Chapter 1

FEEDBACK ANALYSIS

1.1 Motivation

Negative feedback has been widely used in analog integrated circuits to:

1. Reduce the sensitivity of amplifier gain to parameter variations
2. Extend the bandwidth.
3. Reduce non-linearity and distortion
4. Control input and output impedance

As a trade-off, all of these can be achieved at the expense of gain reduction.

1.2 General Feedback Structure

According the general block diagram shown in Fig. 1.1, it is easy to obtain:

$$x_o = Ax_i = A(x_s - x_f) = A(x_s - fx_o) \qquad (1.1)$$

As a result, the overall gain A_f is given by:

$$A_f \equiv \frac{x_o}{x_s} = \frac{A}{1 + Af} \qquad (1.2)$$

where A_f is called the closed-loop gain, A the open-loop gain, Af the loop gain, and (1+Af) is the amount of feedback which measures the reduction factor for the gain.

Note: For negative feedback, it is necessary that x_s and x_f have the same polarity!

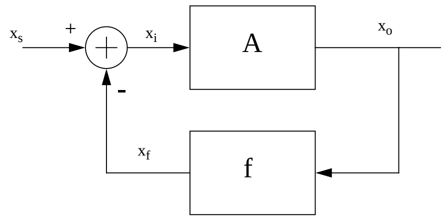


Fig. 1.1 General block diagram of a feedback system

1.2.1 Gain Insensitivity:

Typically, Af >> 1, and therefore,

$$A_f \equiv \frac{x_o}{x_s} \approx \frac{1}{f} \qquad (1.3)$$

That is, the closed-loop gain A_f of the feedback amplifier is dependent almost entirely on the feedback network! Since feedback networks usually consists of passive components, the closed-loop gain can be designed to be very stable and insensitive to the amplifier's parameter variations.

The sensitivity of the closed-loop gain can be obtained by differentiating Eq. 1.2 to get:

$$\frac{dA_f}{A_f} = \frac{dA}{(1+Af)^2}\frac{1}{A_f} = \frac{1}{(1+Af)}\frac{dA}{A} \qquad (1.4)$$

Compared to the percentage change in the open-loop gain (without feedback), the percentage change in the closed-loop gain A_f (with feedback) is reduced by the amount of feedback (1+Af)!

1.2.2 Bandwidth:

Assume that the open-loop gain has a transfer function A(s) with a 3dB bandwidth ω_{-3db}:

$$A(s) = \frac{A_o}{1 + \frac{s}{\omega_{-3db}}} \qquad (1.5)$$

With a feedback factor f, the transfer function of the closed-loop gain becomes:

$$A_f(s) = \frac{A(s)}{1 + A(s)f} = \frac{A_o}{1 + \frac{s}{\omega_{-3db}}} \cdot \frac{1}{1 + \frac{A_0 f}{1 + \frac{s}{\omega_{-3db}}}} = \frac{A_0}{1 + A_0 f} \cdot \frac{1}{1 + \frac{s}{\omega_{-3db}(1 + A_0 f)}} \qquad (1.6)$$

This goes to show that with feedback, the overall bandwidth is increased by the amount of feedback (1+A_0 f) and the closed-loop gain is reduced by the same amount!

1.2.3 Distortion:

Since negative feedback makes the closed-loop gain constant and dependent only on the feedback factor f, it also helps reduce distortion! Distortion can be seen by a change in the slope of the amplifier dc transfer curve and thus the change in gain of the amplifier.

In Fig. 1.2 are the two transfer characteristic curves of an amplifier with and without feedback. Without feedback, the distortion is shown as two different slopes A_1 and A_2. With the feedback, the closed-loop gains A_{f1} and A_{f2} are approximately the same:

$$A_{f_1} = \frac{A_1}{1 + A_1 f} \approx \frac{1}{f} \approx \frac{A_2}{1 + A_2 f} = A_{f_2} \qquad (1.7)$$

The distortion is indeed reduced although the gain is also reduced!

1.3 FEEDBACK ANALYSIS

There are four basic feedback topologies, including

1. Series-shunt (voltage amplifier),
2. Series-series (transconductance amplifier),
3. Shunt-shunt (transresistance amplifier), and
4. Shunt-series (current amplifier).

For each topology, we will learn how to calculate the overall small-signal parameters (gain, input resistance, and output resistance) first in the *ideal* situation and then in the *practical* situation.

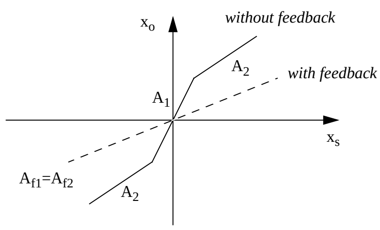


Fig. 1.2 An amplifier's transfer curve with and without feedback

1.4 Series-Shunt Feedback Analysis (Voltage Amplifier)

1.4.1 Ideal Situation:

Ideally, the feedback network does not have any loading effect on the basic amplifier as shown in Fig. 1.3

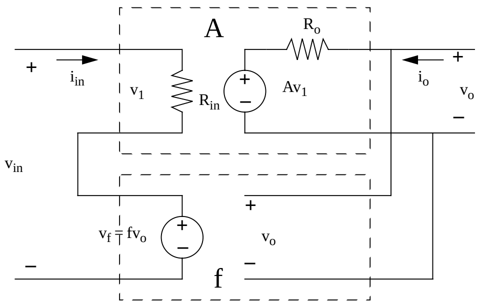


Fig. 1.3 Ideal feedback network without any loading effect

1.4.1.1 Gain:

It is easy to show that:

$$v_o = Av_1 = A(v_{in} - v_f) = A(v_{in} - fv_o) \quad (1.8)$$

As a result, the overall voltage gain A_f is given by:

$$A_f \equiv \frac{v_o}{v_{in}} = \frac{A}{1 + Af} \quad (1.9)$$

1.4.1.2 Input Resistance:

$$R_{in_f} \equiv \frac{v_{in}}{i_{in}} = \frac{v_{in}}{v_1 / R_{in}} = R_{in}\frac{v_1 + fv_o}{v_1} = R_{in}\frac{v_1 + fAv_1}{v_1} = R_{in}(1 + Af) \quad (1.10)$$

1.4.1.3 Output Resistance:

$$v_o = Av_1 + i_oR_o = -Afv_o + i_oR_o \quad (1.11)$$

As a result,

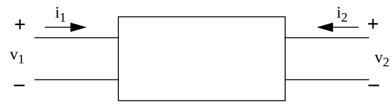
$$R_{o_f} \equiv \frac{v_o}{i_o} = \frac{R_o}{(1 + Af)} \quad (1.12)$$

1.4.2 Practical Situation:

1.4.2.1 Theory:

In practice, feedback network will inevitably load the basic amplifier and therefore change the original small-signal parameters. To understand the feedback theory, it is convenient to use an appropriate two-port network to represent the basic amplifier and the feedback network. (A complete summary and review of the four possible representations of a two-port network can be found from Appendix B in Sedra & Smith, "Microelectronic Circuits," pp. B.1 - B.7).

Since the basic amplifier and the feedback network have the same *input current* and the same *output voltage*, it is most convenient to use the *hybrid h-parameter* two-port representation shown in Fig. 1.4, which has *input current* and *output voltage* as the *independent variables*.



$$v_1 = h_{11}i_1 + h_{12}v_2$$

$$i_2 = h_{21}i_1 + h_{22}v_2$$

Fig. 1.4 Hybrid h-parameter two-port representation

where

$$h_{11} = \left.\frac{v_1}{i_1}\right|_{v_2 = 0} \quad (1.13)$$

$$h_{12} = \left.\frac{v_1}{v_2}\right|_{i_1 = 0} \quad (1.14)$$

$$h_{21} = \left.\frac{i_2}{i_1}\right|_{v_2 = 0} \quad (1.15)$$

$$h_{22} = \left.\frac{i_2}{v_2}\right|_{i_1 = 0} \quad (1.16)$$

This hybrid h-parameter representation can be used in the series-shunt feedback configuration as shown in Fig. 1.5, where h_a's and h_f's represent the h-parameters for the basic amplifier and the feedback network, respectively.

For such a typical amplifier, it is always valid to assume that:

$$h_{12a} \ll h_{12f} \quad (1.17)$$

$$h_{21f} \ll h_{21a} \quad (1.18)$$

As a result, the circuit can be much simplified as shown in Fig. 1.6, in which the basic amplifier is modified to include all the loading effects. The small-signal parameters for this open-loop circuit with loading, compared to those in the ideal circuit of Fig. 1.3, are easily obtained to be:

$$R_{in}' = h_{11a} + h_{11f} \quad (1.19)$$

$$R_o' = \frac{1}{h_{22a} + h_{22f}} \quad (1.20)$$

$$A' \equiv \frac{v_2}{v_{in}} = \frac{-h_{21a}R_o'}{R_{in}'} \quad (1.21)$$

and

$$f = h_{12f} \quad (1.22)$$

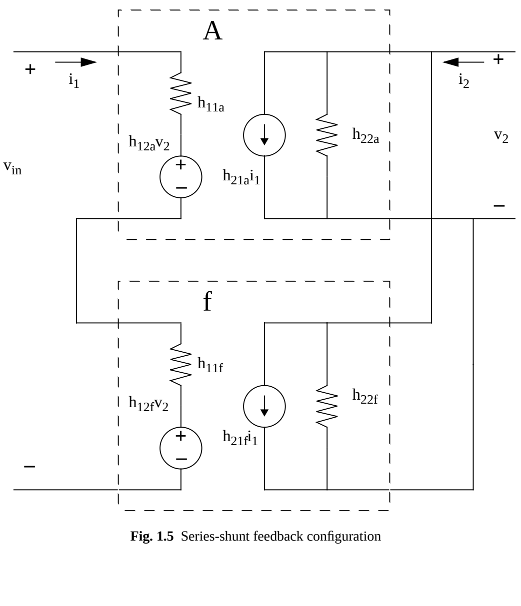


Fig. 1.5 Series-shunt feedback configuration

Note that the input loading h_{11f} is connected in *series* with the input and the output loading h_{22f} is connected in *parallel* with the output. These two loading parameters and the feedback factor h_{12f} can be found using Eqs. 1.13-1.15.

More specifically, regarding the feedback network, the loading h_{11f} is the output resistance with the *input shorted* ($v_2 = 0$), and the output loading h_{22f} is the input resistance with the *output opened* ($i_1 = 0$). Similarly, the feedback factor f is the voltage gain, which is defined as the ratio of the output voltage and the input voltage.

Since the circuit of Fig. 1.6 is in the same form of an ideal feedback configuration, the closed-loop small-signal parameters (A_f, R_{inf}, R_{of}) can be calculated using the ideal feedback equations derived in Eqs. 1.9, 1.11, and 1.12.

1.4.2.2 Procedure:

To take into account the loading effect of the feedback network,

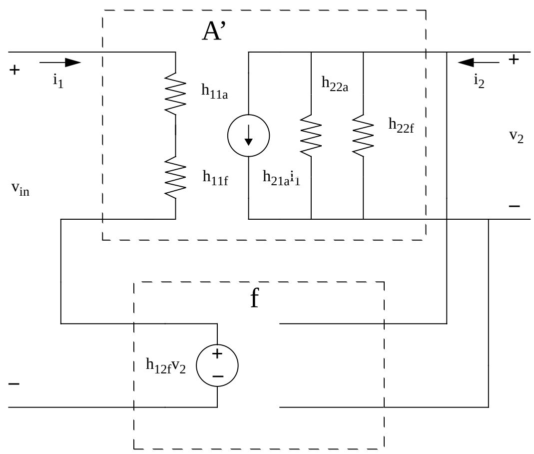


Fig. 1.6 Simplified version of series-shunt feedback configuration including loading efffect

i) Break the loop; determine and include *appropriate loading*

ii) Calculate the feedback factor f and the *open-loop* small-signal parameters *with loading A', R_{in}', R_o'*.

iii) Calculate the closed-loop small-signal parameters by applying the following formulas:

$$A_f \equiv \frac{A'}{1 + A'f} \tag{1.23}$$

$$R_{in_f} = R_{in}'(1 + A'f) \tag{1.24}$$

$$R_{o_f} = \frac{R_o'}{(1 + A'f)} \tag{1.25}$$

1.4.2.3 Appropriate Loading:

Refer to Figs. 1.7a and 1.7b,

i) For *input* loading R_f, *short* the input of the feedback network and calculate the output resistance of the feedback network (*series* loading) (Fig. 1.7a).

ii) For *output* loading R_{Lf}, *open* the output of the feedback network and calculate the input resistance of the feedback network (*parallel* loading) (Fig. 1.7b).

1.4.2.4 Open-Loop Small-Signal Parameters With Loading:

Refer to Figs. 1.7c and 1.7d,

i) With the loading R_f in series with the input and R_{Lf} in parallel with the output of the basic amplifier, the open-loop small-signal parameters with loading can be determined as A', R_{in}', and R_o', (Fig. 1.7c) where:

$$A' \equiv \frac{v_o'}{v_{in}'} \tag{1.26}$$

ii) The feedback factor f can be obtained by applying a test voltage v_o' at the input of the feedback network and measuring the open-circuit voltage v_f' at the output of the feedback network (Fig. 1.7d) and

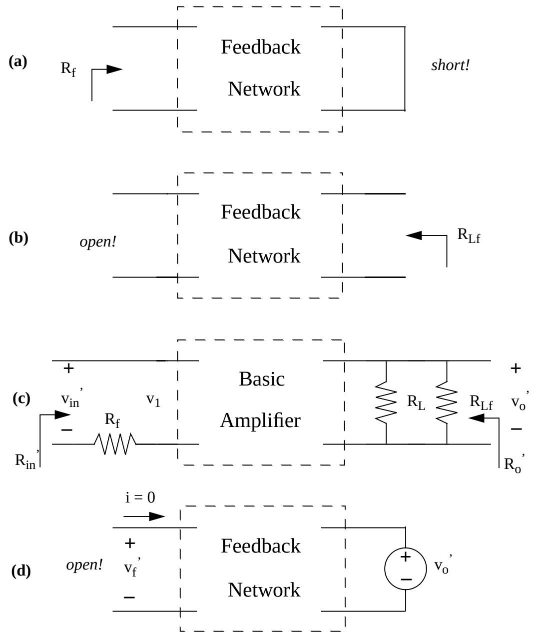


Fig. 1.7 Calculation of small-signal parameters for series-shunt feedback configuration

$$f \equiv \frac{v_f'}{v_o'} \tag{1.27}$$

1.4.3 Example:

As an example, consider the series-shunt feedback amplifier shown in Fig. 1.8a.

From Figs. 1.8b and 1.8c, the loading R_f and R_{Lf} are given by:

$$R_f = R_E \parallel R_F \tag{1.28}$$

$$R_{L_f} = R_E + R_F \tag{1.29}$$

From Fig. 1.8d,

$$A' = \frac{v_o'}{v_{in}'} = A \cdot \frac{R_L'}{R_L' + R_o} \cdot \frac{v_1}{v_{in}} = A \cdot \frac{R_L'}{R_L' + R_o} \cdot \frac{R_{in}}{R_{in} + R_f} \tag{1.30}$$

where

$$R_L' = R_L \parallel R_{L_f} = R_L \parallel (R_E + R_F) \tag{1.31}$$

and

$$R_{in}' = R_{in} + R_f = R_{in} + (R_E \parallel R_F) \tag{1.32}$$

$$R_o' = R_o \parallel R_L \parallel R_{L_f} = R_o \parallel R_L \parallel (R_E + R_F) \tag{1.33}$$

From Fig. 1.8e,

$$f = \frac{v_f'}{v_o'} = \frac{R_E}{R_E + R_F} \tag{1.34}$$

Put all together, the closed-loop small-signal parameters A_f, R_{inf}, and R_{of} can be readily obtained as:

$$A_f = \frac{A'}{1 + A'f} \tag{1.35}$$

$$R_{in_f} = R_{in}'(1 + A'f) \tag{1.36}$$

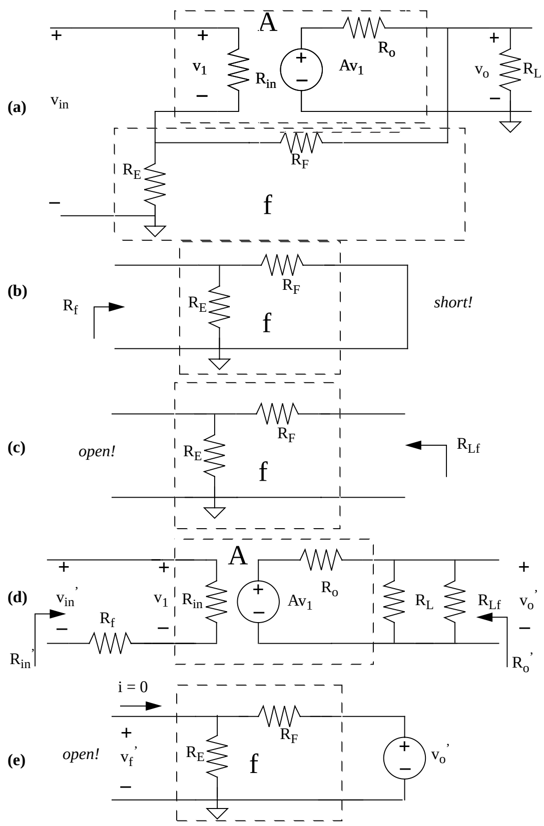


Fig. 1.8 Example of a series-shunt feedback circuit

$$R_{o_f} = \frac{R_o'}{(1 + A'f)} \tag{1.37}$$

where A’, R_{in}’, R_o’, and f are given by Eqs. 1.30, 1.32, 1.33, and 1.34, respectively.

1.5 Series-Series Feedback Analysis (Transconductance Amplifier)

1.5.1 Ideal Situation**:**

Ideally, the feedback network does not have any loading effect on the basic amplifier as shown in Fig. 1.9.

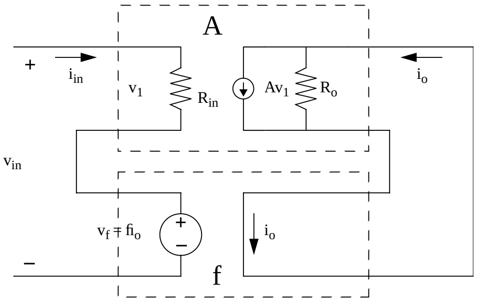


Fig. 1.9 Ideal series-series feedback configuration

1.5.1.1 Gain:

It is easy to show that:

$$i_o = Av_1 = A(v_{in} - v_f) = A(v_{in} - fi_o) \tag{1.38}$$

As a result, the overall transconductance gain A_f is given by:

$$A_f \equiv \frac{i_o}{v_{in}} = \frac{A}{1 + Af} \tag{1.39}$$

1.5.1.2 Input Resistance:

$$R_{in_f} \equiv \frac{v_{in}}{i_{in}} = \frac{v_{in}}{v_1 / R_{in}} = R_{in}\frac{v_1 + fv_o}{v_1} = R_{in}\frac{v_1 + fAv_1}{v_1} = R_{in}(1 + Af) \tag{1.40}$$

1.5.1.3 Output Resistance:

$$v_o = (i_o - Av_1)R_o = i_oR_o(1 + Af) \tag{1.41}$$

As a result,

$$R_{o_f} \equiv \frac{v_o}{i_o} = R_o(1 + Af) \tag{1.42}$$

1.5.2 Practical Situation**:**

1.5.2.1 Procedure:

To take into account the loading effect of the feedback network,

i) Break the loop; determine and include *appropriate loading*

ii) Calculate the feedback factor f and the *open-loop* small-signal parameters *with loading A’, R_{in}’, R_o’*.

iii) Calculate the closed-loop small-signal parameters by applying the following formulas:

$$A_f \equiv \frac{A'}{1 + A'f} \tag{1.43}$$

$$R_{in_f} = R_{in}'(1 + A'f) \tag{1.44}$$

$$R_{o_f} = R_o'(1 + A'f) \tag{1.45}$$

1.5.2.2 Appropriate Loading:

Refer to Figs. 1.10a and 1.10b,

i) For *input* loading R_f, *open* the input of the feedback network and calculate the output resistance of the feedback network (*series* loading) (Fig. 1.10a).

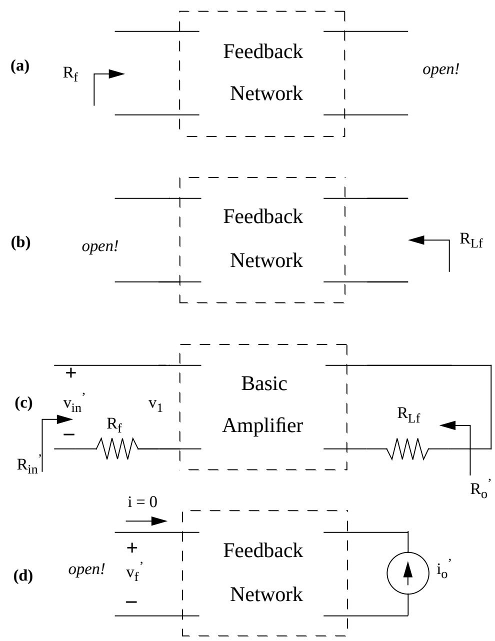


Fig. 1.10 Calculation of small-signal parameters for series-series feedback configuration

ii) For *output* loading R_{Lf}, *open* the output of the feedback network and calculate the input resistance of the feedback network (*series* loading) (Fig. 1.10b).

1.5.2.3 Open-Loop Small-Signal Parameters With Loading:

Refer to Figs. 1.10c and 1.10d,

i) With the loading R_f in series with the input and R_{Lf} in parallel with the output of the basic amplifier, the open-loop small-signal parameters with loading can be determined as A’, R_{in}’, and R_o’, (Fig. 1.10c) where:

$$A' \equiv \frac{i_o'}{v_{in}'} \tag{1.46}$$

ii) The feedback factor f can be obtained by applying a test current i_o’ at the input of the feedback network and measuring the open-circuit voltage v_f’ at the output of the feedback network (Fig. 1.10d) and

$$f \equiv \frac{v_f'}{i_o'} \tag{1.47}$$

1.5.3 Example**:**

As an example, consider the series-series feedback amplifier shown in Fig. 1.11a. Since the sensed output is current, the open-loop transconductance gain A and the feedback factor f are defined as the A = i_o / v_{in} and f = v_f / i_o, respectively.

From Figs. 1.11b and 1.11c, the loading R_f and R_{Lf} are given by:

$$R_f = R_{E_1} \parallel (R_{E_2} + R_F) \tag{1.48}$$

$$R_{L_f} = R_{E_2} \parallel (R_{E_1} + R_F) \tag{1.49}$$

The open-loop gain with loading A’ can be obtained from Fig. 1.12a,

$$A' = \frac{i_o'}{v_{in}'} = \frac{g_{m_1}(R_{C_1} \parallel r_{\pi_2})}{1 + g_{m_1}R_f} \cdot g_{m_2}(R_{C_2} \parallel R_{\pi_2}) \cdot \frac{g_{m_3}}{1 + g_{m_3}R_{L_f}} \propto \frac{R_{C_1}}{R_f} \cdot g_{m_2}R_{C_2} \cdot \frac{1}{R_{L_f}} \tag{1.50}$$

where

$$R_{\pi_2} = r_{\pi_2} + (1 + \beta_2)R_{L_f} \tag{1.51}$$

The open-loop input and output resistance with loading are given by:

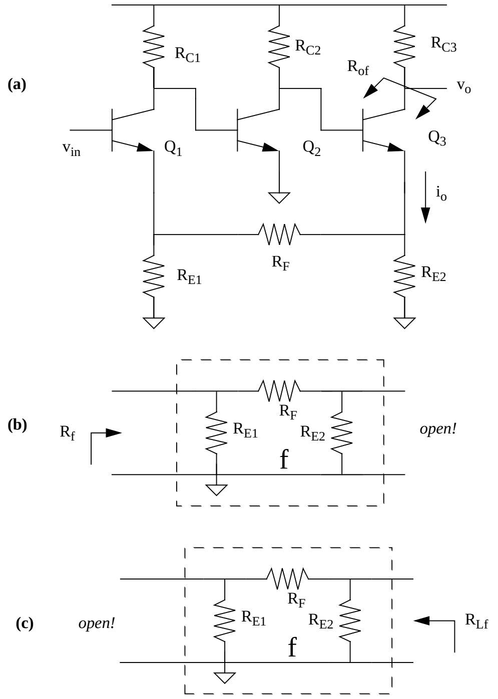


Fig. 1.11 Example of a series-series feedback circuit

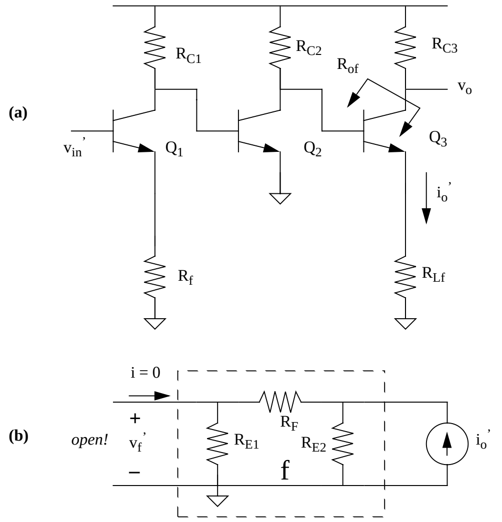


Fig. 1.12 Equivalent circuits to calculate the open-loop gain and the feedback factor for Fig.1.11

$$R_{in}' = r_{\pi_1} + (1 + \beta_1)R_f \tag{1.52}$$

$$R_o' = r_{o_3}\{1 + g_{m_3}(R_{L_f} \| r_{\pi_3})\} \tag{1.53}$$

From Fig. 1.12b,

$$f = \frac{v_f'}{i_o'} = \frac{R_{E_1}R_{E_2}}{R_{E_1} + R_{E_2} + R_F} \tag{1.54}$$

Put all together, the closed-loop small-signal parameters A_f, R_{inf}, and R_{of} can be readily obtained as:

$$A_f = \frac{A'}{1 + A'f} \tag{1.55}$$

$$R_{in_f} = R_{in}'(1 + A'f) \tag{1.56}$$

$$R_{o_f} = R_o'(1 + A'f) \tag{1.57}$$

where A', R_{in}', R_o', and f are given by Eqs. 1.50, 1.52, 1.53, and 1.54, respectively.

1.6 Shunt-Shunt Feedback Analysis (Transimpedance Amplifier)

1.6.1 Ideal Situation:

Figure 1.13 shows an ideal shunt-shunt feedback network.

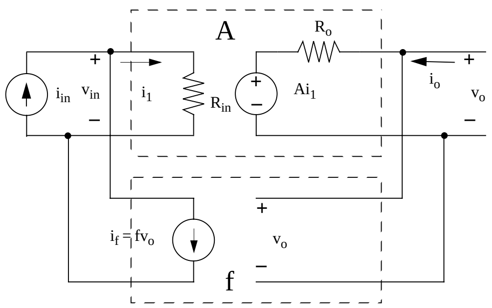


Fig. 1.13 Ideal shunt-shunt feedback configuration

1.6.1.1 Gain:

It is easy to show that:

$$v_o = Ai_1 = A(i_{in} - i_f) = A(i_{in} - fv_o) \tag{1.58}$$

As a result, the overall transimpedance gain A_f is given by:

$$A_f \equiv \frac{v_o}{i_{in}} = \frac{A}{1 + Af} \tag{1.59}$$

1.6.1.2 Input Resistance:

$$i_{in} = i_1 + i_f = i_1 + fv_o = i_1(1 + Af) \tag{1.60}$$

Therefore,

$$R_{in_f} \equiv \frac{v_{in}}{i_{in}} = \frac{v_{in}}{i_1(1 + Af)} = \frac{R_{in}}{(1 + Af)} \tag{1.61}$$

1.6.1.3 Output Resistance:

Open-circuit the input,

$$v_o = Ai_1 + i_oR_o = -Afv_o + i_oR_o \tag{1.62}$$

It follows that

$$R_{o_f} \equiv \frac{v_o}{i_o} = \frac{R_o}{(1 + Af)} \tag{1.63}$$

1.6.2 Practical Situation**:**

1.6.2.1 Procedure:

To take into account the loading effect of the feedback network,

i) Break the loop; determine and include *appropriate loading*

ii) Calculate the feedback factor f and the *open-loop* small-signal parameters *with loading A', R_{in}', R_o'.*

iii) Calculate the closed-loop small-signal parameters by applying the following formulas:

$$A_f \equiv \frac{A'}{1 + A'f} \tag{1.64}$$

$$R_{in_f} = \frac{R_{in}'}{(1 + A'f)} \tag{1.65}$$

$$R_{o_f} = \frac{R_o'}{(1 + A'f)} \tag{1.66}$$

1.6.2.2 Appropriate Loading:

Refer to Figs. 1.14a and 1.14b,

i) For *input* loading R_f, *short* the input of the feedback network and calculate the output resistance of the feedback network (*parallel* loading) (Fig. 1.14a).

ii) For *output* loading R_{Lf}, *short* the output of the feedback network and calculate the input resistance of the feedback network (*parallel* loading) (Fig. 1.14b).

1.6.2.3 Open-Loop Small-Signal Parameters With Loading:

Refer to Figs. 1.14c and 1.14d,

i) With the loading R_f in parallel with the input and R_{Lf} in parallel with the output of the basic open-loop amplifier, the open-loop small-signal parameters with loading can be determined as A', R_{in}', and R_o', (Fig. 1.14c) where:

$$A' \equiv \frac{v_o'}{i_{in}'} \tag{1.67}$$

ii) The feedback factor f can be obtained by applying a test voltage v_o' at the input of the feedback network and measuring the short-circuit current i_f' at the output of the feedback network (Fig. 1.14d) and

$$f \equiv \frac{i_f'}{v_o'} \tag{1.68}$$

1.6.3 Example:

As an example, consider the shunt-shunt feedback amplifier shown in Fig. 1.15a.

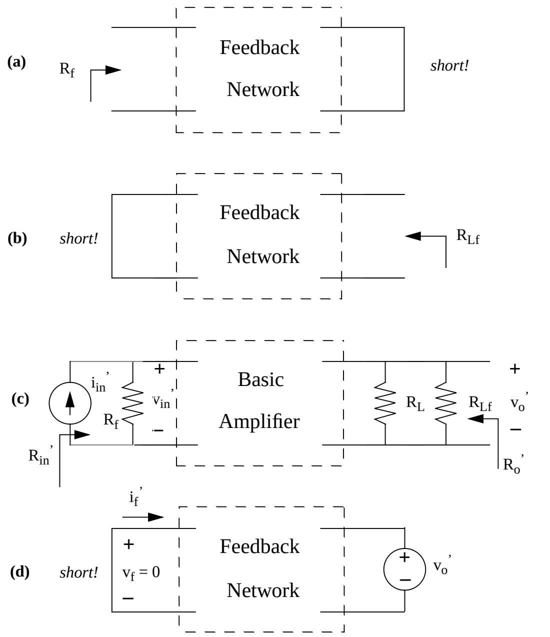


Fig. 1.14 Calculation of small-signal parameters for shunt-shunt feedback configuration

Since the input is shunted, an input current source is desired. If a voltage source is given, it is *necessary* to Nortonize, ie. convert the voltage source to a current source $i_{in} = v_{in} / R_S$ in parallel with the source resistor R_S as shown in Fig. 1.15b.

From Figs. 1.16a and 1.16b, the loading R_f and R_{Lf} are given by:

$$R_f = R_F \tag{1.69}$$

$$R_{L_f} = R_F \tag{1.70}$$

Figure 1.16c shows the open-loop circuit with loading, from which it can be found that

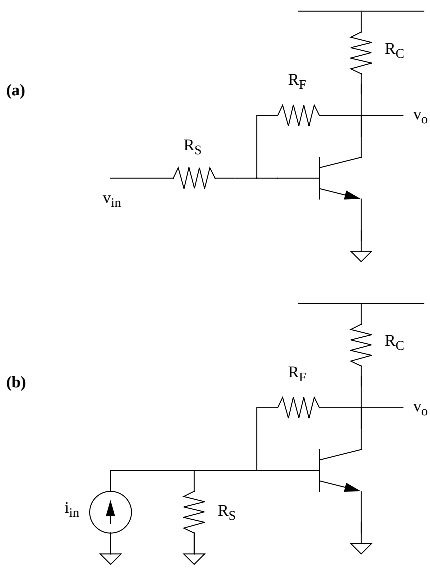


Fig. 1.15 Example of a shunt-shunt feedback amplifier

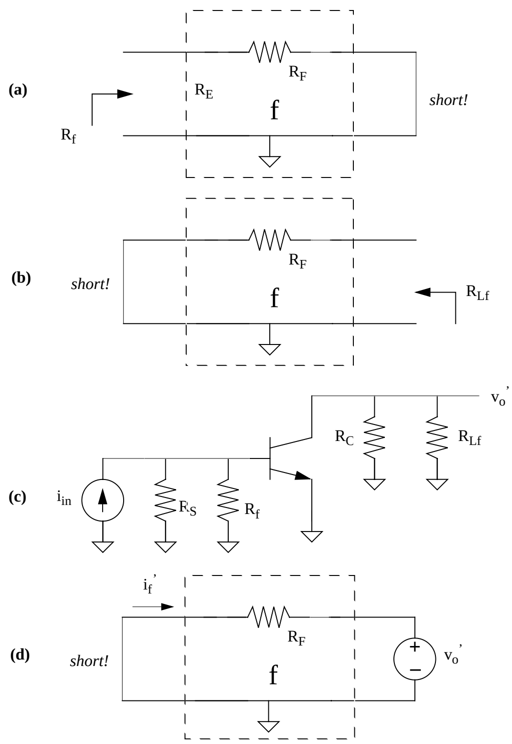


Fig. 1.16 Small-signal parameters for the shunt-shunt feedback amplifier in Fig. 1.15

$$A' = \frac{v_o'}{i_{in}'} = -g_m \cdot (R_S \parallel R_f \parallel r_\pi) \cdot (r_o \parallel R_C \parallel R_{L_f}) \quad (1.71)$$

and

$$R_{in}' = (R_S \parallel R_f \parallel r_\pi) \quad (1.72)$$

$$R_o' = (r_o \parallel R_C \parallel R_{L_f}) \quad (1.73)$$

From Fig. 1.16d,

$$f = \frac{i_f'}{v_o'} = -\frac{1}{R_F} \quad (1.74)$$

Put all together, the closed-loop small-signal parameters A_f, R_{inf}, and R_{of} can be readily obtained as:

$$A_f = \frac{A'}{1 + A'f} \quad (1.75)$$

$$R_{in_f} = \frac{R_{in}'}{(1 + A'f)} \quad (1.76)$$

$$R_{o_f} = \frac{R_o'}{(1 + A'f)} \quad (1.77)$$

where A', R_{in}', R_o', and f are given by Eqs. 1.71-1.74, respectively.

Note: As mentioned earlier, if the original circuit has a voltage source as the input, it needs to be converted to a current source before the feedback analysis can be carried out. In this case, the voltage gain of the original circuit can be readily obtained as follows:

$$\frac{v_o}{v_{in}} = \frac{v_o}{i_{in}R_S} = A_f \cdot \frac{1}{R_S} = \frac{A'}{1 + A'f} \cdot \frac{1}{R_S} \quad (1.78)$$

1.7 Shunt-Series Feedback Analysis (Current Amplifier)

1.7.1 Ideal Situation:

Figure 1.17 illustrates an ideal shunt-series feedback configuration.

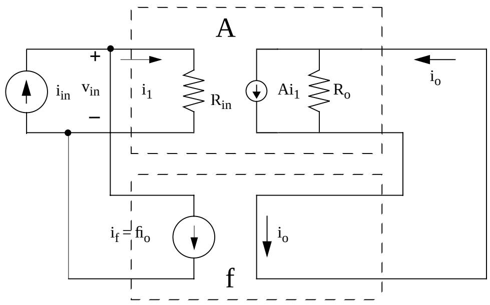


Fig. 1.17 Ideal shunt-series feedback configuration

1.7.1.1 Gain:

It is easy to show that:

$$i_o = Ai_1 = A(i_{in} - i_f) = A(i_{in} - fi_o) \quad (1.79)$$

As a result, the overall current gain A_f is given by:

$$A_f \equiv \frac{i_o}{i_{in}} = \frac{A}{1 + Af} \quad (1.80)$$

1.7.1.2 Input Resistance:

Similar to the shunt-shunt feedback, the input resistance can be found to be:

$$R_{in_f} \equiv \frac{v_{in}}{i_{in}} = \frac{v_{in}}{i_1(1 + Af)} = \frac{R_{in}}{(1 + Af)} \quad (1.81)$$

1.7.1.3 Output Resistance:

Similar to the series-series feedback configuration, the output resistance is given by:

$$R_{o_f} \equiv \frac{v_o}{i_o} = R_o(1 + Af) \quad (1.82)$$

1.7.2 Practical Situation:

1.7.2.1 Procedure:

To take into account the loading effect of the feedback network,

i) Break the loop; determine and include *appropriate loading*

ii) Calculate the feedback factor f and the *open-loop* small-signal parameters *with loading A', R_{in}', R_o'*.

iii) Calculate the closed-loop small-signal parameters by applying the following formulas:

$$A_f \equiv \frac{A'}{1 + A'f} \quad (1.83)$$

$$R_{in_f} = \frac{R_{in}'}{(1 + A'f)} \quad (1.84)$$

$$R_{o_f} = R_o'(1 + A'f) \quad (1.85)$$

1.7.2.2 Appropriate Loading:

Refer to Figs. 1.18a and 1.18b,

i) For *input* loading R_f, *open* the input of the feedback network and calculate the output resistance of the feedback network (*series* loading) (Fig. 1.18a).

ii) For *output* loading R_{Lf}, *short* the output of the feedback network and calculate the input resistance of the feedback network (*parallel* loading) (Fig. 1.18b).

1.7.2.3 Open-Loop Small-Signal Parameters With Loading:

Refer to Figs. 1.18c and 1.18d,

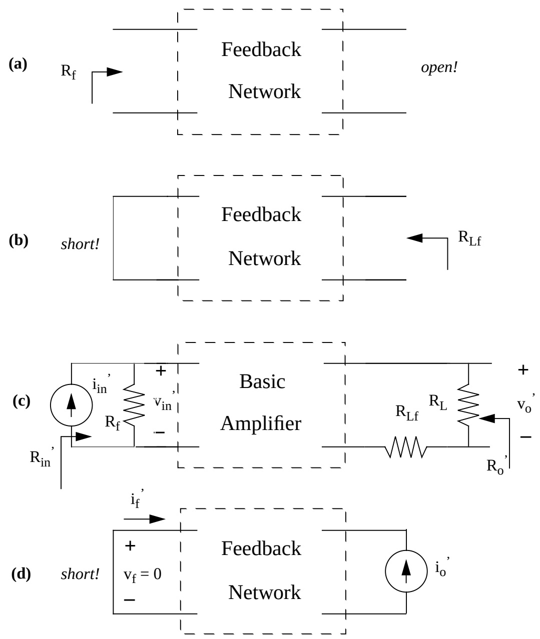


Fig. 1.18 Calculation of small-signal parameters for shunt-series feedback configuration

i) With the loading R_f in series with the input and R_{Lf} in parallel with the output of the basic amplifier, the open-loop small-signal parameters with loading can be determined as A', R_{in}', and R_o', (Fig. 1.18c) where:

$$A' \equiv \frac{i_o'}{i_{in}'} \tag{1.86}$$

ii) The feedback factor f can be obtained by applying a test current i_o' at the input of the feedback network and measuring the short-circuit current i_f' at the output of the feedback network (Fig. 1.18d) and

$$f \equiv \frac{i_f'}{i_o'} \tag{1.87}$$

1.7.3 Example:

As an example, consider the shunt-series feedback configuration (current feedback pair) shown in Fig. 1.19a. From Figs. 1.19b and 1.19c, the loading R_f and R_{Lf} are given by:

$$R_f = R_E + R_F \tag{1.88}$$

$$R_{L_f} = R_E \parallel R_F \tag{1.89}$$

The open-loop current gain with loading A' can be obtained from Fig. 1.20a,

$$A' = \frac{i_o'}{i_{in}'} = -g_{m_1} \cdot (R_S \parallel R_f \parallel r_\pi) \cdot (R_{C_1} \parallel R_{\pi_2}) \cdot \frac{g_{m_2}}{1 + g_{m_2} R_{L_f}} \propto -g_{m_1} \cdot (R_S \parallel R_f \parallel r_\pi) \cdot \frac{R_{C_1}}{R_{L_f}} \tag{1.90}$$

where

$$R_{\pi_2} = r_{\pi_2} + (1 + \beta_2) R_{L_f} \tag{1.91}$$

The open-loop input and output resistance with loading are given by:

$$R_{in}' = (R_S \parallel R_f \parallel r_\pi) \tag{1.92}$$

$$R_o' = r_{o_2}\{1 + g_{m_2}(R_{L_f} \parallel r_{\pi_2})\} \tag{1.93}$$

From Fig. 1.20b,

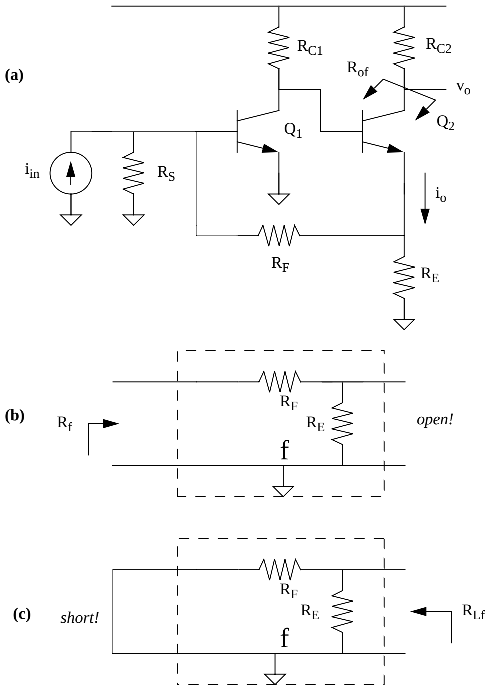


Fig. 1.19 Example of a shunt-series feedback amplifier

$$f = \frac{i_f'}{i_o'} = -\frac{R_E}{R_E + R_F} \tag{1.94}$$

Put all together, the closed-loop small-signal parameters A_f, R_{inf}, and R_{of} can be readily obtained as:

$$A_f = \frac{A'}{1 + A'f} \tag{1.95}$$

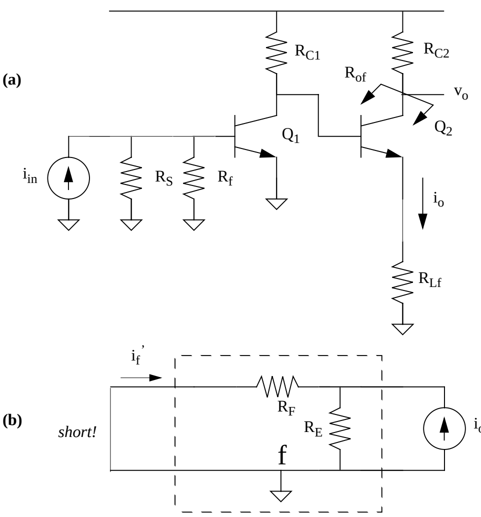


Fig. 1.20 Equivalent circuits to calculate the open-loop gain and the feedback factor for Fig. 1.19

$$R_{in_f} = \frac{R_{in}'}{(1 + A'f)} \tag{1.96}$$

$$R_{o_f} = R_o'(1 + A'f) \tag{1.97}$$

where A', R_{in}', R_o', and f are given by Eqs. 1.90, 1.92, 1.93, and 1.94, respectively.

1.8 Local Series-Series Feedback

Consider the series-series feedback amplifier shown in Fig. 1.21a. From Fig. 1.21b, the loading R_f and R_{Lf} are given by:

$$R_f = R_E \tag{1.98}$$

$$R_{L_f} = R_E \tag{1.99}$$

The open-loop gain with loading A' can be obtained from Fig. 1.21c,

$$A' = \frac{i_o'}{v_{in}'} = \frac{g_m r_\pi}{r_\pi + R_f} \tag{1.100}$$

The open-loop input and output resistance with loading are given by:

$$R_{in}' = r_\pi + R_f \tag{1.101}$$

$$R_o' = r_o + R_{L_f} \propto r_o \tag{1.102}$$

From Fig. 1.21d,

$$f = \frac{v_f'}{i_o'} = R_E \tag{1.103}$$

Put all together, we obtain:

$$1 + A'f = 1 + \frac{g_m r_\pi}{r_\pi + R_E} \cdot R_E = 1 + g_m (r_\pi \parallel R_E) \tag{1.104}$$

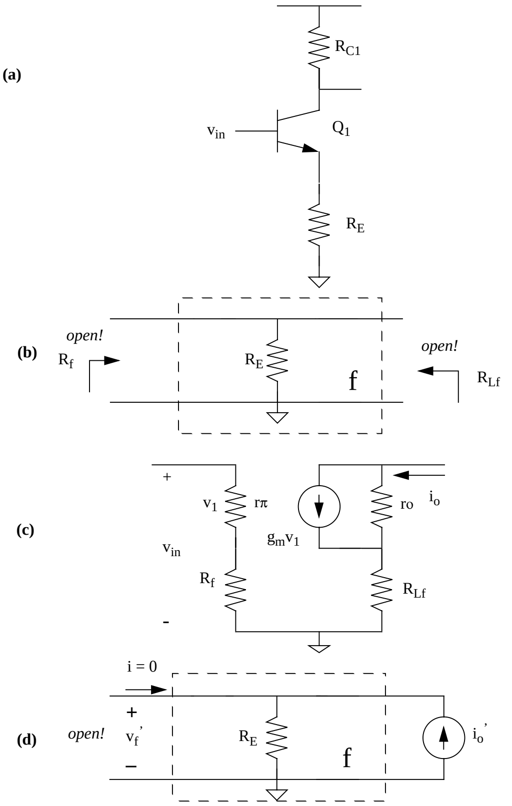


Fig. 1.21 Local series-series feedback amplifier

It follows that

$$A_f = \frac{A'}{1 + A'f} = \frac{g_m r_\pi}{r_\pi + R_E} \cdot \frac{1}{1 + g_m(r_\pi \parallel R_E)} \propto \frac{g_m}{1 + g_m R_E} \quad (1.105)$$

and

$$R_{in_f} = R_{in}'(1 + A'f) = (r_\pi + R_f) \cdot \{1 + g_m(r_\pi \parallel R_E)\} = r_\pi + (1 + \beta)R_E \quad (1.106)$$

$$R_{o_f} = R_o'(1 + A'f) = r_o \cdot \{1 + g_m(r_\pi \parallel R_E)\} \quad (1.107)$$

Note that all these results are exactly the same as we expected (either by inspection or by direct small-signal analysis).

1.9 Appendix on Input Resistance of Voltage-Controlled Shunt-Shunt Feedback

This is to illustrate how the input resistance of a shunt-shunt feedback shown in Fig. 1.22a should be calculated *given a voltage source as the input* (as opposed to an input current source).

Clearly, from Fig. 1.22b,

$$R_{in} \equiv R_S + R_{in}' \quad (1.108)$$

If we want to use feedback analysis, it is necessary to Nortonize the input source and thus to include the source resistance R_S in the feedback loop. Denote the closed-loop input resistance as R_{if}, we have:

$$R_{if} \equiv R_S \parallel R_{in}' \quad (1.109)$$

From Eqs. 1.108 and 1.109, the original input resistance is given by:

$$R_{in} \equiv R_S + R_{in}' = R_S + \frac{R_S R_{if}}{R_S - R_{if}} \quad (1.110)$$

In general, $R_S >> R_{if}$ and therefore,

$$R_{in} = R_S + \frac{R_S R_{if}}{R_S - R_{if}} \approx R_S + R_{if} \quad (1.111)$$

To verify the procedure, let's compare the result with that obtained using small-signal analysis in Fig. 1.22b, it can be shown that:

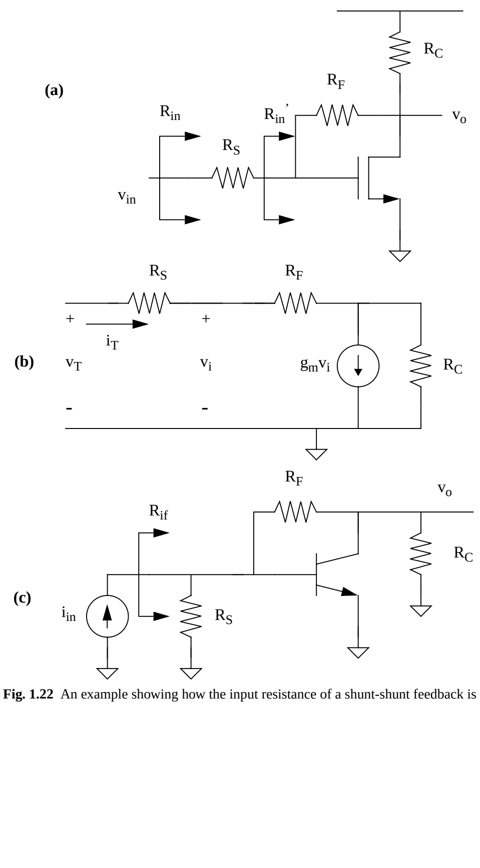


Fig. 1.22 An example showing how the input resistance of a shunt-shunt feedback is calculated

$$i_o = i_T - g_m(v_T - i_T R_S) = i_T(1 + g_m R_S) - g_m v_T \quad (1.112)$$

$$v_T = i_T - g_m(v_T - i_T R_S) = i_T(R_F + R_S) + i_o R_C = i_T[R_F + R_S + R_C(1 + g_m R_S)] - g_m v_T R_C$$

As a result,

$$R_{in} \equiv \frac{v_T}{i_T} = \frac{R_F + R_S + R_C(1 + g_m R_S)}{(1 + g_m R_C)} = R_S + \frac{R_F + R_C}{(1 + g_m R_C)} \quad (1.113)$$

Now, using the feedback analysis, we can obtain:

$$A' = \frac{v_o'}{i_{in}'} = -g_m \cdot (R_S \parallel R_F) \cdot (R_C \parallel R_F) \quad (1.114)$$

and

$$R_{in}' = (R_S \parallel R_F) \quad (1.115)$$

$$f = \frac{i_f'}{v_o'} = -\frac{1}{R_F} \quad (1.116)$$

Put all together, the closed-loop small-signal parameter R_{inf} can be obtained as:

$$R_{in_f} = \frac{R_{in}'}{(1 + A'f)} = \frac{(R_S \parallel R_F)}{1 + \frac{g_m \cdot (R_S \parallel R_F) \cdot (R_C \parallel R_F)}{R_F}} = \frac{(R_S \parallel R_F)R_F}{R_F + g_m \cdot (R_S \parallel R_F) \cdot (R_C \parallel R_F)} \quad (1.117)$$

It follows that:

$$R_{in}' = \frac{R_S R_{if}}{R_S - R_{if}} = \frac{\frac{R_S(R_S \parallel R_F)R_F}{R_F + g_m \cdot (R_S \parallel R_F) \cdot (R_C \parallel R_F)}}{\frac{[R_F + g_m \cdot (R_S \parallel R_F) \cdot (R_C \parallel R_F)]R_S - (R_S \parallel R_F)R_F}{R_F + g_m \cdot (R_S \parallel R_F) \cdot (R_C \parallel R_F)}} \quad (1.118)$$

$$R_{in}' = \frac{R_S(R_S \parallel R_F)R_F}{[R_F + g_m \cdot (R_S \parallel R_F) \cdot (R_C \parallel R_F)]R_S - (R_S \parallel R_F)R_F} = \frac{R_S R_F}{R_F + R_S + g_m \cdot (R_C \parallel R_F)R_S - R_F} \quad (1.119)$$

$$R_{in}' = \frac{R_S R_F}{R_S + g_m \cdot (R_C \parallel R_F) R_S} = \frac{R_F}{1 + g_m \cdot (R_C \parallel R_F)} \tag{1.120}$$

For $g_m R_F >> 1$, which is typically true,

$$R_{in}' \approx \frac{(R_C + R_F)R_F}{R_F + g_m \cdot (R_C \cdot R_F)} = \frac{(R_C + R_F)}{1 + g_m \cdot R_C} \tag{1.121}$$

Using the result in Eq. 1.110, the original input resistance Rin is found to be:

$$R_{in} \equiv R_S + R_{in}' = R_S + \frac{R_F + R_C}{(1 + g_m R_C)} \tag{1.122}$$

which is exactly the same as given by Eq. 1.113 that was obtained using the small-signal analysis.

Note: If the loop gain A'f >> 1, the simplified result can be approximated directly from Eq. 1.117 to be:

$$R_{in_f} \approx \frac{(R_S \parallel R_F)}{\frac{g_m \cdot (R_S \parallel R_F) \cdot (R_C \parallel R_F)}{R_F}} = \frac{R_F + R_C}{g_m R_C} \approx R_{in}' \tag{1.123}$$

which is consistent with the assumption $R_S >> R_{if}$ and verifies our prediction in Eq. 1.111:

$$R_{in} = R_S + R_{in}' \approx R_S + R_{if} \tag{1.124}$$