3.6 - DEVICE MISMATCH IN DIFFERENTIAL AMPLIFIERS

INTRODUCTION

Objective

The objective of this presentation is:

1.) Characterize the dependence of bias circuits on the power supply

2.) Introduce circuits that have various degrees of power supply independence

Outline

- Characterization of power supply dependence
- Simple bias circuits
- Bootstrapped bias circuits
- Temperature characterization of bias circuits

Objective

The objective of this presentation is:

1.) Illustrate the method of analyzing mismatches

2.) Analyze the input current and voltage offsets for differential amplifiers

Outline

- The general approach to analyzing mismatches
- Input voltage and current offsets of BJT differential amplifiers
- Input voltage offsets of MOS differential amplifiers
- Summary

MISMATCH ANALYSIS METHODS

General Method

Suppose that two performances, p_1 and p_2, can be written can be written as

$$p_1 = f_1(x_1, y_1, z_1,...) \quad \text{and} \quad p_2 = f_2(x_2, y_2, z_2,...)$$

Ideally, y_1 should be equal to y_2, but in practice their difference could be expressed as

$$\text{Error} = e(p_1, p_2) = f(x_1, y_1, z_1,..., x_2, y_2, z_2,...)$$

Now assume that x_1,y_1, z_1,... and x_2,y_2, z_2,...) can be expressed in terms of their difference and average values. We illustrate only for x_1 and x_2,

$$\Delta x = x_1 - x_2 \quad \text{and} \quad x = \frac{x_1+x_2}{2}$$

We can solve for x_1 and x_2 in terms of Δx and x as follows,

$$x_1 = x + 0.5\Delta x \quad \text{and} \quad x_2 = x - 0.5\Delta x$$

Now the error can be express as

$$e(p_1, p_2) = f(x+0.5\Delta x, x-0.5\Delta x)$$

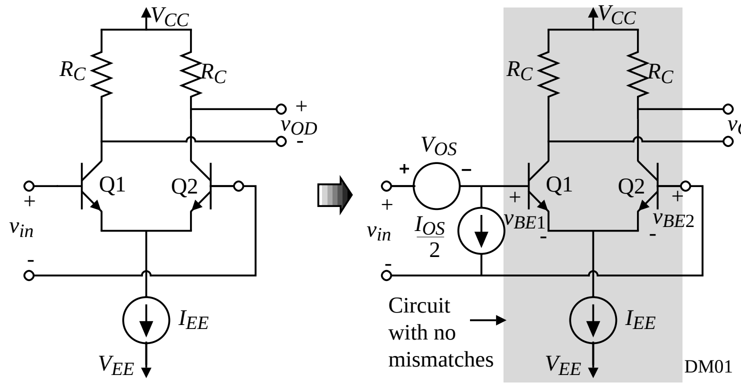
This expressing can generally be simplified by assuming that $\Delta x << x$ and using the following approximations,

$$\frac{1}{1-\varepsilon} \approx 1+\varepsilon \quad \text{or} \quad \frac{1}{1+\varepsilon} \approx 1-\varepsilon$$

and neglecting higher power values of ε, i.e. ε^2

INPUT VOLTAGE AND CURRENT OFFSETS OF THE BJT DIFFERENTIAL AMPLIFIER

Model for Input Offset Voltage and Current



where

$$V_{OS} = V_{BE1}\text{-}V_{BE2} = V_t\ln\left(\frac{I_{C1}}{I_{s1}}\right) - V_t\ln\left(\frac{I_{C2}}{I_{s2}}\right) = V_t\ln\left(\frac{I_{C1}}{I_{C2}}\frac{I_{s2}}{I_{s1}}\right)$$

How does I_s depend upon the semiconductor parameters?

$$I_{s1} = \frac{qn_i^2\bar{D}_n}{N_A W_{B1}(V_{CB})}A_1 = \frac{qn_i^2\bar{D}_n}{Q_{B1}(V_{CB})}A_1 \quad \text{and} \quad I_{s2} = \frac{qn_i^2\bar{D}_n}{N_A W_{B2}(V_{CB})}A_2 = \frac{qn_i^2\bar{D}_n}{Q_{B2}(V_{CB})}A_2$$

where $W_B(V_{CB})$ is the base width as a function of V_{CB}, N_A is the acceptor density in the base and A is the emitter area.

BJT Input Offset Voltage - Continued

In order for the output voltage to be zero, it is required that

$$I_{C1}R_{C1} = I_{C2}R_{C2} \qquad \Rightarrow \qquad \frac{I_{C1}}{I_{C2}} = \frac{R_{C2}}{R_{C1}}$$

Combining these two relationships gives

$$V_{OS} = V_t \ln\left[\left(\frac{R_{C2}}{R_{C1}}\right)\left(\frac{A_2}{A_1}\right)\left(\frac{Q_{B1}(V_{CB})}{Q_{B2}(V_{CB})}\right)\right]$$

Making the following definitions,

$$\Delta R_C = R_{C1}\text{-}R_{C2},\ R_C = \frac{R_{C1}+R_{C2}}{2},\ \Delta A = A_1\text{-}A_2,\ A = \frac{A_1+A_2}{2},\ \Delta Q_B = Q_{B1}\text{-}Q_{B2},\ \text{and}\ Q_B = \frac{Q_{B1}+Q_{B2}}{2}$$

which gives

$$R_{C1}=R_C+\frac{\Delta R_C}{2},\ R_{C2}=R_C-\frac{\Delta R_C}{2},\ A_1=A+\frac{\Delta A}{2},\ A_2=A-\frac{\Delta A}{2},\ Q_{B1}=Q_B+\frac{\Delta Q_B}{2},\ \text{and}\ Q_{B2}=Q_B-\frac{\Delta Q_B}{2}$$

Substituting these values into the expression for V_{OS} gives,

$$V_{OS} = V_t \ln\left[\left(\frac{R_C - \frac{\Delta R_C}{2}}{R_C + \frac{\Delta R_C}{2}}\right)\left(\frac{A - \frac{\Delta A}{2}}{A + \frac{\Delta A}{2}}\right)\left(\frac{Q_B + \frac{\Delta Q_B}{2}}{Q_B - \frac{\Delta Q_B}{2}}\right)\right] \approx V_t \ln\left[\left(1-\frac{\Delta R_C}{R_C}\right)\left(1-\frac{\Delta A}{A}\right)\left(1+\frac{\Delta Q_B}{Q_B}\right)\right]$$

if $\Delta R_C << R_C$, $\Delta A << A$, and $\Delta Q_B << Q_B$

Expanding the logarithm and neglecting higher order terms gives

$$V_{OS} \approx V_t\left(-\frac{\Delta R_C}{R_C} - \frac{\Delta A}{A} + \frac{\Delta Q_B}{Q_B}\right) = V_t\left(-\frac{\Delta R_C}{R_C} - \frac{\Delta I_s}{I_s}\right) \quad \text{where} \quad \frac{\Delta I_s}{I_s} = \frac{\Delta A}{A} - \frac{\Delta Q_B}{Q_B}$$

Example 1 - Calculation of Input Voltage Offset for a BJT Differential Amplifier

Find the value of the input offset voltage for a BJT differential amplifier at room temperature if the standard deviations of the resistor match and saturation current match are 1% and 5%, respectively. Assume that the standard deviations are correlated. Repeat this example if the standard deviations are not correlated.

Solution

For the correlated case we have,

$$V_{OS} = V_t\left(-\frac{\Delta R_C}{R_C} - \frac{\Delta I_s}{I_s}\right) = 0.026(-0.01-0.05) \approx -1.5\text{mV}$$

(Magnitude, not sign is important since the polarity of the mismatch is not known.)

When the variation in R_C and I_s are uncorrelated then we get,

$$V_{OS} = V_t\sqrt{\left(\frac{\Delta R_C}{R_C}\right)^2 + \left(\frac{\Delta I_s}{I_s}\right)^2} = 0.026\sqrt{(0.01)^2+(0.05)^2} = (0.026)0.051 \approx 1.3\text{mV}$$

Temperature Dependence of the Input Offset Voltage

The temperature dependence of V_{OS} is found by examining the temperature dependence of

$$V_{OS} = V_t\left(-\frac{\Delta R_C}{R_C} - \frac{\Delta I_s}{I_s}\right)$$

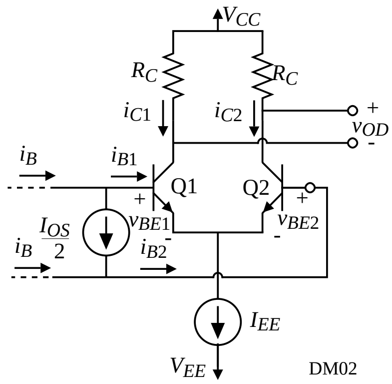
While I_s and R_C have reasonably large temperature dependence, the temperature dependence of their difference can be neglected. Therefore assuming a value of V_{OS} = 2mV, we get

$$\frac{dV_{OS}}{dT} = \frac{V_{OS}}{T} \approx \frac{2\text{mV}}{300} = 6.67\mu\text{V}/°\text{C}$$

Offset voltage can be cancelled using external circuitry but to cancel the temperature drift requires the external circuitry to have the same temperature dependence.

Input Offset Current of the BJT Differential Amplifier

Consider the following model based on the previous circuit,



The input offset current of the BJT differential amplifier can be written as,

$$I_{B1} = I_B + 0.5I_{OS} \quad \text{and} \quad I_{B2} = I_B - 0.5I_{OS} \quad \Rightarrow \quad I_{OS} = I_{B2} - I_{B1} = \frac{I_{C2}}{\beta_{F2}} - \frac{I_{C1}}{\beta_{F1}}$$

Defining $\quad \Delta I_C = I_{C2} - I_{C1},\ I_C = \dfrac{I_{C1}+I_{C2}}{2},\ \Delta\beta_F = \beta_{F2} - \beta_{F1},$ and $\beta_F = \dfrac{\beta_{F1}+\beta_{F2}}{2}$

which gives $\quad I_{C1} = I_C - \dfrac{\Delta I_C}{2},\ I_{C2} = I_C + \dfrac{\Delta I_C}{2},\ \beta_{F1} = \beta_F - \dfrac{\Delta\beta_F}{2},$ and $\beta_{F2} = \beta_F + \dfrac{\Delta\beta_F}{2}$

Input Offset Current of the BJT Differential Amplifier - Continued

Combining the previous expressions into the function for I_{OS} gives,

$$I_{OS} = \left(\frac{I_C + \frac{\Delta I_C}{2}}{\beta_F + \frac{\Delta\beta_F}{2}} - \frac{I_C - \frac{\Delta I_C}{2}}{\beta_F - \frac{\Delta\beta_F}{2}} \right) \approx \frac{I_C}{\beta_F}\left(\frac{\Delta I_C}{I_C} - \frac{\Delta\beta_F}{\beta_F}\right) \qquad \text{when } \Delta I_C << I_C \text{ and } \Delta\beta_F << \beta_F$$

Recalling that for the output voltage to be zero that $\dfrac{I_{C1}}{I_{C2}} = \dfrac{R_{C2}}{R_{C1}}$, then

$$\frac{1 - \frac{\Delta I_C}{2I_C}}{1 + \frac{\Delta I_C}{2I_C}} = \frac{1 + \frac{\Delta R_C}{2R_C}}{1 - \frac{\Delta R_C}{2R_C}} \quad \Rightarrow \quad \left(1 - \frac{\Delta I_C}{2I_C}\right)\left(1 - \frac{\Delta I_C}{2I_C}\right) = \left(1 + \frac{\Delta R_C}{2R_C}\right)\left(1 + \frac{\Delta R_C}{2R_C}\right) \quad \Rightarrow \quad -\frac{\Delta I_C}{I_C} = \frac{\Delta R_C}{R_C}$$

Therefore,

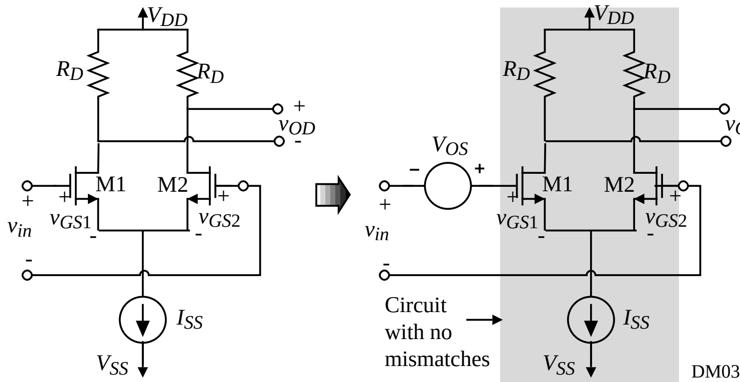
$$I_{OS} = -\frac{I_C \Delta R_C}{\beta_F R_C} - \frac{I_C \Delta\beta_F}{\beta_F \beta_F} = -\frac{I_C}{\beta_F}\left(\frac{\Delta R_C}{R_C} - \frac{\Delta\beta_F}{\beta_F}\right)$$

Typically $\Delta\beta_F/\beta_F$ is about 10% and $\Delta R_C/R_C$ is about 1% giving

$$I_{OS} = -\frac{I_C}{\beta_F}(0.01+0.1) = -0.11 I_B \approx 1.1\mu\text{A assuming that } I_B = 10\mu\text{A}$$

INPUT VOLTAGE OFFSET OF MOS DIFFERENTIAL AMPLIFIER

Model for Input Offset Voltage



where for v_{in} = 0,

$$V_{OS} = V_{GS1}-V_{GS2} = \sqrt{\frac{2I_{D1}}{\beta_1}} + V_{T1} - \sqrt{\frac{2I_{D2}}{\beta_2}} + V_{T2} \quad \text{where } \beta = \frac{K'W}{L}$$

Define,

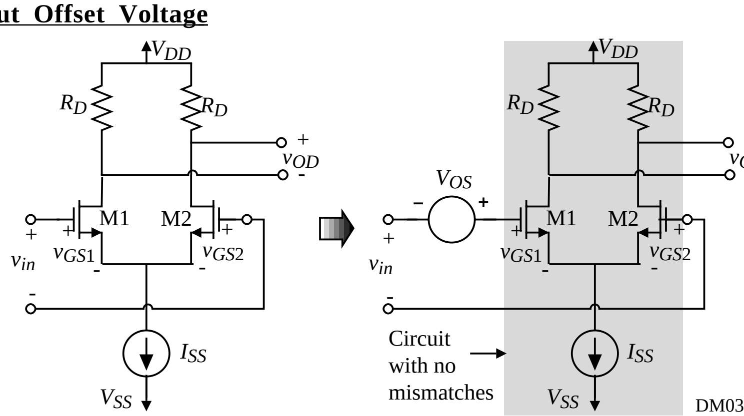
$$\Delta I_D = I_{D1}-I_{D2},\ I_D = \frac{I_{D1}+I_{D2}}{2},\ \Delta\beta = \beta_1-\beta_2,\ \beta = \frac{\beta_1+\beta_2}{2},\ \Delta V_T = V_{T1}-V_{T2} \text{ and } V_T = \frac{V_{T1}+V_{T2}}{2}$$

Gives,

$$I_{D1} = I_D + \frac{\Delta I_D}{2},\ I_{D2} = I_D - \frac{\Delta I_D}{2},\ \beta_1 = \beta + \frac{\Delta\beta}{2},\ \beta_2 = \beta - \frac{\Delta\beta}{2},\ V_{T1} = V_T + \frac{\Delta V_T}{2} \text{ and } V_{T2} = V_T - \frac{\Delta V_T}{2}$$

INPUT VOLTAGE OFFSET OF MOS DIFFERENTIAL AMPLIFIER

Model for Input Offset Voltage



where for v_{in} = 0,

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Define,

$$\Delta I_D = I_{D1}-I_{D2},\ I_D = \frac{I_{D1}+I_{D2}}{2},\ \Delta\beta = \beta_1-\beta_2,\ \beta = \frac{\beta_1+\beta_2}{2},\ \Delta V_T = V_{T1}-V_{T2} \text{ and } V_T = \frac{V_{T1}+V_{T2}}{2}$$

Gives,

$$I_{D1} = I_D + \frac{\Delta I_D}{2},\ I_{D2} = I_D - \frac{\Delta I_D}{2},\ \beta_1 = \beta + \frac{\Delta\beta}{2},\ \beta_2 = \beta - \frac{\Delta\beta}{2},\ V_{T1} = V_T + \frac{\Delta V_T}{2} \text{ and } V_{T2} = V_T - \frac{\Delta V_T}{2}$$

Example 2 - Calculation of Input Voltage Offset for a MOS Differential Amplifier

Find the value of the input offset voltage for a MOS differential amplifier at room temperature if the standard deviations of the the resistor match and beta match are 1% and 5%, respectively. Assume that the threshold voltage deviation is normalized to the value of V_{GS}-V_T = 1.5V and is 10%. Assume that the standard deviations are correlated. Repeat this example if the standard deviations are not correlated.

Solution

For the correlated case we have,

$$\frac{V_{OS}}{V_{GS}-V_T} = \frac{\Delta V_T}{V_{GS}-V_T} - \frac{\Delta R_D}{2R_D} - \frac{\Delta\beta}{2\beta} = \frac{0.10}{1.5} - \frac{0.01}{2} - \frac{0.05}{2} = 0.0367$$

$\therefore$ V_{OS} = 0.0367x1.5V = 55mV

When the variation in R_D, β and V_T are uncorrelated then we get,

$$\frac{V_{OS}}{V_{GS}-V_T} = \sqrt{\left(\frac{\Delta V_T}{V_{GS}-V_T}\right)^2 + \left(\frac{\Delta R_D}{2R_D}\right)^2 + \left(\frac{\Delta\beta}{2\beta}\right)^2} = 0.0714$$

$\therefore$ V_{OS} = 0.0714x1.5V = 107mV

Temperature Dependence of the MOS Input Offset Voltage

$$V_{OS} = \Delta V_T - \sqrt{\frac{2I_D}{\beta}}\left[\frac{\Delta R_D}{2R_D} + \frac{\Delta\beta}{2\beta}\right]$$

While R_D and V_T have a strong temperature dependence, the temperature dependence of there matching can be ignored.

The temperature dependence of β is

$$\beta(T) = \frac{K'(T)W}{L} = \beta(T_0)\left(\frac{T}{T_0}\right)^{-1.5} \quad \Rightarrow \quad \frac{d\beta}{dT} = -\frac{1.5\beta(T_0)(T/T_0)^{-1.5}}{T} = \frac{-1.5\beta(T)}{T}$$

$$\frac{dV_{OS}}{dT} = \frac{d}{dT}\left(\sqrt{\frac{2I_D}{\beta}}\left[\frac{\Delta R_D}{2R_D} + \frac{\Delta\beta}{2\beta}\right]\right) = \frac{-1}{2\beta}\left(\sqrt{\frac{2I_D}{\beta}}\left[\frac{\Delta R_D}{2R_D} + \frac{\Delta\beta}{2\beta}\right]\right)\frac{d\beta}{dT} = \frac{3}{4T}\sqrt{\frac{2I_D}{\beta}}\left[\frac{\Delta R_D}{2R_D} + \frac{\Delta\beta}{2\beta}\right]$$

At room temperature and a current of 100µA and β = 200, we get,

$$\frac{dV_{OS}}{dT} = \frac{1\cdot 3}{4\cdot 300}\left[\frac{0.01}{2} + \frac{0.05}{2}\right] = 75\mu V/°C$$

Comparison of BJT and MOS Offset Voltages

BJT:

$$V_{OS}(\text{BJT}) \propto V_t = \frac{I_C}{g_m(\text{BJT})}$$

MOS:

$$V_{OS}(\text{MOS}) \propto V_{GS} - V_T = \frac{(V_{GS}-V_T)^2}{V_{GS}-V_T} = \frac{\frac{2I_D}{\beta}}{V_{GS}-V_T} = \frac{I_D}{0.5\beta(V_{GS}-V_T)} = \frac{I_D}{g_m(\text{MOS})}$$

Assuming that the bias currents are the same, the offsets are greater for the MOS differential amplifier because the transconductances are smaller.

What is the key concept here?

When amplifier imperfections (offset, noise, nonlinearity, etc.) are reflected to the input of the amplifier, the larger the gain of the amplifier, the smaller the reflected imperfections.

SUMMARY

- Mismatch analysis between two components can be performed by defining a difference and average value of the components and expressing the mismatch in terms of the difference and average value.
- BJT differential amplifiers have both a voltage and current input offset.
- Cancellation of the input offset is difficult because of temperature dependence of the offset.
- The MOS differential amplifier has only input voltage offset but this offset is larger than the BJT differential amplifier
- The temperature dependence of the MOSFET input offset voltage is about 10 times that of the BJT
- To reduce the value of the amplifier imperfection reflected to the input of the amplifier, make the gain of the amplifier as large as possible.