6.2 noise analysis

Why do you need noise analysis ?

Equivalent input noise voltage of amp:

determines

the minimum input voltage of amp that can be amplified

(the min detectable signal)

1. What is noise ?
2. Noise of resistor
3. kT / C noise
4. Noise BW
5. MOSFET noise
6. Equivalent input noise voltage (concept, MOSFET)

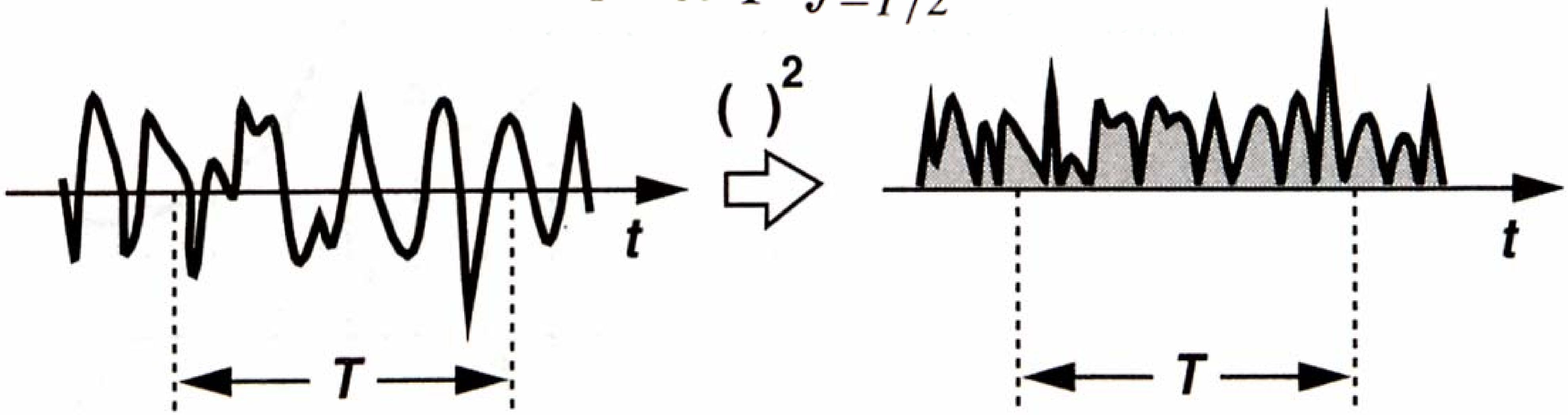
What is noise ?

Noise: random process

(instantaneous value in time domain cannot be predicted)

Noise: Average power (P_{av}) can be predicted

$$P_{av} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} x^2(t)dt$$



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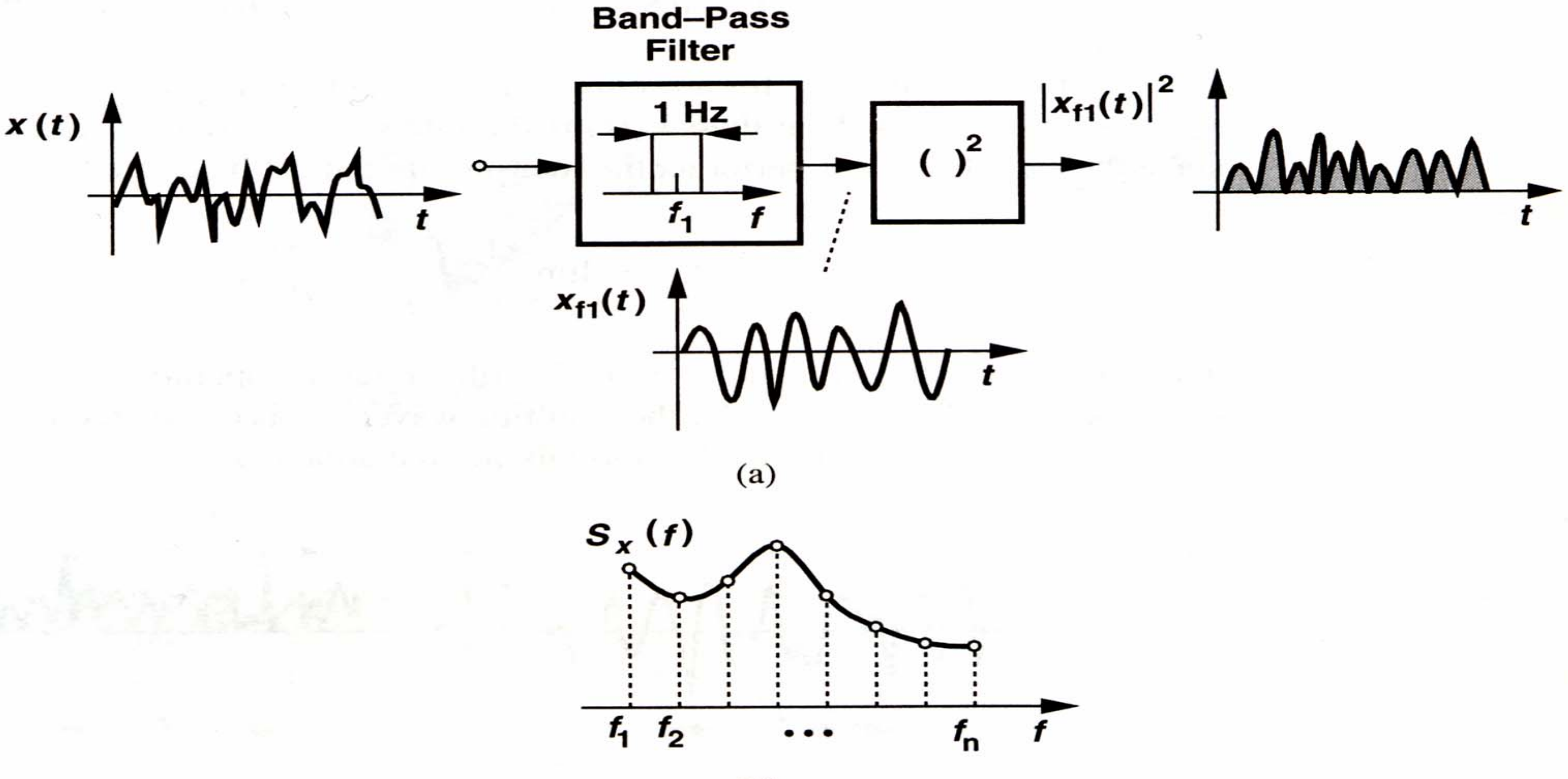
Power spectral density (PSD) $S_x(f)$:

Average power in one-Hertz bandwidth around frequency f



(a)

(b)

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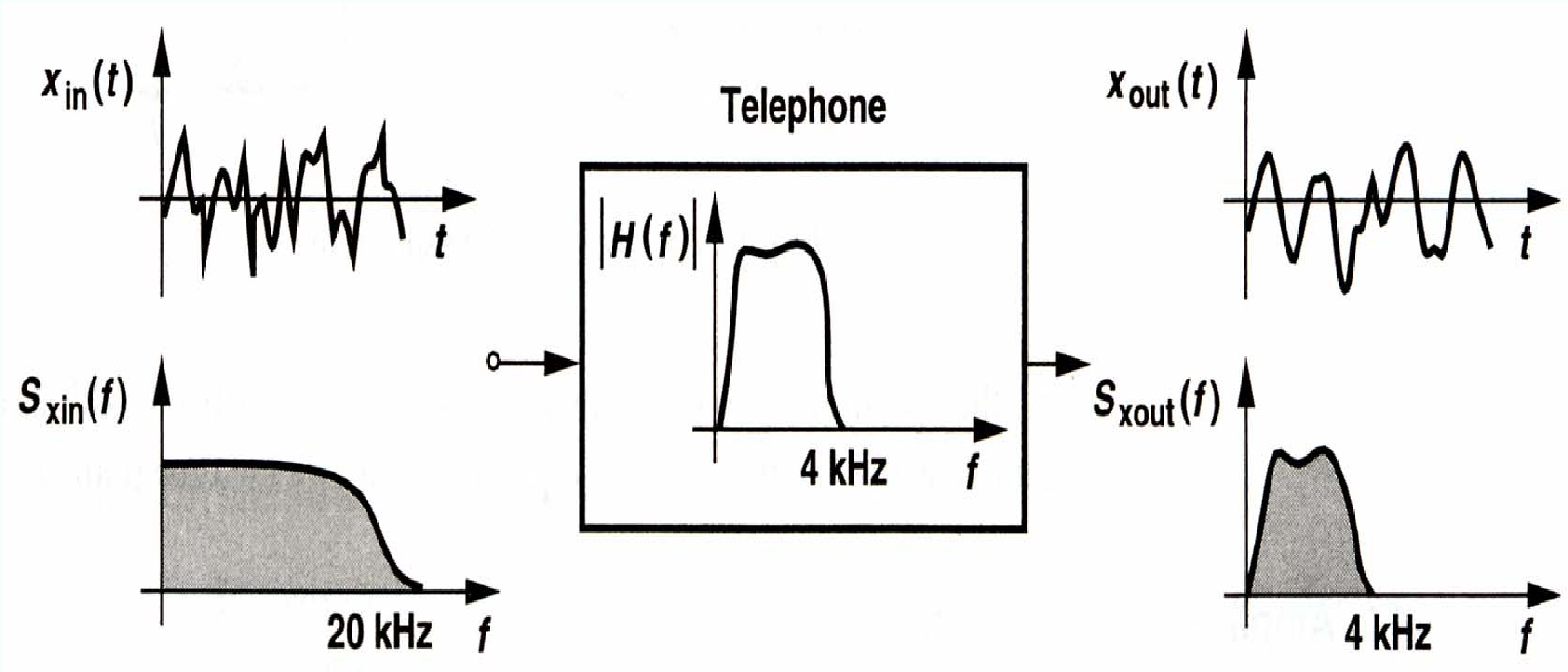
What is noise ?

LTI (linear time invariant) system with transfer function H(f)

$$S_Y(f) = S_X(f)|H(f)|^2$$

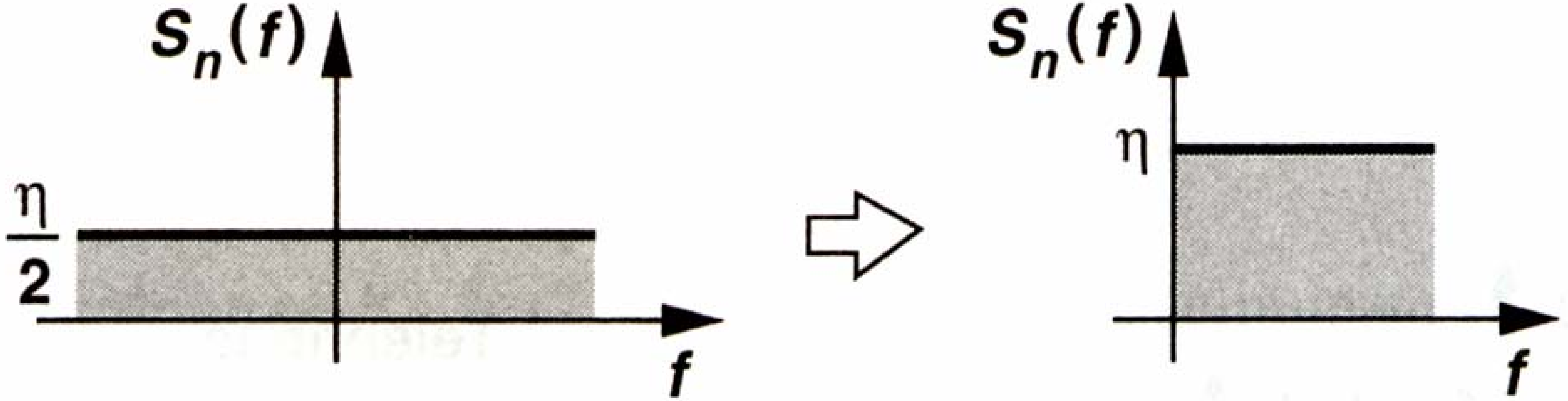


Razavi

What is noise ?

Power spectral density (PSD) $S_n(f)$: Even function of f

$$|V_n|^2 \ (Power) = \int S_n(f)\, df$$



Two-sided PSD

One-sided PSD

Can use either of these two
(keep power the same)

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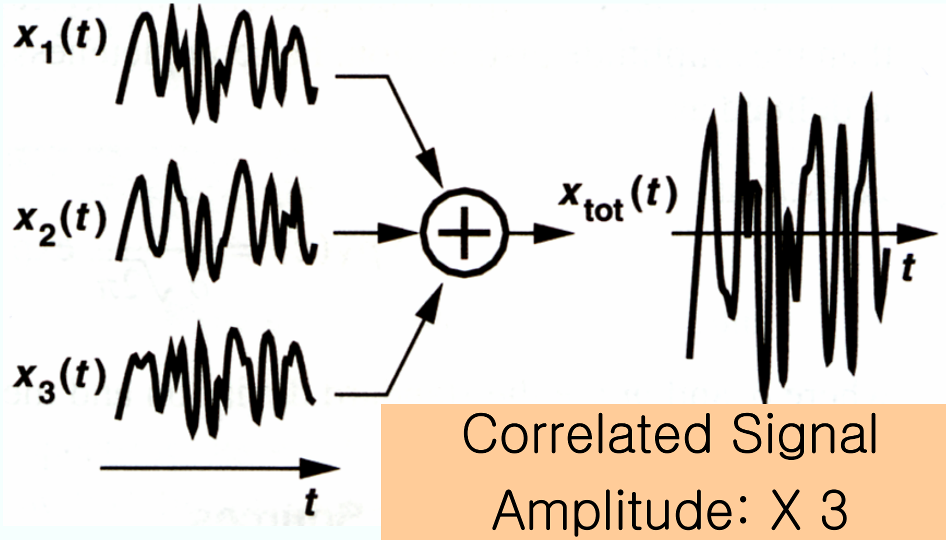
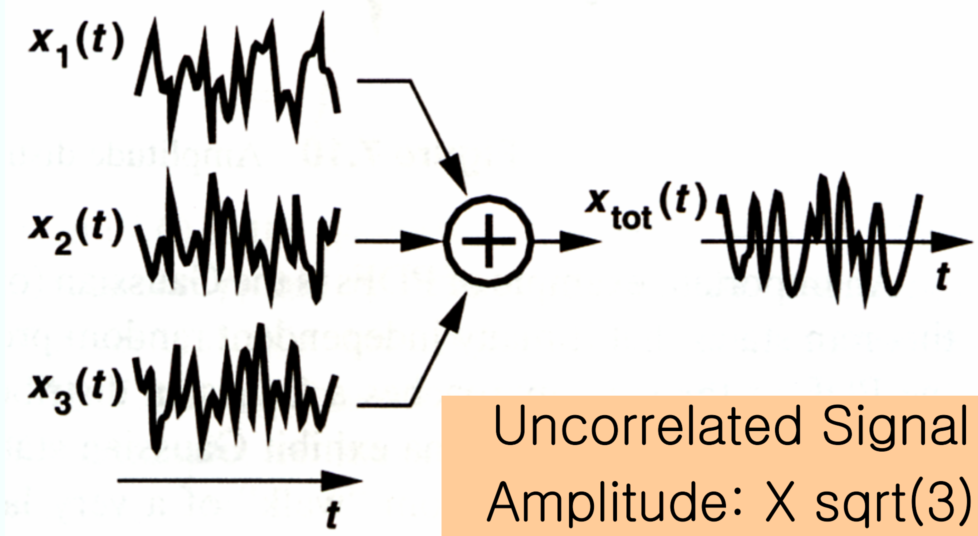
What is noise ?

Correlated vs Uncorrelated signals



Correlated Signal
Amplitude: X 3



Uncorrelated Signal
Amplitude: X sqrt(3)

$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} [x_1(t) + x_2(t)]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} x_1^2(t)dt + \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} x_2^2(t)dt$$

$$+ \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} 2x_1(t)x_2(t)dt$$

$$= P_{av1} + P_{av2} + \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} 2x_1(t)x_2(t)dt,$$

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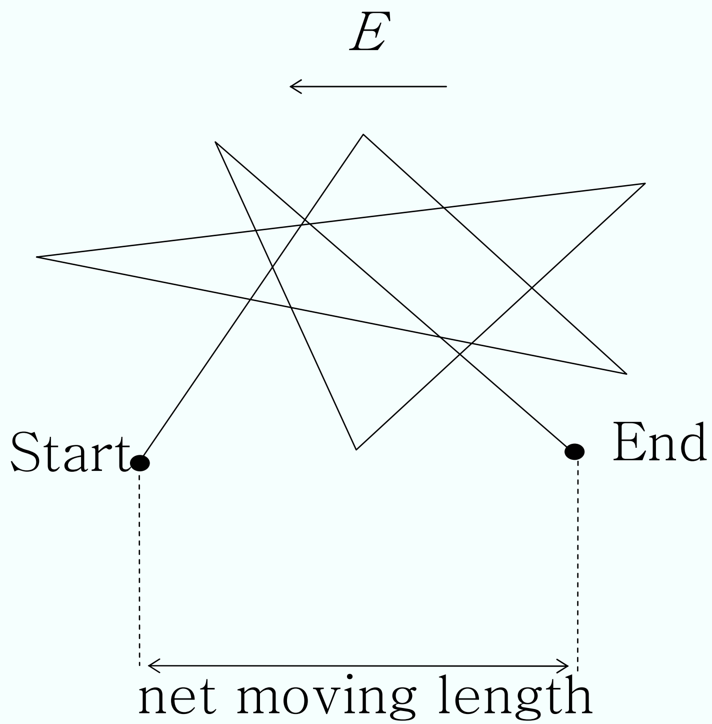
6.2.1 thermal noise of resistor

Thermal energy of an electron : *k T*

➔ Random motion



$$\frac{1}{2} \cdot m \cdot v_{th}^2 = \frac{3}{2} \cdot kT$$

Fig 6.2.1 Motion of an electron inside a resistor

Noise of resistor

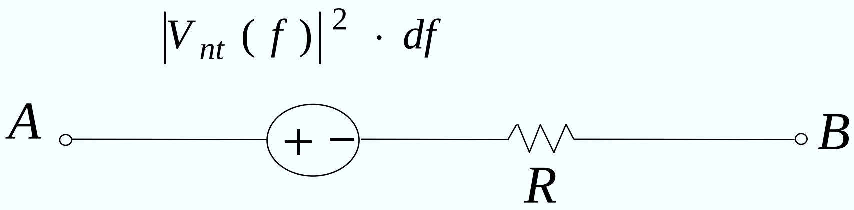
$$\overline{v_{nt}^2} \triangleq \lim_{T\to\infty} \frac{1}{T} \cdot \int_0^T \{v_{nt}(t)\}^2 \cdot dt = \int_0^\infty |V_{nt}(f)|^2 \cdot df$$

$$|V_{nt}(f)|^2 = 4kTR$$

1-sided PSD

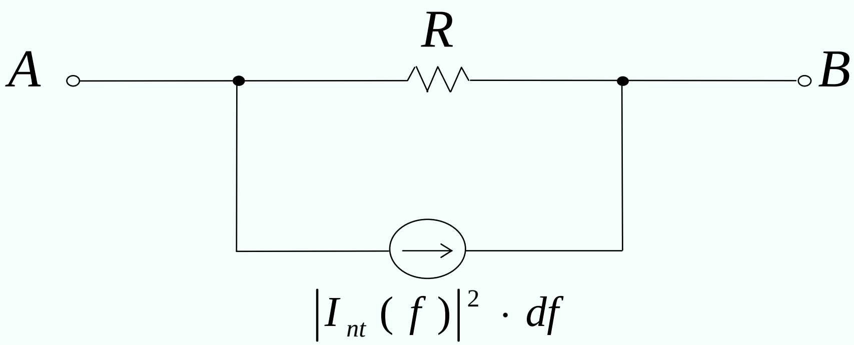
$$|I_{nt}(f)|^2 = \frac{|V_{nt}(f)|^2}{R^2} = \frac{4kT}{R}$$

Noise of resistor



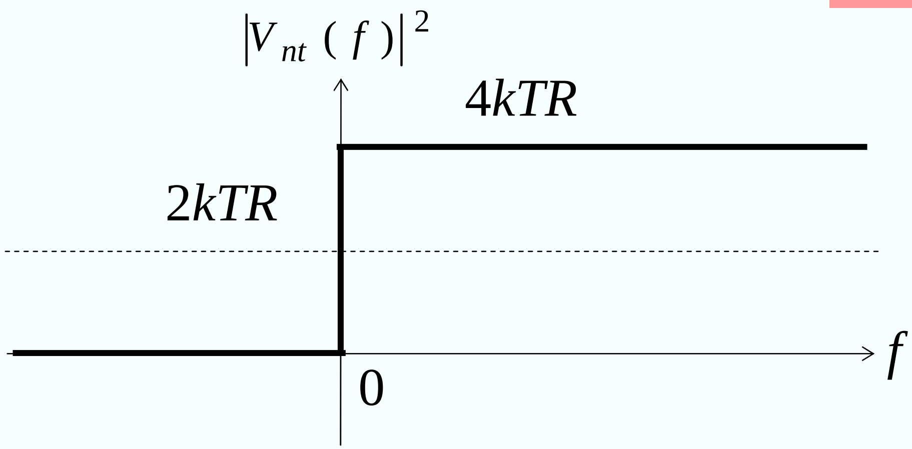
(a) Thevenin form

$$|V_{nt}(f)|^2 = 4kTR$$



(b) Norton form

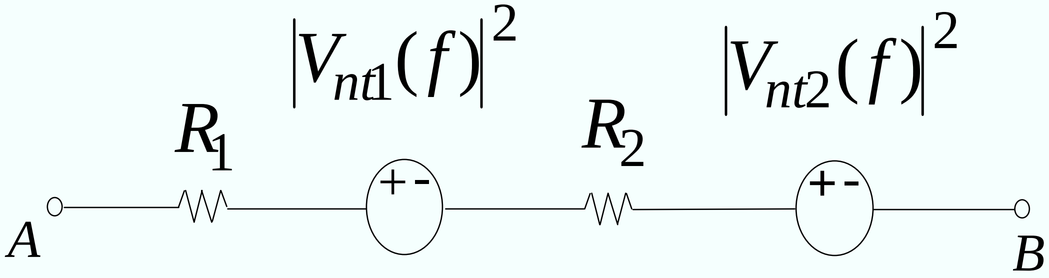
$$|I_{nt}(f)|^2 = \frac{|V_{nt}(f)|^2}{R^2} = \frac{4kT}{R}$$

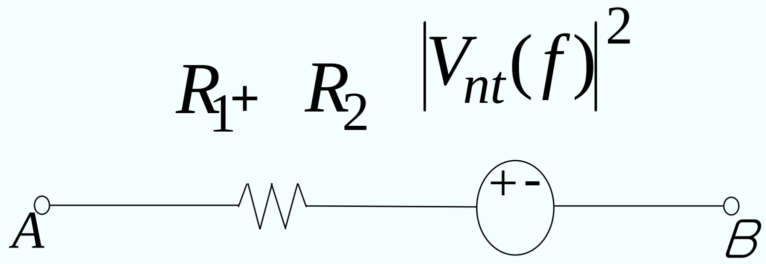


(c) Frequency spectrum of thermal noise voltage variance

(Solid: one-sided PSD, $f > 0$ only, Dotted: two-sided PSD)

Noise of resistor





$$\overline{v_{nt}^2} = \lim_{T\to\infty} \frac{1}{T}\int_0^T (v_{nt1}(t) + v_{nt2}(t) - 0)^2 \cdot dt$$

$$\lim_{T\to\infty} \frac{1}{T}\int_0^T (v_{nt1}(t) \cdot v_{nt2}(t)) \cdot dt = 0$$ ← uncorrelated

$$\overline{v_{nt}^2} = \overline{v_{nt1}^2} + \overline{v_{nt2}^2}$$

$$|V_{nt}(f)|^2 = |V_{nt1}(f)|^2 + |V_{nt2}(f)|^2 = 4kT\cdot(R_1 + R_2)$$

6.2.2 kT/C noise

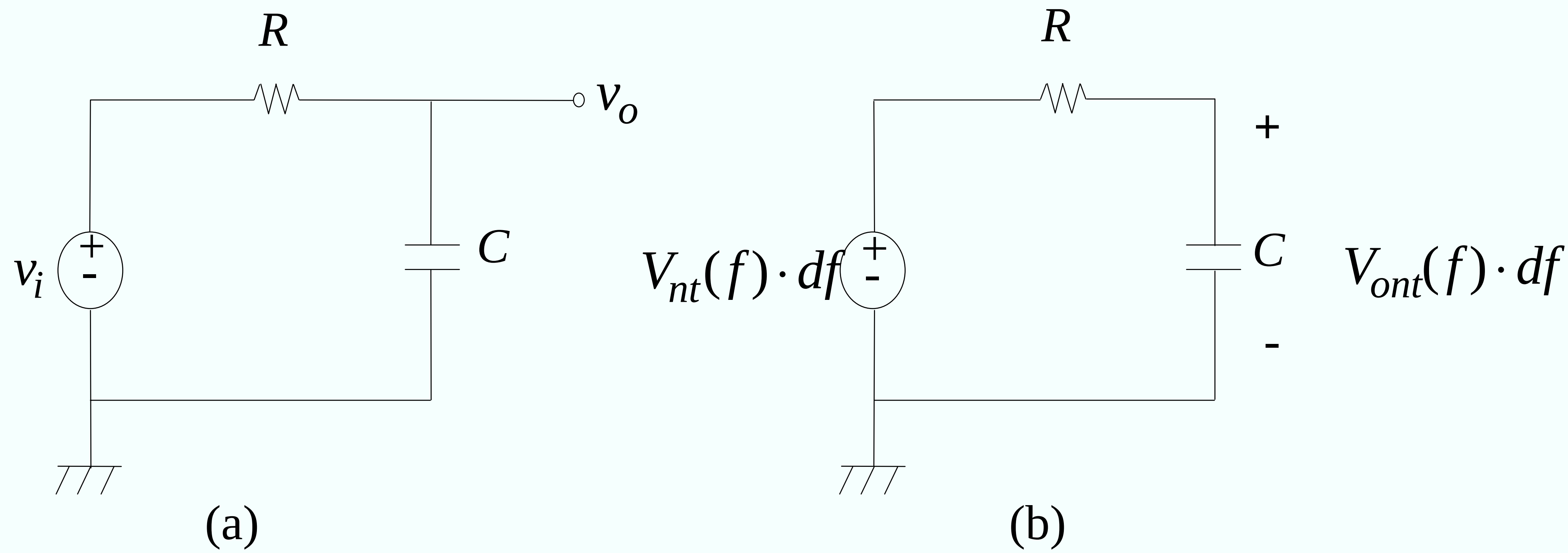
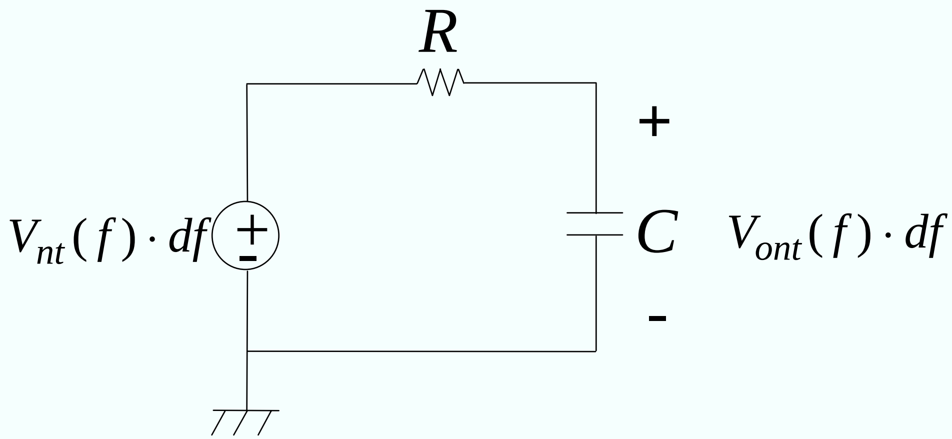


Fig **6.2.4 Output noise spectrum of** RC low pass filter

kT/C noise

$$H(f) = \frac{1}{1 + j \cdot \frac{f}{f_c}}$$



$$V_{ont}(f) = V_{nt}(f) \cdot H(f)$$

$$\overline{v_{ont}^2} = \int_0^\infty |V_{ont}(f)|^2 \, df = \int_0^\infty |H(f)|^2 \cdot |V_{nt}(f)|^2 \cdot df$$

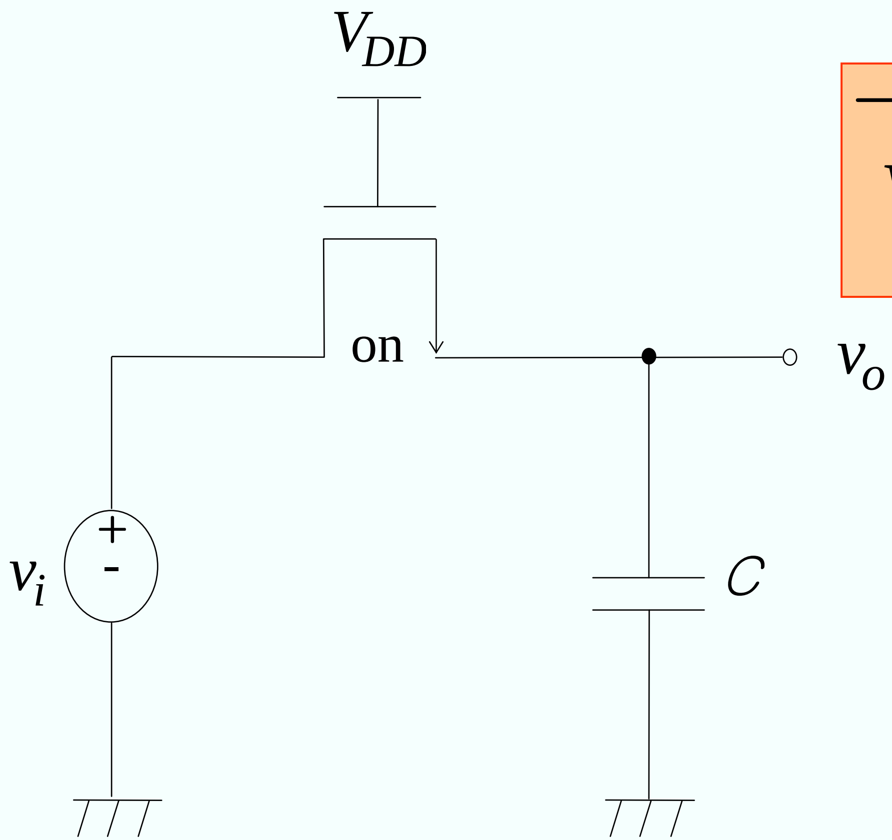
$$= 4kTR \cdot \int_0^\infty |H(f)|^2 \, df = 4kTR \cdot \int_0^\infty \frac{1}{1 + (f/f_c)^2} \, df$$

$$\overline{v_{ont}^2} = \frac{kT}{C}$$

$$\overline{v_{ont}} = \sqrt{\frac{kT}{C}}$$

N_e (Number of electrons) $= \sqrt{kTC}/q$

kT/C noise



$$\overline{v_{ont}^2} = \frac{kT}{C}$$

F i g **6.2.5** Sampling circuit using MOS switch

Noise bandwidth : *NBW*

$$\overline{v_{ont}^2} = \left|V_{ont}(0)\right|^2 \cdot NBW = 4kTR \cdot NBW = \frac{kT}{C}$$

$$NBW = \frac{\int_0^\infty \left|H(f)\right|^2 df}{\left|H(0)\right|^2} = \frac{\pi}{2} \cdot f_c$$

Single-pole

$$H(f) = \frac{1}{1 + j \cdot \dfrac{f}{f_c}}$$

6.2.3 Noise model of MOS transistor

MOSFET thermal noise

$$\left| I_{dnt}(f) \right|^2 = \frac{4kT}{R_{avg}}$$

← Drain noise current spectral density

$$R_{avg} = \frac{L}{W \cdot \sigma_{avg}}$$

← Average channel resistance

$$\sigma_{avg} = \frac{1}{L} \cdot \int_0^L \sigma(y)dy = \frac{1}{L} \cdot \int_0^L \mu_n \cdot q_N(y)dy$$

$$q_N(y) = C_{ox} \cdot (V_{GS} - V_{TH}) \cdot \sqrt{1-(y/L)}$$

MOSFET noise

$$\sigma_{avg} = \frac{1}{L} \cdot \int_0^L \sigma(y)dy = \frac{1}{L} \cdot \int_0^L \mu_n \cdot q_N(y)dy$$

$$\sigma_{avg} = \frac{2}{3} \cdot \mu_n C_{ox} \cdot (V_{GS} - V_{TH}) \qquad R_{avg} = \frac{3}{2} \cdot \frac{L}{W} \cdot \frac{1}{\mu_n C_{ox}(V_{GS} - V_{TH})}$$

$$|I_{dnt}(f)|^2 = \frac{4kT}{R_{avg}}$$

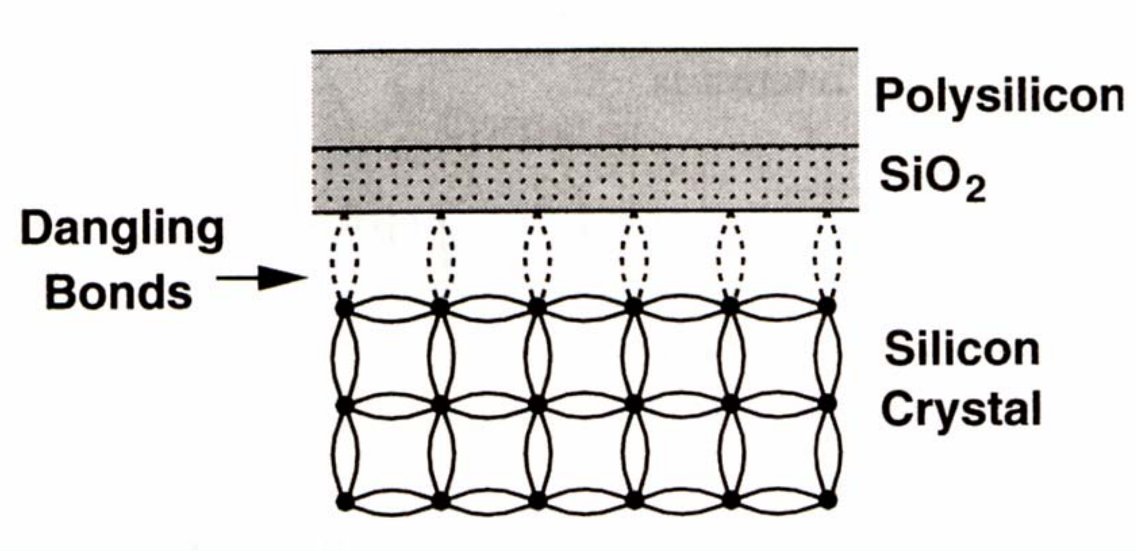
$$|I_{dnt}(f)|^2 = \left(\frac{8kT}{3}\right) \cdot \mu_n C_{ox} \cdot \frac{W}{L} \cdot (V_{GS} - V_{TH})$$

$$|I_{dnt}(f)|^2 = \left(\frac{8kT}{3}\right) \cdot g_m = 4kT \cdot \frac{2}{3} \cdot g_m$$

Equiv noise R = 1.5/gm

Short channel: 2/3 ➔ 2.5 at 0.25um

MOSFET flicker noise



$$\left|I_{dn}(f)\right|^2 = \left|I_{dnt}(f)\right|^2 + \left|I_{dnf}(f)\right|^2$$

$$\left|I_{dn}(f)\right|^2 = \frac{8kT}{3} \cdot g_m + \frac{KF \cdot I_D^{AF}}{f}$$

MOSFET flicker noise models

$$\left|I_{dn}(f)\right|^2 = \frac{8kT}{3} \cdot g_m + \frac{KF \cdot I_D^{AF}}{f}$$

$$\left|I_{dn}(f)\right|^2 = \frac{8kT}{3} \cdot g_m + \frac{K}{W\,L\,C_{ox} \cdot f} \cdot g_m^2$$

MOSFET noise

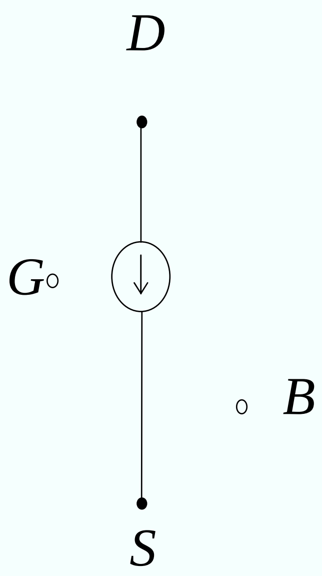
$$\overline{i_{dnf}^{2}(f)} = \frac{KF \cdot I_D^{AF}}{WLC_{ox} \cdot f^{EF}}$$ Flicker noise

$$\overline{i_{dnt}^{2}(f)} = \frac{8}{3} \cdot kT \cdot (g_m + g_{mb} + g_{ds})$$ Thermal noise

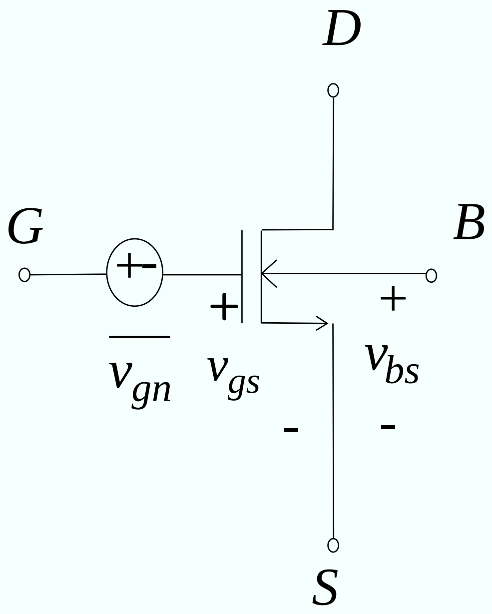
Example of noise model parameter values

	NMOS	PMOS
KF	3.76×10^{-25}	7.58×10^{-25}
AF	9.70×10^{-1}	1.67
EF	1.31	1.12

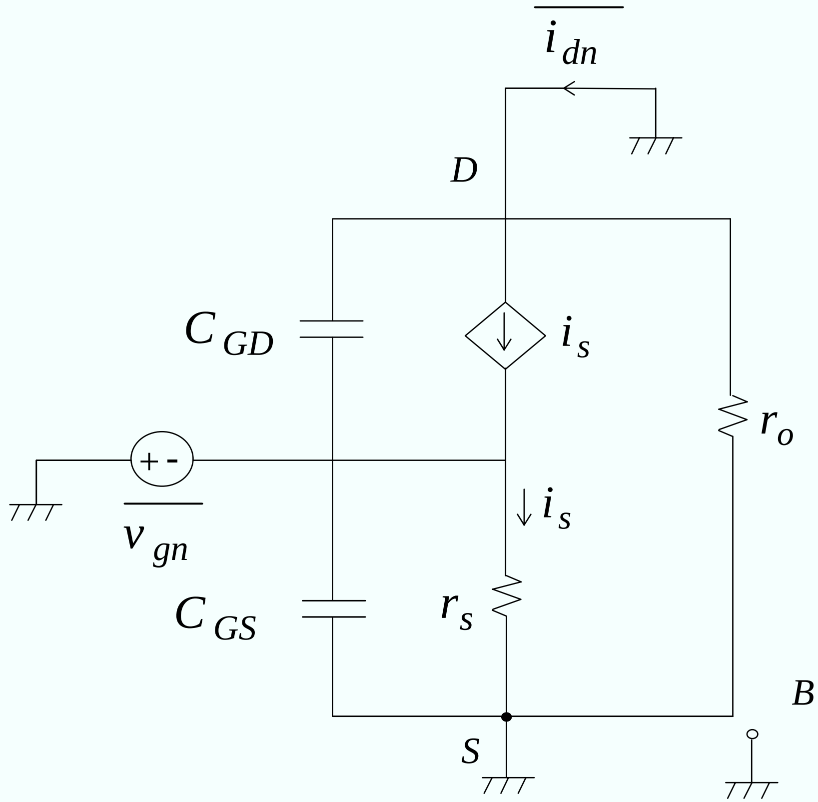
Equivalent noise voltage of MOSFET



(a)



(b)



$$\overline{i_{dn}} = -(g_m + j2\pi f \cdot C_{GD}) \cdot \overline{v_{gn}}$$

$$\overline{i_{dn}} \approx -g_m \cdot \overline{v_{gn}}$$

$$\overline{v_{gn}^2} = \frac{\overline{i_{dn}^2}}{g_m^2}$$

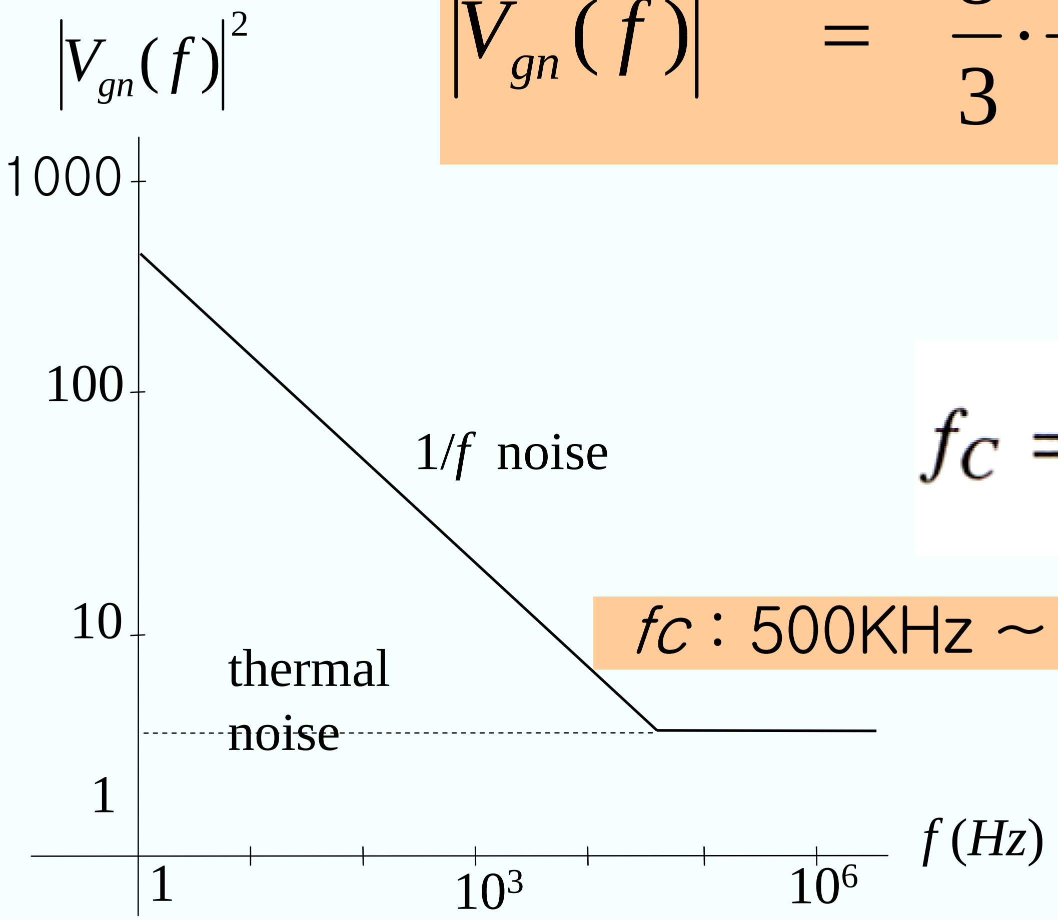
$$\overline{v_{gn}^2} = \frac{\overline{i_{dn}^2}}{g_m^2}$$

$$\overline{i_{dn}} = \sqrt{\overline{i_{dn}^2}} = \sqrt{\int_0^\infty |I_{dn}(f)|^2\, df}$$

$$|V_{gn}(f)|^2 = \frac{|I_{dn}(f)|^2}{g_m^2} = \frac{8}{3} \cdot \frac{kT}{g_m} + \frac{KF \cdot I_D^{AF}}{f \cdot g_m^2}$$

$$|V_{gn}(f)|^2 = \frac{8}{3} \cdot \frac{kT}{g_m} + \frac{K}{W\, L\, C_{ox} \cdot f}$$

$$|V_{gn}(f)|^2 = \frac{8}{3}\cdot\frac{kT}{g_m} + \frac{K}{WLC_{ox}\cdot f}$$

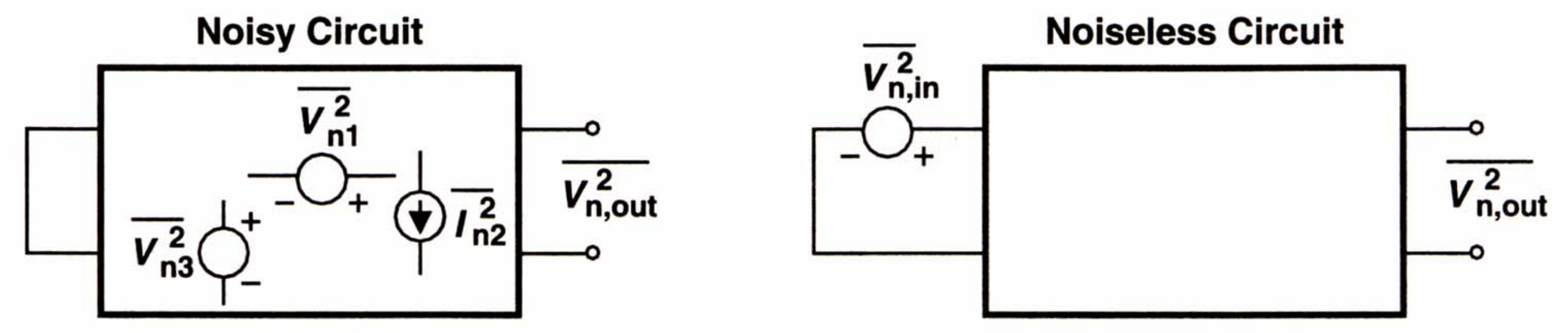


$$f_C = \frac{K}{C_{ox}WL} g_m \frac{3}{8kT}$$

fc : 500KHz ~ 1MHz for submicron

Fig. **6.2.7** MOSFET 1/*f* noise spectrum

Equivalent input noise voltage



Equivalent input referred noise voltage determines

Min input voltage that can be amplified
(min detectable input voltage)

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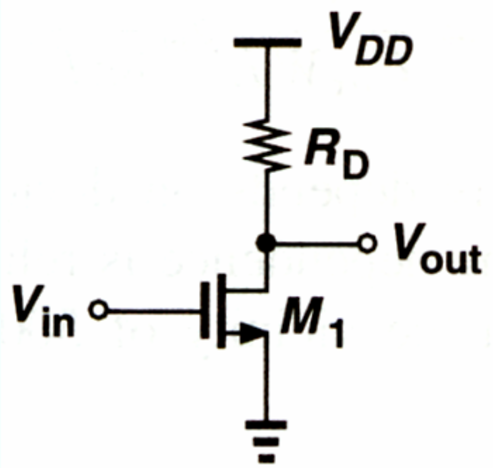
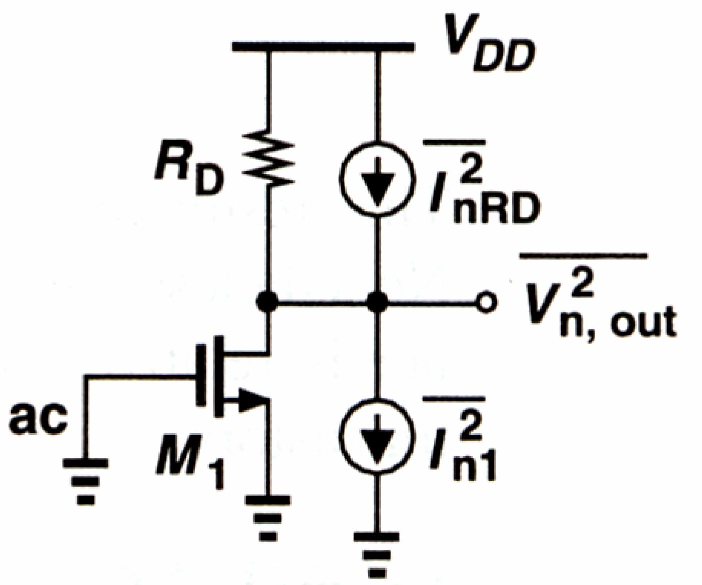
6.2.4 Input referred noise voltage of single stage amp



$$\overline{V_{n,out}^2} = \left(4kT\frac{2}{3}g_m + \frac{K}{C_{ox}WL}\cdot\frac{1}{f}\cdot g_m^2 + \frac{4kT}{R_D}\right)R_D^2$$



$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{A_v^2}$$

$$= \left(4kT\frac{2}{3}g_m + \frac{K}{C_{ox}WL}\cdot\frac{1}{f}\cdot g_m^2 + \frac{4kT}{R_D}\right)R_D^2\frac{1}{g_m^2R_D^2}$$

$$= 4kT\frac{2}{3g_m} + \frac{K}{C_{ox}WL}\cdot\frac{1}{f} + \frac{4kT}{g_m^2R_D}.$$

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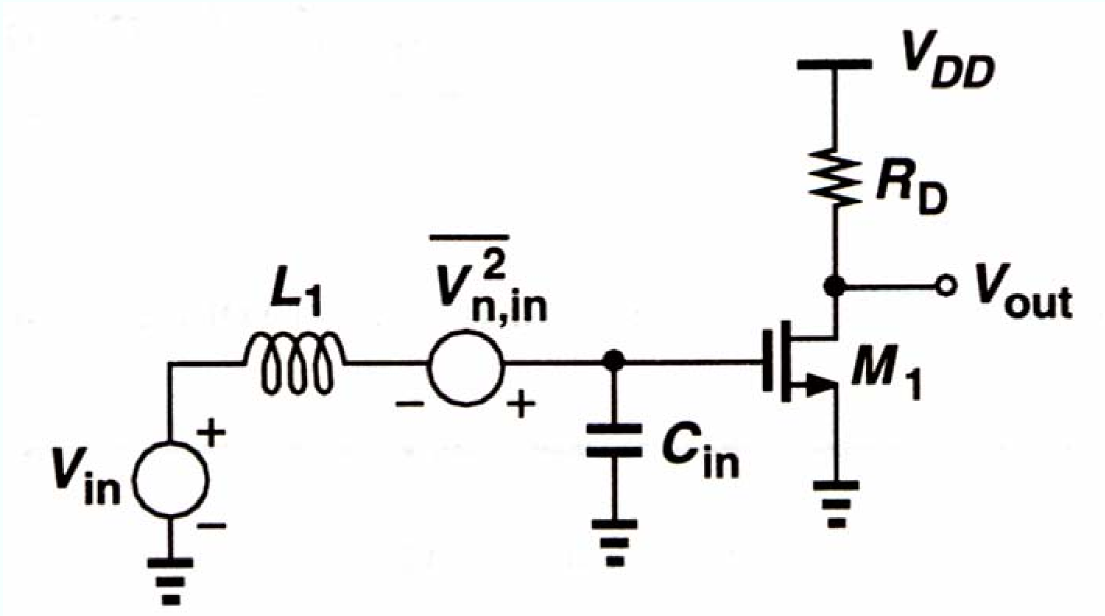
Equivalent input noise voltage

If only the noise voltage is attached at input,
for an infinite source impedance ➔ output noise = 0 : problem!



Source Impedance Zs = infinity (current source signal input)
➔
Noise output = 0 even with a non-zero input noise voltage $V_{n,in}$

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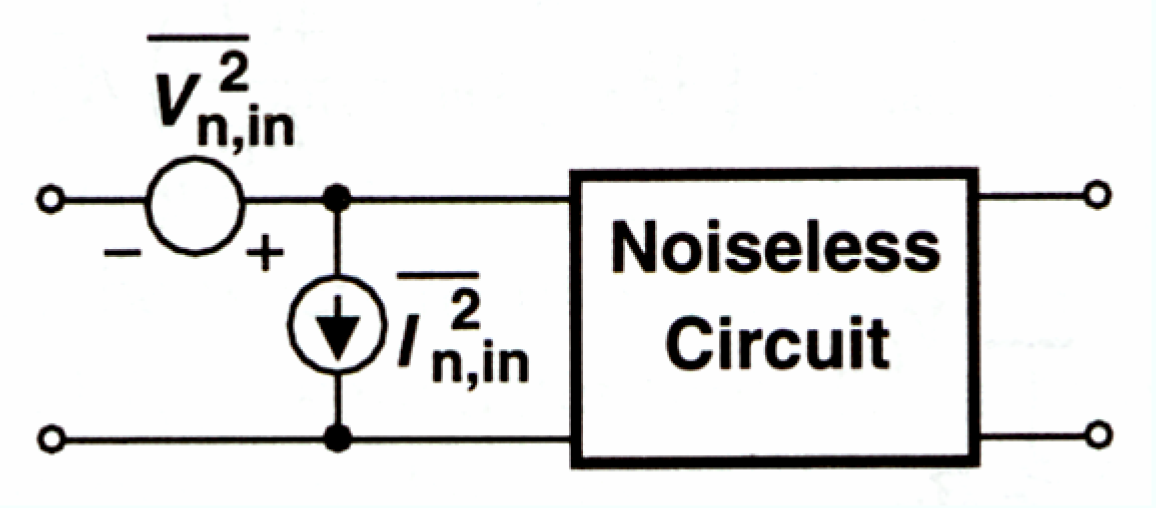
Equivalent input noise voltage

For a general case of source impedance, both a noise voltage and a noise current must be used.



$$I_{n.in} \quad and \quad V_{n.in} \quad : \quad correlated$$

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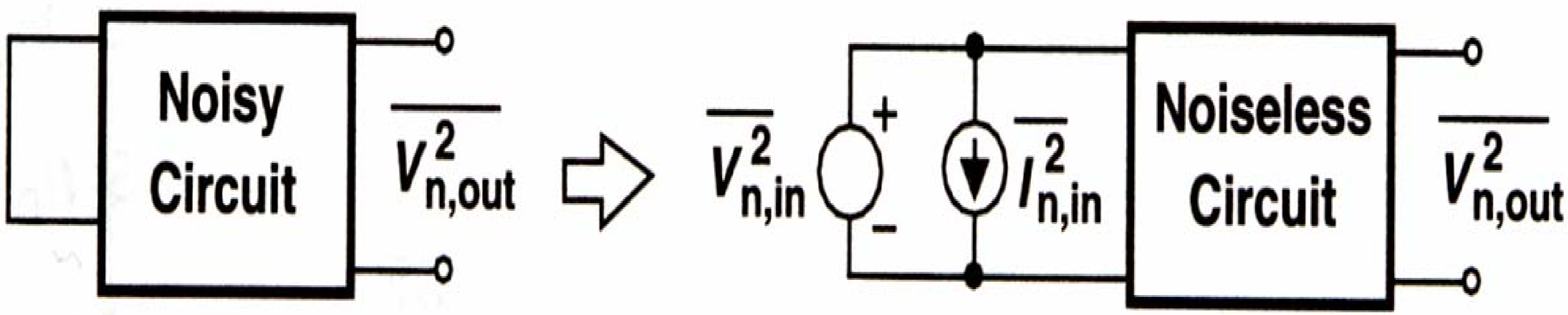
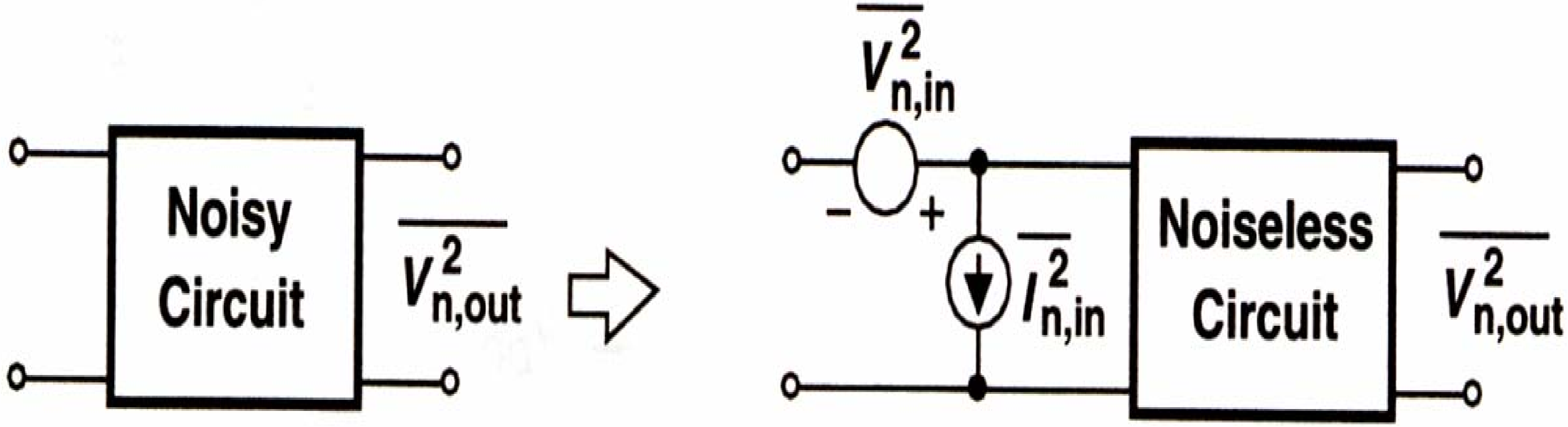
Equivalent input noise voltage



$$V_{n.in} \quad can\ be\ calculated\ with \quad Z_S = 0$$



$$I_{n.in}\ can\ be\ calculated\ with\ Z_S = \infty$$

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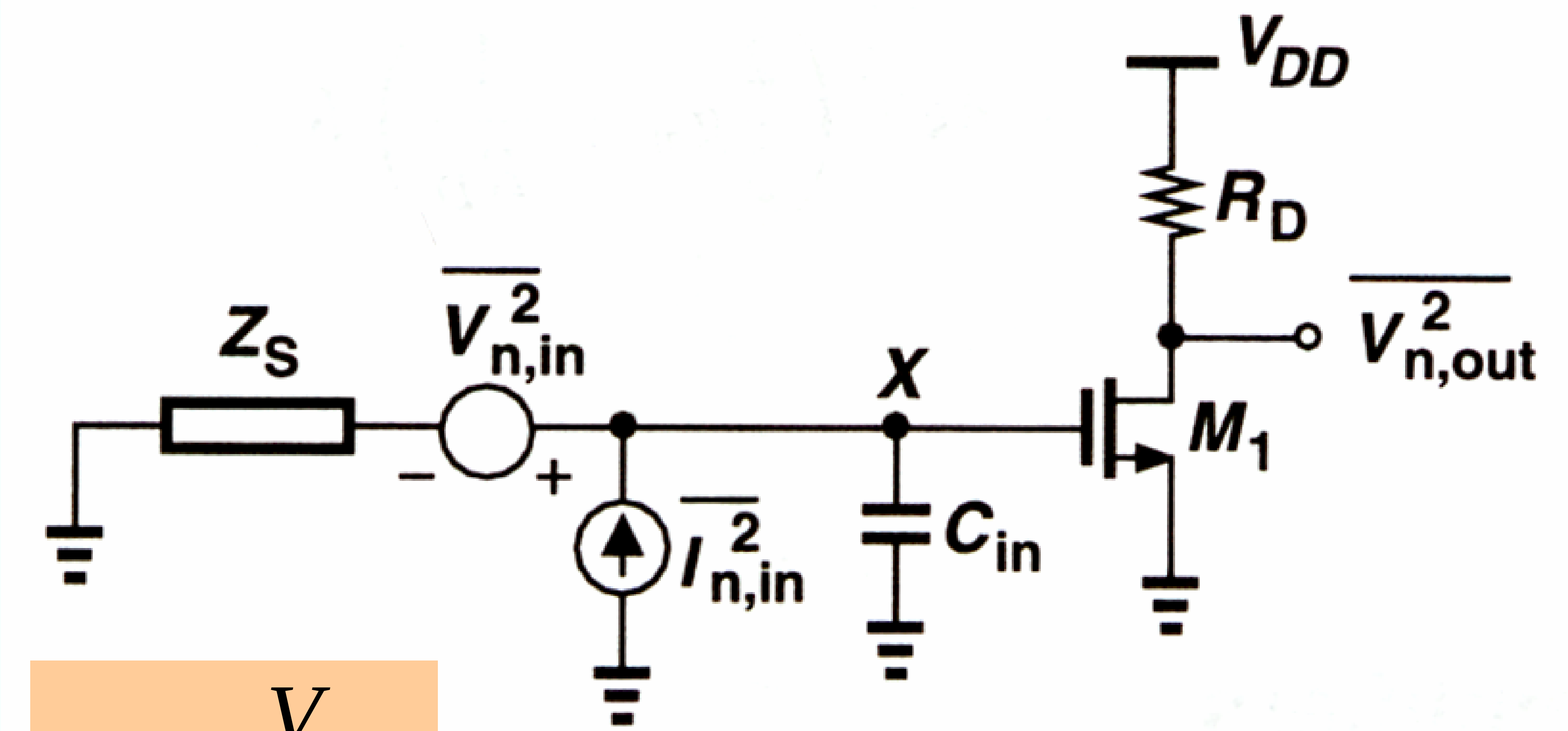
Equivalent input noise voltage



$$I_{n.in} = \frac{V_{n.in}}{Z_{in}}$$

$I_{n.in}$ $V_{n.in}$: *correlated*

Holds for CS amp

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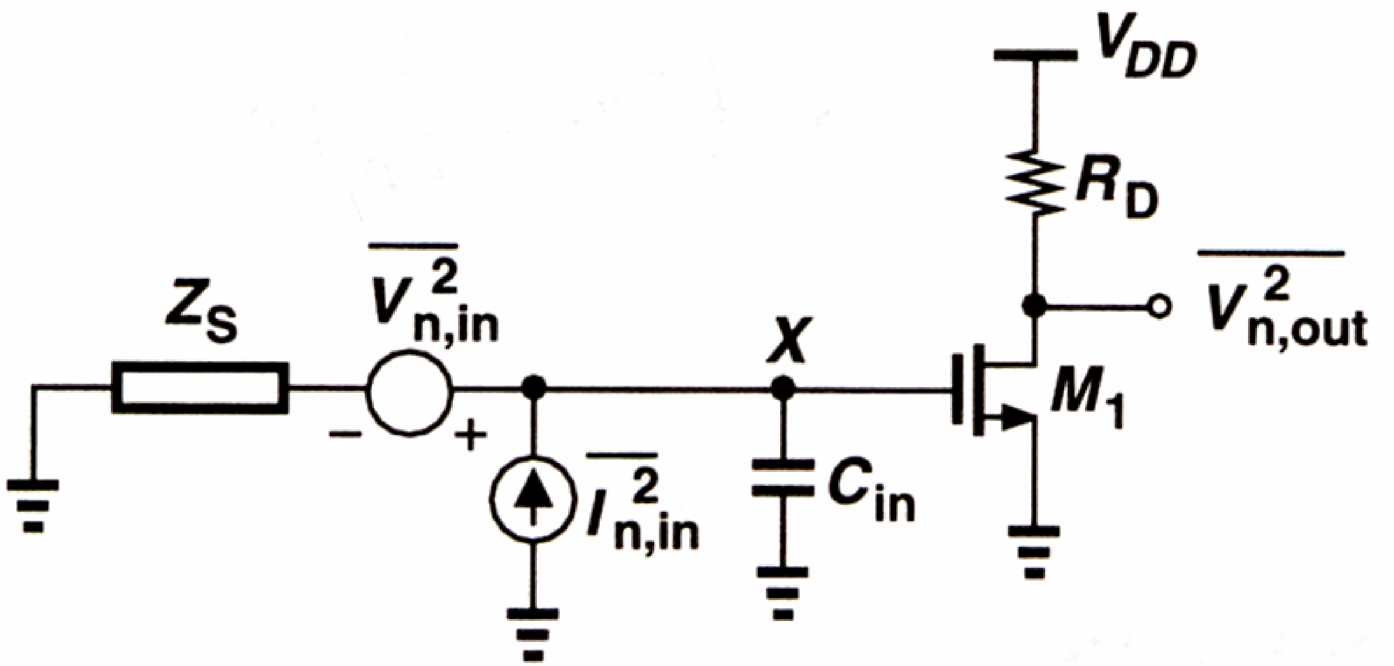
Equivalent input noise voltage



$$\overline{V^2_{n,in}} = \frac{\overline{V^2_{n,out}}}{A_v^2}$$

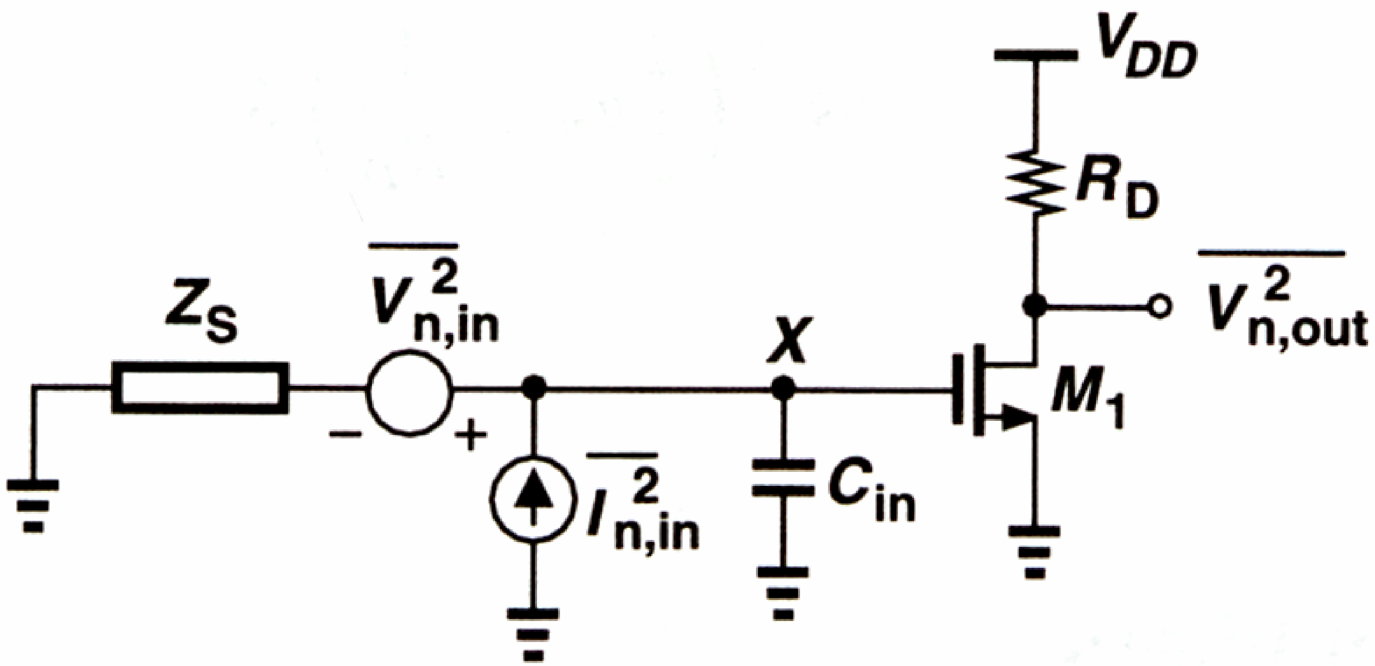
$$= \left(4kT\frac{2}{3}g_m + \frac{K}{C_{ox}WL}\cdot\frac{1}{f}\cdot g_m^2 + \frac{4kT}{R_D}\right)R_D^2\frac{1}{g_m^2R_D^2}$$

$$= 4kT\frac{2}{3g_m} + \frac{K}{C_{ox}WL}\cdot\frac{1}{f} + \frac{4kT}{g_m^2R_D}.$$

$V_{n.in}$ *can be calculated with* $Z_S = 0$

Razavi

Equivalent input noise voltage



$I_{n.in}$ *can be calculated with* $Z_S = \infty$

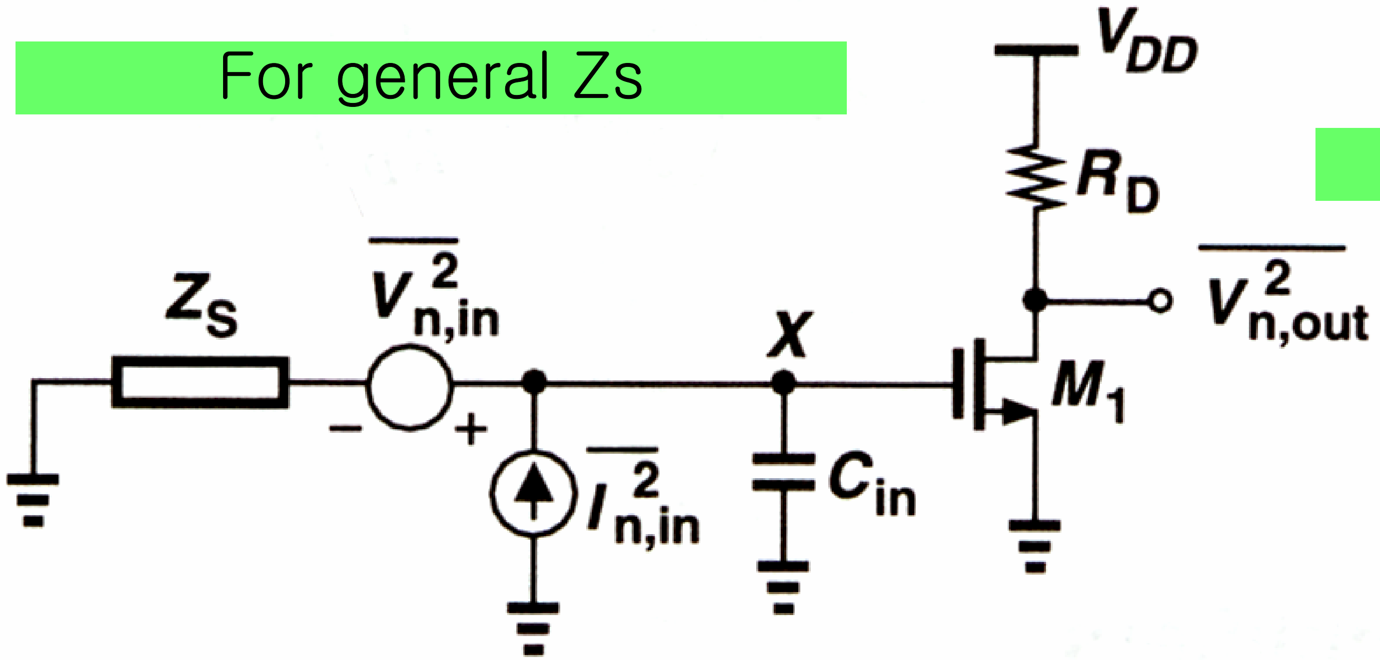
$$I_{n.in} = \frac{V_{n.in}}{Z_{in}} = sC_{in} \cdot V_{n.in}$$

$V_{n.X} = V_{n.in}$ *as desired*

gives the same Vn.out

Equivalent input noise voltage

For general Zs



$V_{n.in}$ $I_{n.in}$

Not double counted

$$V_{n,X} = V_{n,in}\frac{\frac{1}{C_{in}s}}{\frac{1}{C_{in}s}+Z_S} + I_{n,in}\frac{\frac{Z_S}{C_{in}s}}{\frac{1}{C_{in}s}+Z_S}$$

$$= \frac{V_{n,in} + I_{n,in}Z_S}{Z_S C_{in}s + 1}.$$

$$I_{n.in} = \frac{V_{n.in}}{Z_{in}} = sC_{in}\cdot V_{n.in}$$

$V_{n.X} = V_{n.in}$ *as desired*

gives the same Vn.out

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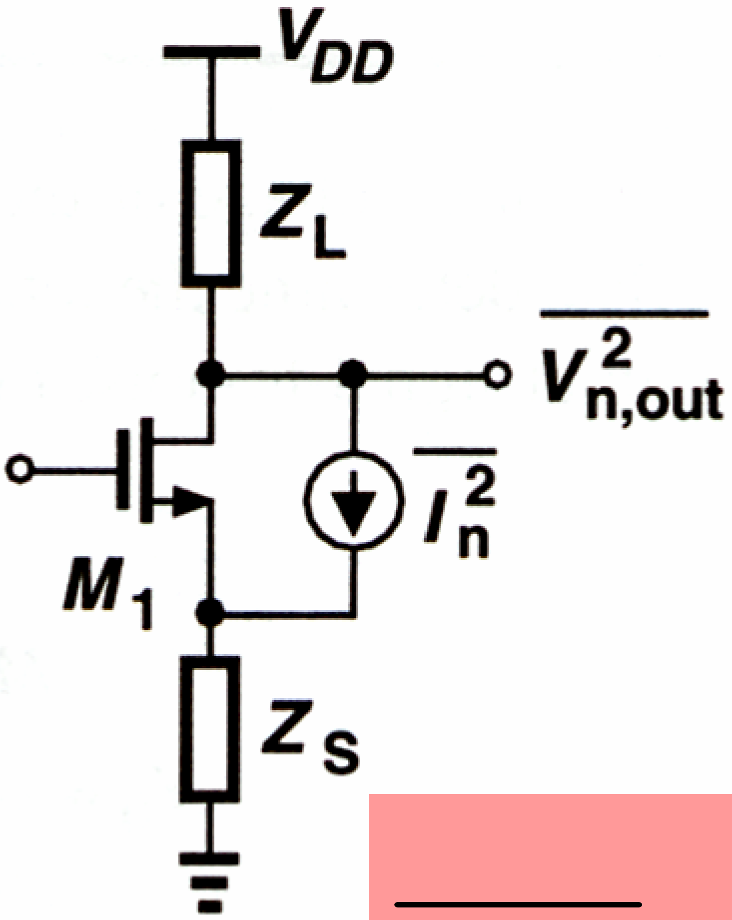
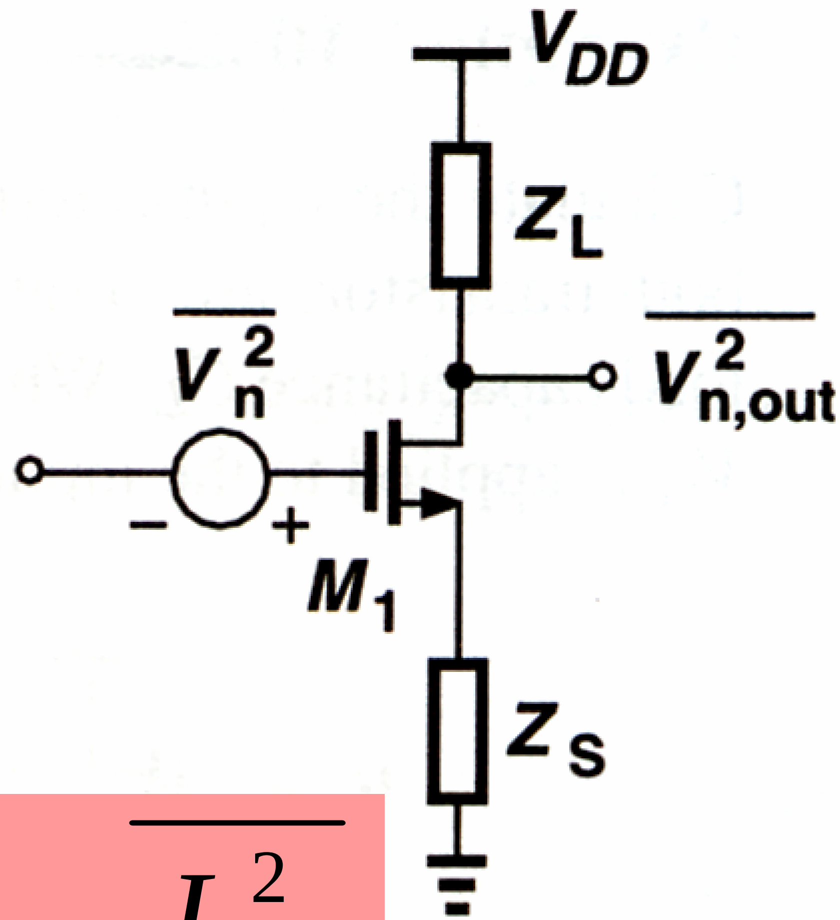
Equivalent input noise voltage





$$\overline{V_n^2} = \frac{\overline{I_n^2}}{g_m^2}$$

At low frequency

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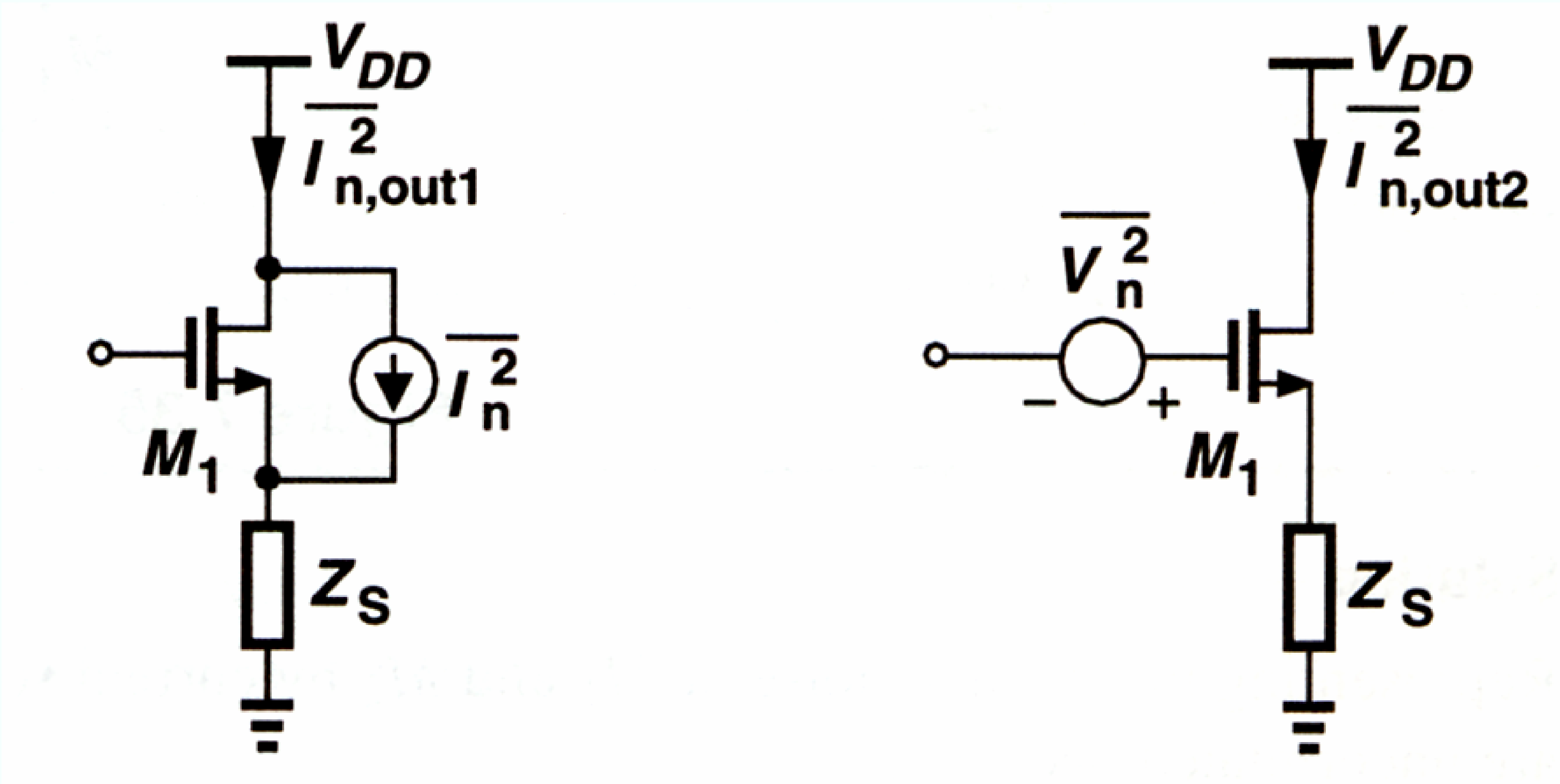
Equivalent input noise voltage



Proof of $\overline{V_n^2} = \dfrac{\overline{I_n^2}}{g_m^2}$ At low frequency

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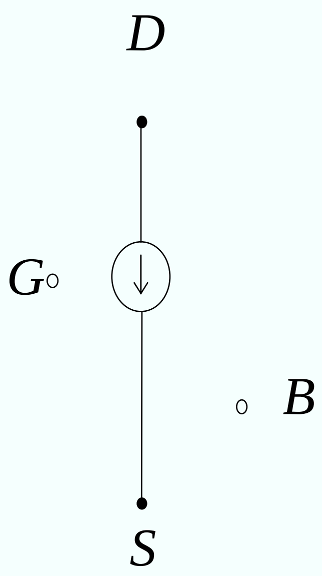
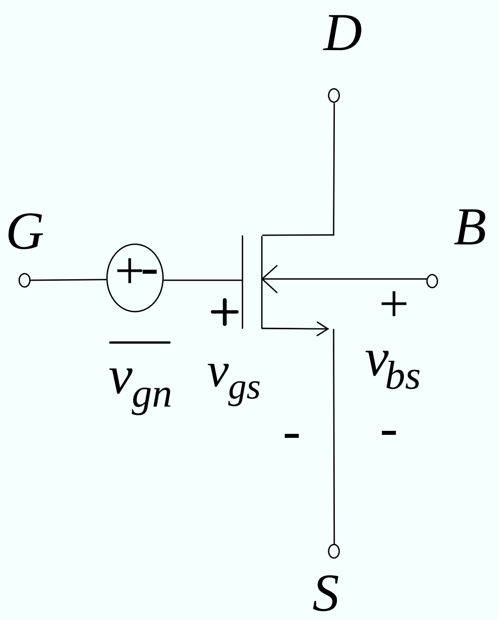
Equivalent input noise voltage

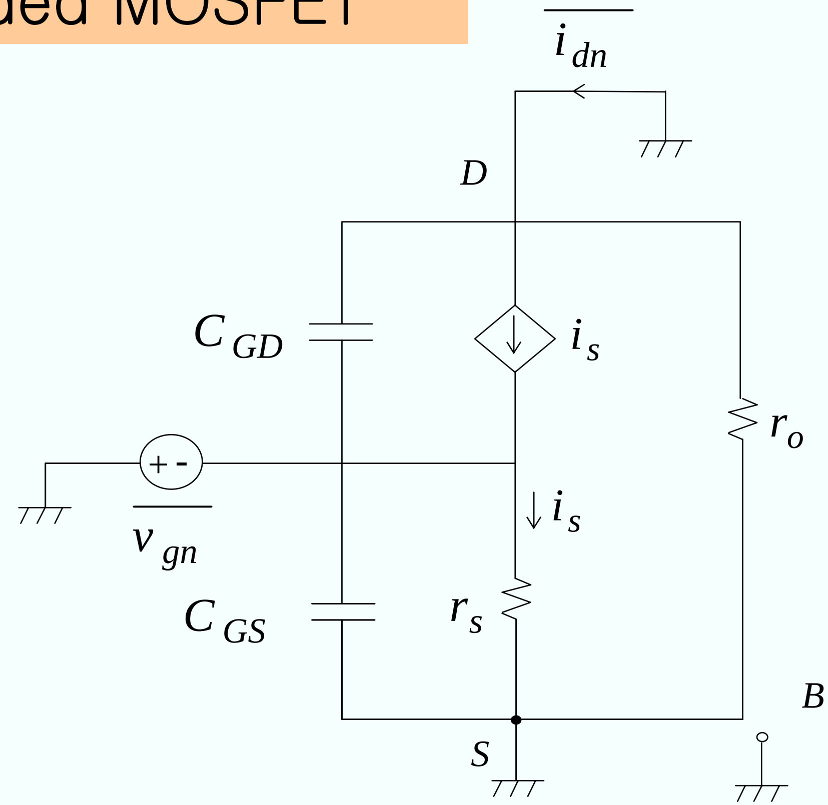
Simple proof for a source grounded MOSFET



(a)



(b)



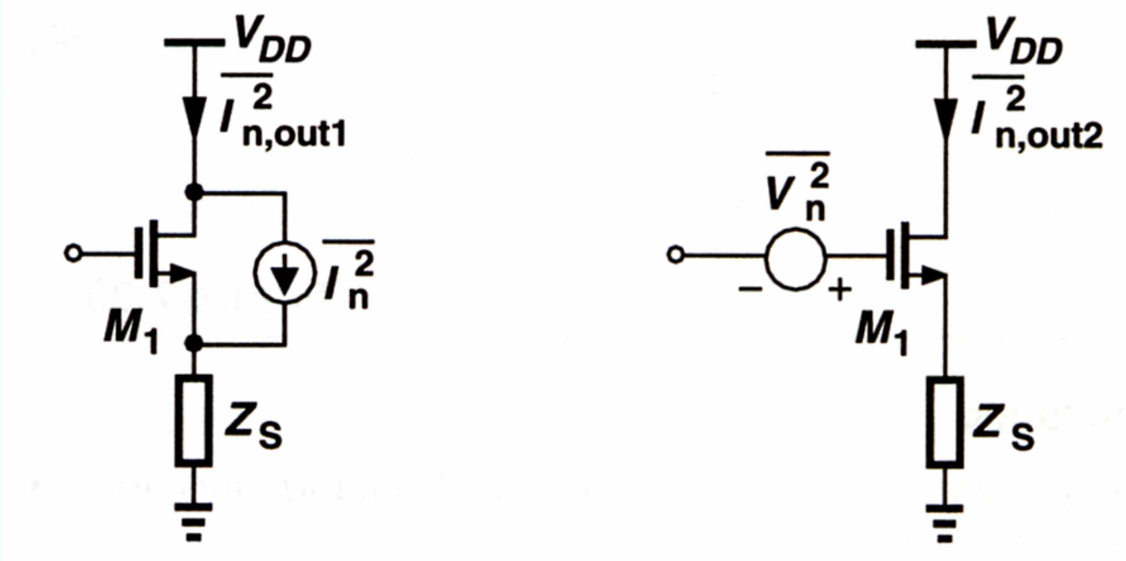
$$\overline{i_{dn}} = -(g_m + j2\pi f \cdot C_{GD}) \cdot \overline{v_{gn}}$$

$$\overline{i_{dn}} \approx -g_m \cdot \overline{v_{gn}}$$

$$\overline{v_{gn}^2} = \frac{\overline{i_{dn}^2}}{g_m^2}$$

Razavi

Equivalent input noise voltage



Proof of $\overline{V_n^2} = \dfrac{\overline{I_n^2}}{g_m^2}$

At low frequency,

$g_{mb}=0$

$Vg = 0$

$$I_{n,out1} = \frac{I_n}{Z_S(g_m + 1/r_O) + 1}$$

$$I_{n,out2} = \frac{g_m V_n}{Z_S(g_m + 1/r_O) + 1}$$

➔ $\overline{V_n^2} = \dfrac{\overline{I_n^2}}{g_m^2}$

$$I_{n.out2} = \frac{V_n}{\dfrac{1}{g_m} + \dfrac{1}{\dfrac{1}{Z_s} + \dfrac{1}{r_o}}} \times \frac{\dfrac{1}{Z_s}}{\dfrac{1}{Z_s} + \dfrac{1}{r_o}}$$

Razavi