Chapter 1

NOISE IN INTEGRATED CIRCUITS

1.1 Introduction

For many high-performance analog circuits and systems, in particular those with high dynamic ranges or signal-to-noise ratios (SNRs), it is critical to estimate the noise contribution from the devices and the systems themselves. The main reason is that such noise contribution, together with interference noise, will determine the minimum input signal levels - a level below which the input signal can no longer be recognized or detected, and thus place a fundamental limit on the achievable dynamic range.

This chapter will discuss how noise in integrated circuits can be estimated and be minimized through appropriate circuit design techniques. It is important to note that we are only dealing with inherent noise, which is due to fundamental properties of the devices, and not interference noise, which is the results of interaction with the outside world.

1.2 Noise Sources

1.2.1 Shot Noise

Shot noise is due to random fluctuations in a current flow and occurs in diodes and bipolar transistors. The mean-square value is given by:

$$\overline{i^2} = \sigma^2 = \overline{(I - I_D)^2} = 2qI_D\Delta f \qquad (1.1)$$

where I_D is the average current and Δf is the bandwidth.

From Eq. 1.1, shot noise has a white spectral density (frequency distribution), i.e. the spectral density is constant as a function of frequency. Moreover, the mean-square value is linearly proportional to both the average current and the bandwidth.

The amplitude of shot noise varies randomly with time, and its probability-density function has a Gaussian distribution as shown in Fig. 1.1. In other words, the probabilities that the amplitude of the noise signal is less than one and three standard deviations (σ and 3σ) are 68% and 99.7%, respectively.

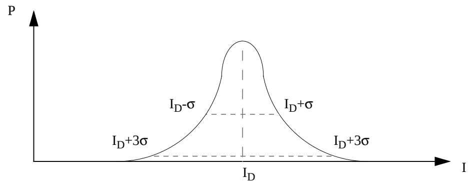

Fig. 1.1 Probability-density function for a diode current due to shot noise

1.2.2 Thermal Noise

Thermal noise or Johnson noise or Nyquist noise is contributed by random thermal excitation of charge carriers and occurs in any passive resistor. The noise signal can be represented by a series voltage source with a mean-square value of:

$$\overline{v^2} = 4kTR\Delta f \qquad (1.2)$$

or, equivalently, a parallel current source with a mean-square value of:

$$\overline{i^2} = 4kT\frac{1}{R}\Delta f \qquad (1.3)$$

Like shot noise, it has a white spectral density and a Gaussian amplitude distribution. However, its mean-square value does not depend on the current itself but depends on the absolute temperature and the resistance of the conductor.

It is useful to note that at room temperature, the thermal noise spectral density contributed by a 1-kΩ resistor is approximately 16 x 10^{-18} V^2/Hz or 4 nV/$Hz^{1/2}$.

1.2.3 Flicker Noise (1/f Noise)

Flicker noise is due to traps or imperfection in semiconductors which capture and release carriers randomly. This noise source only occurs when a dc current is flowing.

The spectral density of flicker noise is given by:

$$\overline{i^2} = K\frac{I^a}{f^b}\Delta f \qquad (1.4)$$

where a, b, and K are constants with $0.5 < a < 2$, $0.8 < b < 1.3$, and K being found empirically. Unlike those of thermal noise and shot noise, the frequency distribution of flicker noise is not white, and the amplitude variation is generally non-Gaussian.

Typically, flicker noise is significant in MOS transistors but can be neglected in bipolar transistors.

1.3 Noise Models in IC Elements

1.3.1 Junction Diode

The noise sources in junction diodes are modeled as shown in Fig. 1.2, where r_s is the physical series resistor of the diode, r_d is the small-signal resistance,

$$r_d = \frac{kT}{qI_D} \qquad (1.5)$$

v_s is the thermal noise contributed by the resistor r_s,

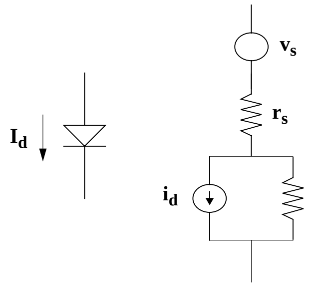

Fig. 1.2 Noise model for a junction diode

$$\overline{v_s^2} = 4kTr_s\Delta f \qquad (1.6)$$

and i_s is the combination of shot noise and flicker noise associated with the current I_D,

$$\overline{i_d^2} = 2qI_D\Delta f + K\frac{I_D^a}{f^b}\Delta f \qquad (1.7)$$

1.3.2 Bipolar Transistor

The noise sources in bipolar transistors can be modeled as shown in Fig. 1.3, where v_b is the thermal noise contributed by the base resistor r_b,

$$\overline{v_b^2} = 4kTr_b\Delta f \qquad (1.8)$$

i_b is the combination of the shot noise and flicker noise contributed by the base-emitter junction diode, exactly as in Eq. 1.7,

$$\overline{i_b^2} = 2qI_B\Delta f + K\frac{I_B^a}{f^b}\Delta f \qquad (1.9)$$

and i_c is the shot noise associated with the current I_C,

$$\overline{i_c^2} = 2qI_C\Delta f \qquad (1.10)$$

Note that the collector series resistor r_c also contributes thermal noise as r_b does. However, since it is in *series* with a high-impedance node, its effect is negligibly small.

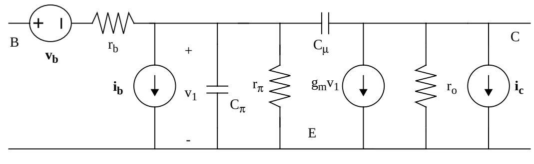

Fig. 1.3 A complete noise model for a npn transistor

1.3.3 MOSFET Transistor

The noise sources in MOS transistors can be modeled as shown in Fig. 1.4, where i_d is contributed by the channel thermal noise and the flicker noise,

$$\overline{i_d^2} = 4kT\left(\frac{2}{3}g_m\right)\Delta f + K\frac{I_D^a}{f^b}\Delta f \tag{1.11}$$

i_g is the shot noise generated by the leakage gate current I_G,

$$\overline{i_g^2} = 2qI_G\Delta f \tag{1.12}$$

and v_g is the thermal noise contributed by the distributed gate resistance R_G,

$$\overline{v_g^2} = 4kTR_G\Delta f \tag{1.13}$$

In practice, the noise contribution of the gate series resistance is usually minimized by interdigitating the polysilicon gate to reduce the gate resistance, in which case the gate resistance can be estimated as,

$$R_G = \frac{1}{Kn^2}R_{sh}\frac{W}{L} \tag{1.14}$$

where R_{sh} is the poly sheet resistance, W/L is the device aspect ratio, n is the number of the gate fingers, and K is 3 or 12 depending on whether the gate fingers are contacting at one end or both ends, respectively.

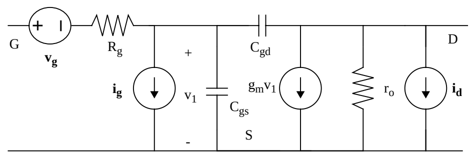

Fig. 1.4 A complete noise model for a MOS transistor

Typically, the channel thermal noise is the most dominant noise source, and more advanced and complicated model can be used,

$$\overline{i_d^2} = 4kT(\gamma g_{d0})\Delta f \tag{1.15}$$

where g_{d0} is the zero-bias drain conductance of the device and γ is the coefficient of the channel thermal noise. For long-channel devices,

$$g_{d0} = g_m \tag{1.16}$$

and

$$\frac{2}{3} \le \gamma \le 1 \tag{1.17}$$

As a good estimate, $\gamma = 2/3$ is used when the device is operated in the saturation region, and $\gamma = 1$ is used when the device is in the linear region.

For short-channel devices, it has been proven that g_{d0} is much larger than g_m, and γ is much larger than 2/3 even for saturation devices, and as a result, the thermal noise is much larger than that predicted by the standard model given by Eq. 1.11.

Although it is beyond the scope of this course, it is also interesting to note that at very high frequencies, there exists another significant noise source due to the current induced by the gate capacitor coupling. To model this noise source, an equivalent physical resistance r_g given by,

$$r_g = \frac{\delta}{5g_{d0}} \tag{1.18}$$

where δ is the gate noise coefficient and equals to 4/3 for long-channel devices in saturation, and its associated thermal noise source v_{rg},

$$\overline{v_{rg}^2} = 4kTr_g\Delta f \tag{1.19}$$

can be included in series with the gate physical series resistance R_G and the noise source v_g as described in Eqs. 1.13 and 1.14.

1.3.4 Capacitors & Inductors

Capacitors and inductors are noiseless except for their parasitic resistance for which the thermal noise model for physical resistors given in Eqs. 1.2 and 1.3 can be readily used.

1.4 Circuit Noise Calculations

Given the noise models for all components as described in the last section, the noise performance of a circuit can be estimated.

The main idea is to approximate each noise source by a separate and independent sinusoidal source with the rms value being equal to the rms value of the noise in a small bandwidth Δf and to apply the conventional small-signal analysis techniques to find out its sinusoidal output response. The mean-square value of the sinusoidal output response is corresponding to the mean-square value of the output noise in bandwidth Δf.

After contribution of each noise source is calculated, the total mean-square value of the output noise is estimated by summing the mean-square values of all the individual sources. In practice, the noise sources are most likely independent on each other, and this method gives a good estimate of the circuit noise performance.

As an example, let's calculate the total thermal noise contribution by two resistors connected in parallel as shown in Fig. 1.5. The thermal noise current contributed by each resistor is given by:

$$\overline{i_d^2} = 4kT\frac{1}{R}\Delta f \tag{1.20}$$

Replacing each noise source with a sinusoidal current source, the total sinusoidal current source becomes

$$i_{dT} = i_{d1} + i_{d2} \tag{1.21}$$

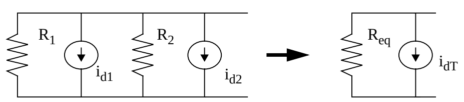

Fig. 1.5 An example for calculation of circuit noise performance

Since there is no correlation between the two noise sources of the resistors, the mean-square value of the total sinusoidal signal and thus of the total noise current is given by,

$$\overline{i_{dT}^2} = \overline{i_{d1}^2} + \overline{i_{d2}^2} = 4kT\frac{1}{R_1}\Delta f + 4kT\frac{1}{R_1}\Delta f = 4kT\frac{1}{R_{eq}}\Delta f \tag{1.22}$$

where R_{eq} is the equivalent parallel resistance,

$$R_{eq} = R_1 \parallel R_2 \tag{1.23}$$

which would be, as it should be, consistent with the result obtained if the two resistors are combined before the calculation.

It is important to note that the polarity of the noise sources are irrelevant since they are random signals. Only their mean-square values are of interest.

1.5 Equivalent Input Noise Sources

1.5.1 General Representation

As mentioned earlier, the main reason for estimating the total noise of a circuit is to determine what is the minimum input signal, mostly referred to as the minimum detectable signal (MDS), that can be applied to the circuit without significant performance degradation. For this purpose, it is more meaningful to be able to estimate the equivalent input-referred noise signal as opposed to the equivalent output-referred noise signal.

Given any source resistance R_S, it would be much simpler and much more convenient to obtain such an input-referred noise signal if the noisy circuit can be represented by a noiseless network with equivalent input noise voltage and current sources v_{eq} and i_{eq} as shown in Fig. 1.6.

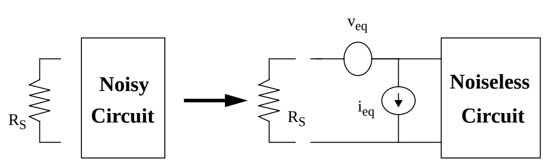

Fig. 1.6 Representation of a two-port network with equivalent input noise sources

1.5.1.1 For $R_S = 0$

For very small source resistance R_S, the input current source i_{eq} is shorted and only the input voltage source v_{eq} contributes noise at the output.

1.5.1.2 For Large R_S

For infinitely large R_S, the input voltage source v_{eq} has negligibly small effect and only the current source i_{eq} contributes significant noise at the output.

1.5.1.3 For Finite R_S

For R_S not being too small nor too large, both the input current source i_{eq} and the input voltage source v_{eq} contribute noise at the output and need to be determined.

To find the equivalent input *current* source i_{eq}, *open-circuit* the input and equate the output noise of both networks.

Similarly, to find the equivalent input *voltage* source v_{eq}, *short-circuit* the input and equate the output noise of both networks.

1.5.2 Equivalent Noise Sources for Bipolar Transistors

Using the procedure outlined in the previous section, the equivalent input noise sources for bipolar transistors can be calculated as follows.

Consider the two networks shown in Fig. 1.7, in which the top one has all the noise sources for a bipolar transistor as discussed earlier and the bottom one is the equivalent noiseless representation with equivalent input noise sources. For simplicity, let us assume here that C_μ is negligibly small and that the base resistance r_b is much smaller than Z_π, where Z_π is defined as

$$Z_\pi = r_\pi \parallel \frac{1}{j\omega C_\pi} \tag{1.24}$$

Short-circuit the input and equate the output current of the two networks to obtain:

$$i_o = g_m v_b + g_m i_b r_b + i_c = g_m v_{eq} \tag{1.25}$$

As a result,

$$v_{eq} = v_b + i_b r_b + \frac{i_c}{g_m} \approx v_b + \frac{i_c}{g_m} \tag{1.26}$$

Since the noise sources v_b, i_b, and i_c are independent, it follows that:

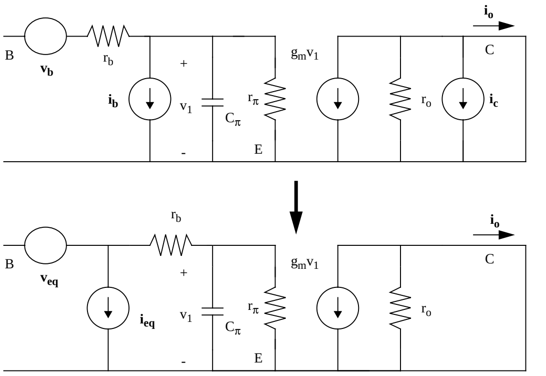

Fig. 1.7 Representation of bipolar transistors with equivalent input noise generators

$$\overline{v_{eq}^2} = \overline{v_b^2} + \frac{\overline{i_c^2}}{g_m^2} = 4kTr_b\Delta f + \frac{2qI_C}{g_m^2}\Delta f = 4kT\left(r_b + \frac{1}{2g_m}\right)\Delta f \tag{1.27}$$

Equivalently, the equivalent total input voltage noise source can be considered to be contributed merely by an equivalent input resistance R_{eq} given by

$$R_{eq} = r_b + \frac{1}{2g_m} \tag{1.28}$$

Similarly, *open-circuit* the input and equate the output current of the two networks to obtain:

$$i_o = g_m i_b Z_\pi + i_c = g_m i_{eq} Z_\pi \tag{1.29}$$

As a result,

$$i_{eq} = i_b + \frac{i_c}{\beta(j\omega)} \tag{1.30}$$

where $\beta(j\omega)$ is the device current gain as a function of frequency

$$\beta(j\omega) \equiv g_m Z_\pi = g_m\left(r_\pi \parallel \frac{1}{j\omega C_\pi}\right) = \frac{\beta}{1 + j\frac{\omega}{\omega_{-3dB}}} = \frac{\beta}{1 + j\frac{\omega}{\omega_T}\beta} \tag{1.31}$$

ω_{-3dB} and ω_T are the 3dB bandwidth and the transition frequency of the transistor, respectively,

$$\omega_{-3dB} = \frac{1}{r_\pi C_\pi} = \frac{g_m}{\beta C_\pi} = \frac{\omega_T}{\beta} \tag{1.32}$$

Since the noise sources i_b and i_c are independent, it follows that:

$$\overline{i_{eq}^2} = \overline{i_b^2} + \frac{\overline{i_c^2}}{|\beta(j\omega)|^2} = \left(2qI_B + K\frac{I_B^a}{f^b} + \frac{2qI_C}{|\beta(j\omega)|^2}\right)\Delta f \tag{1.33}$$

Here, the equivalent total input current noise can be considered to come from the shot noise of an equivalent base current I_{eq} given by,

$$I_{eq} = I_B + K'\frac{I_B^a}{f^b} + \frac{I_C}{|\beta(j\omega)|^2} \tag{1.34}$$

1.5.3 Equivalent Noise Sources for MOS Transistors

Consider the two networks shown in Fig. 1.8 for MOS transistors. The same analysis that was applied to calculate the equivalent input noise sources for bipolar transistors can be used here. Alternatively, since there exist so much similarities between the two models, it would be much simpler to apply directly the results for bipolar transistors with appropriate parameter replacement to account for the difference between bipolar and MOS transistors.

For example, r_b used for bipolar transistors should be replaced by the gate series resistance R_G for MOS transistors. Applying appropriate noise sources for MOS transistors to Eq. 1.27, it can be easily shown that:

$$\overline{v_{eq}^2} = \overline{v_g^2} + \frac{\overline{i_d^2}}{g_m^2} = 4kT\left(R_G + \frac{2}{3}\frac{1}{g_m} + K''\frac{I_D^a}{g_m^2 f^b}\right)\Delta f \tag{1.35}$$

Similar to bipolar transistors, the equivalent total input voltage noise source can be considered to be contributed merely by an equivalent input resistance R_{eq} given by

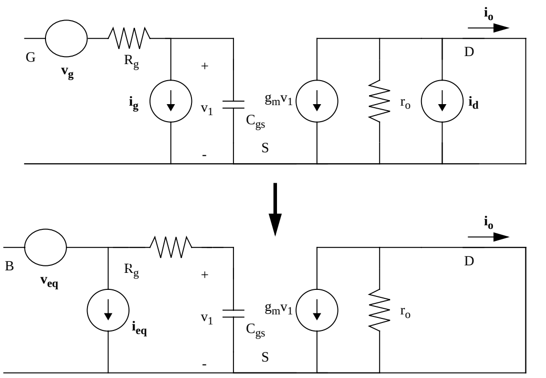

Fig. 1.8 Representation of MOS transistors with equivalent input noise generators

$$R_{eq} = R_G + \frac{2}{3}\frac{1}{g_m} + K''\frac{I_D^a}{f^b} \tag{1.36}$$

In practice, the contribution due to flicker noise to the equivalent input voltage noise for MOS transistors has been experimentally found to be approximated as,

$$\overline{v_{eq}^2}\Big|_{1/f} = K\frac{I_D^a}{g_m^2 f^b}\Delta f \approx \frac{K_f}{WLC_{ox}f} \tag{1.37}$$

where the flicker noise coefficient K_f is typically around $3\times10^{-12}\ V^2pF$. As such, the equivalent input voltage noise given in Eq. 1.35 can be rewritten as

$$\overline{v_{eq}^2} = 4kT\left(R_G + \frac{2}{3}\frac{1}{g_m} + \frac{K_f'}{WLC_{ox}f}\right)\Delta f \tag{1.38}$$

with K_f' being defined as

$$K_f' = \frac{K_f}{4kT} \tag{1.39}$$

Similarly, the equivalent input current noise can be obtained as:

$$i_{eq} = i_g + \frac{i_d}{\beta(j\omega)} \tag{1.40}$$

where $\beta(j\omega)$ is the current gain as a function of frequency,

$$\beta(j\omega) \equiv g_m Z_{gs} = g_m \frac{1}{j\omega C_{gs}} \tag{1.41}$$

It follows that:

$$\overline{i_{eq}^2} = \overline{i_g^2} + \frac{\overline{i_d^2}}{|\beta(j\omega)|^2} = \left\{2qI_G + \frac{1}{|\beta(j\omega)|^2}\left(4kT\frac{2}{3}g_m + K\frac{I_D^a}{f^b}\right)\right\}\Delta f \tag{1.42}$$

1.5.4 Observations

At this point, the following interesting and useful observations can be made.

1. For bipolar transistors, the equivalent input current noise increases at low frequencies because of flicker noise. As comparison, for MOS transistors, at low frequencies, the equivalent input current noise is dependent only on the gate leakage current and thus is extremely small. As a result, *for applications with a large source resistance R_S, MOS devices have much lower noise than bipolar* counterparts.

2. *On the other hand*, the equivalent input voltage noise of MOS is typically larger than that of bipolar transistors because MOS devices have smaller transconductance for a given bias current. It follows that *bipolar devices become more preferable for applications with small source resistance R_S*.

3. For both MOS and bipolar transistors, the equivalent input *current* noise increases at high frequencies because of the degradation in the current gain.

4. For both MOS and bipolar transistors, the equivalent input *voltage* noise can be minimized by maximizing the device transconductance, which corresponds to maximizing either the device size or the bias current.

5. *On the contrary*, for both MOS and bipolar transistors, the equivalent input *current* noise is proportional to the device bias current and can only be minimized by minimizing the bias current.

1.6 Noise Performance of Feedback Circuits

1.6.1 Ideal Feedback Circuits

Figure 1.9 shows a noisy main amplifier A connected to a feedback network f. Ideally, the feedback network would not contribute any loading or extra noise to the main amplifier. In that case, the equivalent input noise sources for the closed-loop circuit v_{eq} and i_{eq} can be easily determined from the equivalent input noise generators for the main amplifiers v_{ia} and i_{ia} as before.

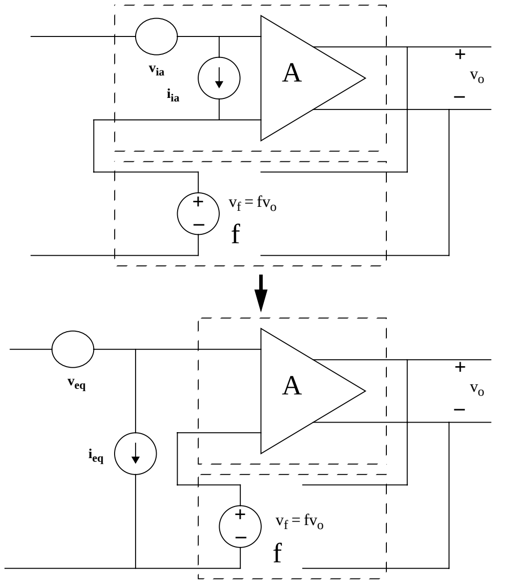

Fig. 1.9 Equivalent input noise generators for a noisy amplifier with *ideal* feedback

It is quite obvious that when the input of the two circuits in Fig. 1.9 is short-circuited, their outputs are equal if and only if

$$\overline{v_{eq}^2} = \overline{v_{ia}^2} \tag{1.43}$$

Similarly, when the input of the two circuits is open-circuited, their outputs can only be equal if

$$\overline{i_{eq}^2} = \overline{i_{ia}^2} \tag{1.44}$$

Therefore, the noise of the whole circuit is the same as the noise of the main amplifier, and the ideal feedback does not affect the circuit noise performance.

1.6.2 Practical Feedback Circuits

In practice, the feedback network will inevitably introduce extra loading and extra noise to the whole circuit as shown in Fig. 1.10. Including the thermal noise of the resistors in the feedback network v_e and v_f and using the equivalent input noise generators for the main amplifiers v_{ia} and i_{ia}, the equivalent input noise sources for the closed-loop circuit v_{eq} and i_{eq} can be determined as follows.

Short-circuit the input and equate the output of the two circuits in Fig. 1.10 to obtain by superposition

$$v_{eq} = v_{ia} + i_{ia}R_f + \frac{R_E}{R_F + R_E}v_f + \frac{R_F}{R_F + R_E}v_e \tag{1.45}$$

where R_f is the input loading given by,

$$R_f = \frac{R_E R_F}{R_F + R_E} \tag{1.46}$$

Assuming that the noise sources are independent, it follows that

$$\overline{v_{eq}^2} = \overline{v_{ia}^2} + \overline{i_{ia}^2}R_f^2 + 4kTR_f\Delta f \tag{1.47}$$

Similarly, open-circuit the inputs and equate the output of the two circuits to get

$$\overline{i_{eq}^2} = \overline{i_{ia}^2} \tag{1.48}$$

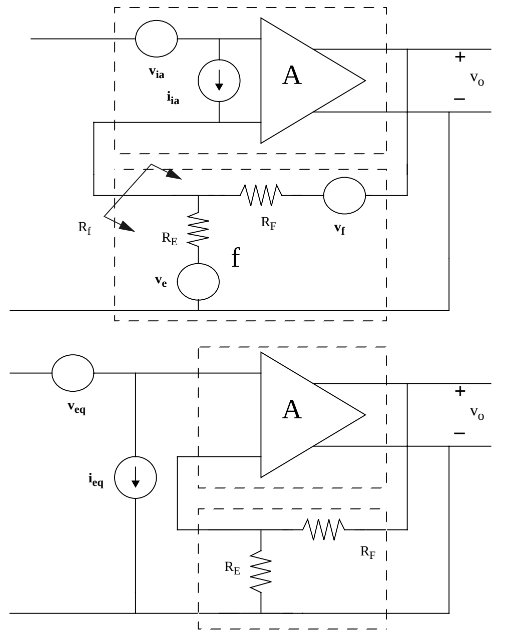

Fig. 1.10 Equivalent input noise generators for a noisy amplifier with *practical* feedback

Therefore, the input voltage noise of the closed-loop amplifier consists of both the noise of the main amplifier and the noise from the feedback. However, the input current noise is not affected by the feedback noise source. This may be somewhat surprising but should be expected since for such a series feedback configuration, the output voltage signal or noise of the feedback network cannot modify the input current.

1.7 Noise Performance of Other Transistor Configurations

1.7.1 Common-Base Circuits

Consider the two configurations shown in Fig. 1.11. The top one is exactly the same as the one in Fig. 1.7 with the equivalent noise sources for bipolar transistors in the common-emitter configuration v_{eq} and i_{eq} except for the orientation. The bottom one is the common-base configuration with its equivalent input noise sources v_{eqB} and i_{eqB} to be determined.

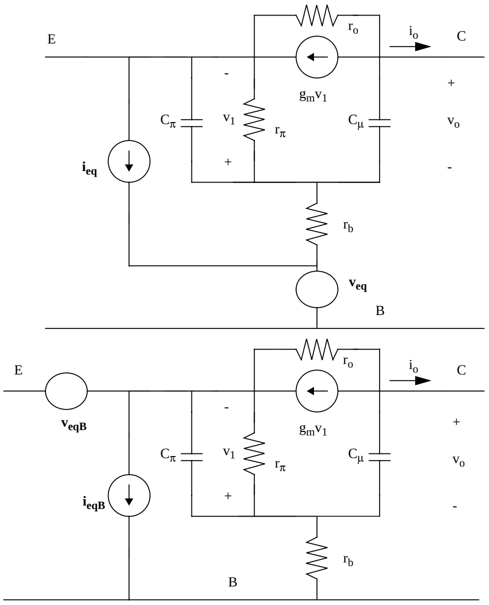

Fig. 1.11 Equivalent input noise generators for a common-base configuration

Short-circuit the input and equate the output noise of the two circuits to obtain:

$$\overline{v^2_{eqB}} = \overline{v^2_{eq}} \tag{1.49}$$

Similarly, open-circuit the input and equate the output noise of the two configurations to get

$$\overline{i^2_{eq}} = \overline{i^2_{ia}} \tag{1.50}$$

It goes to show that the equivalent input noise sources are the same for both common-emitter and common-base configurations. However, since the current gain for the common-base is close to unity, current noise from the output will be unattenuated when referred to the input. For this reason, the common-emitter configuration is in general more preferable.

1.7.2 Emitter Follower Configuration

Since a bipolar transistor connected in the emitter-follower configuration is itself a series-series feedback network, the results discussed earlier for feedback circuits can be applied to determine its equivalent input noise sources.

More specifically, the input voltage noise source can be proved to be,

$$\overline{v^2_{eqE}} = \overline{v^2_{eq}} + 4kTR_E\Delta f \tag{1.51}$$

and the input current noise is given by

$$\overline{i^2_{eqE}} = \overline{i^2_{eq}} \tag{1.52}$$

where R_E is the load of the emitter follower, v_{eq} and i_{eq} are the standard equivalent noise sources of bipolar transistors as described in Fig. 1.7.

Similar to the problem of unity current gain in the common-base configuration, the problem with the emitter-follower configuration is that the voltage gain is no more than unity. As a result, voltage noise from the output when referred to input will be unattenuated.

1.7.3 Differential Pairs

There are two different ways to represent the equivalent input noise sources for differential pairs as illustrated in Fig. 1.12. In one way, two noise sources are used at each input, and in the other, the two voltage noise sources are combined into one and placed at only one input while a current noise source is still used at each input. In terms of the voltage noise source at one end v_{seq}, the combined voltage noise source v_{deq} is given by,

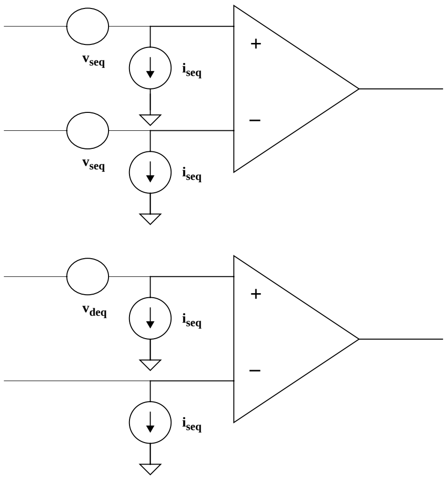

Fig. 1.12 Equivalent input noise sources for differential pairs

$$\overline{v^2_{deq}} = 2\overline{v^2_{seq}} \tag{1.53}$$

The equivalent noise sources v_{seq} and i_{seq} at each input are contributed by each individual device and can be calculated separately using the techniques developed so far.

As an example, let us consider the differential pair shown in Fig. 1.13. The noise sources for Q_1 are shown in Fig. 1.13 with r_{E1} being the load resistor seen from its emitter. By superposition, the transistor Q_2 can be assumed noiseless when the noise sources for Q_1 are calculated.

Since the small-signal resistor r_{E1} is not a physical resistor, it does not contribute any noise. As a result, the equivalent input noise sources can be readily calculated using the results obtained for the standard common-emitter configuration.

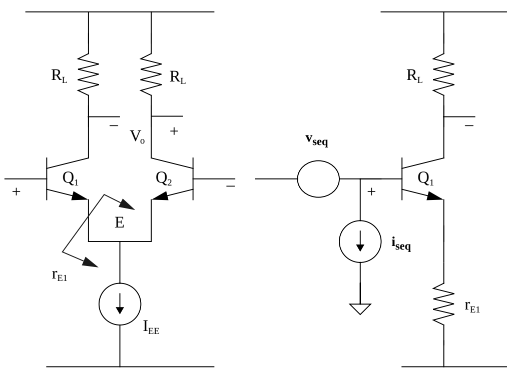

Fig. 1.13 The differential pair used as an example for noise calculation

The input voltage noise source v_{seq} can be proved to be

$$\overline{v^2_{seq}} = \overline{v^2_{eq}} \tag{1.54}$$

and the input current noise is given by

$$\overline{i^2_{seq}} = \overline{i^2_{eq}} \tag{1.55}$$

where v_{eq} and i_{eq} are the standard equivalent noise sources of bipolar transistors in the common-emitter configuration as described earlier in Fig. 1.7.

In practice, the bias current source also contains noise. However, since the noise itself is a common-mode signal, it does not affect the output differential signal and thus can be typically neglected in the noise calculation.

1.8 Noise in Operational Amplifiers

In general, low-noise amplifiers need to be as simple and contains as few devices as possible. However, in many applications, a cascade of many amplifier stages as shown in Fig. 1.14 is indispensable to achieve enough gain. In that case, each additional stage would inevitably contribute more noise to the total input-referred noise.

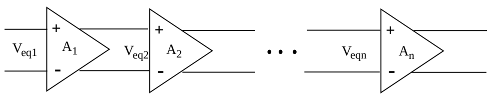

Fig. 1.14 Noise calculation for a cascade of many amplifier stages

In Fig. 1.14, if the gain and the input-referred voltage noise of each amplifier are assumed to be A_1, V_{eq1}, A_2, V_{eq2},..., A_n, and V_{eqn}, respectively, the total input-referred voltage noise v_{eqT} can be estimated as

$$V_{eq_T} = V_{eq_1} + \frac{V_{eq_2}}{A_1} + \frac{V_{eq_3}}{A_1 \cdot A_2} + \ldots + \frac{V_{eq_n}}{A_1 \ldots A_{(n-1)}} \tag{1.56}$$

By making the gain of the first amplifier stage A_1 high, the noise contribution of the subsequent stages can be neglected, and the total input-referred voltage noise can be approximated as merely the noise from the first stage. As such, only the noise contribution of the first stage is of importance, and the noise analysis of the whole system is much simplified to that of the first stage.

As an example, let us estimate the noise for the 741 op amp. Since the input stage has some voltage gain and current gain, using the result discussed above, it is sufficient to estimate the noise contribution from the input stage and estimate it as the noise for the whole amplifier.

Figure 1.15 shows the simplified schematic of the input stage. To simplify the problem, it is desirable to estimate separately the noise contribution of the active load devices (Q_5, Q_6, R_1 and R_2), that of the level-shift devices (Q_3 and Q_4), and that of the input devices (Q_1, Q_2 and Q_8). Note that the noise contribution by the resistor R_3 in the original schematic can be neglected because of its large resistance value. The noise contribution by Q_7 can be easily included but ignored here for simplicity.

For the active load devices, the total contribution to the output current noise can be calculated using the equivalent schematic with noise sources shown in Fig. 1.16. Since the equivalent resistance

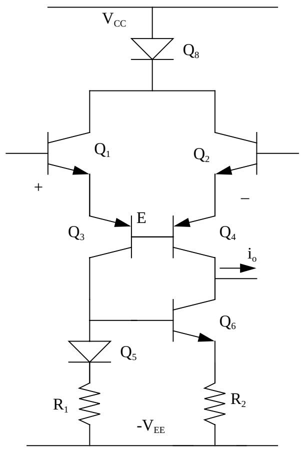

Fig. 1.15 Simplified schematic for the 741 input stage for noise calculation

R_{eq5} is much smaller than R_{eq6}, the contribution of the current noise i_{eq6} to the output can be neglected, and the total equivalent voltage noise at the base of Q_6 can be estimated as,

$$\overline{v^2_{eq(5,6)}} = \overline{v^2_{eq6}} + \overline{v^2_{R1}} + \overline{v^2_{b5}} + \overline{i^2_{d5}} \times \frac{1}{g^2_{m5}} = 4kT\left(r_{b6} + \frac{1}{2g_{m6}} + R_2 + R_1 + r_{b5} + \frac{1}{2g_{m5}}\right)\Delta f \tag{1.57}$$

Referring this equivalent voltage noise to the output, its contribution to the output current noise is given by,

$$\frac{\overline{i^2_{oL}}}{\Delta f} = \frac{\overline{v^2_{eq(5,6)}}}{\Delta f} \times G^2_{m6} = 4kT\left(r_{b6} + \frac{1}{2g_{m6}} + R_2 + R_1 + r_{b5} + \frac{1}{2g_{m5}}\right) \times \left(\frac{g_{m6}}{1 + g_{m6} \times R_2}\right)^2 \tag{1.58}$$

Using the real values with $I_{C5} = I_{C6} = 12\ \mu A$, $R_1 = R_3 = 1\ k\Omega$, and $r_{b5} = r_{b6} = 500\ \Omega$, the current noise contribution can be evaluated to become $8.6 \times 10^{-24}\ A^2/Hz$. Equivalently, this current noise can be considered to be contributed by the thermal noise of a 1.94-kΩ resistor.

Referring to the input, the input-referred noise contribution of the active load is given by,

$$\frac{\overline{v^2_{ieqL}}}{\Delta f} = \frac{\overline{i^2_{oL}}}{\Delta f} \times \frac{1}{G^2_{m1}} = \overline{i^2_{oL}} \times \frac{4}{g^2_{m1}} = 1.6x10^{-16}(V^2/Hz) = 4kT(9.64k\Omega) \tag{1.59}$$

For the level-shift devices Q_3 and Q_4, the input-referred noise is given by,

$$\begin{aligned} \frac{\overline{v^2_{ieq3}}}{\Delta f} &= \frac{\overline{v^2_{eq3}}}{\Delta f} \times \left(\frac{v_{in}}{v_{E3}}\right)^2 + \frac{\overline{i^2_{eq3}}}{\Delta f} \times \left(\frac{1/2}{g_{m1}/2}\right)^2 = \frac{\overline{v^2_{eq3}}}{\Delta f} \times 4 + \frac{\overline{i^2_{eq3}}}{\Delta f} \times \left(\frac{1}{g_{m1}}\right)^2 \\ &= 4kT\left(4r_{b3} + \frac{2}{g_{m3}}\right) + 2qI_{B3} \times \left(\frac{1}{g_{m1}}\right)^2 = 10.5x10^{-17}(V^2/Hz) = 4kT(6.33k\Omega) \end{aligned} \tag{1.60}$$

where a current gain of 50 has been assumed for Q_3. Note that the contribution of the current noise is negligibly small because of the small base current.

Finally, for the input devices Q_1 and Q_2, the noise contribution is readily given as,

$$\frac{\overline{v^2_{eq1}}}{\Delta f} = 4kT\left(r_{b1} + \frac{1}{2g_{m1}}\right) = 2.63x10^{-17}(V^2/Hz) = 4kT(1.58k\Omega) \tag{1.61}$$

Ignoring the contribution by the diode-connected transistor Q_8 because it represents a common-mode signal and putting all the results together, the total input-referred voltage noise can be estimated as,

$$\frac{\overline{v^2_{iT}}}{\Delta f} = \frac{\overline{v^2_{ieqL}}}{\Delta f} + 2\frac{\overline{v^2_{ieq3}}}{\Delta f} + 2\frac{\overline{v^2_{eq1}}}{\Delta f} = 4kT(25.46k\Omega) \tag{1.62}$$

which corresponds to a rms value approximately of $20\ nV/Hz^{1/2}$.

The total input-referred current noise can be estimated in the same way. However, since the current gain of the input devices is as large as 250, the total input-referred current noise can be approximated to be contributed only by the input devices Q_1 and Q_2, which is,

$$\frac{\overline{i^2_{iT}}}{\Delta f} = 2qI_{B1} = 2q(48nA) \tag{1.63}$$

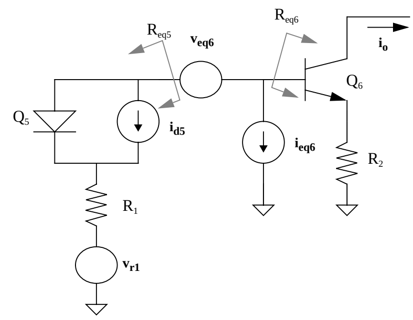

Fig. 1.16 Noise contribution of the active load for the 741 input stage

where a current gain of 250 has been assumed for Q_1.

As a result, the total noise sources for the whole 741 op amp can be estimated by the noise sources of the input stage v_{iT} and i_{iT} as given by Eqs. 1.62 - 1.63 and as illustrated in Fig. 1.17.

As another example, let us calculate the noise performance of a CMOS single-stage amplifier shown in Fig. 1.18. At low frequencies, the current noise is dominated by the gate leakage current and thus can be neglected. Therefore, only voltage noise sources are to be considered.

Similar to the noise analysis for 741 op amp, the total input-referred noise can be easily derived to be,

$$\begin{aligned} \overline{v^2_{eqT}} &= \overline{v^2_{eq1}} + \overline{v^2_{eq2}} + (\overline{v^2_{eq3}} + \overline{v^2_{eq4}}) \times \left(\frac{g_{m3}}{g_{m1}}\right)^2 \\ &= \frac{2K_{fP}}{W_1L_1C_{ox}f}\left(1 + \frac{K_{fN}\mu_N L_1^2}{K_{fP}\mu_P L_3^2}\right)\Delta f + 4kT\frac{4}{3g_{m1}}\left(1 + \frac{g_{m3}}{g_{m1}}\right)\Delta f \end{aligned} \tag{1.64}$$

To minimize the contribution from both the flicker and thermal noise, it is desirable to design the amplifier so that the transconductance of the input devices is much larger than that of the load devices. In addition, it is quite interesting to note that the noise contribution due to the flicker noise

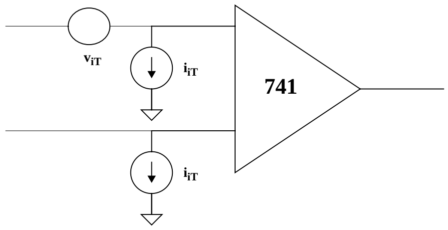

Fig. 1.17 Equivalent input noise sources for the 741 op amp

does not depend on the width of the load devices and can be minimized if the WL product of the input devices is maximized while minimizing the ratio L_1/L_3.

1.9 Noise Terminologies

1.9.1 Noise Bandwidth

Consider a circuit with a white equivalent input noise spectral density given by

$$\frac{\overline{v_i^2}}{\Delta f} = S_i(f) = S_{io} \tag{1.65}$$

and the magnitude squared of the voltage gain $|A_v(jf)|^2$ as shown in Fig. 1.19a.

The total output noise is given by,

$$\overline{v_o^2} = \int_0^\infty S_i(f)|A_v(jf)|^2 df = S_{io}\int_0^\infty |A_v(jf)|^2 df \tag{1.66}$$

It is quite useful to be able to represent the gain-frequency characteristic in Fig. 1.19a by an equivalent "brick-wall" transfer function as shown in Fig. 1.19b, which by definition has the same low-frequency gain as the original but has an abrupt band edge at the frequency f_N and would result in the same output noise as the original. The frequency f_N is then referred to as the noise bandwidth.

The output noise contribution by the equivalent transfer function can be obtained as,

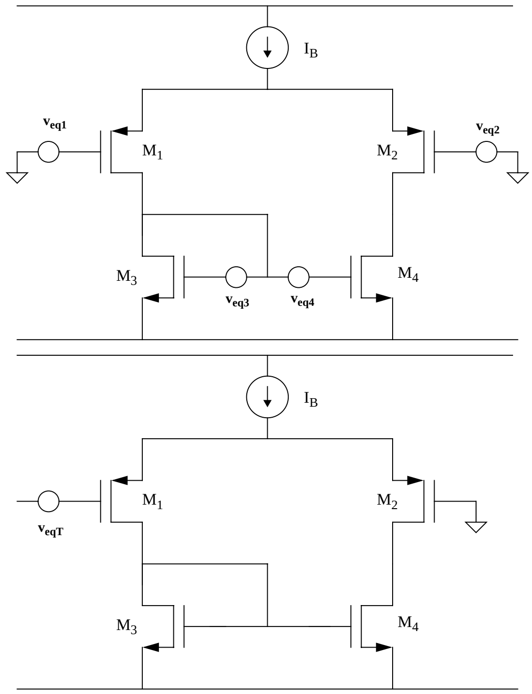

Fig. 1.18 Equivalent noise sources for a CMOS single-stage amplifier

$$\overline{v_o^2} = \int_0^\infty S_i(f)|A_v(jf)|^2 df = A_o^2\int_0^{f_N} S_i(f)df = A_o^2 S_{io} f_N \tag{1.67}$$

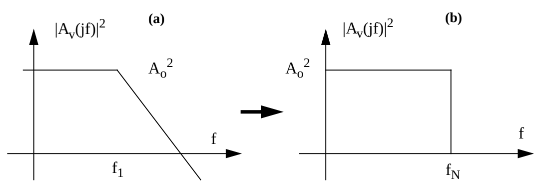

Fig. 1.19 Definition and representation of noise bandwidth

As a result, by definition, the noise bandwidth f_N can be determined as,

$$f_N = \frac{1}{A_o^2}\int_0^\infty |A_v(jf)|^2 df \tag{1.68}$$

The advantage for doing so is that the noise bandwidth only depends on the frequency response of the circuit and that once the noise bandwidth is determined, the total output noise can be easily obtained simply by multiplying it with the input noise spectral and the low-frequency gain as given by Eq. 1.67.

For a simple amplifier with a single dominant pole at f_1, that is,

$$A_v(jf) = \frac{A_o}{1 + j\frac{f}{f_1}} \tag{1.69}$$

the noise bandwidth becomes

$$f_N = \frac{1}{A_o^2}\int_0^\infty |A_v(jf)|^2 df = \int_0^\infty \left|\frac{1}{1 + j\frac{f}{f_1}}\right|^2 df = \frac{\pi}{2} f_1 \tag{1.70}$$

1.9.2 Noise Figure

Noise performance of a circuit or noise degradation due to a circuit can be specified by its f noise figure F, which is defined as,

$$F = \frac{(S/N)_i}{(S/N)_o} = \frac{S_i N_o}{N_i S_o} = \frac{N_o}{GN_i} = \frac{GN_i + N_{o,ckt}}{GN_i} = 1 + \frac{N_{o,ckt}}{GN_i} = 1 + \frac{N_{i,ckt}}{N_i} \tag{1.71}$$

where G is the power gain of the circuit, S_i, S_o, N_i, and N_o are the signal and noise power levels at the input and output, respectively. The input noise power N_i is due to the source resistance, $N_{i,ckt}$ is the input-referred noise of the circuit, and the output noise power N_o includes both the output-referred noise of the circuit $N_{o,ckt}$ and the output-referred noise due to the source resistance GN_i.

It is necessary that the bandwidth is specified in order to calculate the noise power and thus the noise figure. Spot noise figure is defined as the noise figure for a very small bandwidth Δf at a frequency f where $\Delta f << f$.

Consider a typical circuit representation with equivalent noise sources shown in Fig. 1.20. The output noise power due to the circuit itself is obtained as

$$N_{o,ckt} = \frac{A^2\overline{v_1^2}}{R_L} = \frac{A^2}{R_L}\left[\overline{v_{eq}^2}\frac{|z_i|^2}{|z_i + R_S|^2} + \overline{i_{eq}^2}\frac{R_S^2|z_i|^2}{|z_i + R_S|^2}\right] \tag{1.72}$$

The output-referred noise power due to the source resistance is given by

$$N_{o,R_S} = \frac{A^2}{R_L}\left[\overline{v_s^2}\frac{|z_i|^2}{|z_i + R_S|^2}\right] \tag{1.73}$$

It follows that the noise figure becomes

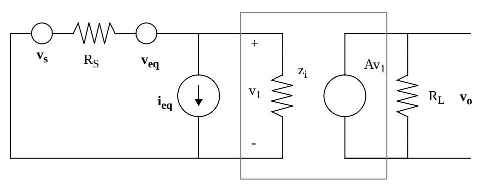

Fig. 1.20 Typical representation of an amplifier with its equivalent input noise generators

$$F = 1 + \frac{N_{o,\,ckt}}{N_{o,\,R_S}} = 1 + \frac{\overline{v_{eq}^2}\frac{|z_i|^2}{|z_i + R_S|^2} + \overline{i_{eq}^2}\frac{R_S^2|z_i|^2}{|z_i + R_S|^2}}{\overline{v_s^2}\frac{|z_i|^2}{|z_i + R_S|^2}} = 1 + \frac{\overline{v_{eq}^2}}{4kTR_S\Delta f} + \frac{\overline{i_{eq}^2}}{4kT\frac{1}{R_S}\Delta f} \tag{1.74}$$

For small values of the source resistance R_S, the equivalent voltage noise source dominates, and the noise figure F decreases with R_S. On the other hand, for large values of the source resistance R_S, the equivalent current noise source becomes dominant, and the noise figure F increases with R_S. As such, there exists an optimal value of R_S that minimizes the noise figure F given by

$$R_{S,\,opt}^2 = \frac{\overline{v_{eq}^2}}{\overline{i_{eq}^2}} \tag{1.75}$$

As an illustration, consider the noise figure of a bipolar transistor at low frequencies. The optimal value of the source resistance is

$$R_{S,\,opt}^2 = \frac{\overline{v_{eq}^2}}{\overline{i_{eq}^2}} = \frac{4kT\left(r_b + \frac{1}{2g_m}\right)}{2qI_B} = \frac{\beta_F}{g_m^2}(1 + 2g_m r_b) \tag{1.76}$$

with which the minimum noise figure becomes

$$F_{min} = 1 + \sqrt{\frac{1 + 2g_m r_b}{\beta_F}} \tag{1.77}$$

Typically, this corresponds to an optimal source resistance of $R_{S,opt}$ = 570 Ω and F_{min} = 1.22 = 0.9 dB for a current I_C = 1 mA, β_F = 100 and r_b = 50.

1.9.3 Noise Temperature

As an alternative to the noise figure, noise performance of a circuit can be represented by its noise temperature, which is defined as the temperature of the source resistance at which the output noise due to the circuit is equal to the output noise due to the source resistance, i.e. F = 2 = 3 dB.

For the circuit representation in Fig. 1.20, the noise temperature T_n can be shown to be

$$T_n = T(F - 1) \tag{1.78}$$

where F is the noise figure in the linear scale and T is the actual temperature at which the noise performance is evaluated.

At the room temperature, T = 300 K, a noise figure F = 2 = 3 dB corresponds to a noise temperature of T_n = 300 K, and similarly, a noise temperature of T_n = 30 K corresponds to a noise figure of F = 1.1 = 0.4 dB.