National Technological University

IC 570-CA/EECS 140

Spring 2005

Midterm

Instructions

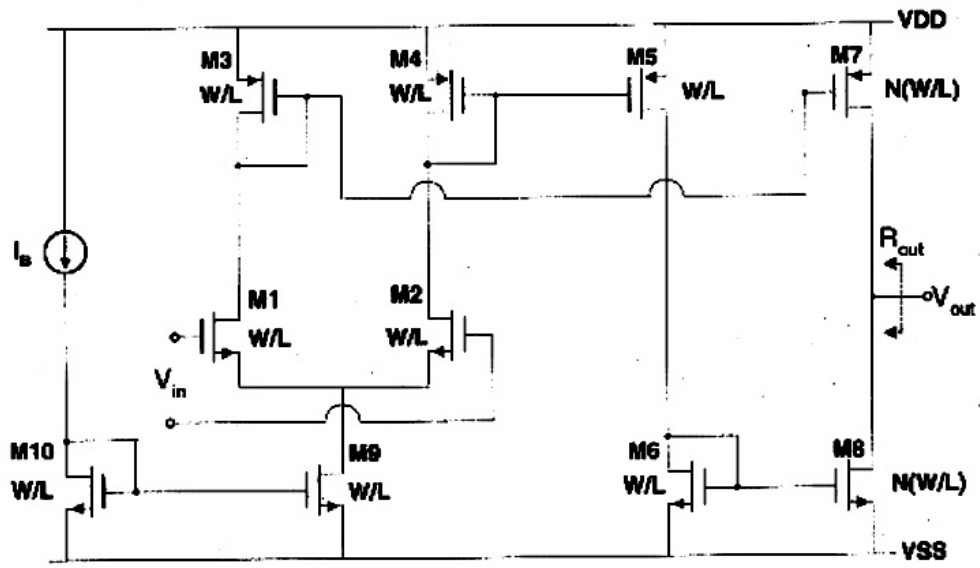
- **You may use one sheet (8.5" x 11") of you own notes. No other materials can be used.**
- **You can use a calculator.**
- **There are two problems. Answer all of them.**
- **The total marks for this midterm is 100.**
- **The time allocated for this midterm is one and a half hours.**
- **There are a total of 9 pages, including this page.**
- **READ THE QUESTIONS CAREFULLY.**

Name : Solutions

1	
2	
Total	

Question 1 : Current Mirror Amplifier (60 marks)

The figure below shows a current mirror amplifier. The bulks of all the transistors are connected to their respective sources. You may assume that transistor pairs are perfectly matched.



a. Suppose the input, V_{in}, is grounded (i.e. gates of M1 and M2 are connected to ground). Determine the DC bias values for the following variables listed on the next page. You may neglect channel length modulation for this. Leave your answers in symbolic form in terms of I_B, K_n, K_p, etc. (26 marks)

$I_{D10} = I_B$ (KCL)

$I_{D9} = I_{D10} = I_B$ (current mirror)

$I_{D1} = I_{D2} = \frac{I_B}{2}$ (Symmetrically matched diff pairs & KCL)

$I_{D3} = I_{D1} = \frac{I_B}{2}$ (KCL)

$I_{D4} = I_{D2} = \frac{I_B}{2}$ (KCL)

$I_{D5} = I_{D4} = \frac{I_B}{2}$ (current mirror)

$I_{D6} = I_{D5} = \frac{I_B}{2}$ (KCL)

$I_{D7} = N I_{D3} = N\frac{I_B}{2}$ (current mirror)

$I_{D8} = N I_{D6} = N\frac{I_B}{2}$ (current mirror)

I_{D1}	$I_B/2$
I_{D2}	$I_B/2$
I_{D6}	$I_B/2$
I_{D8}	$N\, I_B/2$
V_{DSAT1}	$\sqrt{\frac{I_B/2}{\left(\frac{k_n}{2}\right)\left(\frac{W}{L}\right)}}$
V_{DSAT2}	$\sqrt{\frac{I_B/2}{\left(\frac{k_n}{2}\right)\left(\frac{W}{L}\right)}}$
V_{DSAT3}	$-\sqrt{\frac{I_B/2}{\left(\frac{k_p}{2}\right)\left(\frac{W}{L}\right)}}$
V_{DSAT4}	$-\sqrt{\frac{I_B/2}{\left(\frac{k_p}{2}\right)\left(\frac{W}{L}\right)}}$
V_{DSAT5}	$-\sqrt{\frac{I_B/2}{\left(\frac{k_p}{2}\right)\left(\frac{W}{L}\right)}}$
V_{DSAT6}	$\sqrt{\frac{I_B/2}{\left(\frac{k_n}{2}\right)\left(\frac{W}{L}\right)}}$
V_{DSAT7}	$-\sqrt{\frac{I_B/2}{\left(\frac{k_p}{2}\right)\left(\frac{W}{L}\right)}}$
V_{DSAT8}	$\sqrt{\frac{I_B/2}{\left(\frac{k_n}{2}\right)\left(\frac{W}{L}\right)}}$
V_{DSAT9}	$\sqrt{\frac{I_B}{\left(\frac{k_n}{2}\right)\left(\frac{W}{L}\right)}}$

(b) Determine the output voltage range with all transistors remaining active.
(5 marks)

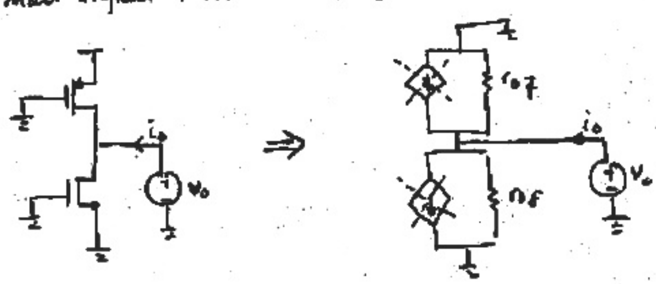
$$\underbrace{V_{SS} + V_{dsat8}}_{\text{For M8 to remain active.}} \leq V_o \leq \underbrace{V_{DD} - |V_{dsat7}|}_{\text{For M7 to remain active}}$$

ANS: $V_{SS} + V_{dsat8} \leq V_o \leq V_{DD} - |V_{dsat7}|$

(c) Determine the output resistance, R_{out}.
(5 marks)

$$R_{out} = \left.\frac{v_o}{i_o}\right|_{v_i=0} = r_{o7} \| r_{o8}$$

Small signal model becomes



ANS: $R_{out} = r_{o7} \| r_{o8}$

(d) Determine the differential mode G_m, i.e

$$G_m = \left.\frac{i_{out}}{v_{in,diff}}\right|_{v_{out}=0}$$

(12 marks)

- For differential mode input, $v_{i1} = \frac{v_d}{2}$, $v_{i2} = -\frac{v_d}{2}$, $v_{in} = v_d$
- Since M_1, M_2, M_3 & M_4 are fully symmetrical, source of M_1 & M_2 is an a-c ground.
- Also, for common source amplifier $g_m r_o \gg 1 \Rightarrow r_o \gg \frac{1}{g_m}$ (ie $r_o \| \frac{1}{g_m} \approx \frac{1}{g_m}$)

  and $i_d \approx g_m v_i$
- We have $g_{m1} = g_{m2} = g_{m1,2}$

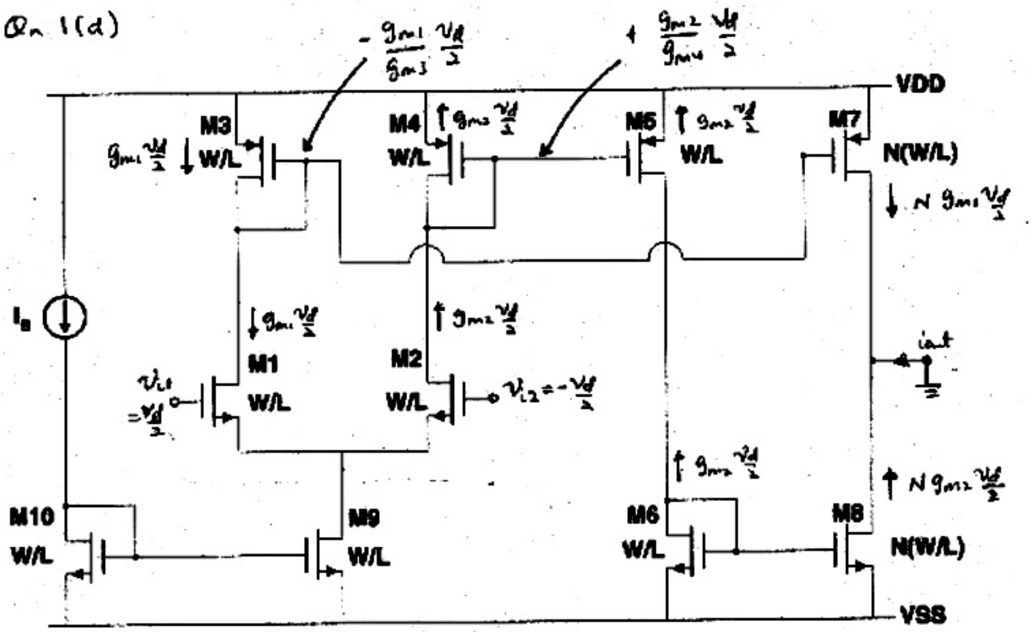
$\therefore$ Working from $M_1 \to M_3 \to M_7$ and $M_2 \to M_4 \to M_5 \to M_6 \to M_8$
(See attached diagram on next page).

$$i_{out} = -\Big[\underbrace{N g_{m1} \frac{v_d}{2}}_{i_{d7}} + \underbrace{N g_{m2} \frac{v_d}{2}}_{|i_{d8}|}\Big]$$

$$\Rightarrow G_m = -N g_{m1,2} \quad \#$$

ANS : $G_m = -N g_{m1,2}$

Qn 1(d)



Small Signal currents for finding Gm.

(e) Determine the common mode voltage gain, A_{cm} :

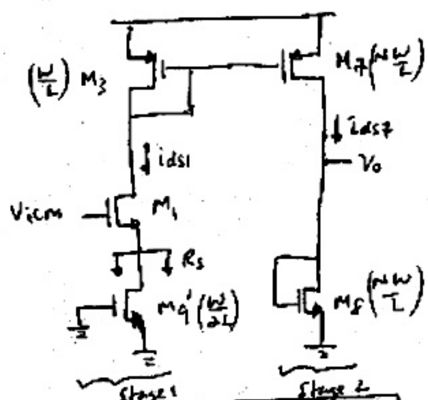
$$A_{cm} = \frac{v_{out}}{v_{in,cm}}$$

(12 marks)

For common mode input, the circuit is symmetric

$M_1 \rightarrow M_3 \rightarrow M_7 \rightarrow M_8$ vs $M_2 \rightarrow M_4 \rightarrow M_5 \rightarrow M_6$

So $V_{O8} = V_{O6} = V_{G6} \doteq V_{G8} \Rightarrow M_8$ is diode connected in common mode. Using half circuit technique,



Since M_9 is shared by the 2 half circuits. $\Rightarrow R_s = 2r_{o9}$

M_1 is a common source amplifier with degeneration.

$$\therefore \; i_{ds1} \approx \frac{g_{m1}}{1+g_{m1}(2r_{o9})} v_{icm}$$

M_3 & M_7 forms a current mirror;

$$i_{ds7} \approx N\, i_{ds1}$$

$$i_{ds8} \approx i_{ds7} \quad \text{(KCL)}$$

$$\therefore v_o \approx i_{ds8}\left(\frac{1}{g_{m8}} \,\|\, r_{o8}\right)$$

$$\approx i_{ds8}\left(\frac{1}{g_{m8}}\right) \qquad \left(\frac{1}{g_{m8}} \ll r_{o8}\right)$$

$$= \frac{g_{m1} N}{g_{m8}(1+2g_{m1}r_{o9})} v_{icm}.$$

$\Rightarrow$ **ANS :** $A_{cm} = \frac{v_o}{v_{icm}} = \frac{1}{1+2\,g_{m1} r_{o9}}$

since $N g_{m1} = g_{m8}$. #

Alternatively,

$$A_{cm} = A_{cm1} \times A_{cm2}$$

$$= \frac{g_{m1}\left(\frac{1}{g_{m3}}\right)}{1+g_{m1}(2r_{o9})} \times \frac{g_{m7}}{g_{m8}}$$

$$= \frac{1}{1+g_{m1}(2r_{o9})}$$

since $g_{m8} = N g_{m1}$

$g_{m7} = N g_{m3}$.

Question 2

In the Fig 2, all devices have identical W/L's. Assume the bulk of all transistors are connected to ground.

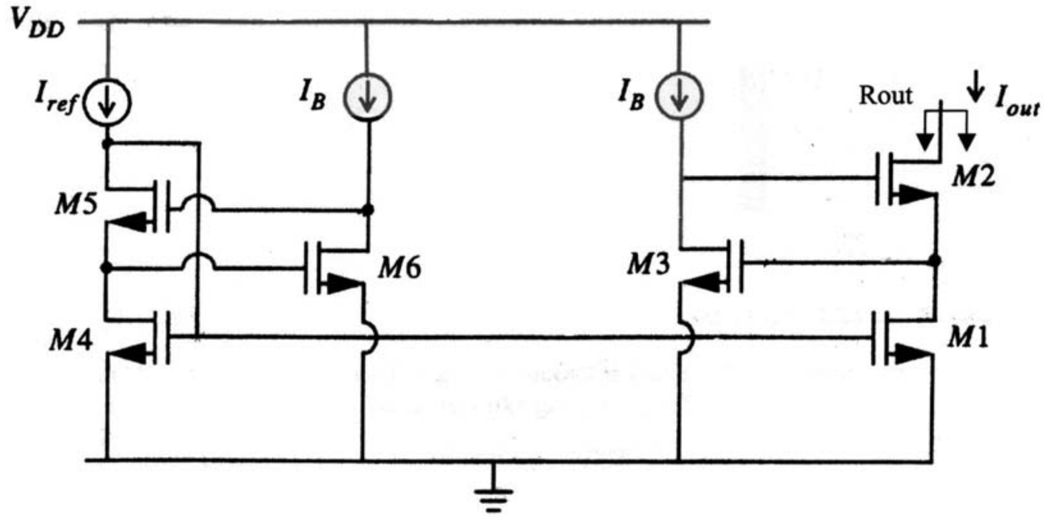


Fig 2.

(a). Assuming all devices biased in the active (i.e. saturation) region, determine the nominal bias values for the following. Leave your answers as expressions involving V_t's, k', I_B, I_{ref} and W/L. (14 marks)

I_{out}	I_{ref}
V_{GS1}	$V_{T1} + \left[\frac{I_{ref}}{(\frac{k}{2})(\frac{W}{L})}\right]^{1/2}$
V_{GS2}	$V_{T2} + \left[\frac{I_{ref}}{(\frac{k}{2})(\frac{W}{L})}\right]^{1/2}$
V_{GS3}	$V_{T3} + \left[\frac{I_B}{(k/2)(\frac{W}{L})}\right]^{1/2}$
V_{GS4}	$V_{T4} + \left[\frac{I_{ref}}{(k/2)(\frac{W}{L})}\right]^{1/2}$
V_{GS5}	$V_{T5} + \left[\frac{I_{ref}}{(k/2)(\frac{W}{L})}\right]^{1/2}$
V_{GS6}	$V_{T6} + \left[\frac{I_B}{(k/2)(\frac{W}{L})}\right]^{1/2}$

(b) Determine the minimum output voltage that keeps all devices in the active region.

$$V_{G2} = V_{T3} + \Delta V_3 + V_{T2} + \Delta V_2 \qquad \Delta V_2 = \left[\frac{I_{ref}}{\left(\frac{k}{2}\right)\left(\frac{W}{L}\right)}\right]^{1/2}$$

$$\text{Min } V_{out} = V_{G2} \quad V_{T2} \qquad \Delta V_3 \quad \left[\frac{I_B}{\left(\frac{k}{2}\right)\left(\frac{W}{L}\right)}\right]^{1/2}$$

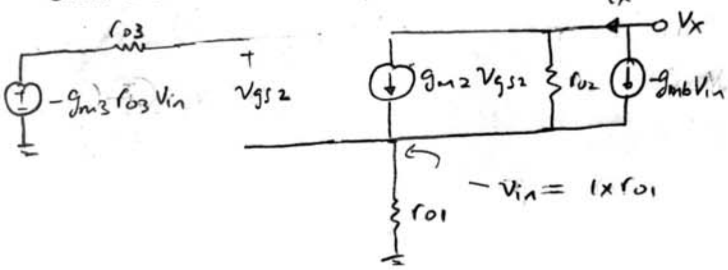
$$V_{T3} + \Delta V_3 + \Delta V_2$$

ANS : $V_{T3} + \left[\frac{I_B}{\frac{k}{2}\left(\frac{W}{L}\right)}\right]^{1/2} + \left[\frac{I_{ref}}{\left(\frac{k}{2}\right)\left(\frac{W}{L}\right)}\right]^{1/2}$

(c) Determine R_{out} for this circuit assuming all devices are active. Express your answer as a formula involving small signal parameters like g_{m1-6}, r_{o1-6}, etc.

Small signal analysis

M_3 acts like inverting amplifier with gain $-g_{m3} r_{o3}$



$$v_{gs2} \quad -g_{m3} r_{o3} V_{in} - V_{in}$$

$$-V_{in}(\ + g_{m3} r_{o3})$$

$$x\, r_o \quad + g_{m3} r_{o3})$$

$$x = g_{m2} V_{gs2} + \frac{V_x - V_{in}}{r_{o2}} \quad g_{mb} V_{in}$$

$$-g_{m2} r_o\ (\ + g_{m3} r_{o3})\ x + \frac{V_x}{r_{o2}} \quad \frac{x\, r_{o1}}{r_{o2}} \quad g_{mb} I_x r_{o1}$$

$$\left(1 + g_{m2} r_{o1}(\ + g_{m3} r_{o3}) + \frac{r_{o1}}{r_{o2}} + g_{mb} r_{o1}\right) \quad \frac{V_x}{r_{o2}}$$

$$R_{out} = \frac{V_x}{x} = r_{o2}(\ + g_{m2} r_{o1}(\ + g_{m3} r_{o3}) + g_{mb} r_{o1}) + r_o$$

ANS : $r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2} (1 + g_{m3} r_{o3}) + g_{mb} r_{o1} r_{o2}$

$$r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2} (1 + g_{m3}\, r_{o3}) + g_{mb} r_{o1} r_{o2}$$

(d) Briefly explain the purpose of M5 and M6 in this circuit.

M5 and M6 (similiar to M3 & M1) forces $V_{d4} = V_{d1}$

Since $V_{GS4} = V_{GS1}$ and now $V_{DS4} = V_{DS1}$, we have

$I_{out} = I_{ref}$