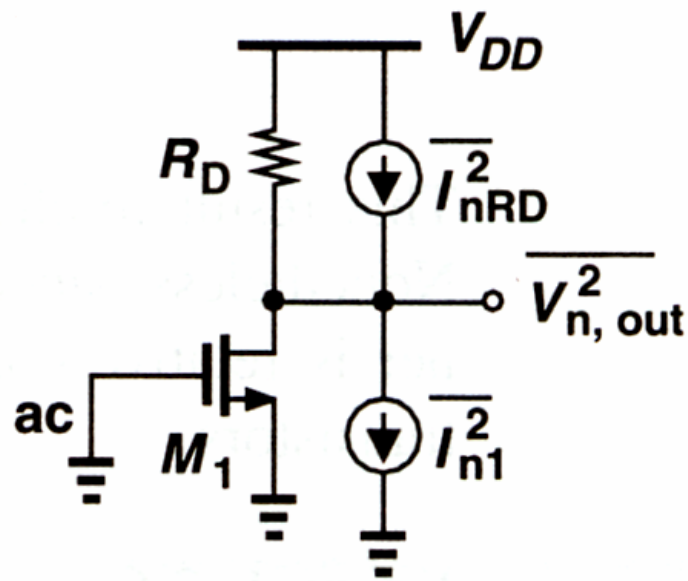
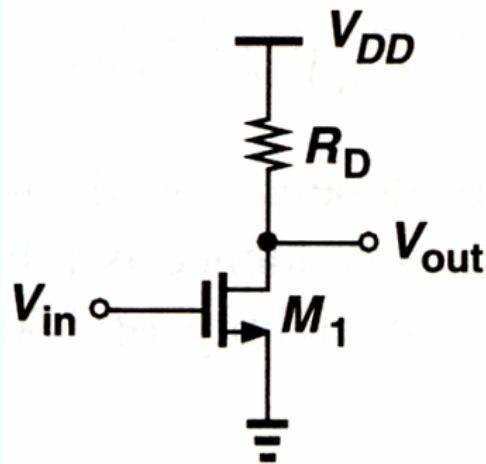


Equivalent input noise voltage

- CS amp
- CG amp
- CD amp
- CMOS inverter amp
- Cascode amp
- Differential pair (R load, current source load, active load)



$$\overline{V_{n,out}^2} = \left(4kT \frac{2}{3} g_m + \frac{K}{C_{ox} WL} \cdot \frac{1}{f} \cdot g_m^2 + \frac{4kT}{R_D} \right) R_D^2$$

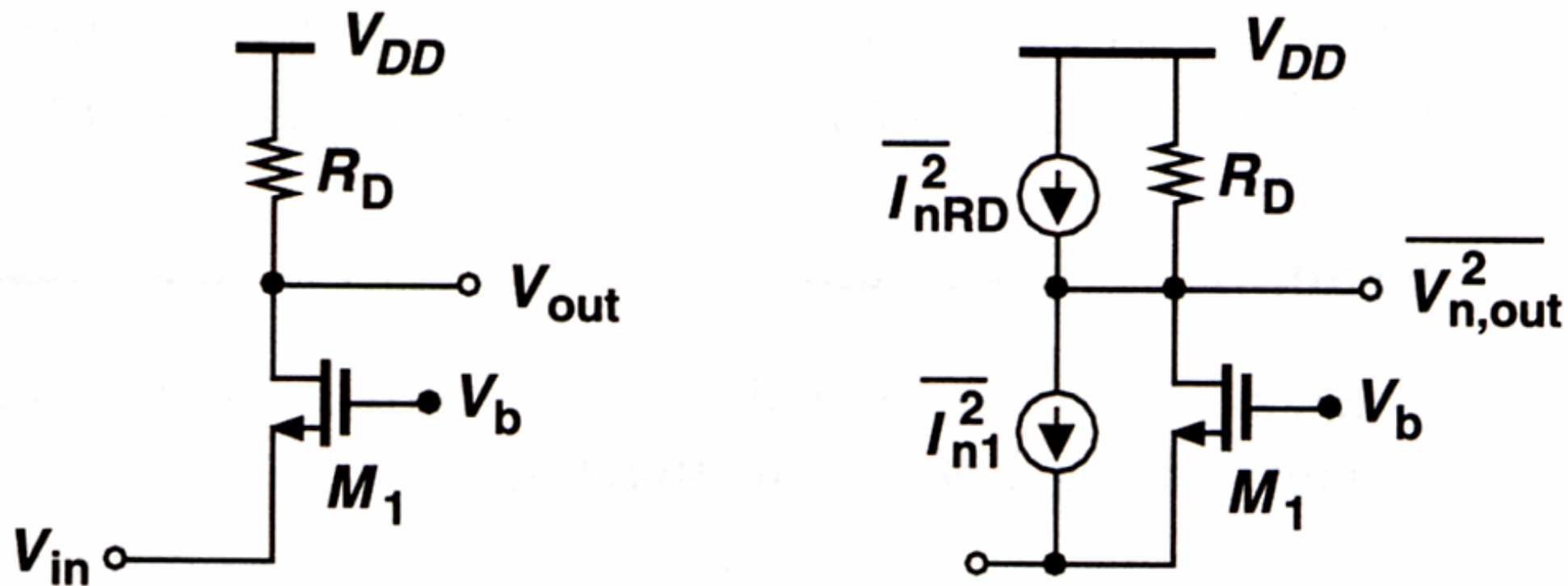
$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{A_v^2}$$

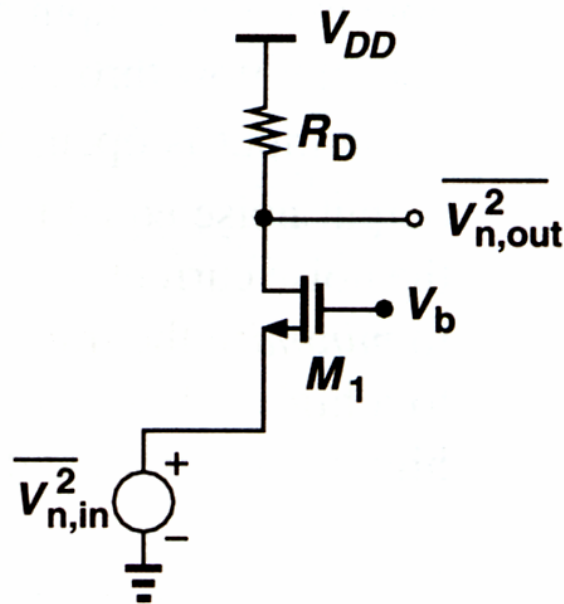
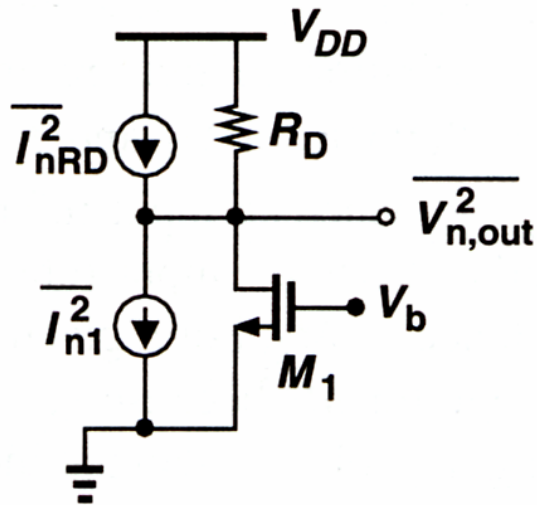
$$= \left(4kT \frac{2}{3} g_m + \frac{K}{C_{ox} WL} \cdot \frac{1}{f} \cdot g_m^2 + \frac{4kT}{R_D} \right) R_D^2 \frac{1}{g_m^2 R_D^2}$$

$$= 4kT \frac{2}{3g_m} + \frac{K}{C_{ox} WL} \cdot \frac{1}{f} + \frac{4kT}{g_m^2 R_D}$$

$$I_{n.in} = \frac{V_{n.in}}{Z_{in}} = 0 \quad \text{since} \quad Z_{in} = \infty$$

At low frequency only

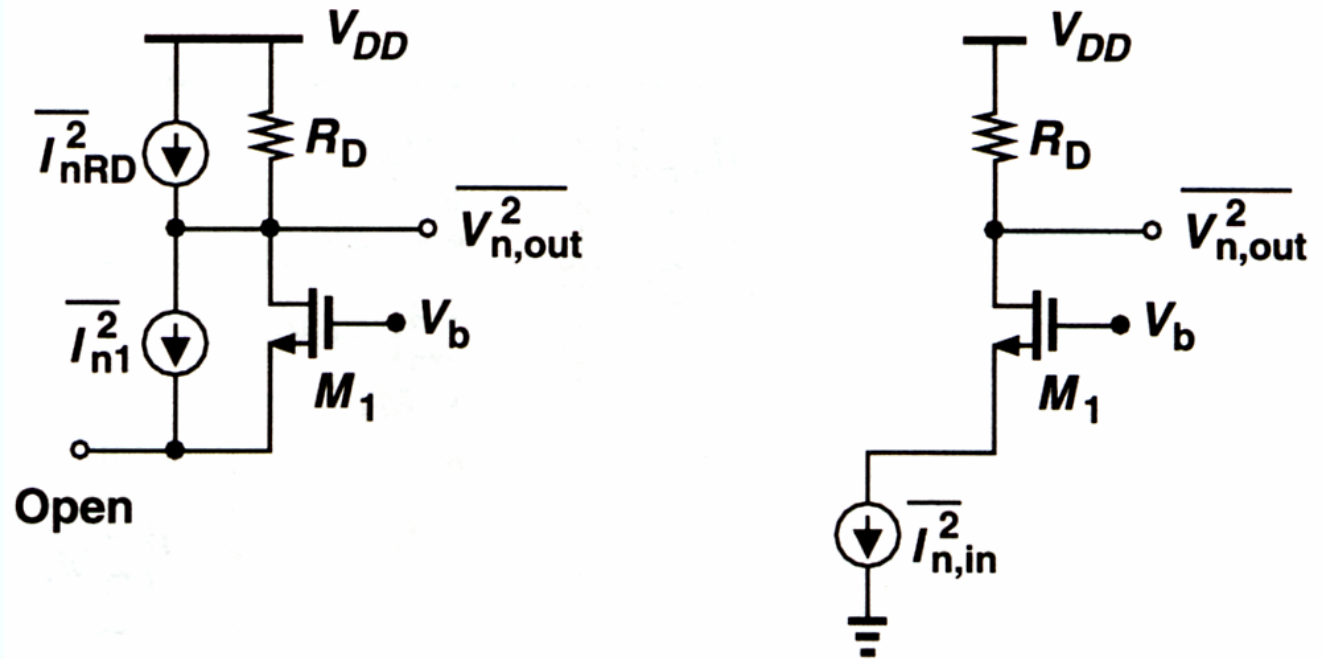




$V_{n.in}$ can be calculated with $Z_S = 0$

$$\left(4kT \frac{2}{3} g_m + \frac{4kT}{R_D}\right) R_D^2 = \overline{V_{n,in}^2} (g_m + g_{mb})^2 R_D^2$$

$$\overline{V_{n,in}^2} = \frac{4kT (2g_m/3 + 1/R_D)}{(g_m + g_{mb})^2}$$



$$\overline{I_{n,in}^2} R_D^2 = 4kTR_D$$

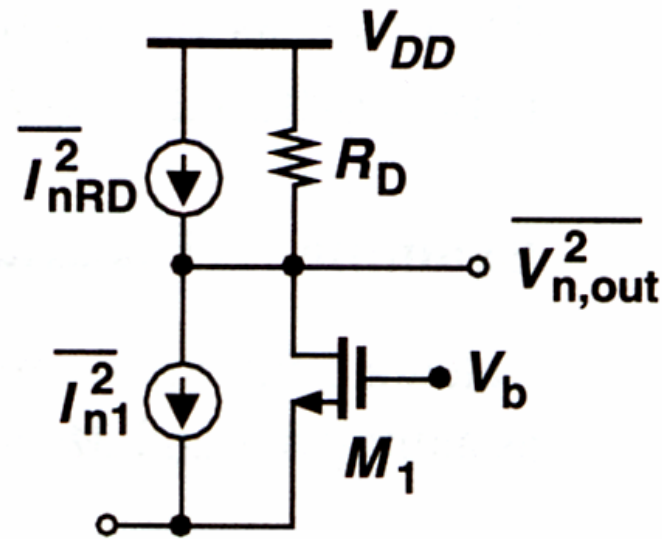
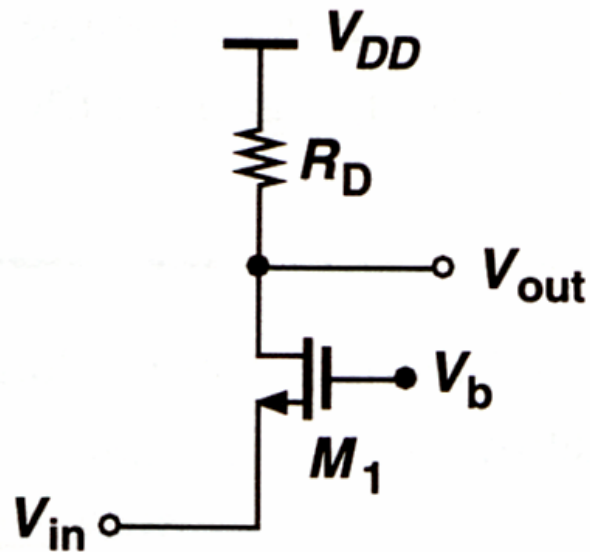
$$\overline{I_{n,in}^2} = \frac{4kT}{R_D}$$

$$\overline{V_{n,in}^2} = \frac{4kT(2g_m/3 + 1/R_D)}{(g_m + g_{mb})^2}$$

$I_{n.in}$ can be calculated with $Z_S = \infty$

$$I_{n.in} \neq \frac{V_{n.in}}{Z_{in}}$$

Equality does not hold with CG amp

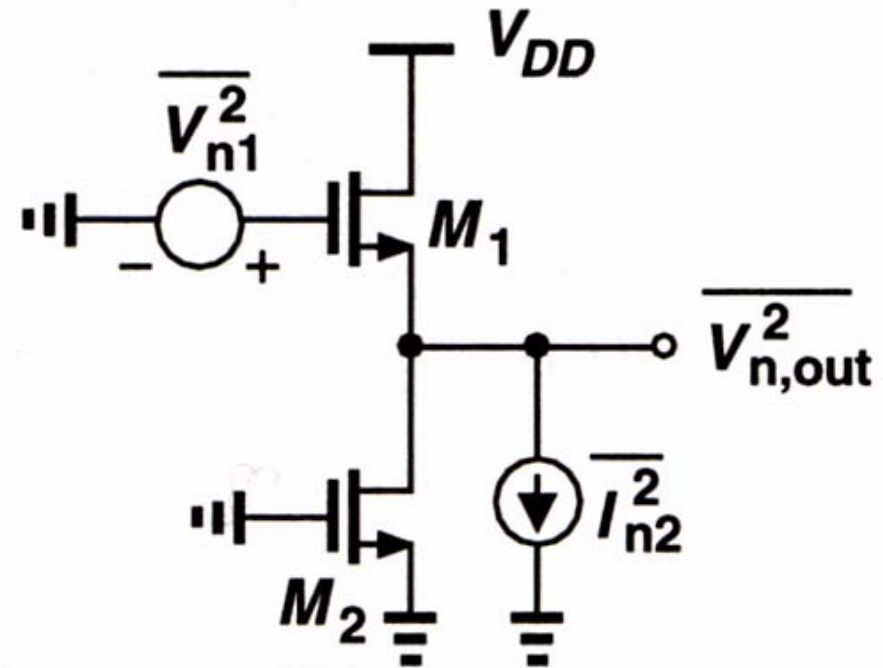
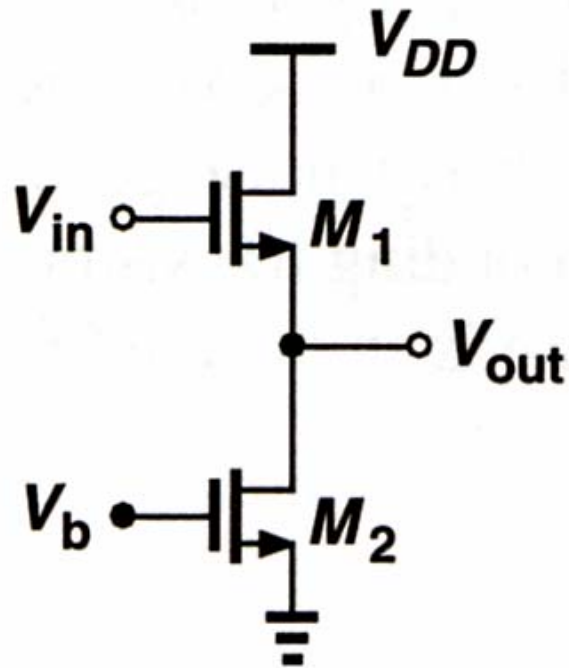


CG amp : current gain = 1



directly reflects load noise current into input noise current($I_{n,in}$)

$$\overline{I_{n,in}^2} = \frac{4kT}{R_D}$$



$$\overline{V_{n,out}^2}|_{M2} = \overline{I_{n2}^2} \left(\frac{1}{g_{m1}} \parallel \frac{1}{g_{mb1}} \parallel r_{O1} \parallel r_{O2} \right)^2$$

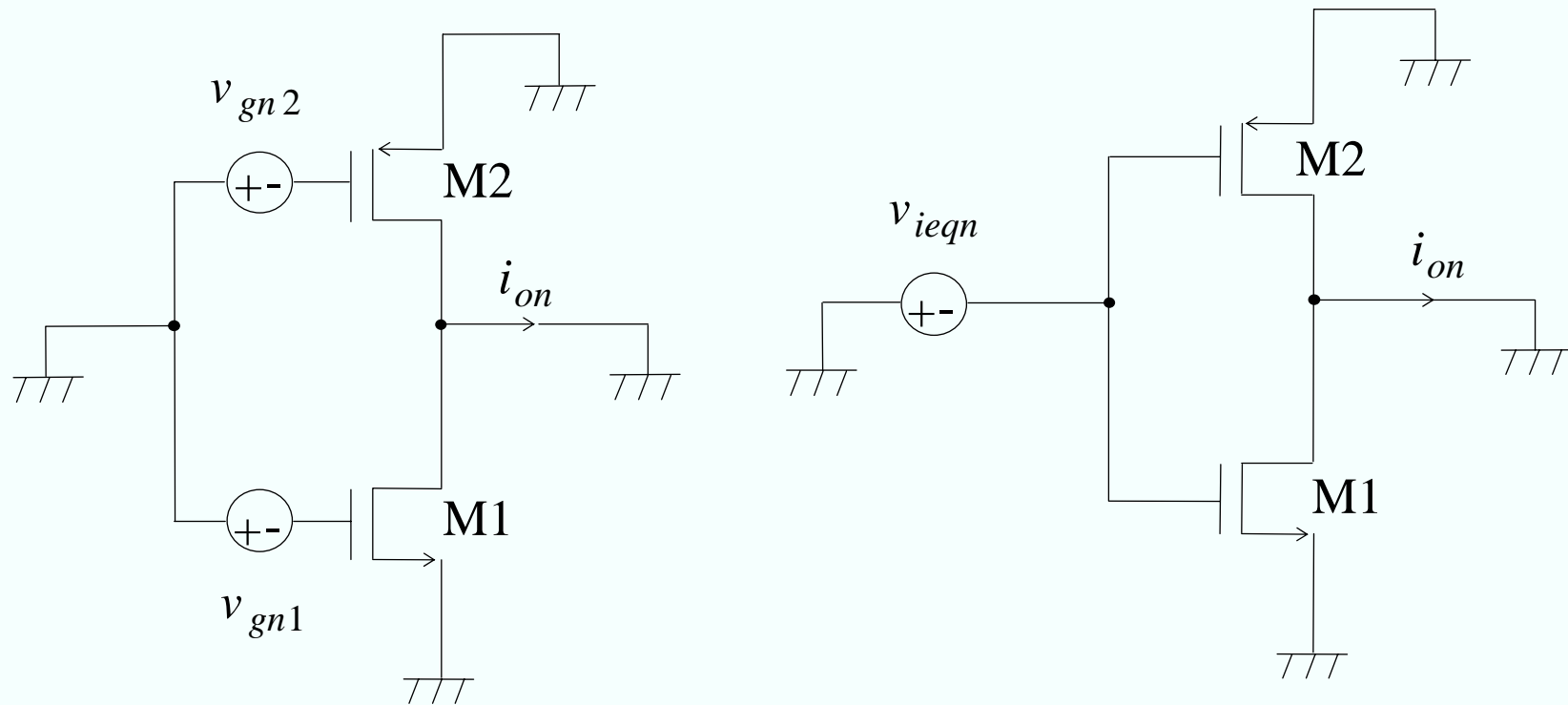
$$A_v = \frac{\frac{1}{g_{mb1}} \parallel r_{O1} \parallel r_{O2}}{\frac{1}{g_{mb1}} \parallel r_{O1} \parallel r_{O2} + \frac{1}{g_{m1}}} \approx 1$$

$$\begin{aligned} \overline{V_{n,in}^2} &= \overline{V_{n1}^2} + \frac{\overline{V_{n,out}^2}|_{M2}}{A_v^2} \\ &= 4kT \frac{2}{3} \left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2} \right) \end{aligned}$$

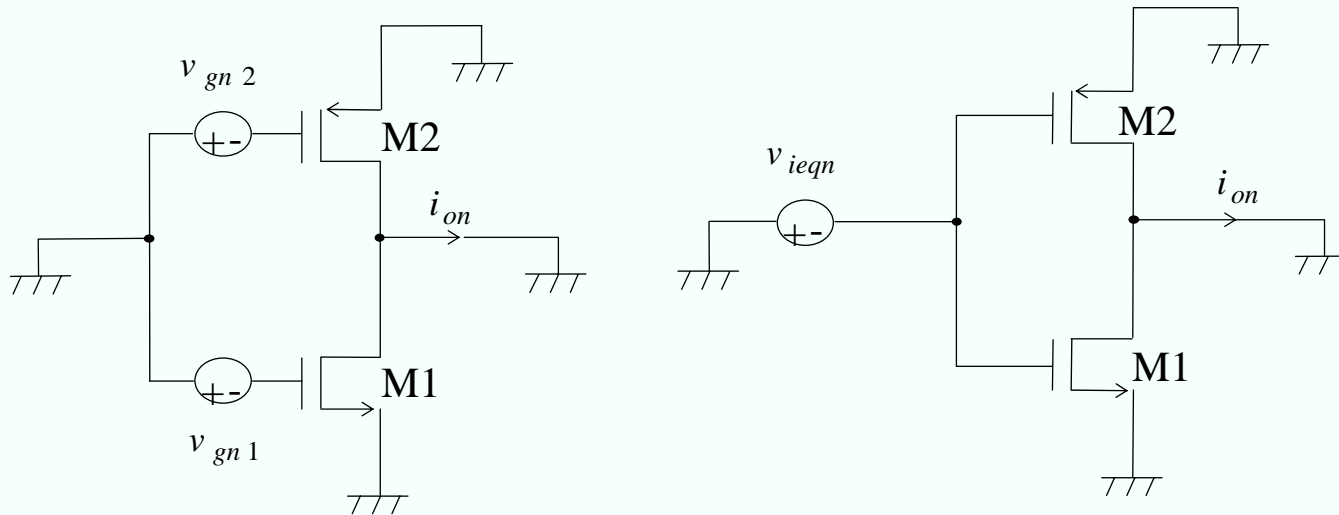
$In.in = 0 \leftarrow$ high Z_{in}

At low frequency only

CMOS inverter amp (push-pull CS amp)



CMOS inverter amp (push-pull CS amp)



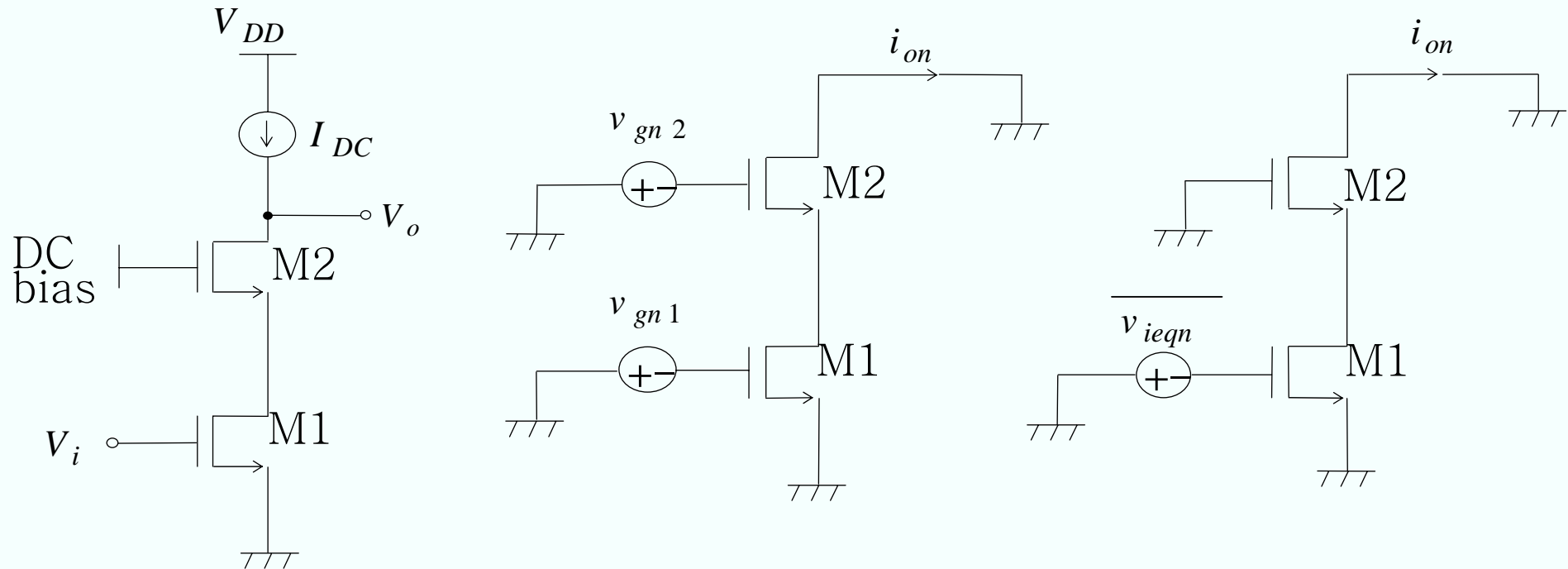
$$i_{on} = g_{m1} \cdot v_{gn1} + g_{m2} \cdot v_{gn2}$$

$$\overline{i_{on}^2} = g_{m1}^2 \cdot \overline{v_{gn1}^2} + g_{m2}^2 \cdot \overline{v_{gn2}^2}$$

$$i_{on} = g_{m1} \cdot v_{ieqn} + g_{m2} \cdot v_{ieqn} = (g_{m1} + g_{m2}) \cdot v_{ieqn}$$

$$\overline{i_{on}^2} = (g_{m1} + g_{m2})^2 \cdot \overline{v_{ieqn}^2}$$

$$\overline{v_{ieqn}^2} = \frac{g_{m1}^2 \cdot \overline{v_{gn1}^2} + g_{m2}^2 \cdot \overline{v_{gn2}^2}}{(g_{m1} + g_{m2})^2}$$

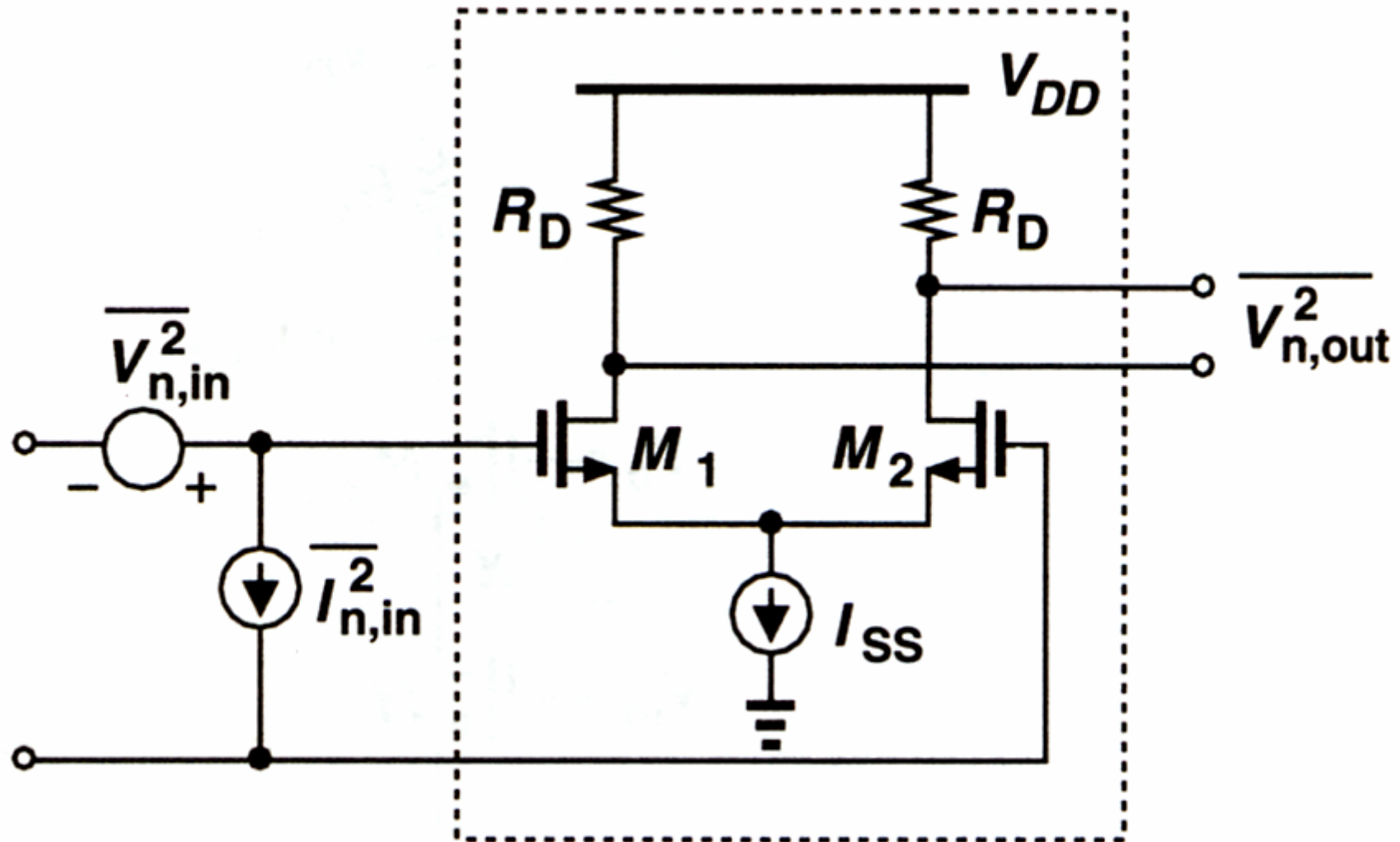


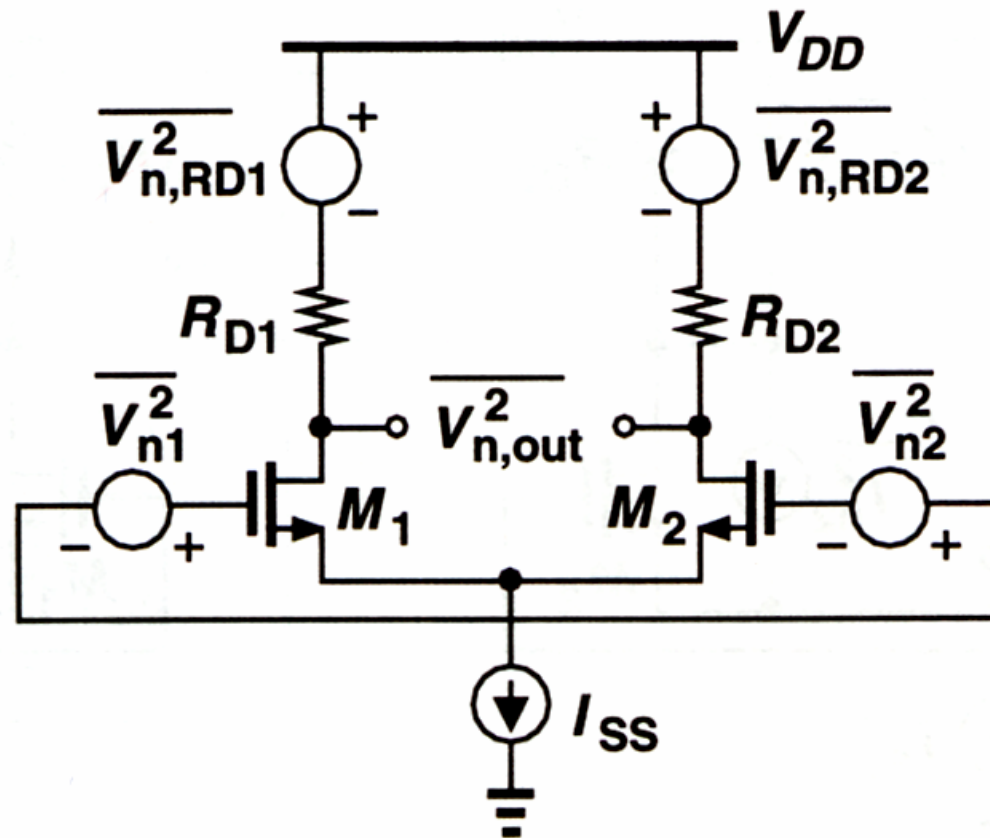
$$i_{on} = -g_{m1} \cdot (-v_{gn1}) + \frac{(-v_{gn2})}{r_{s2} + r_{o1} \parallel r_{o2}} \cdot \frac{r_{o2}}{r_{o1} + r_{o2}} \approx g_{m1} v_{gn1} - \frac{v_{gn2}}{r_{o1}} \approx g_{m1} v_{gn1}$$

$$\overline{i_{on}^2} \approx (g_{m1})^2 \cdot \overline{v_{gn1}^2}$$

$$\overline{v_{ieqn}} = \overline{v_{gn1}}$$

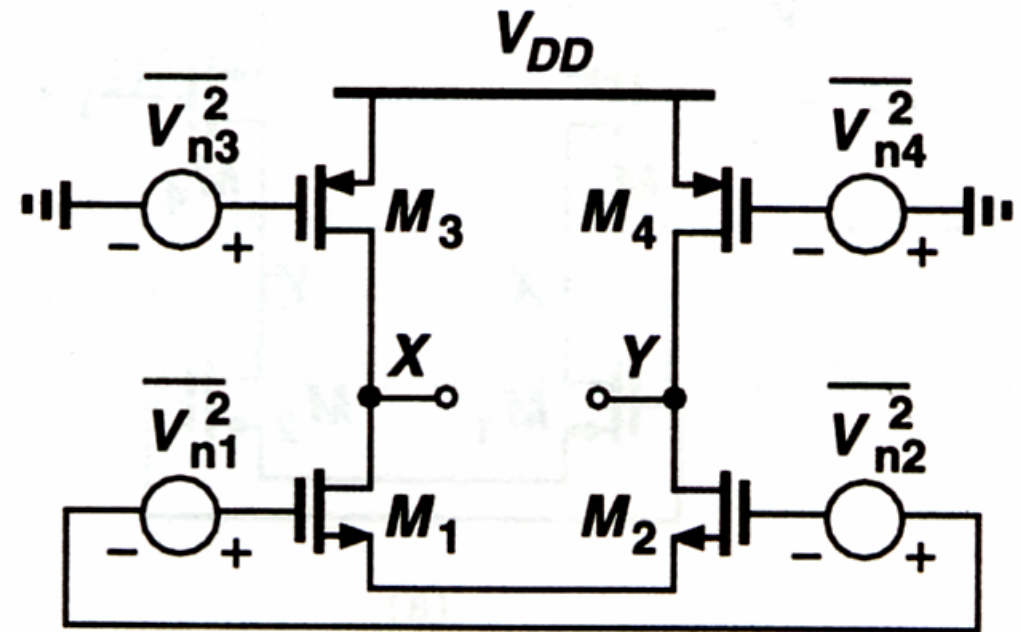
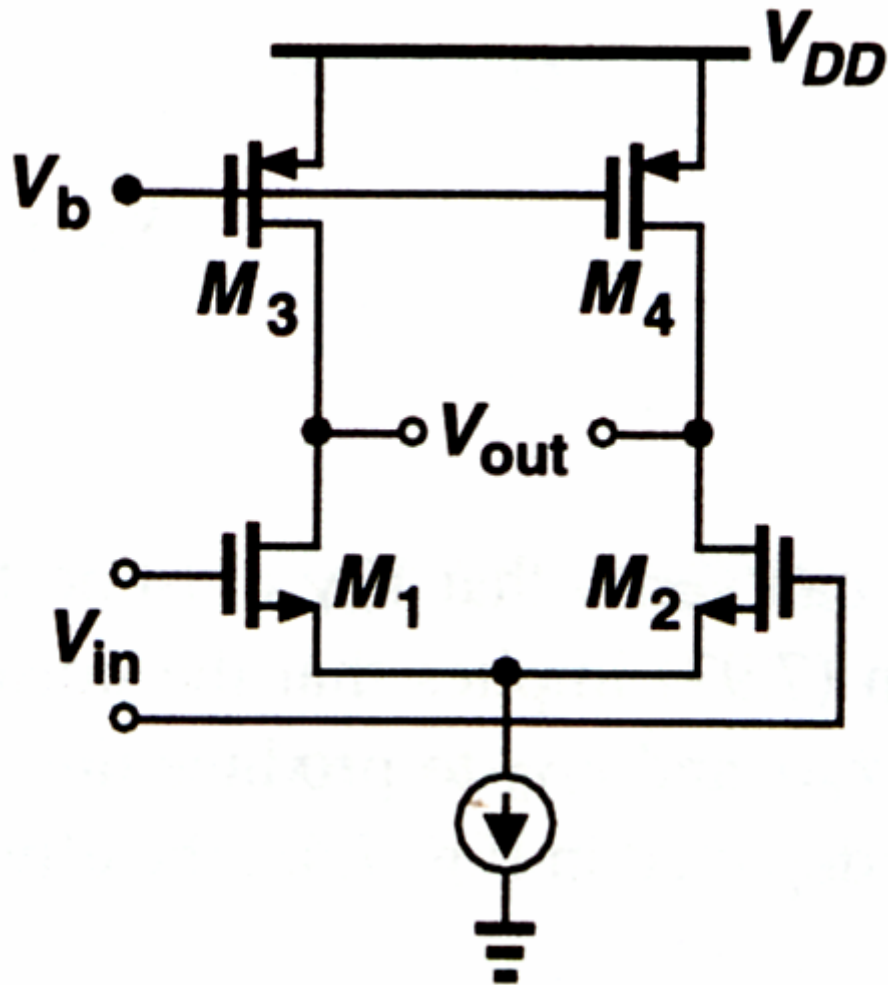
Cascode TR(M2)
 \Rightarrow no effect in output noise





$$\overline{V_{n,in,tot}^2} = 8kT \left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_D} \right) + \frac{2K}{C_{ox} W L} \frac{1}{f}$$

2 X CS amp input noise spectrum



Neglecting r_o 's

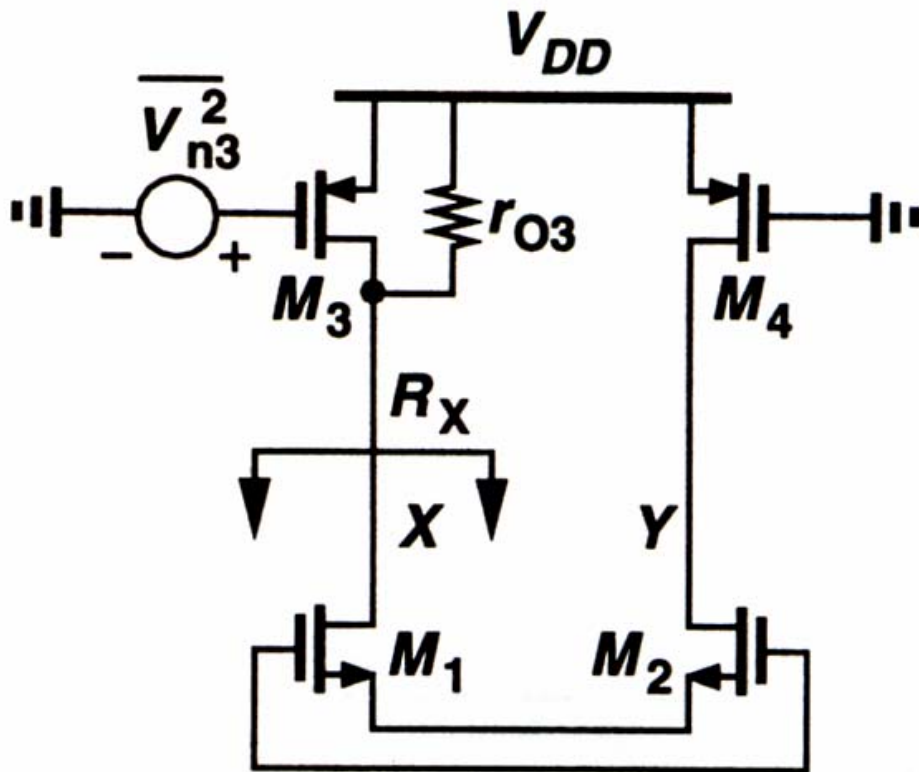
$$\overline{i_{dn}^2} \approx (g_{m1})^2 \cdot \overline{v_{gn1}^2} + (g_{m3})^2 \cdot \overline{v_{gn3}^2}$$

$$\overline{i_{dn}^2} = (g_{m1})^2 \cdot \overline{v_{n.in}^2}$$

$$\overline{v_{n.in}^2} = \overline{v_{gn1}^2} + \frac{(g_{m3})^2}{(g_{m1})^2} \cdot \overline{v_{gn3}^2}$$

Alternative derivation

currents flowing through r_{O3} and R_X by I_{nA} and I_{nB}



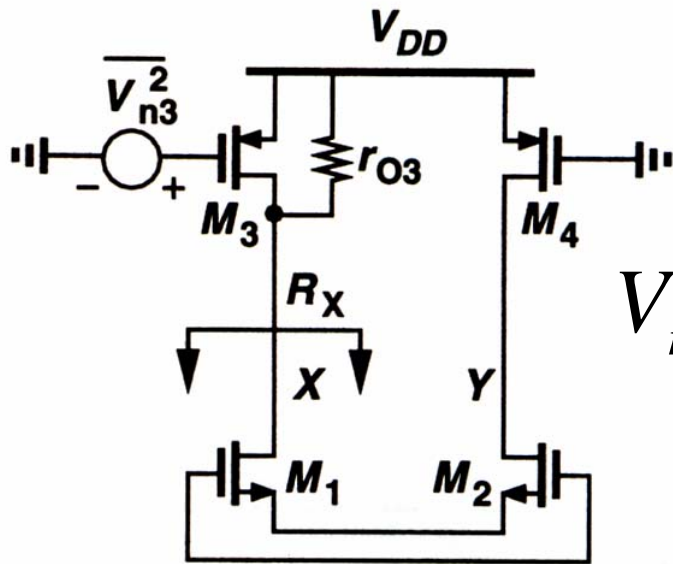
$$R_X = r_{O4} + 2r_{O1}$$

$$I_{nA} = g_{m3} V_{n3} \frac{r_{O4} + 2r_{O1}}{2r_{O4} + 2r_{O1}}$$

$$I_{nB} = g_{m3} V_{n3} \frac{r_{O3}}{2r_{O4} + 2r_{O1}}$$

$$V_{nX} - V_{nY} = I_{nA} \cdot r_{O3} - I_{nB} \cdot r_{O4} = g_{m3} V_{n3} \cdot (r_{O1} \parallel r_{O3})$$

Alternative derivation



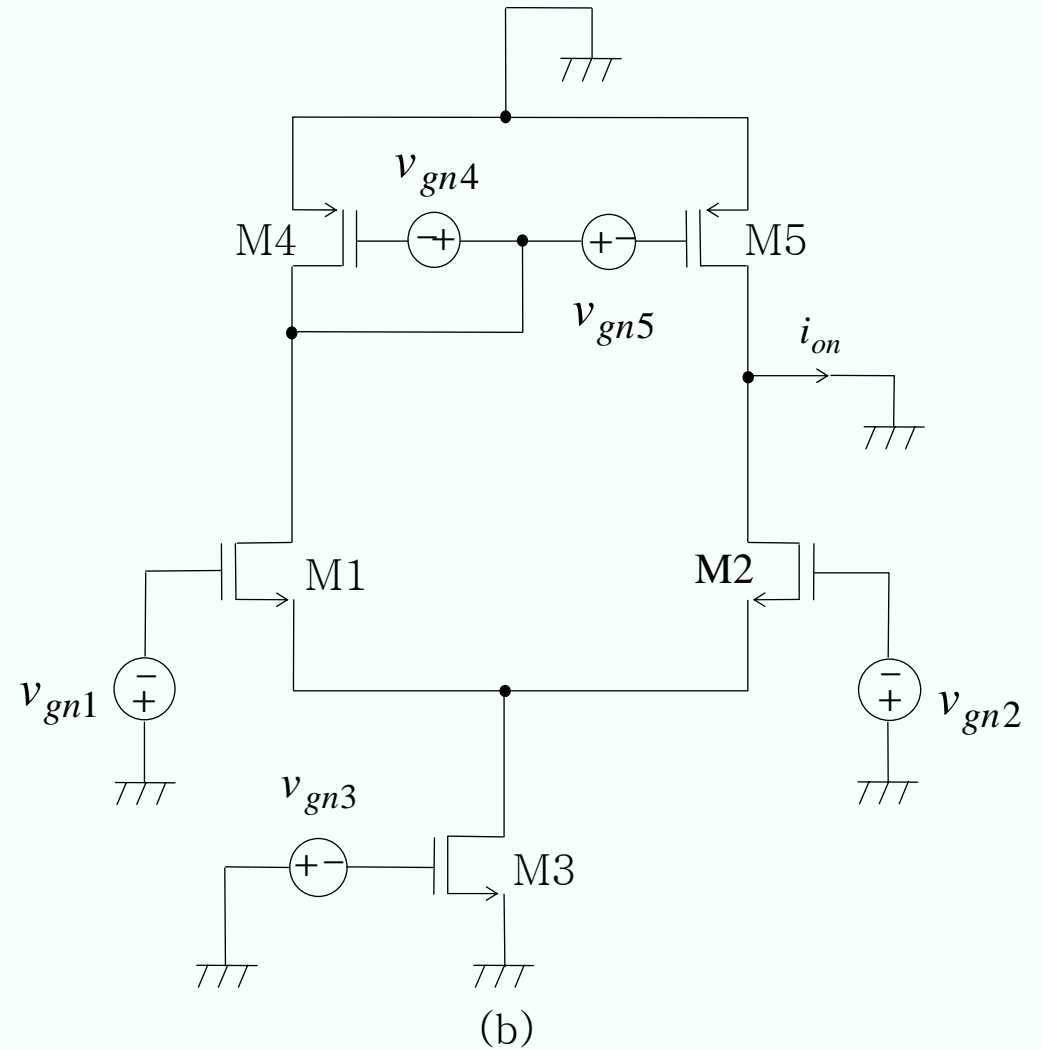
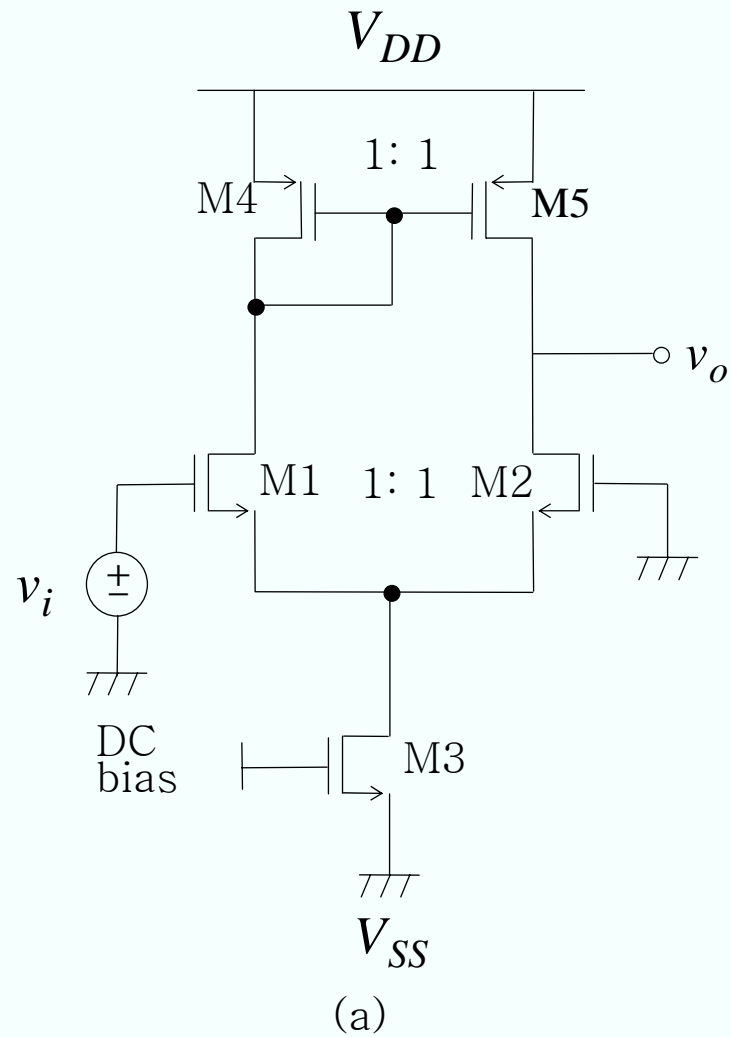
$$V_{nX} - V_{nY} = g_{m3} V_{n3} \cdot (r_{o1} \parallel r_{o3})$$

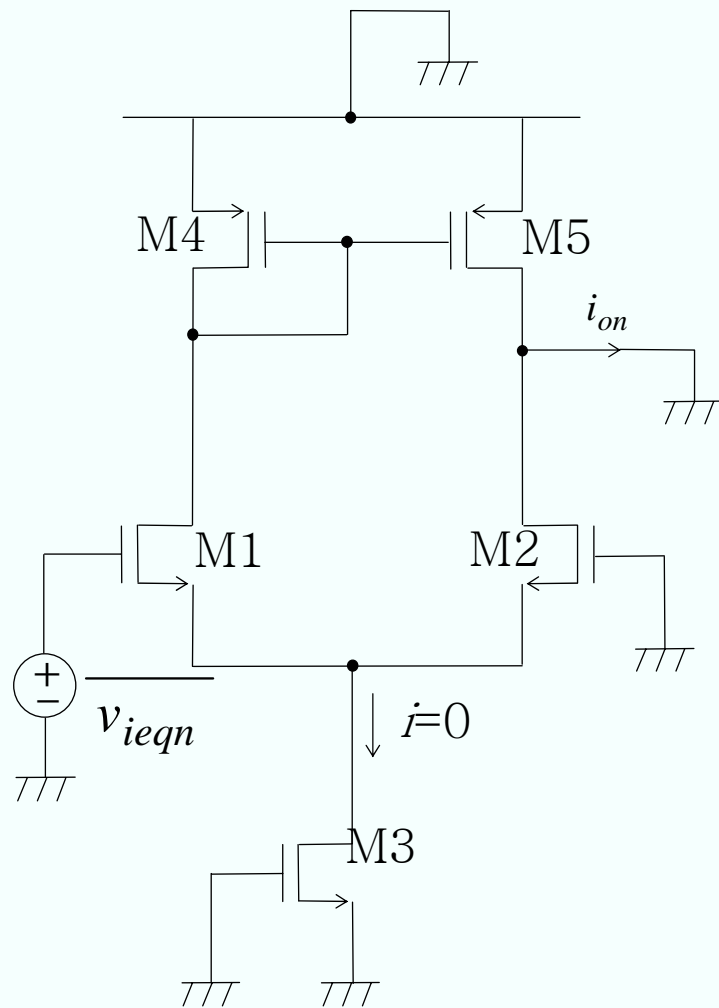
$$V_{nX} - V_{nY} = g_{m1} \cdot (r_{o1} \parallel r_{o3}) \cdot (V_{g1} - V_{g2})$$

$$\overline{V_{n,in}^2} = 2\overline{V_{n1}^2} + 2\frac{g_{m3}^2}{g_{m1}^2}\overline{V_{n3}^2}$$

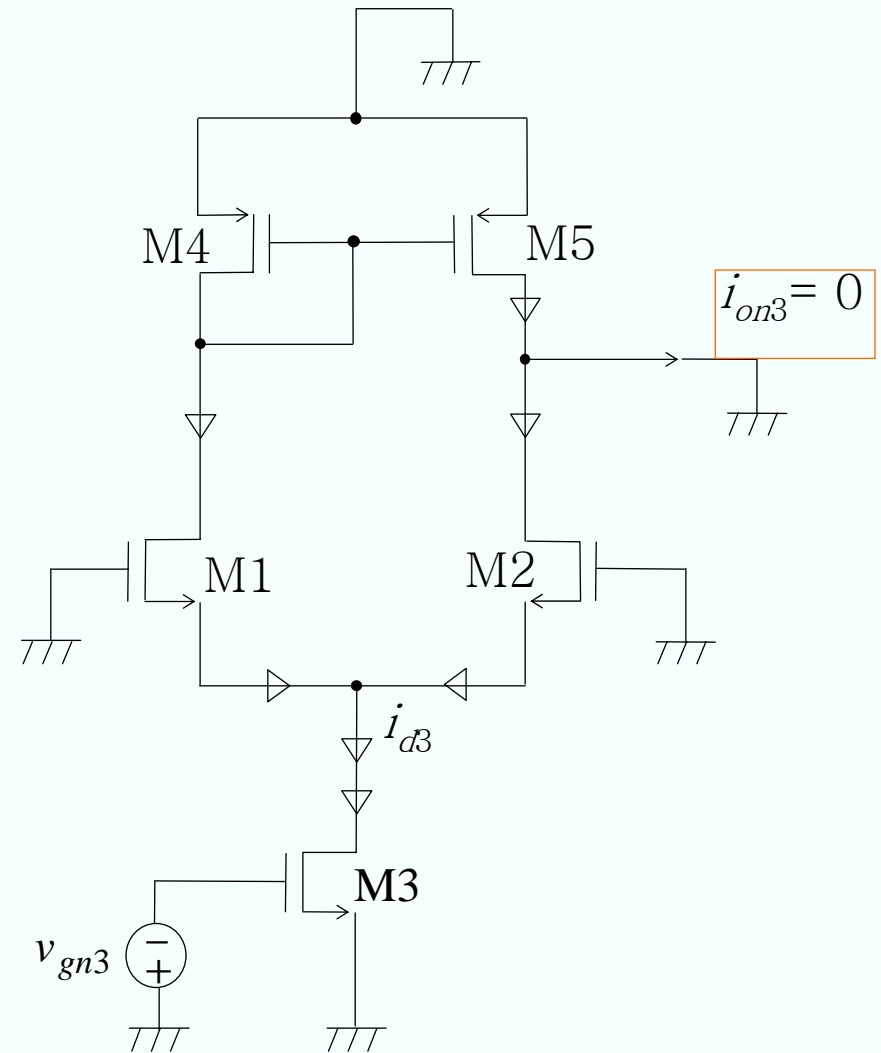
$$\overline{V_{n,in}^2} = 8kT \left(\frac{2}{3g_{m1}} + \frac{2g_{m3}}{3g_{m1}^2} \right) + \frac{2K_N}{C_{ox}(WL)_1 f} + \frac{2K_P}{C_{ox}(WL)_3 f} \frac{g_{m3}^2}{g_{m1}^2}$$

6.2.5 Equivalent input noise voltage of diff pair with active load

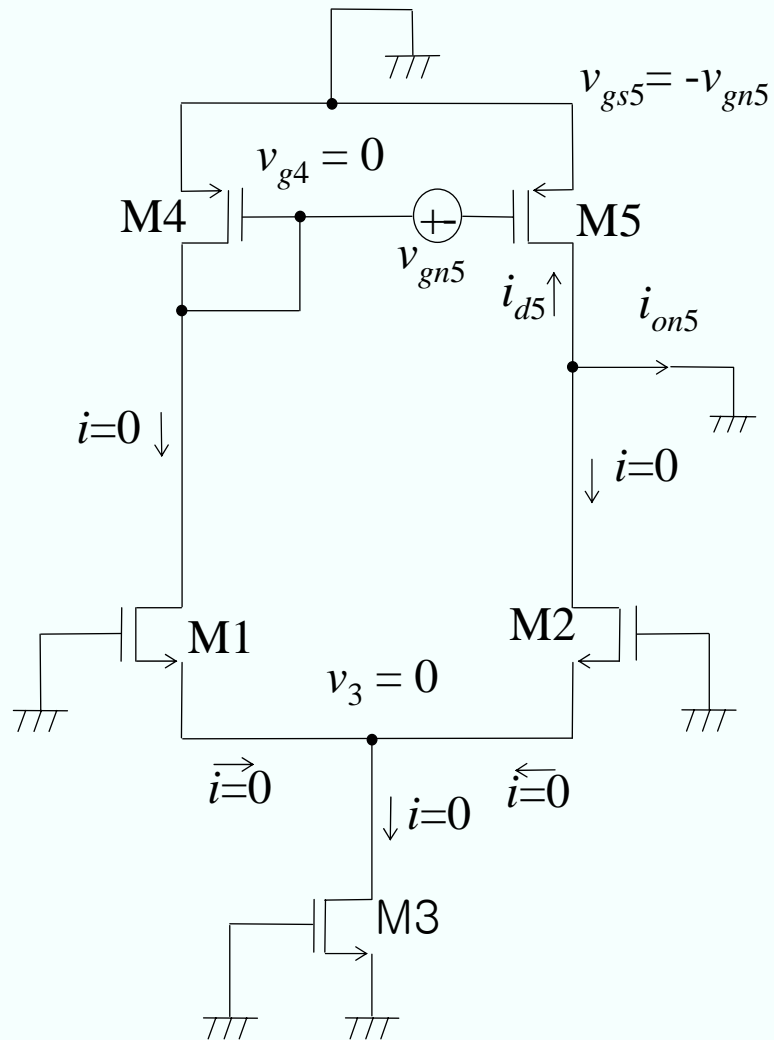




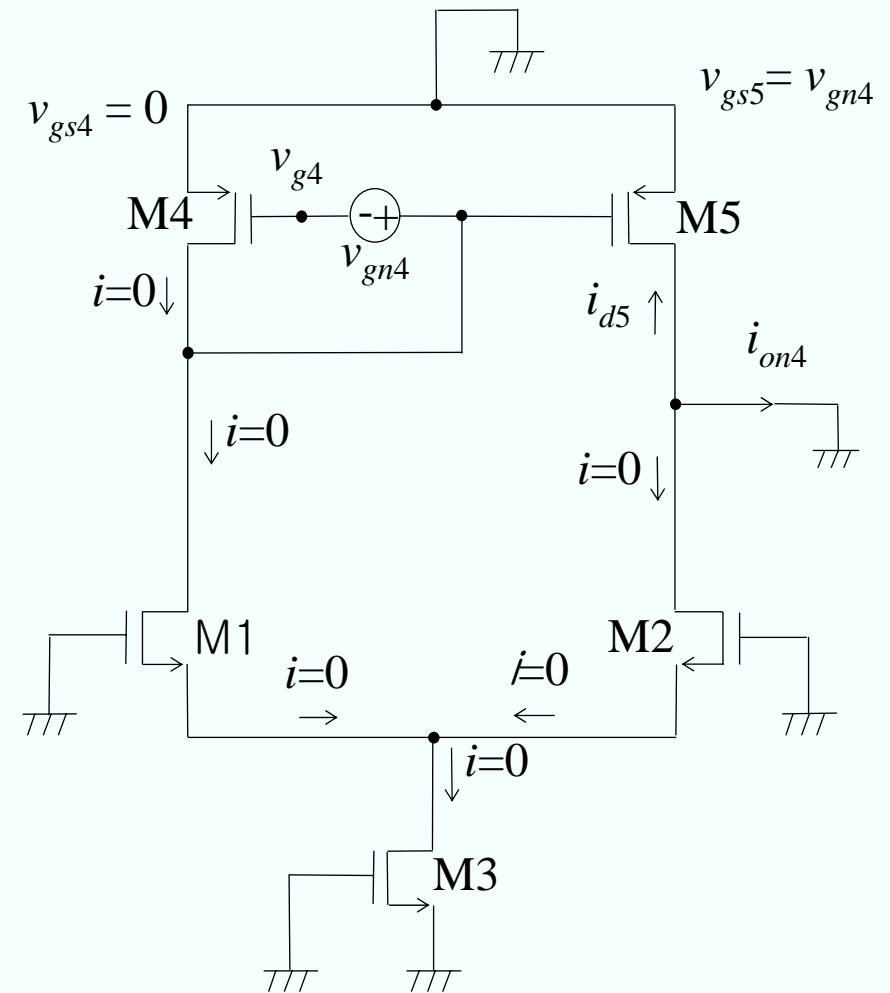
$$i_{on} = -g_{m1}v_{ieqn}$$



$$i_{on3} = 0$$



$$i_{on5} = -i_{d5} = g_{m5} v_{gn5}$$



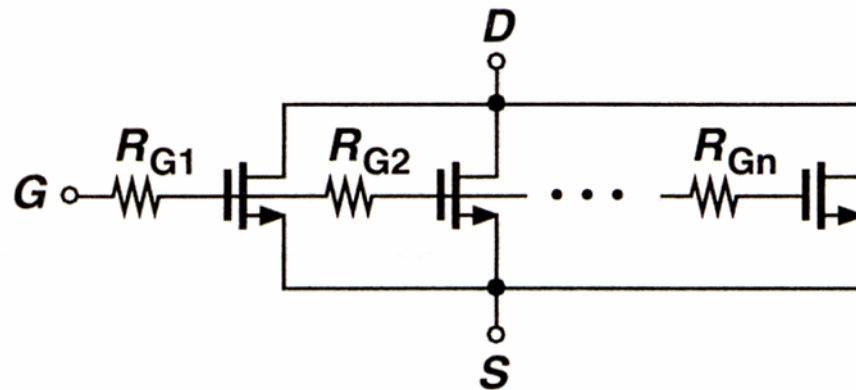
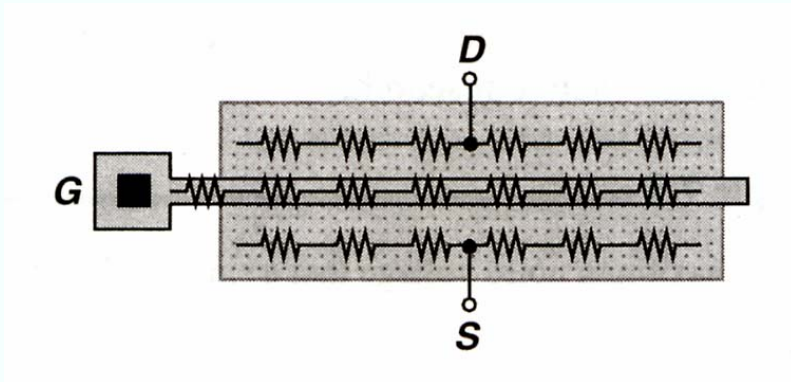
$$i_{on4} = -i_{d5} = -g_{m5} v_{gn4}$$

$$i_{on} = i_{on1} + i_{on2} + i_{on3} + i_{on4} + i_{on5} = g_{m1} \cdot (-v_{gn1} + v_{gn2}) + g_{m5} \cdot (-v_{gn4} + v_{gn5})$$

$$\overline{i_{on}^2} = g_{m1}^2 \cdot (\overline{v_{gn1}^2} + \overline{v_{gn2}^2}) + g_{m5}^2 \cdot (\overline{v_{gn4}^2} + \overline{v_{gn5}^2})$$

$$\overline{i_{on}^2} = g_{m1}^2 \cdot \overline{v_{ieqn}^2}$$

$$\overline{v_{ieqn}^2} = \overline{v_{gn1}^2} + \overline{v_{gn2}^2} + \left(\frac{g_{m5}}{g_{m1}} \right)^2 \cdot (\overline{v_{gn4}^2} + \overline{v_{gn5}^2})$$



$$R_{G1} + R_{G2} + \dots + R_{Gn} = R_G$$

Compute the equivalent input noise voltage of a MOSFET due to the series resistance of gate material.

Assume the total resistance of gate is R_G , $W \gg L$, MOSFET is operating in saturation region. Use the distributed property of gate material.

Ignore intrinsic thermal noise of MOSFET.

$$g_{m,i} = g_m \cdot \frac{1}{n}$$

$$V_{n,i} = V_{n,1} + V_{n,2} + \dots + V_{n,i} = i \cdot V_{n,1}$$

$$I_{n,total} = \frac{g_m}{n} \cdot \{V_{n,1} + (V_{n,1} + V_{n,2}) + (V_{n,1} + V_{n,2} + V_{n,3}) + \dots + (V_{n,1} + V_{n,2} + V_{n,3} + \dots + V_{n,n})\}$$

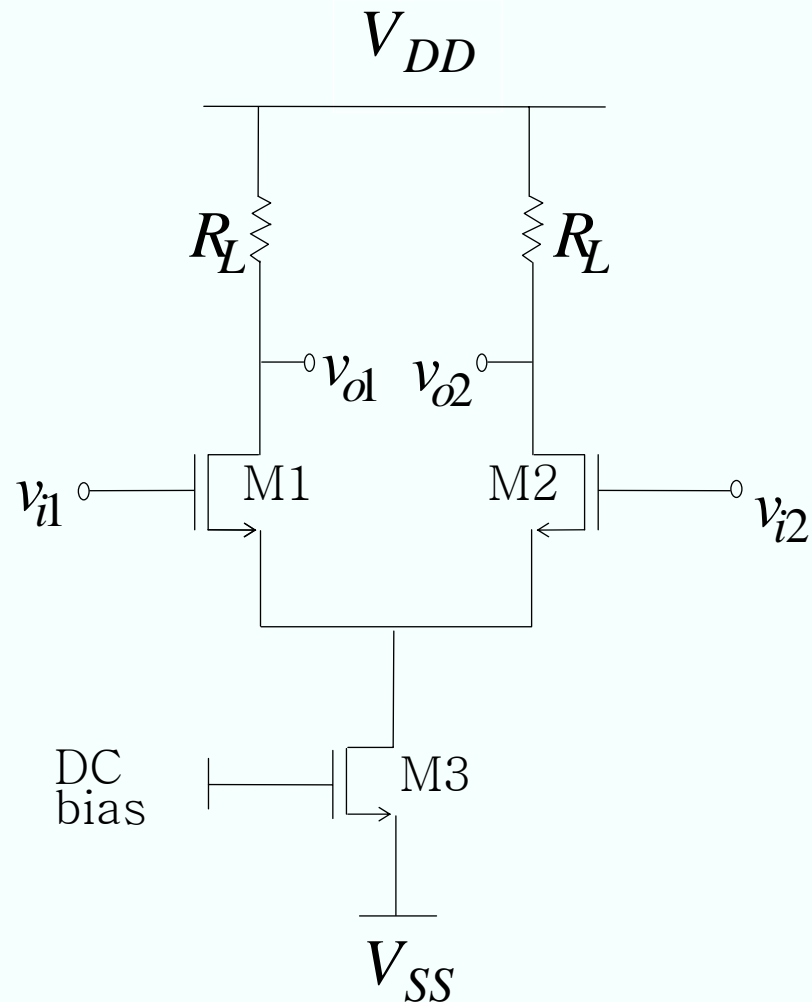
$$= \frac{g_m}{n} \cdot \{n \cdot V_{n,1} + (n-1) \cdot V_{n,2} + (n-2) \cdot V_{n,3} + \dots + 1 \cdot V_{n,n}\} \text{ correlation considered}$$

$$\overline{I_{n,total}^2} = \left(\frac{g_m}{n}\right)^2 \cdot \{n^2 + (n-1)^2 + (n-2)^2 + \dots + 1^2\} \cdot \overline{V_{n,1}^2}$$

$$= \left(\frac{g_m}{n}\right)^2 \cdot \frac{n(n+1)(2n+1)}{6} \cdot 4kT \frac{R_G}{n}$$

As n approaches ∞ ,

$$\overline{I_{n,total}^2} = 4kT \frac{R_G}{3}$$



Calculate the rms spectrum (V/sqrt(Hz)) of equivalent input thermal noise voltage and compare it with the SPICE value at 1MHz.

(set $KF = 0$ in the model parameter set)

```
noise analysis of CMOS differential amplifier with active load
m1 4 1 3 8 nmos w=20u l=1.2u ad=72p pd=22.4u as=72p ps=22.4u
m2 5 2 3 8 nmos w=20u l=1.2u ad=72p pd=22.4u as=72p ps=22.4u
m3 3 6 8 8 nmos w=10u l=1.2u ad=36p pd=12.4u as=36p ps=12.4u
m4 4 4 7 7 nmos w=20u l=1.2u ad=72p pd=22.4u as=72p ps=22.4u
m5 5 4 7 7 nmos w=20u l=1.2u ad=72p pd=22.4u as=72p ps=22.4u
m2 6 6 8 8 nmos w=10u l=1.2u ad=36p pd=12.4u as=36p ps=12.4u
iss 7 6 100u
vi1 1 0 dc 0 ac 1
vi2 2 0 dc 0
vdd 7 0 dc 2.5
vss 8 0 dc -2.5
.ac dec 10 1 100mega
.noise v(5) vi1
.print noise onoise inoise
. probe vm(1) vm(5)
. model nmos nmos tox=200e-10 uo=500 vto=0.8 gamma=0.8
+ lambda=0.08 cgdo=300p cgso=300p cj=2.75e-4 cjsw=1.9e-10
+ ld=0.2u af=1 kf=5e-30
```

```
. model pmos pmos tox=200e-10 uo=200 vto=-0.8 gamma=0.8
+ lambda=0.08 cgdo=300p cgso=300p cj=2.75e-4 cjsw=1.9e-10
+ ld=0.2u af=1 kf=5e-30
. model pmos pmos tox=200e-10 uo=200 vto=-0.8 gamma=0.8
+ lambda=0.1 cgdo=300p cgso=300p cj=2.75e-4 cjsw=1.9e-10
+ ld=0.2u af=1 kf=1e-30
. end
```

Fig 6.2.11 SPICE netlist to calculate the equivalent input noise voltage of CMOS diff pair with active load

* differential pair (R load)

mn1 outb in ns vss n w=10u l=0.35u ad=10.5p as=10.5p pd=11.05u ps=11.05u

mn2 out inb ns vss n w=10u l=0.35u ad=10.5p as=10.5p pd=11.05u ps=11.05u

mn3 ns vbn vss vss n w=10u l=0.35u ad=10.5p as=10.5p pd=11.05u ps=11.05u

rL1 vdd outb 20K

rL2 vdd out 20K

mn6 vbn vbn vss vss n w=10u l=0.35u ad=10.5p as=10.5p pd=11.05u ps=11.05u

CL1 out vss 0.1p

CL2 outb vss 0.1p

iss vdd vbn 100u

vdd vdd vss dc 3.3

vss vss 0 dc 0