{-3})(198)}$

$\underline{g_m' = 0.60\times10^{-3}\ \text{A/V}}$

Reduced by $\approx 12\%$

7.42

a.

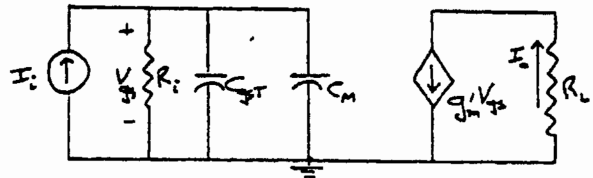


$I_0 = g_m V_{gs}$ and $V_{gs} = I_i R_i - g_m V_{gs} r_S$

so $V_{gs} = \dfrac{I_i R_i}{1+g_m r_S}$

Then $\underline{A_i = \dfrac{I_0}{I_i} = \dfrac{g_m R_i}{1+g_m r_S}}$

b. As an approximation, consider



In this case

$A_i = \dfrac{I_0}{I_i} = g_m' R_i \cdot \dfrac{1}{1+sR_i(C_{gsT}+C_M)}$

where $C_M = C_{gdT}(1+g_m' R_L)$ and $g_m' = \dfrac{g_m}{1+g_m r_s}$

c. As r_S increases, C_M decreases, so the bandwidth increases, but the current gain magnitude decreases.

7.43

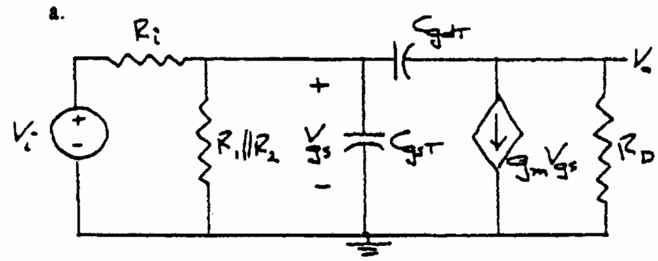
$V_{GS} = \left(\dfrac{R_2}{R_1+R_2}\right)V_{DD} = \left(\dfrac{225}{225+500}\right)(10)$

$V_{GS} = 3.10$ V

$g_m = 2K_n(V_{GS}-V_{TN}) = 2(1)(3.10-2)$

$g_m = 2.2\ mA/V$

a.



b. $C_M = C_{gdT}(1+g_m R_D) = (1)[1+(2.2)(5)]$

$\underline{C_M = 12\ \text{pF}}$

c. $\tau = (R_i\|R_1\|R_2)(C_{gsT}+C_M)$

$R_i\|R_1\|R_2 = 1\|500\|225 = 1\|155 = 0.994\ \text{k}\Omega$

$\tau = (0.994\times10^3)(5+12)\times10^{-12} = 1.69\times10^{-8}$ s

$f_H = \dfrac{1}{2\pi\tau} = \dfrac{1}{2\pi(1.69\times10^{-8})} \Rightarrow \underline{f_H = 9.42\ \text{MHz}}$

$A_v = \dfrac{-g_m V_{gs} R_D}{V_i}$ and

$V_{gs} = \dfrac{R_1\|R_2}{R_1\|R_2+R_i}\cdot V_i = \dfrac{155}{155+1}\cdot V_i = 0.994V_i$

$A_v = -(2.2)(5)(0.994) \Rightarrow \underline{A_v = -10.9}$

7.44

$R_{TH} = R_1 \| R_2 = 33\|22 = 13.2 \text{ k}\Omega$

$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(5) = \left(\frac{22}{22+33}\right)(5) = 2 \text{ V}$

$I_{BQ} = \frac{2 - 0.7}{13.2 + (121)(4)} = 0.00261 \text{ mA}$

$I_{CQ} = 0.314 \text{ mA}$

$r_\pi = \frac{(120)(0.026)}{0.314} = 9.94 \text{ k}\Omega$

$g_m = \frac{0.314}{0.026} = 12.1 \text{ mA/V}$

$r_0 = \frac{100}{0.314} = 318 \text{ k}\Omega$

a. $f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$

$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{12.1 \times 10^{-3}}{2\pi(600 \times 10^6)}$

$C_\pi + C_\mu = 3.21 \text{ pF};\ C_\mu = 1 \text{ pF} \Rightarrow C_\pi = 2.21 \text{ pF}$

$C_M = C_\mu[1 + g_m(r_0\|R_C\|R_L)]$

$= (1)[1 + (12.1)(318\|4\|5)]$

$C_M = 27.7 \text{ pF}$

b. $\tau = R_{eq}(C_\pi + C_M)$

$R_{eq} = R_1\|R_2\|R_S\|r_\pi = 33\|22\|2\|r_\pi$

$= 1.74\|9.94 \text{ k}\Omega \Rightarrow R_{eq} = 1.48 \text{ k}\Omega$

$\tau = (1.48 \times 10^3)(2.21 + 27.7) \times 10^{-12}$

$\tau = 4.43 \times 10^{-8} \text{ s}$

$f_H = \frac{1}{2\pi\tau} = \frac{1}{2\pi(4.43 \times 10^{-8})}$

$\Rightarrow \underline{f_H = 3.59 \text{ MHz}}$

$V_0 = -g_m V_\pi (r_0\|R_C\|R_L)$

$V_\pi = \left(\frac{R_1\|R_2\|r_\pi}{R_1\|R_2\|r_\pi + R_S}\right)V_i$

$R_1\|R_2\|r_\pi = 33\|22\|9.94 = 5.67 \text{ k}\Omega$

$V_\pi = \left(\frac{5.67}{5.67 + 2}\right)V_i = 0.739 V_i$

$r_0\|R_C\|R_L = 318\|4\|5 = 2.18 \text{ k}\Omega$

$A_v = -(12.1)(0.739)(2.18)$

$\underline{A_v = -19.5}$

7.45

$R_{TH} = R_1\|R_2 = 40\|5 = 4.44 \text{ k}\Omega$

$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\frac{5}{5+40}\right)(10) = 1.11 \text{ V}$

$I_{BQ} = \frac{1.11 - 0.7}{4.44 + (121)(0.5)} = 0.00631 \text{ mA}$

$I_{CQ} = 0.758 \text{ mA}$

$r_\pi = \frac{(120)(0.026)}{0.758} = 4.12 \text{ k}\Omega$

$g_m = \frac{0.758}{0.026} = 29.2 \text{ mA/V}$

$r_0 = \infty$

$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$

$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{29.2 \times 10^{-3}}{2\pi(250 \times 10^6)}$

$C_\pi + C_\mu = 18.6 \text{ pF};\ C_\mu = 3 \text{ pF} \Rightarrow C_\pi = 15.6 \text{ pF}$

a. $C_M = C_\mu[1 + g_m(R_C\|R_L)]$

$C_M = 3[1 + (29.2)(5\|2.5)] \Rightarrow C_M = 149 \text{ pF}$

For upper frequency:

$\tau_H = R_{eq}(C_\pi + C_M)$

$R_{eq} = r_\pi\|R_1\|R_2\|R_S = 4.12\|40\|5\|0.5$

$R_{eq} = 0.405 \text{ k}\Omega$

$\tau_H = (0.405 \times 10^3)(15.6 + 149) \times 10^{-12}$

$= 6.67 \times 10^{-8} \text{ s}$

$f_H = \frac{1}{2\pi\tau_H} \Rightarrow \underline{f_H = 2.39 \text{ MHz}}$

For lower frequency:

$\tau_L = R_{eq}C_{C1}$

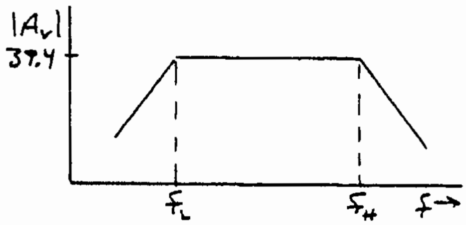
$R_{eq} = R_S + R_1\|R_2\|r_\pi = 0.5 + 40\|5\|4.12$

$R_{eq} = 2.64 \text{ k}\Omega$

$\tau_L = (2.64 \times 10^3)(4.7 \times 10^{-6}) = 1.24 \times 10^{-2} \text{ s}$

$f_L = \frac{1}{2\pi\tau_L} \Rightarrow \underline{f_L = 12.8 \text{ Hz}}$

b.



$V_0 = -g_m V_\pi (R_C\|R_L)$

$V_\pi = \left(\frac{R_1\|R_2\|r_\pi}{R_1\|R_2\|r_\pi + R_S}\right)V_i$

$V_\pi = \left(\frac{2.14}{2.14 + 0.5}\right)V_i = 0.8106 V_i$

$|A_v| = (29.2)(0.8106)(5\|2.5)$

$\underline{|A_v| = 39.4}$

7.46

$$I_D = K_p(V_{SG} + V_{TP})^2 = \frac{9 - V_{SG}}{R_S}$$

$$(2)(1.2)(V_{SG}^2 - 4V_{SG} + 4) = 9 - V_{SG}$$

$$2.4V_{SG}^2 - 8.6V_{SG} + 0.6 = 0$$

$$V_{SG} = \frac{8.6 \pm \sqrt{(8.6)^2 - 4(2.4)(0.6)}}{2(2.4)}$$

$$V_{SG} = 3.51 \text{ V}$$

$$g_m = 2K_p(V_{SG} + V_{TP}) = 2(2)(3.51 - 2)$$

$$g_m = 6.04 \ mA/V$$

$$I_D = (2)(3.51 - 2)^2 = 4.56 \text{ mA}$$

$$r_0 = \frac{1}{\lambda I_0} = \frac{1}{(0.01)(4.56)} \Rightarrow r_0 = 21.9 \text{ k}\Omega$$

a. $C_M = C_{gdT}(1 + g_m(r_0 \| R_D))$

$$C_M = (1)[1 + (6.04)(21.9\|1)] \Rightarrow \underline{C_M = 6.78 \text{ pF}}$$

b. $\tau_H = (R_i \| R_G)(C_{gsT} + C_M)$

$$\tau_H = (2\|100) \times 10^3 (10 + 6.78) \times 10^{-12}$$

$$\tau_H = 3.29 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau_H} \rightarrow \underline{f_H = 4.84 \text{ MHz}}$$

$$V_0 = -g_m(r_0 \| R_D) \cdot V_{gs}$$

$$V_{gs} = \left(\frac{R_G}{R_G + R_i}\right) V_i = \left(\frac{100}{102}\right) V_i$$

$$A_v = -(6.04)\left(\frac{100}{102}\right)(21.9\|1)$$

$$\underline{A_v = -5.66}$$

7.47

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(20) - 10 = \left(\frac{22}{22 + 8}\right)(20) - 10$$

$$V_G = 4.67 \text{ V}$$

$$I_D = \frac{10 - V_{SG} - 4.67}{R_S} = K_p(V_{SG} + V_{TP})^2$$

$$5.33 - V_{SG} = (1)(0.5)(V_{SG}^2 - 4V_{SG} + 4)$$

$$0.5V_{SG}^2 - V_{SG} - 3.33 = 0$$

$$V_{SG} = \frac{1 \pm \sqrt{1 + 4(0.5)(3.33)}}{2(0.5)} \Rightarrow V_{SG} = 3.77 \text{ V}$$

$$g_m \doteq 2K_p(V_{SG} + V_{TP}) = 2(1)(3.77 - 2)$$

$$g_m = 3.54 \ mA/V$$

b. $C_M = C_{gdT}(1 + g_m(R_D \| R_L))$

$$C_M = (3)[1 + (3.54)(2\|5)] \Rightarrow \underline{C_M = 18.2 \text{ pF}}$$

a. $\tau = R_{eq}(C_{gsT} + C_M)$

$$R_{eq} = R_i \| R_1 \| R_2 = 0.5\|8\|22 = 0.461 \text{ k}\Omega$$

$$\tau = (0.461 \times 10^3)(15 + 18.2) \times 10^{-12}$$

$$= 1.53 \times 10^{-8} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau} \Rightarrow \underline{f_H = 10.4 \text{ MHz}}$$

c. $V_0 = -g_m V_{gs}(R_D \| R_L)$

$$V_{gs} = \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_i}\right) V_i = \left(\frac{5.87}{5.87 + 0.5}\right) V_i$$

$$\Rightarrow V_{gs} = (0.9215) V_i$$

$$A_v = -(3.54)(0.9215)(2\|5)$$

$$\Rightarrow \underline{A_v = -4.66}$$

7.48

$$I_E = 0.5 \text{ mA} \Rightarrow I_{CQ} = \left(\frac{100}{101}\right)(0.5) = 0.495 \text{ mA}$$

$$g_m = \frac{0.495}{0.026} = 19.0 \text{ mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{0.495} = 5.25 \text{ k}\Omega$$

a. Input: From Eq. 7.107b

$$\tau_{P\pi} = \left[\frac{r_\pi}{1 + \beta} \,\Big\|\, R_E \,\Big\|\, R_S\right] C_\pi$$

$$= \left[\frac{5.25}{101} \,\Big\|\, 0.5 \,\Big\|\, 0.05\right] \times 10^3 \times 10 \times 10^{-12}$$

$$= 2.43 \times 10^{-10} \text{ s}$$

$$f_{H\pi} = \frac{1}{2\pi\tau_{P\pi}} \Rightarrow \underline{f_{H\pi} = 656 \text{ MHz}}$$

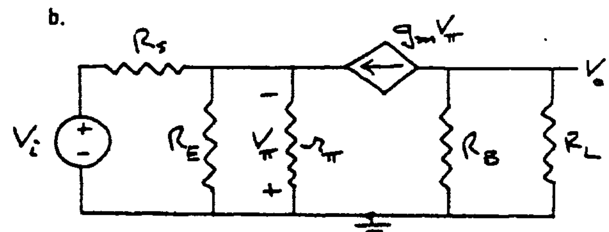
Output: From Eq. 7.108b

$$\tau_{P\mu} = (R_B \| R_L) C_\mu = (100\|1) \times 10^3 \times 10^{-12}$$

$$= 9.90 \times 10^{-10} \text{ s}$$

$$f_{H\mu} = \frac{1}{2\pi\tau_{P\mu}} \Rightarrow \underline{f_{H\mu} = 161 \text{ MHz}}$$

b.



$$V_0 = -g_m V_\pi (R_B \| R_L)$$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{V_i - (-V_\pi)}{R_S} = 0$$

$$V_\pi\left[g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S}\right] = \frac{-V_i}{R_S}$$

$$V_\pi\left[19 + \frac{1}{5.25} + \frac{1}{0.5} + \frac{1}{0.05}\right] = \frac{-V_i}{0.05}$$

$$V_\pi(41.19) = -V_i(20)$$

$$V_\pi = -(0.4856)V_i$$

$$\frac{V_0}{V_i} = -(19)(-0.4856)(100\|1)$$

$\underline{A_v = 9.14}$

c. $\tau = C_L(R_L \| R_B) = (15 \times 10^{-12})(1\|100) \times 10^3$

$\tau = 1.485 \times 10^{-8}$ s

$f = \dfrac{1}{2\pi\tau} \rightarrow f = 10.7$ MHz

Since $f < f_{H\mu} \Rightarrow 3dB$ freq. dominated by C_L.

7.49

$$I_{EQ} = \frac{20 - 0.7}{10} = 1.93 \text{ mA}$$

$$I_{CQ} = \left(\frac{100}{101}\right)(1.93) = 1.91 \text{ mA}$$

$$g_m = \frac{1.91}{0.026} = 73.5 \text{ mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{1.91} = 1.36 \text{ k}\Omega$$

a. Input:

$$\tau_{P\pi} = \left[\frac{r_\pi}{1+\beta} \,\Big\|\, R_E \,\Big\|\, R_S\right] \cdot C_\pi$$

$$= \left[\frac{1.36}{101} \,\Big\|\, 10 \,\Big\|\, 1\right] \times 10^3 \times 10 \times 10^{-12}$$

$\tau_{P\pi} = 1.327 \times 10^{-10}$ s

$f_{P\pi} = \dfrac{1}{2\pi\tau_{P\pi}} \Rightarrow \underline{f_{P\pi} = 1.20 \text{ GHz}}$

output:

$\tau_{P\mu} = (R_C \| R_L)C_\mu = (6.5\|5) \times 10^3 \times 10^{-12}$

$\tau_{P\mu} = 2.826 \times 10^{-9}$ s

$f_{P\mu} = \dfrac{1}{2\pi\tau_{P\mu}} \rightarrow \underline{f_{P\mu} = 56.3 \text{ MHz}}$

b. $V_0 = -g_m V_\pi (R_C \| R_L)$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{V_i - (-V_\pi)}{R_S} = 0$$

$$V_\pi\left(g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S}\right) = -\frac{V_i}{R_S}$$

$$V_\pi\left[73.5 + \frac{1}{1.36} + \frac{1}{10} + \frac{1}{1}\right] = \frac{-V_i}{(1)}$$

$$V_\pi(75.34) = -V_i \Rightarrow V_\pi = -(0.01327)V_i$$

$$V_0 = -(73.5)(-0.01327)(6.5\|5)V_i$$

$\underline{A_v = 2.76}$

c. $\tau = C_L(R_L \| R_C) = (15 \times 10^{-12})(6.5\|5) \times 10^3$

$\tau = 4.24 \times 10^{-8}$ s

$f = \dfrac{1}{2\pi\tau} \rightarrow f = 3.75$ MHz

Since $f < f_{P\mu}$, $3dB$ frequency is dominated by C_L.

7.50

$$V_{GS} + I_D R_S = 5$$

$$I_D = \frac{5 - V_{GS}}{R_S} = K_n(V_{GS} - V_{TN})^2$$

$$5 - V_{GS} = (3)(10)(V_{GS}^2 - 2V_{GS} + 1)$$

$$30V_{GS}^2 - 59V_{GS} + 25 = 0$$

$$V_{GS} = \frac{59 \pm \sqrt{(59)^2 - 4(30)(25)}}{2(30)} \Rightarrow V_{GS} = 1.35 \text{ V}$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(3)(1.35 - 1)$$

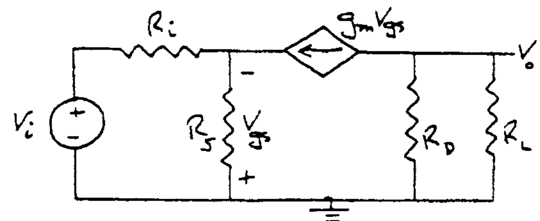
$$g_m = 2.1 \, mA/V$$

On the output:

$\tau_{P\mu} = (R_D \| R_L)C_{gdT} = (5\|4) \times 10^3 \times 4 \times 10^{-12}$

$\tau_{P\mu} = 8.89 \times 10^{-9}$ s

$f_{P\mu} = \dfrac{1}{2\pi\tau_{P\mu}} \rightarrow \underline{f_{P\mu} = 17.9 \text{ MHz}}$



$$V_0 = -g_m V_{gs}(R_D \| R_L)$$

$$g_m V_{gs} + \frac{V_{gs}}{R_S} + \frac{V_i - (-V_{gs})}{R_i} = 0$$

$$V_{gs}\left(g_m + \frac{1}{R_S} + \frac{1}{R_i}\right) = -\frac{V_i}{R_i}$$

$$V_{gs}\left(2.1 + \frac{1}{10} + \frac{1}{2}\right) = -\frac{V_i}{2}$$

$$V_{gs} = -(0.185)V_i$$

$$A_v = \frac{V_0}{V_i} = (2.1)(0.185)(5\|4)$$

$\underline{A_v = 0.863}$

7.51

dc analysis

$$I_D = \frac{V^+ - V_{SG}}{R_S} = K_p(V_{SG} + V_{TP})^2$$

$$5 - V_{SG} = (1)(4)(V_{SG} - 0.8)^2$$

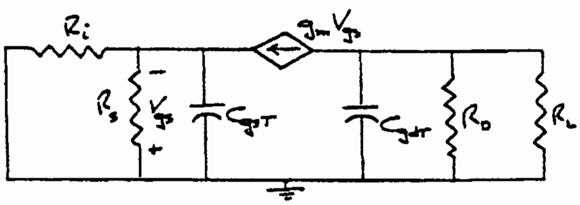
$$= 4(V_{SG}^2 - 1.6V_{SG} + 0.64)$$

$$4V_{SG}^2 - 5.4V_{SG} - 2.44 = 0$$

$$V_{SG} = \frac{5.4 \pm \sqrt{(5.4)^2 + 4(4)(2.44)}}{2(4)} = 1.707$$

$$g_m = 2K_p(V_{SG} + V_{TP}) = 2(1)(1.707 - 0.8)$$

$$g_m = 1.81\ mA/V$$



3 · dB frequency due to C_{gsT} : $R_{eq} = \frac{1}{g_m} \| R_S \| R_i$

$$f_A = \frac{1}{2\pi R_{eq} \cdot C_{gsT}}$$

$$R_{eq} = \frac{1}{1.81} \| 4 \| 0.5 = 0.246\ \text{k}\Omega$$

$$f_A = \frac{1}{2\pi(246)(4 \times 10^{-12})} = 162\ \text{MHz}$$

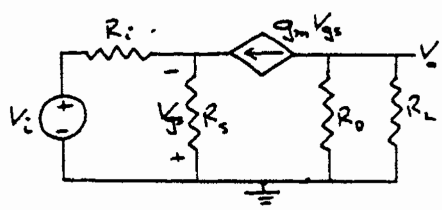
3 − dB frequency due to C_{gdT}

$$f_B = \frac{1}{2\pi(R_D \| R_L)C_{gdT}}$$

$$= \frac{1}{2\pi(2\|4) \times 10^3 \times 10^{-12}}$$

$\underline{f = 119\ \text{MHz}}$

Midband gain



$$V_{gs} = \frac{-\frac{1}{g_m} \| R_S}{\frac{1}{g_m} \| R_S + R_i} \cdot V_i = \frac{-\frac{1}{1.81} \| 4}{\frac{1}{1.81} \| 4 + 0.5} \cdot V_i$$

$$= -0.492V_i$$

$$V_0 = -g_m V_{gs}(R_D \| R_L)$$

$$A_v = (0.492)(1.81)(4\|2) \Rightarrow \underline{A_v = 1.19}$$

7.52

$$r_\pi = \frac{(120)(0.026)}{1.02} = 3.06\ \text{k}\Omega$$

$$g_m = 39.2\ \text{mA/V}$$

a. Input: $f_{H\pi} = \frac{1}{2\pi\tau_\pi}$

$$\tau_\pi = [R_S \| R_2 \| R_3 \| r_\pi](C_\pi + 2C_\mu)$$

$$= 0.1\|20.5\|28.3\|3.06 = 0.096\ \text{k}\Omega$$

$$\tau_\pi = (96)(12 + 2(2)) \times 10^{-12} = 1.536 \times 10^{-9}\ \text{s}$$

$$f_{H\pi} = \frac{1}{2\pi(1.536 \times 10^{-9})} = \underline{103.6\ \text{MHz}}$$

Output: $f_{H\mu} = \frac{1}{2\pi\tau_\mu}$

$$\tau_\mu = (R_C \| R_L)C_\mu$$

$$= (5\|10) \times 10^3 \times 2 \times 10^{-12}$$

$$= 6.67 \times 10^{-9}$$

$$f_{H\mu} = \frac{1}{2\pi(6.67 \times 10^{-9})} = \underline{23.9\ \text{MHz}}$$

b. $A = g_m(R_C \| R_L)\left[\frac{R_2 \| R_3 \| r_\pi}{R_2 \| R_3 \| r_\pi + R_S}\right]$

$$R_2 \| R_3 \| r_\pi = 20.5\|28.3\|3.06 = 2.43\ \text{k}\Omega$$

$$A = (39.2)(5\|10)\left[\frac{2.43}{2.43 + 0.1}\right] \Rightarrow \underline{A = 125.5}$$

c. $C_L = 15$ pF $> C_\mu \Rightarrow C_L$ dominates frequency response.

Chapter 8

Exercise Solutions

E8.1

For $V_{DS} = 0$, $I_D(\max) = \frac{24}{20} = \underline{1.2\text{ A} = I_D(\max)}$

For $I_D = 0 \Rightarrow \underline{V_{DS}(\max) = 24\text{ V}}$

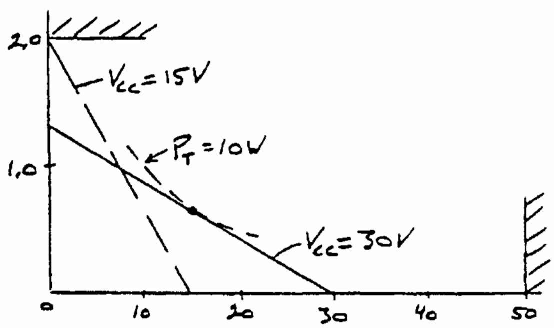
Maximum power when

$V_{DS} = \frac{V_{DS}(\max)}{2} = 12$ V and

$I_D = \frac{I_D(\max)}{2} = 0.6$ A

$\Rightarrow \underline{P_D(\max) = (12)(0.6) = 7.2\text{ Watts}}$

E8.2



a. $V_{CC} = 30$ V, $V_{CE} = 30 - I_C R_C$, $I_C V_{CE} = 10$

Maximum power at $V_{CE} = \frac{1}{2} V_{CC} = 15$

$I_C = \frac{10}{V_{CE}} = \frac{10}{15} = \frac{2}{3}$

So $15 = 30 - \frac{2}{3} R_L \Rightarrow \underline{R_L = 22.5\ \Omega}$

$\Rightarrow$ Maximum Power = 10 W

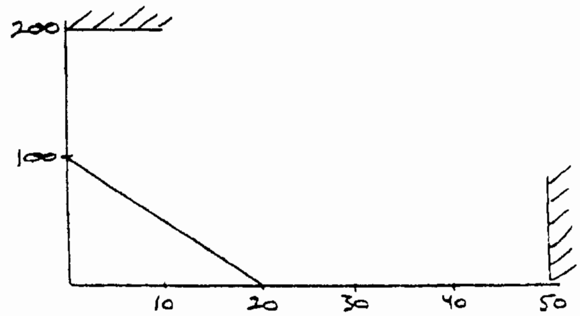
b. $V_{CC} = 15$ V, $I_{C,\max} = 2$ A

$V_{CE} = 15 - I_C R_L$

$0 = 15 - 2R_L \Rightarrow \underline{R_L = 7.5\ \Omega}$

$\Rightarrow$ Maximum Power = (1)(7.5) = 7.5 W

E8.3



Maximum power at center of load line

$P_{\max} = (0.05)(10) \Rightarrow \underline{P_{\max} = 0.5\text{ W}}$

E8.4

Power = $i_D \cdot v_{DS} = (1)(12) = 12$ watts

c. $T_{sink} = T_{amb} + P \cdot \theta_{snk-amb}$

$T_{sink} = 25 + (12)(4) \Rightarrow \underline{T_{sink} = 73°\text{C}}$

b. $T_{case} = T_{sink} + P \cdot \theta_{case-snk}$

$T_{case} = 73 + (12)(1) \Rightarrow \underline{T_{case} = 85°\text{C}}$

a. $T_{dev} = T_{case} + P \cdot \theta_{dev-case}$

$T_{dev} = 85 + (12)(3) \Rightarrow \underline{T_{dev} = 121°\text{C}}$

E8.5

$$\theta_{dev-case} = \frac{T_{j,\max} - T_{amb}}{P_{D,rated}} = \frac{200 - 25}{50} = 3.5°\text{C/W}$$

$$P_{D,\max} = \frac{T_{j,\max} - T_{amb}}{\theta_{dev-case} + \theta_{case-snk} + \theta_{snk-amb}}$$

$$= \frac{200 - 25}{3.5 + 0.5 + 2}$$

$\Rightarrow \underline{P_{D,\max} = 29.2\text{ W}}$

$T_{case} = T_{amb} + P_{D,\max}(\theta_{case-snk} + \theta_{snk-amb})$

$= 25 + (29.2)(0.5 + 2)$

$\Rightarrow \underline{T_{case} = 98°\text{C}}$

E8.6

a. $P_Q = V_{CEQ} \cdot I_{CQ} = (7.5)(7.5)$

$\underline{P_Q = 56.3\ \text{mW}}$

b. $\overline{P_L} = \frac{1}{2} \cdot \frac{V_P^2}{R_L} = \frac{1}{2} \cdot \frac{(6.5)^2}{1} \Rightarrow \underline{\overline{P_L} = 21.1\ \text{mW}}$

$\overline{P_S} = (15)(7.5) \Rightarrow \overline{P_S} = 113\ \text{mW}$

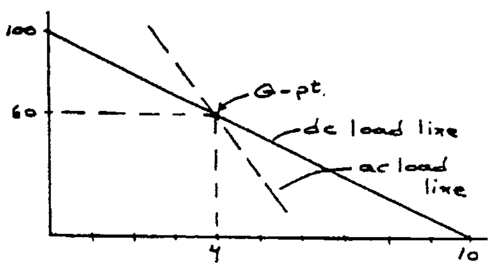
$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{21.1}{113} \Rightarrow \underline{\eta = 18.7\%}$

$\underline{\overline{P_Q} = 56.3 - 21.1 = 35.2\ \text{mW}}$

E8.7

a. $I_{DQ} = \frac{10-4}{0.1} \Rightarrow \underline{I_{DQ} = 60\ \text{mA}}$

b.



$v_{ds} = -\left(\frac{9}{10}\right)(60)(0.050) = -2.7\ \text{V}$

$\Rightarrow v_{DS}(\text{min}) = 4 - 2.7 = 1.3\ \text{V}$

So maximum swing is determined by drain-to-source voltage.

$V_{PP} = 2 \times (2.5) = 5.0\ \text{V}$

c. $\overline{P_L} = \frac{1}{2} \cdot \frac{V_P^2}{R_L} = \frac{1}{2} \cdot \frac{(2.5)^2}{0.1} \Rightarrow \underline{\overline{P_L} = 31.25\ \text{mW}}$

$\overline{P_S} = V_{DD} \cdot I_{DQ} = (10)(60) = 600\ \text{mW}$

$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{31.25}{600} \Rightarrow \underline{\eta = 5.2\%}$

E8.8

a. $\overline{P_L} = \frac{1}{2} \cdot \frac{V_P^2}{R_L}$

$\Rightarrow V_P = \sqrt{2R_L\overline{P_L}} = \sqrt{2(8)(25)} \Rightarrow V_P = 20\ \text{V}$

$\Rightarrow V_{CC} = \frac{20}{0.8} \Rightarrow \underline{V_{CC} = 25\ \text{V}}$

b. $I_P = \frac{V_P}{R_L} = \frac{20}{8} \Rightarrow \underline{I_P = 2.5\ \text{A}}$

c. $\overline{P_Q} = \frac{V_{CC}V_P}{\pi R_L} - \frac{V_P^2}{4R_L}$

$\overline{P_Q} = \frac{(25)(20)}{\pi(8)} - \frac{(20)^2}{4(8)} = 19.9 - 12.5$

$\Rightarrow \underline{\overline{P_Q} = 7.4\ \text{W}}$

d. $\eta = \frac{\pi V_P}{4V_{CC}} = \frac{\pi}{4} \cdot \frac{20}{25} \Rightarrow \underline{\eta = 62.8\%}$

E8.9

a. $\overline{P_L} = \frac{1}{2} \cdot \frac{V_P^2}{R_L} = \frac{(4)^2}{2(0.1)} \Rightarrow \underline{\overline{P_L} = 80\ \text{mW}}$

b. $I_P = \frac{V_P}{R_L} = \frac{4}{0.1} \Rightarrow \underline{I_P = 40\ \text{mA}}$

c. $\overline{P_Q} = \frac{V_{CC}V_P}{\pi R_L} - \frac{V_P^2}{4R_L}$

$\overline{P_Q} = \frac{(5)(4)}{\pi(0.1)} - \frac{(4)^2}{4(0.1)} = 63.7 - 40$

$\Rightarrow \underline{\overline{P_Q} = 23.7\ \text{mW}}$

d. $\eta = \frac{\pi V_P}{4V_{CC}} = \frac{\pi}{4} \cdot \frac{4}{5} \Rightarrow \underline{\eta = 62.8\%}$

E8.10

a. $v_I = v_O + v_{GSn} - \frac{V_{BB}}{2}$

$\frac{dv_I}{dv_O} = 1 + \frac{dv_{GSn}}{dv_O}$

$i_{Dn} = K_n(v_{GSn} - V_{TN})^2$

$v_{GSn} = \sqrt{\frac{i_{Dn}}{K_n}} + V_{TN}$

$\frac{dv_{GSn}}{dv_o} = \frac{dv_{GSn}}{di_{Dn}} \cdot \frac{di_{Dn}}{dv_o}$

So $\frac{dv_{GSn}}{di_{Dn}} = \frac{1}{\sqrt{K_n}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{i_{Dn}}}$

At $v_O = 0$, $i_{Dn} = 0.050$ A

So $\frac{dv_{GSn}}{di_{Dn}} = \frac{1}{\sqrt{0.2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{0.050}} = 5$

$i_{Dn} = i_L + i_{Dp}$

For a small change in $v_O \rightarrow \Delta i_L = \Delta i_{Dn} - (-\Delta i_{Dp})$

So $\Delta i_{Dn} = \frac{1}{2}\Delta i_L$

or

$\frac{di_{Dn}}{dv_O} = \frac{1}{2} \cdot \frac{di_L}{dv_O} = \frac{1}{2} \cdot \frac{1}{R_L} = \frac{1}{2} \cdot \frac{1}{20} = 0.025$

Then $\frac{dv_{GSn}}{dv_O} = (5)(0.025) = 0.125$

Then $\frac{dv_I}{dv_O} = 1 + 0.125 = 1.125$

and $A_v = \frac{dv_O}{dv_I} = \frac{1}{1.125} \Rightarrow \underline{A_v = 0.889}$

b. For $v_O = 5$ V, $i_L = 0.25\ \text{A} = i_{Dn}$, and $i_{Dp} = 0$

$$\frac{dv_{GSn}}{dv_O} = \frac{dv_{GSn}}{di_{Dn}} \cdot \frac{di_{Dn}}{dv_O}$$

$$\frac{dv_{GSn}}{di_{Dn}} = \frac{1}{\sqrt{K_n}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{i_{Dn}}} = \frac{1}{\sqrt{0.2}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{0.25}} = 2.24$$

$$\frac{di_{Dn}}{dv_O} = \frac{di_L}{dv_O} = \frac{1}{20} = 0.05$$

$$\frac{dv_{GSn}}{dv_O} = (2.24)(0.05) = 0.112$$

$$\frac{dv_I}{dv_O} = 1 + 0.112 = 1.112$$

$$A_v = \frac{dv_O}{dv_I} = \frac{1}{1.112} \Rightarrow \underline{A_v = 0.899}$$

E8.11

a. $I_{CQ} \cong \frac{1}{2} \cdot \left(\frac{2V_{CC}}{R_L}\right) = \frac{V_{CC}}{R_L} = \frac{12}{1.5} = 8 \text{ mA}$

$$R_{TH} = R_1 \| R_2$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{8}{75} = 0.107 \text{ mA} = \frac{V_{TH} - V_{BE}}{R_{TH} + (1+\beta)R_E}$$

Let $R_{TH} = (1+\beta)R_E = (76)(0.1) = 7.6 \text{ k}\Omega$

$$0.107 = \frac{\frac{1}{R_1} \cdot (7.6)(12) - 0.7}{7.6 + 7.6}$$

$$\frac{1}{R_1} \cdot (91.2) = 2.33 \Rightarrow \underline{R_1 = 39.1 \text{ k}\Omega}$$

$$\frac{39.1 R_2}{39.1 + R_2} = 7.6 \Rightarrow (39.1 - 7.6)R_2 = (7.6)(39.1)$$

$$\Rightarrow \underline{R_2 = 9.43 \text{ k}\Omega}$$

b. $\overline{P_L} = \frac{1}{2} \cdot (0.9 I_{CQ})^2 R_L = \frac{1}{2}[(0.9)(8)]^2(1.5)$

$$\Rightarrow \underline{\overline{P_L} = 38.9 \text{ mW}}$$

$$\overline{P_S} = V_{CC} I_{CQ} = (12)(8) = 96 \text{ mW}$$

$$\overline{P_Q} = \overline{P_S} - \overline{P_L} = 96 - 38.9 \Rightarrow \underline{\overline{P_Q} = 57.1 \text{ mW}}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{38.9}{96} \Rightarrow \underline{\eta = 40.5\%}$$

E8.12

a. $R_b = r_\pi + (1+\beta)R'_E$

and $R'_E = a^2 R_L = (10)^2(8) = 800 \ \Omega$

$R_i = 1.5 \text{ k}\Omega = R_{TH} \| R_b$

$$I_Q = \frac{V_{CC}}{a^2 R_L} = \frac{18}{(10)^2(8)} = 22.5 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{22.5} = 0.116 \text{ k}\Omega$$

$$R_b = 0.116 + (101)(0.8) = 80.9 \text{ k}\Omega$$

$$1.5 = R_{TH} \| 80.9 = \frac{R_{TH}(80.9)}{R_{TH} + (80.9)}$$

$$\Rightarrow (80.9 - 1.5)R_{TH} = (1.5)(80.9)$$

$$\Rightarrow R_{TH} = 1.53 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

$$I_{BQ} = \frac{I_Q}{\beta} = \frac{22.5}{100} = 0.225 \text{ mA}$$

$$I_{BQ} = \frac{V_{TH} - 0.7}{R_{TH}}$$

$$\Rightarrow \frac{1}{R_1}(1.53)(18) = (0.225)(1.53) + 0.7$$

$$\Rightarrow \underline{R_1 = 26.4 \text{ k}\Omega}$$

$$\frac{26.4 R_2}{26.4 + R_2} = 1.53$$

$$(26.4 - 1.53)R_2 = (1.53)(26.4)$$

$$\Rightarrow \underline{R_2 = 1.62 \text{ k}\Omega}$$

b. $v_E = 0.9 V_{CC} = (0.9)(18) = 16.2 \text{ V}$

$$i_E = 0.9 I_{CQ} = (0.9)(22.5) = 20.25 \text{ mA}$$

$$v_O = \frac{v_E}{a} = \frac{16.2}{10} \Rightarrow \underline{V_P = 1.62 \text{ V}}$$

$$i_O = a i_E = (10)(20.25) \Rightarrow \underline{I_P = 203 \text{ mA}}$$

$$\overline{P_L} = \frac{1}{2}(1.62)(0.203) \Rightarrow \underline{\overline{P_L} = 0.164 \text{ W}}$$

E8.13

a. $I_C = I_{SQ} \exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_{SQ}}\right)$

$$V_{BE} = (0.026)\ln\left(\frac{5 \times 10^{-3}}{2 \times 10^{-13}}\right) = 0.6225 \text{ V}$$

$$\Rightarrow V_{D1} = V_{D2} = 0.6225$$

$$I_{Bias} = I_D = I_{SD} \exp\left(\frac{0.6225}{0.026}\right)$$

$$= 5 \times 10^{-13} \exp\left(\frac{0.6225}{0.026}\right)$$

$$\underline{I_{Bias} = 12.5 \text{ mA}}$$

b. $V_0 = 2\text{ V},\ i_L = \dfrac{2}{0.075} = 26.7\text{ mA}$

1st approximation:

$i_{Cn} \cong 26.7\text{ mA},\ i_{Bn} = 0.444\text{ mA}$

$V_{BE} = (0.026)\ln\left(\dfrac{26.7\times10^{-3}}{2\times10^{-13}}\right) = 0.6661$

$I_D = 12.5 - 0.444 = 12.056\text{ mA}$

$V_D = (0.026)\ln\left(\dfrac{12.056\times10^{-3}}{5\times10^{-13}}\right) = 0.6216$

$2V_D = 1.243\text{ V}$

$V_{EB} = 2V_D - V_{BE} = 0.5769$

$i_{Cp} = 2\times10^{-13}\exp\left(\dfrac{0.5769}{0.026}\right) = 0.866\text{ mA}$

2nd approximation:

$i_{En} = i_L + i_{Cp} = 26.7 + 0.866 \cong \underline{27.6\text{ mA} = i_{En}}$

$i_{Cn} = \left(\dfrac{60}{61}\right)(27.6) \Rightarrow \underline{i_{Cn} = 27.1\text{ mA}}$

$i_{Bn} = 0.452\text{ mA}$

$I_D = 12.5 - 0.452 \Rightarrow \underline{I_D = 12.05\text{ mA}}$

$V_{BEn} = (0.026)\ln\left(\dfrac{27.1\times10^{-3}}{2\times10^{-13}}\right)$

$\Rightarrow \underline{V_{BEn} = 0.6664\text{ V}}$

$V_D = (0.026)\ln\left(\dfrac{12.05\times10^{-3}}{5\times10^{-13}}\right) = 0.6215\text{ V}$

$2V_{DD} = 1.243\text{ V}$

$V_{EB} = 1.243 - 0.6664 \Rightarrow \underline{V_{EBp} = 0.5766\text{ V}}$

$i_{Cp} = 2\times10^{-13}\exp\left(\dfrac{0.5766}{0.026}\right) \Rightarrow \underline{i_{Cp} = 0.856\text{ mA}}$

c. $V_0 = 10\text{ V},\ i_L = \dfrac{10}{0.075} = 133\text{ mA}$

$i_{En} \cong i_L = 133\text{ mA} \Rightarrow \underline{i_{CN} = 131\text{ mA}}$

$i_{Bn} = 2.18\text{ mA} \Rightarrow I_D = 12.5 - 2.18$

$\Rightarrow \underline{I_D = 10.3\text{ mA}}$

$V_D = (0.026)\ln\left(\dfrac{10.3\times10^{-3}}{5\times10^{-13}}\right) = 0.6175$

$2V_{DD} = 1.235\text{ V}$

$V_{BEn} = (0.026)\ln\left(\dfrac{131\times10^{-3}}{2\times10^{-13}}\right)$

$\Rightarrow \underline{V_{BEn} = 0.7074\text{ V}}$

$V_{EBp} = 1.235 - 0.7074 \Rightarrow \underline{V_{EBp} = 0.5276\text{ V}}$

$i_{Cp} = 2\times10^{-13}\exp\left(\dfrac{0.5276}{0.026}\right)$

$\Rightarrow \underline{i_{Cp} = 0.130\text{ mA}}$

E8.14

a. $v_I = 0 = v_O,\ v_{B3} = 0.7\text{ V}$

$I_{R1} = \dfrac{12 - 0.7}{R_1} = \dfrac{11.3}{0.25} \Rightarrow I_{R1} = 45.2\text{ mA}$

If transistors are matched, then

$i_{E1} = i_{E3}$

$i_{R1} = i_{E1} + i_{B3} = i_{E1} + \dfrac{i_{E3}}{1+\beta}$

$i_{R1} = i_{E1}\left(1 + \dfrac{1}{1+\beta}\right) = i_{E1}\left(1 + \dfrac{1}{41}\right)$

$i_{E1} = \dfrac{45.2}{1.024} \Rightarrow \underline{i_{E1} = i_{E2} = 44.1\text{ mA}}$

$i_{B1} = i_{B2} = \dfrac{i_{E1}}{1+\beta} = \dfrac{44.1}{41}$

$\Rightarrow \underline{i_{B1} = i_{B2} = 1.08\text{ mA}}$

b. For $v_I = 5\text{ V} \Rightarrow v_O = 5\text{ V}$

$i_O = \dfrac{5}{8} \Rightarrow \underline{i_O = 0.625\text{ A}}$

$i_{E3} \cong 0.625\text{ A},\ i_{B3} = \dfrac{0.625}{41} \Rightarrow i_{B3} = 15.2\text{ mA}$

$v_{B3} = 5.7\text{ V} \Rightarrow i_{R1} = \dfrac{12 - 5.7}{0.25} = 25.2\text{ mA}$

$i_{E1} = 25.2 - 15.2 \Rightarrow \underline{i_{E1} = 10.0\text{ mA}}$

$\Rightarrow \underline{i_{B1} = \dfrac{10}{41} = 0.244\text{ mA}}$

$v_{B4} = 5 - 0.7 = 4.3\text{ V}$

$I_{R2} = \dfrac{4.3 - (-12)}{0.25} = \underline{65.2\text{ mA} \cong i_{E2}}$

$\underline{i_{B2} = \dfrac{65.2}{41} = 1.59\text{ mA}}$

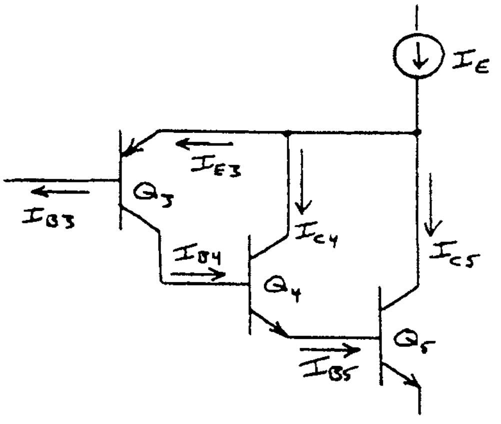
$i_I = i_{B2} - i_{B1} = 1.59 - 0.244 \Rightarrow \underline{i_I = 1.35\text{ mA}}$

c. $A_I = \dfrac{i_O}{i_I} = \dfrac{625}{1.35} \Rightarrow \underline{A_I = 463}$

From Equation (8.54)

$A_I = \dfrac{(1+\beta)R}{2R_L} = \dfrac{(41)(250)}{2(8)} = \underline{641}$

E8.15



$$I_E = I_{E3} + I_{C4} + I_{C5}$$
$$= I_{E3} + I_{C4} + \beta_5 I_{B5}$$
$$= I_{E3} + \beta_4 I_{B4} + \beta_5(1+\beta_4)I_{B4}$$
$$I_E = (1+\beta_3)I_{B3} + \beta_4\beta_3 I_{B3} + \beta_5(1+\beta_4)\beta_3 I_{B3}$$

If β_4 and β_5 are large, then

$$I_E \cong \beta_3\beta_4\beta_5 I_{B3}$$

So that composite current gain is

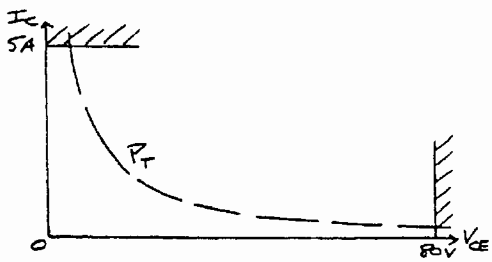
$$\underline{\beta \cong \beta_3\beta_4\beta_5}$$

Chapter 8

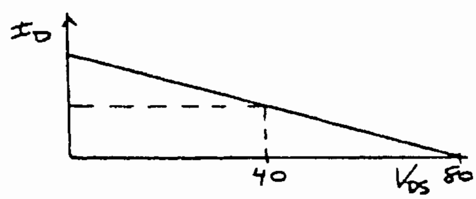
Problem Solutions

8.1

a.



b. *i.* $V_{DD} = 80$ V

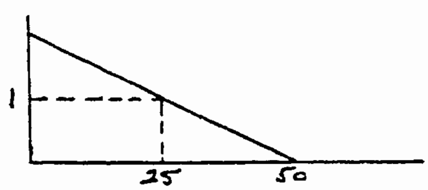


Maximum power at $V_{DS} = \dfrac{V_{DD}}{2} = 40$ V

$$I_D = \frac{P_T}{V_{DS}} = \frac{25}{40} = 0.625 \text{ A}$$

$$R_D = \frac{80 - 40}{0.625} \Rightarrow \underline{R_D = 64\ \Omega}$$

ii. $V_{DD} = 50$ V



Maximum power at $V_{DS} = \dfrac{V_{DD}}{2} = 25$ V

$$I_D = \frac{P_T}{V_{DS}} = \frac{25}{25} = 1 \text{ A}$$

$$R_D = \frac{50 - 25}{1} \Rightarrow \underline{R_D = 25\ \Omega}$$

8.2

a. $P_Q(\text{max}) = I_{CQ} \cdot \dfrac{V_{CC}}{2}$

So $I_{CQ} = \dfrac{2P_Q(\text{max})}{V_{CC}} = \dfrac{2(20)}{24} = 1.67$ A

$$R_L = \frac{V_{CC} - (V_{CC}/2)}{I_{CQ}} = \frac{24 - 12}{1.67} \Rightarrow \underline{R_L = 7.2\ \Omega}$$

$$I_B = \frac{I_{CQ}}{\beta} = \frac{1.67}{80} \Rightarrow 20.8 \text{ mA}$$

$$R_B = \frac{24 - 0.7}{20.8} \Rightarrow \underline{R_B = 1.12 \text{ k}\Omega}$$

b. $|A_v| = g_m R_L = \dfrac{I_{CQ} \cdot R_L}{V_T} = \dfrac{(1.67)(7.2)}{0.026} = 462$

$V_0(\text{max}) = 12 \text{ V} \Rightarrow V_P = \dfrac{V_0(\text{max})}{A_v} = \dfrac{12}{462}$

$$\Rightarrow \underline{V_P \cong 26 \text{ mV}}$$

8.3

a. For maximum power delivered to the load, set

$$V_{CEQ} = \frac{V_{CC}}{2}$$

Set $V_{CC} = 25 \text{ V} = V_{CE(sus)}$

Then $I_{Cm} = \dfrac{V_{CC}}{R_L} = \dfrac{25}{0.1}$

$$I_{Cm} = 250 \text{ mA} < I_{C,\text{max}}$$

$$I_{CQ} = \frac{25 - 12.5}{0.1} = 125 \text{ mA}$$

$$P_Q(\text{max}) = I_{CQ} \cdot \frac{V_{CC}}{2} = (0.125)(12.5)$$

$$= 1.56 \text{ W} < P_{D,\text{max}}$$

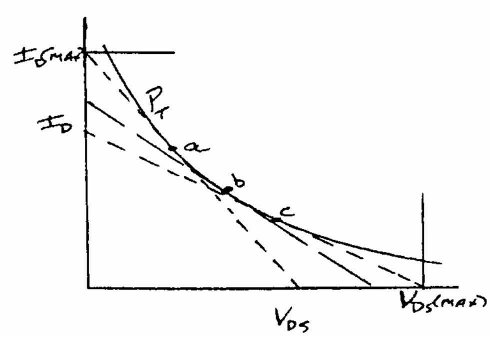
$$I_{BQ} = \frac{125}{100} = 1.25 \text{ mA}$$

$$R_B = \frac{25 - 0.7}{1.25} \Rightarrow \underline{R_B = 19.4 \text{ k}\Omega}$$

b. $P_L(\text{max}) = \dfrac{1}{2} \cdot I_{CQ}^2 \cdot R_L = \dfrac{1}{2}(0.125)^2(100)$

$$\Rightarrow \underline{P_L(\text{max}) = 0.781 \text{ W}}$$

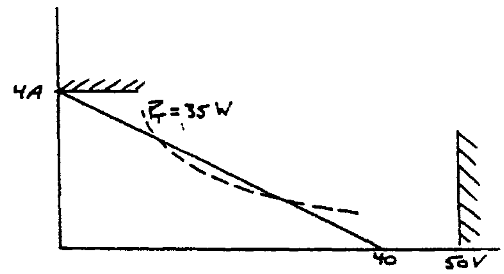
8.4



Point (b): Maximum power delivered to load.
Point (a): Will obtain maximum signal current output.
Point (c): Will obtain maximum signal voltage output.

8.5

a.



b. $V_{GG} = 5$ V, $I_D = 0.25(5-4)^2 = 0.25A$,
$V_{DS} = 37.5$ V, $P = 9.375$ W

$V_{GG} = 6$ V, $I_D = 0.25(6-4)^2 = 1.0$ A,
$V_{DS} = 30$ V, $P = 30$ W

$V_{GG} = 7$ V, $I_D = 0.25(7-4)^2 = 2.25$ A,
$V_{DS} = 17.5$ V, $P = 39.375$ W

$V_{GG} = 8$ V, $I_D = 0.25[2(8-4)V_{DS} - V_{DS}^2]$

$$= \frac{40 - V_{DS}}{10} \Rightarrow V_{DS} = 2.92$$

$I_D = 3.71$ A, $\underline{P = 10.8\text{ W}}$

$V_{GG} = 9$ V, $I_D = 0.25[2(9-4)V_{DS} - V_{DS}^2]$

$$= \frac{40 - V_{DS}}{10} \Rightarrow V_{DS} = 1.88 \text{ V}$$

$I_D = 3.81$ A, $\underline{P = 7.16\text{ W}}$

c. Yes, at $V_{GG} = 7$ V, $P = 39.375$ W $> P_{D,max} =$ 35 W

8.6

a. Set $V_{DSQ} = \frac{V_{DD}}{2} = 25$ V

$$I_{DQ} = \frac{50-25}{20} = 1.25 \text{ A}$$

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$\sqrt{\frac{1.25}{0.2}} + 4 = V_{GS} = 6.5 \text{ V}$$

$$V_{GS} = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD}$$

Let $R_1 + R_2 = 100$ kΩ

$$6.5 = \left(\frac{R_2}{100}\right)(50) \Rightarrow \underline{R_2 = 13 \text{ k}\Omega}$$

$$\underline{R_1 = 87 \text{ k}\Omega}$$

b. $P_D = I_{DQ}V_{DSQ} = (1.25)(25) \Rightarrow \underline{P_D = 31.25\text{ W}}$

c. $I_{D,max} = 2I_{DQ} \Rightarrow \underline{I_{D,max} = 2.5\text{ A}}$

$V_{DS,max} = V_{DD} \Rightarrow \underline{V_{DS,max} = 50\text{ V}}$

$\underline{P_{D,max} = 31.25\text{ W}}$

d. $\left|\frac{V_0}{V_i}\right| = g_m R_L$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(1.25)} = 1 \text{ A/V}$$

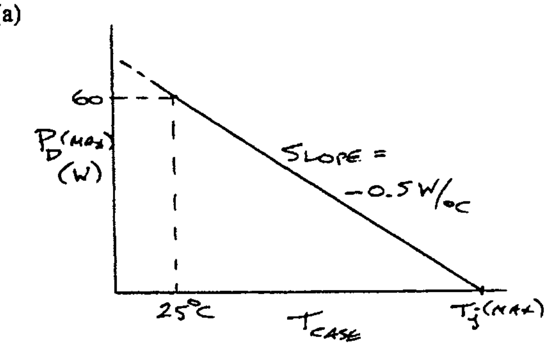
$$|V_0| = (1)(20)(0.5) = 10 \text{ V}$$

$$\overline{P_L} = \frac{1}{2}\cdot\frac{V_0^2}{R_L} = \frac{1}{2}\cdot\frac{(10)^2}{20} \Rightarrow \underline{\overline{P_L} = 2.5\text{ W}}$$

$$\overline{P_Q} = 31.25 - 2.5 \Rightarrow \underline{\overline{P_Q} = 28.75\text{ W}}$$

8.7

(a)



(b) $P_D = P_{D,max} - (Slope)(T_j - 25)$

At $P_D = 0$, $T_{j,max} = \frac{60}{0.5} + 25 \Rightarrow$ $\underline{T_{j,max} = 145°C}$

(c) $P_{D,max} = \frac{T_{j,max} - T_{case}}{\theta_{dev-amb}}$

or

$$\theta_{dev-amb} = \frac{145-25}{60} \Rightarrow \underline{\theta_{dev-amb} = 2\,°C/W}$$

8.8

$$P_{D,rated} = \frac{T_{j,max} - T_{amb}}{\theta_{dev-case}}$$

or

$$\theta_{dev-case} = \frac{T_{j,max} - T_{amb}}{P_{D,rated}} = \frac{150-25}{50} = 2.5°\text{C/W}$$

Then

$$T_{dev} - T_{amb} = P_D(\theta_{dev-case} + \theta_{case-amb})$$

$$150 - 25 = P_D(2.5 + \theta_{case-amb})$$

$$\Rightarrow \underline{125 = P_D(2.5 + \theta_{case-amb})}$$

8.9

$$P_D = I_D \cdot V_{DS} = (4)(5) = 20 \text{ W}$$

$$T_{dev} - T_{amb} = P_D(\theta_{dev-case} + \theta_{case-snk} + \theta_{snk-amb})$$

$$T_{dev} - 25 = 20(1.75 + 0.8 + 3) = 111$$

$$\Rightarrow \underline{T_{dev} = 136^\circ\text{C}}$$

$$T_{dev} - T_{case} = P_D \cdot \theta_{dev-case} = (20)(1.75) = 35$$

$$T_{case} = T_{dev} - 35 = 136 - 35 \Rightarrow \underline{T_{case} = 101^\circ\text{C}}$$

$$T_{case} - T_{sink} = P_D \cdot \theta_{case-snk} = (20)(0.8) = 16^\circ\text{C}$$

$$T_{sink} = T_{case} - 16 = 101 - 16 \Rightarrow \underline{T_{sink} = 85^\circ\text{C}}$$

8.10

$$T_{dev} - T_{amb} = P_D(\theta_{dev-case} + \theta_{case-amb})$$

$$200 - 25 = 25(3 + \theta_{case-amb})$$

$$\Rightarrow \underline{\theta_{case-amb} = 4^\circ\text{C/W}}$$

8.11

$$\theta_{dev-case} = \frac{T_{j,max} - T_{amb}}{P_{D,rated}} = \frac{175 - 25}{15} = 10^\circ\text{C/W}$$

$$P_D = \frac{T_{j,max} - T_{amb}}{\theta_{dev-case} + \theta_{case-snk} + \theta_{snk-amb}}$$

$$= \frac{175 - 25}{10 + 1 + 4} \Rightarrow \underline{P_D = 10 \text{ W}}$$

8.12

$$\eta = \frac{\overline{P_L}}{\overline{P_S}}$$

$$\overline{P_S} = V_{CC} \cdot I_Q$$

$$\overline{P_L} = V_P \cdot I_P = \left(\frac{V_{CC}}{2}\right)(I_Q)$$

$$\eta = \frac{\frac{1}{2} \cdot V_{CC} \cdot I_Q}{V_{CC} \cdot I_Q} \Rightarrow \underline{\eta = 50\%}$$

8.13

a. Neglect base currents.

$$v_O(\max) = V^+ - V_{CE}(\text{sat}) = 10 - 0.2 = 9.8 \text{ V}$$

$$i_L(\max) = I_Q = \frac{9.8}{R_L} = \frac{9.8}{1} \Rightarrow \underline{I_Q = 9.8 \text{ mA}}$$

$$R = \frac{0 - 0.7 - (-10)}{9.8} \Rightarrow \underline{R = 949\ \Omega}$$

$$i_{E1}(\max) = 2I_Q \Rightarrow \underline{i_{E1}(\max) = 19.6 \text{ mA}}$$

$$\underline{i_{E1}(\min) = 0}$$

$$\underline{i_L(\max) = I_Q = 9.8 \text{ mA}}$$

$$\underline{i_L(\min) = -I_Q = -9.8 \text{ mA}}$$

b. $\overline{P_L} = \frac{1}{2}(i_L(\max))^2 R_L = \frac{1}{2}(9.8)^2(1)$

$$\Rightarrow \overline{P_L} = 48.02 \text{ mW}$$

$$\overline{P_S} = I_Q(V^+ - V^-) + I_Q(0 - V^-)$$

$$= 9.8(20) + 9.8(10) \Rightarrow \overline{P_S} = 294 \text{ mW}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{48.02}{294} \Rightarrow \underline{\eta = 16.3\%}$$

8.14

a. $I_Q(\min) = \frac{v_O(\max)}{R_L} = \frac{10}{0.1} \Rightarrow \underline{I_Q(\min) = 100 \text{ mA}}$

$$R = \frac{0 - 0.7 - (-12)}{100} \Rightarrow \underline{R = 113\ \Omega}$$

b. $P_{Q1} = I_Q \cdot V_{CE1} = (100)(12) \Rightarrow \underline{P_{Q1} = 1.2 \text{ W}}$

$$P(\text{source}) = 2I_Q(12) = \underline{2.4 \text{ W}}$$

c. $\overline{P_L} = \frac{1}{2} \cdot \frac{V_P^2}{R_L} = \frac{(10)^2}{2(100)} = 0.5 \text{ W}$

$$\overline{P_S} = 1.2 + 2.4 = 3.6 \text{ W}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{0.5}{3.6} \Rightarrow \underline{\eta = 13.9\%}$$

8.15

$$\overline{P_L} = \frac{V_P^2}{R_L} = \frac{(V^+)^2}{R_L}$$

$$\overline{P_S} = \frac{1}{2} \cdot \frac{(V^+)^2}{R_L} + \frac{1}{2} \cdot \frac{(V^-)^2}{R_L}, \quad V^- = -V^+$$

So $\overline{P_S} = \frac{(V^+)^2}{R_L}$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} \Rightarrow \underline{\eta = 100\%}$$

8.16

(a) $V_{DS} \geq V_{DS}(sat) = V_{GS} - V_{TN} = V_{GS}$

$V_{DS} = 10 - V_O(\max)$ and $I_D = I_L = K_n(V_{GS})^2$

$$\frac{V_O(\max)}{R_L} = K_n(V_{GS})^2$$

$$V_{GS} = \sqrt{\frac{V_O(\max)}{R_L \cdot K_n}}$$

So

$$10 - V_O(\max) = \sqrt{\frac{V_O(\max)}{R_L \cdot K_n}} = \sqrt{\frac{V_O(\max)}{(5)(0.4)}}$$

$$[10 - V_O(\max)]^2 = \frac{V_O(\max)}{2}$$

$$100 - 20V_O(\max) + V_O^2(\max) = \frac{V_O(\max)}{2}$$

$V_0^2(\max) - 20.5V_0(\max) + 100 = 0$

$$V_0(\max) = \frac{20.5 \pm \sqrt{(20.5)^2 - 4(100)}}{2}$$

$\Rightarrow \underline{V_0(\max) = 8 \text{ V}}$

$i_L = \frac{8}{5} \Rightarrow \underline{i_L = 1.6 \text{ mA}}$

$V_{GS} = \sqrt{\frac{i_L}{K_n}} = \sqrt{\frac{1.6}{0.4}} = 2 V$

$\Rightarrow \underline{v_I = 10 \text{ V}}$

b. $\overline{P_L} = \frac{1}{2} \cdot \frac{(8)^2}{5} = 6.4 \text{ mW}$

$\overline{P_S} = \frac{20(1.6)}{\pi} = 10.2 \text{ mW}$

$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{6.4}{10.2} \Rightarrow \underline{\eta = 62.7\%}$

8.17

$v_O = i_L R_L$ and $i_L = i_D = K_n(v_{GS} - V_{TN})^2$

or $i_L = K_n(v_{GS})^2$ and $v_{GS} = v_I - v_O$

Then

$v_O = K_n R_L (v_I - v_O)^2$ or $v_O = 2(v_I - v_O)^2$

$$\frac{dv_O}{dv_I} = 2.2(v_I - v_O)\left(1 - \frac{dv_O}{dv_I}\right)$$

$$\frac{dv_O}{dv_I}[1 + 4(v_I - v_O)] = 4(v_I - v_O)$$

or $\frac{dv_O}{dv_I} = \frac{4(v_I - v_O)}{1 + 4(v_I - v_O)}$

For $v_I = 10$ V, $v_O = 8$ V $\Rightarrow \frac{dv_O}{dv_I} = \frac{4(10-8)}{1+4(10-8)}$

$\Rightarrow \underline{\frac{dv_O}{dv_I} = 0.889}$

At $v_I = 0$, $v_O = 0 \Rightarrow \frac{dv_O}{dv_I} = 0$

At $v_I = 1$, $v_O = 0.5 \Rightarrow \frac{dv_O}{dv_I} = 0.667$

8.18

a. $V_{BE} = V_T \ln\left(\frac{i_C}{I_S}\right) = (0.026)\ln\left(\frac{5 \times 10^{-3}}{5 \times 10^{-13}}\right)$

$V_{BE} = \frac{V_{BB}}{2} = 0.5987$ V

$\Rightarrow \underline{V_{BB} = 1.1973 \text{ V}}$

$P_Q = i_C \cdot v_{CE} = (5)(10) \Rightarrow \underline{P_Q = 50 \text{ mW}}$

b. $v_O = -8$ V

$i_L = \frac{-8}{0.1} \Rightarrow \underline{i_L = -80 \text{ mA}}$

$\underline{i_{Cp} \approx 80 \text{ mA}}$

$v_{EB} = V_T \ln\left(\frac{i_{Cp}}{I_S}\right) = (0.026)\ln\left(\frac{80 \times 10^{-3}}{5 \times 10^{-13}}\right)$

$v_{EB} = 0.6708$ V

$v_I = \frac{V_{BB}}{2} - v_{EB} + v_O = 0.5987 - 0.6708 - 8$

$\Rightarrow \underline{v_I = -8.072 \text{ V}}$

$v_{BE} = V_{BB} - v_{EB} = 1.1973 - 0.6708 = 0.5265$ V

$i_{Cn} = I_S \exp\left(\frac{v_{BE}}{V_T}\right) = 5 \times 10^{-13} \exp\left(\frac{0.5265}{0.026}\right)$

$\Rightarrow \underline{i_{Cn} = 0.311 \text{ mA}}$

$P_L = i_L^2 R_L = (80)^2(0.1) \Rightarrow \underline{P_L = 640 \text{ mW}}$

$P_{Qn} = i_{Cn} \cdot v_{CE} = (0.311)(10 - (-8))$

$\Rightarrow \underline{P_{Qn} = 5.60 \text{ mW}}$

$P_{Qp} = i_{Cp} \cdot v_{EC} = (80)(2) \Rightarrow \underline{P_{Qp} = 160 \text{ mW}}$

8.19

(a) $i_{Dn} = K_n(v_{GSn} - V_{TN})^2$

$\sqrt{\frac{0.5}{2}} + 2 = v_{GSn} = 2.5 \text{ V} = \frac{V_{BB}}{2}$

$\Rightarrow \underline{V_{BB} = 5.0 \text{ V}}$

$P_n = (0.5)(10) \Rightarrow \underline{P_n = P_p = 5 \text{ mW}}$

(b) $V_{DS} = V_{GS} - V_{TN} \Rightarrow V_{DS} = V_{GS} - 2$

$V_{DS} = 10 - v_O(\max)$

and

$$V_{GS} = \sqrt{\frac{i_L}{K_n}} + V_{TN} = \sqrt{\frac{v_O(\max)}{R_L K_n}} + 2$$

$$= \sqrt{\frac{v_O(\max)}{(2)(1)}} + 2$$

so

$$10 - v_O(\max) = \sqrt{\frac{v_O(\max)}{2}} + 2 - 2 = \sqrt{\frac{v_O(\max)}{2}}$$

so $\underline{v_O(\max) = 8 \text{ V}}$

$i_{Dn} = i_L = \frac{8}{1} \Rightarrow \underline{i_{Dn} = i_L = 8 \text{ mA}}$

$V_{GS} = \sqrt{\frac{8}{2}} + 2 \Rightarrow V_{GS} = 4$ V

Then $v_I = v_O + V_{GS} - \frac{V_{BB}}{2} = 8 + 4 - 2.5$

$\Rightarrow \underline{v_I = 9.5 \text{ V}}$

$v_{SGp} = v_O - \left(v_I - \frac{V_{BB}}{2}\right) = 8 - (9.5 - 2.5)$

$v_{SGp} = 1 \text{ V} \Rightarrow M_p$ cutoff $\Rightarrow \underline{i_{Dp} = 0}$

$P_L = i_L^2 R_L = (8)^2(1) \Rightarrow \underline{P_L = 64 \text{ mW}}$

$P_{Mn} = i_{Dn} \cdot v_{DS} = (8)(10-8) \Rightarrow \underline{P_{Mn} = 16 \text{ mW}}$

$P_{Mp} = i_{Dp} \cdot v_{SD} \Rightarrow \underline{P_{Mp} = 0}$

8.20

a. $v_O = 24 \text{ V} \Rightarrow i_L = \frac{24}{8} \Rightarrow \underline{i_L \approx i_N = 3 \text{ A}}$

$i_{Bn} = \frac{3}{41} \Rightarrow i_{Bn} = 73.2 \text{ mA}$

For $i_D = 25$ mA $\Rightarrow i_{R1} = 25 + 73.2 = 98.2$ mA

$$V_{BE} = V_T \ln\left(\frac{i_N}{I_S}\right) = (0.026)\ln\left(\frac{3}{6 \times 10^{-12}}\right) = 0.7004 \text{ V}$$

Then $98.2 = \frac{30 - (24 + 0.7)}{R_1} \Rightarrow R_1 = \frac{5.3}{98.2}$

$\Rightarrow \underline{R_1 = 53.97 \ \Omega}$

$$V_D = (0.026)\ln\left(\frac{25 \times 10^{-3}}{6 \times 10^{-12}}\right) = 0.5759 \text{ V}$$

$$V_{EB} = 2V_D - V_{BE} = 2(0.5759) - 0.7004 = 0.4514 \text{ V}$$

$$i_P = I_S \exp\left(\frac{V_{EB}}{V_T}\right) = (6 \times 10^{-12})\exp\left(\frac{0.4514}{0.026}\right)$$

$\Rightarrow \underline{i_P = 0.208 \text{ mA}}$

b. Neglecting base current

$i_D \approx \frac{30 - 0.6}{R_1} = \frac{30 - 0.6}{53.97} \Rightarrow \underline{i_D \approx 545 \text{ mA}}$

$$V_D = (0.026)\ln\left(\frac{0.545}{6 \times 10^{-12}}\right) = 0.656 \text{ V}$$

Approximation for i_D is okay.

Diodes and transistors matched $\Rightarrow \underline{i_N = i_P = 545 \text{ mA}}$

8.21

(a) $I_{D1} = K_1(V_{GS1} - V_{TN})^2$

$V_{GS1} = \sqrt{\frac{5}{5}} + 1 = 2\ V$

$I_{D3} = K_3(V_{GS3} - V_{TN})^2$

$200 = K_3(2-1)^2 \Rightarrow \underline{K_{n3} = K_{p4} = 200\ \mu A/V^2}$

(b) $v_I + V_{SG4} + V_{GS3} - V_{GS1} = v_O$

For v_O large, $i_L = i_1 = K_{n1}(V_{GS1} - V_{TN})^2$

$$V_{GS1} = \sqrt{\frac{i_L}{K_{n1}}} + V_{TN} = \sqrt{\frac{v_O}{R_L K_{n1}}} + V_{TN}$$

So $v_I + 2 + 2 - \left(\sqrt{\frac{v_O}{(0.5)(5)}} + 1\right) = v_O$

$$v_I = v_O + \sqrt{\frac{v_O}{2.5}} - 3$$

$$\frac{dv_I}{dv_I} = 1 = \frac{dv_O}{dv_I} + \frac{1}{2} \cdot \frac{1}{\sqrt{2.5 v_O}} \cdot \frac{dv_O}{dv_I}$$

$$1 = \frac{dv_O}{dv_I}\left[1 + \frac{1}{2\sqrt{2.5 v_O}}\right]$$

For $v_O = 5$ V :

$$1 = \frac{dv_O}{dv_I}\left[1 + \frac{1}{2\sqrt{2.5(5)}}\right] = \frac{dv_O}{dv_I}(1.1414)$$

$\Rightarrow \underline{\frac{dv_O}{dv_I} = 0.876}$

8.22

$v_O = v_I + \frac{V_{BB}}{2} - V_{GS}$ and $V_{GS} = \sqrt{\frac{I_{Dn}}{K_n}} + V_{TN}$

For $v_O \approx 0$, $I_{Dn} = I_{DQ} + i_L = I_{DQ} + \frac{v_O}{R_L}$

Then

$$v_O = v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ} + (v_O/R_L)}{K_n}}$$

or

$$v_O = v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}} \cdot \sqrt{1 + \frac{v_O}{I_{DQ}R_L}}$$

For v_O small,

$$v_O \cong v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}} \cdot \left(1 + \frac{1}{2} \cdot \frac{v_O}{I_{DQ}R_L}\right)$$

$$v_O\left[1 + \frac{1}{2} \cdot \sqrt{\frac{I_{DQ}}{K_n}} \cdot \frac{1}{I_{DQ}R_L}\right] = v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}}$$

Now

$$\frac{dv_O}{dv_I} = \frac{1}{\left[1 + \frac{1}{2} \cdot \sqrt{\frac{I_{DQ}}{K_n}} \cdot \frac{1}{I_{DQ}R_L}\right]} = 0.95$$

So $\frac{1}{2} \cdot \sqrt{\frac{I_{DQ}}{K_n}} \cdot \frac{1}{I_{DQ}R_L} = \frac{1}{0.95} - 1 = 0.0526$

For $R_L = 0.1\ k\Omega$, then $\frac{1}{\sqrt{K_n I_{DQ}}} = 0.01052$

Or $\sqrt{K_n I_{DQ}} = 95.1$

We can write

$g_m = 2\sqrt{K_n I_{DQ}} = 190\ mA/V$

This is the required transconductance for the output transistor. This implies a very large transistor.

8.23

$A_v = -g_m R_L$

So $-12 = -g_m(2) \Rightarrow g_m = 6 \text{ mA/V} = \dfrac{I_{CQ}}{V_T}$

$I_{CQ} = (6)(0.026) \Rightarrow I_{CQ} = 0.156 \text{ mA}$

But for maximum symmetrical swing, set

$$I_{CQ} = \frac{V_{CC}}{R_L} = \frac{10}{2} = 5 \text{ mA} \Rightarrow |A_v| > 12$$

Maximum power to the load:

$$\overline{P_L}(\max) = \frac{1}{2}\cdot\frac{V_{CC}^2}{R_L} = \frac{(10)^2}{2(2)} \Rightarrow \underline{\overline{P_L}(\max) = 25 \text{ mW}}$$

$\overline{P_S} = V_{CC}\cdot I_{CQ} = (10)(5) = 50 \text{ mW}$

So $\underline{\eta = 50\%}$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{5}{180} = 0.0278 \text{ mA}$$

$R_i = R_{TH} = 6 \text{ k}\Omega$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E$

Set $R_E = 20\ \Omega$

$V_{TH} = (0.0278)(6) + 0.7 + (181)(0.0278)(0.020)$

$V_{TH} = 0.967 \text{ V}$

$$V_{TH} = \frac{1}{R_1}\cdot R_{TH}\cdot V_{CC}$$

$0.967 = \dfrac{1}{R_1}(6)(10) \Rightarrow \underline{R_1 = 62.0 \text{ k}\Omega}$

$\underline{R_2 = 6.64 \text{ k}\Omega}$

8.24

$$I_{CQ} = \frac{V_{CC}}{R_L} = \frac{15}{1} = 15 \text{ mA}$$

$$I_{BQ} = \frac{15}{100} = 0.15 \text{ mA}$$

$$\overline{P_L}(\max) = \frac{1}{2}\cdot\frac{V_{CC}^2}{R_L} = \frac{(15)^2}{2(1)} \Rightarrow \underline{\overline{P_L}(\max) = 112.5 \text{ mW}}$$

Let $R_{TH} = 10 \text{ k}\Omega$

$V_{TH} = I_{BQ}R_{TH} + V_{BE} + (1+\beta)I_{BQ}R_E$

$= (0.15)(10) + 0.7 + (101)(0.15)(0.1)$

$$V_{TH} = 3.715 = \frac{1}{R_1}\cdot R_{TH}\cdot V_{CC} = \frac{1}{R_1}\cdot(10)(15)$$

$\underline{R_1 = 40.4 \text{ k}\Omega}$

$\underline{R_2 = 13.3 \text{ k}\Omega}$

8.25

$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)V_{CC} = \left(\frac{1.55}{1.55+0.73}\right)(10)$$

$= 6.80 \text{ V}$

$R_{TH} = R_1 \| R_2 = 0.73 \| 1.55 = 0.496 \text{ k}\Omega$

$$I_{BQ} = \frac{V_{TH} - V_{BE}}{R_{TH} + (1+\beta)R_E} = \frac{6.80 - 0.70}{0.496 + (26)(0.02)}$$

$I_{BQ} = 6.0 \text{ mA}$, $I_{CQ} = 150 \text{ mA}$

$A_v = -g_m R_L'$ and $R_L' = a^2 R_L = (3)^2(8) = 72\ \Omega$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{150}{0.026} \Rightarrow 5.77 \text{ A/V}$$

$A_v = -(5.77)(72) = -415$

$|V_0|' = |A_v|\cdot V_i = (415)(0.017) = 7.06 \text{ V}$

$$V_0 = \frac{7.06}{3} = 2.35 \text{ V}$$

$$\overline{P_L} = \frac{1}{2}\cdot\frac{V_0^2}{R_L} = \frac{(2.35)^2}{2(8)} \Rightarrow \underline{\overline{P_L} = 345 \text{ mW}}$$

$\overline{P_S} = I_{CQ}\cdot V_{CC} = (0.15)(10) = 1.5 \text{ W}$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{0.345}{1.5} \Rightarrow \underline{\eta = 23\%}$$

8.26

a. Assuming the maximum power is being delivered, then

$$V_0'(\text{peak}) = 36 \text{ V} \Rightarrow V_0 = \frac{36}{4} = 9 \text{ V}$$

$$\Rightarrow V_{rms} = \frac{9}{\sqrt{2}} \Rightarrow \underline{V_{rms} = 6.36 \text{ V}}$$

b. $V_0 = \dfrac{36}{\sqrt{2}} \Rightarrow \underline{V_0 = 25.5 \text{ V}}$

c. Secondary $I_{rms} = \dfrac{R_L}{V_{rms}} = \dfrac{2}{6.36} \Rightarrow \underline{I_{rms} = 0.314 \text{ A}}$

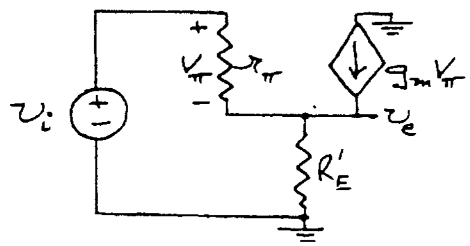
Primary $I_P = \dfrac{0.314}{4} \Rightarrow \underline{I_P = 78.6 \text{ mA}}$

d. $\overline{P_S} = I_{CQ}\cdot V_{CC} = (0.15)(36) = 5.4 \text{ W}$

$$\eta = \frac{2}{5.4} \Rightarrow \underline{\eta = 37\%}$$

8.27

a.



$$\nu_e = \left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right) R'_E = V_\pi \left(\frac{1}{r_\pi} + g_m\right) R'_E$$

$$= V_\pi \left(\frac{1+\beta}{r_\pi}\right) R'_E$$

$\nu_i = V_\pi + \nu_e \Rightarrow V_\pi = \nu_i - \nu_e$

$$\nu_e = (\nu_i - \nu_e)\left(\frac{1+\beta}{r_\pi}\right) R'_E$$

$$\frac{\nu_e}{\nu_i} = \frac{\frac{1+3}{r_\pi}\cdot R'_E}{1 + \frac{1+\beta}{r_\pi}\cdot R'_E} = \underline{\frac{(1+\beta)R'_E}{r_\pi + (1+\beta)R'_E} = \frac{\nu_e}{\nu_i}}$$

$\underline{\text{where } R'_E = \left(\frac{n_1}{n_2}\right)^2 R_L}$

$$\nu_0 = \frac{\nu_e}{\left(\frac{n_1}{n_2}\right)} \text{ so } \nu_e = \nu_0\left(\frac{n_1}{n_2}\right)$$

$$\underline{\text{so } \frac{\nu_0}{\nu_i} = \frac{1}{\left(\frac{n_1}{n_2}\right)} \cdot \frac{(1+\beta)R'_E}{r_\pi + (1+\beta)R'_E}}$$

b. $\overline{P_L} = \frac{1}{2}\cdot I_P^2 R_L,\ a = \frac{n_1}{n_2},\ I_{CQ} = \frac{I_P}{a}$

so $\overline{P_L} = \frac{1}{2}\cdot a^2 I_{CQ}^2 R_L$

$\overline{P_S} = I_{CQ}\cdot V_{CC}$

For $\eta = 50\%$:

$$\frac{\overline{P_L}}{\overline{P_S}} = 0.5 = \frac{\frac{1}{2}\cdot a^2 I_{CQ}^2 R_L}{I_{CQ}\cdot V_{CC}} = \frac{a^2 I_{CQ} R_L}{2V_{CC}}$$

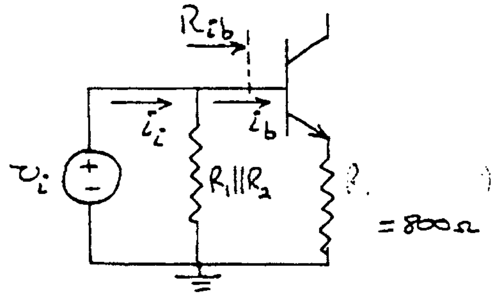
$$\text{so } a^2 = \frac{V_{CC}}{I_{CQ}\cdot R_L} = \frac{V_{CC}}{(0.1)(50)} \Rightarrow \underline{a^2 = \frac{V_{CC}}{5}}$$

c. $R_0 = \frac{r_\pi}{1+\beta} = \frac{\beta V_T}{(1+\beta)I_{CQ}} = \frac{49(0.026)}{(50)(0.1)}$

$\Rightarrow \underline{R_0 = 0.255\ \Omega}$

8.28

a. With a 10:1 transformer ratio, we need a current gain of 8 through the transistor.



$i_e = (1+\beta)i_b$ and $i_b = \left(\frac{R_1\|R_2}{R_1\|R_2 + R_{ib}}\right)i_i$

so we need

$$\frac{i_e}{i_i} = 8 = (1+\beta)\left(\frac{R_1\|R_2}{R_1\|R_2 + R_{ib}}\right)$$

where

$R_{ib} = r_\pi + (1+\beta)R'_L \approx (1+\beta)R'_L$

$= (101)(0.8) = 80.8$

Then $8 = (101)\left(\frac{R_1\|R_2}{R_1\|R_2 + 80.8}\right)$

$\frac{R_1\|R_2}{R_1\|R_2 + 80.8} = 0.0792$ or $R_1\|R_2 = 6.95\ \text{k}\Omega$

Set

$\frac{2V_{CC}}{2I_{CQ}} = R'_L \Rightarrow I_{CQ} = \frac{V_{CC}}{R'_L} = \frac{12}{0.8} = 15\ \text{mA}$

$I_{BQ} = \frac{15}{100} = 0.15\ \text{mA}$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}$

$\frac{1}{R_1}\cdot R_{TH}\cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}$

$\frac{1}{R_1}(6.95)(12) = (0.15)(6.95) + 0.7$

$\Rightarrow \underline{R_1 = 47.9\ \text{k}\Omega}$ then $\underline{R_2 = 8.13\ \text{k}\Omega}$

b. $I_e = 0.9I_{CQ} = 13.5\ \text{mA} = \frac{I_L}{a} \Rightarrow I_L = 135\ \text{mA}$

$\overline{P_L} = \frac{1}{2}(0.135)^2(8) \Rightarrow \underline{\overline{P_L} = 72.9\ \text{mW}}$

$\overline{P_S} = V_{CC}I_{CQ} = (12)(15) \Rightarrow \underline{\overline{P_S} = 180\ \text{mW}}$

$\eta = \frac{\overline{P_L}}{\overline{P_S}} \Rightarrow \underline{\eta = 40.5\%}$

8.29

a. $V_P = \sqrt{2R_L \overline{P_L}}$

$V_P = \sqrt{2(8)(1)} = 5.66$ V = peak output voltage

$I_P = \frac{V_P}{R_L} = \frac{5.66}{8} = 0.708$ A = peak output current

Set $V_e = 0.9V_{CC} = aV_P$ to minimize distortion

Then $a = \frac{(0.9)(18)}{5.66} \Rightarrow \underline{a = 2.86}$

b. Now

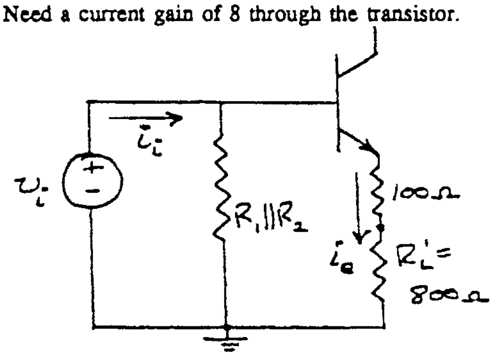
$I_{CQ} = \frac{1}{0.9}\left(\frac{I_P}{a}\right) = \frac{1}{0.9}\left(\frac{0.708}{2.86}\right) \Rightarrow I_{CQ} = 0.275$ A

Then $P_Q = V_{CC}I_Q = (18)(0.275)$

$\Rightarrow \underline{P_Q = 4.95 \text{ W}}$ Power rating of transistor

8.30

a. Need a current gain of 8 through the transistor.



$\frac{i_b}{i_i} = 8 = (1+\beta)\left(\frac{R_1||R_2}{R_1||R_2 + R_{ib}}\right)$

where $R_{ib} \approx (1+\beta)(0.9) = 90.9$ kΩ

$\frac{8}{101} = \left(\frac{R_1||R_2}{R_1||R_2 + 90.9}\right) = 0.0792$

or $R_1||R_2 = 7.82$ kΩ

Set

$\frac{2V_{CC}}{2I_{CQ}} = 0.9 \text{ k}\Omega \Rightarrow I_{CQ} = \frac{12}{0.9} = 13.3$ mA

$I_{BQ} = \frac{13.3}{100} = 0.133$ mA

Then

$\frac{1}{R_1}(7.82)(12) = (0.133)(7.82) + 0.7$

$\Rightarrow \underline{R_1 = 53.9 \text{ k}\Omega}$ and $\underline{R_2 = 9.15 \text{ k}\Omega}$

b. $I_e = (0.9)I_{CQ} = 12 \text{ mA} = \frac{I_L}{a} \Rightarrow I_L = 120$ mA

$\overline{P_L} = \frac{1}{2}(0.12)^2(8) \Rightarrow \underline{\overline{P_L} = 57.6 \text{ mW}}$

$\overline{P_S} = V_{CC}I_{CQ} = (12)(13.3) \Rightarrow \underline{\overline{P_S} = 159.6 \text{ mW}}$

$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{57.6}{159.6} \Rightarrow \underline{\eta = 36.1\%}$

8.31

a. All transistors are matched.

$3 \text{ mA} = i_{E1} + i_{B3} = \left(\frac{1+\beta}{\beta}\right)i_C + \frac{i_C}{\beta}$

$3 = \left(\frac{61}{60} + \frac{1}{60}\right)i_C \Rightarrow \underline{i_C = 2.90 \text{ mA}}$

b. For $v_O = 6$ V, let $\underline{R_L = 200\ \Omega}$.

$i_O = \frac{6}{200} = 0.03 \text{ A} = 30 \text{ mA} \cong i_{E3}$

$i_{B3} = \frac{30}{61} = 0.492$ mA

$i_{E1} = 3 - 0.492 = 2.508$ mA

$i_{B1} = \frac{2.508}{61} \Rightarrow \underline{i_{B1} = 41.11\ \mu\text{A}}$

$i_{E2} \cong 3 \text{ mA} \Rightarrow i_{B2} = \frac{3}{61} \Rightarrow 49.18\ \mu$A

$i_I = i_{B2} - i_{B1} = 49.18 - 41.11 \Rightarrow i_I = 8.07\ \mu$A

Current gain

$A_i = \frac{30 \times 10^{-3}}{8.07 \times 10^{-6}} \Rightarrow \underline{A_i = 3.72 \times 10^3}$

$V_{BE3} = V_T \ln\left(\frac{i_{E3}}{I_S}\right) = (0.026)\ln\left(\frac{30 \times 10^{-3}}{5 \times 10^{-13}}\right)$

$V_{BE3} = 0.6453$ V

$V_{EB1} = V_T \ln\left(\frac{i_{E1}}{I_S}\right) = (0.026)\ln\left(\frac{2.508 \times 10^{-3}}{5 \times 10^{-13}}\right)$

$V_{EB1} = 0.5807$ V

$v_I = v_O + V_{BE3} - V_{EB1} = 6 + 0.6453 - 0.5807$

$v_I = 6.0646$ V

Voltage gain

$A_v = \frac{v_O}{v_I} = \frac{6}{6.0646} \Rightarrow \underline{A_v = 0.989}$

8.32

a. For $i_O = 1$ A, $I_{B3} \cong \frac{1}{50} \Rightarrow 20$ mA

We can then write

$$\frac{10 - V_{EB1}}{R_1} = 2\left[\frac{10 - (v_{O,max} + V_{BE3})}{R_1} - 20\right]$$

If, for simplicity, we assume $V_{EB1} = V_{BE3} = 0.7$ V, then

$$\frac{10 - V_{BE}}{R_1} = \frac{2v_{O,max}}{R_1} + 40$$

If we assume $v_{O,max} = 4$ V, then

$$\frac{9.3}{R_1} = \frac{2(4)}{R_1} + 40$$

which yields $\underline{R_1 = R_2 = 32.5\ \Omega}$

b. For $v_I = 0$,

$$I_{E1} = \frac{9.3}{32.5} \Rightarrow \underline{I_{E1} = 0.286\text{ A} = I_{E2}}$$

Since $I_{S3,4} = 10 I_{S1,2}$, then

$$\underline{I_{E3} = I_{E4} = 2.86\text{ A}}$$

c. We can write

$$R_O = \frac{1}{2}\left\{\frac{r_{\pi3} + R_1 \left\| \frac{r_{\pi1}}{1+\beta_1}\right.}{1+\beta_3}\right\}$$

Now $r_{\pi3} = \frac{\beta_3 V_T}{I_{C3}} = \frac{(50)(0.026)}{2.86} = 0.4545\ \Omega$

$r_{\pi1} = \frac{\beta_1 V_T}{I_{C1}} = \frac{(120)(0.026)}{0.286} = 10.91\ \Omega$

So

$$R_O = \frac{1}{2}\left\{\frac{0.4545 + 32.5 \left\| \frac{10.91}{121}\right.}{51}\right\}$$

$$32.5 \left\| \frac{10.91}{121}\right. = 32.5 \| 0.0902 = 0.0900$$

Then

$$R_O = \frac{1}{2}\left\{\frac{0.4545 + 0.0900}{51}\right\} \text{ or } \underline{R_O = 0.00534\ \Omega}$$

8.33

$$R_i = \frac{1}{2}\{r_{\pi1} + (1+\beta)[R_1 \| (r_{\pi3} + (1+\beta)R_L)]\}$$

$i_{C1} \approx 7.2$ mA and $i_{C3} \approx 7.2$ mA

Then $r_\pi = \frac{(60)(0.026)}{7.2} = 0.217$ kΩ

So

$$R_i = \frac{1}{2}\{0.217 + (61)[2 \| (0.217 + (61)(0.1))]\}$$

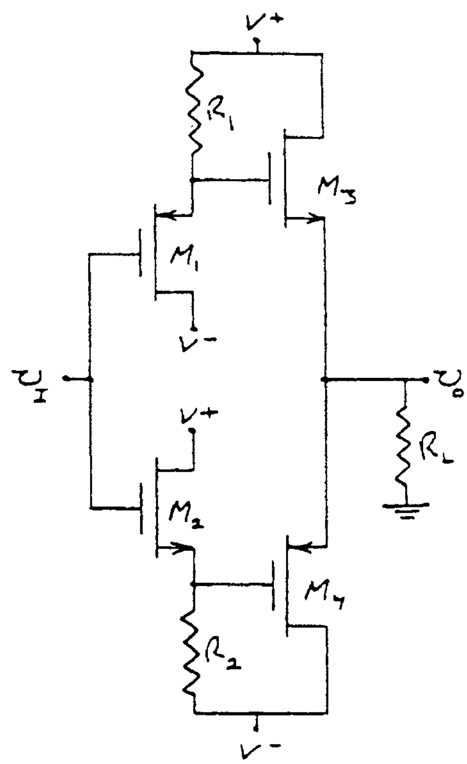
$$= \frac{1}{2}\{0.217 + 61[2 \| 6.32]\}$$

or

$$R_i = 46.4\text{ k}\Omega$$

8.34

a.



b. $I_1 = K_1(V_{SG} + V_{TP})^2 = \frac{V^+ - V_{SG}}{R_1}$

$5 = 10(V_{SG} - 2)^2 \Rightarrow V_{SG} = 2.707$ V

$5 = \frac{10 - 2.707}{R_1} \Rightarrow \underline{R_1 = R_2 = 1.46\text{ k}\Omega}$

c. $R_L = 100\ \Omega$ For a sinusoidal output signal:

$$\overline{P_L} = \frac{1}{2}\cdot\frac{(\nu_O)^2}{R_L} = \frac{1}{2}\cdot\frac{(5)^2}{0.1} \Rightarrow \underline{\overline{P_L} = 125\text{ mW}}$$

$$i_{D3} \approx \frac{(\nu_O)}{R_L} = \frac{(5)}{0.1} \Rightarrow i_{D3} = 50\text{ mA}$$

$$V_{GS3} = \sqrt{\frac{50}{10}} + 2 = 4.236\text{ V}$$

$$I_1 = \frac{10-(4.236+5)}{1.46} \Rightarrow \underline{I_{D1} = 0.523\text{ mA}}$$

$$V_{SG1} = \sqrt{\frac{0.523}{10}} + 2 = 2.229\text{ V}$$

$$\nu_I = 5 + 4.236 - 2.229 \Rightarrow \nu_I = 7.007\text{ V}$$

$$I_{D2} = \frac{(\nu_I - V_{SG}) - (-10)}{1.46} = 10(V_{SG}-2)^2$$

$$\frac{17.007 - V_{SG}}{1.46} = 10(V_{SG}^2 - 4V_{SG} + 4)$$

$$14.6V_{SG}^2 - 57.4V_{SG} + 41.4 = 0$$

$$V_{SG} = \frac{57.4 \pm \sqrt{(57.4)^2 - 4(14.6)(41.4)}}{2(14.6)}$$

$$V_{SG2} = 2.98\text{ V}$$

$$I_{D2} = 10(2.98-2)^2 \Rightarrow \underline{I_{D2} = 9.60\text{ mA}}$$

$$V_{G4} = \nu_I - V_{GS2} = 7 - 2.98 = 4.02\text{ V}$$

$$V_{SG4} = 5 - 4.02 = 0.98\text{ V} \Rightarrow \underline{I_{D4} = 0}$$

8.35

For $\nu_O = 0$

$$I_Q = I_{C3} + I_{C2} + I_{E1}$$

$$I_{B3} = I_{E2} = \left(\frac{1+\beta_n}{\beta_n}\right)I_{C2} = \frac{I_{C3}}{\beta_n}$$

$$I_{C3} = (1+\beta_n)I_{C2}$$

$$I_{B2} = I_{C1} = \left(\frac{\beta_p}{1+\beta_p}\right)I_{E1} = \frac{I_{C2}}{\beta_n}$$

$$I_{C2} = \beta_n\left(\frac{\beta_p}{1+\beta_p}\right)I_{E1}$$

$$I_{C3} = (1+\beta_n)\beta_n\left(\frac{\beta_p}{1+\beta_p}\right)I_{E1}$$

$$I_Q = (1+\beta_n)\beta_n\left(\frac{\beta_p}{1+\beta_p}\right)I_{E1} + \beta_n\left(\frac{\beta_p}{1+\beta_p}\right)I_{E1} + I_{E1}$$

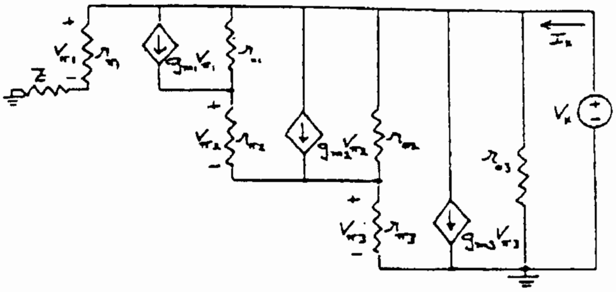
$$= (51)(50)\left(\frac{10}{11}\right)I_{E1} + (50)\left(\frac{10}{11}\right)I_{E1} + I_{E1}$$

$$I_Q = 2318.18I_{E1} + 45.45I_{E1} + I_{E1}$$

$$I_{E1} = 1.692\ \mu\text{A} \Rightarrow \underline{I_{C1} = 1.534\ \mu\text{A}}$$

$$I_{C2} = (50)\left(\frac{10}{11}\right)(1.692) \Rightarrow I_{C2} = 76.9\ \mu\text{A}$$

$$I_{C3} = (51)(50)\left(\frac{10}{11}\right)(1.692) \Rightarrow \underline{I_{C3} = 3.92\text{ mA}}$$



Because of $r_{\pi 1}$ and Z, neglect effect of r_o. Then neglecting r_{o1}, r_{o2}, and r_{o3}, we find

$$I_X = g_{m3}V_{\pi 3} + g_{m2}V_{\pi 2} + g_{m1}V_{\pi 1} + \frac{V_X}{r_{\pi 1} + Z}$$

Now

$$V_{\pi 1} = \left(\frac{r_{\pi 1}}{r_{\pi 1} + Z}\right)V_X,\quad V_{\pi 2} \cong g_{m1}V_{\pi 1}r_{\pi 2}$$

and

$$V_{\pi 3} = (g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2})r_{\pi 3}$$
$$= [g_{m1}V_{\pi 1} + g_{m2}(g_{m1}V_{\pi 1}r_{\pi 2})]r_{\pi 3}$$

$$V_{\pi 3} = \left(\frac{r_{\pi 1}}{r_{\pi 1} + Z}\right)[g_{m1} + g_{m1}g_{m2}r_{\pi 2}]r_{\pi 3}\cdot V_X$$

$$V_{\pi 3} = \frac{(\beta_1 + \beta_1\beta_2)r_{\pi 3}}{r_{\pi 1} + Z}\cdot V_X$$

and

$$V_{\pi 2} = g_{m1}\left(\frac{r_{\pi 1}}{r_{\pi 1} + Z}\right)r_{\pi 2}V_X = \left(\frac{\beta_1 r_{\pi 2}}{r_{\pi 1} + Z}\right)V_X$$

Then

$$I_X = \frac{(\beta_1 + \beta_1\beta_2)\beta_3}{r_{\pi 1} + Z}\cdot V_X + \frac{\beta_1\beta_2}{r_{\pi 1} + Z}\cdot V_X + \frac{\beta_1}{r_{\pi 1} + Z}\cdot V_X + \frac{V_X}{r_{\pi 1} + Z}$$

Then

$$R_o = \frac{V_X}{I_X} = \frac{r_{\pi 1} + Z}{1 + \beta_1 + \beta_1\beta_2 + (\beta_1 + \beta_1\beta_2)\beta_3}$$

$$r_{\pi 1} = \frac{(10)(0.026)}{1.534} = 0.169\text{ M}\Omega$$

$$Z = 25\text{ k}\Omega$$

Then

$$R_0 = \frac{169 + 25}{1 + (10) + (10)(50) + [10 + (10)(50)](50)}$$

$$R_0 = \frac{194}{26,011} = 0.00746 \text{ k}\Omega$$

or $\underline{R_0 = 7.46\ \Omega}$

8.36

a. Neglect base currents.

$$V_{BB} = 2V_D = 2V_T \ln\left(\frac{I_{Bias}}{I_S}\right)$$

$$= 2(0.026)\ln\left(\frac{5 \times 10^{-3}}{10^{-13}}\right)$$

$$\Rightarrow \underline{V_{BB} = 1.281 \text{ V}}$$

$$V_{BE1} + V_{EB3} = V_{BB}$$

$$I_{E1} = I_{E3} + I_{C2}$$

$$I_{B2} = I_{C3} = \left(\frac{\beta_p}{1+\beta_p}\right) I_{E3}$$

$$I_{C2} = \beta_n I_{B2} = \beta_n \left(\frac{\beta_p}{1+\beta_p}\right) I_{E3}$$

$$I_{E1} = I_{E3} + \beta_n \left(\frac{\beta_p}{1+\beta_p}\right) I_{E3}$$

$$I_{E1} = I_{E3}\left[1 + \beta_n\left(\frac{\beta_p}{1+\beta_p}\right)\right]$$

$$\left(\frac{1+\beta_n}{\beta_n}\right) I_{C1} = \left(\frac{1+\beta_p}{\beta_p}\right) I_{C3}\left[1 + \beta_n\left(\frac{\beta_p}{1+\beta_p}\right)\right]$$

$$V_{BE1} = V_T \ln\left[\frac{I_{C1}}{I_S}\right],\ V_{EB3} = V_T \ln\left[\frac{I_{C3}}{I_S}\right]$$

$$(1.01) I_{C1} = \left(\frac{21}{20}\right) I_{C3}\left[1 + (100)\left(\frac{20}{21}\right)\right]$$

$$= I_{C3}\left[\frac{21}{20} + 100\right] = 101.05 I_{C3}$$

$$I_{C1} = 100.05 I_{C3}$$

$$V_T \ln\left(\frac{100.05 I_{C3}}{I_S}\right) + V_T \ln\left(\frac{I_{C3}}{I_S}\right) = V_{BB}$$

$$V_T \ln\left(\frac{100.05 I_{C3}^2}{I_S^2}\right) = V_{BB}$$

$$\frac{100.05 I_{C3}^2}{I_S^2} = \exp\left(\frac{V_{BB}}{V_T}\right)$$

$$I_{C3} = \frac{I_S}{\sqrt{100.05}}\sqrt{\exp\left(\frac{V_{BB}}{V_T}\right)} = \underline{0.4997 \text{ mA} = I_{C3}}$$

Then $I_{E3} = 0.5247$ mA

Now

$$I_{C1} = 100.05 I_{C3} = \underline{50 \text{ mA} = I_{C1}}$$

$$I_{C2} = (100)\left(\frac{20}{21}\right)(0.5247) = \underline{49.97 \text{ mA} = I_{C2}}$$

b. $v_0 = 10 \text{ V} \Rightarrow i_{E1} \approx \frac{10}{100} = 0.10 \text{ A} = i_{C1}$

$$i_{B1} = \frac{100}{100} = 1 \text{ mA}$$

$$V_{BB} = 2(0.026)\ln\left(\frac{4 \times 10^{-3}}{10^{-13}}\right) = 1.269 \text{ V}$$

$$V_{BE1} = (0.026)\ln\left(\frac{0.1}{10^{-13}}\right) = 0.7184$$

$$V_{EB3} = 1.269 - 0.7184 = 0.5506 \text{ V}$$

$$I_{C3} = 10^{-13}\exp\left(\frac{0.5506}{0.026}\right) = 0.157 \text{ mA}$$

$$\overline{P_L} = \frac{v_0^2}{R_L} = \frac{(10)^2}{100} \Rightarrow \underline{\overline{P_L} = 1 \text{ W}}$$

$$P_{Q1} = i_{C1} \cdot v_{CE1} = (0.1)(12 - 10) \Rightarrow \underline{P_{Q1} = 0.2 \text{ W}}$$

$$P_{Q3} = i_{C3} \cdot v_{EC3} = (0.157)(10 - [0.7 - 12])$$

$$\Rightarrow \underline{P_{Q3} = 3.34 \text{ mW}}$$

$$i_{C2} = (100)(i_{C3}) = (100)(0.157) = 15.7 \text{ mA}$$

$$P_{Q2} = i_{C2} \cdot v_{CE2} = (15.7)(10 - [-12])$$

$$\Rightarrow \underline{P_{Q2} = 0.345 \text{ W}}$$

8.37

a. $V_{BB} = 3(0.026)\ln\left(\frac{10 \times 10^{-3}}{2 \times 10^{-12}}\right)$

$$\Rightarrow \underline{V_{BB} = 1.74195 \text{ V}}$$

$$V_{BE1} + V_{BE2} + V_{EB3} = V_{BB}$$

$$I_{C1} \approx \frac{I_{C2}}{\beta_n},\quad I_{C3} \approx \frac{I_{C2}}{\beta_n^2}$$

$$V_T \ln\left(\frac{I_{C1}}{I_S}\right) + V_T \ln\left(\frac{I_{C2}}{I_S}\right) + V_T \ln\left(\frac{I_{C3}}{I_S}\right) = V_{BB}$$

$$V_T \ln\left[\frac{I_{C2}^3}{\beta_n^3 I_S^3}\right] = V_{BB}$$

$$I_{C2} = \beta_n I_S \sqrt[3]{\exp\left(\frac{V_{BB}}{V_T}\right)}$$

$$= (20)(2 \times 10^{-12})\sqrt[3]{\exp\left(\frac{1.74195}{0.026}\right)}$$

$I_{C2} = 0.20$ A. $I_{C1} \approx 10$ mA, $I_{C3} \approx 0.5$ mA

$$V_{BE1} = (0.026)\ln\left(\frac{10 \times 10^{-3}}{2 \times 10^{-12}}\right)$$

$$\Rightarrow \underline{V_{BE1} = 0.58065 \text{ V}}$$

$$V_{BE2} = (0.026)\ln\left(\frac{0.2}{2\times10^{-12}}\right)$$

$$\Rightarrow \underline{V_{BE2} = 0.6585\ \text{V}}$$

$$V_{EB3} = (0.026)\ln\left(\frac{0.5\times10^{-3}}{2\times10^{-12}}\right)$$

$$\Rightarrow \underline{V_{EB3} = 0.50276\ \text{V}}$$

b. $\overline{P_L} = 10\ \text{W} = \frac{1}{2}\cdot\frac{V_0^2}{R_L} = \frac{1}{2}\cdot\frac{V_0^2}{20}$

$$\Rightarrow V_0(\text{max}) = 20\ \text{V}$$

For $v_0(\text{max})$:

$$P_L = \frac{v_0^2}{R_L} = \frac{(20)^2}{20} \Rightarrow \underline{P_L = 20\ \text{W}}$$

$$i_0(\text{max}) = -\frac{20}{20} = -1\ \text{A}$$

$$i_{C5} + i_{C4} + i_{E3} = -i_0(\text{max}) = 1\ \text{A}$$

$$i_{C5} + \frac{i_{C5}}{\beta_n}\cdot\left(\frac{1+\beta_p}{\beta_p}\right) + \frac{i_{C4}}{\beta_n}\cdot\left(\frac{1+\beta_p}{\beta_p}\right) = 1$$

$$i_{C5}\left[1 + \frac{1}{\beta_n}\cdot\left(\frac{1+\beta_p}{\beta_p}\right) + \left\{\frac{1}{\beta_n}\cdot\left(\frac{1+\beta_p}{\beta_p}\right)\right\}^2\right] = 1$$

$$i_{C5}\left[1 + \frac{1}{20}\cdot\left(\frac{6}{5}\right) + \left(\frac{1}{20}\cdot\frac{6}{5}\right)^2\right]$$

$$i_{C5}[1 + 0.06 + 0.0036] = 1 \Rightarrow \underline{i_{C5} = 0.940\ \text{A}}$$

$$\underline{i_{C4} = 0.0564\ \text{A}}$$

$$\underline{i_{E3} = 3.38\ \text{mA}}$$

$$i_{C2} = 2.82\ \text{mA}$$

$$V_{EB3} = (0.026)\ln\left(\frac{2.82\times10^{-3}}{2\times10^{-12}}\right) = 0.5477\ \text{V}$$

$$V_{BE1} + V_{BE2} = 1.74195 - 0.5477 = 1.1942$$

$$V_T\ln\left(\frac{I_{C2}}{\beta_n I_S}\right) + V_T\ln\left(\frac{I_{C2}}{I_S}\right) = 1.1942$$

$$I_{C2} = \sqrt{\beta_n}\cdot I_S\sqrt{\exp\left(\frac{1.1942}{0.026}\right)}$$

$$= \sqrt{20}(18.83)\ \text{mA}$$

$$I_{C2} = 84.2\ \text{mA}$$

Chapter 9

Exercise Solutions

E9.1

$A_{CL} = -\frac{R_2}{R_1} = \frac{-100\ k\Omega}{10\ k\Omega} \Rightarrow \underline{A_{CL} = -10}$

$v_I = 0.25\ V \Rightarrow \underline{v_O = -2.5\ V}$

$i_1 = \frac{v_I}{R_1} = \frac{0.25}{10\ k\Omega} = 0.025\ mA \Rightarrow \underline{i_1 = 25\ \mu A}$

$\underline{i_2 = i_1 = 25\ \mu A}$

$\underline{R_i = R_1 = 10\ k\Omega}$

E9.2

$A_{CL} = -\frac{R_2}{R_1} = -15$

$R_i = \underline{R_1 = 20\ k\Omega} \Rightarrow R_2 = (15)(20\ k\Omega)$

$\underline{R_2 = 300\ k\Omega}$

E9.3

(a) $A_v = \frac{-R_2}{R_1 + R_S}$

$A_v(\min) = \frac{-100}{19 + 1.3} = -4.926$

$A_v(\max) = \frac{-100}{19 + 0.7} = -5.076$

so $\underline{4.926 \le |A_v| \le 5.076}$

(b) $i_1(\max) = \frac{0.1}{19 + 0.7} = 5.076\ \mu A$

$i_1(\min) = \frac{0.1}{19 + 1.3} = 4.926\ \mu A$

so $\underline{4.926 \le i_1 \le 5.076\ \mu A}$

(c) Maximum current specification is violated.

E9.4

We can write

$A_{CL} = -\frac{R_2}{R_1}\left(1 + \frac{R_3}{R_4}\right) - \frac{R_3}{R_1}$

$R_i = \underline{R_1 = 10\ k\Omega}$

Want $A_{CL} = -50$ Set $\underline{R_2 = R_3 = 50\ k\Omega}$

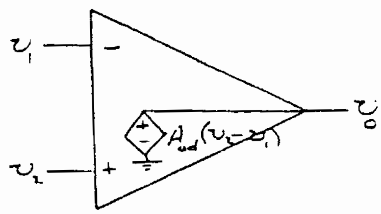
$A_{CL} = -50 = -5\left(1 + \frac{R_3}{R_4}\right) - 5$

$1 + \frac{R_3}{R_4} = 9 \Rightarrow \frac{R_3}{R_4} = 8 = \frac{50}{R_4}$

$\underline{R_4 = 6.25\ k\Omega}$

E9.5

$v_O = A_d(v_2 - v_1) \qquad A_d = 10^3$



a. $v_2 = 0,\ v_O = 5$

$v_1 = -\frac{v_O}{A_d} = -\frac{5}{10^3} \Rightarrow \underline{v_1 = -5\ mV}$

b. $v_1 = 5.\ v_O = -10$

$\frac{v_O}{A_d} = v_2 - v_1$

$\frac{-10}{10^3} = v_2 - 5 \Rightarrow \underline{v_2 = 4.99\ V}$

c. $v_1 = 0.001,\ v_2 = -0.001$

$v_O = 10^3(-0.001 - 0.001)$

$\underline{v_O = -2\ V}$

d. $v_2 = 3,\ v_O = 3$

$v_O = A_d(v_2 - v_1)$

$\frac{v_O}{A_d} = v_2 - v_1$

$\frac{3}{10^3} = 3 - v_1 \Rightarrow \underline{v_1 = 2.997\ V}$

E9.6

We have

$A_{CL} = -\frac{R_2}{R_1} \cdot \frac{1}{\left[1 + \frac{1}{A_d}\left(1 + \frac{R_2}{R_1}\right)\right]}$

$\underline{R_i = R_1 = 25\ k\Omega}$ Let $\frac{R_2}{R_1} = x$

$-12 = -\frac{x}{1 + \frac{1}{5 \times 10^3}(1 + x)}$

$= \frac{-x}{1.0002 + \frac{x}{5 \times 10^3}}$

$12\left(1.0002 + \frac{x}{5 \times 10^3}\right) = x$

$12.0024 = x - (2.4 \times 10^{-3})x$

$x = \frac{12.0024}{0.9976} = 12.0313 = \frac{R_2}{25\ k\Omega}$

$\underline{R_2 = 300.78\ k\Omega}$

E9.7

$$v_O = -\left(\frac{R_4}{R_1}v_{I1} + \frac{R_4}{R_2}v_{I2} + \frac{R_4}{R_3}v_{I3}\right)$$

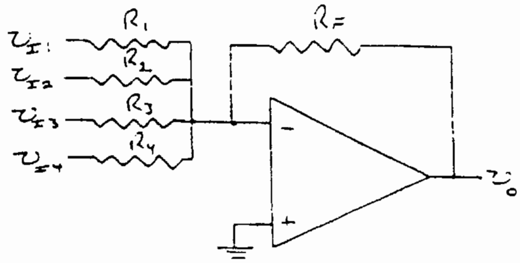
$$v_O = -\left[\left(\frac{40}{10}\right)(250) + \left(\frac{40}{20}\right)(200) + \left(\frac{40}{30}\right)(75)\right]$$

$$v_O = -[1000 + 400 + 100]$$

$$\underline{v_O = -1500\ \mu\text{V} = -1.5\ \text{mV}}$$

E9.8

$$v_O = -\left(\frac{R_F}{R_1}v_{I1} + \frac{R_F}{R_2}v_{I2} + \frac{R_F}{R_3}v_{I3} + \frac{R_F}{R_4}v_{I4}\right)$$



We need

$$\frac{R_F}{R_1} = 7,\ \frac{R_F}{R_2} = 14,\ \frac{R_F}{R_3} = 3.5,\ \frac{R_F}{R_4} = 10$$

Set $R_F = 280\ \text{k}\Omega$

$$\text{Then } R_1 = \frac{280}{7} = 40\ \text{k}\Omega$$

$$R_2 = \frac{280}{14} = 20\ \text{k}\Omega$$

$$R_3 = \frac{280}{3.5} = 80\ \text{k}\Omega$$

$$R_4 = \frac{280}{10} = 28\ \text{k}\Omega$$

E9.9

$$|v_O| = \frac{v_{I1} + v_{I2} + v_{I3}}{3} = \frac{R_F}{R}(v_{I1} + v_{I2} + v_{I3})$$

$$\frac{R_F}{R} = \frac{1}{3} \Rightarrow R_1 = R_2 = R_3 \equiv R = 1\ M\Omega$$

$$\text{Then } R_F = \frac{1}{3}\ M\Omega = 333\ k\Omega$$

E9.10

$$A_v = \frac{v_O}{v_I} = \left(1 + \frac{R_2}{R_1}\right) = 5$$

so that $\frac{R_2}{R_1} = 4$

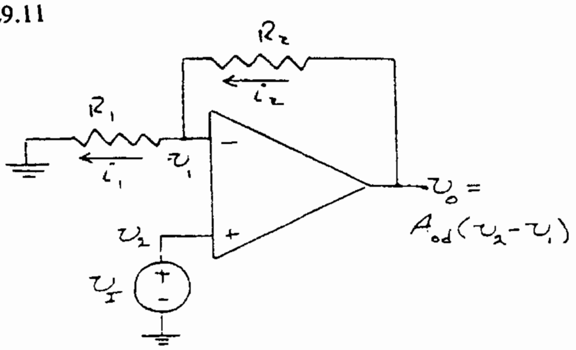
For $v_O = 10$ V, $v_I = 2$ V

Then $i_1 = \frac{2}{R_1} = 50\ \mu\text{A} \Rightarrow \underline{R_1 = 40\ \text{k}\Omega}$

Then $\underline{R_2 = 160\ \text{k}\Omega}$ we find

$$i_2 = \frac{v_O - v_I}{R_2} = \frac{10 - 2}{160} = 50\ \mu\text{A}$$

E9.11



$$v_O = A_{od}(v_2 - v_1) = A_{od}(v_I - v_1)$$

$$\frac{v_O}{A_{od}} - v_I = -v_1 \text{ or } v_1 = v_I - \frac{v_O}{A_{od}}$$

$$i_1 = \frac{v_1}{R_1} = i_2 \text{ and } i_2 = \frac{v_O - v_1}{R_2}$$

$$\text{Then } v_1\left(\frac{1}{R_1}\right) = \frac{v_O - v_1}{R_2}$$

$$v_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v_O}{R_2}$$

$$v_O\left(1 + \frac{R_2}{R_1}\right)v_1 = \left(1 + \frac{R_2}{R_1}\right)\left(v_I - \frac{v_O}{A_{od}}\right)$$

$$\text{So } A_v = \frac{v_O}{v_I} = \frac{\left(1 + \frac{R_2}{R_1}\right)}{1 + \frac{1}{A_{od}}\left(1 + \frac{R_2}{R_1}\right)}$$

E9.12

For $v_{I2} = 0$, $v_2 = \left(\frac{R_b}{R_b + R_a}\right)v_{I1}$ and

$$v_O(v_{I1}) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_b}{R_b + R_a}\right)v_{I1}$$

$$= \left(1 + \frac{70}{5}\right)\left(\frac{50}{50 + 25}\right)v_{I1}$$

$$= 10v_{I1}$$

For $v_{I1} = 0$,

$$v_2 = \left(\frac{R_a}{R_b + R_a}\right)v_{I2}$$

$$v_O(v_{I2}) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_a}{R_b + R_a}\right)v_{I2}$$

$$= \left(1 + \frac{70}{5}\right)\left(\frac{25}{25 + 50}\right)v_{I2}$$

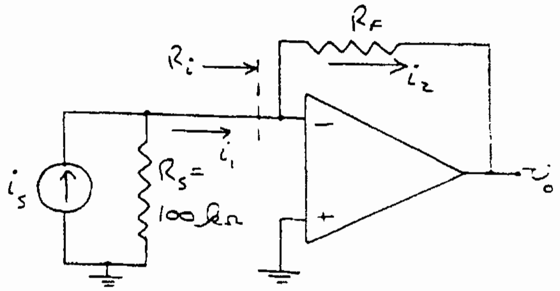
$$= 5v_{I2}$$

Then

$$v_O = v_O(v_{I1}) + v_O(v_{I2})$$

$$\underline{v_O = 10v_{I1} + 5v_{I2}}$$

E9.13



$R_S \gg R_i$ so $i_1 = i_2 = i_S = 100\ \mu A$

$v_O = -i_S R_F$

We want $-10 = -(100 \times 10^{-6}) R_F$

$\Rightarrow \underline{R_F = 100\ k\Omega}$

E9.14

We may note that

$\frac{R_3}{R_2} = \frac{3}{1.5} = 2$ and $\frac{R_F}{R_1} = \frac{20}{10} = 2$

so that

$$\frac{R_3}{R_2} = \frac{R_F}{R_1}$$

Then

$$i_L = \frac{-v_I}{R_2} = \frac{-(-3)}{1.5\ k\Omega}$$

$\Rightarrow \underline{i_L = 2\ mA}$

$v_L = i_L Z_L = (2 \times 10^{-3})(200) = 0.4\ V$

$i_4 = \frac{v_L}{R_2} = \frac{0.4}{1.5\ k\Omega} = 0.267\ mA$

$i_3 = i_4 + i_L = 0.267 + 2 = 2.267\ mA$

$v_O = i_3 R_3 + v_L = (2.267 \times 10^{-3})(3 \times 10^3) - 0.4$

$\Rightarrow \underline{v_O = 7.2\ V}$

E9.15

We want $i_L = 1$ mA when $v_I = -5$ V

$i_L = \frac{-v_I}{R_2} \Rightarrow R_2 = \frac{-v_I}{i_2} = \frac{-(-5)}{10^{-3}}$

$\Rightarrow \underline{R_2 = 5\ k\Omega}$

$v_L = i_L Z_L = (10^{-3})(500)$

$\Rightarrow v_L = 0.5\ V$

$i_4 = \frac{v_L}{R_2} = \frac{0.5}{5\ k\Omega} \Rightarrow i_4 = 0.1\ mA$

$i_3 = i_4 + i_L = 0.1 + 1 = 1.1\ mA$

If op-amp is biased at ± 10 V, output must be limited to ≈ 8 V.

So $v_O = i_3 R_3 + v_L$

$8 = (1.1 \times 10^{-3}) R_3 + 0.5$

$\Rightarrow R_3 = 6.82\ k\Omega$

Let $\underline{R_3 = 7.0\ k\Omega}$

Then we want

$\frac{R_3}{R_2} = \frac{R_F}{R_1} = \frac{7}{5} = 1.4$

Can choose $\underline{R_1 = 10\ k\Omega}$ and $\underline{R_F = 14\ k\Omega}$

E9.16

Refer to Fig. 9.24

$R_i = 2R_1 = 5\ k\Omega$

Let $\underline{R_1 = R_3 = 2.5\ k\Omega}$

Set $R_2 = R_4$

Differential Gain $= \frac{v_O}{v_i} = \frac{R_2}{R_1} = 100 = \frac{R_2}{2.5\ k\Omega}$

$\Rightarrow \underline{R_2 = R_4 = 250\ k\Omega}$

E9.17

We have the general relation that

$v_O = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{[R_4/R_3]}{1 + [R_4/R_3]}\right) v_{I2} - \frac{R_2}{R_1} v_{I1}$

$R_1 = R_3 = 10\ k\Omega,\ R_2 = 20\ k\Omega,\ R_4 = 21\ k\Omega$

$v_O = \left(1 + \frac{20}{10}\right)\left(\frac{[21/10]}{1 + [21/10]}\right) v_{I2} - \left(\frac{20}{10}\right) v_{I1}$

$v_O = 2.0323 v_{I2} - 2.0 v_I$

a. $v_{I1} = 1,\ v_{I2} = -1$

$v_O = -2.0323 - 2.0 \Rightarrow \underline{v_O = -4.032\ V}$

b. $v_{I1} = v_{I2} = 1$ V

$v_O = 2.0323 - 2.0 \Rightarrow \underline{v_O = 0.0323\ V}$

c. $v_{cm} = v_{I1} = v_{I2}$ so common-mode gain

$\underline{A_{cm} = \frac{v_O}{v_{cm}} = 0.0323}$

d. $CMRR_{dB} = 20 \log_{10}\left(\frac{A_d}{A_{cm}}\right)$

$A_d = \frac{2.0323}{2} - (2.0)\left(-\frac{1}{2}\right) = 2.016$

$CMRR_{dB} = 20 \log_{10}\left(\frac{2.016}{0.0323}\right) = 35.9\ dB$

E9.18

$$v_O = -\frac{R_4}{R_3}\left(1+\frac{2R_2}{R_1}\right)(v_{I1}-v_{I2})$$

Differential gain (magnitude) $= \frac{R_4}{R_3}\left(1+\frac{2R_2}{R_1}\right)$

Minimum Gain $\Rightarrow$ Maximum $R_1 = 1 + 50 = 51$ kΩ

So $A_d = \frac{20}{20}\left(1+\frac{2(100)}{51}\right) \Rightarrow A_d = 4.92$

Maximum Gain $\Rightarrow$ Minimum $R_1 = 1$ kΩ

$$A_d = \frac{20}{20}\left(1+\frac{2(100)}{1}\right) \Rightarrow A_d = 201$$

Range of Differential Gain = $\underline{4.92-201}$

E9.19

a. $i_1 = \frac{v_{I1}-v_{I2}}{R_1}$

$v_{O1} = v_{I1} + i_1 R_2'$, $v_{O2} = v_{I2} - i_1 R_2$ and

$$v_O = \frac{R_4}{R_3}(v_{O2}-v_{O1})$$

$$v_O = \frac{R_4}{R_3}[v_{I2} - i_1 R_2 - v_{I1} - i_1 R_2']$$

$$v_O = \frac{R_4}{R_3}[(v_{I2}-v_{I1}) - i_1(R_2+R_2')]$$

$$v_O = \frac{R_4}{R_3}\left[(v_{I2}-v_{I1}) - \left(\frac{v_{I2}-v_{I1}}{R_1}\right)(R_2+R_2')\right]$$

For common-mode input $v_{I2} = v_{I1}$

$\Rightarrow v_O = 0 \Rightarrow$ Common Gain = 0, $CMRR = \infty$

b. $A_d(\min) \Rightarrow R_2'$ min, R_1 max

$$A_d = \left(\frac{20}{20}\right)\left[1+\frac{100+95}{51}\right] = 4.82$$

$$A_d(\max) = \left(\frac{20}{20}\right)\left[1+\frac{100+105}{1}\right] = 206$$

c. $CMRR = \left|\frac{A_d}{A_{cm}}\right|$

$A_{cm} = 0 \Rightarrow \underline{CMRR = \infty}$

E9.20

Differential Gain $= \frac{R_4}{R_3}\left(1+\frac{2R_2}{R_1}\right)$

Let $R_3 = R_4$ so the difference amplifier gain is unity.

Minimum Gain $\Rightarrow$ Maximum R_1

So $\left(1+\frac{2R_2}{R_1(\max)}\right) = 2$

We want $2R_2 = R_1(\max)$

Maximum Gain $\Rightarrow$ Minimum R_1

So $\left(1+\frac{2R_2}{R_1(\min)}\right) = 1000$ or $2R_2 = 999R_1(\min)$

If $R_2 = 50$ kΩ, let $R_1(\min) = 100\ \Omega$ fixed resistor

and let $R_1(\max) = \underbrace{100\text{ k}\Omega}_{\text{pot}} + 100\ \Omega = 100.1$

Then actual differential gain is in the range of

$\underline{1.999 - 1001}$

E9.21

Time constant $= \tau = R_1C_2 = (10^4)(0.1\times10^{-6})$

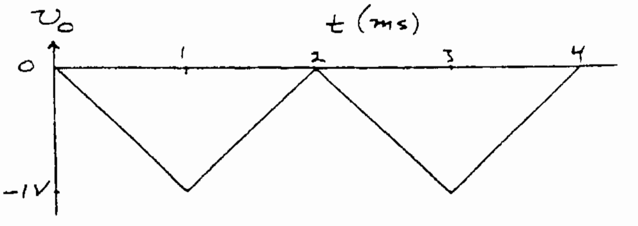
$= 1$ m sec

$0 \le t \le 1 \Rightarrow v_O = \frac{-1}{R_1C_2}\times t$

At $t = 1$ m sec $\Rightarrow v_O = -1$ V

$0 \le t \le 2 \Rightarrow v_O = -1 + \frac{1}{R_1C_2}\times(t-1)$

At $t = 2$ m sec $\Rightarrow v_O = -1 + \frac{(2-1)}{1} = 0$



E9.22

End of 1st pulse:

$$v_O = \frac{-1}{\tau}\times t\Big|_0^{10\ \mu\text{sec}} = \frac{-10\times10^{-6}}{\tau}$$

After 10 pulses:

$$v_O = -5 = \frac{-(10)(10\times10^{-6})}{\tau}$$

So $\tau = \frac{100\times10^{-6}}{5} = \underline{20\ \mu\text{sec} = \tau}$

$\tau = R_1C_2 = 20\ \mu$sec $= 20\times10^{-6}$

For example,

$\underline{C_2 = 0.01\times10^{-6} = 0.01\ \mu\text{F}} \Rightarrow \underline{R_1 = 2\text{ k}\Omega}$

E9.23

$$v_O = v_{I1} + 10v_{I2} - 25v_{I3} - 80v_{I4}$$

From Figure 9.37, v_{I3} input to R_1, v_{I4} input to R_2, v_{I1} input to R_A, and v_{I2} input to R_B.

From Equation (9.87)

$$\frac{R_F}{R_1} = 25 \text{ and } \frac{R_F}{R_2} = 80$$

Set $\underline{R_F = 500\text{ k}\Omega}$, then $\underline{R_1 = 20\text{ k}\Omega}$, and $\underline{R_2 = 6.25\text{ k}\Omega}$.

Also $\left(1 + \frac{R_F}{R_N}\right)\left(\frac{R_P}{R_A}\right) = 1$

and $\left(1 + \frac{R_F}{R_N}\right)\left(\frac{R_P}{R_B}\right) = 10$

where $R_N = R_1 \| R_2 = 20\|6.25 = 4.76\text{ k}\Omega$

and $R_P = R_A \| R_B \| R_C$

We find that $\frac{R_A}{R_B} = 10$

Let $\underline{R_A = 200\text{ k}\Omega}$, $\underline{R_B = 20\text{ k}\Omega}$

Now $\left(1 + \frac{500}{4.76}\right)\left(\frac{R_P}{R_A}\right) = 1 = (106)\left(\frac{R_P}{200}\right)$

Then $R_P = 1.89\text{ k}\Omega$

$R_A \| R_B = 200\|20 = 18.2\text{ k}\Omega$

So $R_P = 1.89 = \frac{18.2R_C}{18.2 + R_C} \Rightarrow \underline{R_C = 2.11\text{ k}\Omega}$

E9.25

$$v_{01} = \left[\frac{R - \Delta R}{(R - \Delta R) + (R + \Delta R)} - \frac{R + \Delta R}{(R + \Delta R) + (R - \Delta R)}\right]V^+$$

$$= \left[\frac{R - \Delta R}{2R} - \frac{R + \Delta R}{2R}\right]V^+$$

$$= \left(\frac{R - \Delta R - R - \Delta R}{2R}\right)V^+$$

$$v_{01} = -\left(\frac{\Delta R}{R}\right)V^+$$

For $V^+ = 3.5$ V, $\Delta R = 50$, $R = 10 \times 10^3$

$$v_{01} = -\left(\frac{50}{10^4}\right)(3.5) = -1.75 \times 10^{-2}$$

Need an amplifier with a gain of

$$A_d = \frac{v_O}{v_i} = \frac{5}{-1.75 \times 10^{-2}} \Rightarrow A_d = -285.7$$

Use instrumentation amplifier, Fig. 9-25.

Connect v_{01} to v_{I1} and $(-v_{01})$ to v_{I2}.

$$|A_d| = \left(\frac{R_4}{R_3}\right)\left(1 + \frac{2R_2}{R_1}\right) = 285.7$$

Let $\underline{R_4 = 150\text{ k}\Omega}$, $\underline{R_3 = 10\text{ k}\Omega}$ Then $\frac{R_2}{R_1} = 9.02$

Let $\underline{R_2 = 100\text{ k}\Omega}$, $\underline{R_1 = 11.1\text{ k}\Omega}$

E9.26

$$v_{01} = \left[\frac{1}{2} - \frac{R}{R + R(1+\delta)}\right]V^+$$

$$= \left[\frac{R + R(1+\delta) - 2R}{2(R + R(1+\delta))}\right]V^+$$

$$= \frac{R\delta}{2R(2+\delta)} \times V^+$$

$$v_{01} \approx \left(\frac{\delta}{4}\right)V^+$$

$V^+ = 5$ For $\delta = 0.01$

$$v_{01} = \left(\frac{0.01}{4}\right)(5) = 0.0125$$

Need a gain

$$A_d = \frac{v_O}{v_{01}} = \frac{5}{0.0125} = 400$$

Use an instrumentation amplifier

$$A_d = 400 = \left(\frac{R_4}{R_3}\right)\left(1 + \frac{2R_2}{R_1}\right)$$

Let $\underline{R_4 = 150\text{ k}\Omega}$, $\underline{R_3 = 10\text{ k}\Omega}$, then $\frac{R_2}{R_1} = 12.8$

Let $\underline{R_2 = 150\text{ k}\Omega}$, $\underline{R_1 = 11.7\text{ k}\Omega}$

Chapter 9

Problem Solutions

9.1

$A_v = -\frac{200}{20} = -10$

and

$R_i = 20\ k\Omega$

for each case

9.2

a. $A_v = -\frac{100}{10} = -10$

$R_i = R_1 = 10\ k\Omega$

b. $A_v = -\frac{100\|100}{10} = -5$

$R_i = R_1 = 10\ k\Omega$

c. $A_v = -\frac{100}{10\|10} = -20$

$R_i = 10\|10 = 5\ k\Omega$

9.3

$A_v = -\frac{R_2}{R_1} = -12 \Rightarrow R_2 = 12R_1$

$R_i = R_1 = 25\ k\Omega$

$\Rightarrow R_2 = (12)(25) = 300\ k\Omega$

9.4

$A_v = -\frac{R_2}{R_1} = -8 \Rightarrow R_2 = 8R_1$

For $v_I = -1$, $i_1 = \frac{1}{R_1} = 15\ \mu A \Rightarrow R_1 = 66.7\ k\Omega$

$\Rightarrow R_2 = 533.3\ k\Omega$

9.5

$A_v = -\frac{R_2}{R_1} = -30 \Rightarrow R_2 = 30R_1$

Set $R_2 = 1\ M\Omega$

$\Rightarrow R_1 = 33.3\ k\Omega$

9.6

a. $A_v = \frac{R_2}{R_1} \Rightarrow \frac{1.05R_2}{0.95R_1} = 1.105\left(\frac{R_2}{R_1}\right)$

$\frac{0.95R_2}{1.05R_1} = 0.905\left(\frac{R_2}{R_1}\right)$

Deviation in gain is +10.5% and −9.5%

b. $A_v \Rightarrow \frac{1.01R_2}{0.99R_1} = 1.02\left(\frac{R_2}{R_1}\right)$

$\Rightarrow \frac{0.99R_2}{1.01R_1} = 0.98\left(\frac{R_2}{R_1}\right)$

Deviation in gain = ±2%

9.7

(a) $A_v = \frac{v_O}{v_I} = \frac{-15}{1} = -15$

$v_O = -15v_I \Rightarrow v_O = -150\sin\omega t\ (mV)$

(b) $i_2 = i_1 = \frac{v_I}{R_1} = 10\sin\omega t\ (\mu A)$

$i_L = \frac{v_O}{R_L} \Rightarrow i_L = -37.5\sin\omega t\ (\mu A)$

$i_O = i_L - i_2$

$i_O = -47.5\sin\omega t\ (\mu A)$

9.8

$A_v = -\frac{R_2}{R_1 + R_S}$

$A_v = -30 \pm 2.5\% \Rightarrow 29.25 \le |A_v| \le 30.75$

So $\frac{R_2}{R_1 + 2} = 29.25$ and $\frac{R_2}{R_1 + 1} = 30.75$

We have $29.25(R_1 + 2) = 30.75(R_1 + 1)$

Which yields $R_1 = 18.5\ k\Omega$ and $R_2 = 599.6\ k\Omega$

For $v_I = 25\ mV$, then

$0.731 \le |v_O| \le 0.769\ V$

9.9

$v_{O1} = -\left(\frac{50}{10}\right)v_I = (-5)(0.15) \Rightarrow v_{O1} = -0.75\ V$

$v_O = -\frac{150}{25}\cdot v_{O1} = (-6)(-0.75) \Rightarrow v_O = 4.5\ V$

$i_1 = i_2 = \frac{0.15}{10} \Rightarrow i_1 = i_2 = 15\ \mu A$

$i_3 = i_4 = \frac{v_{O1}}{R_3} = -\frac{0.75}{25} \Rightarrow i_3 = i_4 = -30\ \mu A$

First op-amp must sink 15 + 30 = 45 μA

9.10

(a) $A_v = -\frac{R_2}{R_1} = -\frac{22}{1} \Rightarrow \underline{A_v = -22}$

(b) $A_v = -\frac{R_2}{R_1} \cdot \frac{1}{\left[1 + \frac{1}{A_{od}}\left(1 + \frac{R_2}{R_1}\right)\right]}$

$= -\frac{22}{1} \cdot \frac{1}{\left[1 + \frac{1}{1.5x10^5}\left(1 + \frac{22}{1}\right)\right]} = -\frac{22}{1} \cdot \frac{1}{1.000153} \Rightarrow$

$\underline{A_v = -21.99663}$

(c) $|A_v| = 22 - 1\% = 22 - 0.22 = 21.78$

Then $21.78 = \frac{22}{1} \cdot \frac{1}{\left[1 + \frac{1}{A_{od}}(23)\right]} \Rightarrow \underline{A_{od} = 2277}$

9.11

$A_v = -\frac{R_2}{R_1}\left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2}\right)$

a. $-10 = -\frac{R_2}{100}\left(1 + \frac{100}{100} + \frac{100}{R_2}\right)$

$10 = \frac{2R_2}{100} + 1 \Rightarrow \underline{R_2 = 450 \text{ k}\Omega}$

b. $100 = \frac{2R_2}{100} + 1 \Rightarrow \underline{R_2 = 4.95 \text{ M}\Omega}$

9.12

a. $A_v = -\frac{R_2}{R_1}\left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2}\right)$

$R_1 = 500 \text{ k}\Omega$

$80 = \frac{R_2}{500}\left(1 + \frac{R_3}{R_4} + \frac{R_3}{R_2}\right)$

Set $\underline{R_2 = R_3 = 500 \text{ k}\Omega}$

$80 = 1\left(1 + \frac{500}{R_4} + 1\right) = 2 + \frac{500}{R_4}$

$\Rightarrow \underline{R_4 = 6.41 \text{ k}\Omega}$

b. For $v_I = -0.05$ V

$i_1 = i_2 = \frac{-0.05}{500 \text{ k}\Omega} \Rightarrow \underline{i_1 = i_2 = -0.1\ \mu\text{A}}$

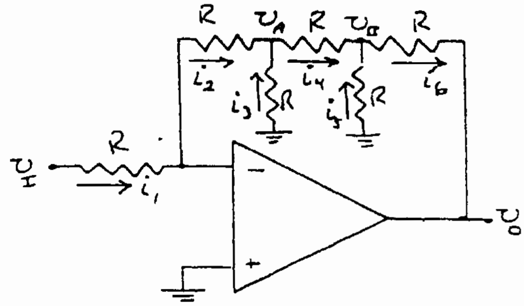
$v_X = -i_2 R_2 = -(-0.1 \times 10^{-6})(500 \times 10^3)$

$= 0.05$

$i_4 = -\frac{v_X}{R_4} = -\frac{0.05}{6.41} \Rightarrow \underline{i_4 = -7.80\ \mu\text{A}}$

$i_3 = i_2 + i_4 = -0.1 - 7.80 \Rightarrow \underline{i_3 = -7.90\ \mu\text{A}}$

9.13



$i_1 = \frac{v_I}{R} = i_2$

$v_A = -i_2 R = -\left(\frac{v_I}{R}\right)R = -v_I$

$i_3 = -\frac{v_A}{R} = \frac{v_I}{R}$

$i_4 = i_2 + i_3 = -\frac{v_A}{R} - \frac{v_A}{R} = -\frac{2v_A}{R} = \frac{2v_I}{R}$

$v_B = v_A - i_4 R = -v_I - \left(\frac{2v_I}{R}\right)(R) = -3v_I$

$i_5 = -\frac{v_B}{R} = -\frac{(-3v_I)}{R} = \frac{3v_I}{R}$

$i_6 = i_4 + i_5 = \frac{2v_I}{R} + \frac{3v_I}{R} = \frac{5v_I}{R}$

$v_O = v_B - i_6 R = -3v_I - \left(\frac{5v_I}{R}\right)R$

$\Rightarrow \underline{\frac{v_O}{v_I} = -8}$

From Figure 9.11 $\Rightarrow \underline{A_v = -3}$

9.14

(a) $A_v = -\frac{R_2}{R_1} \cdot \frac{1}{\left[1 + \frac{1}{A_{od}}\left(1 + \frac{R_2}{R_1}\right)\right]}$

$= -\frac{50}{10} \cdot \frac{1}{\left[1 + \frac{1}{2x10^5}\left(1 + \frac{50}{10}\right)\right]} \Rightarrow \underline{A_v = -4.99985}$

(b) $v_O = -(4.99985)(100x10^{-3}) \Rightarrow$

$\underline{v_O = -499.985\, mV}$

(c) Error $= \frac{0.5 - 0.499985}{0.5} x100\% \Rightarrow \underline{0.003\%}$

9.15

a. From Equation (9.23)

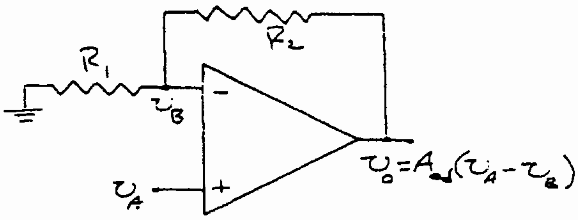
$$A_v = -\frac{R_2}{R_1} \cdot \frac{1}{\left[1 + \frac{1}{A_{od}}\left(1 + \frac{R_2}{R_1}\right)\right]}$$

$$= -\frac{100}{100} \cdot \frac{1}{\left[1 + \frac{1}{10^3}\left(1 + \frac{100}{100}\right)\right]} = -0.9980$$

Then $v_O = A_v \cdot v_I = (-0.9980)(2)$

$\Rightarrow \underline{v_O = -1.9960}$

b.



$v_O = A_{od}(v_A - v_B)$

$$\frac{v_B}{R_1} = \frac{v_O - v_B}{R_2} \Rightarrow v_B\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v_O}{R_2}$$

$$v_B = \frac{v_O}{\left(1 + \frac{R_2}{R_1}\right)}$$

Then $v_O = A_{od}v_A - \dfrac{A_{od}v_O}{\left(1 + \frac{R_2}{R_1}\right)}$

$$v_O\left[1 + \frac{A_{od}}{\left(1 + \frac{R_2}{R_1}\right)}\right] = A_{od}v_A$$

$$v_O\left[\frac{\left(1 + \frac{R_2}{R_1}\right) + A_{od}}{\left(1 + \frac{R_2}{R_1}\right)}\right] = A_{od}v_A$$

$$v_O = \frac{A_{od}\left(1 + \frac{R_2}{R_1}\right)v_A}{A_{od} + \left(1 + \frac{R_2}{R_1}\right)}$$

$$v_O = \frac{\left(1 + \frac{R_2}{R_1}\right)v_A}{1 + \frac{1}{A_{od}}\left(1 + \frac{R_2}{R_1}\right)}$$

So

$$v_O = \frac{\left(1 + \frac{10}{10}\right)\left(\frac{v_I}{2}\right)}{1 + \frac{1}{10^3}\left(1 + \frac{10}{10}\right)} = 0.9980v_I$$

For $v_I = 2$ V

$\underline{v_O = 1.9960 \text{ V}}$

9.16

(a) $i_1 = \frac{v_I}{R_1} = i_2 = -\frac{v_O}{R_2} \Rightarrow \frac{v_O}{v_I} = -\frac{R_2}{R_1}$

(b) $i_2 = i_1 = \frac{v_I}{R_1} = i_3 + \frac{v_O}{R_L} = i_3 + \frac{1}{R_L}\left(-\frac{R_2}{R_1}\cdot v_I\right)$

Then $\underline{i_3 = \frac{v_I}{R_1}\left(1 + \frac{R_2}{R_L}\right)}$

9.17

$$V_{X,\max} = \left(\frac{R_3\|R_1}{R_3\|R_1 + R_4}\right)\cdot V^+ = \left(\frac{0.1\|1}{0.1\|1 + 10}\right)(10) \Rightarrow$$

$V_{X,\max} = 0.09008\, V$

$$|v_O| = \frac{R_2}{R_1}\cdot V_{X,\max}$$

$$10 = \frac{R_2}{R_1}(0.09008) \Rightarrow \frac{R_2}{R_1} = 111$$

So $\underline{R_2 = 111\, k\Omega}$

9.18

$v_O = A_{OL}(v_2 - v_1)$

a. $A_{OL} = \dfrac{1}{[1-(-1)] \times 10^{-3}} \Rightarrow \underline{A_{OL} = 500}$

b. $1 = 500(v_2 - 1 \times 10^{-3}) \Rightarrow \underline{v_2 = 3 \text{ mV}}$

c. $5 = 500(1 - v_1) \Rightarrow \underline{v_1 = 0.99 \text{ V}}$

d. $v_O = 500(-1 - (-1)) \Rightarrow \underline{v_O = 0}$

e. $-3 = 500(v_2 - (-0.5)) \Rightarrow \underline{v_2 = -0.506 \text{ V}}$

9.19

a. $v_O = -\dfrac{R_F}{R_1}\cdot v_{I1} - \dfrac{R_F}{R_2}\cdot v_{I2} - \dfrac{R_F}{R_3}\cdot v_{I3}$

$$v_O = -\frac{80}{20}(0.5) - \frac{80}{40}(-1) - \frac{80}{60}(2)$$

$= -4(0.5) - 2(-1) - 1.33(2)$

$\underline{v_O = -2.667 \text{ V}}$

b. $-5.2 = -4(1) - 2(0.25) - 1.33v_{I3}$

$\underline{v_{I3} = 0.525}$

9.20

$$v_O = -8v_{I1} - 2v_{I2} - 5v_{I3}$$
$$= -\frac{R_F}{R_1}v_{I1} - \frac{R_F}{R_2}v_{I2} - \frac{R_F}{R_3}v_{I3}$$
$$\frac{R_F}{R_1} = 8 \quad \frac{R_F}{R_2} = 2 \quad \frac{R_F}{R_3} = 5$$

Let $R_F = 500\ \text{k}\Omega \Rightarrow R_1 = 62.5\ \text{k}\Omega$
$R_2 = 250\ \text{k}\Omega$
$R_3 = 100\ \text{k}\Omega$

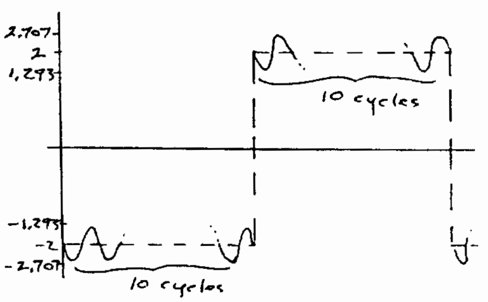
9.21

$$v_O = -4v_{I1} - 0.5v_{I2} = -\frac{R_F}{R_1}v_{I1} - \frac{R_F}{R_2}v_{I2}$$
$$\frac{R_F}{R_1} = 4 \quad \frac{R_F}{R_2} = 0.5$$
$\Rightarrow R_1$ is the smallest resistor

$$|i| = 100\ \mu\text{A} = \frac{v_I}{R_1} = \frac{2}{R_1} \Rightarrow \underline{R_1 = 20\ \text{k}\Omega}$$
$$\Rightarrow \underline{R_F = 80\ \text{k}\Omega}$$
$$\Rightarrow \underline{R_2 = 160\ \text{k}\Omega}$$

9.22

$$v_{I1} = (0.05)\sqrt{2}\sin(2\pi f t) = 0.0707\sin(2\pi f t)$$
$$f = 1\ kHz \Rightarrow T = \frac{1}{10^3} \Rightarrow 1\ ms$$
$$v_{I2} \Rightarrow T_2 = \frac{1}{100} \Rightarrow 10\ ms$$
$$v_O = -\frac{R_F}{R_1}\cdot v_{I1} - \frac{R_F}{R_2}\cdot v_{I2} = -\frac{10}{1}\cdot v_{I1} - \frac{10}{5}\cdot v_{I2}$$
$$v_O = -(10)(0.0707\sin(2\pi f t)) - (2)(\pm 1\ V)$$
$$v_O = -0.707\sin(2\pi f t) - (\pm 2\ V)$$



9.23

$$v_O = -\frac{R_F}{R_1}\cdot v_{I1} - \frac{R_F}{R_2}\cdot v_{I2} - \frac{R_F}{R_3}\cdot v_{I3}$$
$$v_O = -\frac{20}{10}\cdot v_{I1} - \frac{20}{5}\cdot v_{I2} - \frac{20}{2}\cdot v_{I3}$$
$$K\sin\omega t = -2v_{I1} - 4[2 + 100\sin\omega t] - 0$$
Set $\underline{v_{I1} = -4}$

9.24

a.

$$v_O = -\frac{R_F}{R_3}\cdot a_3(-5) - \frac{R_F}{R_2}\cdot a_2(-5) - \frac{R_F}{R_1}\cdot a_1(-5) - \frac{R_F}{R_0}\cdot a_0(-5)$$

So $v_O = \frac{R_F}{10}\left[\frac{a_3}{2} + \frac{a_2}{4} + \frac{a_1}{8} + \frac{a_0}{16}\right](5)$

b. $v_O = 2.5 = \frac{R_F}{10}\cdot\frac{1}{2}\cdot 5 \Rightarrow \underline{R_F = 10\ \text{k}\Omega}$

c. i. $v_O = \frac{10}{10}\cdot\frac{1}{16}\cdot 5 \Rightarrow \underline{v_O = 0.3125\ \text{V}}$

ii. $v_O = \frac{10}{10}\left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right](5)$

$\Rightarrow \underline{v_O = 4.6875\ \text{V}}$

9.25

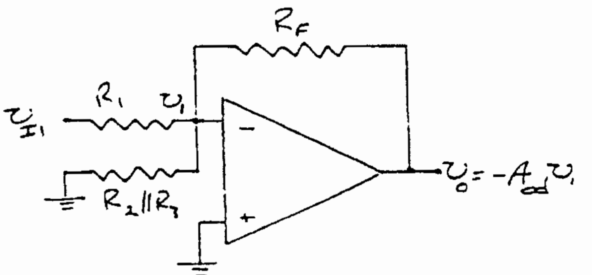
(a) $v_{O1} = -\frac{10}{1}\cdot v_{I1}$

$$v_O = -\frac{20}{1}\cdot v_{O1} - \frac{20}{1}\cdot v_{I2} = -(20)(-10)v_{I1} - (20)v_{I2}$$
$$\underline{v_O = 200v_{I1} - 20v_{I2}}$$

(b) $v_{I1} = 1 + 2\sin\omega t\ (mV)$
$v_{I2} = -10\ mV$
Then $v_O = 200(1 + 2\sin\omega t) - 20(-10)$
So $\underline{v_O = 0.4 + 0.4\sin\omega t\ (V)}$

9.26

For one-input



$$v_1 = -\frac{v_O}{A_{od}}$$
$$\frac{v_{I1} - v_1}{R_1} = \frac{v_1}{R_2\|R_3} + \frac{v_1 - v_O}{R_F}$$
$$\frac{v_{I1}}{R_1} = v_1\left[\frac{1}{R_1} + \frac{1}{R_2\|R_3} + \frac{1}{R_F}\right] - \frac{v_O}{R_F}$$
$$= -\frac{v_O}{A_{od}}\left[\frac{1}{R_1} + \frac{1}{R_2\|R_3} + \frac{1}{R_F}\right] - \frac{v_O}{R_F}$$
$$= -v_O\left\{\frac{1}{A_{od}R_F} + \frac{1}{R_F} + \frac{1}{A_{od}}\left(\frac{1}{R_1} + \frac{1}{R_2\|R_3}\right)\right\}$$
$$= -\frac{v_O}{R_F}\left\{\frac{1}{A_{od}} + 1 + \frac{1}{A_{od}}\cdot\frac{R_F}{R_1\|R_2\|R_3}\right\}$$

$$v_O = -\frac{R_F}{R_1} \cdot v_{I1} \cdot \left\{ \frac{1}{1 + \frac{1}{A_{od}}\left(1 + \frac{R_F}{R_P}\right)} \right\}$$

where $R_P = R_1 \| R_2 \| R_3$

Therefore, for three-inputs

$$v_O = \frac{-1}{1 + \frac{1}{A_{od}}\left(1 + \frac{R_F}{R_P}\right)} \times \left(\frac{R_F}{R_1} \cdot v_{I1} + \frac{R_F}{R_2} \cdot v_{I2} + \frac{R_F}{R_3} \cdot v_{I3}\right)$$

9.27

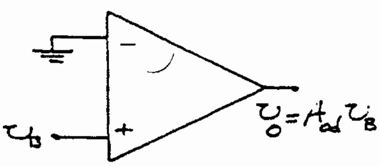
$$A_v = \left(1 + \frac{R_2}{R_1}\right) = 10$$

$$\frac{R_2}{R_1} = 9$$

$$|i| = \frac{v_I}{R_1} = \frac{0.8}{R_1} = 100\ \mu\text{A}$$

$$\Rightarrow \underline{R_1 = 8\ \text{k}\Omega},\ \underline{R_2 = 72\ \text{k}\Omega}$$

9.28



$$v_B = \left(\frac{1}{1 + 500}\right) v_I, \quad v_O = A_{od}\left(\frac{1}{501}\right) v_I$$

a. $2.5 = A_{od}\left(\frac{1}{501}\right)(5) \Rightarrow \underline{A_{od} = 250.5}$

b. $v_O = 5000\left(\frac{1}{501}\right)(5) \Rightarrow \underline{v_O = 49.9\ \text{V}}$

9.29

$$v_O = \left(1 + \frac{50}{50}\right)\left[\left(\frac{20}{20 + 40}\right) v_{I2} + \left(\frac{40}{20 + 40}\right) v_{I1}\right]$$

$$\underline{v_O = 1.33 v_{I1} + 0.667 v_{I2}}$$

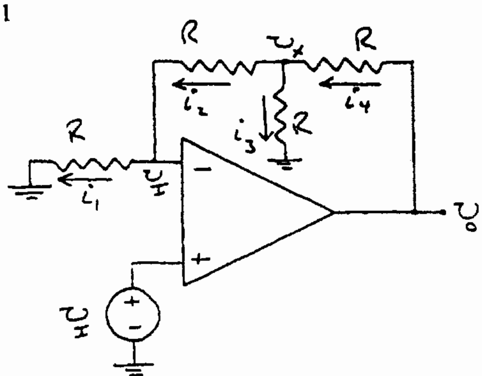
9.30

$$v_O = \left(1 + \frac{100}{50}\right) \times \left[\left(\frac{10\|40}{10\|40 + 20}\right) v_{I1} + \left(\frac{10\|20}{10\|20 + 40}\right) v_{I2}\right]$$

$$v_O = 3\left[\left(\frac{8}{8 + 20}\right) v_{I1} + \left(\frac{6.67}{6.67 + 40}\right) v_{I2}\right]$$

$$\underline{v_O = 0.857 v_{I1} + 0.429 v_{I2}}$$

9.31



$$i_1 = \frac{v_I}{R} = i_2$$

$$v_X = i_2 R + v_I = \left(\frac{v_I}{R}\right) R + v_I = 2v_I$$

$$i_3 = \frac{v_X}{R} = \frac{2v_I}{R}$$

$$i_4 = i_2 + i_3 = \frac{v_I}{R} + \frac{2v_I}{R} = \frac{3v_I}{R}$$

$$v_O = i_4 R + v_X = \left(\frac{3v_I}{R}\right) R + 2v_I$$

$$\underline{\frac{v_O}{v_I} = 5}$$

9.32

(a) $\frac{v_O}{v_I} = 1$

(b) From Exercise 9.11

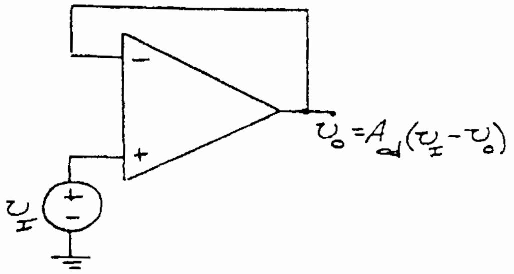
$$\frac{v_O}{v_I} = \frac{\left(1 + \frac{R_2}{R_1}\right)}{\left[1 + \frac{1}{A_{od}}\left(1 + \frac{R_2}{R_1}\right)\right]}$$

But $R_2 = 0, R_1 = \infty$

$$\frac{v_O}{v_I} = \frac{1}{1 + \frac{1}{A_{od}}} = \frac{1}{1 + \frac{1}{1.5\text{x}10^5}} \Rightarrow \underline{\frac{v_O}{v_I} = 0.99999}$$

(b) Want $\frac{v_O}{v_I} = 0.990 = \frac{1}{1 + \frac{1}{A_{od}}} \Rightarrow \underline{A_{od} = 99}$

9.33



$$v_O = A_{od}(v_I - v_O)$$

$$\left(\frac{1}{A_{od}} + 1\right)v_O = v_I$$

$$v_O = \frac{v_I}{\left(1 + \frac{1}{A_{od}}\right)}$$

$A_{od} = 10^4$; $\frac{v_O}{v_I} = 0.99990$

$A_{od} = 10^3$; $\frac{v_O}{v_I} = 0.9990$

$A_{od} = 10^2$; $\frac{v_O}{v_I} = 0.990$

$A_{od} = 10$; $\frac{v_O}{v_I} = 0.909$

9.34

$$v_{OA} = \left(1 + \frac{R_2}{R_1}\right)v_I$$

$v_{O1} = \left(1 + \frac{R_2}{R_1}\right)v_I$, $v_{O2} = -\left(1 + \frac{R_2}{R_1}\right)v_I$

So $v_{O1} = -v_{O2}$

9.35

(a) $i_L = \frac{v_I}{R_1}$

(b) $v_{O1} = i_L R_L + v_I = i_L R_L + i_L R_1$

$v_{O1}(\max) \cong 8\ V = i_L(1+9) = 10 i_L$

So $i_L(\max) \cong 0.8\ mA$

Then $v_I(\max) \cong i_L R_1 = (0.8)(9) \Rightarrow$ $v_I(\max) \cong 7.2\ V$

9.36

(a) $v_X = \left(\frac{20}{20+40}\right)\cdot v_I = \left(\frac{20}{60}\right)(6) = 2$

$v_O = 2\ V$

(b) Same as (a)

(c) $v_X = \left(\frac{6}{6+48}\right)(6) = 0.666\ V$

$v_O = \left(1 + \frac{10}{10}\right)\cdot v_X \Rightarrow$ $v_O = 1.33\ V$

9.37

a. $R_{in} = \frac{v_1}{i_1}$ and $\frac{v_1 - v_O}{R_F} = i_1$ and $v_O = -A_{od}v_1$

So $i_1 = \frac{v_1 - (-A_{od}v_1)}{R_F} = \frac{v_1(1 + A_{od})}{R_F}$

Then $R_{in} = \frac{v_1}{i_1} = \frac{R_F}{1 + A_{od}}$

b. $i_1 = \left(\frac{R_S}{R_S + R_{in}}\right)i_S$ and $v_O = -A_{od}\cdot\frac{R_F}{1 + A_{od}}\cdot i_1$

So $v_O = -R_F\left(\frac{A_{od}}{1 + A_{od}}\right)\left(\frac{R_S}{R_S + R_{in}}\right)i_S$

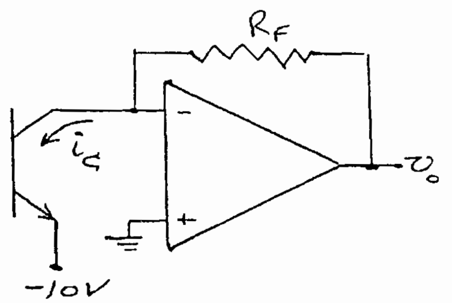
$$R_{in} = \frac{R_F}{1 + A_{od}} = \frac{10}{1001} = 0.009990$$

$$v_O = -R_F\left(\frac{1000}{1001}\right)\left(\frac{R_S}{R_S + 0.009990}\right)i_S$$

Want $\left(\frac{1000}{1001}\right)\left(\frac{R_S}{R_S + 0.009990}\right) \geq 0.990$

which yields $R_S \geq 1.099\ k\Omega$

9.38



$v_O = i_C R_F$, $0 \leq i_C \leq 8\ mA$

For $v_O(\max) = 8\ V$, Then

$R_F = 1\ k\Omega$

9.39

$i = \frac{v_I}{R}$ so $1 = \frac{10}{R} \Rightarrow R = 10\ k\Omega$

In the ideal op-amp, R_1 has no influence.

Output voltage : $v_O = \left(1 + \frac{R_2}{R}\right)v_I$

v_O must remain within the bias voltages of the op-amp; the larger the R_2, the smaller the range of input voltage v_I in which the output is valid.

9.40

$$i_L = \frac{-v_I}{R_2} \Rightarrow 10 = -\frac{(-10)}{R_2} \Rightarrow \underline{R_2 = 1\ \text{k}\Omega}$$

$$i_4 = \frac{v_L}{R_2} \text{ and } v_L = i_L Z_L = (0.010)(100) = 1\ \text{V}$$

$$i_4 = \frac{1}{1\ \text{k}\Omega} = 1\ \text{mA}$$

$$i_3 = i_4 + i_L = 1 + 10 = 11\ \text{mA}$$

For $v_O(\max) \approx 12\ \text{V} = i_3 R_3 + v_L = (11)R_3 + 1$

$\Rightarrow \underline{R_3 = 1\ \text{k}\Omega}$

We need

$$\frac{R_3}{R_2} = \frac{R_F}{R_1} = \frac{1\ \text{k}\Omega}{1\ \text{k}\Omega} \Rightarrow \underline{R_1 = R_F}$$

9.41

(a) $i_1 = i_2$ and $i_2 = \frac{v_X}{R_2} + i_D$, $v_X = -i_2 R_F$

Then $i_1 = -i_1\left(\frac{R_F}{R_2}\right) + i_D$

Or $\underline{i_D = i_1\left(1 + \frac{R_F}{R_2}\right)}$

(b) $R_1 = \frac{v_I}{i_1} = \frac{5}{1} \Rightarrow \underline{R_1 = 5\,k\Omega}$

$$12 = (1)\left(1 + \frac{R_F}{R_2}\right) \Rightarrow \underline{\frac{R_F}{R_2} = 11}$$

For example, $\underline{R_2 = 5\,k\Omega,\ R_F = 55\,k\Omega}$

9.42

(1) $I_X = \frac{V_X}{R_2} + \frac{V_X - v_O}{R_3}$

(2) $\frac{V_X}{R_1} + \frac{V_X - v_O}{R_F} = 0$

From (2) $v_O = V_X\left(1 + \frac{R_F}{R_1}\right)$

Then (1) $I_X = V_X\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{1}{R_3}\cdot V_X\left(1 + \frac{R_F}{R_1}\right)$

$$\frac{I_X}{V_X} = \frac{1}{R_o} = \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_3} - \frac{R_F}{R_1 R_3} = \frac{1}{R_2} - \frac{R_F}{R_1 R_3}$$

$$= \frac{R_1 R_3 - R_2 R_F}{R_1 R_2 R_3}$$

or

$$\underline{R_o = \frac{R_1 R_2 R_3}{R_1 R_3 - R_2 R_F}}$$

Note: If $\frac{R_F}{R_1 R_3} = \frac{1}{R_2} \Rightarrow R_2 R_F = R_1 R_3$

then $R_o = \infty$, which corresponds to an ideal current source.

9.43

$$A_d = \frac{R_2}{R_1} = \frac{R_4}{R_3} = 5$$

Minimum resistance seen by v_{I1} is R_1.

Set $\underline{R_1 = R_3 = 25\ \text{k}\Omega}$ Then $\underline{R_2 = R_4 = 125\ \text{k}\Omega}$

$$i_L = \frac{v_O}{R_L} \Rightarrow v_O = i_L R_L = (0.5)(5) = 2.5\ \text{V}$$

$$v_O = 5(v_{I2} - v_{I1})$$

$$2.5 = 5(v_{I2} - 2) \Rightarrow \underline{v_{I2} = 2.5\ \text{V}}$$

9.44

a. From superposition:

$$v_{O1} = -\frac{R_2}{R_1}\cdot v_{I1}$$

$$v_{O2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)v_{I2}$$

Setting $v_{I1} = v_{I2} = v_{cm}$

$$v_O = v_{O1} + v_{O2} = \left[\left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{1 + \frac{R_3}{R_4}}\right) - \frac{R_2}{R_1}\right]v_{cm}$$

$$A_{cm} = \frac{v_O}{v_{cm}} = \frac{R_4}{R_3}\cdot\left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{1 + \frac{R_3}{R_4}}\right) - \frac{R_2}{R_1}$$

$$= \frac{\frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) - \frac{R_2}{R_1}\left(1 + \frac{R_4}{R_3}\right)}{\left(1 + \frac{R_4}{R_3}\right)}$$

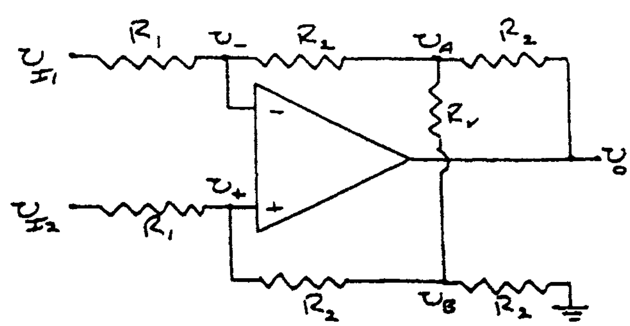
$$\underline{A_{cm} = \frac{\frac{R_4}{R_3} - \frac{R_2}{R_1}}{\left(1 + \frac{R_4}{R_3}\right)}}$$

b. Max. $|A_{cm}| \Rightarrow$ Min. $\frac{R_4}{R_3}$ and Max. $\frac{R_2}{R_1}$

$$\text{Max. } |A_{cm}| = \frac{\frac{47.5}{10.5} - \frac{52.5}{9.5}}{1 + \frac{47.5}{10.5}} = \frac{4.5238 - 5.5263}{1 + 4.5238}$$

$\Rightarrow \underline{|A_{cm}|_{\max} = 0.1815}$

9.45



$$\frac{v_{I1}-v_A}{R_1+R_2}=\frac{v_A-v_B}{R_V}+\frac{v_A-v_O}{R_2} \quad (1)$$

$$\frac{v_{I2}-v_B}{R_1+R_2}=\frac{v_B-v_A}{R_V}+\frac{v_B}{R_2} \quad (2)$$

$$v_-=\left(\frac{R_1}{R_1+R_2}\right)v_A+\left(\frac{R_2}{R_1+R_2}\right)v_{I1} \quad (3)$$

$$v_+=\left(\frac{R_1}{R_1+R_2}\right)v_B+\left(\frac{R_2}{R_1+R_2}\right)v_{I2} \quad (4)$$

Now $v_-=v_+ \Rightarrow R_1v_A+R_2v_{I1}=R_1v_B+R_2v_{I2}$

So that $v_A=v_B+\frac{R_2}{R_1}(v_{I2}-v_{I1})$

$$\frac{v_{I1}}{R_1+R_2}=v_A\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)-\frac{v_B}{R_V}-\frac{v_O}{R_2} \quad (1)$$

$$\frac{v_{I2}}{R_1+R_2}=v_B\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)-\frac{v_A}{R_V} \quad (2)$$

Then

$$\frac{v_{I1}}{R_1+R_2} \quad (1)$$
$$=v_B\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)-\frac{v_B}{R_V}-\frac{v_O}{R_2}+\left(\frac{R_2}{R_1}\right)\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)(v_{I2}-v_{I1})$$

$$\frac{v_{I2}}{R_1+R_2}=v_B\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)-\frac{1}{R_V}\left[v_B+\frac{R_2}{R_1}(v_{I2}-v_{I1})\right] \quad (2)$$

Substitute (1)-(2)

$$\frac{1}{R_1+R_2}(v_{I1}-v_{I2})=\left(\frac{R_2}{R_1}\right)\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)(v_{I2}-v_{I1})-\frac{v_O}{R_2}+\frac{1}{R_V}\cdot\frac{R_2}{R_1}(v_{I2}-v_{I1})$$

$$\frac{v_O}{R_2}=(v_{I2}-v_{I1})\left\{\left(\frac{R_2}{R_1}\right)\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)+\frac{1}{R_1+R_2}+\frac{1}{R_V}\cdot\frac{R_2}{R_1}\right\}$$

$$v_O=(v_{I2}-v_{I1})\left(\frac{R_2}{R_1}\right)\left\{\frac{R_2}{R_1+R_2}+\frac{R_2}{R_V}+1+\frac{R_1}{R_1+R_2}+\frac{R_2}{R_V}\right\}$$

$$\underline{v_O=\frac{2R_2}{R_1}\left(1+\frac{R_2}{R_V}\right)(v_{I2}-v_{I1})}$$

9.46

$$v_{O1}=\left(1+\frac{R_2}{R_1}\right)v_{I1}-\frac{R_2}{R_1}\cdot v_{I2}$$
$$=\left(1+\frac{50}{10}\right)(-25\sin\omega t)-\frac{50}{10}(25\sin\omega t)$$

$\underline{v_{O1}=-275\sin\omega t\text{ mV}}$

$$v_{O2}=\left(1+\frac{R_2}{R_1}\right)v_{I2}-\frac{R_2}{R_1}\cdot v_{I1}$$
$$=\left(1+\frac{50}{10}\right)(25\sin\omega t)-\frac{50}{10}(-25\sin\omega t)$$

$\underline{v_{O2}=275\sin\omega t\text{ mV}}$

$$v_O=\frac{R_4}{R_3}\left(1+\frac{2R_2}{R_1}\right)(v_{I2}-v_{I1})$$
$$=\frac{30}{20}\left(1+2\left[\frac{50}{10}\right]\right)(25-[-25])\sin\omega t$$

$\underline{v_O=825\sin\omega t\text{ mV}}$

Current in R_1 and R_2:

$$i_1=\frac{v_{I1}-v_{I2}}{R_1}=\frac{(-25-25)\sin\omega t\text{ mV}}{10\text{ k}\Omega}$$

$\underline{|i_1|=5\sin\omega t\ \mu\text{A}}$

Current in bottom R_3 and R_4:

$$i_3=\frac{v_{O2}}{R_3+R_4}=\frac{275\sin\omega t\text{ mV}}{(20+30)\text{ k}\Omega}$$

$\underline{|i_3|=5.5\sin\omega t\ \mu\text{A}}$

Current in top R_3 and R_4:

$$i_3'=\frac{v_{O1}-\left(\frac{R_4}{R_3+R_4}\right)v_{O2}}{R_3}$$
$$=\frac{\left[-275-\left(\frac{30}{30+20}\right)(275)\right]\sin\omega t\text{ mV}}{20\text{ k}\Omega}$$

$\underline{|i_3'|=22\sin\omega t\ \mu\text{A}}$

9.47

$$v_O=\frac{30}{20}\left(1+\frac{2(50)}{R_1}\right)(25-(-25))\sin\omega t\text{ mV}$$

$$|v_O|=(1.5)(50)\left(1+\frac{100}{R_1}\right)\text{ mV}$$

For

$$|v_O|=0.1\text{ V}=(1.5)(0.050)\left(1+\frac{100}{R_1}\right)$$

$\Rightarrow \underline{R_1=300\text{ k}\Omega}$

For

$|v_O| = 5 \text{ V} = (1.5)(0.050)\left(1 + \frac{100}{R_1}\right)$

$\Rightarrow \underline{R_1 = 1.52 \text{ k}\Omega}$

So $\underline{R_{1f} = 1.52 \text{ k}\Omega} \Rightarrow$ $\underline{\text{Potentiometer} \approx 300 \text{ k}\Omega}$

9.48

$$A_d = \frac{R_4}{R_3}\left(1 + \frac{2R_2}{R_1}\right)$$

For $A_d(\max)$, $R_1 = R_1(\min) = R_{1f}$

$$200 = \frac{R_4}{R_3}\left(1 + \frac{2R_2}{R_{1f}}\right)$$

For $A_d(\min)$, $R_1 = R_1(\max) \approx 50 \text{ k}\Omega$

$$0.5 = \frac{R_4}{R_3}\left(1 + \frac{2R_2}{50}\right)$$

Let $\underline{\frac{R_4}{R_3} = 0.4}$

Then $0.5 = 0.4\left(1 + \frac{2R_2}{50}\right) \Rightarrow \underline{R_2 = 6.25 \text{ k}\Omega}$

and

$200 = (0.4)\left(1 + \frac{2(6.25)}{R_{1f}}\right) \Rightarrow \underline{R_{1f} = 25.05 \ \Omega}$

9.49

For a common-mode gain, $v_{cm} = v_{I1} = v_{I2}$
Then

$$v_{01} = \left(1 + \frac{R_2}{R_1}\right)v_{cm} - \frac{R_2}{R_1}v_{cm} = v_{cm}$$

$$v_{02} = \left(1 + \frac{R_2}{R_1}\right)v_{cm} - \frac{R_2}{R_1}v_{cm} = v_{cm}$$

From Problem 9.41, we can write

$$A_{cm} = \frac{\frac{R_4}{R_3} - \frac{R_4}{R_3'}}{\left(1 + \frac{R_4}{R_3}\right)}$$

$R_3 = R_4 = 20 \text{ k}\Omega$, $R_3' = 20 \text{ k}\Omega \pm 5\%$

$$A_{cm} = \frac{1 - \frac{20}{R_3'}}{(1+1)} = \frac{1}{2}\left(1 - \frac{20}{R_3'}\right)$$

For $R_3' = 20 \text{ k}\Omega - 5\% = 19 \text{ k}\Omega$

$$A_{cm} = \frac{1}{2}\left(1 - \frac{20}{19}\right) = -0.0263$$

For $R_3' = 20 \text{ k}\Omega + 5\% = 21 \text{ k}\Omega$

$$A_{cm} = \frac{1}{2}\left(1 - \frac{20}{21}\right) = 0.0238$$

So $\underline{|A_{cm}|_{\max} = 0.0263}$

9.50

a. $v_O = \frac{1}{R_1C_2} \cdot \int v_I(t')\,dt'$

$\int 0.5 \sin \omega t \, dt = -\frac{0.5}{\omega} \cos \omega t$

$|v_O| = 0.5 = \frac{1}{R_1C_2} \cdot \frac{(0.5)}{\omega} = \frac{0.5}{2\pi R_1C_2 f}$

$$f = \frac{1}{2\pi R_1C_2} = \frac{1}{2\pi(50 \times 10^3)(0.1 \times 10^{-6})}$$

$\Rightarrow \underline{f = 31.8 \text{ Hz}}$

Output signal lags input signal by 90°

b. i. $f = \frac{0.5}{2\pi(50 \times 10^3)(0.1 \times 10^{-6})} \Rightarrow \underline{f = 15.9 \text{ Hz}}$

ii. $f = \frac{0.5}{(0.1)(2\pi)(50 \times 10^3)(0.1 \times 10^{-6})}$

$\Rightarrow \underline{f = 159 \text{ Hz}}$

9.51

(a) $v_O = -\frac{1}{RC}\int v_I(t')dt'$

$v_O = -\frac{1}{0.2}(0.5)(2) \Rightarrow \underline{v_O = -5 V}$

(b) $-15 = -\frac{1}{0.2}(0.5)t \Rightarrow \underline{t = 6 s}$

9.52

a. $$\frac{v_O}{v_I} = \frac{-R_2 \| \frac{1}{j\omega C_2}}{R_1} = -\frac{R_2 \cdot \frac{1}{j\omega C_2}}{R_1\left(R_2 + \frac{1}{j\omega C_2}\right)}$$

$$\underline{\frac{v_O}{v_I} = -\frac{R_2}{R_1} \cdot \frac{1}{1 + j\omega R_2C_2}}$$

b. $\underline{\frac{v_O}{v_I} = -\frac{R_2}{R_1}}$

c. $\underline{f = \frac{1}{2\pi R_2C_2}}$

9.53

a. $$\frac{v_O}{v_I} = \frac{-R_2}{R_1 + \frac{1}{j\omega C_1}} = -\frac{R_2(j\omega C_1)}{1 + j\omega R_1C_1}$$

$$\underline{\frac{v_O}{v_I} = -\frac{R_2}{R_1} \cdot \frac{j\omega R_1C_1}{1 + j\omega R_1C_1}}$$

b. $\underline{\frac{v_O}{v_I} = -\frac{R_2}{R_1}}$

c. $\underline{f = \frac{1}{2\pi R_1C_1}}$

9.54

Assuming the Zener diode is in breakdown,

$$v_O = -\frac{R_2}{R_1}\cdot V_Z = -\frac{1}{1}(6.8) \Rightarrow \quad v_O = -6.8\,V$$

$$i_2 = \frac{0 - v_O}{R_2} = \frac{0-(-6.8)}{1} \Rightarrow \quad i_2 = 6.8\,mA$$

$$i_Z = \frac{10 - V_Z}{R_S} - i_2 = \frac{10-6.8}{5.6} - 6.8 \Rightarrow \quad i_Z = -6.2\,mA!!!$$

Circuit is not in breakdown. Now

$$\frac{10-0}{R_S + R_1} = i_2 = \frac{10}{5.6+1} \Rightarrow \quad \underline{i_2 = 1.52\,mA}$$

$$v_O = -i_2 R_2 = -(1.52)(1) \Rightarrow \quad \underline{v_O = -1.52\,V}$$

$$\underline{i_Z = 0}$$

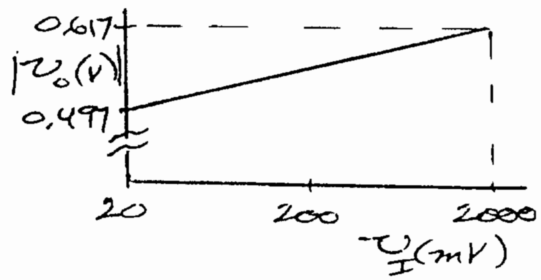
9.55

$$v_O = -V_T \ln\left(\frac{v_I}{I_S R_1}\right) = -(0.026)\ln\left[\frac{v_I}{(10^{-14})(10^4)}\right] \Rightarrow$$

$$v_O = -0.026\ln\left(\frac{v_I}{10^{-10}}\right)$$

For $v_I = 20\,mV$, $|v_O| = 0.497\,V$

For $v_I = 2\,V$, $|v_O| = 0.617\,V$



9.56

$$v_O = \left(\frac{333}{20}\right)(v_{O1} - v_{O2}) = 16.65(v_{O1} - v_{O2})$$

$$v_{O1} = -v_{BE1} = -V_T \ln\left(\frac{i_{C1}}{I_S}\right)$$

$$v_{O2} = -v_{BE2} = -V_T \ln\left(\frac{i_{C2}}{I_S}\right)$$

$$v_{O1} - v_{O2} = -V_T \ln\left(\frac{i_{C1}}{i_{C2}}\right) = V_T \ln\left(\frac{i_{C2}}{i_{C1}}\right)$$

$$i_{C2} = \frac{v_2}{R_2}, \quad i_{C1} = \frac{v_1}{R_1}$$

So $v_{O1} - v_{O2} = V_T \ln\left(\frac{v_2}{R_2}\cdot\frac{R_1}{v_1}\right)$

Then

$$v_O = (16.65)(0.026)\ln\left(\frac{v_2}{v_1}\cdot\frac{R_1}{R_2}\right)$$

$$v_O = 0.4329\ln\left(\frac{v_2}{v_1}\cdot\frac{R_1}{R_2}\right)$$

$$\ln(x) = \log_e(x) = [\log_{10}(x)]\cdot[\log_e(10)]$$

$$= 2.3026\log_{10}(x)$$

Then

$$\underline{v_O \cong (1.0)\log_{10}\left(\frac{v_2}{v_1}\cdot\frac{R_1}{R_2}\right)}$$

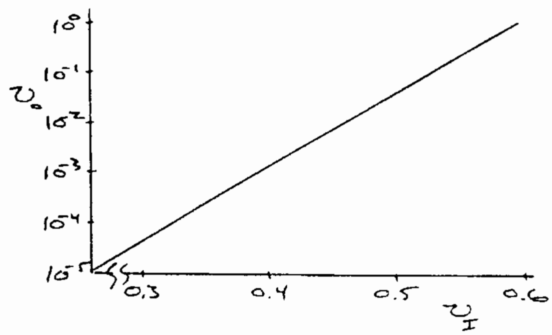
9.57

$$v_O = -I_S R\left(e^{v_I/V_T}\right) = -(10^{-14})(10^4)e^{v_I/V_T}$$

$$|v_O| = (10^{-10})e^{v_I/0.026}$$

For $v_I = 0.30\,V$, $|v_O| = 1.03x10^{-5}\,V$

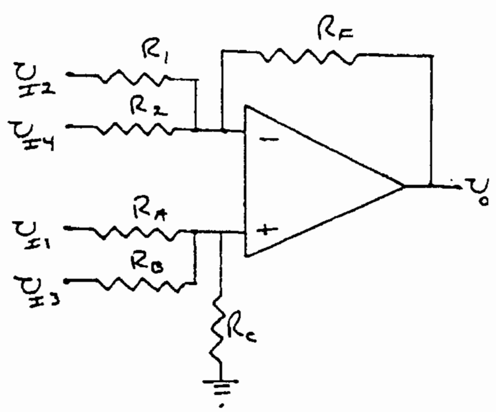
For $v_I = 0.60\,V$, $|v_O| = 1.05\,V$



9.58

$$v_O = 2v_{I1} - 10v_{I2} + 3v_{I3} - v_{I4}$$

From Figure 9. 37



From Equation (9.110), we can write

$$v_O = -\frac{R_F}{R_1}\cdot v_{I2} - \frac{R_F}{R_2}\cdot v_{I4} + \left(1 + \frac{R_F}{R_N}\right)\left[\frac{R_P}{R_A}\cdot v_{I1} + \frac{R_P}{R_B}\cdot v_{I3}\right]$$

where $R_N = R_1 \| R_2$, $R_P = R_A \| R_B \| R_C$

Then $\frac{R_F}{R_1} = 10$; $\frac{R_F}{R_2} = 1$

Set $R_F = 500$ kΩ, then $R_1 = 50$ kΩ, $R_2 = 500$ kΩ

Now $R_N = R_1 \| R_2 = 50 \| 500 = 45.45$ kΩ

$$\left(1 + \frac{R_F}{R_N}\right) = \left(1 + \frac{500}{45.45}\right) = 12$$

Then $(12)\left(\frac{R_P}{R_A}\right) = 2$; $(12)\left(\frac{R_P}{R_B}\right) = 3$

Now $\frac{12(R_P/R_A)}{12(R_P/R_B)} = \frac{2}{3} = \frac{R_B}{R_A}$

R_A is the largest resistor

Set $\underline{R_A = 500 \text{ k}\Omega}$, then $\underline{R_B = 333.3 \text{ k}\Omega}$

Then $\frac{12R_P}{R_A} = 2 = \frac{12R_P}{500} = 2 \Rightarrow R_P = 83.33$ kΩ

$R_P = R_A \| R_B \| R_C$ and $R_A \| R_B = 500 \| 333.3$

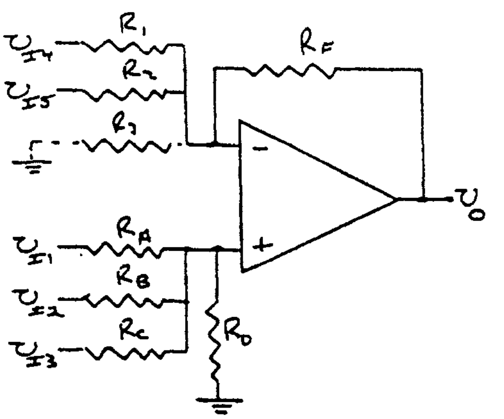
$= 200$ kΩ

Then $200 \| R_C = 83.33$ so $\underline{R_C = 142.8 \text{ k}\Omega}$

9.59

$v_O = 6v_{I1} + 3v_{I2} + 5v_{I3} - v_{I4} - 2v_{I5}$

From Figure 9.37



$$v_O = -\frac{R_F}{R_1} \cdot v_{I4} - \frac{R_F}{R_2} \cdot v_{I5} + \left(1 + \frac{R_F}{R_N}\right)\left[\frac{R_P}{R_A} \cdot v_{I1} + \frac{R_P}{R_B} \cdot v_{I2} + \frac{R_P}{R_C} \cdot v_{I3}\right]$$

where

$R_N = R_1 \| R_2$, $R_P = R_A \| R_B \| R_C \| R_D$

Then $\frac{R_F}{R_1} = 1$; $\frac{R_F}{R_2} = 2$

Let $R_F = 250$ kΩ, then $R_1 = 250$ kΩ, $R_2 = 125$ kΩ

Then $R_N = R_1 \| R_2 = 250 \| 125 = 83.33$ kΩ

$$\left(1 + \frac{R_F}{R_N}\right) = \left(1 + \frac{250}{83.33}\right) = 4$$

Now $4\left(\frac{R_P}{R_A}\right) = 6$; $4\left(\frac{R_P}{R_B}\right) = 3$; $4\left(\frac{R_P}{R_C}\right) = 5$

$$\frac{4(R_P/R_A)}{4(R_P/R_B)} = \frac{6}{3} = 2 = \frac{R_B}{R_A}$$

$$\frac{4(R_P/R_C)}{4(R_P/R_B)} = \frac{5}{3} = 1.667 = \frac{R_B}{R_C}$$

Set $R_B = 250$ kΩ, then

$R_A = 125$ kΩ, $R_C = 150$ kΩ

Then $\frac{4R_P}{R_A} = 6 = \frac{4R_P}{125} = 6 \Rightarrow R_P = 187.5$ kΩ

$\Rightarrow$ won't work since

$R_P = R_A \| R_B \| R_C \| R_D > R_A$ and R_C

Add a resistor R_3 in parallel with R_1 and R_2 to decrease R_N (but with zero input to R_3).

Set $R_D = \infty \Rightarrow R_P = R_A \| R_B \| R_C = 53.57$ kΩ

Then

$$\left(1 + \frac{R_F}{R_N}\right) \cdot \frac{R_P}{R_A} = 6 = \left(1 + \frac{R_F}{R_N}\right) \cdot \left(\frac{53.57}{125}\right)$$

$$\Rightarrow \frac{R_F}{R_N} = 13.0$$

So

$$R_N = \frac{250}{13} = 19.23 = R_1 \| R_2 \| R_3 = 83.33 \| R_3$$

So $\underline{R_3 = 25 \text{ k}\Omega}$

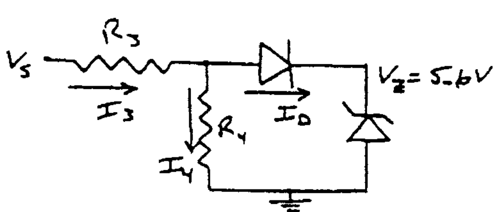
9.60

$$\frac{V_O}{V_Z} = \left(1 + \frac{R_2}{R_1}\right)$$

$$\frac{9}{5.6} = 1 + \frac{R_2}{R_1} \Rightarrow \underline{\frac{R_2}{R_1} = 0.607}$$

$$I_F = \frac{V_O - V_Z}{R_F}$$

Set $I_F = 0.8$ mA $= \frac{9 - 5.6}{R_F} \Rightarrow \underline{R_F = 4.25 \text{ k}\Omega}$



$V_Z' = 5.6 + 0.7 = 6.3$ V

$I_4 = \frac{V_Z'}{R_4} = \frac{6.3}{R_4}$, $I_3 = \frac{V_S - V_Z'}{R_3}$

If $V_S = 10$ V, $I_3 = \frac{10 - 6.3}{R_3} = \frac{3.7}{R_3}$

Want $I_{D1} = 0.1$ mA; if we set $I_4 = 0.1$ mA $= \frac{6.3}{R_4}$

$\Rightarrow \underline{R_4 = 63 \text{ k}\Omega}$

Then $I_3 = 0.2$ mA $= \frac{3.7}{R_3} \Rightarrow \underline{R_3 = 18.5 \text{ k}\Omega}$

9.61

For $I_Z = 1$ mA

$$I_Z = \frac{V_0 - V_Z}{R_1} = \frac{10 - 5.6}{R_1} = 1 \text{ mA} \Rightarrow \underline{R_1 = 4.4\text{ k}\Omega}$$

$$V_Z = \left(\frac{R_3}{R_2 + R_3}\right) \cdot V_0 \Rightarrow 5.6 = \frac{R_3}{R_2 + R_3} \cdot 10$$

$$= \frac{10}{\left(1 + \frac{R_2}{R_3}\right)}$$

$$1 + \frac{R_2}{R_3} = \frac{10}{5.6} \Rightarrow \underline{\frac{R_2}{R_3} = 0.786}$$

Let $\frac{V_0}{R_2 + R_3} = 1 \text{ mA} = \frac{10}{R_2 + R_3} = 1$ mA

$\Rightarrow R_2 + R_3 = 10$ kΩ

$R_2 = 0.786R_3 \Rightarrow 0.786R_3 + R_3 = 10$

$\Rightarrow \underline{R_3 = 5.6\text{ k}\Omega}$, $\underline{R_2 = 4.4\text{ k}\Omega}$

Let $\frac{V_{in} - V_0}{R_4} = 2 \text{ mA} = \frac{12 - 10}{R_4} \Rightarrow \underline{R_4 = 1\text{ k}\Omega}$

In this case, the op-amp must supply the load current.

9.62

For $v_{O1} - v_{O2} = 0$ at $T = 250°$ K, set

$R_1 = R_2 = R_2 = R_T$ @ $250°$ K $= 12$ kΩ

Assuming v_{O1} and v_{O2} look into an open circuit.

$$v_{O2} = \left(\frac{R_T}{R_T + R_2}\right)V^+ \text{ and}$$

$$v_{O1} = \left(\frac{R_3}{R_1 + R_3}\right) \cdot V^+ = \frac{V^+}{2}$$

As temperature increases, R_T decreases. Let $R_T = 12(1 - \delta)$ where δ is positive.

$R_T = 10$ kΩ @ $300° \Rightarrow \underline{\delta = 0.1667}$

$$v_{O2} = \left(\frac{12(1-\delta)}{12(1-\delta) + 12}\right)V^+$$

Consider

$$v_{O2} - v_{O1} = \left(\frac{12(1-\delta)}{12(1-\delta)+12} - \frac{1}{2}\right)(10)$$

$$= \left(\frac{12(1-\delta) - 6[(1-\delta)+1]}{12[(1-\delta)+1]}\right)(10)$$

$$= \left(\frac{6(1-\delta) - 6}{12[(1-\delta)+1]}\right)(10) = \frac{60\delta}{12[(1-\delta)+1]}$$

Now consider $v_{O1} - v_{O2} = \frac{5\delta}{2-\delta} \approx \frac{5}{2} \cdot \delta$

Connect v_{O1} to the v_{I2} terminal of the instrumentation amplifier and v_{O2} to the v_{I1} terminal.

Now $v_O = \frac{R_4}{R_3}\left(1 + \frac{2R_2}{R_1}\right)\left(\frac{5}{2}\right) \cdot \delta$

For $\delta = 0.1667$, $v_O = 5$

$$\frac{5}{0.1667} = 30 = \frac{R_4}{R_3}\left(1 + \frac{2R_2}{R_1}\right)\left(\frac{5}{2}\right) \text{ or}$$

$$\frac{R_4}{R_3}\left(1 + \frac{2R_2}{R_1}\right) = 12$$

Set $\underline{\frac{R_4}{R_3} = 4}$ Then $\underline{\frac{2R_2}{R_1} = 3}$ If $R_2 = 30$ kΩ

$R_1 = 20$ kΩ

Resistor R_1 can be a fixed resistor in series with a potentiometer for more precise control.

9.63

Using the bridge circuit shown in Figure 9.44

$$v_{O1} = \left[\frac{1}{2} - \frac{R}{R + R(1+\delta)}\right]V^+ = \left[\frac{1}{2} - \frac{1}{2+\delta}\right](10)$$

$$= \left(\frac{2 + \delta - 2}{2(2+\delta)}\right)(10)$$

$$\Rightarrow v_{O1} \cong \frac{10\delta}{4} = 2.5\delta$$

Connect v_{O1} to an instrumentation amplifier.

$$v_O = \frac{R_4}{R_3}\left(1 + \frac{2R_2}{R_1}\right)(2.5\delta)$$

When $\delta = 0.02$, $v_O = 5$

$$\frac{5}{(2.5)(0.02)} = 100 = \frac{R_4}{R_3}\left(1 + \frac{2R_2}{R_1}\right)$$

Set $\underline{\frac{R_4}{R_3} = 10}$ Then $\underline{\frac{2R_2}{R_1} = 9}$

Chapter 10

Exercise Solutions

E10.1

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on})}{R_1} = \frac{10 - 0.7}{15}$$

$\underline{I_{REF} = 0.62 \text{ mA}}$

$$I_0 = \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{0.62}{1 + \frac{2}{75}}$$

$\underline{I_0 = 0.604 \text{ mA}}$

E10.2

For $I_0 = 0.75$ mA

$$I_{REF} = I_0\left(1 + \frac{2}{\beta}\right) = (0.75)\left(1 + \frac{2}{100}\right)$$

$\underline{I_{REF} = 0.765 \text{ mA}}$

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_1}$$

$$R_1 = \frac{5 - 0.7 - (-5)}{0.765}$$

$\underline{R_1 = 12.2 \text{ k}\Omega}$

E10.3

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_1} = \frac{5 - 0.7 - (-5)}{12}$$

$I_{REF} = 0.775$ mA

$$I_0 = \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{0.775}{1 + \frac{2}{75}} = 0.754 \text{ mA}$$

$\Delta I_0 = (0.02)(0.754) = 0.0151$ mA

and $\Delta I_0 = \frac{1}{r_0}\Delta V_{CE2} \Rightarrow r_0 = \frac{\Delta V_{CE2}}{\Delta I_0}$

$$r_0 = \frac{4}{0.0151} = 265 \text{ k}\Omega = \frac{V_A}{I_0}$$

$\Rightarrow V_A = (265)(0.754) \Rightarrow \underline{V_A \cong 200 \text{ V}}$

E10.4

$$I_{REF} = \frac{V^+ - 2V_{BE}(\text{on})}{R_1} = \frac{9 - 2(0.7)}{12}$$

$\underline{I_{REF} = 0.6333 \text{ mA}}$

$$I_0 = \frac{I_{REF}}{1 + \frac{2}{\beta(1+\beta)}} = \frac{0.6333}{1 + \frac{2}{75(76)}} = 0.6331 \text{ mA}$$

$\underline{I_0 = 0.6331 \text{ mA} = I_{C1}}$

$I_{B1} = I_{B2} = \frac{I_0}{\beta} \Rightarrow \underline{I_{B1} = I_{B2} = 8.44\ \mu\text{A}}$

$I_{E3} = I_{B1} + I_{B2} \Rightarrow \underline{I_{E3} = 16.88\ \mu\text{A}}$

$I_{B3} = \frac{I_{E3}}{1+\beta} \Rightarrow \underline{I_{B3} = 0.222\ \mu\text{A}}$

E10.5

$$I_{REF} = \frac{10 - (0.7)(2)}{12} = 0.717 \text{ mA}$$

$I_0 \approx I_{REF} = 0.717$

$r_0 = \frac{V_A}{I_0} = \frac{100}{0.717} \Rightarrow \underline{r_0 = 139 \text{ k}\Omega}$

$\Delta I_0 = \frac{1}{r_0}\Delta V_{CE2} = \frac{4}{139}$

$\Rightarrow \underline{\Delta I_0 = 0.0288 \text{ mA}}$

E10.6

$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \frac{2}{\beta(1+\beta)}\right)} = \frac{0.50}{\left(1 + \frac{2}{50(51)}\right)}$$

$\Rightarrow \underline{I_0 = 0.4996 \text{ mA}}$

$I_{B3} = \frac{I_0}{\beta} \Rightarrow \underline{I_{B3} = 9.99\ \mu\text{A}}$

$I_{E3} = \left(\frac{1+\beta}{\beta}\right) I_{C3} \Rightarrow \underline{I_{E3} = 0.5096 \text{ mA}}$

$$I_{C2} = \frac{I_{E3}}{\left(1 + \frac{2}{\beta}\right)} = \frac{0.5096}{\left(1 + \frac{2}{50}\right)}$$

$\Rightarrow \underline{I_{C2} = 0.490 \text{ mA} = I_{C1}}$

$I_{B1} = I_{B2} = \frac{I_{C2}}{\beta} \Rightarrow \underline{I_{B1} = I_{B2} = 9.80\ \mu\text{A}}$

E10.7

$$I_0 R_E = V_T \ln\left(\frac{I_{REF}}{I_0}\right)$$

$$R_E = \frac{V_T}{I_0}\ln\left(\frac{I_{REF}}{I_0}\right) = \frac{0.026}{0.025}\ln\left(\frac{0.75}{0.025}\right)$$

$\Rightarrow \underline{R_E = 3.54 \text{ k}\Omega}$

$R_1 = \frac{5 - 0.7}{0.75} \Rightarrow \underline{R_1 = 5.73 \text{ k}\Omega}$

$V_{BE1} - V_{BE2} = I_0 R_E = (0.025)(3.54)$

$\Rightarrow \underline{V_{BE1} - V_{BE2} = 88.5 \text{ mV}}$

E10.8

$I_{REF} = \frac{5 - 0.7 - (-5)}{12} \Rightarrow \underline{I_{REF} = 0.775 \text{ mA}}$

$$I_0 R_E = V_T \ln\left(\frac{I_{REF}}{I_0}\right)$$

$$I_0(6) = (0.026)\ln\left(\frac{0.775}{I_0}\right)$$

$\Rightarrow \underline{I_0 \cong 16.6\ \mu\text{A}}$

E10.9

$$I_0 R_E = V_T \ln\left(\frac{I_{REF}}{I_0}\right)$$
$$R_E = \frac{0.026}{0.025}\ln\left(\frac{0.70}{0.025}\right) \Rightarrow R_E = 3.47\text{ k}\Omega$$
$$g_{m2} = \frac{I_0}{V_T} = \frac{0.025}{0.026} \Rightarrow g_{m2} = 0.962\text{ mA/V}$$
$$r_{\pi 2} = \frac{\beta V_T}{I_0} = \frac{(150)(0.026)}{0.025} = 156\text{ k}\Omega$$
$$r_{02} = \frac{V_A}{I_0} = \frac{100}{0.025} = 4000\text{ k}\Omega$$

$$R'_E = R_E \| r_{\pi 2} = 3.47\|156 = 3.39\text{ k}\Omega$$
$$R_0 = r_{02}(1 + g_{m2}R'_E) = 4000[1 + (0.962)(3.39)]$$
$$R_0 = 17,045\text{ k}\Omega$$
$$dI_0 = \frac{1}{R_0}\cdot dV_{C2} = \frac{3}{17,045}$$
$$\Rightarrow \underline{dI_0 = 0.176\ \mu\text{A}}$$

E10.10

$$I_{REF} = I_R + I_{BR} + I_{B1} + \ldots + I_{BN}$$
$$I_R = I_{01} = I_{02} = \ldots = I_{0N}$$
$$\text{and } I_{BR} = I_{B1} = I_{B2} = \ldots = I_{BN} = \frac{I_{01}}{\beta}$$
$$I_{REF} = I_{01} + (N+1)\left(\frac{I_{01}}{\beta}\right) = I_{01}\left(1 + \frac{N+1}{\beta}\right)$$

$$\text{So } I_{01} = I_{02} = \ldots = I_{0N} = \frac{I_{REF}}{1 + \frac{N+1}{\beta}}$$

$$\frac{I_{01}}{I_{REF}} = 0.90 = \frac{1}{1 + \frac{N+1}{50}}$$
$$1 + \frac{N+1}{50} = \frac{1}{0.9}$$
$$N + 1 = \left(\frac{1}{0.9} - 1\right)(50)$$
$$N = \left(\frac{1}{0.9} - 1\right)(50) - 1$$
$$N = 4.55 \Rightarrow \underline{N = 4}$$

E10.11

a. From Equation (10.52),

$$V_{GS1} = \frac{\sqrt{\frac{3}{12}}}{1 + \sqrt{\frac{3}{12}}} \times 10 + \left(\frac{1 - \sqrt{\frac{3}{12}}}{1 + \sqrt{\frac{3}{12}}}\right) \times (1.8)$$

$$V_{GS1} = \left(\frac{0.5}{1 + 0.5}\right)(10) + \left(\frac{1 - 0.5}{1 + 0.5}\right) \times (1.8)$$
$$V_{GS1} = 3.93\text{ V also } V_{DS1} = 3.93\text{ V}$$
$$I_{REF} = (12)(0.020)[3.93 - 1.8]^2[1 + (0.01)(3.93)]$$
$$\Rightarrow \underline{I_{REF} = 1.13\text{ mA}}$$

b. $I_0 = I_{REF} \times \frac{(W/L)_2}{(W/L)_1} \times \frac{(1 + \lambda V_{DS2})}{(1 + \lambda V_{DS1})}$

$$I_0 = (1.13) \times \left(\frac{6}{12}\right) \times \frac{[1 + (0.01)(2)]}{[1 + (0.01)(3.93)]}$$
$$\Rightarrow \underline{I_0 = 0.555\text{ mA}}$$

c. For $V_{DS2} = 6$ V

$$\Rightarrow \underline{I_0 = 0.576\text{ mA}}$$

E10.12

$$K_{n1}(V_{GS1} - V_{TN})^2 = K_{n3}(V_{GS3} - V_{TN})^2$$
$$V_{GS1} - 2 = \left(\sqrt{\frac{0.10}{0.25}}\right)(V_{GS3} - 2)$$
$$V_{GS1} - 2 = (0.632)(V_{GS3} - 2)$$
$$V_{GS3} = 10 - V_{GS1}$$
$$V_{GS1} - 2 = (0.632)(10 - V_{GS1}) - (0.632)(2)$$
$$1.632V_{GS1} = 7.056 \Rightarrow V_{GS1} = 4.32\text{ V}$$

$$I_{REF} = K_{n1}(V_{GS1} - V_{TN})^2 = (0.25)(4.32 - 2)^2 \Rightarrow$$
$$\underline{I_{REF} = 1.35\ mA}$$
$$I_O = 3K_{n2}(V_{GS1} - V_{TN})^2 = 3(0.25)(4.32 - 2)^2$$
$$I_O = 3I_{REF} \Rightarrow \underline{I_O = 4.04\ mA}$$

E10.13

$$V_{DS}(sat) = 1V = V_{GS2} - V_{TN} = V_{GS2} - 2$$
$$\Rightarrow V_{GS2} = 3V$$
$$I_O = K_{n2}(V_{GS2} - V_{TN})^2 = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2$$
$$0.20 = (0.020)\left(\frac{W}{L}\right)_2 (3-2)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_2 = 10}$$
$$I_{REF} = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2$$
$$V_{GS1} = V_{GS2}$$
$$0.5 = (0.020)\left(\frac{W}{L}\right)_1 (3-2)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 25}$$
$$V_{GS3} = V^+ - V_{GS1} = 10 - 3 = 7\ V$$
$$I_{REF} = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right)_3 (V_{GS3} - V_{TN})^2$$
$$0.5 = (0.020)\left(\frac{W}{L}\right)_3 (7-2)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_3 = 1}$$

E10.14

a. $I_{REF} = K_n(V_{GS} - V_{TN})^2$

$$0.020 = 0.080(V_{GS} - 1)^2$$

$\underline{V_{GS} = 1.5\text{ V}}$ all transistors

b. $V_{G4} = V_{GS3} + V_{GS1} + V^- = 1.5 + 1.5 - 5 = -2 \text{ V}$

$V_{S4} = V_{G4} - V_{GS4} = -2 - 1.5 = -3.5 \text{ V}$

$V_{D4}(\text{min}) = V_{S4} + V_{DS4}(\text{sat})$

and $V_{DS4}(sat) = V_{GS4} - V_{TN} = 1.5 - 1 = 0.5\,V$

So $V_{D4}(\text{min}) = -3.5 + 0.5$

$\Rightarrow \underline{V_{D4}(\text{min}) = -3.0 \text{ V}}$

c. $R_0 = r_{04} + r_{02}(1 + g_m r_{04})$

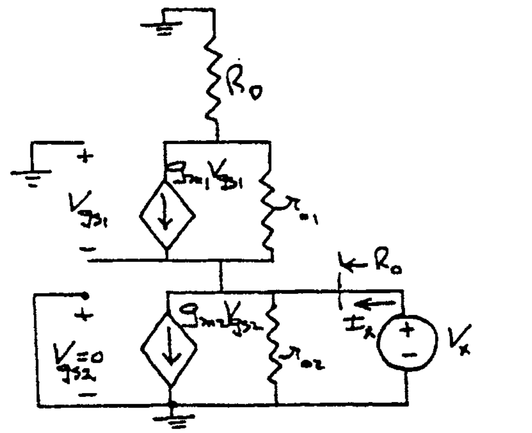
$$r_{02} = r_{04} = \frac{1}{\lambda I_0} = \frac{1}{(0.02)(0.020)} = 2500 \text{ k}\Omega$$

$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.080)(1.5 - 1) \Rightarrow$

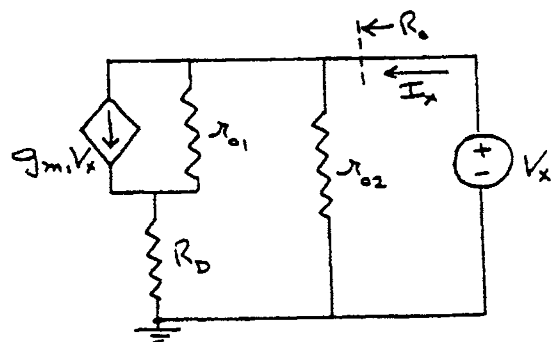
$g_m = 0.080\,mA/V$

$R_0 = 2500 + 2500(1 + (0.080)(2500))$

$\Rightarrow \underline{R_0 = 505 \text{ M}\Omega}$



$V_{gs2} = 0,\ V_{gs1} = -V_X$



$$I_X = \frac{V_X}{r_{02}} + \frac{V_X - V_1}{r_{01}} + g_{m1}V_X \qquad (1)$$

$$\frac{V_1}{R_D} + \frac{V_1 - V_X}{r_{01}} = g_{m1}V_X \qquad (2)$$

$$V_1 = \frac{V_X\left(\frac{1}{r_{01}} + g_{m1}\right)}{\frac{1}{R_D} + \frac{1}{r_{01}}}$$

$$\frac{I_X}{V_X} = \frac{1}{R_0} = \frac{1}{r_{02}} + \frac{1}{r_{01}} + g_{m1} - \frac{\frac{1}{r_{01}}\left(\frac{1}{r_{01}} + g_{m1}\right)}{\frac{1}{R_D} + \frac{1}{r_{01}}}$$

$$= \frac{1}{r_{02}} + \left(\frac{1}{r_{01}} + g_{m1}\right)\left[1 - \frac{\frac{1}{r_{01}}}{\frac{1}{R_D} + \frac{1}{r_{01}}}\right]$$

$$= \frac{1}{r_{02}} + \left(\frac{1}{r_{01}} + g_{m1}\right)\left(\frac{\frac{1}{R_D}}{\frac{1}{R_D} + \frac{1}{r_{01}}}\right)$$

E10.15

$I_{REF} = 0.20 = K_{n1}(V_{GS1} - V_{TN})^2 = 0.15(V_{GS1} - 1)^2$

$\Rightarrow \underline{V_{GS1} = V_{GS2} = 2.15\,V}$

$I_O = K_{n2}(V_{GS2} - V_{TN})^2 = \frac{0.15}{2}(2.15 - 1)^2 \Rightarrow$

$\underline{I_O = 0.10\,mA}$

$I_O = K_{n3}(V_{GS3} - V_{TN})^2$

$0.10 = 0.15(V_{GS3} - 1)^2 \Rightarrow \underline{V_{GS3} = 1.82\,V}$

E10.16

All transistors are identical

$\Rightarrow \underline{I_0 = I_{REF} = 250\ \mu\text{A}}$

$I_{REF} = K_n(V_{GS} - V_{TN})^2$

$0.25 = 0.20(V_{GS} - 1)^2$

$\Rightarrow \underline{V_{GS} = 2.12 \text{ V}}$

E10.17

For Q_2: $v_{DS}(\text{min}) = |V_P| = 2 \text{ V}$

$\Rightarrow V_S(\text{min}) = v_{DS}(\text{min}) - 5 = 2 - 5$

$\Rightarrow \underline{V_S(\text{min}) = -3 \text{ V}}$

$I_0 = I_{DSS2}(1 + \lambda v_{DS2}) = 0.5(1 + (0.15)(2))$

$\Rightarrow \underline{I_0 = 0.65 \text{ mA}}$

$$I_0 = I_{DSS1}\left(1 - \frac{v_{GS1}}{V_{P1}}\right)^2$$

$$0.65 = 0.80\left(1 - \frac{v_{GS1}}{-2}\right)^2$$

$\frac{v_{GS1}}{-2} = 0.0986 \Rightarrow v_{GS1} = -0.197 \text{ V}$

$v_{GS1} = V_I - V_S$

$-0.197 = V_I - (-3) \Rightarrow \underline{V_I(\text{min}) = -3.2 \text{ V}}$

For $R_D \ll r_{01}$

$$\Rightarrow \frac{1}{R_0} \cong \frac{1}{r_{02}} + \left(\frac{1}{r_{01}} + g_{m1}\right)$$

For Q_1 :

$$g_{m1} = \frac{2I_{DSS1}}{|V_P|}\left(1 - \frac{V_{GS1}}{V_P}\right) = \frac{2(0.8)}{2}\left(1 - \frac{-0.197}{-2}\right)$$

$g_{m1} = 0.721$ mA/V

$$r_0 = \frac{1}{\lambda I_0} = \frac{1}{(0.15)(0.65)} = 10.3 \text{ k}\Omega$$

$$\frac{1}{R_0} = \frac{1}{10.3} + \frac{1}{10.3} + 0.721 = 0.915$$

$\Rightarrow \underline{R_0 = 1.09 \text{ k}\Omega}$

E10.18

For Q_1 : $i_D = I_{DSS1}(1 + \lambda v_{DS1})$

For Q_2 : $i_D = I_{DSS2}\left(1 - \frac{v_{GS2}}{V_P}\right)^2 (1 + \lambda v_{DS2})$

$v_{GS2} = -v_{DS1}$

and $v_{DS2} = V_{DS} - v_{DS1}$

So

$$I_{DSS1}(1 + \lambda v_{DS1}) = I_{DSS2}\left[1 - \frac{-v_{DS1}}{V_P}\right]^2 [1 + \lambda(V_{DS} - v_{DS1})]$$

$I_{DSS1} = I_{DSS2}$

$$[1 + (0.1)v_{DS1}] = \left[1 - \frac{v_{DS1}}{2}\right]^2 [1 + (0.1)(3) - (0.1)v_{DS1}]$$

$$1 + 0.1v_{DS1} = (1 - v_{DS1} + 0.25v_{DS1}^2)(1.3 - 0.1v_{DS1})$$

This becomes

$$0.025v_{DS1}^3 - 0.425v_{DS1}^2 + 1.5v_{DS1} - 0.3 = 0$$

We find $\underline{v_{DS1} = 0.212 \text{ V}}$, $\underline{v_{DS2} = 2.79 \text{ V}}$,

$\underline{v_{GS2} = -0.212 \text{ V}}$

$i_D = I_{DSS1}(1 + \lambda v_{DS1}) = 2[1 + (0.1)(0.212)]$

$\underline{i_D = 2.04 \text{ mA}}$

$R_0 = r_{02} + r_{01}(1 + g_{m2}r_{02})$

$$g_m = \frac{2I_{DSS}}{-V_P}\left(1 - \frac{v_{GS2}}{V_P}\right) = \frac{2(2)}{2}\left(1 - \frac{-0.212}{-2}\right)$$

$g_m = 1.79$ mA/V

$$r_{02} = r_{04} = \frac{1}{\lambda I_{DSS}} = \frac{1}{(0.1)(2)} = 5 \text{ k}\Omega$$

$R_0 = 5 + 5[1 + (1.79)(5)]$

$\Rightarrow \underline{R_0 = 54.8 \text{ k}\Omega}$

E10.19

a. $$I_{REF} = I_S \exp\left(\frac{V_{EB2}}{V_T}\right)$$

$$V_{EB2} = V_T \ln\left(\frac{I_{REF}}{I_S}\right) = (0.026)\ln\left(\frac{0.5 \times 10^{-3}}{10^{-12}}\right)$$

$\Rightarrow \underline{V_{EB2} = 0.521 \text{ V}}$

b. $$R_1 = \frac{5 - 0.521}{0.5} \Rightarrow \underline{R_1 = 8.96 \text{ k}\Omega}$$

c. From Equation (10.72)

$$I_{S0}\left[\exp\left(\frac{V_I}{V_T}\right)\right]\left(1 + \frac{V_{CE0}}{V_{AN}}\right) = I_{REF} \times \frac{\left(1 + \frac{V_{EC2}}{V_{AP}}\right)}{\left(1 + \frac{V_{EB2}}{V_{AP}}\right)}$$

$$10^{-12}\left[\exp\left(\frac{V_I}{V_T}\right)\right]\left(1 + \frac{2.5}{100}\right) = (0.5 \times 10^{-3})\frac{\left(1 + \frac{2.5}{100}\right)}{\left(1 + \frac{0.521}{100}\right)}$$

$$1.03 \times 10^{-12} \exp\left(\frac{V_I}{V_T}\right) = 5.125 \times 10^{-4}$$

$$\exp\left(\frac{V_I}{V_T}\right) = 4.976 \times 10^{8}$$

$\Rightarrow \underline{V_I = 0.521 \text{ V}}$

d.

$$A_v = \frac{-\left(\frac{1}{V_T}\right)}{\frac{1}{V_{AN}} + \frac{1}{V_{AP}}} = \frac{-\frac{1}{0.026}}{\frac{1}{100} + \frac{1}{100}} = \frac{-38.46}{0.01 + 0.01}$$

$\Rightarrow \underline{A_v = -1923}$

E10.20

a. $$V_{EB2} = (0.026)\ln\left(\frac{0.1 \times 10^{-3}}{5 \times 10^{-14}}\right)$$

$\Rightarrow \underline{V_{EB2} = 0.557 \text{ V}}$

b. $$R_1 = \frac{5 - 0.557}{0.1} \Rightarrow \underline{R_1 = 44.4 \text{ k}\Omega}$$

c. $$I_{S0}\left[\exp\left(\frac{V_I}{V_T}\right)\right]\left(1 + \frac{V_{CE0}}{V_{AN}}\right) = I_{REF} \times \left(\frac{1 + \frac{V_{EC2}}{V_{AP}}}{1 + \frac{V_{EB2}}{V_{AP}}}\right)$$

$$5 \times 10^{-14}\left[\exp\left(\frac{V_I}{V_T}\right)\right]\left(1+\frac{2.5}{100}\right)$$
$$= (0.1 \times 10^{-3})\left(\frac{1+\frac{2.5}{100}}{1+\frac{0.557}{100}}\right)$$

$$(5.125 \times 10^{-14})\exp\left(\frac{V_I}{V_T}\right) = 1.019 \times 10^{-4}$$
$$\exp\left(\frac{V_I}{V_T}\right) = 1.988 \times 10^{9}$$
$\Rightarrow \underline{V_I = 0.557\text{ V}}$

d. $A_v = \dfrac{-\frac{1}{0.026}}{\frac{1}{100}+\frac{1}{100}} \Rightarrow \underline{A_v = -1923}$

E10.21

a. $I_{REF} = K_{p1}(V_{SG}+V_{TP})^2$

$0.25 = 0.20(V_{SG}-1)^2$

$\Rightarrow \underline{V_{SG} = 2.12\text{ V}}$

b. From Equation (10.89)

$$V_{DSO} = V_o = \frac{\left[1+\lambda_p(V^+ - V_{SG})\right]}{\lambda_n+\lambda_p} - \frac{K_n(V_I - V_{TN})^2}{I_{REF}(\lambda_n+\lambda_p)}$$

$$5 = \frac{1+(0.015)(10-2.12)}{0.030} - \frac{(0.2)(V_I-1)^2}{0.25(0.030)}$$

$0.15 = 1.12 - 0.8(V_I-1)^2$

$\Rightarrow V_I = 2.10\text{ V}$

c. $A_v = \dfrac{-2K_n(V_I - V_{TN})}{I_{REF}(\lambda_n+\lambda_p)}$

$$A_v = -\frac{2(0.2)(2.10-1.0)}{0.25(0.030)}$$

$\Rightarrow \underline{A_v = -58.7}$

E10.22

(a) $I_{REF} = K_{p1}(V_{SG}+V_{TP})^2$

$80 = 50(V_{SG}-1)^2 \Rightarrow \underline{V_{SG} = 2.26\,V}$

(b) $V_{DSo} = V_o = \dfrac{\left[1+\lambda_p(V^+ - V_{SG})\right]}{\lambda_n+\lambda_p} - \dfrac{K_n(V_I - V_{TN})^2}{I_{REF}(\lambda_n+\lambda_p)}$

$$5 = \frac{[1+(0.015)(10-2.26)]}{0.030} - \frac{(50)(V_I-1)^2}{(80)(0.030)}$$

$20.83(V_I-1)^2 = 32.2 \Rightarrow \underline{V_I = 2.24\text{ V}}$

(c) $A_v = \dfrac{-2K_n(V_I - V_{TN})}{I_{REF}(\lambda_n+\lambda_p)} = \dfrac{-2(50)(2.24-1)}{(80)(0.030)} \Rightarrow$

$\underline{A_v = -51.7}$

E10.23

$g_m = \dfrac{I_{CQ}}{V_T} = \dfrac{0.8}{0.026} = 30.8\text{ mA/V}$

$r_0 = r_{02} = \dfrac{V_A}{I_{CQ}} = \dfrac{80}{0.8} = 100\text{ k}\Omega$

a. $V_0 = -g_m V_{\pi 1}(r_0 \| r_{02}),\quad V_{\pi 1} = V_i$

$A_v = -g_m(r_0\|r_{02}) = -(30.8)[100\|100]$

$\Rightarrow \underline{A_v = -1540}$

b. $A_v = -g_m(r_0\|r_{02}\|R_L)$

$A_v = -\dfrac{1540}{2} = -770$

$-770 = -(30.8)(50\|R_L) \Rightarrow (50\|R_L) = 25$

$\Rightarrow \underline{R_L = 50\text{ k}\Omega}$

E10.24

a. $g_m = \dfrac{I_{C0}}{V_T} = \dfrac{0.5}{0.026} \Rightarrow \underline{g_m = 19.2\text{ mA/V}}$

$r_0 = \dfrac{V_{AN}}{I_{CQ}} = \dfrac{120}{0.5} \Rightarrow \underline{r_0 = 240\text{ k}\Omega}$

$r_{02} = \dfrac{V_{AP}}{I_{CQ}} = \dfrac{80}{0.5} \Rightarrow \underline{r_{02} = 160\text{ k}\Omega}$

b. $A_v = -g_m(r_0\|r_{02}\|R_L) = -(19.2)[240\|160\|50]$

$\Rightarrow \underline{A_v = -631}$

E10.25

(a) Neglecting effect of λ and R_L

$I_O = I_{REF} = K_n\left(V_{IQ} - V_{TN}\right)^2$

$0.40 = 0.25(V_{IQ} - 1)^2$

Then $\underline{V_{IQ} = 2.26\ \text{V}}$

b. $r_0 = r_{02} = \dfrac{1}{\lambda I_0} = \dfrac{1}{(0.02)(0.4)} = 125\ \text{k}\Omega$

$g_m = 2K_n\left(V_{IQ} - V_{TN}\right) = 2(0.25)(2.26 - 1)$

$= 0.63\ \text{mA/V}$

$A_v = -g_m(r_0 \| r_{02}) = -(0.63)(125\|125)$

$\Rightarrow \underline{A_v = -39.4}$

c. $A_v = -g_m(r_0\|r_{02}\|R_L)$

$-\dfrac{39.4}{2} = -(0.63)(62.5\|R_L)$

$\Rightarrow 62.5\|R_L = 31.25 \Rightarrow \underline{R_L = 62.5\ \text{k}\Omega}$

E10.26

M_1 and M_2 identical $\Rightarrow I_0 = I_{REF}$

a. $I_O = K_n\left(V_I - V_{YN}\right)^2$

$0.25 = 0.2(V_T - 1)^2$

$V_I = 2.12\ \text{V}$

$g_m = 2K_n\left(V_I - V_{TN}\right) = 2(0.2)(2.12 - 1)$

$\Rightarrow \underline{g_m = 0.448\ \text{mA/V}}$

$r_{0n} = \dfrac{1}{\lambda_n I_0} = \dfrac{1}{(0.01)(0.25)} \Rightarrow \underline{r_{0n} = 400\ \text{k}\Omega}$

$r_{0p} = \dfrac{1}{\lambda_p I_0} = \dfrac{1}{(0.02)(0.25)} \Rightarrow \underline{r_{0p} = 200\ \text{k}\Omega}$

b. $A_v = -g_m(r_0\|r_{02}\|R_L)$

$A_v = -(0.448)[400\|200\|100]$

$\Rightarrow \underline{A_v = -25.6}$

Chapter 10

Problem Solutions

10.1

a. $I_1 = I_2 = \dfrac{0 - 2V_\gamma - V^-}{R_1 + R_2}$

$2V_\gamma + I_2 R_2 = V_{BE} + I_C R_3$

$2V_\gamma + \dfrac{R_2}{R_1 + R_2}(-2V_\gamma - V^-) = V_{BE} + I_C R_3$

$\underline{I_C = \dfrac{1}{R_3}\left\{2V_\gamma - (2V_\gamma + V^+)\left(\dfrac{R_2}{R_1 + R_2}\right) - V_{BE}\right\}}$

b. $V_\gamma = V_{BE}$ and $R_1 = R_2$

$I_C = \dfrac{1}{R_3}\left\{2V_\gamma - \dfrac{1}{2}(2V_\gamma + V^-) - V_{BE}\right\}$

or $\underline{I_C = \dfrac{-V^-}{2R_3}}$

c. $I_C = 2 \text{ mA} = \dfrac{-(-10)}{2R_3} \Rightarrow \underline{R_3 = 2.5 \text{ k}\Omega}$

$I_1 = I_2 = 2 \text{ mA} = \dfrac{-2(0.7) - (-10)}{R_1 + R_2}$

$\Rightarrow R_1 + R_2 = 4.3 \text{ k}\Omega$

$\Rightarrow \underline{R_1 = R_2 = 2.15 \text{ k}\Omega}$

10.2

$I_{C2} = \dfrac{I_{REF}}{1 + \frac{2}{\beta}} = \dfrac{1}{1 + \frac{2}{50}}$

$\underline{I_{C1} = I_{C2} = 0.962 \text{ mA}}$

$I_{B1} = I_{B2} = \dfrac{I_{C2}}{\beta} \Rightarrow \underline{I_{B1} = I_{B2} = 0.0192 \text{ mA}}$

10.3

(a) $I_{REF} = \dfrac{V^+ - V_{BE}(on) - V^-}{R_1}$

or

$R_1 = \dfrac{15 - 0.7 - (-15)}{0.5} \Rightarrow \underline{R_1 = 58.6\ k\Omega}$

(b) $R_1 = \dfrac{V^+ - V_{BE}(on) - V^-}{I_{REF}} = \dfrac{0 - 0.7 - (-15)}{0.5} \Rightarrow$

$\underline{R_1 = 28.6\ k\Omega}$

Advantage: Requires smaller resistance.

(c) For part (a):

$I_O(\max) = \dfrac{29.3}{(58.6)(0.95)} = 0.526\ mA$

$I_O(\min) = \dfrac{29.3}{(58.6)(1.05)} = 0.476\ mA$

$\Delta I_O = 0.526 - 0.476 = 0.05\ mA \Rightarrow \pm 5\%$

For part (b): $I_O(\max) = \dfrac{14.3}{(28.6)(0.95)} = 0.526\ mA$

$I_O(\min) = \dfrac{14.3}{(28.6)(1.05)} = 0.476\ mA$

$\Delta I_O = 0.05\ mA \Rightarrow \underline{\pm 5\%}$

10.4

a. $I_{REF} = I_0\left(1 + \dfrac{2}{\beta}\right) = 2\left(1 + \dfrac{2}{100}\right)$

or $I_{REF} = 2.04$ mA

$R_1 = \dfrac{15 - 0.7}{2.04} \Rightarrow \underline{R_1 = 7.01 \text{ k}\Omega}$

b. $r_0 = \dfrac{V_A}{I_0} = \dfrac{80}{2} = 40 \text{ k}\Omega$

$\dfrac{\Delta I_0}{\Delta V_{CE}} = \dfrac{1}{r_0} \Rightarrow \Delta I_0 = \left(\dfrac{1}{40}\right)(9.3) = 0.2325 \text{ mA}$

$\dfrac{\Delta I_0}{I_0} = \dfrac{0.2325}{2} \Rightarrow \underline{\dfrac{\Delta I_0}{I_0} = 11.6\%}$

10.5

$I_{REF} = I_0\left(1 + \dfrac{2}{\beta}\right) = (0.5)\left(1 + \dfrac{2}{25}\right)$

$= \underline{0.54 \text{ mA} = I_{REF}}$

$R_1 = \dfrac{5 - 0.7}{0.54} \Rightarrow \underline{R_1 = 7.96 \text{ k}\Omega}$

10.6

a. $I_{REF} = \dfrac{5 - 0.7}{18} = 0.239 \text{ mA}$

$I_0 = \dfrac{0.239}{1 + \frac{2}{50}} \Rightarrow \underline{I_0 = 0.230 \text{ mA}}$

b. $r_0 = \dfrac{V_A}{I_0} = \dfrac{50}{0.230} = 217 \text{ k}\Omega$

$\Delta I_0 = \dfrac{1}{r_0} \cdot \Delta V_{EC} = \left(\dfrac{1}{217}\right)(1.3) = 0.00599 \text{ mA}$

$\Rightarrow \underline{I_0 = 0.236 \text{ mA}}$

c. $\Delta I_0 = \left(\dfrac{1}{217}\right)(3.3) = 0.0152 \text{ mA}$

$\Rightarrow \underline{I_0 = 0.245 \text{ mA}}$

10.7

a. $I_{REF} = 1 = \dfrac{5 - 0.7 - (-5)}{R_1}$

$\Rightarrow \underline{R_1 = 9.3 \text{ k}\Omega}$

b. $I_O = 2I_{REF} \Rightarrow \underline{I_O = 2 \text{ mA}}$

c. For $V_{EC2}(\min) = 0.7 \Rightarrow R_{C2} = \dfrac{5 - 0.7}{2}$

$\Rightarrow \underline{R_{C2} = 2.15 \text{ k}\Omega}$

10.8

$I_O = nI_{C1}$

$$I_{REF} = I_{C1} + I_{B1} + I_{B2} = I_{C1} + \frac{I_{C1}}{\beta} + \frac{I_O}{\beta}$$

$$I_{REF} = I_{C1}\left(1 + \frac{1}{\beta} + \frac{n}{\beta}\right) = I_{C1}\left(1 + \frac{1+n}{\beta}\right)$$

$$= \frac{I_O}{n}\left(1 + \frac{1+n}{\beta}\right)$$

$$\text{or } I_O = \underline{\frac{nI_{REF}}{\left(1 + \frac{1+n}{\beta}\right)}}$$

10.9

Using the results of Problem 10-8,

$$2 = \frac{2I_{REF}}{1 + \frac{3}{50}} \Rightarrow \underline{I_{REF} = 1.06 \text{ mA}}$$

$$R_1 = \frac{5 - 0.7}{1.06} \Rightarrow \underline{R_1 = 4.06 \text{ k}\Omega}$$

10.10

First approximation – BE area of Q_2 is 3 times that of Q_1.

$$R_1 \cong \frac{5 - 0.7 - (-5)}{0.5} \Rightarrow R_1 = 18.6 \, k\Omega$$

Second approximation – take into account I_C vs V_{BE} variation.

For Q_1:

$$\frac{I_{C1}}{I_{C2}} = \exp\left(\frac{V_{BE1} - V_{BE2}}{V_T}\right)$$

or

$$V_{BE1} - V_{BE2} = V_T \ln\left(\frac{I_{C1}}{I_{C2}}\right)$$

$$0.7 - V_{BE2} = 0.026 \ln\left(\frac{1}{0.5}\right) \Rightarrow V_{BE} = 0.682\, V \text{ for}$$

$I_C = 0.5\, mA$

Then

$$R_1 = \frac{5 - 0.682 - (-5)}{0.5} \Rightarrow \underline{R_1 = 18.64 \, k\Omega}$$

Now

$$I_{S1} = \frac{I_C}{\exp\left(\frac{V_{BE}}{V_T}\right)} = \frac{1x10^{-3}}{\exp\left(\frac{0.7}{0.026}\right)} \Rightarrow I_{S1} = 2.03x10^{-15}\ A$$

For Q_2:

$$I_{S2} = \frac{I_{C2}}{\exp\left(\frac{V_{BE2}}{V_T}\right)} = \frac{1.5x10^{-3}}{\exp\left(\frac{0.682}{0.026}\right)} = 6.09x10^{-15}\ A$$

10.11

$I_2 = 2I_1$ and $I_3 = 3I_1$

(a) $I_2 = 1.0\, mA$, $I_3 = 1.5\, mA$

(b) $I_1 = 0.25\, mA$, $I_3 = 0.75\, mA$

(c) $I_1 = 0.167\, mA$, $I_2 = 0.333\, mA$

10.12

a. $I_O = I_{C1}$ and $I_{REF} = I_{C1} + I_{B3} = I_{C1} + \dfrac{I_{E3}}{1+\beta}$

$$I_{E3} = I_{B1} + I_{B2} + \frac{V_{BE}}{R_2} = \frac{2I_{C1}}{\beta} + \frac{V_{BE}}{R_2}$$

$$I_{REF} = I_{C1} + \frac{2I_{C1}}{\beta(1+\beta)} + \frac{V_{BE}}{(1+\beta)R_2}$$

$$I_{REF} - \frac{V_{BE}}{(1+\beta)R_2} = I_O\left(1 + \frac{2}{\beta(1+\beta)}\right)$$

$$\underline{I_O = \frac{I_{REF} - \frac{V_{BE}}{(1+\beta)R_2}}{\left(1 + \frac{2}{\beta(1+\beta)}\right)}}$$

b. $I_{REF} = (0.70)\left(1 + \dfrac{2}{(80)(81)}\right) + \dfrac{0.7}{(81)(10)}$

$I_{REF} = 0.700216 + 0.000864$

$\underline{I_{REF} = 0.7011 \text{ mA}} = \dfrac{10 - 2(0.7)}{R_1}$

$\Rightarrow \underline{R_1 = 12.27 \text{ k}\Omega}$

10.13

a. $I_{O i} = I_{CR}$ and $I_{REF} = I_{CR} + I_{BS} = I_{CR} + \dfrac{I_{ES}}{1+\beta}$

$$I_{ES} = I_{BR} + I_{B1} + I_{B2} + \ldots + I_{BN} = (1+N)I_{BR}$$

$$= \frac{(1+N)I_{CR}}{\beta}$$

$$\text{Then } I_{REF} = I_{CR} + \frac{(1+N)I_{CR}}{\beta(1+\beta)}$$

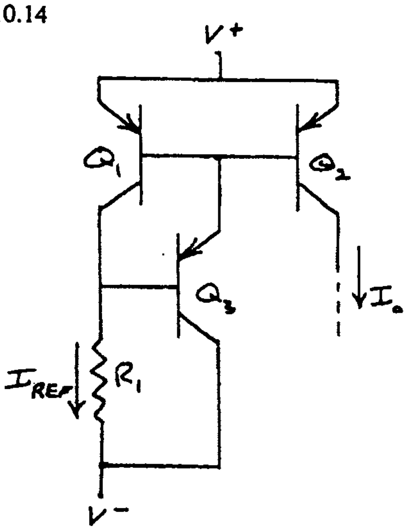
$$\text{or } I_{O i} = \underline{\frac{I_{REF}}{\left(1 + \frac{(1+N)}{\beta(1+\beta)}\right)}}$$

b. $I_{REF} = (0.5)\left[1 + \dfrac{6}{(50)(51)}\right] = 0.5012 \text{ mA}$

$$R_1 = \frac{5 - 2(0.7) - (-5)}{0.5012}$$

$\Rightarrow \underline{R_1 = 17.16 \text{ k}\Omega}$

10.14

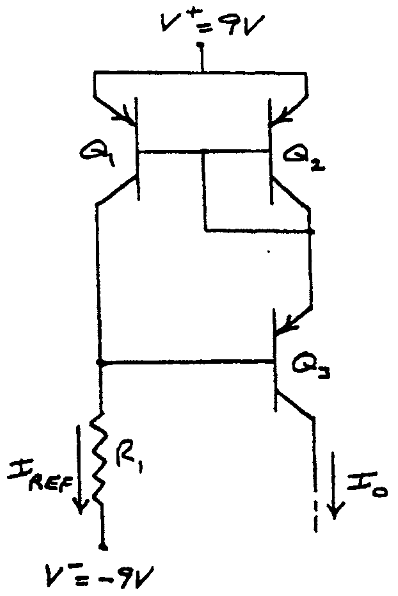


$$I_{REF} = I_0\left(1 + \frac{2}{\beta(1+\beta)}\right) = (0.5)\left[1 + \frac{2}{(50)(51)}\right]$$

$\Rightarrow \underline{I_{REF} = 0.5004 \text{ mA}}$

$$R_1 = \frac{5 - 2(0.7) - (-5)}{0.5004} \Rightarrow \underline{R_1 = 17.19 \text{ k}\Omega}$$

10.15



$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \frac{2}{\beta(2+\beta)}\right)}$$

For $I_0 = 0.8$ mA

$$I_{REF} = (0.8)\left(1 + \frac{2}{25(27)}\right)$$

$\Rightarrow \underline{I_{REF} = 0.8024 \text{ mA}}$

$$R_1 = \frac{18 - 2(0.7)}{0.8024} \Rightarrow \underline{R_1 = 20.69 \text{ k}\Omega}$$

10.16

The analysis is exactly the same as in the text. We have

$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \frac{2}{\beta(2+\beta)}\right)}$$

10.17

$I_0 = 2$ mA, $I_{B2} = \frac{2}{75} = 0.0267$ mA

$I_{C1} = 1$ mA, $I_{B1} = \frac{1}{75} = 0.0133$ mA

$I_{E3} = I_{B1} + I_{B2} = 0.0133 + 0.0267 = 0.04$ mA

$I_{B3} = \frac{I_{E3}}{1+\beta} = \frac{0.04}{76} = 0.000526$ mA

$I_{REF} = I_{C1} + I_{B3} \Rightarrow \underline{I_{REF} = 1.000526 \approx 1 \text{ mA}}$

$R_1 = \frac{10 - 2(0.7)}{I_{REF}} = \frac{8.6}{1} \Rightarrow \underline{R_1 = 8.6 \text{ k}\Omega}$

10.18

a. We have

$R_0 \cong \frac{\beta r_{03}}{2}$

$r_{03} = \frac{V_A}{I_0} \approx \frac{V_A}{I_{REF}} = \frac{80}{0.5} = 160 \text{ k}\Omega$

Then

$R_0 \approx \frac{(80)(160)}{2} \Rightarrow \underline{R_0 \approx 6.4 \text{ M}\Omega}$

b. $\Delta I_0 = \frac{1}{R_0} \cdot \Delta V_C = \frac{5}{6.4} \Rightarrow \underline{\Delta I_0 = 0.781\ \mu\text{A}}$

10.19

$V_{BE} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$

$0.7 = (0.026)\ln\left(\frac{10^{-3}}{I_S}\right) \Rightarrow I_S = 2.03 \times 10^{-15}$ A

At 2 mA, $V_{BE} = (0.026)\ln\left(\frac{2 \times 10^{-3}}{2.03 \times 10^{-15}}\right)$

$= 0.718$ V

$R_1 = \frac{15 - 0.718}{2} \Rightarrow \underline{R_1 = 7.14 \text{ k}\Omega}$

$R_E = \frac{V_T}{I_0}\ln\left(\frac{I_{REF}}{I_0}\right) = \frac{0.026}{0.050} \cdot \ln\left(\frac{2}{0.050}\right)$

$\Rightarrow \underline{R_E = 1.92 \text{ k}\Omega}$

10.20

a. $I_{REF} \approx \frac{10-0.7}{20} = 0.465$ mA

Let $V^- = 0$

$V_{BE} \cong V_T \ln\left(\frac{I_{REF}}{I_S}\right)$

$0.7 = (0.026)\ln\left(\frac{10^{-3}}{I_S}\right) \Rightarrow I_S = 2.03 \times 10^{-15}$ A

Then

$V_{BE} \cong (0.026)\ln\left(\frac{0.465 \times 10^{-3}}{2.03 \times 10^{-15}}\right) = 0.680$ V

Then

$I_{REF} \cong \frac{10-0.680}{20} \Rightarrow \underline{I_{REF} = 0.466 \text{ mA}}$

b. $R_E = \frac{V_T}{I_O}\ln\left(\frac{I_{REF}}{I_O}\right) = \frac{0.026}{0.10}\cdot\ln\left(\frac{0.466}{0.10}\right)$

$\Rightarrow \underline{R_E = 400\Omega}$

10.21

(a) $I_{REF} = \frac{V^+ - V_{BE1} - V^-}{R_1} = \frac{5-0.7-(-5)}{100} \Rightarrow$

$\underline{I_{REF} = 93\ \mu A}$

$I_O R_E = V_T \ln\left(\frac{I_{REF}}{I_O}\right) \Rightarrow I_O(10) = 0.026\ln\left(\frac{93x10^{-3} mA}{I_O}\right)$

By trial and error, $\underline{I_O \cong 6.8\ \mu A}$

$R_o = r_{o2}(1 + g_{m2}R_E')$

Now

$r_{o2} = \frac{30}{6.8} = 4.41\ M\Omega$

$g_{m2} = \frac{0.0068}{0.026} = 0.262\ mA/V$

$r_{\pi 2} = \frac{(100)(0.026)}{0.0068} = 382\ k\Omega$

So

$R_E' = r_{\pi 2}\|R_E = 382\|10 = 9.74\ k\Omega$

Then

$R_o = 4.41[1+(0.262)(9.74)] \Rightarrow \underline{R_o = 15.7\ M\Omega}$

(d) $V_{BE1} - V_{BE2} = I_O R_E = (0.0068)(10) \Rightarrow$

$\underline{V_{BE1} - V_{BE2} = 0.068\ V}$

10.22

$\Delta I_O = \frac{1}{R_O}\cdot\Delta V_C$

$R_O = r_{O2}(1 + g_{m2}R_E')$

$r_{O2} = \frac{V_A}{I_O} = \frac{80}{17.4} = 4.6$ MΩ

$g_{m2} = \frac{I_O}{V_T} = \frac{0.0174}{0.026} = 0.669$ mA/V

$r_{\pi 2} = \frac{(80)(0.026)}{0.0174} = 119.5$ kΩ

$R_E' = R_E\|r_{\pi 2} = 7\|119.5$

$R_E' = 6.61$ kΩ

$R_O = (4.6)[1 + (0.669)(6.61)] \Rightarrow R_O = 24.9$ MΩ

Now

$\Delta I_O = \left(\frac{1}{24.9}\right)(5) \Rightarrow \underline{\Delta I_O = 0.201\ \mu A}$

10.23

$R_O = r_{O2}(1 + g_{m2}R_E')$ where $R_E' = R_E\|r_{\pi 2}$

$r_{O2} = \frac{V_A}{I_O} = \frac{75}{25} = 3$ MΩ

$g_{m2} = \frac{I_O}{V_T} = \frac{0.025}{0.026} = 0.962$ mA/V

$r_{\pi 2} = \frac{\beta V_T}{I_O} = \frac{(80)(0.026)}{0.025} = 83.2$ kΩ

$R_E = \frac{V_T}{I_O}\ln\left(\frac{I_{REF}}{I_O}\right) = \frac{0.026}{0.025}\cdot\ln\left(\frac{0.75}{0.025}\right)$

$= 3.54$ kΩ

$R_E' = 3.54\|83.2 = 3.40$ kΩ

$R_O = 3[1 + (0.962)(3.4)] = 12.8$ MΩ

$\Delta I_O = \frac{1}{R_O}\cdot\Delta V_{C2} = \frac{3}{12.8} = 0.234\ \mu$A

So

$\frac{\Delta I_O}{I_O} = \frac{0.234}{25} \Rightarrow \underline{0.936\%}$

10.24

Let $\underline{R_1 = 5\ k\Omega}$, Then

$I_{REF} = \frac{12-0.7-(-12)}{5} \Rightarrow I_{REF} = 4.66\ mA$

Now

$I_O R_E = V_T \ln\left(\frac{I_{REF}}{I_O}\right) \Rightarrow$

$R_E = \frac{0.026}{0.10}\ln\left(\frac{4.66}{0.10}\right) \Rightarrow \underline{R_E \cong 1\ k\Omega}$

10.25

$$I_{REF} \approx \frac{10 - 0.7 - (-10)}{40} = 0.4825 \text{ mA}$$

$$V_{BE} \cong V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$0.7 = (0.026)\ln\left(\frac{10^{-3}}{I_S}\right) \Rightarrow I_S = 2.03 \times 10^{-15} \text{ A}$$

Now

$$V_{BE} = (0.026)\ln\left(\frac{0.4825 \times 10^{-3}}{2.03 \times 10^{-15}}\right) = 0.681 \text{ V}$$

$\underline{V_{BE1} = 0.681 \text{ V}}$

So

$$I_{REF} \cong \frac{10 - 0.681 - (-10)}{40}$$

$\Rightarrow \underline{I_{REF} = 0.483 \text{ mA}}$

$$I_0 R_E = V_T \ln\left(\frac{I_{REF}}{I_0}\right)$$

$$I_0(12) = (0.026)\ln\left(\frac{0.483}{I_0}\right)$$

By trial and error,

$\Rightarrow \underline{I_0 \cong 8.7\ \mu\text{A}}$

$$V_{BE2} = V_{BE1} - I_0 R_E = 0.681 - (0.0087)(12)$$

$\Rightarrow \underline{V_{BE2} = 0.5766 \text{ V}}$

10.26

$$V_{BE1} + I_{REF}R_{E1} = V_{BE2} + I_0 R_{E2}$$

$$V_{BE1} - V_{BE2} = I_0 R_{E2} - I_{REF}R_{E1}$$

For matched transistors

$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$V_{BE2} = V_T \ln\left(\frac{I_0}{I_S}\right)$$

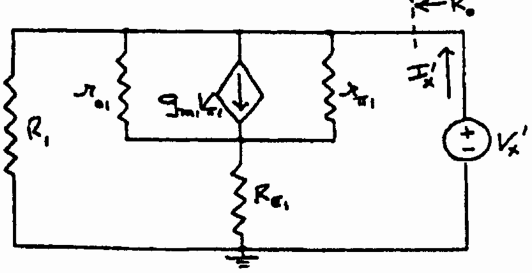
Then

$$\underline{V_T \ln\left(\frac{I_{REF}}{I_0}\right) = I_0 R_{E2} - I_{REF}R_{E1}}$$

Output resistance looking into the collector of Q_2 is increased.

10.27

a.



$$I_X' = \frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} + \frac{V_{\pi1}}{r_{01}} + \frac{V_X'}{R_1} \quad (1)$$

$$V_X' = V_{\pi1} + \left(\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} + \frac{V_{\pi1}}{r_{01}}\right)R_{E1} \quad (2)$$

$$V_X' = V_{\pi1}\left[1 + \left(\frac{1}{r_{\pi1}} + g_{m1} + \frac{1}{r_{01}}\right)R_{E1}\right]$$

$$I_{REF} \cong \frac{10 - 0.7}{13.6 + 5} = 0.5 \text{ mA}$$

$$r_{\pi1} = \frac{(50)(0.026)}{0.5} = 2.6 \text{ k}\Omega$$

$$r_{01} = \frac{75}{0.5} = 150 \text{ k}\Omega$$

$$g_{m1} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

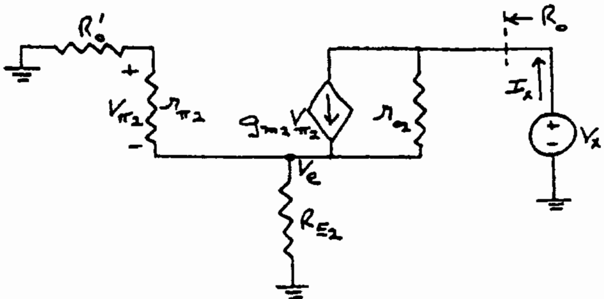
$$V_X' = V_{\pi1}\left[1 + \left(\frac{1}{2.6} + 19.23 + \frac{1}{150}\right)(5)\right]$$

$$\Rightarrow V_{\pi1} = V_X'(0.01009)$$

Then $I_X' = V_X'(0.01009)\left[\frac{1}{2.6} + 19.23 + \frac{1}{150}\right] + \frac{V_X'}{13.6}$

$$I_X' = V_X'(0.1980) + V_X'(0.0735)$$

$$\Rightarrow R_0' = \frac{V_X'}{I_X'} = 3.683 \text{ k}\Omega$$



$$I_X = \frac{V_X - V_e}{r_{02}} + g_{m2}V_{\pi2}$$

$$V_e = I_X\left[R_{E2} \| (r_{\pi2} + R_0')\right]$$

$$V_{\pi2} = -\left(\frac{r_{\pi2}}{r_{\pi2} + R_0'}\right)V_e$$

Then

$$I_X = \frac{V_X}{r_{02}} - \frac{I_X}{r_{02}}\left[R_{E2} \| (r_{\pi2} + R_0')\right] - g_{m2}\left(\frac{r_{\pi2}}{r_{\pi2} + R_0'}\right)(I_X)\left[R_{E2} \| (r_{\pi2} + R_0')\right]$$

$$R_{E1} = R_{E2} \Rightarrow I_{REF} = I_0 \Rightarrow r_{\pi2} = 2.6 \text{ k}\Omega$$

$$r_{02} = 150 \text{ k}\Omega$$

$$g_{m2} = 19.23 \text{ mA/V}$$

$$I_X = \frac{V_X}{150} - \frac{I_X}{150}[5\|(2.6+3.68)]$$
$$-(19.23)\left(\frac{2.6}{2.6+3.68}\right)(I_X)[5\|(2.6+3.68)]$$
$$I_X = V_X(0.00666) - I_X(0.01853) - I_X(22.13)$$
$$I_X(23.148) = V_X(0.00666)$$
$$\Rightarrow R_0 = \frac{V_X}{I_X} = 3.48\ \text{M}\Omega$$

b. When $R_{E1} = R_{E2} = 0$

$R_0 \cong r_{02} = 150\ \text{k}\Omega$

10.28

Assume all transistors are matched.

a.

$$2V_{BE1} = V_{BE3} + I_0 R_E$$
$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$
$$V_{BE3} = V_T \ln\left(\frac{I_0}{I_S}\right)$$

$$2V_T \ln\left(\frac{I_{REF}}{I_S}\right) - V_T \ln\left(\frac{I_0}{I_S}\right) = I_0 R_E$$
$$V_T\left[\ln\left(\frac{I_{REF}}{I_S}\right)^2 - \ln\left(\frac{I_0}{I_S}\right)\right] = I_0 R_E$$
$$V_T \ln\left(\frac{I_{REF}^2}{I_0 I_S}\right) = I_0 R_E$$

b. $V_{BE} = 0.7$ V at 1 mA $\Rightarrow 10^{-3} = I_S \exp\left(\frac{0.7}{0.026}\right)$

or $I_S = 2.03 \times 10^{-15}$ A

V_{BE} at 0.1 mA

$$\Rightarrow V_{BE} = (0.026)\ln\left(\frac{0.1\times 10^{-3}}{2.03\times 10^{-15}}\right) = 0.640\ \text{V}$$

Since $I_0 = I_{REF}$, then

$$V_{BE} = I_0 R_E \Rightarrow R_E = \frac{0.640}{0.1}$$

or $R_E = 6.4\ \text{k}\Omega$

10.29

$$I_{REF} = \frac{10-0.7}{R_1} = 0.5 \Rightarrow R_1 = 18.6\ \text{k}\Omega$$
$$I_{02}R_{E2} = V_T \ln\left(\frac{I_{REF}}{I_{01}}\right)$$
$$R_{E2} = \frac{0.026}{0.010}\cdot\ln\left(\frac{0.50}{0.01}\right) \Rightarrow R_{E2} = 10.17\ \text{k}\Omega$$
$$R_{E3} = \frac{0.026}{0.030}\cdot\ln\left(\frac{0.50}{0.03}\right) \Rightarrow R_{E3} = 2.438\ \text{k}\Omega$$

$$V_{BE2} = 0.7 - I_{02}R_{E2} = 0.7 - (0.01)(10.17)$$
$$\Rightarrow V_{BE2} = 0.598\ \text{V}$$
$$V_{BE3} = 0.7 - I_{03}R_{E3} = 0.7 - (0.03)(2.438)$$
$$\Rightarrow V_{BE3} = 0.627\ \text{V}$$

10.30

(a) $V_{BE1} = V_{BE2}$

$$I_{REF} = \frac{V^+ - 2V_{BE1} - V^-}{R_1 + R_2}$$

Now

$$2V_{BE1} + I_{REF}R_2 = V_{BE3} + I_O R_E$$

or

$$I_O R_E = 2V_{BE1} - V_{BE3} + I_{REF}R_2$$

We have

$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right) \text{ and } V_{BE3} = V_T \ln\left(\frac{I_O}{I_S}\right)$$

(b) Let $R_1 = R_2$ and $I_O = I_{REF} \Rightarrow V_{BE1} = V_{BE3} \equiv V_{BE}$

Then

$$V_{BE} = I_O R_E - I_{REF}R_2 = I_O(R_E - R_2)$$

so

$$I_{REF} = I_O = \frac{V^+ - V^- - 2I_O(R_E - R_2)}{2R_2}$$
$$= \frac{V^+ - V^-}{2R_2} - I_O\left(\frac{R_E}{R_2}\right) + I_O$$

Then

$$I_O = \frac{V^+ - V^-}{2R_E}$$

(c) Want $I_O = 0.5\ mA$

So $R_E = \dfrac{5-(-5)}{2(0.5)} \Rightarrow R_E = 10\ k\Omega$

$$2R_2 = \frac{5 - 2(0.7) - (-5)}{0.5} = 17.2\ k\Omega$$

Then $R_1 = R_2 = 8.6\ k\Omega$

10.31

a. $I_{REF} = \dfrac{20 - 0.7 - 0.7}{12} = 1.55$ mA

$I_{01} = 2I_{REF} = 3.1$ mA

$I_{02} = I_{REF} = 1.55$ mA

$I_{03} = 3I_{REF} = 4.65$ mA

b. $V_{CE1} = -I_{01}R_{C1} - (-10) = -(3.1)(2) + 10$

$\Rightarrow V_{CE1} = 3.8$ V

$V_{EC2} = 10 - I_{02}R_{C2} = 10 - (1.55)(3)$

$\Rightarrow V_{EC2} = 5.35$ V

$V_{EC3} = 10 - I_{03}R_{C3} = 10 - (4.65)(1)$

$\Rightarrow V_{EC3} = 5.35$ V

10.32

a. Ist approximation

$I_{REF} \cong \frac{20 - 1.4}{8} = 2.325$ mA

Now $V_{BE} - 0.7 = (0.026)\ln\left(\frac{2.32}{1}\right)$

$\Rightarrow V_{BE} = V_{EB} = 0.722$ V

Then 2nd approximation

$I_{REF} \cong \frac{20 - 2(0.722)}{8} = 2.32$ mA

$I_{01} = 2I_{REF} = 4.64$ mA

$I_{02} = I_{REF} = 2.32$ mA

$I_{03} = 3I_{REF} = 6.96$ mA

b. At the edge of saturation, $V_{CE} = V_{BE} = 0.722$ V

$R_{C1} = \frac{0 - 0.722 - (-10)}{4.64} \Rightarrow \underline{R_{C1} = 2.0\text{ k}\Omega}$

$R_{C2} = \frac{10 - 0.722}{2.32} \Rightarrow \underline{R_{C2} = 4.0\text{ k}\Omega}$

$R_{C3} = \frac{10 - 0.722}{6.96} \Rightarrow \underline{R_{C3} = 1.33\text{ k}\Omega}$

10.33

Ist approximation

$I_{REF} = \frac{10 - 0.7}{6.3 + 3} = 1$ mA

$\Rightarrow V_B = 0.7$ V as assumed

$V_{RER} = I_{REF} \cdot R_{ER} = (1)(3) = 3$ V

$V_{RE1} = 3\text{ V} \Rightarrow R_{E1} = \frac{V_{RE1}}{I_{01}} = \frac{3}{1} \Rightarrow \underline{R_{E1} = 3\text{ k}\Omega}$

$V_{RE2} = 3\text{ V} \Rightarrow R_{E2} = \frac{V_{RE2}}{I_{02}} = \frac{3}{2} \Rightarrow \underline{R_{E2} = 1.5\text{ k}\Omega}$

$V_{RE3} = 3\text{ V} \Rightarrow R_{E3} = \frac{V_{RE3}}{I_{03}} = \frac{3}{4} \Rightarrow \underline{R_{E3} = 0.75\text{ k}\Omega}$

$I_{01} = 1$ mA

$I_{02} = 2$ mA

$I_{03} = 4$ mA

10.34

$V_{GS} = V_{TN1} + \sqrt{\frac{I_{REF}}{K_{n1}}} = 1 + \sqrt{\frac{200}{250}} = 1.89\ V = V_{DS1}$

$\frac{I_0}{I_{REF}} = \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}}$

a. $V_{DS2} = 2$ V

$I_0 = (200)\left[\frac{1 + (0.02)(2)}{1 + (0.02)(1.89)}\right] \Rightarrow \underline{I_0 \cong 200\ \mu\text{A}}$

b. $V_{DS2} = 4$ V

$I_0 = (200)\left[\frac{1 + (0.02)(4)}{1 + (0.02)(1.89)}\right] \Rightarrow \underline{I_0 \cong 208\ \mu\text{A}}$

c. $V_{DS2} = 6$ V

$I_0 = (200)\left[\frac{1 + (0.02)(6)}{1 + (0.02)(1.89)}\right] \Rightarrow \underline{I_0 \cong 216\ \mu\text{A}}$

10.35

(a) $V_{GS} = V_{TN1} + \sqrt{\frac{I_{REF}}{K_{n1}}} = 1 + \sqrt{\frac{0.5}{0.5}} = 2\ V$

$I_O = K_{n2}\left(\sqrt{\frac{I_{REF}}{K_{n1}}}\right)^2 = K_{n2}\left(\frac{I_{REF}}{K_{n1}}\right)$

$I_0(\max) = (0.5)(1.05)\left(\frac{0.5}{0.5}\right) \Rightarrow I_0(\max) = 0.525$ mA

$I_0(\min) = (0.5)(0.95)\left(\frac{0.5}{0.5}\right) \Rightarrow I_0(\min) = 0.475$ mA

So

$0.475 \le I_0 \le 0.525$ mA

(b) $I_O = K_{n2}\left[\sqrt{\frac{I_{REF}}{K_{n1}}} + V_{TN1} - V_{TN2}\right]^2$

$I_0(\min) = (0.5)\left[\sqrt{\frac{0.5}{0.5}} + 1 - 1.05\right]^2$

$\Rightarrow I_0(\min) = 0.451$ mA

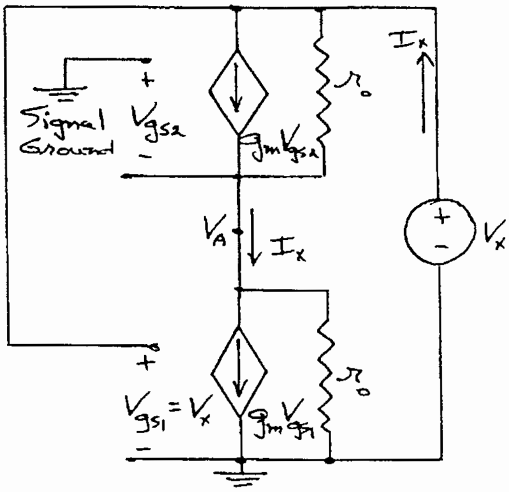
$I_0(\max) = (0.5)\left[\sqrt{\frac{0.5}{0.5}} + 1 - 0.95\right]^2$

$\Rightarrow I_0(\max) = 0.551$ mA

So

$0.451 \le I_0 \le 0.551$ mA

10.36



(1) $I_x = \dfrac{V_x - V_A}{r_o} + g_m V_{gs2}$

(2) $I_x = \dfrac{V_A}{r_o} + g_m V_{gs1}$

$V_{gs1} = V_x$, $V_{gs2} = -V_A$

So

(1) $I_x = \dfrac{V_x}{r_o} - V_A\left(\dfrac{1}{r_o} + g_m\right)$

(2) $I_x = \dfrac{V_A}{r_o} + g_m V_x \Rightarrow V_A = r_o\left[I_x - g_m V_x\right]$

Then

$$I_x = \frac{V_x}{r_o} - r_o\left(I_x - g_m V_x\right)\left(\frac{1}{r_o} + g_m\right)$$

$$I_x = \frac{V_x}{r_o} - r_o\left[\frac{I_x}{r_o} + g_m I_x - \frac{g_m}{r_o}\cdot V_x - g_m^2 V_x\right]$$

$$I_x = \frac{V_x}{r_o} - I_x - g_m r_o I_x + g_m V_x + g_m^2 r_o V_x$$

$$I_x\left[2 + g_m r_o\right] = V_x\left[\frac{1}{r_o} + g_m + g_m^2 r_o\right]$$

Since $g_m >> \dfrac{1}{r_o}$

$I_x\left[2 + g_m r_o\right] \cong V_x\left(g_m\right)\left(1 + g_m r_o\right)$

Then

$$\frac{V_x}{I_x} = R_o = \frac{2 + g_m r_o}{g_m\left(1 + g_m r_o\right)}$$

Usually, $g_m r_o >> 2$, so that

$$R_o \cong \frac{1}{g_m}$$

10.37

(a) $V_{DS}(sat) = V_{GS} - V_{TN}$

or $V_{GS} = V_{DS}(sat) + V_{TN} = 0.2 + 0.8 = 1.0$

$$I_D = \frac{k_n'}{2}\left(\frac{W}{L}\right)\left(V_{GS} - V_{TN}\right)^2$$

$$50 = 48\left(\frac{W}{L}\right)(0.2)^2 \Rightarrow \underline{\left(\frac{W}{L}\right) = 26}$$

(b) $V_{GS5} - V_{TN} = 2\left(V_{GS} - V_{TN}\right)$

$V_{GS5} = 0.8 + 2(0.2) \Rightarrow \underline{V_{GS5} = 1.2\ V}$

(c) $V_{D1}(\min) = 2V_{DS}(sat) = 2(0.2) \Rightarrow$

$\underline{V_{D1}(\min) = 0.4\ V}$

10.38

$V_{DS2}(sat) = 2\ V = V_{GS2} - V_{TN2} = V_{GS2} - 1.5 \Rightarrow$

$V_{GS2} = 3.5\ V$

$$I_O = \left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_2 \left(V_{GS2} - V_{TN2}\right)^2$$

$$250 = (20)\left(\frac{W}{L}\right)_2 (3.5 - 1.5)^2 \Rightarrow$$

$$\underline{\left(\frac{W}{L}\right)_2 = 3.125}$$

$$I_{REF} = \left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_1 \left(V_{GS2} - V_{TN1}\right)$$

$$100 = (20)\left(\frac{W}{L}\right)_2 (3.5 - 1.5)^2 \Rightarrow$$

$$\underline{\left(\frac{W}{L}\right)_1 = 1.25}$$

Now $V_{GS3} = 10 - V_{GS2} = 10 - 3.5 = 6.5\ V$

So $100 = (20)\left(\dfrac{W}{L}\right)_3 (6.5 - 1.5)^2 \Rightarrow$

$$\underline{\left(\frac{W}{L}\right)_3 = 0.2}$$

10.39

a. From Equation (10.50),

$$V_{GS1} = V_{GS2} = \left(\frac{\sqrt{\frac{5}{25}}}{1 + \sqrt{\frac{5}{25}}}\right)(5) + \left(\frac{1 - \sqrt{\frac{5}{25}}}{1 + \sqrt{\frac{5}{25}}}\right)(0.5)$$

$$= \left(\frac{0.447}{1 + 0.447}\right)(5) + \left(\frac{1 - 0.447}{1 + 0.447}\right)(0.5)$$

$V_{GS1} = V_{GS2} = 1.74$ V

$I_{REF} \cong K_{n1}(V_{GS1} - V_{TN})^2 = (18)(25)(1.74 - 0.5)^2 \Rightarrow$
$\underline{I_{REF} = 0.692\ mA}$

b. $I_O = \left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2 (1 + \lambda V_{DS2})$

$I_O = (18)(15)(1.74 - 0.5)^2[1 + (0.02)(2)]$
$= (415)(104) \quad \Rightarrow \underline{I_O = 0.432\ \text{mA}}$

c. $I_O = (415)[1 + (0.02)(4)]$

$\Rightarrow \underline{I_O = 0.448\ \text{mA}}$

10.40

(a) $I_{REF} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_1 (V_{SG1} + V_{TP})^2$
$= \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$

But $V_{SG3} = 3 - V_{SG1}$
So

$25(V_{SG1} - 0.4)^2 = 5(3 - V_{SG1} - 0.4)^2$
which yields $\underline{V_{SG1} = 1.08\ V}$ and $\underline{V_{SG3} = 1.92\ V}$

$I_{REF} = 20(25)(1.08 - 0.4)^2 \Rightarrow \underline{I_{REF} = 231\ \mu A}$

$\frac{I_O}{I_{REF}} = \frac{(W/L)_2}{(W/L)_1} = \frac{15}{25} = 0.6$

Then $\underline{I_O = (0.6)(231) = 139\ \mu A}$

(b) $V_{DS2}(sat) = 1.08 - 0.4 = 0.68\ V$
$V_R = 3 - 0.68 = 2.32 = I_O R$
then
$R = \frac{2.32}{0.139} \Rightarrow \underline{R = 16.7\ k\Omega}$

10.41

$V_{SD2}(sat) = 0.25 = V_{SG} + V_{TP} = V_{SG} - 0.4 \Rightarrow$
$V_{SG2} = 0.65\ V$

$I_O = \frac{k_p'}{2}\left(\frac{W}{L}\right)_2 (V_{SG2} + V_{TP})^2$

$25 = \frac{40}{2}\left(\frac{W}{L}\right)_2 (0.65 - 0.4)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_2 = 20}$

$I_{REF} = 75\ \mu A = \frac{(W/L)_1}{(W/L)_2} \cdot I_O \Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 60}$

$I_{REF} = \frac{k_p'}{2}\left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$

$V_{SG3} = 3 - 0.65 = 2.35\ V$
Then

$75 = \frac{40}{2}\left(\frac{W}{L}\right)_3 (2.35 - 0.4)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_3 = 0.986}$

10.42

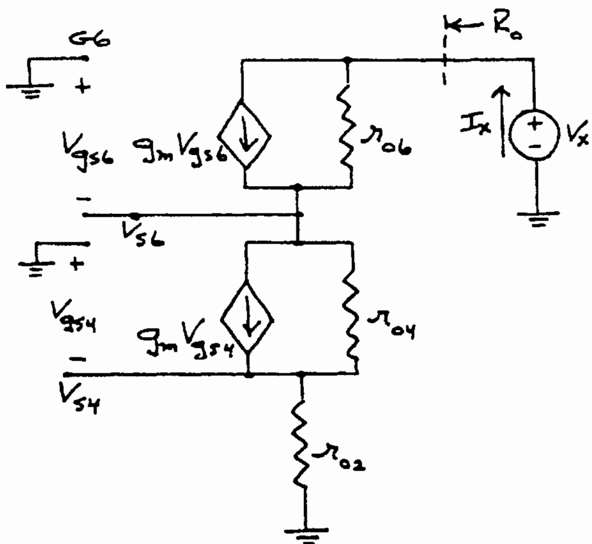
a. $I_{REF} = K_n(V_{GS} - V_{TN})^2$

$100 = 100(V_{GS} - 2)^2 \Rightarrow V_{GS} = 3\ \text{V}$
For $V_{D4} = -3\ \text{V}$, $\underline{I_O \approx 100\ \mu\text{A}}$

b. $R_o = r_{o4} + r_{o2}(1 + g_m r_{o4})$

$r_{o2} = r_{o4} = \frac{1}{\lambda I_O} = \frac{1}{(0.02)(0.1)} = 500\ \text{k}\Omega$
$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.1)(3 - 2) = 0.2\ mA/V$
$R_o = 500 + 500[1 + (0.2)(500)]$
$R_o = 51\ \text{M}\Omega$
$\Delta I_O = \frac{1}{R_o} \cdot \Delta V_{D4} = \frac{6}{51} \Rightarrow \underline{\Delta I_O = 0.118\ \mu\text{A}}$

10.43



$V_{gs4} = -I_X r_{o2}$
$V_{S6} = (I_X - g_m V_{gs4}) r_{o4} + I_X r_{o2}$
$= (I_X + g_m I_X r_{o2}) r_{o4} + I_X r_{o2}$
$V_{S6} = I_X[r_{o2} + (1 + g_m r_{o2}) r_{o4}] = -V_{gs6}$

$I_X = g_m V_{gs6} + \frac{V_X - V_{SG}}{r_{o6}} = \frac{V_X}{r_{o6}} - V_{S6}\left(g_m + \frac{1}{r_{o6}}\right)$

$I_X = \frac{V_X}{r_{o6}} - I_X\left(g_m + \frac{1}{r_{o6}}\right)[r_{o2} + (1 + g_m r_{o2}) r_{o4}]$

$I_X\left\{1 + \left(g_m + \frac{1}{r_{o6}}\right)[r_{o2} + (1 + g_m r_{o2}) r_{o4}]\right\} = \frac{V_X}{r_{o6}}$

$\frac{V_X}{I_X} = R_o = r_{o6} + (1 + g_m r_{o6})[r_{o2} + (1 + g_m r_{o2}) r_{o4}]$
$I_O \approx I_{REF} = 0.2\ \text{mA} = 0.2(V_{GS} - 1)^2$
$V_{GS} = 2\ \text{V}$

$g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.2)(2-1) = 0.4\ mA/V$

$r_{02} = r_{04} = r_{06} = \dfrac{1}{\lambda I_0} = \dfrac{1}{(0.02)(0.2)} = 250\ \text{k}\Omega$

$R_0 = 250 + [1 + (0.4)(250)]$

$\times \{250 + [1 + (0.4)(250)](250)\}$

$R_0 = 2575750\ \text{k}\Omega$

$\Rightarrow \underline{R_0 = 2.58 \times 10^9\ \Omega}$

10.44

$$\frac{k_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2 = \frac{k_n'}{2}\left(\frac{W}{L}\right)_3 (V_{GS3} - V_{TN})^2$$

$$= \frac{k_p'}{2}\left(\frac{W}{L}\right)_4 (V_{SG4} + V_{TP})^2$$

(1) $50(20)(V_{GS1} - 0.5)^2 = 50(5)(V_{GS3} - 0.5)^2$

(2) $50(20)(V_{GS1} - 0.5)^2 = 20(10)(V_{SG4} - 0.5)^2$

(3) $V_{SG4} + V_{GS3} + V_{GS1} = 6$

From (1) $4(V_{GS1} - 0.5)^2 = (V_{GS3} - 0.5)^2 \Rightarrow$

$V_{GS3} = 2(V_{GS1} - 0.5) + 0.5$

From (2) $5(V_{GS1} - 0.5)^2 = (V_{GS4} - 0.5)^2 \Rightarrow$

$V_{GS4} = \sqrt{5}(V_{GS1} - 0.5) + 0.5$

Then (3) becomes

$\sqrt{5}(V_{GS1} - 0.5) + 0.5 + 2(V_{GS1} - 0.5) + 0.5 + V_{GS1} = 6$

which yields $V_{GS1} = 1.36\,V$ and

$V_{GS3} = 2.22\,V$, $V_{SG4} = 2.42\,V$

Then

$$I_{REF} = \frac{k_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2 = 50(20)(1.36 - 0.5)^2$$

or $\underline{I_{REF} = I_O = 0.740\ mA}$

$V_{GS1} = V_{GS2} = 1.36\,V$

$V_{DS2}(sat) = V_{GS2} - V_{TN} = 1.36 - 0.5 \Rightarrow$

$\underline{V_{DS2}(sat) = 0.86\,V}$

10.45

$V_{DS2}(sat) = 0.5\,V = V_{GS2} - V_{TN} = V_{GS2} - 0.5 \Rightarrow$

$V_{GS2} = 1\,V$

$$I_O = 50\ \mu A = \frac{k_n'}{2}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2$$

$$= 50\left(\frac{W}{L}\right)_2 (1 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_2 = 4}$$

$V_{GS1} = V_{GS2} = 1\,V \Rightarrow$

$$I_{REF} = 150 = \frac{k_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2$$

$$= 50\left(\frac{W}{L}\right)_1 (1 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 12}$$

$V_{GS3} + V_{SG4} + V_{GS1} = 6$

$2V_{GS3} = 6 - 1 = 5\,V \Rightarrow V_{GS3} = 2.5\,V$

$$I_{REF} = 150 = 50\left(\frac{W}{L}\right)_3 (2.5 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_3 = 0.75}$$

$$I_{REF} = \frac{k_p'}{2}\left(\frac{W}{L}\right)_4 (V_{SG4} + V_{TP})^2$$

$$150 = 20\left(\frac{W}{L}\right)_4 (2.5 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_4 1.88}$$

10.46

a. As a first approximation

$I_{REF} = 80 = 80(V_{GS1} - 1)^2 \Rightarrow V_{GS1} = 2\text{ V}$

Then $V_{DS1} \cong 2(2) = 4\text{ V}$

The second approximation

$80 = 80(V_{GS1} - 1)^2[1 + (0.02)(4)]$

Or $\dfrac{80}{86.4} = (V_{GS1} - 1)^2 \Rightarrow V_{GS1} = 1.962$

Then

$I_O = K_n(V_{GS1} - V_{TN})^2(1 + \lambda_n V_{GS1})$

$= 80(1.962 - 1)^2[1 + (0.02)(1.962)]$

Or $\underline{I_O = 76.94\ \mu\text{A}}$

b. From a PSpice analysis, $I_O = 77.09\ \mu$A for $V_{D3} = -1$ V and $I_O = 77.14\ \mu$A for $V_{D3} = 3$ V. The change is $\Delta I_O \approx 0.05\ \mu$A or 0.065%.

10.47

a. For a first approximation,

$I_{REF} = 80 = 80(V_{GS4} - 1)^2 \Rightarrow V_{GS4} = 2\text{ V}$

As a second approximation

$I_{REF} = 80 = 80(V_{GS4} - 1)^2[1 + (0.02)(2)]$

Or $V_{GS4} = 1.98\text{ V} = V_{GS1}$

$I_O = K_n(V_{GS2} - V_{TN})^2(1 + \lambda V_{GS2})$

To a very good approximation

$I_O = 80\ \mu$A

b. From a PSpice analysis, $I_O = 80.00\ \mu$A for $V_{D3} = -1$ V and the output resistance is $R_o = 76.9\ \text{M}\Omega$. Then

$$\Delta I_O = \frac{1}{R_o} \cdot V_{D3} = \frac{4}{76.9} = 0.052\ \mu\text{A}$$

or a change of 0.065%.

10.48

(a) $K_{n1} = \frac{k_n'}{2}\left(\frac{W}{L}\right)_1 = 50(5) = 250\ \mu A/V^2$

$$R = \frac{1}{\sqrt{K_{n1}I_{D1}}}\left(1 - \sqrt{\frac{(W/L)_1}{(W/L)_2}}\right)$$

$$= \frac{1}{\sqrt{(0.25)(0.05)}}\left(1 - \sqrt{\frac{5}{50}}\right) = (8.94)(0.684)$$

$R = 6.11\ k\Omega$

(b) $V^+ - V^- = V_{SD3}(sat) + V_{GS1}$

$V_{SD3}(sat) = V_{SG3} + V_{TP}$

$I_{D1} = 50 = 20(5)(V_{SG3} - 0.5)^2 \Rightarrow \quad V_{SG3} = 1.21\ V$

Then

$V_{SD3}(sat) = 1.21 - 0.5 = 0.71\ V$

Also

$I_{D1} = 50 = 50(5)(V_{GS1} - 0.5)^2 \Rightarrow \quad V_{GS1} = 0.947\ V$

Then

$\underline{(V^+ - V^-)_{\min} = 0.71 + 0.947 = 1.66\ V}$

(c) $I_{O1} = 25 = 50\left(\frac{W}{L}\right)_5 (0.947 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_5 = 2.5$

$I_{O2} = 75 = 20\left(\frac{W}{L}\right)_6 (1.21 - 0.5)^2 \Rightarrow \quad \left(\frac{W}{L}\right)_6 = 7.44$

10.49

$V_{GS3} = \frac{1}{3}(5) = 1.667$ V

$$I_{REF} = \left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_3 (V_{GS3} - V_{TN})^2$$

$$100 = (20)\left(\frac{W}{L}\right)_3 (1.667 - 1)^2$$

$$\Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \left(\frac{W}{L}\right)_5 = 11.2$$

$$I_{O1} = \left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_1 (V_{GS3} - V_{TN})^2$$

Or $\frac{I_{REF}}{I_{O1}} = \frac{\left(\frac{W}{L}\right)_3}{\left(\frac{W}{L}\right)_1}$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{I_{O1}}{I_{REF}}\right)\left(\frac{W}{L}\right)_3 = \left(\frac{0.2}{0.1}\right)(11.2)$$

$$\Rightarrow \left(\frac{W}{L}\right)_1 = 22.4$$

And $\left(\frac{W}{L}\right)_2 = \left(\frac{I_{O2}}{I_{REF}}\right)\left(\frac{W}{L}\right)_3 = \left(\frac{0.3}{0.1}\right)(11.2)$

$$\Rightarrow \left(\frac{W}{L}\right)_2 = 33.6$$

10.50

$$I_{REF} = \frac{24 - V_{SGP} - V_{GSN}}{R}$$

Also

$I_{REF} = 40(1)(V_{GSN} - 1.2)^2$

$I_{REF} = 18(1)(V_{SGP} - 1.2)^2$

Then

$\sqrt{40}(V_{GSN} - 1.2) = \sqrt{18}(V_{SGP} - 1.2)$

which yields

$$V_{SGP} = \frac{6.325}{4.243}(V_{GSN} - 1.2) + 1.2$$

Then

$$\left[0.040(V_{GSN} - 1.2)^2\right]\cdot R = 24 - V_{GSN} - 1.49(V_{GSN} - 1.2) - 1.2$$

which yields

$V_{GSN} = 2.69\ V$ and $V_{SGP} = 3.42\ V$

Now

$$I_{REF} = \frac{24 - 3.42 - 2.69}{200} \Rightarrow \quad I_{REF} = 89.5\ \mu A$$

$I_1 = \frac{89.5}{5} = 17.9\ \mu A$

$I_2 = (1.25)(89.5) = 112\ \mu A$

$I_3 = (0.8)(89.5) = 71.6\ \mu A$

$I_4 = 4(89.5) = 358\ \mu A$

10.51

We have $V_{GSN} = 2.69\ V$ and $V_{SGP} = 3.42\ V$

So

$$I_{REF} = \frac{10 - 2.69 - 3.42}{R} = \frac{3.89}{200} \Rightarrow \quad I_{REF} = 19.45\ \mu A$$

Then

$I_1 = (0.2)(19.45) = 3.89\ \mu A$

$I_2 = (1.25)(19.45) = 24.3\ \mu A$

$I_3 = (0.8)(19.45) = 15.56\ \mu A$

$I_4 = 4(19.45) = 77.8\ \mu A$

10.52

For $v_{GS} = 0$, $i_D = I_{DSS}(1 + \lambda v_{DS})$

a. $V_D = -5$ V, $v_{DS} = 5$

$i_D = (2)[1 + (0.05)(5)]$

$\Rightarrow \underline{i_D = 2.5 \text{ mA}}$

b. $V_D = 0$, $v_{DS} = 10$

$i_D = (2)[1 + (0.05)(10)]$

$\Rightarrow \underline{i_D = 3 \text{ mA}}$

c. $V_D = 5$ V, $v_{DS} = 15$ V

$i_D = (2)[1 + (0.05)(15)]$

$\Rightarrow \underline{i_D = 3.5 \text{ mA}}$

10.54

$$I_0 = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2 = 4\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$\frac{V_{GS}}{V_P} = 1 - \sqrt{\frac{2}{4}} = 0.293$$

So $V_{GS} = (0.293)(-4) = -1.17$ V

Then $I_0 = \frac{V_S}{R}$ and $V_S = -V_{GS}$

$$R = \frac{-V_{GS}}{I_0} = -\frac{(-1.17)}{2} \Rightarrow \underline{R = 0.585\ k\Omega}$$

10.55

a. $I_{REF} = I_{S1}\exp\left(\frac{V_{EB1}}{V_T}\right)$

or $V_{EB1} = V_T \ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026)\ln\left(\frac{1\times10^{-3}}{5\times10^{-13}}\right)$

$\Rightarrow \underline{V_{EB1} = 0.5568}$

b. $R_1 = \frac{5 - 0.5568}{1} \Rightarrow \underline{R_1 = 4.44\ k\Omega}$

c. From Equation (10.72) and letting $V_{CE0} = V_{EC2} = 2.5$ V

$$10^{-12}\exp\left(\frac{V_I}{V_T}\right)\left[1+\frac{2.5}{120}\right] = 10^{-3}\left(\frac{1+\frac{2.5}{80}}{1+\frac{0.5568}{80}}\right)$$

$$1.0208x10^{-12}\exp\left(\frac{V_I}{V_T}\right) = (10^{-3})\left(\frac{1.03125}{1.00696}\right)$$

Then

$V_I = 0.026\ln(1.003613x10^9)$

So $V_I = 0.5389\,V$

d. $A_v = \frac{-(1/V_T)}{(1/V_{AN}) + (1/V_{AP})}$

$$A_v = \frac{-\frac{1}{0.026}}{\frac{1}{120}+\frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125}$$

$\underline{A_v = -1846}$

10.56

a. $V_{BE} = V_T \ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026)\ln\left(\frac{0.5\times10^{-3}}{10^{-12}}\right)$

$\Rightarrow \underline{V_{BE} = 0.5208}$

b. $R_1 = \frac{5 - 0.5208}{0.5} \Rightarrow \underline{R_1 = 8.96\ k\Omega}$

c. From Equation (10.72) applies with slight modifications

$$I_{S0}\exp\left(\frac{V_{EB0}}{V_T}\right)\left[1 + \frac{V_{EC0}}{V_{AP}}\right] = I_{REF}\cdot\left(\frac{1 + \frac{V_{CE2}}{V_{AN}}}{1 + \frac{V_{BE2}}{V_{AN}}}\right)$$

$$(5\times10^{-13})\left[\exp\left(\frac{V_{EB0}}{V_T}\right)\right]\left(1 + \frac{2.5}{80}\right)$$

$$= (0.5\times10^{-3})\cdot\left(\frac{1 + \frac{2.5}{120}}{1 + \frac{0.5208}{120}}\right)$$

$$5.15625\times10^{-13}\exp\left(\frac{V_{EB0}}{V_T}\right) = (0.5\times10^{-3})\cdot\frac{1.02083}{1.00434}$$

$V_{EB0} = 0.5384 \Rightarrow V_I = 5 - 0.5384$

$\Rightarrow \underline{V_I = 4.462\ V}$

d. $A_v = \frac{-(1/V_T)}{(1/V_{AN}) + (1/V_{AP})}$

$$A_v = \frac{-\frac{1}{0.026}}{\frac{1}{120}+\frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125}$$

$\underline{A_v = -1846}$

10.57

Ignore $(W/L)_3 = 5$ specification

a. M_1 and M_2 matched, so we must have

$V_{SD2} = V_{SG} = V_{SG3} = V_{DS0} = 2.5$ V

For M_1 and M_3:

$$I_{REF} = \left(\frac{1}{2}\mu_p C_{ox}\right)\left(\frac{W}{L}\right)_1 (V_{SG} + V_{TP})^2(1 + \lambda_p V_{SD})$$

$$100 = 10\left(\frac{W}{L}\right)_1 (2.5 - 1)^2[1 + (0.02)(2.5)]$$

$$\Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 4.23 = \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_2}$$

For M_0:

$$I_O = \left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_0 (V_{GS} - V_{TN})^2(1 + \lambda_n V_{DS})$$

$$100 = 20\left(\frac{W}{L}\right)_0 (2 - 1)^2[1 + (0.02)(2.5)]$$

$$\Rightarrow \underline{\left(\frac{W}{L}\right)_0 = 4.76}$$

b. $r_{on} = r_{op} = \dfrac{1}{\lambda I_0} = \dfrac{1}{(0.02)(0.1)} = 500\ \text{k}\Omega$

$$g_m = 2\sqrt{K_n I_O} = 2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right)_o I_O}$$
$$= 2\sqrt{(0.02)(4.76)(0.1)}$$
$$g_m = 0.195\ \text{mA/V}$$
$$A_v = -g_m(r_{on}\|r_{op}) = -(0.195)(500\|500)$$
$$\Rightarrow \underline{A_v = -48.75}$$

10.58

a. $I_{REF} = K_{p1}(V_{SG} + V_{TP})^2$

$$100 = 100(V_{SG} - 1)^2 \Rightarrow \underline{V_{SG} = 2\ \text{V}}$$

b. $V_{DS0} = V_{DS2} = 5\ \text{V}$

From Equation (10.87)

$$100(V_I - 1)^2 = 100[1 + (0.02)(5)] \times [1 - (0.02)(2)][1 - (0.02)(5)]$$
$$(V_I - 1)^2 = (1.1)(0.96)(0.90) = 0.9504$$
$$\Rightarrow \underline{V_I = 1.975\ \text{V}}$$

c. $A_v = -g_m(r_{on}\|r_{op})$

$$r_{on} = r_{op} = \frac{1}{\lambda I_{REF}} = \frac{1}{(0.02)(0.1)} = 500\ \text{k}\Omega$$
$$g_m = 2\sqrt{K_n I_{REF}} = 2\sqrt{(0.1)(0.1)} = 0.2\ mA/V$$
$$A_v = -(0.2)(500\|500)$$
$$\Rightarrow \underline{A_v = -50}$$

10.59

a. Using the results of problem 10.27, we find the resistance looking into the collector of Q_2 to be

$$R_o = r_{o2}\left[1 + \frac{R_E\|(r_{\pi 2} + R_0')}{r_{o2}} + g_{m2}\left(\frac{r_{\pi 2}}{r_{\pi 2} + R_0'}\right)[R_E\|(r_{\pi 2} + R_0')]\right]$$

where R_0' is the resistance from the base of Q_2 toward Q_1. We found

$$\frac{1}{R_0'} = \frac{1}{R_1} + \frac{\dfrac{1}{r_{\pi 1}} + g_{m1} + \dfrac{1}{r_{o1}}}{1 + \left(\dfrac{1}{r_{\pi 1}} + g_{m1} + \dfrac{1}{r_{o1}}\right)(R_E)}$$

b. $A_v = -g_{m0}(r_0\|R_L\|R_o)$

10.60

Output resistance of Wilson source

$$R_0 \cong \frac{\beta r_{03}}{2}$$

Then

$$A_v = -g_m(r_0\|R_0)$$

$$r_{03} = \frac{V_{AP}}{I_{REF}} = \frac{80}{0.2} = 400\ \text{k}\Omega$$
$$r_0 = \frac{V_{AN}}{I_{REF}} = \frac{120}{0.2} = 600\ \text{k}\Omega$$
$$g_m = \frac{I_{REF}}{V_T} = \frac{0.2}{0.026} = 7.69\ \text{mA/V}$$

$$A_v = -7.69\left[600 \,\Big\|\, \frac{(80)(400)}{2}\right] = -7.69[600\|16,000]$$
$$\Rightarrow \underline{A_v = -4447}$$

10.61

a. $g_m(M_o) = 2\sqrt{K_n I_{REF}}$

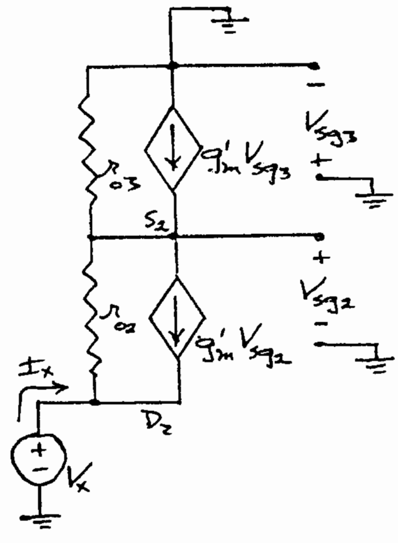
$$g_m(M_0) = 2\sqrt{(0.25)(0.2)}$$
$$\Rightarrow \underline{g_m(M_0) = 0.447\ \text{mA/V}}$$
$$r_{on} = \frac{1}{\lambda_n I_{REF}} = \frac{1}{(0.02)(0.2)} \Rightarrow \underline{r_{on} = 250\ \text{k}\Omega}$$
$$r_{op} = \frac{1}{\lambda_p I_{REF}} = \frac{1}{(0.03)(0.2)} \Rightarrow \underline{r_{op} = 167\ \text{k}\Omega}$$

b. $A_v = -g_m(r_{on}\|r_{op}) = -(0.447)(250\|167)$

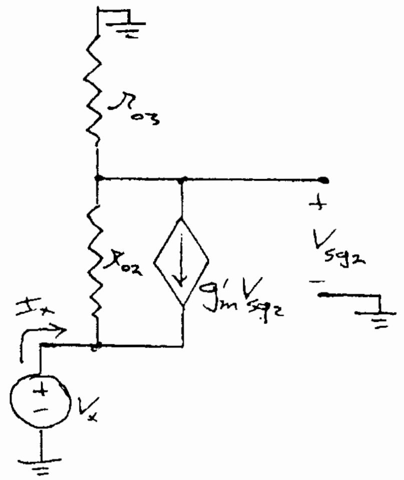
$$\Rightarrow \underline{A_v = -44.8}$$

c. $R_L = 250\|167 = r_{on}\|r_{op}$ or $\underline{R_L = 100\ \text{k}\Omega}$

10.62



Since $V_{sg3} = 0$, the circuit becomes



$$I_x = -g_m' V_{sg2} + \frac{V_x - V_{sg2}}{r_{02}} \text{ and } V_{sg2} = I_x r_{o3}$$

Then

$$I_x\left(1 + g_m' r_{o3} + \frac{r_{o3}}{r_{o2}}\right) = \frac{V_x}{r_{o2}}$$

so that

$$\frac{V_x}{I_x} = R_o = r_{o2}\left(1 + g_m' r_{o3} + \frac{r_{o3}}{r_{o2}}\right)$$

or

$$R_o = r_{o2} + r_{o3}(1 + g_m' r_{o2})$$

$$A_v = \frac{v_o}{v_i} = -g_{m1}(r_{o1} \| R_o)$$

Now

$$g_{m1} = 2\sqrt{(0.050)(20)(0.10)} = 0.632 \ mA/V$$

$$r_{o1} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.02)(0.10)} = 500 \ k\Omega$$

$$g_m' = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.020)(80)(0.1)} = 0.80 \ mA/V$$

$$r_{o2} = r_{o3} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.020)(0.1)} = 500 \ k\Omega$$

Then

$$R_o = 500 + 500[1 + (0.8)(500)] \Rightarrow 201 \ M\Omega$$

$$A_v = -(0.632)(500 \| 201000) \Rightarrow \underline{A_v = -315}$$

10.63

$$A_v = -g_{m1}(R_{o2} \| R_{o3})$$

From the results of JFETs:

$$R_{o2} = r_{o1} + r_{o2}(1 + g_m' r_{o1})$$

From results of Problem 10.62

$$R_{o3} = r_{o3} + r_{o4}(1 + g_m' r_{o3})$$

We find

$$g_{m1} = 2\sqrt{(0.05)(20)(0.08)} = 0.566 \ mA/V$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.02)(0.08)} = 625 \ k\Omega$$

$$r_{o3} = r_{o4} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.02)(0.08)} = 625 \ k\Omega$$

$$g_m' = 2\sqrt{(0.02)(40)(0.08)} = 0.506 \ mA/V$$

Then

$$R_{o2} = 625 + 625[1 + (0.506)(625)] \Rightarrow 199 \ M\Omega$$

$$R_{o3} = 625 + 625[1 + (0.506)(625)] \Rightarrow 199 \ M\Omega$$

Then

$$A_v = -(0.566)[199000 \| 199000] \Rightarrow \underline{A_v = -56{,}317}$$

Chapter 11

Exercise Solutions

E11.1

$V_d = V_1 - V_2$

$= 2 + 0.005\sin\omega t - (0.5 - 0.005\sin\omega t)$

$\Rightarrow \underline{V_d = 1.5 + 0.010\sin\omega t\ (\text{V})}$

$V_{cm} = \dfrac{V_1 + V_2}{2}$

$= \dfrac{2 + 0.005\sin\omega t + 0.5 - 0.005\sin\omega t}{2}$

$\Rightarrow \underline{V_{cm} = 1.25\ \text{V}}$

E11.2

$v_E = -V_{BE}(\text{on}) \Rightarrow \underline{v_E = -0.7\ \text{V}}$

$I_{C1} = I_{C2} = 0.5\ \text{mA}$

$v_{C1} = v_{C2} = 10 - (0.5)(10)$

$\Rightarrow \underline{v_{C1} = v_{C2} = 5\ \text{V}}$

E11.3

For $v_1 = v_2 = +4$ V

$\Rightarrow$ Minimum $v_{C1} = v_{C2} = 4$ V

$I_{C1} = I_{C2} = \dfrac{I_Q}{2} = 1\ \text{mA}$

$R_C = \dfrac{10 - 4}{1} \Rightarrow \underline{R_C = 6\ \text{k}\Omega}$

E11.4

$\dfrac{i_{C2}}{I_Q} = \dfrac{1}{1 + \exp\left(\dfrac{v_d}{V_T}\right)} = 0.99$

$1 + \exp\left(\dfrac{v_d}{V_T}\right) = \dfrac{1}{0.99}$

$\exp\left(\dfrac{v_d}{V_T}\right) = \dfrac{1}{0.99} - 1$

$v_d = V_T \ln\left[\dfrac{1}{0.99} - 1\right]$

$\Rightarrow \underline{v_d = -119.5\ \text{mV}}$

E11.6

a. $v_1 = v_2 = 0 \Rightarrow \underline{v_E = 0.7\ \text{V}}$

$\Delta V_{RC} = (0.25)(8) = 2$ V

$\Rightarrow v_{C1} = v_{C2} = -3$ V

$\Rightarrow \underline{v_{EC1} = 3.7\ \text{V}}$

b. $v_1 = v_2 = 2.5\ \text{V} \Rightarrow \underline{v_E = 3.2\ \text{V}}$

$\Rightarrow \underline{v_{EC1} = 6.2\ \text{V}}$

c. $v_1 = v_2 = -2.5\ \text{V} \Rightarrow \underline{v_E = -1.8\ \text{V}}$

$\Rightarrow \underline{v_{EC1} = 1.2\ \text{V}}$

E11.7

Let $I_Q = 1\,mA$, then $I_{CQ1} = I_{CQ2} = 0.5\,mA$

$g_{m1} = g_{m2} = \dfrac{0.5}{0.026} = 19.23\,mA/V$

At v_{C2},

$A_d = \dfrac{v_{c2}}{v_d} = \dfrac{1}{2} g_m R_{C2}$

So, $150 = \dfrac{1}{2}(19.23)R_{C2} \Rightarrow \underline{R_{C2} = 15.6\,k\Omega}$

At v_{C1},

$A_d = \dfrac{v_{c1}}{v_d} = -\dfrac{1}{2} g_m R_{C1}$

So, $-100 = -\dfrac{1}{2}(19.23)R_{C1} \Rightarrow \underline{R_{C1} = 10.4\,k\Omega}$

If $V^+ = +10V$ and $V^- = -10V$, dc biasing is OK.

E11.8

a. Diff. Gain $A_d = \dfrac{I_Q R_C}{4V_T}$

For $v_1 = v_2 = 5$ V $\Rightarrow$ Minimum collector voltage

$v_{C2} = 5$ V

$\Rightarrow \dfrac{I_Q}{2} \cdot R_C = 15 - 5 = 10$ V

or $\underline{I_Q R_C = 20\ \text{V}}$ for max. A_d

Then

$A_d = \dfrac{20}{2(0.026)} \Rightarrow \underline{A_d(\text{max}) = 192}$

b. If $I_Q = 0.5$ mA, $R_C = 40$ kΩ

$$A_{cm} = \frac{-\left(\dfrac{I_Q R_C}{2V_T}\right)}{\left[1 + \dfrac{(1+\beta) I_Q R_o}{V_T \beta}\right]}$$

$$\text{Then } A_{cm} = \frac{-\left(\dfrac{20}{2(0.026)}\right)}{\left[1 + \dfrac{(201)(0.5)(100)}{(0.026)(200)}\right]}$$

$\Rightarrow \underline{A_{cm} = -0.199}$

and $CMRR_{dB} = 20\log_{10}\left(\dfrac{192}{0.199}\right)$

$\Rightarrow \underline{CMRR_{dB} = 59.7\ \text{dB}}$

E11.9

For $v_1 = v_2 = 5$ V $\Rightarrow$ min $v_{C1} = v_{C2} = 5$ V

So $I_{C1}R_C = 10 - 5 = 0.25R_C \Rightarrow \underline{R_C = 20\text{ k}\Omega}$

$$A_d = \frac{I_Q R_C}{4V_T} = \frac{(0.5)(20)}{4(0.026)} \Rightarrow A_d = 96.2 \quad \text{Let } I_Q = 0.5\,mA$$

$CMRR_{dB} = 95$ db $\Rightarrow CMRR = 5.62 \times 10^4$

$$\Rightarrow A_{cm} = \frac{96.2}{5.62 \times 10^4} \Rightarrow |A_{cm}| = 1.71 \times 10^{-3}$$

$$|A_{cm}| = \frac{\left(\dfrac{I_Q R_C}{2V_T}\right)}{\left[1 + \dfrac{(1+\beta)I_Q R_o}{V_T \beta}\right]} = 1.71 \times 10^{-3}$$

$$\frac{\left[\dfrac{(0.5)(20)}{2(0.026)}\right]}{\left[1 + \dfrac{(201)(0.5)R_o}{(0.026)(200)}\right]} = 1.71 \times 10^{-3}$$

$1 + 19.3R_o = 1.12 \times 10^5 \Rightarrow R_o = 5.83 \times 10^3$ kΩ

$= 5.83$ MΩ

We have $R_o = r_{o4}[1 + g_{m2}(R_2 \| r_{\pi 2})]$

$$r_{o4} = \frac{V_A}{I_Q} = \frac{125}{0.5} = 250\text{ k}\Omega$$

$$g_{m2} = \frac{I_Q}{V_T} = \frac{0.5}{0.026} = 19.2\text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta V_T}{I_Q} = \frac{(200)(0.026)}{0.5} = 10.4\text{ k}\Omega$$

$5830 = 250[1 + g_{m2}(R_2 \| r_{\pi 2})]$

$19.2(R_2 \| r_{\pi 2}) = 22.3$

$(R_2 \| r_{\pi 2}) = 1.16$ kΩ

$$\frac{R_2(10.4)}{R_2 + 10.4} = 1.16$$

$R_2(10.4 - 1.16) = (1.16)(10.4)$

$\Rightarrow \underline{R_2 = 1.31\text{ k}\Omega}$

$$I_Q R_2 - I_1 R_3 = V_T \ln\left(\frac{I_1}{I_Q}\right) \qquad \text{Let } I_1 = 1\,mA$$

$$(0.5)(1.31) - (1)R_3 = (0.026)\ln\left(\frac{1}{0.5}\right)$$

$\Rightarrow \underline{R_3 = 0.637\text{ k}\Omega}$

If $V_{BE}(Q_3) \cong 0.7$ V

$$R_1 + R_3 = \frac{10 - 0.7 - (-10)}{1} = 19.3$$

$\Rightarrow \underline{R_1 \cong 18.7\text{ k}\Omega}$

E11.10

a. $v_O = A_d v_d + A_{cm} v_{cm}$

$v_d = v_1 - v_2 = 0.505 \sin \omega t - 0.495 \sin \omega t$

$= 0.01 \sin \omega t$

$$v_{cm} = \frac{v_1 + v_2}{2} = \frac{0.505 \sin \omega t + 0.495 \sin \omega t}{2}$$

$= 0.50 \sin \omega t$

$v_O = (60)(0.01 \sin \omega t) + (0.5)(0.5 \sin \omega t)$

$\Rightarrow \underline{v_O = 0.85 \sin \omega t\text{ (V)}}$

b.

$v_d = v_1 - v_2$

$= 0.5 + 0.005 \sin \omega t - (0.5 - 0.005 \sin \omega t)$

$= 0.01 \sin \omega t$

$$v_{cm} = \frac{v_1 + v_2}{2}$$

$$= \frac{0.5 + 0.005 \sin \omega t + 0.5 - 0.005 \sin \omega t}{2}$$

$= 0.5$

$v_O = (60)(0.01 \sin \omega t) + (0.5)(0.5)$

$\Rightarrow \underline{v_O = 0.25 + 0.6 \sin \omega t\text{ (V)}}$

E11.11

a. $$I_{B1} = I_{B2} = \frac{I_Q/2}{(1+\beta)} = \frac{1}{151}$$

$\Rightarrow \underline{I_{B1} = I_{B2} = 6.62\ \mu\text{A}}$

b. $$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(150)(0.026)}{1} = 3.9\text{ k}\Omega$$

$R_{id} = 2r_\pi = 2(3.9) = 7.8$ kΩ

$$I_b = \frac{V_d}{R_{id}} = \frac{10 \sin \omega t\text{ (mV)}}{7.8\text{ k}\Omega}$$

$\Rightarrow \underline{I_b = 1.28 \sin \omega t\ (\mu\text{A})}$

c. $R_{icm} \cong 2(1+\beta)R_o = 2(151)(50) \Rightarrow 15.1$ MΩ

$$I_b = \frac{V_{cm}}{R_{icm}} = \frac{3 \sin \omega t}{15.1\text{ M}\Omega} \Rightarrow \underline{I_b = 0.199 \sin \omega t\ (\mu\text{A})}$$

E11.12

$$\frac{i_{D1}}{I_Q} = \frac{1}{2} + \sqrt{\frac{K_n}{2I_Q}} \cdot v_d \sqrt{1 - \left(\frac{K_n}{2I_Q}\right) v_d^2}$$

Using the parameters in Example 11.11,
$K_n = 0.5\,mA/V^2$, $I_Q = 1\,mA$, then

$$\frac{i_{D1}}{I_Q} = 0.90 = \frac{1}{2} + \sqrt{\frac{0.5}{2(1)}} \cdot v_d \sqrt{1 - \left(\frac{0.5}{2(1)}\right) v_d^2}$$

By trial and error,
$\underline{v_d = 0.894\,V}$

E11.13

$$I_1 = \frac{10 - V_{GS4}}{R_1} = K_{n3}(V_{GS4} - V_{TN})^2$$

$10 - V_{GS4} = (0.1)(80)(V_{GS4} - 0.8)^2$

$10 - V_{GS4} = 8(V_{GS4}^2 - 1.6V_{GS4} + 0.64)$

$8V_{GS4}^2 - 11.8V_{GS4} - 4.88 = 0$

$$V_{GS4} = \frac{11.8 \pm \sqrt{(11.8)^2 + 4(8)(4.88)}}{2(8)} = 1.81\text{ V}$$

$$I_1 = I_Q = \frac{10 - 1.81}{80} = 0.102 \text{ mA}$$
$$I_{D1} = I_{D2} = \frac{0.102}{2} = 0.051 \text{ mA}$$
$$= K_{n1}(V_{GS1} - V_{TN})^2$$
$$0.051 = 0.050(V_{GS1} - 0.8)^2 \Rightarrow V_{GS1} = 1.81 \text{ V}$$

$$v_{O1} = v_{O2} = 5 - (0.051)(40) = 2.96 \text{ V}$$
Max v_{cm}: $V_{DS1}(sat) = V_{GS1} - V_{TN}$
$$= 1.81 - 0.8 = 1.01 \text{ V}$$
$$v_{cm}(\max) = v_{O1} - V_{DS1}(\text{sat}) + V_{GS1}$$
$$= 2.96 - 1.01 + 1.81$$
$$v_{cm}(\max) = 3.76 \text{ V}$$

Min v_{cm}: $V_{DS4}(sat) = V_{GS4} - V_{TN}$
$$= 1.81 - 0.8 = 1.01 \text{ V}$$
$$v_{cm}(\min) = V_{GS1} + V_{DS4}(\text{sat}) - 5$$
$$= 1.81 + 1.01 - 5$$
$$v_{cm}(\min) = -2.18 \text{ V}$$
$$\underline{-2.18 \le v_{cm} \le 3.76 \text{ V}}$$

E11.14

$$g_f(\max) = \sqrt{\frac{K_n I_Q}{2}} = \sqrt{\frac{(1)(2)}{(2)}} \Rightarrow$$
$$\underline{g_f(\max) = 1\, mA/V}$$
$$A_d = g_f R_D = (1)(5) \Rightarrow \underline{A_d = 5}$$

E11.15

$$A_d = g_f R_D$$
$$8 = g_f(4) \Rightarrow \underline{g_f(\max) = 2\, mA/V}$$
$$g_f(\max) = \sqrt{\frac{K_n I_Q}{2}}$$
$$(2)^2 = K_n\left(\frac{4}{2}\right) \Rightarrow \underline{K_n = 2\, mA/V^2}$$

E11.16

From Example 11-10, $I_Q = 0.588$ mA
$$A_d = \sqrt{\frac{K_{n2} I_Q}{2}} \cdot R_D = \sqrt{\frac{(0.1)(0.588)}{2}} \cdot (16)$$
$$\Rightarrow \underline{A_d = 2.74}$$

For M_4, $R_O = \frac{1}{\lambda_4 I_Q} = \frac{1}{(0.02)(0.588)} \Rightarrow R_O = 85 \text{ k}\Omega$
$$g_m = 2K_n(V_{GS2} - V_{TN}) = 2(0.1)(2.71 - 1)$$
$$= 0.342 \text{ mA/V}$$

$$A_{cm} = \frac{-g_m R_D}{1 + 2g_m R_O} = \frac{-(0.342)(16)}{1 + 2(0.342)(85)}$$
$$\Rightarrow \underline{A_{cm} = -0.0925}$$
$$CMRR_{dB} = 20 \log_{10}\left(\frac{2.74}{0.0925}\right)$$
$$\Rightarrow \underline{CMRR_{dB} = 29.4 \text{ dB}}$$

E11.17

$$CMRR = \frac{1}{2}\left[1 + 2\sqrt{2K_n I_Q} \cdot R_o\right]$$
$$CMRR_{dB} = 60 \text{ dB} \Rightarrow CMRR = 1000$$
$$1000 = \frac{1}{2}\left[1 + 2\sqrt{2(0.1)(0.2)} \cdot R_O\right]$$
$$2000 = 1 + 0.4R_O$$
$$\Rightarrow \underline{R_O \cong 5 \text{ M}\Omega}$$

E11.18

$$R_o = r_{o4} + r_{o2}(1 + g_{m4} r_{o4})$$
Assume $I_{REF} = I_O = 100\,\mu A$ and $\lambda = 0.01 V^{-1}$
$$r_{o2} = r_{o4} = \frac{1}{\lambda I_D} = \frac{1}{(0.01)(0.1)} \Rightarrow 1\, M\Omega$$
Let K_n(all devices) $= 0.1\, mA/V^2$

Then
$$g_{m4} = 2\sqrt{K_n I_D} = 2\sqrt{(0.1)(0.1)} = 0.2\, mA/V$$

$$R_o = 1000 + 1000(1 + (0.2)(1000)) \Rightarrow 202\, M\Omega$$
Now
$$V_{GS1} = V_{GS2} = \sqrt{\frac{I_D}{K_n}} + V_{TN} = \sqrt{\frac{0.05}{0.1}} + 1 = 1.71V$$
$$V_{DS1}(sat) = V_{GS1} - V_{TN} = 1.71 - 1 = 0.71V$$
So
$$v_{o1}(\min) = +4 - V_{GS1} + V_{DS1}(sat) = 4 - 1.71 + 0.71$$
$$v_{o1}(\min) = 3V = 10 - I_D R_D = 10 - (0.05)R_D \Rightarrow$$
$$\underline{R_D = 140\, k\Omega}$$
For a one-sided output, the differential gain is:
$$A_d = \frac{1}{2} g_{m1} R_D \quad \text{where} \quad g_{m1} = 2\sqrt{K_n I_D}$$
$$= 2\sqrt{(0.1)(0.05)} = 0.141\, mA/V$$
$$A_d = \frac{1}{2}(0.141)(140) \Rightarrow \underline{A_d = 9.87}$$
The common-mode gain is:
$$A_{cm} = \frac{\sqrt{2K_n I_Q} \cdot R_D}{1 + 2\sqrt{2K_n I_Q} \cdot R_o} = \frac{\sqrt{2(0.1)(0.1)} \cdot (140)}{1 + 2\sqrt{2(0.1)(0.1)} \cdot (202000)}$$
$$\Rightarrow \underline{A_{cm} = 0.000347}$$
Then
$$CMRR_{dB} = 20\log_{10}\left|\frac{A_d}{A_{cm}}\right| \Rightarrow \underline{CMRR_{dB} = 89.1\, dB}$$

E11.19

a. $I_{B5} = \frac{I_Q}{\beta(1+\beta)} = \frac{0.5}{(180)(181)} \Rightarrow 15.3 \text{ nA}$

So $\underline{I_0 = 15.3 \text{ nA}}$

b. For a balanced condition

$V_{EC4} = V_{EC3} = V_{EB3} \Rightarrow \underline{V_{EC4} = 0.7 \text{ V}}$

$V_{CE2} = V_{C2} - V_{E2} = (10 - 0.7) - (-0.7)$

$\Rightarrow \underline{V_{CE2} = 10 \text{ V}}$

E11.20

a. $g_f = \frac{I_Q}{4V_T} = \frac{0.5}{4(0.026)} = 4.81 \text{ mA/V}$

$r_{02} = \frac{V_{A2}}{I_{C2}} = \frac{125}{0.25} = 500 \text{ k}\Omega$

$r_{04} = \frac{V_{A4}}{I_{C4}} = \frac{85}{0.25} = 340 \text{ k}\Omega$

$A_d = 2g_f(r_{02} \| r_{04}) = 2(4.81)(500 \| 340)$

$\Rightarrow \underline{A_d = 1947}$

b. $A_d = 2g_f(r_{02} \| r_{04} \| R_L)$

$A_d = 2(4.81)[500 \| 340 \| 100]$

$\Rightarrow \underline{A_d = 644}$

c. $r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(150)(0.026)}{0.25} = 15.6 \text{ k}\Omega$

$R_{id} = 2r_\pi \Rightarrow \underline{R_{id} = 31.2 \text{ k}\Omega}$

d. $R_0 = r_{02} \| r_{04} = 500 \| 340 \Rightarrow \underline{R_0 = 202 \text{ k}\Omega}$

E11.21

$A_d = 2g_f(r_{02} \| r_{04})$

$g_f = \frac{I_Q}{4V_T} = \frac{0.2}{4(0.026)} = 1.92 \text{ mA/V}$

$r_{02} = \frac{V_{A2}}{I_{C2}} = \frac{120}{0.1} = 1200 \text{ k}\Omega$

$r_{04} = \frac{V_{A4}}{I_{C4}} = \frac{80}{0.1} = 800 \text{ k}\Omega$

$A_d = 2(1.92)(1200 \| 800) \Rightarrow \underline{A_d = 1843}$

E11.22

$P = (I_Q + I_{REF})(5 - (-5))$

$10 = (0.1 + I_{REF})(10) \Rightarrow \quad I_{REF} = 0.9 \, mA$

$R_1 = \frac{5 - 0.7 - (-5)}{I_{REF}} = \frac{9.3}{0.9} \Rightarrow \quad \underline{R_1 = 10.3 \, k\Omega}$

$I_Q R_E = V_T \ln\left(\frac{I_{REF}}{I_Q}\right)$

$R_E = \frac{0.026}{0.1} \ln\left(\frac{0.9}{0.1}\right) \Rightarrow \quad \underline{R_E = 0.571 \, k\Omega}$

$r_{o2} = \frac{V_{A2}}{I_{C2}} = \frac{125}{0.05} \Rightarrow 2.5 \, M\Omega$

$r_{o4} = \frac{V_{A4}}{I_{C4}} = \frac{85}{0.05} \Rightarrow 1.7 \, M\Omega$

$g_m = \frac{0.05}{0.026} = 1.923 \, mA/V$

$A_d = g_m(r_{o2} \| r_{o4} \| R_L) = (1.923)(2500 \| 1700 \| 90) \Rightarrow$

$\underline{A_d = 159}$

E11.23

a. $R_0 = r_{02} \| r_{04}$

$r_{02} = \frac{120}{0.1} = 1.2 \text{ M}\Omega$

$r_{04} = \frac{80}{0.1} = 0.8 \text{ M}\Omega$

$R_0 = 1.2 \| 0.8 \Rightarrow \underline{R_0 = 0.48 \text{ M}\Omega}$

b. $A_d(\text{open circuit}) = 2g_f(r_{02} \| r_{04})$

$A_d(\text{with load}) = 2g_f(r_{02} \| r_{04} \| R_L)$

For $A_d(\text{with load}) = \frac{1}{2} A_d(\text{open circuit})$

$\Rightarrow R_L = (r_{02} \| r_{04}) \Rightarrow \underline{R_L = 0.48 \text{ M}\Omega}$

E11.24

$A_d = 2\sqrt{\frac{2K_n}{I_Q}} \cdot \frac{1}{(\lambda_2 + \lambda_4)}$

$= 2\sqrt{\frac{2(0.1)}{0.1}} \cdot \frac{1}{(0.01 + 0.015)} \Rightarrow$

$\underline{A_d = 113}$

E11.25

For the MOSFET, $I_D = 25 \, \mu\text{A}$

$25 = 20(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.12 \text{ V}$

$g_{m1} = 2K_{n1}(V_{GS} - V_{TN}) = 2(20)(2.12 - 1)$

$\Rightarrow \underline{g_{m1} - 44.8 \, \mu\text{A/V}}, \quad \underline{r_0 = \infty}$

For the Bipolar, $I_Q = 100 - 25 = 75 \, \mu\text{A}$

$r_{\pi 2} = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.075} \Rightarrow \underline{r_{\pi 2} = 34.7 \text{ k}\Omega}$

$g_{m2} = \frac{I_{CQ}}{V_T} = \frac{75}{0.026}$

$\Rightarrow \underline{g_{m2} = 2.88 \text{ mA/V}}, \quad \underline{r_{02} = \infty}$

$g_m^C = \frac{g_{m1}(1 + g_{m2} r_\pi)}{(1 + g_{m1} r_\pi)}$

$= \frac{(44.8)[1 + (2.88)(34.7)]}{1 + (0.0448)(34.7)}$

$= \frac{(44.8)(100.9)}{2.55}$

$\Rightarrow \underline{g_m^C = 1.77 \text{ mA/V}}$

E11.26

From Figure 11.41

$r_{04} = \dfrac{80}{0.5} = 160 \text{ k}\Omega$

$R_0 \cong \beta r_{04} = (150)(160) \text{ k}\Omega \Rightarrow \underline{R_0 = 24 \text{ M}\Omega}$

From Figure 11.42

$r_{06} = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.0125)(0.5)} \Rightarrow \underline{r_{06} = 160 \text{ k}\Omega}$

$0.5 = 0.5(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2 \text{ V}$

$g_{m6} = 2K_n(V_{GS} - V_{TN}) = 2(0.5)(2-1) = 1 \, mA/V$

$r_{04} = 160 \text{ k}\Omega$

$R_0 = (g_{m6})(r_{06})(\beta r_{04}) = (1)(160)(150)(160)$

$\Rightarrow \underline{R_0 = 3,840 \text{ M}\Omega}$

E11.27

From Equation (11.103)

$R_i = \dfrac{2(1+\beta)\beta V_T}{I_Q} = \dfrac{2(121)(120)(0.026)}{0.5}$

$\Rightarrow \underline{R_i = 1.51 \text{ M}\Omega}$

$r_{\pi 11} = \dfrac{\beta V_T}{I_Q} = \dfrac{(120)(0.026)}{0.5} = 6.24 \text{ k}\Omega$

$R'_E = r_{\pi 11} \| R_3 = 6.24 \| 0.1 = 0.0984 \text{ k}\Omega$

$g_{m11} = \dfrac{I_Q}{V_T} = \dfrac{0.5}{0.026} = 19.23 \text{ mA/V}$

$r_{011} = \dfrac{V_A}{I_Q} = \dfrac{120}{0.5} = 240 \text{ k}\Omega$

Then

$R_{C11} = r_{011}(1 + g_{m11} R'_E)$

$= 240[1 + (19.23)(0.0984)]$

$= 694 \text{ k}\Omega$

$r_{\pi 8} = \dfrac{\beta V_T}{I_{C8}} = \dfrac{(120)(0.026)}{2} = 1.56 \text{ k}\Omega$

$R_{b8} = r_{\pi 8} + (1+\beta)R_4 = 1.56 + (121)(5)$

$= 607 \text{ k}\Omega$

Then

$R_{L7} = R_{C11} \| R_{b8} = 694 \| 607 = 324 \text{ k}\Omega$

Then

$A_v = \left(\dfrac{I_Q}{2V_T}\right) R_{L7} = \left[\dfrac{0.5}{2(0.026)}\right](324)$

$\Rightarrow \underline{A_v = 3115}$

$R_0 = R_4 \left\| \left(\dfrac{r_{\pi 8} + Z}{1+\beta}\right)\right.$

$Z = R_{C11} \| R_{C7}$

$R_{C7} = \dfrac{V_A}{I_Q} = \dfrac{120}{0.5} = 240 \text{ k}\Omega$

$Z = 694 \| 240 = 178 \text{ k}\Omega$

$R_0 = 5 \left\| \left(\dfrac{1.56 + 178}{121}\right)\right. = 5 \| 1.48$

$\Rightarrow \underline{R_0 = 1.14 \text{ k}\Omega}$

E11.28

$A_v = \left(\dfrac{I_Q}{2V_T}\right) R_{L7}$

$10^3 = \left(\dfrac{0.5}{2(0.026)}\right) R_{L7}$

$\Rightarrow \underline{R_{L7} = 104 \text{ k}\Omega}$

E11.29

a. $R_1 = \frac{10 - 0.7 - (-10)}{I_1} = \frac{19.3}{0.6} \Rightarrow \underline{R_1 = 32.2 \text{ k}\Omega}$

$I_{C1} = I_{C2} = 0.1 \text{ mA} \Rightarrow I_Q \cong 0.2 \text{ mA}$

$R_2 = \frac{V_T}{I_Q} \cdot \ln\left(\frac{I_1}{I_Q}\right) = \frac{0.026}{0.2} \cdot \ln\left(\frac{0.6}{0.2}\right)$

$\Rightarrow \underline{R_2 = 143 \ \Omega}$

$I_{R6} = I_1 = 0.6 \text{ mA} \Rightarrow \underline{R_3 = 0}$

$V_{02} = V_{CE2} + V_E = 4 - 0.7 = 3.3 \text{ V}$

$R_C = \frac{10 - 3.3}{I_{C2}} = \frac{6.7}{0.1} \Rightarrow \underline{R_C = 67 \text{ k}\Omega}$

$V_{E4} = V_{02} - 2V_{BE} = 3.3 - 2(0.7) = 1.9 \text{ V}$

$R_4 = \frac{1.9}{I_{R4}} = \frac{1.9}{0.6} \Rightarrow \underline{R_4 = 3.17 \text{ k}\Omega}$

$V_{03} = V_{CE4} + V_{E4} = 3 + 1.9 = 4.9$

$R_5 = \frac{10 - 4.9}{I_{R4}} = \frac{5.1}{0.6} \Rightarrow \underline{R_5 = 8.5 \text{ k}\Omega}$

$V_{E5} = V_{03} - V_{BE} = 4.9 - 0.7 = 4.2$

$R_6 = \frac{4.2 - 0.7}{I_{R6}} = \frac{3.5}{0.6} \Rightarrow \underline{R_6 = 5.83 \text{ k}\Omega}$

$R_7 = \frac{0 - (-10)}{I_{R7}} = \frac{10}{5} \Rightarrow \underline{R_7 = 2 \text{ k}\Omega}$

b. $R_{i2} = r_{\pi 3} + (1 + \beta) r_{\pi 4}$

$r_{\pi 4} = \frac{\beta V_T}{I_{R4}} = \frac{(100)(0.026)}{0.6} = 4.33 \text{ k}\Omega$

$r_{\pi 3} \cong \frac{\beta^2 V_T}{I_{R4}} = \frac{(100)^2(0.026)}{0.6} = 433 \text{ k}\Omega$

$R_{i2} = 433 + (101)(4.33) \Rightarrow \underline{R_{i2} = 870 \text{ k}\Omega}$

$R_{i3} = r_{\pi 5} + (1 + \beta)[R_6 + r_{\pi 6} + (1 + \beta) R_7]$

$r_{\pi 5} = \frac{\beta V_T}{I_{R6}} = \frac{(100)(0.026)}{0.6} = 4.33 \text{ k}\Omega$

$r_{\pi 6} = \frac{\beta V_T}{I_{R7}} = \frac{(100)(0.026)}{5} = 0.52 \text{ k}\Omega$

$R_{i3} = 4.33 + (101)[5.83 + 0.52 + (101)(2)]$

$\Rightarrow \underline{R_{i3} = 21.0 \text{ M}\Omega}$

c. $A_d = A_{d1} \cdot A_2 \cdot A_3$

$A_{d1} = g_f (R_C \| R_{i2})$

$g_f = \frac{I_Q}{4V_T} = \frac{0.2}{4(0.026)} = 1.92 \text{ mA/V}$

$A_{d1} = (1.92)(67 \| 870) = 119$

$A_2 = \left(\frac{I_{R4}}{2V_T}\right) R_5 = \frac{0.6}{2(0.026)}(8.5) = 98.1$

$A_3 \cong 1$

$A_d = (119)(98.1)(1)$

$\Rightarrow \underline{A_d = 11,674}$

Chapter 11

Problem Solutions

11.1

a. $I_E = I_{C1} + I_{C2} = 4 \text{ mA} = \dfrac{-0.7-(-8.7)}{R_E}$

So $R_E = \dfrac{8.0}{4} = 2 \text{ k}\Omega = R_E$

Then

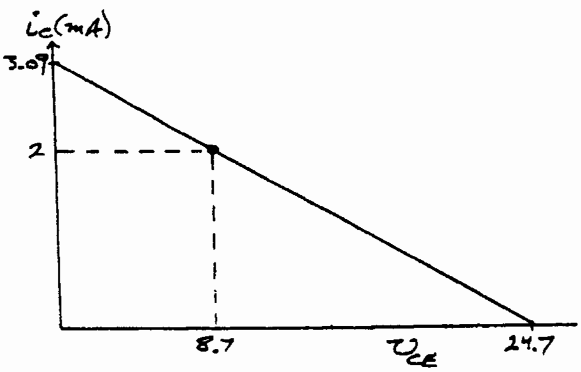
$R_C = \dfrac{16 - V_{02}}{I_{C2}} = \dfrac{16-8}{2} \Rightarrow R_C = 4 \text{ k}\Omega$

b. Neglecting base currents

$16 = I_C R_C + V_{CE2} + 2I_C R_E - 8.7$

$V_{CE2} = 24.7 - I_C(R_C + 2R_E) = 24.7 - I_C(8)$

For $I_C = 2$ mA, $V_{CEQ} = 8.7$ V



c. $v_{cm}(\max)$ for $V_{CB} = 0 \Rightarrow v_{cm}(\max) = 8$ V

$v_{cm}(\min)$ for Q_1 and Q_2 at edge of cutoff

$\Rightarrow v_{cm}(\min) = -8$ V

11.2

$P = (I_1 + I_{C4})(V^+ - V^-)$

$I_1 \cong I_{C4}$ so

$1.2 = 2I_1(6) \Rightarrow \quad I_1 = I_{C4} = 0.1\, mA$

$R_1 = \dfrac{3-0.7-(-3)}{0.1} \Rightarrow \quad R_1 = 53\, k\Omega$

For $v_{CM} = +1V \Rightarrow V_{C1} = V_{C2} = 1V \Rightarrow$

$R_C = \dfrac{3-1}{0.05} \Rightarrow \quad R_C = 40\, k\Omega$

One-sided output

$A_d = \dfrac{1}{2} g_m R_C$ where $g_m = \dfrac{0.05}{0.026} = 1.923\, mA/V$

Then

$A_d = \dfrac{1}{2}(1.923)(40) \Rightarrow \quad A_d = 38.5$

11.3

a. $I_1 = \dfrac{10 - 2(0.7)}{8.5} \Rightarrow I_1 = 1.01$ mA

$I_{C2} = \dfrac{I_1}{1 + \dfrac{2}{\beta(1+\beta)}} = \dfrac{1.01}{1 + \dfrac{2}{(100)(101)}}$

$\Rightarrow I_{C2} \cong 1.01$ mA

$I_{C4} = \left(\dfrac{100}{101}\right)\left(\dfrac{1.01}{2}\right) \Rightarrow I_{C4} \cong 0.50$ mA

$V_{CE2} = (0 - 0.7) - (-5) \Rightarrow V_{CE2} = 4.3$ V

$V_{CE4} = [5 - (0.5)(2)] - (-0.7) \Rightarrow V_{CE4} = 4.7$ V

b. For $V_{CE4} = 2.5 \text{ V} \Rightarrow V_{C4} = -0.7 + 2.5 = 1.8$ V

$I_{C4} = \dfrac{5-1.8}{2} \Rightarrow I_{C4} = 1.6$ mA

$I_{C2} + \left(\dfrac{1+\beta}{\beta}\right)(2I_{C4}) = \left(\dfrac{101}{100}\right)(2)(1.6)$

$\Rightarrow I_{C2} = 3.23$ mA

$I_1 \approx I_{C2} = 3.23$ mA

$R_1 = \dfrac{10-2(0.7)}{3.23} \Rightarrow R_1 = 2.66 \text{ k}\Omega$

11.4

a. $0 = 0.7 + \dfrac{I_E}{2}(2) + I_E(85) - 5$

$I_E = \dfrac{5-0.7}{85+1} \Rightarrow I_E = 0.050$ mA

$I_{C1} = I_{C2} = \left(\dfrac{\beta}{1+\beta}\right)\left(\dfrac{I_E}{2}\right) = \left(\dfrac{100}{101}\right)\left(\dfrac{0.050}{2}\right)$

Or $I_{C1} = I_{C2} = 0.0248$ mA

$V_{CE1} = V_{CE2} = [5 - I_{C1}(100)] - (-0.7)$

So $V_{CE1} = V_{CE2} = 3.22$ V

b. $v_{cm}(\max)$ for $V_{CB} = 0$ and

$V_C = 5 - I_{C1}(100) = 2.52$ V

So $v_{cm}(\max) = 2.52$ V

$v_{cm}(\min)$ for Q_1 and Q_2 at the edge of cutoff

$\Rightarrow v_{cm}(\min) = -4.3$ V

(c) Differential-mode half circuits

$-\dfrac{v_d}{2} = V_\pi + \left(\dfrac{V_\pi}{r_\pi} + g_m V_\pi\right) \cdot R_E$

$= V_\pi\left[1 + \dfrac{(1+\beta)}{r_\pi} R_E\right]$

Then

$$V_\pi = \frac{-(v_d/2)}{\left[1+\frac{(1+\beta)}{r_\pi}R_E\right]}$$

$$v_o = -g_m V_\pi R_C \Rightarrow A_d = \frac{1}{2}\cdot\frac{\beta R_C}{r_\pi + (1+\beta)R_E}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.0248} = 105\ k\Omega$$

Then

$$A_d = \frac{1}{2}\cdot\frac{(100)(100)}{105+(101)(2)} \Rightarrow \underline{A_d = 16.3}$$

11.5

a. i. $\underline{(v_{O1} - v_{O2}) = 0}$

ii. $I_{C1} = I_{C2} = 1$ mA

$$v_{O1} - v_{O2} = [V^+ - I_{C1}R_{C1}] - [V^+ - I_{C2}R_{C2}]$$
$$= I_C(R_{C2} - R_{C1}) = (1)(7.9 - 8)$$
$$\Rightarrow \underline{v_{O1} - v_{O2} = -0.1\text{ V}}$$

b. $I_O = (I_{S1} + I_{S2})\exp\left(\frac{v_{BE}}{V_T}\right)$

So $\exp\left(\frac{v_{BE}}{V_T}\right) = \frac{2\times 10^{-3}}{10^{-13} + 1.1\times 10^{-13}}$

$= 9.524 \times 10^9$

$I_{C1} = I_{S1}\exp\left(\frac{v_{BE}}{V_T}\right) = (10^{-13})(9.524\times 10^9)$

$\Rightarrow I_{C1} = 0.952$ mA

$I_{C2} = (1.1\times 10^{-13})(9.524\times 10^9)$

$\Rightarrow I_{C2} = 1.048$ mA

i. $v_{O1} - v_{O2} = I_{C2}R_{C2} - I_{C1}R_{C1}$

$\Rightarrow v_{O1} - v_{O2} = (1.048 - 0.952)(8)$

$\Rightarrow \underline{v_{O1} - v_{O2} = 0.768\text{ V}}$

ii. $v_{O1} - v_{O2} = (1.048)(7.9) - (0.952)(8)$

$v_{O1} - v_{O2} = 8.279 - 7.616$

$\Rightarrow \underline{v_{O1} - v_{O2} = 0.663\text{ V}}$

11.6

From Equation (11.12(b))

$$i_{C2} = \frac{I_Q}{1+e^{v_d/V_T}}$$

$$0.90 = \frac{1}{1+e^{v_d/V_T}}$$

So $e^{v_d/V_T} = \frac{1}{0.90} - 1 = 0.111$

$v_d = V_T \ln(0.111) = (0.026)\ln(0.111)$

$\Rightarrow \underline{v_d = -0.0571\text{ V}}$

11.7

For $v_{CM} = 3.5\,V$ and a maximum peak-to-peak swing in the output voltage of $2\,V$, we need the quiescent collector voltage to be

$V_C = 3.5 + 1 = 4.5\,V$

Assume the bias is $\pm 10\,V$, and $I_Q = 0.5\,mA$.

Then $I_C = 0.25\,mA$

Now $R_C = \frac{10-4.5}{0.25} \Rightarrow \underline{R_C = 22\ k\Omega}$

In this case, $r_\pi = \frac{(100)(0.026)}{0.25} = 10.4\ k\Omega$

Then

$A_d = \frac{(100)(22)}{2(10.4+0.5)} = 101$ So gain specification is met.

For $CMRR_{dB} = 80\,dB \Rightarrow$

$$CMRR = 10^4 = \frac{1}{2}\left[1+\frac{(1+\beta)I_Q R_o}{V_T\beta}\right]$$
$$= \frac{1}{2}\left[1+\frac{(101)(0.5)R_o}{(0.026)(100)}\right] \Rightarrow$$

$\underline{R_o = 1.03\ M\Omega}$

Need to use a Modified Widlar current source.

$R_o = r_o\left[1 + g_m(R_{E1}\| r_\pi)\right]$

If $V_A = 100\,V$, then $r_o = \frac{100}{0.5} = 200\ k\Omega$

$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2\ k\Omega$

$g_m = \frac{0.5}{0.026} = 19.23\ mA/V$

Then

$1030 = 200\left[1 + (19.23)(R_{E1}\| r_\pi)\right] \Rightarrow$

$R_{E1}\| r_\pi = 0.216\ k\Omega = R_{E1}\| 5.2 \Rightarrow$

$\underline{R_{E1} = 225\ \Omega}$

Also let $R_{E2} = 225\ \Omega$ and $I_{REF} \cong 0.5\ mA$

11.8

a. For $v_1 = v_2 = 0$ and neglecting base currents

$$R_E = \frac{-0.7-(-10)}{0.15} \Rightarrow \underline{R_E = 62\text{ k}\Omega}$$

b. Using the small-signal equivalent circuit shown in Figure 11.8, but including the R_B resistors, we have

$$\frac{V_{\pi1}}{r_\pi} + g_m V_{\pi1} + g_m V_{\pi2} + \frac{V_{\pi2}}{r_\pi} = \frac{V_e}{R_E}$$

$$(V_{\pi1}+V_{\pi2})\left(\frac{1+\beta}{r_\pi}\right) = \frac{V_e}{R_E}$$

Now $\dfrac{v_1 - V_e}{R_B + r_\pi} = \dfrac{V_{\pi1}}{r_\pi}$ and $\dfrac{v_2 - V_e}{R_B + r_\pi} = \dfrac{V_{\pi2}}{r_\pi}$

Then

$$V_{\pi1} = \left(\frac{r_\pi}{R_B + r_\pi}\right)(v_1 - V_e)$$

and $V_{\pi2} = \left(\dfrac{r_\pi}{R_B + r_\pi}\right)(v_2 - V_e)$

Substituting, we find

$$(v_1 + v_2 - 2V_e)\left(\frac{r_\pi}{R_B + r_\pi}\right)\left(\frac{1+\beta}{r_\pi}\right) = \frac{V_e}{R_E}$$

or

$$(v_1 + v_2 - 2V_e)\left(\frac{1+\beta}{R_B + r_\pi}\right) = \frac{V_e}{R_E}$$

Solving for V_e,

$$V_e = \frac{v_1 + v_2}{2 + \dfrac{R_B + r_\pi}{(1+\beta)R_E}}$$

Now $v_{02} = -g_m V_{\pi2} R_C$

$$= -g_m R_C \left(\frac{r_\pi}{R_B + r_\pi}\right)(v_2 - V_e)$$

Substituting for V_e,

$$v_{02} = \frac{-\beta R_C}{R_B + r_\pi}\left[v_2 - \frac{v_1 + v_2}{2 + \dfrac{R_B + r_\pi}{(1+\beta)R_E}}\right]$$

$$= \frac{-\beta R_C}{R_B + r_\pi}\left[\frac{v_2\left(1 + \dfrac{R_B + r_\pi}{(1+\beta)R_E}\right) - v_1}{2 + \dfrac{R_B + r_\pi}{(1+\beta)R_E}}\right]$$

Now $v_1 = v_{cm} + \dfrac{v_d}{2}$

$v_2 = v_{cm} - \dfrac{v_d}{2}$

Substituting and rearranging terms, we obtain

$$v_{02} = \frac{-\beta R_C}{R_B + r_\pi}\left\{\frac{v_d}{2} - v_{cm}\left[\frac{1}{1 + \dfrac{2R_E(1+\beta)}{R_B + r_\pi}}\right]\right\}$$

$$A_d = \frac{v_{02}}{v_d} = \frac{\beta R_C}{2(r_\pi + R_B)}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.075} = 34.7\text{ k}\Omega$$

$$A_d = \frac{(100)(50)}{2(34.7+0.5)} \Rightarrow \underline{A_d = 71.0}$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B}\left[\frac{1}{1 + \dfrac{2R_E(1+\beta)}{r_\pi + R_B}}\right]$$

$$= -\frac{(100)(50)}{34.7+0.5}\left[\frac{1}{1 + \dfrac{2(62)(101)}{34.7+0.5}}\right]$$

$\Rightarrow \underline{A_{cm} = -0.398}$

$$CMRR_{dB} = 20\log_{10}\left|\frac{71.0}{0.398}\right| \Rightarrow \underline{CMRR_{dB} = 45.0\text{ dB}}$$

c. $R_{id} = 2(r_\pi + R_B)$

$R_{id} = 2(34.7+0.5) \Rightarrow \underline{R_{id} = 70.4\text{ k}\Omega}$

Common-mode input resistance

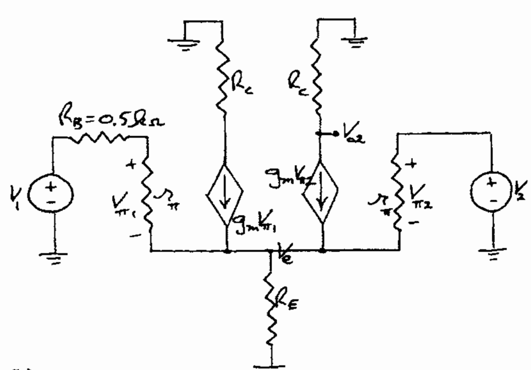
$$R_{icm} = \frac{1}{2}[r_\pi + R_B + 2(1+\beta)R_E]$$

$$= \frac{1}{2}[34.7 + 0.5 + 2(101)(62)]$$

$\Rightarrow \underline{R_{icm} = 6.28\text{ M}\Omega}$

11.9

(a) $R_E = \dfrac{-0.7-(-10)}{0.25} \Rightarrow \underline{R_E = 37.2\text{ k}\Omega}$



(b)

$$\frac{V_{\pi1}}{r_\pi} + g_m V_{\pi1} + \frac{V_{\pi2}}{r_\pi} + g_m V_{\pi2} = \frac{V_e}{R_E}$$

or (1) $\left(\dfrac{1+\beta}{r_\pi}\right)(V_{\pi1} + V_{\pi2}) = \dfrac{V_e}{R_E}$

$$\frac{V_{\pi1}}{r_\pi} = \frac{V_1 - V_e}{R_B + r_\pi} \Rightarrow V_{\pi1} = \left(\frac{r_\pi}{r_\pi + R_B}\right)(V_1 - V_e)$$

$V_{\pi 2} = V_2 - V_e$

Then

$$(1)\ \left(\frac{1+\beta}{r_\pi}\right)\left[\frac{r_\pi}{r_\pi + R_B}(V_1 - V_e) + (V_2 - V_e)\right] = \frac{V_e}{R_E}$$

From this, we find

$$V_e = \frac{V_1 + \frac{r_\pi + R_B}{r_\pi}\cdot V_2}{\left[\frac{r_\pi + R_B}{R_E(1+\beta)} + 1 + \frac{r_\pi + R_B}{r_\pi}\right]}$$

Now

$V_o = -g_m V_{\pi 2} R_C = -g_m R_C (V_2 - V_e)$

We have

$$r_\pi = \frac{(120)(0.026)}{0.125} \cong 25\ k\Omega\ ,\ g_m = \frac{0.125}{0.026} = 4.81\ mA/V$$

(i) Set $V_1 = \frac{V_d}{2}$ and $V_2 = -\frac{V_d}{2}$

Then

$$V_e = \frac{\frac{V_d}{2}\left(1 - \left(\frac{25+0.5}{25}\right)\right)}{\left[\frac{25+0.5}{(37.2)(121)} + 1 + \frac{25+0.5}{25}\right]} = \frac{\frac{V_d}{2}(-0.02)}{2.026}$$

So

$V_e = -0.00494 V_d$

Now

$$V_o = -(4.81)(50)\left(-\frac{V_d}{2} - (-0.00494)V_d\right) \Rightarrow$$

$$\underline{A_d = \frac{V_o}{V_d} = 119}$$

(ii) Set $V_1 = V_2 = V_{cm}$

Then

$$V_e = \frac{V_{cm}\left(1 + \frac{25+0.5}{25}\right)}{\left[\frac{25+0.5}{(37.2)(121)} + 1 + \frac{25+0.5}{25}\right]} = \frac{V_{cm}(2.02)}{2.02567}$$

$V_e = V_{cm}(0.9972)$

Then

$V_o = -(4.81)(50)[V_{cm} - V_{cm}(0.9972)]$

or

$$\underline{A_{cm} = \frac{V_o}{V_{cm}} = -0.673}$$

11.10

a. Neglecting base currents

$$I_1 = I_3 = 400\ \mu A \Rightarrow R_1 = \frac{30 - 0.7}{0.4}$$

$\Rightarrow \underline{R_1 = 73.25\ k\Omega}$

$V_{CE1} = 10\ V \Rightarrow V_{C1} = 9.3\ V$

$$R_C = \frac{15 - 9.3}{0.2} \Rightarrow \underline{R_C = 28.5\ k\Omega}$$

b. $r_\pi = \frac{(100)(0.026)}{0.2} = 13\ k\Omega$

$$r_0(Q_3) = \frac{50}{0.4} = 125\ k\Omega$$

Using the results from problem 11.9, we have

$$A_d = \frac{\beta R_C}{2(r_\pi + R_B)} = \frac{(100)(28.5)}{2(13+10)} \Rightarrow \underline{A_d = 62}$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B}\left\{\frac{1}{1 + \frac{2r_0(1+\beta)}{r_\pi + R_B}}\right\}$$

$$= -\frac{(100)(28.5)}{13+10}\left\{\frac{1}{1 + \frac{2(125)(101)}{13+10}}\right\}$$

$\Rightarrow \underline{A_{cm} = -0.113}$

$$CMRR_{dB} = 20\log_{10}\left(\frac{62}{0.113}\right)$$

$\Rightarrow \underline{CMRR_{dB} = 54.8\ \text{dB}}$

c. $R_{id} = 2(r_\pi + R_B) = 2(13+10) \Rightarrow \underline{R_{id} = 46\ k\Omega}$

$$R_{icm} = \frac{1}{2}[r_\pi + R_B + 2(1+\beta)r_0]$$

$$= \frac{1}{2}[13 + 10 + 2(101)(125)]$$

$\Rightarrow \underline{R_{icm} = 12.6\ M\Omega}$

11.11

From Equation (11.18)

$v_0 = v_{C2} - v_{C1} = g_m R_C v_d$

$$g_m = \frac{I_{CQ}}{V_T}$$

For $I_Q = 2$ mA, $I_{CQ} = 1$ mA

Then $g_m = \frac{1}{0.026} = 38.46$ mA/V

Now

$2 = (38.46)R_C(0.015)$

So $\underline{R_C = 3.47\ k\Omega}$

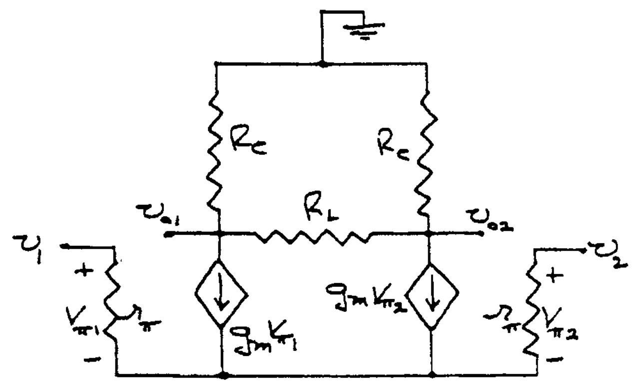
Now $V_C = V^+ - I_C R_C = 10 - (1)(3.47)$

$= 6.53$ V

For $V_{CB} = 0 \Rightarrow \underline{v_{cm}(\max) = 6.53\ V}$

11.12

The small-signal equivalent circuit is



A KVL equation: $v_1 = V_{\pi1} - V_{\pi2} + v_2$

$$v_1 - v_2 = V_{\pi1} - V_{\pi2}$$

A KCL equation

$$\frac{V_{\pi1}}{r_\pi} + g_m V_{\pi1} + \frac{V_{\pi2}}{r_\pi} + g_m V_{\pi2} = 0$$

$$(V_{\pi1} + V_{\pi2})\left(\frac{1}{r_\pi} + g_m\right) = 0 \Rightarrow V_{\pi1} = -V_{\pi2}$$

Then $v_1 - v_2 = 2V_{\pi1} \Rightarrow V_{\pi1} = \frac{1}{2}(v_1 - v_2)$

and $V_{\pi2} = -\frac{1}{2}(v_1 - v_2)$

At the v_{01} node:

$$\frac{v_{01}}{R_C} + \frac{v_{01} - v_{02}}{R_L} + g_m V_{\pi1} = 0$$

$$v_{01}\left(\frac{1}{R_C} + \frac{1}{R_L}\right) - v_{02}\left(\frac{1}{R_L}\right) = \frac{1}{2}g_m(v_2 - v_1) \qquad (1)$$

At the v_{02} node:

$$\frac{v_{02}}{R_C} + \frac{v_{02} - v_{01}}{R_L} + g_m V_{\pi2} = 0$$

$$v_{02}\left(\frac{1}{R_C} + \frac{1}{R_L}\right) - v_{01}\left(\frac{1}{R_L}\right) = \frac{1}{2}g_m(v_1 - v_2) \qquad (2)$$

From (1) :

$$v_{02} = v_{01}\left(1 + \frac{R_L}{R_C}\right) - \frac{1}{2}g_m R_L(v_2 - v_1)$$

Substituting into (2)

$$v_{01}\left(1 + \frac{R_L}{R_C}\right)\left(\frac{1}{R_C} + \frac{1}{R_L}\right)$$
$$- \frac{1}{2}g_m R_L(v_2 - v_1)\left(\frac{1}{R_C} + \frac{1}{R_L}\right)$$
$$- v_{01}\left(\frac{1}{R_L}\right) = \frac{1}{2}g_m(v_1 - v_2)$$

$$v_{01}\left(\frac{1}{R_C} + \frac{R_L}{R_C^2} + \frac{1}{R_C}\right)$$
$$= \frac{1}{2}g_m(v_1 - v_2)\left[1 - \left(\frac{R_L}{R_C} + 1\right)\right]$$

$$\frac{v_{01}}{R_C}\left(2 + \frac{R_L}{R_C}\right) = -\frac{1}{2}g_m\left(\frac{R_L}{R_C}\right)(v_1 - v_2)$$

For $v_1 - v_2 = v_d$

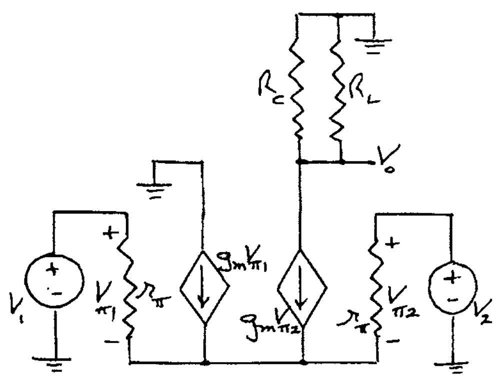
$$A_{v1} = \frac{v_{01}}{v_d} = \frac{-\frac{1}{2}g_m R_L}{\left(2 + \frac{R_L}{R_C}\right)}$$

From symmetry: $A_{v2} = \dfrac{v_{02}}{v_d} = \dfrac{\frac{1}{2}g_m R_L}{\left(2 + \frac{R_L}{R_C}\right)}$

Then $A_v = \dfrac{v_{02} - v_{01}}{v_d} = \dfrac{g_m R_L}{\left(2 + \frac{R_L}{R_C}\right)}$

11.13

The small-signal equivalent circuit is



KVL equation: $v_1 = V_{\pi1} - V_{\pi2} + v_2$ or

$$v_1 - v_2 = V_{\pi1} - V_{\pi2}$$

KCL equation:

$$\frac{V_{\pi1}}{r_\pi} + g_m V_{\pi1} + g_m V_{\pi2} + \frac{V_{\pi2}}{r_\pi} = 0$$

$$(V_{\pi1} + V_{\pi2})\left(\frac{1}{r_\pi} + g_m\right) = 0 \Rightarrow V_{\pi1} = -V_{\pi2}$$

Then $v_1 - v_2 = -2V_{\pi2}$ or $V_{\pi2} = -\frac{1}{2}(v_1 - v_2)$

Now $v_0 = -g_m V_{\pi2}(R_C \| R_L)$

$$= \frac{1}{2}g_m(R_C\|R_L)(v_1 - v_2)$$

For $v_1 - v_2 \equiv v_d$

$$\Rightarrow A_d = \frac{v_0}{v_d} = \frac{1}{2}g_m(R_C\|R_L)$$

11.14

We have

$V_{C2} = -g_m V_{\pi 2} R_C = -g_m (V_{b2} - V_e) R_C$

and

$V_{C1} = -g_m V_{\pi 1} R_C = -g_m (V_{b1} - V_e) R_C$

Then

$V_0 = V_{C2} - V_{C1}$

$= -g_m (V_{b2} - V_e) R_C - [-g_m (V_{b1} - V_e) R_C]$

$= g_m R_C (V_{b1} - V_{b2})$

Differential gain

$A_d = \dfrac{V_0}{V_{b1} - V_{b2}} = g_m R_C$

Common-mode gain

$A_{cm} = 0$

11.15

(a) $v_{cm} = 3\,V \Rightarrow V_{C1} = V_{C2} = 3\,V$

Then $R_C = \dfrac{10-3}{0.1} \Rightarrow R_C = 70\,k\Omega$

(b) $CMRR_{dB} = 75\,dB \Rightarrow CMRR = 5623$

Now

$CMRR = \dfrac{1}{2}\left[1 + \dfrac{(1+\beta) I_Q R_o}{\beta V_T}\right]$

$5623 = \dfrac{1}{2}\left[1 + \dfrac{(151)(0.2) R_o}{(150)(0.026)}\right] \Rightarrow R_o = 1.45\,M\Omega$

Use a Widlar current source.

$R_o = r_o [1 + g_m R_E']$

Let V_A of current source transistor be $100\,V$.

Then

$r_o = \dfrac{100}{0.2} = 500\,k\Omega,\ g_m = \dfrac{0.2}{0.026} = 7.69\,mA/V$

$r_\pi = \dfrac{(150)(0.026)}{0.2} = 19.5\,k\Omega$

So

$1450 = 500[1 + (7.69) R_E'] \Rightarrow R_E' = 0.247\,k\Omega$

Now

$R_E' = R_E \| r_\pi \Rightarrow 0.247 = R_E \| 19.5 \Rightarrow R_E = 250\,\Omega$

Then

$I_Q R_E = V_T \ln\left(\dfrac{I_{REF}}{I_Q}\right)$

$(0.2)(0.250) = (0.026)\ln\left(\dfrac{I_{REF}}{(0.2)}\right) \Rightarrow I_{REF} = 1.37\,mA$

Then

$R_1 = \dfrac{10 - 0.7 - (-10)}{1.37} \Rightarrow R_1 = 14.1\,k\Omega$

11.16

$A_d = 180,\ CMRR_{dB} = 85$ dB

$CMRR = 17,783 = \left|\dfrac{A_d}{A_{cm}}\right| = \dfrac{180}{A_{cm}}$

$\Rightarrow |A_{cm}| = 0.01012$

Assume the common-mode gain is negative.

$v_0 = A_d v_d + A_{cm} v_{cm}$

$= 180 v_d - 0.01012 v_{cm}$

$v_0 = 180(2 \sin \omega t)$ mV $- (0.01012)(2 \sin \omega t)$ V

$v_0 = 0.36 \sin \omega t - 0.02024 \sin \omega t$

Ideal Output: $v_0 = 0.360 \sin \omega t$ (V)

Actual Output: $v_0 = 0.340 \sin \omega t$ (V)

11.17

At terminal A.

$R_{THA} = R_A \| R = \dfrac{R(1+\delta)\cdot R}{R(1+\delta)+R} = \dfrac{R(1+\delta)}{2+\delta} \cong \dfrac{R}{2} = 5\,k\Omega$

Variation in R_{TH} is not significant

$V_{THA} = \left(\dfrac{R_A}{R_A + R}\right) V^+ = \dfrac{R(1+\delta)(5)}{R(1+\delta)+R} = \dfrac{5(1+\delta)}{2+\delta}$

At terminal B.

$R_{THB} = R \| R = \dfrac{R}{2} = 5\,k\Omega$

$V_{THB} = \left(\dfrac{R}{R+R}\right) V^+ = 2.5\,V$

From Eq. (11.27)

$V_O = \dfrac{-\beta R_C (V_2 - V_1)}{2(r_\pi + R_B)}$ where $V_2 = V_{THB}$ and $V_1 = V_{THA}$

$R_B = 5\,k\Omega$, $r_\pi = \dfrac{(120)(0.026)}{0.25} = 12.5\,k\Omega$

So

$V_O = \dfrac{-(120)(3)(V_2 - V_1)}{2(12.5+5)} = -10.3(V_2 - V_1)$

We can find $V_2 - V_1 = V_{THB} - V_{THA}$

$V_{THB} - V_{THA} = 2.5 - \left[\dfrac{5(1+\delta)}{2+\delta}\right]$

$= \dfrac{2.5(2+\delta) - 5(1+\delta)}{2+\delta} = \dfrac{2.5\delta - 5\delta}{2+\delta}$

$\cong \dfrac{-2.5\delta}{2} = -1.25\delta$

Then

$V_O = -(10.3)(-1.25)\delta = 12.9\delta$

So for $-0.01 \le \delta \le 0.01$

We have

$-0.129 \le V_{O2} \le 0.129\,V$

11.18

a. $R_{id} = 2r_\pi$

$$r_\pi = \frac{(180)(0.026)}{0.2} = 23.4 \text{ k}\Omega$$

So

$\underline{R_{id} = 46.8 \text{ k}\Omega}$

b. Assuming $r_\mu \to \infty$, then

$$R_{icm} \cong [(1+\beta)R_0] \,\Big\|\, \left[(1+\beta)\left(\frac{r_0}{2}\right)\right]$$

$$r_0 = \frac{125}{0.2} = 625 \text{ k}\Omega$$

$$R_{icm} = [(181)(1)] \| [(181)(0.3125)]$$

$$= 181 \| 56.56$$

$\Rightarrow \underline{R_{icm} = 43.1 \text{ M}\Omega}$

11.19

a. For $I_1 = 1$ mA, $V_{BE3} = 0.7$ V

$$R_1 = \frac{20 - 0.7}{1} \Rightarrow \underline{R_1 = 19.3 \text{ k}\Omega}$$

$$R_2 = \frac{V_T}{I_Q} \cdot \ln\left(\frac{I_1}{I_Q}\right) = \frac{0.026}{0.1} \cdot \ln\left(\frac{1}{0.1}\right)$$

$\Rightarrow \underline{R_2 = 0.599 \text{ k}\Omega}$

b. $r_{\pi 4} = \dfrac{(180)(0.026)}{0.1} = 46.8 \text{ k}\Omega$

$$g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$r_{04} = \frac{100}{0.1} \Rightarrow 1 \text{ M}\Omega$$

From Chapter 10

$$R_0 = r_{04}[1 + g_m(R_E \| r_{\pi 4})]$$

$$R_E \| r_{\pi 4} = 0.599 \| 46.8 = 0.591$$

$$R_0 = (1)[1 + (3.846)(0.591)] = 3.27 \text{ M}\Omega$$

$$r_{01} = \frac{100}{0.05} \Rightarrow 2 \text{ M}\Omega$$

$$R_{icm} \cong [(1+\beta)R_0] \,\Big\|\, \left[(1+\beta)\left(\frac{r_{01}}{2}\right)\right]$$

$$= [(181)(3.27)] \| [(181)(1)]$$

$$= 592 \| 181$$

$\Rightarrow \underline{R_{icm} = 139 \text{ M}\Omega}$

(c) From Eq. (11.32(b))

$$A_{cm} = \frac{-g_m R_C}{1 + \dfrac{2(1+\beta)R_o}{r_\pi + R_B}}$$

$$g_m = \frac{0.05}{0.026} = 1.923 \, mA/V$$

$$r_\pi = \frac{(180)(0.026)}{0.05} = 93.6 \, k\Omega$$

$R_B = 0$

Then

$$A_{cm} = \frac{-(1.923)(50)}{1 + \dfrac{2(181)(3270)}{93.6}} \Rightarrow \underline{A_{cm} = -0.00760}$$

11.20

$$A_{d1} = g_{m1}(R_1 \| r_{\pi 3})$$

$$g_{m1} = \frac{I_{Q1}/2}{V_T} = 19.23 I_{Q1}$$

$$r_{\pi 3} = \frac{\beta V_T}{I_{Q2}/2} = \frac{2(100)(0.026)}{I_{Q2}} = \frac{5.2}{I_{Q2}}$$

$$A_{d2} = \frac{g_{m3}R_2}{2}, \quad g_{m3} = \frac{I_{Q2}/2}{V_T} = 19.23 I_{Q2}$$

Then

$$30 = \frac{(19.23)I_{Q2}}{2} \cdot R_2 \Rightarrow I_{Q2}R_2 = 3.12 \, V$$

Maximum $v_{o2} - v_{o1} = \pm 18 \, mV$ for linearity

$v_{o3}(\max) = (\pm 18)(30) \, mV \Rightarrow \pm 0.54 \, V$

so $I_{Q2}R_2 = 3.12 \, V$ is OK.

From A_{d1}:

$$20 = 19.23 I_{Q1}(R_1 \| r_{\pi 3})$$

$$= 19.23 I_{Q1} \left(\frac{R_1\left(\dfrac{5.2}{I_{Q2}}\right)}{R_1 + \left(\dfrac{5.2}{I_{Q2}}\right)} \right)$$

$$20 = \frac{19.23 I_{Q1} R_1 (5.2)}{I_{Q2} R_1 + 5.2}$$

Let $\dfrac{I_{Q1}}{2} \cdot R_1 = 5V \Rightarrow I_{Q1}R_1 = 10V$

Then

$$20 = \frac{19.23(10)(5.2)}{I_{Q2}R_1 + 5.2} \Rightarrow I_{Q2}R_1 = 44.8 \, V$$

Now

$$I_{Q1}R_1 = 10 \Rightarrow R_1 = \frac{10}{I_{Q1}}$$

So

$$I_{Q2}\left(\frac{10}{I_{Q1}}\right) = 44.8 \Rightarrow \frac{I_{Q2}}{I_{Q1}} = 4.48$$

Let $\underline{I_{Q1} = 100 \, \mu A, \, I_{Q2} = 448 \, \mu A}$

Then

$I_{Q2}R_2 = 3.12 \Rightarrow \underline{R_2 = 6.96 \, k\Omega}$

$I_{Q1}R_1 = 10 \Rightarrow \underline{R_1 = 100 \, k\Omega}$

11.21

a. $I_1 = \frac{20 - V_{GS3}}{50} = 0.25(V_{GS3} - 2)^2$

$20 - V_{GS3} = 12.5(V_{GS3}^2 - 4V_{GS3} + 4)$

$12.5V_{GS3}^2 - 49V_{GS3} + 30 = 0$

$V_{GS3} = \frac{49 \pm \sqrt{(49)^2 - 4(12.5)(30)}}{2(12.5)}$

$\Rightarrow \underline{V_{GS3} = 3.16 \text{ V}}$

$I_1 = \frac{20 - 3.16}{50} \Rightarrow \underline{I_1 = I_Q = 0.337 \text{ mA}}$

$I_{D1} = \frac{I_Q}{2} \Rightarrow \underline{I_{D1} = 0.168 \text{ mA}}$

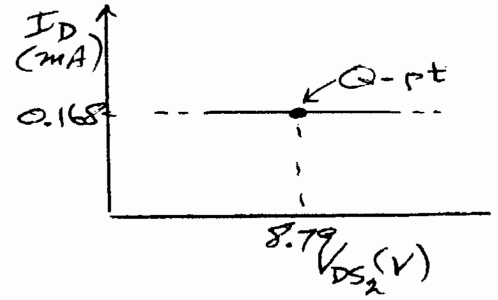
$0.168 = 0.25(V_{GS1} - 2)^2 \Rightarrow V_{GS1} = 2.82 \text{ V}$

$V_{DS4} = -2.82 - (-10) \Rightarrow \underline{V_{DS4} = 7.18 \text{ V}}$

$V_{D1} = 10 - (0.168)(24) = 5.97 \text{ V}$

$V_{DS1} = 5.97 - (-2.82) \Rightarrow \underline{V_{DS1} = 8.79 \text{ V}}$

(b)



(c)Max $v_{CM} \Rightarrow V_{DS1} = V_{DS2} = V_{DS}(sat) = V_{GS1} - V_{TN}$
$2.82 - 2 = 0.82\,V$

Now $V_{D1} = 10 - (0.168)(24) = 5.97\,V$

$V_S(\max) = 5.97 - V_{DS1}(sat) = 5.97 - 0.82$

$V_S(\max) = 5.15\,V$

$v_{CM}(\max) = V_S(\max) + V_{GS1} = 5.15 + 2.82$

$\underline{v_{CM}(\max) = 7.97\,V}$

$v_{CM}(\min) = V^- + V_{DS4}(sat) + V_{GS1}$

$V_{DS4}(sat) = V_{GS4} - V_{TN} = 3.16 - 2 = 1.16\,V$

Then

$v_{CM}(\min) = -10 + 1.16 + 2.82 \Rightarrow$

$\underline{v_{CM}(\min) = -6.02\,V}$

11.22

a. $I_{D1} = I_{D2} = 120\ \mu\text{A} = 100(V_{GS1} - 1.2)^2$

$\Rightarrow V_{GS1} = V_{GS2} = 2.30 \text{ V}$

For $v_1 = v_2 = -5.4$ V and $V_{DS1} = V_{DS2} = 12$ V

$\Rightarrow V_0 = -5.4 - 2.30 + 12 = 4.3 \text{ V}$

$R_D = \frac{10 - 4.3}{0.12} \Rightarrow \underline{R_D = 47.5 \text{ k}\Omega}$

$I_Q = I_{D1} + I_{D2} \Rightarrow \underline{I_Q = I_1 = 240\ \mu\text{A}}$

$I_1 = 240 = 200(V_{GS3} - 1.2)^2 \Rightarrow V_{GS3} = 2.30 \text{ V}$

$R_1 = \frac{20 - 2.3}{0.24} \Rightarrow \underline{R_1 = 73.75 \text{ k}\Omega}$

b. $r_{04} = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(0.24)} = 416.7 \text{ k}\Omega$

$\Delta I_Q = \frac{1}{r_{04}} \cdot \Delta V_{DS} = \frac{5.4}{416.7} \Rightarrow \underline{\Delta I_Q \cong 13\ \mu\text{A}}$

11.23

a. $R_D = \frac{10 - 7}{0.5} \Rightarrow \underline{R_D = 6 \text{ k}\Omega}$

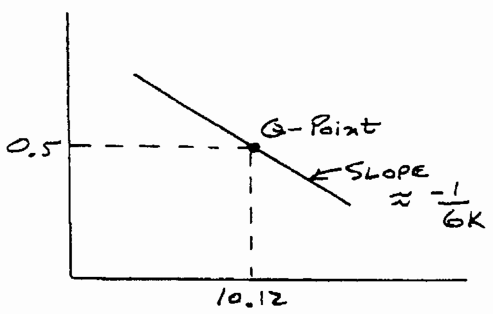
$I_Q = I_{D1} + I_{D2} \Rightarrow \underline{I_Q = 1 \text{ mA}}$

b. $10 = I_D(6) + V_{DS} - V_{GS}$

and $V_{GS} = \sqrt{\frac{I_D}{K_n}} + V_{TN}$

For $I_D = 0.5$ mA, $V_{GS} = \sqrt{\frac{0.5}{0.4}} + 2 = 3.12$ V

and $V_{DS} = 10.12$



Load line is actually nonlinear.

c. Maximum common-mode voltage when M_1 and M_2 reach the transition point, or

$V_{DS}(sat) = V_{GS} - V_{TN} = 3.12 = 2 = 1.12\,V$

Then

$v_{cm} = v_{02} - v_{DS}(\text{sat}) + V_{GS} = 7 - 1.12 + 3.12$

Or $\underline{v_{cm}(\max) = 9 \text{ V}}$

Minimum common-mode voltage, voltage across I_Q becomes zero.

So $v_{cm}(\min) = -10 + 3.12$

$\Rightarrow \underline{v_{cm}(\min) = -6.88 \text{ V}}$

11.24

a. $I_{D1} = I_{D2} = 0.5$ mA

$v_{01} - v_{02} = [V^+ - I_{D1}R_{D1}] - [V^+ - I_{D2}R_{D2}]$

$v_{01} - v_{02} = I_{D2}R_{D2} - I_{D1}R_{D1} = I_D(R_{D2} - R_{D1})$

i. $R_{D1} = R_{D2} = 6\ k\Omega$, $\underline{v_{O1} - v_{O2} = 0}$

ii. $R_{D1} = 6\ k\Omega$, $R_{D2} = 5.9\ k\Omega$

$v_{O1} - v_{O2} = (0.5)(5.9 - 6)$

$\Rightarrow \underline{v_{O1} - v_{O2} = -0.05\ V}$

b. $K_{n1} = 0.4\ mA/V^2$, $K_{n2} = 0.44\ mA/V^2$

$V_{GS1} = V_{GS2}$

$I_Q = (K_{n1} + K_{n2})(V_{GS} - V_{TN})^2$

$1 = (0.4 + 0.44)(V_{GS} - V_{TN})^2$

$\Rightarrow (V_{GS} - V_{TN})^2 = 1.19$

$I_{D1} = (0.4)(1.19) = 0.476$ mA

$I_{D2} = (0.44)(1.19) = 0.524$ mA

i. $R_{D1} = R_{D2} = 6\ k\Omega$

$v_{O1} - v_{O2} = (0.524 - 0.476)(6)$

$\Rightarrow \underline{v_{O1} - v_{O2} = 0.288\ V}$

ii. $R_{D1} = 6\ k\Omega$, $R_{D2} = 5.9\ k\Omega$

$v_{O1} - v_{O2} = (0.524)(5.9) - (0.476)(6)$

$= 3.0916 - 2.856$

$\Rightarrow \underline{v_{O1} - v_{O2} = 0.236\ V}$

11.25

(a) From Equation (11.51)

$$\frac{i_{D2}}{I_Q} = \frac{1}{2} - \sqrt{\frac{K_n}{2I_Q}} \cdot v_d \sqrt{1 - \left(\frac{K_n}{2I_Q}\right) v_d^2}$$

$$0.90 = 0.50 - \sqrt{\frac{0.1}{2(0.25)}} \cdot v_d \sqrt{1 - \left[\frac{0.1}{2(0.25)}\right] v_d^2}$$

$$-0.40 = -(0.4472) v_d \sqrt{1 - (0.2) v_d^2}$$

$$0.8945 = v_d \sqrt{1 - (0.2) v_d^2}$$

Square both sides

$$0.80 = v_d^2 (1 - [0.2] v_d^2)$$

$$(0.2)(v_d^2)^2 - v_d^2 + 0.80 = 0$$

$$v_d^2 = \frac{1 \pm \sqrt{1 - 4(0.2)(0.80)}}{2(0.2)} = 4V^2 \text{ or } 1V^2$$

Then $v_d = 2$ V or 1 V

But $|v_d|_{max} = \sqrt{\frac{I_Q}{k_n}} = \sqrt{\frac{0.25}{0.1}} = 1.58$

So $\underline{v_d = 1\ V}$

b. From part (a), $\underline{v_{d,max} = 1.58\ V}$

11.26

$$A_d = \frac{g_m R_D}{2}$$

For $v_{CM} = 2.5\,V$

$$I_{D1} = I_{D2} = \frac{I_Q}{2} = 0.25\ mA$$

Let $V_{D1} = V_{D2} = 3\,V$, then $R_D = \frac{10 - 3}{0.25} \Rightarrow$

$\underline{R_D = 28\ k\Omega}$

Then $100 = \frac{g_m(28)}{2} \Rightarrow g_m = 7.14\ mA/V$

And $g_m = 2\sqrt{\frac{k_n'}{2}\left(\frac{W}{L}\right) I_D}$

$$7.14 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.25)} \Rightarrow$$

$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 1274$ (Extremely large transistors to meet the gain requirement.)

Need $|A_{CM}| = 0.10$

From Eq.(11.64(b))

$$|A_{CM}| = \frac{g_m R_D}{1 + 2 g_m R_o}$$

So $0.10 = \frac{(7.14)(28)}{1 + 2(7.14) R_o} \Rightarrow R_o = 140\ k\Omega$

For the basic 2-transistor current source

$$R_o = r_o = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(0.5)} = 200\ k\Omega$$

This current source is adequate to meet common-mode gain requirement.

11.27

a. $I_S = \frac{-V_{GS1} - (-5)}{R_S}$

and $I_S = 2I_D = 2K_n(V_{GS1} - V_{TN})^2$

$$\frac{5 - V_{GS1}}{20} = 2(0.050)(V_{GS1} - 1)^2$$

$$5 - V_{GS1} = 2(V_{GS1}^2 - 2V_{GS1} + 1)$$

$$2V_{GS1}^2 - 3V_{GS1} - 3 = 0$$

$$V_{GS1} = \frac{3 \pm \sqrt{(3)^2 + 4(2)(3)}}{2(2)} \Rightarrow V_{GS1} = 2.186\ V$$

$I_S = \frac{5 - 2.186}{20} \Rightarrow \underline{I_S = 0.141\ mA}$

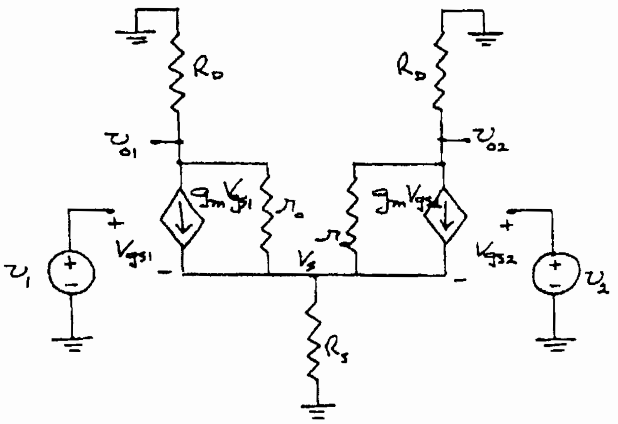
$I_{D1} = I_{D2} = \frac{I_S}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.0704\ mA}$

$v_{O2} = 5 - (0.0704)(25) \Rightarrow \underline{v_{O2} = 3.24\ V}$

b. $g_m = 2K_n(V_{GS} - V_{TN}) = 2(0.05)(2.186 - 1)$

$g_m = 0.119$ mA/V

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.0704)} = 710\ \text{k}\Omega$$



$V_{gs1} = \nu_1 - V_S,\ \ V_{gs2} = \nu_2 - V_S$

$$\frac{\nu_{01}}{R_D} + g_m V_{gs1} + \frac{\nu_{01} - V_S}{r_0} = 0$$

$$\nu_{01}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) + g_m(\nu_1 - V_S) - \frac{V_S}{r_0} = 0 \qquad (1)$$

$$\frac{\nu_{02}}{R_D} + g_m V_{gs2} + \frac{\nu_{02} - V_S}{r_0} = 0$$

$$\nu_{02}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) + g_m(\nu_2 - V_S) - \frac{V_S}{r_0} = 0 \qquad (2)$$

$$g_m V_{gs1} + \frac{\nu_{01} - V_S}{r_0} + \frac{\nu_{02} - V_S}{r_0} + g_m V_{gs2} = \frac{V_S}{R_S}$$

$$g_m(\nu_1 - V_S) + \frac{\nu_{01}}{r_0} + \frac{\nu_{02}}{r_0} - \frac{2V_S}{r_0} + g_m(\nu_2 - V_S) = \frac{V_S}{R_S}$$

$$g_m(\nu_1 + \nu_2) + \frac{\nu_{01}}{r_0} + \frac{\nu_{02}}{r_0} = V_S\left\{2g_m + \frac{2}{r_0} + \frac{1}{R_S}\right\} \qquad (3)$$

From (1)

$$\nu_{01} = \frac{V_S\left(g_m + \frac{1}{r_0}\right) - g_m\nu_1}{\left(\frac{1}{R_D} + \frac{1}{r_0}\right)}$$

Then

$$g_m(\nu_1 + \nu_2) + \frac{V_S\left(g_m + \frac{1}{r_0}\right) - g_m\nu_1}{r_0\left(\frac{1}{R_D} + \frac{1}{r_0}\right)} + \frac{\nu_{02}}{r_0} = V_S\left\{2g_m + \frac{2}{r_0} + \frac{1}{R_S}\right\} \qquad (3)$$

$$g_m(\nu_1 + \nu_2)r_0\left(\frac{1}{R_D} + \frac{1}{r_0}\right) + V_S\left(g_m + \frac{1}{r_0}\right) - g_m\nu_1 + \nu_{02}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) = V_S\left\{2g_m + \frac{2}{r_0} + \frac{1}{R_S}\right\}\cdot r_0\left(\frac{1}{R_D} + \frac{1}{r_0}\right)$$

$$g_m(\nu_1 + \nu_2)\left(1 + \frac{r_0}{R_D}\right) - g_m\nu_1 + \nu_{02}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) = V_S\left\{\left(2g_m + \frac{2}{r_0} + \frac{1}{R_S}\right)\left(1 + \frac{r_0}{R_D}\right) - \left(g_m + \frac{1}{r_0}\right)\right\}$$

$$g_m\left(\nu_1\cdot\frac{r_0}{R_D} + \nu_2 + \nu_2\cdot\frac{r_0}{R_D}\right) + \nu_{02}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) = V_S\left\{2g_m + \frac{2}{r_0} + \frac{1}{R_S} + 2g_m\cdot\frac{r_0}{R_D} + \frac{2}{R_D} + \frac{r_0}{R_S R_D} - g_m - \frac{1}{r_0}\right\}$$

$$g_m\left(\nu_1\cdot\frac{r_0}{R_D} + \nu_2 + \nu_2\cdot\frac{r_0}{R_D}\right) + \nu_{02}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) = V_S\left\{2g_m + \frac{1}{r_0} + \frac{1}{R_S}\left(1 + \frac{r_0}{R_D}\right) + \frac{2}{R_D}(1 + g_m r_0)\right\} \qquad (4)$$

Then substituting into (2),

$$\nu_{02}\left(\frac{1}{R_D} + \frac{1}{r_0}\right) + g_m\nu_2 = V_S\left(g_m + \frac{1}{r_0}\right)$$

Substitute numbers:

$$(0.119)\left[\nu_1\frac{710}{25} + \nu_2 + \nu_2\frac{710}{25}\right] + \nu_{02}\left[\frac{1}{25} + \frac{1}{710}\right] = V_S\left\{0.119 + \frac{1}{710} + \frac{1}{20}\left(1 + \frac{710}{25}\right) + \frac{2}{25}[1 + (0.119)(710)]\right\} \qquad (4)$$

$$(0.119)[28.4\nu_1 + 29.4\nu_2] + (0.0414)\nu_{02} = V_S\{0.1204 + 1.470 + 6.8392\} = V_S(8.4296)$$

or

$$V_S = 0.4010\nu_1 + 0.4150\nu_2 + 0.00491\nu_{02}$$

Then

$$\nu_{02}\left(\frac{1}{25} + \frac{1}{710}\right) + (0.119)\nu_2 = V_S\left(0.119 + \frac{1}{710}\right) \qquad (2)$$

$$\nu_{02}(0.0414) + \nu_2(0.119) = (0.1204)[0.401\nu_1 + 0.4150\nu_2 + 0.00491\nu_{02}]$$

$$\nu_{02}(0.0408) = (0.04828)\nu_1 - (0.0690)\nu_2$$

$$\nu_{02} = (1.183)\nu_1 - (1.691)\nu_2$$

Now $\quad v_1 = v_{cm} + \dfrac{v_d}{2}$

$$v_2 = v_{cm} - \frac{v_d}{2}$$

So

$$v_{02} = (1.183)\left(v_{cm} + \frac{v_d}{2}\right) - (1.691)\left(v_{cm} - \frac{v_d}{2}\right)$$

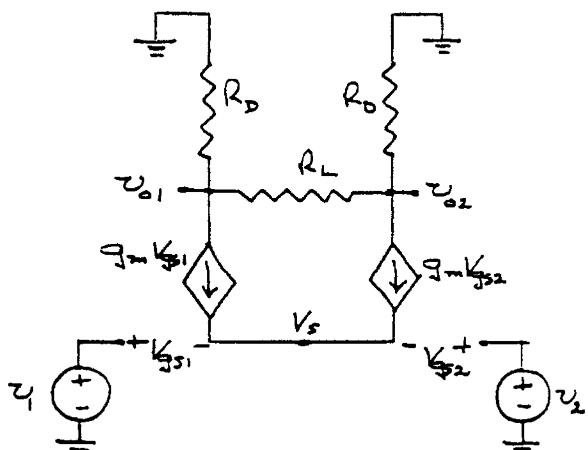
Or $v_{02} = 1.437 v_d - 0.508 v_{cm}$

$\Rightarrow \underline{A_d = 1.437}, \quad \underline{A_{cm} = -0.508}$

$$CMRR_{dB} = 20\log_{10}\left(\frac{1.437}{0.508}\right)$$

$\Rightarrow \underline{CMRR_{dB} = 9.03 \text{ dB}}$

11.28



KVL:

$v_1 = V_{gs1} - V_{gs2} + v_2$

So $v_1 - v_2 = V_{gs1} - V_{gs2}$

KCL:

$g_m V_{gs1} + g_m V_{gs2} = 0 \Rightarrow V_{gs1} = -V_{gs2}$

So $V_{gs1} = \dfrac{1}{2}(v_1 - v_2), \quad V_{gs2} = -\dfrac{1}{2}(v_1 - v_2)$

Now

$$\frac{v_{02}}{R_D} + \frac{v_{02} - v_{01}}{R_L} = -g_m V_{gs2} = v_{02}\left(\frac{1}{R_D} + \frac{1}{R_L}\right) - \frac{v_{01}}{R_L} \qquad (1)$$

$$\frac{v_{01}}{R_D} + \frac{v_{01} - v_{02}}{R_L} = -g_m V_{gs1} = v_{01}\left(\frac{1}{R_D} + \frac{1}{R_L}\right) - \frac{v_{02}}{R_L} \qquad (2)$$

From (1): $\quad v_{01} = v_{02}\left(1 + \dfrac{R_L}{R_D}\right) + g_m R_L V_{gs2}$

Substitute into (2):

$$-g_m V_{gs1} = v_{02}\left(1 + \frac{R_L}{R_D}\right)\left(\frac{1}{R_D} + \frac{1}{R_L}\right) + g_m R_L\left(\frac{1}{R_D} + \frac{1}{R_L}\right)V_{gs2} - \frac{v_{02}}{R_L}$$

$$-g_m \cdot (v_1 - v_2) + g_m\left(1 + \frac{R_L}{R_D}\right)\left(\frac{1}{2}\right)(v_1 - v_2) = v_{02}\left(\frac{1}{R_D} + \frac{R_L}{R_D^2} + \frac{1}{R_D}\right)$$

$$\frac{1}{2}g_m\left(\frac{R_L}{R_D}\right)(v_1 - v_2) = \frac{v_{02}}{R_D}\left(2 + \frac{R_L}{R_D}\right)$$

$$\Rightarrow \underline{A_{d2} = \frac{v_{02}}{v_1 - v_2} = \frac{\frac{1}{2}\cdot g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}}$$

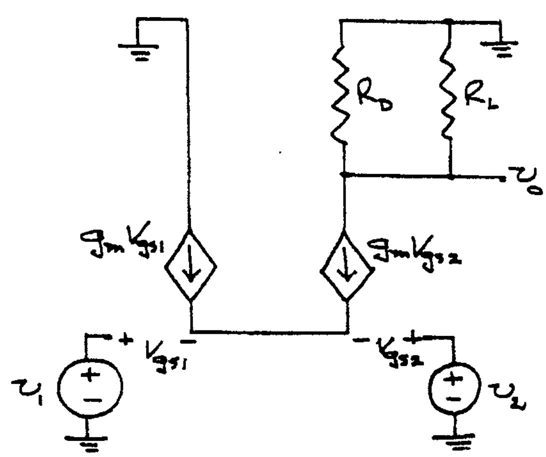
From symmetry

$$\underline{A_{d1} = \frac{v_{01}}{v_1 - v_2} = \frac{-\frac{1}{2}\cdot g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}}$$

Then

$$\underline{A_v = \frac{v_{02} - v_{01}}{v_1 - v_2} = \frac{g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}}$$

11.29



$v_1 - v_2 = V_{gs1} - V_{gs2}$

and $g_m V_{gs1} + g_m V_{gs2} = 0 \Rightarrow V_{gs1} = -V_{gs2}$

Then $v_1 - v_2 = -2V_{gs2}$

Or $V_{gs2} = -\dfrac{1}{2}(v_1 - v_2)$

$v_0 = -g_m V_{gs2}(R_D \| R_L) = \frac{g_m}{2}(R_D \| R_L)(v_1 - v_2)$

Or $A_d = \frac{g_m}{2}(R_D \| R_L)$

11.30

From Equation (11.64(a)), $A_d = \sqrt{\frac{K_n I_Q}{2}} \cdot R_D$

We need $A_d = \frac{2}{0.2} = 10$

Then $10 = \sqrt{\frac{K_n(0.5)}{2}} \cdot R_D$ or $\sqrt{K_n} \cdot R_D = 20$

If we set $R_D = 20\ k\Omega$, then $K_n = 1\ mA/V^2$

For this case $V_D = 10 - (0.25)(20) = 5$ V

$V_{GS} = \sqrt{\frac{0.25}{1}} + 1 = 1.5$ V

$V_{DS}(sat) = V_{GS} - V_{TN} = 1.5 - 1 = 0.5\ V$

Then $v_{cm}(\max) = V_D - V_{DS}(\text{sat}) + V_{GS}$

$= 5 - 0.5 + 1.5$

Or $v_{cm}(\max) = 6$ V

11.31

$V_{d1} = -g_m V_{gs1} R_D = -g_m R_D (V_1 - V_s)$

$V_{d2} = -g_m V_{gs2} R_D = -g_m R_D (V_2 - V_s)$

Now

$V_o = V_{d2} - V_{d1} = -g_m R_D (V_2 - V_s) - (-g_m R_D (V_1 - V_s))$

$V_o = g_m R_D (V_1 - V_2)$

Define $V_1 - V_2 \equiv V_d$

Then

$A_d = \frac{V_o}{V_d} = g_m R_D$

and

$A_{cm} = 0$

11.32

(a) $K_{n1} = K_{n2} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right) = \left(\frac{0.080}{2}\right)(10) = 0.40\ mA/V^2$

$V_{GS1} = V_{GS2} = \sqrt{\frac{I_D}{K_n}} + V_{TN} = \sqrt{\frac{0.1}{0.4}} + 1 = 1.5\ V$

$V_{DS1}(sat) = 1.5 - 1 = 0.5\ V$

For $v_{CM} = +3\ V \Rightarrow V_{D1} = V_{D2} = v_{CM} - V_{GS1} + V_{DS1}(sat)$

$= 3 - 1.5 + 0.5 \Rightarrow V_{D1} = V_{D2} = 2\ V$

$R_D = \frac{10-2}{0.1} \Rightarrow$ $R_D = 80\ k\Omega$

(b) $A_d = \frac{1}{2} g_m R_D$ and $g_m = 2\sqrt{(0.4)(0.1)} = 0.4\ mA/V$

Then $A_d = \frac{1}{2}(0.4)(80) = 16$

$CMRR_{dB} = 45 \Rightarrow CMRR = 177.8 = \frac{16}{A_{cm}}$

So $|A_{cm}| = 0.090$

$|A_{cm}| = \frac{g_m R_D}{1 + 2 g_m R_o}$

$0.090 = \frac{(0.4)(80)}{1 + 2(0.4)R_o} \Rightarrow R_o = 443\ k\Omega$

If we assume $\lambda = 0.01\ V^{-1}$ for the current source transistor, then

$r_o = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(0.2)} = 500\ k\Omega$

So the CMRR specification can be met by a 2-transistor current source.

Let $\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 1$

Then $K_{n3} = K_{n4} = \left(\frac{0.080}{2}\right)(1) = 0.040\ mA/V^2$

and $V_{GS3} = \sqrt{\frac{I_Q}{K_{n3}}} + V_{TN} = \sqrt{\frac{0.2}{0.04}} + 1 = 3.24\ V$

For $v_{CM} = -3\ V$, $V_{D3} = -3 - V_{GS1} = -3 - 1.5 = -4.5\ V$

$\Rightarrow V_{DS3}(\min) = -4.5 - (-10) = 5.5\ V > V_{DS3}(sat)$

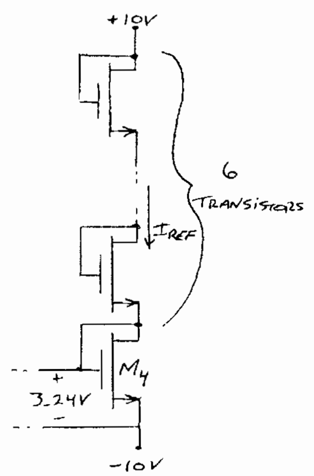
So design is OK.

On reference side: For $\left(\frac{W}{L}\right) \ge 1$, $V_{GS}(\max) = 3.24\ V$

$20 - V_{GS3} = 20 - 3.24 = 16.76\ V$

Then

$\frac{16.67}{3.24} = 5.17 \Rightarrow$ We need six transistors in series.



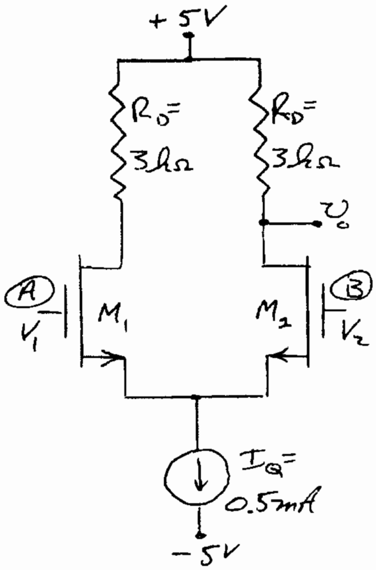
$V_{GS} = \frac{20 - 3.24}{6} = 2.793\ V$

$I_{REF} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$

$0.2 = \left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(2.793 - 1)^2 \Rightarrow$

$\left(\frac{W}{L}\right) = 1.56$ for each of the 6 transistors.

11.33



$$A_d = \frac{1}{2} g_m R_D$$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.25)(0.25)} = 0.50\, mA/V$$

$$A_d = \frac{1}{2}(0.50)(3) = 0.75$$

From Problem 11.17

$$V_1 = V_A = \frac{5(1+\delta)}{2+\delta}, \quad V_2 = V_B = 2.5V$$

and $V_1 - V_2 = 1.25\delta$

Then

$$V_{o2} = A_d \cdot (V_1 - V_2) = (0.75)(1.25\delta) = 0.9375\delta$$

So for $-0.01 \le \delta \le 0.01$

$$-9.375 \le V_{o2} \le 9.375\, mV$$

11.34

From previous results

$$A_{d1} = \frac{v_{o2} - v_{o1}}{v_1 - v_2} = g_{m1} R_1 = \sqrt{2K_{n1} I_{Q1}} \cdot R_1 = 20$$

and

$$A_{d2} = \frac{v_{o3}}{v_{o2} - v_{o1}} = \frac{1}{2} g_{m3} R_2 = \frac{1}{2}\sqrt{2K_{n3} I_{Q2}} \cdot R_2 = 30$$

Set $\dfrac{I_{Q1}R_1}{2} = 5V$ and $\dfrac{I_{Q2}R_2}{2} = 2.5V$

Let $I_{Q1} = I_{Q2} = 0.1\, mA$

Then $R_1 = 100\, k\Omega$, $R_2 = 50\, k\Omega$

Then

$$2\left(\frac{0.06}{2}\right)\left(\frac{W}{L}\right)_1 (0.1) = \left(\frac{20}{100}\right)^2 \Rightarrow$$

$$\underline{\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 6.67}$$

and

$$2\left(\frac{0.060}{2}\right)\left(\frac{W}{L}\right)_3 (0.1) = \left(\frac{2(30)}{50}\right)^2 \Rightarrow$$

$$\underline{\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 240}$$

11.35

a. $$i_{D1} = I_{DSS}\left(1 - \frac{v_{GS1}}{V_P}\right)^2$$

$$i_{D2} = I_{DSS}\left(1 - \frac{v_{GS2}}{V_P}\right)^2$$

$$\sqrt{i_{D1}} - \sqrt{i_{D2}}$$

$$= \sqrt{I_{DSS}}\left(1 - \frac{v_{GS1}}{V_P}\right) - \sqrt{I_{DSS}}\left(1 - \frac{v_{GS2}}{V_P}\right)$$

$$= \frac{\sqrt{I_{DSS}}}{V_P}(v_{GS2} - v_{GS1})$$

$$= -\frac{\sqrt{I_{DSS}}}{V_P} \cdot v_d = \frac{\sqrt{I_{DSS}}}{(-V_P)} \cdot v_d$$

$$i_{D1} + i_{D2} = I_Q \Rightarrow i_{D2} = I_Q - i_{D1}$$

$$\left(\sqrt{i_{D1}} - \sqrt{I_Q - i_{D1}}\right)^2 = \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2$$

$$i_{D1} - 2\sqrt{i_{D1}(I_Q - i_{D1})} + (I_Q - i_{D1}) = \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2$$

Then

$$\sqrt{i_{D1}(I_Q - i_{D1})} = \frac{1}{2}\left[I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2\right]$$

Square both sides

$$i_{D1}^2 - i_{D1} I_Q + \frac{1}{4}\left[I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2\right]^2 = 0$$

$$i_{D1} = \frac{I_Q \pm \sqrt{I_Q^2 - 4\left(\frac{1}{4}\right)\left[I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2\right]^2}}{2}$$

$$i_{D1} =$$

$$\frac{I_Q}{2} \pm \frac{1}{2}\sqrt{I_Q^2 - \left[I_Q^2 - \frac{2I_Q I_{DSS} v_d^2}{(-V_P)^2} + \left(\frac{I_{DSS} v_d^2}{(-V_P)^2}\right)^2\right]}$$

Use + sign

$$i_{D1} = \frac{I_Q}{2} + \frac{1}{2}\sqrt{\frac{2I_Q I_{DSS}}{(-V_P)^2} \cdot v_d^2 - \left(\frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2\right)^2}$$

$$i_{D1} = \frac{I_Q}{2} + \frac{1}{2}\frac{I_Q}{(-V_P)} v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

Or

$$\frac{i_{D1}}{I_Q} = \frac{1}{2} + \left(\frac{1}{-2V_P}\right) \cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

We had

$$i_{D2} = I_Q - i_{D1}$$

Then

$$\frac{i_{D2}}{I_Q} = \frac{1}{2} - \left(\frac{1}{-2V_P}\right)\cdot v_d\sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

b. If $i_{D1} = I_Q$, then

$$1 = \frac{1}{2} + \left(\frac{1}{-2V_P}\right)\cdot v_d\sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

$$|V_P| = v_d\sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

Square both sides

$$|V_P|^2 = v_d^2\left[\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2\right]$$

$$\left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{1}{V_P}\right)^2(v_d^2)^2 - \frac{2I_{DSS}}{I_Q}\cdot v_d^2 + |V_P|^2 = 0$$

$v_d^2 =$

$$\frac{\frac{2I_{DSS}}{I_Q} \pm \sqrt{\left(\frac{2I_{DSS}}{I_Q}\right)^2 - 4\left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{1}{V_P}\right)^2(V_P)^2}}{2\left(\frac{2I_{DSS}}{I_Q}\right)^2\left(\frac{1}{V_P}\right)^2}$$

$$v_d^2 = (V_P)^2\left(\frac{I_Q}{I_{DSS}}\right)$$

Or $|v_d| = |V_P|\left(\frac{I_Q}{I_{DSS}}\right)^{1/2}$

c. For v_d small,

$$i_{D1} \approx \frac{I_Q}{2} + \frac{1}{2}\cdot\frac{I_Q}{(-V_P)}\cdot v_d\sqrt{\frac{2I_{DSS}}{I_Q}}$$

$$g_f = \left.\frac{di_{D1}}{dv_d}\right|_{v_d\to 0} = \frac{1}{2}\cdot\frac{I_Q}{(-V_P)}\cdot\sqrt{\frac{2I_{DSS}}{I_Q}}$$

Or

$$\Rightarrow g_f(\max) = \left(\frac{1}{-V_P}\right)\sqrt{\frac{I_Q I_{DSS}}{2}}$$

11.36

a. $I_Q = I_{D1} + I_{D2} \Rightarrow \underline{I_Q = 1\text{ mA}}$

$v_O = 7 = 10 - (0.5)R_D \Rightarrow \underline{R_D = 6\text{ k}\Omega}$

b. $g_f(\max) = \left(\frac{1}{-V_P}\right)\sqrt{\frac{I_Q\cdot I_{DSS}}{2}}$

$$g_f(\max) = \left(\frac{1}{4}\right)\sqrt{\frac{(1)(2)}{2}}$$

$\Rightarrow \underline{g_f(\max) = 0.25\text{ mA/V}}$

c. $A_d = \frac{g_m R_D}{2} = g_f(\max)\cdot R_D$

$A_d = (0.25)(6) \Rightarrow \underline{A_d = 1.5}$

11.37

a. $I_S = \frac{-V_{GS} - (-5)}{R_S} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$$5 - V_{GS} = (0.8)(20)\left(1 - \frac{V_{GS}}{(-2)}\right)^2$$

$$5 - V_{GS} = 16\left(1 + V_{GS} + \frac{1}{4}V_{GS}^2\right)$$

$$4V_{GS}^2 + 17V_{GS} + 11 = 0$$

$$V_{GS} = \frac{-17 \pm \sqrt{(17)^2 - 4(4)(11)}}{2(4)}$$

$V_{GS} = -0.796$ V

$I_S = \frac{5-(-0.796)}{20} \Rightarrow \underline{I_S = 0.290\text{ mA}}$

$I_{D1} = I_{D2} = \frac{I_S}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.145\text{ mA}}$

$v_{O2} = 5 - (0.145)(25) \Rightarrow \underline{v_{O2} = 1.375\text{ V}}$

b. Taking into account the r_o parameters of Q_1 and Q_2, the analysis is identical to that in problem 11.34.

11.38

Equivalent circuit and analysis is identical to that in problem 11.36.

$$A_{d2} = \frac{\frac{1}{2}\cdot g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}$$

$$A_{d1} = \frac{-\frac{1}{2}\cdot g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}$$

$$A_v = \frac{v_{O2} - v_{O1}}{v_d} = \frac{g_m R_L}{\left(2 + \frac{R_L}{R_D}\right)}$$

11.39

a. Using the results of problem 10.59, the resistance from the base of Q_4 looking toward Q_3:

$$\frac{1}{R_0'} = \frac{1}{r_{01}} + \frac{\left(\frac{1}{r_{\pi 3}} + g_{m3} + \frac{1}{r_{03}}\right)}{\left[1 + \left(\frac{1}{r_{\pi 3}} + g_{m3} + \frac{1}{r_{03}}\right)R_E\right]}$$

$r_{01} = \frac{120}{0.1} = 1200\text{ k}\Omega$, $r_{03} = \frac{80}{0.1} = 800\text{ k}\Omega$

Assume $\beta = 100$

$r_{\pi 3} = \frac{(100)(0.026)}{0.1} = 26\text{ k}\Omega$

$g_{m3} = \frac{0.1}{0.026} = 3.846\text{ mA/V}$

$$\frac{1}{R_o'} = \frac{1}{1200} + \frac{\left(\frac{1}{26} + 3.846 + \frac{1}{800}\right)}{\left[1 + \left(\frac{1}{26} + 3.846 + \frac{1}{800}\right)(1)\right]}$$

$$= \frac{1}{1200} + \frac{3.886}{1 + (3.886)(1)} \Rightarrow R_o' = 1.256 \text{ k}\Omega$$

$$R_o = r_{o2}\left[1 + \frac{R_E \| (r_{\pi 2} + R_o')}{r_{o2}} + g_{m2}\left(\frac{r_{\pi 2}}{r_{\pi 2} + R_o'}\right)\{R_E \| (r_{\pi 2} + R_o')\}\right]$$

$$R_E \| (r_{\pi 2} + R_o') = (1) \| (26 + 1.256)$$

$$= (1) \| (27.256)$$

$$= 0.965 \text{ k}\Omega$$

$$R_o = 1200\left[1 + \frac{0.965}{1200} + (3.846)\left(\frac{26}{26 + 1.256}\right)(0.965)\right]$$

$$\Rightarrow R_o = 5.45 \text{ M}\Omega$$

Then

$$A_v = -g_m(r_{o2} \| R_o)$$

$$r_{o2} = \frac{120}{0.1} = 1200 \text{ k}\Omega$$

$$g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$A_v = -(3.846)[1200 \| 5450]$$

$$\Rightarrow \underline{A_v = -3782}$$

b. For $R = 0$, $r_{o4} = \frac{80}{0.1} = 800 \text{ k}\Omega$

$$A_v = -g_m(r_{o2} \| r_{o4})$$

$$= -(3.846)[1200 \| 800]$$

$$\Rightarrow \underline{A_v = -1846}$$

(c) For part (a), $R_o = (5.45 \| 1.2) = 0.983\ M\Omega$

For part (b), $R_o = (1.2 \| 0.8) = 0.48\ M\Omega$

11.40

$$I_{B5} = \frac{I_{E5}}{1+\beta} = \frac{I_{B3} + I_{B4}}{1+\beta} = \frac{I_{C3} + I_{C4}}{\beta(1+\beta)}$$

Now $I_{C3} + I_{C4} \approx I_Q$

So $I_{B5} \approx \frac{I_Q}{\beta(1+\beta)}$

$$I_{B6} = \frac{I_{E6}}{1+\beta} = \frac{I_{Q1}}{\beta(1+\beta)}$$

For balance, we want $I_{B6} = I_{B5}$

So that $\underline{I_{Q1} = I_Q}$

11.41

a. $A_d = g_m(r_{o2} \| r_{o4})$

$$r_{o2} = \frac{V_{A2}}{I_{C2}} = \frac{150}{0.4} = 375 \text{ k}\Omega$$

$$r_{o4} = \frac{V_{A4}}{I_{C4}} = \frac{100}{0.4} = 250 \text{ k}\Omega$$

$$g_m = \frac{I_{C2}}{V_T} = \frac{0.4}{0.026} = 15.38 \text{ mA/V}$$

$$A_d = (15.38)(375 \| 250)$$

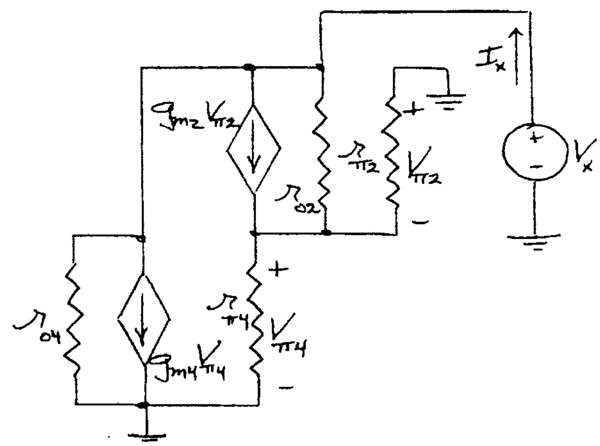
$$\Rightarrow \underline{A_d = 2307}$$

b. $R_L = r_{o2} \| r_{o4} = 375 \| 250$

$$\Rightarrow \underline{R_L = 150 \text{ k}\Omega}$$

11.42

(a) For Q_2, Q_4



(1) $I_x = \frac{V_x - V_{\pi 4}}{r_{o2}} + g_{m2}V_{\pi 2} + g_{m4}V_{\pi 4} + \frac{V_x}{r_{o4}}$

(2) $g_{m2}V_{\pi 2} + \frac{V_x - V_{\pi 4}}{r_{o2}} = \frac{V_{\pi 4}}{r_{\pi 4} \| r_{\pi 2}}$

(3) $V_{\pi 4} = -V_{\pi 2}$

From (2)

$$\frac{V_x}{r_{o2}} = V_{\pi 4}\left[\frac{1}{r_{\pi 4} \| r_{\pi 2}} + \frac{1}{r_{o2}} + g_{m2}\right]$$

Now

$$I_{C4} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{I_Q}{2}\right) = \left(\frac{120}{121}\right)(0.5) = 0.496\,mA$$

$$I_{C2} = \left(\frac{I_Q}{2}\right)\left(\frac{1}{1+\beta}\right)\left(\frac{\beta}{1+\beta}\right) = (0.5)\left(\frac{120}{(121)^2}\right) \Rightarrow$$

$$I_{C2} = 0.0041\,mA$$

So

$$r_{\pi 2} = \frac{(120)(0.026)}{0.0041} = 761\ k\Omega$$

$$g_{m2} = \frac{0.0041}{0.026} = 0.158\ mA/V$$

$$r_{o2} = \frac{100}{0.0041} \Rightarrow 24.4\ M\Omega$$

$$r_{\pi 4} = \frac{(120)(0.026)}{0.496} = 6.29\ k\Omega$$

$$g_{m4} = \frac{0.496}{0.026} = 19.08\ mA/V$$

$$r_{o4} = \frac{100}{0.496} = 202\ k\Omega$$

Now

$$\frac{V_x}{r_{o2}} = V_{\pi 4}\left[\frac{1}{6.29\|761} + \frac{1}{24400} + 0.158\right] \Rightarrow$$

which yields

$$V_{\pi 4} = \frac{V_x}{(0.318)r_{o2}}$$

From (1),

$$I_x = \frac{V_x}{r_{o2}} + \frac{V_x}{r_{o4}} + V_{\pi 4}\left(g_{m4} - g_{m2} - \frac{1}{r_{o2}}\right)$$

$$\frac{I_x}{V_x} = \left[\frac{1}{24400} + \frac{1}{202} + \frac{\left(19.08 - 0.158 - \frac{1}{24400}\right)}{(0.318)(24400)}\right]$$

which yields

$$R_{o2} = \frac{V_x}{I_x} = 135\ k\Omega$$

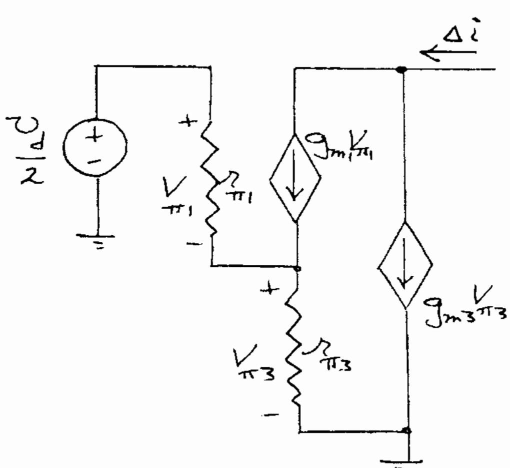
Now

$$r_{o6} = \frac{80}{0.5} = 160\ k\Omega$$

Then

$$R_o = R_{o2}\|r_{o6} = 135\|160 \Rightarrow \underline{R_o = 73.2\ k\Omega}$$

(b) $A_d = g_m^c R_o$ where $g_m^c = \dfrac{\Delta i}{v_d/2}$



$$\Delta i = g_{m1}V_{\pi 1} + g_{m3}V_{\pi 3} \text{ and } V_{\pi 1} + V_{\pi 3} = \frac{v_d}{2}$$

Also $\left(\dfrac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1}\right)r_{\pi 3} = V_{\pi 3}$

So $V_{\pi 1}\left(\dfrac{1+\beta}{r_{\pi 1}}\right)r_{\pi 3} = V_{\pi 3}$

Or

$$V_{\pi 1}\left(\frac{121}{761}\right)(6.29) = V_{\pi 3} \cong V_{\pi 1}$$

Then $2V_{\pi 1} = \dfrac{v_d}{2} \Rightarrow V_{\pi 1} = \dfrac{v_d}{4}$

So

$$\Delta i = (g_{m1} + g_{m3})V_{\pi 1} = (0.158 + 19.08)\left(\frac{v_d}{4}\right) = 9.62\left(\frac{v_d}{2}\right)$$

So

$$g_m^c = \frac{\Delta i}{v_d/2} = 9.62 \Rightarrow A_d = (9.62)(73.2) \Rightarrow$$

$\underline{A_d = 704}$

Now

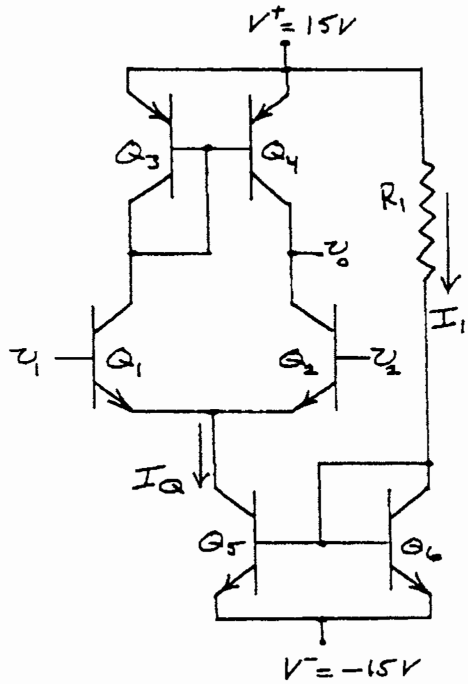
$R_{id} = 2R_i$ where $R_i = r_{\pi 1} + (1+\beta)r_{\pi 3}$

$R_i = 761 + (121)(6.29) = 1522\ k\Omega$

Then

$\underline{R_{id} = 3.044\ M\Omega}$

11.43



a. $g_f = \dfrac{I_Q}{4V_T} \Rightarrow I_Q = g_f(4V_T) = (8)(4)(0.026)$

$\Rightarrow \underline{I_Q = 0.832\ \text{mA}}$

Neglecting base currents,

$R_1 = \frac{30 - 0.7}{0.832} \Rightarrow \underline{R_1 = 35.2\ \text{k}\Omega}$

b. $r_{o4} = r_{o2} = \frac{V_A}{I_{CQ}} = \frac{100}{0.416} = 240\ \text{k}\Omega$

$g_m = \frac{I_{CQ}}{V_T} = \frac{0.416}{0.026} = 16\ \text{mA/V}$

$A_d = g_m(r_{o2} \| r_{o4}) = 16(240 \| 240)$

$\Rightarrow \underline{A_d = 1920}$

$R_{id} = 2r_\pi,\ r_\pi = \frac{(180)(0.026)}{0.416} = 11.25\ \text{k}\Omega$

$\Rightarrow \underline{R_{id} = 22.5\ \text{k}\Omega}$

$R_O = r_{o2} \| r_{o4} \Rightarrow \underline{R_O = 120\ \text{k}\Omega}$

c. Max. common-mode voltage when

$V_{CB} = 0$ for Q_1 and Q_2.

Therefore

$v_{cm}(\max) = V^+ - V_{EB}(Q_3) = 15 - 0.7$

$v_{cm}(\max) = 14.3\ \text{V}$

Min. common-mode voltage when

$V_{CB} = 0$ for Q_5.

Therefore

$v_{cm}(\min) = 0.7 + 0.7 + (-15) = -13.6\ \text{V}$

So $\underline{-13.6 \le v_{cm} \le 14.3\ \text{V}}$

$R_{icm} \cong \frac{1}{2}(1+\beta)(2R_o)$

$R_o = \frac{V_A}{I_Q} = \frac{100}{0.832} = 120\ \text{k}\Omega$

$R_{icm} = (181)(120) \Rightarrow \underline{R_{icm} = 21.7\ \text{M}\Omega}$

11.44

a. $I_0 = I_{B3} + I_{B4} \approx 2\left(\frac{I_Q}{2}\right)\left(\frac{1}{\beta}\right)$

$I_0 = \frac{I_Q}{\beta} = \frac{0.2}{100} \Rightarrow \underline{I_0 = 2\ \mu\text{A}}$

b. $r_{o2} = r_{o4} = \frac{V_A}{I_{CQ}} = \frac{100}{0.1} = 1000\ \text{k}\Omega$

$g_m = \frac{I_{CQ}}{V_T} = \frac{0.1}{0.026} = 3.846\ \text{mA/V}$

$A_d = g_m(r_{o2} \| r_{o4}) = (3.846)(1000 \| 1000)$

$\Rightarrow \underline{A_d = 1923}$

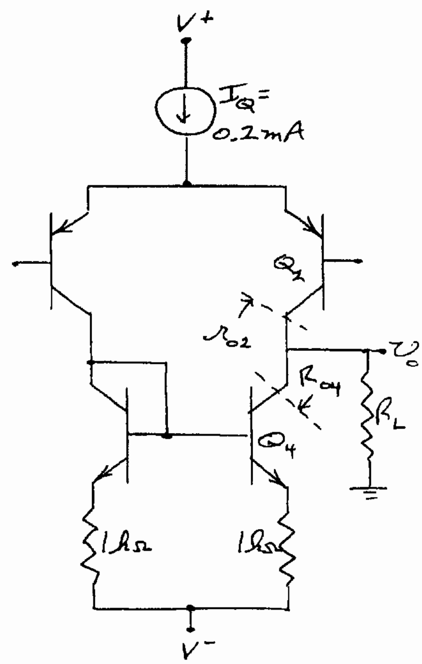
c. $A_d = g_m(r_{o2} \| r_{o4} \| R_L)$

$A_d = (3.846)(1000 \| 1000 \| 250)$

$\Rightarrow \underline{A_d = 641}$

11.45

Let $\beta = 100,\ V_A = 100\,V$



$r_{o2} = \frac{V_A}{I_{CQ}} = \frac{100}{0.1} = 1000\ k\Omega$

$R_{o4} = r_{o4}\left[1 + g_m R_E'\right]$ where $R_E' = r_\pi \| R_E$

Now

$r_\pi = \frac{(100)(0.026)}{0.1} = 26\ k\Omega$

$g_m = \frac{0.1}{0.026} = 3.846\ mA/V$

$R_E' = 26 \| 1 = 0.963\ k\Omega$

Then

$R_{o4} = 1000\left[1 + (3.846)(0.963)\right] = 4704\ k\Omega$

$A_d = g_m(r_{o2} \| R_{o4}) = 3.846(1000 \| 4704) \Rightarrow$

$\underline{A_d = 3172}$

11.46

a. $A_d = g_m(r_{o2} \| r_{o4} \| R_L)$

$g_m = \frac{I_{CQ}}{V_T} = \frac{I_Q}{2V_T}$

$r_{o2} = \frac{V_{A2}}{I_{CQ}} = \frac{125}{I_{CQ}}$

$r_{o4} = \frac{V_{A4}}{I_{CQ}} = \frac{80}{I_{CQ}}$

If $I_Q = 2$ mA, then $g_m = 38.46$ mA/V

$r_{02} = 125$ kΩ. $r_{04} = 80$ kΩ

So $A_d = 38.46[125\|80\|200]$

Or $A_d = 1508$

For each gain of 1000, lower the current level

For $I_Q = 0.60$ mA, $I_{CQ} = 0.30$ mA

$$g_m = \frac{0.3}{0.026} = 11.54 \text{ mA/V}$$

$$r_{02} = \frac{125}{0.3} = 417 \text{ k}\Omega$$

$$r_{04} = \frac{80}{0.3} = 267 \text{ k}\Omega$$

$A_d = 11.54[417\|267\|200] = 1036$

So $I_Q = 0.60$ mA is adequate

b. For $V^+ = 10$ V, $V_{BE} = V_{EB} = 0.6$ V

For $V_{CB} = 0$,

$v_{cm}(\max) = V^+ - 2V_{EB} = 10 - 2(0.6)$

Or $\underline{v_{cm}(\max) = 8.8 \text{ V}}$

11.48

a. From symmetry,

$$V_{GS3} = V_{GS4} = V_{DS3} = V_{DS4} = \sqrt{\frac{0.1}{0.1}} + 1$$

Or $\underline{V_{DS3} = V_{DS4} = 2 \text{ V}}$

$$V_{SG1} = V_{SG2} = \sqrt{\frac{0.1}{0.1}} + 1 = 2 \text{ V}$$

$$V_{SD1} = V_{SD2} = V_{SG1} - (V_{DS3} - 10)$$
$$= 2 - (2 - 10)$$

Or $\underline{V_{SD1} = V_{SD2} = 10 \text{ V}}$

b. $r_{on} = \dfrac{1}{\lambda_n I_{DQ}} = \dfrac{1}{(0.01)(0.1)} \Rightarrow 1 \text{ M}\Omega$

$$r_{op} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.015)(0.1)} \Rightarrow 0.667 \text{ M}\Omega$$

$$g_m = 2K_p(V_{SG} + V_{TP})$$
$$= 2(0.1)(2-1) = 0.2 \; mA/V$$

$$A_d = g_m(r_{on}\|r_{op}) = (0.2)(1000\|667)$$

$\Rightarrow \underline{A_d = 80}$

(c) $I_{D2} = I_{D1} = \dfrac{I_Q}{2} = 0.1 \; mA$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.01)(0.1)} = 1000 \; k\Omega$$

$$r_{o2} = \frac{1}{\lambda_p I_{D2}} = \frac{1}{(0.015)(0.1)} = 667 \; k\Omega$$

$$R_o = r_{o2}\|r_{o4} = 667\|1000 = 400 \; k\Omega$$

11.49

$A_d = g_m(r_{02}\|r_{04})$

$$g_m = 2\sqrt{k_n I_{DQ}} = 2\sqrt{(0.12)(0.075)}$$
$$= 0.1897 \text{ mA/V}$$

$$r_{02} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.015)(0.075)} = 889 \text{ k}\Omega$$

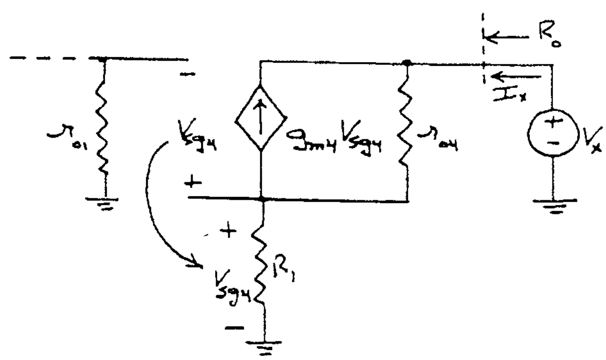
$$r_{04} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.02)(0.075)} = 667 \text{ k}\Omega$$

$A_d = (0.1897)(889\|667)$

$\Rightarrow \underline{A_d = 72.3}$

11.50

Resistance looking into drain of M_4.



$V_{sg4} = I_X R_1$

$$I_X = g_{m4}V_{sg4} = \frac{V_X - V_{sg4}}{r_{04}}$$

$$I_X\left[1 + g_{m4}R_1 + \frac{R_1}{r_{04}}\right] = \frac{V_X}{r_{04}}$$

$$\text{Or } R_0 = r_{04}\left[1 + g_{m4}R_1 + \frac{R_1}{r_{04}}\right]$$

a. $A_d = g_{m2}(r_{o2}\|R_o)$

$$g_{m2} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.080)(0.1)}$$
$$= 0.179 \; mA/V$$

$$r_{o2} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.015)(0.1)} = 667 \; k\Omega$$

$$g_{m4} = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.080)(0.1)}$$
$$= 0.179 \; mA/V$$

$$r_{o4} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 \; k\Omega$$

$$R_0 = 500\left[1 + (0.179)(1) + \frac{1}{500}\right] = 590.5 \text{ k}\Omega$$

$A_d = (0.179)[667\|590.5]$

$\Rightarrow \underline{A_d = 56.06}$

b. When $R_1 = 0$, $R_0 = r_{04} = 500$ kΩ

$A_d = (0.179)[667\|500]$

$\Rightarrow \underline{A_d = 51.15}$

(c) For part (a), $R_o = r_{o2}\|R_o = 667\|590.5 \Rightarrow$

$\underline{R_o = 313\ k\Omega}$

For part (b), $R_o = r_{o2}\|r_{o4} = 667\|500 \Rightarrow$

$\underline{R_o = 286\ k\Omega}$

11.51

(a) $A_d = 100 = g_m(r_{o2}\|r_{o4})$

Let $I_Q = 0.5\ mA$

$$r_{o2} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.02)(0.25)} = 200\ k\Omega$$

$$r_{o4} = \frac{1}{\lambda_p I_D} = \frac{1}{(0.025)(0.25)} = 160\ k\Omega$$

Then

$100 = g_m(200\|160) \Rightarrow g_m = 1.125\ mA/V$

$$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_D}$$

$$1.125 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.25)} \Rightarrow \underline{\left(\frac{W}{L}\right)_n = 31.6}$$

Now $\left(\frac{W}{L}\right)_p$ somewhat arbitrary. Let $\left(\frac{W}{L}\right)_p = 31.6$

11.52

$A_d = g_m(r_{o2}\|r_{o4})$

$P = (I_Q + I_{REF})(V^+ - V^-)$

Let $I_Q = I_{REF}$

Then $0.5 = 2I_Q(3-(-3)) \Rightarrow I_Q = I_{REF} = 0.0417\ mA$

$$r_{o2} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.015)(0.0208)} = 3205\ k\Omega$$

$$r_{o4} = \frac{1}{\lambda_p I_D} = \frac{1}{(0.02)(0.0208)} = 2404\ k\Omega$$

Then

$A_d = 80 = g_m(3205\|2404) \Rightarrow g_m = 0.0582\ mA/V$

$$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_n I_D}$$

$$0.0582 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)_n(0.0208)} \Rightarrow$$

$$\underline{\left(\frac{W}{L}\right)_n = 1.02}$$

11.53

$A_d = g_m(r_{o2}\|R_o)$

Want $A_d = 400$

From Example 11.15, $r_{o2} = 1\ M\Omega$

Assuming that $g_m = 0.283\ mA/V$ for the PMOS from Example 11.15, then $R_o = 285\ M\Omega$.

So

$400 = g_m(1000\|285000) \Rightarrow$

$$g_m = 0.4014\ mA/V = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_1 I_{DQ}}$$

$$0.04028 = \left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)_1(0.1) \Rightarrow$$

$$\underline{\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 10.1}$$

11.54

$A_d = g_m(R_{o4}\|R_{o6})$

where

$R_{o4} = r_{o4} + r_{o2}[1 + g_{m4}r_{o4}]$

$R_{o6} = r_{o6} + r_{o8}[1 + g_{m6}r_{o6}]$

We have

$$r_{o2} = r_{o4} = \frac{1}{(0.015)(0.040)} = 1667\ k\Omega$$

$$r_{o6} = r_{o8} = \frac{1}{(0.02)(0.040)} = 1250\ k\Omega$$

$$g_{m4} = 2\sqrt{\left(\frac{0.060}{2}\right)(15)(0.040)} = 0.268\ mA/V$$

$$g_{m6} = 2\sqrt{\left(\frac{0.025}{2}\right)(10)(0.040)} = 0.141\ mA/V$$

Then

$R_{o4} = 1667 + 1667[1 + (0.268)(1667)] \Rightarrow 748\ M\Omega$

$R_{o6} = 1250 + 1250[1 + (0.141)(1250)] \Rightarrow 222.8\ M\Omega$

(a)

$R_o = R_{o4}\|R_{o6} = 748\|222.8 \Rightarrow \underline{R_o = 172\ M\Omega}$

(b)

$A_d = g_{m4}(R_{o4}\|R_{o6}) = (0.268)(172000) \Rightarrow$

$\underline{A_d = 46096}$

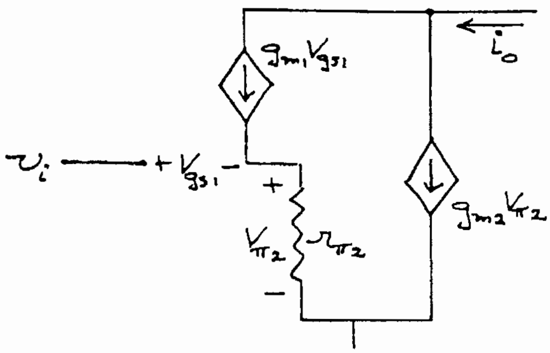
11.55

$$g_{m1} = 2\sqrt{K_n I_{Bias1}} = 2\sqrt{(0.2)(0.25)}$$

$$= 0.447\ \text{mA/V}$$

$$g_{m2} = \frac{I_{CQ}}{V_T} = \frac{0.75}{0.026} = 28.85\ \text{mA/V}$$

$$r_{\pi 2} = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.75} = 4.16\ \text{k}\Omega$$



$i_o = g_{m1}V_{gs1} + g_{m2}V_{\pi2}$

$V_{\pi2} = g_{m1}V_{gs1}r_{o2}$ and $v_i = V_{gs1} + V_{\pi2}$

$i_o = V_{gs1}(g_{m1} + g_{m2} \cdot g_{m1}r_{\pi2})$

$v_i = V_{gs1} + g_{m1}V_{gs1}r_{\pi2}$

and $V_{gs1} = \dfrac{v_i}{1 + g_{m1}r_{\pi2}}$

$i_o = v_i \cdot \dfrac{g_{m1}(1+\beta)}{1 + g_{m1}r_{\pi2}}$

$g_m^C = \dfrac{i_o}{v_i} = \dfrac{g_{m1}(1+\beta)}{1 + g_{m1}r_{\pi2}}$

$= \dfrac{(0.447)(121)}{1 + (0.447)(4.16)}$

$\Rightarrow \underline{g_m^C = 18.9 \text{ mA/V}}$

11.56

$r_o(M_2) = \dfrac{1}{\lambda_n I_{DQ}} = \dfrac{1}{(0.01)(0.2)} = 500 \text{ k}\Omega$

$r_o(Q_2) = \dfrac{V_A}{I_{CQ}} = \dfrac{80}{0.2} = 400 \text{ k}\Omega$

$g_m(M_2) = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(0.2)}$

$= 0.4 \text{ mA/V}$

$A_d = g_m(M_2)[r_o(M_2) \| r_o(Q_2)]$

$= 0.4[500 \| 400]$

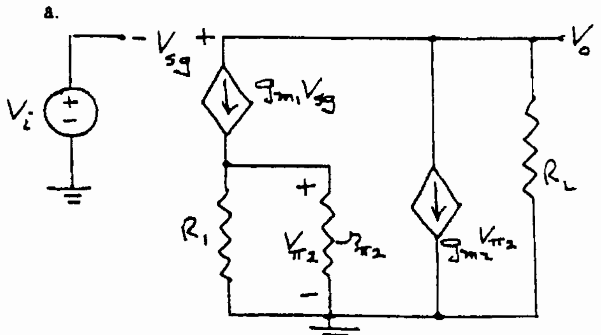
$\Rightarrow \underline{A_d = 88.9}$

If the I_Q current source is ideal,

$\underline{A_{cm} = 0}$ and $\underline{CMRR_{dB} = \infty}$

11.57

a.



b. Assume R_L is capacitively coupled. Then

$I_{CQ} + I_{DQ} = I_Q$

$I_{DQ} = \dfrac{V_{BE}}{R_1} = \dfrac{0.7}{8} = 0.0875 \text{ mA}$

$I_{CQ} = 0.9 - 0.0875 = 0.8125 \text{ mA}$

$g_{m1} = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(0.0875)}$

$\Rightarrow \underline{g_{m1} = 0.592 \text{ mA/V}}$

$g_{m2} = \dfrac{I_{CQ}}{V_T} = \dfrac{0.8125}{0.026} \Rightarrow \underline{g_{m2} = 31.25 \text{ mA/V}}$

$r_{\pi2} = \dfrac{\beta V_T}{I_{CQ}} = \dfrac{(100)(0.026)}{0.8125} \Rightarrow \underline{r_{\pi2} = 3.2 \text{ k}\Omega}$

c. $V_O = (-g_{m1}V_{sg} - g_{m2}V_{\pi2})R_L$

$V_i + V_{sg} = V_O \Rightarrow V_{sg} = V_O - V_i$

$V_{\pi2} = (g_{m1}V_{sg})(R_1 \| r_{\pi2})$

$V_O = -[g_{m1}V_{sg} + g_{m2}g_{m1}V_{sg}(R_1 \| r_{\pi2})]R_L$

$V_O = -(V_O - V_i)[g_{m1} + g_{m2}g_{m1}(R_1 \| r_{\pi2})]R_L$

$A_v = \dfrac{V_O}{V_i} = \dfrac{[g_{m1} + g_{m2}g_{m1}(R_1 \| r_{\pi2})]R_L}{1 + [g_{m1} + g_{m2}g_{m1}(R_1 \| r_{\pi2})]R_L}$

We find

$g_{m1} + g_{m2}g_{m1}(R_1 \| r_{\pi2})$

$= 0.592 + (31.25)(0.592)(8 \| 3.2)$

$= 42.88$

Then $\underline{A_v = \dfrac{(42.88)(R_L)}{1 + (42.88)(R_L)}}$

11.58

a. Assume R_L is capacitively coupled.

$I_{DQ} = \dfrac{0.7}{8} = 0.0875 \text{ mA}$

$I_{CQ} = 1.2 - 0.0875 = 1.11 \text{ mA}$

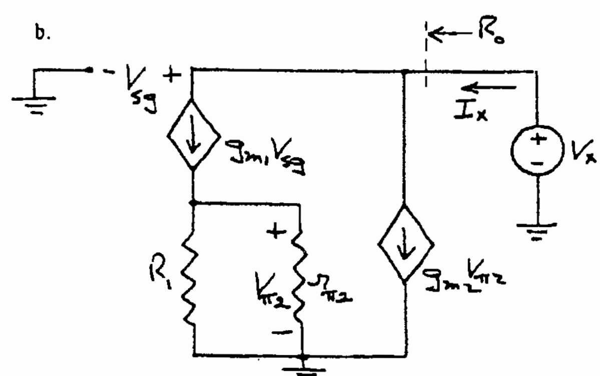
$g_{m1} = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1)(0.0875)}$

$\Rightarrow \underline{g_{m1} = 0.592 \text{ mA/V}}$

$g_{m2} = \dfrac{I_{CQ}}{V_T} = \dfrac{1.11}{0.026} \Rightarrow \underline{g_{m2} = 42.7 \text{ mA/V}}$

$r_{\pi2} = \dfrac{\beta V_T}{I_{CQ}} = \dfrac{(100)(0.026)}{1.11} \Rightarrow \underline{r_{\pi2} = 2.34 \text{ k}\Omega}$

b.



$V_{sg} = V_X$

$I_X = g_{m2}V_{\pi 2} + g_{m1}V_{sg}$

$(g_{m1}V_{sg})(R_1 \| r_{\pi 2}) = V_{\pi 2}$

$I_X = V_X[g_{m1} + g_{m2}g_{m1}(R_1 \| r_{\pi 2})]$

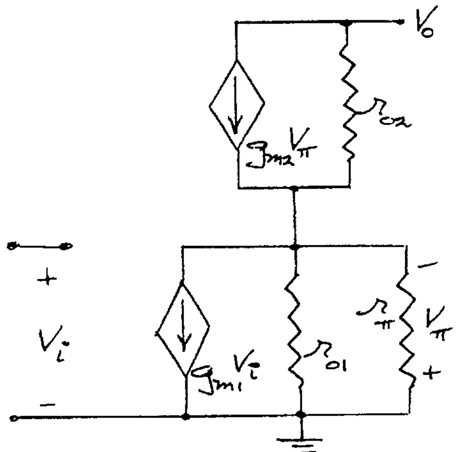
$$R_0 = \frac{V_X}{I_X} = \frac{1}{g_{m1} + g_{m2}g_{m1}(R_1 \| r_{\pi 2})}$$

$$= \frac{1}{0.592 + (0.592)(42.7)(8 \| 2.34)}$$

$\Rightarrow \underline{R_0 = 21.6\ \Omega}$

11.59

(a)



$$(1)\ \ g_{m2}V_\pi + \frac{V_o - (-V_\pi)}{r_{o2}} = 0$$

$$(2)\ \ g_{m2}V_\pi + \frac{V_o - (-V_\pi)}{r_{o2}} = g_{m1}V_i + \frac{-V_\pi}{r_{o1}} + \frac{-V_\pi}{r_\pi}$$

or

$$0 = g_{m1}V_i - V_\pi\left(\frac{1}{r_{o1}} + \frac{1}{r_\pi}\right)$$

Then

$$V_\pi = \frac{g_{m1}V_i}{\left(\frac{1}{r_{o1}} + \frac{1}{r_\pi}\right)}$$

From (1)

$$\left(g_{m2} + \frac{1}{r_{o2}}\right)V_\pi + \frac{V_o}{r_{o2}} = 0$$

$$V_o = -r_{o2}\left(g_{m2} + \frac{1}{r_{o2}}\right)V_\pi = -r_{o2}g_{m1}V_i\frac{\left(g_{m2} + \frac{1}{r_{o2}}\right)}{\left(\frac{1}{r_{o1}} + \frac{1}{r_\pi}\right)}$$

$$A_v = \frac{V_o}{V_i} = \frac{-g_{m1}r_{o2}\left(g_{m2} + \frac{1}{r_{o2}}\right)}{\left(\frac{1}{r_{o1}} + \frac{1}{r_\pi}\right)}$$

Now

$g_{m1} = 2\sqrt{K_n I_Q} = 2\sqrt{(0.25)(0.025)} = 0.158\ mA/V$

$g_{m2} = \frac{I_Q}{V_T} = \frac{0.025}{0.026} = 0.9615\ mA/V$

$r_{o1} = \frac{1}{\lambda I_Q} = \frac{1}{(0.02)(0.025)} = 2000\ k\Omega$

$r_{o2} = \frac{V_A}{I_Q} = \frac{50}{0.025} = 2000\ k\Omega$

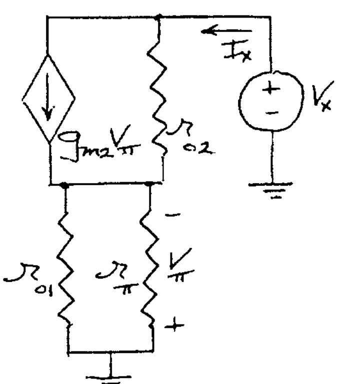
$r_\pi = \frac{\beta V_T}{I_Q} = \frac{(100)(0.026)}{0.025} = 104\ k\Omega$

Then

$$A_v = \frac{-(0.158)(2000)\left(0.9615 + \frac{1}{2000}\right)}{\left(\frac{1}{2000} + \frac{1}{104}\right)} \Rightarrow$$

$\underline{A_v = -30039}$

To find R_o; set $V_i = 0 \Rightarrow g_{m1}V_i = 0$



$$I_x = g_{m2}V_\pi + \frac{V_x - (-V_\pi)}{r_{o2}}$$

$V_\pi = -I_x(r_{o1} \| r_\pi)$

Then

$$I_x = \left(g_{m2} + \frac{1}{r_{o2}}\right)(-I_x)(r_{o1} \| r_\pi) + \frac{V_x}{r_{o2}}$$

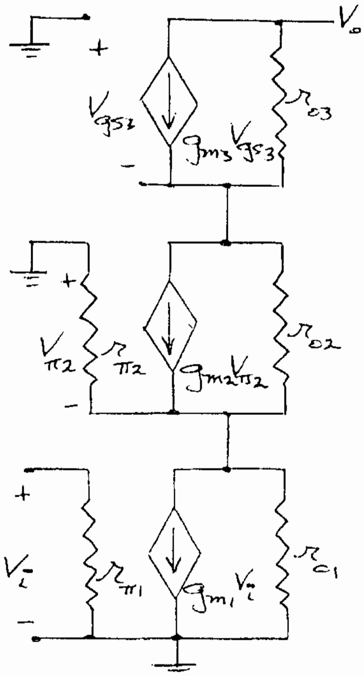
Combining terms,

$$R_o = \frac{V_x}{I_x} = r_{o2}\left[1 + (r_{o1} \| r_\pi)\left(g_{m2} + \frac{1}{r_{o2}}\right)\right]$$

$$= 2000\left[1 + (2000 \| 104)\left(0.9615 + \frac{1}{2000}\right)\right] \Rightarrow$$

$\underline{R_o = 192.2\ M\Omega}$

(b)



(1) $g_{m3}V_{gs3} + \dfrac{V_o - (-V_{gs3})}{r_{o3}} = 0$

(2) $g_{m3}V_{gs3} + \dfrac{V_o - (-V_{gs3})}{r_{o3}} = g_{m2}V_{\pi 2} + \dfrac{-V_{gs3} - (-V_{\pi 2})}{r_{o2}}$

or

$$0 = V_{\pi 2}\left(g_{m2} + \frac{1}{r_{o2}}\right) - \frac{V_{gs3}}{r_{o2}}$$

(3) $\dfrac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} + \dfrac{-V_{gs3} - (-V_{\pi 2})}{r_{o2}} = g_{m1}V_i + \dfrac{(-V_{\pi 2})}{r_{o1}}$

From (2), $V_{\pi 2} = \dfrac{V_{gs3}}{r_{o2}\left(g_{m2} + \dfrac{1}{r_{o2}}\right)}$

Then

(3) $V_{\pi 2}\left(\dfrac{1}{r_{\pi 2}} + g_{m2} + \dfrac{1}{r_{o2}} + \dfrac{1}{r_{o1}}\right) = g_{m1}V_i + \dfrac{V_{gs3}}{r_{o2}}$

or

$$\frac{V_{gs3}}{r_{o2}\left(g_{m2} + \frac{1}{r_{o2}}\right)}\left[\frac{1}{r_{\pi 2}} + g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}}\right] = g_{m1}V_i + \frac{V_{gs3}}{r_{o2}}$$

$$\frac{V_{gs3}}{2000\left(0.9615 + \frac{1}{2000}\right)}\left[\frac{1}{104} + 0.9615 + \frac{1}{2000} + \frac{1}{2000}\right]$$

$$= 0.9615V_i + \frac{V_{gs3}}{2000}$$

Then $V_{gs3} = 1.83x10^5 V_i$

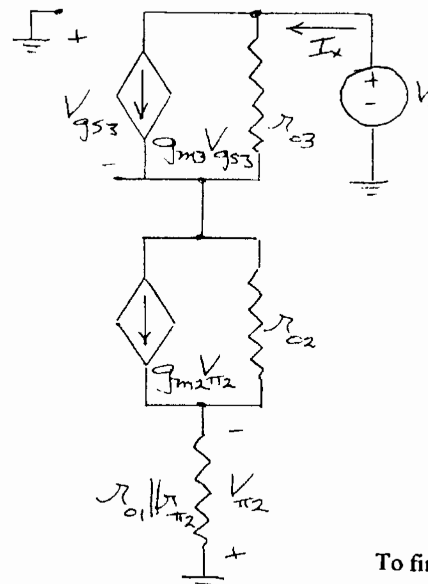
From (1),

$$\left(g_{m3} + \frac{1}{r_{o3}}\right)V_{gs3} = \frac{-V_o}{r_{o3}}$$

or

$$V_o = -2000\left(0.158 + \frac{1}{2000}\right)\left(1.83x10^5\right)V_i$$

$$A_v = \frac{V_o}{V_i} = -5.80x10^7$$



To find R_o

(1) $I_x = g_{m3}V_{gs3} + \dfrac{V_x - (-V_{gs3})}{r_{o3}}$

(2) $g_{m3}V_{gs3} + \dfrac{V_x - (-V_{gs3})}{r_{o3}} = g_{m2}V_{\pi 2} + \dfrac{-V_{gs3} - (-V_{\pi 2})}{r_{o2}}$

(3) $V_{\pi 2} = -I_x(r_{o1} \| r_{\pi 2})$

From (1) $I_x = V_{gs3}\left(g_{m3} + \dfrac{1}{r_{o3}}\right) + \dfrac{V_x}{r_{o3}}$

$$I_x = V_{gs3}\left(0.158 + \frac{1}{2000}\right) + \frac{V_x}{2000}$$

So

$$V_{gs3} = \frac{I_x - \frac{V_x}{2000}}{0.1585}$$

From (2),

$$V_{gs3}\left[g_{m3} + \frac{1}{r_{o3}} + \frac{1}{r_{o2}}\right] + \frac{V_x}{r_{o3}} = V_{\pi 2}\left(g_{m2} + \frac{1}{r_{o2}}\right)$$

$$V_{gs3}\left[0.158 + \frac{1}{2000} + \frac{1}{2000}\right] + \frac{V_x}{2000}$$

$$= V_{\pi 2}\left(0.9615 + \frac{1}{2000}\right)$$

Then

$$\left[\frac{I_x - V_x/2000}{0.1585}\right](0.159) + \frac{V_x}{2000} = -I_x(2000 \| 104)(0.962)$$

We find

$$R_o = \frac{V_x}{I_x} = 6.09x10^{10}\ \Omega$$

11.60

Assume emitter of Q_1 is capacitively coupled to signal ground.

$$I_{CQ} = 0.2\left(\frac{80}{81}\right) = 0.1975\,mA$$

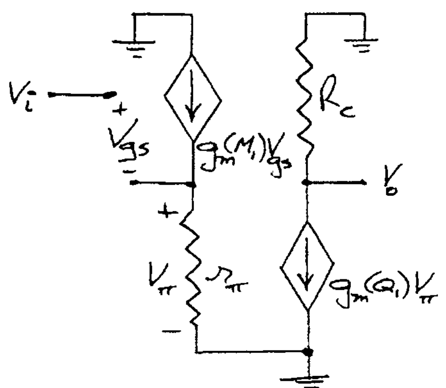
$$I_{DQ} = \frac{0.2}{81} = 0.00247\,mA$$

$$r_\pi = \frac{(80)(0.026)}{0.1975} = 10.5\,k\Omega$$

$$g_m(Q_1) = \frac{0.1975}{0.026} = 7.60\,mA/V$$

$$g_m(M_1) = 2\sqrt{K_n I_D} = 2\sqrt{(0.2)(0.00247)}$$

$$g_m(M_1) = 0.0445\,mA/V$$



$V_i = V_{gs} + V_\pi$ and $V_\pi = g_m(M_1)V_{gs}r_\pi$

or

$$V_{gs} = \frac{V_\pi}{g_m(M_1)r_\pi}$$

Then

$$V_i = V_\pi\left(1 + \frac{1}{g_m(M_1)r_\pi}\right)$$

or

$$V_\pi = \frac{V_i}{\left(1 + \frac{1}{g_m(M_1)r_\pi}\right)}$$

$$V_o = -g_m(Q_1)V_\pi R_C \Rightarrow$$

$$A_v = \frac{V_o}{V_i} = \frac{-g_m(Q_1)R_C}{\left(1 + \frac{1}{g_m(M_1)r_\pi}\right)}$$

Then

$$A_v = \frac{-(7.60)(20)}{\left(1 + \frac{1}{(0.0445)(10.5)}\right)} \Rightarrow$$

$$\underline{A_v = -48.4}$$

11.61

Using the results from Chapter 4 for the emitter-follower:

$$R_0 = R_4 \,\Bigg\|\, \left[\frac{r_{\pi 8} + \dfrac{r_{\pi 9} + r_{o7} + R_{o11}}{1+\beta}}{1+\beta}\right]$$

$$r_{\pi 8} = \frac{\beta V_T}{I_{C8}} = \frac{(100)(0.026)}{1} = 2.6\ \text{k}\Omega$$

$$I_{C9} \approx \frac{I_{C8}}{\beta} = \frac{1}{100} = 0.01\ \text{mA}$$

$$r_{\pi 9} = \frac{(100)(0.026)}{0.01} = 260\ \text{k}\Omega$$

$$r_{o7} = \frac{V_A}{I_Q} = \frac{100}{0.2} = 500\ \text{k}\Omega$$

$$r_{o11} = \frac{V_A}{I_Q} = \frac{100}{0.2} = 500\ \text{k}\Omega$$

$$R_{o11} = r_{o11}\left[1 + g_m R_E'\right],\quad g_m = \frac{0.2}{0.026} = 7.69$$

$$r_{\pi 11} = \frac{(100)(0.026)}{0.2} = 13\ \text{k}\Omega$$

$$R_E' = 0.2\|13 = 0.197\ \text{k}\Omega$$

$$R_{o11} = 500[1 + (7.69)(0.197)] = 1257\ \text{k}\Omega$$

Then

$$R_0 = 5 \,\Bigg\|\, \left[\frac{2.6 + \dfrac{260 + 500 + 1257}{101}}{101}\right]$$

$$= 5\|0.223$$

$$\Rightarrow \underline{R_0 = 0.213\ \text{k}\Omega}$$

11.62

$$R_i = r_{\pi 1} + (1+\beta)r_{\pi 2}$$

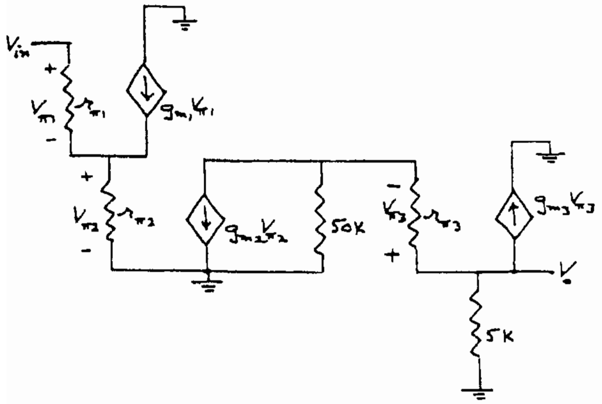
$$r_{\pi 2} = \frac{(100)(0.026)}{0.5} = 5.2\ \text{k}\Omega$$

$$r_{\pi 1} = \frac{(100)(0.026)}{(0.5/100)} = \frac{(100)^2(0.026)}{0.5} = 520\ \text{k}\Omega$$

$$R_i = 520 + (101)(5.2) \Rightarrow \underline{R_i \cong 1.05\ \text{M}\Omega}$$

$$R_0 = 5 \,\Big\|\, \frac{r_{\pi 3} + 50}{101},\quad r_{\pi 3} = \frac{(100)(0.026)}{1} = 2.6\ \text{k}\Omega$$

$$R_0 = 5 \,\Big\|\, \frac{2.6 + 50}{101} = 5\|0.521 \Rightarrow \underline{R_0 = 0.472\ \text{k}\Omega}$$



$$V_0 = -\left(\frac{V_{\pi3}}{r_{\pi3}} + g_{m3}V_{\pi3}\right)(5)$$

$$V_0 = -V_{\pi3}\left(\frac{1+\beta}{r_{\pi3}}\right)(5) \quad (1)$$

$$\frac{V_{\pi3}}{r_{\pi3}} = g_{m2}V_{\pi2} + \frac{(V_0 - V_{\pi3})}{50}$$

$$g_{m2}V_{\pi2} = V_{\pi3}\left(\frac{1}{r_{\pi3}} + \frac{1}{50}\right) - \frac{V_0}{50} \quad (2)$$

$$V_{\pi2} = \left(\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1}\right)r_{\pi2}$$

$$= V_{\pi1}\left(\frac{1+\beta}{r_{\pi1}}\right)r_{\pi2} \quad (3)$$

and

$$V_{in} = V_{\pi1} + V_{\pi2} \quad (4)$$

$$g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

Then

$$V_0 = -V_{\pi3}\left(\frac{101}{2.6}\right)(5) \quad (1)$$

$$\Rightarrow V_{\pi3} = -V_0(0.005149)$$

And

$$19.23V_{\pi2} = -V_0(0.005149)\left(\frac{1}{2.6} + \frac{1}{50}\right) - \frac{V_0}{50} \quad (2)$$

$$= -V_0(0.02208)$$

Or $V_{\pi2} = -V_0(0.001148)$

And

$$V_{\pi1} = V_{in} - V_{\pi2} = V_{in} + V_0(0.001148) \quad (4)$$

So

$$-V_0(0.001148) = [V_{in} + V_0(0.001148)]\left(\frac{101}{520}\right)(5.2) \quad (3)$$

$$-V_0(0.001148) - V_0(0.001159) = V_{in}(1.01)$$

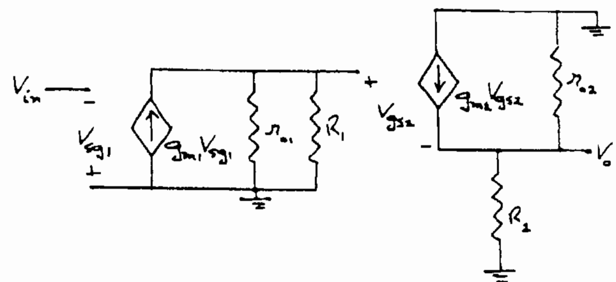
$$\Rightarrow A_v = \frac{V_0}{V_{in}} = -438$$

11.63

$$I_2 = \frac{5}{5} = 1 \text{ mA}$$

$$V_{GS2} = \sqrt{\frac{1}{0.5}} + 0.8 = 2.21 \text{ V}$$

$$I_1 = \frac{2.21 - (-5)}{35} = 0.206 \text{ mA}$$



$$V_0 = (g_{m2}V_{gs2})(R_2\|r_{02})$$

$$V_{gs2} = (g_{m1}V_{sg1})(r_{01}\|R_1) - V_0$$

and $V_{sg1} = -V_{in}$

So

$$V_{gs2} = -(g_{m1}V_{in})(r_{01}\|R_1) - V_0$$

Then

$$V_0 = g_{m2}(R_2\|r_{02})[-(g_{m1}V_{in})(r_{01}\|R_1) - V_0]$$

$$A_v = \frac{V_0}{V_{in}} = \frac{-g_{m2}(R_2\|r_{02})g_{m1}(r_{01}\|R_1)}{1 + g_{m2}(R_2\|r_{02})}$$

$$g_{m2} = 2\sqrt{K_{n2}I_{D2}} = 2\sqrt{(0.5)(1)} = 1.414\ mA/V$$

$$g_{m1} = 2\sqrt{K_{p1}I_{D1}} = 2\sqrt{(0.2)(0.206)} = 0.406\ mA/V$$

$$r_{01} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.01)(0.206)} = 485 \text{ k}\Omega$$

$$r_{02} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.01)(1)} = 100 \text{ k}\Omega$$

$$R_2\|r_{02} = 5\|100 = 4.76 \text{ k}\Omega$$

$$R_1\|r_{01} = 35\|485 = 32.6 \text{ k}\Omega$$

Then

$$A_v = \frac{-(1.414)(4.76)(0.406)(32.6)}{1 + (1.414)(4.76)}$$

So

$\Rightarrow \underline{A_v = -11.5}$

Output Resistance—From the results for a source follower in Chapter 6.

$$R_O = \frac{1}{g_{m2}} \parallel R_2 \parallel r_{o2} = \frac{1}{1.414} \parallel 5 \parallel 100$$

$$= 0.707 \| 4.76$$

So $\underline{R_O = 0.616 \text{ k}\Omega}$

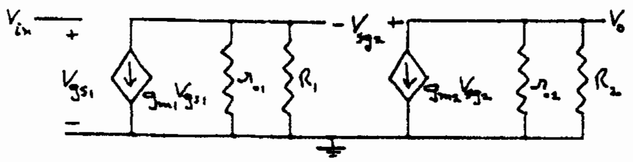
11.64

a. $R_2 = \dfrac{5}{0.5} \Rightarrow \underline{R_2 = 10 \text{ k}\Omega}$

$$V_{SG2} = \sqrt{\frac{I_{D2}}{K_{p2}}} - V_{TP2} = \sqrt{\frac{0.5}{0.25}} + 1 = 2.41\,V$$

$$R_1 = \frac{5 - (-2.41)}{0.1} \Rightarrow \underline{R_1 = 74.1 \text{ k}\Omega}$$

b.



$$V_O = -(g_{m2} V_{sg2})(r_{o2} \| R_2)$$

$$V_{sg2} = V_O - [-(g_{m1} V_{gs1})(r_{o1} \| R_1)]$$

and $V_{gs1} = V_{in}$

$$A_v = \frac{V_O}{V_{in}} = \frac{-(g_{m2})(r_{o2} \| R_2)(g_{m1})(r_{o1} \| R_1)}{1 + (g_{m2})(r_{o2} \| R_2)}$$

$$g_{m1} = 2\sqrt{K_{n1} I_{D1}} = 2\sqrt{(0.1)(0.1)} = 0.2\, mA/V$$

$$g_{m2} = 2\sqrt{K_{p2} I_{D2}} = 2\sqrt{(0.25)(0.5)} = 0.707\, mA/V$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.01)(0.1)} = 1000 \text{ k}\Omega$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.01)(0.5)} = 200 \text{ k}\Omega$$

$$r_{o2} \| R_2 = 200 \| 10 = 9.52 \text{ k}\Omega$$

$$r_{o1} \| R_1 = 1000 \| 74.1 = 69.0 \text{ k}\Omega$$

Then

$$A_v = \frac{-(0.707)(9.52)(0.2)(69)}{1 + (0.707)(9.52)}$$

So

$\Rightarrow \underline{A_v = -12.0}$

$$R_O = \frac{1}{g_{m2}} \parallel R_2 \parallel r_{o2} = \frac{1}{0.707} \parallel 10 \parallel 200$$

$$= 1.414 \| 9.52$$

Or $\underline{R_O = 1.23 \text{ k}\Omega}$

11.65

a. $I_{C2} = 0.25$ mA

$$R = \frac{5-2}{0.25} \Rightarrow \underline{R = 12 \text{ k}\Omega}$$

$$I_{C3} = \frac{v_{O2} - V_{BE}(\text{on})}{R_{E1}} \Rightarrow R_{E1} = \frac{2 - 0.7}{0.5}$$

$\Rightarrow \underline{R_{E1} = 2.6 \text{ k}\Omega}$

$$R_C = \frac{5 - v_{O3}}{I_{C3}} = \frac{5-3}{0.5} \Rightarrow \underline{R_C = 4 \text{ k}\Omega}$$

$$I_{C4} = \frac{[v_{O3} - V_{BE}(\text{on})] - (-5)}{R_{E2}}$$

$$R_{E2} = \frac{3 - 0.7 + 5}{3} \Rightarrow \underline{R_{E2} = 2.43 \text{ k}\Omega}$$

b. Input resistance to base of Q_3,

$$R_{i3} = r_{\pi 3} + (1+\beta) R_{E1}$$

$$r_{\pi 3} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$R_{i3} = 5.2 + (101)(2.6) = 267.8 \text{ k}\Omega$$

$$A_{d1} = \frac{v_{O2}}{v_d} = \frac{1}{2} g_{m2} (R \| R_{i3})$$

$$g_{m2} = \frac{0.25}{0.026} = 9.62 \text{ mA/V}$$

$$A_{d1} = \frac{1}{2}(9.62)(12 \| 267.8) \Rightarrow \underline{A_{d1} = 55.2}$$

Now $\dfrac{v_{O3}}{v_{O2}} = \dfrac{-\beta (R_C \| R_{i4})}{r_{\pi 3} + (1+\beta) R_{E1}}$

where $R_{i4} = r_{\pi 4} + (1+\beta) R_{E2}$

and $\dfrac{v_O}{v_{O3}} = \dfrac{(1+\beta) R_{E2}}{r_{\pi 4} + (1+\beta) R_{E2}}$

$$r_{\pi 4} = \frac{(100)(0.026)}{3} = 0.867 \text{ k}\Omega$$

$$\frac{v_O}{v_{O3}} = \frac{(101)(2.43)}{0.867 + (101)(2.43)} = 0.9965$$

$$R_{i4} = 0.867 + (101)(2.43) = 246.3 \text{ k}\Omega$$

$$r_{\pi 3} = 5.2 \text{ k}\Omega$$

So

$$\frac{v_{O3}}{v_{O2}} = \frac{-(100)(4 \| 246.3)}{5.2 + (101)(2.6)} = -1.47$$

So

$$A_d = \frac{v_O}{v_d} = (55.2)(0.9965)(-1.47)$$

$\Rightarrow \underline{A_d = -80.9}$

c. Using Equation (11.32b)

$$A_{cm1} = \frac{-g_{m2}(R \| R_{i3})}{1 + \dfrac{2(1+\beta)R_o}{r_{\pi 2}}}$$

$$r_{\pi 2} = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$$

$$A_{cm1} = \frac{-(9.62)(12 \| 267.8)}{1 + \dfrac{2(101)(100)}{10.4}} = \underline{-0.0569 = A_{cm1}}$$

Then

$$A_{cm} = \left(\frac{v_O}{v_{O3}}\right)\left(\frac{v_{O3}}{v_{O2}}\right) \cdot A_{cm1}$$

$$= (0.9965)(-1.47)(-0.0569)$$

$$\Rightarrow \underline{A_{cm} = 0.08335}$$

$$CMRR_{dB} = 20\log_{10}\left(\frac{80.9}{0.08335}\right)$$

$$\Rightarrow \underline{CMRR_{dB} = 59.7 \text{ dB}}$$

11.66

a. $R_{C1} = \dfrac{10 - v_{O1}}{I_{C1}} = \dfrac{10-2}{0.1} \Rightarrow \underline{R_{C1} = 80 \text{ k}\Omega}$

$$R_{C2} = \frac{10 - v_{O4}}{I_{C4}} = \frac{10-6}{0.2} \Rightarrow \underline{R_{C2} = 20 \text{ k}\Omega}$$

b. $A_{d1} = \dfrac{v_{O1} - v_{O2}}{v_d} = -g_{m1}(R_{C1} \| r_{\pi 3})$

$$g_{m1} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(180)(0.026)}{0.2} = 23.4 \text{ k}\Omega$$

$$A_{d1} = -(3.846)(80\|23.4) \Rightarrow \underline{A_{d1} = -69.6}$$

$$A_{d2} = \frac{v_{O4}}{v_{O1} - v_{O2}} = \frac{1}{2} g_{m4} R_{C2}$$

$$g_{m4} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$A_{d2} = \frac{1}{2}(7.692)(20) = 76.9$$

Then $A_d = (76.9)(-69.6)$

$$\Rightarrow \underline{A_d = -5352}$$

11.67

a. Neglect the effect of r_O in determining the differential-mode gain.

$$A_{d1} = \frac{v_{O2}}{v_d} = \frac{1}{2} g_{m2}(R_C \| R_{i3})$$

where $R_{i3} = r_{\pi 3} + (1+\beta)R_E$

$$A_2 = \frac{-\beta R_{C2}}{r_{\pi 3} + (1+\beta)R_E}$$

$$I_1 = \frac{12 - 0.7 - (-12)}{R_1} = \frac{23.3}{12} = 1.94 \text{ mA} \approx I_{C5}$$

$$g_{m2} = \frac{\frac{1}{2}\cdot(1.94)}{0.026} = 37.3 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(200)(0.026)}{I_{C3}}$$

$$v_{O2} = 12 - \frac{1}{2}(1.94)(8) = 4.24 \text{ V}$$

$$I_{C3} = \frac{4.24 - 0.7}{3.3} = 1.07 \text{ mA}$$

$$r_{\pi 3} = \frac{(200)(0.026)}{1.07} = 4.86 \text{ k}\Omega$$

$$R_{i3} = 4.86 + (201)(3.3) = 668 \text{ k}\Omega$$

$$A_{d1} = \frac{1}{2}(37.3)[8\|668] = 147.4$$

$$A_2 = \frac{-(200)(4)}{4.86 + (201)(3.3)} = -1.197$$

Then

$$A_d = A_{d1} \cdot A_2 = (147.4)(-1.197) \Rightarrow \underline{A_d = -176}$$

$$R_O = r_{O5} = \frac{V_A}{I_{C5}} = \frac{80}{1.94} = 41.2 \text{ k}\Omega$$

$$A_{cm1} = \frac{-g_{m2}(R_C \| R_{i3})}{1 + \dfrac{2(1+\beta)R_O}{r_{\pi 2}}}$$

$$r_{\pi 2} = \frac{(200)(0.026)}{\frac{1}{2}\cdot(1.94)} = 5.36 \text{ k}\Omega$$

$$A_{cm1} = \frac{-(37.3)(8\|668)}{1 + \dfrac{2(201)(41.2)}{5.36}} = -0.09539$$

$$A_2 = -1.197$$

$$A_{cm} = (-0.09539)(-1.197) \Rightarrow \underline{A_{cm} = 0.114}$$

b. $v_d = v_1 - v_2 = 2.015\sin\omega t - 1.985\sin\omega t$

$$v_d = 0.03\sin\omega t \text{ (V)}$$

$$v_{cm} = \frac{v_1 + v_2}{2} = 2.0\sin\omega t$$

$$v_{O3} = A_d v_d + A_{cm} v_{cm}$$

$$= (-176)(0.03) + (0.114)(2)$$

Or

$$\underline{v_{O3} = -5.052\sin\omega t}$$

Ideal, $A_{cm} = 0$

So

$$v_{O3} = A_d v_d = (-176)(0.03)$$

$$\underline{v_{O3} = -5.28\sin\omega t}$$

c. $R_{id} = 2r_{\pi 2} = 2(5.36) \Rightarrow \underline{R_{id} = 10.72 \text{ k}\Omega}$

$$2R_{icm} \cong 2(1+\beta)R_O \| (1+\beta)r_O$$

$$r_O = \frac{V_A}{I_{C2}} = \frac{80}{\frac{1}{2}\cdot(1.94)} = 82.5 \text{ k}\Omega$$

$$2R_{icm} = [2(201)(41.2)] \| [(201)(82.5)]$$

$$= 16.6 \text{ M}\Omega \| 16.6 \text{ M}\Omega$$

So

$$\Rightarrow \underline{R_{icm} = 4.15 \text{ M}\Omega}$$

11.68

a. $I_1 = \dfrac{24 - V_{GS4}}{R_1} = k_n(V_{GS4} - V_{Th})^2$

$24 - V_{GS4} = (55)(0.2)(V_{GS4} - 2)^2$

$24 - V_{GS4} = 11(V_{GS4}^2 - 4V_{GS4} + 4)$

$11V_{GS4}^2 - 43V_{GS4} + 20 = 0$

$$V_{GS4} = \frac{43 \pm \sqrt{(43)^2 - 4(11)(20)}}{2(11)} = 3.37 \text{ V}$$

$I_1 = \dfrac{24 - 3.37}{55} = 0.375 \text{ mA} = I_Q$

$v_{02} = 12 - \left(\dfrac{0.375}{2}\right)(40) = 4.5 \text{ V}$

$\dfrac{v_{02} - V_{GS3}}{R_5} = I_{D3} = k_n(V_{GS3} - V_{Th})^2$

$4.5 - V_{GS3} = (0.2)(6)(V_{GS3}^2 - 4V_{GS3} + 4)$

$1.2V_{GS3}^2 - 3.8V_{GS3} + 0.3 = 0$

$$V_{GS3} = \frac{3.8 \pm \sqrt{(3.8)^2 - 4(1.2)(0.3)}}{2(1.2)} = 3.09 \text{ V}$$

$I_{D3} = \dfrac{4.5 - 3.09}{6} = 0.235 \text{ mA}$

$g_{m2} = 2\sqrt{K_n I_{D2}} = 2\sqrt{(0.2)\left(\dfrac{0.375}{2}\right)}$

$= 0.387 \text{ mA/V}$

$A_{d1} = \dfrac{1}{2}g_{m2}R_D = \dfrac{1}{2}(0.387)(40)$

$\Rightarrow A_{d1} = 7.74$

$A_2 = \dfrac{-g_{m3}R_{D2}}{1 + g_{m3}R_5}$

$g_{m3} = 2\sqrt{K_n I_{D3}} = 2\sqrt{(0.2)(0.235)}$

$= 0.434 \text{ mA/V}$

$A_2 = \dfrac{-(0.434)(4)}{1 + (0.434)(6)} = -0.482$

So $A_d = A_{d1} \cdot A_2 = (7.74)(-0.482)$

$\Rightarrow \underline{A_d = -3.73}$

$R_0 = r_{05} = \dfrac{1}{\lambda I_Q} = \dfrac{1}{(0.02)(0.375)} = 133 \text{ k}\Omega$

$A_{cm1} = \dfrac{-g_{m2}R_D}{1 + 2g_{m2}R_0} = \dfrac{-(0.387)(40)}{1 + 2(0.387)(133)}$

$= -0.149$

$A_{cm} = (-0.149)(-0.482) \Rightarrow \underline{A_{cm} = 0.0718}$

b. $v_d = v_1 - v_2 = 0.3 \sin \omega t$

$v_{cm} = \dfrac{v_1 + v_2}{2} = 2 \sin \omega t$

$v_{03} = A_d v_d + A_{cm} v_{cm}$

$= (-3.73)(0.3) + (0.0718)(2)$

$\Rightarrow \underline{v_{03} = -0.975 \sin \omega t \text{ (V)}}$

Ideal, $A_{cm} = 0$

$v_{03} = A_d v_d = (-3.73)(0.3)$

Or

$\Rightarrow \underline{v_{03} = -1.12 \sin \omega t \text{ (V)}}$

11.69

The low-frequency, one-sided differential gain is

$A_{v2} = \dfrac{v_{02}}{v_d} = \dfrac{1}{2}g_m R_C\left(\dfrac{r_\pi}{r_\pi + R_B}\right)$

$= \dfrac{\frac{1}{2}\cdot\beta R_C}{r_\pi + R_B}$

$r_\pi = \dfrac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$

$A_{v2} = \dfrac{\frac{1}{2}\cdot(100)(10)}{5.2 + 0.5} \Rightarrow \underline{A_{v2} = 87.7}$

$C_M = C_\mu(1 + g_m R_C)$

$g_m = \dfrac{0.5}{0.026} = 19.23 \text{ mA/V}$

$C_M = 2[1 + (19.23)(10)] \Rightarrow \underline{C_M = 387 \text{ pF}}$

$f_H = \dfrac{1}{2\pi[r_\pi \| R_B](C_\pi + C_M)}$

$= \dfrac{1}{2\pi[5.2\|0.5] \times 10^3 \times (8 + 387) \times 10^{-12}}$

So

$\Rightarrow \underline{f_H = 883 \text{ kHz}}$

11.70

a. From Equation (11.117),

$f_Z = \dfrac{1}{2\pi R_0 C_0} = \dfrac{1}{2\pi(5 \times 10^6)(0.8 \times 10^{-12})}$

Or $\underline{f_Z = 39.8 \text{ kHz}}$

b. From Problem 11.69, $f_H = 883\ kHz$. From Equation (11.116(b)), the low-frequency common-mode gain is

$$A_{cm} = \frac{-g_m R_C}{\left[\left(1 + \frac{R_B}{r_\pi}\right) + \frac{2(1+\beta)R_0}{r_\pi}\right]}$$

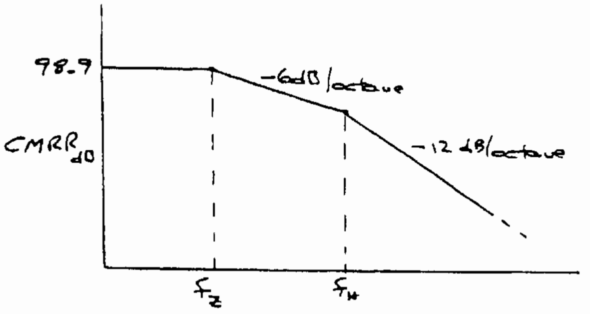
$r_\pi = 5.2 \text{ k}\Omega,\ g_m = 19.23 \text{ mA/V}$

So

$$A_{cm} = \frac{-(19.23)(10)}{\left[\left(1 + \frac{0.5}{5.2}\right) + \frac{2(101)(5 \times 10^6)}{5.2 \times 10^3}\right]}$$

$= -9.9 \times 10^{-4}$

$$CMRR_{dB} = 20\log_{10}\left(\frac{87.7}{9.9 \times 10^{-4}}\right) = 98.9 \text{ dB}$$



11.71

a. From Equation (7.72),

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$

$$g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

Then

$$800 \times 10^6 = \frac{38.46 \times 10^{-3}}{2\pi(C_\pi + C_\mu)}$$

Or

$C_\pi + C_\mu = 7.65 \times 10^{-12}$ F $= 7.65$ pF

And $C_\pi = 6.65$ pF

$$C_M = C_\mu(1 + g_m R_C) = 1[1 + (38.46)(10)] = 386 \text{ pF}$$

$$f_H = \frac{1}{2\pi[r_\pi \| R_B](C_\pi + C_M)}$$

$$r_\pi = \frac{(120)(0.026)}{1} = 3.12 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi[3.12\|1] \times 10^3 \times (6.65 + 386) \times 10^{-12}}$$

Or

$\underline{f_H = 535 \text{ kHz}}$

b. From Equation (11.117),

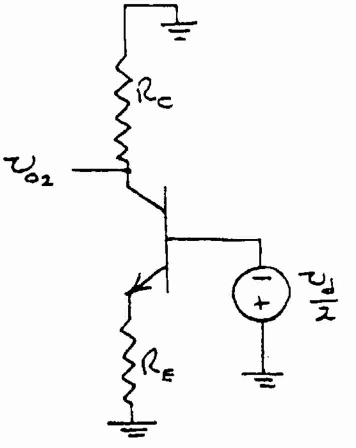
$$f_Z = \frac{1}{2\pi R_0 C_0} = \frac{1}{2\pi(10 \times 10^6)(10^{-12})}$$

Or

$\underline{f_Z = 15.9 \text{ kHz}}$

11.72

The differential-mode half circuit is:



$$v_{02} = \frac{g_m\left(\frac{v_d}{2}\right)R_C}{1 + \left(\frac{1+\beta}{r_\pi}\right)R_E} \text{ or } A_v = \frac{\left(\frac{1}{2}\right)\beta R_C}{r_\pi + (1+\beta)R_E}$$

$$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$A_v = \frac{\left(\frac{1}{2}\right)(100)(10)}{5.2 + (101)R_E} = \frac{500}{5.2 + (101)R_E}$$

a. For $R_E = 0.1$ kΩ : $\underline{A_v = 32.7}$
b. For $R_E = 0.25$ kΩ : $\underline{A_v = 16.4}$

Chapter 12

Exercise Solutions

E12.1

a. $A_f = \dfrac{A}{1+A\beta}$

$1 + A\beta = \dfrac{A}{A_f} \Rightarrow A\beta = \dfrac{A}{A_f} - 1$

$\beta = \dfrac{1}{A_f} - \dfrac{1}{A} = \dfrac{1}{20} - \dfrac{1}{10^4} = 0.05 - 0.0001$

$\Rightarrow \underline{\beta = 0.0499}$

b. $\dfrac{A_f}{(1/\beta)} = \dfrac{20}{(1/0.0499)} = \dfrac{20}{20.040} = \underline{0.998}$

E12.2

$A_f = \dfrac{A}{1+A\beta}$

$A_f + A_f A\beta = A$

$A_f = A(1 - A_f\beta)$

$A = \dfrac{A_f}{1 - A_f\beta} = \dfrac{80}{1-(80)(0.0120)}$

$\underline{A = 2000}$

E12.3

$A_f = \dfrac{A}{1+A\beta}$

$\beta = \dfrac{1}{A_f} - \dfrac{1}{A} = \dfrac{1}{100} - \dfrac{1}{10^6} = 0.01 - 10^{-6}$

$\beta = 0.009999$

$\dfrac{dA_f}{A_f} = \dfrac{1}{(1+\beta A)} \cdot \dfrac{dA}{A} = \left(\dfrac{A_f}{A}\right) \cdot \dfrac{dA}{A}$

$\dfrac{dA_f}{A_f} = \left(\dfrac{100}{10^6}\right)(20)\%$

$\Rightarrow \underline{\dfrac{dA_f}{A_f} = 0.002\%}$

E12.4

$\dfrac{dA_f}{A_f} = \dfrac{1}{(1+\beta A)} \cdot \dfrac{dA}{A} = \left(\dfrac{A_f}{A}\right) \cdot \dfrac{dA}{A}$

$\dfrac{dA}{A} = \dfrac{dA_f}{A_f} \cdot \left(\dfrac{A}{A_f}\right) = (0.001)\left(\dfrac{5 \times 10^5}{100}\right)$

$\Rightarrow \underline{\dfrac{dA}{A} = \pm 5\%}$

E12.5

Bandwidth $= \omega_H(1 + \beta A_0)$

$= \omega_H\left(\dfrac{A_0}{A_f}\right) = (2\pi)(10)\left(\dfrac{10^5}{100}\right)$

$\underline{\omega = (2\pi)(10^4) \text{ rad/sec} \Rightarrow f = 10 \text{ kHz}}$

E12.6

$A_f \cdot f_H = A_0 \cdot f_1$

$A_f = \dfrac{A_0 \cdot f_1}{f_H} = \dfrac{(10^6)(8)}{250 \times 10^3}$

$\Rightarrow \underline{A_f(0) = 32}$

E12.7

a. $V_\epsilon = V_S - V_{fb} = 100 - 99 = 1 \text{ mV}$

$V_0 = A_v V_\epsilon \Rightarrow A_v = \dfrac{5}{0.001} \Rightarrow \underline{A_v = 5000 \text{ V/V}}$

$V_{fb} = \beta V_0 \Rightarrow \beta = \dfrac{V_{fb}}{V_0} = \dfrac{0.099}{5} \Rightarrow \underline{\beta = 0.0198 \text{ V/V}}$

$A_{vf} = \dfrac{A_v}{1+\beta A_v} = \dfrac{5000}{1+(0.0198)(5000)}$

$\Rightarrow \underline{A_{vf} = 50 \text{ V/V}}$

b. $R_{if} = R_i(1+\beta A_v) = (5)[1 + (0.0198)(5000)]$

$\Rightarrow \underline{R_{if} = 500 \text{ k}\Omega}$

$R_{0f} = \dfrac{R_0}{1+\beta A_v} = \dfrac{4}{1+(0.0198)(5000)}$

$\Rightarrow \underline{R_{0f} \Rightarrow 40\ \Omega}$

E12.8

a. $I_\epsilon = I_S - I_{fb} = 100 - 99 = 1\ \mu\text{A}$

$A_i = \dfrac{I_0}{I_\epsilon} = \dfrac{5}{0.001} \Rightarrow \underline{A_i = 5000 \text{ A/A}}$

$\beta = \dfrac{I_{fb}}{I_0} = \dfrac{0.099}{5} \Rightarrow \underline{\beta = 0.0198 \text{ A/A}}$

$A_{if} = \dfrac{A_i}{1 + A_i\beta} = \dfrac{5000}{1+(5000)(0.0198)}$

$\Rightarrow \underline{A_{if} = 50 \text{ A/A}}$

b. $R_{if} = \dfrac{R_i}{1+\beta A_i} = \dfrac{5}{1+(0.0198)(5000)}$

$\Rightarrow \underline{R_{if} \Rightarrow 50\ \Omega}$

$R_{0f} = (1+\beta A_i)R_0 = [1+(0.0198)(5000)](4)$

$\Rightarrow \underline{R_{0f} = 400 \text{ k}\Omega}$

E12.9

$V_\epsilon = V_S - V_{fb} = 100 - 99 = 1 \text{ mV}$

$A_g = \dfrac{I_0}{V_\epsilon} = \dfrac{5 \text{ mA}}{1 \text{ mV}} \Rightarrow \underline{A_g = 5 \text{ A/V}}$

$\beta = \dfrac{V_{fb}}{I_0} = \dfrac{99 \text{ mV}}{5 \text{ mA}} \Rightarrow \underline{\beta = 19.8 \text{ V/A}}$

$A_{gf} = \dfrac{A_g}{1+\beta A_g} = \dfrac{5}{1+(19.8)(5)}$

$\Rightarrow \underline{A_{gf} = 0.05 \text{ A/V} = 50 \text{ mA/V}}$

E12.10

$I_\epsilon = I_S - I_{fb} = 100 - 99 = 1\ \mu A$

$A_z = \dfrac{V_0}{I_\epsilon} = \dfrac{5\ V}{1\ \mu A} \Rightarrow \underline{A_z = 5 \times 10^6\ V/A}$

$\beta = \dfrac{I_{fb}}{V_0} = \dfrac{99\ \mu A}{5\ V} \Rightarrow \underline{\beta = 1.98 \times 10^{-5}\ A/V}$

$$A_{zf} = \frac{A_z}{1+\beta A_z} = \frac{5\times 10^6}{1+(1.98\times 10^{-5})(5\times 10^6)}$$

$\Rightarrow \underline{A_{zf} = 5 \times 10^4\ V/A = 50\ V/mA}$

E12.11

$$A_{vf} = \frac{A_v}{1+\dfrac{A_v}{1+(R_2/R_1)}} = \frac{10^4}{1+\dfrac{10^4}{1+(30/10)}}$$

$\Rightarrow \underline{A_{vf} = 3.9984}$

$$A_{vf} = \frac{10^5}{1+\dfrac{10^5}{1+(30/10)}} = 3.99984$$

$\dfrac{3.99984 - 3.9984}{3.9984} \times 100\% \Rightarrow \underline{0.0360\%}$

E12.12

a. $r_\pi = \dfrac{h_{FE}V_T}{I_{CQ}} = \dfrac{(100)(0.026)}{0.5} = 5.2\ k\Omega$

$g_m = \dfrac{I_{CQ}}{V_T} = \dfrac{0.5}{0.026} = 19.23\ mA/V$

$$A_{vf} = \frac{\left(\dfrac{1}{r_\pi}+g_m\right)R_E}{1+\left(\dfrac{1}{r_\pi}+g_m\right)R_E} = \frac{\left(\dfrac{1}{5.2}+19.23\right)(2)}{1+\left(\dfrac{1}{5.2}+19.23\right)(2)}$$

$= \dfrac{(19.42)(2)}{1+(19.42)(2)} \Rightarrow \underline{A_{vf} = 0.97490}$

$R_{if} = r_\pi + (1+h_{FE})R_E = 5.2 + (101)(2)$

$\Rightarrow \underline{R_{if} = 207.2\ k\Omega}$

$R_{of} = R_E \left\| \dfrac{r_\pi}{1+h_{FE}} = 2 \right\| \dfrac{5.2}{101}$

$\Rightarrow \underline{R_{of} = 0.0502\ k\Omega \Rightarrow 50.2\ \Omega}$

b. $h_{FE} = 150 \Rightarrow r_\pi = 7.8\ k\Omega,\ g_m = 19.23\ mA/V$

$$A_{vf} = \frac{\left(\dfrac{1}{7.8}+19.23\right)(2)}{1+\left(\dfrac{1}{7.8}+19.23\right)(2)} = \frac{(19.36)(2)}{1+(19.36)(2)}$$

$A_{vf} = 0.97482 \Rightarrow \underline{0.0082\%\ \text{change in}\ A_{vf}}$

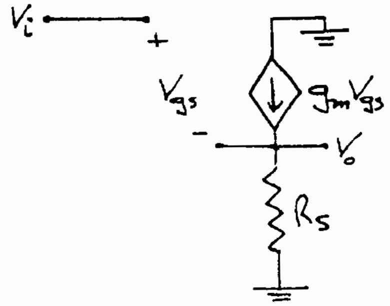
$R_{if} = 7.8 + (101)(2) = 209.8\ k\Omega$

$\Rightarrow \underline{1.25\%\ \text{change in}\ R_{if}}$

$R_{of} = R_E \left\| \dfrac{r_\pi}{1+h_{FE}} = 2 \right\| \dfrac{7.8}{151} = 2 \| 0.0517$

$R_{of} = 50.4\ \Omega \Rightarrow \underline{0.397\%\ \text{change in}\ R_{of}}$

E12.13



$V_0 = (g_m V_{gs})R_S$

$V_i = V_{gs} + g_m R_S V_{gs}$

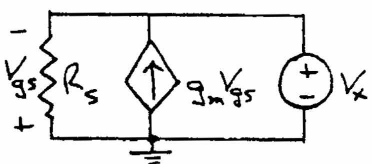
$V_{gs} = \dfrac{V_i}{1+g_m R_S}$

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(0.25)} = 0.447\ mA/V$

$A_{vf} = \dfrac{V_0}{V_i} = \dfrac{g_m R_S}{1+g_m R_S}$

$A_{vf} = \dfrac{(0.447)(5)}{1+(0.447)(5)} \Rightarrow \underline{A_{vf} = 0.691}$

$\underline{R_{if} = \infty}$



$I_X = g_m V_{gs} = \dfrac{V_X}{R_S}$

$V_{gs} = -V_X$

$I_X = V_X\left(g_m + \dfrac{1}{R_S}\right)$

$R_{of} = \dfrac{1}{g_m} \left\| R_S = \dfrac{1}{0.447} \right\| 5$

$\underline{R_{of} = 1.55\ k\Omega}$

E12.15

$$i_o = \left(\frac{h_{FE}}{1+h_{FE}}\right)\left(\frac{R_E}{R_E+\dfrac{r_\pi}{1+h_{FE}}}\right)\cdot i_i$$

$r_\pi = \dfrac{(80)(0.026)}{0.5} = 4.16\ k\Omega$

Then

$\dfrac{r_\pi}{1+h_{FE}} = \dfrac{4.16}{81} = 0.0514\ k\Omega$

Then we want

$$\frac{i_o}{i_i} = 0.95 = \left(\frac{80}{81}\right)\left(\frac{R_E}{R_E + 0.0514}\right)$$

or

$$\left(\frac{R_E}{R_E + 0.0514}\right) = 0.9619$$

which yields

$\underline{R_E(\min) = 1.30\ k\Omega}$

and

$$V^+ = I_E R_E + 0.7 = \left(\frac{81}{80}\right)(0.5)(1.3) + 0.7 \Rightarrow$$

$\underline{V^+(\min) = 1.36\ V}$

E12.19

a. $A_{gf} = \dfrac{h_{FE} \cdot A_g}{1 + (h_{FE} A_g) R_E} = \dfrac{(200)(10^3)}{1 + (200)(10^3)(10^3)}$

$\Rightarrow \underline{A_{gf} = 10^{-3}\ \text{A/V} = 1\ \text{mA/V}}$

b. $A_{gf} = \dfrac{(200)(10^4)}{1 + (200)(10^4)(10^3)} \Rightarrow \underline{A_{gf} \cong 1\ \text{mA/V}}$

Percent change is negligible. $(4.5 \times 10^{-7}\%)$

$$\frac{\dfrac{(200)(10^4)}{1 + (200)(10^4)(10^3)} - \dfrac{(200)(10^3)}{1 + (200)(10^3)(10^3)}}{10^{-3}}$$

$$= \frac{(200)(10^4)\left[1 + (200)(10^3)(10^3)\right]}{(10^{-3})(2 \times 10^9)(2 \times 10^8)} - \frac{(200)(10^3)\left[1 + (200)(10^4)(10^3)\right]}{(10^{-3})(2 \times 10^9)(2 \times 10^8)}$$

$$= \frac{200 \times 10^4 - 200 \times 10^3}{(10^{-3})(2 \times 10^9)(2 \times 10^8)} = 4.5 \times 10^{-9}$$

E12.20

a. $V_G = \left(\dfrac{R_2}{R_1 + R_2}\right)(10) - 5 = \left(\dfrac{20}{20 + 30}\right)(10) - 5$

$V_G = -1\ \text{V}$

$V_S = -1 - V_{GS}$

$$I_D = \frac{V_S - (-5)}{R_S} = K_n (V_{GS} - V_{TN})^2$$

$1 - V_{GS} + 5 = (1.5)(0.4)(V_{GS} - 2)^2$

$4 - V_{GS} = 0.6(V_{GS}^2 - 4V_{GS} + 4)$

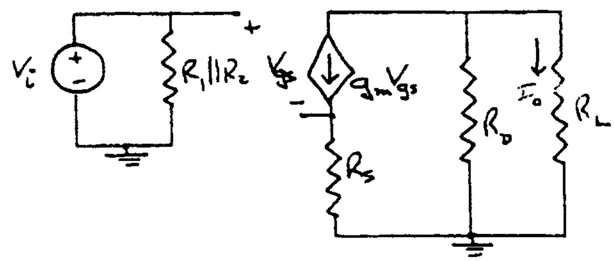
$0.6V_{GS}^2 - 1.4V_{GS} - 1.6 = 0$

$$V_{GS} = \frac{1.4 \pm \sqrt{(1.4)^2 + 4(0.6)(1.6)}}{2(0.6)}$$

$V_{GS} = 3.17\ \text{V}$

$g_m = 2K_n(V_{GS} - V_{TN}) = 2(1.5)(3.17 - 2)$

$g_m = 3.51\ \text{mA/V}$



$$I_o = -\left(\frac{R_D}{R_D + R_L}\right)(g_m V_{gs}) = -\left(\frac{2}{2+2}\right)(3.51)V_{gs}$$

$I_o = -1.76 V_{gs}$

$$V_i = V_{gs} + g_m V_{gs} R_S \Rightarrow V_{gs} = \frac{V_i}{1 + g_m R_S}$$

$$V_{gs} = \frac{V_i}{1 + (3.51)(0.4)} = (0.416)V_i$$

$I_o = -(1.76)(0.416)V_i$

$$\Rightarrow \underline{A_{gf} = \frac{I_o}{V_i} = -0.732\ \text{mA/V}}$$

b. For $K_n = 1\ mA/V^2$

From dc analysis:

$4 - V_{GS} = (1)(0.4)(V_{GS} - 2)^2$

$4 - V_{GS} = 0.4(V_{GS}^2 - 4V_{GS} + 4)$

$0.4V_{GS}^2 - 0.6V_{GS} - 2.4 = 0$

$$V_{GS} = \frac{0.6 \pm \sqrt{(0.6)^2 + 4(0.4)(2.4)}}{2(0.4)}$$

$V_{GS} = 3.31\ \text{V}$

$g_m = 2(1)(3.31 - 2) = 2.62\ \text{mA/V}$

$$I_o = -\left(\frac{2}{2+2}\right)(2.62)V_{gs} = -1.31V_{gs}$$

$$V_{gs} = \frac{V_i}{1 + (2.62)(0.4)} = V_i(0.488)$$

$I_o = -(1.31)(0.488)V_i$

$$A_{gf} = \frac{I_o}{V_i} = -0.639\ \text{mA/V}$$

$$\%\ \text{change} = \frac{0.732 - 0.639}{0.732} \Rightarrow \underline{12.7\%}$$

E12.22

dc analysis:

$$\frac{10 - V_0}{4.7} = I_D + \frac{V_0}{47 + 20} \quad (1)$$

$$I_D = K_n(V_{GS} - V_{TN})^2 \quad (2)$$

$$V_{GS} = \left(\frac{20}{20 + 47}\right)V_0 = 0.2985V_0 \quad (3)$$

$$2.13 - V_0(0.213) = I_D + (0.0149)V_0$$
$$I_D = 2.13 - V_0(0.2279) \quad (1)$$

From (2):

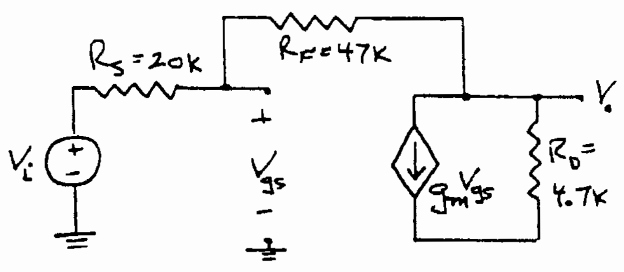
$$2.13 - V_0(0.2279) = 1[(0.2985V_0) - 1.5]^2$$
$$2.13 - V_0(0.2279) = 0.0891V_0^2 - 0.8955V_0 + 2.25$$
$$0.0891V_0^2 - 0.6676V_0 + 0.12 = 0$$

$$V_0 = \frac{0.6676 \pm \sqrt{(0.6676)^2 - 4(0.0891)(0.12)}}{2(0.0891)}$$
$$V_0 = 7.31 \text{ V}$$

$$I_D = \frac{10 - 7.31}{4.7} - \frac{7.31}{67} = 0.572 - 0.109$$
$$I_D = 0.463 \text{ mA}$$
$$V_{GS} = \sqrt{\frac{0.463}{1}} + 1.5 = 2.18$$

a. $g_m = 2K_n(V_{GS} - V_{TN}) = 2(1)(2.18 - 1.5)$

$\Rightarrow g_m = 1.36$ mA/V



$$\frac{V_0}{R_D} + g_m V_{gs} + \frac{V_0 - V_{gs}}{R_F} = 0 \quad (1)$$

$$\frac{V_{gs} - V_i}{R_S} + \frac{V_{gs} - V_0}{R_F} = 0 \quad (2)$$

$$V_{gs}\left(\frac{1}{R_S} + \frac{1}{R_F}\right) = \frac{V_0}{R_F} + \frac{V_i}{R_S}$$
$$V_{gs}\left(\frac{1}{20} + \frac{1}{47}\right) = \frac{V_0}{47} + \frac{V_i}{20}$$

$$V_{gs}(0.0713) = V_0(0.0213) + V_i(0.050)$$
$$V_{gs} = V_0(0.299) + V_i(0.701)$$

From (1):

$$\frac{V_0}{4.7} + (1.36)V_{gs} + \frac{V_0}{47} - \frac{V_{gs}}{47} = 0$$
$$V_0(0.213) + (1.36)V_{gs} + V_0(0.0213) - V_{gs}(0.0213) = 0$$
$$V_0(0.234) + V_{gs}(1.34) = 0$$

$$V_0(0.234) + (1.34)[(V_0)(0.299) + (V_i)(0.701)] = 0$$
$$V_0(0.635) + V_i(0.939) = 0$$
$$\Rightarrow A_{vf} = \frac{V_0}{V_i} = -1.48$$

b. For $K_n = 1.5\, mA/V^2$

From dc analysis:

$$2.13 - V_0(0.2279) = 1.5[(0.2985V_0) - 1.5]^2$$
$$= 1.5[0.0891V_0^2 - 0.8955V_0 + 2.25]$$
$$= 0.1337V_0^2 - 1.343V_0 + 3.375$$

$$0.1337V_0^2 - 1.115V_0 + 1.245 = 0$$
$$V_0 = \frac{1.115 \pm \sqrt{(1.115)^2 - 4(0.1337)(1.245)}}{2(0.1337)}$$
$$V_0 = \frac{1.115 \pm 0.7597}{2(0.1337)} \Rightarrow V_0 = 7.01 \text{ V}$$

$$I_D = \frac{10 - 7.01}{4.7} - \frac{7.01}{67} = 0.636 - 0.105$$
$$I_D = 0.531 \text{ mA}$$
$$V_{GS} = \sqrt{\frac{0.531}{1.5}} + 1.5 = 2.09$$

$$g_m = 2K_n(V_{GS} - V_{TN}) = 2(1.5)(2.09 - 1.5)$$
$$= 1.77 \text{ mA/V}$$

From ac analysis:

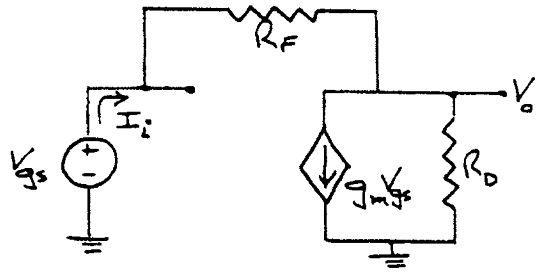
$$\frac{V_0}{4.7} + (1.77)V_{gs} + \frac{V_0}{47} - \frac{V_{gs}}{47} = 0$$
$$V_0(0.213) + (1.77)V_{gs} + V_0(0.0213) - V_{gs}(0.0213) = 0$$
$$V_0(0.234) + V_{gs}(1.75) = 0$$

$$V_0(0.234) + (1.75)[V_0(0.299) + V_i(0.701)] = 0$$
$$V_0(0.757) + V_i(1.23) = 0$$
$$\Rightarrow A_{vf} = \frac{V_0}{V_i} = -1.62$$

$$\% \text{ change} = \frac{1.62 - 1.48}{1.48} \Rightarrow 9.46\%$$

E12.23

a. Input resistance.



$$\frac{V_O}{R_D} + g_m V_{gs} + \frac{V_O - V_{gs}}{R_F} = 0 \qquad (1)$$

$$I_i = \frac{V_{gs} - V_O}{R_F} \qquad (2)$$

So $V_O = V_{gs} - I_i R_F$

$$V_O\left(\frac{1}{R_D} + \frac{1}{R_F}\right) + V_{gs}\left(g_m - \frac{1}{R_F}\right) = 0 \qquad (1)$$

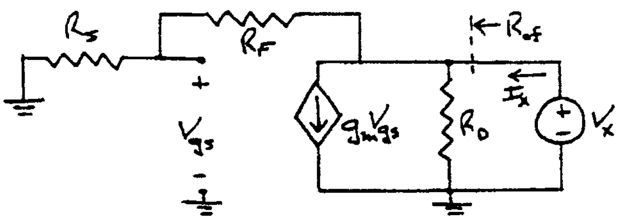
$$[V_{gs} - I_i(47)]\left(\frac{1}{4.7} + \frac{1}{47}\right) + V_{gs}\left(1.36 - \frac{1}{47}\right) = 0$$

$$[V_{gs} - I_i(47)](0.234) + V_{gs}(1.34) = 0$$

$$V_{gs}(1.57) = I_i(11.0)$$

$$\Rightarrow \underline{R_{if} = \frac{V_{gs}}{I_i} = 7.0\ \text{k}\Omega}$$

Output Resistance.



$$I_X = \frac{V_X}{R_D} + g_m V_{gs} + \frac{V_X}{R_S + R_F}$$

$$V_{gs} = \left(\frac{R_S}{R_S + R_F}\right) V_X = \left(\frac{20}{20 + 47}\right) V_X = 0.2985 V_X$$

$$I_X = \frac{V_X}{4.7} + (1.36)(0.2985)V_X + \frac{V_X}{20 + 47}$$

$$I_X = V_X[0.213 + 0.406 + 0.0149]$$

$$R_{Of} = \frac{V_X}{I_X} = 1.58\ \text{k}\Omega$$

b. From part (a)

$$[V_{gs} - I_i(47)](0.234) + V_{gs}\left(1.77 - \frac{1}{47}\right) = 0$$

$$V_{gs}(1.98) = I_i(11)$$

$$\Rightarrow \underline{R_{if} = \frac{V_{gs}}{I_i} = 5.56\ \text{k}\Omega}$$

$$I_X = \frac{V_X}{4.7} + (1.77)(0.2985)V_X + \frac{V_X}{20 + 47}$$

$$I_X = V_X[0.213 + 0.528 + 0.0149]$$

$$\Rightarrow \underline{R_{Of} = \frac{V_X}{I_X} = 1.32\ \text{k}\Omega}$$

E12.25

$$V_{TH} = \left(\frac{5.5}{5.5 + 51}\right)(10) = 0.973\ \text{V}$$

$$R_{TH} = 5.5\|51 = 4.96\ \text{k}\Omega$$

$$I_{BQ} = \frac{0.973 - 0.7}{4.96 + (121)(1)} = 0.00217\ \text{mA}$$

$$I_{CQ} = 0.260\ \text{mA}$$

$$r_\pi = 12\ \text{k}\Omega,\ g_m = 10\ \text{mA/V}$$

$$R_{eq} = R_S\|R_1\|R_2\|r_\pi = 10{,}000\|51\|5.5\|12$$

$$= 3.51\ \text{k}\Omega$$

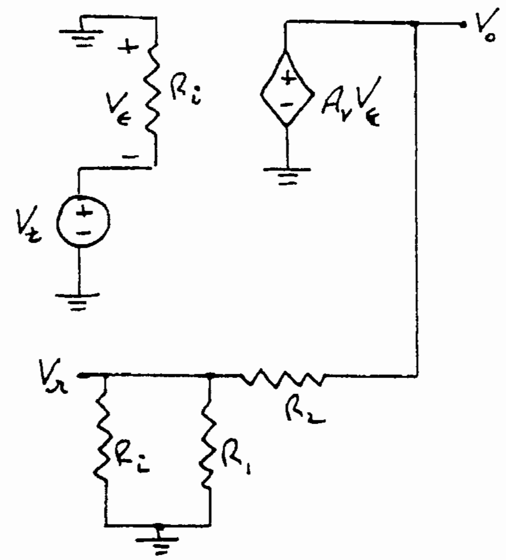
From Equation (12.99(b)):

$$T = (g_m R_C)\left(\frac{R_{eq}}{R_C + R_F + R_{eq}}\right)$$

$$= (10)(10)\left(\frac{3.51}{10 + 82 + 3.51}\right)$$

$$\Rightarrow \underline{T = 3.68}$$

E12.26



$V_\epsilon = -V_t,\ V_O = A_v V_\epsilon = -A_v V_t$

$$V_r = \left(\frac{R_1\|R_i}{R_1\|R_i + R_2}\right) V_O = -\left(\frac{R_1\|R_i}{R_1\|R_i + R_2}\right)(A_v V_t)$$

$$\underline{T = -\frac{V_r}{V_t} = A_v\left(\frac{R_1\|R_i}{R_1\|R_i + R_2}\right)}$$

or

$$\underline{T = \frac{A_v}{1 + \frac{R_2}{R_1\|R_i}}}$$

E12.27

$$T = A_i\beta = \frac{A_{i0}\beta}{\left(1 + j\cdot\frac{f}{f_1}\right)} = \frac{(10^5)(0.01)}{1 + j\cdot\left(\frac{f}{10}\right)}$$

$$|T(f_1)| = 1 = \frac{10^3}{\sqrt{1 + \left(\frac{f'_E}{10}\right)^2}}$$

$$1 + \left(\frac{f'_E}{10}\right)^2 = 10^6$$

$f'_E = 10\sqrt{10^6 - 1} \Rightarrow \underline{f'_E \approx 10^4 \text{ Hz}}$

$$\text{Phase} = \phi = -\tan^{-1}\left(\frac{f'_E}{10}\right) = -\tan^{-1}\left(\frac{10^4}{10}\right)$$
$$= -\tan^{-1}(10^3)$$

$\phi \approx 90°$

Phase Margin $= 180 - 90 \Rightarrow$ $\underline{\text{Phase Margin} = 90°}$

E12.28

$$T = A_i\beta = \frac{A_{i0}\beta}{\left(1 + j\cdot\frac{f}{f_1}\right)\left(1 + j\cdot\frac{f}{f_2}\right)}$$

$$\text{Phase} = -\left[\tan^{-1}\left(\frac{f}{f_1}\right) + \tan^{-1}\left(\frac{f}{f_2}\right)\right]$$

Phase Margin $= 60° \Rightarrow$ Phase $= -120°$

$$-120° = -\left[\tan^{-1}\left(\frac{f}{10^4}\right) + \tan^{-1}\left(\frac{f}{10^5}\right)\right]$$

At $f' = 7.66 \times 10^4$ Hz,

$$\text{Phase} = -\left[\tan^{-1}(7.66) + \tan^{-1}(0.766)\right]$$
$$= -[82.56 + 37.45]$$
$$= -120°$$

$$|T(f')| = 1 = \frac{(10^5)\beta}{\sqrt{1 + (7.66)^2} \times \sqrt{1 + (0.766)^2}}$$

$$1 = \frac{(10^5)\beta}{(7.725)(1.26)} \Rightarrow \underline{\beta = 9.73 \times 10^{-5}}$$

E12.29

$$\text{Phase} = -180° = -3\tan^{-1}\left(\frac{f'}{10^5}\right)$$

or $\tan^{-1}\left(\frac{f'}{10^5}\right) = 60° \Rightarrow f' = 1.732 \times 10^5$ Hz

$$|T(f')| = 1 = \frac{\beta(100)}{\left[\sqrt{1 + \left(\frac{f'}{10^5}\right)^2}\right]^3}$$

$$= \frac{\beta(100)}{\left[\sqrt{1 + (1.732)^2}\right]^3}$$

$\Rightarrow \underline{\beta = 0.08}$

E12.30

Phase Margin $= 60° \Rightarrow$ Phase $= -120°$

$$\text{Phase} = -120° = -3\tan^{-1}\left(\frac{f'}{10^5}\right)$$

$\tan^{-1}\left(\frac{f'}{10^5}\right) = 40° \Rightarrow f' = 0.839 \times 10^5$ Hz

$$|T(f')| = 1 = \frac{\beta(100)}{\left[\sqrt{1 + \left(\frac{f'}{10^5}\right)^2}\right]^3}$$

$$= \frac{\beta(100)}{\left[\sqrt{1 + (0.839)^2}\right]^3}$$

$\Rightarrow \underline{\beta = 0.0222}$

E12.31

The new loop gain function is

$$T'(f) = \frac{10^5}{\left(1+j\cdot\frac{f}{f_{PD}}\right)\left(1+j\cdot\frac{f}{5\times10^5}\right)} \times \frac{1}{\left(1+j\cdot\frac{f}{10^7}\right)\left(1+j\cdot\frac{f}{5\times10^8}\right)}$$

$$\text{Phase} = -\left\{\tan^{-1}\left(\frac{f}{f_{PD}}\right)+\tan^{-1}\left(\frac{f}{5\times10^5}\right) + \tan^{-1}\left(\frac{f}{10^7}\right)+\tan^{-1}\left(\frac{f}{5\times10^8}\right)\right\}$$

For a phase margin 45° ⇒ Phase = −135°, the poles are far apart so this will occur at approximately $f' = 5\times10^5$ Hz. Then

$$|T'(f)| = 1 = \frac{10^5}{\sqrt{1+\left(\frac{5\times10^5}{f_{PD}}\right)^2}\times\sqrt{1+1}\times\sqrt{1}\times\sqrt{1}}$$

$$1 = \frac{10^5}{(1.414)\sqrt{1+\left(\frac{5\times10^5}{f_{PD}}\right)^2}}$$

$$1+\left(\frac{5\times10^5}{f_{PD}}\right)^2 = 5\times10^9$$

$$f_{PD} \cong \frac{5\times10^5}{\sqrt{5\times10^9}} \Rightarrow \underline{f_{PD} = 7.07 \text{ Hz}}$$

E12.32

Phase Margin = 45° ⇒ Phase = −135°

This will occur at approximately $f' = 10^7$ Hz

$$|T'(f)| = 1 = \frac{10^5}{\sqrt{1+\left(\frac{10^7}{f_{PD}}\right)^2}\times\sqrt{2}\times\sqrt{1}}$$

$$1+\left(\frac{10^7}{f_{PD}}\right)^2 = 5\times10^9$$

$$f_{PD} \cong \frac{10^7}{\sqrt{5\times10^9}} \Rightarrow \underline{f_{PD} = 141 \text{ Hz}}$$

E12.33

$$A_f(0) = \frac{A_0}{1+\beta A_0} = \frac{2\times10^5}{1+(0.05)(2\times10^5)}$$

$$\Rightarrow \underline{A_f(0) \cong 20}$$

$$f_C = f_{PD}(1+\beta A_0) = 100[1+(0.05)(2\times10^5)]$$

$$\Rightarrow \underline{f_C \cong 1 \text{ MHz}}$$

Chapter 12

Problem Solutions

12.1

a. $A_f = \dfrac{A}{1+A\beta}$

$80 = \dfrac{10^5}{1+(10^5)\beta} \Rightarrow 1+(10^5)\beta = \dfrac{10^5}{80}$

$\Rightarrow \beta = \dfrac{\frac{10^5}{80}-1}{10^5} \Rightarrow \underline{\beta = 0.01249}$

b. $\dfrac{dA_f}{A_f} = \left(\dfrac{A_f}{A}\right)\cdot\dfrac{dA}{A} = \dfrac{80}{10^5}(-20)$

or

$\underline{\dfrac{dA_f}{A_f} = -0.016\%}$

$A_f = 80 - (0.00016)80 \Rightarrow \underline{A_f \cong 79.99}$

c. $80 = \dfrac{10^3}{1+(10^3)\beta}$

$\beta = \dfrac{\frac{10^3}{80}-1}{10^3} \Rightarrow \underline{\beta = 0.0115}$

$\dfrac{dA_f}{A_f} = \left(\dfrac{80}{10^3}\right)(-20) \Rightarrow \underline{\dfrac{dA_f}{A_f} = -1.6\%}$

$A_f = 80 - (0.016)(80) \Rightarrow \underline{A_f = 78.72}$

12.2

a. $A_f = \dfrac{(A)^3}{1+(A)^3\beta}$

$100 = \dfrac{1000}{1+(1000)\beta} \Rightarrow \beta = \dfrac{\frac{1000}{100}-1}{1000} \Rightarrow \underline{\beta = 0.009}$

b. A goes from 10 to 11 so

$A_f = \dfrac{(11)^3}{1+(11)^3(0.009)} = \dfrac{1331}{1+(1331)(0.009)}$

or

$A_f = 102.55$

so

$\dfrac{\Delta A_f}{A_f} = \dfrac{2.55}{100} \Rightarrow \underline{2.55\% \text{ change}}$

12.3

(a) $V_O = (-10)(-15)(-20)V_\epsilon = -3000V_\epsilon$

$V_\epsilon = \beta V_O + V_S$

So $V_O = -3000(\beta V_O + V_S)$

We find

$A_f = \dfrac{V_O}{V_S} = \dfrac{-3000}{1+3000\beta}$

For $A_f = -120 = \dfrac{-3000}{1+3000\beta} \Rightarrow \underline{\beta = 0.008}$

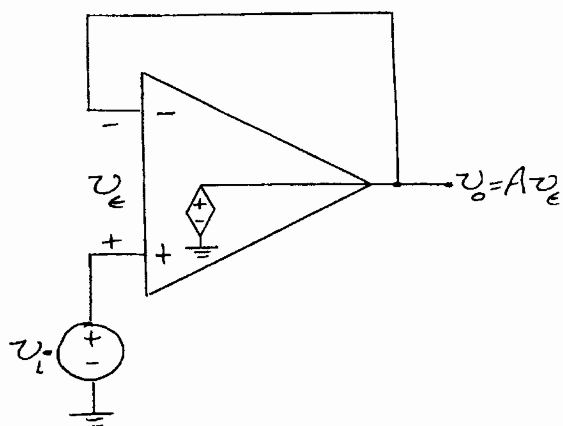
(b) Now $V_O = (-9)(-13.5)(-18)V_\epsilon = -2187V_\epsilon$

Then

$A_f = \dfrac{-2187}{1+2187\beta} = \dfrac{-2187}{1+2187(0.008)} = -118.24$

$\%\text{ change} = \dfrac{120-118.24}{120}x100 \Rightarrow \underline{1.47\%\text{ change}}$

12.4



$v_o = v_i - v_\epsilon \Rightarrow v_\epsilon = v_i - v_o$

Then $v_o = A(v_i - v_o) = Av_i - Av_o$

And

$v_o(1+A) = Av_i$

so

$\dfrac{v_o}{v_i} = \dfrac{A}{1+A} = 0.9998 \Rightarrow \underline{A = 4999}$

12.5

$(10^5)(4) = (50)f_B \Rightarrow \underline{f_B = 8\ kHz}$

12.6

(a) $(50)f_{3-dB} = (10^5)(4) \Rightarrow \underline{f_{3-dB} = 8\ kHz}$

(b) $(10)f_{3-dB} = (10^5)(4) \Rightarrow \underline{f_{3-dB} = 40\ kHz}$

12.7

$(50)(20\times10^3) = 5A_0$ so $\underline{A_0 = 2\times10^5}$

12.8

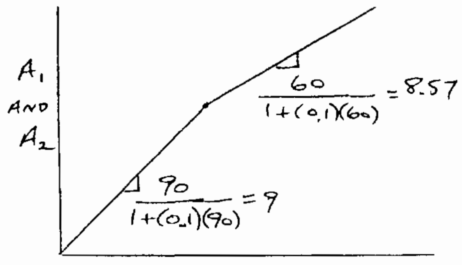
$v_0 = A_1A_2v_i + A_1v_n$

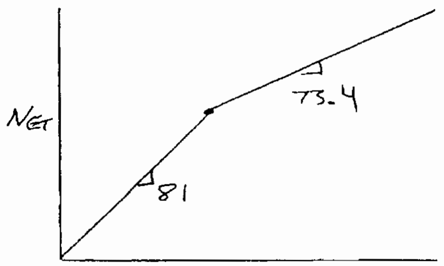
$v_0 = (100)v_i + (1)v_n = (100)(10) + (1)(1)$

$\Rightarrow \underline{\dfrac{S_0}{N_0} = \dfrac{1000}{1} = 1000}$

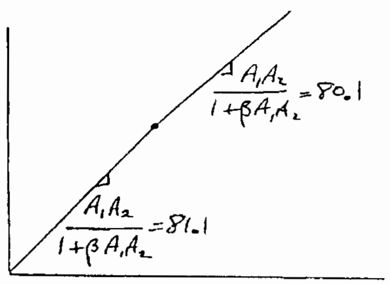
12.9

(a)





(b)



Circuit (b) – less distortion

12.10

(a) Low input $R \Rightarrow$ Shunt input
Low output $R \Rightarrow$ Shunt output
Or a Shunt-Shunt circuit

(b) High input $R \Rightarrow$ Series input
High output $R \Rightarrow$ Series output
Or a Series-Series circuit

(c) Shunt-Series circuit

(d) Series-Shunt circuit

12.11

(a) $R_i(\max) = R_i(1+T) = 10(1+10^4) \Rightarrow$
$R_i(\max) \cong 10^5\ k\Omega$

$$R_i(\min) = \frac{R_i}{1+T} = \frac{10}{1+10^4} \cong 10^{-3}\ k\Omega$$

Or $R_i(\min) = 1\ \Omega$

(b) $R_o(\max) = R_o(1+T) = 1(1+10^4) \Rightarrow$
$R_o(\max) \cong 10^4\ k\Omega$

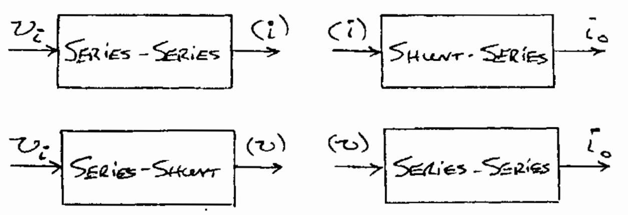
$$R_o(\min) = \frac{R_o}{1+T} = \frac{1}{1+10^4} \cong 10^{-4}\ k\Omega$$

Or $R_o(\min) = 0.1\ \Omega$

12.12

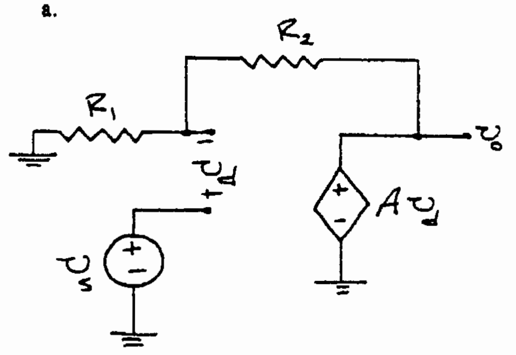
Overall Transconductance Amplifier, $A_g = \frac{i_o}{v_i}$

Series output = current signal and Shunt input = current signal. Also, Shunt output = voltage signal and Series input = voltage signal. Two possible solutions are shown.



12.13

a.



$$\frac{v_S - v_d}{R_1} = \frac{v_O - (v_S - v_d)}{R_2} \text{ and } v_d = \frac{v_O}{A}$$

$$\frac{v_S}{R_1} + \frac{v_S}{R_2} = \frac{v_O}{R_2} + v_d\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \frac{v_O}{R_2} + \frac{v_O}{A}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$v_S\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{v_O}{R_2}\left[1+\frac{1}{A}\left(1+\frac{R_2}{R_1}\right)\right]$$

$$\frac{v_O}{v_S}=\frac{\left(1+\frac{R_2}{R_1}\right)}{1+\frac{1}{A}\left(1+\frac{R_2}{R_1}\right)}$$

which can be written as

$$A_{vf}=\frac{v_O}{v_S}=\frac{A}{1+\left[A/\left(1+\frac{R_2}{R_1}\right)\right]}$$

b. $\beta=\dfrac{1}{1+\frac{R_2}{R_1}}$

c. $20=\dfrac{10^5}{1+(10^5)\beta}$

So $\beta=\dfrac{\frac{10^5}{20}-1}{10^5}\Rightarrow \underline{\beta=0.04999}$

Then $\dfrac{R_2}{R_1}=\dfrac{1}{\beta}-1=\dfrac{1}{0.04999}-1$

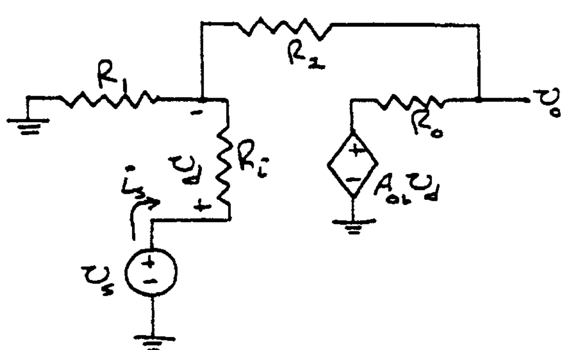
$\Rightarrow \underline{\dfrac{R_2}{R_1}=19.004}$

d. $A\rightarrow 8\times10^4$

$$A_f=\frac{8\times10^4}{1+(8\times10^4)(0.04999)}=19.999$$

$$\frac{\Delta A_f}{A_f}=-\frac{0.001}{20}\Rightarrow \underline{\frac{\Delta A_f}{A_f}=-0.005\%}$$

12.14



$$A_{vf}\approx\left(1+\frac{R_2}{R_1}\right)=20\Rightarrow\frac{R_2}{R_1}=19$$

$v_d=i_SR_i$

$$i_S=\frac{v_S-v_d}{R_1}+\frac{(v_S-v_d)-v_O}{R_2}\qquad(1)$$

$$\frac{v_O-A_{OL}v_d}{R_O}+\frac{v_O-(v_S-v_d)}{R_2}=0\qquad(2)$$

$$v_O\left(\frac{1}{R_O}+\frac{1}{R_2}\right)=\frac{A_{OL}v_d}{R_O}+\frac{(v_S-v_d)}{R_2}$$

$$v_O=\frac{\frac{A_{OL}v_d}{R_O}+\frac{(v_S-v_d)}{R_2}}{\left(\frac{1}{R_O}+\frac{1}{R_2}\right)}$$

From (1):

$$i_S=\frac{v_S-v_d}{R_1}+\frac{v_S-v_d}{R_2}-\frac{\frac{1}{R_2}\cdot\left[\frac{A_{OL}v_d}{R_O}+\frac{(v_S-v_d)}{R_2}\right]}{\left(\frac{1}{R_O}+\frac{1}{R_2}\right)}$$

$$i_S=v_S\left(\frac{1}{R_1}+\frac{1}{R_2}-\frac{\frac{1}{R_2}}{1+\frac{R_2}{R_O}}\right)-v_d\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{\frac{A_{OL}}{R_O}-\frac{1}{R_2}}{1+\frac{R_2}{R_O}}\right)$$

$v_d=i_SR_i$

$$i_S\left\{1+\frac{R_i\left[\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\left(1+\frac{R_2}{R_O}\right)+\frac{A_{OL}}{R_O}-\frac{1}{R_2}\right]}{1+\frac{R_2}{R_O}}\right\}=v_S\left[\frac{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\left(1+\frac{R_2}{R_O}\right)-\frac{1}{R_2}}{1+\frac{R_2}{R_O}}\right]$$

$$i_S\left\{1+\frac{R_2}{R_O}+R_i\left[\frac{1}{R_1}+\frac{R_2}{R_1}\cdot\frac{1}{R_O}+\frac{1}{R_O}+\frac{A_{OL}}{R_O}\right]\right\}=v_S\left[\frac{1}{R_1}+\frac{R_2}{R_1}\cdot\frac{1}{R_O}+\frac{1}{R_O}\right]$$

$$i_S\left\{R_O+R_2+R_i\left[\frac{R_O}{R_1}+\left(1+\frac{R_2}{R_1}\right)+A_{OL}\right]\right\}=v_S\left[\frac{R_O}{R_1}+\left(1+\frac{R_2}{R_1}\right)\right]\qquad(1)$$

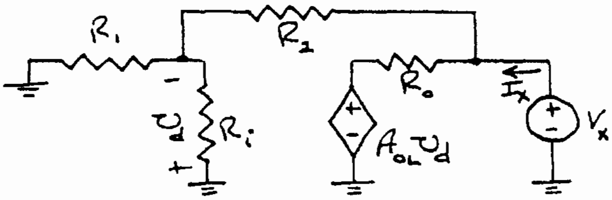
Let $R_2=190\ \text{k}\Omega$, $R_1=10\ \text{k}\Omega$

$$i_S\left\{0.1+190+100\cdot\left[\frac{0.1}{10}+20+10^5\right]\right\}=v_S\left[\frac{0.1}{10}+20\right]$$

$$i_S(1.000219\times10^7)=v_S(20.01)$$

$$R_{if}=\frac{v_S}{i_S}\cong5\times10^5\ \text{k}\Omega\Rightarrow \underline{R_{if}\cong500\ \text{M}\Omega}$$

Output Resistance



$$I_X = \frac{V_X - A_{OL}v_d}{R_0} + \frac{V_X}{R_2 + R_1\|R_i}$$

$$v_d = \frac{-R_1\|R_i}{R_1\|R_i + R_2}\cdot V_X$$

$$\frac{I_X}{V_X} = \frac{1}{R_{0f}} = \frac{1}{R_0} + \frac{A_{OL}\cdot R_1\|R_i}{R_0(R_1\|R_i + R_2)} + \frac{1}{R_2 + R_1\|R_i}$$

$R_1\|R_i = 10\|100 = 9.09$

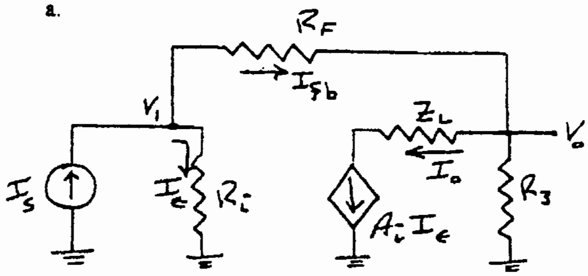
$$\frac{1}{R_{0f}} = \frac{1}{0.1} + \frac{10^5}{0.1}\cdot\left(\frac{9.09}{9.09+190}\right) + \frac{1}{190+9.09}$$

$$= 10 + 4.566\times10^4 + 0.00502$$

$R_{0f} = 2.19\times10^{-5}$ kΩ $\Rightarrow$ $\underline{R_{0f} = 0.0219\ \Omega}$

12.15

a.



Assume that V_1 is at virtual ground.

$V_0 = -I_{fb}R_F$

Now

$$I_{fb} = I_0 + \frac{V_0}{R_3} = I_0 - \frac{I_{fb}R_F}{R_3}$$

$I_{fb} = I_S - I_\epsilon$

and

$$I_0 = A_iI_\epsilon \Rightarrow I_\epsilon = \frac{I_0}{A_i}$$

so

$$I_{fb} = I_S - \frac{I_0}{A_i}$$

From above

$$I_{fb}\left(1+\frac{R_F}{R_3}\right) = I_0$$

$$\left(I_S - \frac{I_0}{A_i}\right)\left(1+\frac{R_F}{R_3}\right) = I_0$$

$$I_S\left(1+\frac{R_F}{R_3}\right) = I_0\left[1+\frac{1}{A_i}\left(1+\frac{R_F}{R_3}\right)\right]$$

or

$$A_{if} = \frac{I_0}{I_S} = \frac{\left(1+\frac{R_F}{R_3}\right)}{\left[1+\frac{1}{A_i}\left(1+\frac{R_F}{R_3}\right)\right]}$$

$$= \frac{A_i}{1+\frac{A_i}{\left(1+\frac{R_F}{R_3}\right)}} = A_{if}$$

b. $$\beta_i = \frac{1}{\left(1+\frac{R_F}{R_3}\right)}$$

c. $$25 = \frac{10^5}{1+(10^5)\beta_i}$$

$$\text{so } \beta_i = \frac{\frac{10^5}{25}-1}{10^5} \Rightarrow \underline{\beta_i = 0.03999}$$

$$\text{so } \frac{R_F}{R_3} = \frac{1}{\beta_i} - 1 = \frac{1}{0.03999} - 1 \Rightarrow \underline{\frac{R_F}{R_3} = 24.0}$$

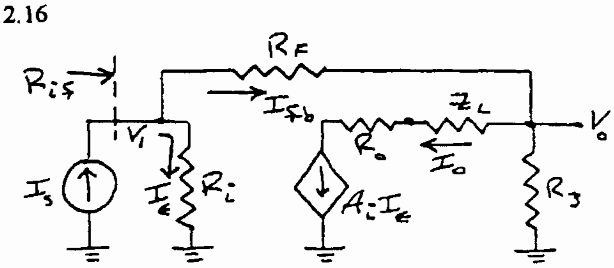
d. $A_i = 10^5 - (0.15)(10^5) = 8.5\times10^4$

$$\text{so } A_{if} = \frac{8.5\times10^4}{1+(8.5\times10^4)(0.03999)} = 24.9989$$

$$\text{so } \frac{\Delta A_{if}}{A_{if}} = -\frac{1.10\times10^{-3}}{25} = -4.41\times10^{-5}$$

$\Rightarrow \underline{-4.41\times10^{-3}\%}$

12.16



$I_S = I_\epsilon + I_{fb}$, $V_1 = I_\epsilon R_i$

$$I_{fb} = I_0 + \frac{V_0}{R_3} \text{ and } V_0 = V_1 - I_{fb}R_F$$

$$I_0 = A_iI_\epsilon \Rightarrow I_\epsilon = \frac{I_0}{A_i}$$

Now

$I_{fb} = A_i I_\epsilon + \frac{1}{R_3}(V_1 - I_{fb}R_F)$

$I_{fb}\left[1 + \frac{R_F}{R_3}\right] = A_i I_\epsilon + \frac{V_1}{R_3}$

$I_{fb} = I_S - I_\epsilon$

$(I_S - I_\epsilon)\left[1 + \frac{R_F}{R_3}\right] = A_i I_\epsilon + \frac{V_1}{R_3}$

$I_S\left[1 + \frac{R_F}{R_3}\right] = I_\epsilon\left[\left(1 + \frac{R_F}{R_3}\right) + A_i\right] + \frac{V_1}{R_3}$

$I_\epsilon = \frac{V_1}{R_i}$

$I_S\left[1 + \frac{R_F}{R_3}\right] = V_1\left\{\frac{1}{R_i} \cdot \left[\left(1 + \frac{R_F}{R_3}\right) + A_i\right] + \frac{1}{R_3}\right\}$

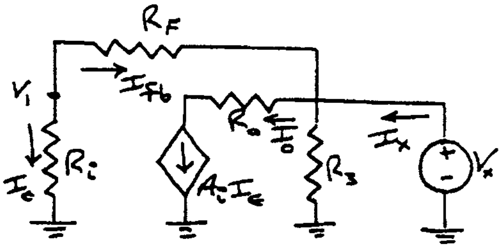
$$R_{if} = \frac{V_1}{I_S} = \frac{\left(1 + \frac{R_F}{R_3}\right)}{\left\{\frac{1}{R_i} \cdot \left[\left(1 + \frac{R_F}{R_3}\right) + A_i\right] + \frac{1}{R_3}\right\}}$$

The $1/R_3$ term in the denominator will be negligible. Using the results of Problem 12.15:

$$R_{if} = \frac{25}{\left\{\frac{1}{2}[(25) + 10^5]\right\}}$$

$R_{if} \cong 5 \times 10^{-4}$ k$\Omega \Rightarrow \underline{R_{if} = 0.5\ \Omega}$

Output Resistance (Let $Z_L = 0$)



$I_X = \frac{V_X}{R_3} + A_i I_\epsilon + \frac{V_X}{R_F + R_i}$

$I_\epsilon = \frac{V_X}{R_F + R_i}$

so

$\frac{I_X}{V_X} = \frac{1}{R_{of}} = \frac{1}{R_3} + \frac{A_i + 1}{R_F + R_i}, \quad \frac{R_F}{R_3} = 24$

Let $R_F = 240$ kΩ, $R_3 = 10$ kΩ

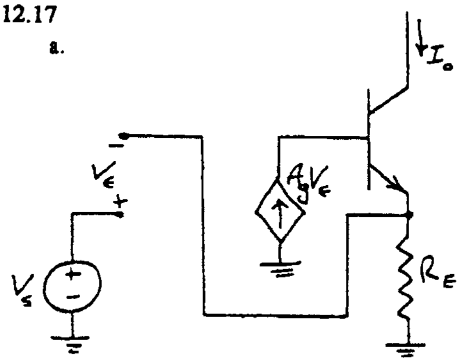
$\frac{1}{R_{of}} = \frac{1}{10} + \frac{10^5 + 1}{240 + 2}$

so $R_{of} \approx \frac{R_F + R_i}{A_i + 1} = \frac{240 + 2}{10^5 + 1}$

$\Rightarrow R_{of} \approx 2.42 \times 10^{-3}$ kΩ or $\underline{R_{of} \approx 2.42\ \Omega}$

12.17

a.



$I_E = \frac{(1 + h_{FE})}{h_{FE}} \cdot I_0 = \frac{V_S - V_\epsilon}{R_E}$

Also $I_0 = h_{FE}(A_g V_\epsilon)$ so $V_\epsilon = \frac{I_0}{h_{FE}A_g}$

Then

$\frac{1 + h_{FE}}{h_{FE}} \cdot I_0 = \frac{V_S}{R_E} - \frac{I_0}{h_{FE}A_g R_E}$

$\left[\frac{1 + h_{FE}}{h_{FE}} + \frac{1}{h_{FE}A_g R_E}\right] I_0 = \frac{V_S}{R_E}$

$\left[\frac{A_g(1 + h_{FE})R_E + 1}{h_{FE}A_g R_E}\right] I_0 = \frac{V_S}{R_E}$

$\frac{I_0}{V_S} = \frac{1}{R_E} \cdot \left[\frac{h_{FE}A_g R_E}{1 + A_g(1 + h_{FE})R_E}\right]$

$\Rightarrow \underline{\frac{I_0}{V_S} \approx \frac{h_{FE}A_g}{1 + (h_{FE}A_g)R_E}}$

b. $\underline{\beta_z = R_E}$

c. $10 = \frac{5 \times 10^5}{1 + (5 \times 10^5)\beta_z}$

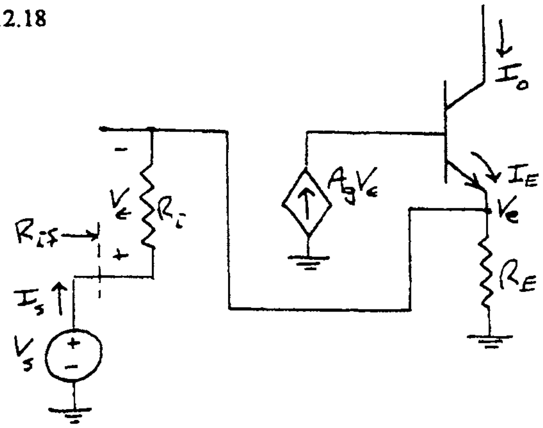
$\beta_z = \frac{\frac{5 \times 10^5}{10} - 1}{5 \times 10^5} \Rightarrow \underline{\beta_z = R_E = 0.099998\text{ k}\Omega}$

d. If $A_g \to 5.5 \times 10^5$ then

$A_{gf} = \frac{5.5 \times 10^5}{1 + (5.5 \times 10^5)(0.099998)} = 10.0000182$

$\frac{\Delta A_{gf}}{A_{gf}} = \frac{1.82 \times 10^{-5}}{10} \Rightarrow \underline{1.82 \times 10^{-4}\%}$

12.18



$I_E = (1+h_{FE})A_g V_\epsilon$, $I_E = \dfrac{V_e}{R_E} - I_S$ and $V_\epsilon = I_S R_i$,

$V_e = V_S - V_\epsilon = V_S - I_S R_i$

Now $(1+h_{FE})A_g I_S R_i = \dfrac{1}{R_E}\cdot(V_S - I_S R_i) - I_S$

$$\left[(1+h_{FE})A_g R_i + \frac{R_i}{R_E} + 1\right] I_S = \frac{V_S}{R_E}$$

$$R_{if} = \frac{V_S}{I_S} = R_E\left[(1+h_{FE})A_g R_i + \frac{R_i}{R_E} + 1\right]$$

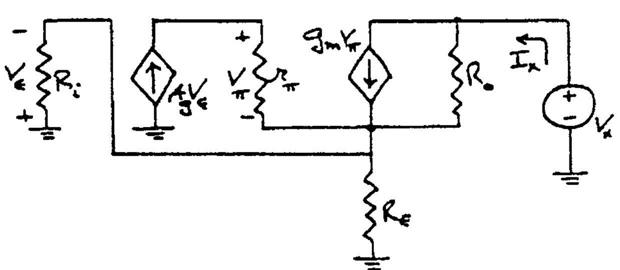
From Problem 12.16:

$(1+h_{FE})A_g \approx h_{FE}A_g = 5\times 10^5$ mS

$R_E \approx 0.1$ kΩ

so $R_{if} = (0.1)\left[(5\times 10^5)(20) + \dfrac{20}{0.1} + 1\right]$

or $\underline{R_{if} = 10^6 \text{ k}\Omega}$



$$\frac{V_\pi}{r_\pi} = A_g V_\epsilon$$

$$I_X = g_m V_\pi + \frac{V_X - (-V_\epsilon)}{R_O} \qquad (1)$$

$$V_\epsilon = -(I_X + A_g V_\epsilon)(R_E \| R_i) \qquad (2)$$

or $V_\epsilon = [1 + A_g(R_E\|R_i)] = -I_X(R_E\|R_i)$

Now

$$I_X = g_m A_g r_\pi V_\epsilon + \frac{V_X}{R_O} + \frac{V_\epsilon}{R_O} \qquad (1)$$

$$I_X = \left(g_m A_g r_\pi + \frac{1}{R_O}\right)\left[\frac{-I_X(R_E\|R_i)}{1 + A_g(R_E\|R_i)}\right] + \frac{V_X}{R_O}$$

$$R_{Of} = \frac{V_X}{I_X}$$

$$= R_O\left\{1 + \left(g_m A_g r_\pi + \frac{1}{R_O}\right)\left[\frac{(R_E\|R_i)}{1 + A_g(R_E\|R_i)}\right]\right\}$$

$g_m r_\pi A_g = h_{FE}A_g = 5\times 10^5$ mS

Let $h_{FE} = 100$ so $A_g = 5\times 10^3$ mS

$R_E\|R_i = 0.1\|20 \approx 0.1$ kΩ

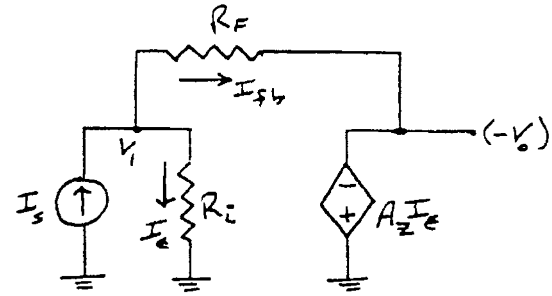
Then

$$R_{Of} = 50\left\{1 + \left(5\times 10^5 + \frac{1}{50}\right)\left[\frac{0.1}{1 + (5\times 10^3)(0.1)}\right]\right\}$$

or $\underline{R_{Of} = 5.04 \text{ M}\Omega}$

12.19

a.



Assuming V_1 is at virtual ground

$(-V_O) = -I_{fb}R_F$ and $(-V_O) = -A_z I_\epsilon \Rightarrow I_\epsilon = \dfrac{V_O}{A_z}$

$I_{fb} = I_S - I_\epsilon$

So $V_O = (I_S - I_\epsilon)R_F = I_S R_F - \left(\dfrac{V_O}{A_z}\right)R_F$

$$V_O\left[1 + \frac{R_F}{A_z}\right] = I_S R_F$$

so $A_{zf} = \dfrac{V_O}{I_S} = \dfrac{R_F}{\left[1 + \dfrac{R_F}{A_z}\right]} = \dfrac{A_z R_F}{A_z + R_F}$

or $A_{zf} = \dfrac{A_z}{1 + A_z\left(\dfrac{1}{R_F}\right)} = \dfrac{A_z}{1 + A_z\beta_g}$

b. $\beta_g = \dfrac{1}{R_F}$

c. $5 \times 10^4 = \dfrac{5 \times 10^6}{1 + (5 \times 10^6)\beta_g}$

$$\beta_g = \frac{\dfrac{5 \times 10^6}{5 \times 10^4} - 1}{5 \times 10^6} \Rightarrow \underline{\beta_g = 1.98 \times 10^{-5}}$$

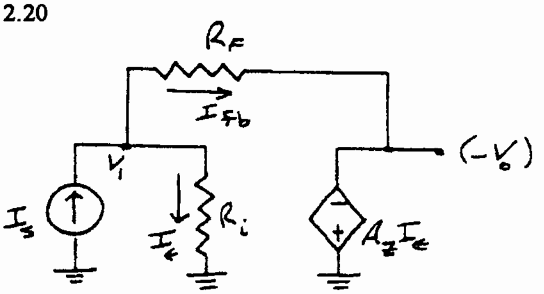
$$R_F = \frac{1}{\beta_g} \Rightarrow \underline{R_F = 50.5 \text{ k}\Omega}$$

d. $A_z = (0.9)(5 \times 10^6) = 4.5 \times 10^6$

$$A_{zf} = \frac{4.5 \times 10^6}{1 + (4.5 \times 10^6)(1.98 \times 10^{-5})} = 4.994 \times 10^4$$

$$\frac{\Delta A_{zf}}{A_{zf}} = -\frac{55.4939}{5 \times 10^4} = -1.11 \times 10^{-3} \Rightarrow \underline{-0.111\%}$$

12.20



$V_1 = I_\epsilon R_i, \quad -V_0 = -A_z I_\epsilon \Rightarrow V_0 = A_z I_\epsilon$

$I_{fb} = I_S - I_\epsilon$ and $-V_0 = V_1 - I_{fb} R_F$

$$-A_z I_\epsilon = V_1 - (I_S - I_\epsilon) R_F$$

$$-A_z\left(\frac{V_1}{R_i}\right) = V_1 - I_S R_F + \left(\frac{V_1}{R_i}\right) R_F$$

$$I_S R_F = V_1\left[1 + \frac{A_z}{R_i} + \frac{R_F}{R_i}\right]$$

$$R_{if} = \frac{V_1}{I_S} = \frac{R_F}{\left[1 + \dfrac{A_z}{R_i} + \dfrac{R_F}{R_i}\right]}$$

Or, using the results from Problem 12.18,

$$R_{if} = \frac{50.5 \times 10^3}{\left[1 + \dfrac{5 \times 10^6}{10 \times 10^3} + \dfrac{50.5 \times 10^3}{10 \times 10^3}\right]}$$

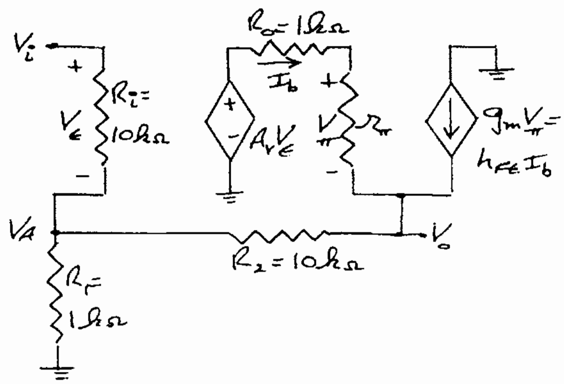
$$= \frac{50.5 \times 10^3}{[1 + 500 + 5.05]}$$

$$\Rightarrow \underline{R_{if} = 99.79\ \Omega}$$

12.21

Assume $I_{CQ} = 0.2\ mA$

Then $r_\pi = \dfrac{(100)(0.026)}{0.2} = 13\ k\Omega$



(1) $\dfrac{V_i - V_A}{R_i} = \dfrac{V_A}{R_1} + \dfrac{V_A - V_o}{R_2} \Rightarrow$

$$\frac{V_i}{R_i} + \frac{V_o}{R_2} = V_A\left(\frac{1}{R_i} + \frac{1}{R_1} + \frac{1}{R_2}\right)$$

Now

$$\frac{V_i}{10} + \frac{V_o}{10} = V_A\left(\frac{1}{10} + \frac{1}{1} + \frac{1}{10}\right) \Rightarrow V_i + V_o = V_A(12)$$

or $V_A = \dfrac{1}{12}(V_i + V_o)$

(2) $\left(\dfrac{A_V V_\epsilon - V_o}{R_o + r_\pi}\right)(1 + h_{FE}) = \dfrac{V_o - V_A}{R_2}$

where $V_\epsilon = V_i - V_A$

Then

$$\left(\frac{A_v(V_i - V_A) - V_o}{R_o + r_\pi}\right)(1 + h_{FE}) = \frac{V_o - V_A}{R_2}$$

we find

$$\frac{A_v V_i(1 + h_{FE})}{R_o + r_\pi} - \frac{V_o(1 + h_{FE})}{R_o + r_\pi} - \frac{V_o}{R_2} = \frac{A_v V_A(1 + h_{FE})}{R_o + r_\pi} - \frac{V_A}{R_2}$$

Then

$$\frac{(5x10^3)(101)V_i}{14} - \frac{V_o(101)}{14} - \frac{V_o}{10}$$

$$= \left(\frac{(5x10^3)(101)}{14} - \frac{1}{10}\right)V_A$$

Rearranging terms, we find

$$\underline{A_{vf} = \frac{V_o}{V_i} = 10.97}$$

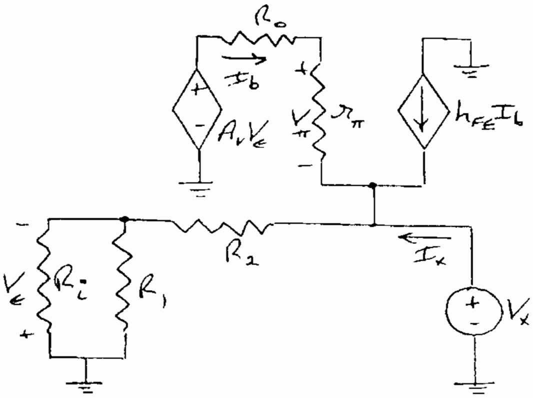
$$R_{if} = \frac{V_i}{I_i} = \frac{V_i}{\left(\dfrac{V_i - V_A}{R_i}\right)} = \left(\frac{V_i}{V_i - V_A}\right) R_i$$

$$V_A = \frac{1}{12}(V_i + V_o) = \frac{1}{12}(V_i + 10.97V_i) = 0.9975V_i$$

Then

$$R_{if} = \left(\frac{1}{1 - 0.9975}\right)(10\ k\Omega) \Rightarrow \underline{R_{if} = 4\ M\Omega}$$

To find the output resistance:



$$I_x + \frac{(A_v V_\epsilon - V_x)(1+h_{FE})}{R_o + r_\pi} = \frac{V_x}{R_2 + R_1\|R_i}$$

$$V_\epsilon = -\left(\frac{R_1\|R_i}{R_1\|R_i + R_2}\right)\cdot V_x$$

Now

$R_1\|R_i = 1\|10 = 0.909$

Then

$V_\epsilon = -0.0833 V_x$

Now

$$I_x = V_x\left\{\left[\frac{(5x10^3)(0.0833)+1}{1+13}\right](101) + \frac{1}{10+0.909}\right\}$$

$$= V_x\{3.012x10^3 + 0.0917\}$$

Or

$$\frac{V_x}{I_x} = R_{of} = 3.32x10^{-4}\ k\Omega \Rightarrow \underline{R_{of} = 0.332\ \Omega}$$

12.22

a. Neglecting base currents

$I_{C2} = 0.5$ mA, $V_{C2} = 12 - (0.5)(22.6) = 0.7$ V

$I_{C1} = 0.5$ mA

$\Rightarrow \underline{v_O = 0}$

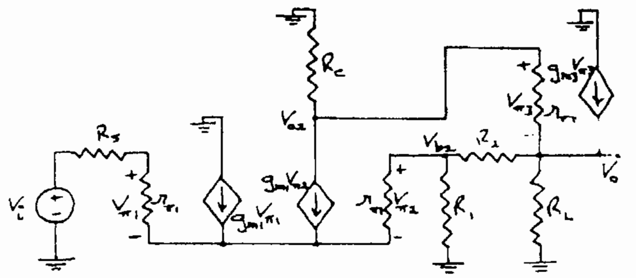
Then $I_{C3} = 2$ mA

b. $r_{\pi 1} = r_{\pi 2} = \dfrac{h_{FE}\cdot V_T}{I_{C1}} = \dfrac{(100)(0.026)}{0.5} = 5.2\ \text{k}\Omega$

$$g_{m1} = g_{m2} = \frac{0.5}{0.026} = 19.23\ \text{mA/V}$$

$$r_{\pi 3} = \frac{(100)(0.026)}{2} = 1.3\ \text{k}\Omega$$

$$g_{m3} = \frac{2}{0.026} = 76.92\ \text{mA/V}$$



$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} + g_{m1}V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi 1}} = 0$$

$$(V_{\pi 1} + V_{\pi 2})\left(\frac{1}{r_{\pi 1}} + g_{m1}\right) = 0$$

$$\Rightarrow V_{\pi 1} = -V_{\pi 2} \qquad (1)$$

$$V_i = \frac{V_{\pi 1}}{r_{\pi 1}}(R_S + r_{\pi 1}) - V_{\pi 2} + V_{b2}$$

or

$$V_i = V_{\pi 1}\left(1 + \frac{R_S}{r_{\pi 1}}\right) - V_{\pi 2} + V_{b2}$$

But $V_{\pi 2} = -V_{\pi 1}$

so

$$V_i = V_{\pi 1}\left(2 + \frac{R_S}{r_{\pi 1}}\right) + V_{b2} \qquad (2)$$

$$\frac{V_{O2}}{R_C} + g_{m1}V_{\pi 2} + \frac{V_{O2} - V_O}{r_{\pi 3}} = 0 \qquad (3)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_O}{R_L} + \frac{V_O - V_{b2}}{R_2}$$

$V_{\pi 3} = V_{O2} - V_O$

so

$$(V_{O2} - V_O)\left(\frac{1+h_{FE}}{r_{\pi 3}}\right) = V_O\left(\frac{1}{R_L} + \frac{1}{R_2}\right) - \frac{V_{b2}}{R_2} \qquad (4)$$

$$\frac{V_{b2} - V_O}{R_2} + \frac{V_{b2}}{R_1} + \frac{V_{\pi 2}}{r_{\pi 1}} = 0 \qquad (5)$$

Substitute numbers into (2), (3), (4) and (5):

$$V_i = -V_{\pi 2}\left(2 + \frac{1}{5.2}\right) + V_{b2}$$

$$V_i = -V_{\pi 2}(2.192) + V_{b2} \qquad (2)$$

$$V_{O2}\left(\frac{1}{22.6} + \frac{1}{1.3}\right) + (19.23)V_{\pi 2} - V_O\left(\frac{1}{1.3}\right) = 0$$

$$V_{O2}(0.8135) + (19.23)V_{\pi 2} - (0.7692)V_O = 0 \qquad (3)$$

$$V_{02}\left(\frac{101}{1.3}\right) = V_0\left(\frac{101}{1.3}+\frac{1}{4}+\frac{1}{50}\right) - V_{b2}\left(\frac{1}{50}\right)$$

$$V_{02}(77.69) = V_0(77.96) - V_{b2}(0.02) \quad (4)$$

$$V_{b2}\left(\frac{1}{50}+\frac{1}{10}\right) - V_0\left(\frac{1}{50}\right) + V_{\pi 2}\left(\frac{1}{5.2}\right) = 0$$

$$V_{b2}(0.120) - V_0(0.020) + V_{\pi 2}(0.1923) = 0 \quad (5)$$

From (2): $V_{b2} = V_i + V_{\pi 2}(2.192)$. Substitute in (4) and (5) to obtain:

$$V_{02}(77.69) = V_0(77.96) - [V_i + V_{\pi 2}(2.192)](0.02) \quad (4')$$

$$[V_i + V_{\pi 2}(2.192)](0.120) - V_0(0.020) + V_{\pi 2}(0.1923) = 0 \quad (5')$$

So we now have the following three equations:

$$V_{02}(0.8135) + (19.23)V_{\pi 2} - (0.7692)V_0 = 0 \quad (3)$$

$$V_{02}(77.69) = V_0(77.96) - V_i(0.02) - V_{\pi 2}(0.04384) \quad (4')$$

$$(0.120)V_i + V_{\pi 2}(0.4553) - V_0(0.020) = 0 \quad (5')$$

From (3): $V_{02} = V_0(0.9455) - V_{\pi 2}(23.64)$. Substitute for V_{02} in (4′) to obtain:

$$(77.69)[V_0(0.9455) - V_{\pi 2}(23.64)] = V_0(77.96) - V_i(0.02) - V_{\pi 2}(0.04384)$$

or

$$0 = V_0(4.504) - V_i(0.02) + V_{\pi 2}(1836.5)$$

Next, solve (5′) for $V_{\pi 2}$:

$$(0.120)V_i + V_{\pi 2}(0.4553) - V_0(0.020) = 0$$

$$V_{\pi 2} = V_0(0.04393) - V_i(0.2636)$$

Finally,

$$0 = V_0(4.504) - V_i(0.02) + (1836.5)[V_0(0.04393) - V_i(0.2636)]$$

$$0 = V_0(85.18) - V_i(484.12)$$

So

$$A_{vf} = \frac{V_0}{V_i} = \frac{484.12}{85.18} \Rightarrow \underline{A_{vf} = 5.68}$$

12.23

a. $R_{TH} = R_1 \| R_2 = 400 \| 75 = 63.2\ \text{k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)V_{CC} = \left(\frac{75}{75+400}\right)(10) = 1.579\ \text{V}$$

$$I_{BQ1} = \frac{1.579-0.7}{63.2+(121)(0.5)} = 0.007106\ \text{mA}$$

$$I_{CQ1} = 0.853\ \text{mA}$$

$$V_{C1} = 10 - (0.853)(8.8) = 2.49\ \text{V}$$

$$I_{C2} \approx \frac{2.49-0.7}{3.6} = 0.497\ \text{mA}$$

$$V_{C2} = 10 - (0.497)(13) = 3.54\ \text{V}$$

$$I_{C3} \approx \frac{3.54-0.7}{1.4} = 2.03\ \text{mA}$$

Then

$$r_{\pi 1} = \frac{(120)(0.026)}{0.853} = 3.66\ \text{k}\Omega$$

$$g_{m1} = \frac{0.853}{0.026} = 32.81\ \text{mA/V}$$

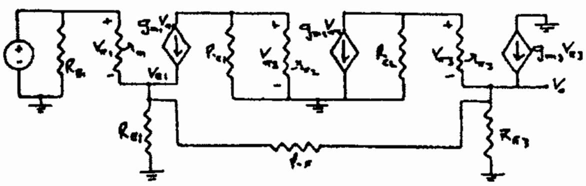
$$r_{\pi 2} = \frac{(120)(0.026)}{0.497} = 6.28\ \text{k}\Omega$$

$$g_{m2} = \frac{0.497}{0.026} = 19.12\ \text{mA/V}$$

$$r_{\pi 3} = \frac{(120)(0.026)}{2.03} = 1.54\ \text{k}\Omega$$

$$g_{m3} = \frac{2.03}{0.026} = 78.08\ \text{mA/V}$$

b.



$$V_i = V_{\pi 1} + V_{e1} \Rightarrow V_{e1} = V_i - V_{\pi 1} \quad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \frac{V_{e1}}{R_{E1}} + \frac{V_{e1}-V_0}{R_F} \quad (2)$$

$$V_{\pi 2} = -(g_{m1}V_{\pi 1})(R_{C1}\|r_{\pi 2}) \quad (3)$$

$$g_{m2}V_{\pi 2} + \frac{V_{\pi 3}+V_0}{R_{C2}} + \frac{V_{\pi 3}}{r_{\pi 3}} = 0 \quad (4)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_0}{R_{E3}} + \frac{V_0 - V_{e1}}{R_F} \quad (5)$$

Substitute numbers in (2), (3), (4) and (5):

$$V_{\pi1}\left(\frac{1}{3.66}+32.81\right)=(V_i-V_{\pi1})\left(\frac{1}{0.5}+\frac{1}{10}\right)-\frac{V_0}{10}$$

or $V_{\pi1}(35.18)=V_i(2.10)-V_0(0.10)$ (2)

$V_{\pi2}=-(32.81)V_{\pi1}(88\|6.28)$

or $V_{\pi2}=-V_{\pi1}(120.2)$ (3)

$$(19.12)V_{\pi2}+\frac{V_{\pi3}}{13}+\frac{V_0}{13}+\frac{V_{\pi3}}{1.54}=0$$

or

$V_{\pi2}(19.12)+V_{\pi3}(0.7263)+V_0(0.07692)=0$ (4)

$$V_{\pi3}\left(\frac{1}{1.54}+78.08\right)=V_0\left(\frac{1}{1.4}+\frac{1}{10}\right)-\frac{V_i-V_{\pi1}}{10}$$

or

$V_{\pi3}(78.73)$

$=V_0(0.8143)-V_i(0.10)+V_{\pi1}(0.10)$ (5)

Now substituting $V_{\pi2}=-V_{\pi1}(120.2)$ in (4):

$(19.12)[-V_{\pi1}(120.2)]+V_{\pi3}(0.7263)$

$+V_0(0.07692)=0$

or

$-V_{\pi1}(2298.2)+V_{\pi3}(0.7263)+V_0(0.07692)=0$

Then

$V_{\pi3}=V_{\pi1}(3164.3)-V_0(0.1059)$

Substituting $V_{\pi3}=V_{\pi1}(3164.3)-V_0(0.1059)$ in (5):

$(78.73)[V_{\pi1}(3164.3)-V_0(0.1059)]$

$=V_0(0.8143)-V_i(0.10)+V_{\pi1}(0.10)$

or

$V_{\pi1}(2.49\times10^5)-V_0(9.152)=-V_i(0.10)$

Then

$V_{\pi1}=V_0(3.674\times10^{-5})-V_i(4.014\times10^{-7})$

Now substituting $V_{\pi1}=V_0(3.674\times10^{-5})$ $-V_i(4.014\times10^{-7})$ in (2):

$(35.18)[V_0(3.674\times10^{-5})-V_i(4.014\times10^{-7})]$

$=V_i(2.10)-V_0(0.10)$

or $V_0(0.1013)=V_i(2.10)$

So $\frac{V_0}{V_i}=20.7$

c. $R_{if}=\frac{V_i}{I_i}$ and $I_i=I_{RB1}+I_{b1}$

$$I_{RB1}=\frac{V_i}{R_{B1}}$$

$$I_{b1}=\frac{V_{\pi1}}{r_{\pi1}}$$

Now

$V_{\pi1}=(20.7V_i)(3.674\times10^{-5})-V_i(4.014\times10^{-7})$

$V_{\pi1}=V_i(7.60\times10^{-4})$

Then

$$R_{if}=\frac{V_i}{\frac{V_i}{63.2}+\frac{V_i(7.60\times10^{-4})}{3.66}}$$

$$=\frac{1}{0.01582+2.077\times10^{-4}}$$

or $\underline{R_{if}=62.4\text{ k}\Omega}$

d. To determine R_{of}:

Equation (1) is modified to $V_{\pi1}+V_{e1}=0$ $(V_i=0)$
Equation (5) is modified to:

$V_{\pi3}(78.73)+I_X=V_0(0.8143)+V_{\pi1}(0.10)$ (5)

Now

$V_{\pi1}(35.18)=-V_0(0.10)$ (2)

$V_{\pi2}=-V_{\pi1}(120.2)$ (3)

$V_{\pi2}(19.12)+V_{\pi3}(0.7263)+V_0(0.07692)=0$ (4)

Now

$V_{\pi1}=-V_0(0.002843)$

so

$V_{\pi2}=-(-V_0)(0.002843)(120.2)$

$V_{\pi2}=V_0(0.3417)$

Then

$V_0(0.3417)(19.12)+V_{\pi3}(0.7263)+V_0(0.07692)=0$

or

$V_{\pi3}=-V_0(9.101)$ (4)

So then

$-V_0(9.101)(78.73)+I_X$

$=V_0(0.8143)+(0.10)(-V_0)(0.002843)$

or

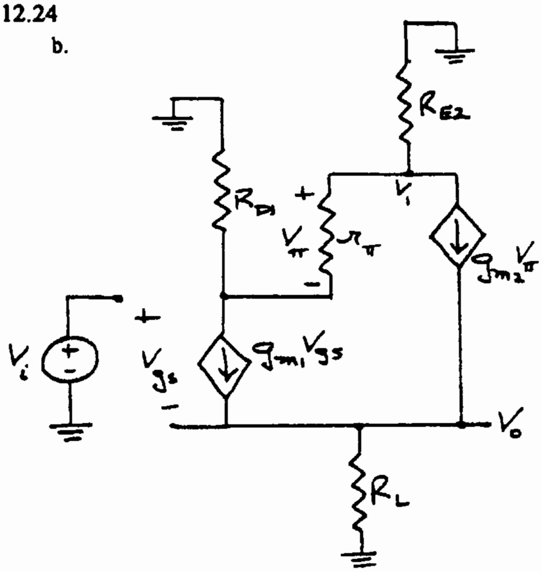
$I_X=V_0(717.3)$ (5)

or

$R_{of}=\frac{V_0}{I_X}=0.00139\text{ k}\Omega\Rightarrow\underline{R_{of}=1.39\ \Omega}$

12.24

b.



$$V_0 = (g_{m1}V_{gs} + g_{m2}V_\pi)R_L \qquad (1)$$

$$V_i = V_{gs} + V_0 \qquad (2)$$

$$\frac{V_\pi}{r_\pi} + g_{m2}V_\pi + \frac{V_1}{R_{E2}} = 0$$

$$\Rightarrow V_\pi\left(\frac{1+h_{FE}}{r_\pi}\right) + \frac{V_1}{R_{E2}} = 0 \qquad (3)$$

$$\frac{V_\pi}{r_\pi} = g_{m1}V_{gs} + \frac{V_1 - V_\pi}{R_{D1}} = 0$$

or

$$V_1 = R_{D1}\left[\frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_{D1}} - g_{m1}V_{gs}\right] \qquad (4)$$

Then

$$V_\pi\left(\frac{1+h_{FE}}{r_\pi}\right) + \frac{R_{D1}}{R_{E2}}\left[\frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_{D1}} - g_{m1}V_{gs}\right] = 0 \qquad (3)$$

$$V_\pi\left\{\left(\frac{1+h_{FE}}{r_\pi}\right) + \frac{R_{D1}}{R_{E2}r_\pi} + \frac{1}{R_{E2}}\right\} = g_{m1}\left(\frac{R_{D1}}{R_{E2}}\right)V_{gs}$$

Let

$$V_\pi \cdot \frac{1}{R_{eq}} = g_{m1}\left(\frac{R_{D1}}{R_{E2}}\right)V_{gs}$$

so $V_\pi = g_{m1}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)V_{gs}$

Then

$$V_0 = \left[g_{m1}V_{gs} + g_{m1}g_{m2}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)V_{gs}\right]R_L \qquad (1)$$

so

$$V_0 = g_{m1}R_L\left[1 + g_{m2}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)\right](V_i - V_0)$$

so

$$A_v = \frac{V_0}{V_i} = \frac{g_{m1}R_L\left[1 + g_{m2}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)\right]}{1 + g_{m1}R_L\left[1 + g_{m2}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)\right]}$$

c. Set $V_i = 0$

$$I_X + g_{m1}V_{gs} + g_{m2}V_\pi = \frac{V_X}{R_L}$$

$$V_{gs} = -V_X$$

From part (b), we have

$$V_\pi = g_{m1}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)V_{gs} = -g_{m1}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)V_X$$

Then

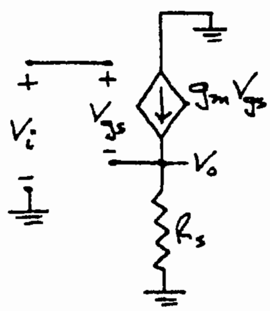
$$\frac{I_X}{V_X} = \frac{1}{R_0} = \frac{1}{R_L} + g_{m1}g_{m2}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)$$

or

$$R_0 = R_L \,\Big\|\, \frac{1}{g_{m1}} \,\Big\|\, \frac{1}{g_{m1}g_{m2}R_{eq}\left(\frac{R_{D1}}{R_{E2}}\right)}$$

12.25

a. $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.5)} = 1\,mA/V$



$$V_0 = (g_m V_{gs})R_S$$

$V_i = V_{gs} + V_0$ so $V_{gs} = V_i - V_0$

Then

$$V_0 = g_m R_S(V_i - V_0)$$
$$A_v = \frac{g_m R_S}{1 + g_m R_S} = \frac{1(2)}{1+(1)(2)} \Rightarrow \underline{A_v = 0.667}$$

To determine R_{0f}

$$I_X + g_m V_{gs} = \frac{V_X}{R_S} \text{ and } V_{gs} = -V_X$$
$$\frac{I_X}{V_X} = \frac{1}{R_{0f}} = g_m + \frac{1}{R_S}$$
$$\text{so } R_{0f} = \frac{1}{g_m} \Big\| R_S = \frac{1}{1} \Big\| 2 \Rightarrow \underline{R_{0f} = 0.667 \text{ k}\Omega}$$

b. For $K_n = 0.8\, mA/V^2$

$$g_m = 2\sqrt{(0.8)(0.5)} = 1.265 \text{ mA/V}$$
$$A_v = \frac{(1.265)(2)}{1+(1.265)(2)} = 0.7167$$
$$\frac{\Delta A_f}{A_f} = \frac{0.7167 - 0.667}{0.667} \Rightarrow \underline{7.45\% \text{ increase}}$$

$$R_{0f} = \frac{1}{1.265} \Big\| 2 = 0.7905\|2$$
$$R_{0f} = 0.5666$$
$$\frac{\Delta R_{0f}}{R_{0f}} = \frac{0.5666 - 0.667}{0.667} \Rightarrow \underline{15.05\% \text{ decrease}}$$

12.26

dc analysis:

$$R_{TH1} = 150\|47 = 35.8 \text{ k}\Omega,$$
$$V_{TH1} = \left(\frac{47}{47+150}\right)(25) = 5.96 \text{ V}$$

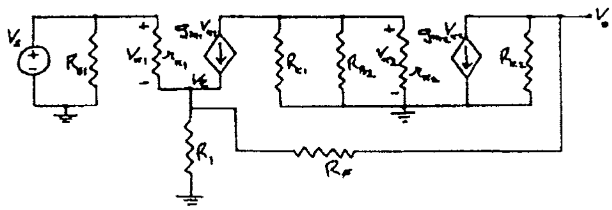
$$R_{TH2} = 33\|47 = 19.4 \text{ k}\Omega,$$
$$V_{TH2} = \left(\frac{33}{33+47}\right)(25) = 10.3 \text{ V}$$

$$I_{B1} = \frac{5.96 - 0.7}{35.8 + (51)(4.8)} = 0.0187 \text{ mA}$$
$$I_{C1} = (50)(0.0187) = 0.935 \text{ mA}$$

$$I_{B2} = \frac{10.3 - 0.7}{19.4 + (51)(4.7)} = 0.03705 \text{ mA}$$
$$I_{C2} = (50)(0.03705) = 1.85 \text{ mA}$$

$$r_{\pi 1} = \frac{(50)(0.026)}{0.935} = 1.39 \text{ k}\Omega;$$
$$r_{\pi 2} = \frac{(50)(0.026)}{1.85} = 0.703 \text{ k}\Omega$$

$$g_{m1} = \frac{0.935}{0.026} = 35.96 \text{ mA/V}$$
$$g_{m2} = \frac{1.85}{0.026} = 71.15 \text{ mA/V}$$



$$V_S = V_{\pi 1} + V_e \quad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \frac{V_e}{R_1} + \frac{V_e - V_0}{R_F} \quad (2)$$

$$g_{m1}V_{\pi 1} + \frac{V_{\pi 2}}{R_{C1}} + \frac{V_{\pi 2}}{R_{B2}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (3)$$

$$g_{m2}V_{\pi 2} + \frac{V_0}{R_{C2}} + \frac{V_0 - V_e}{R_F} = 0 \quad (4)$$

Substitute numerical values in (2), (3) and (4):

$$V_e = V_S - V_{\pi 1} \quad (1)$$

$$\frac{V_{\pi 1}}{1.39} + (35.96)V_{\pi 1}$$
$$= (V_S - V_{\pi 1})\left(\frac{1}{0.1} + \frac{1}{4.7}\right) - V_0\left(\frac{1}{4.7}\right)$$

or

$$V_{\pi 1}(46.89) = V_S(10.213) - V_0(0.2128) \quad (2)$$

$$(35.96)V_{\pi 1} + V_{\pi 2}\left(\frac{1}{10} + \frac{1}{19.4} + \frac{1}{0.703}\right) = 0$$

or

$$(35.96)V_{\pi 1} + V_{\pi 2}(1.574) = 0 \quad (3)$$

$$(71.15)V_{\pi 2} + V_0\left(\frac{1}{4.7} + \frac{1}{4.7}\right)$$
$$- (V_S - V_{\pi 1})\left(\frac{1}{4.7}\right) = 0$$

or

$$(71.15)V_{\pi 2} + V_0(0.4255) - V_S(0.2128)$$
$$+ V_{\pi 1}(0.2128) = 0 \quad (4)$$

From (3) : $V_{\pi 2} = -V_{\pi 1}(22.85)$

Then substitute in (4) :

$$-(71.15)V_{\pi 1}(22.85) + V_0(0.4255)$$
$$- V_S(0.2128) + V_{\pi 1}(0.2128) = 0$$

or

$$-V_{\pi 1}(1625.6) + V_0(0.4255) - V_S(0.2128) = 0$$

From (2) : $V_{\pi 1} = V_S(0.2178) - V_0(0.004538)$

Then

$$-(1625.6)[V_S(0.2178) - V_0(0.004538)]$$
$$+ V_0(0.4255) - V_S(0.2128) = 0$$

or $-V_S(354.3)+V_O(7.802)=0$

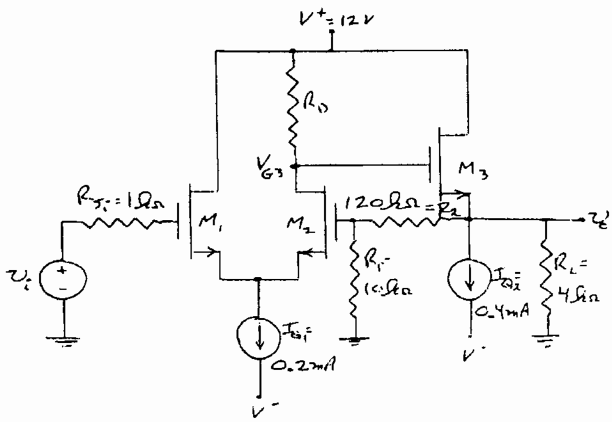
Finally

$$\Rightarrow \frac{V_O}{V_S}=45.4$$

12.27

For example, use the circuit shown in Figure 12.23

12.28



For M_3: $K_{n3}=\frac{k_n'}{2}\cdot\left(\frac{W}{L}\right)_3$ Let $\left(\frac{W}{L}\right)_3=25$

Then $K_{n3}=\left(\frac{0.080}{2}\right)(25)=1\,mA/V^2$

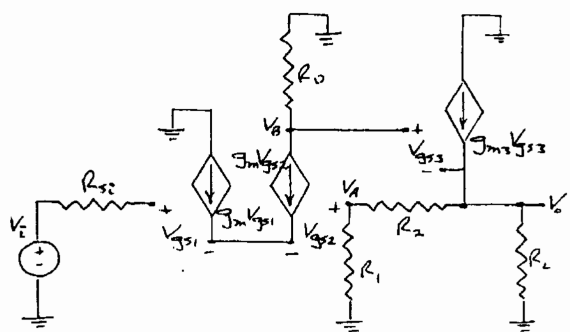
Want $v_o=0$ for $v_i=0$, so that

$I_{D3}=I_{Q2}=0.4=1\cdot(V_{GS3}-V_{TN})^2$

Then $V_{GS3}=\sqrt{\frac{0.4}{1}}+2=2.63\,V$

For $V_{GS3}=2.63\,V \Rightarrow V_{G3}=12-I_{D2}R_D$

Or $2.63=12-(0.1)R_D \Rightarrow$ $\underline{R_D=93.7\,k\Omega}$



$g_{m3}=2\sqrt{K_{n3}I_{D3}}=2\sqrt{(1)(0.4)}=1.26\,mA/V$

$$V_A=\left(\frac{R_1}{R_1+R_2}\right)(V_o)=\left(\frac{10}{120+10}\right)(V_o)=0.0769V_o$$

(Small amount of feedback)

(1) $V_i=V_{gs1}-V_{gs2}+V_A$

(2) $g_mV_{gs1}+g_mV_{gs2}=0 \Rightarrow V_{gs1}=-V_{gs2}$

Then

$V_i=-2V_{gs2}+V_A \Rightarrow V_{gs2}=\frac{1}{2}(V_A-V_i)$

$V_{gs2}=0.03846V_o-0.5V_i$

(3) $V_B=-g_mV_{gs2}R_D=-g_mR_D[0.03846V_o-0.5V_i]$

(4) $V_{gs3}=V_B-V_o$ and $V_o=g_{m3}V_{gs3}[R_L\|(R_1+R_2)]$

So

$V_o=g_{m3}[R_L\|(R_1+R_2)](V_B-V_o)$

Then

$V_o=g_{m3}[R_L\|(R_1+R_2)][-g_mR_D(0.03846V_o-0.5V_i)-V_o]$

Or

$V_o[1+g_{m3}[R_L\|(R_1+R_2)][g_mR_D(0.03846)+1]]$
$\quad=g_{m3}[R_L\|(R_1+R_2)][0.5g_mR_D]\cdot V_i$

Now

$R_L\|(R_1+R_2)=4\|130=3.88$

So

$V_o[1+(1.26)(3.88)[g_m(93.7)(0.03846)+1]]$
$\quad=(1.26)(3.88)(0.5)g_m(93.7)V_i$

Rearranging terms, we find

$$\frac{V_o}{V_i}=\frac{229g_m}{5.89+17.6g_m}=10 \Rightarrow g_m=1.11\,mA/V$$

We have

$$g_m=2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_D} \Rightarrow$$

$$1.11=2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.1)} \Rightarrow$$

$$\underline{\left(\frac{W}{L}\right)_1=\left(\frac{W}{L}\right)_2=77}$$

12.29

Assuming an ideal op-amp, then from Equation (12.58)

$$\frac{I_O}{I_S}=1+\frac{R_1}{R_2}=\frac{20}{0.2}=100$$

Then $\frac{R_1}{R_2}=99$

For example, set $R_2=5\ k\Omega$ and $R_1=495\ k\Omega$

12.30

(a) $I_{C1}=\left(\frac{h_{FE}}{1+h_{FE}}\right)I_{E1}=\left(\frac{100}{101}\right)(0.2)=0.198\,mA$

$V_{C1}=10-(0.198)(40)=2.08\,V$

$I_{E2}=\frac{2.08-0.7}{1}=1.38\,mA$

$I_{C2}=\left(\frac{100}{101}\right)(1.38)=1.37\,mA$

For Q_1:

$$r_{\pi 1} = \frac{(100)(0.026)}{0.198} = 13.1\ k\Omega$$

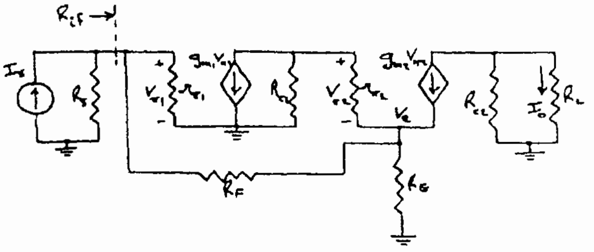
$$g_{m1} = \frac{0.198}{0.026} = 7.62\ mA/V$$

For Q_2:

$$r_{\pi 2} = \frac{(100)(0.026)}{1.37} = 1.90\ k\Omega$$

$$g_{m2} = \frac{1.37}{0.026} = 52.7\ mA/V$$

(b)



$$I_S = \frac{V_{\pi 1}}{R_S} + \frac{V_{\pi 1}}{r_{\pi 1}} + \frac{V_{\pi 1} - V_e}{R_F} \qquad (1)$$

$$g_{m1}V_{\pi 1} + \frac{V_{\pi 2} + V_e}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \qquad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} = \frac{V_e}{R_E} + \frac{V_e - V_{\pi 1}}{R_F} \qquad (3)$$

Substitute numerical values in (1), (2), and (3):

$$I_S = V_{\pi 1}\left(\frac{1}{10} + \frac{1}{13.1} + \frac{1}{10}\right) - V_e\left(\frac{1}{10}\right)$$

$$I_S = V_{\pi 1}(0.2763) - V_e(0.10) \qquad (1)$$

$$(7.62)V_{\pi 1} + V_{\pi 2}\left(\frac{1}{40} + \frac{1}{1.90}\right) + V_e\left(\frac{1}{40}\right) = 0$$

$$(7.62)V_{\pi 1} + V_{\pi 2}(0.5513) + V_e(0.025) = 0 \qquad (2)$$

$$V_{\pi 2}\left(\frac{1}{1.90} + 52.7\right) = V_e\left(\frac{1}{1} + \frac{1}{10}\right) - V_{\pi 1}\left(\frac{1}{10}\right)$$

$$V_{\pi 2}(53.23) = V_e(1.10) - V_{\pi 1}(0.10) \qquad (3)$$

From (3),

$$V_e = V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)$$

Substituting into (1),

$$I_S = V_{\pi 1}(0.2763) - (0.10)\left[V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)\right]$$

or

$$I_S = V_{\pi 1}(0.2672) - V_{\pi 2}(4.839) \qquad (1')$$

and substituting into (2),

$$(7.62)V_{\pi 1} + V_{\pi 2}(0.5513) + (0.025)\left[V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)\right] = 0$$

or

$$(7.622)V_{\pi 1} + V_{\pi 2}(1.761) = 0$$

$$\Rightarrow V_{\pi 1} = -V_{\pi 2}(0.2310) \qquad (2')$$

Then substituting (2') into (1'), we obtain

$$I_S = (0.2672)(-V_{\pi 2})(0.2310) - V_{\pi 2}(4.839)$$

or

$$I_S = -V_{\pi 2}(4.901)$$

Now

$$I_O = -g_{m2}V_{\pi 2}\left(\frac{R_{C2}}{R_{C2} + R_L}\right)$$

$$= -(52.7)\left(\frac{2}{2 + 0.5}\right)V_{\pi 2} = -(42.16)V_{\pi 2}$$

Then

$$I_O = -(42.16)\left(\frac{-I_S}{4.901}\right)$$

or

$$A_{if} = \frac{I_O}{I_S} = 8.60$$

(c) $R_i = \frac{V_{\pi 1}}{I_S}$ and $R_i = R_S \| R_{if}$

We had

$V_{\pi 1} = -V_{\pi 2}(0.2310)$ and $I_S = -V_{\pi 2}(4.901)$

so

$$I_S = -\left(\frac{-V_{\pi 1}}{0.2310}\right)(4.901) = V_{\pi 1}(21.22)$$

Then

$$R_i = \frac{V_{\pi 1}}{I_S} = \frac{1}{21.22} = 0.04713$$

Finally

$$0.04713 = \frac{10R_{if}}{10 + R_{if}} \Rightarrow$$

$$R_{if} = 47.4\ \Omega$$

12.31

(a) Using Figure 12.25

$$I_s = \frac{V_{\pi 1}}{R_S \| R_{B1} \| r_{\pi 1}} + \frac{V_{\pi 1} - V_{e2}}{R_F} \qquad (1)$$

$$g_{m1}V_{\pi 1} + \frac{V_{\pi 2}}{R_{C1} \| R_{B2}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \qquad (2)$$

$$= g_{m1}V_{\pi 1} + \frac{V_{\pi 2}}{R_{C1} \| R_{B2} \| r_{\pi 2}}$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_{e2}}{R_{E2}} + \frac{V_{e2} - V_{\pi 1}}{R_F} \quad (3)$$

$$I_0 = -\left(\frac{R_{C2}}{R_{C2} + R_L}\right)(g_{m2} V_{\pi 2}) \quad (4)$$

$$I_i = \frac{V_{\pi 1}}{R_S \| R_{B1} \| r_{\pi 1} \| R_F} - \frac{V_{e2}}{R_F} \quad (1')$$

so that

$$V_{e2} = \left(\frac{R_F}{R_S \| R_{B1} \| r_{\pi 1} \| R_F}\right) V_{\pi 1} - R_F I_i$$

Then

$$V_{\pi 2}\left(\frac{1+\beta_2}{r_{\pi 2}}\right) = \left(\frac{1}{R_{E2}} + \frac{1}{R_F}\right) \times \left\{\left(\frac{R_F}{R_S \| R_{B1} \| r_{\pi 1} \| R_F}\right) V_{\pi 1} - R_F I_i\right\} - \frac{V_{\pi 1}}{R_F} \quad (3')$$

From (2):

$$V_{\pi 1} = -\frac{V_{\pi 2}}{g_{m1}} \cdot \frac{1}{(R_{C1} \| R_{B2} \| r_{\pi 2})}$$

Then

$$V_{\pi 2}\left(\frac{1+\beta_2}{r_{\pi 2}}\right) = \frac{V_{\pi 2}}{g_{m1} R_F} \cdot \frac{1}{R_{C1} \| R_{B2} \| r_{\pi 2}} \times \left\{1 - \left(1 + \frac{R_F}{R_{E2}}\right)\left(\frac{R_F}{R_S \| R_{B1} \| r_{\pi 1} \| R_F}\right)\right\} - \left(1 + \frac{R_F}{R_{E2}}\right) I_i$$

Solve for $V_{\pi 2}$ and substitute into Equation 4.

(b) $R_{TH1} = 20\|80 = 16\ k\Omega = R_{B1}$

$$V_{TH1} = \left(\frac{20}{100}\right)(10) = 2\,V$$

$$I_{BQ1} = \frac{2-0.7}{16+(101)(1)} = 0.0111\,mA \Rightarrow$$

$$I_{CQ1} = 1.11\,mA$$

$$R_{TH2} = 15\|85 = 12.75\,k\Omega$$

$$V_{TH2} = \left(\frac{15}{15+85}\right)(10) = 1.5\,V$$

$$I_{BQ2} = \frac{1.5-0.7}{12.75+(101)(0.5)} = 0.0126\,mA \Rightarrow$$

$$I_{CQ2} = 1.26\,mA$$

$$g_{m1} = \frac{1.11}{0.026} = 42.69\,mA/V$$

$$g_{m2} = \frac{1.26}{0.026} = 48.46\,mA/V$$

$$r_{\pi 1} = \frac{(100)(0.026)}{1.11} = 2.34\,k\Omega$$

$$r_{\pi 2} = \frac{(100)(0.026)}{1.26} = 2.06\,k\Omega$$

From part (a)

$$V_{\pi 2}\left(\frac{101}{2.06}\right) = \frac{V_{\pi 2}}{(42.69)(10)} \cdot \frac{1}{2\|12.75\|2.06}$$
$$x\left\{1 - \left(1+\frac{10}{0.5}\right)\left(\frac{10}{10000\|16\|2.34\|10}\right)\right\} - \left(1+\frac{10}{0.5}\right) I_i$$

So

$$V_{\pi 2}(49.34) = -(21) I_i$$

or

$$V_{\pi 2} = -0.4256 I_i$$

Now

$$I_o = -\left(\frac{4}{4+4}\right)(48.46)(-0.4256) I_i$$

or

$$A_i = \frac{I_o}{I_i} = 10.3$$

From Example 12.9, computer analysis showed $A_i = 9.58$. The difference in results is usually in the calculation of quiescent currents which leads to slight differences in the small-signal parameter values.

12.32

a. $R_{TH} = 13.5\|38.3 = 9.98\ \text{k}\Omega$

$$V_{TH} = \left(\frac{13.5}{13.5+38.3}\right)(10) = 2.606\ \text{V}$$

$$I_{C1} = \frac{(120)(2.606-0.7)}{9.98+(121)(1)} = 1.75\ \text{mA}$$

$$V_{C1} = 10 - (1.75)(3) = 4.75\ \text{V}$$

$$I_{C2} \approx \frac{4.75-0.7}{8.1} = 0.50\ \text{mA}$$

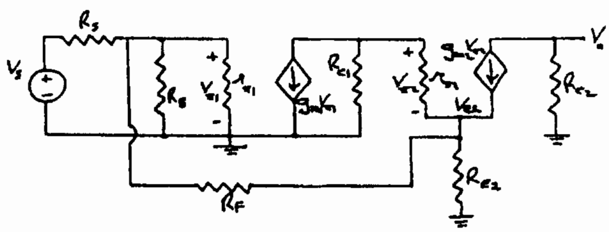
$$r_{\pi 1} = \frac{(120)(0.026)}{1.75} = 1.78\ \text{k}\Omega$$

$$g_{m1} = \frac{1.75}{0.026} = 67.31\ \text{mA/V}$$

$$r_{\pi 2} = \frac{(120)(0.026)}{0.50} = 6.24\ \text{k}\Omega$$

$$g_{m2} = \frac{0.50}{0.026} = 19.23\ \text{mA/V}$$

b.



$$\frac{V_S - V_{\pi 1}}{R_S} = \frac{V_{\pi 1}}{R_B \| r_{\pi 1}} + \frac{V_{\pi 1} - V_{e2}}{R_F} \qquad (1)$$

$$g_{m1} V_{\pi 1} + \frac{V_{\pi 2} + V_{e2}}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \qquad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_{e2}}{R_{E2}} + \frac{V_{e2} - V_{\pi 1}}{R_F} \qquad (3)$$

and

$$V_0 = -(g_{m2} V_{\pi 2}) R_{C2} \qquad (4)$$

Substitute numerical values in (1), (2), and (3):

$$\frac{V_S}{0.6} = V_{\pi 1}\left[\frac{1}{0.6} + \frac{V_{\pi 1}}{9.98 \| 1.78} + \frac{1}{1.2}\right] - \frac{V_{e2}}{1.2}$$

or

$$V_S(1.667) = V_{\pi 1}(4.011) - V_{e2}(0.8333) \qquad (1)$$

$$(67.31)V_{\pi 1} + V_{\pi 2}\left(\frac{1}{3} + \frac{1}{6.24}\right) + \frac{V_{e2}}{3} = 0$$

or

$$V_{\pi 1}(67.31) + V_{\pi 2}(0.4936) + V_{e2}(0.3333) = 0 \qquad (2)$$

$$V_{\pi 2}\left(\frac{1}{6.24} + 19.23\right) = \frac{V_{e2}}{8.1} + \frac{V_{e2}}{1.2} - \frac{V_{\pi 1}}{1.2}$$

or

$$V_{\pi 2}(19.39) = V_{e2}(0.9568) - V_{\pi 1}(0.8333) \qquad (3)$$

From (1)

$$V_{e2} = V_{\pi 1}(4.813) - V_S(2.00)$$

Then

$$V_{\pi 1}(67.31) + V_{\pi 2}(0.4936) + (0.3333)[V_{\pi 1}(4.813) - V_S(2.00)] = 0$$

or

$$V_{\pi 1}(68.91) + V_{\pi 2}(0.4936) - V_S(0.6666) = 0 \qquad (2')$$

and

$$V_{\pi 2}(19.39) = (0.9568)[V_{\pi 1}(4.813) - V_S(2.00)] - V_{\pi 1}(0.8333)$$

or

$$V_{\pi 2}(19.39) = V_{\pi 1}(3.772) - V_S(1.914) \qquad (3')$$

We find

$$V_{\pi 1} = V_S(0.009673) - V_{\pi 2}(0.007163)$$

Then

$$V_{\pi 2}(19.39) = (3.772)[V_S(0.009673) - V_{\pi 2}(0.007163)] - V_S(1.914)$$

$V_{\pi 2}(19.42) = V_S(-1.878)$ or $V_{\pi 2} = -V_S(0.09670)$

so that

$$V_0 = -(19.23)(4)(-V_S)(0.09670)$$

Then

$$\underline{\frac{V_0}{V_S} = 1.86}$$

12.33

Using the circuit from Problem 12.32, we have $R_{if} = \dfrac{V_{\pi 1}}{I_s}$

where $I_S = \dfrac{V_S - V_{\pi 1}}{R_S}$.

From Problem 12.32

$$V_{\pi 1} = V_S(0.009673) - V_{\pi 2}(0.007163)$$
$$= V_S(0.009673) - (0.007163)(-V_S)(0.09670)$$
$$= V_S(0.01037)$$

So

$$R_{if} = \frac{V_S(0.01037)\cdot(0.6)}{V_S - V_S(0.01037)} = 0.00629 \text{ k}\Omega$$

or

$$\underline{R_{if} = 6.29\ \Omega}$$

12.34

$$R_{TH} = 1.4 \| 17.9 = 1.298 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{1.4}{1.4 + 17.9}\right)(10) = 0.7254 \text{ V}$$

$$I_{B1} = \frac{0.7254 - 0.7}{1.298} = 0.0196 \text{ mA}$$

$$I_{C1} = (50)(0.0196) = 0.98 \text{ mA}$$

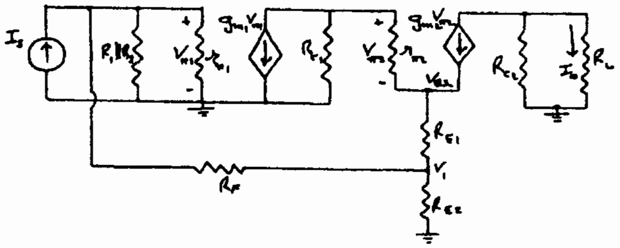
Neglecting dc base currents,

$$V_{B2} = 10 - (0.98)(7) = 3.14 \text{ V}$$

$$I_{E2} = \frac{3.14 - 0.7}{0.25 + 0.5} = 3.25 \text{ mA}$$

$$I_{C2} = \left(\frac{50}{51}\right)(3.25) = 3.19 \text{ mA}$$

$$r_{\pi 1} = \frac{(50)(0.026)}{0.98} = 1.33 \text{ k}\Omega$$
$$g_{m1} = \frac{0.98}{0.026} = 37.7 \text{ mA/V}$$
$$r_{\pi 2} = \frac{(50)(0.026)}{3.19} = 0.408 \text{ k}\Omega$$
$$g_{m2} = \frac{3.19}{0.026} = 123 \text{ mA/V}$$



$$I_S = \frac{V_{\pi 1}}{R_1 \| R_2 \| r_{\pi 1}} + \frac{V_{\pi 1} - V_1}{R_F} \quad (1)$$

$$g_{m1} V_{\pi 1} + \frac{V_{\pi 2}}{r_{\pi 2}} + \frac{V_{\pi 2} + V_{e2}}{R_{C1}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_{e2} - V_1}{R_{E1}} \quad (3)$$

$$\frac{V_{e2} - V_{\pi 1}}{R_{E1}} = \frac{V_1}{R_{E2}} + \frac{V_1 - V_{\pi 1}}{R_F} \quad (4)$$

Enter numerical values in (1), (2), (3) and (4):

$$I_S = \frac{V_{\pi 1}}{17.9 \| 1.4 \| 1.33} + \frac{V_{\pi 1} - V_1}{5}$$

or

$$I_S = V_{\pi 1}(1.722) - V_1(0.20) \quad (1)$$

$$(37.7)V_{\pi 1} + \frac{V_{\pi 2}}{0.408} + \frac{V_{\pi 2} + V_{e2}}{7} = 0$$

or

$$V_{\pi 1}(37.7) + V_{\pi 2}(2.594) + V_{e2}(0.1429) = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{0.408} + (123)V_{\pi 2} = \frac{V_{e2} - V_1}{0.25}$$

or

$$V_{\pi 2}(125.5) = V_{e2}(4) - V_1(4) \quad (3)$$

$$\frac{V_{e2} - V_1}{0.25} = \frac{V_1}{0.50} + \frac{V_1 - V_{\pi 1}}{5}$$

or

$$V_{e2}(4) = V_1(6.20) - V_{\pi 1}(0.20) \quad (4)$$

From (4):

$$V_{e2} = V_1(1.55) - V_{\pi 1}(0.05)$$

Then substituting in (3):

$$V_{\pi 2}(125.5) = (4)[V_1(1.55) - V_{\pi 1}(0.05)] - V_1(4)$$

or

$$V_{\pi 2}(125.5) = V_1(2.20) - V_{\pi 1}(0.20) \quad (3')$$

and substituting in (2):

$$V_{\pi 1}(37.7) + V_{\pi 2}(2.594) + (0.1429)[V_1(1.55) - V_{\pi 1}(0.05)] = 0$$

or

$$V_{\pi 1}(37.69) + V_{\pi 2}(2.594) + V_1(0.2215) = 0$$

Now

$$V_1 = -V_{\pi 1}(170.16) - V_{\pi 2}(11.71)$$

Then substituting in (1):

$$I_S = V_{\pi 1}(1.722) - (0.20)[-V_{\pi 1}(170.16) - V_{\pi 2}(11.71)]$$

or

$$I_S = V_{\pi 1}(35.75) + V_{\pi 2}(2.342)$$

and substituting in (3'):

$$V_{\pi 2}(125.5) = (2.20)[-V_{\pi 1}(170.16) - V_{\pi 2}(11.71)] - V_{\pi 1}(0.20)$$

or $V_{\pi 2}(151.3) = -V_{\pi 1}(374.55)$ so that

$$V_{\pi 1} = -V_{\pi 2}(0.4040)$$

Then

$$I_S = (35.75)[-V_{\pi 2}(0.4040)] + V_{\pi 2}(2.342)$$
$$I_S = -V_{\pi 2}(12.10)$$

$$I_0 = -(g_{m2} V_{\pi 2})\left(\frac{R_{C2}}{R_{C2} + R_L}\right)$$
$$= -(123)\left(\frac{2.2}{2.2 + 2}\right)V_{\pi 2} = -(64.43)V_{\pi 2}$$

or $V_{\pi 2} = -(0.01552)I_0$

Then

$$\frac{I_0}{I_S} = \frac{1}{(0.01552)(12.10)} \Rightarrow \underline{\frac{I_0}{I_S} = 5.33}$$

12.35

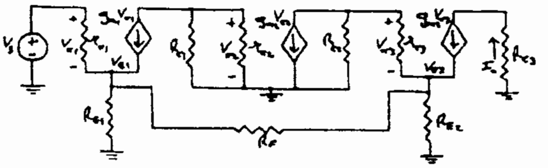
For example, use the circuit shown in Figure P12.30

12.36

$r_{\pi 1} = 6.24\ \text{k}\Omega,\ r_{\pi 2} = 3.12\ \text{k}\Omega,\ r_{\pi 3} = 1.56\ \text{k}\Omega$

$g_{m1} = 19.23\ \text{mA/V},\ g_{m2} = 38.46\ \text{mA/V},$

$g_{m3} = 76.92\ \text{mA/V}$



$$V_S = V_{\pi 1} + V_{e1} \quad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \frac{V_{e1}}{R_{E1}} + \frac{V_{e1} - V_{e3}}{R_F} \quad (2)$$

$$V_{\pi 2} = -g_{m1}V_{\pi 1}(R_{C1}\|r_{\pi 2}) \quad (3)$$

$$g_{m2}V_{\pi 2} + \frac{V_{\pi 3} + V_{e3}}{R_{C2}} + \frac{V_{\pi 3}}{r_{\pi 3}} = 0 \quad (4)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_{e3}}{R_{E2}} + \frac{V_{e3} - V_{e1}}{R_F} \quad (5)$$

Enter numerical values in (2)-(5):

$$\frac{V_{\pi 1}}{6.24} + (19.23)V_{\pi 1} = V_{e1}\left(\frac{1}{0.1} + \frac{1}{0.8}\right) - V_{e3}\left(\frac{1}{0.8}\right)$$

or

$$V_{\pi 1}(19.39) = V_{e1}(11.25) - V_{e3}(1.25) \quad (2)$$

$$V_{\pi 2} = -(19.23)V_{\pi 1}(5\|3.12) = -(36.94)V_{\pi 1} \quad (3)$$

$$(38.46)V_{\pi 2} + V_{\pi 3}\left(\frac{1}{2} + \frac{1}{1.56}\right) + V_{e3}\left(\frac{1}{2}\right) = 0$$

or

$$V_{\pi 2}(38.46) + V_{\pi 3}(1.141) + V_{e3}(0.5) = 0 \quad (4)$$

$$V_{\pi 3}\left(\frac{1}{1.56} + 76.92\right) = V_{e3}\left(\frac{1}{0.1} + \frac{1}{0.8}\right) - V_{e3}\left(\frac{1}{0.8}\right)$$

or

$$V_{\pi 3}(77.56) = V_{e3}(11.25) - V_{e1}(1.25) \quad (5)$$

From (1) $V_{\pi 1} = V_S - V_{e1}$

Then

$$(V_S - V_{e1})(19.39) = V_{e1}(11.25) - V_{e3}(1.25)$$

or

$$V_S(19.39) = V_{e1}(30.64) - V_{e3}(1.25) \quad (2')$$

$$V_{\pi 2} = -V_S(36.94) + V_{e1}(36.94) \quad (3')$$

$$(38.46)[-V_S(36.94) + V_{e1}(36.94)] + V_{\pi 3}(1.141) + V_{e3}(0.5) = 0 \quad (4')$$

From (5): $V_{e3} = V_{\pi 3}(6.894) + V_{e1}(0.1111)$

Then

$$V_S(19.39) = V_{e1}(30.64) - (1.25)[V_{\pi 3}(6.894) + V_{e1}(0.1111)]$$

or

$$V_S(19.39) = V_{e1}(30.50) - V_{\pi 3}(8.6175) \quad (2'')$$

and

$$-V_S(1420.7) + V_{e1}(1420.7) + V_{\pi 3}(1.141) + (0.5)[V_{\pi 3}(6.894) + V_{e1}(0.1111)] = 0$$

or

$$-V_S(1420.7) + V_{e1}(1420.76) + V_{\pi 3}(4.588) = 0 \quad (4'')$$

From $(2'')$:

$$V_{e1} = V_S(0.6357) + V_{\pi 3}(0.2825)$$

Then substituting in $(4'')$:

$$-V_S(1420.7) + (1420.76)[V_S(0.6357) + V_{\pi 3}(0.2825)] + V_{\pi 3}(4.588) = 0$$

$$-V_S(517.5) + V_{\pi 3}(405.95) = 0$$

Now

$I_0 = g_{m3}V_{\pi 3} = 76.92V_{\pi 3}$ or $V_{\pi 3} = I_0(0.0130)$

Then $-V_S(517.5) + I_0(0.0130)(405.95) = 0$

or

$$\frac{I_0}{V_S} = 98.06\ \text{mA/V}$$

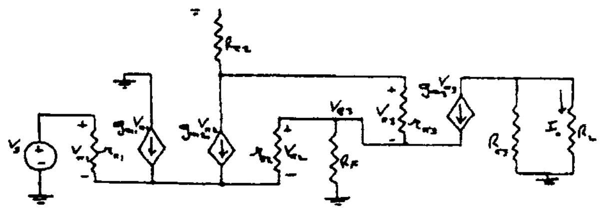
12.38

$$r_{\pi1} = r_{\pi2} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$g_{m1} = g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi3} = \frac{(100)(0.026)}{2} = 1.3 \text{ k}\Omega$$

$$g_{m3} = \frac{2}{0.026} = 76.92 \text{ mA/V}$$



$$\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} + g_{m2}V_{\pi2} + \frac{V_{\pi2}}{r_{\pi2}} = 0 \qquad (1)$$

Since $r_{\pi1} = r_{\pi2}$ and $g_{m1} = g_{m2}$, then $V_{\pi1} = -V_{\pi2}$

$$V_S = V_{\pi1} - V_{\pi2} + V_{e3} = -2V_{\pi2} + V_{e3} \qquad (2)$$

$$g_{m2}V_{\pi2} + \frac{V_{\pi3}}{r_{\pi3}} + \frac{V_{\pi3} + V_{e3}}{R_{C2}} = 0 \qquad (3)$$

$$\frac{V_{\pi3}}{r_{\pi3}} + g_{m3}V_{\pi3} = \frac{V_{e3}}{R_F} + \frac{V_{\pi2}}{r_{\pi2}} \qquad (4)$$

$$I_0 = -\left(\frac{R_{C3}}{R_{C3} + R_L}\right)(g_{m3}V_{\pi3}) \qquad (5)$$

From (2): $V_{e3} = V_S + 2V_{\pi2}$

$$(19.23)V_{\pi2} + \frac{V_{\pi3}}{1.3} + \frac{V_{\pi3}}{18.6} + \frac{1}{18.6}(V_S + 2V_{\pi2}) = 0$$

or

$$(19.23)V_{\pi2} + (0.8230)V_{\pi3} + (0.05376)V_S = 0 \qquad (3')$$

$$V_{\pi3}\left(\frac{1}{1.3} + 76.92\right) = \left(\frac{1}{10}\right)(V_S + 2V_{\pi2}) + \frac{V_{\pi2}}{5.2}$$

or

$$(77.69)V_{\pi3} = (0.3923)V_{\pi2} + (0.1)V_S \qquad (4')$$

$$I_0 = -\left(\frac{2}{2+1}\right)(76.92)V_{\pi3} = -(51.28)V_{\pi3} \qquad (5')$$

From (3'):

$$V_{\pi2} = -(0.04255)V_{\pi3} - (0.002780)V_S$$

Then

$$(77.69)V_{\pi3} = (0.3923)[-(0.04255)V_{\pi3} - (0.002780)V_S] + (0.1)V_S$$

$$(77.71)V_{\pi3} = (0.0989)V_S$$

or

$$V_{\pi3} = (0.001273)V_S$$

so that

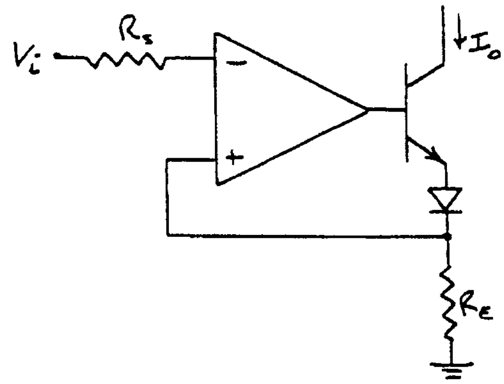
$$I_0 = -(51.28)(0.001273)V_S$$

or

$$\frac{I_0}{V_S} = -(0.0653) \text{ mA/V}$$

12.39

Use the basic circuit shown in Figure 12.27.



For the ideal case

$$\frac{I_0}{V_i} = \frac{1}{R_E}$$

we want

$$\frac{I_0}{V_i} = 10^{-3} \text{ A/V} = 1 \text{ mA/V}$$

Set $R_E = 1$ kΩ

Since the op-amp has a finite gain, finite input resistance, and finite output resistance, the closed-loop gain is slightly less than the ideal. R_E will need to be slightly decreased to increase the gain.

12.40

dc analysis

$$I_E R_E + V_{EB}(\text{on}) + I_B R_B + V_{CC} = ?$$

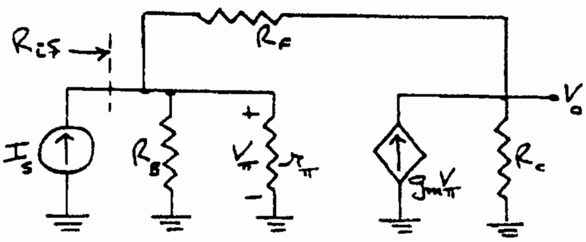
$$I_B = \frac{5 - 0.7}{100 + (51)(0.5)} = 0.0343$$

$$I_C = (50)(0.0343) = 1.71 \text{ mA}$$

Then $r_\pi = \dfrac{(50)(0.026)}{1.71} = 0.760$ kΩ

$$g_m = \frac{1.71}{0.026} = 65.77 \text{ mA/V}$$

a.



To determine R_{if}:

$$I_S + \frac{V_\pi}{R_B \| r_\pi} + \frac{V_0 - (-V_\pi)}{R_F} = 0 \qquad (1)$$

$$g_m V_\pi = \frac{V_0}{R_C} + \frac{V_0 - (-V_\pi)}{R_F} \qquad (2)$$

Now from (2):

$$(65.77)V_\pi - \frac{V_\pi}{10} = V_0\left(\frac{1}{1} + \frac{1}{10}\right)$$

$$(65.67)V_\pi = V_0(1.10)$$

or

$$V_0 = (59.7)V_\pi$$

and from (1):

$$I_S + \frac{V_\pi}{100\|0.760} + \frac{V_\pi}{10} + \frac{V_0}{10} = 0$$

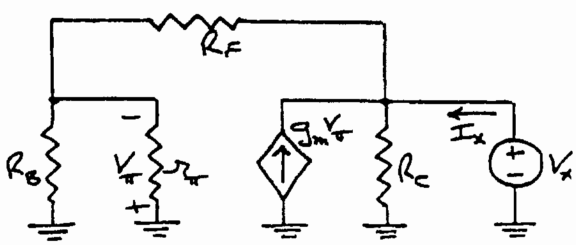
$$I_S + V_\pi(0.8543) + (0.1)(59.7)V_\pi = 0$$

$$I_S = -V_\pi(6.824)$$

Now

$$R_{if} = \frac{(-V_\pi)}{I_S} \Rightarrow \underline{R_{if} = 147\ \Omega}$$

To determine R_{of}:



$$I_X = \frac{V_X}{R_C} + \frac{V_X}{R_F + R_B\|r_\pi} - g_m V_\pi \qquad (3)$$

$$V_\pi = \left(\frac{-(R_B\|r_\pi)}{(R_B\|r_\pi) + R_F}\right)(V_X) \qquad (4)$$

Now

$$V_\pi = \left(\frac{-(100\|0.760)}{(100\|0.760) + 10}\right)(V_X) = -(0.07014)V_X$$

so

$$I_X = V_X\left(\frac{1}{1} + \frac{1}{10.754} + (65.77)(0.07014)\right)$$

$$R_{of} = \frac{V_X}{I_X} \Rightarrow \underline{R_{of} = 175\ \Omega}$$

b. From part (a), we find

$$V_\pi = -\frac{I_S}{6.824}$$

then

$$V_0 = (59.7)\left(\frac{-I_S}{6.824}\right)$$

or

$$\underline{\frac{V_0}{V_S} = -8.75\ \text{k}\Omega}$$

c. If capacitance is finite, a phase shift will be introduced.

12.41

dc analysis: $V_{GS} = V_{DS}$

$$I_D = \frac{V_{DD} - V_{GS}}{R_D} = K_n(V_{GS} - V_{TN})^2$$

$$10 - V_{GS} = (0.20)(8)(V_{GS} - 2)^2$$

$$10 - V_{GS} = 1.6(V_{GS}^2 - 4V_{GS} + 4)$$

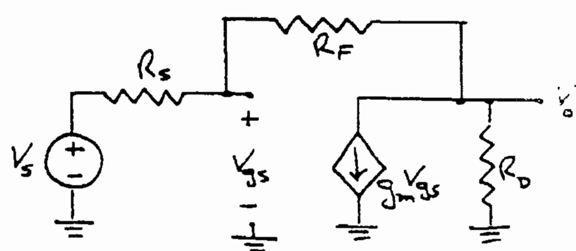
$$1.6V_{GS}^2 - 5.4V_{GS} - 3.6 = 0$$

$$V_{GS} = \frac{5.4 \pm \sqrt{(5.4)^2 + 4(1.6)(3.6)}}{2(1.6)} = 3.95\ \text{V}$$

$$I_D = \frac{10 - 3.95}{8} = 0.756\ \text{mA}$$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.2)(0.756)}$$

$$\Rightarrow g_m = 0.778\ \text{mA/V}$$



a.

$$\frac{V_{gs} - V_S}{R_S} + \frac{V_{gs} - V_0}{R_F} = 0$$

$$V_{gs}\left(\frac{1}{R_S} + \frac{1}{R_F}\right) = \frac{V_S}{R_S} + \frac{V_0}{R_F} \qquad (1)$$

$$\frac{V_0}{R_D} + g_m V_{gs} + \frac{V_0 - V_{gs}}{R_F} = 0$$

$$V_0\left(\frac{1}{R_D} + \frac{1}{R_F}\right) = V_{gs}\left(\frac{1}{R_F} - g_m\right) \qquad (2)$$

So from (1):

$$V_{gs}\left(\frac{1}{10} + \frac{1}{100}\right) = \frac{V_S}{10} + \frac{V_0}{100}$$

or

$$V_{gs}(0.11) = V_S(0.10) + V_0(0.010)$$

$$V_{gs} = V_S(0.909) + V_0(0.0909)$$

Then from (2):

$$V_0\left(\frac{1}{8} + \frac{1}{100}\right) = V_{gs}\left(\frac{1}{100} - 0.778\right)$$

$$V_0(0.135) = V_{gs}(-0.768)$$

$$= (-0.768)[V_S(0.909) + V_0(0.0909)]$$

$$V_0(0.2048) = -V_S(0.6981)$$

so

$$\underline{A_v = \frac{V_0}{V_S} = -3.41}$$

b. We have

$$V_{gs} = V_S(0.909) + V_0(0.0909)$$

$$= V_S(0.909) + (0.0909)(-3.41V_S)$$

$$= 0.599V_S$$

Now

$$A_{zf} = \frac{V_0}{I_S} = \frac{V_0}{\frac{V_S - V_{gs}}{R_S}} = \frac{(-3.41V_S)R_S}{V_S - 0.599V_S}$$

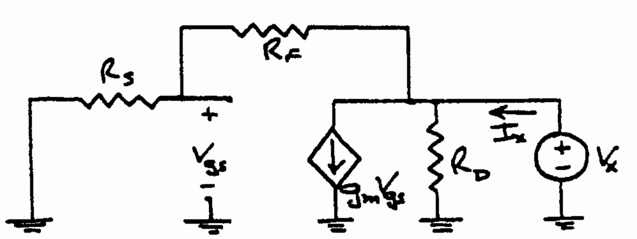
or

$$A_{zf} = \frac{(-3.41)(10)}{0.401} \Rightarrow \underline{A_{zf} = -85.0 \text{ V/ma}}$$

c. $$R_{if} = \frac{V_{gs}}{I_S} = \frac{V_{gs}}{\frac{0.401V_S}{R_S}} = \frac{0.599V_S}{0.401V_S}\cdot(10)$$

$$\Rightarrow \underline{R_{if} = 14.9 \text{ k}\Omega}$$

d.



$$I_X = \frac{V_X}{R_D} + g_m V_{gs} + \frac{V_X}{R_S + R_F}$$

$$V_{gs} = \left(\frac{R_S}{R_S + R_F}\right)V_X = \left(\frac{10}{10 + 100}\right)V_X$$

$$= (0.0909)V_X$$

$$I_X = V_X\left[\frac{1}{8} + (0.778)(0.0909) + \frac{1}{10 + 100}\right]$$

$$\frac{I_X}{V_X} = \frac{1}{R_{0f}} = 0.2048 \Rightarrow \underline{R_{0f} = 4.88 \text{ k}\Omega}$$

12.42

As $g_m \to \infty$, $\frac{V_0}{V_S} = \frac{-R_F}{R_S} = \frac{-100}{10} = -10$

To be within 10% of ideal,

$$\frac{V_0}{V_S} = -10(0.9) = -9$$

From Problem 12.41, we had

$$V_{gs} = V_S(0.909) + V_0(0.0909)$$

$$= V_S(0.909) + (-9V_S)(0.0909)$$

$$= 0.0909V_S$$

Also from Problem 12.41, we had

$$V_0(0.135) = V_{gs}(0.010 - g_m)$$

or

$$(-9V_S)(0.135) = (0.0909)V_S(0.010 - g_m)$$

$$-1.215 = 0.000909 - 0.0909g_m$$

or

$$\underline{g_m = 13.36 \text{ mA/V}}$$

12.43

dc analysis

$$R_{TH} = 24\|150 = 20.7 \text{ k}\Omega$$

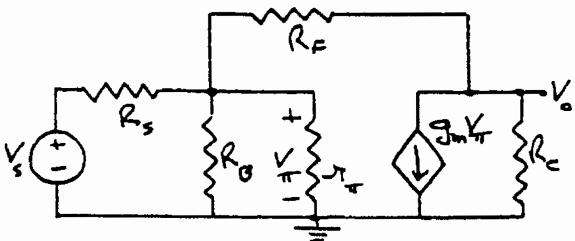
$$V_{TH} = \left(\frac{24}{24 + 150}\right)(12) = 1.655 \text{ V}$$

$$I_{BQ} = \frac{1.655 - 0.7}{20.7 + (151)(1)} = 0.00556 \text{ mA}$$

so $I_{CQ} = 0.834$ mA

$$r_\pi = \frac{(150)(0.026)}{0.834} = 4.68 \text{ k}\Omega$$

$$g_m = \frac{0.834}{0.026} = 32.08 \text{ mA/V}$$



$$\frac{V_S - V_\pi}{R_S} = \frac{V_\pi}{R_B\|r_\pi} + \frac{V_\pi - V_0}{R_F} \quad (1)$$

$$g_m V_\pi + \frac{V_0}{R_C} + \frac{V_0 - V_\pi}{R_F} = 0 \quad (2)$$

From (1):

$$\frac{V_S}{5} = V_\pi\left[\frac{1}{5} + \frac{1}{20.7\|4.68} + \frac{1}{R_F}\right] - \frac{V_0}{R_F}$$

or

$$V_S(0.20) = V_\pi\left(0.4620 + \frac{1}{R_F}\right) - \frac{V_0}{R_F}$$

From (2):

$$\left(32.08 - \frac{1}{R_F}\right)V_\pi + V_0\left(\frac{1}{6} + \frac{1}{R_F}\right) = 0$$

so

$$V_\pi = \frac{-V_0\left(0.1667 + \frac{1}{R_F}\right)}{\left(32.08 - \frac{1}{R_F}\right)} \quad (2)$$

Then

$V_S(0.20)$

$$= \left(0.4620 + \frac{1}{R_F}\right)\left[\frac{-V_0\left(0.1667 + \frac{1}{R_F}\right)}{\left(32.08 - \frac{1}{R_F}\right)}\right] - \frac{V_0}{R_F}$$

Neglect the R_F in the denominator term. Now

$$\frac{V_0}{V_S} = -5 \Rightarrow V_S = -\frac{V_0}{5} = -V_0(0.20)$$

$$-V_0(0.20)(0.20)R_F$$

$$= (0.4620R_F + 1)\left[\frac{-V_0(0.1667R_F + 1)}{32.08R_F}\right] - V_0$$

$$-1.283R_F^2 = -(0.4620R_F + 1)(0.1667R_F + 1) - 32.08R_F$$

$$1.206R_F^2 - 32.71R_F - 1 = 0$$

$$R_F = \frac{32.71 \pm \sqrt{(32.71)^2 + 4(1.206)(1)}}{2(1.206)}$$

so that

$\underline{R_F = 27.2\ \text{k}\Omega}$

12.44

dc analysis

$R_{TH} = 4\|15 = 3.16\ \text{k}\Omega = R_B$

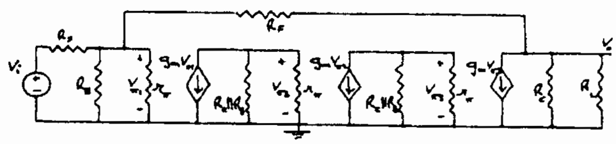
$$V_{TH} = \left(\frac{4}{4+15}\right)12 = 2.526\ \text{V}$$

$$I_{BQ} = \frac{2.526 - 0.7}{3.16 + (181)(4)} = 0.00251$$

$I_{CQ} = 0.452\ \text{mA}$

$$r_\pi = \frac{(180)(0.026)}{0.452} = 10.4\ \text{k}\Omega$$

$$g_m = \frac{0.452}{0.026} = 17.4\ \text{mA/V}$$



$$\frac{V_i - V_{\pi1}}{R_S} = \frac{V_{\pi1}}{R_B\|r_\pi} + \frac{V_{\pi1} - V_0}{R_F} \quad (1)$$

$$g_m V_{\pi1} + \frac{V_{\pi2}}{R_C\|R_B\|r_\pi} = 0 \quad (2)$$

$$g_m V_{\pi2} + \frac{V_{\pi3}}{R_C\|R_B\|r_\pi} = 0 \quad (3)$$

$$g_m V_{\pi3} + \frac{V_0}{R_C} + \frac{V_0}{R_L} + \frac{V_0 - V_{\pi1}}{R_F} = 0 \quad (4)$$

Now

$R_C\|R_B\|r_\pi = 8\|3.16\|10.4 = 1.86\ \text{k}\Omega$

$R_B\|r_\pi = 3.16\|10.4 = 2.42\ \text{k}\Omega$

Now substituting in (2):

$$(17.4)V_{\pi1} + \frac{V_{\pi2}}{1.86} = 0 \text{ or } V_{\pi2} = -(32.36)V_{\pi1}$$

and substituting in (3):

$$(17.4)V_{\pi2} + \frac{V_{\pi3}}{1.86} = 0$$

$$(17.4)[-(32.36)V_{\pi1}] + \frac{V_{\pi3}}{1.86} = 0$$

or $V_{\pi3} = (1047.3)V_{\pi1}$

Substitute numerical values in (1):

$$\frac{V_i}{10} = V_{\pi 1}\left(\frac{1}{10} + \frac{1}{2.42} + \frac{1}{R_F}\right) - \frac{V_0}{R_F}$$

or

$$V_i(0.10) = V_{\pi 1}\left(0.513 + \frac{1}{R_F}\right) - \frac{V_0}{R_F}$$

Substitute numerical values in (4):

$$(17.4)(1047.3)V_{\pi 1} + V_0\left(\frac{1}{8} + \frac{1}{4} + \frac{1}{R_F}\right) - \frac{V_{\pi 1}}{R_F} = 0$$

$$V_{\pi 1}\left(1.822 \times 10^4 - \frac{1}{R_F}\right) + V_0\left(0.375 + \frac{1}{R_F}\right) = 0$$

$$V_{\pi 1} = \frac{-V_0\left(0.375 + \frac{1}{R_F}\right)}{1.822 \times 10^4 - \frac{1}{R_F}}$$

so that

$V_i(0.10)$

$$= \left(0.513 + \frac{1}{R_F}\right)\left[\frac{-V_0\left(0.375 + \frac{1}{R_F}\right)}{1.822 \times 10^4 - \frac{1}{R_F}}\right] - \frac{V_0}{R_F}$$

We have $\frac{V_0}{V_i} = -80$ or $V_i = -\frac{V_0}{80}$

$$-\frac{(0.10)}{80}$$

$$= \left(0.513 + \frac{1}{R_F}\right)\left[\frac{-\left(0.375 + \frac{1}{R_F}\right)}{1.822 \times 10^4 - \frac{1}{R_F}}\right] - \frac{1}{R_F}$$

Neglect the $1/R_F$ term in the denominator.

$$-(0.00125R_F) = -\frac{(0.513R_F + 1)(0.375R_F + 1)}{1.822 \times 10^4 R_F} - 1$$

$22.775R_F^2$

$$= (0.513R_F + 1)(0.375R_F + 1) + 1.822 \times 10^4 R_F$$

We find

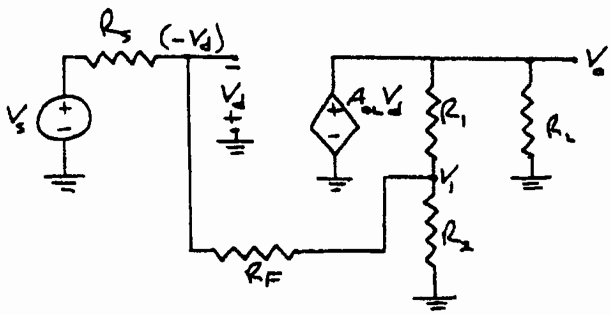
$$22.58R_F^2 - 1.822 \times 10^4 R_F - 1 = 0$$

$$R_F = \frac{1.822 \times 10^4 \pm \sqrt{(1.822 \times 10^4)^2 + 4(22.58)(1)}}{2(22.58)}$$

or

$$\underline{R_F = 0.807\ \text{M}\Omega}$$

12.45



a.

$$\frac{V_S - (-V_d)}{R_S} = \frac{-V_d - V_1}{R_F}$$

or

$$V_d\left(\frac{1}{R_S} + \frac{1}{R_F}\right) + \frac{V_S}{R_S} + \frac{V_1}{R_F} = 0 \qquad (1)$$

$$\frac{V_0 - V_1}{R_1} = \frac{V_1}{R_2} + \frac{V_1 - (-V_d)}{R_F}$$

or

$$\frac{V_0}{R_1} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_F}\right) + \frac{V_d}{R_F} \qquad (2)$$

and $V_0 = A_{OL}V_d$ or $V_d = \frac{V_0}{A_{OL}}$

Substitute numerical values in (1) and (2):

$$\frac{V_0}{10^4}\cdot\left(\frac{1}{5} + \frac{1}{10}\right) + \frac{V_S}{5} + \frac{V_1}{10} = 0$$

or

$$V_0(0.3 \times 10^{-4}) + V_S(0.20) + V_1(0.10) = 0 \qquad (1)$$

$$\frac{V_0}{50} = V_1\left(\frac{1}{50} + \frac{1}{10} + \frac{1}{10}\right) + \frac{V_0}{10^4}\cdot\left(\frac{1}{10}\right)$$

or

$$V_0(0.02 - 10^{-5}) = V_1(0.22) \qquad (2)$$

Then $V_1 = V_0\left(\frac{0.02 - 10^{-5}}{0.22}\right)$

and

$$V_0(0.3 \times 10^{-4}) + V_S(0.20) + (0.10)\left[V_0\left(\frac{0.02 - 10^{-5}}{0.22}\right)\right] = 0$$

$$V_0\left[0.3\times10^{-4}-0.4545\times10^{-5}+0.00909\right]$$
$$+V_S(0.20)=0$$

Then $\dfrac{V_0}{V_S}=\dfrac{-0.20}{9.115\times10^{-3}}\Rightarrow \underline{\dfrac{V_0}{V_S}=-21.94}$

b. $R_{if}=\dfrac{-V_d}{\dfrac{V_S-(-V_d)}{R_S}}=\dfrac{-V_d\cdot R_S}{V_S+V_d}$

Now $V_d=\dfrac{V_0}{A_{0L}}=-\dfrac{21.94V_S}{10^4}$

Then $R_{if}=\dfrac{(21.94\times10^{-4})(5)}{1-21.94\times10^{-4}}$

or $R_{if}=1.099\times10^{-2}\ \text{k}\Omega$

$\Rightarrow \underline{R_{if}=10.99\ \Omega}$

c. Because of the $A_{0L}V_d$ source,

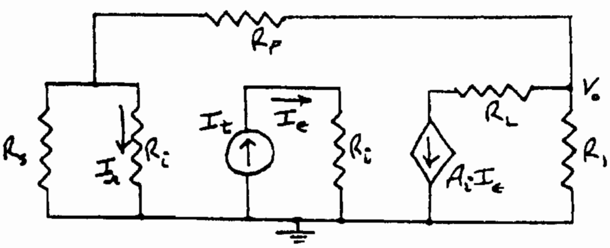
$\underline{R_{0f}=0}$

12.46

For example, use the circuit shown in Figure 12.41

12.47

Break the loop



$I_t=I_e$

Now $A_iI_t+\dfrac{V_0}{R_1}+\dfrac{V_0}{R_F+R_S\|R_i}=0$

$I_r=\left(\dfrac{R_S}{R_S+R_i}\right)\cdot\dfrac{V_0}{R_F+R_S\|R_i}$

or $V_0=I_r\left(\dfrac{R_S+R_i}{R_S}\right)\cdot(R_F+R_S\|R_i)$

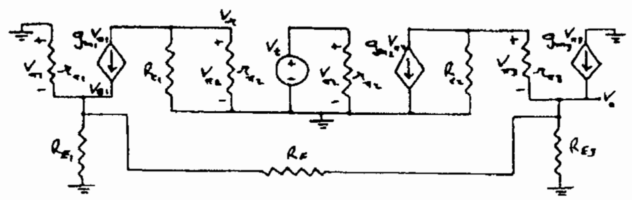
Then

$$A_iI_t+\left(\frac{1}{R_1}+\frac{1}{R_F+R_S\|R_i}\right)\times\left[I_r\left(\frac{R_S+R_i}{R_S}\right)(R_F+R_S\|R_i)\right]=0$$

$T=-\dfrac{I_r}{I_t}\Rightarrow$

$$\underline{T=\frac{A_i}{\left[\dfrac{1}{R_1}+\dfrac{1}{R_F+R_S\|R_i}\right]\left(\dfrac{R_S+R_i}{R_S}\right)(R_F+R_S\|R_i)}}$$

12.48



$$\frac{V_{\pi1}}{r_{\pi1}}+g_{m1}V_{\pi1}=\frac{V_{e1}}{R_{E1}}+\frac{V_{e1}-V_0}{R_F}\qquad(1)$$

$$g_{m1}V_{\pi1}+\frac{V_r}{R_{C1}\|r_{\pi2}}=0$$
$$\Rightarrow V_r=-(g_{m1}V_{\pi1})(R_{C1}\|r_{\pi2})\qquad(2)$$

$V_{\pi2}=V_t$ so that

$$g_{m2}V_t+\frac{V_{\pi3}+V_0}{R_{C2}}+\frac{V_{\pi3}}{r_{\pi3}}=0\qquad(3)$$

$$\frac{V_{\pi3}}{r_{\pi3}}+g_{m3}V_{\pi3}=\frac{V_0}{R_{E3}}+\frac{V_0-V_{e1}}{R_F}\qquad(4)$$

From (4):

$$V_0\left(\frac{1}{R_{E3}}+\frac{1}{R_F}\right)=V_{\pi3}\left(\frac{1}{r_{\pi3}}+g_{m3}\right)+\frac{V_{e1}}{R_F}$$

But $V_{e1}=-V_{\pi1}$

$$\text{so } V_0=\frac{V_{\pi3}\left(\dfrac{1}{r_{\pi3}}+g_{m3}\right)-\dfrac{V_{\pi1}}{R_F}}{\left(\dfrac{1}{R_{E3}}+\dfrac{1}{R_F}\right)}$$

Then

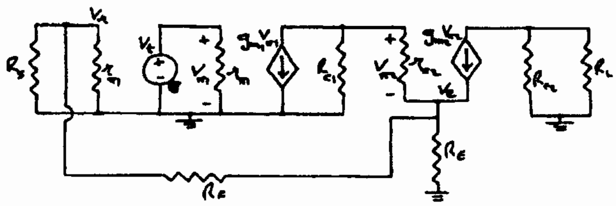
$$V_{\pi1}\left[\left(\frac{1}{r_{\pi1}}+g_{m1}\right)-\left(\frac{1}{R_{E1}}+\frac{1}{R_F}\right)\right]=\frac{-V_{\pi3}\left(\dfrac{1}{r_{\pi3}}+g_{m3}\right)+\dfrac{V_{\pi1}}{R_F}}{R_F\cdot\left(\dfrac{1}{R_{E3}}+\dfrac{1}{R_F}\right)}\qquad(1')$$

and

$$g_{m2}V_t+V_{\pi3}\left(\frac{1}{R_{C2}}+\frac{1}{r_{\pi3}}\right)+\frac{V_{\pi3}\left(\dfrac{1}{r_{\pi3}}+g_{m3}\right)-\dfrac{V_{\pi1}}{R_F}}{R_{C2}\cdot\left(\dfrac{1}{R_{E3}}+\dfrac{1}{R_F}\right)}=0\qquad(3')$$

From (3′), solve for $V_{\pi3}$ and substitute into (1′). Then from (1′), solve for $V_{\pi1}$ and substitute into (2). Then $T=-\dfrac{V_r}{V_t}$.

12.49



$$\frac{V_r}{R_S} + \frac{V_r}{r_{\pi 1}} + \frac{V_r - V_e}{R_F} = 0 \qquad (1)$$

$$g_{m1} V_t + \frac{V_{\pi 2} + V_e}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \qquad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_e}{R_E} + \frac{V_e - V_r}{R_F} \qquad (3)$$

Using the parameters from Problem 12.29, we obtain

$$V_r\left(\frac{1}{10} + \frac{1}{15.8} + \frac{1}{10}\right) - \frac{V_e}{10} = 0$$

or

$$V_r(0.2633) = V_e(0.10) \qquad (1)$$

$$(7.62)V_t + V_{\pi 2}\left(\frac{1}{40} + \frac{1}{2.28}\right) + \frac{V_e}{40} = 0$$

or

$$V_t(7.62) + V_{\pi 2}(0.4636) + V_e(0.025) = 0 \qquad (2)$$

$$V_{\pi 2}\left(\frac{1}{2.28} + 52.7\right) = V_e\left(\frac{1}{1} + \frac{1}{10}\right) - \frac{V_r}{10}$$

or

$$V_{\pi 2}(53.14) = V_e(1.10) - V_r(0.10)$$

Then

$$V_{\pi 2} = V_e(0.0207) - V_r(0.001882) \qquad (3)$$

Substituting in (2):

$$V_t(7.62) + (0.4636)[V_e(0.0207) - V_r(0.001882)] + V_e(0.025) = 0$$

or

$$V_t(7.62) + V_e(0.03460) - V_r(0.0008725) = 0$$

From (1) $V_e = V_r(2.633)$

Then

$$V_t(7.62) + V_r(2.633)(0.03460) - V_r(0.0008725) = 0$$

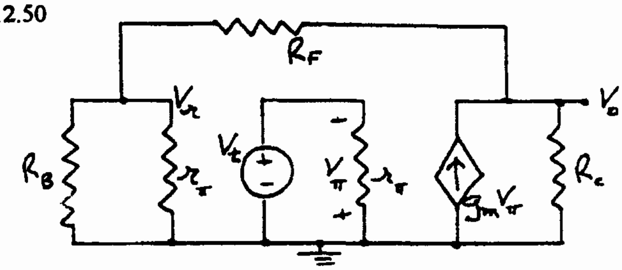
$$V_t(7.62) + V_r(0.09023) = 0$$

or $\dfrac{V_r}{V_t} = -84.45$

Now

$$T = -\frac{V_r}{V_t} \Rightarrow \underline{T = 84.45}$$

12.50



$$V_\pi = -V_t$$

$$g_m V_\pi = \frac{V_0}{R_C} + \frac{V_0}{R_F + R_B \| r_\pi} \qquad (1)$$

and

$$V_r = \left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_F}\right) V_0 \qquad (2)$$

Now

$$(65.77)V_\pi = V_0\left(\frac{1}{1} + \frac{1}{10 + 100 \| 0.760}\right)$$

or $(65.77)V_\pi = V_0(1.0930)$

and

$$V_r = \left(\frac{0.754}{10 + 0.754}\right) V_0 = (0.07011) V_0$$

so $V_0 = (14.26)V_r$

Then $(65.77)(-V_t) = (14.26)V_r(1.0930)$

$\dfrac{V_r}{V_t} = -4.22$ so that $\underline{T = 4.22}$

12.51

a. $$\phi = -\tan^{-1}\left(\frac{f}{5 \times 10^2}\right) - 2\tan^{-1}\left(\frac{f}{10^4}\right)$$

or

$$-180 = -\tan^{-1}\left(\frac{f_{180}}{5 \times 10^2}\right) - 2\tan^{-1}\left(\frac{f_{180}}{10^4}\right)$$

$$\Rightarrow \underline{f_{180} \approx 1.05 \times 10^4 \text{ Hz}}$$

b.

$$|T(f_{180})| = 1$$

$$= \frac{\beta(10^5)}{\sqrt{1 + \left(\frac{1.05 \times 10^4}{5 \times 10^2}\right)^2}\left[1 + \left(\frac{1.05 \times 10^4}{10^4}\right)^2\right]}$$

$$1 = \frac{\beta(10^5)}{(21.02)(2.105)} \qquad \text{or}$$

$$\underline{\beta = 4.42 \times 10^{-4}}$$

12.52

$$A = \frac{5x10^3}{\left(1+j\frac{f}{10^4}\right)\left(1+j\frac{f}{10^5}\right)^2}$$

Phase $= \phi = -\tan^{-1}\left(\frac{f}{10^4}\right) - 2\tan^{-1}\left(\frac{f}{10^5}\right)$

By trial and error, when $f = 1.095x10^5\ Hz$,

$\phi \cong 180°$

For $|T| = 1$ at $f = 1.095x10^5\ Hz$,

$$1 = \frac{\beta(5x10^3)}{\sqrt{1+\left(\frac{f}{10^4}\right)^2} \cdot \left[1+\left(\frac{f}{10^5}\right)^2\right]} \Rightarrow$$

$$1 = \frac{\beta(5x10^3)}{(10.996)(2.199)} \Rightarrow \underline{\beta = 4.84x10^{-3}}$$

12.53

$$\phi = -\tan^{-1}\left(\frac{f}{10^4}\right) - \tan^{-1}\left(\frac{f}{5\times 10^4}\right) - \tan^{-1}\left(\frac{f}{10^5}\right)$$

At $f = 8.1 \times 10^4$ Hz, $\phi = -180.28°$

Determine $|T(f)|$ at this frequency.

$|T|$

$$= \beta(10^3) \times \frac{1}{\sqrt{1+\left(\frac{8.1\times 10^4}{10^4}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{8.1\times 10^4}{5\times 10^4}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{8.1\times 10^4}{10^5}\right)^2}}$$

$$= \frac{\beta(10^3)}{(8.161)(1.904)(1.287)}$$

a. For $\beta = 0.005$

$|T(f)| = 0.250 < 1 \Rightarrow$ Stable

b. For $\beta = 0.05$

$|T(f)| = 2.50 > 1 \Rightarrow$ Unstable

12.54

(b)Phase margin $= 80° \Rightarrow \phi = -100°$

$$\phi = -100 = -2\tan^{-1}\left(\frac{f}{10^3}\right) - \tan^{-1}\left(\frac{f}{5x10^4}\right)$$

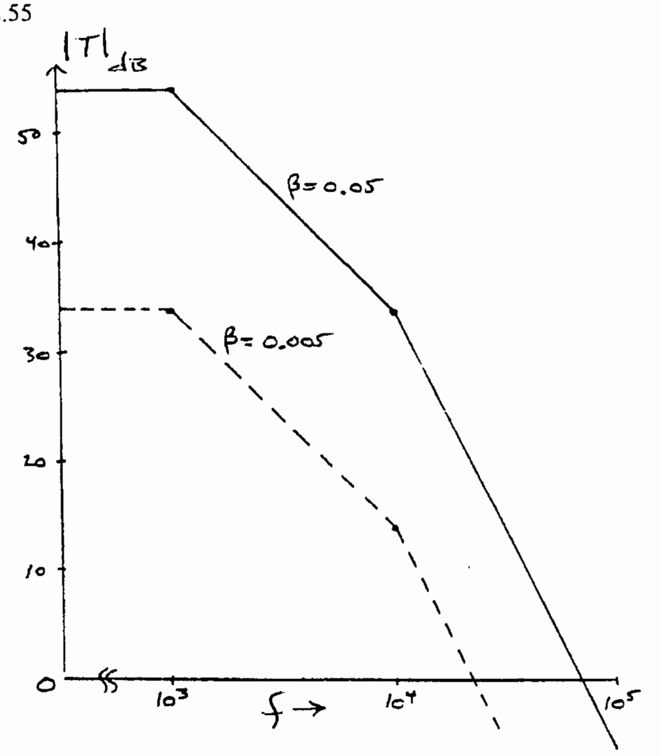
By trial and error, $f = 1.16x10^3\ Hz$

Then

$$|T| = 1 = \frac{\beta(5x10^3)}{\left(\sqrt{1+\left(\frac{1.16x10^3}{10^3}\right)^2}\right)^2 \cdot \sqrt{1+\left(\frac{1.16x10^3}{5x10^4}\right)^2}}$$

$$= \frac{\beta(5x10^3)}{(2.35)(1.00)} \Rightarrow \underline{\beta = 4.7x10^{-4}}$$

12.55



c. For $\beta = 0.005$,

$|T(f)| = 1$ (0 dB) at $f \approx 2.24 \times 10^4$ Hz

Then

$$\phi = -\tan^{-1}\left(\frac{2.24\times 10^4}{10^3}\right) - \tan^{-1}\left(\frac{2.24\times 10^4}{10^4}\right) - \tan^{-1}\left(\frac{2.24\times 10^4}{10^5}\right)$$

$$= -87.44 - 65.94 - 12.63$$

or

$\phi = -166°$ System is stable.

Phase margin $= 14°$

For $\beta = 0.05$,

$|T(f)| = 1$ (0 dB) at $f \approx 7.08 \times 10^4$ Hz

Then

$$\phi = -\tan^{-1}\left(\frac{7.08 \times 10^4}{10^3}\right) - \tan^{-1}\left(\frac{7.08 \times 10^4}{10^4}\right) - \tan^{-1}\left(\frac{7.08 \times 10^4}{10^5}\right)$$

$$= -89.19 - 81.96 - 35.30$$

or

$\phi = -206.45° \Rightarrow$ **System is unstable.**

12.56

$$T = A\beta = \frac{\beta(10^5)}{\left(1 + j\frac{f}{5x10^4}\right)\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{5x10^5}\right)}$$

Phase Margin $= 60° \Rightarrow \phi = -120°$

So

$$-120 = -\tan^{-1}\left(\frac{f}{5x10^4}\right) - \tan^{-1}\left(\frac{f}{10^5}\right) - \tan^{-1}\left(\frac{f}{5x10^5}\right)$$

By trial and error, at $f = 10^5\ Hz$, $\phi \cong -120°$

Then

$$|T| = 1 = \frac{\beta(10^5)}{\sqrt{1+\left(\frac{10^5}{5x10^4}\right)^2} \cdot \sqrt{1+\left(\frac{10^5}{10^5}\right)^2} \cdot \sqrt{1+\left(\frac{10^5}{5x10^5}\right)^2}}$$

$$1 = \frac{\beta(10^5)}{(2.236)(1.414)(1.02)} \Rightarrow \underline{\beta = 3.22x10^{-5}}$$

12.57

a. **Phase Margin** $= 60° \Rightarrow \phi = -120°$

Then

$$\phi = -120° = -2\tan^{-1}\left(\frac{f}{10^3}\right)$$

or $f = 1.732 \times 10^3$ Hz

Then

$$|T(f)| = 1 = \frac{\beta(10^3)}{\left[1 + \left(\frac{1.732 \times 10^3}{10^3}\right)^2\right]}$$

which yields

$\underline{\beta = 4 \times 10^{-3}}$

12.58

$$T(0) = A(0)\beta = (500)(0.6) = 300$$

$$T(f) = \frac{300}{\left(1 + j\frac{f}{10^4}\right)\left(1 + j\frac{f}{10^5}\right)\left(1 + j\frac{f}{10^6}\right)}$$

Find f at which $|T = 1|$

$$1 = \frac{300}{\sqrt{1+\left(\frac{f_1}{10^4}\right)^2} \cdot \sqrt{1+\left(\frac{f_1}{10^5}\right)^2} \cdot \sqrt{1+\left(\frac{f_1}{10^6}\right)^2}}$$

By trial and error, $f_1 = 5.12x10^5\ Hz$

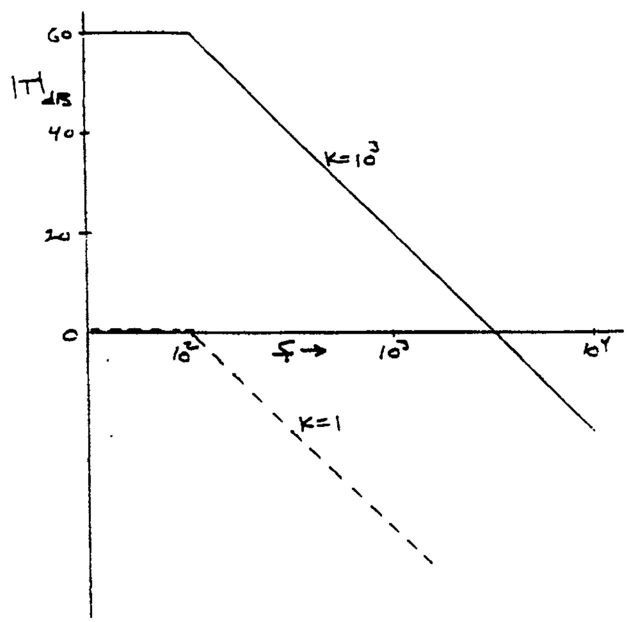
Then

$$\phi = -\tan^{-1}\left(\frac{f_1}{10^4}\right) - \tan^{-1}\left(\frac{f_1}{10^5}\right) - \tan^{-1}\left(\frac{f_1}{10^6}\right)$$

$$= -88.88 - 78.95 - 27.1 = -194.9°$$

System is unstable, Phase margin is not defined.

12.59



12.60

Phase Margin $= 45° \Rightarrow \phi = -135°$

$$\phi = -135°$$

$$= -\tan^{-1}\left(\frac{f}{10^3}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) - \tan^{-1}\left(\frac{f}{10^5}\right) - \tan^{-1}\left(\frac{f}{10^6}\right)$$

At $f = 10^4$ Hz, $\phi = -135.6°$

$$|T| = 1 =$$
$$= \beta(10^3) \times \frac{1}{\sqrt{1+\left(\frac{10^4}{10^3}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{10^4}{10^4}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{10^4}{10^5}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{10^4}{10^6}\right)^2}}$$
$$1 = \frac{\beta(10^3)}{(10.05)(1.414)(1.005)(1.00)}$$

or

$$\underline{\beta = 0.01428}$$

12.61

$$T = 5000 \times \frac{1}{\left(1+j\frac{f}{f_{PD}}\right)} \times \frac{1}{\left(1+j\frac{f}{300\times 10^3}\right)} \times \frac{1}{\left(1+j\frac{f}{2\times 10^6}\right)} \times \frac{1}{\left(1+j\frac{f}{25\times 10^6}\right)}$$

Phase Margin $= 45^\circ \Rightarrow \phi = -135^\circ$ at $f = 300$ kHz

$$-135^\circ = -\tan^{-1}\left(\frac{300\times 10^3}{f_{PD}}\right) - \tan^{-1}\left(\frac{300\times 10^3}{300\times 10^3}\right) - 0 - 0$$
$$= -90^\circ - 45^\circ$$

Now

$|T| = 1 @ f = 300$ kHz

$$|T| = 1 \approx \frac{5000}{\sqrt{1+\left(\frac{300\times 10^3}{f_{PD}}\right)^2}\cdot\sqrt{2}\cdot 1\cdot 1}$$

$$1+\left(\frac{300\times 10^3}{f_{PD}}\right)^2 = \left(\frac{5000}{\sqrt{2}}\right)^2$$
$$f_{PD} \approx \frac{300\times 10^3\sqrt{2}}{5000}$$
$$\Rightarrow \underline{f_{PD} = 84.8 \text{ Hz}}$$

12.62

a. $T(0) = 100$ dB $\Rightarrow T(0) = 10^5$

$$T(f) = \frac{10^5}{\left(1+j\frac{f}{10}\right)\left(1+j\frac{f}{5\times 10^6}\right)\left(1+j\frac{f}{10\times 10^6}\right)}$$

$$|T| = 1 =$$
$$= 10^5 \times \frac{1}{\sqrt{1+\left(\frac{f}{10}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{f}{5\times 10^6}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{f}{10\times 10^6}\right)^2}}$$

By trial and error

$$\underline{f = 0.976 \text{ MHz}}$$

$$\phi = -\tan^{-1}\left(\frac{0.976\times 10^6}{10}\right) - \tan^{-1}\left(\frac{0.976}{5}\right) - \tan^{-1}\left(\frac{0.976}{10}\right)$$
$$= -90^\circ - 11.05^\circ - 5.574^\circ = -106.6^\circ$$

Phase Margin $= 180^\circ - 106.6^\circ = \underline{73.4^\circ}$

b. $f'_{P1} \propto \frac{1}{C_F}$ so $\frac{10}{f'_{P1}} = \frac{75}{20}$

or

$$\underline{f'_{P1} = 2.67 \text{ Hz}}$$

Now

$$|T| = 1 =$$
$$= 10^5 \times \frac{1}{\sqrt{1+\left(\frac{f}{2.67}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{f}{5\times 10^6}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{f}{10\times 10^6}\right)^2}}$$

By trial and error

$$f \approx 2.66\times 10^5 \text{ Hz}$$

then

$$\phi = -\tan^{-1}\left(\frac{2.66\times 10^5}{2.67}\right) - \tan^{-1}\left(\frac{0.266}{5}\right) - \tan^{-1}\left(\frac{0.266}{10}\right)$$
$$= -90^\circ - 3.045^\circ - 1.524^\circ = -94.57^\circ$$

Phase Margin $= 180^\circ - 94.57^\circ = \underline{85.4^\circ}$

12.63

(a) $f_{3-dB} = \frac{1}{2\pi\tau}$ where $\tau = (R_{o1} \| R_{i2})C_i$

$= (500\|1000)x10^3 x2x10^{-12} \Rightarrow \tau = 6.67x10^{-7}\ s$

Then

$f_{3-dB} = \frac{1}{2\pi(6.67x10^{-7})} \Rightarrow \underline{f_{3-dB} = 239\ kHz}$

(b) For

$f_{PD} = 10\ Hz,\quad \tau = \frac{1}{2\pi f_{PD}} = \frac{1}{2\pi(10)} = 0.0159\ s$

Then $\tau = (R_{o1} \| R_{i2})(C_i + C_M)$

$0.0159 = (500\|1000)x10^3 x(C_i + C_M)$

or

$(C_i + C_M) = 4.77x10^{-8} = 2x10^{-12} + C_M \Rightarrow$

$\underline{C_M = 477\ \mu F}$

12.64

Want $f_1 = 12\ MHz$ for a phase margin of 45°

$T_{dB}(0) = 80\ dB \Rightarrow T(0) = 10^4$

Then

$$T(f) = \frac{T(0)}{\left(1 + j\frac{f}{f_{PD}}\right)\left(1 + j\frac{f}{12x10^6}\right)}$$

Set $f = f_1$ and $|T| = 1$

So

$$|T| = 1 = \frac{10^4}{\sqrt{1 + \left(\frac{12x10^6}{f_{PD}}\right)^2} \cdot \sqrt{2}}$$

which yields

$\frac{12x10^6}{f_{PD}} = \frac{10^4}{\sqrt{2}} \Rightarrow \underline{f_{PD} = 1.70\ kHz}$

12.65

$A_0 = 80\ \text{dB} \Rightarrow A_0 = 10^4$

$A_f(0) = \frac{A_0}{1 + \beta A_0}$

or $5 = \frac{10^4}{1 + \beta(10^4)} \Rightarrow \beta \approx 0.2$

Then $T(0) = \beta A_0 = 0.2 \times 10^4$

Inserting a dominate pole

$$\phi = -\tan^{-1}\left(\frac{f}{f_{PD}}\right) - \tan^{-1}\left(\frac{f}{10^6}\right) - \tan^{-1}\left(\frac{f}{10^7}\right)$$

If we want a phase margin of 45°, then

$$-135° \approx -90° - \tan^{-1}\left(\frac{f}{10^6}\right) - \tan^{-1}\left(\frac{f}{10^7}\right)$$

By trial and error, $f \approx 0.845$ MHz

Then

$$|T| = 1 = \frac{0.2 \times 10^4}{\sqrt{1 + \left(\frac{0.845 \times 10^6}{f_{PD}}\right)^2}\sqrt{1 + \left(\frac{0.845}{1}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{0.845}{10}\right)^2}}$$

$$\frac{0.845 \times 10^6}{f_{PD}} \approx \frac{0.2 \times 10^4}{(1.309)(1.0036)}$$

so $\underline{f_{PD} = 555\ \text{Hz}}$

12.66

Assuming a phase margin of 45°,

$$-135° \approx -90° - \tan^{-1}\left(\frac{f}{2 \times 10^6}\right) - \tan^{-1}\left(\frac{f}{25 \times 10^6}\right)$$

By trial and error, $f \approx 1.74$ MHz

Then

$|T| = 1$

$$= 5000 \times \frac{1}{\sqrt{1 + \left(\frac{1.74 \times 10^6}{f_{PD}}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{1.74}{2}\right)^2}} \times \frac{1}{\sqrt{1 + \left(\frac{1.74}{25}\right)^2}}$$

or

$$\frac{1.74 \times 10^6}{f_{PD}} \approx \frac{5000}{(1.325)(1.0024)}$$

so $\underline{f_{PD} = 462\ \text{Hz}}$

Chapter 13

Exercise Solutions

E13.3

$V_{iN}(\max) = V^+ - V_{EB}(\text{on}) = 15 - 0.6 = 14.4$ V

$V_{iN}(\min) \cong 4V_{BE}(\text{on}) + V^+$

$= 4(0.6) - 15 = -12.6$ V

$\underline{-12.6 \le V_{iN}(\text{cm}) \le 14.4 \text{ V}}$

E13.4

a. $V_0(\max) \cong V^+ - 2V_{BE}(\text{on}) = 15 - 2(0.6)$

$V_0(\max) = 13.8$ V

$V_0(\min) = 3V_{BE}(\text{on}) + V^- = 3(0.6) - 15$

$V_0(\min) \cong -13.2$ V

$\underline{-13.2 \le V_0 \le 13.8 \text{ V}}$

b. $V_0(\max) = 5 - 1.2 = 3.8$ V

$V_0(\min) \cong 3V_{BE} + V^- = 3(0.6) - 5 = -3.2$ V

$\underline{-3.2 \le V_0 \le 3.8 \text{ V}}$

E13.5

$I_{C1} = I_{C2} \cong 9.5\ \mu$A

$I_{B1} = I_{B2} = \dfrac{9.5\ \mu\text{A}}{200} = 0.0475\ \mu$A

$\Rightarrow \underline{I_{B1} = I_{B2} = 47.5 \text{ nA}}$

E13.6

$$I_{REF} \cong \frac{15 - 2(0.6) - (-15)}{40} = 0.72 \text{ mA}$$

$$V_{BE} = V_T \ln\left(\frac{I_{REF}}{I_S}\right) = (0.026)\ln\left(\frac{0.72 \times 10^{-3}}{10^{-14}}\right)$$

$= 0.650$ V

So

$$I_{REF} = \frac{30 - 2(0.65)}{40} \Rightarrow \underline{I_{REF} = 0.718 \text{ mA}}$$

$\underline{V_{BE11} = 0.650 \text{ V}}$

$$I_{C10}R_4 = V_T \ln\left(\frac{I_{REF}}{I_{C10}}\right)$$

$$I_{C10}(5) = (0.026)\ln\left(\frac{0.718}{I_{C10}}\right)$$

By trial and error: $I_{C10} = 18.9\ \mu$A

$V_{BE10} = V_{BE11} - I_{C10}R_4 = 0.650 - (0.0189)(5)$

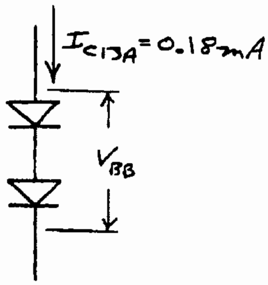
$\Rightarrow \underline{V_{BE10} \cong 0.556 \text{ V}}$

$$I_{C6} \cong \frac{I_{C10}}{2} = \frac{18.9}{2} = 9.45\ \mu\text{A}$$

$$V_{BE6} = V_T \ln\left(\frac{I_{C6}}{I_S}\right) = (0.026)\ln\left(\frac{9.45 \times 10^{-6}}{10^{-14}}\right)$$

$\Rightarrow \underline{V_{BE6} = 0.537 \text{ V}}$

E13.7



$$0.18 \times 10^{-3} = 10^{-14} \exp\left(\frac{V_D}{V_T}\right)$$

$$V_D = V_T \ln\left(\frac{0.18 \times 10^{-3}}{10^{-14}}\right)$$

$$= (0.026)\ln\left(\frac{0.18 \times 10^{-3}}{10^{-14}}\right)$$

$V_D = 0.6140$

$\underline{V_{BB} = 2V_{DD} \cong 1.228 \text{ V}}$

$$I_{C14} = I_{C20} = I_S \exp\left(\frac{V_{BB}/2}{V_T}\right)$$

$$= 3 \times 10^{-14} \exp\left(\frac{0.6140}{0.026}\right)$$

$\underline{I_{C14} = I_{C20} = 0.541 \text{ mA}}$

E13.8

$$I_{REF} = \frac{10 - 0.6 - 0.6 - (-10)}{40}$$

$\Rightarrow \underline{I_{REF} = 0.47 \text{ mA}}$

$$I_{C10}R_4 = V_T \ln\left(\frac{I_{REF}}{I_{C10}}\right)$$

$$I_{C10}(5) = (0.026)\ln\left(\frac{0.47}{I_{C10}}\right)$$

By trial and error:

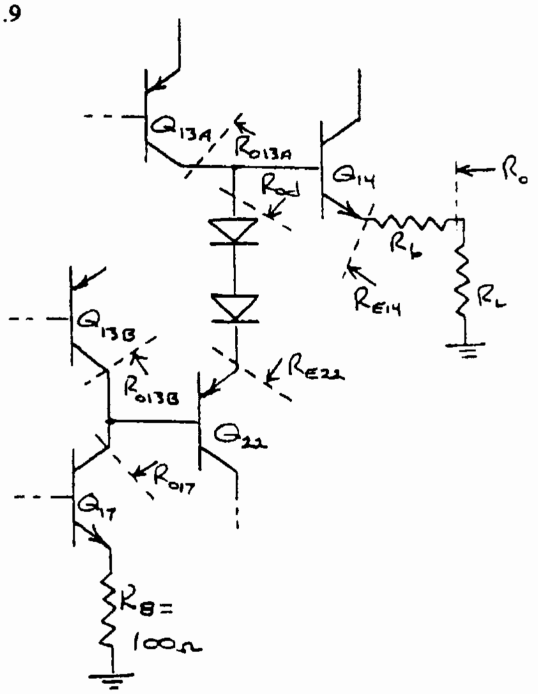
$\Rightarrow \underline{I_{C10} \cong 17.2\ \mu\text{A}}$

$I_{C6} \cong \dfrac{I_{C10}}{2} \Rightarrow \underline{I_{C6} = 8.6\ \mu\text{A}}$

$I_{C13B} = (0.75)I_{REF} \Rightarrow \underline{I_{C13B} = 0.353 \text{ mA}}$

$I_{C13A} = (0.25)I_{REF} \Rightarrow \underline{I_{C13A} = 0.118 \text{ mA}}$

E13.9



$R_0 = R_6 + R_{E14}$

$R_{E14} = \dfrac{r_{\pi 14} + R_{0d} \| R_{013A}}{1 + \beta_n}$

The diode resistance can be found as

$I_D = I_S \exp\left(\dfrac{V_D}{V_T}\right)$

$\dfrac{1}{r_d} = \dfrac{\partial I_D}{\partial V_D} = I_S\left(\dfrac{1}{V_T}\right) \cdot \exp\left(\dfrac{V_D}{V_T}\right) = \dfrac{I_D}{V_T}$

or

$r_d = \dfrac{V_T}{I_D} = \dfrac{V_T}{I_{C13A}} = \dfrac{0.026}{0.18} \Rightarrow 144\ \Omega$

$R_{E22} = \dfrac{r_{\pi 22} + R_{017} \| R_{013B}}{1 + \beta_P}$

$R_{013B} = r_{013B} = 92.6\ \text{k}\Omega$

$R_{017} = r_{017}[1 + g_{m17}(R_8 \| r_{\pi 17})] = 283\ \text{k}\Omega$

From previous calculations

$R_{E22} = 1.51\ \text{k}\Omega$

$R_{0d} = 2r_d + R_{E22} = 2(0.144) + 1.51 = 1.80\ \text{k}\Omega$

$R_{013A} = r_{013A} = 278\ \text{k}\Omega$

$r_{\pi 14} = \dfrac{\beta_n V_T}{I_{C14}} = \dfrac{(200)(0.026)}{5} = 1.04\ \text{k}\Omega$

$R_{E14} = \dfrac{1.04 + 1.8 \| 278}{201} \Rightarrow 14.1\ \Omega$

$R_0 = R_6 + R_{E14} = 27 + 14.1$

$\Rightarrow \underline{R_0 \cong 41\ \Omega}$

E13.10

For Q_6 we have $V_{SG5} = V_{SG6} = 1.06$ V

So

$V_{SD6}(\text{sat}) = 1.06 - 0.5 = 0.56$ V

For M_1 and M_2

$I_D = \dfrac{I_Q}{2} = K_p(V_{SG1} + V_{TP})^2$

$\dfrac{0.0397}{2} = 0.125(V_{SG1} - 0.5)^2$

$\Rightarrow V_{SG1} = 0.898$ V

So maximum input voltage

$= V^+ - V_{SD6}(\text{sat}) - V_{SG1}$

$= 5 - 0.56 - 0.898$

$\Rightarrow \underline{V_{IN}(\text{max}) = 3.54\ \text{V}}$

For M_3,

$K_p = (6.25)(20) = 125\ \mu A / V^2$

$I_{D3} = \dfrac{I_Q}{2} = \dfrac{39.7}{2}\ \mu\text{A}$

$\dfrac{39.7}{2} = 125(V_{GS3} - V_{TN})^2$

$V_{TN} = 0.5 V \Rightarrow$

$V_{GS3} = 0.898$ V

$V_{SD1}(\text{sat}) = 0.898 - 0.5 = 0.398$ V

$V_{IN}(\text{min}) = V^- + V_{GS3} + V_{SD1}(\text{sat}) - V_{SG1}$

$= -5 + 0.898 + 0.398 - 0.898$

$\underline{V_{IN}(\text{min}) = -4.60\ \text{V}}$

$\underline{-4.60 \le V_{IN}(\text{cm}) \le 3.54\ \text{V}}$

E13.11

$V_0(\text{max}) = V^+ - V_{SD8}(\text{sat})$

$V_{SG8} = V_{SG5} = 1.06$ V

$V_{SD8}(\text{sat}) = 1.06 - 0.5 = 0.56$ V

$V_0(\text{max}) = 5 - 0.56 = 4.44$ V

$V_0(\text{min}) = V^- + V_{DS7}(\text{sat})$

$V_{GS7} = 1.06 \Rightarrow V_{DS7}(\text{sat}) = 1.06 - 0.5 = 0.56$

$V_0(\text{min}) = -5 + 0.56 = -4.44$

$\underline{-4.44 \le V_0 \le 4.44\ \text{V}}$

E13.12

(a) For M_5, $K_{p5} = 125\ \mu A/V^2$

$$K_{p5}(V_{SG5} + V_{TP})^2 = \frac{V^+ - V^- - V_{SG5}}{R_{set}}$$

$$0.125(V_{SG5} - 0.5)^2 = \frac{5 + 5 - V_{SG5}}{100}$$

$$12.5(V_{SG5}^2 - V_{SG5} + 0.25) = 10 - V_{SG5}$$

$$12.5V_{SG5}^2 - 11.5V_{SG5} - 6.875 = 0$$

$$V_{SG5} = \frac{11.5 \pm \sqrt{(11.5)^2 + 4(12.5)(6.875)}}{2(12.5)}$$

$$V_{SG5} = 1.33\ \text{V}$$

Then

$$I_{REF} = I_Q = I_{D8} = I_{D7} = \frac{10 - 1.33}{100} \Rightarrow 86.7\ \mu\text{A}$$

$$I_{D1} = I_{D2} = I_{D3} = I_{D4} = \frac{I_Q}{2} = 43.35\ \mu\text{A}$$

(b) $K_{p1} = K_{p2} = 125\ \mu A/V^2$

$$r_{o2} = r_{o4} = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.04335)} = 1153\ k\Omega$$

Input stage gain

$$A_d = \sqrt{2K_{p1}I_Q} \cdot (r_{o2} \| r_{o4})$$

$$= \sqrt{2(0.125)(0.0867)} \cdot (1153 \| 1153) \Rightarrow$$

$$A_d = 84.9$$

Transconductance of M_7

$$g_{m7} = 2\sqrt{K_{n7}I_{D7}} = 2\sqrt{(0.250)(0.0867)}$$

$$= 0.294\ mA/V$$

$$r_{o7} = r_{o8} = \frac{1}{\lambda I_{D7}} = \frac{1}{(0.02)(0.0867)} = 577\ k\Omega$$

Second stage gain

$$A_{v2} = g_{m7}(r_{o7} \| r_{o8}) = (0.294)(577 \| 577) \Rightarrow$$

$$A_{v2} = 84.8$$

Overall gain $= A_d \cdot A_{v2} = (84.9)(84.8) \Rightarrow$

$$A = 7{,}200$$

E13.13

$$I_{D1} = I_{D2} = 25\ \mu A$$

$$g_{m1} = g_{m8} = 2\sqrt{\frac{k_p'}{2}\left(\frac{W}{L}\right)I_{DQ}} = 2\sqrt{\left(\frac{40}{2}\right)(25)(25)} \Rightarrow$$

$$g_{m1} = g_{m8} = 224\ \mu A/V$$

$$g_{m6} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_{DQ}} = 2\sqrt{\left(\frac{80}{2}\right)(25)(25)} \Rightarrow$$

$$g_{m6} = 316\ \mu A/V$$

$$r_{o1} = r_{o6} = r_{o8} = r_{o10} = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(25)} = 2\ M\Omega$$

$$r_{o4} = \frac{1}{\lambda I_{D4}} = \frac{1}{(0.02)(50)} = 1\ M\Omega$$

$$R_{o8} = g_{m8}(r_{o8}r_{o10}) = (224)(2)(2) = 896\ M\Omega$$

$$R_{o6} = g_{m6}(r_{o6})(r_{o4} \| r_{o1}) = 316(2)(1 \| 2) = 421\ M\Omega$$

Then

$$A_d = g_{m1}(R_{o6} \| R_{o8}) = 224(421 \| 896) \Rightarrow$$

$$A_d = 64{,}158$$

E13.14

(a) $A_d = Bg_{m1}(r_{o6} \| r_{o8})$

$$g_{m1} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_{D1}} = 2\sqrt{\left(\frac{80}{2}\right)(20)(50)}$$

$$g_{m1} = 400\ \mu A/V$$

$$r_{o6} = \frac{1}{\lambda_p I_{D6}} = \frac{1}{(0.02)(150)} = 0.333\ M\Omega$$

$$r_{o8} = \frac{1}{\lambda_n I_{D8}} = \frac{1}{(0.02)(150)} = 0.333\ M\Omega$$

$$A_d = 3(400)(0.333 \| 0.333) \Rightarrow \quad A_d = 200$$

(b) $f_{PD} = \dfrac{1}{2\pi R_o(C_L + C_P)}$

where $R_o = r_{o6} \| r_{o8} = 0.333 \| 0.333\ M\Omega$

$$f_{PD} = \frac{1}{2\pi(0.333 \| 0.333)x10^6 x2x10^{-12}} \Rightarrow$$

$$f_{PD} = 477\ kHz$$

$$f_{PD} \cdot A_d = (477x10^3)(200) \Rightarrow 95.4\ MHz$$

E13.15

(a) From Exercise 13.14, $g_{m1} = 400\ \mu A/V$

$$r_{o6} = r_{o8} = r_{o10} = r_{o12} = 0.333\ M\Omega$$

$$g_{m10} = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)I_{D10}} = 2\sqrt{\left(\frac{40}{2}\right)(20)(150)} \Rightarrow$$

$$g_{m10} = 490\ \mu A/V$$

$$g_{m12} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_{D12}} = 2\sqrt{\left(\frac{80}{2}\right)(20)(150)} \Rightarrow$$

$$g_{m12} = 693\ \mu A/V$$

$$R_{o10} = g_{m10}(r_{o10}r_{o6}) = (490)(0.333)(0.333) = 54.3\ M\Omega$$

$$R_{o12} = g_{m12}(r_{o12}r_{o8}) = (693)(0.333)(0.333) = 76.8\ M\Omega$$

$$A_d = Bg_{m1}(R_{o10} \| R_{o12}) = 3(400)(54.3 \| 76.8) \Rightarrow$$

$$A_d = 38{,}172$$

(b) $R_o = R_{o10} \| R_{o12} = 54.3 \| 76.8 = 31.8\ M\Omega$

$$f_{PD} = \frac{1}{2\pi(31.8x10^6)(2x10^{-12})} = 2.50\ kHz$$

$$f_{PD} \cdot A_d = (2.5x10^3)(38{,}172) \Rightarrow 95.4\ MHz$$

E13.16

(a) $A_d = g_{m1}(R_{o6} \| R_{o8})$

From Example 13.10,

$g_{m1} = 316\ \mu A/V$, $R_{o8} = 316\ M\Omega$

Now

$R_{o6} = g_{m6}(r_{o6})(r_{o4} \| r_{o1})$

$r_{o1} = 1\ M\Omega$, $r_{o4} = 0.5\ M\Omega$

$$g_{m6} = \frac{I_{C6}}{V_T} = \frac{50}{0.026} \Rightarrow 1.923\ mA/V$$

$$r_{o6} = \frac{V_{A6}}{I_{C6}} = \frac{80}{50} = 1.6 M\Omega$$

Then

$R_{o6} = (1.923)(1600)(0.5 \| 1) = 1026\ M\Omega$

$A_d = (316)(1026 \| 316) \Rightarrow \underline{A_d = 76{,}343}$

(b) $$f_{PD} = \frac{1}{2\pi(316 \| 1026) x 10^6 x 2 x 10^{-12}} \Rightarrow$$

$\underline{f_{PD} = 329\ Hz}$

$f_{PD} \cdot A_d = (329)(76{,}343) \Rightarrow 25.1\ MHz$

E13.17

$$V^+ - V^- = V_{EB1} + V_{EB6} + V_{BE7} + I_1 R_1$$

$$= 0.6 + 0.6 + 0.6 + (0.24)(8) = 3.72\ \text{V}$$

So

$\underline{V^+ = -V^- = 1.86\ \text{V}}$

E13.18

For Q_7 and R_1

$$V_{SG} = V_{BE7} + I_1 R_1 = 0.6 + I_1(5)$$

For M_8 :

$$I_2 = K_p(V_{SG} + V_{TP})^2$$

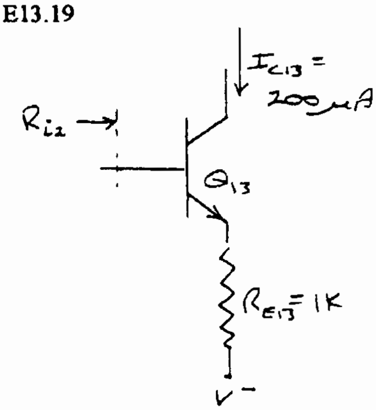
$$I_2 = 0.3(V_{SG} - 1.4)^2$$

By trial and error:

$\underline{V_{SG} = 2.54\ \text{V}}$

$\underline{I_1 = I_2 = 0.388\ \text{mA}}$

E13.19



$$r_{\pi 13} = \frac{\beta V_T}{I_{C13}} = \frac{(200)(0.026)}{0.20}$$

$$= 26\ \text{k}\Omega$$

$$R_{i2} = r_{\pi 13} + (1+\beta)R_{E13} = 26 + 201(1)$$

$$= 227\ \text{k}\Omega$$

$$r_{O10} = \frac{1}{\lambda I_{D10}} = \frac{1}{(0.02)(0.1)} = 500\ \text{k}\Omega$$

$$r_{O12} = \frac{V_A}{I_{C12}} = \frac{50}{0.1} = 500\ \text{k}\Omega$$

$$g_{m12} = \frac{I_{C12}}{V_T} = \frac{0.1}{0.026} = 3.85\ \text{mA/V}$$

$$r_{\pi 12} = \frac{\beta V_T}{I_{C12}} = \frac{(200)(0.026)}{0.1} = 52\ \text{k}\Omega$$

$$R_{act1} = r_{O12}[1 + g_{m12}(r_{\pi 12} \| R_5)]$$

$$= 500[1 + (3.85)(52 \| 0.5)] = 1453\ \text{k}\Omega$$

$$A_d = \sqrt{2K_n I_{Q5}} \cdot (r_{o10} \| R_{act1} \| R_{i2})$$

$$= \sqrt{2(0.6)(0.2)} \cdot (500 \| 1453 \| 227)$$

$= (0.490)(141) \Rightarrow \underline{A_d = 69.1}$

E13.20

For J_6 biased in the saturation region

$$\Rightarrow I_{C3} = I_{DSS} = 300\ \mu\text{A}$$

Q_1, Q_2, Q_3 are matched

$\Rightarrow \underline{I_{C1} = I_{C2} = I_{C3} = 300\ \mu\text{A}}$

Chapter 13

Problem Solutions

13.3

(a) $A_d = g_{m1}(r_{o2} \| r_{o4} \| R_{i6})$

$g_{m1} = \frac{I_{C1}}{V_T} = \frac{20}{0.026} \Rightarrow 0.769\ mA/V$

$r_{o2} = \frac{V_{A2}}{I_{C2}} = \frac{80}{20} = 4\ M\Omega$

$r_{o4} = \frac{V_{A4}}{I_{C2}} = \frac{80}{20} = 4\ M\Omega$

$R_{i6} = r_{\pi6} + (1+\beta_n)[R_1 \| r_{\pi7}]$

$r_{\pi7} = \frac{(120)(0.026)}{0.2} = 15.6\ k\Omega$

$I_{C6} \cong \frac{V_{BE}(on)}{R_1} = \frac{0.6}{20} = 0.030\ mA$

$r_{\pi6} = \frac{(120)(0.026)}{0.030} = 104\ k\Omega$

Then

$R_{i6} = 104 + (121)[20 \| 15.6] \Rightarrow 1.16\ M\Omega$

Then

$A_d = 769(4 \| 4 \| 1.16) \Rightarrow \underline{A_d = 565}$

Now

$V_o = -I_{c7} r_{o7} = -(\beta_n I_{b7}) r_{o7} = -\beta_n r_{o7}\left(\frac{R_1}{R_1 + r_{\pi7}}\right) I_{e6}$

$= -\beta_n(1+\beta_n) r_{o7}\left(\frac{R_1}{R_1 + r_{\pi7}}\right) I_{b6}$ and $I_{b6} = \frac{V_{o1}}{R_{i6}}$

Then

$A_{v2} = \frac{V_o}{V_{o1}} = \frac{-\beta_n(1+\beta_n) r_{o7}}{R_{i6}}\left(\frac{R_1}{R_1 + r_{\pi7}}\right)$

$r_{o7} = \frac{V_A}{I_{C7}} = \frac{80}{0.2} = 400\ k\Omega$

So

$A_{v2} = \frac{-(120)(121)(400)}{1160}\left(\frac{20}{20+15.6}\right) \Rightarrow$

$\underline{A_{v2} = -2813}$

Overall gain $= A_d \cdot A_{v2} = (565)(-2813) \Rightarrow$

$\underline{A = -1.59x10^6}$

(b) $R_{id} = 2r_{\pi1}$ and $r_{\pi1} = \frac{(80)(0.026)}{0.020} = 104\ k\Omega$

$\underline{R_{id} = 208\ k\Omega}$

(c) $f_{PD} = \frac{1}{2\pi R_{eq} C_M}$ and

$C_M = (10)(1+2813) = 28{,}140\ pF$

$R_{eq} = r_{o2} \| r_{o4} \| R_{i6} = 4 \| 4 \| 1.16 = 0.734\ M\Omega$

$f_{PD} = \frac{1}{2\pi(0.734x10^6)(28{,}140x10^{-12})} = 7.71\ Hz$

Gain-Bandwidth Product $= (7.71)(1.59x10^6) \Rightarrow$

$\underline{12.3\ MHz}$

13.4

a. Q_3 acts as the protection device.
b. Same as part (a).

13.5

If we assume $V_{BE}(\text{on}) = 0.7$ V, then

$$V_{in} = 0.7 + 0.7 + 50 + 5$$

So breakdown voltage ≈ <u>56.4 V</u>.

13.6

(a) $I_{REF} = \frac{15 - 0.6 - 0.6 - (-15)}{R_5} = 0.50$

$\underline{\Rightarrow R_5 = 57.6\ k\Omega}$

$I_{C10} R_4 = V_T \ln\left(\frac{I_{REF}}{I_{C10}}\right)$

$R_4 = \frac{0.026}{0.030}\ln\left(\frac{0.50}{0.030}\right) \Rightarrow \underline{R_4 = 2.44\ k\Omega}$

(b) $I_{REF} = \frac{5 - 0.6 - 0.6 - (-5)}{57.6} \Rightarrow \underline{I_{REF} = 0.153\ mA}$

$I_{C10}(2.44) = (0.026)\ln\left(\frac{0.153}{I_{C10}}\right)$

By trial and error, $\underline{I_{C10} \cong 21.1\ \mu A}$

13.7

(a) $I_{REF} \cong 0.50\ mA$

$V_{BE} = V_T \ln\left(\frac{I_{REF}}{I_S}\right) = (0.026)\ln\left(\frac{0.50x10^{-3}}{10^{-14}}\right) \Rightarrow$

$\underline{V_{BE11} = 0.641 V = V_{EB12}}$

Then

$R_5 = \frac{15 - 0.641 - 0.641 - (-15)}{0.50} \Rightarrow \underline{R_5 = 57.4\ k\Omega}$

$R_4 = \frac{0.026}{0.030}\ln\left(\frac{0.50}{0.030}\right) \Rightarrow R_4 = 2.44\ k\Omega$

$V_{BE10} = 0.026\ln\left(\frac{0.030x10^{-3}}{10^{-14}}\right) \Rightarrow V_{BE10} = 0.567\ V$

(b) From Problem 13.6, $I_{REF} \cong 0.15\,mA$

$$V_{BE11} = V_{EB12} = 0.026\ln\left(\frac{0.15x10^{-3}}{10^{-14}}\right) = 0.609\,V$$

Then $I_{REF} = \dfrac{5-0.609-0.609-(-5)}{57.4} \Rightarrow$

$\underline{I_{REF} = 0.153\,mA}$

Then $\underline{I_{C10} \cong 21.1\,\mu A}$ from Problem 13.6

13.8

a. $I_{REF} = \dfrac{5-0.6-0.6-(-5)}{40}$

$\Rightarrow \underline{I_{REF} = 0.22 \text{ mA}}$

$$I_{C10}R_4 = V_T \ln\left(\frac{I_{REF}}{I_{C10}}\right)$$

$$I_{C10}(5) = (0.026)\ln\left(\frac{0.22}{I_{C10}}\right)$$

By trial and error;

$\underline{I_{C10} \cong 14.2\ \mu\text{A}}$

$I_{C6} \cong \dfrac{I_{C10}}{2} \Rightarrow \underline{I_{C6} = 7.10\ \mu\text{A}}$

$I_{C17} = 0.75 I_{REF} \Rightarrow \underline{I_{C17} = 0.165 \text{ mA}}$

$I_{C13A} = 0.25 I_{REF} \Rightarrow \underline{I_{C13A} = 0.055 \text{ mA}}$

b. Using Example 13.4

$r_{\pi 17} = 31.5$ kΩ

$R'_E = 50\|[31.5 + (201)(0.1)] = 50\|51.6$

$= 25.4$ kΩ

$r_{\pi 16} = \dfrac{\beta_n V_T}{I_{C16}}$ and

$$I_{C16} = \frac{0.165}{200} + \frac{(0.165)(0.1)+0.6}{50} = 0.0132 \text{ mA}$$

$r_{\pi 16} = 394$ kΩ

Then

$R_{i2} = 394 + (201)(25.4) \Rightarrow 5.5$ MΩ

$r_{\pi 6} = 732$ kΩ

$g_{m6} = \dfrac{0.00710}{0.026} = 0.273$ mA/V

$r_{o6} = \dfrac{50}{0.0071} = 7.04$ MΩ

Then

$R_{act1} = 7.04[1 + (0.273)(1\|732)] = 8.96$ MΩ

$r_{o4} = \dfrac{50}{0.0071} = 7.04$ MΩ

Then

$$A_d = -\left(\frac{7.1}{0.026}\right)(7.04\|8.96\|5.5)$$

or

$\underline{A_d = -627}$ Gain of differential amp stage

Using Example 13.5, and neglecting the input resistance to the output stage:

$$R_{act2} = \frac{V_A}{I_{C13B}} = \frac{50}{0.165} = 303 \text{ k}\Omega$$

$$A_{v2} = \frac{-(200)(201)(50)(303)}{(5500)[50 + 31.5 + (201)(0.1)]}$$

or

$\underline{A_{v2} = -1090}$ Gain of second stage

13.9

$I_{C10} = 19\ \mu$A

From Equation (13.6)

$$I_{C10} = 2I\left[\frac{\beta_P^2 + 2\beta_P + 2}{\beta_P^2 + 3\beta_P + 2}\right] = 2I\left[\frac{(10)^2 + 2(10) + 2}{(10)^2 + 3(10) + 2}\right]$$

$$= 2I\left[\frac{122}{132}\right]$$

So

$$2I = (19)\left(\frac{132}{122}\right) = 20.56\ \mu\text{A}$$

$\underline{I_{C2} = I = 10.28\ \mu\text{A}}$

$$I_{C9} = \frac{2I}{\left(1+\dfrac{2}{\beta_P}\right)} = \frac{20.56}{\left(1+\dfrac{2}{10}\right)} \Rightarrow \underline{I_{C9} = 17.13\ \mu\text{A}}$$

$$I_{B9} = \frac{I_{C9}}{\beta_P} = \frac{17.13}{10} \Rightarrow \underline{I_{B9} = 1.713\ \mu\text{A}}$$

$$I_{B4} = \frac{I}{(1+\beta_P)} = \frac{10.28}{11} \Rightarrow \underline{I_{B4} = 0.9345\ \mu\text{A}}$$

$$I_{C4} = I\left(\frac{\beta_P}{1+\beta_P}\right) = (10.28)\left(\frac{10}{11}\right)$$

$\Rightarrow \underline{I_{C4} = 9.345\ \mu\text{A}}$

13.10

$V_{B5} - V^- = V_{BE}(\text{on}) + I_{C5}(1)$

$= 0.6 + (0.0095)(1) = 0.6095$

$I_{C7} = \dfrac{0.6095}{50} \Rightarrow I_{C7} = 12.2\ \mu$A

$I_{C8} = I_{C9} = 19\ \mu A$

$I_{REF} = 0.72$ mA

$I_{E13} = I_{REF} = 0.72$ mA

$I_{C14} = 138\ \mu A$

$$\text{Power} = (V^+ - V^-)[I_{C7} + I_{C8} + I_{C9} + I_{REF} + I_{E13} + I_{C14}]$$
$$= 30[0.0122 + 0.019 + 0.019 + 0.72 + 0.72 + 0.138]$$
$$\Rightarrow \underline{\text{Power} = 48.8 \text{ mW}}$$

Current supplied by V^+ and V^-

$$= I_{C7} + I_{C8} + I_{C9} + I_{REF} + I_{E13} + I_{C14}$$
$$= \underline{1.63 \text{ mA}}$$

13.11

(a) $v_{cm}(\min) = -15 + 0.6 + 0.6 + 0.6 + 0.6 = -12.6V$
$v_{cm}(\max) = +15 - .6 = 14.4V$
So $-12.6 \le v_{cm} \le 14.4V$

(b) $v_{cm}(\min) = -5 + 4(0.6) = -2.6V$
$v_{cm}(\max) = 5 - 0.6 = 4.4V$
So $-2.6 \le v_{cm} \le 4.4V$

13.12

If $v_O = V^- = -15$ V, the base voltage of Q_{14} is pulled low, and Q_{18} and Q_{19} are effectively cut off. As a first approximation

$$I_{C14} = \frac{0.6}{0.027} = 22.2 \text{ mA}$$
$$I_{B14} = \frac{22.2}{200} = 0.111 \text{ mA}$$

Then

$$I_{C15} = I_{C13A} - I_{B14} = 0.18 - 0.111 = 0.069 \text{ mA}$$

Now

$$V_{BE15} = V_T \ln\left(\frac{I_{C15}}{I_S}\right)$$
$$= (0.026)\ln\left(\frac{0.069 \times 10^{-3}}{10^{-14}}\right)$$
$$= 0.589 \text{ V}$$

As a second approximation

$$I_{C14} = \frac{0.589}{0.027} \Rightarrow \underline{I_{C14} = 21.8 \text{ mA}}$$
$$I_{B14} = \frac{21.8}{200} = 0.109 \text{ mA}$$

and

$$I_{C15} = 0.18 - 0.109 \Rightarrow \underline{I_{C15} = 0.071 \text{ mA}}$$

13.13

a. **Neglecting base currents:**

$I_D = I_{BIAS}$

Then

$$V_{BB} = 2V_D = 2V_T \ln\left(\frac{I_D}{I_S}\right)$$
$$= 2(0.026)\ln\left(\frac{0.25 \times 10^{-3}}{2 \times 10^{-14}}\right)$$

or

$$\underline{V_{BB} = 1.2089 \text{ V}}$$
$$I_{CN} = I_{CP} = I_S \exp\left(\frac{V_{BB}/2}{V_T}\right)$$
$$= 5 \times 10^{-14} \exp\left(\frac{1.2089}{2(0.026)}\right)$$

So

$$\underline{I_{CN} = I_{CP} = 0.625 \text{ mA}}$$

b. For $v_I = 5$ V, $\underline{v_O \cong 5 \text{ V}}$

$$\underline{i_L \cong \frac{5}{4} = 1.25 \text{ mA}}$$

As a first approximation

$$I_{CN} \approx i_L = 1.25 \text{ mA}$$
$$V_{BEN} = (0.026)\ln\left(\frac{1.25 \times 10^{-3}}{5 \times 10^{-14}}\right) = 0.6225 \text{ V}$$

Neglecting base currents,

$$\underline{V_{BB} = 1.2089 \text{ V}}$$

Then $V_{EBP} = 1.2089 - 0.6225 = 0.5864$ V

$$I_{CP} = 5 \times 10^{-14} \exp\left(\frac{0.5864}{0.026}\right) \Rightarrow I_{CP} = 0.312 \text{ mA}$$

As a second approximation,

$$I_{CN} = i_L + I_{CP} = 1.25 + 0.31 \Rightarrow \underline{I_{CN} \cong 1.56 \text{ mA}}$$
$$V_{BEN} = (0.026)\ln\left(\frac{1.56 \times 10^{-3}}{5 \times 10^{-14}}\right) = 0.62826 \text{ V}$$

$$V_{EBP} = 1.2089 - 0.62826 = 0.5806 \text{ V}$$
$$I_{CP} = 5 \times 10^{-14} \exp\left(\frac{0.5806}{0.026}\right) \Rightarrow \underline{I_{CP} = 0.25 \text{ mA}}$$

13.14

$$R_1 + R_2 = \frac{V_{BB}}{(0.1)I_{BIAS}} = \frac{1.157}{0.018} = 64.28 \text{ k}\Omega$$
$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = (0.026)\ln\left(\frac{(0.9)I_{BIAS}}{I_S}\right)$$
$$= (0.026)\ln\left(\frac{0.162 \times 10^{-3}}{10^{-14}}\right)$$

$V_{BE} = 0.6112$ V

$$V_{BE} = \left(\frac{R_2}{R_1 + R_2}\right) V_{BB}$$

$$0.6112 = \left(\frac{R_2}{64.28}\right)(1.157)$$

So

$\underline{R_2 = 33.96 \text{ k}\Omega}$

Then

$\underline{R_1 = 30.32 \text{ k}\Omega}$

13.15

(a) $A_d = -g_m(r_{o4} \| r_{o6} \| R_{i2})$

From example 13.4

$g_m = \dfrac{9.5}{0.026} = 365\ \mu A/V$, $r_{o4} = 5.26\ M\Omega$

Now

$r_{o6} = r_{o4} = 5.26\ M\Omega$

Assuming $R_E = 0$, we find

$R_{i2} = r_{\pi 16} + (1+\beta_n)R_E'$

$= 329 + (201)(50\|9.63) \Rightarrow 1.95\ M\Omega$

Then

$A_d = -(365)(5.26\|5.26\|1.95) \Rightarrow \underline{A_d = -409}$

(b) From Equation (13.20),

$$A_{v2} = \frac{-\beta_n(1+\beta_n)R_9(R_{act2}\|R_{i3}\|R_{017})}{R_{i2}\{R_9 + [r_{\pi 17} + (1+\beta_n)R_8]\}}$$

For $R_8 = 0$, $R_{i2} = 1.95\ M\Omega$

Using the results of Example 13.5

$$A_{v2} = \frac{-200(201)(50)(92.6\|4050\|92.6)}{(1950)\{50 + 9.63\}} \Rightarrow$$

$\underline{A_{v2} = -792}$

13.16

Let $I_{C10} = 40\ \mu$A, then $I_{C1} = I_{C2} = 20\ \mu$A. Using Example 13.5,

$R_{i2} = 4.07$ MΩ

$$r_{\pi 6} = \frac{(200)(0.026)}{0.020} = 260 \text{ k}\Omega$$

$$g_{m6} = \frac{0.020}{0.026} = 0.769 \text{ mA/V}$$

$$r_{06} = \frac{50}{0.02} \Rightarrow 2.5 \text{ M}\Omega$$

Then

$R_{act1} = 2.5[1 + (0.769)(1\|260)] = 4.42$ MΩ

$$r_{06} = \frac{50}{0.02} \Rightarrow 2.5 \text{ M}\Omega$$

Then

$$A_d = -\left(\frac{I_{CQ}}{V_T}\right)(r_{04}\|R_{act1}\|R_{i2})$$

$$= -\left(\frac{20}{0.026}\right)(2.5\|4.42\|4.07)$$

So

$\underline{A_d = -882}$

13.17

Now

$R_{e14} = \dfrac{r_{\pi 14} + R_{01}}{1 + \beta_P}$ and $R_0 = R_6 + R_{e14}$

Assume series resistance of Q_{18} and Q_{19} is small. Then

$R_{01} = r_{013A}\|R_{e22}$

where $R_{e22} = \dfrac{r_{\pi 22} + R_{017}\|r_{013B}}{1 + \beta_P}$

and $R_{017} = r_{017}[1 + g_{m17}(R_8\|r_{\pi 17})]$

Using results from Example 13.6,

$r_{\pi 17} = 9.63$ kΩ $\quad r_{\pi 22} = 7.22$ kΩ

$g_{m17} = 20.8$ mA/V $\quad r_{017} = 92.6$ kΩ

Then

$R_{017} = 92.6[1 + (20.8)(0.1\|9.63)] = 283$ kΩ

$r_{013B} = \dfrac{50}{0.54} = 92.6$ kΩ

Then

$R_{e22} = \dfrac{7.22 + 283\|92.6}{51} = 1.51$ kΩ

$R_{01} = r_{013A}\|R_{e22} = 278\|1.51 = 1.50$ kΩ

$r_{\pi 14} = \dfrac{(50)(0.026)}{2} = 0.65$ kΩ

Then

$R_{e14} = \dfrac{0.65 + 1.50}{51} = 0.0422$ kΩ

or

$R_{e14} = 42.2\ \Omega$

Then

$R_0 = 42.2 + 27 \Rightarrow \underline{R_0 = 69.2\ \Omega}$

13.18

$$R_{id} = 2\left[r_{\pi 1} + (1+\beta_n)\left(\frac{r_{\pi 3}}{1+\beta_P}\right)\right]$$

Assume $\beta_n = 200$ and $\beta_P = 10$

Then

$$r_{\pi 1} = \frac{(200)(0.026)}{0.0095} = 547 \text{ k}\Omega$$

$$r_{\pi 3} = \frac{(10)(0.026)}{0.0095} = 27.4 \text{ k}\Omega$$

Then

$$R_{id} = 2\left[547 + \frac{(201)(27.4)}{11}\right]$$

or

$$R_{id} = 2.095 \text{ M}\Omega$$

13.19

We can write

$$A(f) = \frac{A_0}{\left(1+j\frac{f}{f_{PD}}\right)\left(1+j\frac{f}{f_1}\right)} = \frac{356,796}{\left(1+j\frac{f}{5.43}\right)\left(1+j\frac{f}{f_1}\right)}$$

Phase:

$$\phi = -\tan^{-1}\left(\frac{f}{5.43}\right) - \tan^{-1}\left(\frac{f}{f_1}\right)$$

For a phase margin $= 70°$, $\phi = -110°$

So

$$-110° = -\tan^{-1}\left(\frac{f}{5.43}\right) - \tan^{-1}\left(\frac{f}{f_1}\right)$$

Assuming $f \gg 5.43$, we have

$$\tan^{-1}\left(\frac{f}{f_1}\right) = 20° \Rightarrow \frac{f}{f_1} = 0.364$$

At this frequency, $|A(f)| = 1$, so

$$1 = \frac{356,796}{\sqrt{1+\left(\frac{f}{5.43}\right)^2}\cdot\sqrt{1+(0.364)^2}} = \frac{335,275}{\sqrt{1+\left(\frac{f}{5.43}\right)^2}}$$

or $\frac{f}{5.43} = 335,275 \Rightarrow f = 1.82$ MHz

Then, second pole at

$$f_1 = \frac{f}{0.364} \Rightarrow \underline{f_1 = 5 \text{ MHz}}$$

13.20

a. Original g_{m1} and g_{m2}

$$K_{p1} = K_{p2} = \left(\frac{W}{L}\right)\left(\frac{\mu_p C_{ox}}{2}\right) = (12.5)(10)$$
$$= 125 \ \mu A/V^2$$

So

$$g_{m1} = g_{m2} = 2\sqrt{K_{p1}\left(\frac{I_Q}{2}\right)} = 2\sqrt{(0.125)(10)}$$
$$= 0.09975 \text{ mA/V}$$

If $\left(\frac{W}{L}\right)$ is increased to 50, then

$$K_{p1} = K_{p2} = (50)(10) = 500 \ \mu A/V^2$$

So

$$g_{m1} = g_{m2} = 2\sqrt{(0.5)(0.0199)} = 0.1995 \text{ mA/V}$$

b. Gain of first stage

$$A_d = g_{m1}(r_{o2}\|r_{o4}) = (0.1995)(5025\|5025)$$

or

$$\underline{A_d = 501}$$

Voltage gain of second stage remains the same, or

$$A_{v2} = 251$$

Then $A_v = A_d \cdot A_{v2} = (501)(251)$

or

$$\underline{A_d = 125,751}$$

13.22

a. $K_p = (10)(20) = 200 \ \mu A/V^2 = 0.2 \ mA/V^2$

$$I_{REF} = I_{SET} = \frac{10 - V_{SG} - (-10)}{200} = k_P(V_{SG} - 1.5)^2$$

$$20 - V_{SG} = (0.2)(200)(V_{SG}^2 - 3V_{SG} + 2.25)$$

$$40V_{SG}^2 - 119V_{SG} + 70 = 0$$

$$V_{SG} = \frac{119 \pm \sqrt{(119)^2 - 4(40)(70)}}{2(40)}$$
$$\Rightarrow \underline{V_{SG} = 2.17 \text{ V}}$$

Then

$$I_{REF} = \frac{20 - 2.17}{200} \Rightarrow \underline{I_{REF} = 89.2 \ \mu A}$$

M_5, M_6, M_8 matched transistors so that

$$\underline{I_Q = I_{D7} = I_{REF} = 89.2 \ \mu A}$$

b. **Small-signal voltage gain of input stage:**

$A_d = \sqrt{2K_{p1}I_Q} \cdot (r_{o2} \| r_{o4})$

$r_{02} = \dfrac{1}{\lambda_P I_D} = \dfrac{1}{(0.02)\left(\dfrac{89.2}{2}\right)} = 1.12 \text{ M}\Omega$

$r_{04} = \dfrac{1}{\lambda_n I_D} = \dfrac{1}{(0.01)\left(\dfrac{89.2}{2}\right)} = 2.24 \text{ M}\Omega$

Then

$A_d = \sqrt{2(200)(89.2)} \cdot (1.12 \| 2.24)$

or

$\underline{A_d = 141}$

Small-signal voltage gain of second stage:

$A_{\nu 2} = g_{m7}(r_{07} \| r_{08})$

$K_{n7} = (20)(20) = 400\ \mu A/V^2$

So

$g_{m7} = 2\sqrt{K_{n7}I_{D7}} = 2\sqrt{(0.4)(0.0892)}$

$= 0.378 \text{ mA/V}$

$r_{08} = \dfrac{1}{\lambda_P I_{D7}} = \dfrac{1}{(0.02)(0.0892)} = 561 \text{ k}\Omega$

$r_{07} = \dfrac{1}{\lambda_n I_{D7}} = \dfrac{1}{(0.01)(0.0892)} = 1121 \text{ k}\Omega$

So

$A_{\nu 2} = (0.378)(1121 \| 561) \Rightarrow \underline{A_{\nu 2} = 141}$

Then overall voltage gain

$A_\nu = A_d \cdot A_{\nu 2} = (141)(141) \Rightarrow \underline{A_\nu = 19,881}$

13.23

Small-signal voltage gain of input stage:

$A_d = \sqrt{2K_{p1}I_Q} \cdot (r_{o2} \| r_{o4})$

$K_{p1} = (10)(10) = 100\ \mu A/V^2$

$r_{02} = \dfrac{1}{\lambda_P\left(\dfrac{I_Q}{2}\right)} = \dfrac{1}{(0.01)\left(\dfrac{0.2}{2}\right)} = 1000 \text{ k}\Omega$

$r_{04} = \dfrac{1}{\lambda_n\left(\dfrac{I_Q}{2}\right)} = \dfrac{1}{(0.005)\left(\dfrac{0.2}{2}\right)} = 2000 \text{ k}\Omega$

Then

$A_d = \sqrt{2(0.1)(0.2)} \cdot (1000 \| 2000)$

or

$\underline{A_d = 133}$

Small-signal voltage gain of second stage:

$A_{\nu 2} = g_{m7}(r_{07} \| r_{08})$

$K_{n7} = (20)(20) = 400\ \mu A/V^2$

So

$g_{m7} = 2\sqrt{K_{n7}I_{D7}} = 2\sqrt{(0.4)(0.2)}$

$= 0.566 \text{ mA/V}$

$r_{08} = \dfrac{1}{\lambda_P I_{D7}} = \dfrac{1}{(0.01)(0.2)} = 500 \text{ k}\Omega$

$r_{07} = \dfrac{1}{\lambda_n I_{D7}} = \dfrac{1}{(0.005)(0.2)} = 1000 \text{ k}\Omega$

So

$A_{\nu 2} = (0.566)(1000 \| 500) \Rightarrow \underline{A_{\nu 2} = 189}$

Then overall voltage gain is

$A_\nu = A_d \cdot A_{\nu 2} = (133)(189) \Rightarrow \underline{A_\nu = 25,137}$

13.24

$$f_{PD} = \frac{1}{2\pi R_{eq} C_i}$$

where $R_{eq} = r_{04} \| r_{02}$ and $C_i = C_1(1 + |A_{\nu 2}|)$

We can find that

$A_{\nu 2} = 251$ and $r_{04} = r_{02} = 5.025 \text{ M}\Omega$

Now

$$R_{eq} = 5.025 \| 5.025 = 2.51 \text{ M}\Omega$$

and

$$C_i = 12(1 + 251) = 3024 \text{ pF}$$

So

$$f_{PD} = \frac{1}{2\pi(2.51 \times 10^6)(3024 \times 10^{-12})}$$

or

$\underline{f_{PD} = 21.0 \text{ Hz}}$

13.25

$$f_{PD} = \frac{1}{2\pi R_{eq} C_i}$$

where $R_{eq} = r_{04} \| r_{02}$

From Problem 13.22,

$r_{02} = 1.12 \text{ M}\Omega$, $r_{04} = 2.24 \text{ M}\Omega$ and $A_{\nu 2} = 141$

So

$$8 = \frac{1}{2\pi(1.12\|2.24) \times 10^6 \times C_i}$$

or

$C_i = 2.66 \times 10^{-8} = C_1(1 + |A_{v2}|) = C_1(142)$

or

$\underline{C_1 = 188 \text{ pF}}$

13.26

$R_0 = r_{07}\|r_{08}$

We can find that

$r_{07} = r_{08} = 2.52 \text{ M}\Omega$

Then

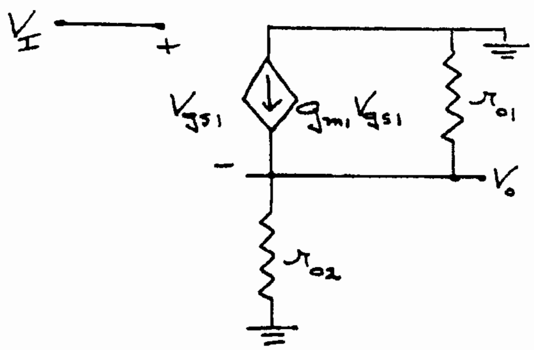
$R_0 = 2.52\|2.52$

or

$\underline{R_0 = 1.26 \text{ M}\Omega}$

13.27

a.



$V_0 = (g_{m1}V_{gs1})(r_{01}\|r_{02})$

$V_I = V_{gs1} + V_0$

Then $V_0 = g_{m1}(r_{01}\|r_{02})(V_I - V_0)$

or

$$\underline{A_v = \frac{g_{m1}(r_{01}\|r_{02})}{1 + g_{m1}(r_{01}\|r_{02})}}$$

b. $I_X + g_{m1}V_{gs1} = \dfrac{V_X}{r_{02}} + \dfrac{V_X}{r_{01}}$ and $V_{gs1} = -V_X$

$$\underline{R_0 = \frac{1}{g_{m1}} \,\Big\|\, r_{01} \,\Big\|\, r_{02}}$$

13.28

(a) $A_d = g_{m1}(R_{o6}\|R_{o8})$

$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.025)} \Rightarrow 224\ \mu A/V$

$g_{m1} = g_{m8}$

$g_{m6} = 2\sqrt{(0.5)(0.025)} \Rightarrow 224\ \mu A/V$

$$r_{o1} = r_{o6} = r_{o8} = r_{o10} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(25)} = 2.67\ M\Omega$$

$$r_{o4} = \frac{1}{\lambda I_{D4}} = \frac{1}{(0.015)(50)} \Rightarrow 1.33\ M\Omega$$

Now

$R_{o8} = g_{m8}(r_{o8}r_{o10}) = (224)(2.67)(2.67) = 1597\ M\Omega$

$R_{o6} = g_{m6}(r_{o6})(r_{o4}\|r_{o1}) = (224)(2.67)(2.67\|1.33) \Rightarrow$

$R_{o6} = 531\ M\Omega$

Then

$A_d = (224)(531\|1597) \Rightarrow \underline{A_d = 89{,}264}$

(b) $R_o = R_{o6}\|R_{o8} = 531\|1597 \Rightarrow \underline{R_o = 398\ M\Omega}$

(c) $$f_{PD} = \frac{1}{2\pi R_o C_L} = \frac{1}{2\pi(398x10^6)(5x10^{-12})} \Rightarrow$$

$\underline{f_{PD} = 80\ Hz}$

$GBW = (89{,}264)(80) \Rightarrow \underline{GBW = 7.14\ MHz}$

13.29

(a) $$r_{o1} = r_{o8} = r_{o10} = \frac{1}{\lambda_p I_D} = \frac{1}{(0.02)(25)} = 2\ M\Omega$$

$$r_{o6} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.015)(25)} = 2.67\ M\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(50)} = 1.33\ M\Omega$$

$$g_{m1} = 2\sqrt{\left(\frac{35}{2}\right)\left(\frac{W}{L}\right)_1(25)} = 41.8\sqrt{\left(\frac{W}{L}\right)_1} = g_{m8}$$

$$g_{m6} = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_6(25)} = 63.2\sqrt{\left(\frac{W}{L}\right)_6}$$

$R_o = R_{o6}\|R_{o8} = [g_{m6}(r_{o6})(r_{o4}\|r_{o1})]\|[g_{m8}(r_{o8}r_{o10})]$

Define $X_1 = \sqrt{\left(\frac{W}{L}\right)_1}$ and $X_6 = \sqrt{\left(\frac{W}{L}\right)_6}$

Then $R_o = [63.2X_6(2.67)(1.33\|2)]\|[41.8X_1(2)(2)]$

$$= 134.8X_6\|167.2X_1 = \frac{22{,}539X_1X_6}{134.8X_6 + 167.2X_1}$$

$$A_d = g_{m1}R_o = (41.8X_1)\left(\frac{22{,}539X_1X_6}{134.8X_6 + 167.2X_1}\right)$$

$= 10{,}000$

Now $$X_6 = \sqrt{\left(\frac{W}{L}\right)_6} = \sqrt{\frac{1}{2.2}\left(\frac{W}{L}\right)_1} = 0.674X_1$$

We then find

$X_1^2 = \left(\frac{W}{L}\right)_1 = 4.06 = \left(\frac{W}{L}\right)_p$

and

$\left(\frac{W}{L}\right)_n = 1.85$

13.30

Let $V^+ = 5V, V^- = -5V$

$P = I_T(10) = 3 \Rightarrow I_T = 0.3\ mA$

$\Rightarrow I_{REF} = 0.1\ mA = 100\ \mu A$

$r_{o1} = r_{o8} = r_{o10} = \frac{1}{(0.02)(50)} = 1\ M\Omega$

$r_{o6} = \frac{1}{(0.015)(50)} = 1.33\ M\Omega$

$r_{o4} = \frac{1}{(0.015)(100)} = 0.667\ M\Omega$

$g_{m1} = 2\sqrt{\left(\frac{35}{2}\right)\left(\frac{W}{L}\right)_1 (50)} = 59.2X_1 = g_{m8}$

where $X_1 = \sqrt{\left(\frac{W}{L}\right)_1}$

Assume all width-to-length ratios are the same.

$g_{m6} = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)(50)} = 89.4X_1$

Now

$R_o = R_{o6} \| R_{o8} = [g_{m6}(r_{o6})(r_{o4} \| r_{o1})] \| [g_{m8}(r_{o8} r_{o10})]$

$= [89.4X_1(1.33)(0.667 \| 1)] \| [59.2X_1(1)(1)]$

$= [47.6X_1] \| [59.2X_1] = \frac{(47.6X_1)(59.2X_1)}{47.6X_1 + 59.2X_1}$

So $R_o = 26.4X_1$

Now

$A_d = g_{m1}R_o = (59.2X_1)(26.4X_1) = 25{,}000$

So that $X_1^2 = \frac{W}{L} = 16$ for all transistors

13.31

(a) $A_d = Bg_{m1}(r_{o6} \| r_{o8})$

$r_{o6} = r_{o8} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(90)} = 0.741\ M\Omega$

$g_{m1} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right) I_{D1}} = 2\sqrt{(500)(30)} = 245\ \mu A/V$

$A_d = (3)(245)(0.741 \| 0.741) \Rightarrow A_d = 272$

(b) $R_o = r_{o6} \| r_{o8} = 0.741 \| 0.741 \Rightarrow R_o = 371\ k\Omega$

(c) $f_{PD} = \frac{1}{2\pi R_o C} = \frac{1}{2\pi(371x10^3)(5x10^{-12})} \Rightarrow$

$f_{PD} = 85.8\ kHz$

$GBW = (272)(85.8x10^3) \Rightarrow GBW = 23.3\ MHz$

13.32

(a) $r_{o6} = \frac{1}{(0.02)(2.5)(40)} = 0.5\ M\Omega$

$r_{o8} = \frac{1}{(0.015)(2.5)(40)} = 0.667\ M\Omega$

$A_d = Bg_{m1}(r_{o6} \| r_{o8})$

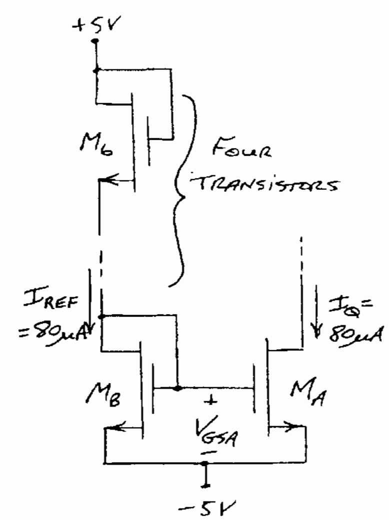
$400 = (2.5)g_{m1}(0.5 \| 0.667) \Rightarrow g_{m1} = 560\ \mu A/V$

$g_{m1} = 560 = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)(40)} \Rightarrow \left(\frac{W}{L}\right) = 49$

Assume all (W/L) ratios are the same except for M_5 and M_6. $\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = 122.5$

(b) Assume the bias voltages are

$V^+ = 5V, V^- = -5V$.



Assume $\left(\frac{W}{L}\right)_A = \left(\frac{W}{L}\right)_B = 49$

$I_Q = \left(\frac{80}{2}\right)(49)(V_{GSA} - 0.5)^2 = 80 \Rightarrow V_{GSA} = 0.702\ V$

Then

$I_{REF} = 80 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_C (V_{GSC} - 0.5)^2$

For four transistors

$$V_{GSC} = \frac{10-0.702}{4} = 2.325\,V$$

$$80 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_C (2.325-0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_C = 0.60$$

(c) $f_{3-dB} = \dfrac{1}{2\pi R_o C}$ $\quad R_o = 0.5\|0.667 = 0.286\ M\Omega$

$$f_{3-dB} = \frac{1}{2\pi(286x10^3)(3x10^{-12})} = 185\ kHz$$

$$GBW = (400)(185x10^3) \Rightarrow 74\ MHz$$

13.33

(a) From previous results, we can write

$$R_{o10} = g_{m10}(r_{o10}r_{o6})$$

$$R_{o12} = g_{m12}(r_{o12}r_{o8})$$

$$A_d = Bg_{m1}(R_{o10}\|R_{o12})$$

Now

$$r_{o10} = r_{o6} = \frac{1}{\lambda_p B(I_Q/2)} = \frac{1}{(0.02)(2.5)(40)} = 0.5\ M\Omega$$

$$r_{o12} = r_{o8} = \frac{1}{\lambda_n B(I_Q/2)} = \frac{1}{(0.015)(2.5)(40)} = 0.667\ M\Omega$$

Assume all transistors have the same width-to-length ratios except for M_5 and M_6.

Let $\left(\frac{W}{L}\right) = X^2$

Then

$$g_{m10} = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_{10}(I_{DQ10})} = 2\sqrt{\left(\frac{35}{2}\right)X^2(2.5)(40)}$$

$$= 83.67X$$

$$g_{m12} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_{12}(I_{DQ12})} = 2\sqrt{\left(\frac{80}{2}\right)X^2(2.5)(40)}$$

$$= 126.5X$$

$$g_{m1} = 2\sqrt{\left(\frac{80}{2}\right)X^2(40)} = 80X$$

Then

$$R_{o10} = (83.67X)(0.5)(0.5) = 20.9X\ M\Omega$$

$$R_{o12} = (126.5X)(0.667)(0.667) = 56.3X\ M\Omega$$

We want

$$20{,}000 = (2.5)(80X)[20.9X\|56.3X]$$

$$= 200X\left[\frac{(20.9X)(56.3X)}{20.9X+56.3X}\right] = 3048X^2$$

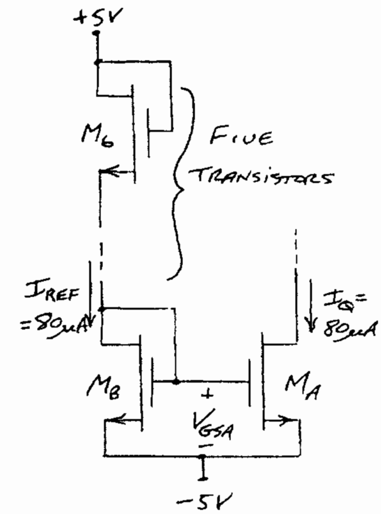
Then

$$X^2 = 6.56 = \left(\frac{W}{L}\right)$$

Then

$$\left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_5 = (2.5)(6.56) = 16.4$$

(b) Assume bias voltages are $V^+ = 5V$, $V^- = -5V$



Assume $\left(\frac{W}{L}\right)_A = \left(\frac{W}{L}\right)_B = 6.56$

$$I_Q = 80 = \left(\frac{80}{2}\right)(6.56)(V_{GSA}-0.5)^2 \Rightarrow$$

$$V_{GSA} = 1.052\,V$$

Need 5 transistors in series

$$V_{GSC} = \frac{10-1.052}{5} = 1.79\,V$$

Then

$$I_{REF} = 80 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_C (1.79-0.5)^2 \Rightarrow$$

$$\left(\frac{W}{L}\right)_C = 1.20$$

(c) $f_{3-dB} = \dfrac{1}{2\pi R_o C}$ where $R_o = R_{o10}\|R_{o12}$

Now

$$R_{o10} = 20.9\sqrt{6.56} = 53.5\ M\Omega$$

$$R_{o12} = 56.3\sqrt{6.56} = 144\ M\Omega$$

Then

$$R_o = 53.5\|144 = 39\ M\Omega$$

$$f_{3-dB} = \frac{1}{2\pi(39x10^6)(3x10^{-12})} = 1.36\ kHz$$

$$GBW = (20{,}000)(1.36x10^3) \Rightarrow GBW = 27.2\ MHz$$

13.34

$A_d = g_m(M_2)\cdot[r_{o2}(M_2)\|r_{o2}(Q_2)]$

$g_m(M_2) = 2\sqrt{\left(\frac{40}{2}\right)(25)(100)} = 447\ \mu A/V$

$r_{o2}(M_2) = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500\ k\Omega$

$r_{o2}(Q_2) = \frac{V_A}{I_{CQ}} = \frac{120}{0.1} = 1200\ k\Omega$

Then

$A_d = 447(0.5\|1.2) \Rightarrow \underline{A_d = 158}$

13.35

$A_d = g_m(M_2)\cdot[r_{o2}(M_2)\|r_{o2}(Q_2)]$

$g_m(M_2) = 2\sqrt{\left(\frac{80}{2}\right)(25)(100)} = 632\ \mu A/V$

$r_{o2}(M_2) = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(0.1)} = 667\ k\Omega$

$r_{o2}(Q_2) = \frac{V_A}{I_{CQ}} = \frac{80}{0.1} = 800\ k\Omega$

$A_d = (632)(0.667\|0.80) \Rightarrow \underline{A_d = 230}$

13.36

$I_{REF} = 200\ \mu A \qquad K_n = K_p = 0.5\ mA/V^2$

$\lambda_n = \lambda_p = 0.015\ V^{-1}$

$A_d = g_{m1}(R_{o6}\|R_{o8})$

where

$R_{o8} = g_{m8}(r_{o8}r_{o10})$

$R_{o6} = g_{m6}(r_{o6})(r_{o4}\|r_{o1})$

Now

$g_{m8} = 2\sqrt{K_p I_{D8}} = 2\sqrt{(0.5)(0.1)} = 0.447\ mA/V$

$r_{o8} = \frac{1}{\lambda_p I_{D8}} = \frac{1}{(0.015)(0.1)} = 667\ k\Omega$

$r_{o10} = \frac{1}{\lambda_p I_{D8}} = 667\ k\Omega$

$g_{m6} = \frac{I_{C6}}{V_T} = \frac{0.1}{0.026} = 3.846\ mA/V$

$r_{o6} = \frac{V_A}{I_{C6}} = \frac{80}{0.1} = 800\ k\Omega$

$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(0.2)} = 333\ k\Omega$

$r_{o1} = \frac{1}{\lambda_p I_{D1}} = \frac{1}{(0.015)(0.1)} = 667\ k\Omega$

$g_{m1} = 2\sqrt{K_p I_{D1}} = 2\sqrt{(0.5)(0.1)} = 0.447\ mA/V$

So

$R_{o8} = (0.447)(667)(667) \Rightarrow 198.9\ M\Omega$

$R_{o6} = (3.846)(800)(333\|667) \Rightarrow 683.4\ M\Omega$

Then

$A_d = 447(198.9\|683.4) \Rightarrow \underline{A_d = 68{,}865}$

13.37

Assume biased at $V^+ = 10\ V$, $V^- = -10\ V$.

$P = 3I_{REF}(20) = 10 \Rightarrow I_{REF} = 167\ \mu A$

$A_d = g_{m1}(R_{o6}\|R_{o8}) = 25{,}000$

$k_n' = 80\ \mu A/V^2$, $k_p' = 35\ \mu A/V^2$

$\lambda_n = 0.015\ V^{-1}$, $\lambda_p = 0.02\ V^{-1}$

Assume $\left(\frac{W}{L}\right)_p = 2.2\left(\frac{W}{L}\right)_n$

$R_{o8} = g_{m8}(r_{o8}r_{o10})$

$R_{o6} = g_{m6}(r_{o6})(r_{o4}\|r_{o1})$

$r_{o8} = \frac{1}{\lambda_p I_{D8}} = \frac{1}{(0.02)(83.3)} = 0.60\ M\Omega$

$r_{o10} = \frac{1}{\lambda_p I_{D8}} = 0.60\ M\Omega$

$g_{m8} = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_8 I_{D8}} = 2\sqrt{\left(\frac{35}{2}\right)(2.2)X^2(83.3)}$

$= 113.3X$

where $X^2 = \left(\frac{W}{L}\right)_n$

$r_{o6} = \frac{V_A}{I_{C6}} = \frac{80}{83.3} = 0.960\ M\Omega$

$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(167)} = 0.40\ M\Omega$

$r_{o1} = \frac{1}{\lambda_p I_{D1}} = \frac{1}{(0.02)(83.3)} = 0.60\ M\Omega$

$g_{m6} = \frac{I_{C6}}{V_T} = \frac{83.3}{0.026} = 3204\ \mu A/V$

$g_{m1} = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_1 I_{D1}} = 2\sqrt{\left(\frac{35}{2}\right)(2.2)X^2(83.3)}$

$= 113.3X$

Now

$R_{o6} = (3204)(0.960)[0.40\|0.60] = 738\ M\Omega$

$R_{o8} = (113.3X)(0.60)(0.60) = 40.8X\ M\Omega$

Then

$A_d = 25{,}000 = (113.3X)[738\|40.8X]$

$= (113.3X)\left[\frac{30{,}110X}{738 + 40.8X}\right]$

which yields $X = 2.48$

or

$$X^2 = 6.16 = \left(\frac{W}{L}\right)_n$$

and

$$\left(\frac{W}{L}\right)_p = (2.2)(6.16) = 12.3$$

13.38

For $v_{cm}(\max)$, assume $V_{CB}(Q_5) = 0$. Then

$$V_S = 15 - 0.6 - 0.6 = 13.8 \text{ V}$$

$$I_{D9} = I_{D10} = \frac{0.236}{2} = 0.118 \text{ mA}$$

Using parameters given in Example 13.11

$$V_{SG} = \sqrt{\frac{I_{D9}}{K_p}} - V_{TP} = \sqrt{\frac{0.118}{0.20}} + 1.4 = 2.17 V$$

Then

$$v_{cm}(\max) = 13.8 - 2.17 \Rightarrow v_{cm}(\max) = 11.6 \text{ V}$$

For

$v_{cm}(\min)$, assume

$$V_{SD}(M_9) = V_{SD}(sat) = V_{SG} + V_{TP}$$
$$= 2.17 - 1.4 = 0.77 \text{ V}$$

Now

$$V_{D10} = I_{D10}(0.5) + 0.6 + I_{D10}(0.5) - 15$$
$$= 0.118 + 0.6 - 15 \Rightarrow V_{D10} = -14.28 \text{ V}$$

Then

$$v_{cm}(\min) = -14.28 + V_{SD}(\text{sat}) - V_{SG}$$
$$= -14.28 + 0.77 - 2.17 = -15.68 \text{ V}$$

Then, common-mode voltage range

$$\underline{-15.68 \le v_{cm} \le 11.6}$$

Or, assuming the input is limited to ± 15 V, then

$$\underline{-15 \le v_{cm} \le 11.6 \text{ V}}$$

13.39

For $I_1 = I_2 = 300\ \mu$A,

$$V_{SG} = V_{BE} + (0.3)(8) = 0.6 + 2.4 = 3.0 \text{ V}$$

Then

$$I_2 = K_p(V_{SG} + V_{TP})^2$$
$$0.3 = K_p(3 - 1.4)^2$$
$$\Rightarrow \underline{K_p = 0.117\ mA/V^2}$$

13.40

For $V_{CB} = 0$ for both Q_6 and Q_7, then

$$V_S = 0.6 + 0.6 + V_{SG} + (-V_S)$$

So $2V_S = 1.2 + V_{SG}$

Now

$$0.6 + I_2R_1 = V_{SG} = \sqrt{\frac{I_1}{K_p}} + V_{TP} \quad \text{and} \quad I_1 = I_2$$

Also $I_1 = I_2 = K_p(V_{SG} + V_{TP})^2$ so

$$0.6 + (0.25)(8)(V_{SG} - 1.4)^2 = V_{SG}$$
$$0.6 + 2(V_{SG}^2 - 2.8V_{SG} + 1.96) = V_{SG}$$
$$2V_{SG}^2 - 6.6V_{SG} + 4.52 = 0$$
$$V_{SG} = \frac{6.6 \pm \sqrt{(6.6)^2 - 4(2)(4.52)}}{2(2)} = 2.33 \text{ V}$$

Then $2V_S = 1.2 + 2.33 = 3.53$ and

$$\underline{V_S = 1.765 \text{ V}}$$

13.41

$$I_{C5} = I_{C4} = 300\ \mu\text{A}$$

Using the parameters from Examples 13.12 and 13.13, we have

$$R_{i2} = r_{\pi 13} = \frac{\beta_n V_T}{I_{C13}} = \frac{(200)(0.026)}{0.3} = 17.3 \text{ k}\Omega$$

$$A_d = \sqrt{2K_n I_{Q5}} \cdot (R_{i2}) = \sqrt{2(0.6)(0.3)} \cdot (17.3)$$

or

$$\underline{A_d = 10.38}$$

Now

$$g_{m13} = \frac{I_{C13}}{V_T} = \frac{0.3}{0.026} = 11.5 \text{ mA/V}$$

$$r_{o13} = \frac{V_A}{I_{C13}} = \frac{50}{0.3} = 167 \text{ k}\Omega$$

Then

$$|A_{v2}| = g_{m13} \cdot r_{o13} = (11.5)(167)$$

or

$$\underline{|A_{v2}| = 1917}$$

Overall gain:

$$\underline{|A_v| = (10.38)(1917) = 19,895}$$

13.42

Assuming the resistances looking into Q_4 and into the output stage are very large, we have

$$|A_{v2}| = \frac{\beta R_{o13}}{r_{\pi 13} + (1+\beta)R_{E13}}$$

where $R_{o13} = r_{o13}[1 + g_{m13}(R_{E13} \| r_{\pi 13})]$

$I_{C13} = 300\ \mu A,\ r_{o13} = \dfrac{50}{0.3} = 167\ k\Omega$

$g_{m13} = \dfrac{0.3}{0.026} = 11.5\ mA/V$

$r_{\pi13} = \dfrac{(200)(0.026)}{0.3} = 17.3\ k\Omega$

So

$R_{o13} = (167)[1 + (11.5)(1\|17.3)] \Rightarrow 1.98\ M\Omega$

Then

$$|A_{v2}| = \frac{(200)(1980)}{17.3 + (201)(1)} = 1814$$

Now

$C_i = C_1(1 + |A_{v2}|) = 12[1 + 1814]$

$\Rightarrow C_i = 21,780\ pF$

$f_{PD} = \dfrac{1}{2\pi R_{eq} C_i}$

$R_{eq} = R_{i2}\|r_{o12}\|r_{o10}$

Neglecting R_3,

$r_{o10} = \dfrac{1}{\lambda I_{D10}} = \dfrac{1}{(0.02)(0.15)} = 333\ k\Omega$

Neglecting R_5,

$r_{o12} = \dfrac{50}{0.15} = 333\ k\Omega$

$R_{i2} = r_{\pi13} + (1+\beta)R_{E13} = 17.3 + (201)(1)$

$= 218\ k\Omega$

Then

$$f_{PD} = \frac{1}{2\pi[218\|333\|333] \times 10^3 \times (21,780) \times 10^{-12}}$$

or

$\underline{f_{PD} = 77.4\ Hz}$

Unity-Gain Bandwidth
Gain of first stage:

$A_d = \sqrt{2K_n I_{Q5}} \cdot (R_{i2}\|r_{o12}\|r_{o10})$

$= \sqrt{2(0.6)(0.3)} \cdot (218\|333\|333)$

$= (0.6)(218\|333\|333)$

or $A_d = 56.6$

Overall gain:

$A_v = (56.6)(1814) = 102,672$

Then unity-gain bandwidth $= (77.4)(102,672)$

$\Rightarrow \underline{7.95\ MHz}$

13.43

Since $V_{GS} = 0$ in J_6, $I_{REF} = I_{DSS}$

$\Rightarrow \underline{I_{DSS} = 0.8\ mA}$

13.44

a. $R_{i2} = r_{\pi5} + (1+\beta)[r_{\pi6} + (1+\beta)R_E]$

$r_{\pi6} = \dfrac{(100)(0.026)}{0.2} = 13\ k\Omega$

$I_{C5} \cong \dfrac{I_{C6}}{\beta} = \dfrac{200\ \mu A}{100} = 2\ \mu A$

So

$r_{\pi5} = \dfrac{(100)(0.026)}{0.002} = 1300\ k\Omega$

Then

$R_{i2} = 1300 + (101)[13 + (101)(0.3)]$

or

$\underline{R_{i2} = 5.67\ M\Omega}$

b. $A_v = g_{m2}(r_{o2}\|r_{o4}\|R_{i2})$

$g_{m2} = \dfrac{2}{V_P} \cdot \sqrt{I_D \cdot I_{DSS}} = \dfrac{2}{3} \cdot \sqrt{(0.1)(0.2)}$

$= 0.0943\ mA/V$

$r_{o2} = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.02)(0.1)} = 500\ k\Omega$

$r_{o4} = \dfrac{V_A}{I_{C4}} = \dfrac{5.0}{0.1} = 500\ k\Omega$

Then

$A_v = (0.0943)[500\|500\|5670]$

or

$\underline{A_v = 22.6}$

13.45

a. Need $V_{SD}(Q_E) \ge V_{SD}(sat) = V_P$
For minimum bias ± 3 V

Set $\underline{V_P = 3\ V}$ and $\underline{V_{ZK} = 3\ V}$

$I_{REF2} = \dfrac{V_{ZK} - V_{D1}}{R_3}$

so that $R_3 = \dfrac{3 - 0.6}{0.1} \Rightarrow \underline{R_3 = 24\ k\Omega}$

Set bias in $Q_E = I_{REF2} + I_{Z2} = 0.1 + 0.1 = 0.2$ mA

Therefore,

$\underline{I_{DSS} = 0.2\ mA}$

b. Neglecting base currents

$$I_{01} = I_{REF1} = 0.5 \text{ mA} = \frac{12 - 0.6}{R_4}$$

so that

$$\underline{R_4 = 22.8 \text{ k}\Omega}$$

13.46

a. We have

$$g_{m2} = \frac{2}{|V_P|} \cdot \sqrt{I_D \cdot I_{DSS}} = \frac{2}{4} \cdot \sqrt{(0.5)(1)}$$

$$= 0.354 \text{ mA/V}$$

$$r_{02} = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.5)} = 100 \text{ k}\Omega$$

$$r_{04} = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$g_{m4} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi 4} = \frac{(200)(0.026)}{0.5} = 10.4 \text{ k}\Omega$$

So

$$R_{04} = r_{04}[1 + g_{m4}(r_{\pi 4} \| R_2)]$$
$$= 200[1 + (19.23)(10.4 \| 0.5)]$$
$$= 2035 \text{ k}\Omega$$

$$|A_d| = g_{m2}(r_{02} \| R_{04} \| R_L)$$

For $R_L \to \infty$

$$|A_d| = 0.354(100 \| 2035) = 33.7$$

With these parameter values, gain can never reach 500.

b. Similarly for this part, gain can never reach 700.

Chapter 14

Exercise Solutions

E14.1

$v_{ICM}(\max) = V^+ - V_{SD1}(sat) - V_{SG1}$

$v_{ICM}(\min) = V^- + V_{DS4}(sat) + V_{SD1}(sat) - V_{SG1}$

We have:

$I_{REF} = 100\ \mu A,\ k_n' = 80\ \mu A/V^2,\ k_p' = 40\ \mu A/V^2,$

$\left(\frac{W}{L}\right) = 25$

For M_1:

$I_D = 50 = \left(\frac{40}{2}\right)(25)(V_{SG1} + V_{TP})^2$

So $50 = 500(V_{SG1} - 0.5)^2 \Rightarrow V_{SG1} = 0.816\,V$

$V_{SD1}(sat) = 0.816 - 0.5 = 0.316\,V$

Then

$v_{CM}(\max) = V^+ - 0.316 - 0.816 = V^+ - 1.13\,V$

For M_4:

$I_D = 100 = \left(\frac{80}{2}\right)(25)(V_{GS4} - V_{TN})^2$

So $100 = 1000(V_{GS4} - 0.5)^2 \Rightarrow V_{GS4} = 0.816\,V$

$V_{DS4}(sat) = 0.816 - 0.5 = 0.316\,V$

$v_{CM}(\min) = V^- + 0.316 + 0.316 - 0.816 = V^- - 0.184$

So

$\underline{V^- - 0.184 \le v_{CM} \le V^+ - 1.13\,V}$

E14.2

$v_o(\max) = V^+ - V_{SD8}(sat) - V_{SD10}(sat)$

$v_o(\min) = V^- + V_{DS4}(sat) + V_{DS6}(sat)$

Now

$V_{SG8} = V_{SG10} = \sqrt{\frac{50}{(40/2)(25)}} + 0.5 = 0.816\,V$

$V_{SD8}(sat) = V_{SD10}(sat) = 0.316\,V$

So $v_o(\max) = V^+ - 0.316 - 0.316 = V^+ - 0.632$

Also

$V_{GS6} = \sqrt{\frac{50}{(80/2)(25)}} + 0.5 = 0.724\,V$

$V_{GS4} = \sqrt{\frac{100}{(80/2)(25)}} + 0.5 = 0.816\,V$

$V_{DS6}(sat) = 0.724 - 0.5 = 0.224\,V$

$V_{DS4}(sat) = 0.816 - 0.5 = 0.316\,V$

So $v_o(\min) = V^- + 0.316 + 0.224 = V^- + 0.54$

Then

$\underline{V^- + 0.54 \le v_o \le V^+ - 0.632\,V}$

E14.3

a. $A_{CL} = \dfrac{-50}{\left[1 + \left(\frac{1}{5 \times 10^4}\right)(51)\right]}$

$\Rightarrow \underline{A_{CL} = -49.949}$

b. $\dfrac{dA_{CL}}{A_{CL}} = 10 \times \dfrac{51}{5 \times 10^4} \Rightarrow \underline{\dfrac{dA_{CL}}{A_{CL}} = 0.0102\%}$

$A_{CL} = \dfrac{-50}{\left[1 + \frac{51}{4.5 \times 10^4}\right]} \Rightarrow \underline{A_{CL} = -49.943}$

E14.4

$A_{CL}(\text{ideal}) = -\frac{500}{20} = -25$

Within 0.1% $\Rightarrow -25 + (0.001)(25)$

$\Rightarrow A_{CL} = -24.975$

$-24.975 = \dfrac{-25}{\left[1 + \frac{26}{A_{OL}}\right]}$

$\dfrac{26}{A_{OL}} = \dfrac{-25}{-24.975} - 1 = 0.0010$

$\underline{A_{OL} = 25,974}$

E14.5

a. $A_{CL} = \dfrac{A_{CL}(\infty)}{1 + \left[\frac{A_{CL}(\infty)}{A_{OL}}\right]}$

$A_{CL}(\infty) = 1 + \frac{R_2}{R_1} = 1 + \frac{495}{5} = 100$

$A_{CL} = \dfrac{100}{1 + \frac{100}{10^5}} \Rightarrow \underline{A_{CL} = 99.90}$

$\underline{A_{CL}(\infty) = 100}$

b. $\dfrac{dA_{CL}}{A_{CL}} = 10 \times \dfrac{100}{10^5} = \underline{0.01\%}$

$A_{CL} = 99.90 - (0.0001)(99.90)$

$\Rightarrow \underline{A_{CL} = 99.89}$

E14.6

$\dfrac{A_{CL}(\infty) - A_{CL}}{A_{CL}(\infty)} = 1 - \dfrac{A_{CL}}{A_{CL}(\infty)} = 1 - \dfrac{1}{1 + \frac{A_{CL}(\infty)}{A_{CL}}}$

So $0.001 = \dfrac{1 + \frac{A_{CL}(\infty)}{A_{CL}} - 1}{1 + \frac{A_{CL}(\infty)}{A_{CL}}} = \dfrac{\frac{A_{CL}(\infty)}{A_{CL}}}{1 + \frac{A_{CL}(\infty)}{A_{CL}}}$

$$0.001 = 0.999 \cdot \frac{A_{CL}(\infty)}{A_{OL}}$$

$$A_{CL}(\infty) = \frac{0.001}{0.999} \cdot A_{OL} = \frac{0.001}{0.999} \cdot (10^4)$$

$$\Rightarrow \underline{A_{CL}(\infty) = 10.010}$$

or

$$A_{CL} = (1 - 0.001)(10.010)$$

$$\Rightarrow \underline{A_{CL} = 10.0}$$

E14.7

a. For $R_0 = 0$

$$\frac{1}{R_{if}} = \frac{1}{10} + \frac{1}{10}(1 + 10^4) = 0.1 + 10^3$$

$$\Rightarrow \underline{R_{if} = 10^{-3}\ \text{k}\Omega = 1\ \Omega}$$

b. For $R_0 = 10\ \text{k}\Omega$

$$\frac{1}{R_{if}} = \frac{1}{10} + \frac{1}{10} \times \left[\frac{1 + 10^4 + 1}{1 + 1 + 1}\right] \cong 0.1 + \frac{10^4}{3(10)}$$

$$R_{if} = 3 \times 10^{-3}\ \text{k}\Omega$$

$$\Rightarrow \underline{R_{if} = 3\ \Omega}$$

E14.8

$$\frac{i_I}{i_1} = \left(\frac{R_{if}}{R_i}\right)$$

a. $\frac{i_I}{i_1} = \frac{0.1}{10^4} = \underline{1 \times 10^{-5}}$

b. $\frac{i_I}{i_1} = \frac{10}{10^4} = \underline{1 \times 10^{-3}}$

E14.9

$$R_{if} = \frac{40(1 + 10^4) + 99\left(1 + \frac{40}{1}\right)}{1 + \frac{99}{1}}$$

$$\cong \frac{4 \times 10^5 + 4.059 \times 10^3}{100}$$

$$R_{if} = 4.04 \times 10^3\ \text{k}\Omega \Rightarrow \underline{R_{if} = 4.04\ \text{M}\Omega}$$

E14.10

Voltage follower $R_2 = 0,\ R_1 = \infty$

$$R_{if} = R_i(1 + A_{OL}) = 10(1 + 5 \times 10^5)$$

$$\cong 5 \times 10^6\ \text{k}\Omega \Rightarrow \underline{R_{if} = 5000\ \text{M}\Omega}$$

E14.11

$$1 + \frac{R_2}{R_1} = 100$$

a. $\frac{1}{R_{0f}} = \frac{1}{100}\left[\frac{10^5}{100}\right] = 10$

$$\Rightarrow \underline{R_{0f} = 0.1\ \Omega}$$

b. $\frac{1}{R_{0f}} = \frac{1}{10}\left[\frac{10^5}{100}\right] = 10^2$

$$R_{0f} = 10^{-2}\ \text{k}\Omega \Rightarrow \underline{R_{0f} = 10\ \Omega}$$

E14.12

From Equation (14.43)

$$A_{CL}(f) = \frac{A_{CL0}}{1 + j \cdot \frac{f}{f_{PD}(A_0/A_{CL0})}}$$

$$= \frac{25}{1 + j\ \frac{f}{(50)(10^4/25)}} = \frac{25}{1 + j \cdot \frac{f}{2 \times 10^4}}$$

a. For $f = 2$ kHz

$$\frac{v_O}{v_I} = 25 \Rightarrow \underline{v_O(\text{peak}) = 1.25\ \text{mV}}$$

b. $f = 20$ kHz

$$\frac{v_O}{v_I} = \frac{1}{\sqrt{2}} \cdot 25 \Rightarrow \underline{v_O(\text{peak}) = 0.884\ \text{mV}}$$

c. $f = 100$ kHz

$$\frac{v_O}{v_I} = \frac{25}{\sqrt{1 + (100/20)^2}} = \frac{25}{5.099} = 4.90$$

$$\Rightarrow \underline{v_O = 0.245\ \text{mV}}$$

E14.13

Full-scale response $= 1 \times 5 = 5$ V

$$t = \frac{5}{2} \Rightarrow \underline{t = 2.5\ \mu s}$$

E14.14

a. $FPBW = \frac{SR}{2\pi V_0(\max)} = \frac{0.63 \times 10^6}{2\pi(1)}$

$$FPBW = 1.0 \times 10^5 \Rightarrow \underline{FPBW = 100\ \text{kHz}}$$

b. $FPBW = \frac{0.63 \times 10^6}{2\pi(10)} = 1.0 \times 10^4$

$$\Rightarrow \underline{FPBW = 10\ \text{kHz}}$$

E14.15

$$f_{3dB} = \frac{f_T}{A_{CL0}} = \frac{(10^5)(10)}{50} \Rightarrow 20 \text{ kHz}$$

$$f_{max} = f_{3dB} = \frac{SR}{2\pi V_0(\max)}$$

$$V_0(\max) = \frac{SR}{2\pi f_{3dB}} = \frac{0.8 \times 10^6}{2\pi(20 \times 10^3)}$$

$$\Rightarrow \underline{V_0(\max) = 6.37 \text{ V}}$$

E14.16

$$|V_{OS}| = \left|V_T \ln\left(\frac{I_{S2}}{I_{S1}}\right)\right| = (0.026)\ln\left(\frac{1.85 \times 10^{-14}}{2 \times 10^{-14}}\right)$$

$$\Rightarrow \underline{V_{OS} = 2.03 \text{ mV}}$$

E14.17

We need

$i_{C1} = i_{C2}$, $v_{EC3} = v_{EC4} = 0.6$ V, and

$v_{CE1} = v_{CE2} = 10$ V

By Equation (14.60(a))

$$i_{C1} = I_{S1}\left[\exp\left(\frac{v_{BE1}}{V_T}\right)\right]\left(1 + \frac{10}{50}\right)$$

$$= I_{S3}\left[\exp\left(\frac{v_{EB3}}{V_T}\right)\right]\left(1 + \frac{0.6}{50}\right)$$

By Equation (14.60(b))

$$i_{C2} = I_{S2}\left[\exp\left(\frac{v_{BE2}}{V_T}\right)\right]\left(1 + \frac{10}{50}\right)$$

$$= I_{S4}\left[\exp\left(\frac{v_{EB4}}{V_T}\right)\right]\left(1 + \frac{0.6}{50}\right)$$

$I_{S1} = I_{S2}$, take the ratio:

$$\exp\left(\frac{v_{BE1} - v_{BE2}}{V_T}\right) = \frac{I_{S3}}{I_{S4}}$$

$$v_{BE1} - v_{BE2} = V_{OS} = V_T \ln\left(\frac{I_{S3}}{I_{S4}}\right)$$

$$= 0.026 \cdot \ln(1.05)$$

$$\Rightarrow \underline{V_{OS} = 1.27 \text{ mV}}$$

E14.18

$$V_{OS} = \frac{1}{2}\cdot\sqrt{\frac{I_Q}{2K_n}}\cdot\left(\frac{\Delta K_n}{K_n}\right)$$

$$0.020 = \frac{1}{2}\cdot\sqrt{\frac{150}{2(50)}}\cdot\left(\frac{\Delta K_n}{50}\right)$$

$$\Rightarrow \underline{\Delta K_n = 1.63\ \mu A/V^2}$$

$$\Rightarrow \frac{\Delta K_n}{K_n} = \frac{1.63}{50} \Rightarrow \underline{3.26\%}$$

E14.19

$$\text{Want } \left(\frac{R_5}{R_5 + R_4}\right)V^+ = 5 \text{ mV}$$

$$R_5 \ll R_4 \text{ so } \frac{R_5}{R_4} \times V^+ = 0.005$$

$$R_5 = \frac{(0.005)(100)}{10} = 0.05 \text{ k}\Omega$$

$$\Rightarrow \underline{R_5 = 50\ \Omega}$$

E14.20

$R_1' = 25\|1 = 0.9615$ kΩ

$R_2' = 75\|1 = 0.9868$ kΩ

For $I_Q = 100\ \mu$A $\Rightarrow i_{C1} = i_{C2} = 50\ \mu$A

From Equation (14.75)

$$(0.026)\ln\left(\frac{50 \times 10^{-6}}{10^{-14}}\right) + (0.050)(0.9615)$$

$$= (0.026)\ln\left(\frac{i_{C2}}{I_{S4}}\right) + (0.050)(0.9868)$$

$$0.58065 + 0.048075$$

$$= (0.026)\ln\left(\frac{i_{C2}}{I_{S4}}\right) + 0.04934$$

$$\ln\left(\frac{i_{C2}}{I_{S4}}\right) = 22.284$$

$$\frac{50 \times 10^{-6}}{I_{S4}} = 4.7625 \times 10^9$$

$$\underline{I_{S4} \cong 1.05 \times 10^{-14} \text{ A}}$$

E14.21

From Equation (14.79)

$$v_O = I_{B1}R_2 - I_{B2}R_3\left(1 + \frac{R_2}{R_1}\right)$$

For $v_O = 0$

$$0 = (1.1 \times 10^{-6})(100 \text{ k}\Omega) - (1.0 \times 10^{-6})R_3\left(1 + \frac{100}{10}\right)$$

$$R_3(11) = (1.1)(100 \text{ k}\Omega) \Rightarrow \underline{R_3 = 10 \text{ k}\Omega}$$

E14.22

a. $v_O = I_{B1}R_3 = (10^{-6})(200 \times 10^3)$

$$\Rightarrow \underline{v_O = 0.20 \text{ V}}$$

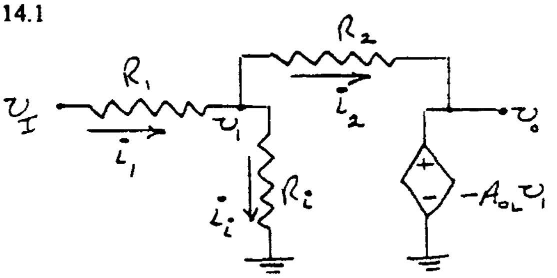
b. $R_4 = R_1\|R_2\|R_3 = 100\|50\|200$

$$\Rightarrow \underline{R_4 = 28.6 \text{ k}\Omega}$$

Chapter 14

Problem Solutions

14.1



$$\frac{v_I - v_1}{R_1} = \frac{v_1 - v_O}{R_2} + \frac{v_1}{R_i} \text{ and } v_O = -A_{OL}v_1$$

so that $v_1 = -\dfrac{v_O}{A_{OL}}$

$$\frac{v_I}{R_1} + \frac{v_O}{R_2} = v_1\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i}\right)$$

So

$$\frac{v_I}{R_1} = -v_O\left[\frac{1}{R_2} + \frac{1}{A_{OL}}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i}\right)\right]$$

Then

$$\frac{v_O}{v_I} = \frac{-(1/R_1)}{\left[\frac{1}{R_2} + \frac{1}{A_{OL}}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_i}\right)\right]} = A_{CL}$$

From Equation (14.20) for $R_L = \infty$ and $R_0 = 0$

$$\frac{1}{R_{if}} = \frac{1}{R_i} + \frac{1}{R_2} \cdot \frac{(1 + A_{OL})}{1}$$

a. For $R_i = 1\ \text{k}\Omega$

$$A_{CL} = \frac{-(1/20)}{\left[\frac{1}{100} + \frac{1}{10^3}\left(\frac{1}{20} + \frac{1}{100} + \frac{1}{1}\right)\right]} = \frac{-0.05}{[0.01 + 1.06 \times 10^{-3}]}$$

or

$$\Rightarrow \underline{A_{CL} = -4.52}$$

$$\frac{1}{R_{if}} = \frac{1}{1} + \frac{1 + 10^3}{100} \Rightarrow \underline{R_{if} = 90.8\ \Omega}$$

b. For $R_i = 10\ \text{k}\Omega$

$$A_{CL} = \frac{-(1/20)}{\left[\frac{1}{100} + \frac{1}{10^3}\left(\frac{1}{20} + \frac{1}{100} + \frac{1}{10}\right)\right]} = \frac{-0.05}{[0.01 + 1.6 \times 10^{-4}]}$$

or

$$\Rightarrow \underline{A_{CL} = -4.92}$$

$$\frac{1}{R_{if}} = \frac{1}{10} + \frac{1 + 10^3}{100} \Rightarrow \underline{R_{if} = 98.9\ \Omega}$$

c. For $R_i = 100\ \text{k}\Omega$

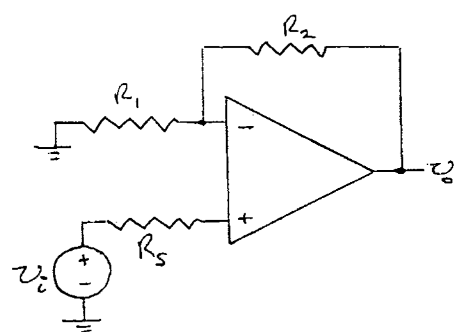
$$A_{CL} = \frac{-(1/20)}{\left[\frac{1}{100} + \frac{1}{10^3}\left(\frac{1}{20} + \frac{1}{100} + \frac{1}{100}\right)\right]} = \frac{-0.05}{[0.01 + 7 \times 10^{-5}]}$$

or

$$\Rightarrow \underline{A_{CL} = -4.965}$$

$$\frac{1}{R_{if}} = \frac{1}{100} + \frac{1 + 10^3}{100} \Rightarrow \underline{R_{if} = 99.8\ \Omega}$$

14.2



$$A_{CL} = \frac{v_o}{v_i} = \frac{\left(1 + \frac{R_2}{R_1}\right)}{\left[1 + \frac{1}{A_{OL}}\left(1 + \frac{R_2}{R_1}\right)\right]}$$

For the ideal:

$$\left(1 + \frac{R_2}{R_1}\right) = \frac{0.10}{0.002} = 50$$

$$v_o(actual) = (0.10)(1 - 0.001) = 0.0999$$

So

$$\frac{0.0999}{0.002} = \frac{50}{1 + \frac{1}{A_{OL}}(50)} = 49.95$$

which yields

$$\underline{A_{OL} = 1000}$$

14.3

$$A_{vf1} = \frac{v_{o1}}{v_1} = \frac{-\left(\frac{A_{OL}}{R_o} - \frac{1}{R_2}\right)}{\left(\frac{1}{R_L} + \frac{1}{R_o} + \frac{1}{R_2}\right)}$$

Or

$$v_{o1} = \frac{-\left(\frac{5x10^3}{1} - \frac{1}{100}\right)}{\left(\frac{1}{10} + \frac{1}{1} + \frac{1}{100}\right)} \cdot v_1 = \frac{-(4.99999x10^3)}{1.11} \cdot v_1$$

$$v_{o1} = -4.504495x10^3 \cdot v_1$$

Now

$$\frac{i_1}{v_1} = \frac{v_i - v_1}{R_1 v_1} \equiv K$$

Then

$$v_i - v_1 = KR_1 v_1$$

which yields

$$v_1 = \frac{v_i}{KR_1 + 1}$$

Now

$$K = \frac{1}{10} + \frac{1}{100}\left[\frac{1 + 5x10^3 + \frac{1}{10}}{1 + \frac{1}{10} + \frac{1}{100}}\right]$$

$$= (0.1) + (0.01)\left[\frac{5.0011x10^3}{1.11}\right] = 45.15495$$

Then

$$v_1 = \frac{v_i}{(45.15495)(10) + 1} = \frac{v_i}{452.5495}$$

We find

$$v_{o1} = -4.504495x10^3\left[\frac{v_i}{452.5495}\right]$$

Or

$$A_{vf1} = \frac{v_{o1}}{v_i} = -9.9536$$

For the second stage, $R_L = \infty$

$$v_{o2} = \frac{-\left(\frac{5x10^3}{1} - \frac{1}{100}\right)}{\left(\frac{1}{1} + \frac{1}{100}\right)} \cdot v_1' = -4.950485x10^3 \cdot v_1'$$

$$K \equiv \frac{1}{10} + \frac{1}{100}\left[\frac{1 + 5x10^3}{1 + \frac{1}{100}}\right] = 49.61485$$

$$v_1' = \frac{v_{o1}}{KR_1 + 1} = \frac{v_{o1}}{(49.61485)(10) + 1} = \frac{v_{o1}}{497.1485}$$

Then

$$\frac{v_{o2}}{v_{o1}} = \frac{-4.950485x10^3}{497.1485} = -9.95776$$

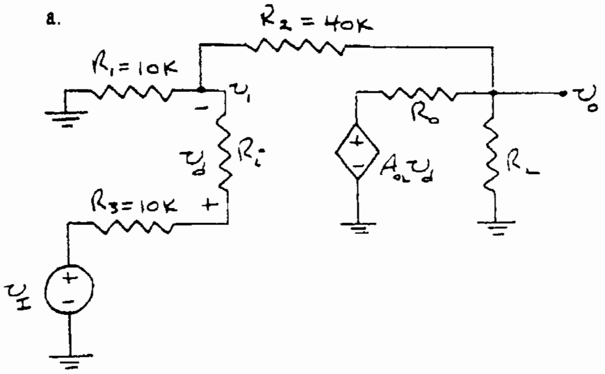
So

$$A_{vf} = \frac{v_{o2}}{v_i} = (-9.9536)(-9.95776) \Rightarrow$$

$$A_{vf} = 99.12$$

14.4

a.



$$\frac{v_1 - v_I}{R_3 + R_i} + \frac{v_1}{R_1} + \frac{v_1 - v_O}{R_2} = 0 \quad (1)$$

$$v_1\left[\frac{1}{R_3 + R_i} + \frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{v_O}{R_2} + \frac{v_I}{R_3 + R_i}$$

$$\frac{v_O}{R_L} + \frac{v_O - A_{OL}v_d}{R_O} + \frac{v_O - v_1}{R_2} = 0 \quad (2)$$

or

$$v_O\left[\frac{1}{R_L} + \frac{1}{R_O} + \frac{1}{R_2}\right] = \frac{v_1}{R_2} + \frac{A_{OL}v_d}{R_O}$$

$$v_d = \left(\frac{v_I - v_1}{R_3 + R_i}\right) \cdot R_i \quad (3)$$

So substituting numbers:

$$v_1\left[\frac{1}{10 + 20} + \frac{1}{10} + \frac{1}{40}\right] = \frac{v_O}{40} + \frac{v_I}{10 + 20} \quad (1)$$

or

$$v_1[0.15833] = v_O[0.025] + v_I[0.03333]$$

$$v_O\left[\frac{1}{1} + \frac{1}{0.5} + \frac{1}{40}\right] = \frac{v_1}{40} + \frac{(10^4)v_d}{0.5} \quad (2)$$

or

$$v_O[3.025] = v_1[0.025] + (2 \times 10^4)v_d$$

$$v_d = \left(\frac{v_I - v_1}{10 + 20}\right) \cdot 20 = 0.6667(v_I - v_1) \quad (3)$$

So

$$v_O[3.025] = v_1[0.025] + (2 \times 10^4)(0.6667)(v_I - v_1)$$

or (2)

$$v_O[3.025] = 1.333 \times 10^4 v_I - 1.333 \times 10^4 v_1$$

From (1):

$v_1 = v_O(0.1579) + v_I(0.2105)$

Then

$v_O[3.025] = 1.333 \times 10^4 v_I$

$\qquad - 1.333 \times 10^4[v_O(0.1579) + v_I(0.2105)]$

$v_O[2.1078 \times 10^3] = v_I[1.0524 \times 10^4]$

or

$$\underline{A_{CL} = \frac{v_O}{v_I} = 4.993}$$

To find R_{if}: Use Equation (14.27)

$$i_I\left(1 + \frac{0.5}{1} + \frac{0.5}{40}\right) = v_1\left\{\left(\frac{1}{10} + \frac{1}{40}\right)\left(1 + \frac{0.5}{1} + \frac{0.5}{40}\right) - \frac{0.5}{(40)^2}\right\} - \frac{(10^3)v_d}{40}$$

$i_I(1.5125) = v_1\{(0.125)(1.5125) - 0.0003125\} - 25v_d$

or

$i_I(1.5125) = v_I\{0.18875\} - 25v_d$

Now

$v_d = i_I R_i = i_I(20)$ and $v_1 = v_I - i_I(20)$

So

$i_I(1.5125) = [v_I - i_I(20)] \cdot [0.18875] - 25i_I(20)$

$i_I[505.3] = v_I(0.18875)$

or

$\frac{v_I}{i_I} = 2677$ kΩ

Now $R_{if} = 10 + 2677 \Rightarrow \underline{R_{if} = 2.687\ \text{M}\Omega}$

To determine R_{of}: Using Equation (14.36)

$$\frac{1}{R'_{of}} = \frac{1}{R_O} \cdot \left[\frac{A_{OL}}{1 + \frac{R_2}{R_1 \| R_i}}\right] = \frac{1}{0.5} \cdot \left[\frac{10^3}{1 + \frac{40}{10\|20}}\right]$$

or $R'_{of} = 3.5\ \Omega$

Then $R_{of} = 1\ \text{k}\Omega \| 3.5\ \Omega$

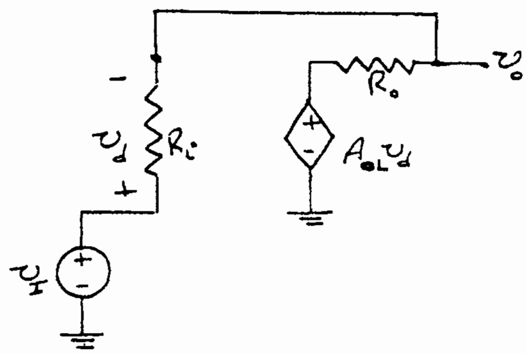
$\Rightarrow \underline{R_{of} = 3.49\ \Omega}$

b. Using Equation (14.16)

$$\frac{dA_{CL}}{A_{CL}} = (-10)\left(\frac{5}{10^3}\right) \Rightarrow \underline{\frac{dA_{CL}}{A_{CL}} = -(0.05)\%}$$

14.5

a.



$$\frac{v_O - A_{OL}v_d}{R_O} + \frac{v_O - v_I}{R_i} = 0 \text{ and } v_d = v_I - v_O$$

So

$$\frac{v_O}{R_O} - \frac{A_{OL}}{R_O} \cdot (v_I - v_O) + \frac{v_O}{R_i} - \frac{v_I}{R_i} = 0$$

$$v_O\left[\frac{1}{R_O} + \frac{A_{OL}}{R_O} + \frac{1}{R_i}\right] = v_I\left[\frac{1}{R_i} + \frac{A_{OL}}{R_O}\right]$$

$$v_O\left[\frac{1}{0.2} + \frac{(10^4)}{0.2} + \frac{1}{100}\right] = v_I\left[\frac{1}{100} + \frac{(10^4)}{0.2}\right]$$

$v_O[5.000501 \times 10^4] = v_I[5.000001 \times 10^4]$

So $\underline{A_{CL} = \frac{v_O}{v_I} = 0.9999}$

b. Set $v_I = 0$

$$i_O = \frac{v_O - A_{OL}v_d}{R_O} + \frac{v_O}{R_i} \text{ and } v_d = -v_O$$

$$i_O = v_O\left[\frac{1}{R_O} + \frac{A_{OL}}{R_O} + \frac{1}{R_i}\right]$$

Then

$$\frac{1}{R_{Of}} = \frac{1}{R_O} + \frac{A_{OL}}{R_O} + \frac{1}{R_i}$$

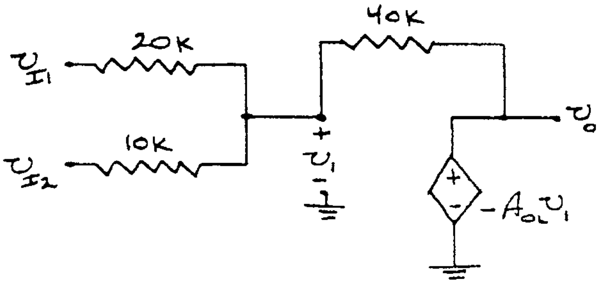
or

$$\frac{1}{R_{Of}} = \frac{1}{0.2} + \frac{(10^4)}{0.2} + \frac{1}{100}$$

which yields

$$\underline{R_{Of} \cong 0.02\ \Omega}$$

14.6



$$\frac{v_{I1}-v_1}{20}+\frac{v_{I2}-v_1}{10}=\frac{v_1-v_O}{40}$$

$$\frac{v_{I1}}{20}+\frac{v_{I2}}{10}+\frac{v_O}{40}=v_1\left[\frac{1}{20}+\frac{1}{10}+\frac{1}{40}\right]$$

and $v_O=-A_{OL}v_1$ so that $v_1=-\dfrac{v_O}{A_{OL}}$

Then

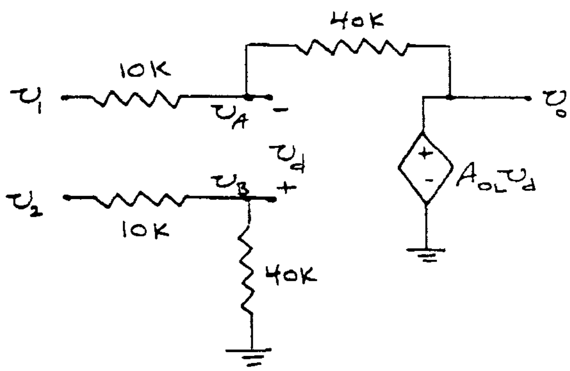
$$v_{I1}(0.05)+v_{I2}(0.10)=-v_O\left\{\frac{1}{40}+\frac{1}{2\times10^3}\cdot\left(\frac{7}{40}\right)\right\}$$

$$=-v_O\left[2.50875\times10^{-2}\right]$$

$$\Rightarrow \underline{v_O=-1.993v_{I1}-3.986v_{I2}}$$

$$\frac{\Delta v_O}{v_O}=\frac{2-1.993}{2}\Rightarrow \underline{\frac{\Delta v_O}{v_O}=0.35\%}$$

14.7



$$v_B=\left(\frac{40}{40+10}\right)v_2=\left(\frac{4}{5}\right)v_2=0.8v_2 \qquad (1)$$

$$\frac{v_1-v_A}{10}=\frac{v_A-v_O}{40}$$

$$\frac{v_1}{10}+\frac{v_O}{40}=v_A\left(\frac{1}{10}+\frac{1}{40}\right)$$

$$v_1(0.1)+v_O(0.025)=v_A(0.125) \qquad (2)$$

$$v_O=A_{OL}v_d=A_{OL}(v_B-v_A) \qquad (3)$$

or

$$v_O=A_{OL}[0.8v_2-v_A]$$

$$\frac{v_O}{A_{OL}}-0.8v_2=-v_A$$

$$\Rightarrow \underline{v_A=0.8v_2-\frac{v_O}{A_{OL}}}$$

Then

$$v_1(0.1)+v_O(0.025)=(0.125)\left[0.8v_2-\frac{v_O}{A_{OL}}\right]$$

$$v_1(0.1)-v_2(0.1)=-v_O\left[0.025+\frac{0.125}{10^3}\right]$$

$$=-v_O\left[2.5125\times10^{-2}\right]$$

$$\Rightarrow \underline{A_d=\frac{v_O}{v_2-v_1}=3.9801}$$

$$\Rightarrow \frac{\Delta A_d}{A_d}=\frac{0.0199}{4}\Rightarrow \underline{0.4975\%}$$

14.8

a. Considering the second op-amp and Equation (14.20), we have

$$\frac{1}{R_{if2}}=\frac{1}{10}+\frac{1}{0.1}\cdot\left[\frac{1+100}{1+\frac{1}{0.1}}\right]=0.10+\frac{101}{(0.1)(11)}$$

So $R_{if2}=0.0109$ kΩ

The effective load on the first op-amp is then

$$R_{L1}=0.1+R_{if2}=0.1109\text{ k}\Omega$$

Again using Equation (14.20), we have

$$\frac{1}{R_{if}}=\frac{1}{10}+\frac{1}{1}\cdot\frac{1+100+\frac{1}{0.1109}}{1+\frac{1}{0.1109}+\frac{1}{1}}=0.10+\frac{110.017}{11.017}$$

so that

$$\underline{R_{if}=99.1\ \Omega}$$

b. To determine R_{of}:
For the first op-amp, we can write, using Equation (14.36)

$$\frac{1}{R_{of1}}=\frac{1}{R_O}\cdot\left[\frac{A_{OL}}{1+\frac{R_2}{R_1\|R_i}}\right]=\frac{1}{1}\cdot\left[\frac{100}{1+\frac{40}{1\|10}}\right]$$

which yields $R_{of1}=0.021$ kΩ

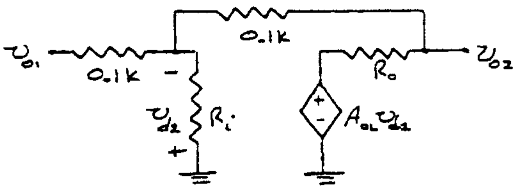
For the second op-amp, then

$$\frac{1}{R_{Of}} = \frac{1}{R_O} \cdot \left[\frac{A_{OL}}{1 + \dfrac{R_2}{(R_1 + R_{Of1}) \| R_i}} \right]$$

$$= \frac{1}{1} \cdot \left[\frac{100}{1 + \dfrac{0.10}{(0.121) \| 10}} \right]$$

or $\underline{R_{Of} = 18.4\ \Omega}$

c. To find the gain, consider the second op-amp.



$$\frac{v_{01} - (-v_{d2})}{0.1} + \frac{v_{d2}}{R_i} = \frac{-v_{d2} - v_{02}}{0.1} \qquad (1)$$

$$\frac{v_{01}}{0.1} + v_{d2}\left(\frac{1}{0.1} + \frac{1}{10} + \frac{1}{0.1}\right) = -\frac{v_{02}}{0.1}$$

or

$$v_{01}(10) + v_{d2}(20.1) = -v_{02}(10)$$

$$\frac{v_{02} - A_{OL}v_{d2}}{R_O} + \frac{v_{02} - (-v_{d2})}{0.1} = 0 \qquad (2)$$

$$\frac{v_{02}}{1} - v_{d2}\left(\frac{100}{1} - \frac{1}{0.1}\right) + \frac{v_{02}}{0.1} = 0$$

$$v_{02}(11) - v_{d2}(90) = 0$$

or

$$v_{d2} = v_{02}(0.1222)$$

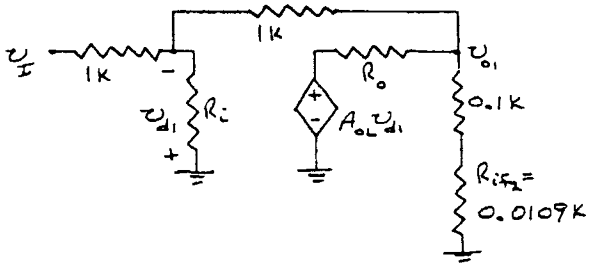
Then Equation (1) becomes

$$v_{01}(10) + v_{02}(0.1222)(20.1) = -v_{02}(10)$$

or

$$v_{01} = -v_{02}(1.246)$$

Now consider the first op-amp.



$$\frac{v_I - (-v_{d1})}{1} + \frac{v_{d1}}{R_i} = \frac{-v_{d1} - v_{01}}{1} \qquad (1)$$

$$v_I(1) + v_{d1}\left(\frac{1}{1} + \frac{1}{10} + \frac{1}{1}\right) = -v_{01}(1)$$

or

$$v_I(1) + v_{d1}(2.1) = -v_{01}(1)$$

$$\frac{v_{01}}{0.1109} + \frac{v_{01} - A_{OL}v_{d1}}{R_O} + \frac{v_{01} - (-v_{d1})}{1} = 0 \qquad (2)$$

$$v_{01}\left(\frac{1}{0.1109} + \frac{1}{1} + \frac{1}{1}\right) - v_{d1}\left(\frac{100}{1} - \frac{1}{1}\right) = 0$$

$$v_{01}(11.017) - v_{d1}(99) = 0$$

or

$$v_{d1} = v_{01}(0.1113)$$

Then Equation (1) becomes

$$v_I(1) + v_{01}(0.1113)(2.1) = -v_{01}$$

or $v_I = -v_{01}(1.234)$

We had $v_{01} = -v_{02}(1.246)$

So $v_I = v_{02}(1.246)(1.234)$

or $\underline{\dfrac{v_{02}}{v_I} = 0.650}$

d. Ideal $\dfrac{v_{02}}{v_I} = 1$

So ratio of actual to ideal = 0.650.

14.9

The open loop gain can be written as

$$A_{0L}(f) = \frac{A_0}{\left(1 + j \cdot \dfrac{f}{f_{PD}}\right)\left(1 + j \cdot \dfrac{f}{5 \times 10^6}\right)}$$

where $A_0 = 2 \times 10^5$.
The closed-loop response is

$$A_{CL} = \frac{A_{0L}}{1 + \beta A_{0L}}$$

At low frequency,

$$100 = \frac{2 \times 10^5}{1 + \beta(2 \times 10^5)}$$

So that $\beta = 9.995 \times 10^{-3}$.
Assuming the second pole is the same for both the open-loop and closed-loop, then

$$\phi = -\tan^{-1}\left(\frac{f}{f_{PD}}\right) - \tan^{-1}\left(\frac{f}{5 \times 10^6}\right)$$

For a phase margin of 80°, $\phi = -100°$.
So

$$-100 = -90 - \tan^{-1}\left(\frac{f}{5 \times 10^6}\right)$$

or

$$f = 8.816 \times 10^5 \text{ Hz}$$

Then

$$|A_{0L}| = 1$$

$$= \frac{2 \times 10^5}{\sqrt{1 + \left(\dfrac{8.816 \times 10^5}{f_{PD}}\right)^2}\sqrt{1 + \left(\dfrac{8.816 \times 10^5}{5 \times 10^6}\right)^2}}$$

or

$$\frac{8.816 \times 10^5}{f_{PD}} \cong 1.9696 \times 10^5$$

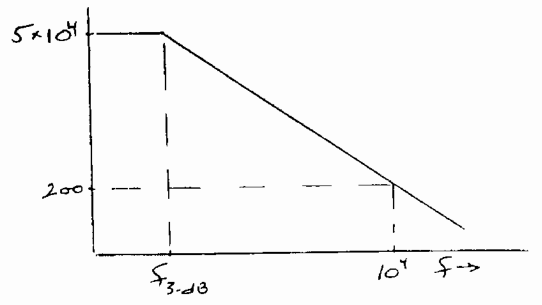
or

$$\underline{f_{PD} = 4.48 \text{ Hz}}$$

14.10

(a) 1st stage
$(10)f_{3-dB} = 1\ MHz \Rightarrow f_{3-dB} = 100\ kHz$
2nd stage
$(50)f_{3-dB} = 1\ MHz \Rightarrow f_{3-dB} = 20\ kHz$
Bandwidth of overall system $\cong 20\ kHz$
(b) If each stage has the same gain, so
$K^2 = 500 \Rightarrow K = 22.36$
Then bandwidth of each stage
$(22.36)f_{3-dB} = 1\ MHz \Rightarrow f_{3-dB} = 44.7\ kHz$

14.11



$$A = \frac{A_o}{1 + j\dfrac{f}{f_{3-dB}}}$$

$$|A| = \frac{A_o}{\sqrt{1 + \left(\dfrac{f}{f_{3-dB}}\right)^2}} \Rightarrow$$

$$200 = \frac{5x10^4}{\sqrt{1 + \left(\dfrac{10^4}{f_{3-dB}}\right)^2}} \Rightarrow \underline{f_{3-dB} = 40\ Hz}$$

Then

$$f_T = (5x10^4)(40) \Rightarrow \underline{f_T = 2\ MHz}$$

14.12

$(5x10^4)f_{PD} = 10^6 \Rightarrow \underline{f_{PD} = 20\ Hz}$

$(25)f_{3-dB} 10^6 \Rightarrow \underline{f_{3-dB} = 40\ kHz}$

$$A_v = \frac{A_{vo}}{1 + j\dfrac{f}{f_{3-dB}}} \Rightarrow |A_v| = \frac{25}{\sqrt{1 + \left(\dfrac{f}{40x10^3}\right)^2}}$$

At $f = 0.5 f_{3-dB} = 20\ kHz$

$$|A_v| = \frac{25}{\sqrt{1 + (0.5)^2}} = 22.36$$

At $f = 2 f_{3-dB} = 80\ kHz$

$$|A_v| = \frac{25}{\sqrt{1 + (2)^2}} = 11.18$$

14.13

$$(20x10^3) \cdot |A_f|_{MAX} = 10^6 \Rightarrow \underline{|A_f|_{MAX} = 50}$$

14.14

From Equation (14.55),

$$FPBW = \frac{SR}{2\pi V_{P0}} = \frac{10 \times 10^6}{2\pi(10)}$$

or

$$\underline{FPBW = f_{max} = 159 \text{ kHz}}$$

14.15

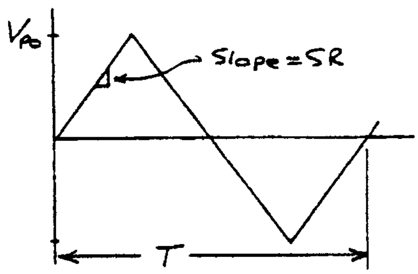
a. Using Equation (14.55),

$$V_{P0} = \frac{8 \times 10^6}{2\pi(250 \times 10^3)}$$

or

$\underline{V_{P0} = 5.09 \text{ V}}$

b.



Period $T = \frac{1}{f} = \frac{1}{250 \times 10^3} = 4 \times 10^{-6}$ s

One-fourth period = 1 μs

Slope = $\frac{V_{P0}}{1\ \mu\text{s}} = SR = 8$ V/μs

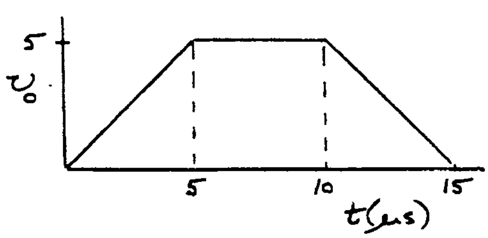
$\Rightarrow \underline{V_{P0} = 8 \text{ V}}$

14.16

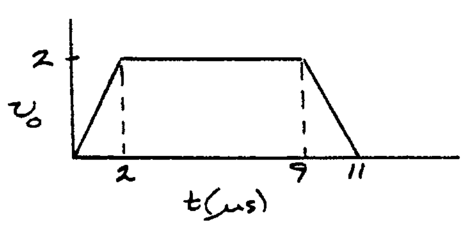
For input (a), maximum output is 5 V.

$SR = 1$ V/μs

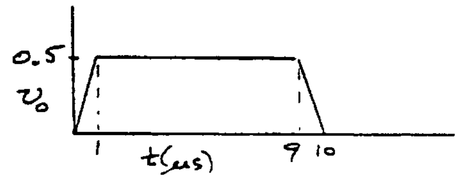
so



For input (b), maximum output is 2 V.

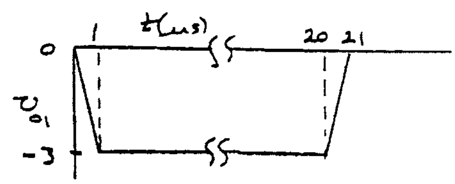


For input (c), maximum output is 0.5 V so the output is

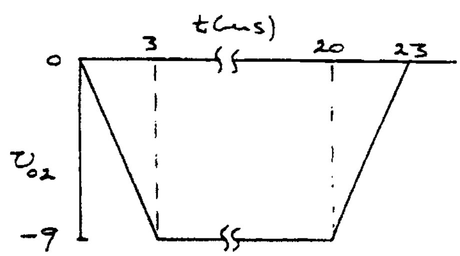


14.17

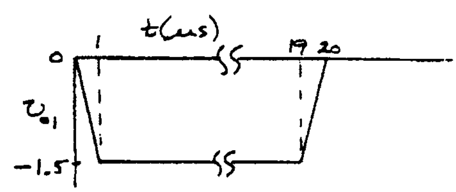
For input (a), max $|v_{O1}| = 3$ V.



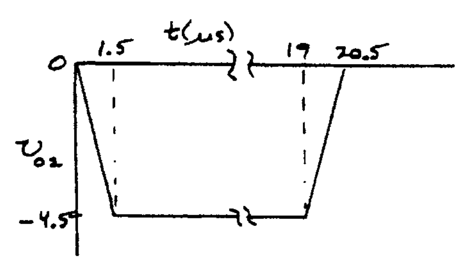
Then $|v_{O2}|_{max} = 3(3) = 9$ V



For input (b), max $|v_{O1}| = 1.5$ V.



Then $|v_{O2}|_{max} = 3(1.5) = 4.5$ V



14.18

$f_{MAX} = 20\ kHz,\ \ SR = 0.8\ V/\mu s$

$$V_{po} = \frac{SR}{2\pi f_{MAX}} = \frac{0.8x10^6}{2\pi(20x10^3)} \Rightarrow$$

$\underline{V_{po} = 6.37\ V}$

14.19

$$I_1 = I_{S1}\exp\left(\frac{V_{BE1}}{V_T}\right),\quad I_2 = I_{S2}\exp\left(\frac{V_{BE2}}{V_T}\right)$$

Want $I_1 = I_2$, so

$$\frac{I_1}{I_2} = 1 = \frac{5x10^{-14}(1+x)\exp\left(\frac{V_{BE1}}{V_T}\right)}{5x10^{-14}(1-x)\exp\left(\frac{V_{BE2}}{V_T}\right)}$$

$$= \frac{(1+x)}{(1-x)}\exp\left(\frac{V_{BE1}-V_{BE2}}{V_T}\right)$$

Or

$$\frac{1+x}{1-x} = \exp\left(\frac{V_{BE2}-V_{BE1}}{V_T}\right) = \exp\left(\frac{V_{OS}}{V_T}\right)$$

$$= \exp\left(\frac{0.0025}{0.026}\right) = 1.10$$

Now

$1 + x = (1-x)(1.10) \Rightarrow$

$x = 0.0476 \Rightarrow 4.76\%$

14.20

From Equation (14.62),

$$\left(\frac{1+\frac{v_{CE1}}{V_{AN}}}{1+\frac{v_{EB}}{V_{AP}}}\right) = \frac{I_{S3}}{I_{S4}}\cdot\left(\frac{1+\frac{v_{CE2}}{V_{AN}}}{1+\frac{v_{EC4}}{V_{AP}}}\right)$$

For $v_{CE2} = 0.6$ V, then $v_{EC4} = 5$ V. We have $v_{CE1} = 5$ V so

$$\left(\frac{1+\frac{5}{80}}{1+\frac{0.6}{80}}\right) = \frac{I_{S3}}{I_{S4}}\cdot\left(\frac{1+\frac{0.6}{80}}{1+\frac{5}{80}}\right)$$

or

$$\frac{I_{S3}}{I_{S4}} = \frac{(1.0625)^2}{(1.0075)^2} = 1.112$$

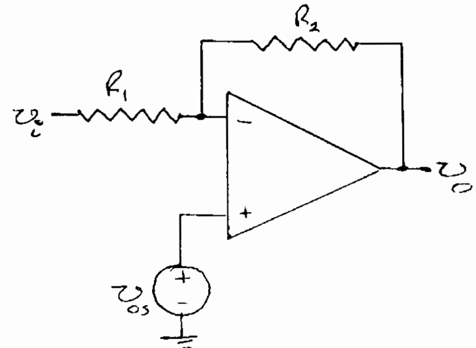
So

$I_{S3} = (10^{-14})(1.112)$

or

$\underline{I_{S3} = 1.112 \times 10^{-14}\ \text{A}}$

14.21



By superposition:

$$v_o(v_i) = -\frac{R_2}{R_1}\cdot v_i = -50v_i$$

$$v_o(v_{os}) = \left(1+\frac{R_2}{R_1}\right)\cdot v_{os} = 51v_{os}$$

So

$v_o = v_o(v_i) + v_o(v_{os}) = -50v_i + 51v_{os}$

For $v_i = 20\ mV$ and $v_{os} + 2.5\ mV$

$v_o = -50(0.02) + 51(0.0025) = -0.8725\ V$

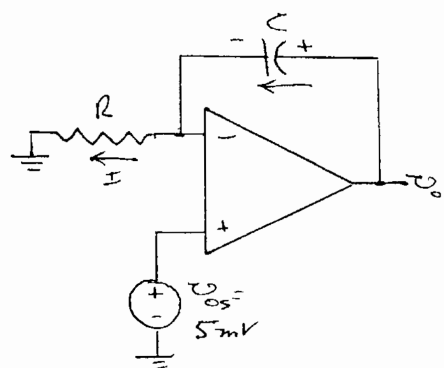
For $v_i = 20\ mV$ and $v_{os} = -2.5\ mV$

$v_o = -50(0.02) + 51(-0.0025) = -1.1275\ V$

So

$-1.1275 \le v_o \le -0.8725\ V$

14.22



$$I = \frac{0.5x10^{-3}}{10^4} = 5x10^{-8}\ A$$

Also

$$I = C\frac{dV_o}{dt} \Rightarrow V_o = \frac{1}{C}\int_0^t I dt = \frac{I}{C}\cdot t$$

Then

$$5 = \frac{5x10^{-8}}{10x10^{-6}}t \Rightarrow \underline{t = 10^3\ s}$$

14.23

a.

$$|v_{01}| = 10\left(1+\frac{100}{10}\right) \text{ or } \underline{|v_{01}| = 110 \text{ mV}}$$

Then

$$|v_{02}| = |v_{01}|(5) + 10\left(1+\frac{50}{10}\right) = (110)(5) + (10)(6)$$

or

$$\underline{|v_{02}| = 610 \text{ mV}}$$

14.24

v_O due to v_I

$$v_O = (0.5)\left(1+\frac{1}{1.1}\right) = 0.9545 \text{ V}$$

Wiper arm at $V^+ = 10$ V, (using superposition)

$$v_1 = \left(\frac{R_1\|R_5}{R_1\|R_5 + R_4}\right)(10) = \left(\frac{0.0909}{0.0909+10}\right)(10)$$

$$= 0.090$$

$$\text{Then } v_{01} = -\left(\frac{1}{1}\right)(0.090) = -0.090$$

Wiper arm in center, $v_1 = 0$ and $v_{02} = 0$

Wiper arm at $V^- = -10$ V, $v_1 = -0.090$
So

$$v_{03} = 0.090$$

Finally, total output v_O: (from superposition)

Wiper arm at V^+,

$$\underline{v_O = 0.8645 \text{ V}}$$

Wiper arm in center,

$$\underline{v_O = 0.9545 \text{ V}}$$

Wiper arm at V^-,

$$\underline{v_O = 1.0445 \text{ V}}$$

14.25

a. $R_1' = R_2' = 0.5\|25 = 0.490$ kΩ

or

$$\underline{R_1' = R_2' = 490\ \Omega}$$

b. From Equation (14.75),

$$(0.026)\ln\left(\frac{125\times10^{-6}}{2\times10^{-14}}\right) + (0.125)R_1'$$
$$= (0.026)\ln\left(\frac{125\times10^{-6}}{2.2\times10^{-14}}\right) + (0.125)R_2'$$

$$0.586452 + (0.125)R_1' = 0.583974 + (0.125)R_2'$$
$$0.002478 = (0.125)(R_2' - R_1')$$
So $R_2' - R_1' = 0.0198$ kΩ $\Rightarrow 19.8\ \Omega$

Then

$$\frac{R_2(1-x)R_x}{R_2+(1-x)R_x} - \frac{R_1 x R_x}{R_1 + xR_x} = 0.0198$$
$$\frac{(0.5)(1-x)(50)}{(0.5)+(1-x)(50)} - \frac{(0.5)(50)x}{(0.5)+x(50)} = 0.0198$$
$$\frac{25(1-x)}{50.5-50x} - \frac{25x}{0.5+50x} = 0.0198$$

$$\frac{(0.5+50x)(25-25x)-(25x)(50.5-50x)}{(50.5-50x)(0.5+50x)} = 0.0198$$

$$25\{0.5 - 0.5x + 50x - 50x^2 - 50.5x + 50x^2\}$$
$$= 0.0198\{25.25 + 2525x - 25x - 2500x^2\}$$

$$25\{0.5 - x\} = 0.0198\{25.25 + 2500x - 2500x^2\}$$
$$0.5 - x = 0.019998 + 1.98x - 1.98x^2$$
$$1.98x^2 - 2.98x + 0.48 = 0$$
$$x = \frac{2.98 \pm \sqrt{(2.98)^2 - 4(1.98)(0.48)}}{2(1.98)}$$

So

$$\underline{x = 0.183}$$

and

$$\underline{1 - x = 0.817}$$

14.26

$$R_1' = R_1\|15 = 0.5\|15 = 0.4839 \text{ k}\Omega$$
$$R_2' = R_2\|35 = 0.5\|35 = 0.4930 \text{ k}\Omega$$

From Equation (14.75),

$$(0.026)\ln\left(\frac{i_{C1}}{I_{S3}}\right) + i_{C1}R_1' = (0.026)\ln\left(\frac{i_{C2}}{I_{S4}}\right) + i_{C2}R_2'$$
$$(0.026)\ln\left(\frac{i_{C1}}{i_{C2}}\right) = i_{C2}R_2' - i_{C1}R_1'$$
$$(0.026)\ln\left(\frac{i_{C1}}{i_{C2}}\right) = i_{C2}R_2'\left[1 - \frac{i_{C1}}{i_{C2}}\cdot\frac{R_1'}{R_2'}\right]$$
$$(0.026)\ln\left(\frac{i_{C1}}{i_{C2}}\right) = i_{C2}(0.4930)\left[1 - (0.9815)\left(\frac{i_{C1}}{i_{C2}}\right)\right]$$

By trial and error:

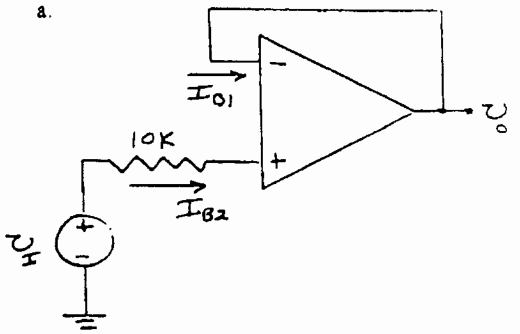
$\underline{i_{C1} = 252\ \mu A}$ and $\underline{i_{C2} = 248\ \mu A}$

or

$$\underline{\frac{i_{C1}}{i_{C2}} = 1.0155}$$

14.27

a.

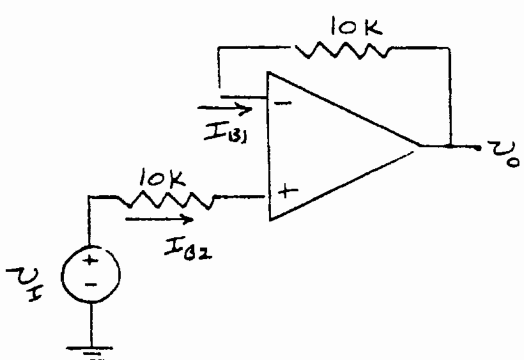


For $I_{B2} = 1\ \mu A$, then $v_O = -(10^{-6})(10^4)$

or

$$\underline{v_O = -0.010\ V}$$

b. If a 10 kΩ resistor is included in the feedback loop



Now $v_O = -I_{B2}(10) + I_{B1}(10) = 0$

Circuit is compensated if $I_{B1} = I_{B2}$.

14.28

From Equation (14.83), we have

$$v_O = R_2 I_{OS}$$

where $R_2 = 40$ kΩ and $I_{OS} = 3\ \mu A$.
Then

$$v_O = (40 \times 10^3)(3 \times 10^{-6})$$

or

$$\underline{v_O = 0.12\ V}$$

14.29

a. Assume all bias currents are in the same direction and into each op-amp.

$v_{O1} = I_{B1}(100\ k\Omega) = (10^{-6})(10^5) \Rightarrow \underline{v_{O1} = 0.1\ V}$

Then

$$v_{O2} = v_{O1}(-5) + I_{B1}(50\ k\Omega)$$
$$= (0.1)(-5) + (10^{-6})(5 \times 10^4)$$
$$= -0.5 + 0.05$$

or

$$\underline{v_{O2} = -0.45\ V}$$

b. Connect $R_3 = 10\|100 = 9.09$ kΩ resistor to noninverting terminal of first op-amp, and $R_3 = 10\|50 = 8.33$ kΩ resistor to noninverting terminal of second op-amp.

14.30

a. For a constant current through a capacitor,

$$v_O = \frac{1}{C}\int_0^t I\,dt$$

or $v_O = \dfrac{0.1 \times 10^{-6}}{10^{-6}} \cdot t \Rightarrow \underline{v_O = (0.1)t}$

b. At $t = 10$ s, $v_O = 1$ V

c. Then

$$v_O = \frac{100 \times 10^{-12}}{10^{-6}} \cdot t \Rightarrow v_O = (10^{-4})t$$

At $t = 10$ s, $\underline{v_O = 1\ mV}$

14.31

a. Assume all bias currents are into the op-amp.

$$v_{O1} = I_{B1}(50\ k\Omega) = (10 \times 10^{-6})(50 \times 10^3)$$

or

$$\underline{v_{O1} = v_{O2} = 0.5\ V}$$

$$v_{O3} = (-1)(v_{O1}) + (10 \times 10^{-6})(20 \times 10^3)$$

or

$$\underline{v_{O3} = -0.3\ V}$$

b. $R_A = 10\|50 \Rightarrow \underline{R_A = 8.33\ k\Omega}$

$R_B = 20\|20 \Rightarrow \underline{R_B = 10\ k\Omega}$

c. Assume the worst case offset current, that is, $I_{OS} = I_{B1} - I_{B2}$ or $I_{OS} = I_{B2} - I_{B1}$.
From Equation (14.83),

$$v_{01} = R_2 I_{OS} = (50 \times 10^3)(2 \times 10^{-6})$$

or

$$\underline{v_{01} = v_{02} = 0.1 \text{ V}}$$

$$v_{03} = (-1)v_{01} - I_{OS}R_2$$
$$= (-1)(0.1) - (2 \times 10^{-6})(20 \times 10^3)$$

or

$$\underline{v_{03} = -0.14 \text{ V}}$$

14.32

a. Using Equation (14.79),

Circuit (a),

$$v_0 = (0.8 \times 10^{-6})(50 \times 10^3) - (0.8 \times 10^{-6})(25 \times 10^3)\left(1 + \frac{50}{50}\right)$$

or

$$\underline{v_0 = 0}$$

Circuit (b),

$$v_0 = (0.8 \times 10^{-6})(50 \times 10^3) - (0.8 \times 10^{-6})(10^3)\left(1 + \frac{50}{50}\right)$$
$$= 4 \times 10^{-2} - 1.6$$

or

$$\underline{v_0 = -1.56 \text{ V}}$$

b. Assume $I_{B1} = 0.7\ \mu$A and $I_{B2} = 0.9\ \mu$A, then using Equation (14.79):

Circuit (a),

$$v_0 = (0.7 \times 10^{-6})(50 \times 10^3) - (0.9 \times 10^{-6})(25 \times 10^3)\left(1 + \frac{50}{50}\right)$$
$$= 0.035 - 0.045$$

or

$$\underline{v_0 = -0.010 \text{ V}}$$

Circuit (b),

$$v_0 = (0.7 \times 10^{-6})(50 \times 10^3) - (0.9 \times 10^{-6})(10^6)\left(1 + \frac{50}{50}\right)$$
$$= 0.035 - 1.8$$

or

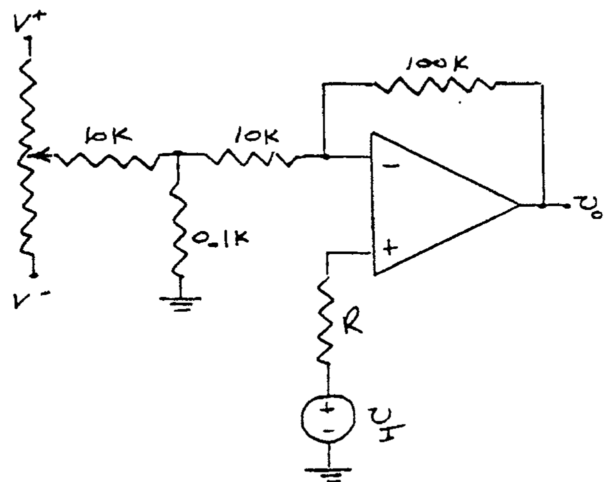
$$\underline{v_0 = -1.765 \text{ V}}$$

14.33

a. If $R = 0$,

$$v_{0,max} = \left(1 + \frac{100}{10}\right)V_{OS} + I_B(100 \text{ k}\Omega)$$
$$= (11)(10 \times 10^{-3}) + (2 \times 10^{-6})(100 \times 10^3)$$
$$v_{0,max} = 0.110 + 0.20$$
$$\Rightarrow \underline{v_{0,max} = 0.310 \text{ V}}$$

b.



$R = 10.1 \| 100 = \underline{9.17 \text{ k}\Omega = R}$

14.34

a. $\left(\dfrac{R_i}{R_i + R_2}\right)(15) = 0.010$ V

$$\frac{15}{15 + R_2} = 0.0006667$$
$$15(1 - 0.0006667) = 0.0006667 R_2$$

Then

$$\underline{R_2 = 22.48 \text{ M}\Omega}$$

b. $R_1 = R_i \| R_F = 15 \| 10 \Rightarrow \underline{R_1 = 6 \text{ k}\Omega}$

14.35

a. Assume the offset voltage polarities are such as to produce the worst case values, but the bias currents are in the same direction.

Use superposition:
Offset voltages

$$|v_{O1}| = \left(1 + \frac{100}{10}\right)(10) = \underline{110 \text{ mV} = |v_{O1}|}$$

$$|v_{O2}| = (5)(110) + \left(1 + \frac{50}{10}\right)(10)$$

$$\Rightarrow \underline{|v_{O2}| = 610 \text{ mV}}$$

Bias Currents:

$$v_{O1} = I_B(100 \text{ k}\Omega) = (2 \times 10^{-6})(100 \times 10^3) = 0.2 \text{ V}$$

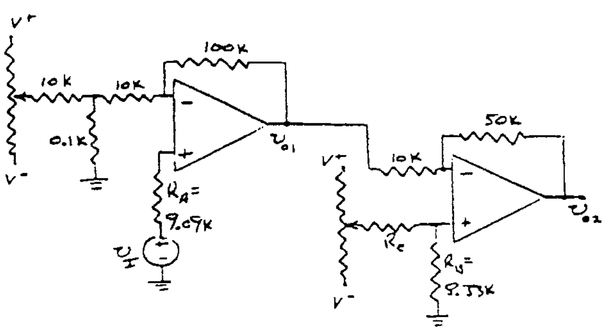
Then

$$v_{O2} = (-5)(0.2) + (2 \times 10^{-6})(50 \times 10^3) = -0.9 \text{ V}$$

Worst case: v_{O1} is positive and v_{O2} is negative, then

$$\underline{v_{O1} = 0.31 \text{ V}} \text{ and } \underline{v_{O2} = -1.51 \text{ V}}$$

b. Compensation network:



If we want

$$\left(\frac{R_B}{R_B + R_C}\right)V^+ = 20 \text{ mV and } V^+ = 10 \text{ V}$$

$$\left(\frac{8.33}{8.33 + R_C}\right)(10) = 0.020$$

or

$$\underline{R_C \cong 4.15 \text{ M}\Omega}$$

14.36

Assume bias currents are in same direction, but assume polarity of offset voltages are such as to produce the worst case output.

a. Let $I_{B1} = 5.5\ \mu\text{A}$, $I_{B2} = 4.5\ \mu\text{A}$

Bias Current Effects:

$$v_{O1} = I_{B1}(50 \text{ k}\Omega) = 0.275 \text{ V} \Rightarrow v_{O2} = 0.275 \text{ V}$$

$$v_{O3} = I_{B1}(20 \text{ k}\Omega) - v_{O1} \Rightarrow v_{O3} = -0.165 \text{ V}$$

Offset Voltage Effects:

$$v_{O1} = (5)\left(1 + \frac{50}{10}\right) = 30 \text{ mV} \Rightarrow v_{O2} = 30 \text{ mV}$$

$$v_{O3} = -v_{O1} - 5\left(1 + \frac{20}{20}\right) \Rightarrow v_{O3} = -40 \text{ mV}$$

Total Effect:

$$\underline{v_{O1} = 0.305 \text{ V}} \text{ and } \underline{v_{O2} = 0.305 \text{ V}}$$

$$\underline{v_{O3} = -0.205 \text{ V}}$$

14.37

For circuit (a), effect of bias current:

$$v_O = (50 \times 10^3)(100 \times 10^{-9}) \Rightarrow 5 \text{ mV}$$

Effect of offset voltage

$$v_O = (2)\left(1 + \frac{50}{50}\right) = 4 \text{ mV}$$

So net output voltage is $\underline{v_O = 9 \text{ mV}}$

For circuit (b), effect of bias current:

Let $I_{B2} = 550$ nA, $I_{B1} = 450$ nA, then from Equation (14.79),

$$v_O = (450 \times 10^{-9})(50 \times 10^3) - (550 \times 10^{-9})(10^6)\left(1 + \frac{50}{50}\right)$$

$$= 2.25 \times 10^{-2} - 1.1$$

or

$$v_O = -1.0775 \text{ V}$$

If the offset voltage is negative, then

$$v_O = (-2)(2) = -4 \text{ mV}$$

So the net output voltage is

$$\underline{v_O = -1.0815 \text{ V}}$$

14.38

a. At $T = 25°C$, $V_{OS} = 2$ mV so the output voltage for each circuit is

$\underline{v_O = 4 \text{ mV}}$

b. For $T = 50°C$, the offset voltage for is

$V_{OS} = 2 \text{ mV} + (0.0067)(25) = 2.1675 \text{ mV}$

so the output voltage for each circuit is

$\underline{v_O = 4.335 \text{ mV}}$

14.39

a. At $T = 25°C$, $V_{OS} = 1$ mV, then

$$v_{O1} = (1)\left(1 + \frac{50}{10}\right) \Rightarrow \underline{v_{O1} = 6 \text{ mV}}$$

and

$$v_{O2} = v_{O1}\left(1 + \frac{60}{20}\right) + (1)\left(1 + \frac{60}{20}\right)$$
$$= 6(4) + (1)(4) \Rightarrow \underline{v_{O2} = 28 \text{ mV}}$$

b. At $T = 50°C$, $V_{OS} = 1 + (0.0033)(25) = 1.0825$ mV, then

$$v_{O1} = (1.0825)(6) \Rightarrow \underline{v_{O1} = 6.495 \text{ mV}}$$

and

$$v_{O2} = (6.495)(4) + (1.0825)(4)$$

or

$$\underline{v_{O2} = 30.31 \text{ mV}}$$

14.40

25°C; $I_B = 500$ nA, $I_{OS} = 200$ nA

50°C, $I_B = 500 \text{ nA} + (8 \text{ nA/°C})(25°C) = 700 \text{ nA}$

$I_{OS} = 200 \text{ nA} + (2 \text{ nA/°C})(25°C) = 250 \text{ nA}$

a. Circuit (a): For I_B, bias current cancellation, $\underline{v_O = 0}$

Circuit (b): For I_B, Equation (14.79),

$$v_O = (500 \times 10^{-9})(50 \times 10^3) - (500 \times 10^{-9})(10^6)\left(1 + \frac{50}{50}\right)$$
$$= 0.025 - 1.00 \Rightarrow \underline{v_O = -0.975 \text{ V}}$$

b. Due to offset bias currents.

Circuit (a):

$$v_O = (200 \times 10^{-9})(50 \times 10^3) \Rightarrow \underline{v_O = 0.010 \text{ V}}$$

Circuit (b):

Let $I_{B2} = 600$ nA

$I_{B1} = 400$ nA

Then

$$v_O = (400 \times 10^{-9})(50 \times 10^3) - (600 \times 10^{-9})(10^6)\left(1 + \frac{50}{50}\right)$$
$$= 0.020 - 1.20 \Rightarrow \underline{v_O = -1.18 \text{ V}}$$

c. Circuit (a): Due to I_B, $\underline{v_O = 0}$

Circuit (b): Due to I_B,

$$v_O = (700 \times 10^{-9})(50 \times 10^3) - (700 \times 10^{-9})(10^6)\left(1 + \frac{50}{50}\right)$$
$$= 0.035 - 1.40 \Rightarrow \underline{v_O = -1.365 \text{ V}}$$

Circuit (a): Due to I_{OS},

$$v_O = (250 \times 10^{-9})(50 \times 10^3) \Rightarrow \underline{v_O = 0.0125 \text{ V}}$$

Circuit (b): Due to I_{OS},

Let $I_{B2} = 825$ nA

$I_{B1} = 575$ nA

Then

$$v_O = (575 \times 10^{-9})(50 \times 10^3) - (825 \times 10^{-9})(10^6)\left(1 + \frac{50}{50}\right)$$
$$= 0.02875 - 1.65 \Rightarrow \underline{v_O = -1.62 \text{ V}}$$

14.41

25°C; $I_B = 2\ \mu$A, $I_{OS} = 0.2\ \mu$A

50°C, $I_B = 2\ \mu\text{A} + (0.020\ \mu\text{A/°C})(25°C) = 2.5\ \mu\text{A}$

$I_{OS} = 0.2\ \mu\text{A} + (0.005\ \mu\text{A/°C})(25°C) = 0.325\ \mu\text{A}$

a. Due to I_B: (Assume bias currents into op-amp).

$$v_{O1} = I_B(50 \text{ k}\Omega) = (2 \times 10^{-6})(50 \times 10^3) \Rightarrow \underline{v_{O1} = 0.10 \text{ V}}$$

$$v_{O2} = v_{O1}\left(1 + \frac{60}{20}\right) + I_B(60 \text{ k}\Omega) - I_B(50 \text{ k}\Omega)\left(1 + \frac{60}{20}\right)$$
$$= (0.1)(4) + (2 \times 10^{-6})(60 \times 10^3) - (2 \times 10^{-6})(50 \times 10^3)(4)$$

or

$\underline{v_{O2} = 0.12 \text{ V}}$

b. Due to I_{OS}:

1st op-amp. Let $I_{B1} = 2.1\ \mu\text{A}$

2nd op-amp. Let $I_{B1} = 2.1\ \mu\text{A}$

$I_{B2} = 1.9\ \mu\text{A}$

$$v_{O1} = I_{B1}(50\text{ k}\Omega) = (2.1\times10^{-6})(50\times10^3)$$

$$\Rightarrow \underline{v_{O1} = 0.105 \text{ V}}$$

$$v_{O2} = v_{O1}\left(1+\frac{60}{20}\right) + I_{B1}(60\text{ k}\Omega) - I_{B2}(50\text{ k}\Omega)\left(1+\frac{60}{20}\right)$$

$$= (0.105)(4) + (2.1\times10^{-6})(60\times10^3) - (1.9\times10^{-6})(50\times10^3)(4)$$

or

$\underline{v_{O2} = 0.166 \text{ V}}$

c. Due to I_B:

$$v_{O1} = (2.5\times10^{-6})(50\times10^3) \Rightarrow \underline{v_{O1} = 0.125 \text{ V}}$$

$$v_{O2} = v_{O1}\left(1+\frac{60}{20}\right) + I_B(60\text{ k}\Omega) - I_B(50\text{ k}\Omega)\left(1+\frac{60}{20}\right)$$

$$= (0.125)(4) + (2.5\times10^{-6})(60\times10^3) - (2.5\times10^{-6})(50\times10^3)(4)$$

or

$\underline{v_{O2} = 0.15 \text{ V}}$

Due to I_{OS}:

Let $I_{B1} = 2.6625\ \mu\text{A}$

$I_{B2} = 2.3375\ \mu\text{A}$

$$v_{O1} = I_{B1}(50\text{ k}\Omega) = (2.6625\times10^{-6})(50\times10^3)$$

$$\Rightarrow \underline{v_{O1} = 0.133 \text{ V}}$$

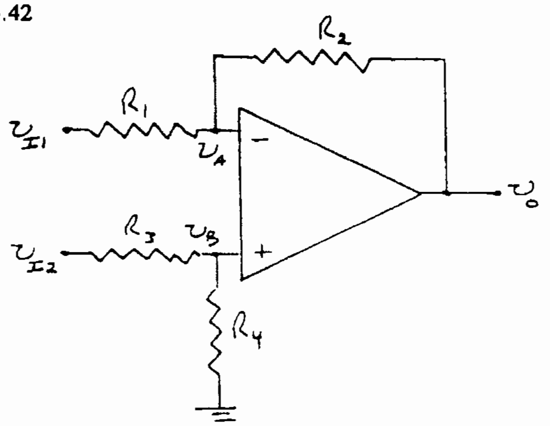
$$v_{O2} = v_{O1}\left(1+\frac{60}{20}\right) + I_{B1}(60\text{ k}\Omega) - I_{B2}(50\text{ k}\Omega)\left(1+\frac{60}{20}\right)$$

$$= (0.133)(4) + (2.6625\times10^{-6})(60\times10^3) - (2.3375\times10^{-6})(50\times10^3)(4)$$

or

$\underline{v_{O2} = 0.224 \text{ V}}$

14.42



$$v_B = \left(\frac{R_4}{R_3+R_4}\right)v_{I2} \text{ and } v_O(v_{I2}) = v_B\left(1+\frac{R_2}{R_1}\right)$$

or

$$v_O(v_{I2}) = \left(\frac{R_4}{R_3+R_4}\right)\left(1+\frac{R_2}{R_1}\right)v_{I2}$$

For v_{I1},

$$v_O(v_{I1}) = -\frac{R_2}{R_1}\cdot v_I$$

Then

$$v_O = \left(\frac{R_4}{R_3+R_4}\right)\left(1+\frac{R_2}{R_1}\right)v_{I2} - \frac{R_2}{R_1}\cdot v_{I1}$$

We can write $v_{I2} = V_{cm} + \frac{V_d}{2}$ and $v_{I1} = V_{cm} - \frac{V_d}{2}$. Then

$$v_O = \left(\frac{R_4}{R_3+R_4}\right)\left(1+\frac{R_2}{R_1}\right)\left(V_{cm}+\frac{V_d}{2}\right) - \frac{R_2}{R_1}\cdot\left(V_{cm}-\frac{V_d}{2}\right)$$

Common-mode gain

$$A_{cm} = \frac{v_O}{V_{cm}} = \left(\frac{R_4}{R_3+R_4}\right)\left(1+\frac{R_2}{R_1}\right) - \frac{R_2}{R_1}$$

Differential mode gain

$$A_d = \frac{v_O}{V_d} = \frac{1}{2}\left[\left(\frac{R_4}{R_3+R_4}\right)\left(1+\frac{R_2}{R_1}\right) + \frac{R_2}{R_1}\right]$$

Then

$$CMRR = \left|\frac{A_d}{A_{cm}}\right| = \frac{\frac{1}{2}\cdot\left[\left(\frac{R_4}{R_3+R_4}\right)\left(1+\frac{R_2}{R_1}\right)+\frac{R_2}{R_1}\right]}{\left[\left(\frac{R_4}{R_3+R_4}\right)\left(1+\frac{R_2}{R_1}\right)-\frac{R_2}{R_1}\right]}$$

$$CMRR = \frac{\frac{1}{2}\cdot\left[\frac{R_4}{R_3}\cdot\frac{1}{\left(1+\frac{R_4}{R_3}\cdot\right)}\cdot\left(1+\frac{R_2}{R_1}\right)+\frac{R_2}{R_1}\right]}{\left[\frac{R_4}{R_3}\cdot\frac{1}{\left(1+\frac{R_4}{R_3}\cdot\right)}\cdot\left(1+\frac{R_2}{R_1}\right)-\frac{R_2}{R_1}\right]}$$

Minimum $CMRR$ ⇒Maximum denominator

⇒ maximum $\frac{R_4}{R_3}$ and minimum $\frac{R_2}{R_1}$. Then

$$\frac{R_4}{R_3} = \frac{(1.02)(50)}{(0.98)(10)} = 5.204$$
$$\frac{R_2}{R_1} = \frac{(0.98)(50)}{(1.02)(10)} = 4.804$$

Then

$$CMRR = \frac{\frac{1}{2}\left[\frac{5.204}{6.204}\cdot(5.804)+(4.804)\right]}{\left[\frac{5.204}{6.204}\cdot(5.804)-(4.804)\right]} = \frac{\frac{1}{2}\cdot(9.6725)}{(0.06447)}$$

$CMRR = 75.0 \Rightarrow CMRR_{dB} = 20\log_{10}(75.0)$
$\Rightarrow \underline{CMRR_{dB} = 37.5 \text{ dB}}$

14.43

Use the results of Problem 14.42:

Let $\frac{R_4}{R_3} = \frac{1+x}{1-x}\cdot\left(\frac{50}{10}\right) \approx (1+2x)(5)$

Let $\frac{R_2}{R_1} = \frac{1-x}{1+x}\cdot\left(\frac{50}{10}\right) \approx (1-2x)(5)$

Then

$$CMRR = \frac{\frac{1}{2}\left[\frac{(1+2x)5}{6+10x}\cdot(6-10x)+(1-2x)(5)\right]}{\left[\frac{(1+2x)5}{6+10x}\cdot(6-10x)-(1-2x)(5)\right]}$$
$$= \frac{\frac{1}{2}[30+10x-100x^2+30-10x-100x^2]}{[30+10x-100x^2-(30-10x-100x^2)]}$$
$$= \frac{\frac{1}{2}\cdot[60-200x^2]}{20x} = \frac{30-100x^2}{20x}$$

a. For $CMRR_{dB} = 90$ dB ⇒ $CMRR = 31,623$
x will be small, neglect the x^2 term. Then

$$20x = \frac{30}{31,623} \Rightarrow x = 0.0000474 = \underline{0.00474\%}$$

b. For $CMRR_{dB} = 60$ dB ⇒ $CMRR = 1000$. Then

$$20x = \frac{30}{1000} \Rightarrow x = 0.0015 = \underline{0.15\%}$$

Chapter 15

Exercise Solutions

E15.1

$$f_{3dB} = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_{3dB}} = \frac{1}{2\pi(10^4)} = 1.59 \times 10^{-5}$$

Let $C = 0.01\ \mu\text{F} \Rightarrow \underline{R = 1.59\ \text{k}\Omega}$

Then

$$C_1 = 0.03546\ \mu\text{F}$$
$$C_2 = 0.01392\ \mu\text{F}$$
$$C_3 = 0.002024\ \mu\text{F}$$

$$|T| = \frac{1}{\sqrt{1+\left(\frac{f}{f_{3d}}\right)^6}} = \frac{1}{\sqrt{1+\left(\frac{20}{10}\right)^6}}$$

$|T| = 0.124$ or $|T| = -18.1$ dB

E15.2

$$f_{3dB} = \frac{1}{2\pi RC} \Rightarrow RC = \frac{1}{2\pi f_{3dB}}$$

$$RC = \frac{1}{2\pi(50 \times 10^3)} = 3.18 \times 10^{-6}$$

Let $C = 0.001\ \mu\text{F} = 1\ \text{nF} \Rightarrow \underline{R = 3.18\ \text{k}\Omega}$

Then

$$R_1 = 2.94\ \text{k}\Omega$$
$$R_2 = 3.44\ \text{k}\Omega$$
$$R_3 = 1.22\ \text{k}\Omega$$
$$R_4 = 8.31\ \text{k}\Omega$$

$$|T| = 0.01 = \frac{1}{\sqrt{1+\left(\frac{f_{3-dB}}{f}\right)^8}}$$

$$1+\left(\frac{f_{3-dB}}{f}\right)^8 = \left(\frac{1}{0.01}\right)^2 = 10^4$$

$$\left(\frac{f_{3-dB}}{f}\right)^2 \cong 10 \Rightarrow f = \frac{f_{3dB}}{\sqrt{10}}$$

$$\Rightarrow \underline{f \cong 15.8\ \text{kHz}}$$

E15.3

1-pole $|T| = \frac{1}{\sqrt{1+\left(\frac{12}{10}\right)^2}} \Rightarrow -3.87$ dB

2-pole $|T| = \frac{1}{\sqrt{1+\left(\frac{12}{10}\right)^4}} \Rightarrow -4.88$ dB

3-pole $|T| = \frac{1}{\sqrt{1+\left(\frac{12}{10}\right)^6}} \Rightarrow -6.0$ dB

4-pole $|T| = \frac{1}{\sqrt{1+\left(\frac{12}{10}\right)^8}} \Rightarrow -7.24$ dB

E15.4

$$R_{eq} = \frac{1}{f_C C}$$

or $f_C C = \frac{1}{R_{eq}} = \frac{1}{5 \times 10^6} = 2 \times 10^{-7}$

If $\underline{C = 10\ \text{pF}} \Rightarrow \underline{f_C = 20\ \text{kHz}}$

E15.5

Low-frequency gain: $T = -\frac{C_1}{C_2} = -\frac{30}{5} = \underline{-6}$

$$f_{3dB} = \frac{f_C C_2}{2\pi C_F} = \frac{(100 \times 10^3)(5 \times 10^{-12})}{2\pi(12 \times 10^{-12})}$$

$$\Rightarrow \underline{f_{3dB} = 6.63\ \text{kHz}}$$

E15.6

$$f_0 = \frac{1}{2\pi\sqrt{3}RC}$$

$$RC = \frac{1}{2\pi f_0\sqrt{3}} = \frac{1}{2\pi(15 \times 10^3)\sqrt{3}} = 6.13 \times 10^{-6}$$

Let $\underline{C = 0.001\ \mu\text{F} = 1\ \text{nF}}$

Then $\underline{R = 6.13\ \text{k}\Omega}$ so $\underline{R_2 = 8R = 49\ \text{k}\Omega}$

E15.7

$$f_0 = \frac{1}{2\pi\sqrt{6}RC} = \frac{1}{2\pi\sqrt{6}(10^4)(100 \times 10^{-12})}$$

$$\Rightarrow \underline{f_0 \cong 65\ \text{kHz}}$$

$$R_2 = 29R = 29(10^4)$$

$$\Rightarrow \underline{R_2 = 290\ \text{k}\Omega}$$

E15.8

$$f_0 = \frac{1}{2\pi RC} \Rightarrow C = \frac{1}{2\pi f_0 R}$$

$$C = \frac{1}{2\pi(800)(10^4)} \Rightarrow \underline{C \cong 0.02\ \mu\text{F}}$$

$$R_2 = 2R_1 = 2(10) \Rightarrow \underline{R_2 = 20\ \text{k}\Omega}$$

E15.9

$$f_0 = \frac{1}{2\pi\sqrt{L \cdot \left(\frac{C_1 C_2}{C_1 + C_2}\right)}} = \frac{1}{2\pi\sqrt{(10^{-6})\left[\frac{(10^{-9})^2}{2 \times 10^{-9}}\right]}}$$

$$\Rightarrow \underline{f_0 = 7.12 \text{ MHz}}$$

$$\frac{C_2}{C_1} = g_m R$$

$$g_m = \frac{C_2}{C_1} \cdot \frac{1}{R} = \frac{1}{4 \times 10^3} \Rightarrow \underline{g_m = 0.25 \text{ mA/V}}$$

We have

$$g_m = 2\left(\frac{k'}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{Th})$$

$k' \cong 20\ \mu\text{A/V}^2$, $V_{GS} - V_{Th} \cong 1$ V

$$\text{So } \frac{W}{L} = \frac{0.25 \times 10^{-3}}{(20 \times 10^{-6})(1)} = 12.5$$

and a value of $W/L = 12.5$ is certainly reasonable.

E15.10

$$V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right) V_H$$

$$2 = \left(\frac{R_1}{R_1 + 20}\right)(12)$$

$$2(R_1 + 20) = 12 R_1$$

$$40 = 10 R_1 \Rightarrow \underline{R_1 = 4 \text{ k}\Omega}$$

E15.11

$$V_{TH} = -\left(\frac{R_1}{R_2}\right) V_L$$

$$0.10 = -\left(\frac{R_1}{R_2}\right)(-10) \Rightarrow \frac{R_1}{R_2} = 0.010$$

Let $\underline{R_1 = 0.10 \text{ k}\Omega}$ then $\underline{R_2 = 10 \text{ k}\Omega}$

E15.12

a. $$V_S = \left(\frac{R_2}{R_1 + R_2}\right) V_{REF} = \left(\frac{10}{1 + 10}\right)(2)$$

$$\underline{V_S = 1.82 \text{ V}}$$

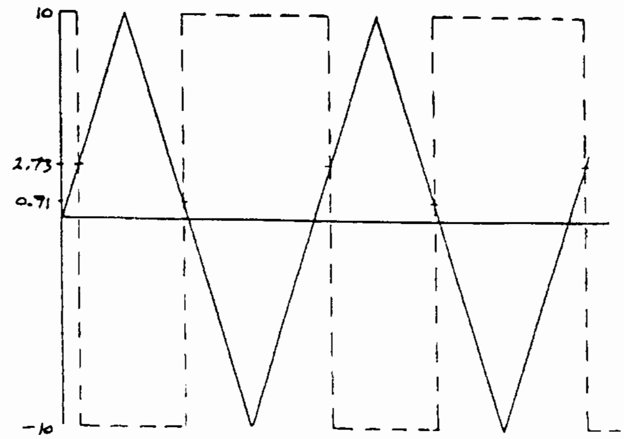
$$V_{TH} = V_S + \left(\frac{R_1}{R_1 + R_2}\right) V_H = 1.82 + \left(\frac{1}{1 + 10}\right)(10)$$

$$\underline{V_{TH} = 2.73 \text{ V}}$$

$$V_{TL} = V_S + \left(\frac{R_1}{R_1 + R_2}\right) V_L = 1.82 + \left(\frac{1}{1 + 10}\right)(-10)$$

$$\underline{V_{TL} = 0.91 \text{ V}}$$

b.



E15.13

$$V_S = \left(1 + \frac{R_1}{R_2}\right) V_{REF}$$

$$V_{TH} = V_S - \left(\frac{R_1}{R_2}\right) V_L \text{ and } V_{TL} = V_S - \left(\frac{R_1}{R_2}\right) V_H$$

$$\text{Hysteresis Width} = V_{TH} - V_{TL} = \left(\frac{R_1}{R_2}\right)(V_H - V_L)$$

$$2.5 = \left(\frac{R_1}{R_2}\right)(5 - [-5]) = 10\left(\frac{R_1}{R_2}\right)$$

$$\text{So } \underline{\frac{R_1}{R_2} = 0.25}$$

Then

$$V_S = -1 = \left(1 + \frac{R_1}{R_2}\right) V_{REF} = (1 + 0.25) V_{REF}$$

$$\Rightarrow \underline{V_{REF} = -0.8 \text{ V}}$$

Then

$$V_{TH} = -1 - (0.25)(-5) \Rightarrow \underline{V_{TH} = 0.25 \text{ V}}$$

$$V_{TL} = -1 - (0.25)(5) \Rightarrow \underline{V_{TL} = -2.25 \text{ V}}$$

E15.14

$$V_{TH} - V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right)(V_H - V_L)$$

$$0.10 = \left(\frac{R_1}{R_1 + R_2}\right)(10 - [-10])$$

$$1 + \frac{R_2}{R_1} = \frac{20}{0.10} = 200 \Rightarrow \underline{\frac{R_2}{R_1} = 199}$$

$$V_S = \left(\frac{R_2}{R_1 + R_2}\right) V_{REF}$$

$$V_{REF} = \left(1 + \frac{R_1}{R_2}\right) V_S = \left(1 + \frac{1}{199}\right)(1)$$

$$\Rightarrow \underline{V_{REF} = 1.005 \text{ V}}$$

$$I = \frac{V_H - V_{BE}(\text{on}) - V_\gamma}{R + 0.1}$$

$$R + 0.1 = \frac{10 - 0.7 - 0.7}{0.2} = 43 \text{ k}\Omega$$

$$\underline{R = 42.9 \text{ k}\Omega}$$

E15.15

At $t = 0^-$, let $v_O = -5$ so $v_X = -2.5$. For $t > 0$

$$v_X = 10 + (-2.5 - 10)\exp\left(\frac{-t}{\tau_X}\right)$$

When $v_X = 5.0$, output switches

$$5.0 = 10 - 12.5\exp\left(-\frac{t_1}{\tau_X}\right)$$

$$\exp\left(-\frac{t_1}{\tau_X}\right) = \frac{10-5}{12.5} = \frac{5.0}{12.5}$$

$$\exp\left(+\frac{t_1}{\tau_X}\right) = \frac{12.5}{5.0} \Rightarrow t_1 = \tau_X \cdot \ln\left(\frac{12.5}{5.0}\right)$$

$$\Rightarrow t_1 = \tau_X(0.916)$$

During the next part of the cycle

$$v_X = -5 + (5 - [-5])\exp\left(-\frac{t}{\tau_X}\right)$$

When $v_X = -2.5$, output switches

$$-2.5 = -5 + 10\exp\left(-\frac{t_2}{\tau_X}\right)$$

$$\exp\left(-\frac{t_2}{\tau_X}\right) = \frac{5-2.5}{10} = \frac{2.5}{10}$$

$$\exp\left(+\frac{t_2}{\tau_X}\right) = \frac{10}{2.5} \Rightarrow t_2 = \tau_X \cdot \ln\left(\frac{10}{2.5}\right)$$

$$\Rightarrow t_2 = \tau_X(1.39)$$

$$\text{Period} = t_1 + t_2 = T = [(0.916) + (1.39)]\tau_X$$

$$= 2.31\tau_X$$

$$\Rightarrow \underline{\text{Frequency} = \frac{1}{2.31\tau_X}}$$

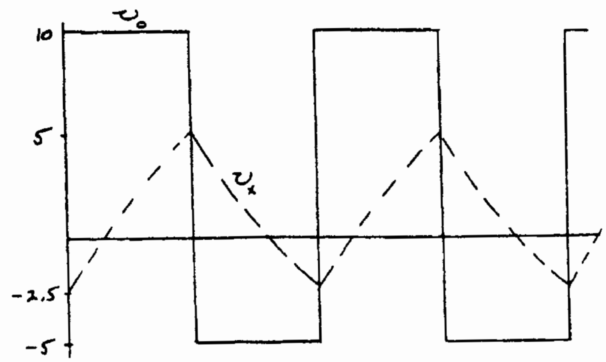
$$\tau_X = (50 \times 10^3)(0.01 \times 10^{-6}) = 5 \times 10^{-4} \text{ s}$$

$$\Rightarrow \underline{f = 866 \text{ Hz}}$$

$$\text{Duty cycle} = \frac{t_1}{t_1 + t_2} \times 100\%$$

$$= \frac{(0.916)}{(0.916) + (1.39)} \times 100\%$$

$$\Rightarrow \underline{\text{Duty cycle} = 39.7\%}$$



E15.16

$$v_X = \left(\frac{R_1}{R_1 + R_2}\right)v_O = \left(\frac{10}{10+20}\right)v_O = \frac{1}{3}v_O$$

$$t = 0, \; v_X = -\frac{10}{3}.$$

$$v_X = 10 + \left(-\frac{10}{3} - 10\right)\exp\left(-\frac{t}{\tau_X}\right)$$

Output switches when $v_X = \frac{10}{3}$

$$\frac{10}{3} = 10 - 13.33\exp\left(-\frac{t_1}{\tau_X}\right)$$

$$\exp\left(-\frac{t_1}{\tau_X}\right) = \frac{10 - 3.33}{13.33} = \frac{6.67}{13.33}$$

$$\exp\left(+\frac{t_1}{\tau_X}\right) = \frac{13.33}{6.67} \cong 2$$

$$t_1 = \tau_X \ln(2) = (0.693)\tau_X$$

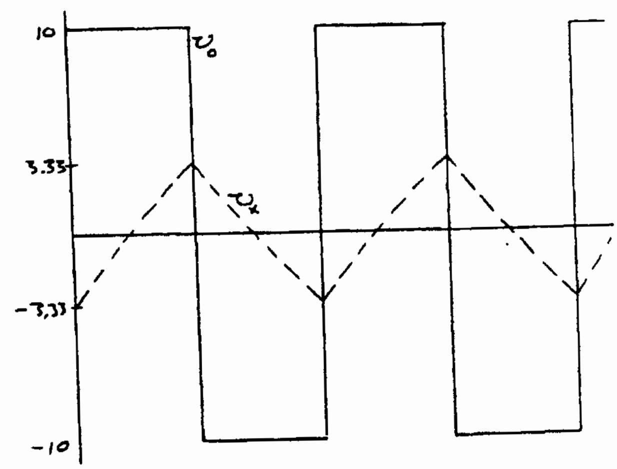
$$T = 2(0.693)\tau_X$$

$$f = \frac{1}{2(0.693)\tau_X}$$

$$\tau_X = R_X C_X = (10^4)(0.1 \times 10^{-6}) = 1 \times 10^{-3}$$

$$\Rightarrow \underline{f = 722 \text{ Hz}}$$

$$\Rightarrow \underline{\text{Duty cycle} = 50\%}$$



E15.17

a. $\tau_X = R_X C_X$

$$v_Y = \left(\frac{R_1}{R_1 + R_2}\right) v_0 = \left(\frac{10}{10+90}\right)(12) = 1.2 \text{ V}$$

$$\beta = \frac{R_1}{R_1 + R_2} = 0.10$$

$$T = \tau_X \ln\left[\frac{1 + V_\gamma/V_P}{1-\beta}\right] = \tau_X \ln\left[\frac{1 + \frac{0.7}{12}}{1-(0.10)}\right]$$

$$T = 50 \times 10^{-6} = \tau_X \ln[1.18] = (0.162)\tau_X$$

$$R_X = \frac{50 \times 10^{-6}}{(0.1 \times 10^{-6})(0.162)} \Rightarrow \underline{R_X = 3.09 \text{ k}\Omega}$$

b. Recovery time

$$v_X = V_P + (-1.2 - V_P)\exp\left(-\frac{t}{\tau_X}\right)$$

When $v_X = V_\gamma$, $t = t_2$

$$0.7 = 12 + (-1.2 - 12)\exp\left(-\frac{t_2}{\tau_X}\right)$$

$$\exp\left(-\frac{t_2}{\tau_X}\right) = \frac{12 - 0.7}{13.2} = 0.856$$

$$t_2 = \tau_X \ln\left(\frac{1}{0.856}\right) = (0.155)\tau_X$$

$$\tau_X = (3.09 \times 10^3)(0.1 \times 10^{-6}) = 3.09 \times 10^{-4}$$

$$\Rightarrow \underline{t_2 = 47.9\ \mu s}$$

E15.18

$$\beta = \left(\frac{R_1}{R_1 + R_2}\right) = \frac{20}{20+40} = 0.333$$

$$\tau_X = R_X C_X = (10^4)(0.01 \times 10^{-6}) = 1 \times 10^{-4}$$

$$T = \tau_X \ln\left(\frac{1 + V_\gamma/V_P}{1-\beta}\right) = (1 \times 10^{-4})\ln\left[\frac{1 + \frac{0.7}{8}}{1 - 0.333}\right]$$

$$\Rightarrow \underline{T = 48.9\ \mu s}$$

Recovery time

$$0.7 = 8 + (-2.66 - 8)\exp\left(-\frac{t_2}{\tau_X}\right)$$

$$\exp\left(-\frac{t_2}{\tau_X}\right) = \frac{8 - 0.7}{10.66} = 0.685$$

$$t_2 = \tau_X \ln\left(\frac{1}{0.685}\right)$$

$$\Rightarrow \underline{t_2 = 37.8\ \mu s}$$

E15.19

$$f = \frac{1}{0.693(R_A + 2R_B)C}$$

$$= \frac{1}{(0.693)[20 + 2(80)] \times 10^3 \times (0.01 \times 10^{-6})}$$

$$\Rightarrow \underline{f = 802 \text{ Hz}}$$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$= \frac{20 + 80}{20 + 2(80)} \times 100\%$$

$\Rightarrow$ Duty cycle = 55.6%

E15.20

$$f = \frac{1}{(0.693)(R_A + R_B)C}$$

$$R_A + R_B = \frac{1}{(0.693)fC}$$

Let $C = 0.01\ \mu\text{F}$, $f = 1$ kHz

$$R_A + R_B = \frac{1}{(0.693)(10^3)(0.01 \times 10^{-6})} = 1.44 \times 10^5$$

$$\text{Duty cycle} = 55 = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$55 = \frac{(1.44 \times 10^5)(100)}{(1.44 \times 10^5) + R_B}$$

$$R_B = \frac{(1.44 \times 10^5)(100 - 55)}{55}$$

$$\Rightarrow \underline{R_B = 118 \text{ k}\Omega} \text{ so } \underline{R_A = 26 \text{ k}\Omega}$$

E15.21

a. $\overline{P} = \frac{1}{2}\cdot\frac{V_P^2}{R_L}$

$$V_P = \sqrt{2R_L\overline{P}} = \sqrt{2(8)(1)} \Rightarrow \underline{V_P = 4 \text{ V}}$$

$$I_P = \frac{V_P}{R_L} = \frac{4}{8} \Rightarrow \underline{I_P = 0.5 \text{ A}}$$

b. $V_{CE} = 12 - 4 = 8$ V

$I_C \approx 0.5$ A

So $P = I_C \cdot V_{CE} = (0.5)(8) \Rightarrow \underline{P = 4 \text{ W}}$

E15.22

a. $\frac{v_{01}}{v_I} = \left(1 + \frac{R_2}{R_1}\right) = \left(1 + \frac{30}{20}\right) = 2.5$

$$\frac{v_{02}}{v_I} = -\frac{R_4}{R_3} = -\frac{50}{20} = -2.5$$

(b) $\overline{P} = \frac{1}{2}\cdot\frac{V_L^2}{R_L} = \frac{1}{2}\cdot\frac{[12-(-12)]^2}{1.2} = 240\, mW$

Or

$\underline{\overline{P} = 0.24\, W}$

c. $\frac{12}{2.5} = \underline{V_{pi} = 4.8 \text{ V}}$

E15.23

$$\text{Line regulation} = \frac{dV_O}{dV^+} = \frac{dV_O}{dV_Z}\cdot\frac{dV_Z}{dV^+}$$

Now

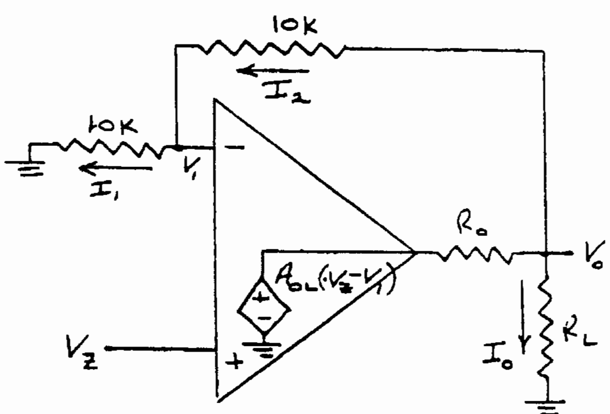
$$\frac{dV_O}{dV_Z} = \left(1+\frac{10}{10}\right) = 2$$

$$\frac{dV_Z}{dV^+} = \left(\frac{r_Z}{r_Z+R_1}\right) = \frac{10}{10+4400} = 0.00227$$

So Line regulation $= (2)(0.00227) = 0.00454$

$\underline{0.454\%}$

E15.24



$$\frac{V_1}{10} = \frac{V_O - V_1}{10} \Rightarrow V_1\left(\frac{1}{10}+\frac{1}{10}\right) = \frac{V_O}{10}$$

$$V_1\left(\frac{2}{10}\right) = \frac{V_O}{10} \Rightarrow V_O = 2V_1 \Rightarrow V_1 = \frac{V_O}{2}$$

$$\frac{V_O - V_1}{10} + \frac{V_O}{R_L} + \frac{V_O - A_{OL}(V_Z - V_1)}{R_O} = 0$$

$$\frac{V_O}{10} + \frac{V_O}{R_L} + \frac{V_O}{R_O} - \frac{A_{OL}V_Z}{R_O} = \frac{V_1}{10} - \frac{A_{OL}V_1}{R_O}$$

$$= \frac{V_O}{2(10)} - \frac{A_{OL}V_O}{2R_O}$$

$$\frac{V_O}{10} + I_O + \frac{V_O}{0.5} - \frac{1000(6.3)}{0.5} = \frac{V_O}{20} - \frac{(1000)V_O}{2(0.5)}$$

$V_O[0.10 + 2.0 - 0.05 + 1000] + I_O = 12,600$

$V_O(1002.05) + I_O = 12,600$

For $I_O = 1$ mA $\Rightarrow V_O = 12.5732$

For $I_O = 100$ mA $\Rightarrow V_O = 12.4744$

$$\text{Load reg} = \frac{V_O(\text{NL}) - V_O(\text{FL})}{V_O(\text{NL})} \times 100\%$$

$$= \frac{12.5732 - 12.4744}{12.5732} \times 100\%$$

$\underline{\text{Load reg} = 0.786\%}$

E15.25

a. $$I_{C3} = \frac{V_Z - 3V_{BE}(\text{on})}{R_1 + R_2 + R_3}$$

$$I_{C3} = \frac{5.6 - 3(0.6)}{3.9 + 3.4 + 0.576} = \frac{3.8}{7.88}$$

$\Rightarrow \underline{I_{C3} = 0.482 \text{ mA}}$

$$I_{C4}R_4 = V_T \ln\left(\frac{I_{C3}}{I_{C4}}\right)$$

$$I_{C4}(0.1) = (0.026)\ln\left(\frac{0.482}{I_{C4}}\right)$$

By trial and error

$\underline{I_{C4} = 0.213 \text{ mA}}$

$V_{B7} = 2(0.6) + (0.482)(3.9)$

$\Rightarrow \underline{V_{B7} = 3.08 \text{ V}}$

b. $$\left(\frac{R_{13}}{R_{13} + R_{12}}\right)V_O = V_{B8} = V_{B7}$$

$$\left(\frac{2.23}{2.23 + R_{12}}\right)(5) = 3.08$$

$(2.23)(5) = (3.08)(2.23) + (3.08)R_{12}$

$11.15 = 6.868 = 3.08R_{12}$

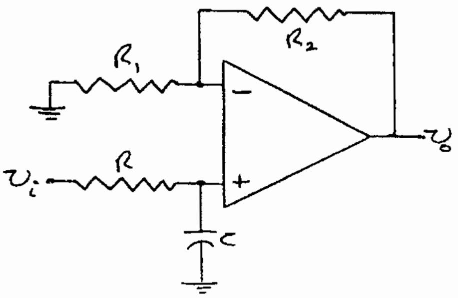
$\Rightarrow \underline{R_{12} = 1.39 \text{ k}\Omega}$

Chapter 15

Problem Solutions

15.1

(a) For example:

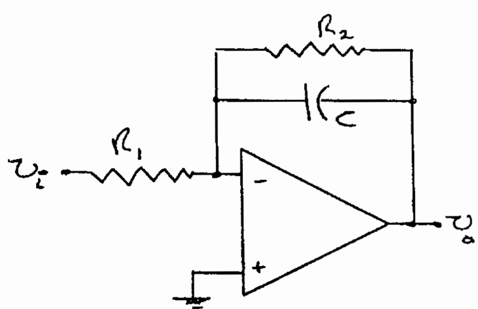


Low-Frequency: $\frac{v_o}{v_i} = \left(1 + \frac{R_2}{R_1}\right) = 10 \Rightarrow \frac{R_2}{R_1} = 9$

Corner Frequency:

$f = \frac{1}{2\pi RC} = 5x10^3 \Rightarrow RC = 3.18x10^{-5}$

(b) For Example:



$$\frac{v_o}{v_i} = \frac{-R_2 \| \frac{1}{j\omega C}}{R_1} = -\frac{R_2}{R_1} \cdot \frac{1}{1 + j\omega R_2 C}$$

So, set

$\frac{R_2}{R_1} = 15 \Rightarrow$ For example, $R_1 = 10\,k\Omega$, $R_2 = 150\,k\Omega$

$$R_2 C = \frac{1}{2\pi f_{3\text{-}dB}} = \frac{1}{2\pi(15x10^3)} = 1.06x10^{-5}$$

Then $C = 70.7\,pF$

15.2

(a) $|A_v| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3\text{-}dB}}\right)^2}} = \frac{1}{\sqrt{1 + (2)^2}} = 0.447 \Rightarrow$

$|A_v| = -7\,dB$

(b) $|A_v| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3\text{-}dB}}\right)^2}} = \frac{1}{\sqrt{1 + (2)^4}} = 0.2425 \Rightarrow$

$|A_v| = -12.3\,dB$

(c) $|A_v| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3\text{-}dB}}\right)^2}} = \frac{1}{\sqrt{1 + (2)^6}} = 0.1240 \Rightarrow$

$|A_v| = -18.1\,dB$

15.3

Using Figure 15.9(a)

$$f_{3-dB} = \frac{1}{2\pi RC}$$

So

$$RC = \frac{1}{2\pi f_{3-dB}} = \frac{1}{2\pi(10 \times 10^3)} = 1.59 \times 10^{-5}$$

For example, $C = 0.001\ \mu\text{F}$, $R = 15.9\ \text{k}\Omega$

so that $R_3 = 11.2\ \text{k}\Omega$ and $R_4 = 22.4\ \text{k}\Omega$

15.4

Use Figure 15.10(b)

$$f_{3-dB} = \frac{1}{2\pi RC}$$

or

$$RC = \frac{1}{2\pi(50 \times 10^3)} = 3.18 \times 10^{-6}$$

For example, let $C = 100$ pF Then $R = 31.8\ \text{k}\Omega$

And $R_1 = 8.97\ \text{k}\Omega$

$R_2 = 22.8\ \text{k}\Omega$

$R_3 = 157\ \text{k}\Omega$

From Equation (15.26)

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{f_{3dB}}{f}\right)^6}}$$

We find

f kHz	$\|T\|$
30	0.211
35	0.324
40	0.456
45	0.589

15.5

From Equation (15.7),

$$T(s) = \frac{Y_1Y_2}{Y_1Y_2 + Y_4(Y_1+Y_2+Y_3)}$$

For a high-pass filter, let $Y_1 = Y_2 = sC$,

$$Y_3 = \frac{1}{R_3}, \text{ and } Y_4 = \frac{1}{R_4}$$

Then

$$T(s) = \frac{s^2C^2}{s^2C^2 + \frac{1}{R_4}\left(sC + sC + \frac{1}{R_3}\right)}$$
$$= \frac{1}{1 + \frac{1}{sR_4C}\left(2 + \frac{1}{sR_3C}\right)}$$

Define $\tau_3 = R_3C$ and $\tau_4 = R_4C$

$$T(s) = \frac{1}{1 + \frac{1}{s\tau_4}\left(2 + \frac{1}{s\tau_3}\right)}$$

Set $s = j\omega$

$$T(j\omega) = \frac{1}{1 + \frac{1}{j\omega\tau_4}\left(2 + \frac{1}{j\omega\tau_3}\right)}$$
$$= \frac{1}{1 - \frac{j}{\omega\tau_4}\left(2 - \frac{j}{\omega\tau_3}\right)}$$
$$= \frac{1}{\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right) - \frac{2j}{\omega\tau_4}}$$

$$|T(j\omega)| = \left\{\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right)^2 + \frac{4}{\omega^2\tau_4^2}\right\}^{-1/2}$$

For a maximally flat filter, we want

$$\left.\frac{d|T|}{d\omega}\right|_{\omega\to\infty} = 0$$

Taking the derivative, we find

$$\frac{d|T(j\omega)|}{d\omega} = -\frac{1}{2}\left\{\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right)^2 + \frac{4}{\omega^2\tau_4^2}\right\}^{-3/2}$$
$$\times \left[2\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right)\left(\frac{2}{\omega^3\tau_3\tau_4}\right) + \frac{4(-2)}{\omega^3\tau_4^2}\right]$$

or

$$\left.\frac{d|T(j\omega)|}{d\omega}\right|_{\omega\to\infty} = 0$$
$$= \left[\left(\frac{4}{\omega^3\tau_3\tau_4}\right)\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right) - \frac{8}{\omega^3\tau_4^2}\right]$$
$$= \frac{4}{\omega^3}\left[\frac{1}{\tau_3\tau_4}\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right) - \frac{2}{\tau_4^2}\right]$$

Then

$$\left.\left[\frac{1}{\tau_3\tau_4}\left(1 - \frac{1}{\omega^2\tau_3\tau_4}\right) - \frac{2}{\tau_4^2}\right]\right|_{\omega\to\infty} = 0$$

So that $\frac{1}{\tau_3} = \frac{2}{\tau_4} \Rightarrow 2\tau_3 = \tau_4$

Then the transfer function can be written as:

$$|T(j\omega)| = \left\{\left[1 - \frac{1}{\omega^2(2\tau_3^2)}\right]^2 + \frac{4}{\omega^2(4\tau_3^2)}\right\}^{-1/2}$$
$$= \left\{1 - \frac{1}{\omega^2\tau_3^2} + \frac{1}{4(\omega^2\tau_3^2)^2} + \frac{1}{\omega^2\tau_3^2}\right\}^{-1/2}$$
$$= \left\{1 + \frac{1}{4(\omega^2\tau_3^2)^2}\right\}^{-1/2}$$

$3 - dB$ frequency

$$2\omega^2\tau_3^2 = 1 \text{ or } \omega = \frac{1}{\sqrt{2}\tau_3} = \frac{1}{\sqrt{2}R_3C}$$

Define

$$\omega = \frac{1}{RC}$$

So that

$$\underline{R_3 = \frac{R}{\sqrt{2}}}$$

We had $2\tau_3 = \tau_4$ or $2(R_3C) = R_4C \Rightarrow R_4 = 2R_3$

So that $\underline{R_4 = \sqrt{2}\cdot R}$

15.6

From Equation (15.25)

$$|T| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_{3-dB}}\right)^{2N}}}$$

-25 dB $\Rightarrow |T| = 0.0562$

$$\frac{f}{f_{3-dB}} = \frac{20}{10} = 2$$

So

$$0.0562 = \frac{1}{\sqrt{1+(2)^{2N}}}$$

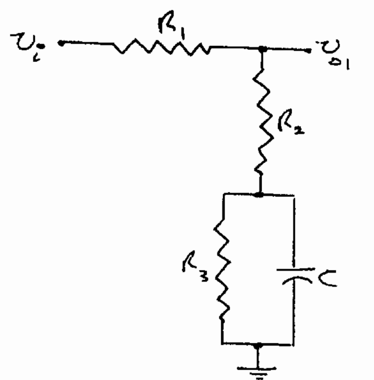
$$1+(2)^{2N} = 316.6 \Rightarrow (2)^{2N} = 315.6$$

$$2N \cdot \ln(2) = \ln(315.6)$$

$$\Rightarrow N = 4.15 \Rightarrow \underline{N=5} \text{ A 5-pole filter}$$

15.7

Consider

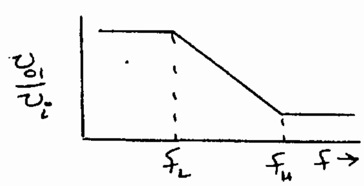


For low-frequency: $\frac{v_o}{v_i} = \frac{R_2+R_3}{R_1+R_2+R_3}$

For high-frequency: $\frac{v_o}{v_i} = \frac{R_2}{R_1+R_2}$

So we need

$$\frac{R_2+R_3}{R_1+R_2+R_3} = 25\left(\frac{R_2}{R_1+R_2}\right)$$

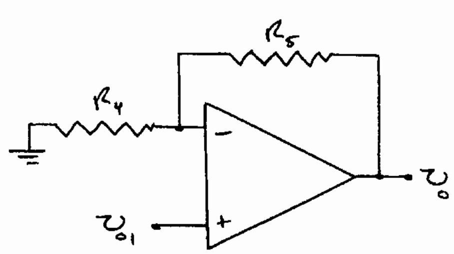


Let $R_1+R_2 = 50\ k\Omega$ and $\underline{R_2 = 1.5\ k\Omega} \Rightarrow$ $\underline{R_1 = 48.5\ k\Omega}$

Then

$$\frac{1.5+R_3}{50+R_3} = 25\left(\frac{1.5}{50}\right) \Rightarrow \underline{R_3 = 144\ k\Omega}$$

Connect the output of this circuit to a non-inverting op-amp circuit.



At low-frequency:

$$v_{o1} = \frac{R_2+R_3}{R_1+R_2+R_3}\cdot v_i = \frac{1.5+144}{48.5+1.5+144}\cdot v_i = 0.75 v_i$$

Need to have $v_o = 25$.

$$v_o = 25 = \left(1+\frac{R_5}{R_4}\right)\cdot v_{o1} = \left(1+\frac{R_5}{R_4}\right)(0.75)v_i \Rightarrow$$

$$\frac{R_5}{R_4} = 32.3$$

To check at high-frequency.

$$v_{o1} = \frac{R_2}{R_1+R_2}v_i = \frac{1.5}{1.5+48.5}v_i = 0.03v_i$$

$$v_o = (1+32.3)v_{o1} = (33.3)(0.03)v_i = (1.0)v_i$$

which meets the design specification

Consider the frequency response.

$$\frac{v_{o1}}{v_i} = \frac{R_2 + R_3 \| \frac{1}{sC}}{R_1+R_2+R_3 \| \frac{1}{sC}}$$

Now

$$R_3 \| \frac{1}{sC} = \frac{R_3}{1+sR_3C}$$

Then, we find

$$\frac{v_{o1}}{v_i} = \frac{R_3+R_2(1+sR_3C)}{R_3+(R_1+R_2)(1+sR_3C)}$$

which can be rearranged as

$$\frac{v_{o1}}{v_i} = \frac{(R_2+R_3)(1+s(R_2\|R_3)C)}{(R_1+R_2+R_3)(1+s(R_3\|(R_1+R_2))C)}$$

So

$$f_L \cong \frac{1}{2\pi(R_2\|R_3)C} = \frac{1}{2\pi(1.5\|144)x10^3C} = \frac{1}{(9.33x10^3)C}$$

$$f_H \cong \frac{1}{2\pi(R_3\|(R_1+R_2))C} = \frac{1}{2\pi(144\|50)x10^3C} = \frac{1}{(2.33x10^5)C}$$

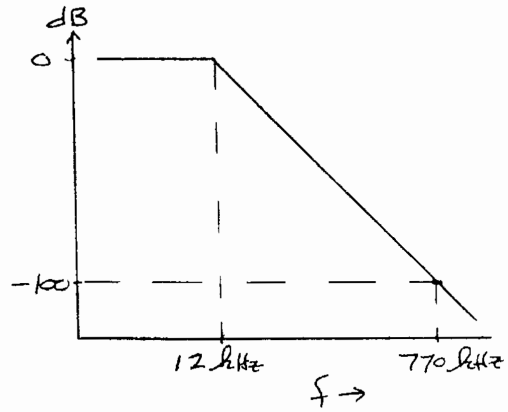
Set

$$25\ kHz = \frac{f_L+f_H}{2} = \frac{1}{2}\left[\frac{1}{(9.33x10^3)C} + \frac{1}{(2.33x10^5)C}\right]$$

Which yields

$\underline{C = 2.23\ nF}$

15.8



$$|A_v| = \frac{1}{\sqrt{1+\left(\frac{f}{f_{3-dB}}\right)^{2N}}}$$

$-100\,dB \Rightarrow 10^{-5}$

So

$$10^{-5} = \frac{1}{\sqrt{1+\left(\frac{770}{12}\right)^{2N}}}$$

or

$$1+(64.2)^{2N} = \left(\frac{1}{10^{-5}}\right)^2 = 10^{10}$$

or

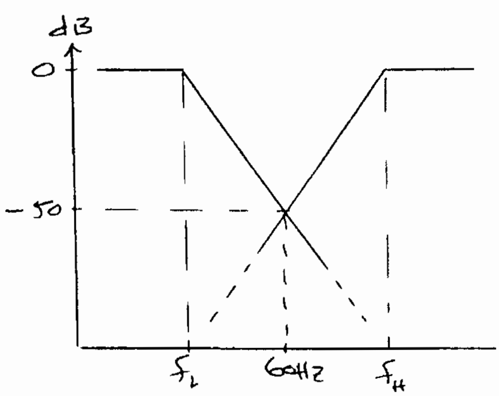
$$(64.2)^{2N} \cong 10^{10}$$

Now

N	Left Side
1	$4.112x10^3$
2	$1.7x10^7$
3	$7x10^{10}$

So, we need a 3rd order filter.

15.9



Low-pass: $-50\,dB \Rightarrow 3.16x10^{-3}$

Then

$$3.16x10^{-3} = \frac{1}{\sqrt{1+\left(\frac{f}{f_L}\right)^4}} = \frac{1}{\sqrt{1+\left(\frac{60}{f_L}\right)^4}}$$

We find $f_L = 3.37\ Hz$

High Pass:

$$3.16x10^{-3} = \frac{1}{\sqrt{1+\left(\frac{f_H}{f}\right)^4}} = \frac{1}{\sqrt{1+\left(\frac{f_H}{60}\right)^4}}$$

We find $f_H = 1067\ Hz$

Bandwidth: $BW = f_H - f_L = 1067 - 3.37 \Rightarrow$

$\underline{BW \cong 1064\ Hz}$

15.10

a.

$$\frac{v_I}{R_4} = -\frac{v_{O2}}{R_3} - \frac{v_O}{R_1 \parallel \left(\frac{1}{sC}\right)} \qquad (1)$$

$$\frac{v_O}{R_2} = -\frac{v_{O1}}{\left(\frac{1}{sC}\right)} \qquad (2)$$

$$\frac{v_{O1}}{R_5} = -\frac{v_{O2}}{R_5} \Rightarrow v_{O1} = -v_{O2} \qquad (3)$$

Then

$$\frac{v_O}{R_2} = +\frac{v_{O2}}{\left(\frac{1}{sC}\right)} \text{ or } v_{O2} = v_O\left(\frac{1}{sR_2C}\right) \qquad (2)$$

And

$$\frac{v_I}{R_4} = -\frac{v_O}{R_3}\cdot\left(\frac{1}{sR_2C}\right) - \frac{v_O}{R_1 \parallel \left(\frac{1}{sC}\right)}$$

$$= -v_O\left[\frac{1}{R_3(sR_2C)} + \frac{1}{\frac{R_1\cdot(1/sC)}{R_1+(1/sC)}}\right] \qquad (1)$$

$$= -v_O\left[\frac{1}{R_3(sR_2C)} + \frac{1+sR_1C}{R_1}\right]$$

$$= -v_O\left[\frac{R_1+(1+sR_1C)(sR_2R_3C)}{(sC)R_1R_2R_3}\right]$$

Then

$$\frac{v_O}{v_I} = -\frac{1}{R_4}\left[\frac{(sC)(R_1R_2R_3)}{R_1+sR_2R_3C+s^2R_1R_2R_3C^2}\right]$$

or

$$A_v(s) = \frac{v_O}{v_I} = \frac{-\frac{1}{R_4}}{\frac{1}{R_1}+sC+\frac{1}{sCR_2R_3}}$$

b. $$A_v(j\omega) = \frac{-\frac{1}{R_4}}{\frac{1}{R_1} + j\omega C + \frac{1}{j\omega C R_2 R_3}}$$

or

$$A_v(j\omega) = \frac{-\frac{1}{R_4}}{\frac{1}{R_1} + j\left[\omega C - \frac{1}{\omega C R_2 R_3}\right]}$$
$$= -\frac{R_1}{R_4} \cdot \frac{1}{\left\{1 + j\left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right]\right\}}$$

$$|A_v(j\omega)| = \frac{R_1}{R_4} \cdot \frac{1}{\left\{1 + \left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right]^2\right\}^{1/2}}$$

$|A_v|_{max}$ when $\left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right] = 0$

Then

$$|A_v|_{max} = \frac{R_1}{R_4} = \frac{85}{3} \Rightarrow \underline{|A_v|_{max} = 28.3}$$

Now

$$\omega R_1 C\left[1 - \frac{1}{\omega^2 C^2 R_2 R_3}\right] = 0 \text{ or } \omega = \frac{1}{C\sqrt{R_2 R_3}}$$

Then

$$f = \frac{1}{2\pi C\sqrt{R_2 R_3}} = \frac{1}{2\pi(0.1 \times 10^{-6})\sqrt{(300)^2}}$$

So

$$\underline{f = 5.305 \text{ kHz}}$$

To find the two $3 - dB$ frequencies,

$$\left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right] = \pm 1$$
$$\omega^2 R_1 R_2 R_3 C^2 - R_1 = \pm\omega R_2 R_3 C$$

$$\omega^2(85 \times 10^3)(300)^2(0.1 \times 10^{-6})^2 - 85 \times 10^3$$
$$= \pm\omega(300)^2(0.1 \times 10^{-6})$$
$$\omega^2(7.65 \times 10^{-5}) - 85 \times 10^3 = \pm\omega(9 \times 10^{-3})$$
$$\omega^2(7.65 \times 10^{-5}) \pm \omega(9 \times 10^{-3}) - 85 \times 10^3 = 0$$

$$\omega = \frac{\pm(9 \times 10^{-3})}{2(7.65 \times 10^{-5})} \pm \frac{\sqrt{(9 \times 10^{-3})^2 + 4(7.65 \times 10^{-5})(85 \times 10^{-3})}}{2(7.65 \times 10^{-5})}$$

We find $f = 5.315$ kHz and $f = 5.296$ kHz

15.11

a.

$$\frac{v_I - v_A}{R} = \frac{v_A}{\left(\frac{1}{sC}\right)} \quad (1)$$

$$\frac{v_I - v_B}{R} = \frac{v_B - v_O}{R} \quad (2)$$

and $v_A = v_B$

So

$$\frac{v_I}{R} = v_A\left(\frac{1}{R} + sC\right) = v_A\left(\frac{1 + sRC}{R}\right) \quad (1)$$

or

$$v_A = \frac{v_I}{1 + sRC}$$

Then

$$v_I + v_O = 2v_B = 2v_A = \frac{2v_I}{1 + sRC} \quad (2)$$

$$v_O = v_I\left[\frac{2}{1 + sRC} - 1\right] = v_I\left[\frac{1 - sRC}{1 + sRC}\right]$$

Now

$$\frac{v_O}{v_I} = A(j\omega) = \frac{1 - j\omega RC}{1 + j\omega RC}$$
$$|A| = \frac{\sqrt{1 + \omega^2 R^2 C^2}}{\sqrt{1 + \omega^2 R^2 C^2}} \Rightarrow |A| = 1$$

Phase:

$$\phi = -2\tan^{-1}(\omega RC)$$

b. $RC = (10^4)(15.9 \times 10^{-9}) = 1.59 \times 10^{-4}$

f	ϕ
0	0
10^2	−11.4
5×10^3	−53.1
$1/2\pi RC = 10^3$ Hz	−90°
5×10^3	−157
10^4	−169

15.12

a. $\frac{V_i}{R_1} + \frac{V_i - V_0}{R_2 \| (1/sC)} = 0$

$$\frac{V_i}{R_1} + \frac{V_i - V_0}{\left[\frac{R_2}{1 + sR_2C}\right]} = 0$$

$$\frac{R_2}{R_1} \cdot \frac{1}{1 + sR_2C}(V_i) + V_i = V_0$$

$$\frac{V_0}{V_i} = \frac{R_2 + R_1(1 + sR_2C)}{R_1(1 + sR_2C)}$$

$$= \frac{(R_2 + R_1)[1 + s(R_1 \| R_2)C]}{R_1(1 + sR_2C)}$$

$$\Rightarrow \frac{V_0}{V_i} = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{1 + s(R_1 \| R_2)C}{(1 + sR_2C)}\right]$$

$$\Rightarrow f_{3dB_1} = \frac{1}{2\pi R_2 C}$$

$$\Rightarrow f_{3dB_2} = \frac{1}{2\pi (R_1 \| R_2) C}$$

b. $\frac{V_i}{R_1 \| (1/sC)} + \frac{V_i - V_0}{R_2} = 0$

$$\frac{V_i}{\left(\frac{R_1}{1 + sR_1C}\right)} + \frac{V_i}{R_2} = \frac{V_0}{R_2}$$

$$V_i\left[\frac{R_2}{R_1} \cdot (1 + sR_1C) + 1\right] = V_0$$

$$\frac{V_i}{R_1} \cdot [R_2 + R_1 + sR_1R_2C] = V_0$$

$$\frac{V_0}{V_i} = \frac{R_2 + R_1}{R_1} \cdot [1 + s(R_1 \| R_2)C]$$

$$\Rightarrow \frac{V_0}{V_i} = \left(1 + \frac{R_2}{R_1}\right)[1 + s(R_1 \| R_2)C]$$

$$\Rightarrow f_{3dB} = \frac{1}{2\pi (R_1 \| R_2) C}$$

15.13

a. $\frac{V_i}{R_1 + (1/sC_1)} = \frac{-V_0}{R_2 \| (1/sC_2)}$

$$V_i\left(\frac{sC_1}{1 + sR_1C_1}\right) = -V_0\left(\frac{1 + sR_2C_2}{sC_2}\right)$$

$$\frac{V_0}{V_i} = \frac{-sR_2C_1}{(1 + sR_1C_1)(1 + sR_2C_2)}$$

$$= \frac{-sR_2C_1}{1 + sR_1C_1 + sR_2C_2 + s^2R_1R_2C_1C_2}$$

$$\frac{V_0}{V_i} = -\frac{R_2}{R_1} \times$$

$$\times \left[\frac{sC_1}{\frac{1}{R_1} + sC_1\left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1}\right) + s^2R_2C_1C_2}\right]$$

or

$$T(s) = \frac{V_0}{V_i}$$

$$= -\frac{R_2}{R_1} \cdot \left[\frac{1}{\frac{1}{sR_1C_1} + \left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1}\right) + sR_2C_2}\right]$$

b.

$$|T(j\omega)| = -\frac{R_2}{R_1} \times$$

$$\times \frac{1}{\left\{\left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1}\right)^2 + \left(\omega R_2C_2 - \frac{1}{\omega R_1C_1}\right)^2\right\}^{1/2}}$$

when $\left(\omega R_2C_2 - \frac{1}{\omega R_1C_1}\right) = 0$, we want

$$|T(j\omega)| = 50 = \frac{R_2}{R_1} \cdot \frac{1}{\left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1}\right)}$$

At the $3 - dB$ frequencies, we want

$$\left(\omega R_2C_2 - \frac{1}{\omega R_1C_1}\right) = \pm\left(1 + \frac{R_2}{R_1} \cdot \frac{C_2}{C_1}\right)$$

For $f = 5$ kHz, use + sign and for $f = 200$ Hz, use − sign.

$$\omega_1 = 2\pi(200) = 1257$$

$$\omega_2 = 2\pi(5 \times 10^3) = 3.142 \times 10^4$$

Define $\tau_2 = R_2C_2$ and $\tau_1 = R_1C_1$
Then

$$50 = \frac{R_2}{R_1} \cdot \frac{1}{1 + \frac{\tau_2}{\tau_1}} \quad (1)$$

$$\left(\omega_2\tau_2 - \frac{1}{\omega_2\tau_1}\right) = +\left(1 + \frac{\tau_2}{\tau_1}\right) \quad (2)$$

$$\left(\omega_1\tau_2 - \frac{1}{\omega_1\tau_1}\right) = -\left(1 + \frac{\tau_2}{\tau_1}\right) \quad (3)$$

From (2)

$$\frac{\omega_2^2\tau_1\tau_2 - 1}{\omega_2\tau_1} = \frac{\tau_1 + \tau_2}{\tau_1}$$

or

$$\omega_2\tau_1\tau_2 - \frac{1}{\omega_2} = \tau_1 + \tau_2$$

$$\tau_1(\omega_2\tau_2 - 1) = \frac{1}{\omega_2} + \tau_2$$

So

$$\tau_1 = \frac{\frac{1}{\omega_2} + \tau_2}{\omega_2\tau_2 - 1}$$

Substituting into (3), we find

$$\omega_1\tau_2 - \frac{1}{\omega_1\left[\frac{\frac{1}{\omega_2}+\tau_2}{\omega_2\tau_2 - 1}\right]} = -\left[1 + \frac{\tau_2(\omega_2\tau_2 - 1)}{\frac{1}{\omega_2}+\tau_2}\right]$$

$$\omega_1\tau_2\left[\frac{1}{\omega_2}+\tau_2\right] - \frac{1}{\omega_1}(\omega_2\tau_2 - 1)$$
$$= -\left[\left(\frac{1}{\omega_2}+\tau_2\right) + \tau_2(\omega_2\tau_2 - 1)\right]$$

$$\frac{\omega_1}{\omega_2}\cdot\tau_2 + \omega_1\tau_2^2 - \frac{\omega_2}{\omega_1}\cdot\tau_2 + \frac{1}{\omega_1}$$
$$= -\frac{1}{\omega_2} - \tau_2 - \omega_2\tau_2^2 + \tau_2$$

$$(\omega_1+\omega_2)\tau_2^2 + \left(\frac{\omega_1}{\omega_2} - \frac{\omega_2}{\omega_1}\right)\tau_2 + \left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right) = 0$$
$$(3.2677\times 10^4)\tau_2^2 - 24.96\tau_2 + 8.273\times 10^{-4} = 0$$

$$\tau_2 = \frac{24.96}{2(3.2677\times 10^4)}$$
$$\pm \frac{\sqrt{(24.96)^2 - 4(3.2677\times 10^4)(8.273\times 10^{-4})}}{2(3.2677\times 10^4)}$$

Since ω_2 is large, τ_2 should be small so use minus sign:

$$\tau_2 = 3.47\times 10^{-5}$$

Then

$$\tau_1 = \frac{3.18\times 10^{-5} + 3.47\times 10^{-5}}{9.09\times 10^{-2}} \Rightarrow \tau_1 = 7.32\times 10^{-4}$$

Now

$$50 = \frac{R_2}{R_1}\cdot\frac{1}{1+\frac{3.47\times 10^{-5}}{7.32\times 10^{-4}}}$$

Then

$$\frac{R_2}{R_1} = 52.37 \text{ or } \underline{R_2 = 524\ \text{k}\Omega}$$

Also

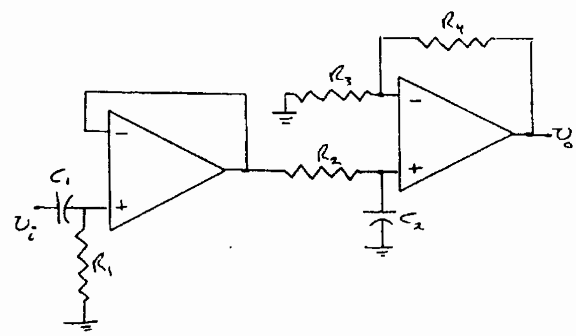
$\tau_1 = R_1C_1$ so that $C_1 = 0.0732\ \mu$F

$\tau_2 = R_2C_2$ so that $C_2 = 66.3$ pF

15.14

Gain = 10 dB $\Rightarrow$ Gain = 3.162

For example, we may have



Want $\frac{R_4}{R_3} = 2.162$

For example, let $R_3 = 50$ kΩ,

$R_4 = 108$ kΩ

$$f_1 = \frac{1}{2\pi R_1C_1} = 200$$

So

$$R_1C_1 = \frac{1}{2\pi(200)} = 0.796\times 10^{-3}$$

For example, let $R_1 = 200$ kΩ ⇒A large input resistance

$C_1 = 0.00398\ \mu$F

$$f_2 = \frac{1}{2\pi R_2C_2} = 50\times 10^3$$
$$\Rightarrow R_2C_2 = \frac{1}{2\pi(50\times 10^3)} = 3.18\times 10^{-6}$$

For example, let

$R_2 = 10$ kΩ and $C_2 = 318$ pF

15.15

$f_C = 100$ kHz

$$R_{eq} = \frac{1}{f_C C}$$

a. For $C = 1$ pF, $R_{eq} = 10$ MΩ
b. For $C = 10$ pF, $R_{eq} = 1$ MΩ
c. For $C = 30$ pF, $R_{eq} = 333$ kΩ

15.16

a. From Equation (15.28),

$$Q = \frac{V_1 - V_2}{R_{eq}} \cdot T_C$$

and $f_C = 100$ kHz so that $T_C = \frac{1}{100 \times 10^3} \Rightarrow 10\ \mu s$

Now

$$R_{eq} = \frac{1}{f_C C} = \frac{1}{(100 \times 10^3)(10 \times 10^{-12})} \Rightarrow 1\ M\Omega$$

So

$$Q = \frac{(2-1)(10 \times 10^{-6})}{10^6} = 10 \times 10^{-12}\ C$$

or

$$\underline{Q = 10\ pC}$$

b. $I_{eq} = \frac{Q}{T_C} = \frac{10 \times 10^{-12}}{10 \times 10^{-6}}$ or $\underline{I_{eq} = 1\ \mu A}$

c.

$Q = CV$ so find the time that V_0 reaches 99% of its full value.

$V_0 = V_1\left(1 - e^{-t/\tau}\right)$ where $\tau = RC$

Then $0.99 = 1 - e^{-t/\tau}$ or $e^{-t/\tau} = 0.01$

or $t = \tau \ln(100)$

$$\tau = RC = (10^3)(10 \times 10^{-12}) = 10^{-8}\ s$$

Then

$$\underline{t = 4.61 \times 10^{-8}\ s}$$

15.17

Low frequency gain $= -10 \Rightarrow \frac{C_1}{C_2} = 10$

$f_{3dB} = 10 \times 10^3\ Hz = \frac{f_C C_2}{2\pi C_F}$

Set

$$f_C = 10 f_{3dB} = 100\ kHz$$

Then

$$\frac{C_2}{C_F} = \frac{2\pi(10 \times 10^3)}{100 \times 10^3} = 0.628$$

The largest capacitor is C_1, so let

$$\underline{C_1 = 30\ pF}$$

Then

$$\underline{C_2 = 3\ pF}$$

and

$$\underline{C_F = 4.78\ pF}$$

15.18

a. Time constant $= R_{eq} \cdot C_F = \tau$ where

$$R_{eq} = \frac{1}{f_C C_1} = \frac{1}{(100 \times 10^3)(5 \times 10^{-12})} = 2 \times 10^6\ \Omega$$

Then

$$\tau = (2 \times 10^6)(30 \times 10^{-12})$$

or

$$\underline{\tau = 60\ \mu s}$$

b. $\nu_0 = -\frac{1}{\tau}\int \nu_I \cdot dt$

or

$$\Delta\nu_0 = \frac{(1)T_C}{\tau},\quad T_C = \frac{1}{f_C}$$

So

$$\Delta\nu_0 = \frac{1}{(60 \times 10^{-6})(100 \times 10^3)}$$

or

$$\underline{\Delta\nu_0 = 0.167\ V}$$

c. Now $\Delta\nu_0 = 13 = N(0.167)$

or

$\underline{N = 78}$ clock pulses

15.19

Using Equation (15.41)

$$f_0 = \frac{1}{2\pi\sqrt{3}RC} = \frac{1}{2\pi\sqrt{3}(4 \times 10^3)(10 \times 10^{-9})}$$

or

$$\underline{f_0 = 2.3\ kHz}$$

$\frac{R_2}{R} = 8$ so that $R_2 = 8(4 \times 10^3)$

$$\Rightarrow \underline{R_2 = 32\ k\Omega}$$

15.20

a. $\nu_1 = \frac{R}{R + (1/sC_V)} \cdot \nu_0 = \left(\frac{sRC_V}{1 + sRC_V}\right) \cdot \nu_0$

$$\nu_2 = \frac{R}{R + \frac{1}{sC}} \cdot \nu_1 = \left(\frac{sRC}{1 + sRC}\right) \cdot \nu_1$$

$$\nu_3 = \frac{R}{R + \frac{1}{sC}} \cdot \nu_2 = \left(\frac{sRC}{1 + sRC}\right) \cdot \nu_2$$

$$\nu_0 = -\frac{R_2}{R} \cdot \nu_3$$

Then

$$v_0 = -\frac{R_2}{R}\left(\frac{sRC}{1+sRC}\right)^2\left(\frac{sRC_V}{1+sRC_V}\right)v_0$$

Set $s = j\omega$

$$1 = -\frac{R_2}{R}\left(\frac{-\omega^2R^2C^2}{1+2j\omega RC-\omega^2R^2C^2}\right)\left(\frac{j\omega RC_V}{1+j\omega RC_V}\right)$$

The real part of the denominator must be zero.

$$1-\omega^2R^2C^2-2\omega^2R^2CC_V=0$$

so

$$\underline{\omega_0 = \frac{1}{R\sqrt{C(C+2C_V)}}}$$

b. $$f_{0,\max} = \frac{1}{2\pi(10^4)\sqrt{(10^{-11})(10^{-11}+2[10^{-11}])}}$$

$f_{0,\max} = 919$ kHz

$$f_{0,\min} = \frac{1}{2\pi(10^4)\sqrt{(10^{-11})(10^{-11}+2[50\times10^{-12}])}}$$

$f_{0,\min} = 480$ kHz

15.21

From Equation (15.46)

$$f_0 = \frac{1}{2\pi\sqrt{6}RC}$$

so $R = \dfrac{1}{2\pi\sqrt{6}(80\times10^3)(100\times10^{-12})}$ or

$\underline{R = 8.12\text{ k}\Omega}$

We need

$$\frac{R_2}{R} = 29$$

so that

$\underline{R_2 = 236\text{ k}\Omega}$

15.22

$$\frac{v_0-v_1}{\frac{1}{sC}} = \frac{v_1}{R}+\frac{v_1-v_2}{\frac{1}{sC}} \qquad (1)$$

or $(v_0-v_1)sC = \dfrac{v_1}{R}+(v_1-v_2)sC$

$$\frac{v_1-v_2}{\frac{1}{sC}} = \frac{v_2}{R}+\frac{v_2}{\frac{1}{sC}+R} \qquad (2)$$

or $(v_1-v_2)sC = \dfrac{v_2}{R}+\dfrac{v_2(sC)}{1+sRC}$

$$\frac{v_2}{\frac{1}{sC}+R} = -\frac{v_0}{R_2} \qquad (3)$$

or $\dfrac{v_2sC}{1+sRC} = -\dfrac{v_0}{R_2}$

so

$$v_2 = \frac{-v_0}{sR_2C}(1+sRC)$$

From (2)

$$v_1(sC) = v_2\left[sC+\frac{1}{R}+\frac{sC}{1+sRC}\right]$$

or

$$v_1 = -\frac{v_0(1+sRC)}{sR_2C}\cdot\left[1+\frac{1}{sRC}+\frac{1}{1+sRC}\right]$$

From (1)

$$v_0(sC) = v_1\left[sC+\frac{1}{R}+sC\right]-v_2(sC)$$

Then

$$v_0 = \left[2+\frac{1}{sRC}\right]\left[\frac{-v_0(1+sRC)}{sR_2C}\right]\times \left[\frac{1+sRC}{sRC}+\frac{1}{1+sRC}\right]+\frac{v_0}{sR_2C}\cdot(1+sRC)$$

$$-1 = \left[\frac{1+2sRC}{sRC}\right]\left[\frac{1+sRC}{sR_2C}\right]\left[\frac{(1+sRC)^2+sRC}{(sRC)(1+sRC)}\right] - \frac{1+sRC}{sR_2C}$$

$$-1 = \frac{(1+2sRC)(1+2sRC+s^2R^2C^2+sRC)}{(sRC)^2(sR_2C)} - \frac{(1+sRC)(sRC)^2}{(sRC)^2(sR_2C)}$$

Set $s = j\omega$

$$-1 = \frac{(1+2j\omega RC)(1+3j\omega RC+\omega^2R^2C^2)}{(-\omega^2R^2C^2)(j\omega R_2C)} - \frac{(1+j\omega RC)(-\omega^2R^2C^2)}{(-\omega^2R^2C^2)(j\omega R_2C)}$$

The real part of the numerator must be zero.

$$1-\omega^2R^2C^2-6\omega^2R^2C^2+\omega^2R^2C^2=0$$

$$6\omega^2R^2C^2=1$$

so that

$$\underline{\omega_0 = \frac{1}{\sqrt{6}RC}}$$

Condition for oscillation:

$$-1 = \frac{2j\omega RC+3j\omega RC-2j\omega^3R^3C^3+j\omega^3R^3C^3}{(-\omega^2R^2C^2)(j\omega R_2C)}$$

$$1 = \frac{5-\omega^2R^2C^2}{(\omega RC)(\omega R_2C)}$$

But

$$\omega = \omega_0 = \frac{1}{\sqrt{6}RC}$$

Then

$$1 = \frac{5 - \frac{1}{6}}{\frac{(RC)(R_2C)}{6R^2C^2}} = \frac{\left(5 - \frac{1}{6}\right)(6R^2C^2)}{RR_2C^2}$$

$$1 = \frac{\left(\frac{29}{6}\right)(6R)}{R_2} \text{ or } \underline{\frac{R_2}{R} = 29}$$

15.23

a.

$$v_{01} = \left(1 + \frac{R_{F1}}{R_{A1}}\right)\left(\frac{\frac{1}{sC_1}}{\frac{1}{sC_1} + R_1}\right) \cdot v_0 \quad (1)$$

$$v_{02} = \left(1 + \frac{R_{F2}}{R_{A2}}\right)\left(\frac{\frac{1}{sC_2}}{\frac{1}{sC_2} + R_2}\right) \cdot v_{01} \quad (2)$$

$$v_{03} = \left(\frac{R_{A3} \,\|\, \frac{1}{sC_3}}{R_{A3} \,\|\, \frac{1}{sC_3} + R_3}\right) \cdot v_{02} \quad (3)$$

$$v_0 = -\frac{R_{F3}}{R_{A3}} \cdot v_{03} \quad (4)$$

With all resistors equal and all capacitors equal, we find:

$$v_{01} = (2)\left(\frac{1}{1 + sRC}\right)v_0 \quad (1)$$

$$v_{02} = (2)\left(\frac{1}{1 + sRC}\right)v_{01} \quad (2)$$

$$v_{03} = (2)\left(\frac{\frac{R}{1+sRC}}{\frac{R}{1+sRC} + R}\right)v_{02}$$

$$= \left[\frac{R}{R + R(1 + sRC)}\right]v_{02}$$

$$v_{03} = \left(\frac{1}{2 + sRC}\right)v_{02} \quad (3)$$

and

$$v_0 = -v_{03} \quad (4)$$

Then

$$v_0 = -\left(\frac{1}{2 + sRC}\right)(2)\left(\frac{1}{1 + sRC}\right)(2)\left(\frac{1}{1 + sRC}\right)v_0$$

Let $s = j\omega$

$$(2 + j\omega RC)(1 + j\omega RC)(1 + j\omega RC) = -4$$

$$(2 + j\omega RC)(1 + 2j\omega RC - \omega^2R^2C^2) = -4 \quad \textbf{(A)}$$

The imaginary term on the left must be zero.

$$4j\omega RC + j\omega RC - j\omega^3R^3C^3 = 0$$

$$\omega RC(5 - \omega^2R^2C^2) = 0$$

or

$$\underline{\omega = \frac{\sqrt{5}}{RC}} \text{ (Not the same as in book)}$$

15.24

a.

$$\frac{v_0 - v_{01}}{R} = \frac{v_{01}}{\left(\frac{1}{sC}\right)} + \frac{v_{01} - v_{02}}{R} \quad (1)$$

$$\frac{v_{01} - v_{02}}{R} = \frac{v_{02}}{\left(\frac{1}{sC}\right)} + \frac{v_{02} - v_{03}}{R} \quad (2)$$

$$\frac{v_{02} - v_{03}}{R} = \frac{v_{03}}{\left(\frac{1}{sC}\right)} + \frac{v_{03}}{R} \quad (3)$$

$$v_0 = -\frac{R_F}{R} \cdot v_{03} \quad (4)$$

We can write the equations as

$$v_0 - v_{01} = v_{01}(sRC) + v_{01} - v_{02} \quad (1)$$

$$v_{01} - v_{02} = v_{02}(sRC) + v_{02} - v_{03} \quad (2)$$

$$v_{02} - v_{03} = v_{03}(sRC) + v_{03} \quad (3)$$

and

$$v_0 = -\frac{R_F}{R} \cdot v_{03} \quad (4)$$

Combining terms, we find

$$v_0 = v_{01}(2 + sRC) - v_{02} \quad (1)$$

$$v_{01} = v_{02}(2 + sRC) - v_{03} \quad (2)$$

$$v_{02} = v_{03}(2 + sRC) \quad (3)$$

and

$$v_0 = -\frac{R_F}{R} \cdot v_{03} \quad (4)$$

Combining Equations (3) and (2)

$$v_{01} = v_{03}(2 + sRC)^2 - v_{03} = v_{03}\left[(2 + sRC)^2 - 1\right] \quad (2)$$

Then Equation (1) is

$$v_0 = v_{03}\left[(2 + sRC)^2 - 1\right](2 + sRC) - v_{03}(2 + sRC)$$

Using Equation (4), we find

$$-\frac{R_F}{R}\cdot v_{O3} = v_{O3}\{[(2+sRC)^2-1](2+sRC) \\ -(2+sRC)\}$$

To find the frequency of oscillation, set $s = j\omega$ and set the imaginary part of the right side of the equation to zero.

We will have

$$-\frac{R_F}{R} = (2+j\omega RC)[4+4j\omega RC-\omega^2R^2C^2-1-1]$$

Then

$$j\omega RC(2-\omega^2R^2C^2)+8j\omega RC = 0$$

or

$$j\omega RC[2-\omega^2R^2C^2+8] = 0$$

Then the frequency of oscillation is

$$\underline{f_0 = \frac{1}{2\pi}\cdot\frac{\sqrt{10}}{RC}}$$

The condition to sustain oscillations is determined from

$$-\frac{R_F}{R} = 2[2-\omega^2R^2C^2]-4\omega^2R^2C^2$$

or

$$-\frac{R_F}{R} = 4-6\omega^2R^2C^2$$

Setting $\omega^2 = \frac{10}{R^2C^2}$, we have

$$-\frac{R_F}{R} = 4-6(10)$$

or

$$\underline{\frac{R_F}{R} = 56}$$

b. For $R = 5$ kΩ and $f_0 = 5$ kHz, we find

$$C = \frac{\sqrt{10}}{2\pi(5\times10^3)(5\times10^3)} \Rightarrow \underline{C = 0.02\ \mu\text{F}}$$

and $R_F = 56(5) \Rightarrow \underline{R_F = 280\text{ k}\Omega}$

15.25

a. We can write

$$v_A = \left(\frac{R_1}{R_1+R_2}\right)v_0 \text{ and } v_B = \left(\frac{Z_p}{Z_p+Z_s}\right)v_0$$

where $Z_p = R_B \parallel \frac{1}{sC_B} = \frac{R_B}{1+sR_BC_B}$

and $Z_s = R_A + \frac{1}{sC_A} = \frac{1+sR_AC_A}{sC_A}$

Setting $v_A = v_B$, we have

$$\frac{R_1}{R_1+R_2} = \frac{\frac{R_B}{1+sR_BC_B}}{\frac{R_B}{1+sR_BC_B}+\frac{1+sR_AC_A}{sC_A}}$$

$$\frac{R_1}{R_1+R_2} = \frac{sR_BC_A}{sR_BC_A+(1+sR_AC_A)(1+sR_BC_B)} \quad (1)$$

To find the frequency of oscillation, set $s = j\omega$ and set the real part of the denominator on the right side of Equation (1) equal to zero.

The denominator term is

$$j\omega R_BC_A+(1+j\omega R_AC_A)(1+j\omega R_BC_B)$$

or

$$j\omega R_BC_A+1+j\omega R_AC_A+j\omega R_BC_B \\ -\omega^2R_AR_BC_AC_B \quad (2)$$

Then from (2), we must have

$$1-\omega_0^2R_AR_BC_AC_B = 0$$

or

$$\underline{f_0 = \frac{1}{2\pi\sqrt{R_AR_BC_AC_B}}}$$

b. To find the condition for sustained oscillation, combine Equations (1) and (2). Then

$$\frac{R_1}{R_1+R_2} = \frac{j\omega R_BC_A}{j\omega R_BC_A+j\omega R_AC_A+j\omega R_BC_B}$$

or

$$1+\frac{R_2}{R_1} = 1+\frac{R_A}{R_B}+\frac{C_B}{C_A}$$

Then

$$\underline{\frac{R_2}{R_1} = \frac{R_A}{R_B}+\frac{C_B}{C_A}}$$

15.26

a. We can write

$$v_A = \left(\frac{R_1}{R_1 + R_2}\right) v_0$$

and

$$v_B = \left(\frac{R \| sL}{R \| sL + R + sL}\right) v_0$$

Setting $v_A = v_B$, we have

$$\frac{R_1}{R_1 + R_2} = \left[\frac{\frac{sRL}{R + sL}}{\frac{sRL}{R + sL} + R + sL}\right] \cdot v_0$$

$$\frac{R_1}{R_1 + R_2} = \frac{sRL}{sRL + (R + sL)^2} \qquad (1)$$

To find the frequency of oscillation, set $s = j\omega$ and set the real part of the denominator on the right side of Equation (1) equal to zero.

The denominator term is:

$$j\omega RL + (R + j\omega L)^2$$

or

$$j\omega RL + R^2 + 2j\omega RL - \omega^2 L^2 \qquad (2)$$

Then

$$R^2 - \omega_0^2 L^2 = 0$$

or

$$f_0 = \frac{1}{2\pi} \cdot \sqrt{\frac{R}{L}}$$

b. To find the condition for sustained oscillations, combine Equations (1) and (2).

$$\frac{R_1}{R_1 + R_2} = \frac{j\omega RL}{j\omega RL + 2j\omega RL} = \frac{1}{3}$$

Then

$$1 + \frac{R_2}{R_1} = 3$$

so that

$$\frac{R_2}{R_1} = 2$$

15.27

From Equation (15.52(b))

$$f_0 = \frac{1}{2\pi RC}$$

or

$$RC = \frac{1}{2\pi f} = \frac{1}{2\pi(80 \times 10^3)}$$

$$RC \cong 2 \times 10^{-6}$$

Set $R = 20$ kΩ and $C = 100$ pF

We must have

$$\frac{R_2}{R_1} = 2$$

Set $R_2 = 40$ kΩ and $R_1 = 20$ kΩ, for example.

15.28

From Equation (15.59)

$$f_0 = \frac{1}{2\pi\sqrt{L\left(\frac{C_1 C_2}{C_1 + C_2}\right)}}$$

and from Equation (15.61)

$$\frac{C_2}{C_1} = g_m R$$

Now,

$$g_m = 2\sqrt{k_n I_{DQ}} = 2\sqrt{(0.5)(1)} = 1.414 \text{ mA/V}$$

We have $C_1 = 0.01\ \mu\text{F}$, $R = 4$ kΩ, $f_0 = 400$ kHz

So

$$C_2 = g_m R C_1 = (1.414)(4)(0.01)$$

or

$$C_2 = 0.0566\ \mu\text{F}$$

and

$$400 \times 10^3 = \frac{1}{2\pi\sqrt{L\left[\frac{(0.01)(0.0566)}{0.01 + 0.0566}\right] \times 10^{-6}}}$$

$$L(8.5 \times 10^{-9}) = \left[\frac{1}{2\pi(400 \times 10^3)}\right]^2 = 1.58 \times 10^{-13}$$

Then

$$L = 18.6\ \mu\text{H}$$

15.29

$$V_\pi = -V_0$$

$$\frac{V_0}{\left(\frac{1}{sC_2}\right)} + \frac{V_0}{R_L} + \frac{V_0 - V_1}{\left(\frac{1}{sC_1}\right)} = g_m V_\pi = -g_m V_0$$

$$V_0\left[sC_2 + sC_1 + \frac{1}{R_L} + g_m\right] = V_1(sC_1) \quad (1)$$

$$\frac{V_1}{sL} + \frac{V_0 - V_1}{\left(\frac{1}{sC_1}\right)} + g_m V_\pi = 0 \quad (2)$$

$$V_1\left(\frac{1}{sL} + sC_1\right) = V_0(sC_1 + g_m)$$

$$V_1 = \frac{V_0(sC_1 + g_m)}{\left(\frac{1}{sL} + sC_1\right)}$$

Then

$$V_0\left[s(C_1 + C_2) + \frac{1}{R_L} + g_m\right] = \frac{V_0(sC_1)(sC_1 + g_m)}{\left(\frac{1}{sL} + sC_1\right)}$$

$$\left[s(C_1 + C_2) + \frac{1}{R_L} + g_m\right]\left(\frac{1}{sL} + sC_1\right) = sC_1(sC_1 + g_m)$$

$$\frac{C_1 + C_2}{L} + s^2C_1(C_1 + C_2) + \frac{1}{sR_LL} + \frac{sC_1}{R_L} + sg_mC_1 + \frac{g_m}{sL} = s^2C_1^2 + sg_mC_1$$

$$\frac{C_1 + C_2}{L} + s^2C_1C_2 + \frac{1}{sR_LL} + \frac{sC_1}{R_L} + \frac{g_m}{sL} = 0$$

Set $s = j\omega$

$$\frac{C_1 + C_2}{L} - \omega^2C_1C_2 + \frac{1}{j\omega R_LL} + \frac{j\omega C_1}{R_L} + \frac{g_m}{j\omega L} = 0$$

Then

$$\omega^2 = \frac{C_1 + C_2}{C_1C_2L} \Rightarrow \omega_0 = \sqrt{\frac{C_1 + C_2}{C_1C_2L}}$$

and

$$\frac{g_m}{\omega L} + \frac{1}{\omega R_LL} = \frac{\omega C_1}{R_L}$$

Then

$$\frac{g_m}{L} + \frac{1}{R_LL} = \frac{(C_1 + C_2)C_1}{C_1C_2LR_L}$$

$$g_m + \frac{1}{R_L} = \frac{C_1 + C_2}{C_2R_L}$$

$$g_mR_L + 1 = \frac{C_1}{C_2} + 1 \text{ or } \frac{C_1}{C_2} = g_mR_L$$

15.30

a.

$$\frac{V_0}{sL_1} + \frac{V_0}{R} + g_mV_\pi + \frac{V_0}{\frac{1}{sC} + sL_2} = 0 \quad (1)$$

$$V_\pi = \left(\frac{sL_2}{\frac{1}{sC} + sL_2}\right)V_0 \quad (2)$$

Then

$$V_0\left\{\frac{1}{sL_1} + \frac{1}{R} + \frac{sC}{1 + s^2L_2C} + \frac{g_m(s^2L_2C)}{1 + s^2L_2C}\right\} = 0$$

$$\left\{\frac{R(1 + s^2L_2C) + (sL_1)(1 + s^2L_2C)}{(sRL_1)(1 + s^2L_2C)} + \frac{s^2RL_1C + g_m(sRL_1)(s^2L_2C)}{(sRL_1)(1 + s^2L_2C)}\right\} = 0$$

Set $s = j\omega$. Both real and imaginary parts of the numerator must be zero.

$$R(1 - \omega^2L_2C) + j\omega L_1(1 - \omega^2L_2C) - \omega^2RL_1C + (j\omega g_mRL_1)(-\omega^2L_2C) = 0$$

Real part:

$$R(1 - \omega^2L_2C) - \omega^2RL_1C = 0$$

$$R = \omega^2RC(L_1 + L_2)$$

or

$$\omega_0 = \frac{1}{\sqrt{C(L_1 + L_2)}}$$

b. **Imaginary part:**

$$j\omega L_1(1 - \omega^2L_2C) - j\omega g_mRL_1(\omega^2L_2C) = 0$$

$$L_1 = \omega^2L_1L_2C + g_mRL_1(\omega^2L_2C)$$

Now $\omega^2 = \frac{1}{(L_1 + L_2)}$

$$1 = \frac{1}{C(L_1 + L_2)}[L_2C + g_mRL_2C]$$

$$1 = \frac{L_2}{L_1 + L_2}(1 + g_mR) \Rightarrow \frac{L_1}{L_2} = (1 + g_mR)$$

or

$$\frac{L_1}{L_2} = g_mR$$

15.31

$$\omega_0 = 2\pi(800 \times 10^3) = \frac{1}{\sqrt{C(L_1 + L_2)}}$$

or

$$C(L_1 + L_2) = 3.96 \times 10^{-14}$$

Also $\frac{L_1}{L_2} = g_m R$

For example, if $R = 1\ k\Omega$, then $\frac{L_1}{L_2} = (20)(1) = 20$

So

$$L_1 = 20L_2$$

Then

$C(21L_2) = 3.96 \times 10^{-14}$ or $CL_2 = 1.89 \times 10^{-15}$

If $\underline{C = 0.01\ \mu F}$

then $\underline{L_2 = 0.189\ \mu H}$

and $\underline{L_1 = 3.78\ \mu H}$

15.32

$$\frac{v_0 - v_1}{\left(\frac{1}{sC}\right)} = \frac{v_1}{R} + \frac{v_1 - v_B}{R} \quad (1)$$

and

$$\frac{v_B}{\left(\frac{1}{sC}\right)} + \frac{v_B - v_1}{R} = 0 \quad (2)$$

or

$$v_B\left(sC + \frac{1}{R}\right) = \frac{v_1}{R} \Rightarrow v_1 = v_B(1 + sRC)$$

From (1)

$$v_0(sC) = v_1\left(sC + \frac{2}{R}\right) - \frac{v_B}{R}$$

or

$$v_0(sRC) = v_B(1 + sRC)(2 + sRC) - v_B$$
$$= v_B[(1 + sRC)(2 + sRC) - 1]$$

Now

$$T(s) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{sRC}{(1 + sRC)(2 + sRC) - 1}\right]$$
$$= \left(1 + \frac{R_2}{R_1}\right)\left[\frac{sRC}{2 + 3sRC + s^2R^2C^2 - 1}\right]$$

or

$$\underline{T(s) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{sRC}{s^2R^2C^2 + 3sRC + 1}\right]}$$

$$T(j\omega) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{j\omega RC}{1 - \omega^2R^2C^2 + 3j\omega RC}\right]$$

Frequency of oscillation:

$$\underline{f_0 = \frac{1}{2\pi RC}}$$

Condition for oscillation:

$$1 = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{j\omega RC}{3j\omega RC}\right]$$

or

$$\underline{\frac{R_2}{R_1} = 2}$$

15.33

$$\frac{v_0 - v_1}{sL} = \frac{v_1}{R} + \frac{v_1 - v_B}{R} \quad (1)$$

$$v_B = \left(\frac{sL}{R + sL}\right)v_1 \quad (2)$$

or

$$v_1 = \left(\frac{R + sL}{sL}\right)v_B$$

Then

$$\frac{v_0}{sL} = v_1\left(\frac{1}{sL} + \frac{2}{R}\right) - \frac{v_B}{R}$$

or

$$\frac{v_0}{sL} = \left(\frac{R + sL}{sL}\right)\left(\frac{1}{sL} + \frac{2}{R}\right)v_B - \frac{v_B}{R}$$
$$= v_B\left\{\left(\frac{R + sL}{sL}\right)\left(\frac{R + 2sL}{sRL}\right) - \frac{1}{R}\right\} \quad (1)$$

Then

$$v_B = \frac{v_0}{sL} \cdot \frac{1}{\left\{\frac{(R + sL)(R + 2sL) - (sL)^2}{(sL)(sRL)}\right\}}$$

Now

$$T(s) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{sRL}{R^2 + 3sRL + 2s^2L^2 - s^2L^2}\right)$$

or

$$\underline{T(s) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{sRL}{s^2L^2 + 3sRL + R^2}\right)}$$

And

$$T(j\omega) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{j\omega RL}{R^2 - \omega^2L^2 + 3j\omega RL}\right)$$

Frequency of oscillation: $\underline{f_0 = \frac{R}{2\pi L}}$

Condition for oscillation:

$$1 = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{3}\right)$$

or

$$\underline{\frac{R_2}{R_1} = 2}$$

15.34

From Equation (15.65(b)), the crossover voltage is

$$v_I = -\frac{R_2}{R_1} \cdot V_{REF}$$

Let $R_2 = R_{VAR} + R_F$ where R_{VAR} is the potentiometer and R_F is the fixed resistor.

Let $\underline{V_{REF} = -5\text{ V}}$, $\underline{R_F = 10\text{ k}\Omega}$, and $\underline{R_{VAR} = 40\text{ k}\Omega}$

Then we have

$$v_I = -\frac{R_F}{R_1} \cdot V_{REF} = -\left(\frac{10}{50}\right)(-5) = 1\text{ V}$$

and

$$v_I = -\left(\frac{50}{50}\right)(-5) = 5\text{ V}$$

15.35

$$i_{max} = \frac{10}{R_1 + R_2} = 0.1 \Rightarrow R_1 + R_2 = 100\text{ k}\Omega$$

Hysteresis width

$$\Delta V = V_{TH} - V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right)(V_H - V_L)$$

or $0.1 = \left(\frac{R_1}{100}\right)(20)$

so that

$$\underline{R_1 = 0.5\text{ k}\Omega}$$

$$\underline{R_2 = 99.5\text{ k}\Omega}$$

15.36

a. $V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right)V_H = \left(\frac{10}{10 + 40}\right)(10)$

so $\underline{V_{TH} = 2\text{ V}}$

$V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right)V_L = \left(\frac{10}{10 + 40}\right)(-10)$

so $\underline{V_{TL} = -2\text{ V}}$

b. $v_I = 5\sin\omega t$

15.37

a. Upper crossover voltage when $v_O = +V_P$.

Now

$$v_B = \left(\frac{R_1}{R_1 + R_2}\right)(+V_P)$$

and

$$v_A = \left(\frac{R_A}{R_A + R_B}\right)V_{REF} + \left(\frac{R_B}{R_A + R_B}\right)V_{TH}$$

$v_A = v_B$ so that

$$\left(\frac{R_1}{R_1 + R_2}\right)V_P = \left(\frac{R_A}{R_A + R_B}\right)V_{REF} + \left(\frac{R_B}{R_A + R_B}\right)V_{TH}$$

or

$$V_{TH} = \left(\frac{R_A + R_B}{R_1 + R_2}\right)\left(\frac{R_1}{R_B}\right)V_P - \left(\frac{R_A}{R_B}\right)V_{REF}$$

Lower crossover voltage when $v_O = -V_P$

So

$$V_{TL} = -\left(\frac{R_A + R_B}{R_1 + R_2}\right)\left(\frac{R_1}{R_B}\right)V_P - \left(\frac{R_A}{R_B}\right)V_{REF}$$

b. $V_{TH} = \left(\frac{10 + 20}{5 + 20}\right)\left(\frac{5}{20}\right)(10) - \left(\frac{10}{20}\right)(2)$

or $\underline{V_{TH} = 2\text{ V}}$

and

$$V_{TL} = -\left(\frac{10 + 20}{5 + 20}\right)\left(\frac{5}{20}\right)(10) - 1 \Rightarrow \underline{V_{TL} = -4\text{ V}}$$

15.38

a. $\frac{v_B}{R_1} = \frac{V_{REF} - v_B}{R_3} + \frac{v_O - v_B}{R_2}$

$$v_B\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V_{REF}}{R_3} + \frac{v_O}{R_2}$$

$V_{TH} = v_B$ when $v_O = +V_P$ and $V_{TL} = v_B$ when $v_O = -V_P$

So

$$V_{TH} = \frac{\frac{V_{REF}}{R_3} + \frac{V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

and

$$V_{TL} = \frac{\frac{V_{REF}}{R_3} - \frac{V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

b.

$$V_S = \frac{V_{REF}}{R_3\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$-5 = \frac{-10}{10\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{10}\right)}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5} - \frac{1}{10} = 0.10$$

$$\Delta V_T = V_{TH} - V_{TL} = \frac{\frac{2V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$0.2 = \frac{2(12)}{R_2(0.10 + 0.10)}$$

So $\underline{R_2 = 600\ \text{k}\Omega}$

Then

$$\frac{1}{R_1} + \frac{1}{R_2} = 0.10$$

$$\frac{1}{R_1} + \frac{1}{600} = 0.10 \Rightarrow \underline{R_1 = 10.17\ \text{k}\Omega}$$

c. $V_{TH} = -5 + 0.1 = -4.9$

$V_{TL} = -5 - 0.1 = -5.1$

15.39

a. If the saturated output voltage is $|V_P| < 6.2$ V, then the circuit behaves as a comparator

where $|\nu_0| < 6.2$ V.

If the saturated output voltage is $|V_P| > 6.2$ V, the output will flip to either $+V_P$ or $-V_P$ and the input has no control.

b. Same as part (a) except the curve at $\nu_I \approx 0$ will have a finite slope.

c.

Circuit works as a comparator as long as $\nu_{01} < 8.7$ V and $\nu_{02} > -3.7$ V. Otherwise the input has no control.

15.40

a. Switching point is when $\nu_0 = 0$. Then

$$\nu_+ = \nu_I \equiv V_S = \left(\frac{R_2}{R_1 + R_2}\right) V_{REF}$$

V_{TH} occurs when $\nu_0 = V_H$, then by superposition

$$\nu_+ = V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right) V_H + \left(\frac{R_2}{R_1 + R_2}\right) V_{REF}$$

or

$$V_{TH} = V_S + \left(\frac{R_1}{R_1 + R_2}\right) V_H$$

V_{TL} occurs when $\nu_0 = V_L$, then by superposition

$$\nu_+ = V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right) V_L + \left(\frac{R_2}{R_1 + R_2}\right) V_{REF}$$

or

$$V_{TL} = V_S + \left(\frac{R_1}{R_1 + R_2}\right) V_L$$

b. For $V_{TH} = 2$ V and $V_{TL} = 1$ V, then $V_S = 1.5$ V

Now

$$2 = 1.5 + \left(\frac{10}{10 + R_2}\right)(10)$$

$$\frac{0.5}{10} = \frac{10}{10 + R_2} \Rightarrow \underline{R_2 = 190\ \text{k}\Omega}$$

$$\text{Now } V_S = 1.5 = \left(\frac{190}{10 + 190}\right) V_{REF}$$

so

$$\underline{V_{REF} = 1.579\ \text{V}}$$

15.41

a. Switching point when $\nu_0 = 0$.

Now

$$\nu_+ = V_{REF} = \left(\frac{R_2}{R_1 + R_2}\right) \nu_I \text{ where } \nu_I = V_S.$$

Then

$$V_S = \left(\frac{R_1 + R_2}{R_2}\right) V_{REF} = \left(1 + \frac{R_1}{R_2}\right) V_{REF}$$

Now upper crossover voltage for ν_I occurs when $\nu_0 = V_L$ and $\nu_+ = V_{REF}$. Then

$$\frac{V_{TH} - V_{REF}}{R_1} = \frac{V_{REF} - V_L}{R_2}$$

$$\text{or } V_{TH} = -\frac{R_1}{R_2} \cdot V_L + V_{REF}\left(1 + \frac{R_1}{R_2}\right)$$

$$\text{or } V_{TH} = V_S - \frac{R_1}{R_2} \cdot V_L$$

Lower crossover voltage for ν_I occurs when $\nu_0 = V_H$ and $\nu_I = V_{REF}$. Then

$$\frac{V_H - V_{REF}}{R_2} = \frac{V_{REF} - V_{TL}}{R_1}$$

$$\text{or } V_{TL} = -\frac{R_1}{R_2} \cdot V_H + V_{REF}\left(1 + \frac{R_1}{R_2}\right)$$

$$\text{or } V_{TL} = V_S - \frac{R_1}{R_2} \cdot V_H$$

b. For $V_{TH} = -1$ and $V_{TL} = -2$, $V_S = -1.5$ V.

Then $V_{TL} = V_S - \frac{R_1}{R_2} \cdot V_H \Rightarrow -2 = -1.5 - \frac{R_1}{20}(12)$
so that $\underline{R_1 = 0.833 \text{ k}\Omega}$

Now

$$V_S = \left(1 + \frac{R_1}{R_2}\right) V_{REF}$$

$$-1.5 = \left(1 + \frac{0.833}{20}\right) V_{REF}$$

which gives

$$\underline{V_{REF} = -1.44 \text{ V}}$$

15.42

a. $V_H = 5.6 + 0.7 = 6.3$ V and $V_L = -6.3$ V

From Equation (15.72(b)),

$$V_{TH} = -\left(\frac{R_1}{R_2}\right) V_L$$

and from Equation (15.75(b)),

$$V_{TL} = -\left(\frac{R_1}{R_2}\right) V_H$$

$$V_{TH} - V_{TL} = 1 = -\left(\frac{R_1}{R_2}\right)(V_L - V_H)$$

$$= +\left(\frac{R_1}{R_2}\right)(2)(6.3)$$

so $\frac{R_1}{R_2} = 0.07937$

Then $R_2 = \frac{1}{0.07937} \Rightarrow \underline{R_2 = 12.6 \text{ k}\Omega}$

Now $V_{TH} = -(0.07937)(-6.3) = 0.5$,
$V_{TL} = -0.5$

b. For v_O high, we have

$I = I_D + I_R$, I is fixed for a given R.
Assume v_I varies between ± 12 V

$$I_R(\text{max}) = \frac{6.3 - (-12)}{13.6} = 1.35 \text{ mA}$$

$$I_R(\text{min}) = \frac{6.3 - 0.5}{13.6} = 0.426 \text{ mA}$$

$$I_R(\text{avg}) = \frac{1.35 + 0.426}{2} = 0.888 \text{ mA}$$

Then we want

$I = I_D(\text{avg}) + I_R(\text{avg}) = 1 + 0.888 = 1.888$ mA
Then

$$R = \frac{12 - 6.3}{1.888} \Rightarrow \underline{R = 3.02 \text{ k}\Omega}$$

15.43

a. $v_O = V_{REF} + 2V_\gamma$

$5 = V_{REF} + 2(0.7)$
or

$$\underline{V_{REF} = 3.6 \text{ V}}$$

b. $V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right)(V_{REF} + 2V_\gamma)$

$$0.5 = \left(\frac{R_1}{R_1 + R_2}\right)(5)$$

or $1 + \frac{R_2}{R_1} = 10 \Rightarrow \frac{R_2}{R_1} = 9$
For example, let $R_2 = 90$ kΩ and $R_1 = 10$ kΩ

c. For $v_I = 10$ V, and v_O is in its low state. D_1 is on and D_2 is off.

$$\frac{v_I - (v_1 + 0.7)}{100} + \frac{V_{REF} - v_1}{1} = \frac{v_1 - v_O}{1}$$

For $v_1 = -0.7$, then

$$\frac{10 - 0}{100} + \frac{3.6 - (-0.7)}{1} = \frac{-0.7 - v_O}{1}$$

or

$$\underline{v_O = -5.1 \text{ V}}$$

15.44

For $v_O = \text{High} = (V_{REF} + 2V_\gamma)$. Then switching point is when

$$v_I = v_B = \left(\frac{R_1}{R_1 + R_2}\right) v_O$$

or $V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right)(V_{REF} + 2V_\gamma)$

Lower switching point is when

$v_1 = v_B = \left(\frac{R_1}{R_1 + R_2}\right) v_O$ and $v_O = -(V_{REF} + 2V_\gamma)$

so

$$V_{TL} = -\left(\frac{R_1}{R_1 + R_2}\right)(V_{REF} + 2V_\gamma)$$

15.45

By symmetry, inverting terminal switches about zero.

Now, for v_O low, upper diode is on.

$V_{REF} - v_1 = v_1 - v_O$
$v_O = 2v_1 - V_{REF}$ where $v_1 = -V_\gamma$
so

$$\underline{v_O = -(V_{REF} + 2V_\gamma)}$$

Similarly, in the high state

$$\underline{v_O = (V_{REF} + 2V_\gamma)}$$

Switching occurs when non-inverting terminal is zero.

So for v_O low,

$$\frac{V_{TH}-0}{R_1}=\frac{0-[-(V_{REF}+2V_\gamma)]}{R_2}$$

or $V_{TH}=\frac{R_1}{R_2}\cdot(V_{REF}+2V_\gamma)$

By symmetry

$$V_{TL}=-\frac{R_1}{R_2}\cdot(V_{REF}+2V_\gamma)$$

15.46

$f_0 = 5$ kHz and 50% duty cycle.

From Equation (15.88)

$$f=\frac{1}{2.2R_XC_X}$$

so

$$R_XC_X=\frac{1}{2.2(5\times10^3)}=9.09\times10^{-5}$$

Let $C_X = 0.01\ \mu$F. Then $R_X = 9.09$ kΩ.

Also let $R_1 = R_2 = 10$ kΩ

15.47

Switching point occurs when

$$v_X=\left(\frac{R_1}{R_1+R_2}\right)V_P=\left(\frac{30}{30+10}\right)(10)$$

$$\Rightarrow v_X=\pm7.5\ \text{V}$$

b. Duty cycle = 50%

From Equation (15.83(a)), we can write

$$v_X=V_P+\left(-\frac{3}{4}V_P-V_P\right)e^{-t/\tau_X}$$

At time $t=t_1$ (one-half period) $v_X=\frac{3}{4}\cdot V_P$

So

$$\frac{3}{4}\cdot V_P=V_P-\frac{7}{4}\cdot V_Pe^{-t_1/\tau_X}$$

$$1=7e^{-t_1/\tau_X}$$

or $t_1=\tau_X\ln(7)$

One period is $T=2\tau_X\ln(7)=3.89\tau_X$ or the frequency is

$$f=\frac{1}{3.89R_XC_X}$$

Then

$$f=\frac{1}{(3.89)(10^4)(0.1\times10^{-6})}\Rightarrow \underline{f=257\ \text{Hz}}$$

15.48

Only change from Problem (15.47) is that maximum output is ±15 V and the v_X switching voltages are ±11.25 V.

15.49

$$t_1=1.1R_XC_X=(1.1)(10^4)(0.1\times10^{-6})$$

$$\Rightarrow t_1=1.1\ \text{ms}$$

$$0<t<t_1,\ v_Y=10\left(1-e^{-t/\tau_Y}\right)$$

$$\tau_Y=R_YC_Y=(2\times10^3)(0.02\times10^{-6})$$

$$=4\times10^{-5}\ \text{s}$$

Now $\frac{t_1}{\tau_Y}=2.75$

$\Rightarrow C_Y$ completely charges during each cycle.

15.50

a. Switching voltage

$$v_X=\left(\frac{R_1+R_3}{R_1+R_3+R_2}\right)\cdot V_P=\left(\frac{10+10}{10+10+10}\right)(\pm10)$$

So $v_X=\pm6.667$ V

Using Equation (15.83(b))

$$v_X=V_P+\left(-\frac{2}{3}V_P-V_P\right)e^{-t_1/\tau_X}=\frac{2}{3}V_P$$

Then $1-\frac{5}{3}\cdot e^{-t_1/\tau_X}=\frac{2}{3}$

$\frac{1}{3}=\frac{5}{3}\cdot e^{-t_1/\tau_X}$ or $t_1=\tau_X\ln(5)$

$$t_1=\frac{T}{2}=\frac{1}{2f}=\frac{1}{2(500)}\Rightarrow t_1=0.001\ \text{s}$$

$$10^{-3}=\tau_X\ln(5)\Rightarrow\tau_X=6.21\times10^{-4}$$

$$=R_X(0.01\times10^{-6})$$

So $\underline{R_X=62.1\ \text{k}\Omega}$

b. Switching voltage

$$v_X=\left(\frac{R_1}{R_1+R_3+R_2}\right)(\pm V_P)$$

$$=\left(\frac{10}{10+10+10}\right)(\pm V_P)=\frac{1}{3}\cdot(\pm V_P)$$

Using Equation (15.83(b))

$$v_X=V_P+\left(-\frac{1}{3}V_P-V_P\right)e^{-t_1/\tau_X}=\frac{1}{3}V_P$$

Then $1-\frac{4}{3}\cdot e^{-t_1/\tau_X}=\frac{1}{3}$

$\frac{2}{3}=\frac{4}{3}\cdot e^{-t_1/\tau_X}$

$$t_1=\tau_X\ln(2)=(6.21\times10^{-4})\ln(2)=4.30\times10^{-4}\ \text{s}$$

$$T=2t_1=8.6\times10^{-4}\ \text{s}$$

$$f=\frac{1}{T}\Rightarrow\underline{f=1.16\ \text{kHz}}$$

15.51

From Equation (15.92)

$$T = \tau_X \ln\left(\frac{1+\left(\frac{V_\gamma}{V_P}\right)}{1-\beta}\right)$$

where $\beta = \frac{R_1}{R_1+R_2} = \frac{10}{10+25} = 0.2857$

so

$$100 = \tau_X \ln\left[\frac{1+\frac{0.7}{5}}{1-0.2857}\right]$$

so $\tau_X = 213.9\ \mu s = R_X C_X$

For example, $R_X = 10\ k\Omega$, $C_X = 0.0214\ \mu F$

$$v_Y = \left(\frac{R_1}{R_1+R_2}\right)V_P = \left(\frac{10}{10+25}\right)(5) = 1.43\ V$$

and $v_X = 0.7$ V

To trigger the circuit, v_Y must be brought to a voltage less than v_X.

Therefore minimum triggering pulse is -0.73 V.

Using Equation (15.82) for $T < t < T'$

$v_X = V_P + (-0.2857V_P - V_P)e^{-t'/\tau_X}$

Recovery period is when $v_X = V_\gamma = 0.7$ V.

$0.7 = 5 + (-6.43)e^{-t'/\tau_X}$

$6.43e^{-t'/\tau_X} = 4.3$

or $t' = \tau_X \ln(1.495)$

$\tau_X = 213.9\ \mu s$

so

$\underline{t' = T' - T = 86.1\ \mu s}$

15.52

From Equation (15.92), the pulse width

$$T = \tau_X \ln\left(\frac{1+\left(\frac{V_\gamma}{V_P}\right)}{1-\beta}\right)$$

where $\beta = \frac{R_1}{R_1+R_2} = \frac{20}{20+20} = 0.5$

$\tau_X = R_X C_X = (50 \times 10^3)(0.1 \times 10^{-6}) = 5$ ms

So $T = 5\ln\left[\frac{1+\frac{0.7}{10}}{1-0.5}\right] \Rightarrow \underline{T = 3.80\ ms}$

Recovery time $\approx 0.4\tau_X = \underline{2\ ms}$

15.53

a. From Equation (15.95)

$T = 1.1RC$

For $T = 60$ s $= 1.1$RC

then $RC = 54.55$ s

For example, let

$\underline{C = 50\ \mu F}$ and $\underline{R = 1.09\ M\Omega}$

b. Recovery time: capacitor is discharged by current through the discharge transistor.

If $V^+ = 5$ V, then $I_B \cong \frac{5-0.7}{100} = 0.043$ mA

If $\beta = 100$, $I_C = 4.3$ mA

$V_C = \frac{1}{C}\int I_C\, dt = \frac{I_C}{C}\cdot t$

Capacitor has charged to $\frac{2}{3}\cdot V^+ = 3.33$ V

So that $t = \frac{V_C \cdot C}{I_C} = \frac{(3.33)(50 \times 10^{-6})}{4.3 \times 10^{-3}}$

So recovery time $\underline{t \approx 38.7\ ms}$

15.54

$T = 1.1RC$

$5 \times 10^{-6} = 1.1RC$

so $RC = 4.545 \times 10^{-6}$ s

For example, let

$\underline{C = 100\ pF}$ and $\underline{R = 45.5\ k\Omega}$

From Problem (15.53), recovery time

$$t \cong \frac{V_C \cdot C}{I_C} = \frac{(3.33)(100 \times 10^{-12})}{4.3 \times 10^{-3}}$$

or

$\underline{t = 77.4\ ns}$

15.55

From Equation (15.102),

$$f = \frac{1}{(0.693)(20+2(20)) \times 10^3 \times (0.1 \times 10^{-6})}$$

or $\underline{f = 240.5\ Hz}$

Duty cycle $= \frac{20+20}{20+2(20)} \times 100\% = \underline{66.7\%}$

15.56

$$f = \frac{1}{(0.693)(R_A + 2R_B)C}$$

$R_A = R_1 = 10\ k\Omega$, $R_B = R_2 + xR_3$

So $10\ k\Omega \le R_B \le 110\ k\Omega$

$$f_{min} = \frac{1}{(0.693)(10 + 2(110)) \times 10^3 \times (0.01 \times 10^{-6})}$$
$$= 627 \text{ Hz}$$
$$f_{max} = \frac{1}{(0.693)(10 + 2(10)) \times 10^3 \times (0.01 \times 10^{-6})}$$
$$= 4.81 \text{ kHz}$$

So $627 \text{ Hz} \le f \le 4.81 \text{ kHz}$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

Now

$$\frac{10 + 10}{10 + 2(10)} \times 100\% = \underline{66.7\%}$$

and

$$\frac{10 + 110}{10 + 2(110)} \times 100\% = \underline{52.2\%}$$

So $52.2 \le \text{Duty cycle} \le 66.7\%$

15.57

$1 \text{ k}\Omega \le R_A \le 51 \text{ k}\Omega$

$1 \text{ k}\Omega \le R_B \le 51 \text{ k}\Omega$

$$f_{min} = \frac{1}{(0.693)(1 + 2(51)) \times 10^3 \times (0.01 \times 10^{-6})}$$
$$= 1.40 \text{ kHz}$$
$$f_{max} = \frac{1}{(0.693)(51 + 2(1)) \times 10^3 \times (0.01 \times 10^{-6})}$$
$$= 2.72 \text{ kHz}$$

or $1.40 \text{ kHz} \le f \le 2.72 \text{ kHz}$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$\frac{1 + 51}{1 + 2(51)} \times 100\% = \underline{50.5\%}$$

or

$$\frac{51 + 1}{51 + 2(1)} \times 100\% = \underline{98.1\%}$$

or $\underline{50.5\% \le \text{Duty cycle} \le 98.1\%}$

15.58

a. $I_{E3} = I_{E4} = \dfrac{V^+ - 3V_{EB}}{R_{1A} + R_{1B}}$

Assume $V_{EB} = 0.7$

$$I_{E3} = I_{E4} = \frac{22 - 3(0.7)}{25 + 25} = 0.398 \text{ mA}$$

Now

$$I_{C3} = I_{C4} = I_{C5} = I_{C6} = \left(\frac{20}{21}\right)(0.398)$$

$\underline{I_{C3} = I_{C4} = I_{C5} = I_{C6} = 0.379 \text{ mA}}$

$$I_{C1} = I_{C2} = \frac{0.398}{21}\left(\frac{20}{21}\right) \Rightarrow \underline{I_{C1} = I_{C2} = 0.018 \text{ mA}}$$

b. $\underline{I_D = 0.398 \text{ mA}}$, current in D_1 and D_2

$$V_{BB} = 2V_D = 2V_T \ln\left(\frac{I_D}{I_S}\right)$$
$$= 2(0.026)\ln\left(\frac{0.398 \times 10^{-3}}{10^{-13}}\right)$$

or $V_{BB} = 1.149 \text{ V} = V_{BE7} + V_{EB8}$

Now

$$I_{C7} \approx I_{C4} + I_{C9} + I_{E8}$$
$$I_{C4} = 0.379 \text{ mA}$$
$$I_{B9} = I_{C8} = \left(\frac{20}{21}\right)I_{E8}$$

So

$$I_{E8} = 1.05 I_{B9} = 1.05\left(\frac{I_{C9}}{100}\right)$$
$$I_{C7} = I_{C4} + \left(\frac{100}{1.05}\right)I_{E8} + I_{E8}$$
$$= I_{C4} + (96.24)\left(\frac{21}{20}\right)I_{C8}$$

So $I_{C7} = 0.379 \text{ mA} + 101 I_{C8}$

and

$$V_{BE7} = V_T \ln\left(\frac{I_{C7}}{I_S}\right);\ V_{EB8} = V_T \ln\left(\frac{I_{C8}}{I_S}\right)$$

Then

$$1.149 = 0.026\left[\ln\left(\frac{I_{C7}}{I_S}\right) + \ln\left(\frac{I_{C8}}{I_S}\right)\right]$$
$$44.19 = \ln\left[\frac{I_{C8}(0.379 \times 10^{-3}) + 101 I_{C8}}{(10^{-13})^2}\right]$$
$$(10^{-13})^2 \exp(44.19) = 101 I_{C8}^2 + 3.79 \times 10^{-4} I_{C8}$$

$$1.554 \times 10^{-7} = 101 I_{C8}^2 + 3.79 \times 10^{-4} I_{C8}$$
$$I_{C8} = \frac{-3.79 \times 10^{-4}}{2(101)} \pm \frac{\sqrt{(3.79 \times 10^{-4})^2 + 4(101)(1.554 \times 10^{-7})}}{2(101)}$$

$\underline{I_{C8} = 37.4\ \mu\text{A}}$

$I_{C7} = 0.379 + 101(0.0374) \Rightarrow \underline{I_{C7} = 4.16 \text{ mA}}$

$$I_{C9} = 4.16 - 0.379 - 0.0374\left(\frac{21}{20}\right)$$

$\underline{I_{C9} = 3.74 \text{ mA}}$

c. $P = (0.398 + 0.398 + 4.16)(22) \Rightarrow \underline{P = 109 \text{ mW}}$

15.59

a. From Figure 15.47, 3.7 W to the load

b. $V^+ \approx 19$ V

c. $\overline{P} = \frac{1}{2}\frac{V_P^2}{R_L}$

or

$V_P = \sqrt{2R_L\overline{P}} = \sqrt{2(10)(3.7)} \Rightarrow V_P = 8.6$ V

15.60

$\overline{P} = \frac{1}{2}\frac{V_P^2}{R_L}$

so $V_P = \sqrt{2R\overline{P}} = \sqrt{2(10)(20)} = 20$ V

peak-to-peak output voltage

Maximum output voltage of each op-amp = ±10 V. Current is (20/10) = 2 A. Bias op-amps at ±12 V.

For A_1, $\frac{v_{01}}{v_I} = \left(1 + \frac{R_2}{R_1}\right) = 15 \Rightarrow \frac{R_2}{R_1} = 14$

For A_2, $\left|\frac{v_{02}}{v_I}\right| = \frac{R_4}{R_3} = 15$

For example, let $R_1 = R_3 = 10$ kΩ, and $R_2 = 140$ kΩ and $R_4 = 150$ kΩ.

15.61

a. $v_{01} = iR_2 + v_I$ where $i = \frac{v_I}{R_1}$

Then

$v_{01} = v_I\left(1 + \frac{R_2}{R_1}\right)$

Now

$v_{02} = -iR_3 = -v_I\left(\frac{R_3}{R_1}\right)$

So

$v_L = v_{01} - v_{02} = v_I\left(1 + \frac{R_2}{R_1}\right) - \left[-v_I\left(\frac{R_3}{R_1}\right)\right]$

$$\underline{A_v = \frac{v_L}{v_I} = 1 + \frac{R_2}{R_1} + \frac{R_3}{R_1}}$$

b. Want $A_v = 10 \Rightarrow \frac{R_2}{R_1} + \frac{R_3}{R_1} = 9$

Also want $\left(1 + \frac{R_2}{R_1}\right) = \frac{R_3}{R_1}$

Then $\frac{R_2}{R_1} + \left(1 + \frac{R_2}{R_1}\right) = 9$ so $\frac{R_2}{R_1} = 4$

For $R_1 = 50$ kΩ, $\underline{R_2 = 200 \text{ k}\Omega}$

and

$\frac{R_3}{R_1} = 5$ so $\underline{R_3 = 250 \text{ k}\Omega}$

c. $\overline{P} = \frac{1}{2}\frac{V_P^2}{R_L}$

or

$V_P = \sqrt{2R_L\overline{P}} = \sqrt{2(20)(10)} = 20$ V

So peak values of output voltages are

$|v_{01}| = |v_{02}| = 10$ V

Peak load current $= \frac{20}{20} = 1$ A

15.62

a. $v_{01} = \left(1 + \frac{R_2}{R_1}\right)v_I$

$v_{02} = -\frac{R_4}{R_3}\cdot v_{01} = -\frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right)v_I$

$v_L = v_{01} - v_{02} = \left(1 + \frac{R_2}{R_1}\right)\left(1 + \frac{R_4}{R_3}\right)v_I$

so

$$\underline{A_v = \left(1 + \frac{R_2}{R_1}\right)\left(1 + \frac{R_4}{R_3}\right)}$$

b. Want $\left(1 + \frac{R_2}{R_1}\right)v_I = |v_{01}| = |v_{02}| \Rightarrow \underline{R_4 = R_3}$

Then

$\left(1 + \frac{R_2}{R_1}\right)(2) = 15 \Rightarrow \underline{\frac{R_2}{R_1} = 6.5}$

c. $\overline{P} = \frac{1}{2}\cdot\frac{V_P^2}{R_L}$

or

$V_P = \sqrt{2R_L\overline{P}} = \sqrt{2(8)(50)} = 28.3$ V

= peak-to-peak load voltage

Then

$\underline{|v_{01}| = |v_{02}| = 14.15 \text{ V}}$

Load current $= \frac{28.3}{8} = \underline{3.54 \text{ A}}$

15.63

Line regulation $= \frac{\Delta V_0}{\Delta V^+}$

Now

$\Delta I = \frac{\Delta V^+}{R_1}$ and $\Delta V_Z = r_Z \cdot \Delta I$ and $\Delta V_0 = 10\Delta V_Z$

So

$\Delta V_0 = 10 \cdot r_Z \cdot \frac{\Delta V^+}{R_1}$

So

Line regulation $= \frac{\Delta V_0}{\Delta V^+} = \frac{10(15)}{9300}$

$\Rightarrow 1.61\%$

15.64

$$R_{of} = -\frac{\Delta V_O}{\Delta I_O}$$

So $R_{of} = \dfrac{-(-10 \times 10^{-3})}{1}$

or

$\underline{R_{of} = 10\ \text{m}\Omega}$

15.65

For $V_O = 8$ V

$$V^+(\text{min}) = V_O + I_O(\text{max})R_{11} + V_{BE11} + V_{BE10} + V_{EB5}$$

This assumes $V_{BC5} = 0$.

Then

$$V^+(\text{min}) = 8 + (0.1)(1.9) + 0.6 + 0.6 + 0.6$$

$\underline{V^+(\text{min}) = 9.99\ \text{V}}$

15.66

a. $I_{C3} = I_{C5} = \dfrac{V_Z - 3V_{BE}(\text{npn})}{R_1 + R_2 + R_3}$

$$I_{C3} = I_{C5} = \frac{6.3 - 3(0.6)}{0.576 + 3.4 + 3.9} = 0.571\ \text{mA}$$

$$I_{C8} = \frac{1}{2}\left(\frac{0.6}{2.84}\right) = 0.106\ \text{mA}$$

Neglecting current in Q_9, total collector current and emitter current in Q_5 is

$0.571 + 0.106 = 0.677$

Now

$$I_{Z2}R_4 + V_{EB4} = V_{EB5}$$

$$V_{EB4} = V_T \ln\left(\frac{I_{Z2}}{I_S}\right)$$

$$V_{EB5} = V_T \ln\left(\frac{I_{C5}}{2I_S}\right)$$

Then $I_{Z2}R_4 = V_T \ln\left(\dfrac{I_{C5}}{2I_{Z2}}\right)$

$$R_4 = \frac{0.026}{0.25} \cdot \ln\left(\frac{0.677}{2(0.25)}\right)$$

or

$\underline{R_4 = 31.5\ \Omega}$

b. From Example 15.16, $V_{B7} = 3.43\,V$. Then

$$\left(\frac{R_{13}}{R_{12} + R_{13}}\right)V_O = V_{B8} = V_{B7}$$

or

$$\left(\frac{2.23}{2.23 + R_{12}}\right)(12) = 3.43$$

$$3.43(2.23 + R_{12}) = (2.23)(12)$$

which yields

$\underline{R_{12} = 5.57\ \text{k}\Omega}$

15.67

Line regulation $= \dfrac{\Delta V_O}{\Delta V^+}$

Now

$$\Delta V_{B7} = \Delta I_{C3} \cdot R_1$$

and $\left(\dfrac{R_{13}}{R_{12} + R_{13}}\right)(\Delta V_O) = \Delta V_{B7} = \Delta I_{C3}R_1$

and $\Delta I_{C3} = \dfrac{\Delta V_Z}{R_1 + R_2 + R_3} = \dfrac{\Delta I_Z \cdot r_Z}{R_1 + R_2 + R_3}$

and $\Delta I_Z = \dfrac{\Delta V^+}{r_O}$ where $r_O = \dfrac{V_A}{I_Z}$

Then

$$(0.4288)(\Delta V_O) = \Delta I_{C3}(3.9)$$

$$= (3.9)\Delta I_Z\left(\frac{0.015}{7.876}\right)$$

$$r_O = \frac{50}{0.571} = 87.6\ \text{k}\Omega$$

Then

$$(0.4288)(\Delta V_O) = (0.00743)\left(\frac{\Delta V^+}{87.6}\right)$$

So

$\underline{\dfrac{\Delta V_O}{\Delta V^+} = 0.0198\%}$

15.68

a. $I_Z = \dfrac{25 - 5}{R_1 + r_Z} = 10$

So $R_1 + r_Z = \dfrac{20}{10} = 2\ \text{k}\Omega = R_1 + 0.01$

$\Rightarrow \underline{R_1 = 1.99\ \text{k}\Omega}$

b. In the ideal case;

$$\left(\frac{R_3 + R_4}{R_2 + R_3 + R_4}\right)V_O = V_Z$$

$$\left(\frac{2 + 1}{2 + 1 + 1}\right)V_O = 5 \Rightarrow V_O = 6.67\ \text{V}$$

and

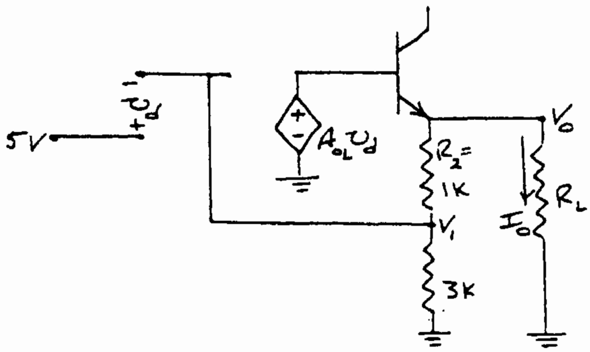
$$\left(\frac{R_4}{R_2 + R_3 + R_4}\right)V_O = V_Z$$

$$\left(\frac{1}{2 + 1 + 1}\right)V_O = 5 \Rightarrow V_O = 20\ \text{V}$$

So

$\underline{6.67 \le V_O \le 20\ \text{V}}$

c.



$V_1 = \frac{3}{4} \cdot V_0$ so $v_d = 5 - \frac{3}{4} \cdot V_0$
and $V_0 = A_{0L} v_d - V_{BE}$
and $V_{BE} = V_T \ln\left(\frac{I_0}{I_S}\right)$

Now

$$V_0 = A_{0L}\left(5 - \frac{3}{4} \cdot V_0\right) - V_{BE}$$

$$V_0\left(1 + \frac{3}{4} \cdot A_{0L}\right) = 5A_{0L} - V_{BE}$$

$$V_0 = \frac{5A_{0L} - V_{BE}}{1 + \frac{3}{4} \cdot A_{0L}}.$$

$$\text{Load regulation} = \frac{V_0(\text{NL}) - V_0(\text{FL})}{V_0(\text{NL})}$$

$$= \frac{\frac{5A_{0L} - V_{BE}(\text{NL})}{(1 + \frac{3}{4}A_{0L})} - \frac{5A_{0L} - V_{BE}(\text{FL})}{(1 + \frac{3}{4}A_{0L})}}{\frac{5A_{0L} - V_{BE}(\text{NL})}{(1 + \frac{3}{4}A_{0L})}}$$

$$= \frac{V_{BE}(\text{FL}) - V_{BE}(\text{NL})}{5A_{0L} - V_{BE}(\text{NL})} = \frac{V_T \ln\left(\frac{I_0(\text{FL})}{I_0(\text{NL})}\right)}{5A_{0L} - V_{BE}(\text{NL})}$$

$$I_0(\text{FL}) = 1\ \text{A},\quad I_0(\text{NL}) = \frac{V_0}{4\ \text{k}\Omega} = \frac{6.67}{4\ \text{k}\Omega} = 1.67\ \text{mA}$$

$$\text{Load regulation} = \frac{(0.026)\ln\left(\frac{1}{1.67 \times 10^{-3}}\right)}{5(10^4) - 0.7}$$

$\Rightarrow$ $\underline{3.33 \times 10^{-4}\%}$

15.69

$$I_E = \frac{V_Z}{R_2} = \frac{5.6}{5} = 1.12\ \text{mA}$$

$$I_0 = \frac{\beta}{1+\beta} \cdot I_E = \left(\frac{100}{101}\right)(1.12)$$

$\Rightarrow$ $\underline{I_0 = 1.109\ \text{mA}}$ Load current

For

$$V_{BC} = 0 \Rightarrow V_0 = 20 - V_Z - 0.6$$
$$= 20 - 5.6 - 0.6$$

or

$$V_0 = 13.8\ \text{V}$$

Then

$$R_L = \frac{V_0}{I_0} = \frac{13.8}{1.109} \Rightarrow R_L = 12.4\ \text{k}\Omega$$

So

$\underline{0 \le R_L \le 12.4\ \text{k}\Omega}$

Chapter 16

Exercise Solutions

E16.1

a. Driver in nonsaturation:

$$i_D = \frac{V_{DD} - v_O}{R_D} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)\left[2(v_I - V_{TN})v_O - v_O^2\right]$$

$$\frac{5-(0.15)}{R_D} = \frac{35}{2}(5)\left[2(5-0.8)(0.15)-(0.15)^2\right]$$

$$\frac{4.85}{R_D} = 87.5[1.2375]$$

$$\Rightarrow \underline{R_D = 44.8\text{ k}\Omega}$$

b. From Equation (16-10):

$$\left(\frac{0.035}{2}\right)(5)(44.8)(V_{It}-0.8)^2 + (V_{It}-0.8) - 5 = 0$$

$$3.920(V_{It}-0.8)^2 + (V_{It}-0.8) - 5 = 0$$

$$V_{It} - 0.8 = \frac{-1 \pm \sqrt{1+4(3.92)(5)}}{2(3.92)}$$

$$V_{It} - 0.8 = 1.0 \Rightarrow \underline{V_{It} = 1.8\text{ V}}$$

$$\underline{V_{0t} = 1.0\text{ V}}$$

E16.2

a. i. $v_I = 0 \Rightarrow v_0 = 4\text{ V}$

ii. $v_I = 4\text{ V}$, driver in nonsaturation

$$\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_L (v_{GSL} - V_{TNL})^2$$

$$= \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_D \left[2(v_{GSD} - V_{TND})v_{DSD} - v_{DSD}^2\right]$$

$$2(5 - v_0 - 1)^2 = (16)\left[2(4-1)v_0 - v_0^2\right]$$

$$16 - 8v_0 + v_0^2 = 8(6v_0 - v_0^2)$$

$$9v_0^2 - 56v_0 + 16 = 0$$

$$v_0 = \frac{56 \pm \sqrt{(56)^2 - 4(9)(16)}}{2(9)}$$

$$v_0 = 0.30\text{ V}$$

b. $P = i_D \cdot V_{DD}$

$$i_D = \frac{35}{2}(2)(5-0.30-1)^2 = 479\ \mu\text{A}$$

$$P = (0.479)(5)$$

$$\Rightarrow \underline{P = 2.4\text{ mW}}$$

E16.3

$$P = i_D \cdot V_{DD} \Rightarrow i_D = \frac{750}{5} = 150\ \mu\text{A}$$

$$150 = \frac{35}{2}\left(\frac{W}{L}\right)_L (5-0.2-0.8)^2$$

$$150 = 280\left(\frac{W}{L}\right)_L \Rightarrow \underline{\left(\frac{W}{L}\right)_L = 0.536}$$

$$i_D = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_D \left[2(v_I - V_{TND})v_O - v_O^2\right]$$

$$150 = \frac{35}{2}\left(\frac{W}{L}\right)_D \left[2(4.2-0.8)(0.2)-(0.2)^2\right]$$

$$150 = 23.1 \cdot \left(\frac{W}{L}\right)_D \Rightarrow \underline{\left(\frac{W}{L}\right)_D = 6.49}$$

E16.4

a. Load in saturation; driver in nonsaturation:

$$\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_L (v_{GSL} - V_{TNL})^2$$

$$= \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_D \left[2(v_{GSD} - V_{TND})v_{DSD} - v_{DSD}^2\right]$$

$$2(-[-1.5])^2 = (6)\left[2(5-0.7)v_0 - v_0^2\right]$$

$$4.5 = 6(8.6v_0 - v_0^2)$$

$$6v_0^2 - 51.6v_0 + 4.5 = 0$$

$$v_0 = \frac{51.6 \pm \sqrt{(51.6)^2 - 4(6)(4.5)}}{2(6)}$$

$$v_0 = 0.0881\text{ V}$$

b. Load

$v_{Ot} = V_{DD} + V_{TNL}$ Equation (16.26(b))

$$= 5 - 1.5 \Rightarrow v_{0t} = 3.5\text{ V}$$

From Equation (16 − 28(b)):

$$\sqrt{\frac{K_D}{K_L}}(v_{It} - V_{TND}) = -V_{TNL}$$

$$\sqrt{\frac{6}{2}}(v_{It} - 0.7) = -(-1.5) = 1.5$$

Load:

$\underline{v_{It} = 1.57\text{ V}}$

$\underline{v_{0t} = 3.5\text{ V}}$

Driver:

$\underline{v_{It} = 1.57\text{ V}}$

$\underline{v_{0t} = 0.87\text{ V}}$

c. $i_D = \frac{35}{2} \cdot (2)(1.5)^2 = 78.75\ \mu\text{A}$

$$P = I_D \cdot V_{DD} = (78.75)(5)$$

$$\Rightarrow \underline{P = 394\ \mu\text{W}}$$

E16.5

$P = i_D \cdot V_{DD} \Rightarrow I_D = \frac{350}{5} = 70\ \mu A$

$i_D = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_L (-V_{TNL})^2$

$70 = \frac{35}{2} \cdot \left(\frac{W}{L}\right)_L (2)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_L = 1}$

$i_D = \frac{35}{2} \cdot \left(\frac{W}{L}\right)_D [2(5 - 0.8)(0.05) - (0.05)^2]$

$70 = 7.31 \cdot \left(\frac{W}{L}\right)_D \Rightarrow \underline{\left(\frac{W}{L}\right)_D = 9.58}$

E16.6

From Equation (16 − 35) :

$V_{IH} = 0.85 + \frac{5 - 0.85}{16} \cdot \left\{\frac{1 + 2(16)}{\sqrt{1 + 3(16)}} - 1\right\}$

$\Rightarrow \underline{V_{IH} = 1.81\ V}$

Then from Equation (16-34):

$V_{OLU} = \frac{(5 - 0.85) + 16(1.81 - 0.85)}{(1 + 2(16))}$

$V_{OLU} = 0.591\ V$

$V_{IL} = 0.85$

$V_{OHU} = 4.15$

So

$NM_L = V_{IL} - V_{OLU} = 0.85 - 0.591$

$\Rightarrow \underline{NM_L = 0.259\ V}$

$NM_H = V_{OHU} - V_{IH} = 4.15 - 1.81$

$\Rightarrow \underline{NM_H = 2.34\ V}$

E16.7

From Equation (16-38):

$V_{IL} = 1 + \frac{1.7}{\sqrt{(5)(6)}} \Rightarrow \underline{V_{IL} = 1.31\ V}$

Then from Equation (16-37):

$V_{OHU} = (5 - 1.7) + (5)(1.31 - 1)$

$= 4.85\ V$

From Equation (16-41):

$V_{IH} = 1 + \frac{2(1.7)}{\sqrt{(3)(5)}} \Rightarrow \underline{V_{IH} = 1.88\ V}$

Then from Equation (16-40):

$V_{OLU} = \frac{1.88 - 1}{2} \Rightarrow V_{OLU} = 0.44\ V$

$NM_L = V_{IL} - V_{OLU} = 1.31 - 0.44$

$\Rightarrow \underline{NM_L = 0.87\ V}$

$NM_H = V_{OHU} - V_{IH} = 4.85 - 1.88$

$\Rightarrow \underline{NM_H = 2.97\ V}$

E16.8

a. i. A = logic 1 = 10 V, B = logic 0

"A" driver in nonsaturation, "B" driver off

$\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_L (-V_{TNL})^2$

$= \left(\frac{k'}{2}\right)\left(\frac{W}{L}\right)_D \left[2(v_I - V_{TND})V_{OL} - V_{OL}^2\right]$

$2(3)^2 = (10)[2(10 - 1.5)V_{OL} - V_{OL}^2]$

$9 = 5(17V_{OL} - V_{OL}^2)$

$5V_{OL}^2 - 85V_{OL} + 9 = 0$

$V_{OL} = \frac{85 \pm \sqrt{(85)^2 - 4(5)(9)}}{2(5)}$

$\Rightarrow \underline{V_{OL} = 0.107\ V}$

ii. $A = B$ = logic 1

$\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_L (-V_{TNL})^2$

$= 2\left(\frac{k'}{2}\right)\left(\frac{W}{L}\right)_D \left[2(v_I - V_{TND})V_{OL} - V_{OL}^2\right]$

$2(3)^2 = (2)(10)[2(10 - 1.5)V_{OL} - V_{OL}^2]$

$9 = 10(17V_{OL} - V_{OL}^2)$

$10V_{OL}^2 - 170V_{OL} + 9 = 0$

$V_{OL} = \frac{170 \pm \sqrt{(170)^2 - 4(10)(9)}}{2(10)}$

$\Rightarrow \underline{V_{OL} = 0.0531\ V}$

b. Both cases.

$i_D = \frac{35}{2} \cdot (2)(3)^2 = 315\ \mu A \Rightarrow P = i_D \cdot V_{DD}$

$\Rightarrow \underline{P = 3.15\ mW}$

E16.9

$P = i_D \cdot V_{DD} \Rightarrow i_D = \frac{800}{5} = 160\ \mu A$

$i_D = 160 = \frac{35}{2} \cdot \left(\frac{W}{L}\right)_L (1.4)^2 = 34.3\left(\frac{W}{L}\right)_L$

$\Rightarrow \underline{\left(\frac{W}{L}\right)_L = 4.66}$

$i_D = 160 = \frac{35}{2} \cdot \left(\frac{W}{L}\right)_D [2(5 - 0.8)(0.12) - (0.12)^2]$

$\Rightarrow \underline{\left(\frac{W}{L}\right)_D = 9.20}$

E16.10

$P = i_D \cdot V_{DD} \Rightarrow i_D = \frac{800}{5} = 160\ \mu A$

$i_D = 160 = \frac{35}{2} \cdot \left(\frac{W}{L}\right)_L (1.4)^2$

$\Rightarrow \underline{\left(\frac{W}{L}\right)_L = 4.66}$

$i_D = 160\ \mu A$

$$= \frac{35}{2}\cdot\frac{1}{3}\cdot\left(\frac{W}{L}\right)_D [2(5-0.8)(0.12)-(0.12)^2]$$

$$\Rightarrow \left(\frac{W}{L}\right)_D = 27.6$$

E16.11

a. From the load transistor:

$$I_{DL} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_L (V_{GSL}-V_{TNL})^2$$

$$= \frac{35}{2}(0.5)(5-0.15-0.7)^2$$

or

$$I_{DL} = 150.7\ \mu A$$

Maximum v_O occurs when either A or B is high and C is high. For the two NMOS in series, the effective k_N is cut in half, so

$$I_{DL} = \frac{1}{2}\left[\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_D\right][2(V_{GSD}-V_{TND})V_{DS}-V_{DS}^2]$$

or

$$150.7 = \frac{1}{2}\left[\frac{35}{2}\cdot\left(\frac{W}{L}\right)_D\right][2(5-0.7)(0.15)-(0.15)^2]$$

which yields

$$\left(\frac{W}{L}\right)_D = 13.6$$

b. $P = i_D \cdot V_{DD} = (150.7)(5) \Rightarrow P = 753\ \mu W$

E16.12

a. $v_O(\max)$ occurs when $A = B = 1$ and $C = D = 0$ or $A = B = 0$ and $C = D = 1$

$$\left(\frac{W}{L}\right)_L (-V_{TNL})^2 = \frac{1}{2}\cdot\left(\frac{W}{L}\right)_D [2(v_I-V_{TND})v_O - v_O^2]$$

$$(0.5)(1.2)^2 = \frac{1}{2}\cdot\left(\frac{W}{L}\right)_D [2(5-0.7)(0.15)-(0.15)^2]$$

$$0.72 = (0.634)\left(\frac{W}{L}\right)_D \Rightarrow \left(\frac{W}{L}\right)_D = 1.14$$

b. $$i_D = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_L (-V_{TNL})^2 = \left(\frac{35}{2}\right)(0.5)[-(-1.2)]^2$$

$i_D = 12.6\ \mu A$

$P = i_D \cdot V_{DD} = (12.6)(5) \Rightarrow P = 63\ \mu W$

E16.13

a. $K_n/K_p = 1$

$$V_{It} = \frac{10-2+(1)(2)}{1+1} \Rightarrow V_{It} = 5\ V$$

$$V_{OPt} = V_{It} + |V_{TP}| = 5+2 \Rightarrow V_{OPt} = 7\,V$$

$$V_{ONt} = V_{It} - V_{TN} = 5-2 \Rightarrow V_{ONt} = 3\,V$$

b. $K_n/K_p = 0.5$

$$\Rightarrow V_{It} = \frac{10-2+\sqrt{0.5}(2)}{1+\sqrt{0.5}} \Rightarrow V_{It} = 5.52\ V$$

$V_{OPt} = 7.52\ V$

$V_{ONt} = 3.52\ V$

c. $K_n/K_p = 2$

$$\Rightarrow V_{It} = \frac{10-2+\sqrt{2}(2)}{1+\sqrt{2}} \Rightarrow V_{It} = 4.49\ V$$

$V_{OPt} = 6.49\ V$

$V_{ONt} = 2.49\ V$

E16.14

a. $K_n = K_p = 50\ \mu A/V^2$

$V_{It} = 2.5\ V$

$$i_D(\max) = K_n(V_{It}-V_{TN})^2 = 50(2.5-0.8)^2$$

$$\Rightarrow i_D(\max) = 145\ \mu A$$

b. $K_n = K_p = 200\ \mu A/V^2$

$V_{It} = 2.5\ V$

$i_D(\max) = (200)(2.5-0.8)^2$

$$\Rightarrow i_D(\max) = 578\ \mu A$$

E16.15

$P = f\cdot C_L \cdot V_{DD}^2$

$(0.10\times 10^{-6}) = f(0.5\times 10^{-12})(3)^2$

$f = 2.22\times 10^4\ \text{Hz} \Rightarrow f = 22.2\ \text{kHz}$

E16.16

a. $K_n/K_p = 200/80 = 2.5$

$$\Rightarrow V_{It} = \frac{10-2+\sqrt{2.5}(2)}{1+\sqrt{2.5}} \Rightarrow V_{It} = 4.32\ V$$

$V_{OPt} = 6.32\ V$

$V_{ONt} = 2.32\ V$

b. $$V_{IL} = 2 + \frac{10-2-2}{2.5-1}\cdot\left[2\sqrt{\frac{2.5}{2.5+3}}-1\right]$$

$$\Rightarrow V_{IL} = 3.39\ V$$

$$V_{OHU} = \frac{1}{2}\{(1+2.5)(3.39)+10-(2.5)(2)+2\}$$

$V_{OHU} = 9.43\ V$

$$V_{IH} = 2 + \frac{10-2-2}{2.5-1}\cdot\left[\frac{2(2.5)}{\sqrt{3(2.5)+1}}-1\right]$$

$$\Rightarrow V_{IH} = 4.86\ V$$

$$V_{OLU} = \frac{(4.86)(1+2.5)-10-(2.5)(2)+2}{2(2.5)}$$

$V_{OLU} = 0.802\ V$

c. $NM_L = V_{IL} - V_{OLU} = 3.39 - 0.802$

$\Rightarrow \underline{NM_L = 2.59\text{ V}}$

$NM_H = V_{OHU} - V_{IH} = 9.43 - 4.86$

$\Rightarrow \underline{NM_H = 4.57\text{ V}}$

E16.17

a. $V_{It} = \dfrac{5 - 2 + (1)(6.8)}{1 + 1}$

$V_{It} = 1.9\text{ V}$

$V_{OPt} = 3.9\text{ V}$

$V_{ONt} = 1.1\text{ V}$

b. $V_{IL} = 0.8 + \dfrac{3}{8} \cdot [5 - 2 - 0.8]$

$\Rightarrow \underline{V_{IL} = 1.63\text{ V}}$

$V_{OHU} = \dfrac{1}{2}\{2(1.63) + 5 - 0.8 + 2\}$

$V_{OHU} = 4.73\text{ V}$

$V_{IH} = 0.8 + \dfrac{5}{8}(5 - 2 - 0.8)$

$\Rightarrow \underline{V_{IH} = 2.18\text{ V}}$

$V_{OLU} = \dfrac{1}{2}\{2(2.18) - 5 - 0.8 + 2\}$

$V_{OLU} = 0.275\text{ V}$

c. $NM_L = V_{IL} - V_{OLU} = 1.63 - 0.275$

$\Rightarrow \underline{NM_L = 1.35\text{ V}}$

$NM_H = V_{OHU} - V_{IH} = 4.73 - 2.18$

$\Rightarrow \underline{NM_H = 2.55\text{ V}}$

E16.18

a. $A = 0 \Rightarrow M_{PA}$ (assume zero resistance)

$$K_n = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_N$$

$$K_p = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_P = \frac{1}{2}\cdot\left(\frac{k_n'}{2}\right)\cdot 8\left(\frac{W}{L}\right)_N$$

$$K_p = 2K_n \Rightarrow \frac{K_n}{K_p} = \frac{1}{2}$$

From Equation (16-55),

$$V_{It} = \frac{5 - 1 + \sqrt{0.5}(1)}{1 + \sqrt{0.5}} \Rightarrow V_{It} = 2.76\text{ V}$$

$V_{OPt} = 3.76\text{ V}$

$V_{ONt} = 1.76\text{ V}$

b. From Equation (16-58(b))

$$i_D(peak) = K_n(V_{It} - V_{TN})^2$$

$$50 = \frac{35}{2}\left(\frac{W}{L}\right)_N (2.76 - 1)^2 = 54.2\left(\frac{W}{L}\right)_N$$

$$\Rightarrow \underline{\left(\frac{W}{L}\right)_N = 0.923}$$

$$\underline{\left(\frac{W}{L}\right)_P = (8)(0.923) = 7.38}$$

E16.19

Want $K_{n,eff} = K_{p,eff}$

$$\left(\frac{k_n'}{2}\right)\cdot\frac{1}{2}\left(\frac{W}{L}\right)_N = \left(\frac{k_p'}{2}\right)\cdot 2\left(\frac{W}{L}\right)_P = \frac{1}{2}\cdot\frac{k_n'}{2}\cdot 2\left(\frac{W}{L}\right)_P$$

$$\Rightarrow \underline{\left(\frac{W}{L}\right)_N = 2\left(\frac{W}{L}\right)_P}$$

E16.20

Want $K_{n,eff} = K_{p,eff}$

$$\left(\frac{k_n'}{2}\right)\cdot 3\left(\frac{W}{L}\right)_N = \left(\frac{k_p'}{2}\right)\cdot\frac{1}{3}\left(\frac{W}{L}\right)_P = \frac{1}{2}\cdot\frac{k_n'}{2}\cdot\frac{1}{3}\left(\frac{W}{L}\right)_P$$

$$3\left(\frac{W}{L}\right)_N = \frac{1}{6}\cdot\left(\frac{W}{L}\right)_P$$

Or

$$\Rightarrow \underline{\frac{(W/L)_p}{(W/L)_N} = 18}$$

E16.21

Want $K_{n,eff} = K_{p,eff}$

$$\left(\frac{k_n'}{2}\right)\cdot\frac{1}{3}\left(\frac{W}{L}\right)_N = \left(\frac{k_p'}{2}\right)\cdot 3\left(\frac{W}{L}\right)_P = \frac{1}{2}\cdot\frac{k_n'}{2}\cdot 3\left(\frac{W}{L}\right)_P$$

$$\frac{1}{3}\left(\frac{W}{L}\right)_N = \frac{3}{2}\cdot\left(\frac{W}{L}\right)_P$$

Or

$$\Rightarrow \underline{\frac{(W/L)_p}{(W/L)_N} = \frac{2}{9}}$$

E16.22

NMOS:

M_{NA}, M_{NB} in series $\Rightarrow \left(\dfrac{W}{L}\right) = 2$

M_{ND}, M_{NE} in parallel $\Rightarrow \left(\dfrac{W}{L}\right) = 1$

M_{NC} in series with $M_{ND} \| M_{NE} \Rightarrow \left(\dfrac{W}{L}\right) = 2$

Effective composite $\left(\dfrac{W}{L}\right) = 1$ for each side.

PMOS:

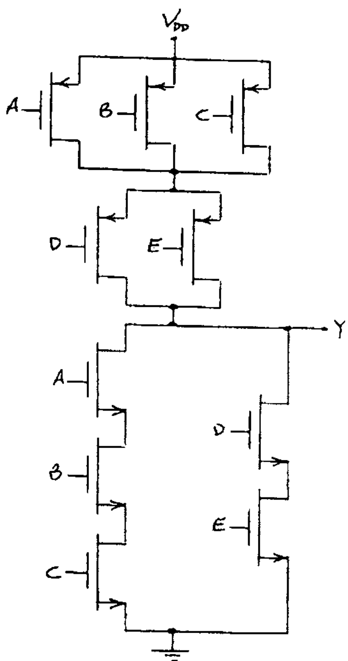
Want the effective composite $\left(\frac{W}{L}\right)$ of each side to be 2.

M_{PA}, M_{PC} in series $\Rightarrow \left(\frac{W}{L}\right) = 4$

M_{PA}, M_{PB} in parallel $\Rightarrow \left(\frac{W}{L}\right)_B = 4$

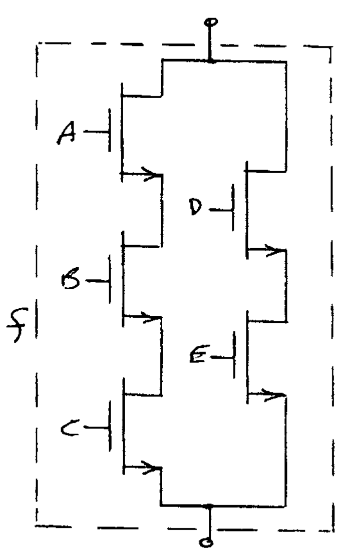
M_{PD}, M_{PE} in series $\Rightarrow \left(\frac{W}{L}\right) = 8$

E16.23



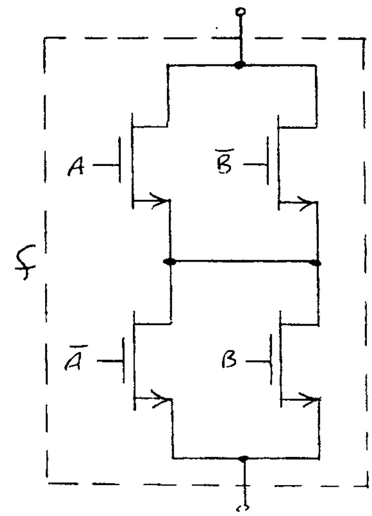
E16.24

The NMOS part of the circuit is:



E16.25

The NMOS part of the circuit is:



E16.26

a. $v_I = \phi = 5\text{ V} \Rightarrow \underline{v_O = 4\text{ V}}$
b. $v_I = 3\text{ V},\ \phi = 5\text{ V} \Rightarrow \underline{v_O = 3\text{ V}}$
c. $v_I = 4.2\text{ V},\ \phi = 5\text{ V} \Rightarrow \underline{v_O = 4\text{ V}}$
d. $v_I = 5\text{ V},\ \phi = 3\text{ V} \Rightarrow \underline{v_O = 2\text{ V}}$

E16.27

(a) $v_I = 8V, \phi = 10V \Rightarrow v_{GSD} = 8V$

M_D in nonsaturation

$$K_D\left[2(v_{GSD} - V_{TND})v_O - v_O^2\right]$$

$$K_L\left[V_{DD} - v_O - V_{TNL}\right]^2$$

$$\frac{K_D}{K_L}\left[2(8-2)(0.5)-(0.5)^2\right] = \left[10-0.5-2\right]^2$$

$$\Rightarrow \underline{\frac{K_D}{K_L} = 9.78}$$

(b) $v_I = \phi = 8V \Rightarrow v_{GSD} = 6V$

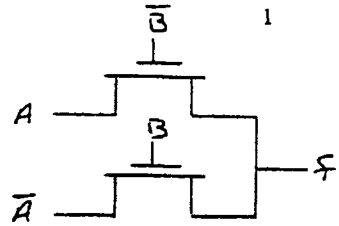
$$\frac{K_D}{K_L}\left[2(6-2)(0.5)-(0.5)^2\right] = \left[10-0.5-2\right]^2$$

$$\Rightarrow \underline{\frac{K_D}{K_L} = 15}$$

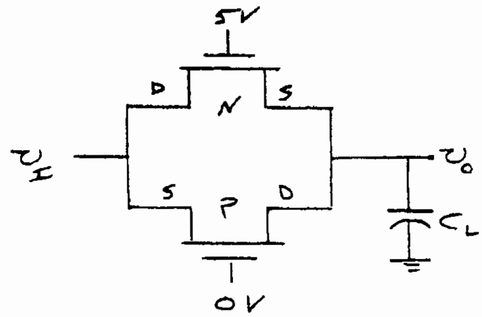
E16.28

Exclusive-OR

A	B	f
0	0	0
1	0	1
0	1	1
1	1	0



E16.29

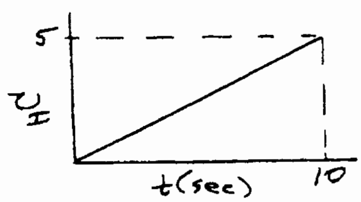


NMOS conducting for $0 \le v_I \le 4.2$ V

$\Rightarrow$NMOS Conducting: $0 \le t \le 8.4$ s
NMOS Cutoff : $8.4 \le t \le 10$ s

PMOS cutoff for $0 \le v_I \le 1.2$ V

$\Rightarrow$PMOS Cutoff : $0 \le t \le 2.4$ s
PMOS Conducting: $2.4 \le t \le 10$ s



E16.30

(a) $1K \Rightarrow 32x32$ array
Each row and column requires a 5-bit word $\Rightarrow$
6 transistors per row and column, $\Rightarrow$
$32x6 + 32x6 = 384$ transistors plus buffer transistors.
(b) $4K \Rightarrow 64x64$ array
Each row and column requires a 6-bit word $\Rightarrow$
7 transistors per row and column $\Rightarrow$
$64x7 + 64x7 = 896$ transistors plus buffer transistors.
(c) $16K \Rightarrow 128x128$ array
Each row and column requires a 7-bit word $\Rightarrow$
8 transistors per row and column $\Rightarrow$
$128x8 + 128x8 = 2048$ transistors plus buffer transistors.

E16.31

$16K \Rightarrow 16384$ cells
Total Power $= 125\,mW = (2.5)I_T$
$\Rightarrow I_T = 50\,mA$

Then, for each cell, $I = \dfrac{50\,mA}{16384} \Rightarrow I = 3.05\,\mu A$

Now, $I \cong \dfrac{V_{DD}}{R}$ or $R = \dfrac{V_{DD}}{I} = \dfrac{2.5}{3.05} \Rightarrow$

$\underline{R = 0.82\,M\Omega}$

E16.32

From Equation (16.93)

$$\frac{(W/L)_{nA}}{(W/L)_{n1}} = \frac{2(V_{DD}V_{TN}) - 3V_{TN}^2}{(V_{DD} - 2V_{TN})^2}$$

$$= \frac{2(2.5)(0.4) - 3(0.4)^2}{(2.5 - 2(0.4))^2} = 0.526$$

From Equation (16.95)

$$\frac{(W/L)_p}{(W/L)_{nB}} = \frac{k_n'}{k_p'} \cdot \frac{2(V_{DD}V_{TN}) - 3V_{TN}^2}{(V_{DD} + V_{TP})^2}$$

$$= (2.5)\left[\frac{2(2.5)(0.4) - 3(0.4)^2}{(2.5 - 0.4)^2}\right] = 1.31$$

So $\left(\dfrac{W}{L}\right)$ of transmission gate device must be < 0.526 times the $\left(\dfrac{W}{L}\right)$ of the NMOS transistors in the inverter cell. The $\left(\dfrac{W}{L}\right)$ of the PMOS transistors must be < 1.31 times the $\left(\dfrac{W}{L}\right)$ of the transmission gate devices. Then the $\left(\dfrac{W}{L}\right)$ of the PMOS devices must be < 0.689 times $\left(\dfrac{W}{L}\right)$ of NMOS devices in cell.

E16.33

Initial voltage across the storage capacitor
$= V_{DD} - V_{TN} = 3 - 0.5 = 2.5\,V$.
Now

$-I = C\dfrac{dV}{dt}$ or $V = -\dfrac{I}{C} \cdot t + K$

where $K = 2.5\,V$, $t = 1.5\,ms$, $V = \dfrac{2.5}{2} = 1.25\,V$, and $C = 0.05\,pF$. Then

$$1.25 = 2.5 - \frac{I(1.5x10^{-3})}{(0.05x10^{-12})} \Rightarrow$$

$\underline{I = 4.17x10^{-11}\,A \Rightarrow I = 41.7\,pA}$

Chapter 16

Problem Solutions

16.1

(a) $\Delta V_{TN} = \frac{\sqrt{2e \in_s N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$

$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85x10^{-14})}{450x10^{-8}} = 7.67x10^{-8}$

$\sqrt{2e \in_s N_a}$

$= \left[2(1.6x10^{-19})(11.7)(8.85x10^{-14})(8x10^{15})\right]^{1/2}$

$= 5.15x10^{-8}$

Then

$\Delta V_{TN} = \frac{5.15x10^{-8}}{7.67x10^{-8}}\cdot\left[\sqrt{2(0.343) + V_{SB}} - \sqrt{2(0.343)}\right]$

For $V_{SB} = 1\,V$:

$\Delta V_{TN} = 0.671\left[\sqrt{1.686} - \sqrt{0.686}\right] \Rightarrow \underline{\Delta V_{TN} = 0.316\,V}$

For $V_{SB} = 2\,V$:

$\Delta V_{TN} = 0.671\left[\sqrt{2.686} - \sqrt{0.686}\right] \Rightarrow \underline{\Delta V_{TN} = 0.544\,V}$

(b) For $V_{GS} = 2.5$ V, $V_{DS} = 5$ V, transistor biased in the saturation region.

$I_D = K_n(V_{GS} - V_{TN})^2$

For $V_{SB} = 0$,

$I_D = 0.2(2.5 - 0.8)^2 = 0.578$ mA

For $V_{SB} = 1$,

$I_D = 0.2(2.5 - [0.8 + 0.316])^2 = 0.383$ mA

For $V_{SB} = 2$,

$I_D = 0.2(2.5 - [0.8 + 0.544])^2 = 0.267$ mA

16.2

(a) $I_D = \frac{V_{DD} - v_O}{R_D} = K_n\left[2(V_{GS} - V_{TN})v_O - v_O^2\right]$

$\frac{5 - (0.1)}{40x10^3} = K_n\left[2(5 - 0.8)(0.1) - (0.1)^2\right]$

or $K_n = 1.476x10^{-4}\,A/V^2 = \frac{8x10^{-5}}{2}\left(\frac{W}{L}\right)$

So that $\underline{\left(\frac{W}{L}\right) = 3.69}$

b. From Equation (16.10),

$K_n R_D [V_{It} - V_{TN}]^2 + [V_{It} - V_{TN}] - V_{DD} = 0$

$(0.1476)(40)[V_{It} - 0.8]^2 + [V_{It} - 0.8] - 5 = 0$

or $[V_{It} - 0.8] = \frac{-1 \pm \sqrt{(1)^2 + 4(0.1476)(40)(5)}}{2(0.1476)(40)}$

or $[V_{It} - 0.8] = 0.839$

So that $\underline{V_{It} = 1.64\text{ V}}$

$P = I_D(\max)\cdot V_{DD}$

and $I_D(\max) = \frac{5 - (0.1)}{40} = 0.1225$ mA

or $\underline{P = 0.6125\text{ mW}}$

16.3

a. From Equation (16.10), the transistor point is found from

$K_n R_D (V_{It} - V_{TN})^2 + (V_{It} - V_{TN}) - V_{DD} = 0$

$K_n = 50\,\mu A/V^2$, $R_D = 20\,k\Omega$, $V_{TN} = 0.8\,V$

$(0.05)(20)(V_{It} - V_{TN})^2 + (V_{It} - V_{TN}) - 5 = 0$

$V_{It} - V_{TN} = \frac{-1 \pm \sqrt{1 + 4(0.05)(20)(5)}}{2(0.05)(20)}$

$= 1.79$ V So $V_{It} = 2.59$ V

$V_{Ot} = 1.79$ V

Output voltage for $v_I = 5$ V is determined from Equation (16.12):

$v_O = 5 - (0.05)(20)[2(5 - 0.8)v_O - v_O^2]$

$v_O^2 - 9.4v_O + 5 = 0$

So $v_O = \frac{9.4 \pm \sqrt{(9.4)^2 - 4(1)(5)}}{2(1)} = 0.566$ V

b. For $R_D = 200$ kΩ,

$(V_{It} - V_{TN}) = \frac{-1 \pm \sqrt{1 + 4(0.05)(200)(5)}}{2(0.05)(200)}$

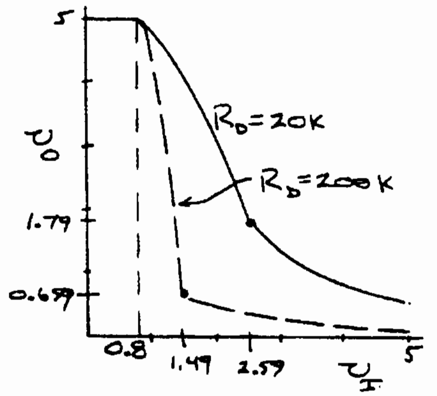
$= 0.659$ V So $V_{It} = 1.46$ V

$V_{Ot} = 0.659$ V

$v_O = 5 - (0.05)(200)[2(5 - 0.8)v_O - v_O^2]$

or $10v_O^2 - 85v_O + 5 = 0$

$v_O = \frac{85 \pm \sqrt{(85)^2 - 4(10)(5)}}{2(10)} = 0.0592$ V



16.4

$P = i_D \cdot V_{DD}$

$1 = i_D(5) \Rightarrow i_D = 0.2\ mA$

Now

$i_D = K_n\left[2(v_I - V_{TN})v_O - v_O^2\right]$

$0.2 = K_n\left[2(5-0.8)(0.2)-(0.2)^2\right]$

or $K_n = 0.122\ mA/V^2 = \left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right) \Rightarrow$

$\underline{\left(\frac{W}{L}\right) = 3.05}$

Also

$v_O = V_{DD} - K_n R_D\left[2(v_I - V_{TN})v_O - v_O^2\right]$

$0.2 = 5-(0.122)R_D\left[2(5-0.8)(0.2)-(0.2)^2\right] \Rightarrow$

$\underline{R_D = 24\ k\Omega}$

16.5

From Equation (16.21):

$$V_{It} = \frac{10 - 2 + 2\left(1 + \sqrt{\frac{200}{50}}\right)}{1 + \sqrt{\frac{200}{50}}}$$

or $\underline{V_{It} = 4.67\text{ V}}$

$V_{Ot} = V_{It} - V_{TND} = 4.67 - 2 \Rightarrow \underline{V_{Ot} = 2.67\,V}$

From Equation (16.23):

$200[2(8-2)v_O - v_O^2] = 50[10 - v_O - 2]^2$

$4[12v_O - v_O^2] = [8 - v_O]^2 = [64 - 16v_O + v_O^2]$

$5v_O^2 - 64v_O + 64 = 0$

$$v_O = \frac{64 \pm \sqrt{(64)^2 - 4(5)(64)}}{2(5)}$$

or $\underline{v_O = 1.09\text{ V}}$

16.6

(a) From Equation (16.23)

$\frac{K_D}{K_L}\left[2(3-0.5)(0.25)-(0.25)^2\right] = (3-0.25-0.5)^2$

$\Rightarrow \underline{\frac{K_D}{K_L} = 4.26}$

(b) $\frac{K_D}{K_L}\left[2(2.5-0.5)(0.25)-(0.25)^2\right] = (3-0.25-0.5)^2$

$\Rightarrow \underline{\frac{K_D}{K_L} = 5.06}$

(c) $i_D = K_L(V_{GSL} - V_{TNL})^2 = K_L(V_{DD} - v_O - V_{TNL})^2$

$= \left(\frac{0.080}{2}\right)(1)(3-0.25-0.5)^2 \Rightarrow$

$i_D = 0.203\ mA$

$P = i_D \cdot V_{DD} = (0.203)(3) \Rightarrow \underline{P = 0.608\ mW}$

for both parts (a) and (b).

16.7

(a) $P = 0.4\ mW = i_D \cdot V_{DD} = i_D(3) \Rightarrow$

$i_D = 0.133\ mA$

$i_D = K_L(V_{DD} - v_O - V_{TNL})^2$

$0.133 = \left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)_L (3-0.1-0.5)^2 = (0.2304)\left(\frac{W}{L}\right)_L$

So $\underline{\left(\frac{W}{L}\right)_L = 0.577}$

$\frac{K_D}{K_L}\left[2(2.5-0.5)(0.1)-(0.1)^2\right] = (3-0.1-0.5)^2$

$\Rightarrow \frac{K_D}{K_L} = 14.8$ so that $\underline{\left(\frac{W}{L}\right)_D = 8.54}$

$$V_{It} = \frac{3-0.5+0.5\left(1+\sqrt{14.8}\right)}{1+\sqrt{14.8}}$$

or

$\underline{V_{It} = 1.02\,V}$, $\underline{V_{Ot} = 0.52\,V}$

(b) $NM_L = V_{IL} - V_{OLU}$

$NM_H = V_{OHU} - V_{IH}$

From Equation (16.35)

$$V_{IH} = 0.5 + \frac{(3-0.5)}{14.8}\left\{\frac{(1+2(14.8))}{\sqrt{1+3(14.8)}} - 1\right\} \Rightarrow$$

$V_{IH} = 1.10\,V$

$V_{OHU} = 3.0 - 0.5 = 2.5\,V$

$NM_H = V_{OHU} - V_{IH} = 2.5 - 1.10 \Rightarrow \underline{NM_H = 1.40\,V}$

$V_{IL} = V_{TND} = 0.5\,V$

$$V_{OLU} = \frac{(V_{DD} - V_{TNL}) + \frac{K_D}{K_L}(V_I - V_{TND})}{1 + 2\left(\frac{K_D}{K_L}\right)}$$

$$= \frac{(3-0.5)+14.8(1.1-0.5)}{1+2(14.8)} \Rightarrow$$

$V_{OLU} = 0.372\,V$

Then $NM_L = V_{IL} - V_{OLU} = 0.5 - 0.372 \Rightarrow$

$\underline{NM_L = 0.128\,V}$

16.8

We have

$$\frac{K_D}{K_L}\left[2(v_I - V_{TND})v_O - v_O^2\right] = (V_{DD} - v_O - V_{TNL})^2$$

$$\frac{(W/L)_D}{(W/L)_L}\left[2(V_{DD} - V_{TN} - V_{TN})(0.08V_{DD}) - (0.08V_{DD})^2\right]$$

$$= (V_{DD} - 0.08V_{DD} - V_{TN})^2$$

$$\frac{(W/L)_D}{(W/L)_L}\left[2(V_{DD} - 2(0.2)V_{DD})(0.08V_{DD}) - 0.0064V_{DD}^2\right]$$

$$= \left[(0.92 - 0.2)V_{DD}\right]^2 = 0.5184V_{DD}^2$$

$$\frac{(W/L)_D}{(W/L)_L}[0.096] = 0.5184 \Rightarrow \underline{\frac{(W/L)_D}{(W/L)_L} = 5.4}$$

16.9

$V_{OH} = V_B - V_{GS} = \text{Logic 1}$

So

(a) $V_B = 4\,V \Rightarrow V_{OH} = 3\,V$

(b) $V_B = 5\,V \Rightarrow V_{OH} = 4\,V$

(c) $V_B = 6\,V \Rightarrow V_{OH} = 5\,V$

(d) $V_B = 7\,V \Rightarrow V_{OH} = 6\,V$

For $v_I = V_{OH}$

$$K_D\left[2(v_I - V_T)v_O - v_O^2\right] = K_L\left[V_B - v_O - V_T\right]^2$$

Then

(a) $(1)\left[2(3-1)V_{OL} - V_{OL}^2\right] = (0.4)\left[4 - V_{OL} - 1\right]^2 \Rightarrow$

$\underline{V_{OL} = 0.657\,V}$

(b) $(1)\left[2(4-1)V_{OL} - V_{OL}^2\right] = (0.4)\left[5 - V_{OL} - 1\right]^2 \Rightarrow$

$\underline{V_{OL} = 0.791\,V}$

(c) $(1)\left[2(5-1)V_{OL} - V_{OL}^2\right] = (0.4)\left[6 - V_{OL} - 1\right]^2 \Rightarrow$

$\underline{V_{OL} = 0.935\,V}$

(d) $(1)\left[2(6-1)V_{OL} - V_{OL}^2\right] = (0.4)\left[7 - V_{OL} - 1\right]^2 \Rightarrow$

$\underline{V_{OL} = 1.08\,V}$

16.10

a. For load $V_{Ot} = V_{DD} + V_{TNL} = 5 - 2 = 3\,V$

$$\sqrt{\frac{K_D}{K_L}}\cdot(V_{It} - V_{TND}) = -V_{TNL}$$

$$\sqrt{\frac{500}{100}}(V_{It} - 0.8) = -(-2)$$

$\left.\begin{array}{l}\Rightarrow V_{It} = 1.69V \\ V_{Ot} = 3\text{ V}\end{array}\right\}$ Load

Driver: $V_{Ot} = V_{It} - V_{TND} = 1.69 - 0.8 = 0.89\,V$

$\left.\begin{array}{l}V_{It} = 1.69\text{ V} \\ V_{Ot} = 0.89\text{ V}\end{array}\right\}$ Driver

b. From Equation (16.29(b)):

$$\frac{500}{100}\cdot\left[2(5-0.8)v_O - v_O^2\right] = [-(-2)]^2$$

$$5v_O^2 - 42v_O + 4 = 0$$

$$v_O = \frac{42 \pm \sqrt{(42)^2 - 4(5)(4)}}{2(5)} \Rightarrow \underline{v_O = 0.0963\text{ V}}$$

c. $i_D = K_L(-V_{TNL})^2 = 100[-(-2)]^2 \Rightarrow$

$i_D = 400\,\mu A$

16.11

$$\left(\frac{500}{50}\right)\left[2(3-0.5)(0.1) - (0.1)^2\right] = (-V_{TNL})^2$$

So

$(-V_{TNL})^2 = 4.9 \Rightarrow \underline{V_{TNL} = -2.21\,V}$

16.12

(a) $P = i_D \cdot V_{DD}$

$150 = i_D \cdot (3) \Rightarrow i_D = 50\,\mu A$

$i_D = K_L(-V_{TNL})^2$

$$50 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_L[-(-1)]^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_L = 1.25}$$

$$\frac{K_D}{K_L}\left[2(3-0.5)(0.1) - (0.1)^2\right] = [-(-1)]^2$$

$$\frac{K_D}{K_L} = \frac{(W/L)_D}{(W/L)_L} = 2.04 \Rightarrow \underline{\left(\frac{W}{L}\right)_D = 2.55}$$

For the Load:

$V_{Ot} = V_{DD} + V_{TNL} = 3 - 1 \Rightarrow \underline{V_{Ot} = 2\,V}$

$\sqrt{2.04}(V_{It} - 0.5) = [-(-1)] \Rightarrow \underline{V_{It} = 1.20\,V}$

For the Driver:

$V_{Ot} = V_{It} - V_{TND} = 1.20 - 0.5 \Rightarrow \underline{V_{Ot} = 0.70\,V}$

$\underline{V_{It} = 1.20\,V}$

(b) $NM_L = V_{IL} - V_{OLU}$

$NM_H = V_{OHU} - V_{IH}$

$$V_{IL} = 0.5 + \frac{[-(-1)]}{\sqrt{(2.04)(1+2.04)}} = 0.902\ V$$

$$V_{IH} = 0.5 + \frac{2[-(-1)]}{\sqrt{3(2.04)}} = 1.31\ V$$

Then $V_{OHU} = (3-1) + (2.04)(0.902 - 0.5) = 2.82\ V$

$$V_{OLU} = \frac{(1.31 - 0.5)}{2} = 0.405\ V$$

$NM_L = 0.902 - 0.405 \Rightarrow \underline{NM_L = 0.497\ V}$

$NM_H = 2.82 - 1.31 \Rightarrow \underline{NM_H = 1.51\ V}$

16.13

a. From Equation (16.29(b)):

$$\left(\frac{W}{L}\right)_D [2(2.5 - 0.5)(0.05) - (0.05)^2] = \left(\frac{W}{L}\right)_L [-(-1)]^2$$

$$\left(\frac{W}{L}\right)_L = 1$$

Then $\underline{\left(\frac{W}{L}\right)_D = 5.06}$

b. $i_D = \left(\frac{80}{2}\right)(1)[-(-1)]^2$

or $\underline{i_D = 40\ \mu A}$

$P = i_D \cdot V_{DD} = (40)(2.5)$

$\Rightarrow \underline{P = 100\ \mu W}$

16.14

a. i. $v_I = 0.5\ V \Rightarrow i_D = 0 \Rightarrow \underline{P = 0}$

ii. $v_I = 5\ V$, From Equation (16.12),

$v_O = 5 - (0.1)(20)[2(5 - 1.5)v_O - v_O^2]$

$2v_O^2 - 15v_O + 5 = 0$

$$v_O = \frac{15 \pm \sqrt{(15)^2 - 4(2)(5)}}{2(2)} \Rightarrow \underline{v_O = 0.35\ V}$$

$$i_D = \frac{5 - 0.35}{20} = 0.2325\ mA$$

$P = i_D \cdot V_{DD} = (0.2325)(5) \Rightarrow \underline{P = 1.16\ mW}$

b. i. $v_I = 0.25\ V \Rightarrow i_D = 0 \Rightarrow \underline{P = 0}$

ii. $v_I = 4.3\ V$, From Equation (16.23),

$100[2(4.3 - 0.7)v_O - v_O^2] = 10[5 - v_O - 0.7]^2$

$10[7.2v_O - v_O^2] = 18.49 - 8.6v_O + v_O^2$

Then

$11v_O^2 - 80.6v_O + 18.49 = 0$

$$v_O = \frac{80.6 \pm \sqrt{(80.6)^2 - 4(11)(18.49)}}{2(11)}$$

$\Rightarrow \underline{v_O = 0.237\ V}$

Then

$i_D = 10[5 - 0.237 - 0.7]^2 = 165\ \mu A$

$P = i_D \cdot V_{DD} = (165)(5) \Rightarrow \underline{P = 825\ \mu W}$

c. i. $v_I = 0.03\ V \Rightarrow i_D = 0 \Rightarrow \underline{P = 0}$

ii. $v_I = 5\ V$

$i_D = K_L(-V_{TNL})^2 = (10)[-(-2)]^2 = 40\ \mu A$

$P = i_D \cdot V_{DD} = (40)(5) \Rightarrow \underline{P = 200\ \mu W}$

16.15

From Equation (16.35)

$$V_{IH} = 0.8 + \frac{5 - (0.8)}{10} \cdot \left\{\frac{1 + 2(10)}{\sqrt{1 + 3(10)}} - 1\right\}$$

$$V_{IH} = 0.8 + 0.42 \cdot \left\{\frac{21}{5.57} - 1\right\}$$

or

$\underline{V_{IH} = 1.96\ V}$

M_{D2} in non-saturation region. Then

$$K_D[2(v_{GS2} - V_{TN})v_{DS2} - v_{DS2}^2] = K_L[V_{DD} - v_{O2} - V_{TN}]^2$$

$v_{DS2} = v_{O2}$ and $v_{GS2} = V_{IH} = 1.96$

$10[2(1.96 - 0.8)v_{O2} - v_{O2}^2] = [5 - v_{O2} - 0.8]^2$

$23.2v_{O2} - 10v_{O2}^2 = 17.64 - 8.4v_{O2} + v_{O2}^2$

or

$11v_{O2}^2 - 31.6v_{O2} + 17.64 = 0$

$$v_{O2} = \frac{31.6 \pm \sqrt{(31.6)^2 - 4(11)(17.64)}}{2(11)}$$

$\Rightarrow \underline{v_{O2} = 0.758\ V}$

Now M_{D1} in saturation region. Then

$$K_D[v_I - V_{TN}]^2 = K_L[V_{DD} - v_{O1} - V_{TN}]^2$$

$$\sqrt{10}\cdot(v_I - 0.8) = 5 - 1.96 - 0.8 = 2.24$$

Then $\underline{v_I = 1.51 \text{ V}}$

16.16

a. From Equation (16.41),

$$V_{IH} = 0.8 + \frac{2[-(-2)]}{\sqrt{3(4)}} \Rightarrow \underline{V_{IH} = 1.95 \text{ V} = v_{O1}}$$

M_{D2} in non-saturation and M_{L2} in saturation.

$$K_D\left[2(v_{O1} - V_{TND})v_{O2} - v_{O2}^2\right] = K_L(-V_{TNL})^2$$

$$4[2(1.95 - 0.8)v_{O2} - v_{O2}^2] = (1)[-(-2)]^2$$

$$4v_{O2}^2 - 9.2v_{O2} + 4 = 0$$

$$v_{O2} = \frac{9.2 \pm \sqrt{(9.2)^2 - 4(4)(4)}}{2(4)} \Rightarrow \underline{v_{O2} = 0.582 \text{ V}}$$

Both M_{D1} and M_{L1} in saturation region. From Equation (16.28(b)),

$$\sqrt{4}\cdot(v_I - 0.8) = -(-2)$$

or $\underline{v_I = 1.8 \text{ V}}$

b. $V_{IL} = 0.8 + \dfrac{(+2)}{\sqrt{4(1+4)}} = 1.25 \text{ V} = v_{O1}$

M_{D2} in saturation, M_{L2} in non-saturation

$$K_D[v_{O1} - V_{TND}]^2$$
$$= K_L\left[2(-V_{TNL})(5 - v_{O2}) - (5 - v_{O2})^2\right]$$

$$4(1.25 - 0.8)^2 = 2(2)(5 - v_{O2}) - (5 - v_{O2})^2$$

$$(5 - v_{O2})^2 - 4(5 - v_{O2}) + 0.81 = 0$$

$$5 - v_{O2} = \frac{4 \pm \sqrt{(4)^2 - 4(1)(0.81)}}{2(1)} = 0.214 \text{ V}$$

so

$$v_{O2} = 4.786 \text{ V}$$

To find v_I:

$$4(v_{O1} - 0.8)^2 = (1)(-(-2))^2$$

$$v_{O1} - 0.8 = 1$$

$$v_{O1} = 1.8 \text{ V}$$

c. $V_{IH} = 1.95$ V, $V_{IL} = 1.25$ V

16.17

a. i. Neglecting the body effect,

$$v_O = V_{DD} - V_{Th}$$

Assume $V_{DD} = 5$ V, then $\underline{v_O = 4.2 \text{ V}}$

ii. Taking the body effect into account: From Problem 16.1,

$$V_{TN} = V_{TN0} + 0.671\left[\sqrt{0.686 + V_{SB}} - \sqrt{0.686}\right]$$

and $V_{SB} = v_O$

Then

$$v_O = 5 - \left[0.8 + 0.671\left(\sqrt{0.686 + v_O} - \sqrt{0.686}\right)\right]$$

$$v_O = 4.756 - 0.671\sqrt{0.686 + v_O}$$

$$0.671\sqrt{0.686 + v_O} = 4.756 - v_O$$

$$0.450(0.686 + v_O) = 22.62 - 9.51v_O + v_O^2$$

$$v_O^2 - 9.96v_O + 22.3 = 0$$

$$v_O = \frac{9.96 \pm \sqrt{(9.96)^2 - 4(22.3)}}{2} \Rightarrow \underline{v_O = 3.40 \text{ V}}$$

b. PSpice results similar to Figure 16.18(a).

16.18

Results similar to Figure 16.18(b).

16.19

a. M_X on, M_Y cutoff.
From Equation (16.29(b)):

$$\frac{K_D}{K_L}\left[2(5 - 0.8)(0.2) - (0.2)^2\right] = \left[-(-2)\right]^2$$

or $\underline{\dfrac{K_D}{K_L} = 2.44}$

b. For $v_X = v_Y = 5$ V

$$2(2.44)[2(5 - 0.8)v_O - v_O^2] = [-(-2)]^2$$

$$4.88v_O^2 - 41.0v_O + 4 = 0$$

$$v_O = \frac{41 \pm \sqrt{(41)^2 - 4(4.88)(4)}}{2(4.88)}$$

or

$$\underline{v_O = 0.0987 \text{ V}}$$

c. $i_D = \left(\dfrac{80}{2}\right)(1)[-(-2)]^2 = 160\ \mu\text{A}$

$P = (160)(5) \Rightarrow \underline{P = 800\ \mu\text{W}}$

for both parts (a) and (b).

16.20

(a) Maximum value of v_O in low state- when only one input is high, then,

$$\frac{K_D}{K_L}\left[2(3-0.5)(0.1)-(0.1)^2\right]=\left[-(-1)\right]^2$$

$$\frac{K_D}{K_L}=2.04$$

(b) $P=i_D \cdot V_{DD}$

$0.1=i_D(3) \Rightarrow i_D=33.3\ \mu A$

$$i_D=\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_L(-V_{TNL})^2$$

$$33.3=\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_L\left[-(-1)\right]^2 \Rightarrow \left(\frac{W}{L}\right)_L=0.8325$$

Then $\left(\frac{W}{L}\right)_D=1.70$

(c) $3(2.04)\left[2(3-0.5)v_O-v_O^2\right]=\left[-(-1)\right]^2 \Rightarrow$

$v_O=0.0329\ V$

16.21

a. $P=i_D \cdot V_{DD}$

$250=i_D(5) \Rightarrow i_D=50\ \mu\text{A}$

$$i_D=\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_{ML1}\left[-V_{TNL1}\right]^2$$

$$50=\left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_{M_{L1}}\left[-(-2)\right]^2$$

So that $\left(\frac{W}{L}\right)_{M_{L1}}=0.417$

$$\frac{K_D}{K_L}\left[2(v_I-V_{TND})v_O-v_O^2\right]=\left[-V_{TNL}\right]^2$$

$$\frac{K_D}{K_L}\left[2(5-0.8)(0.15)-(0.15)^2\right]=\left[-(-2)\right]^2$$

or $\frac{K_D}{K_L}=3.23 \Rightarrow \left(\frac{W}{L}\right)_{MD1}=1.35$

b. For $v_X=v_Y=0 \Rightarrow v_{O1}=5$ and $v_{O3}=4.2$

Then

$$K_{D2}\left[2(v_{O1}-V_{TND})v_{O2}-v_{O2}^2\right]$$
$$+K_{D3}\left[2(v_{O3}-V_{TND})v_{O2}-v_{O2}^2\right]=K_{L2}\left[-V_{TNL2}\right]^2$$

$K_{D2}\propto 8,\ K_{D3}\propto 8,\ K_{L2}\propto 1$

$$8\left[2(5-0.8)v_{O2}-v_{O2}^2\right]+8\left[2(4.2-0.8)v_{O2}-v_{O2}^2\right]=(1)\left[-(-2)\right]^2$$

$$67.2v_{O2}-8v_{O2}^2+54.4v_{O2}-8v_{O2}^2=4$$

Then

$$16v_O^2-121.6v_O+4=0$$

$$v_{O2}=\frac{121.6\pm\sqrt{(121.6)^2-4(16)(4)}}{2(16)}$$

So $v_{O2}=0.0330\ \text{V}$

16.22

a. We can write

$$K_x\left[2(v_X-V_{TN})v_{DSX}-v_{DSX}^2\right]$$
$$=K_y\left[2(v_Y-v_{DSX}-V_{TN})v_{DSY}-v_{DSY}^2\right]$$
$$=K_L\left[V_{DD}-v_O-V_{TN}\right]^2$$

where $v_O=v_{DSX}+v_{DSY}$

We have

$v_X=v_Y=9.2\,V,\ V_{DD}=10\,V,\ V_{TN}=0.8\,V$

As a good first approximation, neglect the v_{DSX}^2 and v_{DSY}^2 terms. Let $v_O\approx 2v_{DSX}$. Then from the first and third terms in the above equation,

$$9[2(9.2-0.8)v_{DSX}]\cong(1)(10-2v_{DSX}-0.8)^2$$

$$(151.2)v_{DSX}\cong 84.64-36.8v_{DSX}$$

So that $v_{DSX}=0.450\ \text{V}$

From the first and second terms of the above equation,

$$9[2(9.2-0.8)v_{DSX}]\cong 9[2(9.2-v_{DSX}-0.8)v_{DSY}]$$

or

$$(16.8)(0.45)=2(9.2-0.45-0.8)v_{DSY}$$

which yields $v_{DSY}=0.475\ \text{V}$

Then $v_O=v_{DSX}+v_{DSY}=0.450+0.475$

or $v_O=0.925\ \text{V}$

We have $v_{GSX}=9.2\ \text{V}$

and $v_{GSY}=9.2-v_{DSX}=9.2-0.45$

or $v_{GSY}=8.75\ \text{V}$

b. Since v_O is close to ground potential, the body effect will have minimal effect on the results.
From a PSpice analysis:

For part (a) :

$v_{DSX}=0.462\ \text{V},\ v_{DSY}=0.491\ \text{V},\ v_O=0.9536\ \text{V},$

$v_{GSX}=9.2\ \text{V}$, and $v_{GSY}=8.738\ \text{V}$

For part (b) :

$v_{DSX}=0.441\ \text{V},\ v_{DSY}=0.475\ \text{V},\ v_O=0.9154\ \text{V},$

$v_{GSX}=9.2\ \text{V}$, and $v_{GSY}=8.759\ \text{V}$

16.23

a. We can write

$$K_x\left[2(v_X - V_{TNX})v_{DSX} - v_{DSX}^2\right]$$
$$= K_y\left[2(v_Y - v_{DSX} - V_{TNY})v_{DSY} - v_{DSY}^2\right]$$
$$= K_L\left[-V_{TNL}\right]^2$$

From the first and third terms, (neglect v_{DSX}^2).

$$4[2(5-0.8)v_{DSX}] = (1)[-(-1.5)]^2$$

or $\underline{v_{DSX} = 0.067\ \text{V}}$

From the second and third terms, (neglect v_{DSY}^2).

$$4[2(5-0.067-0.8)v_{DSY}] = (1)[-(-1.5)]^2$$

or $\underline{v_{DSY} = 0.068\ \text{V}}$

Now

$v_{GSX} = 5$, $v_{GSY} = 5 - 0.067 \Rightarrow v_{GSY} = 4.933$ V

and $v_0 = v_{DSX} + v_{DSY} \Rightarrow \underline{v_0 = 0.135\ \text{V}}$

Since v_0 is close to ground potential, the body-effect has little effect on the results.

16.24

Complement of (B AND C) OR $A \Rightarrow \overline{(B\cdot C)+A}$

16.25

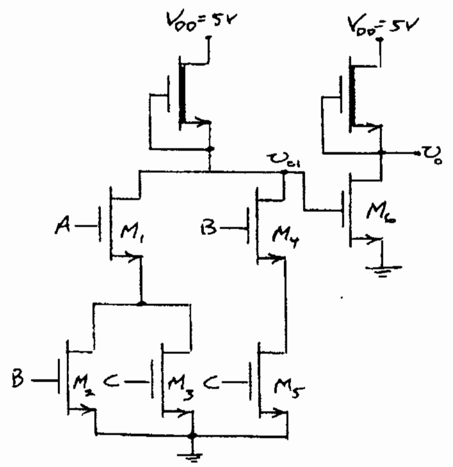
Considering a truth table, we find

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

which shows that the circuit performs the exclusive-OR function.

16.26

(a) Carry-out $= A\bullet(B+C)+B\bullet C$



(b) For $v_{O1} = Low = 0.2\,V$

$$\frac{K_D}{K_L}\left[2(5-0.8)(0.2)-(0.2)^2\right] = \left[-(-1.5)\right]^2 \Rightarrow$$

For $\left(\frac{W}{L}\right)_L = 1$, then $\left(\frac{W}{L}\right)_D = 1.37$

So, for M_6: $\underline{\left(\frac{W}{L}\right)_6 = 1.37}$

To achieve the required composite conduction parameter,

For $M_1 - M_5$: $\underline{\left(\frac{W}{L}\right)_{1-5} = 2.74}$

16.28

a. From Equation (16.55),

$$v_I = V_{It} = \frac{5-0.8+0.8}{1+1} = \underline{V_{It} = 2.5\ \text{V}}$$

p – channel, $V_{OPt} = 2.5 - (-0.8) \Rightarrow \underline{V_{OPt} = 3.3\ \text{V}}$

n – channel, $V_{ONt} = 2.5 - 0.8 \Rightarrow \underline{V_{ONt} = 1.7\ \text{V}}$

c For $v_I = 2$ V, NMOS in saturation and PMOS in nonsaturation. From Equation (16.49),

$$(2-0.8)^2 = \left[2(5-2-0.8)(5-v_0) - (5-v_0)^2\right]$$
$$1.44 = 4.4(5-v_0) - (5-v_0)^2$$

So $(5-v_0)^2 - 4.4(5-v_0) + 1.44 = 0$

$$(5-v_0) = \frac{4.4 \pm \sqrt{(4.4)^2 - 4(1)(1.44)}}{2}$$

or

$$5 - v_0 = 0.356 \Rightarrow \underline{v_0 = 4.64\ \text{V}}$$

By symmetry, for $v_I = 3$ V, $\underline{v_0 = 0.356\ \text{V}}$

16.29

(a) $K_n = \left(\frac{80}{2}\right)(2) = 80\ \mu A/V^2$

$K_p = \left(\frac{40}{2}\right)(4) = 80\ \mu A/V^2$

(i) $$V_{It} = \frac{V_{DD} + V_{TP} + \sqrt{\frac{K_n}{K_p}}\cdot V_{TN}}{1+\sqrt{\frac{K_n}{K_p}}} = \frac{3.3-0.4+(1)(0.4)}{1+1}$$

$\underline{V_{It} = 1.65\,V}$

PMOS:

$V_{Ot} = V_{It} - V_{TP} = 1.65 - (-0.4) \Rightarrow \underline{V_{Ot} = 2.05\,V}$

NMOS:

$V_{Ot} = V_{It} - V_{TN} = 1.65 - (0.4) \Rightarrow \underline{V_{Ot} = 1.25\,V}$

(iii) For $v_O = 0.4\,V$: NMOS: Non-sat: PMOS:Sat

$K_n\left[2(V_{GSN} - V_{TN})V_{DS} - V_{DS}^2\right] = K_p\left[V_{SGP} + V_{TP}\right]^2$

$2(v_I - 0.4)(0.4) - (0.4)^2 = (3.3 - v_I - 0.4)^2 \Rightarrow$

$v_I = 1.89\,V$

For $v_O = 2.9\,V$, By symmetry

$v_I = 1.65 - (1.89 - 1.65) \Rightarrow \underline{v_I = 1.41\,V}$

(b) $K_n = \left(\frac{80}{2}\right)(2) = 80\ \mu A/V^2$

$K_p = \left(\frac{40}{2}\right)(2) = 40\ \mu A/V^2$

(i) $$V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{80}{40}}\cdot(0.4)}{1 + \sqrt{\frac{80}{40}}} \Rightarrow \underline{V_{It} = 1.44\,V}$$

PMOS:

$V_{Ot} = 1.44 - (-0.4) \Rightarrow \underline{V_{Ot} = 1.84\,V}$

NMOS:

$V_{Ot} = 1.44 - 0.4 \Rightarrow \underline{V_{Ot} = 1.04\,V}$

(iii) For $v_O = 0.4\,V$

$(80)\left[2(v_I - 0.4)(0.4) - (0.4)^2\right] = (40)\left[3.3 - v_I - 0.4\right]^2$

$\Rightarrow \underline{v_I = 1.62\,V}$

For $v_O = 2.9\,V$: NMOS:Sat, PMOS:Non-sat

$(80)\left[v_I - 0.4\right]^2 = (40)\left[2(3.3 - v_I - 0.4)(0.4) - (0.4)^2\right]$

$\Rightarrow \underline{v_I = 1.16\,V}$

16.30

a. For $v_{O1} = 0.6 < V_{TN} \Rightarrow \underline{v_{O2} = 5\,V}$

N_1 in nonsaturation and P_1 in saturation. From Equation (16.57),

$[2(\nu_I - 0.8)(0.6) - (0.6)^2] = [5 - \nu_I - 0.8]^2$

$1.2\nu_I - 1.32 = 17.64 - 8.4\nu_I + \nu_I^2$

or

$\nu_I^2 - 9.6\nu_I + 18.96 = 0$

$$\nu_I = \frac{9.6 \pm \sqrt{(9.6)^2 - 4(1)(18.96)}}{2}$$

or

$\underline{\nu_I = 2.78\ \text{V}}$

b. $V_{ONt} \le \nu_{O2} \le V_{OPt}$

From symmetry, $V_{It} = 2.5$ V

$V_{OPt} = 2.5 + 0.8 = 3.3$ V

and $V_{ONt} = 2.5 - 0.8 = 1.7$ V

So $\underline{1.7 \le \nu_{O2} \le 3.3\ \text{V}}$

16.31

a. $V_{ONt} \le \nu_{O1} \le V_{OPt}$

By symmetry, $V_{It} = 2.5$ V

$V_{OPt} = 2.5 + 0.8 = 3.3$ V

and $V_{ONt} = 2.5 - 0.8 = 1.7$ V

So $\underline{1.7 \le \nu_{O1} \le 3.3\ \text{V}}$

b. For $v_{O2} = 0.6 < V_{TN} \Rightarrow \underline{v_{O3} = 5\,V}$

N_2 in nonsaturation and P_2 in saturation. From Equation (16.57),

$[2(\nu_{I2} - 0.8)(0.6) - (0.6)^2] = [5 - \nu_{I2} - 0.8]^2$

$1.2\nu_{I2} - 1.32 = 17.64 - 8.4\nu_{I2} + \nu_{I2}^2$

or

$\nu_{I2}^2 - 9.6\nu_{I2} + 18.96 = 0$

So $\nu_{I2} = \underline{\nu_{O1} = 2.78\ \text{V}}$

For $\nu_{O1} = 2.78$, both N_1 and P_1 in saturation. Then

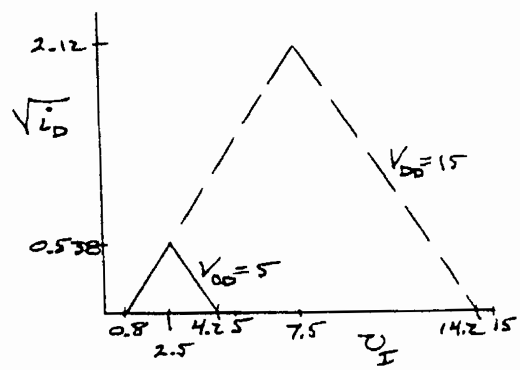
$\underline{\nu_I = 2.5\ \text{V}}$

16.32

a. $\sqrt{i_{peak}} = \sqrt{K_n}(v_I - V_{TN})$

$\sqrt{i_{peak}} = \sqrt{0.1}\cdot(2.5 - 0.8) = 0.538\ (\text{mA})^{1/2}$

b. $\sqrt{i_{peak}} = \sqrt{0.1}\cdot(7.5 - 0.8) = 2.12\ (\text{mA})^{1/2}$



16.33

(a) $K_n = \left(\frac{50}{2}\right)(2) = 50\ \mu A/V^2$

$K_p = \left(\frac{25}{2}\right)(4) = 50\ \mu A/V^2$

$I_{D,peak} = K_n(v_I - V_{TN})^2 = 50(2.5 - 0.8)^2$

or $\underline{I_{D,peak} = 144.5\ \mu A}$

(b) $K_n = 50\ \mu A/V^2$, $K_p = 25\ \mu A/V^2$

From Equation (16.55),

$$V_{It} = \frac{5 - 0.8 + \sqrt{\frac{50}{25}}(0.8)}{1 + \sqrt{\frac{50}{25}}} = 2.21\,V$$

Then

$$I_{D,peak} = K_n(V_{It} - V_{TN})^2 = 50(2.21 - 0.8)^2$$

or $I_{D,peak} = 99.4\ \mu A$

16.34

a. $P = fC_L V_{DD}^2$

For $V_{DD} = 5$ V, $P = (10 \times 10^6)(0.2 \times 10^{-12})(5)^2$

or $\underline{P = 50\ \mu\text{W}}$

For $V_{DD} = 15$ V, $P = (10 \times 10^6)(0.2 \times 10^{-12})(15)^2$

or $\underline{P = 450\ \mu\text{W}}$

b. For $V_{DD} = 5$ V, $P = (10 \times 10^6)(0.2 \times 10^{-12})(5)^2$

or $\underline{P = 50\ \mu\text{W}}$

16.35

(a) $P = f C_L V_{DD}^2 = (150x10^6)(0.4x10^{-12})(5)^2$

$= 1.5x10^{-3}$ W/inverter

Total power: $P_T = (2x10^6)(1.5x10^{-3}) \Rightarrow$

$P_T = 3000\,W$!!!!

(b) For $f = 300\ MHz$

$1.5x10^{-3} = (300x10^6)(0.4x10^{-12})V_{DD}^2 \Rightarrow$

$\underline{V_{DD} = 3.54\,V}$

16.36

(a) For $v_I \cong V_{DD}$, NMOS in nonsaturation

$i_D = K_n\left[2(v_I - V_{TN})v_{DS} - v_{DS}^2\right]$ and $v_{DS} \cong 0$

So $\frac{1}{r_{ds}} = \frac{di_D}{dv_{DS}} \cong K_n\left[2(V_{DD} - V_{TN})\right]$

Or

$$r_{ds} = \frac{1}{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_n \cdot 2(V_{DD} - V_{TN})}$$

or

$$r_{ds} = \frac{1}{k_n'\left(\frac{W}{L}\right)_n \cdot (V_{DD} - V_{TN})}$$

For $v_I \cong 0$, PMOS in nonsaturation

$i_D = K_p\left[2(V_{DD} - v_I + V_{TP})v_{SD} - v_{SD}^2\right]$

and $v_{SD} \cong 0$ for $v_I \cong 0$.

So

$$\frac{1}{r_{sd}} = \frac{di_D}{dv_{SD}} \cong \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_p \cdot 2(V_{DD} + V_{TP})$$

or

$$r_{sd} = \frac{1}{k_p'\left(\frac{W}{L_p}\right) \cdot (V_{DD} + V_{TP})}$$

(b) For $\left(\frac{W}{L}\right)_n = 2$, $\left(\frac{W}{L}\right)_p = 4$

$$r_{ds} = \frac{1}{(50)(2)(5 - 0.8)} \Rightarrow r_{ds} = 2.38\ k\Omega$$

$$r_{sd} = \frac{1}{(25)(4)(5 - 0.8)} \Rightarrow r_{sd} = 2.38\ k\Omega$$

For $\left(\frac{W}{L}\right)_p = 2$,.

$$r_{sd} = \frac{1}{(25)(2)(5 - 0.8)} \Rightarrow r_{sd} = 4.76\ k\Omega$$

Now, for NMOS:

$v_{ds} = i_d r_{ds}$ or $i_d = \frac{v_{ds}}{r_{ds}} = \frac{0.5}{2.38} \Rightarrow$ $\underline{i_d = 0.21\,mA}$

For PMOS:

For $r_{sd} = 2.38\ k\Omega$,

$i_d = \frac{v_{sd}}{r_{sd}} = \frac{0.5}{2.38} \Rightarrow$ $\underline{i_d = 0.21\,mA}$

For $r_{sd} = 4.76\ k\Omega$,

$i_d = \frac{v_{sd}}{r_{sd}} = \frac{0.5}{4.76} \Rightarrow$ $\underline{i_d = 0.105\,mA}$

16.37

From Equation (16.73)

$V_{IL} = 1.5 + \frac{3}{8} \cdot (10 - 1.5 - 1.5) \Rightarrow \underline{V_{IL} = 4.125\text{ V}}$

and Equation (16.72)

$$V_{OHU} = \frac{1}{2} \cdot [2(4.125) + 10 - 1.5 + 1.5]$$

or $\underline{V_{OHU} = 9.125\text{ V}}$

From Equation (16.79)

$V_{IH} = 1.5 + \frac{5}{8} \cdot (10 - 1.5 - 1.5) \Rightarrow \underline{V_{IH} = 5.875\text{ V}}$

and Equation (16.78)

$$V_{OLU} = \frac{1}{2} \cdot [2(5.875) - 10 - 1.5 + 1.5]$$

or $\underline{V_{OLU} = 0.875\text{ V}}$

Now

$$NM_L = V_{IL} - V_{OLU} = 4.125 - 0.875$$

$$\Rightarrow \underline{NM_L = 3.25 \text{ V}}$$

$$NM_H = V_{OHU} - V_{TH} = 9.125 - 5.875$$

$$\Rightarrow \underline{NM_H = 3.25 \text{ V}}$$

16.38

From Equation (16.71)

$$V_{IL} = 1.5 + \frac{(10 - 1.5 - 1.5)}{\left(\frac{100}{50} - 1\right)}\left[2\sqrt{\frac{\frac{100}{50}}{\frac{100}{50} + 3}} - 1\right]$$

$$= 1.5 + 7[2(0.632) - 1]$$

or

$$\underline{V_{IL} = 3.348 \text{ V}}$$

From Equation (16.70)

$$V_{OHU} = \frac{1}{2}\cdot\left\{\left(1 + \frac{100}{50}\right)(3.348) + 10 - \left(\frac{100}{50}\right)(1.5) + 1.5\right\}$$

or $\underline{V_{OHU} = 9.272 \text{ V}}$

From Equation (16.77)

$$V_{IH} = 1.5 + \frac{(10 - 1.5 - 1.5)}{\left(\frac{100}{50} - 1\right)}\left[\frac{2\left(\frac{100}{50}\right)}{\sqrt{3\left(\frac{100}{50}\right) + 1}} - 1\right]$$

$$= 1.5 + 7[1.51 - 1]$$

or

$$\underline{V_{IH} = 5.07 \text{ V}}$$

From Equation (16.76)

$$V_{OLU} = \frac{(5.07)\left(1 + \frac{100}{50}\right) - 10 - \left(\frac{100}{50}\right)(1.5) + 1.5}{2\left(\frac{100}{50}\right)}$$

or $\underline{V_{OLU} = 0.9275 \text{ V}}$

Now $NM_L = V_{IL} - V_{OLU} = 3.348 - 0.9275$

or $\underline{NM_L = 2.42 \text{ V}}$

$NM_H = V_{OHU} - V_{IH} = 9.272 - 5.07$

or $\underline{NM_H = 4.20 \text{ V}}$

16.39

a. $v_A = v_B = 5$ V

N_1 and N_2 on, so $v_{DS1} \approx v_{DS2} \approx 0$ V

P_1 and P_2 off

So we have a $P_3 - N_3$ CMOS inverter. By symmetry, $v_C = 2.5$ V (Transition Point).

b. For $v_A = v_B = v_C \equiv v_I$

Want $K_{n,eff} = K_{p,eff}$

$$\frac{k_n'}{2}\cdot\left(\frac{W}{3L}\right)_n = \frac{k_P'}{2}\cdot\left(\frac{3W}{L}\right)_P$$

With $k_n' = 2k_P'$, then

$$\frac{2}{2}\cdot\frac{1}{3}\cdot\left(\frac{W}{L}\right)_n = \frac{1}{2}\cdot 3\cdot\left(\frac{W}{L}\right)_P$$

Or $\underline{\left(\frac{W}{L}\right)_n = \frac{9}{2}\left(\frac{W}{L}\right)_P}$

c. We have

$$K_n = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_n = \left(\frac{2k_p'}{2}\right)\left(\frac{9}{2}\right)\left(\frac{W}{L}\right)_p$$

$$K_p = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_p$$

Then from Equation (16.55)

$$V_{It} = \frac{5 + (-0.8) + \sqrt{\frac{K_n}{K_p}}\cdot(0.8)}{1 + \sqrt{\frac{K_n}{K_p}}}$$

Now

$$\frac{K_n}{K_p} = (2)\left(\frac{9}{2}\right) = 9$$

Then

$$V_{It} = \frac{5 + (-0.8) + 3(0.8)}{1 + 3} \Rightarrow \underline{V_{It} = 1.65 V}$$

16.40

By definition, NMOS is on if gate voltage is 5 V and is off if gate voltage is 0 V.

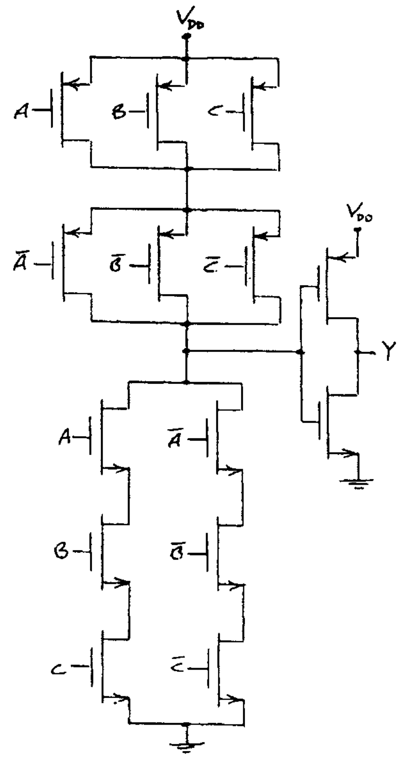
State	N_1	N_2	N_3	N_4	N_5	v_O
1	off	on	off	on	on	0
2	off	off	on	on	off	0
3	on	on	off	off	on	5
4	on	on	off	on	on	0

Logic function $(v_X \text{ OR } v_Y) \otimes (v_X \text{ AND } v_Z)$

Exclusive OR of $(v_X \text{ OR } v_Y)$ with $(v_X \text{ AND } v_Z)$

16.41

(a) A classic design is shown:



$\bar{A}, \bar{B}, \bar{C}$ signals supplied through inverters.

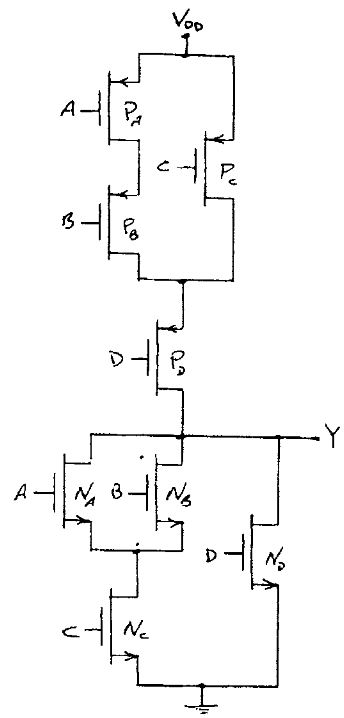
(b) For Inverters, $\left(\frac{W}{L}\right)_n = 1$ and $\left(\frac{W}{L}\right)_p = 2$

For PMOS in Logic function, let $\left(\frac{W}{L}\right)_p = 1$, then

for NMOS in Logic function, $\left(\frac{W}{L}\right)_n = 2.25$

16.42

(a) A classic design is shown:



(b) $\left(\frac{W}{L}\right)_{ND} = 1$, $\left(\frac{W}{L}\right)_{NA,NB,NC} = 2$

$\left(\frac{W}{L}\right)_{PA,PB} = 8$, $\left(\frac{W}{L}\right)_{PC,PD} = 4$

16.43

$\overline{(A \text{ OR } B) \text{ AND } C}$

16.44

Let $\left(\frac{W}{L}\right)_p = 1$ for each PMOS: Composite PMOS

$\left(\frac{W}{L}\right) = 5$. Want composite $\left(\frac{W}{L}\right) = 2.5$ for NMOS,

So that $\left(\frac{W}{L}\right)_n = 5(2.5) = 12.5$ for each NMOS.

16.45

(a)Let $\left(\frac{W}{L}\right)_n = 1$ for each NMOS. Composite $\left(\frac{W}{L}\right)$ of NMOS = 6. Want composite $\left(\frac{W}{L}\right)$ of PMOS = 12. Then $\left(\frac{W}{L}\right)_p = 6(12) = 72$ for each PMOS. Let $\left(\frac{W}{L}\right)_n = 1$ and $\left(\frac{W}{L}\right)_p = 2$ for each transistor in inverter.

(a) For 3-input NOR:
Let $\left(\frac{W}{L}\right)_n = 1$ for each NMOS. Composite $\left(\frac{W}{L}\right)$ of NMOS = 3. Want composite $\left(\frac{W}{L}\right)$ of PMOS = 6.

Then $\left(\frac{W}{L}\right)_p = 3(6) = 18$ for each PMOS.

For 2-input NAND:

Let $\left(\frac{W}{L}\right)_p = 1$ for each PMOS. Composite $\left(\frac{W}{L}\right)$ of PMOS = 2. Want composite $\left(\frac{W}{L}\right)$ of NMOS = 1.

Then $\left(\frac{W}{L}\right)_n = 2$ for each NMOS.

Sizes of PMOS transistors in (b) are substantially less than those in (a).

16.46

By definition:

NMOS off if gate voltage = 0
NMOS on if gate voltage = 5 V
PMOS off if gate voltage = 5 V
PMOS on if gate voltage = 0

State	N_1	P_1	N_A	N_B	N_C	v_{O1}	N_2	P_2	v_{O2}
1	off	on	off	off	off	5	on	off	0
2	on	off	on	off	off	5	on	off	0
3	off	on	off	off	off	5	on	off	0
4	on	off	off	off	on	5	on	off	0
5	off	on	off	off	off	5	on	off	0
6	on	off	off	on	on	0	off	on	5

Logic function is

$$v_{O2} = (v_A \text{ OR } v_B) \text{ AND } v_C$$

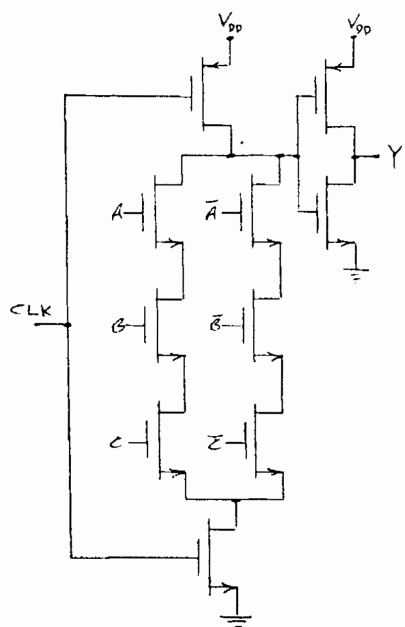
16.47

State	v_{O1}	v_{O2}	v_{O3}
1	5	5	0
2	0	0	5
3	5	5	0
4	5	0	5
5	5	5	0
6	0	5	0

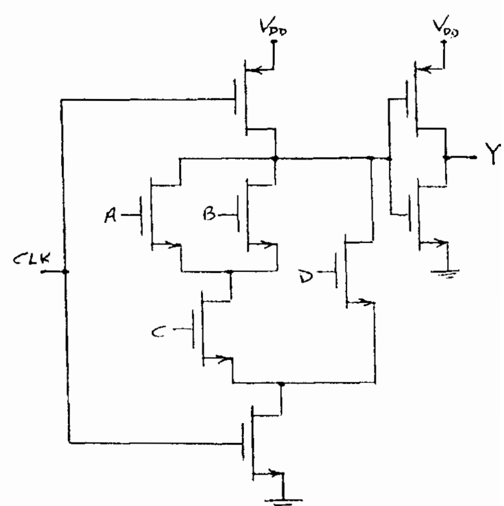
Logic function:

$$v_{O3} = (v_X \text{ OR } v_Z) \text{ AND } v_Y$$

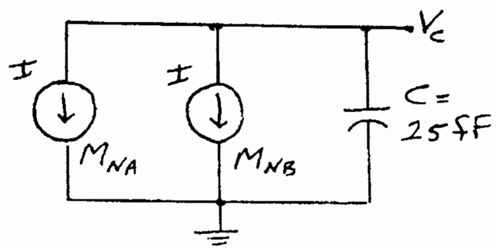
16.48



16.49



16.50



$$2I = -C\frac{dV_C}{dt}$$

So

$$\Delta V_C = -\frac{1}{C}(2I)\cdot t$$

For $\Delta V_C = -0.5\,V$

$$-0.5 = -\frac{2(2x10^{-12})\cdot t}{25x10^{-15}} \Rightarrow \underline{t = 3.125\,ms}$$

16.51

(a)
(i) $v_O = 0$
(ii) $v_O = 4.2\,V$
(iii) $v_O = 2.5\,V$

(b)
(i) $v_O = 0$
(ii) $v_O = 3.2\,V$
(iii) $v_O = 2.5\,V$

16.52

Neglect the body effect.

a. v_{O1}(logic 1) = 4.2 V, v_{O2}(logic 1) = 5 V

b. v_I = 5 V $\Rightarrow$ v_{GS1} = 4.2 V

M_1 in nonsaturation and M_2 in saturation. From Equation (16.23)

$$\left(\frac{W}{L}\right)_D\left[2(v_{GS1} - V_{TND})v_{O1} - v_{O1}^2\right] = \left(\frac{W}{L}\right)_L(V_{DD} - v_{O1} - V_{TNL})^2$$

$$\left(\frac{W}{L}\right)_D[2(4.2 - 0.8)(0.1) - (0.1)^2] = (1)[5 - 0.1 - 0.8]^2$$

Or

$$\left(\frac{W}{L}\right)_D(0.67) = 16.81 \Rightarrow \underline{\left(\frac{W}{L}\right)_D = 25.1}$$

Now

$$v_{O1} = 4.2\text{ V} \Rightarrow v_{GS3} = 4.2\text{ V}$$

M_3 in nonsaturation and M_4 in saturation. From Equation (16.29(b)).

$$\left(\frac{W}{L}\right)_D\left[2(v_{GS3} - V_{TND})v_{O2} - v_{O2}^2\right] = \left(\frac{W}{L}\right)_L[-V_{TNL}]^2$$

$$\left(\frac{W}{L}\right)_D[2(4.2 - 0.8)(0.1) - (0.1)^2] = (2)[-(-1.5)]^2$$

$$\left(\frac{W}{L}\right)_D(0.67) = 2.25$$

Or $\underline{\left(\frac{W}{L}\right)_D = 3.36}$

16.53

For $\phi = 1$, $\bar{\phi} = 0$, then $Y = B$. And for $\phi = 0$, $\bar{\phi} = 1$, then $Y = A$.
A multiplexer.

16.54

A	B	Y	
0	0	1	
0	1	0	
1	0	0	
1	1	0, 1	$\Rightarrow$indeterminate

Without the top transistor, the circuit performs the exclusive-NOR function.

16.55

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

Exclusive-OR function.

16.56

This circuit is referred to as a ratioless circuit. Identical minimum-sized transistors can be used throughout.

When ϕ_1 is low, C_3 is charged to V_{DD}. Then when ϕ_1 is high and ϕ_2 is low, M_6 turns on. If $A = B = 0$, then M_3 and M_4 are off so C_3 remains charged and v_{O1} = high. When ϕ_2 goes high, then v_{O1} is applied to the gates of M_9 and M_{10}. The circuit performs the OR logic function.

16.57

This circuit is referred to as a two-phase ratioed circuit. The same width-to-length ratios between the driver and load transistors must be maintained as discussed previously with the enhancement load inverter.

When ϕ_1 is high, v_{O1} becomes the complement of v_I. When ϕ_2 goes high, then v_O becomes the complement of v_{O1} or is the same as v_I. The circuit is a shift register.

16.58

Let $Q = 0$ and $\overline{Q} = 1$; as S increases, $\overline{Q}$ decreases. When $\overline{Q}$ reaches the transition point of the $M_5 - M_6$ inverter, the flip-flop with change state. From Equation (16.28(b)),

$$V_{It} = \sqrt{\frac{K_L}{K_D}} \cdot (-V_{TNL}) + V_{TND}$$

where $K_L = K_6$ and $K_D = K_5$.

Then

$$V_{It} = \sqrt{\frac{30}{100}} \cdot [-(-2)] + 1 \Rightarrow V_{It} = \overline{Q} = 2.095\,V$$

This is the region where both M_1 and M_3 are biased in the saturation region. Then

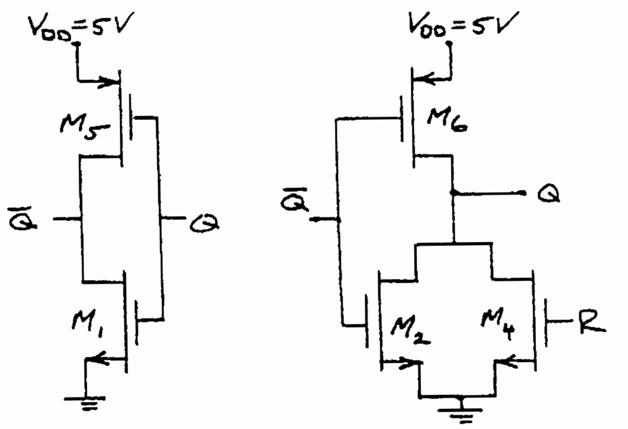
$$S = \sqrt{\frac{K_3}{K_1}} \cdot (-V_{TNL}) + V_{TND} = \sqrt{\frac{30}{200}} \cdot [-(-2)] + 1$$

or $\underline{S = 1.77\,V}$

This analysis neglects the effect of M_2 starting to turn on at the same time.

16.59

Let $v_Y = R$, $v_X = S$, $v_{O2} = Q$, and $v_{O1} = \overline{Q}$. Assume $V_{ThN} = 0.5$ V and $V_{ThP} = -0.5$ V. For $S = 0$, we have the following:



If we want the switching to occur for $R = 2.5$ V, then because of the nonsymmetry between the two circuits, we cannot have Q and $\overline{Q}$ both equal to 2.5 V.

Set $R = Q = 2.5$ V and assume $\overline{Q}$ goes low.

For the $M_1 - M_5$ inverter, M_1 in nonsaturation and M_5 in saturation. Then

$$K_n\left[2(2.5 - 0.5)\overline{Q} - \overline{Q}^2\right] = K_p[2.5 - 0.5]^2$$

Or

$$4\overline{Q} - \overline{Q}^2 = 4\left(\frac{K_p}{K_n}\right)$$

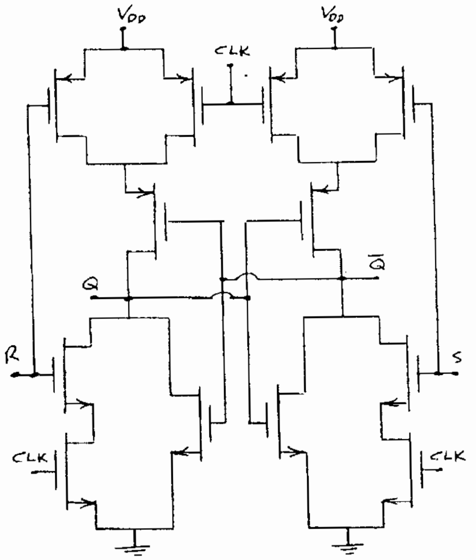
For the other circuit, $M_2 - M_4$ in saturation and M_6 in nonsaturation. Then

$$K_n(2.5 - 0.5)^2 + K_n(\overline{Q} - 0.5)^2$$
$$= K_p\left[2(5 - \overline{Q} - 0.5)(2.5) - (2.5^2)\right]$$

Combining these equations and neglecting the $\overline{Q}^3$ term, we find

$\underline{\overline{Q} = 1.4\,V}$ and $\underline{\frac{K_p}{K_n} = 0.9}$

16.60



16.61

a. Positive edge triggered flip-flop when CLK = 1, output of first inverter is $\overline{D}$ and then $Q = \overline{\overline{D}} = D$.

b. For example, put a CMOS transmission gate between the output and the gate of M_1 driven by a $\overline{\text{CLK}}$ pulse.

16.62

For $J = 1$, $K = 0$, and CLK = 1; this makes $Q = 1$ and $\overline{Q} = 0$.

For $J = 0$, $K = 1$, and CLK = 1, and if $Q = 1$, then the circuit is driven so that $Q = 0$ and $\overline{Q} = 1$.

If initially, $Q = 0$, then the circuit is driven so that there is no change and $Q = 0$ and $\overline{Q} = 1$.

$J = 1$, $K = 1$, and CLK = 1, and if $Q = 1$, then the circuit is driven so that $Q = 0$.

If initially, $Q = 0$, then the circuit is driven so that $Q = 1$.

So if $J = K = 1$, the output changes state.

16.63

For $J = v_X = 1$, $K = v_Y = 0$, and CLK $= v_Z = 1$, then $v_0 = 0$.

For $J = v_X = 0$, $K = v_Y = 1$, and CLK $= v_Z = 1$, then $v_0 = 1$.

Now consider $J = K =$ CLK $= 1$. With $v_X = v_Z = 1$, the output is always $v_0 = 0$. So the output does not change state when $J = K =$ CLK $= 1$. This is not actually a $J - K$ flip-flop.

16.64

$64\,K \Rightarrow 65{,}536$ transistors arranged in a 256x256 array.

(a) Each column and row decoder required 8 inputs.

(b)

(i) Address = 01011110 so input $= a_7\bar{a}_6a_5\bar{a}_4\bar{a}_3\bar{a}_2\bar{a}_1a_0$

(ii) Address = 11101111 so input $= \bar{a}_7\bar{a}_6\bar{a}_5a_4\bar{a}_3\bar{a}_2\bar{a}_1\bar{a}_0$

(c)

(i) Address = 00100111 so input $= a_7a_6\bar{a}_5a_4a_3\bar{a}_2\bar{a}_1\bar{a}_0$

(ii) Address = 01111011 so input $= a_7\bar{a}_6\bar{a}_5\bar{a}_4\bar{a}_3a_2\bar{a}_1\bar{a}_0$

16.65

Put 128 words in a 8x16 array, which means 8 row (or column) address lines and 16 column (or row) address lines.

16.66

Assume the address line is initially uncharged, then

$$I = C\frac{dV_C}{dt} \text{ or } V_C = \frac{1}{C}\int I dt = \frac{I}{C}\cdot t$$

$$\text{Then } t = \frac{V_C \cdot C}{I} = \frac{(2.7)(5.8\text{x}10^{-12})}{250\text{x}10^{-6}} \Rightarrow$$

$$\underline{t = 6.26\text{x}10^{-8}\ s \Rightarrow 62.6\ ns}$$

16.67

(a) $$\frac{5-0.1}{1} = \left(\frac{35}{2}\right)\left(\frac{W}{L}\right)\left[2(5-0.7)(0.1)-(0.1)^2\right]$$

or $\underline{\left(\frac{W}{L}\right) = 0.329}$

(b) $16\,K \Rightarrow 16{,}384$ cells

$$i_D \cong \frac{2}{1} = 2\ \mu A$$

Power per cell $= (2\ \mu A)(2\ V) = 4\ \mu W$

Total Power $= P_T = (4\ \mu W)(16{,}384) \Rightarrow$

$\underline{P_T = 65.5\ mW}$

Standby current $= (2\ \mu A)(16{,}384) \Rightarrow \underline{I_T = 32.8\ mA}$

16.68

$16\,K \Rightarrow 16{,}384$ cells

$P_T = 200\ mW \Rightarrow$ Power per cell

$$= \frac{200}{16{,}384} \Rightarrow 12.2\ \mu W$$

$$i_D = \frac{P}{V_{DD}} = \frac{12.2}{2.5} = 4.88\ \mu A \cong \frac{V_{DD}}{R} = \frac{2.5}{R} \Rightarrow$$

$\underline{R = 0.512\ M\Omega}$

If we want $v_O = 0.1\,V$ for a logic 0, then

$$i_D = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)\left[2(V_{DD}-V_{TN})v_O - v_O^2\right]$$

$$4.88 = \left(\frac{35}{2}\right)\left(\frac{W}{L}\right)\left[2(2.5-0.7)(0.1)-(0.1)^2\right]$$

So $\underline{\left(\frac{W}{L}\right) = 0.797}$

16.69

$Q = 0, \overline{Q} = 1$

So $\underline{\overline{D} = Logic\,1 = 5\,V}$

A very short time after the row has been addressed, D remains charged at $V_{DD} = 5\,V$. Then M_{P3}, M_A, and M_{N1} begin to conduct and D decreases. In steady-state, all three transistors are biased in the nonsaturation region. Then

$$K_{p3}\left[2(V_{SG3}+V_{TP3})V_{SD3} - V_{SD3}^2\right]$$
$$= K_{nA}\left[2(V_{GSA}-V_{TNA})V_{DSA} - V_{DSA}^2\right]$$
$$= K_{n1}\left[2(V_{GS1}-V_{TN1})V_{DS1} - V_{DS1}^2\right]$$

Or

$$K_{p3}\left[2(V_{DD}+V_{TP3})(V_{DD}-D)-(V_{DD}-D)^2\right]$$
$$= K_{nA}\left[2(V_{DD}-Q-V_{TNA})(D-Q)-(D-Q)^2\right]$$
$$= K_{n1}\left[2(V_{DD}-V_{TN1})Q-Q^2\right] \quad (1)$$

Equating the first and third terms:

$$\left(\frac{20}{2}\right)(1)\left[2(5-0.8)(5-D)-(5-D)^2\right]$$

$$=\left(\frac{40}{2}\right)(2)\left[2(5-0.8)Q-Q^2\right] \qquad (2)$$

As a first approximation, neglect the $(5-D)^2$ and Q^2 terms. We find

$$Q=1.25-0.25D \qquad (3)$$

Then, equating the first and second terms of Equation (1):

$$\left(\frac{20}{2}\right)(1)\left[2(5-0.8)(5-D)-(5-D)^2\right]$$

$$=\left(\frac{40}{2}\right)(1)\left[2(5-Q-0.8)(D-Q)-(D-Q)^2\right]$$

Substituting Equation (3), we find as a first approximation: $D=2.14\,V$

Substituting this value of D into equation (2), we find

$$8.4(5-2.14)-(5-2.14)^2=4\left[8.4Q-Q^2\right]$$

We find $\underline{Q=0.50\,V}$

Using this value of Q, we can find a second approximation for D by equating the second and third terms of equation (1). We have

$$20\left[2(4.2-Q)(D-Q)-(D-Q)^2\right]$$

$$=40\left[2(4.2Q)-Q^2\right]$$

Using $Q=0.50\,V$, we find $\underline{D=1.79\,V}$

16.70

Initially M_{N1} and M_A turn on.

M_{N1}, Nonsat; M_A, sat.

$$K_{nA}\left[V_{DD}-Q-V_{TN}\right]^2=K_{n1}\left[2(V_{DD}-V_{TN1})Q-Q^2\right]$$

$$\left(\frac{40}{2}\right)(1)[5-Q-0.8]^2=\left(\frac{40}{2}\right)(2)\left[2(5-0.8)Q-Q^2\right]$$

which yields

$$\underline{Q=0.771\,V}$$

Initially M_{P2} and M_B turn on

Both biased in nonsaturation reagion

$$K_{p2}\left[2(V_{DD}+V_{TP3})(V_{DD}-\overline{Q})-(V_{DD}-\overline{Q})^2\right]$$

$$=K_{nB}\left[2(V_{DD}-V_{TNB})\overline{Q}-\overline{Q}^2\right]$$

$$\left(\frac{20}{2}\right)(4)\left[2(5-0.8)(5-\overline{Q})-(5-\overline{Q})^2\right]$$

$$=\left(\frac{40}{2}\right)(1)\left[2(5-0.8)\overline{Q}-\overline{Q}^2\right]$$

which yields $\underline{\overline{Q}=3.78\,V}$

Note: (W/L) ratios do not satisfy Equation (16.95)

16.71

For Logic 1, v_1:

$(5)(0.05)+(4)(1)=(1+0.05)v_1 \Rightarrow \underline{v_1=4.0476\,V}$

v_2:

$(5)(0.025)+(4)(1)=(1+1.025)v_2 \Rightarrow \underline{v_2=4.0244\,V}$

For Logic 0, v_1:

$(0)(0.05)+(4)(1)=(1+0.05)v_1 \Rightarrow \underline{v_1=3.8095\,V}$

v_2:

$(0)(0.025)+(4)(1)=(1+0.025)v_2 \Rightarrow \underline{v_2=3.9024\,V}$

Chapter 17

Exercise Solutions

E17.1

a. $i_E = \frac{-0.7-(-5)}{R_E} = 1 \text{ mA} \Rightarrow \underline{R_E = 4.3 \text{ k}\Omega}$

$i_{C1} = i_{C2} = 0.5 \text{ mA} = \frac{5-3.5}{R_{C1}}$

$\Rightarrow \underline{R_{C1} = R_{C2} = 3 \text{ k}\Omega}$

b. i. $v_1 = 1 \text{ V}$

$i_E = \frac{(1-0.7)-(-5)}{4.3} \Rightarrow \underline{i_E = 1.23 \text{ mA}}$

$i_{C1} = i_E = v_{01} = 5-(1.23)(3)$

$\Rightarrow \underline{v_{01} = 1.31 \text{ V}}$

$\underline{v_{02} = 5 \text{ V}}$

ii. $v_I = -1 \text{ V}$

$i_E = 1 \text{ mA} \Rightarrow v_{02} = 5-(1)(3)$

$\Rightarrow \underline{v_{02} = 2 \text{ V}}$

$\underline{v_{01} = 5 \text{ V}}$

E17.2

a. Q_R on

$i_E = \frac{1.5-0.7-(-3.5)}{R_E} = 2 \text{ mA}$

$\Rightarrow \underline{R_E = 2.15 \text{ k}\Omega}$

$i_{CR} \approx i_E = 2 \text{ mA} = \frac{3.5-2}{R_{C2}}$

$\Rightarrow \underline{R_{C2} = 0.75 \text{ k}\Omega}$

b. $v_X = v_Y = 2 \text{ V} \Rightarrow Q_1$ and Q_2 on

$i_E = \frac{2-0.7-(-3.5)}{R_E} = \frac{4.8}{2.15} \Rightarrow \underline{i_E = 2.23 \text{ mA}}$

$R_{C1} = \frac{3.5-2}{i_{CXY}} = \frac{1.5}{2.23} \Rightarrow \underline{R_{C1} = 0.673 \text{ k}\Omega}$

E17.3

logic $1 = -0.7$ V

Q_1 and Q_2 on when $v_X = v_Y = -0.7$ V

$i_E = \frac{-0.7-0.7-(-5.2)}{R_E} = 2.5$

$\Rightarrow \underline{R_E = 1.52 \text{ k}\Omega}$

$v_{NOR} = -1.5 \Rightarrow R_{C1} = \frac{0-(-1.5+0.7)}{2.5}$

$\Rightarrow \underline{R_{C1} = 320 \ \Omega}$

$V_R = \frac{-1.5-0.7}{2} \Rightarrow \underline{V_R = -1.1 \text{ V}}$

Q_R on $\Rightarrow i_E = \frac{-1.1-0.7-(-5.2)}{1.52} = 2.24 \text{ mA}$

$R_{C2} = \frac{0-(-1.5+0.7)}{2.24} \Rightarrow \underline{R_{C2} = 357 \ \Omega}$

$R_3 = R_4 = \frac{-0.7-(-5.2)}{2.5} \Rightarrow \underline{R_3 = R_4 = 1.8 \text{ k}\Omega}$

E17.4

$P(i_{CXY} + i_{CR} + i_3 + i_4)(5.2)$

a. $v_X = v_Y =$ logic 1

$\Rightarrow i_{CXY} = 3.22 \text{ mA}$

$i_{CR} = 0$

$i_3 = \frac{-0.7+5.2}{1.5} = 3 \text{ mA}$

$i_4 = \frac{-1.4+5.2}{1.5} = 2.53 \text{ mA}$

$P = (3.22+0+3+2.53)(5.2)$

$\Rightarrow \underline{P = 45.5 \text{ mW}}$

b. $v_X = v_Y =$ logic 0

$\Rightarrow i_{CXY} = 0$

$i_{CR} = 2.92 \text{ mA}$

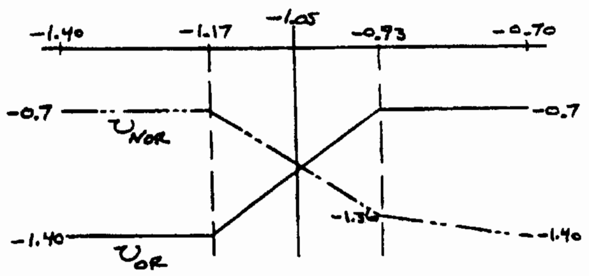
$i_3 = 2.53 \text{ mA}$

$i_4 = 3 \text{ mA}$

$P = (0+2.92+2.53+3)(5.2)$

$\Rightarrow \underline{P = 43.9 \text{ mW}}$

E17.5



$NM_H = -0.70-(-0.93) \Rightarrow \underline{NM_H = 0.23 \text{ V}}$

$NM_L = -1.17-(-1.40) \Rightarrow \underline{NM_L = 0.23 \text{ V}}$

E17.6

$P = I_Q \cdot V_{CC} \Rightarrow 0.2 = I_Q(1.7)$

$\Rightarrow \underline{I_Q = 118\ \mu A}$

Q_R on $\Rightarrow v_0 = 1.7 - I_Q R_C = 1.7 - 0.4$

$\Rightarrow R_C = \dfrac{0.4}{0.118} \Rightarrow \underline{R_C = 3.39\ k\Omega}$

$V_R = \dfrac{1.7 + 1.3}{2} \Rightarrow \underline{V_R = 1.5\ V}$

E17.7

State	A	B	C	Q_{01}	Q_{02}	Q_{03}	Q_1	Q_2	Q_R	v_0
1	0	0	0	off	off	off	off	on	on	0
2	1	0	0	"on"	off	off	off	on	on	0
3	0	1	0	off	on	off	off	on	"off"	1
4	0	0	1	off	off	on	on	off	on	0
5	1	1	0	on	on	off	off	on	off	1
6	1	0	1	on	off	on	on	off	"off"	1
7	0	1	1	off	on	on	on	off	on	0
8	1	1	1	on	on	on	on	off	off	1

$\underbrace{(A \text{ AND } C)}_{\text{true for states 6 and 8}}$ OR $\underbrace{(B \text{ AND } \overline{C})}_{\text{true for states 3 and 5}}$

Output goes high for these 4 states

E17.8

A	B	C	v_0
0	0	0	0
1	0	0	1
0	1	0	1
0	0	1	1
1	1	0	0
1	0	1	0
0	1	1	0
1	1	1	1

$\Rightarrow (A \oplus B) \oplus C$

E17.9

$i_C = \dfrac{5 - 0.1}{1} = 4.9$ mA

$i_B = \dfrac{4.9}{30} = 0.163\ \text{mA} = \dfrac{v_I - 0.7}{R_B} = \dfrac{4.3}{R_B}$

$\Rightarrow \underline{R_B = 26.4\ k\Omega}$

E17.10

(a) $v_X = v_Y = 5V$

$v_1 = V_{BE}(sat) + 2V_\gamma = 0.8 + 2(0.7) = 2.2\ V$

$i_1 = \dfrac{5 - 2.2}{4} = 0.70\ mA$

$i_{RC} = \dfrac{V_{CC} - V_{CE}(sat)}{R_C} = \dfrac{5 - 0.1}{4} = 1.23\ mA$

$P = (i_1 + i_{RC})V_{CC} = (0.70 + 1.23)(5)$

or $\underline{P = 9.65\ mW}$

(b) $v_X = v_Y = 0 \Rightarrow v_1 = 0.70V$

$i_1 = \dfrac{V_{CC} - v_1}{R_1} = \dfrac{5 - 0.70}{4} = 1.08\ mA$

$P = i_1 \cdot V_{CC} = (1.08)(5) \Rightarrow \underline{P = 5.4\ mW}$

E17.11

a. Inputs high $v_1 = 0.8 + 0.7 + 0.8 = 2.3$ V

$i_1 = i_{B1} = \dfrac{5 - 2.3}{4} = 0.675$ mA

$v_{C1} = 0.8 + 0.7 + 0.7 = 1.6$ V

$i_2 = \dfrac{5 - 1.6}{2} = 1.7$ mA

$i_3 = 1.7 + 0.675 = 2.38$ mA

$i_4 = \dfrac{0.8}{10} = 0.08$ mA

$i_{B0} = 2.38 - 0.08 = 2.3$ mA

$i_{RC} = \dfrac{5 - 0.1}{4} = 1.23$ mA

$i'_L = \dfrac{5 - 0.8}{4} = 1.05$ mA

$\beta i_{B0} = N i'_L + i_{RC}$

$(30)(2.3) = N(1.05) + 1.23 \Rightarrow \underline{N = 64}$

b. $I_{C,rated} = 20$ mA

$20 = N i'_L + i_{RC} = N(1.05) + 1.23$

$\Rightarrow \underline{N = 17}$

E17.12

a. $v_X = v_Y = 0.1 \Rightarrow v_1 = 0.8$

$i_1 = \frac{5-0.8}{6} \Rightarrow \underline{i_1 = 0.7 \text{ mA}}$

$\underline{i_2 = i_R = i_B = i_{RC} = 0}$

$\underline{v_0 = 5 \text{ V}}$

b. Same as part (a).

c. $v_X = v_Y = 5 \text{ V} \Rightarrow v_1 = 0.8 + 0.7 + 0.7 = 2.2 \text{ V}$

$i_1 = i_2 = \frac{5-2.2}{6} \Rightarrow \underline{i_1 = i_2 = 0.467 \text{ mA}}$

$i_R = \frac{0.8}{15} \Rightarrow \underline{i_R = 0.053 \text{ mA}}$

$i_B = i_2 - i_R \Rightarrow \underline{i_B = 0.414 \text{ mA}}$

$i_{RC} = \frac{5-0.1}{5} \Rightarrow \underline{i_{RC} = 0.98 \text{ mA}}$

$\underline{v_0 = 0.1 \text{ V}}$

E17.13

a. $v_X = v_Y = 0.1 \text{ V}$

$i_1 = \frac{5-0.8}{8} = 0.525 \text{ mA}$

$P = i_1(5-0.1) = (0.525)(4.9)$

$\Rightarrow \underline{P = 2.57 \text{ mW}}$

b. $v_X = v_Y = 5 \text{ V}, \ v_1 = 2.3 \text{ V}$

$i_1 = \frac{5-2.3}{8} = 0.338 \text{ mA}$

$v_{C1} = 1.6 \text{ V} \Rightarrow i_2 = \frac{5-1.6}{3.6} = 0.944 \text{ mA}$

$i_{RC} = \frac{5-0.1}{6} = 0.817 \text{ mA}$

$P = (i_1 + i_2 + i_{RC})(V_{CC})$

$= (0.338 + 0.944 + 0.817)(5)$

$\underline{P = 10.3 \text{ mW}}$

E17.14

a. $v_X = v_Y = 0.1 \text{ V}$

$v_{B1} = 0.1 + 0.8 = 0.9 \text{ V}$

$i_{B1} = i_1 = \frac{5-0.9}{6} \Rightarrow \underline{i_1 = i_{B1} = 0.683 \text{ mA}}$

$i_{C1} \approx 0$

$i_{B2} = i_{C2} = 0$

$i_{B0} = i_{C0} = 0$

b. $v_X = v_Y = 3.6 \text{ V}$

$v_{B1} = 0.8 + 0.8 + 0.7 = 2.3 \text{ V}$

$i_1 = i_{B1} = \frac{5-2.3}{6} \Rightarrow \underline{i_1 = i_{B1} = 0.45 \text{ mA}}$

$i_{BX} = i_{BY} = i_1 \cdot \beta_R = (0.45)(0.1) = 0.045 \text{ mA}$

$i_{B2} = |i_{C1}| = i_1 + i_{BX} + i_{BY} = 0.45 + 2(0.045)$

$\underline{i_{B2} = |i_{C1}| = 0.54 \text{ mA}}$

$v_{C2} = 0.8 + 0.1 = 0.9$

$i_2 = i_{C2} = \frac{5-0.9}{1.5} \Rightarrow \underline{i_2 = i_{C2} = 2.73 \text{ mA}}$

$i_{E2} = 2.73 + 0.54 = 3.27 \text{ mA}$

$i_4 = \frac{0.8}{1.5} = 0.533 \text{ mA}$

$i_{B0} = 3.27 - 0.533 \Rightarrow \underline{i_{B0} = 2.74 \text{ mA}}$

$i_3 = i_{C0} = \frac{5-0.1}{2.2} \Rightarrow \underline{i_3 = i_{C0} = 2.23 \text{ mA}}$

For Q_2:

$\frac{i_{C2}}{i_{B2}} = \frac{2.73}{0.54} = 5.06 < \beta_F \Rightarrow Q_2$ in saturation

For Q_0:

$\frac{i_{C0}}{i_{B0}} = \frac{2.23}{2.74} = 0.81 < \beta_F \Rightarrow Q_0$ in saturation

E17.15

a. Output low:

$i_L' = \frac{5-(0.1+0.8)}{6} = 0.683$

$i_{C0}(\text{max}) = \beta_F \cdot i_{B0} = N i_L' + i_3$

$(20)(2.74) = N(0.683) + 2.23$

$\Rightarrow \underline{N = 76}$

b. Output high:

Q_1' inverse active

$i_1' = \frac{5-2.3}{6} = 0.45 \text{ mA}$

$i_L' = \beta_R \cdot i_1' = (0.1)(0.45) = 0.045 \text{ mA}$

$v_0(\text{min}) = 2.4 \text{ V} \Rightarrow i_L(\text{max}) = \frac{5-2.4}{2.2} - N \cdot i_L'$

$1.18 = N(0.045)$

$\Rightarrow \underline{N = 26}$

E17.16

a. $v_X = v_Y = 0.1 \text{ V} \Rightarrow v_0(\text{max}) = 3.6 \text{ V}$

Q_1' inverse active

$i_1' = \frac{5-2.3}{6} = 0.45 \text{ mA}$

$i_L' = \beta_R \cdot i_1' = (0.1)(0.45) = 0.045$

$i_{B3} = \frac{0.1}{2} = 0.05 \text{ mA}$

$i_L = (1+\beta_F) i_{B3} = (21)(0.05) = 1.05 \text{ mA}$

$i_L = N i_L' \Rightarrow 1.05 = N(0.045) \Rightarrow \underline{N = 23}$

b. $v_X = v_Y = 3.6$ V, Output low:

$i_1 = \frac{5-2.3}{6} = 0.45$ mA

$i_{B2} = (1+2\beta_R)i_1$

$i_{B2} = 0.54$ mA

$v_{B3} = 0.8 + 0.1 = 0.9$

$i_2 = \frac{5-0.9}{2} = 2.05$ mA

$i_{B0} = 2.05 + 0.54 - \frac{0.8}{1.5} \Rightarrow i_{B0} = 2.06$ mA

$i'_L = \frac{5-0.9}{6} = 0.683$ mA

$\beta_F \cdot i_{B0} = N \cdot i'_L \Rightarrow (20)(2.06) = N(0.683)$

$\Rightarrow \underline{N = 60}$

E17.17

a. $v_X = v_Y = 3.6$ V

$i_{B1} = \frac{5-2.3}{4} = \underline{0.675 \text{ mA} = i_{B1} = |i_{C1}| = i_{B2}}$

$v_{B4} = 0.8 + 0.1 = 0.9$ V

$i_2 = \frac{5-0.9}{1.6} = 2.56$ mA

$i_{B4} = \frac{0.2}{(1+\beta_F)(4)} = \frac{0.2}{(31)(4)} \Rightarrow \underline{i_{B4} = 1.61\ \mu\text{A}}$

$\underline{i_{C4} = 48.3\ \mu\text{A}}$

$\underline{i_{B3} = i_{C3} = 0}$

$\Rightarrow \underline{i_{C2} \approx i_2 = 2.56 \text{ mA}}$

$i_{B0} = 2.45 + 0.675 - \frac{0.8}{1} \Rightarrow \underline{i_{B0} = 2.44 \text{ mA}}$

One load:

$i'_L = i_{C0} = \frac{5-0.9}{4} \Rightarrow \underline{i_{C0}(\text{max}) = 1.03 \text{ mA}}$

b. $v_X = v_Y = 0.1$ V

$i_{B1} = \frac{5-0.9}{4} \Rightarrow \underline{i_{B1} = 1.03 \text{ mA}}$

$\underline{|i_{C1}| = i_{B2} = i_{C2} = 0}$

$\underline{i_{B0} = i_{C0} = 0}$

Output high, $\beta_R = 0 \Rightarrow \underline{i_{B3} = i_{C3} = 0}$

$5 = i_{B4}(1.6) + 0.7 + (31)i_{B4}(4)$

$\Rightarrow \underline{i_{B4} = 0.0342 \text{ mA}}$

$\underline{i_{C4} = 1.03 \text{ mA}}$

E17.18

Q_1 in saturation

$i_{B1} = \frac{5-0.9}{6} \Rightarrow \underline{i_{B1} = 0.683 \text{ mA}}$

$\underline{|i_{C1}| = i_{B2} = i_{C2} = 0}$

$\underline{i_{B0} = i_{C0} = 0}$

$v_{B4} = 0.1 + 0.7 = 0.8$ V

$i_{B4} = \frac{0.1}{(21)(4)} \Rightarrow \underline{i_{B4} = 1.19\ \mu\text{A}}$

$\underline{i_{C4} = 23.8\ \mu\text{A}}$

$\underline{i_{B3} = i_{C3} = 0}$

E17.19

a. $i_{RC} = \frac{5-0.4}{2.25} = 2.04$ mA

$i'_C = \frac{2+2.04}{1+\frac{1}{10}} \Rightarrow \underline{i'_C = 3.67 \text{ mA}}$

$\underline{i'_B = \frac{3.67}{10} = 0.367 \text{ mA}}$

$i_D = 2 - 0.367 \Rightarrow \underline{i_D = 1.63 \text{ mA}}$

b. $i_D = 0 \Rightarrow i'_B = i_B = 2$ mA

$i'_C = \beta i'_B = (10)(2) = 20 \text{ mA} = i_{RC} + i_L$

$i_L = 20 - 2.04 \Rightarrow \underline{i_L(\text{max}) \approx 18 \text{ mA}}$

Chapter 17

Problem Solutions

17.1

a. $v_I = -1.5$ V; Q_1 off, Q_2 on

$i_E = \frac{-0.7-(-3.5)}{5} \Rightarrow \underline{i_E = 0.56 \text{ mA}}$

$i_{C1} = 0 \Rightarrow \underline{v_{01} = 3.5 \text{ V}}$

$i_{C2} = i_E \Rightarrow v_{02} = 3.5 - i_E R_{C2} = 3.5 - (0.56)(2)$

or

$\underline{v_{02} = 2.38 \text{ V}}$

b. $v_I = 1.0$ V; Q_1 on, Q_2 off

$i_E = \frac{(1-0.7)-(-3.5)}{5} \Rightarrow \underline{i_E = 0.76 \text{ mA}}$

$i_{C2} = 0 \Rightarrow \underline{v_{02} = 3.5 \text{ V}}$

c. logic 0 at v_{02} (low level) $= 2.38$ V

Then

$v_{01} = 2.38 = 3.5 - (0.76)R_{C1}$

or

$\underline{R_{C1} = 1.47 \text{ k}\Omega}$

17.2

(a) Q_2 on, $v_E = -1.2 - 0.7 = -1.9\,V$

$i_E = i_{C2} = \frac{-1.9-(-5.2)}{2.5} = 1.32\,mA$

$v_2 = -1V = -i_{C2}R_{C2} = -(1.32)(R_{C2})$

$\underline{R_{C2} = 0.758\,k\Omega}$

(b) Q_1 on, $v_E = -0.7 - 0.7 = -1.40\,V$

$i_E = i_{C1} = \frac{-1.4-(-5.2)}{2.5} = 1.52\,mA$

$v_1 = -1V = -i_{C1}R_{C1} = -(1.52)(R_{C1})$

$\underline{R_{C1} = 0.658\,k\Omega}$

(c) For $v_{in} = -0.7\,V$, Q_1 on, Q_2 off

$\underline{\Rightarrow v_{O1} = -0.70\,V}$

$v_{O2} = -1 - 0.7 \Rightarrow \underline{v_{O2} = -1.7\,V}$

For $v_{in} = -1.7\,V$, Q_1 off, Q_2 on

$\Rightarrow v_{O2} = -0.7\,V$

$v_{O1} = -1 - 0.7 \Rightarrow \underline{v_{O1} = -1.7\,V}$

(d) (i) For $v_{in} = -0.7\,V$, $i_E = 1.52\,mA$

$i_{C4} = \frac{-1.7-(-5.2)}{3} = 1.17\,mA$

$i_{C3} = \frac{-0.7-(-5.2)}{3} = 1.5\,mA$

$P = (i_E + i_{C4} + i_{C3})(5.2) = (1.52 + 1.17 + 1.5)(5.2)$

or $\underline{P = 21.8\,mW}$

(ii) For $v_{in} = -1.7\,V$, $i_E = 1.32\,mA$

$i_{C4} = \frac{-0.7-(-5.2)}{3} = 1.5\,mA$

$i_{C3} = \frac{-1.7-(-5.2)}{3} = 1.17\,mA$

$P = (1.32 + 1.5 + 1.17)(5.2)$

or $\underline{P = 20.7\,mW}$

17.3

a. $I_3 = \frac{3.7-0.7}{0.67+1.33} = 1.5$ mA

$V_R = I_3 R_4 + V_\gamma = (1.5)(1.33) + 0.7$

or

$\underline{V_R = 2.70 \text{ V}}$

b. logic 1 level $= 3.7 - 0.7 \Rightarrow \underline{3.0 \text{ V}}$

For $v_X = v_Y =$ logic 1,

$i_E = \frac{3-0.7}{0.8} = 2.875 \text{ mA} = i_{RC1}$

$v_{B3} = 3.7 - (2.875)(0.21) = 3.10$ V

$\Rightarrow \underline{v_{01}(\text{logic } 0) = 2.4 \text{ V}}$

For $v_X = v_Y =$ logic 0, Q_R on

$i_E = \frac{2.7-0.7}{0.8} = 2.5 \text{ mA} = i_{RC2}$

$v_{B4} = 3.7 - (2.5)(0.24) = 3.1$ V

$\Rightarrow \underline{v_{02}(\text{logic } 0) = 2.4 \text{ V}}$

17.4

$V_R = \frac{\text{logic } 1 + \text{logic } 0}{2} = \frac{1+0}{2} = 0.5$ V

For $i_2 = 1$ mA

$R_5 = \frac{0.5-(-2.3)}{1} \Rightarrow \underline{R_5 = 2.8 \text{ k}\Omega}$

For Q_R on,

$i_E = \frac{V_R - V_{BE} - (-2.3)}{R_E}$

or

$R_E = \frac{0.5 - 0.7 + 2.3}{1} \Rightarrow \underline{R_E = 2.1 \text{ k}\Omega}$

$V_{B2} = V_R + V_{BE} = 0.5 + 0.7 = 1.2$ V

$$i_1 = \frac{1.2 - 1.4 - (-2.3)}{R_2}$$

or

$$R_2 = \frac{1.2 - 1.4 + 2.3}{1} \Rightarrow \underline{R_2 = 2.1\ \text{k}\Omega}$$

$$R_1 = \frac{1.7 - 1.2}{1} \Rightarrow \underline{R_1 = 0.5\ \text{k}\Omega}$$

$$i_3 = \frac{1 - (-2.3)}{R_3} = 3 \Rightarrow \underline{R_3 = 1.1\ \text{k}\Omega}$$

$$i_4 = \frac{0 - (-2.3)}{R_4} = 3 \Rightarrow \underline{R_4 = 0.767\ \text{k}\Omega}$$

For Q_R on,

$v_{OR} = \text{logic } 0 = 0 \text{ V} \Rightarrow v_{B3} = 0.7$ V

$i_E = i_{CR} = 1$ mA

So

$$R_{C2} = \frac{1.7 - 0.7}{1} \Rightarrow \underline{R_{C2} = 1\ \text{k}\Omega}$$

For $v_I = \text{logic } 1 = 1$ V,

$$i_E = \frac{1 - 0.7 - (-2.3)}{2.1} = 1.238 \text{ mA}$$

For $v_{NOR} = \text{logic } 0 = 0$ V,

$$v_{B4} = 0.7 \text{ V}$$

Then

$$R_{C1} = \frac{1.7 - 0.7}{1.238} \Rightarrow \underline{R_{C1} = 0.808\ \text{k}\Omega}$$

17.5

Maximum i_E for $v_I = \text{logic } 1 = 3.3$ V

Then $i_E = 5 \text{ mA} = \dfrac{3.3 - 0.7}{R_E} \Rightarrow \underline{R_E = 0.52\ \text{k}\Omega}$

For $v_{O2} = \text{logic } 1 = 3.3$ V

$$i_{E3} = \frac{3.3 - 0}{R_3} = 5 \text{ mA}$$

or

$$\underline{R_3 = 0.66\ \text{k}\Omega}$$

By symmetry,

$$\underline{R_2 = 0.66\ \text{k}\Omega}$$

For Q_1 on,

$$i_E = i_{RC1} = 5 \text{ mA} = \frac{4 - (2.7 + 0.7)}{R_{C1}}$$

So

$$\underline{R_{C1} = 0.12\ \text{k}\Omega}$$

For Q_R on,

$$i_E = \frac{3 - 0.7}{0.52} = 4.423 \text{ mA} = i_{RC2}$$

and $i_{RC2} = 4.423 = \dfrac{4 - (2.7 + 0.7)}{R_{C2}}$

$$\Rightarrow \underline{R_{C2} = 0.136\ \text{k}\Omega}$$

17.6

Neglecting base currents:

(a) $\underline{I_{E1} = 0, I_{E3} = 0}$

$I_{E5} = \dfrac{5 - 0.7}{2.5} \Rightarrow \underline{I_{E5} = 1.72\ mA}$

$\underline{Y = 0.7\ V}$

(b) $I_{E1} = \dfrac{5 - 0.7}{18} \Rightarrow \underline{I_{E1} = 0.239\ mA}$

$\underline{I_{E3} = 0}$

$I_{E5} = \dfrac{5 - 0.7}{2.5} \Rightarrow \underline{I_{E5} = 1.72\ mA}$

$\underline{Y = 0.7\ V}$

(c) $I_{E1} = I_{E3} = \dfrac{5 - 0.7}{18} \Rightarrow \underline{I_{E1} = I_{E3} = 0.239\ mA}$

$\underline{I_{E5} = 0, Y = 5\ V}$

(d) Same as (c).

17.7

(a) $V_R = -(1)(1) - 0.7 \Rightarrow V_R = -1.7\ V$

(b) Q_R off, then $v_{O1} = Logic\ 1 = -0.7\ V$

Q_R on, then $v_{O1} = -(1)(2) - 0.7 \Rightarrow$

$\underline{v_{O1} = Logic\ 0 = -2.7\ V}$

Q_A / Q_B off, then $\underline{v_{O2} = Logic\ 1 = -0.7\ V}$

Q_A / Q_B on, then $v_{O2} = -(1)(2) - 0.7 \Rightarrow$

$\underline{v_{O2} = Logic\ 0 = -2.7\ V}$

(c) $A = B = Logic\ 0 = -2.7\ V$, Q_R on,

$V_E = -1.7 - 0.7 \Rightarrow \underline{V_E = -2.4\ V}$

$A = B = Logic\ 1 = -0.7\ V$, Q_A / Q_B on,

$V_E = -0.7 - 0.7 \Rightarrow \underline{V_E = -1.4\ V}$

(d) $A = B = Logic\ 1 = -0.7\ V$, Q_A / Q_B on,

$$i_{C3} = \frac{-2.7 - (-5.2)}{1.5} = 1.67\ mA$$

$$i_{C2} = \frac{-0.7 - (-5.2)}{1.5} = 3\ mA$$

$P = (1.67 + 1 + 1 + 1 + 3)(5.2) \Rightarrow \underline{P = 39.9\ mW}$

$A = B = Logic\ 0 = -2.7\ V$

$i_{C3} = 3\ mA, i_{C2} = 1.67\ mA$

$\underline{P = 39.9\ mW}$

17.8

a. AND logic function

b. $\underline{\text{logic } 0 = 0 \text{ V}}$

Q_3 on, $i = \dfrac{5 - (1.6 + 0.7)}{1.2} = 2.25$ mA

$V_2 = (2.25)(0.8) \Rightarrow \underline{\text{logic } 1 = 1.8 \text{ V}}$

c. $i_{E1} = \frac{5 - 0.7}{2.6} \Rightarrow \underline{i_{E1} = 1.65 \text{ mA}}$

$i_{E2} = \frac{5 - (0.7 + 0.7)}{1.2} \Rightarrow \underline{i_{E2} = 3 \text{ mA}}$

$\underline{i_{C1} = 0}, \ \underline{i_{C2} = i_{E2} = 3 \text{ mA}}$

$\underline{V_2 = 0}$

d. $i_{E1} = \frac{5 - (1.8 + 0.7)}{2.6} \Rightarrow \underline{i_{E1} = 0.962 \text{ mA}}$

$i_{E2} = \frac{5 - (1.6 + 0.7)}{1.2} \Rightarrow \underline{i_{E2} = 2.25 \text{ mA}}$

$\underline{i_{C2} = 0}, \ \underline{i_{C1} = i_{E2} = 2.25 \text{ mA}}$

$\underline{V_2 = 1.8 \text{ V}}$

17.9

a. $V_R = \frac{3.5 + 3.1}{2} - 0.7 \Rightarrow \underline{V_R = 2.6 \text{ V}}$

b. For Q_1 on, $v_X = v_Y = \text{logic } 1 = 3.5$ V

$i_E = \frac{3.5 - (0.7 + 0.7)}{12} = 0.175 \text{ mA}$

Want $i_{RC1} = \frac{0.175}{2} = \frac{0.4}{R_{C1}} \Rightarrow \underline{R_{C1} = 4.57 \text{ k}\Omega}$

c. For Q_2 on, $i_E = \frac{2.6 - 0.7}{12} = 0.158$ mA

Want $i_{RC2} = \frac{0.158}{2} = \frac{0.4}{R_{C2}} \Rightarrow \underline{R_{C2} = 5.06 \text{ k}\Omega}$

d. For $v_Y = \text{logic } 1 = 3.5$ V

$i_{R1} = \frac{3.5 - 0.7}{8} = 0.35 \text{ mA}, \ i_E = 0.175 \text{ mA}$

$P = (i_{R1} + i_E)(V_{CC}) = (0.35 + 0.175)(3.5)$

$\Rightarrow \underline{P = 1.84 \text{ mW}}$

17.10

a. logic 1 = 0.2 V

logic 0 = −0.2 V

b. $i_E = \frac{(0 - 0.7) - (-3.1)}{R_E} = 0.8$

So

$\underline{R_E = 3 \text{ k}\Omega}$

c. Want $i_{R1} = \frac{0.8}{2} = \frac{0.4}{R_1} \Rightarrow \underline{R_1 = 1 \text{ k}\Omega}$

d. For $v_X = v_Y = \text{logic } 1 = 0.2$ V

$i_E = \frac{(0.2 - 0.7) - (-3.1)}{3} = 0.867 \text{ mA}$

$i_{R2} = \frac{0.4}{1} \Rightarrow \underline{i_{R2} = 0.4 \text{ mA}}$

$\underline{i_{D2} = 0.467 \text{ mA}}$

e. $i_E = 0.867$ mA

$i_3 = \frac{0.2 - (-3.1)}{3.3} = 1 \text{ mA}$

$i_4 = \frac{-0.2 - (-3.1)}{3.3} = 0.879 \text{ mA}$

$P = (0.867 + 1 + 0.879)(0.9 - (-3.1))$

or

$\underline{P = 11.0 \text{ mW}}$

17.11

a. $i_1 = \frac{(-0.9 - 0.7) - (-3)}{1} \Rightarrow \underline{i_1 = 1.4 \text{ mA}}$

$i_3 = \frac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow \underline{i_3 = 0.14 \text{ mA}}$

$i_4 = \frac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow \underline{i_4 = 0.14 \text{ mA}}$

$i_2 + i_D = i_1 + i_3 = 1.4 + 0.14 = 1.54 \text{ mA}$

$i_2 = \frac{0.4}{0.5} \Rightarrow \underline{i_2 = 0.8 \text{ mA}}$

$\underline{i_D \doteq 0.74 \text{ mA}}$

$\underline{v_0 = -0.4 \text{ V}}$

b. $\underline{i_1 = 1.4 \text{ mA}}$

$i_3 = \frac{(0 - 0.7) - (-3)}{15} \Rightarrow \underline{i_3 = 0.153 \text{ mA}}$

$i_4 = i_3 \Rightarrow \underline{i_4 = 0.153 \text{ mA}}$

$i_2 + i_D = i_4 \Rightarrow \underline{i_2 = 0.153 \text{ mA}}$

$\underline{i_D = 0}$

$v_0 = -(0.153)(0.5) \Rightarrow \underline{v_0 = -0.0765 \text{ V}}$

c. $i_1 = \frac{(0 - 0.7 - 0.7) - (-3)}{1} \Rightarrow \underline{i_1 = 1.6 \text{ mA}}$

$i_3 = \frac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow \underline{i_3 = 0.14 \text{ mA}}$

$i_4 = i_3 \Rightarrow \underline{i_4 = 0.14 \text{ mA}}$

$i_2 + i_D = i_3 \Rightarrow \underline{i_2 = 0.14 \text{ mA}}$

$\underline{i_D = 0.0}$

$v_0 = -(0.14)(0.5) \Rightarrow \underline{v_0 = -0.07 \text{ V}}$

d. $i_1 = \frac{(0 - 0.7 - 0.7) - (-3)}{1} \Rightarrow \underline{i_1 = 1.6 \text{ mA}}$

$i_3 = \frac{(0 - 0.7) - (-3)}{15} \Rightarrow \underline{i_3 = 0.153 \text{ mA}}$

$i_4 = i_3 \Rightarrow \underline{i_4 = 0.153 \text{ mA}}$

$i_2 + i_D = i_1 + i_4 = 1.6 + 0.153 = 1.753 \text{ mA}$

$i_2 = \frac{0.4}{0.5} \Rightarrow \underline{i_2 = 0.8 \text{ mA}}$

$\underline{i_D = 0.953 \text{ mA}}$

$\underline{v_0 = -0.40 \text{ V}}$

17.12

a. i. $A = B = C = D = 0 \Rightarrow Q_1 - Q_4$ cutoff

So

$$V_{DD} = 2I_E R_1 + V_{EB} + I_B R_2$$

and

$$I_B = \frac{I_E}{1+\beta_P} = \frac{I_E}{51}$$

Then

$$2.5 = 2I_E(2) + 0.7 + \frac{I_E}{51}\cdot(15)$$

$$2.5 - 0.7 = I_E \cdot \left(4 + \frac{15}{51}\right)$$

So

$$I_E = 0.419 \text{ mA}$$

and

$Y = 2.5 - 2(0.419)(2) \Rightarrow \underline{Y = 0.824 \text{ V}}$

ii. $A = C = 0,\ B = D = 2.5$ V

Now

$$v_{B5} = v_{B6} = 2.5 - 0.7 = 1.8 \text{ V}$$

and

$$Y = v_{B5} + 0.7 \Rightarrow \underline{Y = 2.5 \text{ V}}$$

b. $Y = (A \text{ OR } B) \text{ AND } (C \text{ OR } D)$

17.13

a. logic 1 = 0 V

logic 0 = −0.4 V

b. $v_{01} = \overline{A \text{ OR } B}$

$v_{02} = \overline{C \text{ OR } D}$

$v_{03} = \overline{v_{01} \text{ OR } v_{02}}$

or

$v_{03} = (A \text{ OR } B) \text{ AND } (C \text{ OR } D)$

17.14

a. For CLOCK = high, I_{DC} flows through the left side of the circuit.. If D is high, I_{DC} flows through the left R resistor pulling $\overline{Q}$ low. If D is low, I_{DC} flows through the right R resistor pulling Q low.

For CLOCK = low, I_{DC} flows through the right side of the circuit maintaining Q and $\overline{Q}$ in their previous state.

b. $P = (I_{DC} + 0.5I_{DC} + 0.1I_{DC} + 0.1I_{DC})(3)$

$P = 1.7I_{DC}(3) = (1.7)(50)(3) \Rightarrow \underline{P = 255\ \mu\text{W}}$

17.15

i. For $v_X = v_Y = 0.1$ V $\Rightarrow \underline{v' = 0.8 \text{ V}}$

$$i_1 = \frac{5-2.2}{8} \Rightarrow \underline{i_1 = 0.525 \text{ mA}}$$

$$\underline{i_3 = i_4 = 0}$$

ii. For $v_X = v_Y = 5$ V,

$$v' = 0.8 + 0.7 + 0.7 \Rightarrow\Rightarrow \underline{v' = 2.2 \text{ V}}$$

$$i_1 = \frac{5-2.2}{8} \Rightarrow \underline{i_1 = 0.35 \text{ mA}}$$

$$i_4 = i_1 - \frac{0.8}{15} \Rightarrow \underline{i_4 = 0.297 \text{ mA}}$$

$$i_3 = \frac{5-0.1}{2.4} \Rightarrow \underline{i_3 = 2.04 \text{ mA}}$$

17.16

a. For $v_X = v_Y = 5$ V, both Q_1 and Q_2 driven into saturation.

$$v_1 = 0.8 + 0.7 + 0.8 \Rightarrow \underline{v_1 = 2.3 \text{ V}}$$

$$i_1 = \frac{5-2.3}{4} \Rightarrow \underline{i_1 = i_{B1} = 0.675 \text{ mA}}$$

$$i_2 = \frac{5-(0.8+0.7+0.1)}{2} \Rightarrow \underline{i_2 = 1.7 \text{ mA}}$$

$$i_4 = i_{B1} + i_2 \Rightarrow \underline{i_4 = 2.375 \text{ mA}}$$

$$i_5 = \frac{0.8}{10} \Rightarrow \underline{i_5 = 0.08 \text{ mA}}$$

$$i_{B2} = i_4 - i_5 \Rightarrow \underline{i_{B2} = 2.295 \text{ mA}}$$

$$i_3 = \frac{5-0.1}{4} \Rightarrow \underline{i_3 = 1.225 \text{ mA}}$$

$$\underline{v_0 = 0.1 \text{ V}}$$

b. $i'_L = \dfrac{5-(0.1+0.7)}{4} = 1.05$ mA

$$i_C(\max) = \beta i_{B2} = N i'_L + i_3$$

$(20)(2.295) = N(1.05) + 1.225$

So

$$\underline{N = 42}$$

17.17

D_X and D_Y off, Q_1 forward active mode

$v_1 = 0.8 + 0.7 + 0.7 = 2.2\,V$

$5 = i_1R_1 + i_2R_2 + v_1$ and $i_1 = (1+\beta)i_2$

So $5 - 2.2 = i_2[(1+\beta)R_1 + R_2]$

Assume $\beta = 25$

$$i_2 = \frac{5-2.2}{(26)(1.75)+2} \Rightarrow \underline{i_2 = 0.0589\,mA}$$

$$i_1 = (1+\beta)i_2 = (26)(0.0589) \Rightarrow \underline{i_1 = 1.53\,mA}$$

$$i_3 = \beta i_2 \Rightarrow \underline{i_3 = 1.47\,mA}$$

$$i_{Bo} = i_2 + i_3 - \frac{0.8}{5} = 0.0589 + 1.47 - 0.16 \Rightarrow$$

$$\underline{i_{Bo} = 1.37\,mA}$$

Q_o in saturation

$$i_{Co} = \frac{5-0.1}{6} \Rightarrow \underline{i_{Co} = 0.817\,mA}$$

17.18

a. i. $v_X = v_Y = 0.1$ V, so Q_1 in saturation.

$$i_1 = \frac{5-(0.1+0.8)}{6} \Rightarrow \underline{i_1 = 0.683 \text{ mA}}$$
$$\Rightarrow \underline{i_{B2} = i_2 = i_4 = i_{B3} = i_3 = 0}$$

ii. $v_X = v_Y = 5$ V, so Q_1 in inverse active mode. Assume Q_2 and Q_3 in saturation.

$$i_1 = \frac{5-(0.8+0.8+0.7)}{6} \Rightarrow \underline{i_1 = i_{B2} = 0.45 \text{ mA}}$$
$$i_2 = \frac{5-(0.8+0.1)}{2} \Rightarrow \underline{i_2 = 2.05 \text{ mA}}$$
$$i_4 = \frac{0.8}{1.5} \Rightarrow \underline{i_4 = 0.533 \text{ mA}}$$

$$i_{B3} = (i_{B2}+i_2) - i_4 = 0.45 + 2.05 - 0.533$$
or
$$\underline{i_{B3} = 1.97 \text{ mA}}$$
$$i_3 = \frac{5-0.1}{2.2} \Rightarrow \underline{i_3 = 2.23 \text{ mA}}$$

b. For Q_3:

$$\frac{i_3}{i_{B3}} = \frac{2.23}{1.97} = 1.13 < \beta$$

For Q_2:

$$\frac{i_2}{i_{B2}} = \frac{2.05}{0.45} = 4.56 < \beta$$

Since $(I_C/I_B) < \beta$, then each transistor is in saturation.

17.19

(a) $v_X = v_Y = Logic\ 1$

$v' = 0.8 + 2(0.7) = 2.2\,V$

$i_1 = \dfrac{5-2.2}{8} = 0.35\,mA$

$i_4 = i_1 - \dfrac{0.8}{15} = 0.35 - 0.0533 = 0.2967\,mA$

$i_3 = \dfrac{5-0.1}{2.4} = 2.04\,mA$

$i_L' = \dfrac{5-(0.1+0.7)}{8} = 0.525\,mA$

Assume $\beta = 25$

Then $(25)(0.2967) = 2.04 + N(0.525)$

So $N = 10.2 \Rightarrow \underline{N = 10}$

(b) Now

$5 = 2.04 + N(0.525)$

So $N = 5.64 \Rightarrow \underline{N = 5}$

17.20

a. For $v_X = v_Y = 5$ V, Q_1 in inverse active mode.

$$i_{B1} = \frac{5-(0.8+0.8+0.7)}{6} = 0.45 \text{ mA}$$
$$i_{B2} = i_{B1} + 2\beta_R i_{B1} = 0.45(1+2[0.1]) = 0.54 \text{ mA}$$

$$i_{C2} = \frac{5-(0.8+0.1)}{2} = 2.05 \text{ mA}$$

$$i_{B3} = (i_{B2}+i_{C2}) - \frac{0.8}{1.5} = 0.54 + 2.05 - 0.533$$
or
$$\underline{i_{B3} = 2.06 \text{ mA}}$$

Now
$$i_L' = \frac{5-(0.1+0.8)}{6} = 0.683 \text{ mA}$$

Then
$$i_{C3}(\max) = \beta_F i_{B3} = N i_L'$$
or $(20)(2.06) = N(0.683)$
$$\Rightarrow \underline{N = 60}$$

b. From above, for v_O high, $I_L' = (0.1)(0.45) = 0.045$ mA. Now

$$I_L'(\max) = (1+\beta_F)\left(\frac{5-4.9}{R_2}\right) = \frac{(21)(0.1)}{2} = 1.05 \text{ mA}$$

So
$$I_L(\max) = N I_L'$$
or $1.05 = N(0.045)$
$$\Rightarrow \underline{N = 23}$$

17.21

(a) $V_{in} = 0.1\,V$: Q_I, Sat: Q_S, Q_O, Cutoff

$i_I = \dfrac{5-(0.1+0.8)}{4} = 1.025\,mA$

$P = i_I(5-0.1) = (1.025)(4.9) \Rightarrow$

$\underline{P = 5.02\,mW}$

(b) $V_{in} = 5\,V$, Q_I, Inverse Active; Q_S, Q_O, Saturation

$v_{BI} = 0.7 + 0.8 + 0.7 = 2.2\,V$

$i_I = \dfrac{5-2.2}{4} = 0.7\,mA$

$i_{BI} = \beta_R \cdot i_I = (0.1)(0.7) = 0.07\,mA$

$V_{out} = 0.7 + 0.1 = 0.8\,V$

$i_2 = \dfrac{5-0.8}{1} = 4.2\,mA$

$P = (i_I + i_{BI} + i_2)(5) = (0.7 + 0.07 + 4.2)(5) \Rightarrow$

$\underline{P = 24.9\,mW}$

17.22

a. $v_X = v_Y = v_Z = 0.1$ V

$$i_{B1} = \frac{5-(0.1+0.8)}{3.9} \Rightarrow \underline{i_{B1} = 1.05 \text{ mA}}$$

Then

$$\underline{i_{C1} = i_{B2} = i_{C2} = i_{B3} = i_{C3} = 0}$$

b. $v_X = v_Y = v_Z = 5$ V

$$i_{B1} = \frac{5-(0.8+0.8+0.7)}{3.9} \Rightarrow \underline{i_{B1} = 0.692 \text{ mA}}$$

Then

$$i_{C1} = i_{B2} = i_{B1}(1+3\beta_R) = (0.692)(1+3[0.5])$$

$$\Rightarrow \underline{i_{C1} = i_{B2} = 1.73 \text{ mA}}$$

$$i_{C2} = \frac{5-(0.1+0.8)}{2} \Rightarrow \underline{i_{C2} = 2.05 \text{ mA}}$$

$$i_{B3} = i_{B2} + i_{C2} - \frac{0.8}{0.8} = 1.73 + 2.05 - 1.0$$

$$\Rightarrow \underline{i_{B3} = 2.78 \text{ mA}}$$

$$i_{R3} = \frac{5-0.1}{2.4} = 2.04 \text{ mA}$$

$$i'_L = \frac{5-(0.1+0.8)}{3.9} = 1.05 \text{ mA}$$

$$i_{C3} = i_{R3} + 5i'_L = 2.04 + (5)(1.05)$$

$$\Rightarrow \underline{i_{C3} = 7.29 \text{ mA}}$$

17.23

a. $v_X = v_Y = v_Z = 2.8$ V, Q_1 biased in the inverse active mode.

$$i_{B1} = \frac{2.8-(0.8+0.8+0.7)}{2} \Rightarrow \underline{i_{B1} = 0.25 \text{ mA}}$$

$$i_{B2} = i_{B1}(1+3\beta_R) = 0.25(1+3[0.3])$$

$$\Rightarrow \underline{i_{B2} = 0.475 \text{ mA}}$$

$$v_{C2} = 0.8 + 0.1 = 0.9 \text{ V}$$

$$i_{B4} = \frac{0.9-(0.7+0.1)}{(1+\beta_F)(0.5)} = \frac{0.1}{(101)(0.5)}$$

$$= 0.00198 \text{ mA} \quad \text{(Negligible)}$$

$$i_{R2} = \frac{5-0.9}{0.9} = 4.56 \text{ mA}$$

$$\Rightarrow \underline{i_{C2} = 4.56 \text{ mA}}$$

$$i_{B3} = i_{B2} + i_{C2} - \frac{0.8}{1} = 0.475 + 4.56 - 0.8$$

$$\Rightarrow \underline{i_{B3} = 4.235 \text{ mA}}$$

b. $v_X = v_Y = v_Z = 0.1$ V

$$i_{B1} = \frac{5-(0.1+0.8)}{2} \Rightarrow \underline{i_{B1} = 2.05 \text{ mA}}$$

From part (a),

$$i'_L = \beta_R \cdot i_{B1} = (0.3)(0.25) = 0.075 \text{ mA}$$

Then

$$i_{B4} = \frac{5i'_L}{1+\beta_F} = \frac{5(0.075)}{101} \Rightarrow \underline{i_{B4} = 0.00371 \text{ mA}}$$

17.24

a. $v_X = v_Y = v_Z = 0.1$ V

$$i_{B1} = \frac{5-(0.1+0.8)}{R_{B1}} + i_{B3}$$

where

$$i_{B3} = \frac{(2-0.7)-(0.9)}{R_{B2}} = \frac{0.4}{1}$$

$$\Rightarrow \underline{i_{B3} = 0.4 \text{ mA}}$$

Then

$$i_{B1} = \frac{1.1}{1} + 0.4 \Rightarrow \underline{i_{B1} = 1.5 \text{ mA}}$$

$$\underline{i_{B2} = 0 = i_{C2}}$$

Q_3 in saturation $i_{C3} = 5i'_L$
For v_0 high,

$$v'_{B1} = 0.8 + 0.7 = 1.5 \text{ V} \Rightarrow Q'_3 \text{ off}$$

$$i'_{B1} = \frac{2-1.5}{1} = 0.5 \text{ mA}$$

$$i'_L = \beta_R i'_{B1} = (0.2)(0.5) = 0.1 \text{ mA}$$

Then

$$\underline{i_{C3} = 0.5 \text{ mA}}$$

b. $v_X = v_Y = v_Z = 2$ V

From part (a),

$$\Rightarrow \underline{i_{B1} = 0.5 \text{ mA}}$$

$$\underline{i_{B3} = 0 = i_{C3}}$$

$$i_{B2} = i_{B1}(1+3\beta_R) = (0.5)(1+3[0.2])$$

$$\underline{i_{B2} = 0.8 \text{ mA}}$$

$i_{C2} = 5i'_L$, and from part (a), $i'_L = 1.5$ mA

So

$$\underline{i_{C2} = 7.5 \text{ mA}}$$

17.25

(a) $I_B + I_D = \frac{5.8-0.7}{10} = 0.51\,mA$

$I_C - I_D = \frac{5-(0.7-0.3)}{1} = 4.6\,mA$

Now

$I_D = 0.51 - I_B = 0.51 - \frac{I_C}{\beta} = 0.51 - \frac{I_C}{50}$

Then

$I_C - I_D = I_C - \left(0.51 - \frac{I_C}{50}\right) = I_C\left(1+\frac{1}{50}\right) - 0.51 = 4.6$

So $\underline{I_C = 5.01\,mA}$

$I_B = \frac{I_C}{\beta} = \frac{5.01}{50} \Rightarrow \underline{I_B = 0.1002\,mA}$

$I_D = 0.51 - 0.1002 \Rightarrow \underline{I_D = 0.4098\,mA}$

$\underline{V_{CE} = 0.4\,V}$

(b) $\underline{I_D = 0, V_{CE} = V_{CE}(sat) = 0.1\,V}$

$I_B = \frac{5.8-0.8}{10} \Rightarrow \underline{I_B = 0.5\,mA}$

$I_C = \frac{5-0.1}{1} \Rightarrow \underline{I_C = 4.9\,mA}$

17.26

a. $v_X = v_Y = 0.4$ V

$v_{B1} = 0.4 + 0.7 \Rightarrow \underline{v_{B1} = 1.1\text{ V}}$

$i_{B1} = \frac{5-1.1}{2.8} \Rightarrow \underline{i_{B1} = 1.39\text{ mA}}$

$v_{B2} = 0.4 + 0.4 \Rightarrow \underline{v_{B2} = 0.8\text{ V}}$

$i_{B2} = i_{C2} = i_{B0} = i_{C0} = i_{B5} = i_{C5}$

$= i_{B3} = i_{C3} = 0$ (No load)

$5 = i_{B4}R_2 + V_{BE} + (1+\beta)i_{B4}R_4$

$i_{B4} = \frac{5-0.7}{0.76 + (31)(3.5)}$

$\Rightarrow \underline{i_{B4} = 0.0394\text{ mA}}$

$i_{C4} = \beta_F i_{B4}$

$\Rightarrow \underline{i_{C4} = 1.18\text{ mA}}$

$v_{B4} = 5 - (0.0394)(0.76)$

$\Rightarrow \underline{v_{B4} = 4.97\text{ V}}$

b. $v_X = v_Y = 3.6$ V

$v_{B1} = 0.7 + 0.7 + 0.3 \Rightarrow \underline{v_{B1} = 1.7\text{ V}}$

$\underline{v_{B2} = 1.4\text{ V}}$

$\underline{v_{B0} = 0.7\text{ V}}$

$\underline{v_{C2} = 1.1\text{ V}}$

$i_{B1} = \frac{5-1.7}{2.8}$

$\Rightarrow \underline{i_{B1} = 1.18\text{ mA}}$

$i_{B2} = i_{B1}(1 + 2\beta_R) = 1.18(1 + 2[0.1])$

$\underline{i_{B2} = 1.42\text{ mA}}$

$i_{B4} = \frac{1.1-0.7}{(31)(3.5)}$

$\Rightarrow \underline{i_{B4} = 0.00369\text{ mA}}$

$i_{R2} = \frac{5-1.1}{0.76} = 5.13\text{ mA}$

$\Rightarrow \underline{i_{C2} \approx 5.13\text{ mA}}$

$i_{B0} \approx i_{B2} + i_{C2}$

$\underline{i_{B0} = 6.55\text{ mA}}$

17.27

a. Assuming the output transistor Q_2 is a Schottky transistor, then

$v_0 = 0.4$ V, $i'_L = \frac{2.5-(0.4+0.3)}{R_{B1}} = 0.5$

Then

$\underline{R_{B1} = 3.6\text{ k}\Omega}$

Then

$i_{B1} = \frac{2.5-(0.7+0.8)}{R_{B1}} = \frac{10}{3.6} = 0.278\text{ mA}$

$i_{B2} = 0.5\text{ mA},\ i_{E1} = 0.5 + \frac{0.7}{0.7} = 1.50\text{ mA}$

$i_{E1} = i_{B1} + i_{C1} \Rightarrow i_{C1} = 1.50 - 0.278 = 1.222\text{ mA}$

and $i_{C1} = \frac{2.5-(0.7+0.1)}{R_{C1}} = 1.222\text{ mA}$

$\Rightarrow \underline{R_{C1} = 1.39\text{ k}\Omega}$

b. $v_X = v_Y = 0.4$ V, $\underline{v_{B1} = 0.7\text{ V}}$

$v_{C2} = 2.5 - 0.7 \Rightarrow \underline{v_{C2} = 1.8\text{ V}}$

All transistor currents are zero.

c. $\underline{v_{B1} = 1.5\text{ V}},\ \underline{v_{C1} = 0.8\text{ V}}$

Currents calculated in part (a).

d. $i_{B2} = 0.5$ mA, $i'_L = 0.5$ mA

$i_{C2}(\max) = \beta i_{B2} = N i'_L$ or $(50)(0.5) = N(0.5)$

So

$\underline{N = 50}$

17.28

a. For $v_X = v_Y = 3.6$ V

$v_{B1} = 3(0.7) = 2.1$

$\Rightarrow i_{B1} = \frac{5-2.1}{10} = 0.29\text{ mA}$

$v_{C1} = 0.7 + 0.7 + 0.4 = 1.8$ V

$\Rightarrow i_{C1} = \frac{5-1.8}{10} = 0.32\text{ mA}$

$i_{B2} = i_{B1} + i_{C1} - \frac{1.4}{15} = 0.29 + 0.32 - 0.0933$

So

$i_{B2} = 0.517$ mA

$v_{C2} = 0.7 + 0.4 = 1.1$ V

$i_{C2} = \frac{5 - 1.1}{4.1} = 0.951$ mA

$i_{B5} = i_{B2} + i_{C2} - \frac{0.7}{4} = 0.517 + 0.951 - 0.175$

or $i_{B5} = 1.293$ mA

For $v_0 = 0.4$ V, $v'_{B1} = 0.4 + 0.7 = 1.1$ V

Then

$i'_{B1} = \frac{1.1 - 0.7}{(31)(15)} = 0.00086$ mA

$i'_L = \frac{5 - 1.1}{10} - 0.00086$ or $i'_L \approx 0.39$ mA

So $i_{C5}(\max) = \beta i_{B5} = N i'_L$

$(30)(1.293) = N(0.39)$

$\Rightarrow \underline{N = 99}$

b. $P = (0.29 + 0.32 + 0.951)(5) + (99)(0.39)(0.4)$

$P = 7.805 + 15.444$ or $\underline{P = 23.2 \text{ mW}}$

(Assumming 99 load circuits which is unreasonably large.)

17.29

a. Assume no load. For v_X = logic 0 = 0.4 V

$i_{E1} = \frac{5 - (0.4 + 0.7)}{40} = 0.0975$ mA

Essentially all of this current goes to ground from V_{CC}.

$P = i_{E1} \cdot V_{CC} = (0.0975)(5)$

$\Rightarrow \underline{P = 0.4875 \text{ mW}}$

b. $i_{R1} = \frac{5 - (3)(0.7)}{40} = 0.0725$ mA

$i_{R2} = \frac{5 - (0.7 + 0.7 + 0.4)}{50} = 0.064$ mA

$i_{R3} = \frac{5 - (0.7 + 0.4)}{15} = 0.26$ mA

$P = (0.0725 + 0.064 + 0.26)(5)$

$\underline{P = 1.98 \text{ mW}}$

c. For $v_0 = 0$, $v_{C7} = 0.7 + 0.4 = 1.1$ V

$i_{R7} = \frac{5 - 1.1}{0.050} \Rightarrow i_{R7} = \underline{78 \text{ mA} \approx i_{SC}}$

17.30

(a) $v_I = v_O = 2.5\,V$; A transient situation

$v_{DS}(M_N) = 2.5 - 0.7 = 1.8\,V$

$v_{GS}(M_N) = 2.5 - 0.7 = 1.8\,V \Rightarrow M_N$ in saturation

$v_{SD}(M_P) = 5 - (2.5 + 0.7) = 1.8\,V$

$v_{SG}(M_P)5 - 2.5 = 2.5\,V \Rightarrow M_P$ in saturation

$i_{DN} = K_n(v_{GSN} - V_{TN})^2 = (0.1)(1.8 - 0.8)^2 \Rightarrow$

$\underline{i_{DN} = 0.1\,mA}$

$i_{DP} = K_p(v_{SGP} + V_{TP})^2 = (0.1)(2.5 - 0.8)^2 \Rightarrow$

$\underline{i_{DP} = 0.289\,mA}$

$i_{C1} = \beta i_{DP} = (50)(0.289) \Rightarrow \underline{i_{C1} = 14.45\,mA}$

$i_{C2} = \beta i_{DN} = (50)(0.1) \Rightarrow \underline{i_{C2} = 5\,mA}$

Difference between i_{E1} and $i_{DN} + i_{C2}$ is a load current.

(b) Assume $i_{C1} = 14.45\,mA$ is a constant

$V_C = \frac{1}{C}\int i_{C1}dt = \frac{i_{C1} \cdot t}{C} \Rightarrow t = \frac{(V_C)(C)}{i_{C1}}$

$t = \frac{(5)(15x10^{-12})}{14.45x10^{-3}} \Rightarrow \underline{t = 5.19\,ns}$

(c) $t = \frac{(5)(15x10^{-12})}{0.289x10^{-3}} \Rightarrow \underline{t = 260\,ns}$

17.31

Let $R_1 = R_2 = 10\,k\Omega$

(a) $\underline{i_{DN} = 0.1\,mA, i_{DP} = 0.289\,mA}$

(Same as Problem 17.30)

$i_{R1} = \frac{0.7}{10} = 0.07\,mA \Rightarrow i_{B1} = 0.289 - 0.07 = 0.219\,mA$

$i_{C1} = (50)(0.219) \Rightarrow \underline{i_{C1} = 10.95\,mA}$

$i_{R2} = \frac{0.7}{10} = 0.07\,mA \Rightarrow i_{B2} = 0.1 - 0.07 = 0.03\,mA$

$i_{C2} = (50)(0.03) \Rightarrow \underline{i_{C2} = 1.5\,mA}$

(b) $t = \frac{(5)(15x10^{-12})}{10.95x10^{-3}} \Rightarrow \underline{t = 6.85\,ns}$

(c) $t = 260\,ns$ (Same as Problem 17.30)

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