Feedback

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Take-home message

Negative feedback for amplifiers improves most performance measures, except noise, at the price of gain and potential stability problems

Negative feedback

Trade high gain for:

a well-specified (but lower) gain

impedances that matches your source and load better

higher bandwidth

lower non-linear distortion

Trade sensitivity to the active element properties for sensitivity to the feedback net (which hopefully it is more well-controlled)

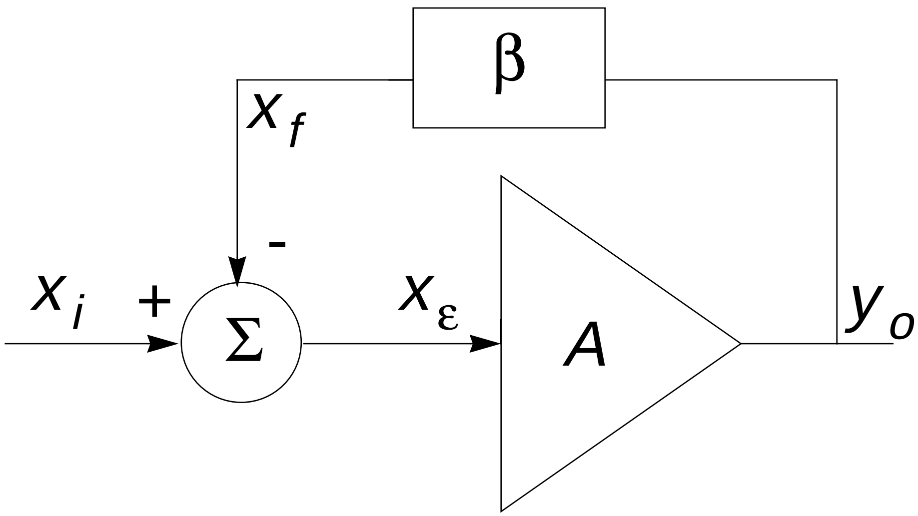
Worry about stability!

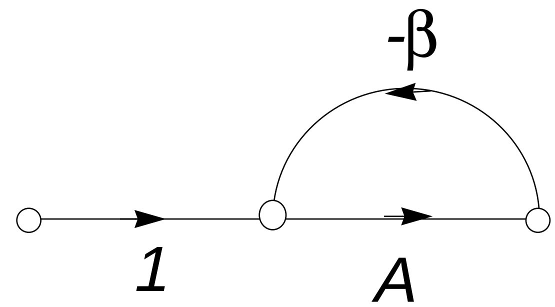
Overview

- *Gain*
  - *Black's feedback model*
  - *Superposition feedback model (Cherry, Nordholt)*
  - *Asymptotic gain*
- *Impedances*
  - *Blackman's impedance relation*
  - *Asymptotic impedance*
  - *Nullor - a general amplification element*
  - *Ideal single-loop feedback configurations*
- *Stability*
  - *Stability criteria*
  - *Feedback of one to four-pole systems*

◆ *Compensation*

Black's feedback model





signal flow graph

$$y_o = \frac{A}{1 + A\beta} x_i \approx \frac{1}{\beta} x_i$$

Book uses H instead of β but β is the most common choice (due to Bode)

Minus sign is used control theory - traditionally no minus sign in electronics!

Signal-flow graphs

A graphical representation of a set of linear equations

Implies a calculation order (source, sink)

Node: represents a specified quantity or a variable.
value = sum of branch outputs entering the node

Branch: represents functional dependence between the variables. Each branch has an arrow. Signals travel only in the branch direction.

Branch transmission: linear quantity relating one node to another. A signal travelling along a branch is multiplied by the transmission of the branch

Reduction of signal-flow graphs

Removal: a node may be removed as long as the transmissions between remaining nodes remain unchanged.

Serial connection: multiply branch transmissions to get total transmission

Parallel: add transmissions to get total transmission

Self loop: divide transmission of branches entering the node by (1-transmission of self loop)

Branch inversion: replace the transmission μ *by its inverse* $1/\mu$

Gain (according to Black's model)

Closed-loop gain

$$A_{CL} = \frac{A}{1 + \beta A}$$

Asymptotic closed-loop gain

$$A_{CL\infty} = \lim_{A \to \infty} A_{CL} = \frac{1}{\beta}$$

Gain definitions

Closed-loop gain $$A_{CL} = \frac{A}{1+\beta A} = \frac{1}{\beta}\left(\frac{\beta A}{1+\beta A}\right)$$

Asymptotic closed-loop gain: $$A_{CL\infty} = \frac{1}{\beta}$$

Loop gain: $$T = -\beta A$$

Return difference:
Amount-of-feedback: $$F = 1 - T = 1 + \beta A$$

Discrepancy: $$\frac{A_{CL}}{A_{CL\infty}} = \frac{-T}{F}$$

Nonideal feedback model due to Cherry & Nordholt

Linear active network with input Q_g; output Q_L.

Choose one element (usually a controlled source) as a reference element. Element input Q_i; controlled output Q_c

Due to superposition we can write:

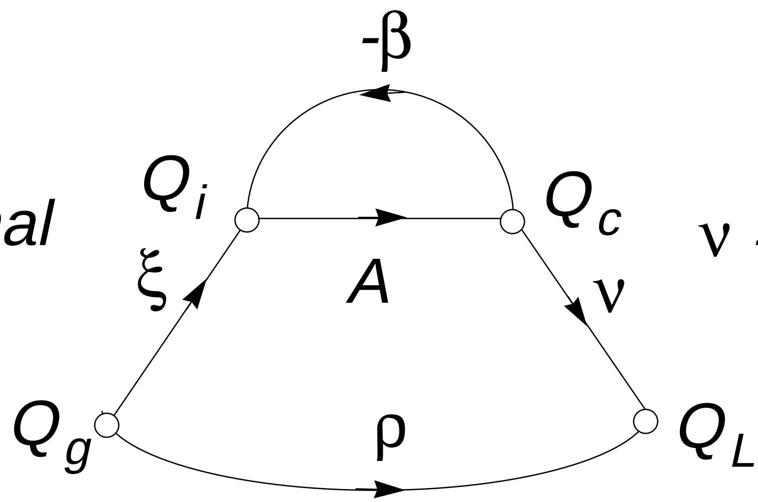
$$\begin{cases} Q_L = \rho Q_g + \nu Q_c \\ Q_i = \xi Q_g - \beta Q_c \end{cases}$$

In addition the reference element adds this equation:

$$Q_c = A Q_i$$

Gain in nonideal model

Signal-flow graph for nonideal feedback model:



ξ *= input signal loss*

ν *= output signal loss*

ρ *= direct transfer*

Closed-loop gain: $A_{CL} = \dfrac{Q_L}{Q_g} = \rho + \dfrac{\nu\xi A}{1 + \beta A}$

Asymptotic gain: $A_{CL\infty} = \rho + \dfrac{\nu\xi}{\beta}$

Gain definitions - nonideal model

Closed-loop gain

$$A_{CL} = \rho + \frac{\xi\nu A}{1+\beta A} = \left(\rho + \frac{\xi\nu}{\beta}\right)\left(\frac{\beta A}{1+\beta A}\right) + \frac{\rho}{1+\beta A}$$

$$A_{CL} \approx \left(\rho + \frac{\xi\nu}{\beta}\right)\left(\frac{\beta A}{1+\beta A}\right)$$

Asymptotic closed-loop gain:

$$A_{CL\infty} = \rho + \frac{\xi\nu}{\beta}$$

Loop gain:

$$T = -\beta A$$

Return difference:
Amount-of-feedback:

$$F = 1 - T = 1 + \beta A$$

Discrepancy:

$$\frac{A_{CL}}{A_{CL\infty}} \approx \frac{-T}{F}$$

Blackman's impedance relation

The impedance between any two terminals, x and y, in an active network with feedback can be expressed as:

$$Z_{xy} = Z^{0}_{xy}\frac{1 + A\beta_{sc}}{1 + A\beta_{oc}}$$

Z^{0}_{xy} *is the impedance with a chosen element removed*
Usually it is the controlled source in the feedback loop

$-A\beta_{sc}$ *is the loop gain with x and y shorted*

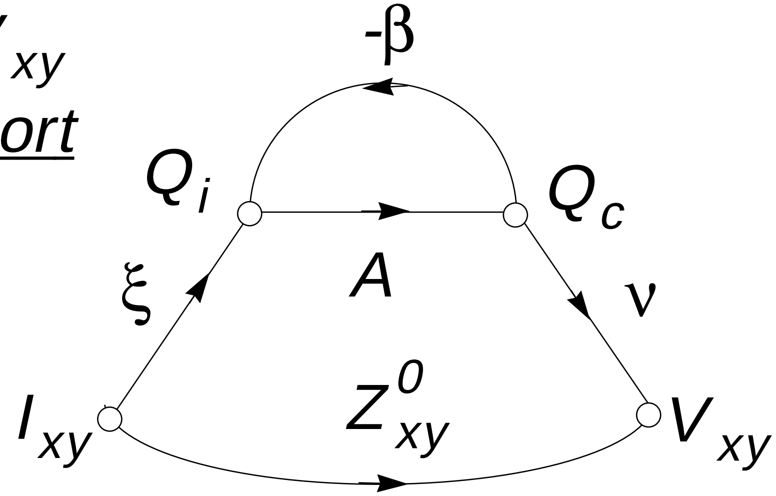
$-A\beta_{oc}$ *is the loop gain with x and y open*

Similarly we have for admittance:

$$Y_{xy} = Y^{0}_{xy}\frac{1 + A\beta_{oc}}{1 + A\beta_{sc}}$$

Blackman's relation as superposition

I_{xy} is the input current and V_{xy} is the response at the same port



$$\begin{cases} V_{xy} = Z_{xy}^0 I_{xy} + \nu Q_c \\ Q_i = \xi I_{xy} - \beta Qc \end{cases}$$

$$Q_c = A Q_i$$

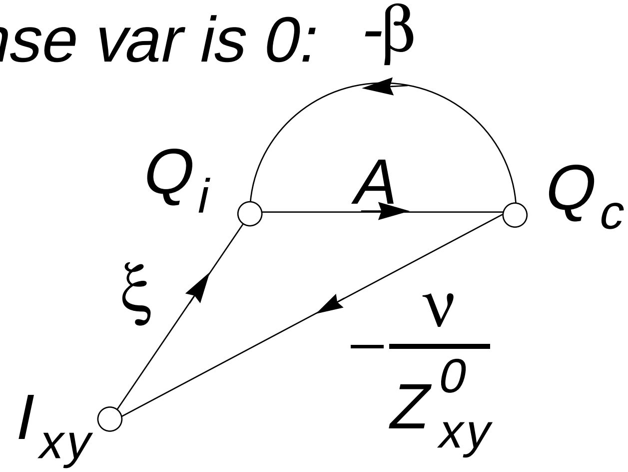
$$Z_{xy} = \frac{V_{xy}}{I_{xy}} = Z_{xy}^0 + \frac{\xi \nu A}{1 + \beta A} = Z_{xy}^0 \frac{1 + \left(\beta + \frac{\upsilon\xi}{Z_{xy}^0}\right) A}{1 + \beta A}$$

The two feedback cases

with xy shorted => response var is 0:

$$\begin{cases} 0 = Z^0_{xy} I_{xy} + \nu Q_c \\ Q_i = \xi I_{xy} - \beta Qc \end{cases}$$

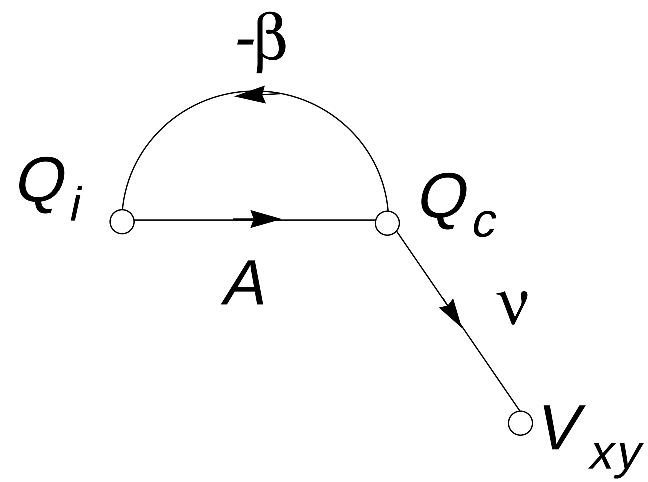
$$Q_c = A Q_i$$



with xy open => input var is 0:

$$\begin{cases} V_{xy} = \nu Q_c \\ Q_i = -\beta Qc \end{cases}$$

$$Q_c = A Q_i$$



Asymptotic impedances

$$Z_{xy\infty} = \lim_{A \to \infty} Z_{xy} = Z_{xy}^{0} \frac{\beta_{sc}}{\beta_{oc}}$$

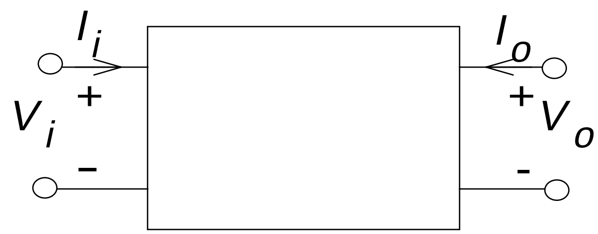
For a single-loop system one of the loop gains is 0 and the other one is large

The asymptotic impedances are then 0 or ∞

For an accurate asymptotic impedance two feedback loops are required

An aside about twoports

Four poles organized in two ports
Net current in each port is zero



Both a circuit element and a model for larger systems

Six matrix formulations: most useful are the transmission version (also called chain parameters):

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_o \\ -I_o \end{bmatrix}$$

$$A = \left.\frac{V_i}{V_o}\right|_{I_0 = 0} \qquad B = \left.\frac{V_i}{I_o}\right|_{V_0 = 0}$$

$$C = \left.\frac{I_i}{V_o}\right|_{I_0 = 0} \qquad D = -\left.\frac{I_i}{I_o}\right|_{V_0 = 0}$$

Two special twoports

Negative impedance converter (NIC):

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} -k_1 & 0 \\ 0 & 1/k_2 \end{bmatrix} \begin{bmatrix} V_o \\ -I_o \end{bmatrix}$$

Converts a positive voltage into a negative one

Gyrator:

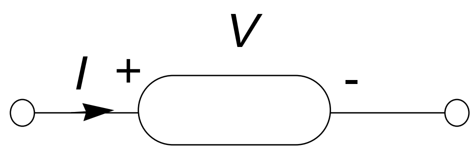
Switches current and voltage

$$\begin{bmatrix} V_i \\ I_i \end{bmatrix} = \begin{bmatrix} 0 & 1/g \\ g & 0 \end{bmatrix} \begin{bmatrix} V_o \\ -I_o \end{bmatrix}$$

Nullor - an ideal amplification element

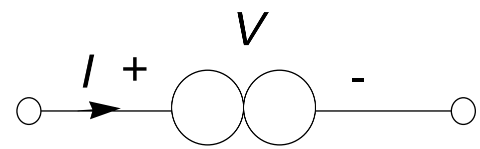
Two weird two-terminal elements:

nullator:



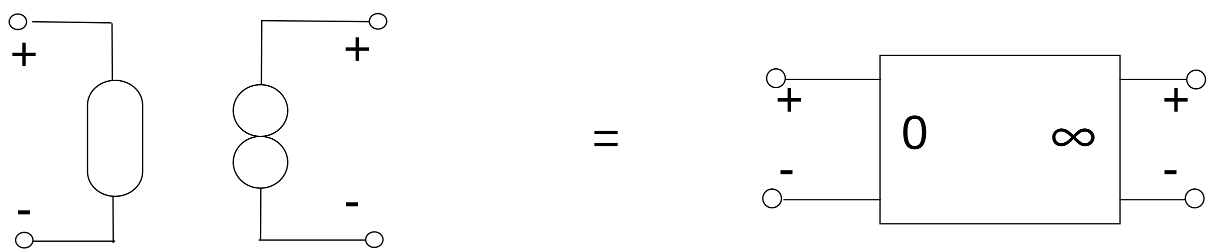
$$I = V = 0$$

norator:



$$V = k_1, I = k_2$$

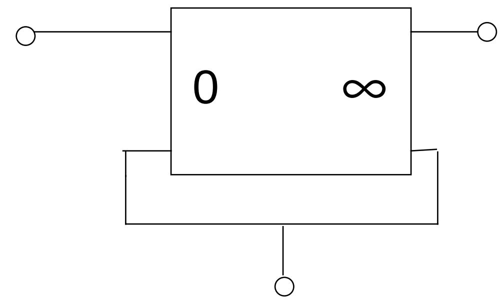
Combined - the nullor twoport:



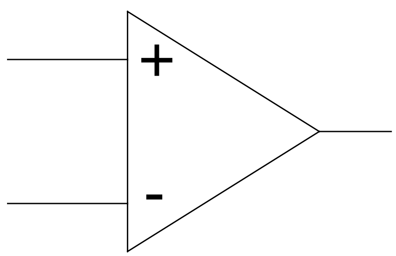
The nullor can be used to describe any active element!

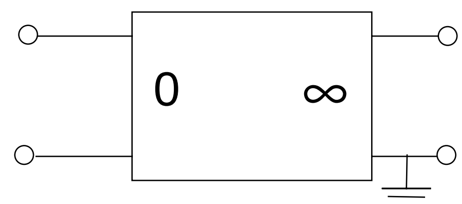
Nullor equivalents of active elements

Transistor:

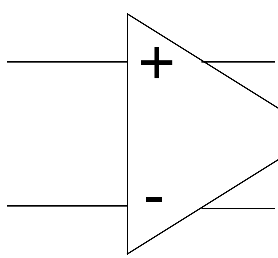


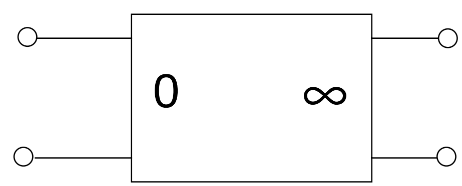
Single-ended opamp:





Double-ended opamp:

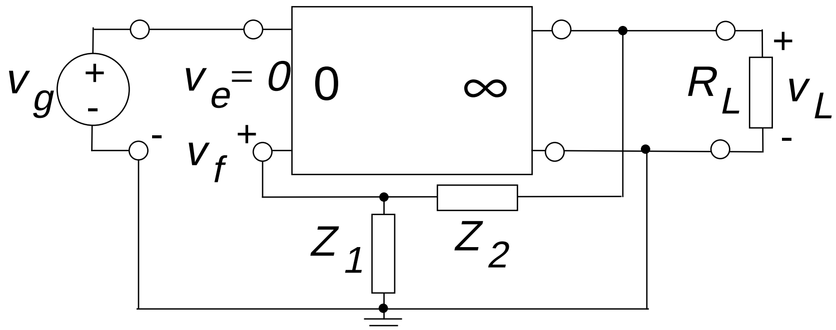




Feedback of ideal amplifier (nullor)

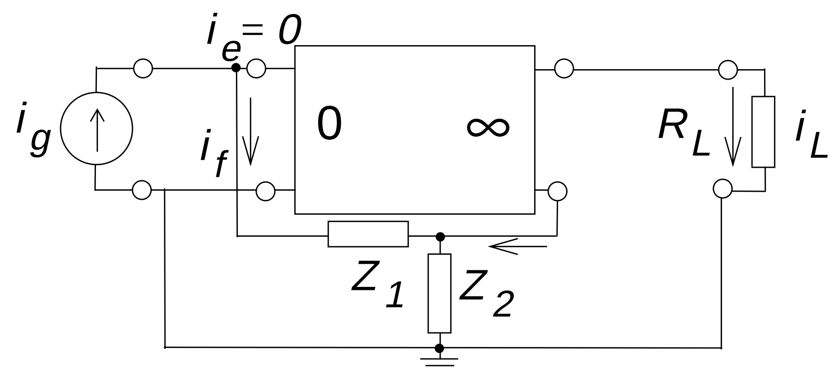
voltage amplifier
voltage in - voltage out = series - shunt

$$A_{CL} = \frac{v_L}{v_g} = 1 + \frac{Z_2}{Z_1}, Z_{in} = \infty, Z_{ut} = 0$$



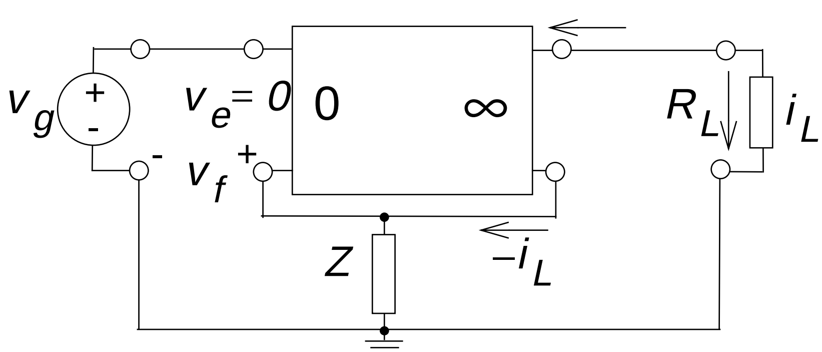
current amplifier
current in - current out = shunt - series

$$A_{CL} = \frac{-i_L}{i_g} = -\left(1 + \frac{Z_1}{Z_2}\right), Z_{in} = 0, Z_{ut} = \infty$$



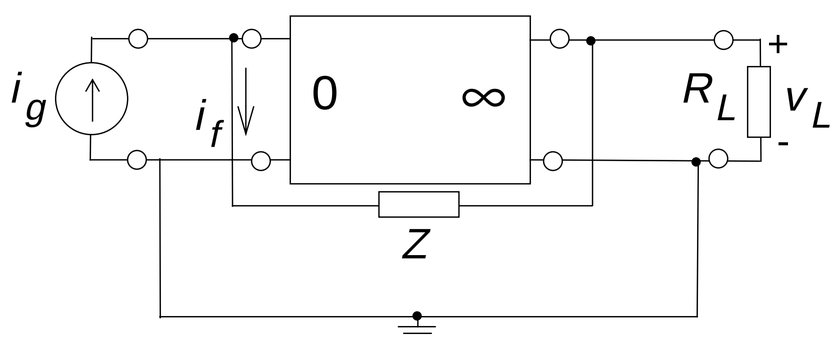
transadmittance amplifier
voltage in - current out = series - series

$$A_{CL} = \frac{-i_L}{v_g} = \frac{1}{Z}, Z_{in} = \infty, Z_{ut} = \infty$$



transimpedance amplifier
current in-voltage out = shunt - shunt

$$A_{CL} = \frac{v_L}{i_g} = -Z, Z_{in} = 0, Z_{ut} = 0$$



How to calculate output impedance

Replace load with test source (let's call it x)

Use a current source for a current output and voltage source for a voltage output

With a voltage input source the output impedance is:

$$Z_{out} = \left.\frac{v_x}{i_x}\right|_{v_g = 0}$$

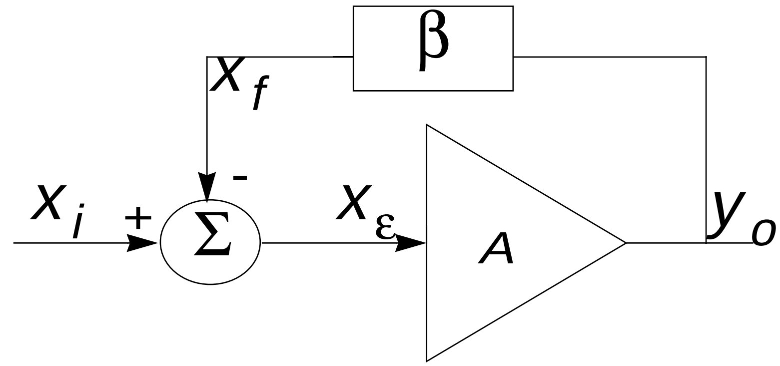
With a current input source the output impedance is:

$$Z_{out} = \left.\frac{v_x}{i_x}\right|_{i_g = 0}$$

Stability

$$A(s) = \frac{N_A(s)}{D_A(s)}$$

$$\beta(s) = \frac{N_\beta(s)}{D_\beta(s)}$$



$$A_{CL}(s) = \frac{N(s)}{D(s)} = \frac{A(s)}{1+\beta(s)A(s)} = \frac{D_\beta(s)N_A(s)}{D_\beta(s)D_A(s)+N_\beta(s)N_A(s)}$$

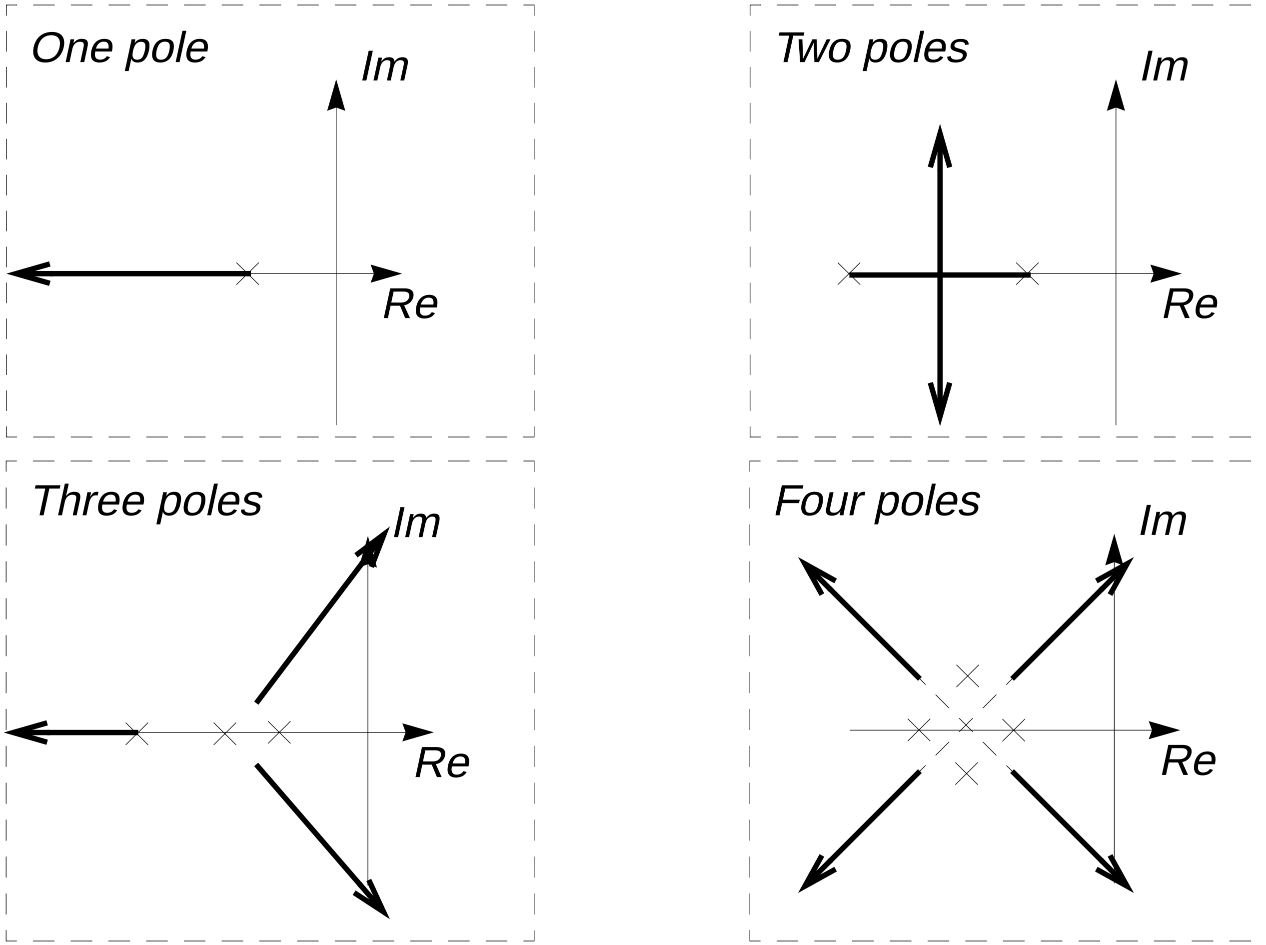
Stability = poles of D(s) have to be in left half plane

Often hard to compute

Offers no design insight!

Asymptotic pole movement

Assume $\beta(s) = \beta$, $A(s) = 1/N_A(s)$ *gives* $D(s) = 1 + \beta N_A(s)$



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Barkhausen's oscillation criteria

$$A_{CL}(s) = \frac{A(s)}{1 + \beta(s)A(s)}$$

Oscillates when poles are on the imaginary axes:

$$1 + \beta(j\omega_0)A(j\omega_0) = 0$$

In polar coordinates:

$$|\beta(j\omega_0)A(j\omega_0)|\, e^{-j\angle\beta(j\omega_0)A(j\omega_0)} = |-1|\, e^{-j180^\circ}$$

Two criteria for oscillation:

$$\begin{cases} |\beta(j\omega_0)A(j\omega_0)| = 1 \\ \varphi_\beta(j\omega_0) + \varphi_A(j\omega_0) = -180^\circ \end{cases}$$

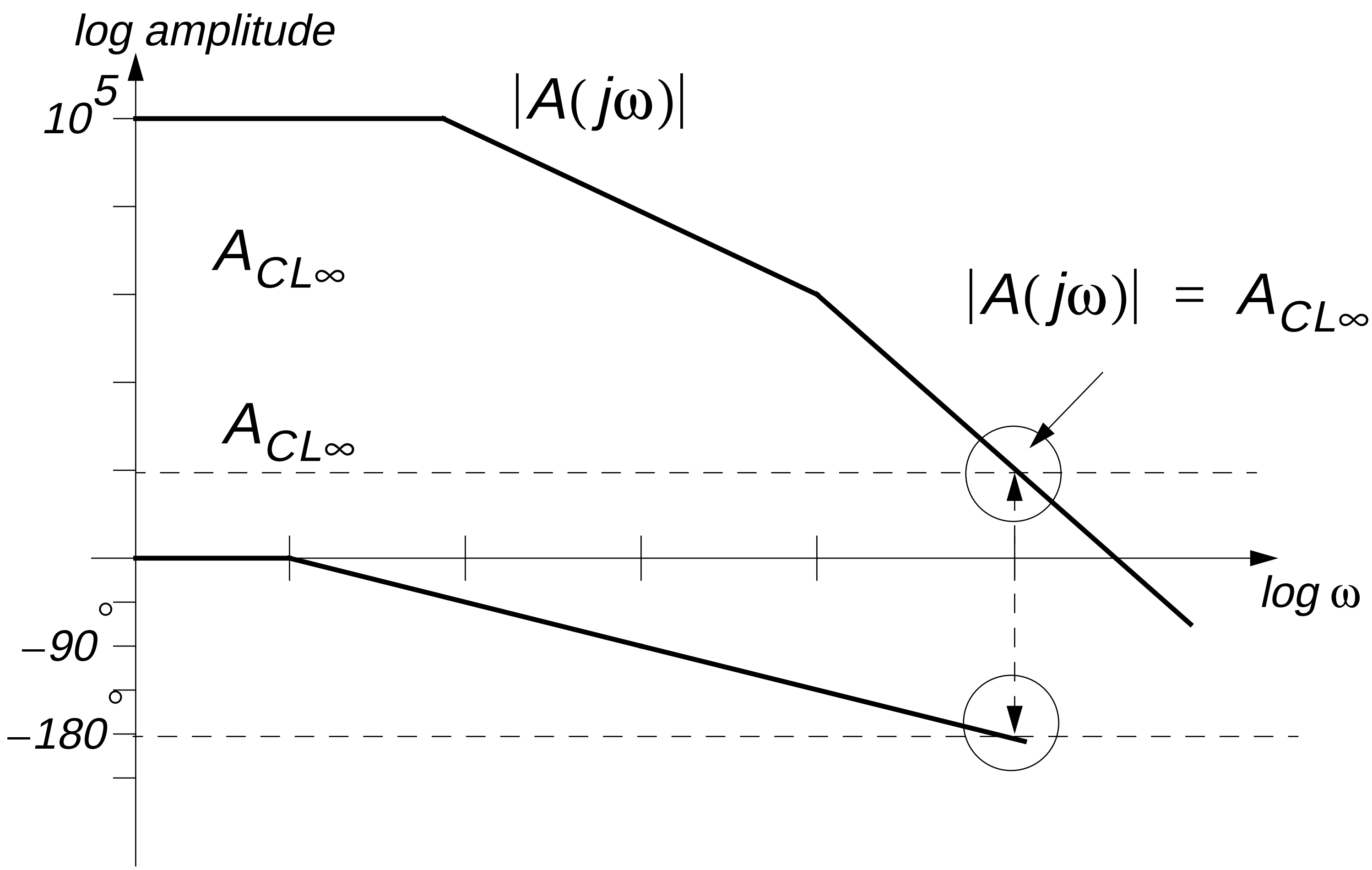
Simplified Barkhausen criteria

If the feedback β is independent of frequency the criteria can be simplified:

$$\begin{cases} |A(j\omega_0)| = \dfrac{1}{\beta} = A_{CL\infty} \\ \varphi_A(j\omega_0) = -180^\circ \end{cases}$$

Instability occurs when the open-loop and closed-loop gain curves intersect where the phase is -180.

A Bode-diagram illustration



Phase and gain margins

When one of the criteria is fulfilled, how large is the margin for the other criteria.

Definition:

ω_u = *the frequency at which the gain is unity (0 dB)*

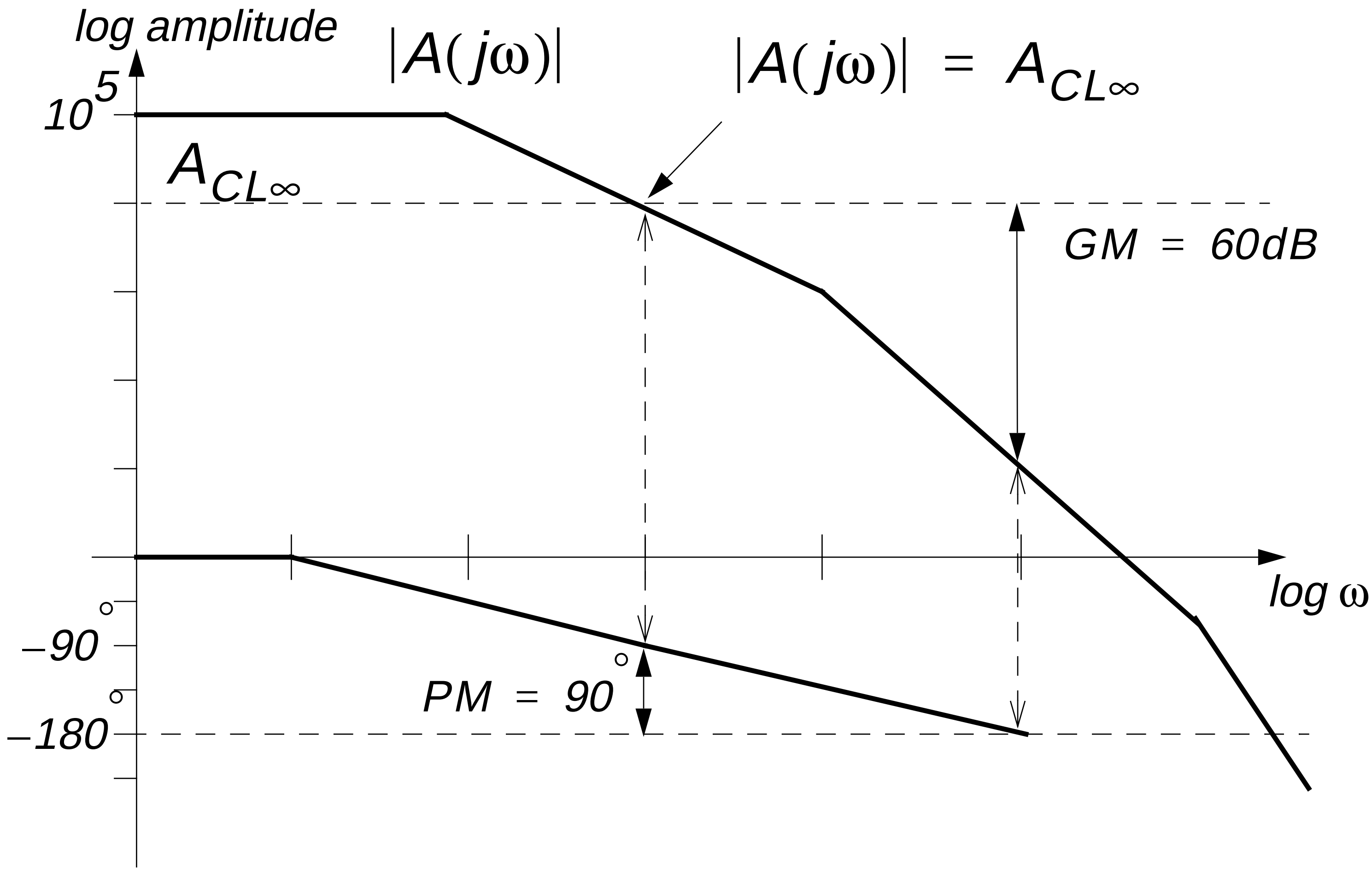
ω_φ = *the frequency at which the phase is -180*

$$PM = \varphi_T(j\omega_u) - (-180^\circ) = \varphi(j\omega_u) + 180^\circ$$

$$GM = \frac{1}{|T(j\omega_\varphi)|}$$

$$GM = 0dB - 20\log|T(j\omega_\varphi)| = -20\log|T(j\omega_\varphi)|$$

A Bode-diagram illustration



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Criteria for <u>avoiding</u> oscillation

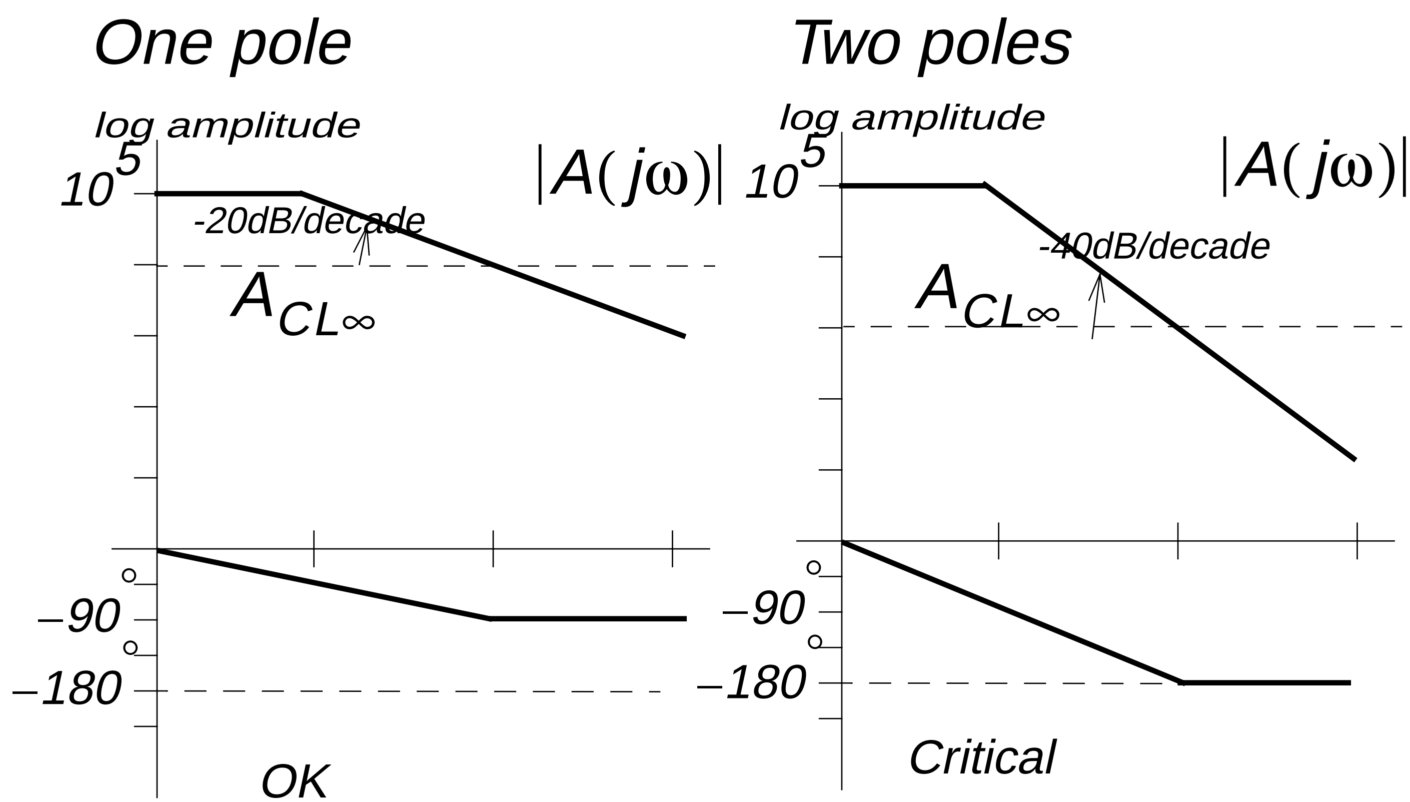
As Barkhausen was originally formulated:

$$\begin{cases} \left|\beta(j\omega_0)A(j\omega_0)\right| = 1 \\ \varphi_\beta(j\omega_0) + \varphi_A(j\omega_0) > -180^\circ \end{cases}$$

Reinterpreted with asymptotic gain:

$$\begin{cases} \left|A(j\omega_0)\right| = \dfrac{1}{\left|\beta(j\omega_0)\right|} = \left|A_{CL\infty}(j\omega_0)\right| \\ \varphi_A(j\omega_0) - \varphi_{CL\infty}(j\omega_0) > -180^\circ \end{cases}$$

Phase criterion may be interpreted from amplitude curve



Phase criterion cont.

For frequencies well above the pole's natural frequency the phase criterion can be interpreted as a gain-curve criterion:

$$slope(|A(j\omega_0)|) - slope(|A_{CL\infty}(j\omega_0)|) > -40dB/decade$$

This relation can be used if the poles are well separated (more than a frequency decade apart)

Compensation - how to optimize the loop gain

- *Tailor either A(s) or β(s) or both*
- *Make A(s)'s phase shift smaller where the gain curves intersect= phase retarding compensation or dominant-pole compensation*
  - *Add a dominant pole*
  - *Move the lowest pole (by cancelling it)*
- *Make β(s) phase shift larger where the gain curves cross.*
- *In opamp design the designer decides A(s), the user decides β(s)*
  - *Design for worst-case, that is follower where β = 1*

Add a dominant pole

Make sure phase margin is large enough (45 degrees) for desired asymptotic gain => place T=1 at lowest uncompensated pole:

$$\omega_c = \frac{\omega_1}{T_0} = \frac{\omega_1}{\beta A_0}$$

Move the lowest pole

Cancel existing lowest pole with a zero and add a new pole such that the distance between the two lowest poles is T0.

Add phase shift to feedback

Conclusion

With negative feedback you trade high gain for:

a well-specified (but lower) gain

impedances that matches your source and load better

higher bandwidth

lower non-linear distortion

You also trade sensitivity to the active element properties for sensitivity to the feedback net (which hopefully it is more well-controlled)

And you have to worry about stability!