

CH8

8.2 Bias circuit

Ideal voltage or current source : no PVT dependence
(PVT: process voltage temperature)

1. Supply voltage dependence of bias circuit
2. Self-bias circuit
3. CMOS band-gap reference

Simple current source (use V_{DD} as reference voltage):
 V_{DD} increase \rightarrow I_{out} increase

$$\Delta I_{out} = \frac{\Delta V_{DD}}{R_1 + 1/g_{m1}} \cdot \frac{(W/L)_2}{(W/L)_1}$$

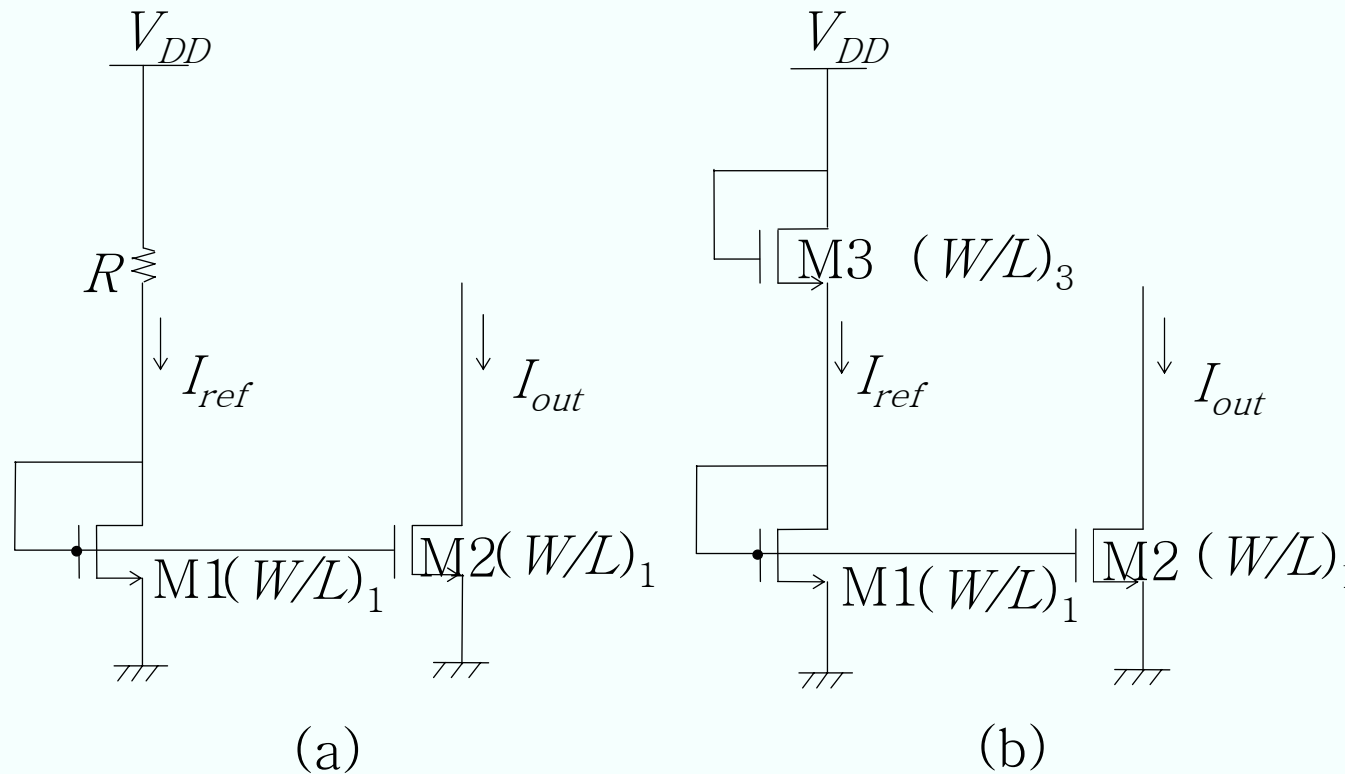
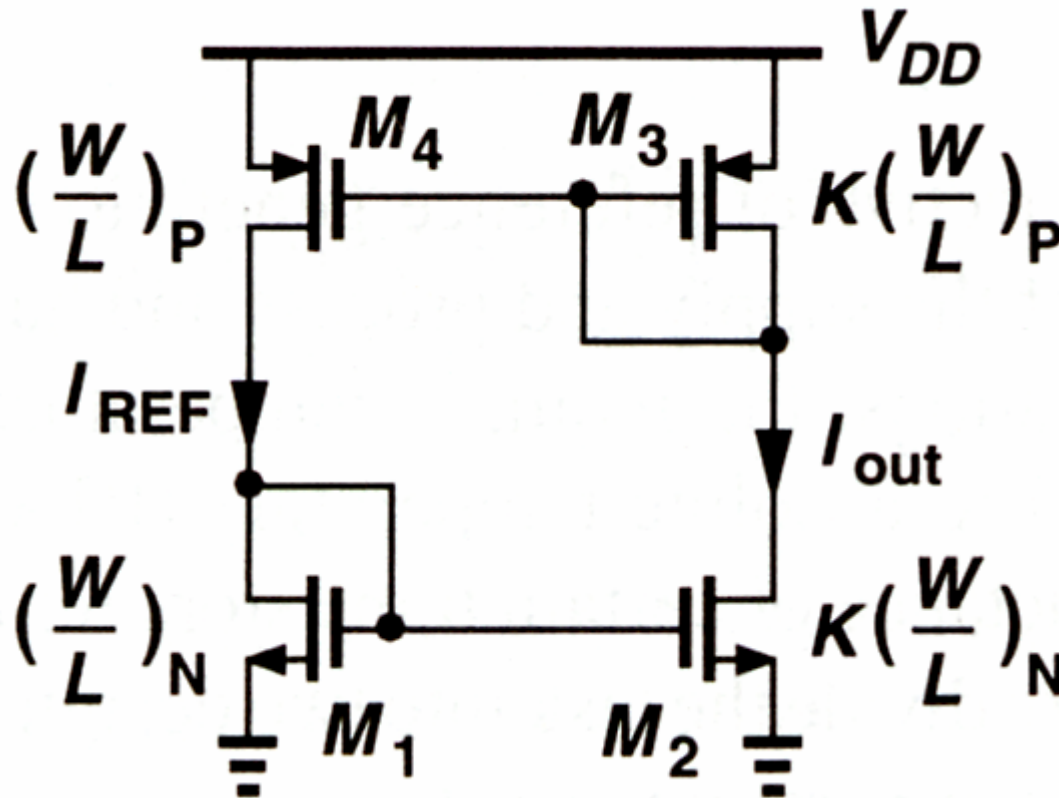


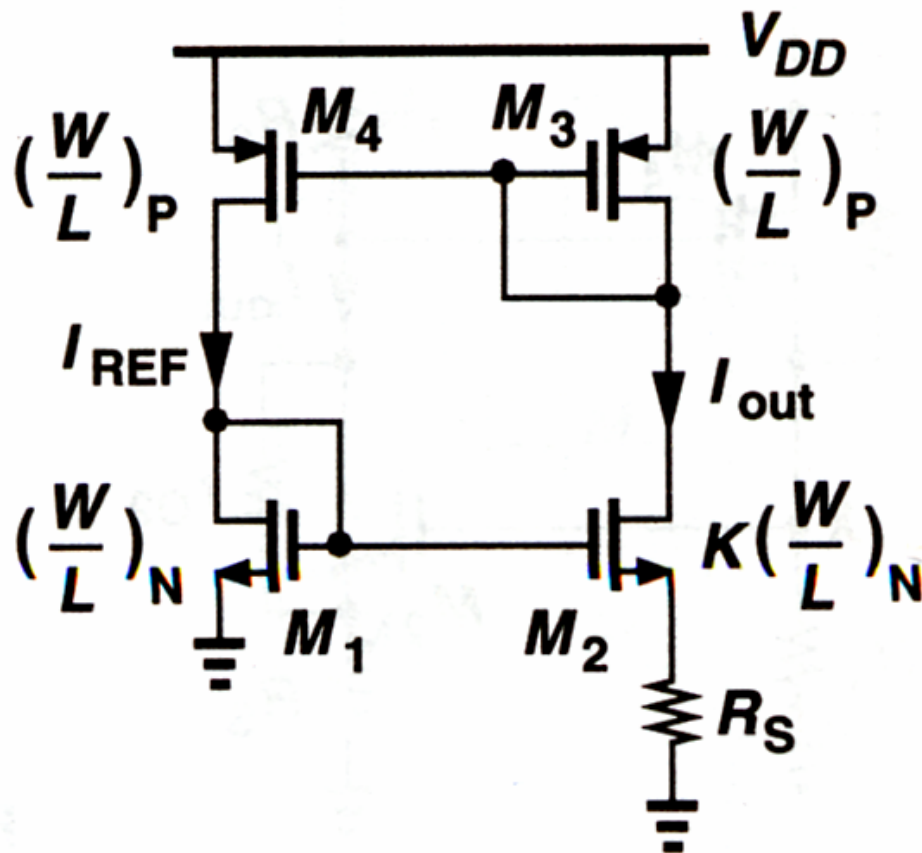
Fig 8.2.1 Simple current bias circuits

A simple self-bias circuit



$$I_{out} = K I_{REF}$$

I_{out} can be any value (including 0),
Cannot be used



I_{out} determined to be a single value

M3, M4 $\rightarrow I_{ref} = I_{out}$

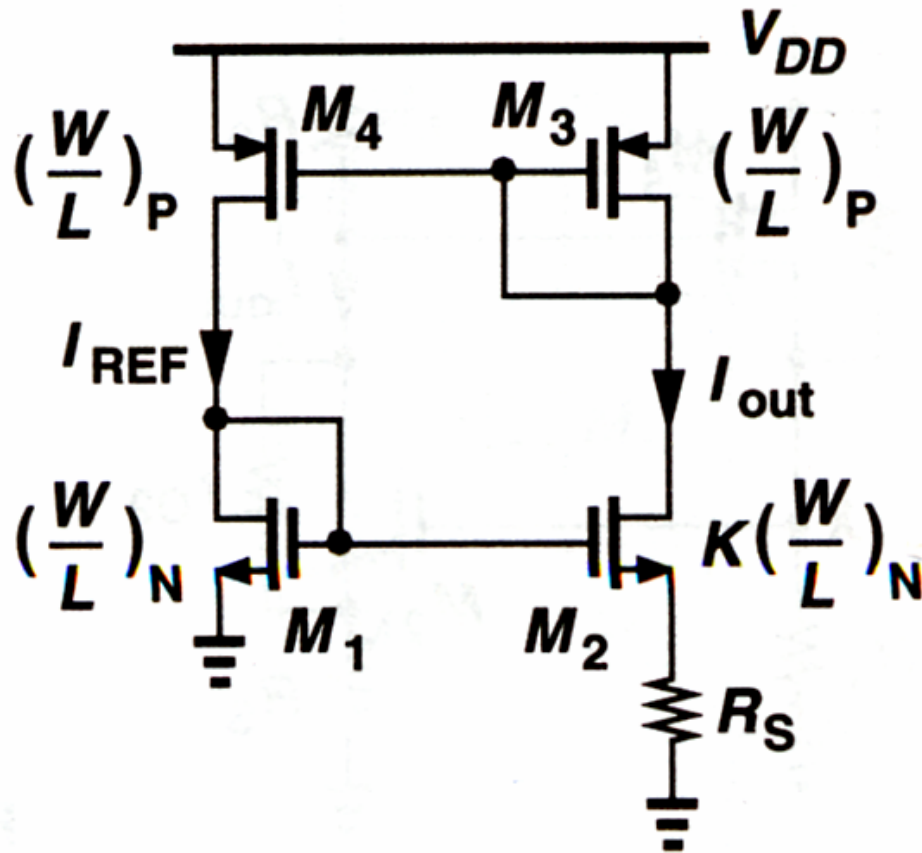
M1, M2, $R_s \rightarrow$

$V_{GS1} = V_{GS2} + I_{out} \times R_s$

$$\sqrt{\frac{2I_{out}}{\mu_n C_{ox} (W/L)_N}} + V_{TH1} = \sqrt{\frac{2I_{out}}{\mu_n C_{ox} K(W/L)_N}} + V_{TH2} + I_{out} R_s$$

I_{out} can be 0

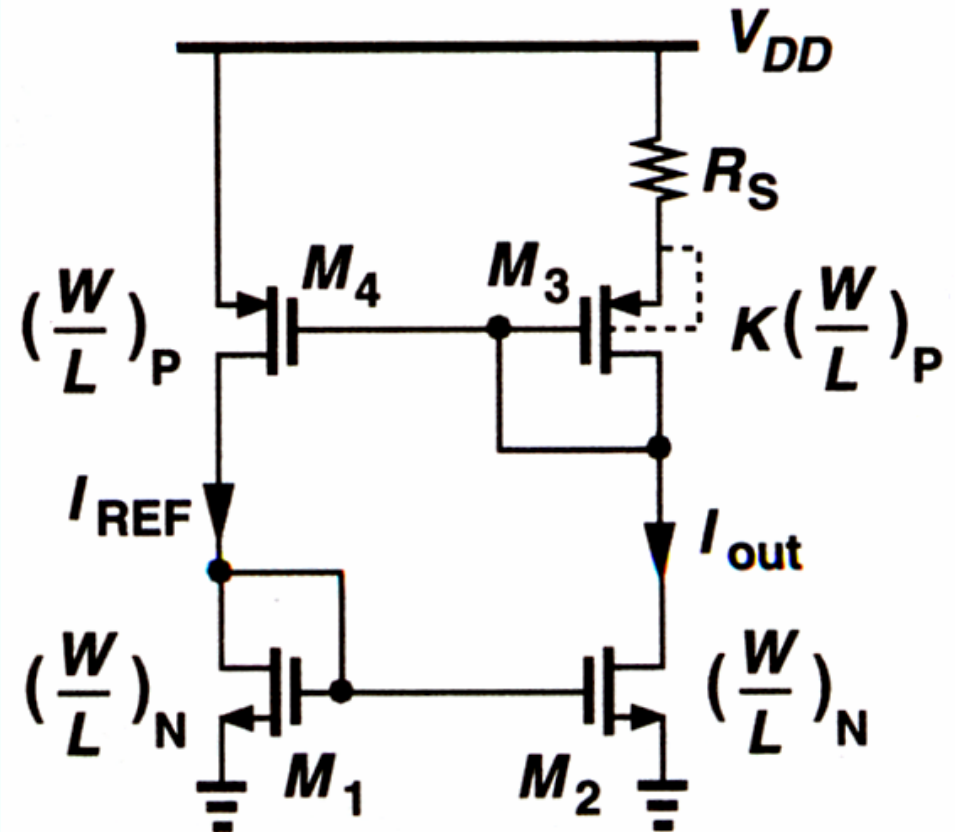
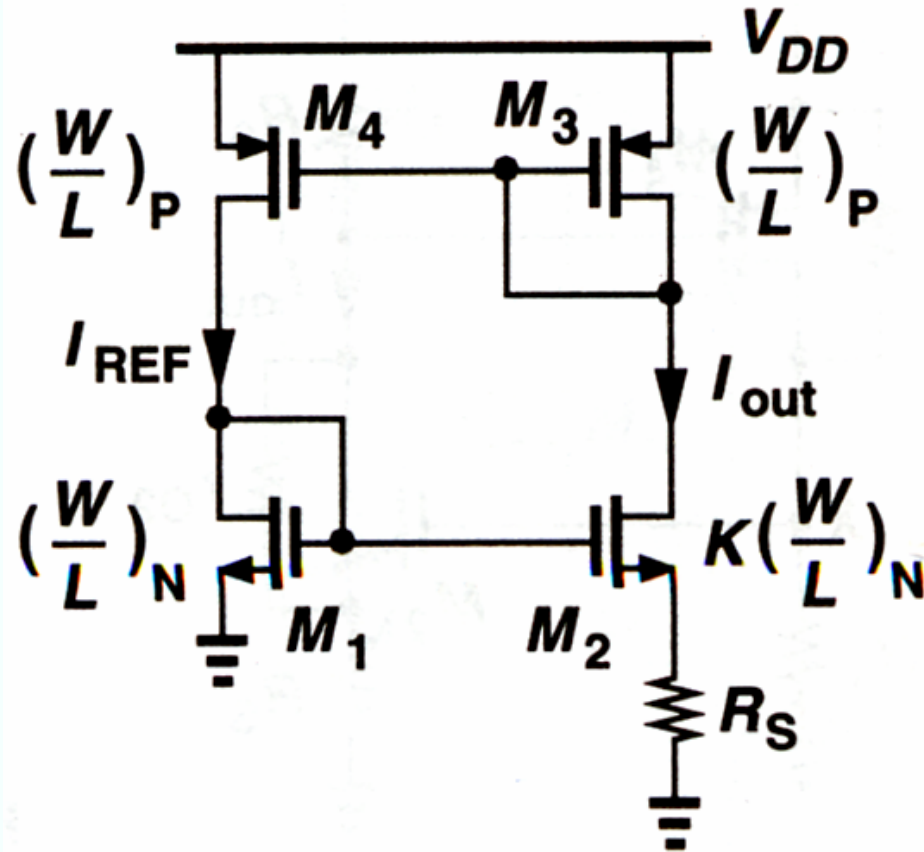
$$I_{out} = \frac{2}{\mu_n C_{ox} (W/L)_N} \cdot \frac{1}{R_s^2} \left(1 - \frac{1}{\sqrt{K}}\right)^2$$



$$I_{out} = \frac{2}{\mu_n C_{ox} (W/L)_N} \cdot \frac{1}{R_S^2} \left(1 - \frac{1}{\sqrt{K}} \right)^2$$

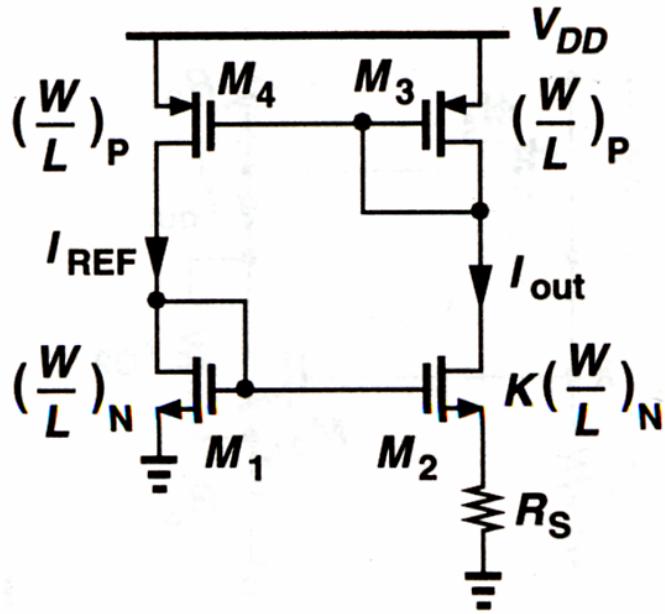
I_{out} can be 0 \rightarrow needs start-up ckt

Body effect on M2 \rightarrow inaccuracy



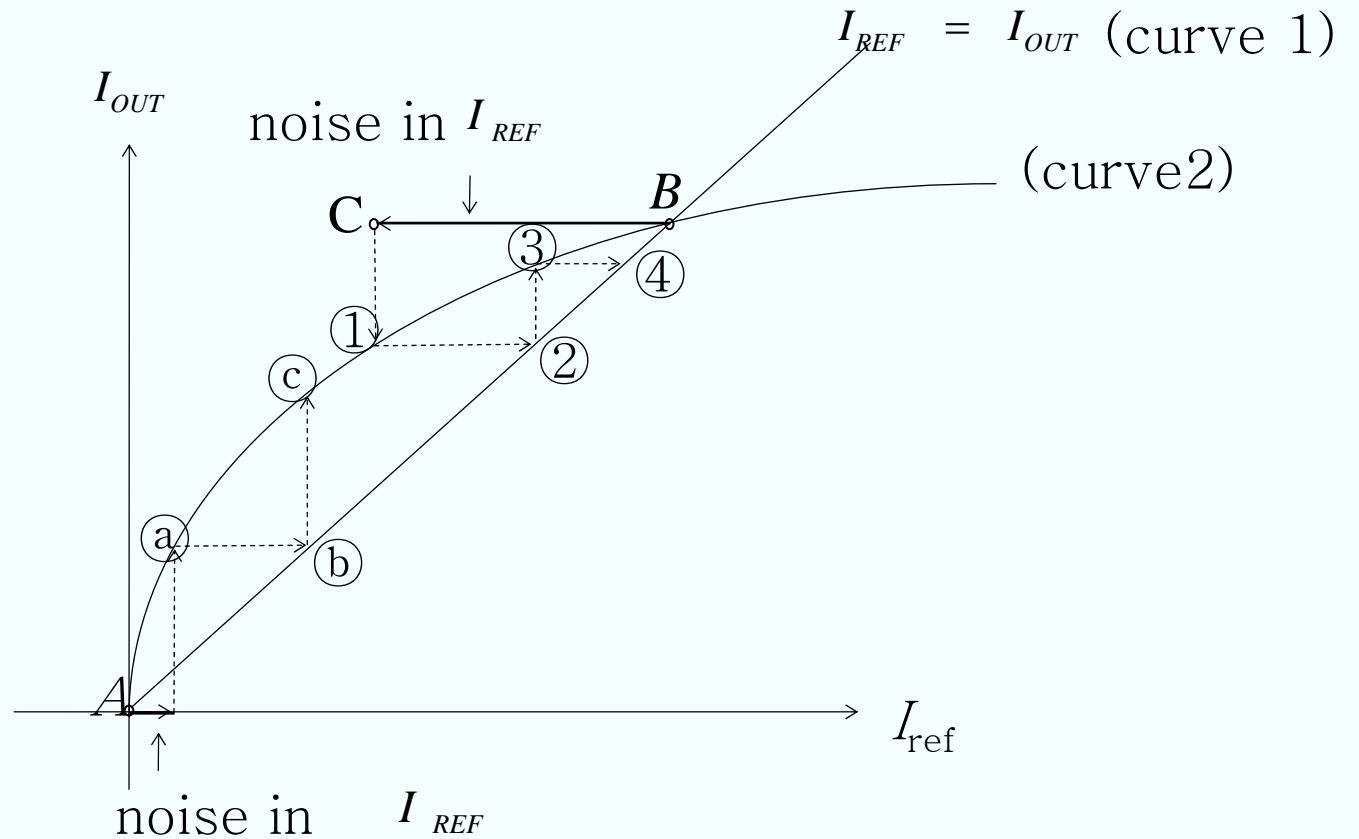
Body effect eliminated

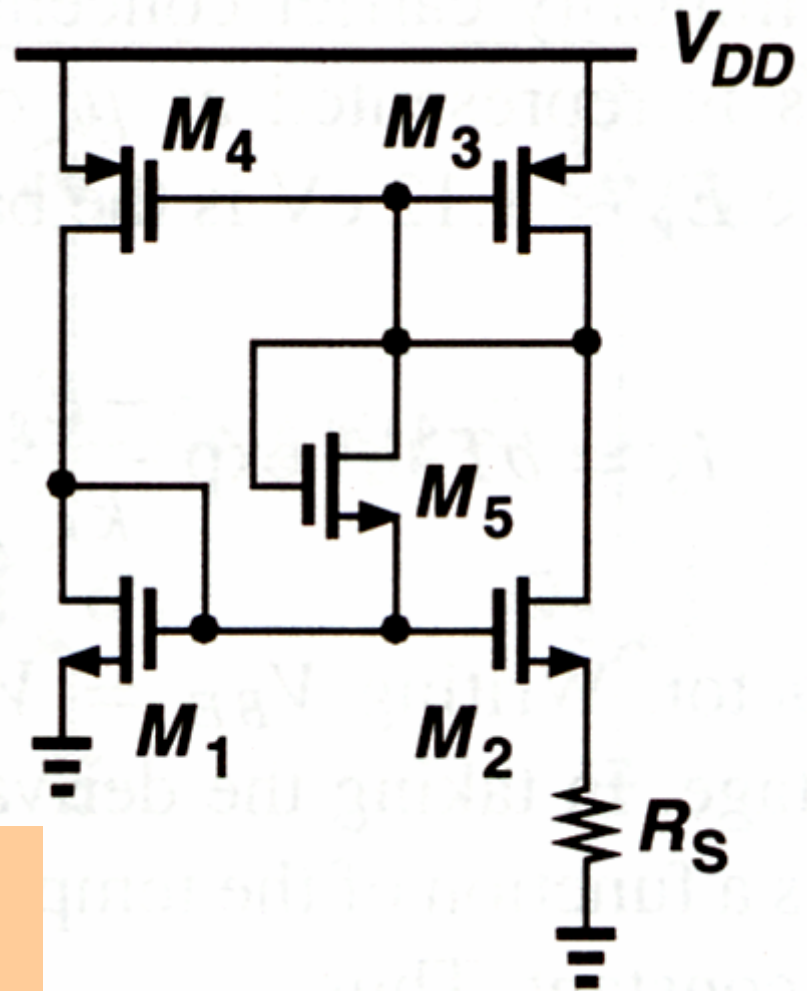
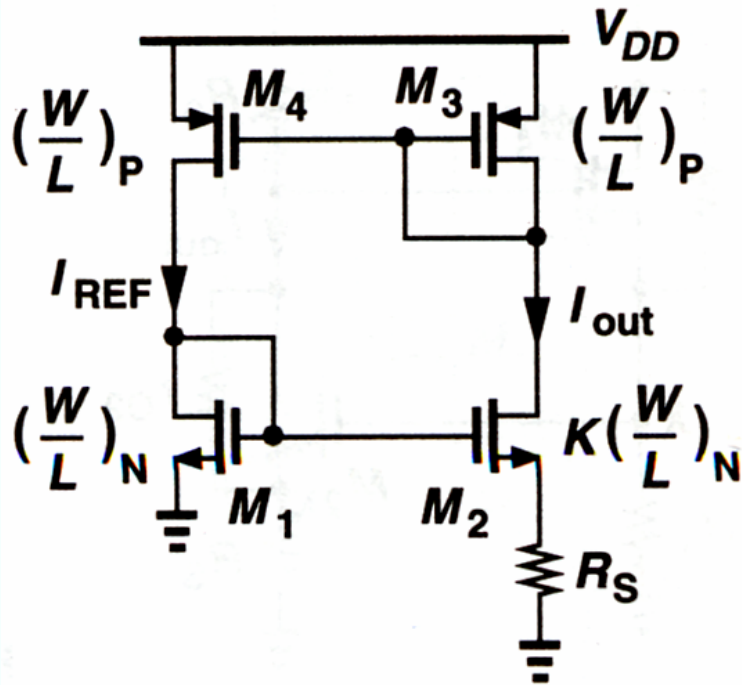
Errata: M_4 , M_2 in diode
 M_3 , M_1 Not in diode



$I_{ref} = I_{out}$ (curve 1)

$$\sqrt{\frac{2I_{out}}{\mu_n C_{ox}(W/L)_N}} + V_{TH1} = \sqrt{\frac{2I_{out}}{\mu_n C_{ox}K(W/L)_N}} + V_{TH2} + I_{out}R_S$$





Eliminate the operating point of $I_{out}=0$
Constraints:

$$(1) \quad V_{TH1} + V_{TH5} + |V_{TH3}| < V_{DD}$$

$$(2) \quad V_{GS1} + V_{TH5} + |V_{GS3}| > V_{DD}$$

➔ Not a good start-up ckt

Supply-insensitive biasing :

cannot use VDD as reference

Use one of the following three as reference instead

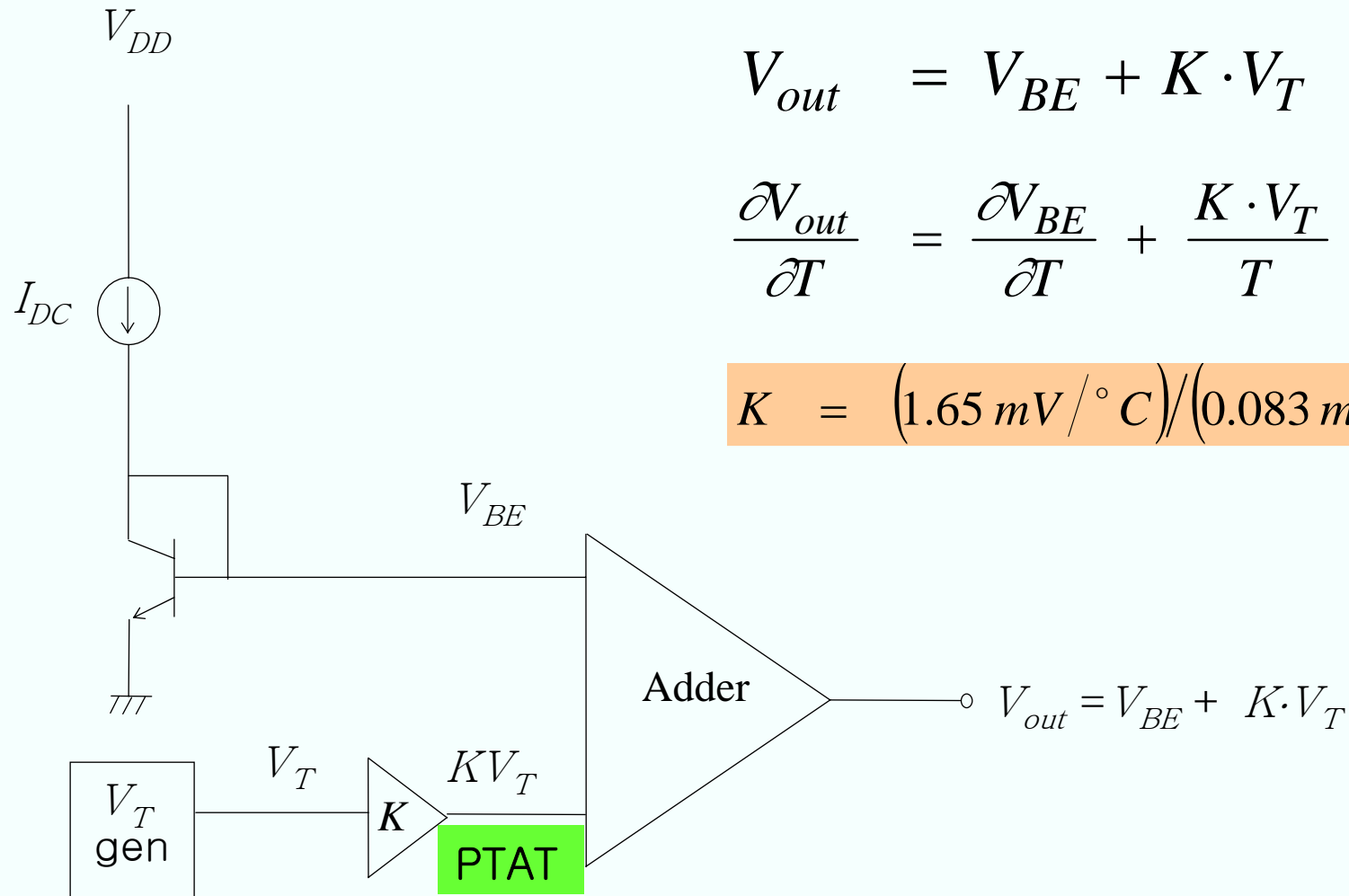
(1) BJT VBE

(2) MOS threshold voltage

(3) Thermal voltage V_T

→ self bias circuit

Band gap reference : PVT insensitive voltage source($\sim 1.2V$)



$$V_{out} = V_{BE} + K \cdot V_T$$

$$\frac{\partial V_{out}}{\partial T} = \frac{\partial V_{BE}}{\partial T} + \frac{K \cdot V_T}{T}$$

$$K = (1.65 \text{ mV}/^\circ\text{C}) / (0.083 \text{ mV}/^\circ\text{C}) = 19.9$$

→ 0 TC

Fig 8.2.9 Band gap reference voltage source

$$V_{BE}(T) = V_T \cdot \ln \left\{ \frac{I_{DC}}{I_S(T)} \right\}$$

→ Assumed DC collector current (constant and PVT independent)

$$I_S = \frac{qA_E D_n n_i^2}{N_A \cdot W_B} \quad n_i^2 = 4 \cdot \left(\frac{2\pi k \sqrt{m_n^* m_p^*}}{h^2} \right)^3 \cdot T^3 \cdot e^{-\frac{E_G}{kT}}$$

$$n_i^2(T) = (1.45 \times 10^{10})^2 \cdot \left(\frac{T}{300} \right)^3 \cdot e^{-\frac{E_G(T)}{kT}} \cdot e^{+\frac{E_G(300)}{300k}}$$

$$I_S(T) = I_S(300) \cdot \left(\frac{T}{300} \right)^{4+m} \cdot e^{-\frac{E_G(T)}{kT}} \cdot e^{+\frac{E_G(300)}{300k}}$$

M = -1.5 due to Temperature dependence of mobility (un)

$$\begin{aligned}
 V_{BE}(T) &= V_T \cdot \ln\left\{\frac{I_{DC}}{I_S(T)}\right\} = V_T \cdot \ln\left\{\frac{I_{DC}}{I_S(300) \cdot \left(\frac{T}{300}\right)^{4+m} \cdot e^{-\frac{E_G(T)}{kT}} \cdot e^{+\frac{E_G(300)}{300k}}}\right\} \\
 &= V_T \cdot \ln\left(\frac{I_{DC}}{I_S(300)}\right) - (4+m) \cdot V_T \cdot \ln\left(\frac{T}{300}\right) + \frac{E_G(T)}{q} - V_T \cdot \frac{E_G(300)}{300k} \\
 &= \frac{T}{300} \cdot \left\{V_{BE}(300) - \frac{E_G(300)}{q} - (4+m) \cdot V_{T300} \cdot \ln\left(\frac{T}{300}\right) + \frac{300}{T} \cdot \frac{E_G(T)}{q}\right\}
 \end{aligned}$$

$$\frac{\partial V_{BE}(T)}{\partial T} = \frac{1}{300} \left\{V_{BE}(300) - \frac{E_G(300)}{q} - (4+m) \cdot V_{T300} \cdot \left\{\ln\frac{T}{300} + 1\right\} + \frac{300}{q} \cdot \frac{\partial E_G(T)}{\partial T}\right\}$$

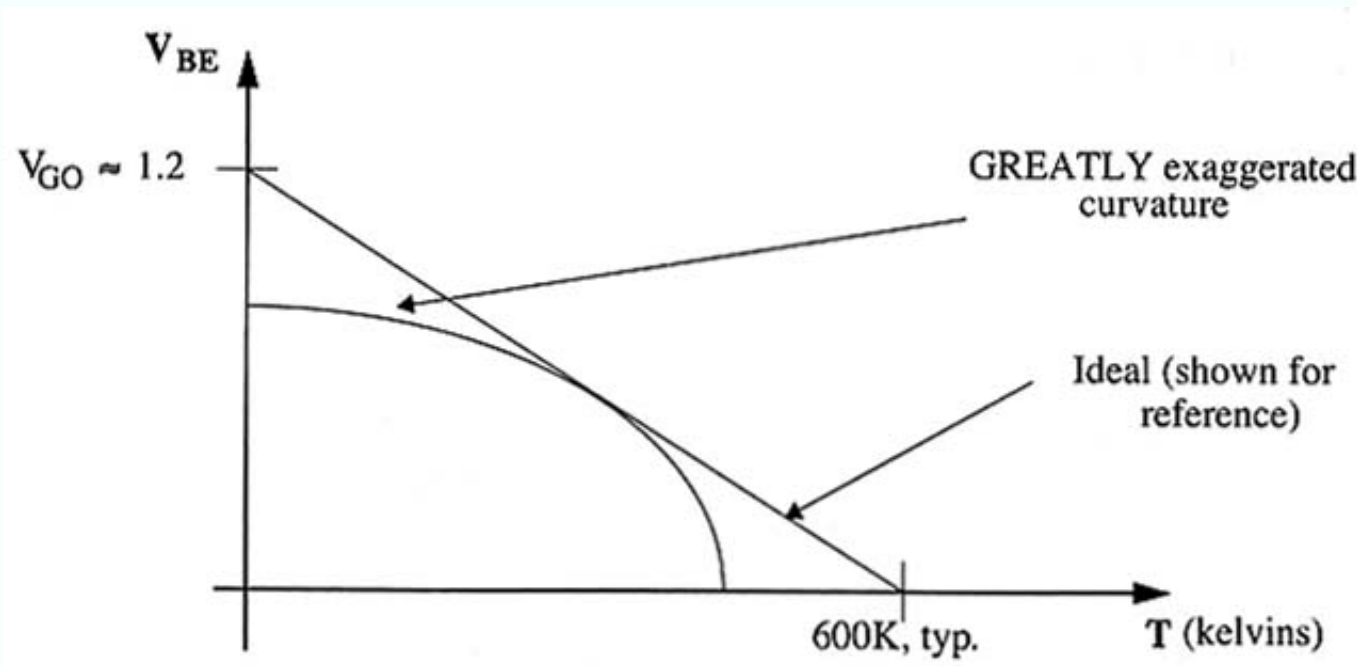
$$\left.\frac{\partial V_{BE}(T)}{\partial T}\right|_{T=300^\circ K} = \frac{1}{300} (0.75 - 1.12 - 0.0625 - 0.051) = -1.61 \text{ mV}/^\circ C$$

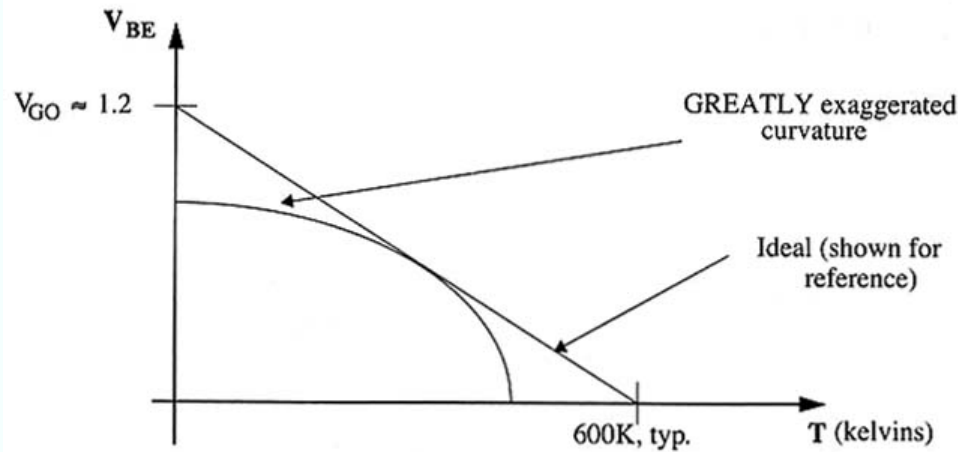
$$\frac{\partial V_{BE}(T)}{\partial T} = \frac{1}{300} \left\{ V_{BE}(300) - \frac{E_G(300)}{q} - (4+m) \cdot V_{T300} \cdot \left\{ \ln \frac{T}{300} + 1 \right\} + \frac{300}{q} \cdot \frac{\partial E_G(T)}{\partial T} \right\}$$

$$\frac{\partial V_{BE}}{\partial T} = \frac{V_{BE} - (4+m)V_T - E_g/q}{T}$$

Razavi

Constant IC assumed

 V_{BE} versus temperature

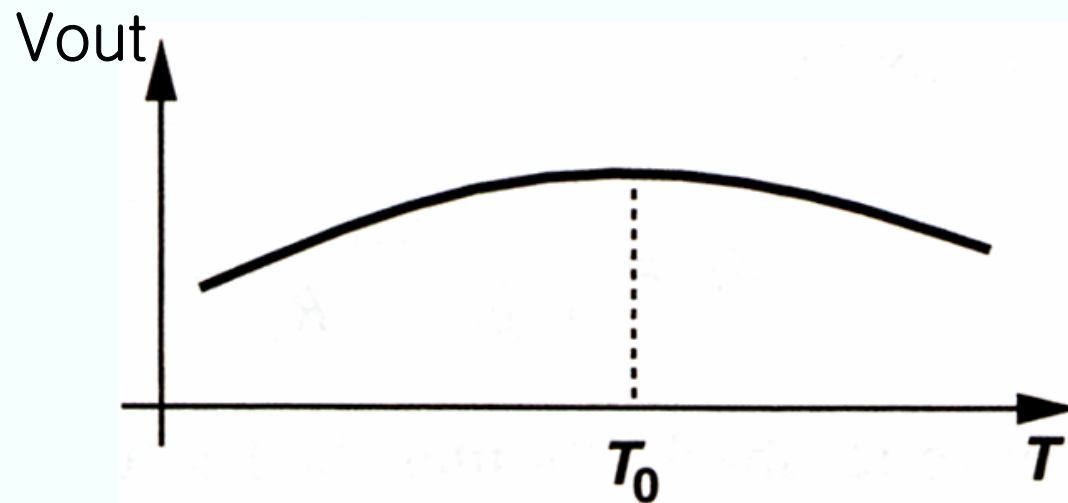


V_{BE} versus temperature

$$\frac{\partial V_{BE}}{\partial T} = \frac{V_{BE} - (4 + m)V_T - E_g/q}{T}$$

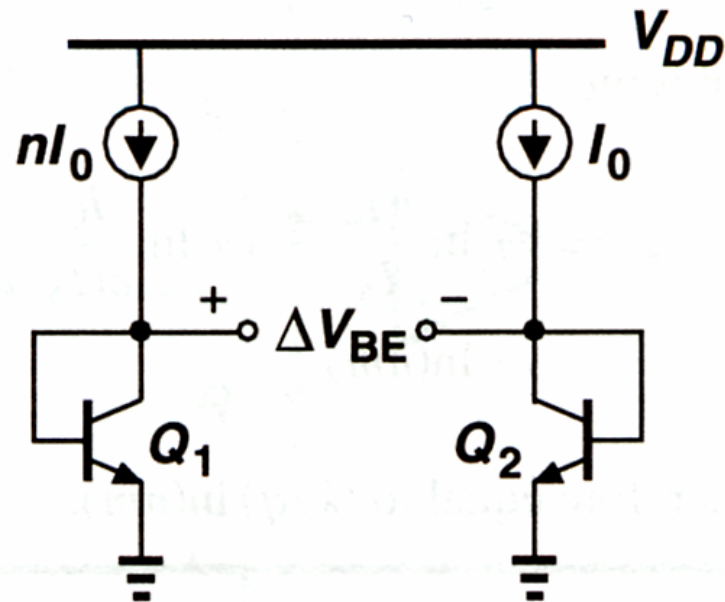
$$V_{out} = V_{BE} + K T$$

$dV_{out} / dT = dV_{BE} / dT + K$
 Changes from negative to 0 and to positive



Curvature in temperature characteristics
 Of $V_{out} (= V_{BE} + K T)$

PTAT voltage (PTAT : proportional to absolute temperature)



$$\begin{aligned}
 \Delta V_{BE} &= V_{BE1} - V_{BE2} \\
 &= V_T \ln \frac{nI_0}{I_{S1}} - V_T \ln \frac{I_0}{I_{S2}} \\
 &= V_T \ln n. \text{ PTAT voltage}
 \end{aligned}$$

PTAT voltage : difference between 2 VBEs of PN junctions with different current density

Bandgap reference : $V_{ref} = V_{BE} + K V_T$

$$\partial V_{BE} / \partial T \approx -1.5 \text{ mV}/^\circ\text{K}$$

$$\partial V_T / \partial T \approx +0.087 \text{ mV}/^\circ\text{K},$$

$$V_{REF} \approx V_{BE} + 17.2 V_T$$

$$\approx 1.25 \text{ V. } \text{Close to bandgap of silicon}$$

$$V_{REF} = V_{BE} + V_T \ln n$$

$$\frac{\partial V_{REF}}{\partial T} = \frac{\partial V_{BE}}{\partial T} + \frac{V_T}{T} \ln n$$

$$\frac{\partial V_{BE}}{\partial T} = \frac{V_{BE} - (4 + m)V_T - E_g/q}{T}$$

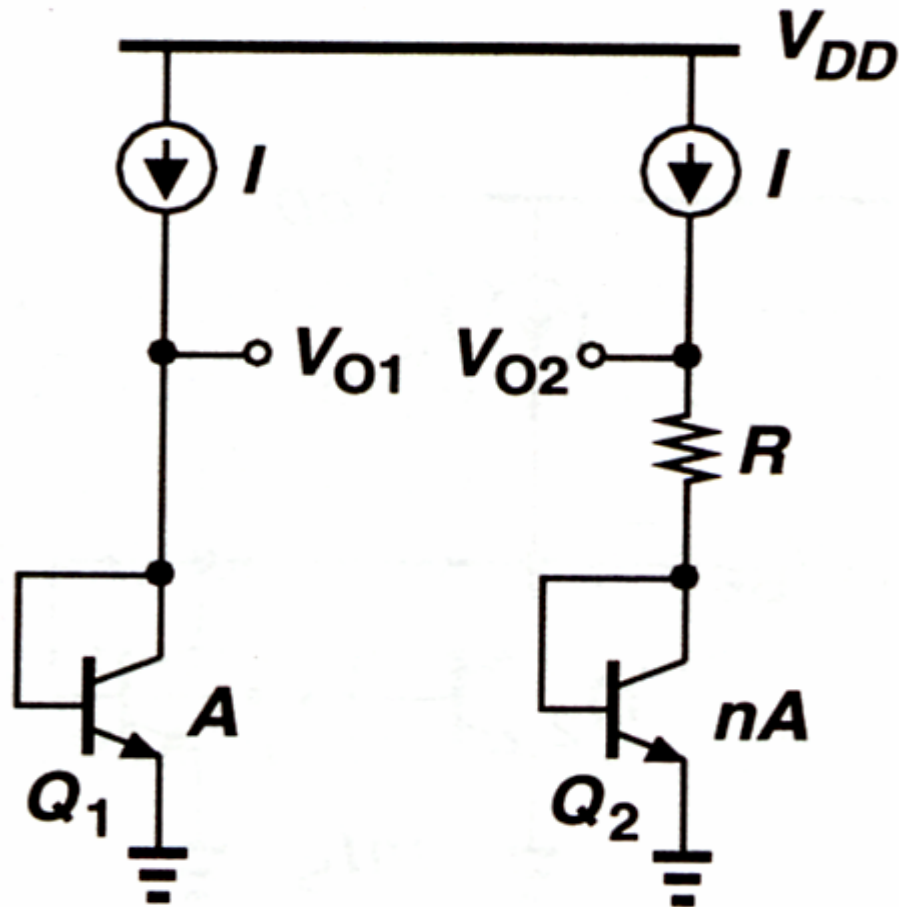
$$\frac{V_{BE} - (4 + m)V_T - E_g/q}{T} = -\frac{V_T}{T} \ln n$$

For 0 TC

$$V_{REF} = \frac{E_g}{q} + (4 + m)V_T$$

Close to bandgap

Conceptual band gap reference :
enforce V_{o1} and V_{o2} to be the same



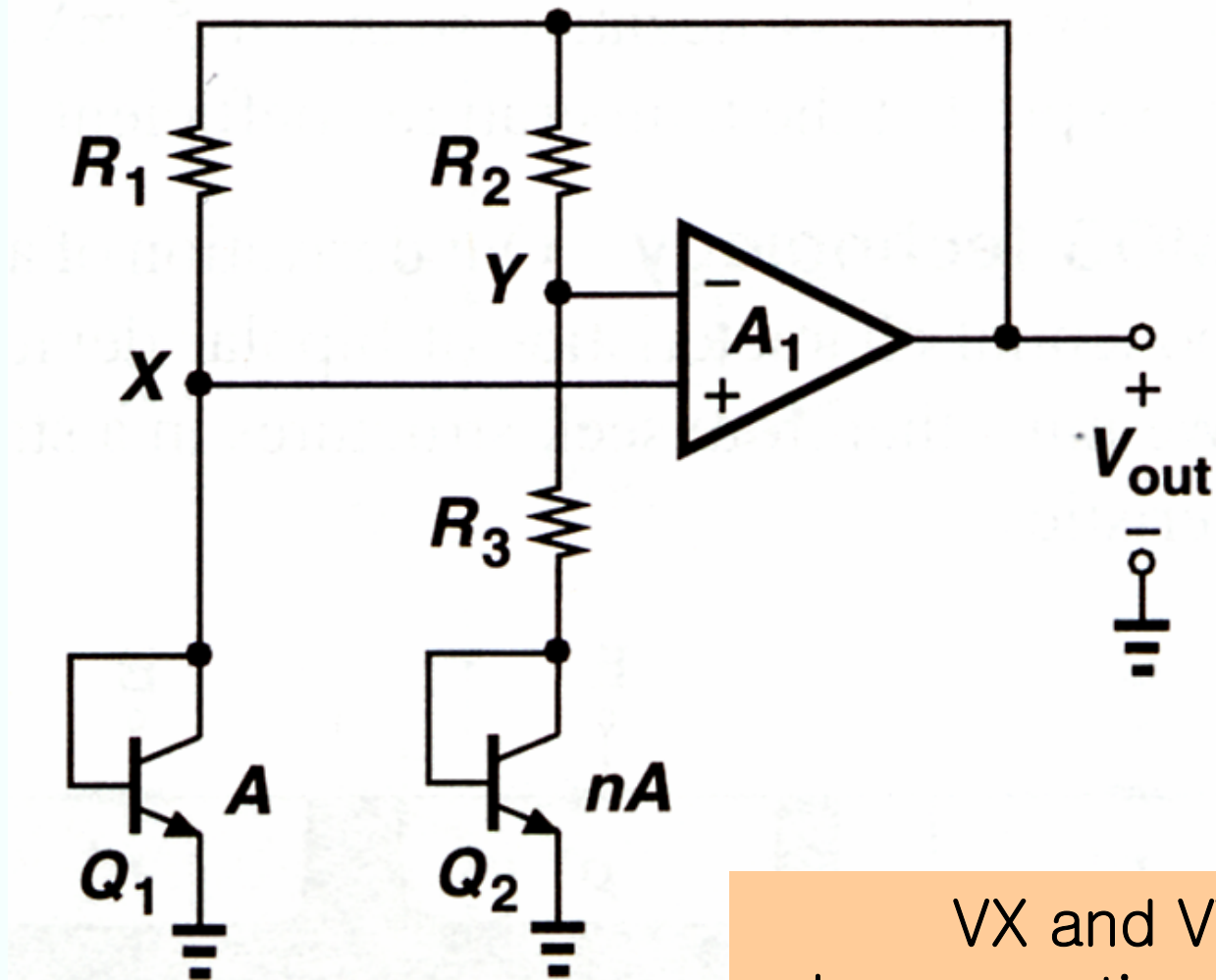
$$V_{BE1} = RI + V_{BE2}$$

$$RI = V_{BE1} - V_{BE2} = V_T \ln n$$

$$\begin{aligned} V_{o2} &= RI + V_{BE2} \\ &= V_T \cdot \ln n + V_{BE2} \end{aligned}$$

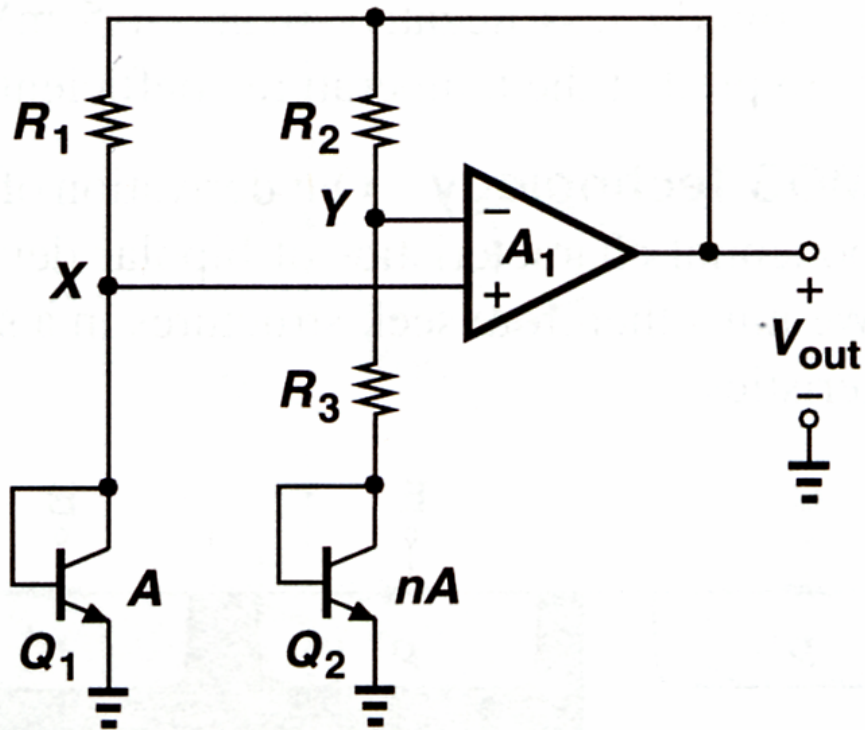
$\ln(n) = 17.2 \rightarrow n = 2.95e7$ (too large, impractical)

Basic band gap reference ckt



V_X and V_Y : kept the same
by a negative feedback OP amp ckt

Basic band gap reference ckt



$$V_{out} = V_{BE2} + \frac{V_T \ln n}{R_3} (R_3 + R_2)$$

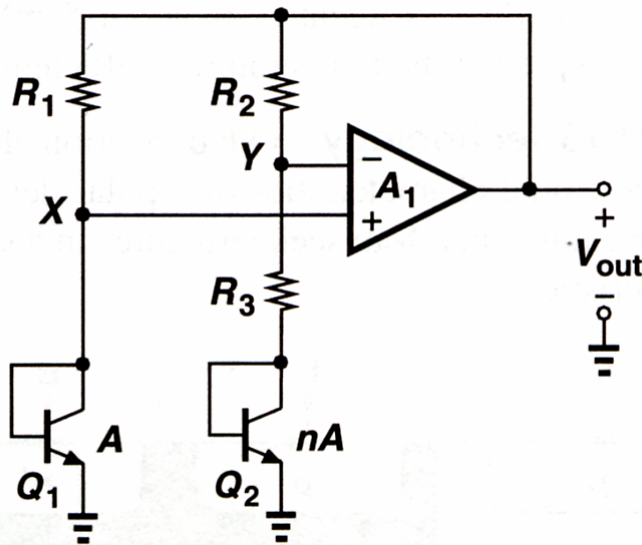
$$= V_{BE2} + (V_T \ln n) \left(1 + \frac{R_2}{R_3} \right)$$

$R_1 = R_2 \rightarrow$ the same current at Q_1, Q_2
 $R_2 > R_3$ to decrease n

$$\ln n \cdot \left(1 + \frac{R_2}{R_3} \right) = 17.2$$

Example: $R_1 = R_2, R_2/R_3 = 4, n = 31$

Basic band gap reference ckt



$$I_{C1} = I_{C2} \approx (V_T \ln n)/R_3$$

$$\frac{\partial V_{BE}}{\partial T} = \frac{\partial V_T}{\partial T} \ln \frac{I_C}{I_S} + V_T \left(\frac{1}{I_C} \frac{\partial I_C}{\partial T} - \frac{1}{I_S} \frac{\partial I_S}{\partial T} \right)$$

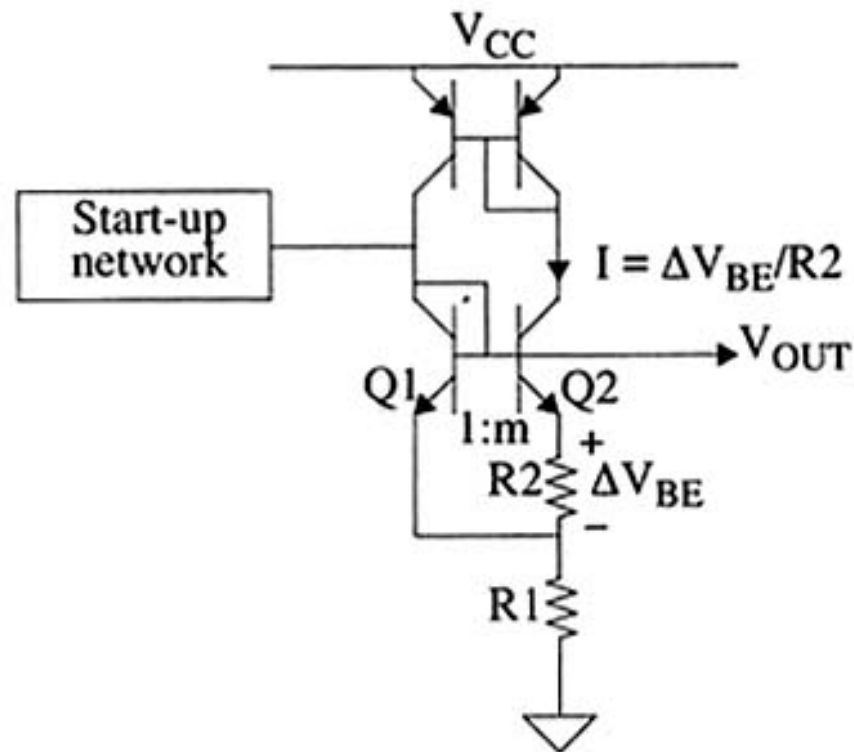
$$\frac{\partial V_{BE}}{\partial T} = \frac{\partial V_T}{\partial T} \ln \frac{I_C}{I_S} + \frac{V_T}{T} - \frac{V_T}{I_S} \frac{\partial I_S}{\partial T}$$

$$\frac{\partial V_{BE}}{\partial T} = \frac{V_{BE} - (3 + m)V_T - E_g/q}{T}$$

PTAT IC

$$\frac{\partial V_{BE}}{\partial T} = \frac{V_{BE} - (4 + m)V_T - E_g/q}{T}$$

Constant IC



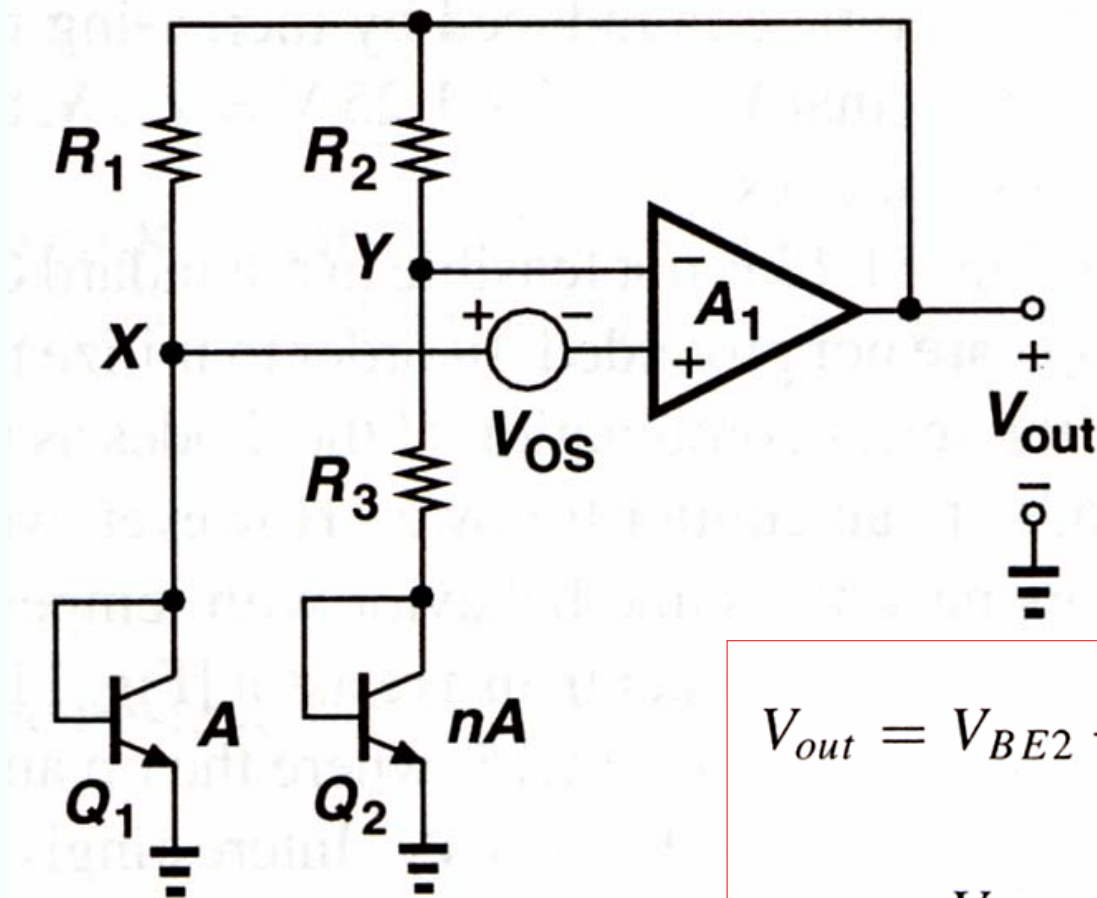
Let $m=8$ and $V_{BE}=0.65V$ at $I_C=100\mu A$, $300K$

$\Delta V_{BE} = V_T \times \ln(8) = 53.8mV \gg \text{offset}$

$R2 = 53.8mV / 100\mu A = 538 \text{ ohm}$

$R1 = (1.2 - 0.65 - 0.0538) / 200\mu A = 0.496V / 200\mu A = 2.48K\text{ohm}$

Effect of Vos (input offset voltage)



$$V_{BE1} - V_{OS} \approx V_{BE2} + I_{C2} \cdot R_3$$

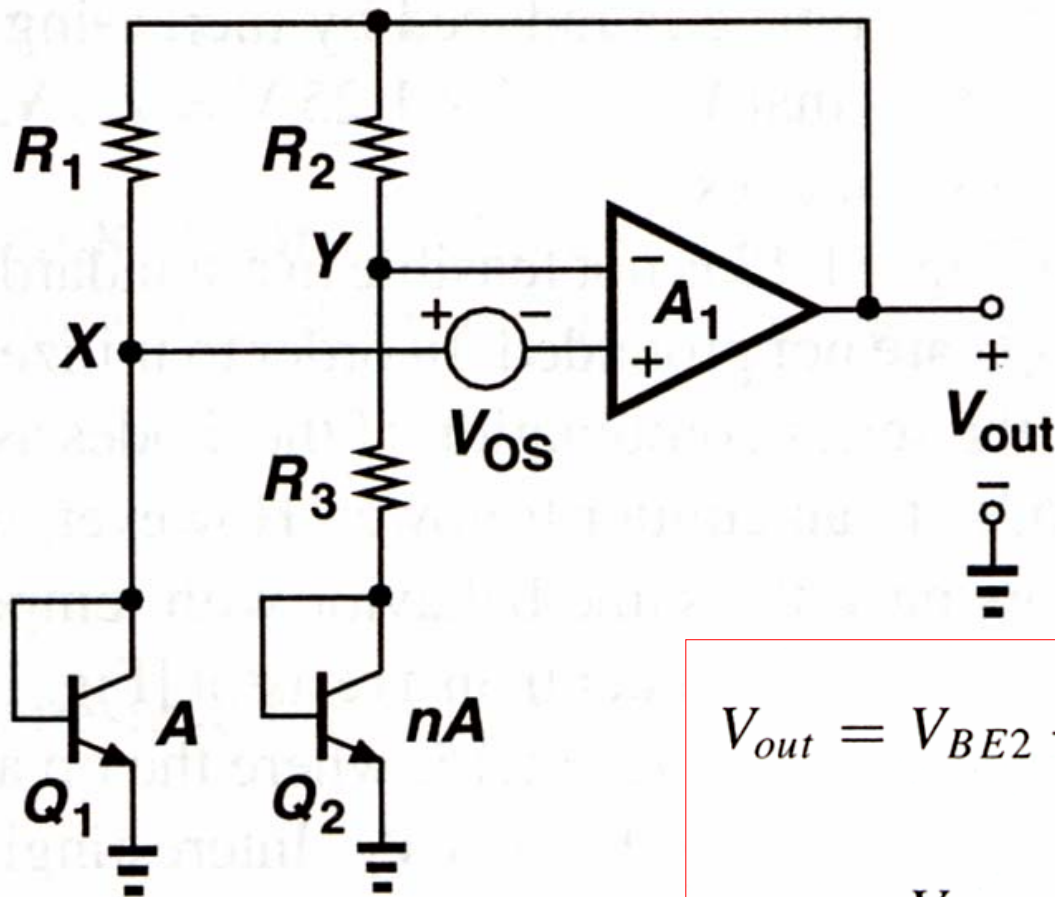
$$\begin{aligned} I_{C2} &= (V_{BE1} - V_{BE2} - V_{OS}) / R_3 \\ &= (V_T \cdot \ln n - V_{OS}) / R_3 \end{aligned}$$

$$V_{out} = V_{BE2} + (R_3 + R_2)I_{C2}$$

$$\begin{aligned} V_{out} &= V_{BE2} + (R_3 + R_2) \frac{V_{BE1} - V_{BE2} - V_{OS}}{R_3} \\ &= V_{BE2} + \left(1 + \frac{R_2}{R_3}\right) (V_T \ln n - V_{OS}), \end{aligned}$$

Vos amplified, Vos varies with temperature

Effect of Vos (input offset voltage), $R_1=R_2$



$$V_{BE1} - V_{OS} \approx V_{BE2} + I_{C2} \cdot R_3$$

$$I_{C2} = (V_{BE1} - V_{BE2} - V_{OS}) / R_3$$

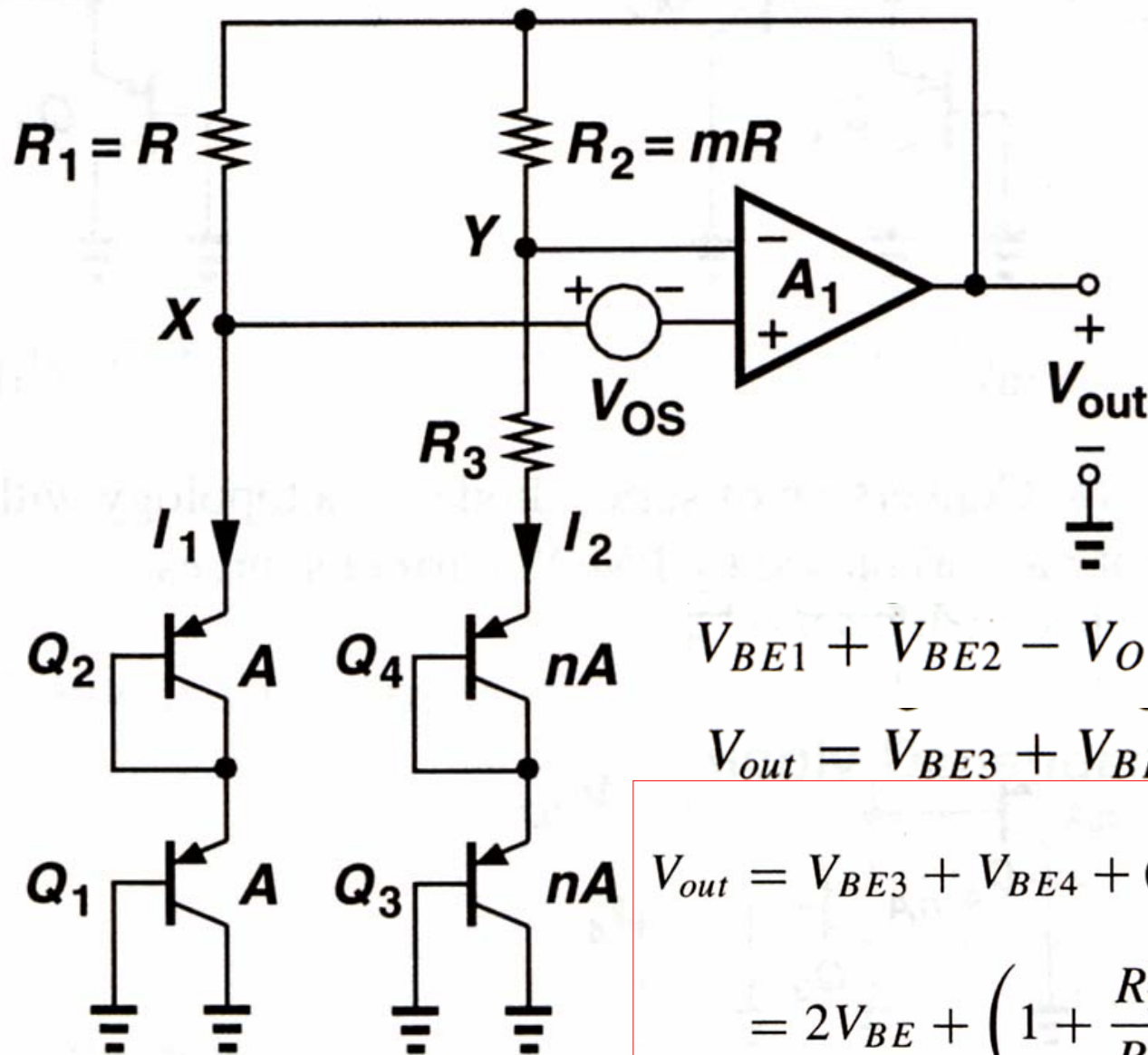
$$= (V_T \cdot \ln n - V_{OS}) / R_3$$

$$V_{out} = V_{BE2} + (R_3 + R_2)I_{C2}$$

$$V_{out} = V_{BE2} + (R_3 + R_2) \frac{V_{BE1} - V_{BE2} - V_{OS}}{R_3}$$

$$= V_{BE2} + \left(1 + \frac{R_2}{R_3}\right) (V_T \ln n - V_{OS}),$$

Vos amplified, Vos varies with temperature



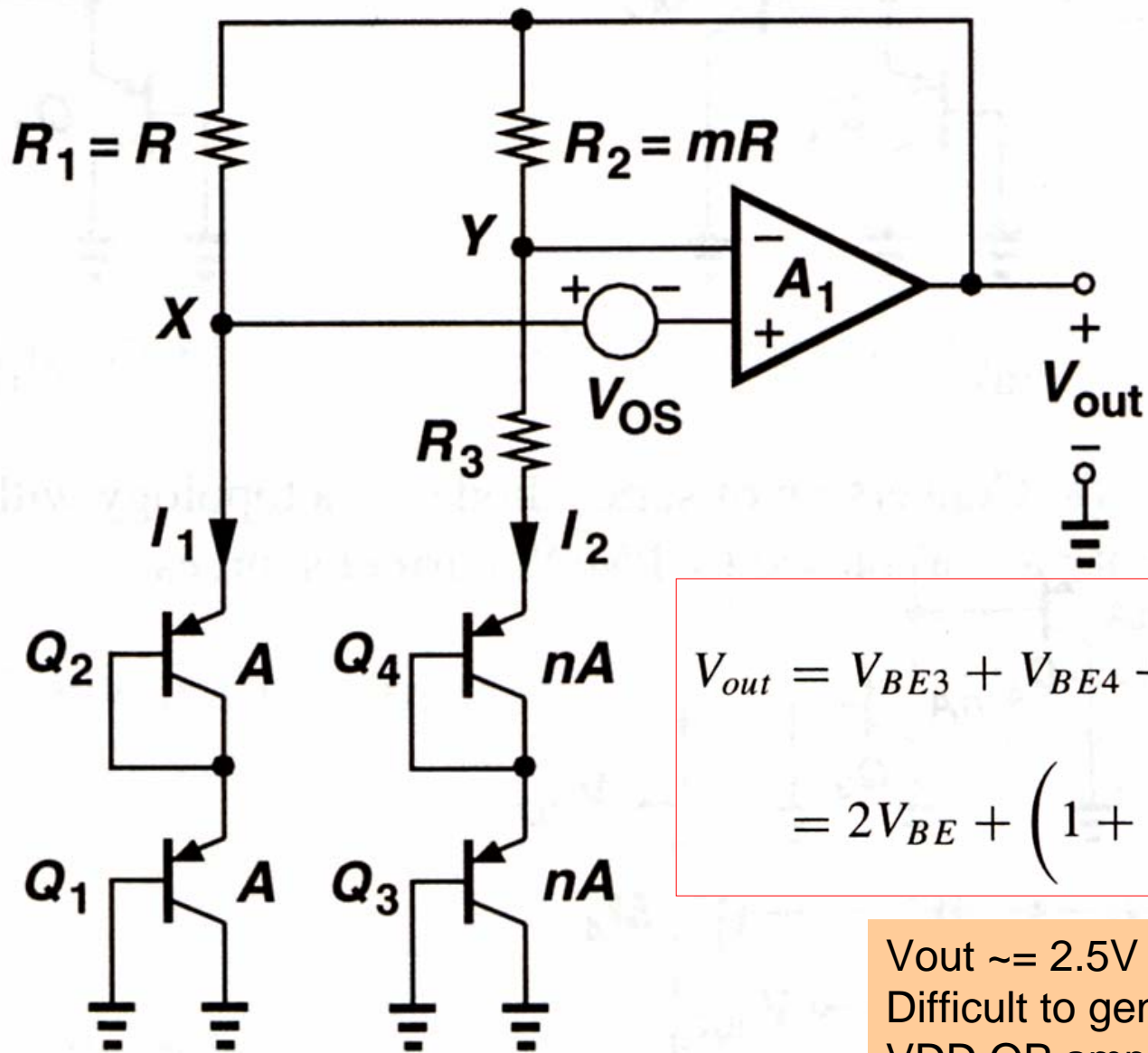
$$I_1 \approx mI_2$$

$$V_{BE1} + V_{BE2} - V_{OS} = V_{BE3} + V_{BE4} + R_3 I_2$$

$$V_{out} = V_{BE3} + V_{BE4} + (R_3 + R_2) I_2$$

$$V_{out} = V_{BE3} + V_{BE4} + (R_3 + R_2) \frac{2V_T \ln(mn) - V_{OS}}{R_3}$$

$$= 2V_{BE} + \left(1 + \frac{R_2}{R_3}\right) [2V_T \ln(mn) - V_{OS}]$$

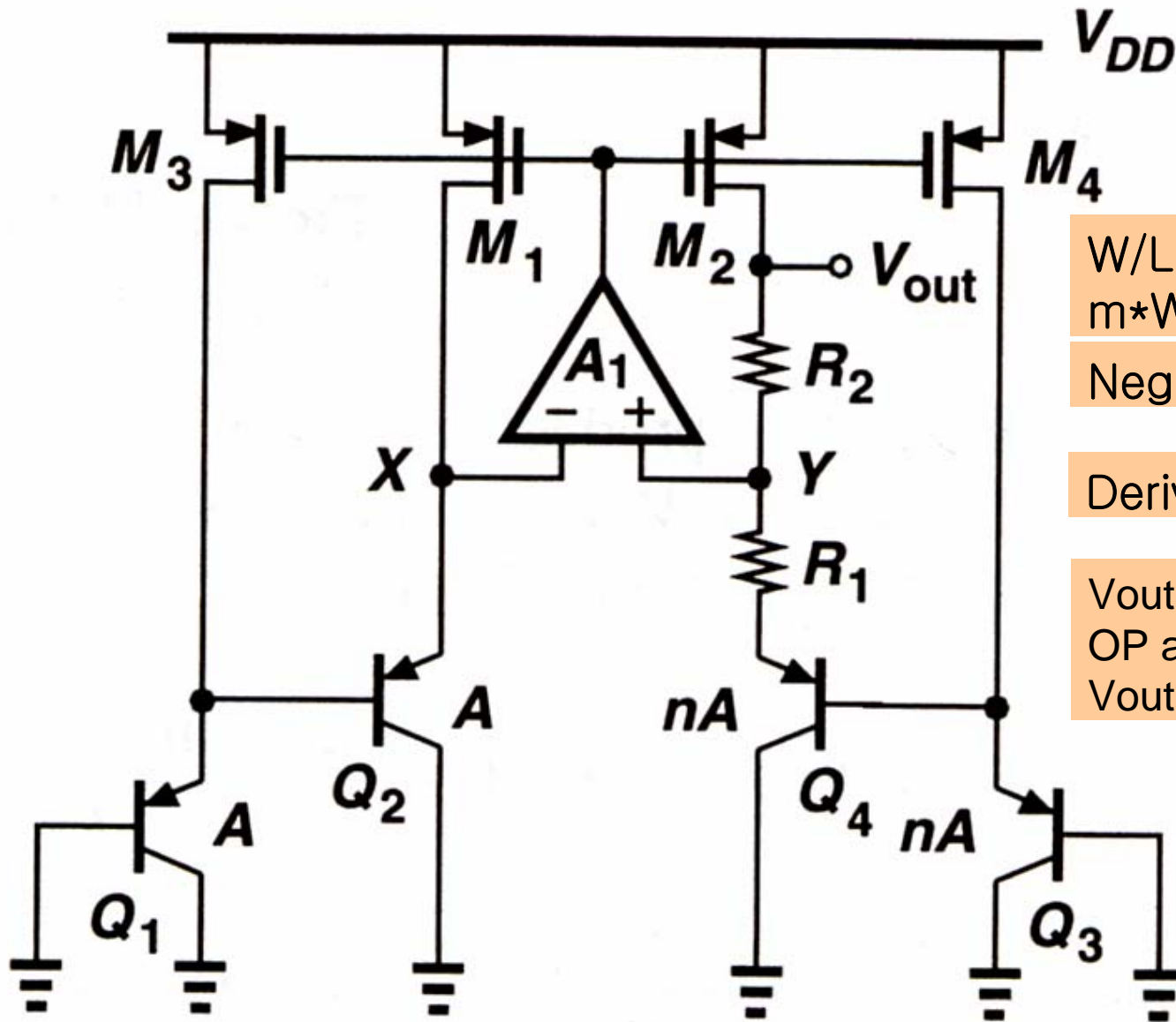


$$V_{out} = V_{BE3} + V_{BE4} + (R_3 + R_2) \frac{2V_T \ln(mn) - V_{OS}}{R_3}$$

$$= 2V_{BE} + \left(1 + \frac{R_2}{R_3}\right) [2V_T \ln(mn) - V_{OS}].$$

$V_{out} \approx 2.5V$

Difficult to generate 2.5V output using low-VDD OP amp → No good at low VDD



$$W/L(M1) = W/L(M3) = m \cdot W/L(M2) = m \cdot W/L(M4)$$

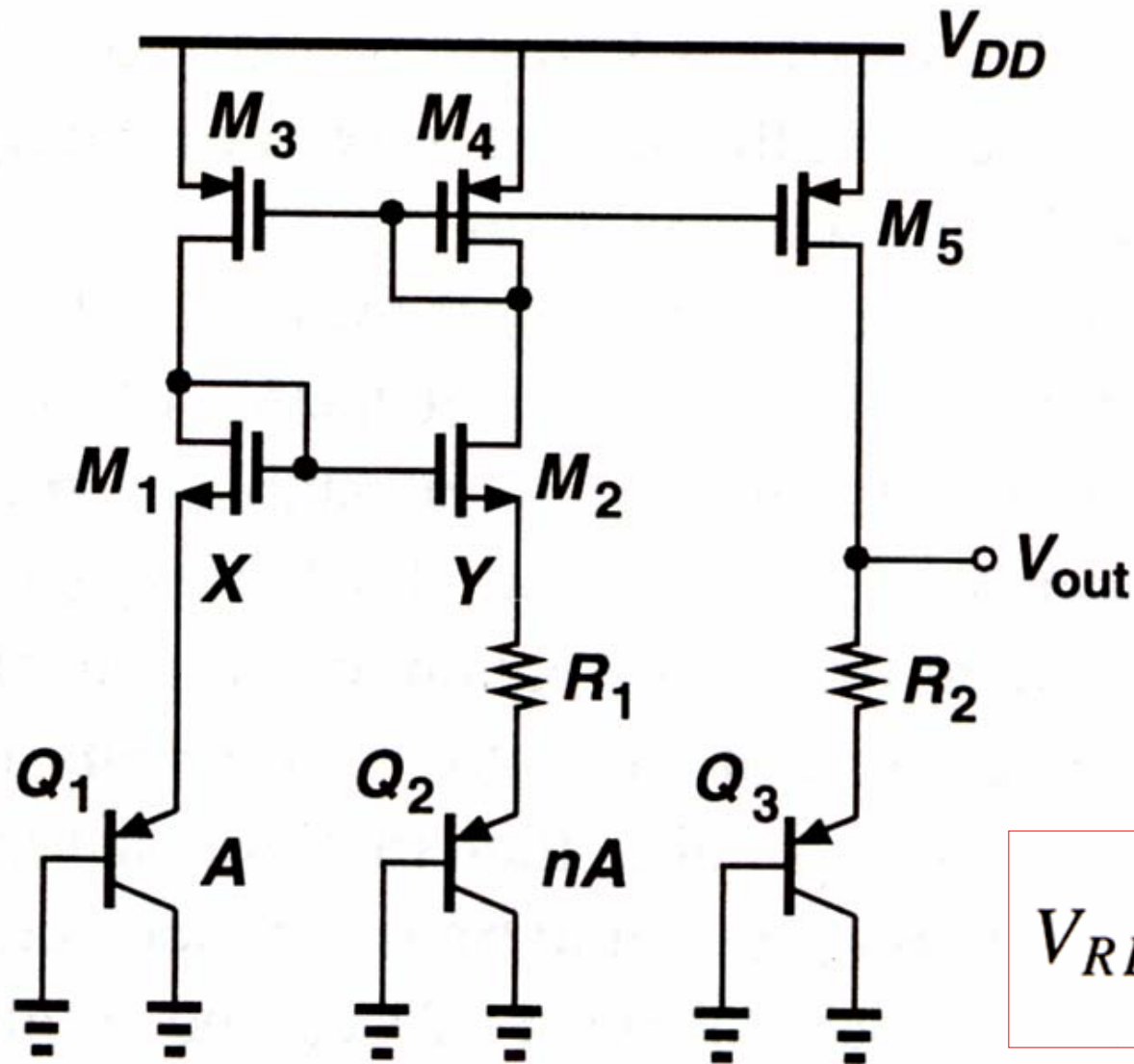
Negative feedback

Derive V_{out} eq. including V_{os}

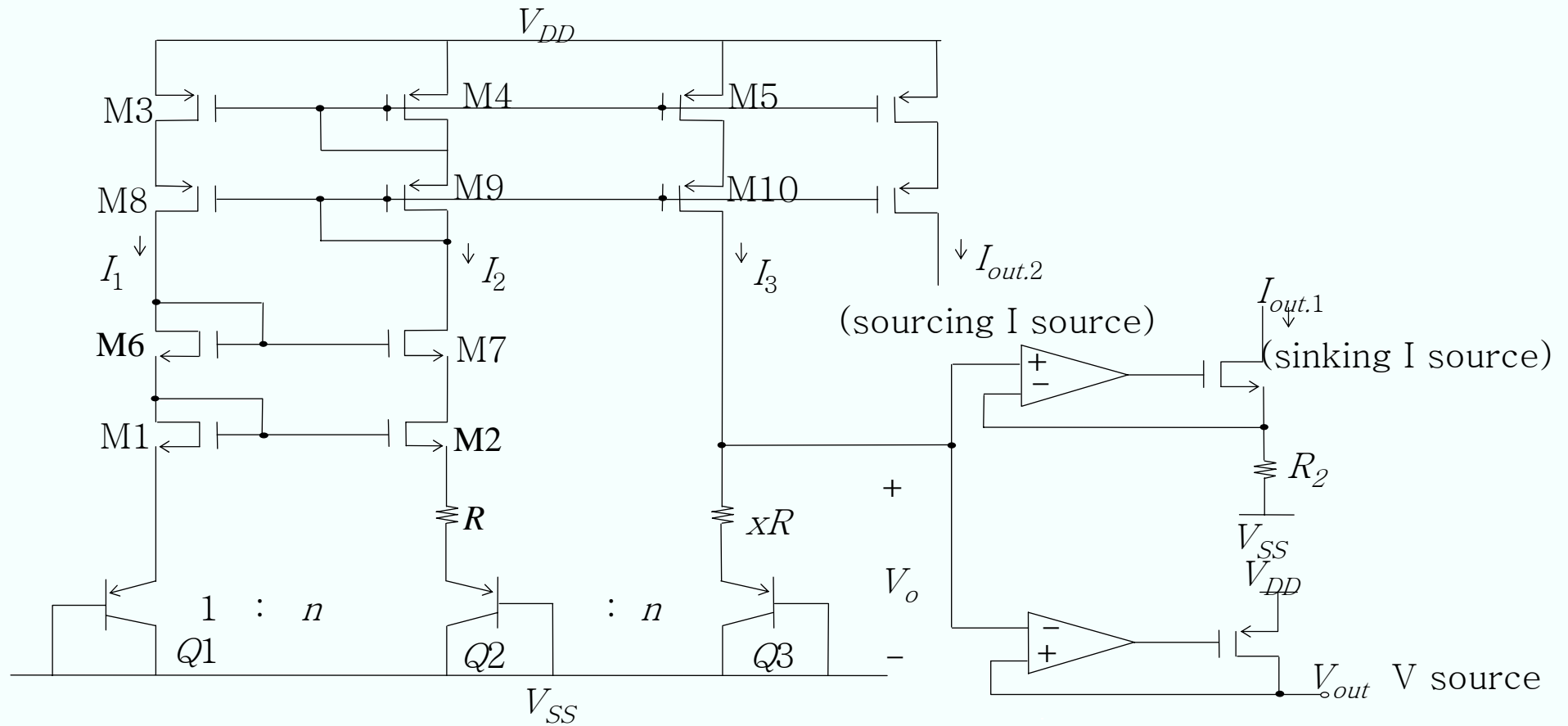
$$V_{out} \approx 2.5V$$

OP amp does not generate

$V_{out}(2.5V) \rightarrow$ OK at low V_{DD}

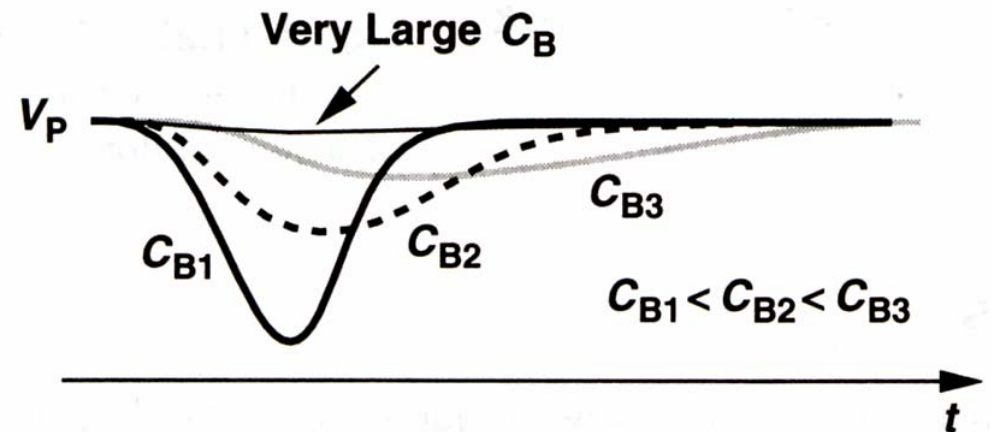
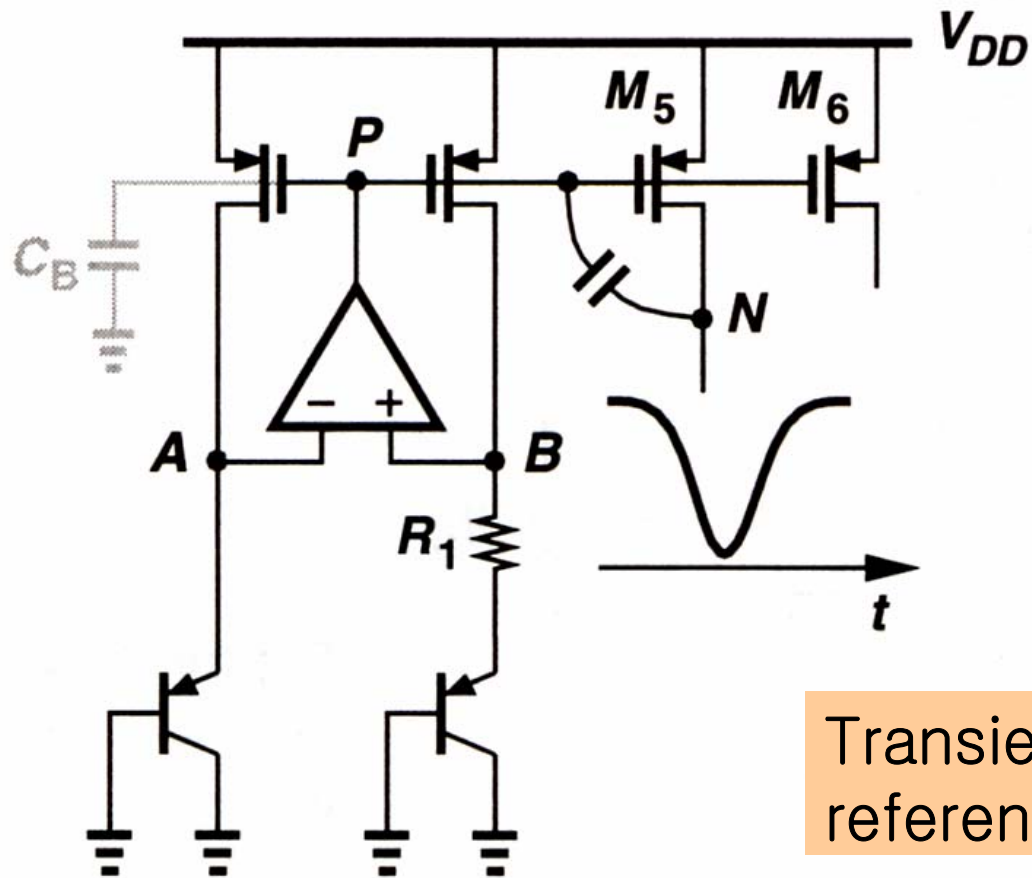


$$V_{REF} = V_{BE3} + \frac{R_2}{R_1} V_T \ln n$$



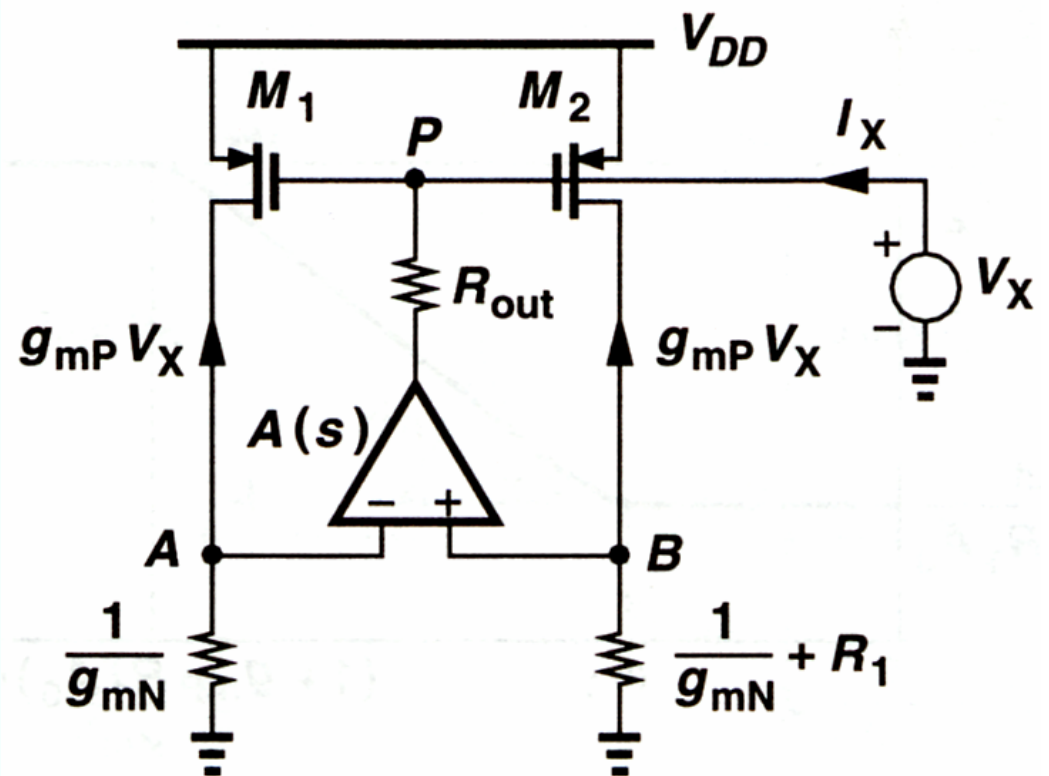
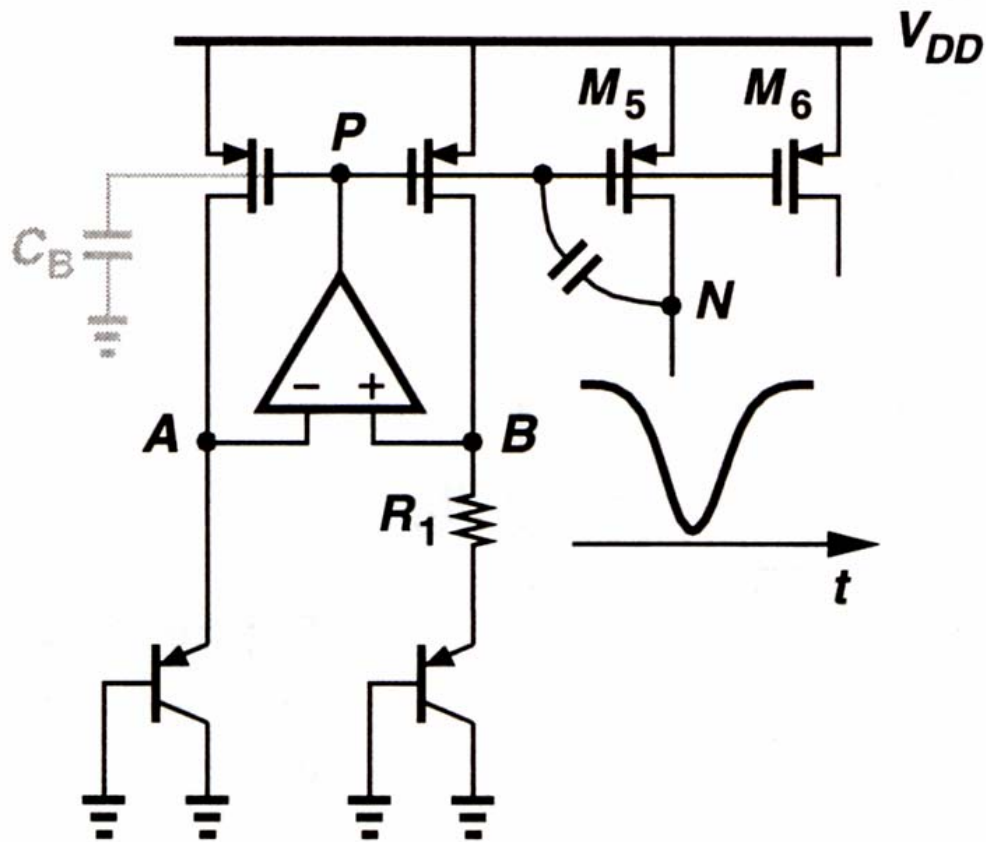
$$I_1 = I_2 \quad I_3 = I_2 \quad I_3 = I_2 = I_1 \approx \frac{V_T \cdot \ln(n)}{R}$$

$$V_o = I_3 \cdot xR + V_{EB3} = V_T \cdot x \cdot \ln(n) + V_{EB3}$$

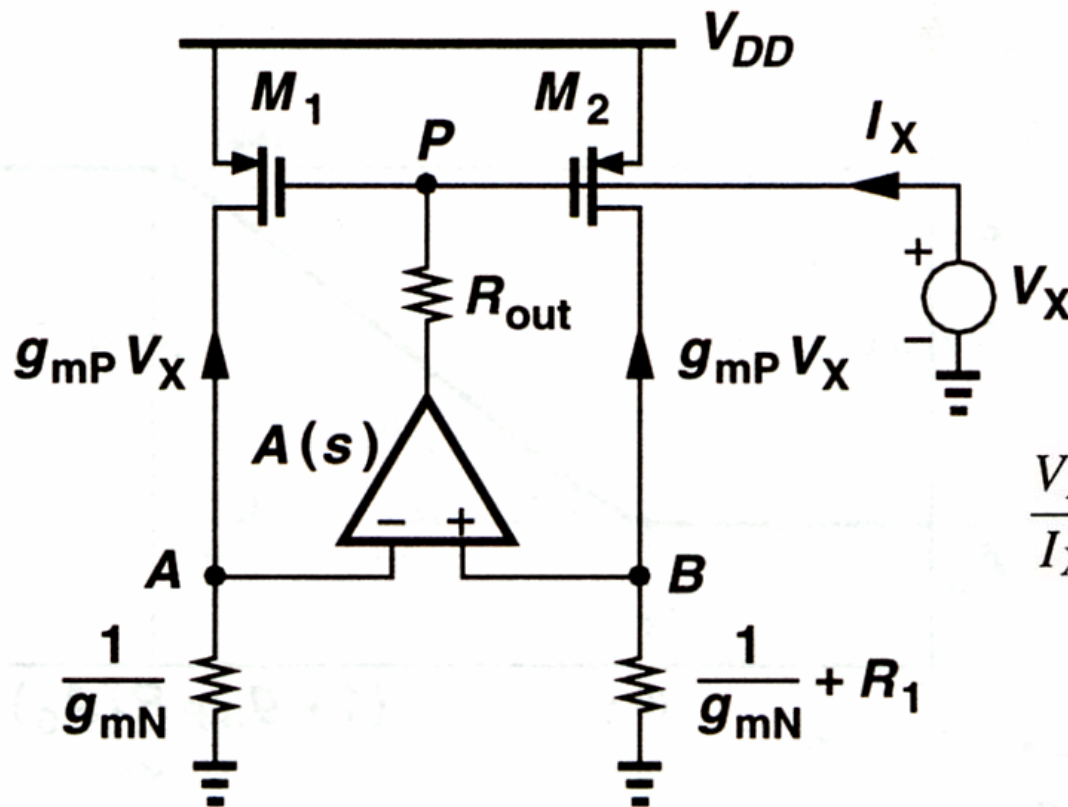


Transients in ckts \rightarrow transients in reference \rightarrow broadcast in circuits

Due to finite Rout of reference

Calculation of R_{out} 

Calculation of Rout



$$V_{AB} = -g_{mP} V_X \frac{1}{g_{mN}} + g_{mP} V_X \left(\frac{1}{g_{mN}} + R_1 \right)$$

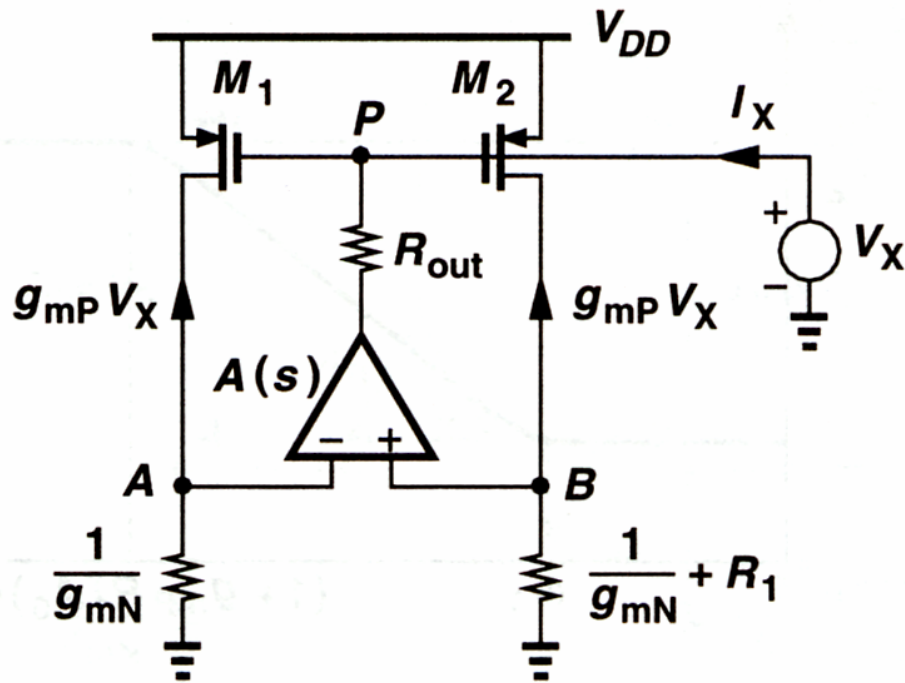
$$= g_{mP} V_X R_1.$$

$$I_X = \frac{V_X + g_{mP} V_X R_1 A(s)}{R_{out}}$$

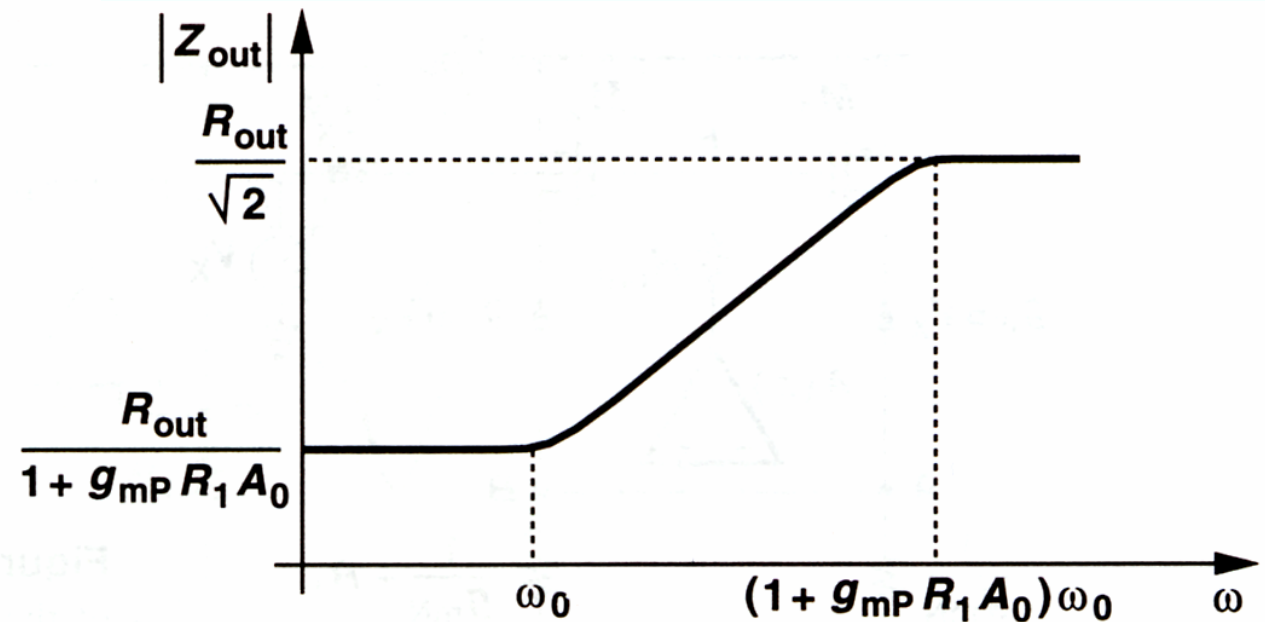
$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + g_{mP} R_1 A(s)}$$

$$= \frac{R_{out}}{1 + g_{mP} R_1 \frac{A_0}{1 + s/\omega_0}}$$

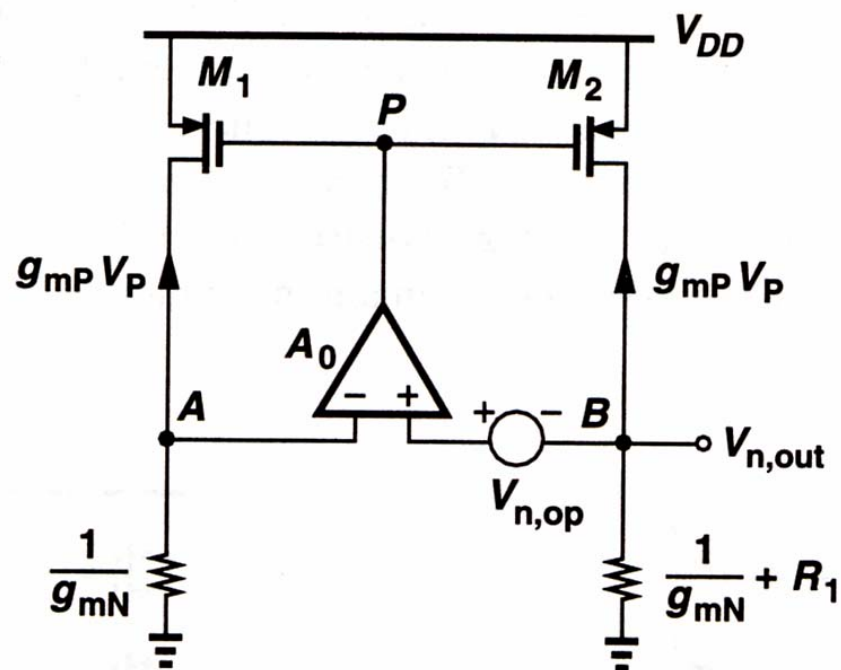
$$= \frac{R_{out}}{1 + g_{mP} R_1 A_0} \frac{1 + \frac{s}{\omega_0}}{1 + \frac{s}{(1 + g_{mP} R_1 A_0)\omega_0}}$$

Calculation of R_{out} =

$$= \frac{R_{out}}{1 + g_{mP} R_1 A_0} \frac{1 + \frac{s}{\omega_0}}{1 + \frac{s}{(1 + g_{mP} R_1 A_0)\omega_0}}$$



$|Z_{out}|$ large at high frequency \rightarrow easier coupling



$$V_P = -g_{mP}^{-1} V_{n,out} / (R_1 + g_{mN}^{-1})$$

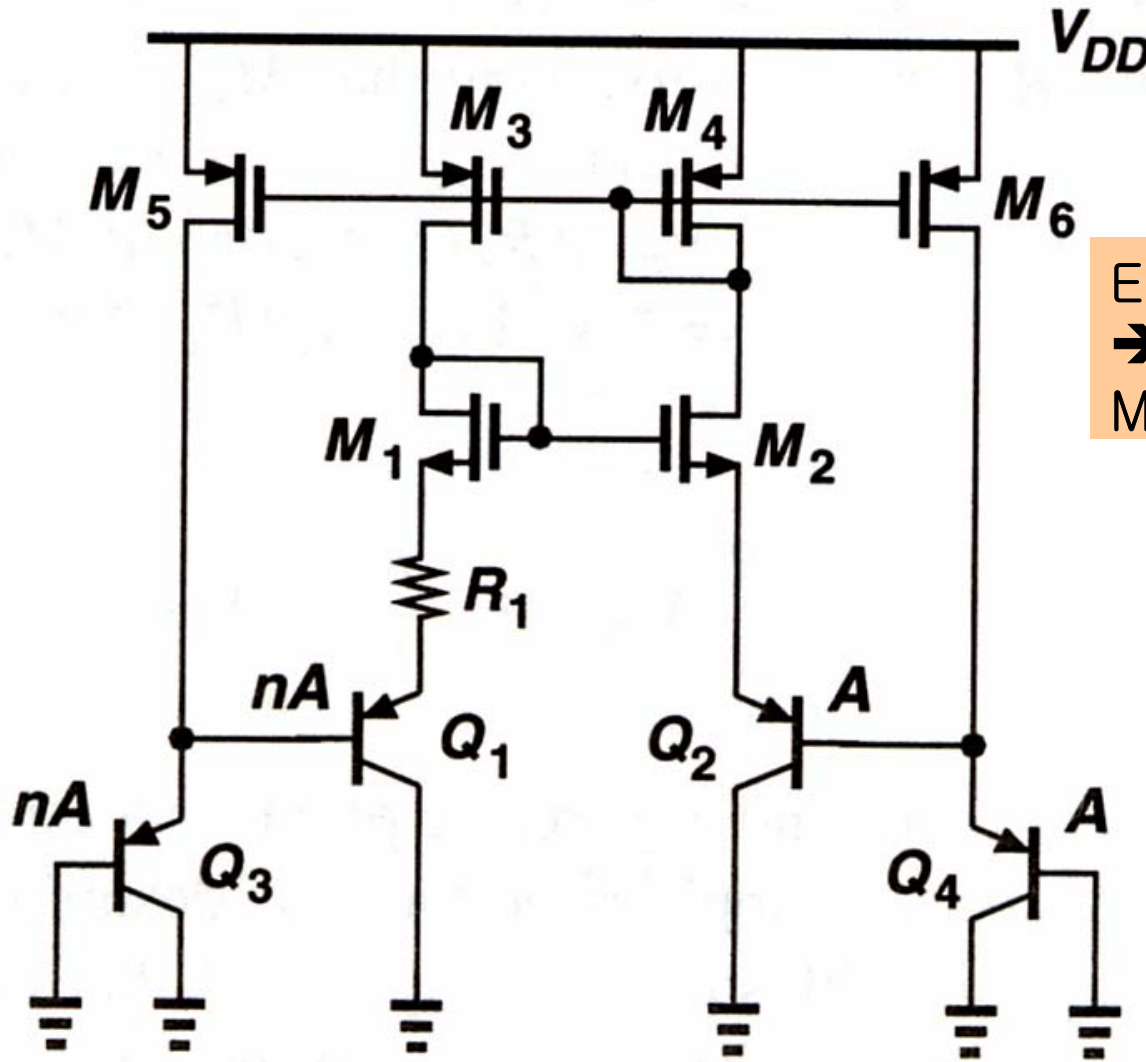
$$\frac{V_{n,out}}{R_1 + g_{mN}^{-1}} \cdot \frac{1}{g_{mN}} - \frac{V_{n,out}}{g_{mP} A_0 (R_1 + g_{mN}^{-1})} = V_{n,op} + V_{n,out}$$

$$g_{mP} A_0 \gg g_{mN} \gg R_1^{-1}$$

$$V_{n,out} \left[\frac{1}{R_1 + g_{mN}^{-1}} \left(\frac{1}{g_{mN}} - \frac{1}{g_{mP} A_0} \right) - 1 \right] = V_{n,op} \quad |V_{n,out}| \approx V_{n,op}$$

Input noise of amp → directly reference out

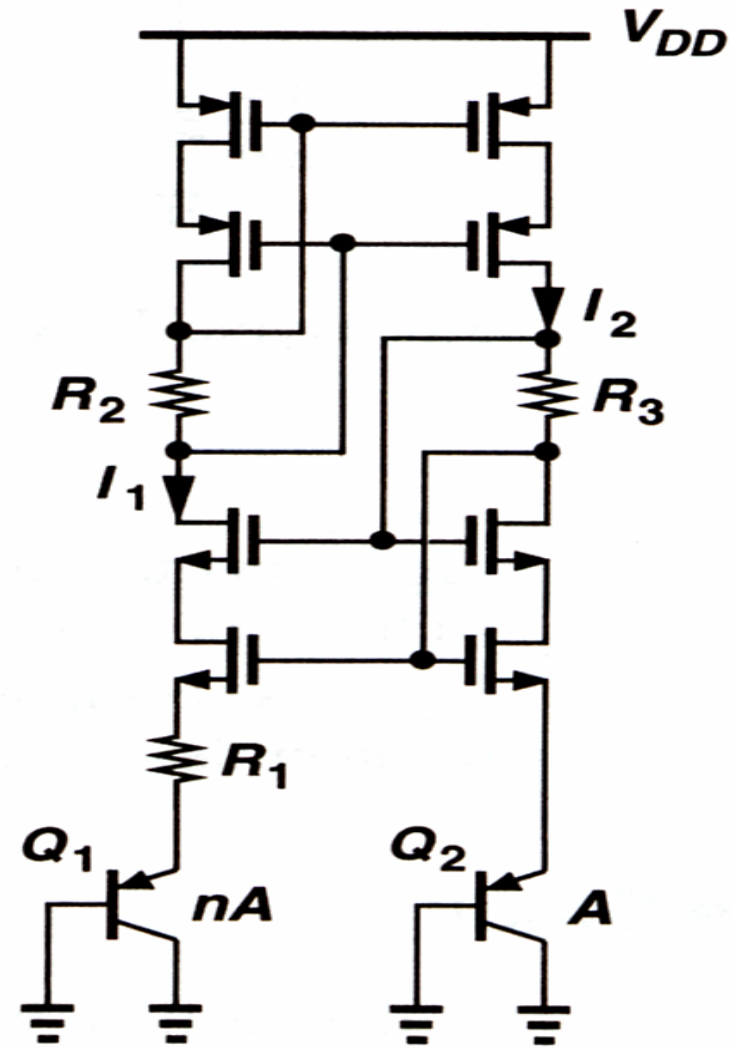
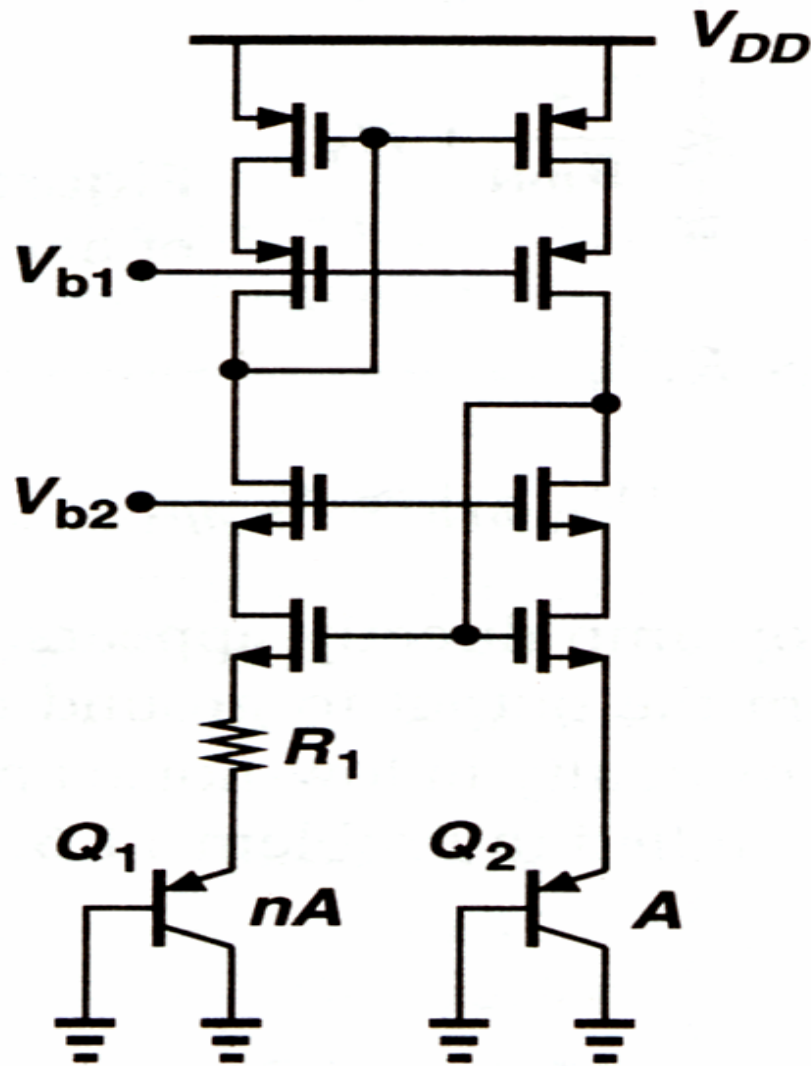
2 VBE used to reduce MOSFET mismatches



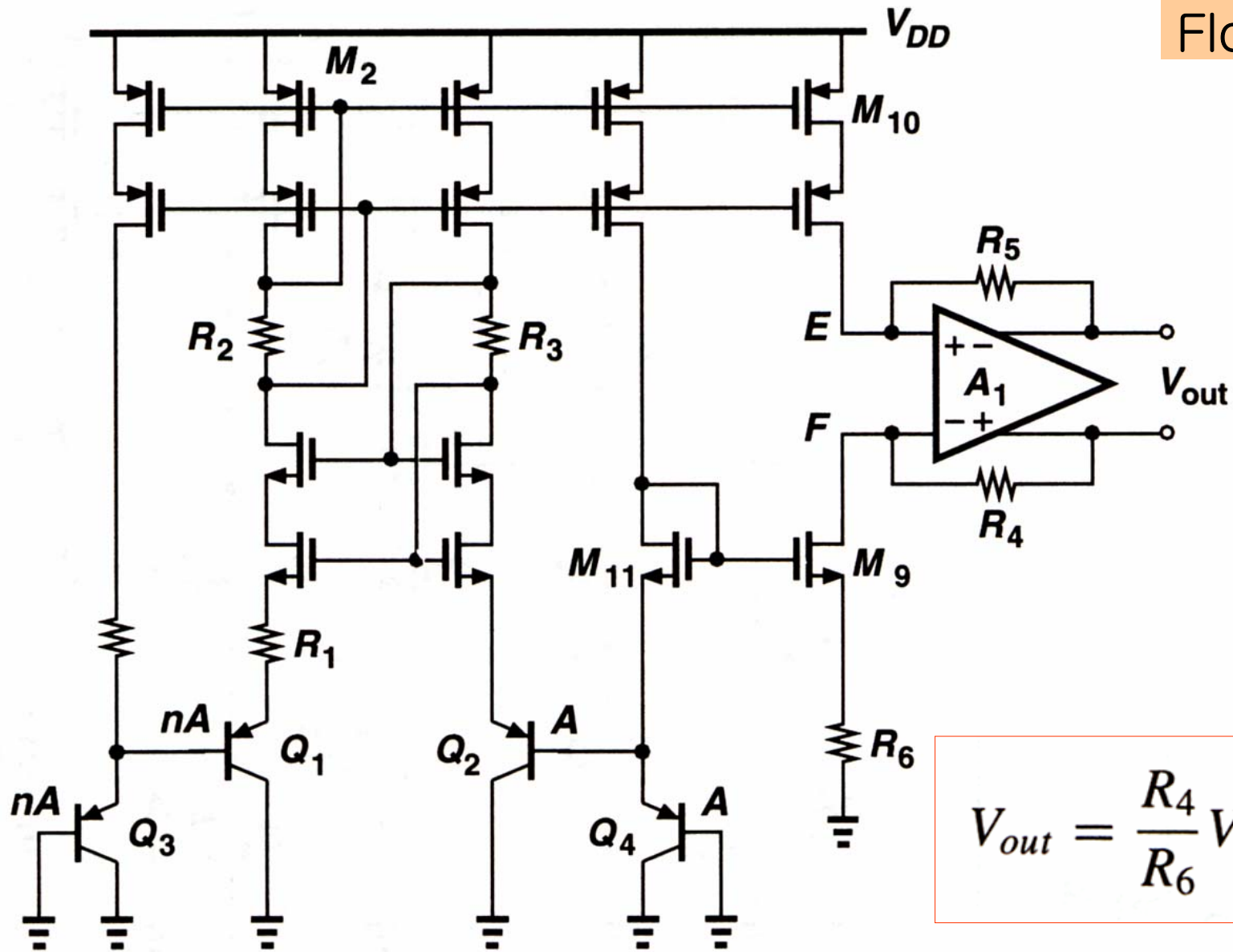
$$V_{OUT} = 2 \times E_{GO}/q \approx 2.5V$$

Errors in MOS diode connections
 → M2, M3 instead of M1, M4
 Must be MOS diodes

Channel length modulation of MOSFET → supply dependence



Floating reference



$$V_{out} = \frac{R_4}{R_6} V_{BE4} + 2 \frac{R_5}{R_1} V_T \ln n$$

Core: floating bandgap reference

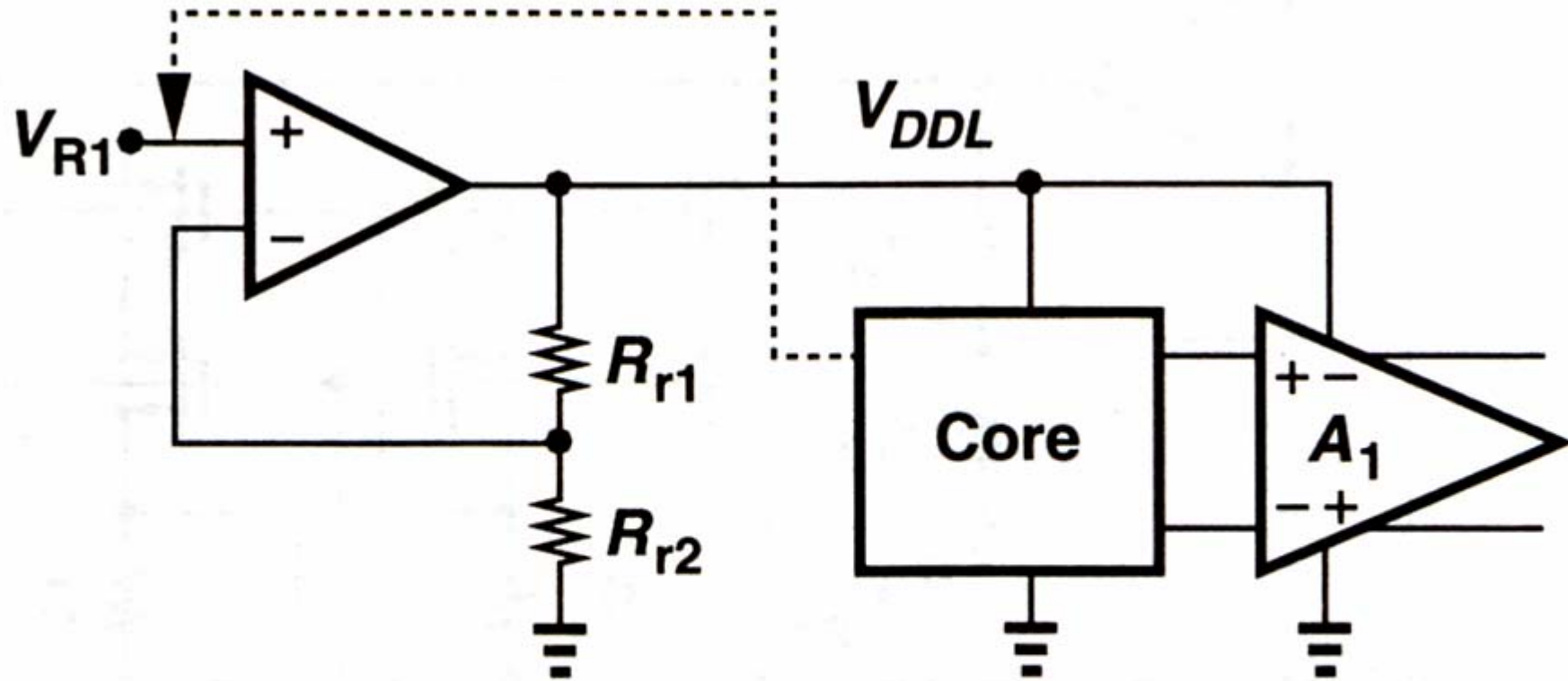
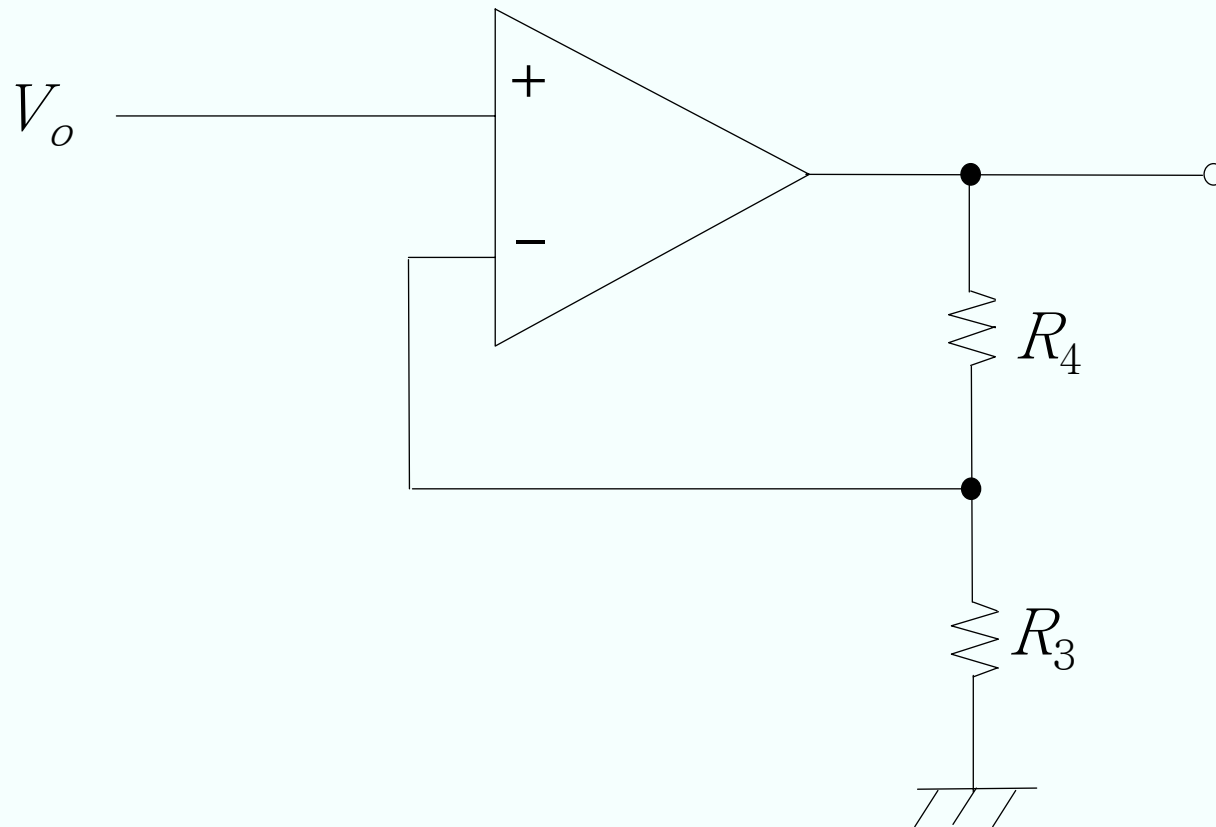


Figure 11.33 Regulation of the supply voltage of the core and op amp to improve supply rejection.



$$V_{out} = V_o \cdot \frac{R_3 + R_4}{R_3}$$

PVT sensitivity of OP amp input offset voltage (V_{os}) → causes problem

Fig 8.2.11 Arbitrary voltage generator from a bandgap source

