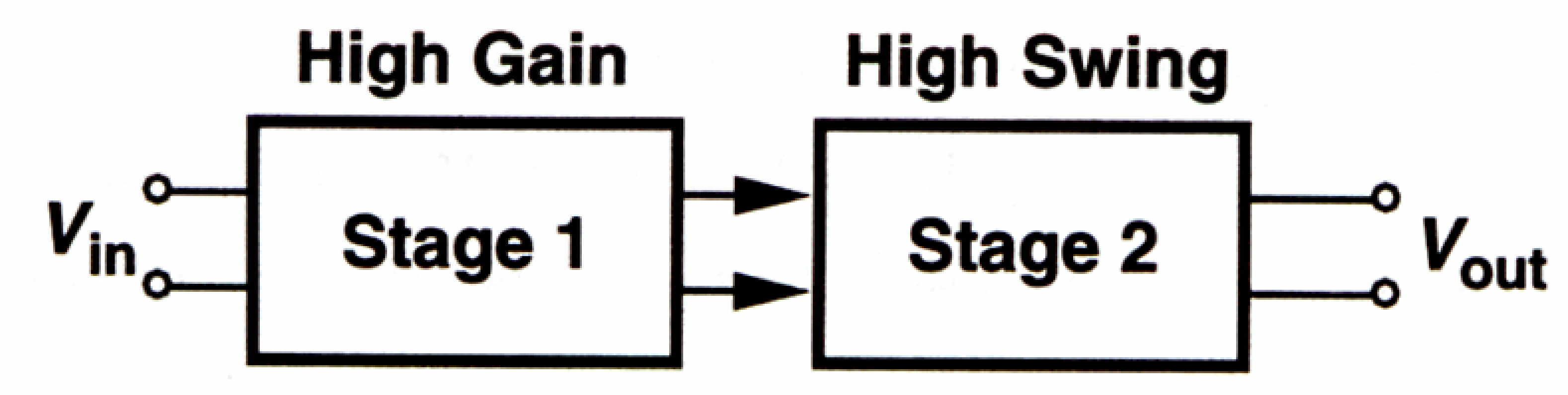
2. 2-stage OP amp

2-stage OP amp

2-stage OP amp : pro → high gain + high output swing



2-stage OP amp :

con → requires frequency compensation (usually Miller(pole-splitting) frequency compensation)

2-stage OP amp

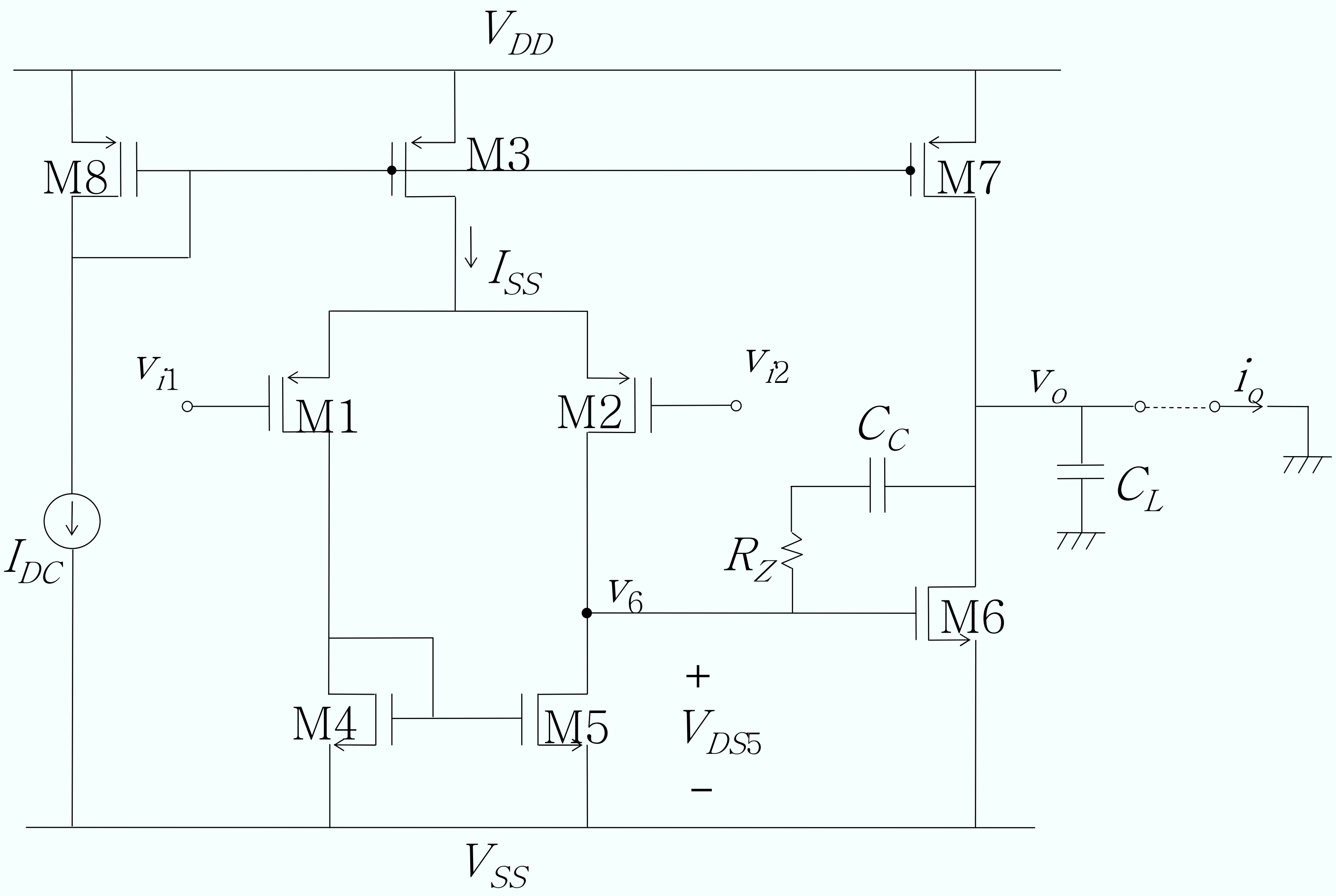


Fig 9.1.1 2-stage CMOS OP amp

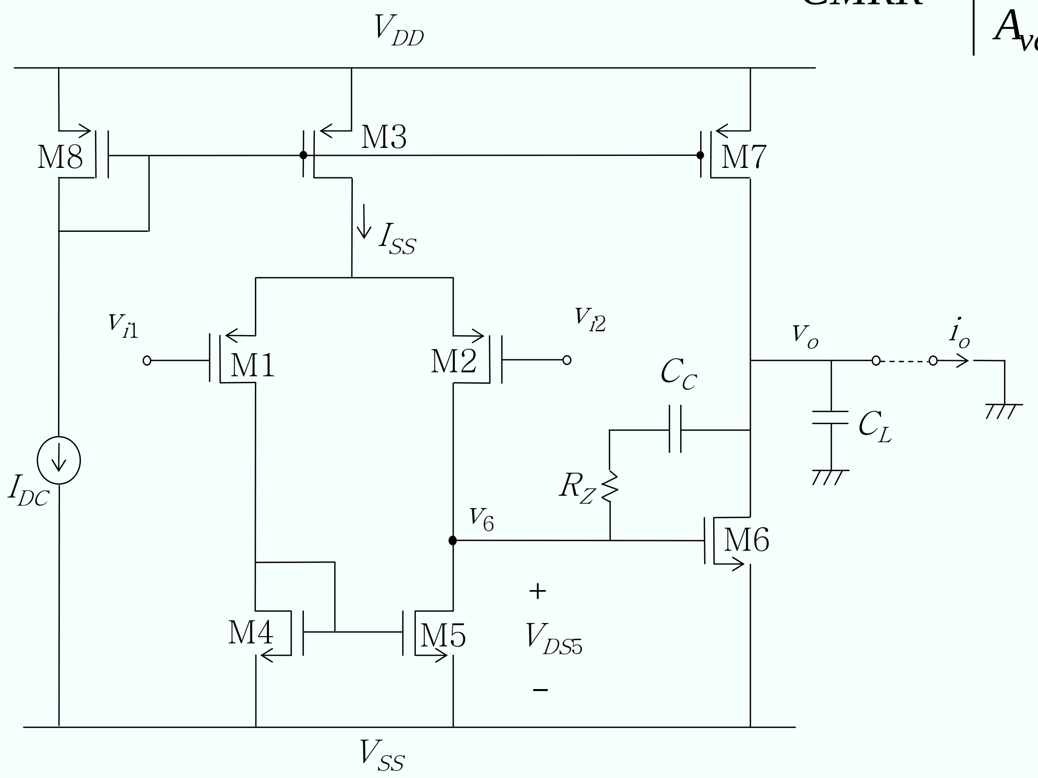
2-stage OP amp

Small-signal gain

$$A_{vd} = A_{vd1} \cdot A_{v2} = -g_{m1} \cdot (r_{o2} \parallel r_{o5}) \cdot g_{m6} \cdot (r_{o6} \parallel r_{o7})$$

$$A_{vc} = \frac{(r_{o2} \parallel r_{o5})}{(2r_{o3}) \cdot (g_{m4} \cdot (r_{o1} \parallel r_{o4}))} \cdot g_{m6} \cdot (r_{o6} \parallel r_{o7})$$

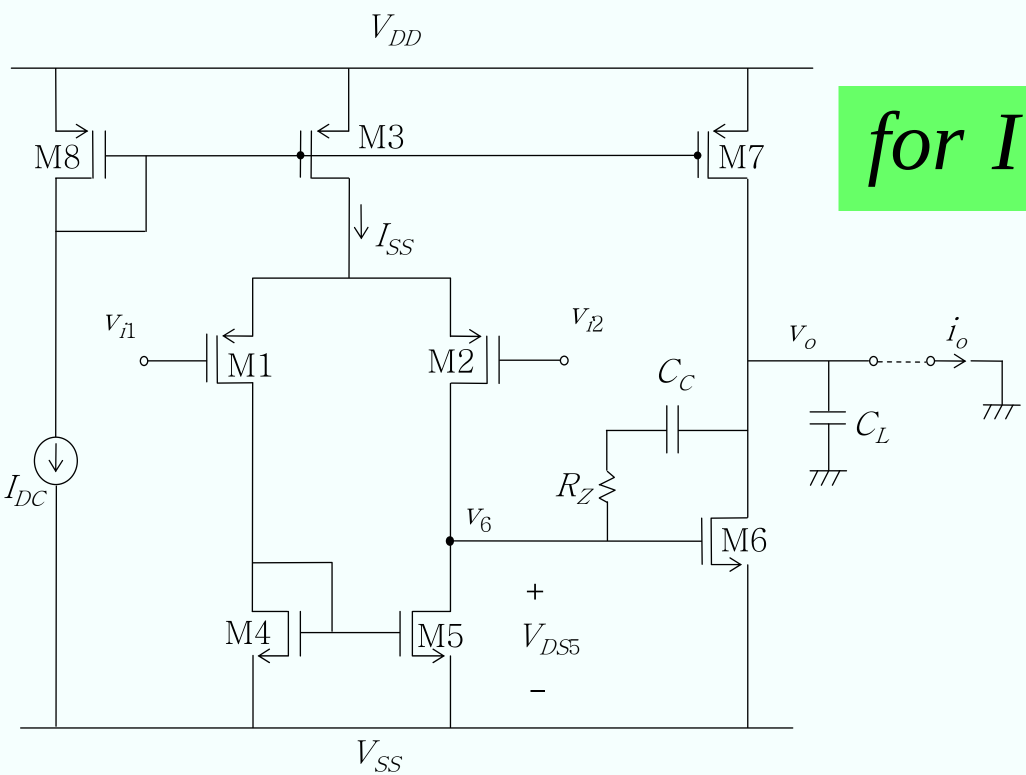
$$CMRR = \left| \frac{A_{vd}}{A_{vc}} \right| = \left| \frac{A_{vd1}}{A_{vc1}} \right| = (2g_{m1}r_{o3}) \cdot (g_{m4} \cdot (r_{o1} \parallel r_{o4}))$$



Systematic input offset voltage

$$V_{I1} = V_{I2} = 0 \quad \rightarrow \quad V_{DS5} = V_{GS4}$$

$$I_{D6} \neq I_{D7} \quad \rightarrow$$ **Systematic offset 발생**



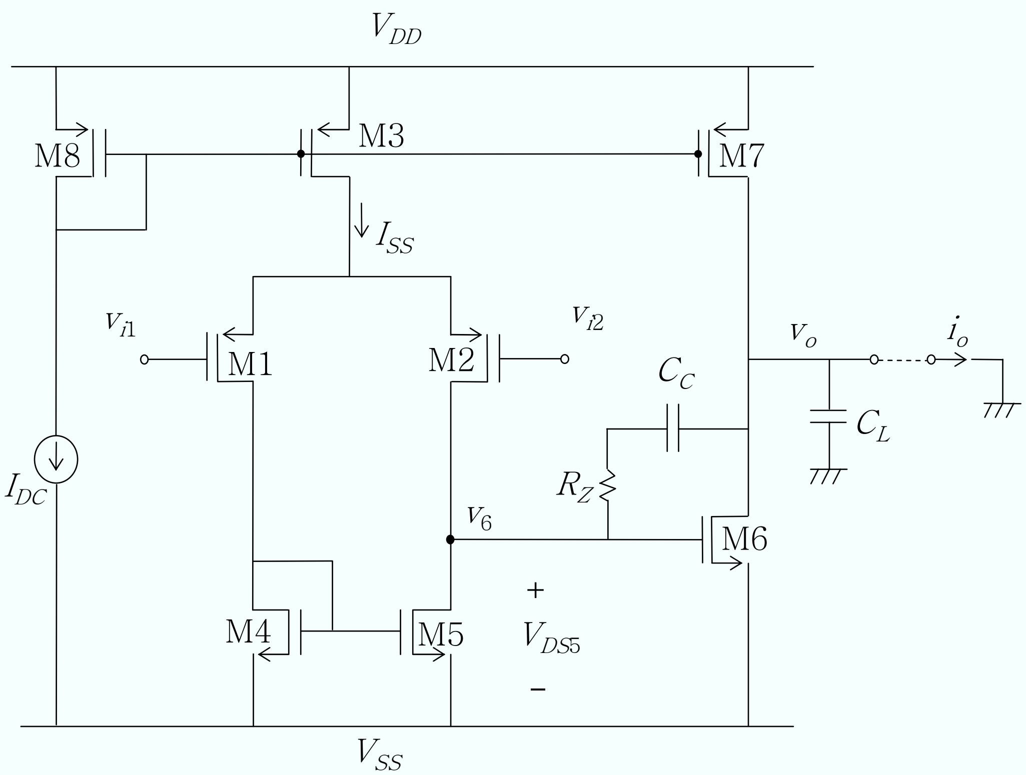
for $I_{D6} = I_{D7}$

$$\frac{(W/L)_6}{(W/L)_4} = \frac{(W/L)_7}{0.5 \cdot (W/L)_3}$$

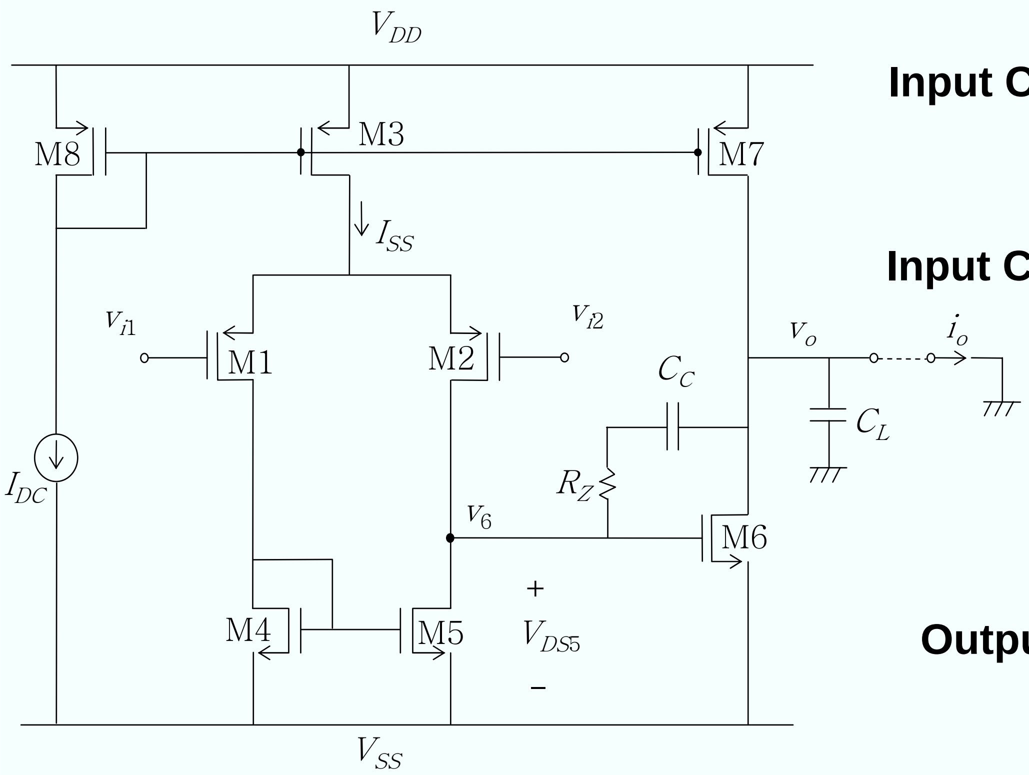
No systematic offset

Random input offset voltage

$$V_{os} = \Delta V_{THn1,2} + \Delta V_{THp4,5} \cdot \frac{g_{m4,5}}{g_{m1,2}} + \frac{(V_{GS} - V_{THn})_{1,2}}{2} \cdot \left\{ -\frac{\Delta (W/L)_{1,2}}{(W/L)_{1,2}} + \frac{\Delta (W/L)_{4,5}}{(W/L)_{4,5}} \right\} + \frac{\Delta V_{GS6}}{A_{vd1}}$$



Active input common mode voltage range & Linear output voltage range



Input CM min: $V_{SS} + V_{THn4} + V_{DSAT\,4} - |V_{THp1}|$

Input CM max: $V_{DD} - |V_{DSAT\,3}| - |V_{DSAT\,1}| - |V_{THp1}|$

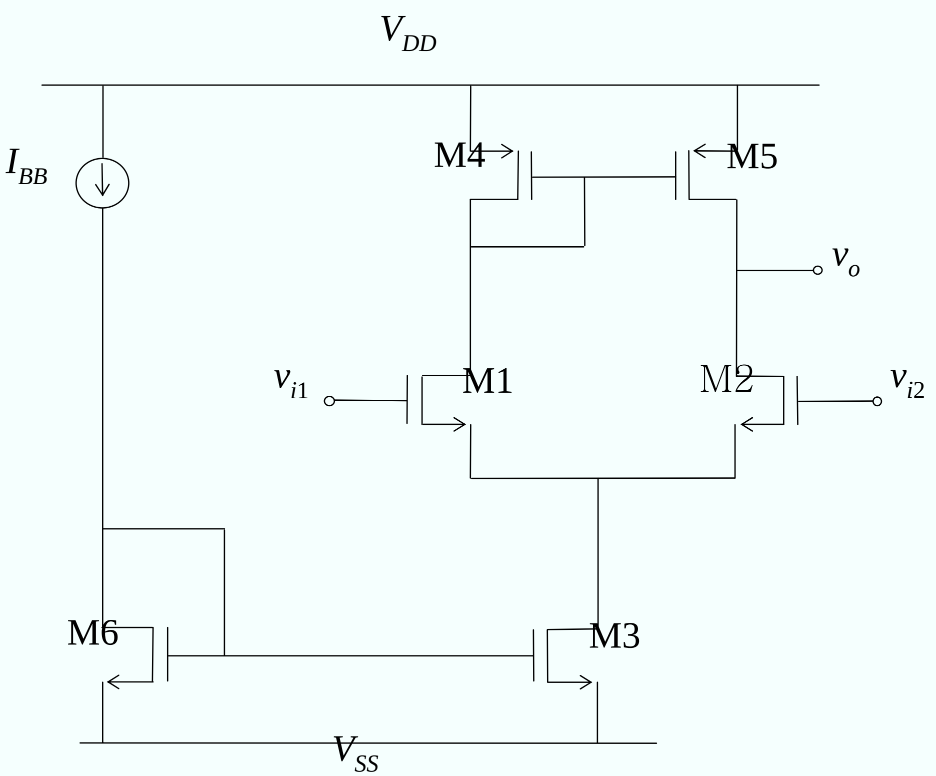
Input CM: shifted toward Vss

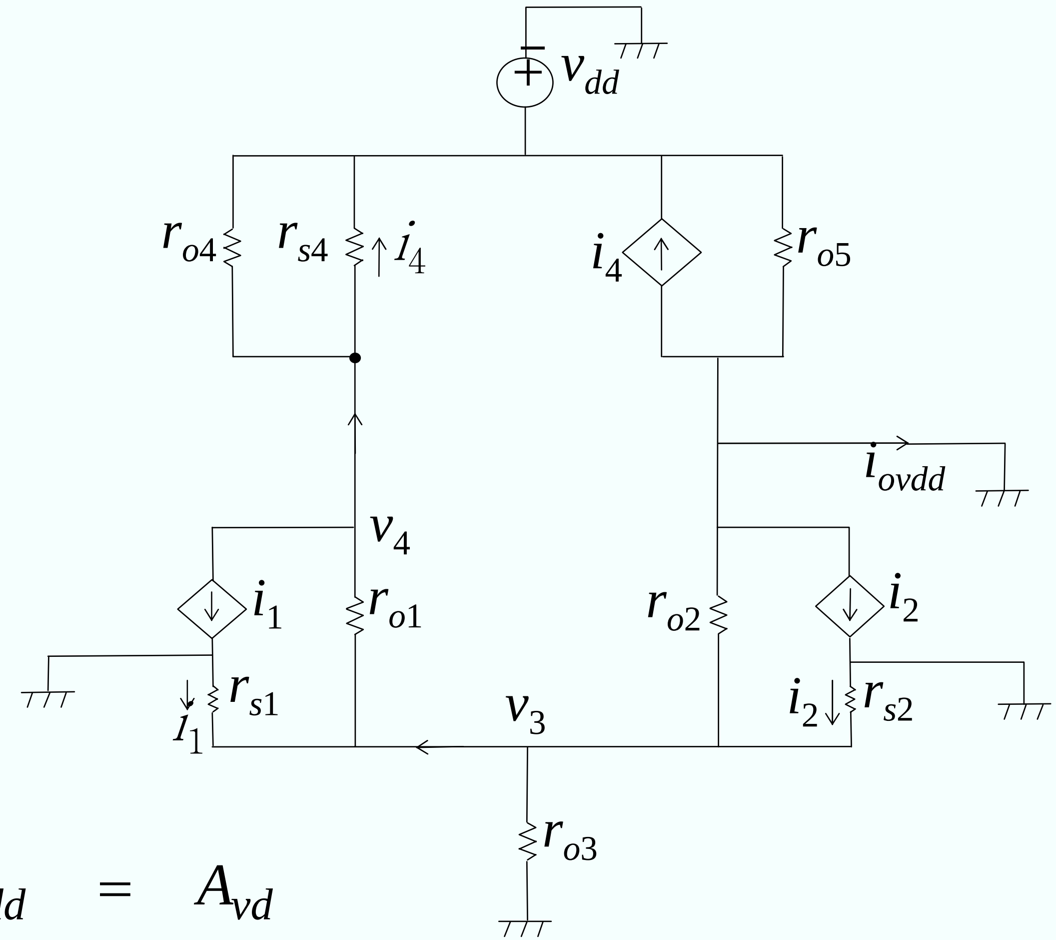
Output min: $(V_{SS} + V_{DSAT\,6})$

Output max: $(V_{DD} - |V_{DSAT\,7}|)$

Large output swing

PSRR (diff pair) review





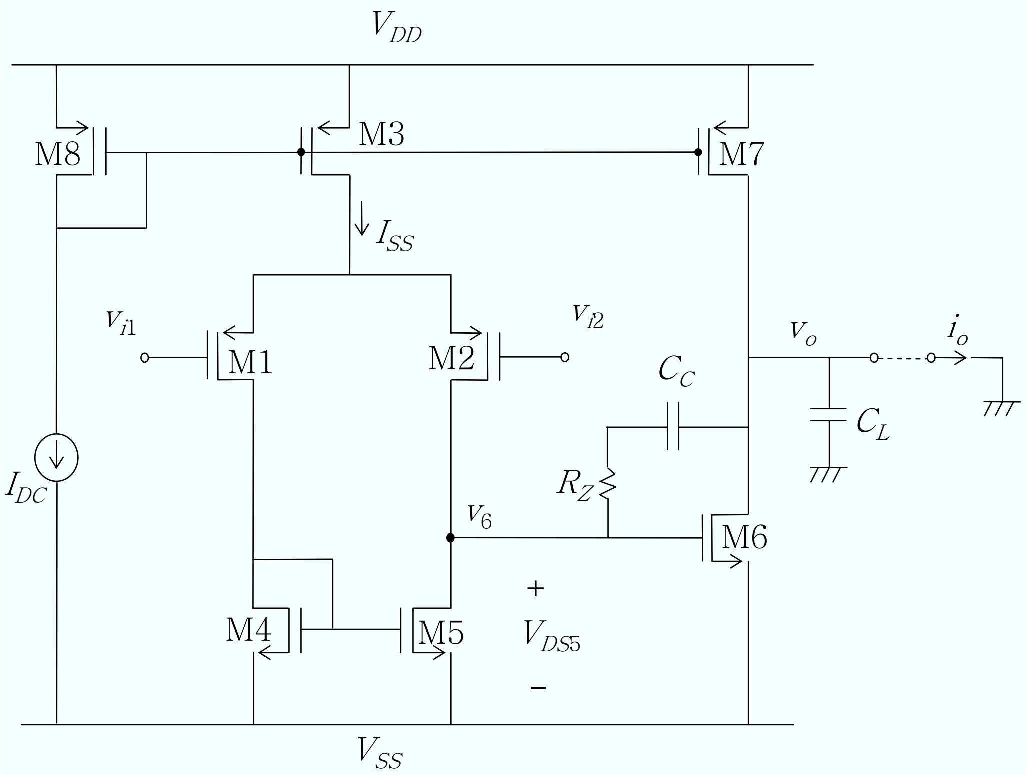
$$v_o = v_{dd} \qquad PSRR^+ \triangleq A_{vd} / A_{vdd} = A_{vd}$$

$$PSRR^- \triangleq \left| \frac{A_{vd}}{A_{vss}} \right| = (2g_{m1}r_{o3}) \cdot \{ g_{m4}(r_{o1} \| r_{o4}) \} = CMRR$$

Good PSRR-

2-stage OP amp

PSRR : not good due to 2nd stage



$$\frac{v_o}{v_{dd}} = \left\{ A_{vc1} \cdot g_{m6} + \frac{1}{r_{o7}} \right\} \cdot (r_{o6} \| r_{o7})$$

$$PSRR^+ = \frac{g_{m1} \cdot (r_{o2} \| r_{o5}) \cdot g_{m6}}{-\frac{g_{m6} \cdot (r_{o2} \| r_{o5})}{2r_{o3} \cdot g_{m4} \cdot (r_{o1} \| r_{o4})} + \frac{1}{r_{o7}}}$$

$$= \frac{1}{-\frac{1}{(CMRR)} + \frac{1}{A_{vd}} \cdot \left(\frac{r_{o6} \| r_{o7}}{r_{o7}} \right)}$$

$$v_o = \frac{r_{o7}}{r_{o6} + r_{o7}} \cdot v_{ss} = \frac{(r_{o6} \| r_{o7})}{r_{o6}} \cdot v_{ss}$$

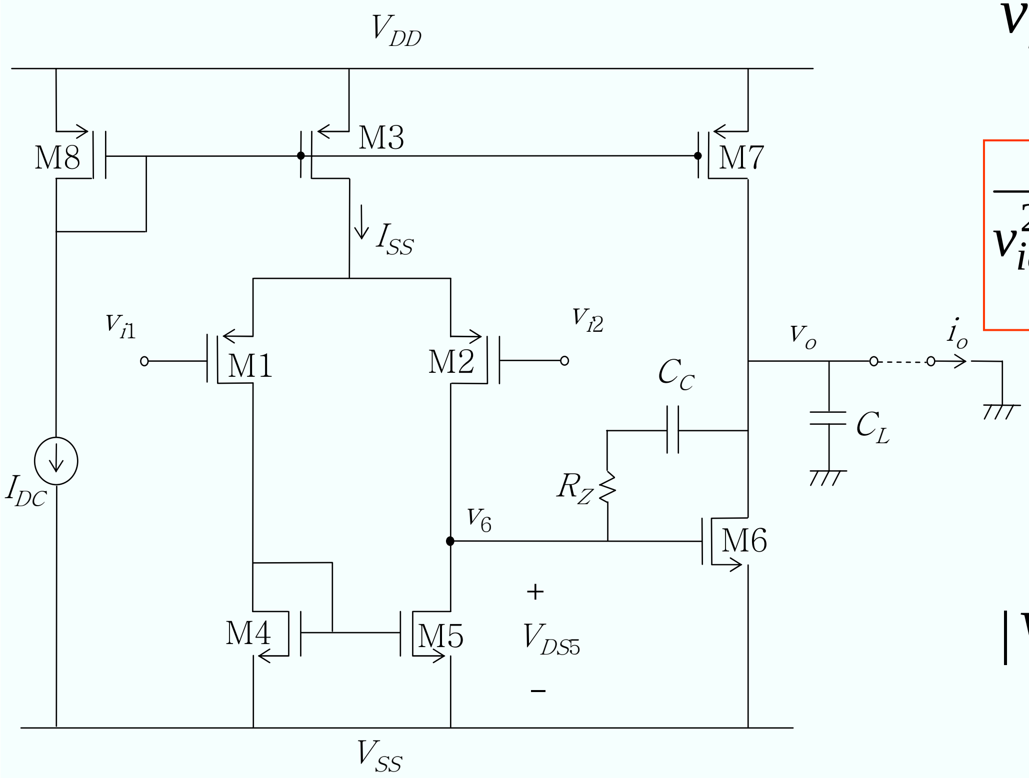
$$PSRR^- = \frac{A_{vd}}{\left(\frac{r_{o6} \| r_{o7}}{r_{o6}} \right)} = A_{vd1} \cdot g_{m6} r_{o6}$$

Both PSRR+, PSRR- : close to Avd

2-stage OP amp

Equivalent input noise voltage

$$\overline{v_{ieqn}^2} = \overline{v_{ieqn1}^2} + \frac{\overline{v_{ieqn2}^2}}{(A_{vd1})^2}$$



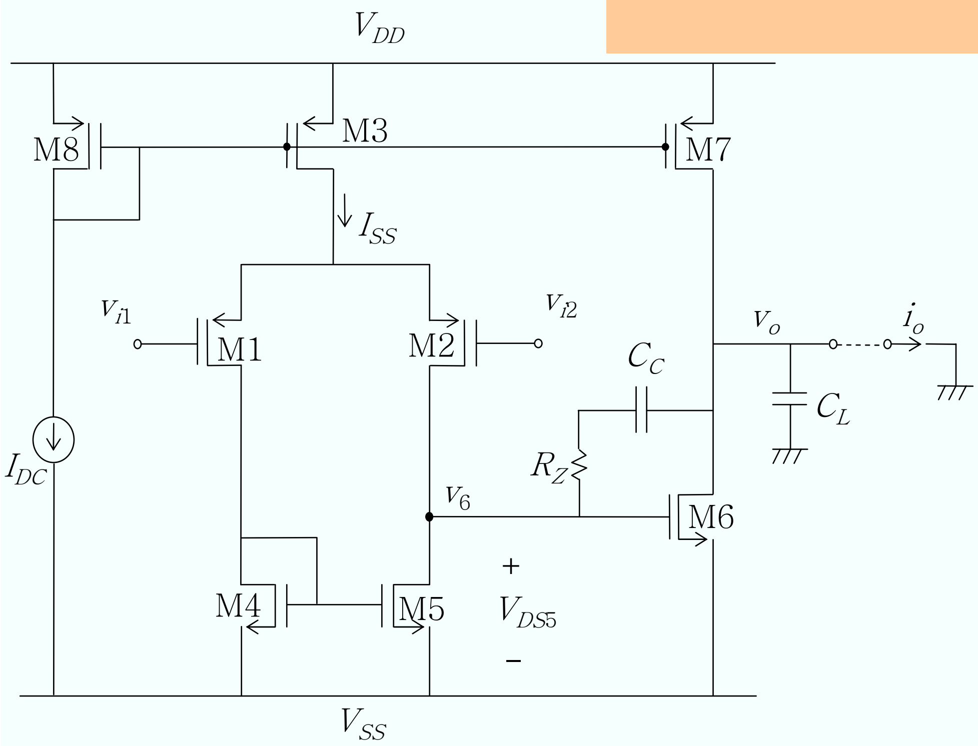
$$\overline{v_{ieqn}^2} = \overline{v_{gn1}^2} + \overline{v_{gn2}^2} + \left(\frac{g_{m5}}{g_{m1}}\right)^2 \cdot \left(\overline{v_{gn4}^2} + \overline{v_{gn5}^2}\right)$$

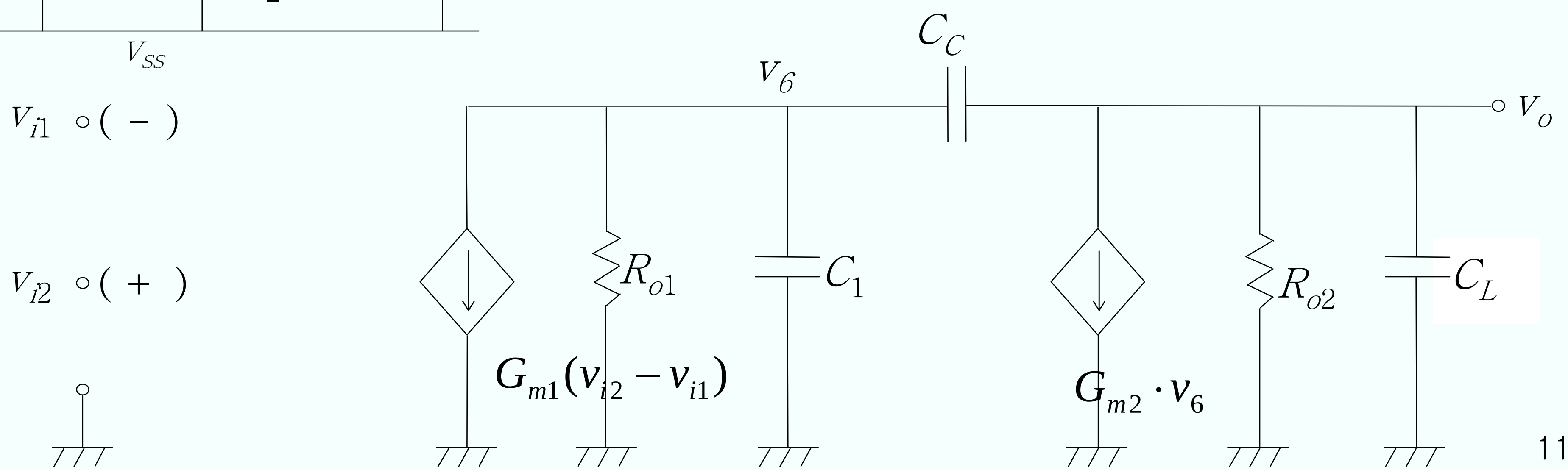
$$|V_{ieqn}(f)|^2 = \frac{16}{3} \cdot kT \cdot \frac{1}{g_{m1}} \cdot \left(1 + \frac{g_{m5}}{g_{m1}}\right)$$

To reduce noise ➔ reduce gm5 ➔ increase VDSAT5

2-stage OP amp

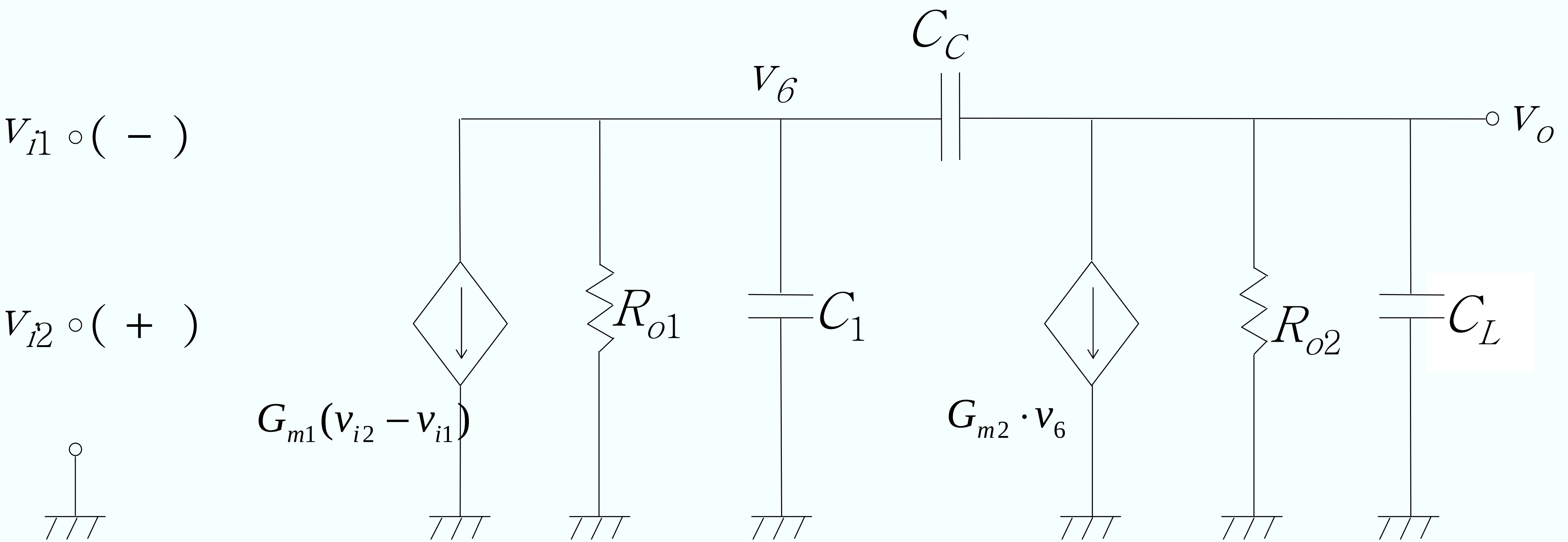
Frequency response





2-stage OP amp

Frequency response

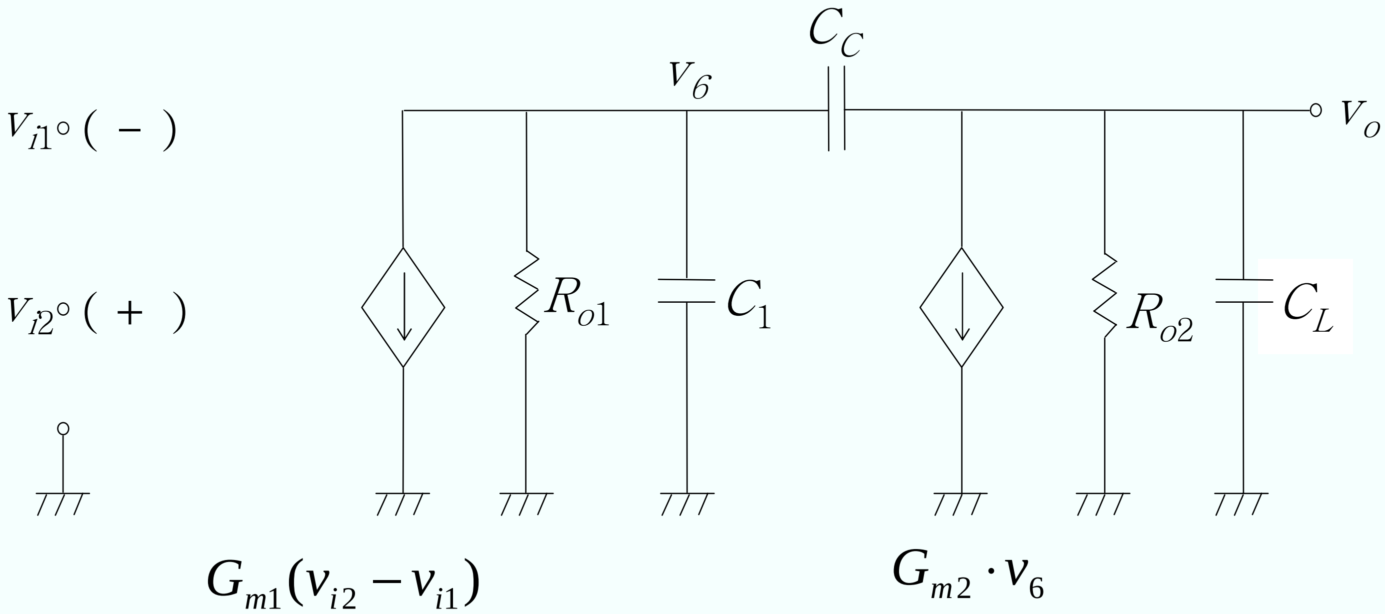


KCL at v6 and vo nodes

$$G_{m1} \cdot (v_{i2} - v_{i1}) + \left\{ s(C_1 + C_C) + \frac{1}{R_{o1}} \right\} \cdot v_6 - sC_C \cdot v_o = 0$$

$$(G_{m2} - sC_C) \cdot v_6 + \left\{ s(C_L + C_C) + \frac{1}{R_{o2}} \right\} \cdot v_o = 0$$

Frequency response



$$A_{dv}(s) = \frac{v_o}{v_{i2} - v_{i1}} = \frac{(G_{m1}R_{o1}) \cdot (G_{m2}R_{o2}) \cdot (1 - sC_C/G_{m2})}{\left[\begin{array}{l} 1 + s \cdot \{C_L R_{o2} + C_1 R_{o1} + C_C \cdot (G_{m2} R_{o2} R_{o1} + R_{o1} + R_{o2})\} \\ + s^2 \cdot \{C_1 C_L + (C_1 + C_L) C_C\} \cdot R_{o1} R_{o2} \end{array}\right]}$$

Frequency response

$$A_{dv}(s) = \frac{v_o}{v_{i2} - v_{i1}} = \frac{(G_{m1}R_{o1}) \cdot (G_{m2}R_{o2}) \cdot (1 - sC_C/G_{m2})}{\left[\begin{array}{l} 1 + s \cdot \{C_L R_{o2} + C_1 R_{o1} + C_C \cdot (G_{m2}R_{o2}R_{o1} + R_{o1} + R_{o2})\} \\ + s^2 \cdot \{C_1 C_L + (C_1 + C_L)\, C_C\} \cdot R_{o1}R_{o2} \end{array}\right]}$$

$$A_{dv}(s) = \frac{A_{dv}(0) \cdot \left(1 - \frac{s}{z_1}\right)}{\left(1 - \frac{s}{p_1}\right) \cdot \left(1 - \frac{s}{p_2}\right)} \approx \frac{A_{dv}(0) \cdot \left(1 - \frac{s}{z_1}\right)}{1 - \frac{s}{p_1} + \frac{s^2}{p_1 p_2}}$$

Dominant pole approximation: $|p_1| << |p_2|$

$$p_1 = \frac{-1}{C_C \cdot (G_{m2}R_{o2}R_{o1} + R_{o1} + R_{o2}) + C_L R_{o2} + C_1 R_{o1}} \approx \frac{-1}{R_{o1} \cdot G_{m2}R_{o2} \cdot C_C}$$

$$p_2 = \frac{+1}{p_1 \cdot \{C_C(C_1 + C_L) + C_1 C_L\} R_{o1}R_{o2}} = \frac{-G_{m2}C_C}{C_C(C_1 + C_L) + C_1 C_L}$$

$$z_1 = +\frac{G_{m2}}{C_C}$$

$$A_{dv}(s) = \frac{A_{dv}(0)\cdot\left(1-\frac{s}{z_1}\right)}{\left(1-\frac{s}{p_1}\right)\cdot\left(1-\frac{s}{p_2}\right)} \approx \frac{A_{dv}(0)\cdot\left(1-\frac{s}{z_1}\right)}{1-\frac{s}{p_1}+\frac{s^2}{p_1 p_2}}$$

Table **9.1.1** Frequency characteristics of 2-stage OP amp for $C_L > C_C >> C_1$

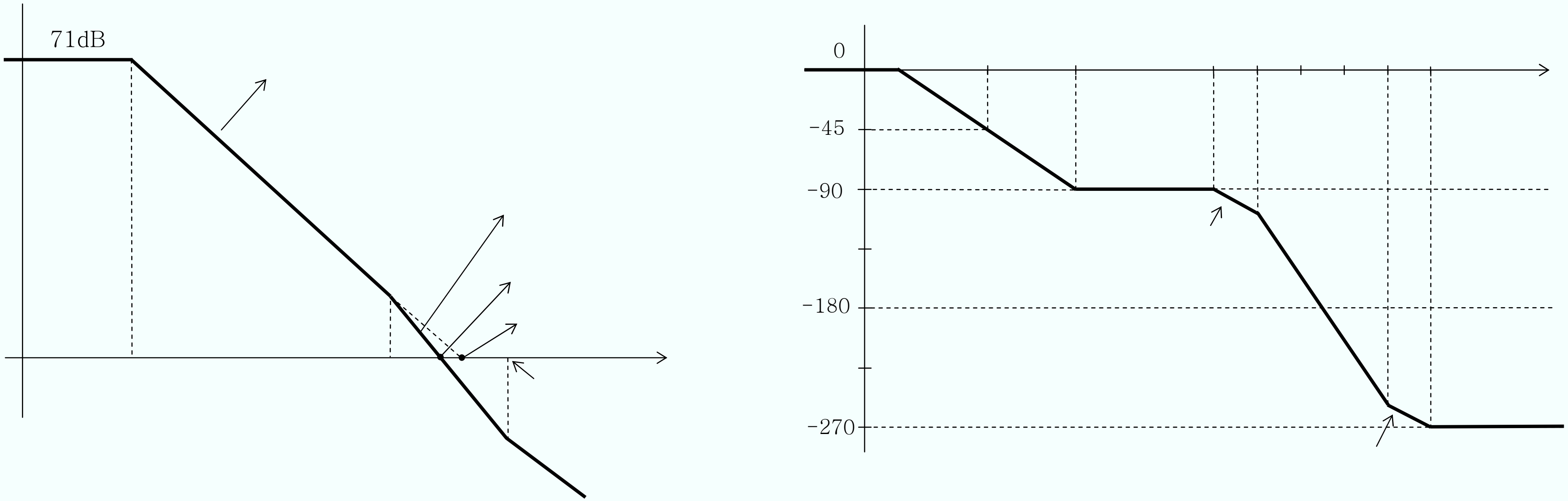
Parameters	$\lvert A_{dv}(0)\rvert$	p_1	p_2	z_1	ω_T
Equations	$G_{m1}R_{o1}\cdot G_{m2}R_{o2}$	$\frac{-1}{R_{o1}\cdot G_{m2}R_{o2}\cdot C_C}$	$-\frac{G_{m2}}{C_L}$	$+\frac{G_{m2}}{C_C}$	$\frac{G_{m1}}{C_C}$

2-stage OP amp

$G_{m1} = 1.76 \times 10^{-3}$ $G_{m2} = 1.94 \times 10^{-3}$ $R_{o1} = 64.8K\Omega$ $R_{o2} = 15.5K\Omega$ $C_L = 10pF$ $C_C = 2pF$

$A_{dv}(0) = 3425(71dB)$ $p_1 = -257\ K \cdot rad/\sec$ $p_2 = -194\ M \cdot rad/\sec$

$z_1 = +970\ M \cdot rad/\sec$ $\omega_T = 880\ M \cdot rad/\sec$



$$ph\{A_{dv}(j\omega_{0dB})\} = -90^\circ - ph\left(1 - \frac{j\omega_{0dB}}{p_2}\right) + ph\left(1 - \frac{j\omega_{0dB}}{z_1}\right) = -178^\circ$$

phase margin(unity-gain fb) = 2 deg ➔ requires frequency compensation

2-stage OP amp

Cases which require frequency compensation

2-stage OP amp: Cascade of two gain stages

Non-dominant pole > Unity gain frequency

$$\frac{1}{R_{o1}C_1} < \frac{G_{m1}R_{o1} \cdot G_{m2}}{C_L}$$

$$G_{m1}R_{o1} \cdot G_{m2}R_{o1} > \frac{C_L}{C_1}$$

If Ro1 is small, even the cascade of two amps does not satisfy the above condition

→ Does not need freqyency compensation

→ It is considered as a single-stage OP amp

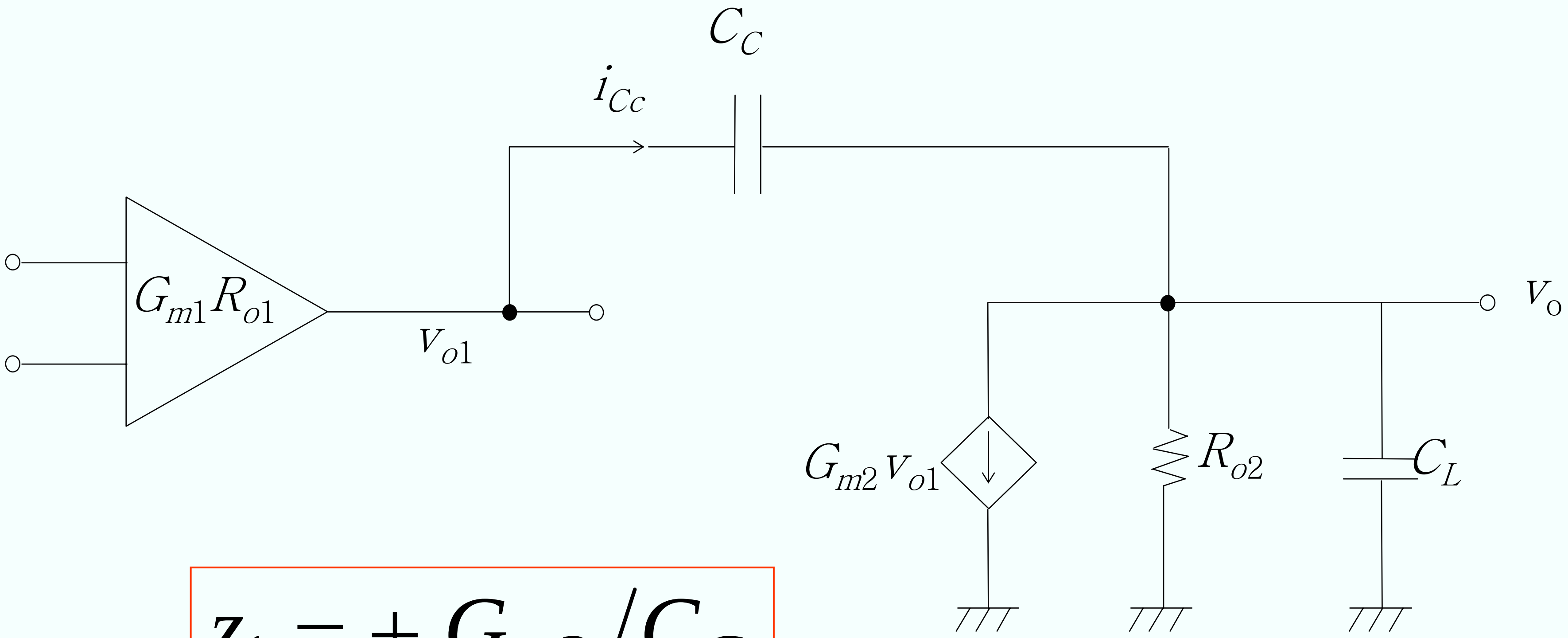
2-stage OP amp

positive real zero

Feed-forward through cap in an inverting amp

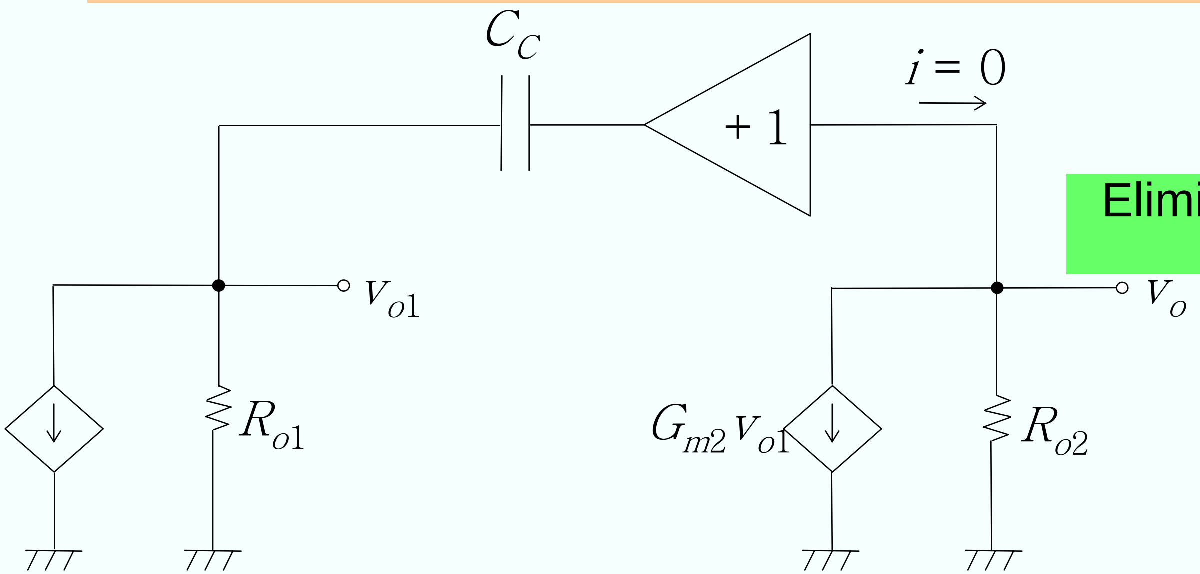
➔ Positive real zero

➔ Degrades phase margin



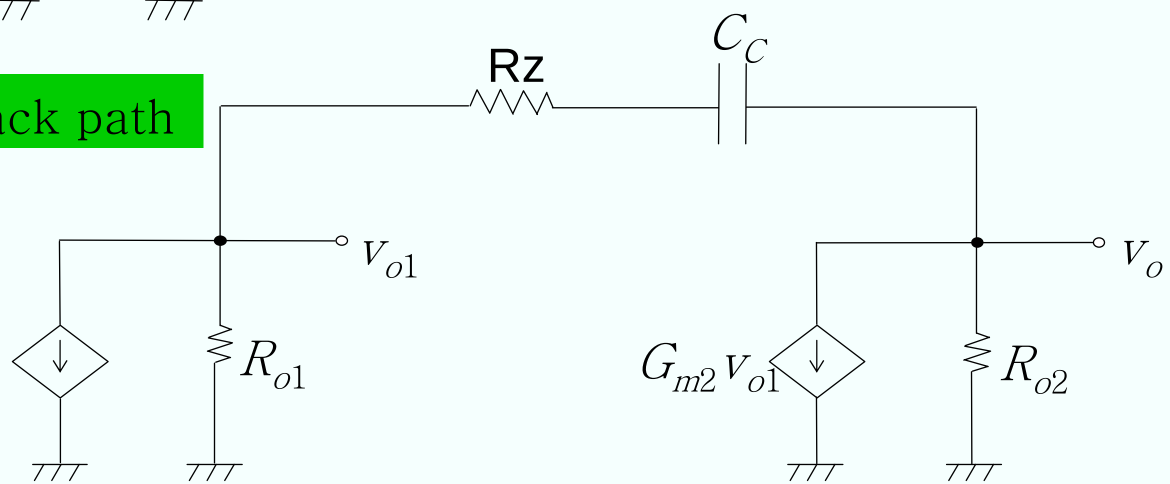
$$z_1 = +G_{m2}/C_C$$

Two methods to eliminate positive real zero



Eliminates feed-forward path ➔ eliminates zero

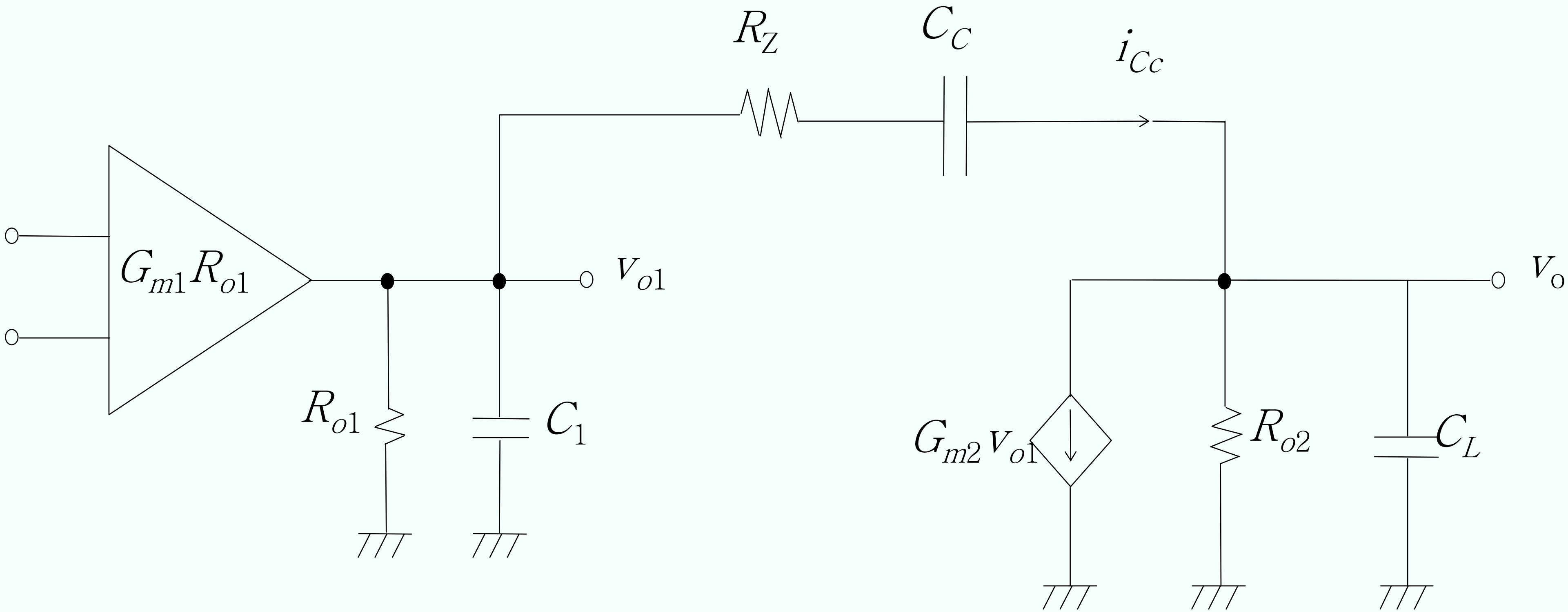
(a) A gain 1 voltage buffer at feedback path



Rz > 1/Gm2 ➔ changes feed-forward polarity ➔ postive real zero changed to negative real zero

(b) Rz (> 1/Gm2) in series with Cc

2-stage OP amp



$$i_{C_c} = \frac{v_{o1}}{R_Z + \frac{1}{sC_C}} = G_{m2}v_{o1}$$

$$z_1 = \frac{G_{m2}}{C_C} \cdot \frac{1}{1 - G_{m2} \cdot R_Z}$$

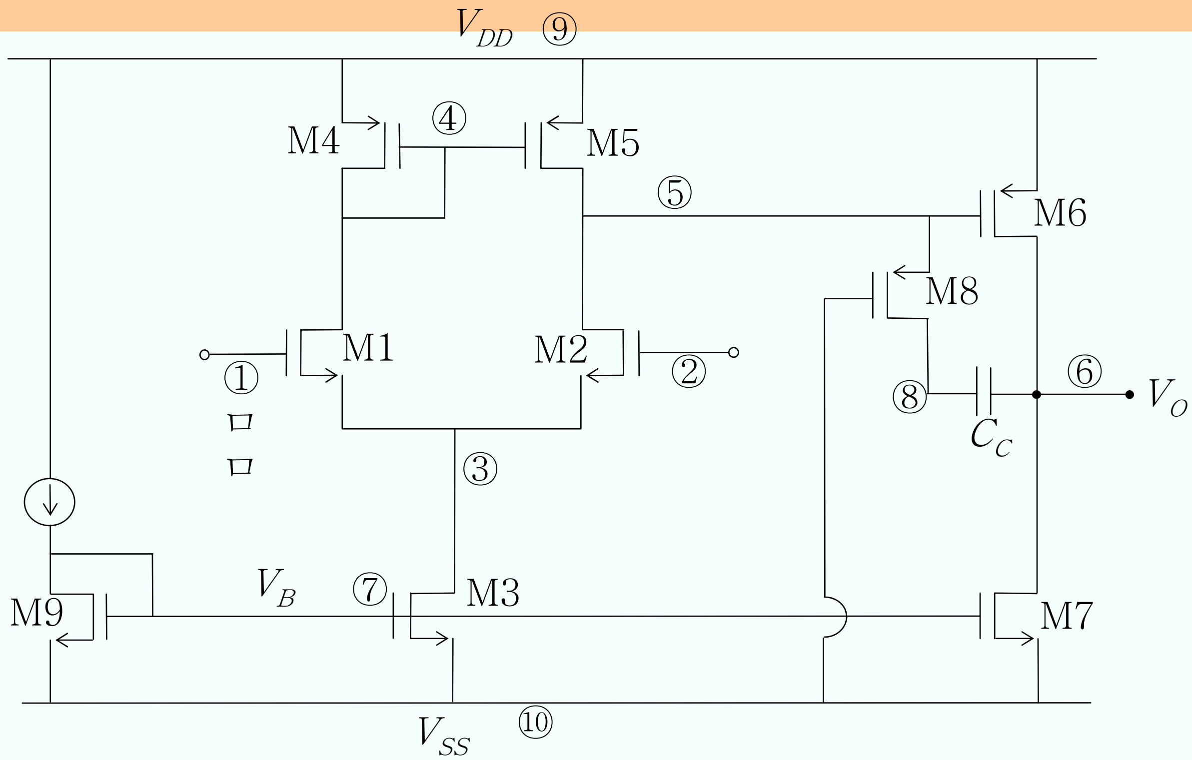
Rz > 1/Gm2 ➔ changes feed-forward polarity ➔ postive real zero changed to negative real zero

Frequency compensation (SPICE simulation results)

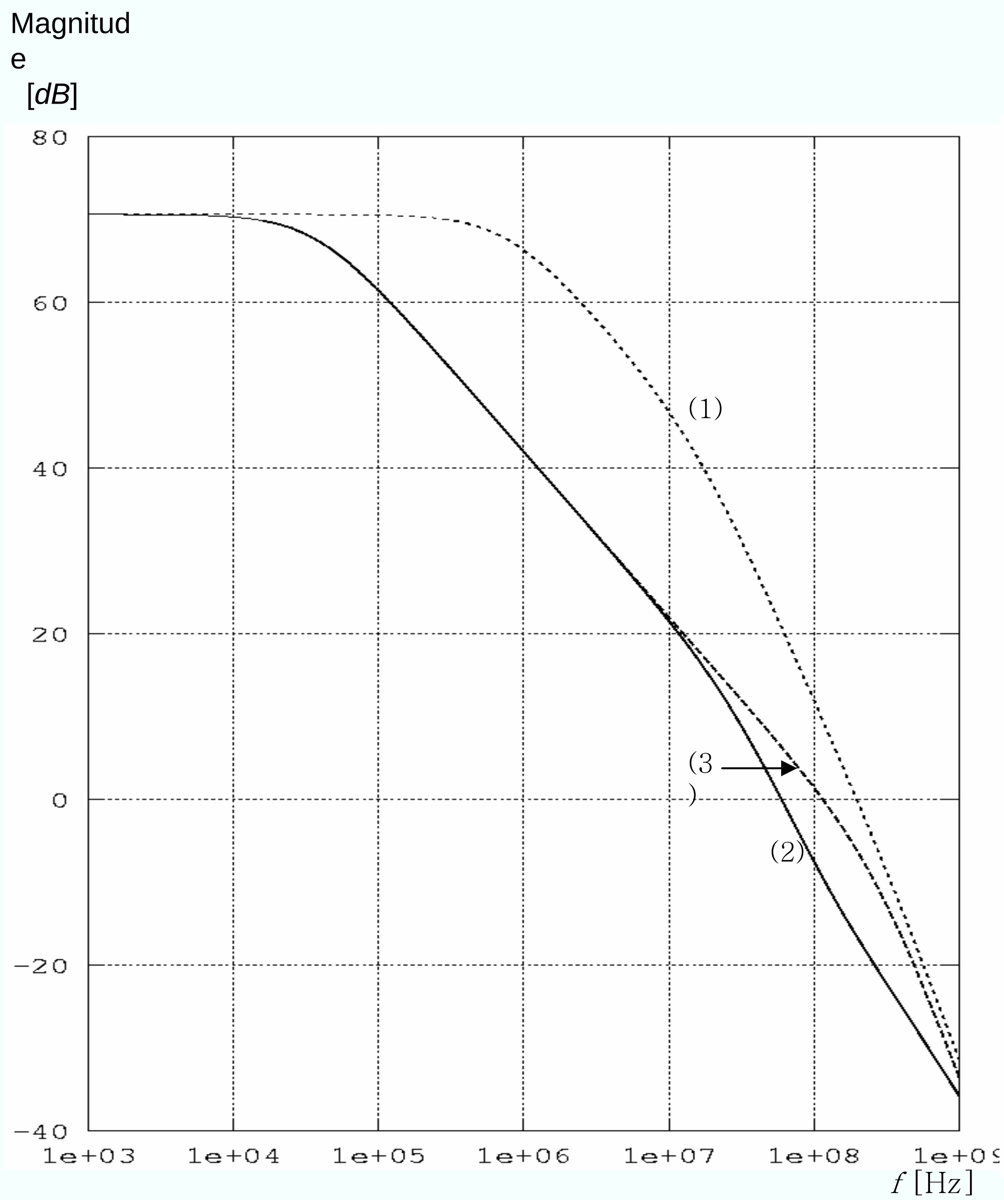
(1) No frequency compensation

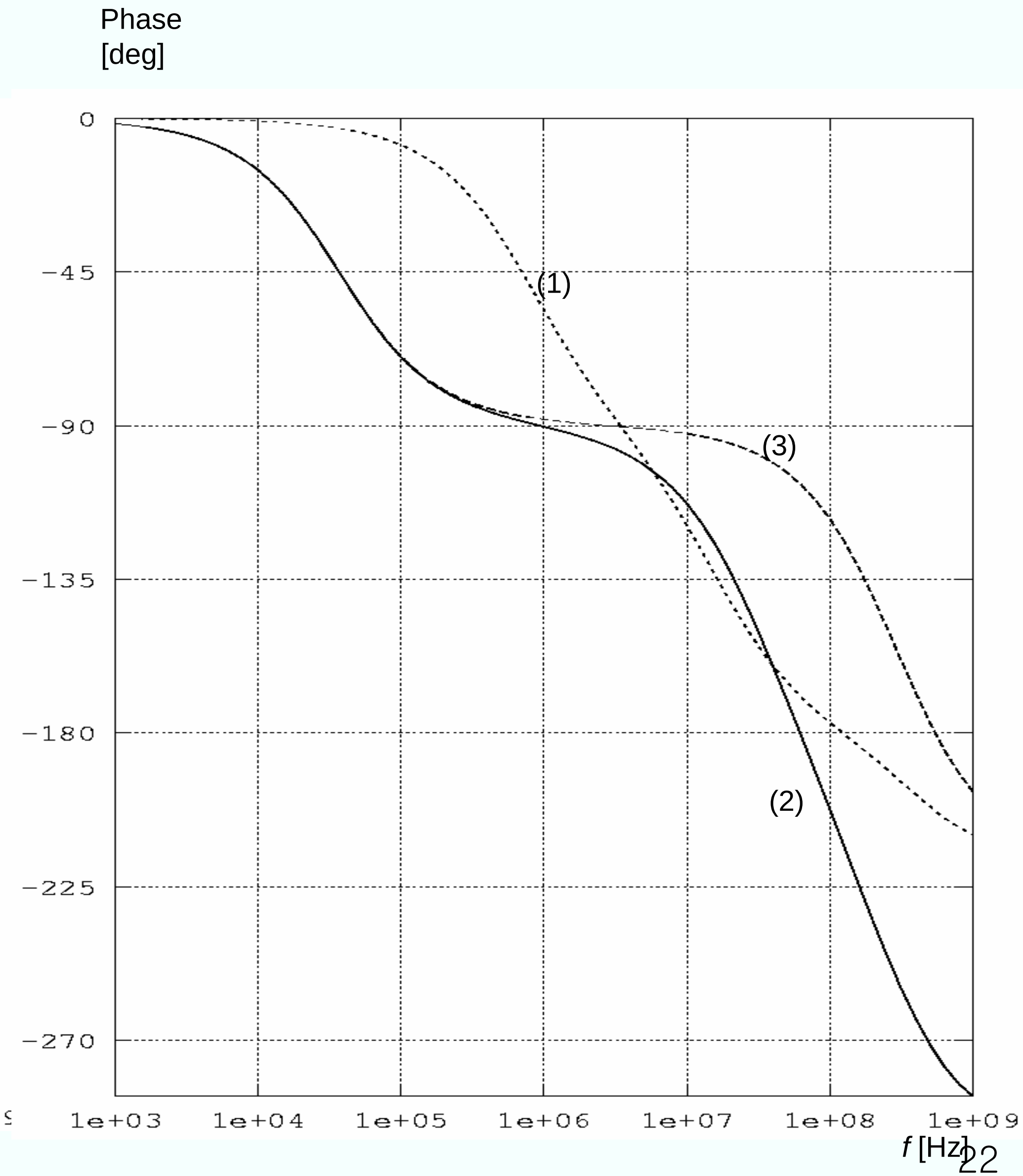
(2) Frequency compensation with a 2*pF* capacitor

(3) Frequency compensation with a series connection of 3.1kohm resistor and a **2*pF*** capacitor



2-stage OP amp (open loop freq response)



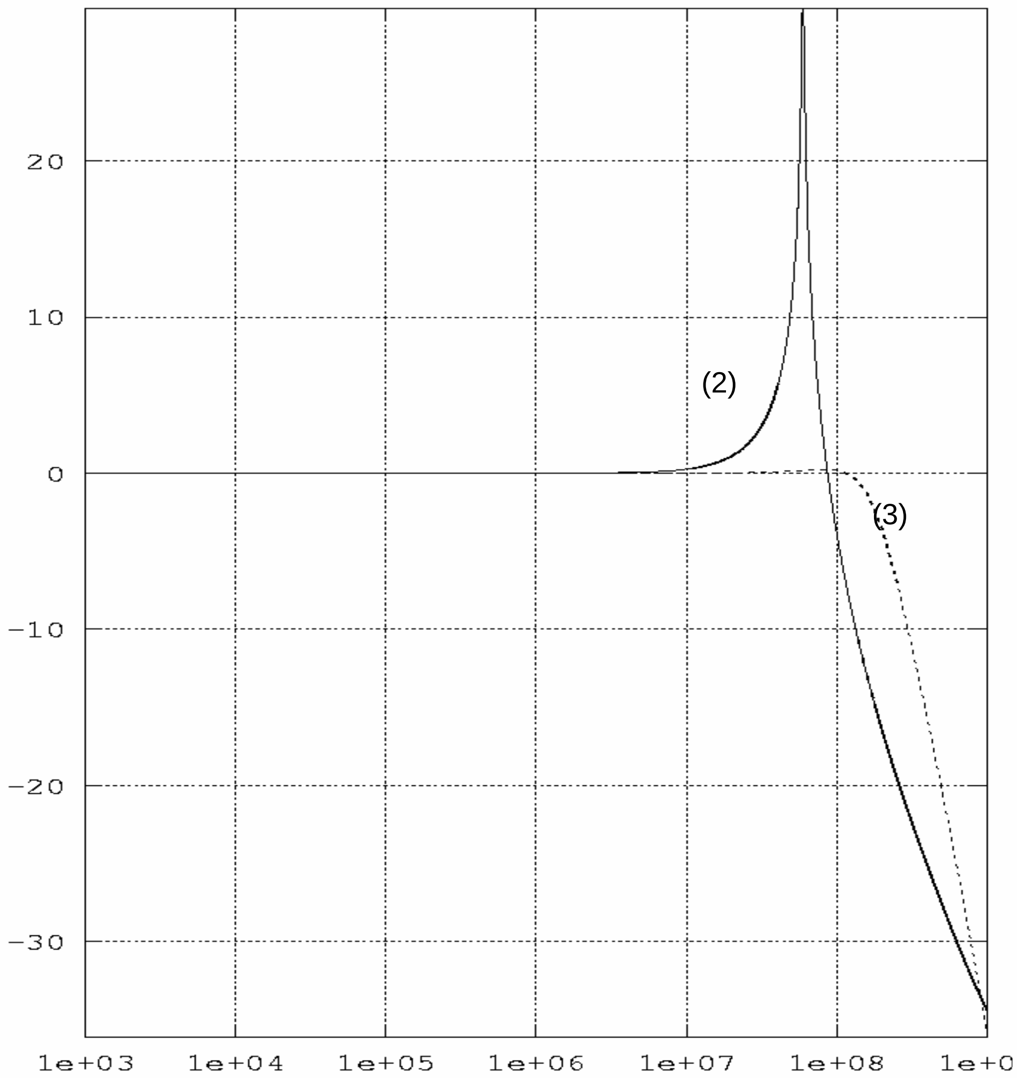


2-stage OP amp (open loop freq response)

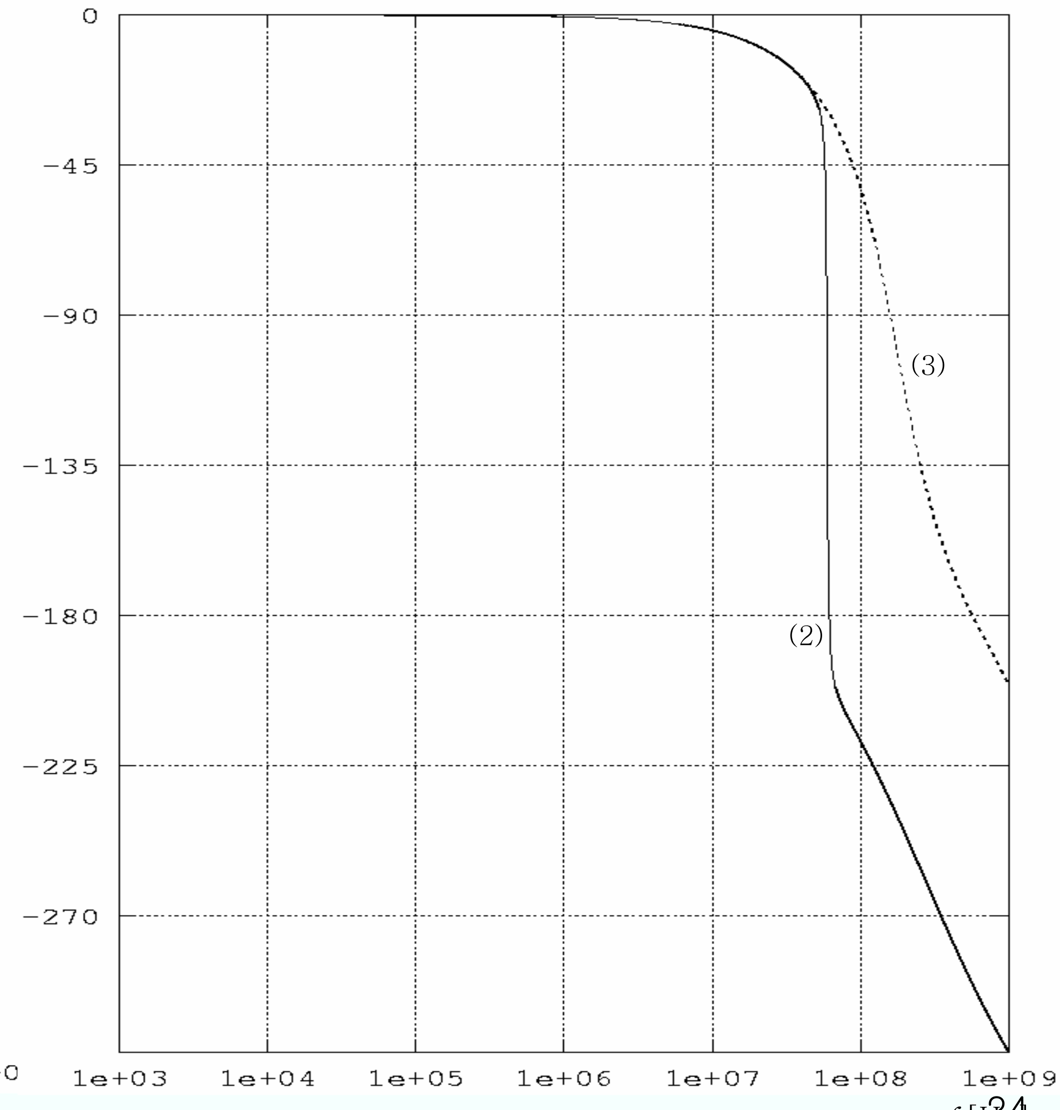
	$\frac{p_1}{2\pi}$	$\frac{p_2}{2\pi}$	$\frac{p_3}{2\pi}$	$\frac{z_1}{2\pi}$	$\frac{\omega_{0dB}}{2\pi}$	phase margin (f=1)
(1) No freq comp	-740 *KHz*	-16.5 *MHz*	-	-	125 *MHz*	-7°
(2) Freq comp with Cc	-37 *KHz*	-30.7 *MHz*	-	+154 *MHz*	59.7 *MHz*	$+1.6^\circ$
(3) Freq comp with Rz, Cc	-37 *KHz*	-30.7 *MHz*	-160 *MHz*	-30.9 *MHz*	115 *MHz*	$+59^\circ$

2-stage OP amp (freq response: unity-gain fb)

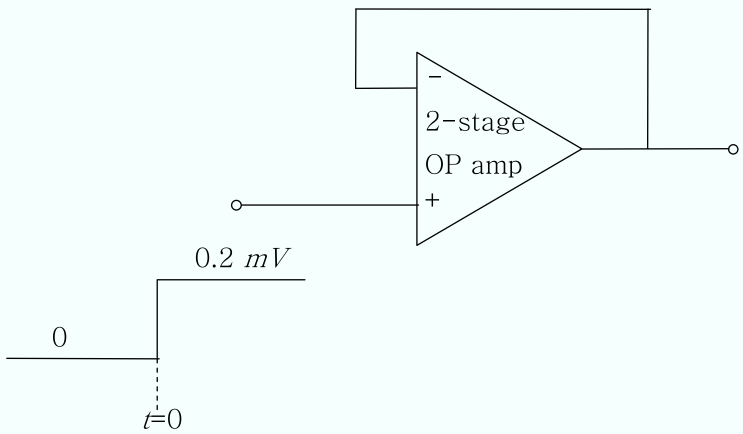
Magnitude [*dB*]

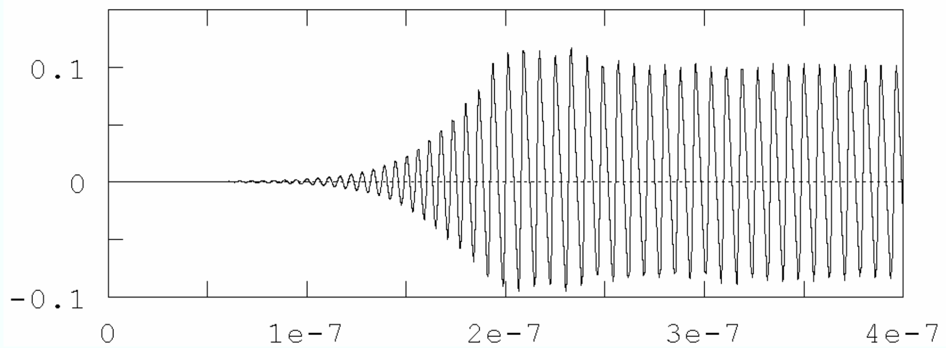


Phase [*dB*]

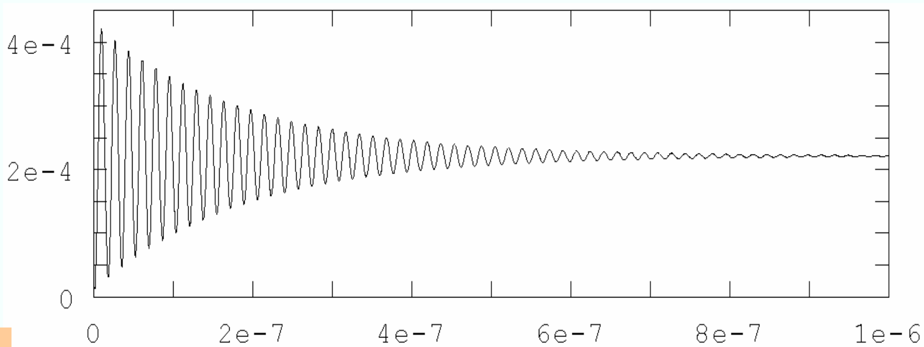


2-stage OP amp (transient response: unity-gain fb)

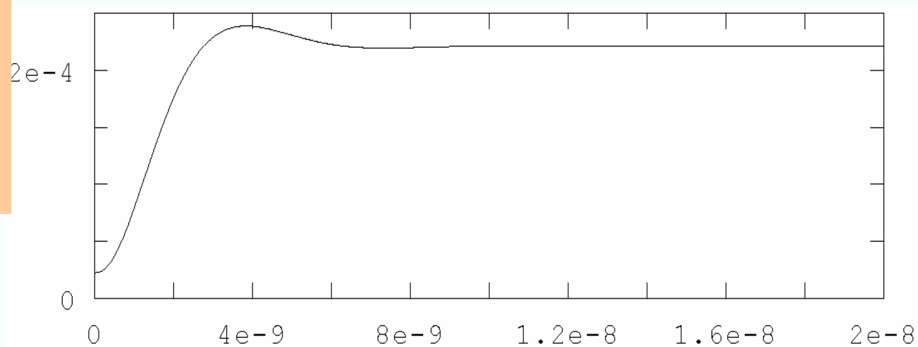




(a)



(b)



(c)

(1) No freq compensation (PM= **-7 deg)**

(2) Frequency compensation with a 2*pF* capacitor (PM = +1.6 deg)

(3) Frequency compensation with a 3.1kohm의 resistor and a 2*pF* capacitor (PM = +59 deg)

2-stage OP amp (freq response: unity-gain fb)

$$A_{dv}(s) = \frac{A_{dv}(0)\cdot\left(1-\frac{s}{z_1}\right)}{\left(1-\frac{s}{p_1}\right)\cdot\left(1-\frac{s}{p_2}\right)}$$

$$A_f(s) = \frac{A_{dv}(0)}{1+A_{dv}(0)}\cdot\frac{1-\frac{s}{z_1}}{1-s\cdot\frac{1}{1+A_{dv}(0)}\cdot\left(\frac{1}{p_1}+\frac{1}{p_2}+\frac{A_{dv}(0)}{z_1}\right)+\frac{s^2}{(1+A_{dv}(0))\cdot p_1\cdot p_2}} \approx \frac{1-\frac{s}{z_1}}{1-s\cdot\left\{\frac{1}{A_{dv}(0)}\cdot\left(\frac{1}{p_1}+\frac{1}{p_2}\right)+\frac{1}{z_1}\right\}+\frac{s^2}{A_{dv}(0)\cdot p_1\cdot p_2}}$$

$$A_f(s) = \frac{1-\frac{s}{z_1}}{1+\frac{s}{\omega_o}\cdot\frac{1}{Q}+\frac{s^2}{\omega_o^2}}$$

$$\omega_o = \sqrt{A_{dv}(0)\cdot p_1\cdot p_2}$$

$$Q = \frac{1}{\frac{-p_1-p_2}{\omega_o}-\frac{\omega_o}{z_1}}$$

Positive real zero: increase Q ➔ less damped

Negative real zero: decrease Q ➔ more damped

2-stage OP amp (step response: unity-gain fb)

$$V_O(s) = \frac{\Delta V_I}{s} \cdot \frac{1 - \frac{s}{z_1}}{1 + \frac{s}{\omega_o} \cdot \frac{1}{Q} + \frac{s^2}{\omega_o^2}} = \Delta V_I \cdot \left(\frac{1}{s} - \frac{\left(s + \frac{\omega_o}{2Q} \right) + \left(\frac{\omega_o}{2Q} + \frac{\omega_o^2}{z_1} \right)}{\left(s + \frac{\omega_o}{2Q} \right)^2 + \omega_o^2 \cdot \left(1 - \frac{1}{4Q^2} \right)} \right)$$

$$v_O(t) = \Delta V_I \cdot u(t) \cdot \left(1 - A_m \cdot e^{-\frac{\omega_0}{2Q} \cdot t} \cdot \sin(\omega_n t + \phi) \right)$$

$$\omega_n = \omega_o \cdot \sqrt{1 - \frac{1}{4Q^2}} \qquad \phi = \tan^{-1}\left(\frac{\sqrt{1 - \frac{1}{4Q^2}}}{\frac{1}{2Q} + \frac{\omega_o}{z_1}} \right) \qquad A_m = \frac{\sqrt{1 + \frac{\omega_o}{Qz_1} + \frac{\omega_o^2}{z_1^2}}}{\sqrt{1 - \frac{1}{4Q^2}}}$$

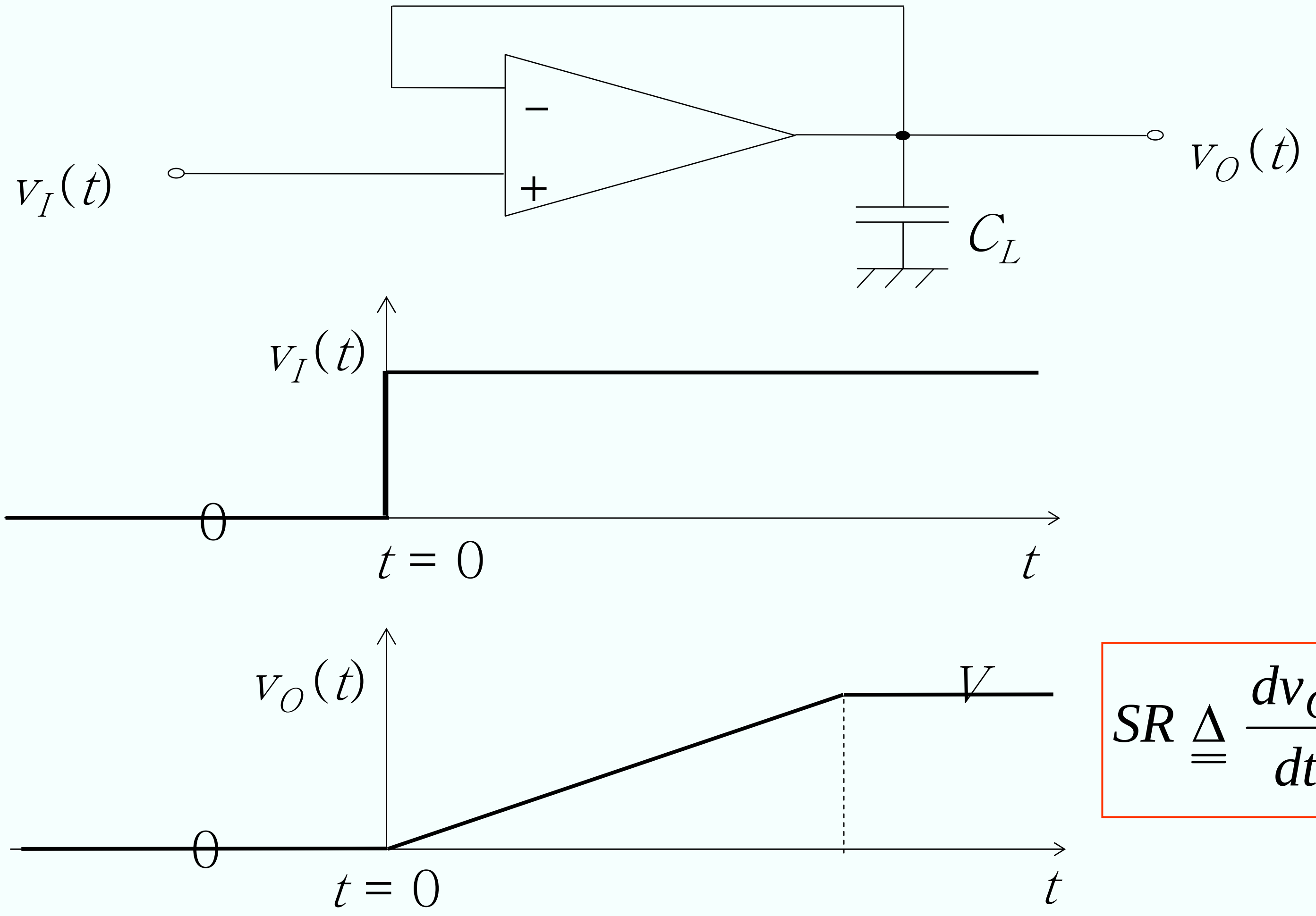
$$t_s = \frac{2Q}{\omega_o} \cdot \ln\left(\frac{A_m}{\varepsilon} \right)$$

increase Q ➔ increase settling time

$$t_s = \frac{1}{\omega_T} \cdot \ln\left(\frac{1}{\varepsilon} \right)$$

Single pole case

Slew rate limitation due to Cc

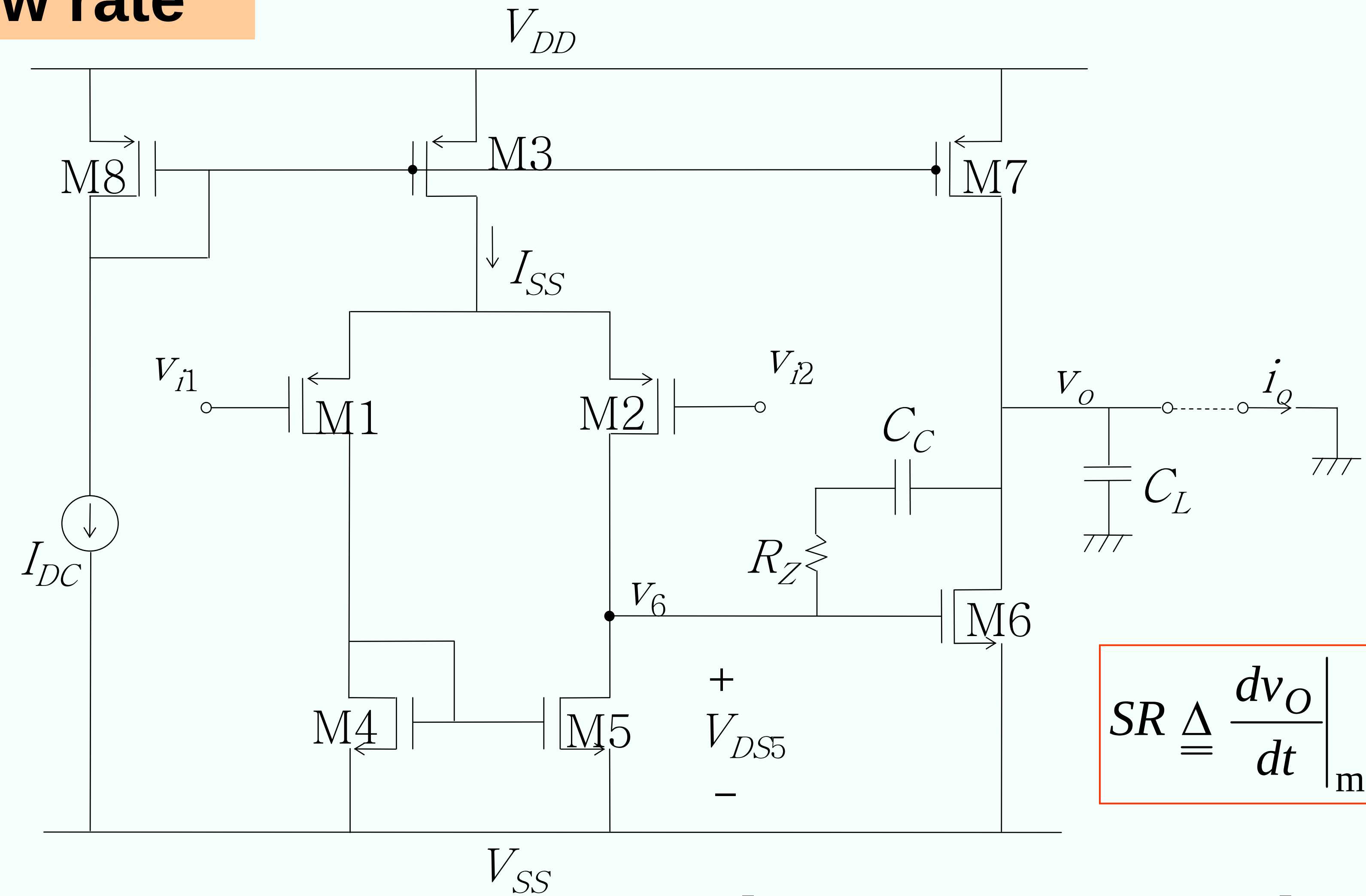


$$SR \triangleq \left.\frac{dv_O}{dt}\right|_{max} = \frac{I_{SS}}{C_C}$$

$$v_O(t) = (Voltage\,across\,C_C) + V_{DS5} + V_{SS} + I_{SS}R_Z \quad = \frac{I_{SS}}{C_C}\cdot t + I_{SS}R_Z + v_O(t=0) \quad = \frac{I_{SS}}{C_C}\cdot t + I_{SS}R_Z$$

2-stage OP amp

Slew rate



$$SR \triangleq \left.\frac{dv_O}{dt}\right|_{\max} = \frac{I_{SS}}{C_C}$$

$$v_O(t) = (Voltage\,across\,C_C) + V_{DS5} + V_{SS} + I_{SS}R_Z \quad = \frac{I_{SS}}{C_C}\cdot t + I_{SS}R_Z + v_O(t=0) \quad = \frac{I_{SS}}{C_C}\cdot t + I_{SS}R_Z$$

2-stage OP amp

FBBW (full power bandwidth): The maximum sine wave frequency with full amplitude without being distorted by slew rate phenomenon

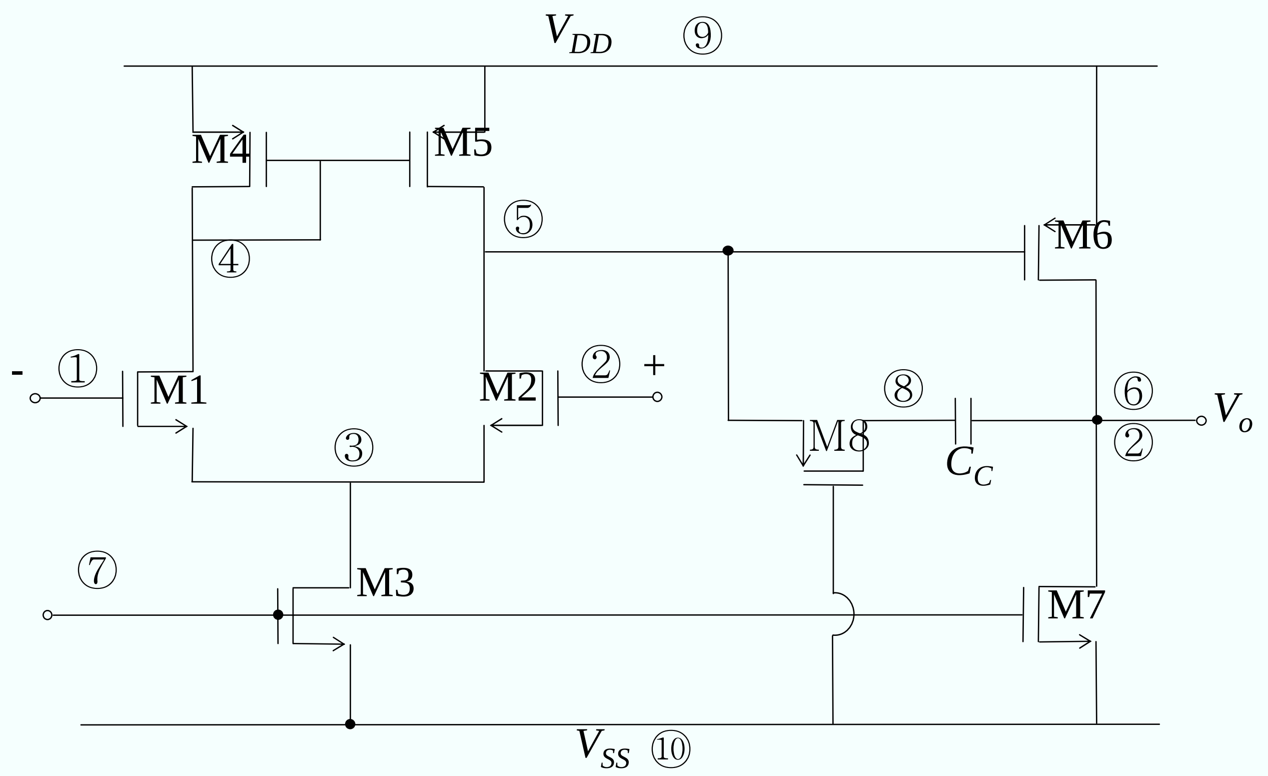
$$v_O(t) = V_m \cdot \sin(2\pi f_{FPBW} \cdot t)$$

$$\left.\frac{dv_O}{dt}\right|_{\max} = 2\pi f_{FPBW} \cdot V_m = SR$$

$$\boxed{f_{FPBW} = \frac{SR}{2\pi V_m}}$$

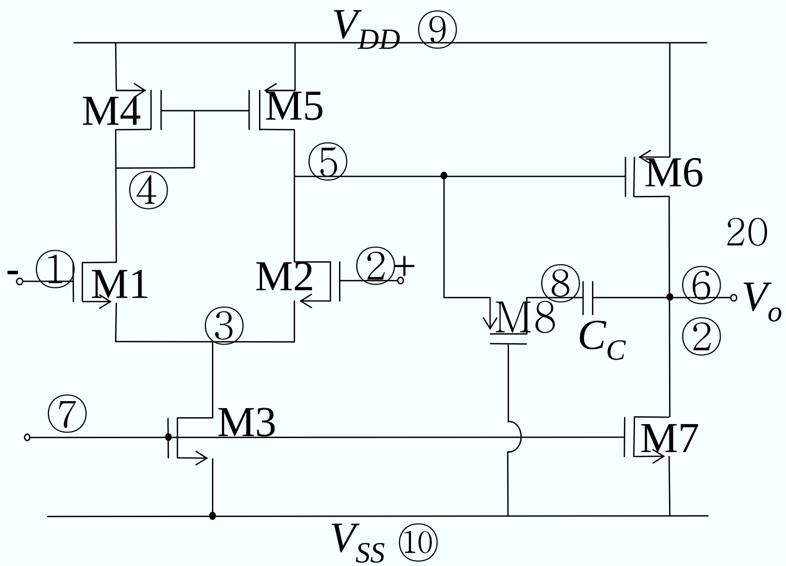
$$SR = \omega_T \cdot \left|V_{GS} - V_{TH}\right|_{1,2}$$

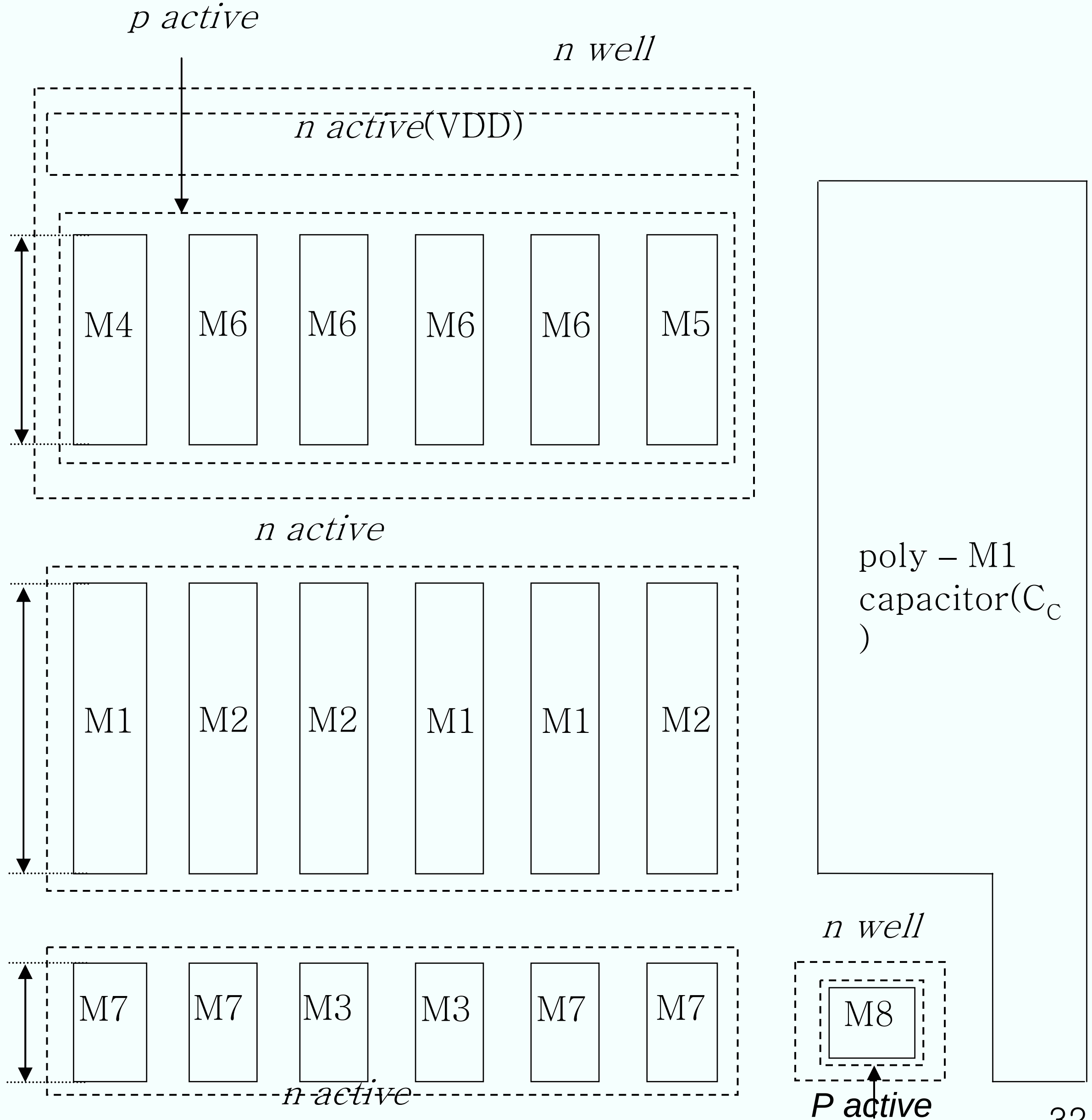
2-stage OP amp (Layout)



transistor	$W(\mu m)$
M1, M2	120
M3	20
M4, M5	20
M6	80
M7	40
M8	7

2-stage OP amp (Layout)





2-stage OP amp (Layout)