

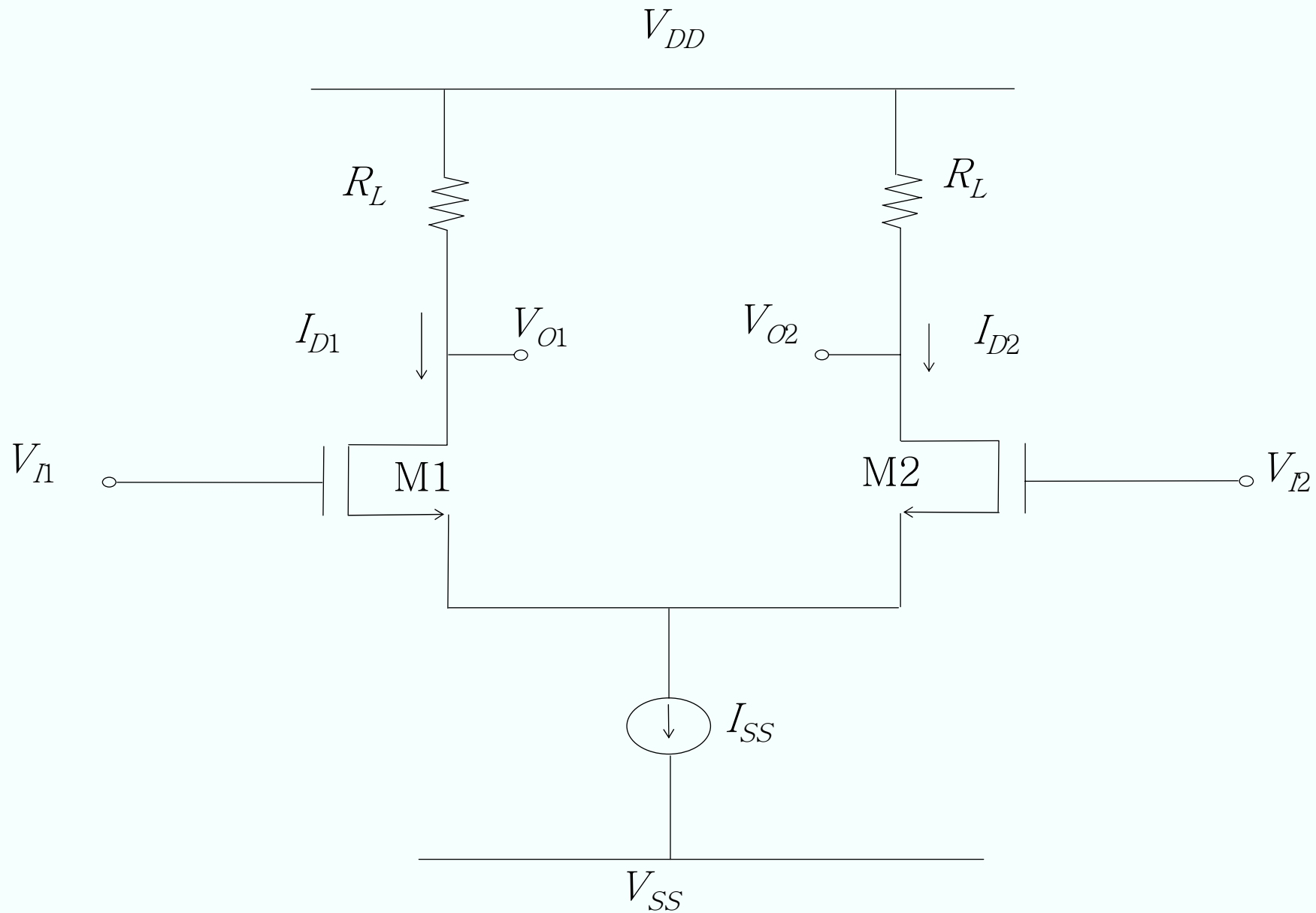
Differential pair with R load

- (1) Large signal transfer curve
- (2) Small signal characteristic (G_{md} , G_{mc} , R_o)
- (3) Active input common mode range
- (4) Input offset voltage
- (5) Common mode to differential conversion due to mismatches

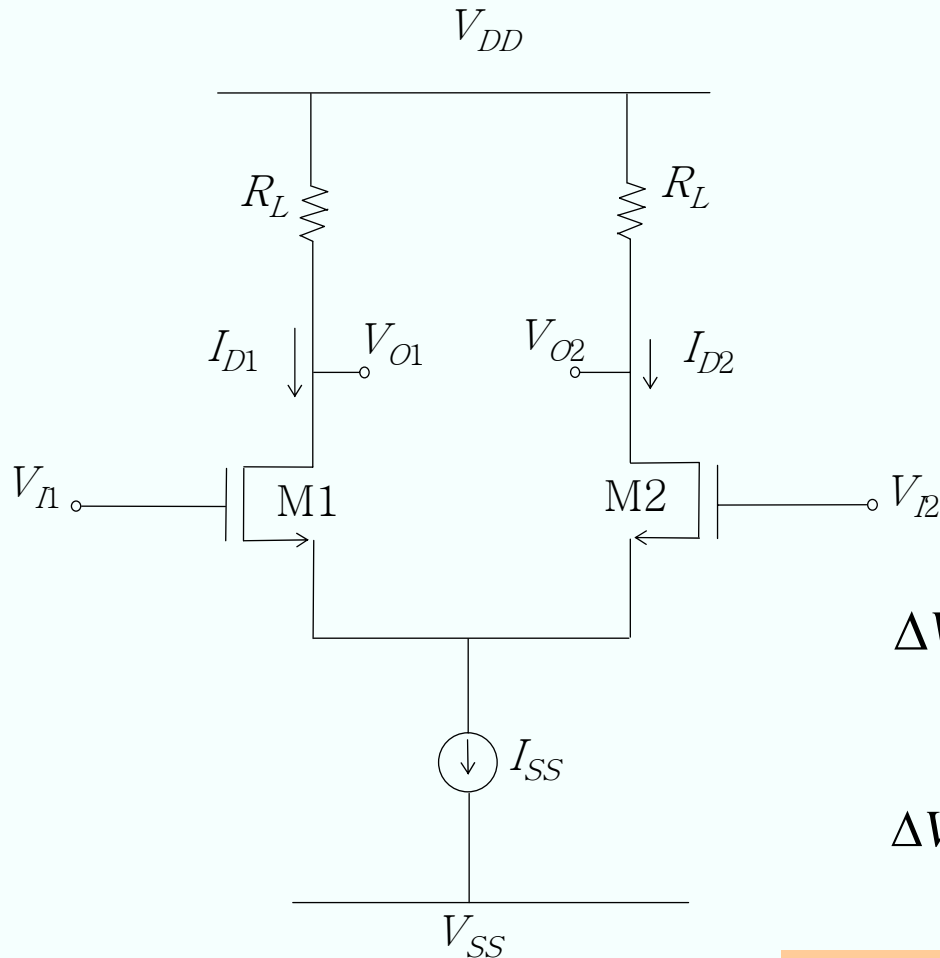
Differential pair with diode load

Differential pair with active load

Differential pair: frequency response



5.1.1 Large signal analysis



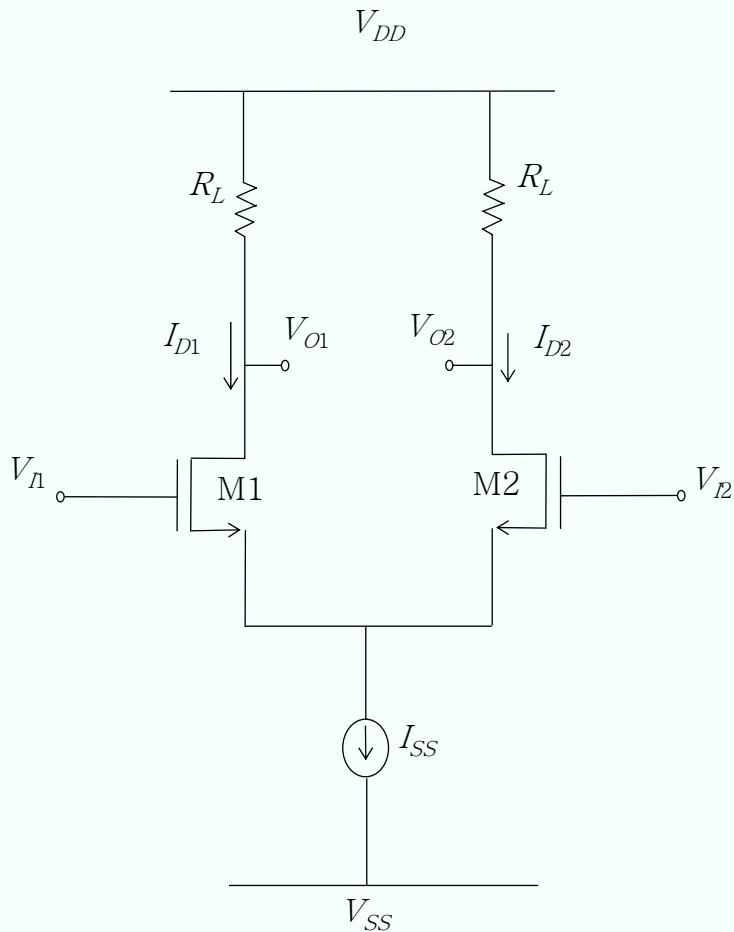
$$I_{D1} = \frac{1}{2} \beta \cdot (V_{GS1} - V_{TH})^2$$

$$I_{D2} = \frac{1}{2} \beta \cdot (V_{GS2} - V_{TH})^2$$

$$\Delta V_O = V_{O1} - V_{O2} = - (I_{D1} - I_{D2}) \cdot R_L$$

$$\Delta V_O = -\beta \cdot \Delta V_I \cdot \left\{ \frac{1}{2} (V_{GS1} + V_{GS2}) - V_{TH} \right\} \cdot R_L$$

$$\Delta V_O = -\beta \cdot \Delta V_I \cdot \{ V_{GS} - V_{TH} \} \cdot R_L$$



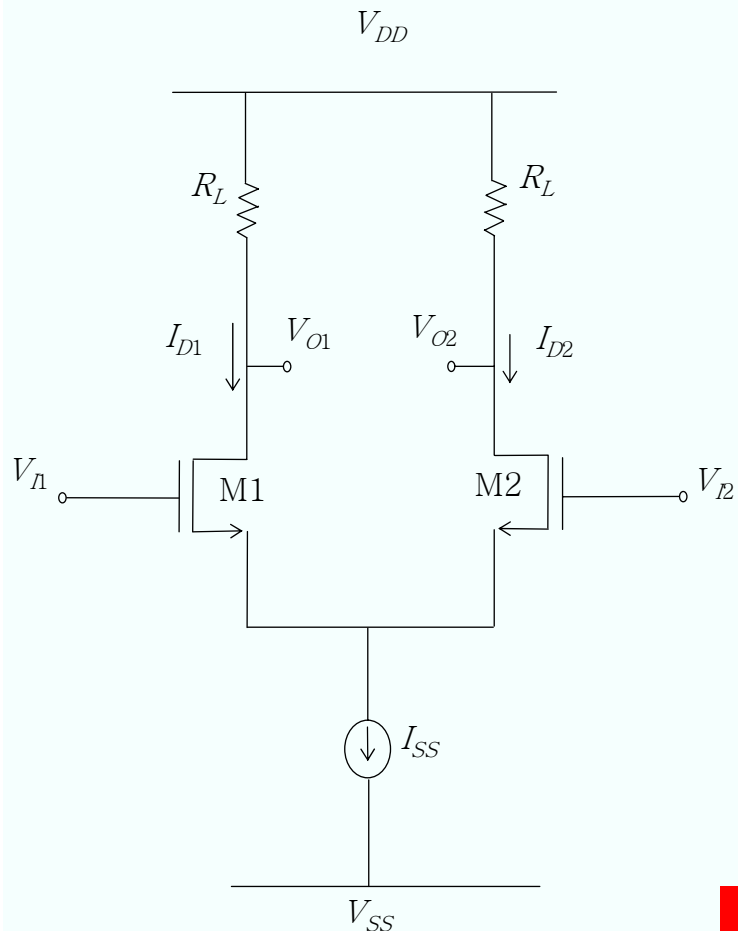
$$I_{D1} + I_{D2} = I_{SS}$$

$$(V_{GS1} - V_{TH})^2 + (V_{GS2} - V_{TH})^2 = \frac{2I_{SS}}{\beta}$$

$$V_{GS1} = V_{GS} + \frac{1}{2} \Delta V_I$$

$$V_{GS2} = V_{GS} - \frac{1}{2} \Delta V_I$$

$$V_{GS} - V_{TH} = \sqrt{\frac{I_{SS}}{\beta} - \frac{1}{4}(\Delta V_I)^2}$$



$$\Delta V_O = -\beta R_L \cdot \Delta V_I \cdot \sqrt{\frac{I_{SS}}{\beta} - \frac{(\Delta V_I)^2}{4}}$$

$$V_{GS1} - V_{TH} = (V_{GS} - V_{TH}) + \frac{\Delta V_I}{2} \geq 0$$

$$V_{GS2} - V_{TH} = (V_{GS} - V_{TH}) - \frac{\Delta V_I}{2} \geq 0$$

$$-\sqrt{\frac{2I_{SS}}{\beta}}$$

$$\leq$$

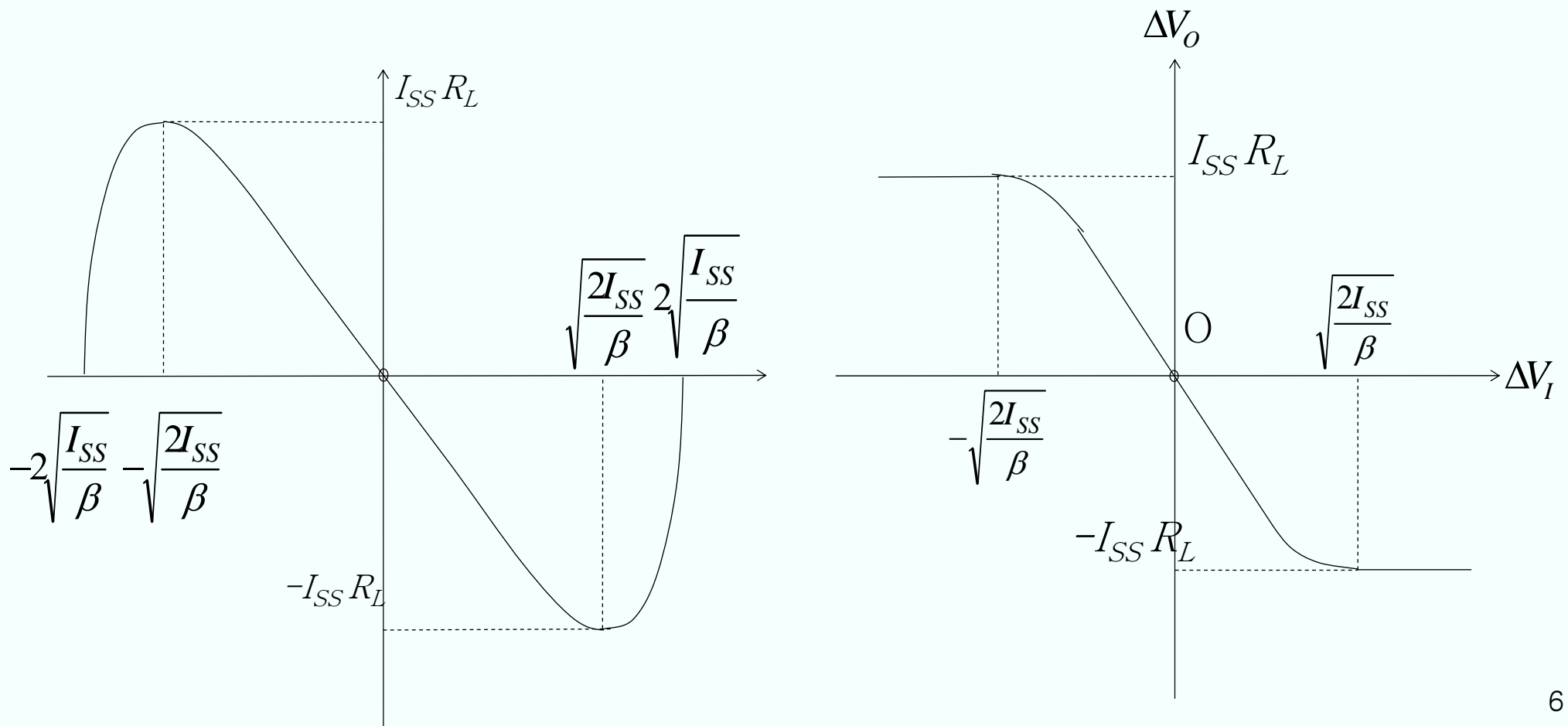
$$\Delta V_I$$

$$\leq$$

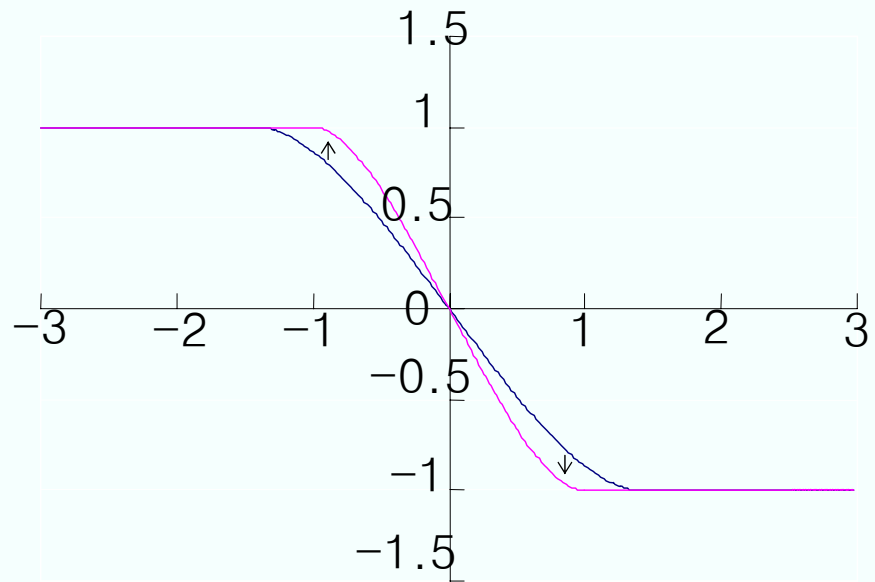
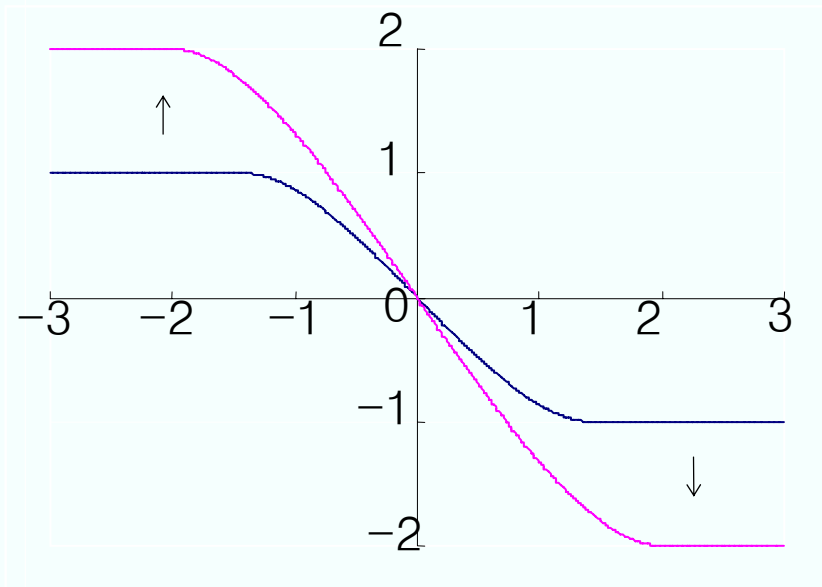
$$\sqrt{\frac{2I_{SS}}{\beta}}$$

$$\Delta V_I|_{\max} = \sqrt{\frac{2I_{SS}}{\beta}} = \sqrt{2} \cdot (V_{GS} - V_{TH})|_{\Delta V_I = 0}$$

$$\Delta V_O = -\beta R_L \cdot \Delta V_I \cdot \sqrt{\frac{I_{SS}}{\beta} - \frac{(\Delta V_I)^2}{4}}$$

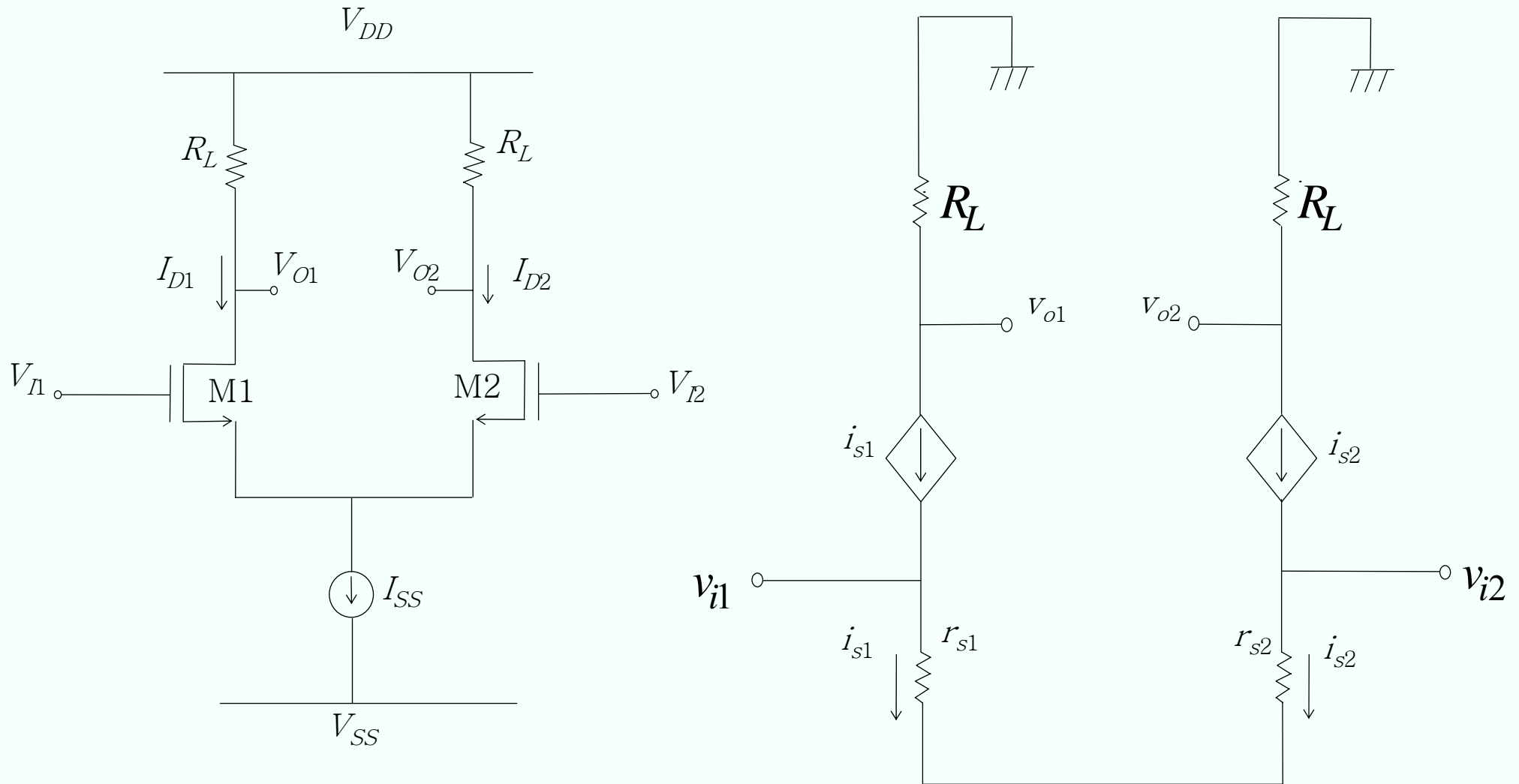


$$A_v = \left. \frac{\partial(V_{O1} - V_{O2})}{\partial(V_{I1} - V_{I2})} \right|_{\Delta V_I=0} = -R_L \cdot \sqrt{\beta I_{SS}} = -g_{m1} \cdot R_L$$

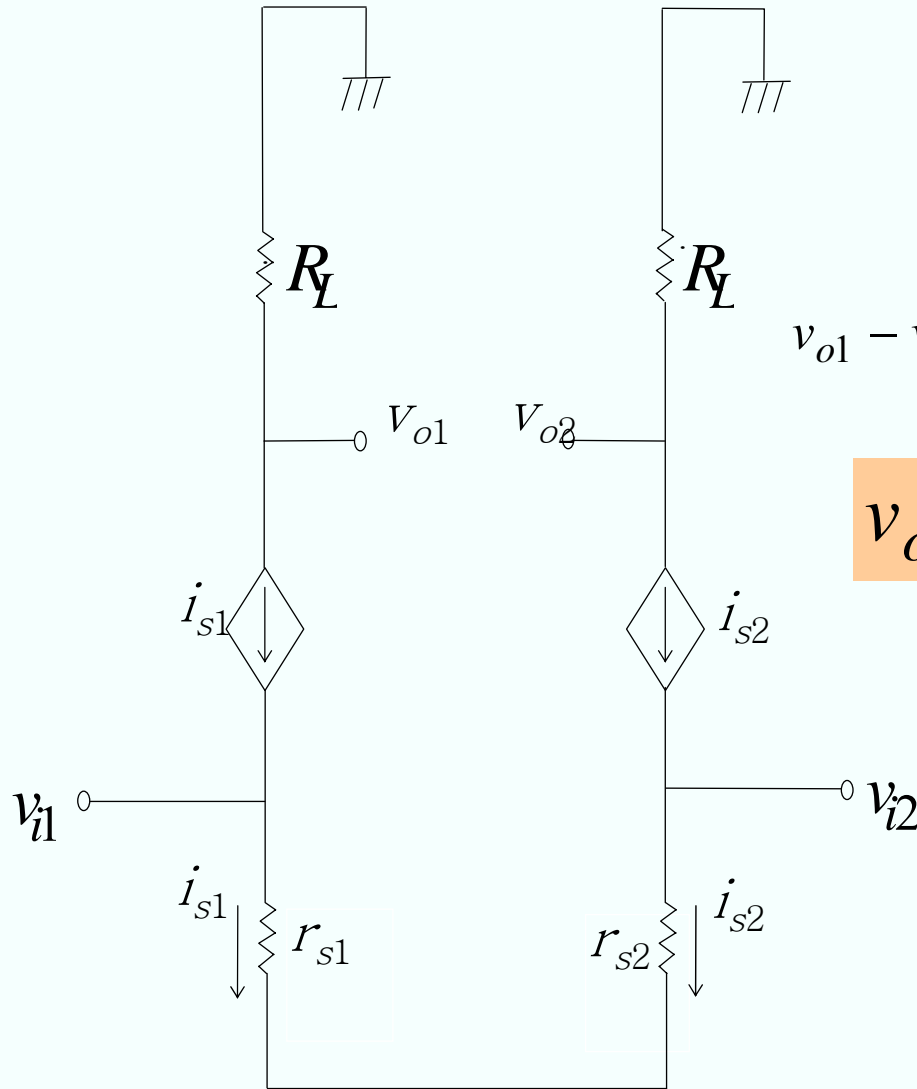
(a) β Inc, I_{SS} Fixed(b) I_{SS} Inc, β Fixed

(*x axis* : $\Delta V_I / (\sqrt{I_{SS} / \beta})$) (*y axis* : $\Delta V_O / (I_{SS} R_L)$)

small signal equivalent circuit



small signal analysis

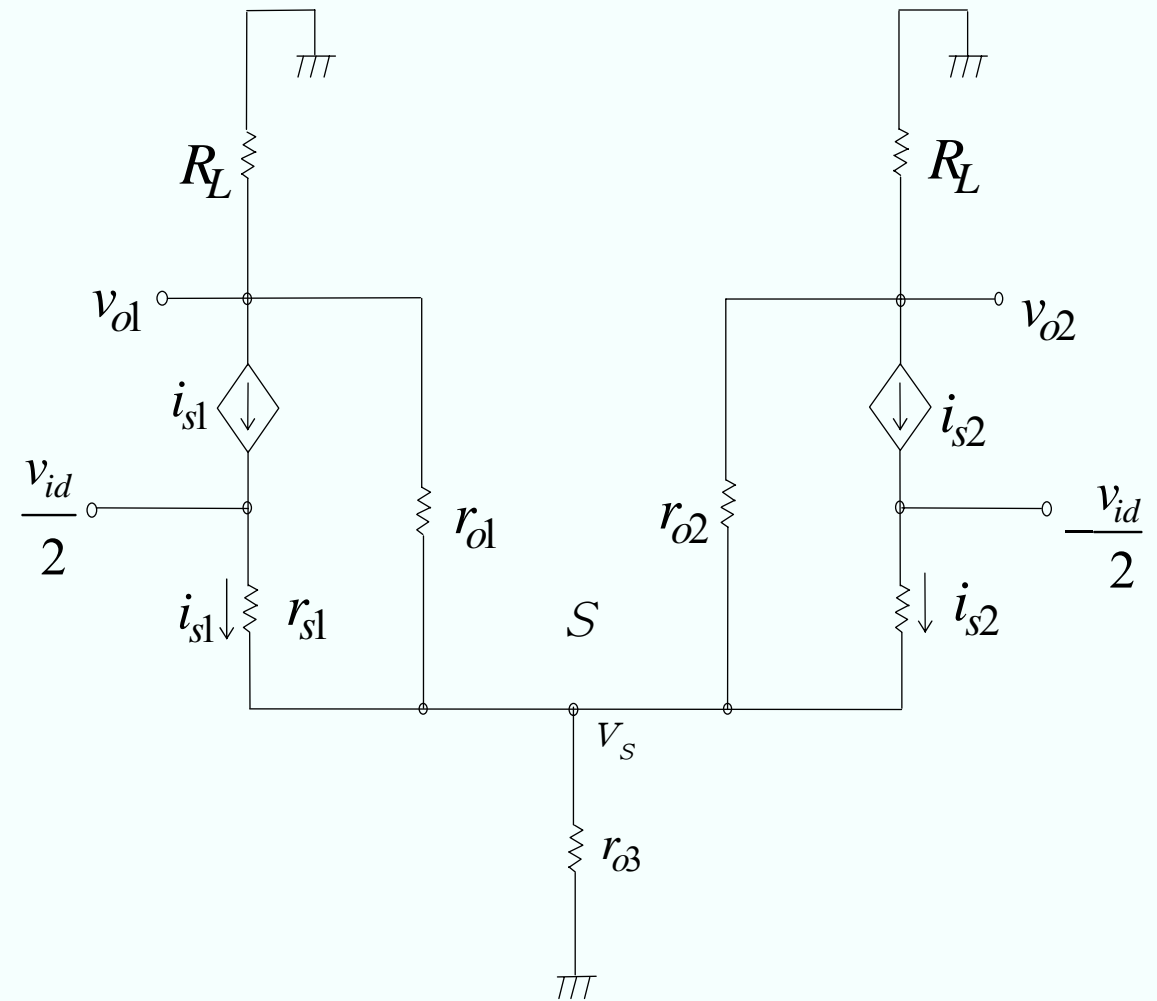
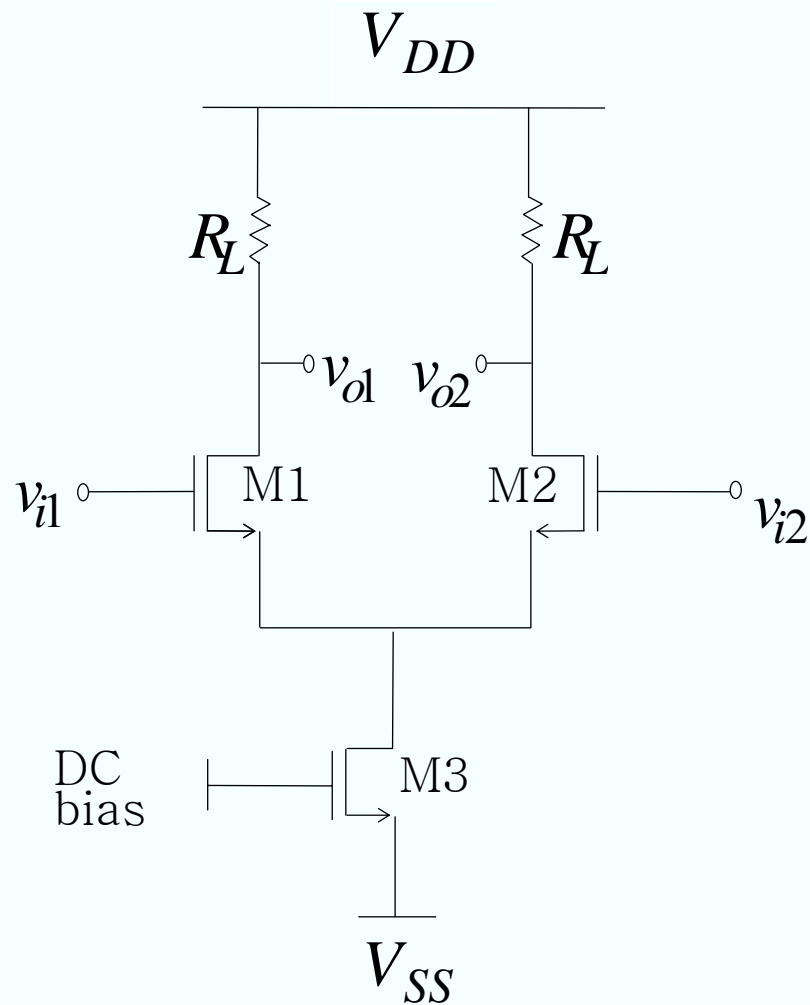


$$i_{s1} = -i_{s2} = \frac{v_{i1} - v_{i2}}{r_{s1} + r_{s2}}$$

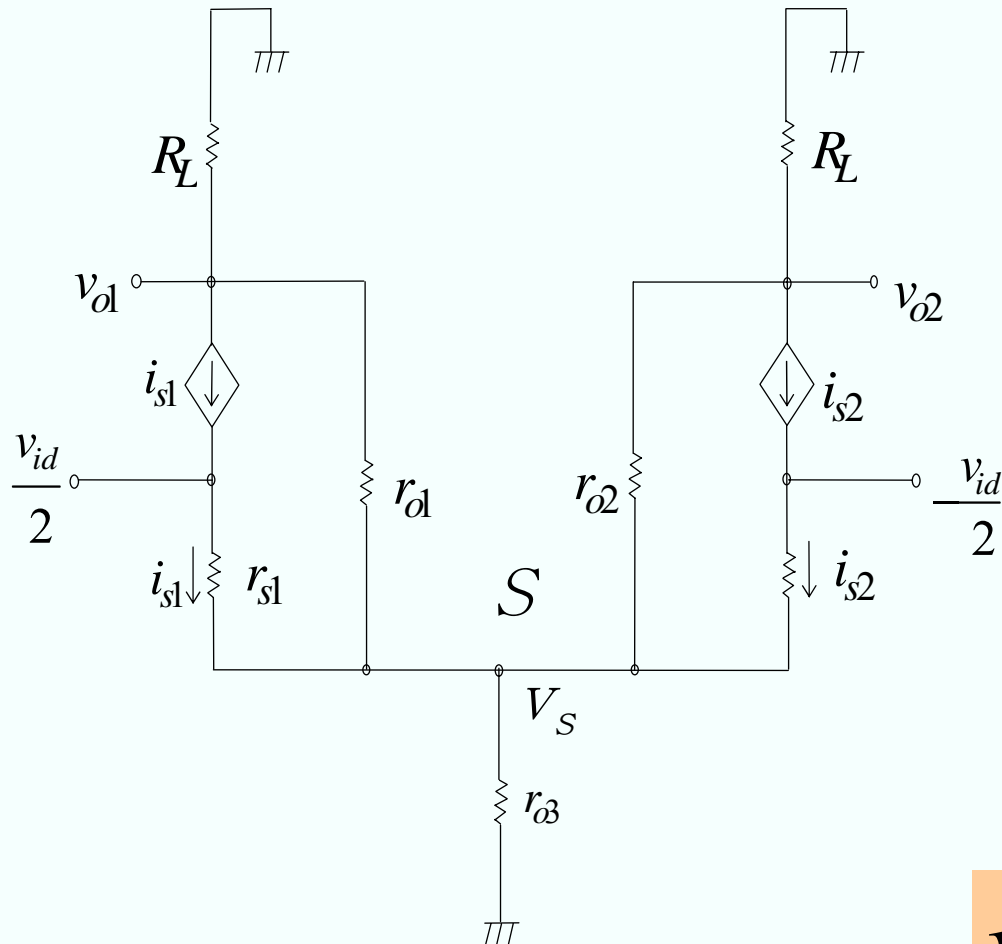
$$v_{o1} - v_{o2} = -i_{s1}R_L + i_{s2}R_L = -\frac{2R_L}{r_{s1} + r_{s2}} \cdot (v_{i1} - v_{i2})$$

$$v_{o1} - v_{o2} = -g_{m1}R_L \cdot (v_{i1} - v_{i2})$$

Small signal analysis using Half-circuit (due to symmetry)



Small signal analysis using Half-circuit (due to symmetry)



Differential input

$$v_{i1} = -v_{i2} = \frac{v_{id}}{2}$$

Due to symmetry $v_S = 0$

KCL at v_{o1} v_{o2} cutset, v_S

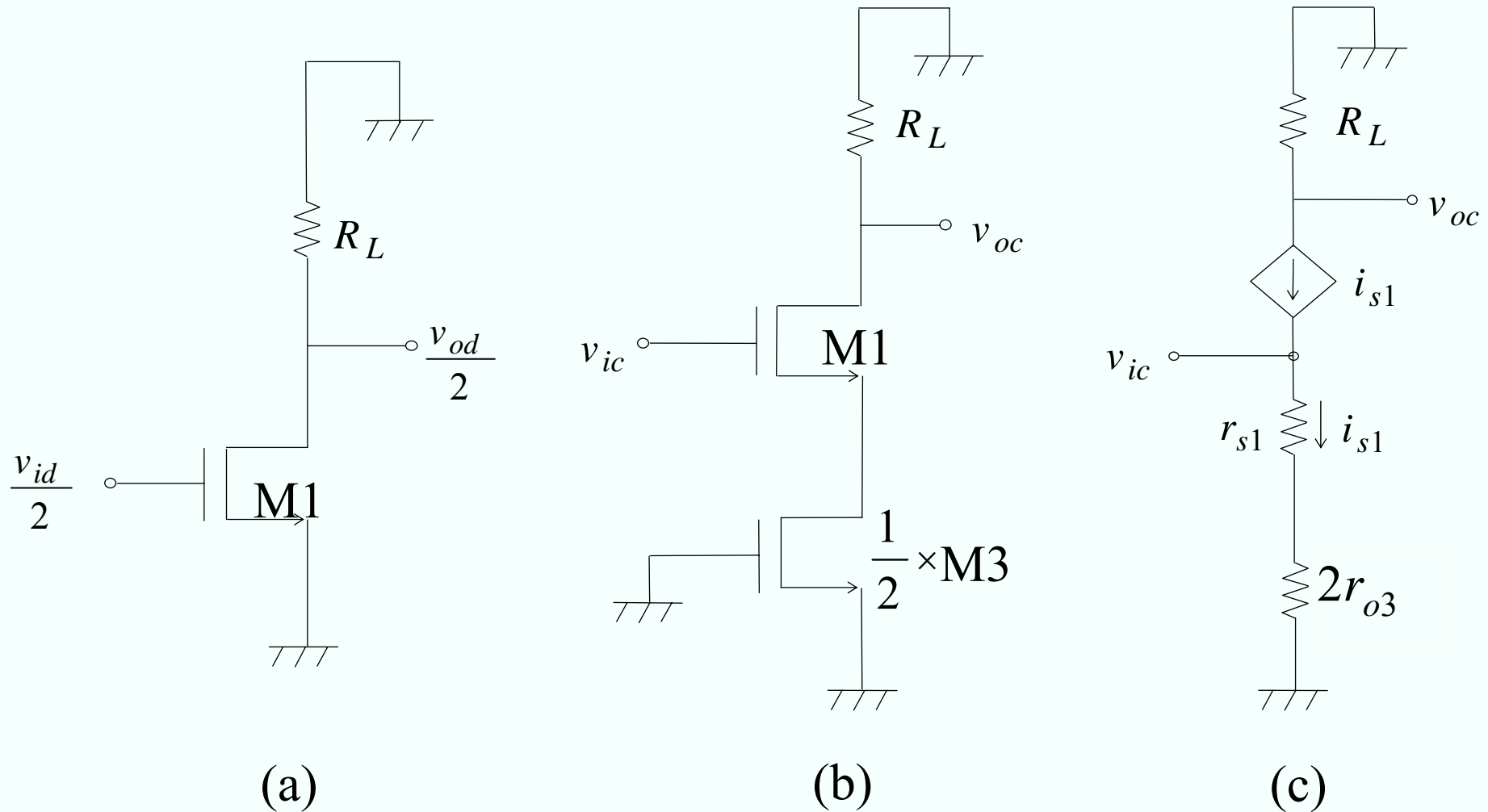
$$-\left(\frac{1}{R_L} + \frac{1}{r_{o1}}\right) \cdot (v_{o1} + v_{o2}) + \left(\frac{2}{r_{s1}} + \frac{2}{r_{o1}}\right) \cdot v_S = 0$$

$$-\frac{1}{r_{o1}} \cdot (v_{o1} + v_{o2}) + \left(\frac{2}{r_{s1}} + \frac{2}{r_{o1}} + \frac{1}{r_{o3}}\right) \cdot v_S = 0$$

$$v_S = 0$$

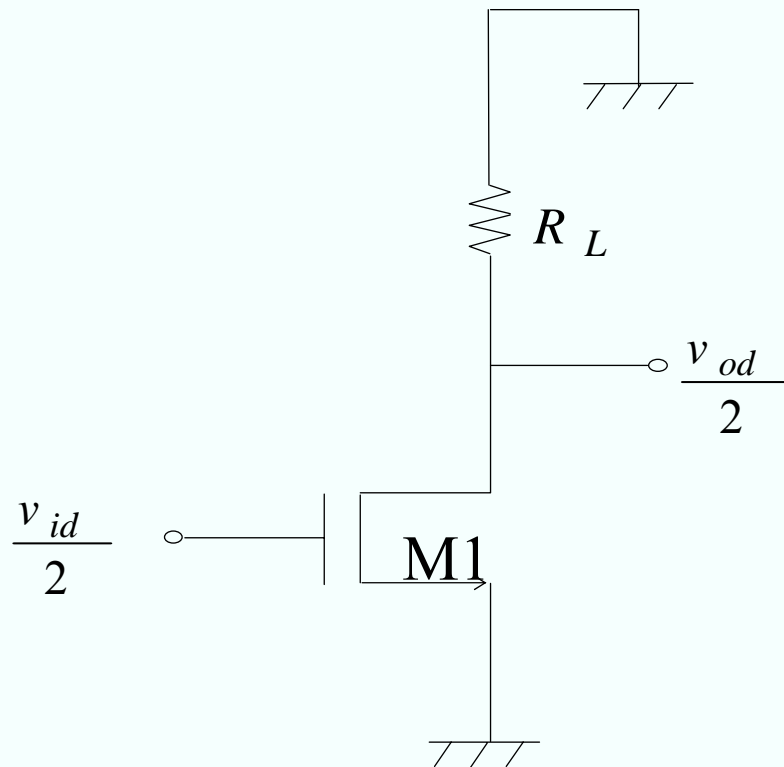
$$v_{o1} + v_{o2} = 0$$

Small signal analysis using Half-circuit (due to symmetry)



Node S : AC ground

Small signal analysis using Half-circuit (due to symmetry)



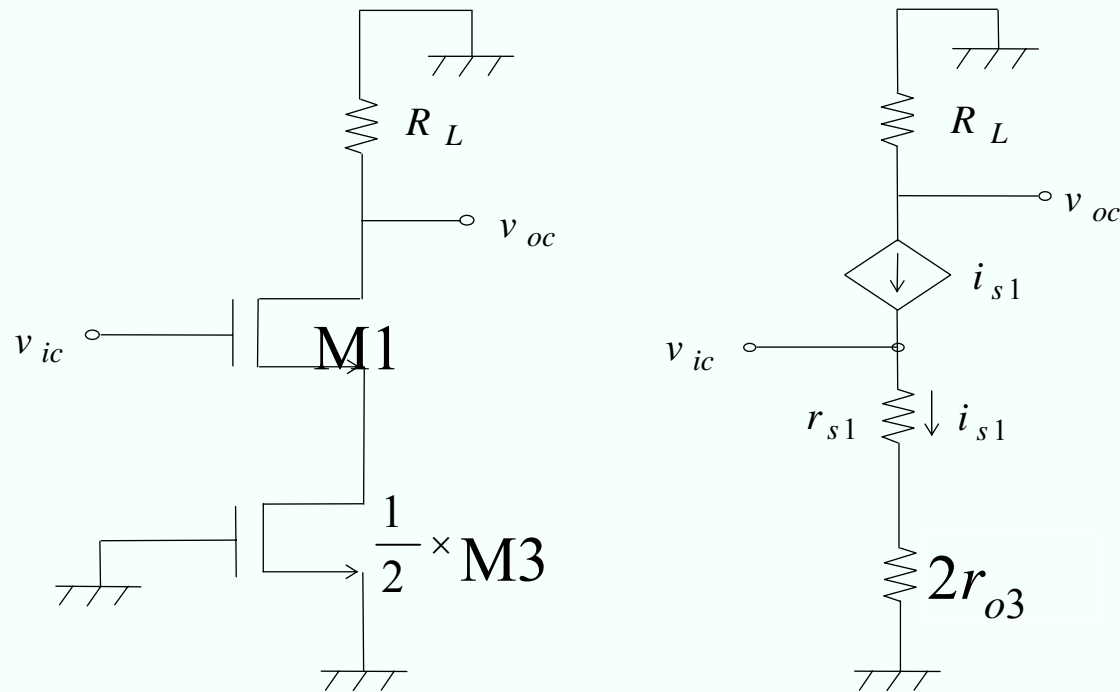
Node S : AC ground

$$A_{vc} = \frac{v_{oc}}{v_{ic}} = -\frac{R_L}{r_{s1} + 2r_{o3}} = -\frac{g_{m1}R_L}{1 + 2g_{m1}r_{o3}}$$

$$CMRR = \left| \frac{A_{vd}}{A_{vc}} \right| = 1 + 2g_{m1}r_{o3} \approx 2g_{m1}r_{o3}$$

$$A_{vd} = \frac{v_{od}}{v_{id}} = -g_{m1} \cdot (R_L \parallel r_{o1}) \approx -g_{m1}R_L \quad (\because R_L \ll r_{o1})$$

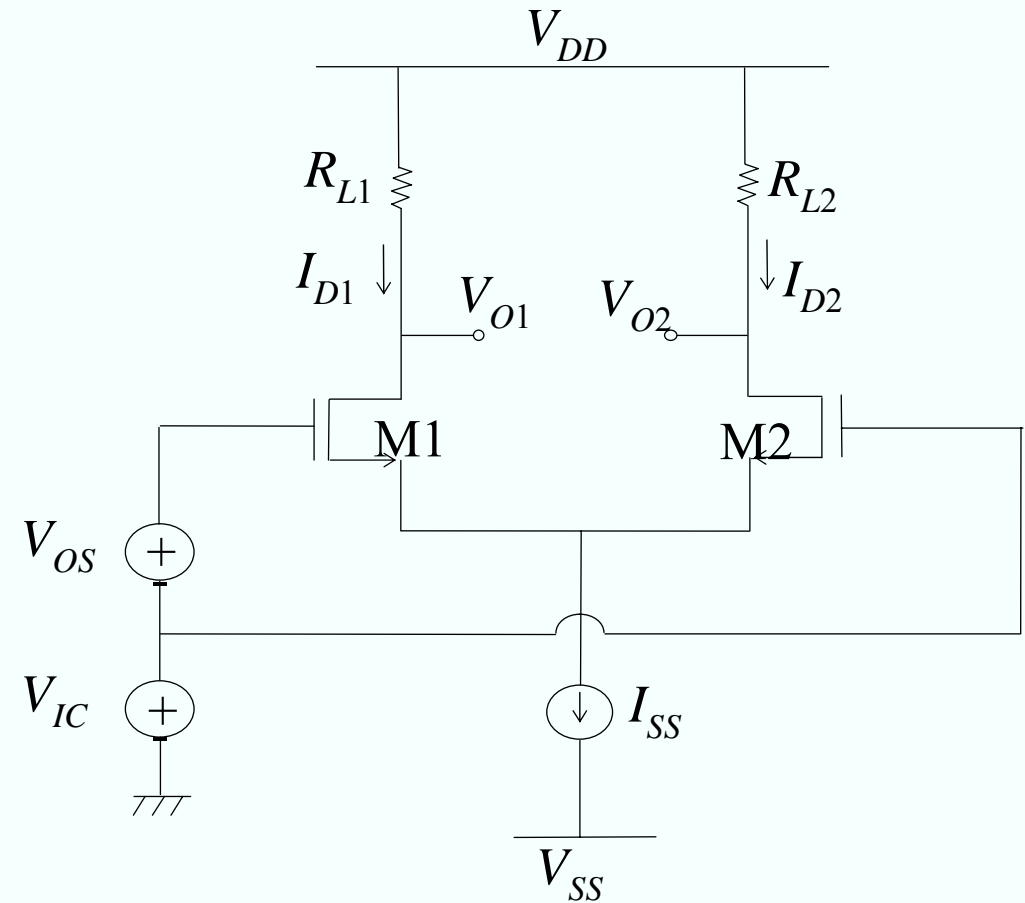
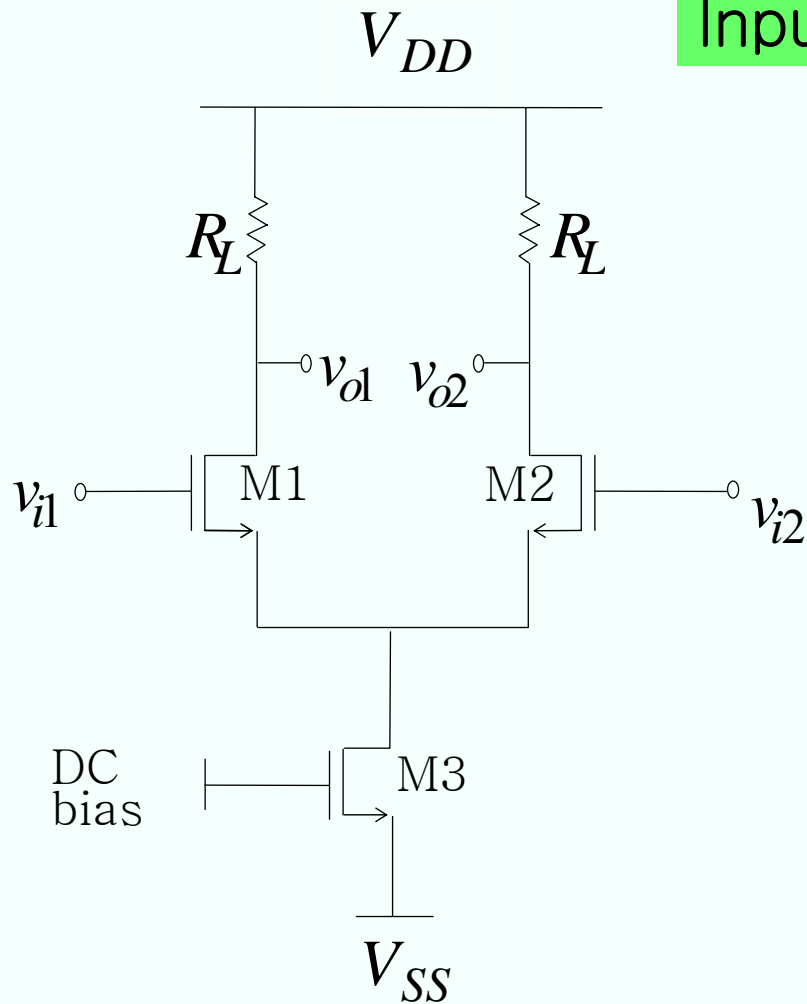
Small signal analysis using Half-circuit (due to symmetry)



$$A_{vc} = \frac{v_{oc}}{v_{ic}} = - \frac{R_L}{r_{s1} + 2r_{o3}} = - \frac{g_{m1} R_L}{1 + 2g_{m1} r_{o3}}$$

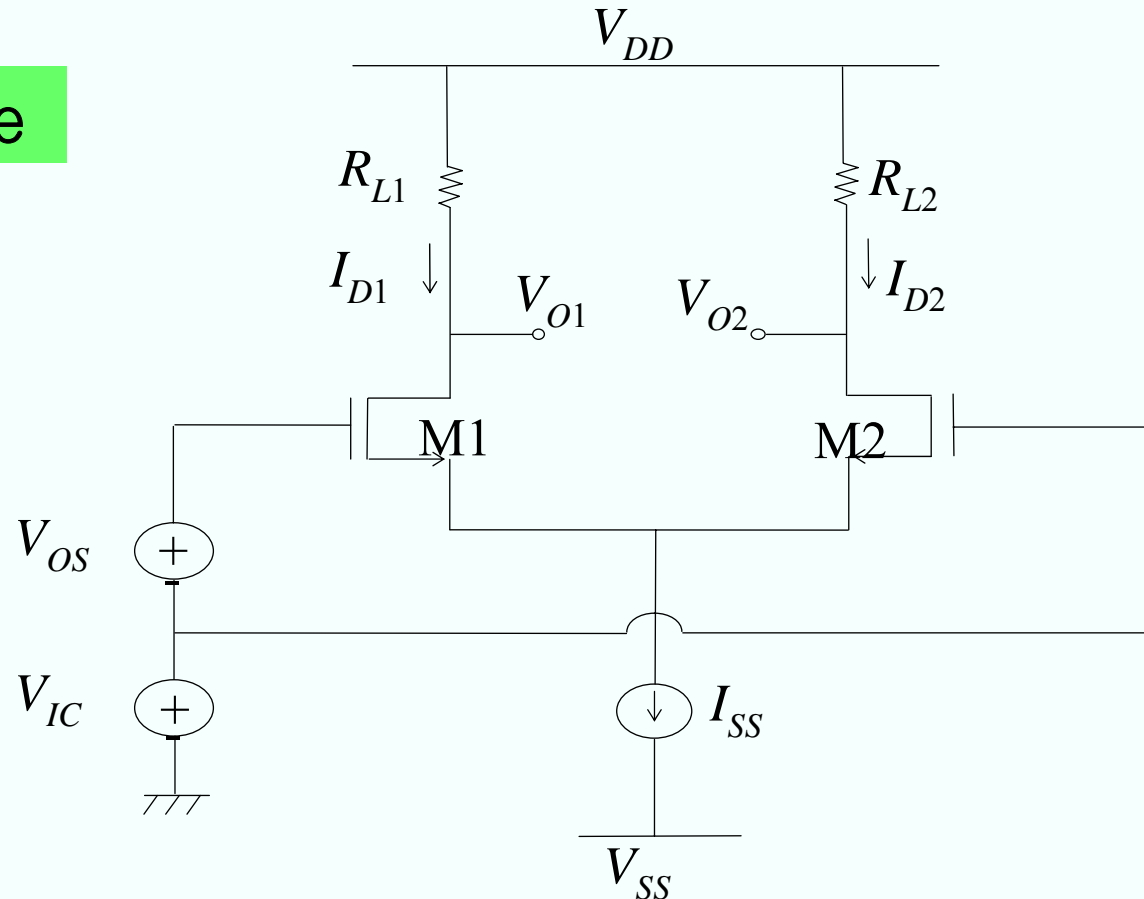
$$CMRR = \left| \frac{A_{vd}}{A_{vc}} \right| = 1 + 2g_{m1} r_{o3} \approx 2g_{m1} r_{o3}$$

Input offset voltage



$$V_{OS} = V_{I1} - V_{I2} \mid v_{o1} = v_{o2}$$

Input offset voltage



$$V_{OS} = V_{GS1} - V_{GS2} = (V_{GS1} - V_{TH1}) - (V_{GS2} - V_{TH2}) + (V_{TH1} - V_{TH2}) = \Delta(V_{GS} - V_{TH}) + \Delta V_{TH}$$

$$V_{OS} = \Delta(V_{GS} - V_{TH}) + \Delta V_{TH}$$

Input offset voltage

$$V_{OS} = \Delta(V_{GS} - V_{TH}) + \Delta V_{TH}$$

$$V_{GS} - V_{TH} = \sqrt{\frac{2I_D}{\mu C_{ox} \frac{W}{L}}}$$

$$V_{OS} = \Delta V_{TH} + \left(\frac{V_{GS} - V_{TH}}{2} \right) \cdot \left\{ \frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)} \right\}$$

$$V_{OS} = \Delta V_{TH} + \frac{I_D}{g_m} \cdot \left\{ \frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)} \right\}$$

Input offset voltage

$$I_{D1}R_{L1} - I_{D2}R_{L2} = 0$$

$$\Delta I_D R_L + I_D \Delta R_L = 0$$

$$\Delta I_D = I_{D1} - I_{D2}, \quad I_D = 0.5(I_{D1} + I_{D2})$$

$$\Delta R_L = R_{L1} - R_{L2}, \quad R_L = 0.5 \cdot (R_{L1} + R_{L2})$$

$$V_{OS} = \Delta V_{TH} + \frac{I_D}{g_m} \cdot \left\{ \frac{\Delta I_D}{I_D} - \frac{\Delta(W/L)}{(W/L)} \right\}$$

$$V_{OS} = \Delta V_{TH} + \frac{I_D}{g_m} \cdot \left\{ -\frac{\Delta R_L}{R_L} - \frac{\Delta(W/L)}{W/L} \right\}$$

Input offset voltage

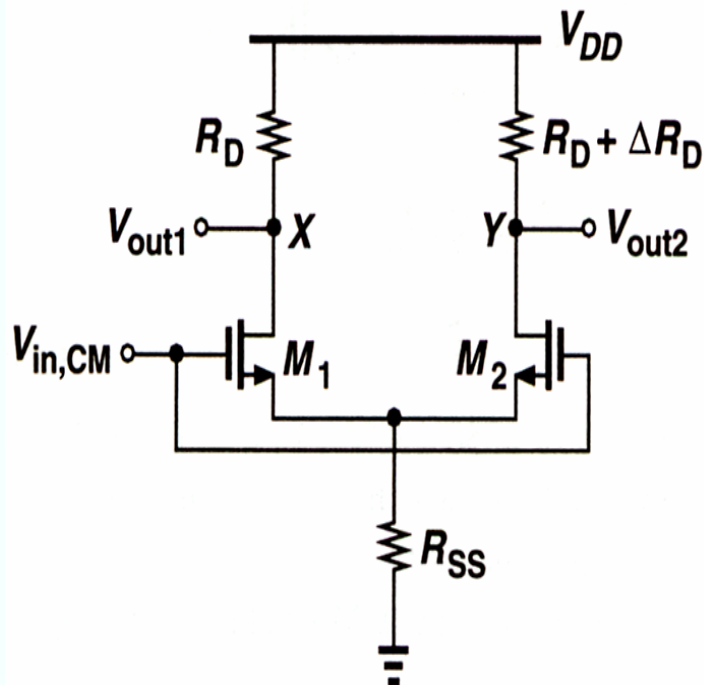
$$V_{OS} = \Delta V_{TH} + \frac{I_D}{g_m} \cdot \left\{ -\frac{\Delta R_L}{R_L} - \frac{\Delta(W/L)}{W/L} \right\}$$

$$V_{OS} = \Delta V_{TH} + \frac{V_{GS} - V_{TH}}{2} \cdot \left\{ -\frac{\Delta R_L}{R_L} - \frac{\Delta(W/L)}{W/L} \right\}$$

$$\sigma_{VOS} = \sqrt{(\sigma_{VTH})^2 + \left(\frac{I_D}{g_m}\right)^2 \cdot \left\{ \left(\frac{\sigma_{RL}}{R_L}\right)^2 + \left(\frac{\sigma_{W/L}}{W/L}\right)^2 \right\}}$$

Common-mode to differential-mode conversion

Due to load mismatch



$$A_{vc} = \frac{v_{oc}}{v_{ic}} = -\frac{R_L}{r_{s1} + 2r_{o3}} = -\frac{g_{m1}R_L}{1 + 2g_{m1}r_{o3}}$$

$$\Delta V_X = -\Delta V_{in,CM} \frac{g_m}{1 + 2g_m R_{SS}} R_D$$

$$\Delta V_Y = -\Delta V_{in,CM} \frac{g_m}{1 + 2g_m R_{SS}} (R_D + \Delta R_D)$$

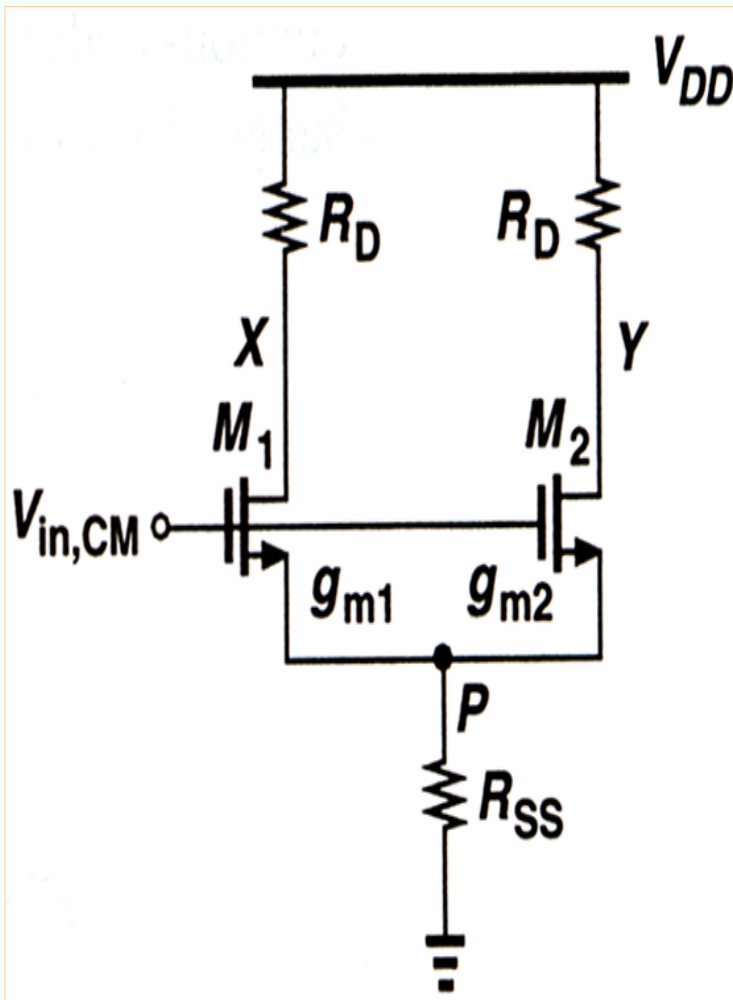
$$\Delta V_X - \Delta V_Y = \Delta V_{in,CM} \cdot \frac{g_m \Delta R_D}{1 + 2g_m R_{SS}}$$

Common-mode to differential-mode conversion :

More significant at high frequency due to reduction of R_{ss} by shunt capacitance $\rightarrow r_{o1}, r_{o2}$ neglected

Common-mode to differential-mode conversion

Due to transistor mismatch



$$(g_{m1} + g_{m2})(V_{in,CM} - V_P)R_{SS} = V_P$$

$$V_P = \frac{(g_{m1} + g_{m2})R_{SS}}{(g_{m1} + g_{m2})R_{SS} + 1} V_{in,CM}$$

$$\begin{aligned} V_X &= -g_{m1}(V_{in,CM} - V_P)R_D \\ &= \frac{-g_{m1}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM} \end{aligned}$$

$$\begin{aligned} V_Y &= -g_{m2}(V_{in,CM} - V_P)R_D \\ &= \frac{-g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM} \end{aligned}$$

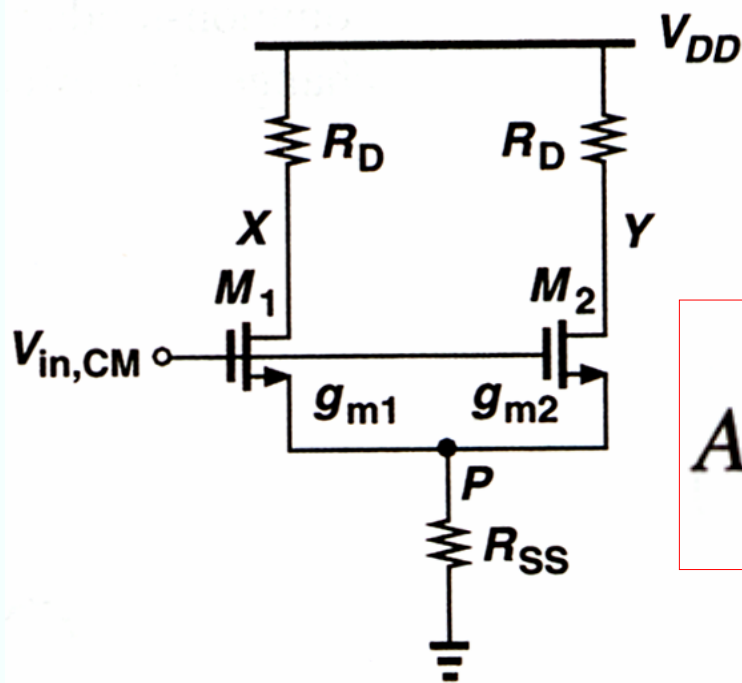
$$V_X - V_Y = -\frac{g_{m1} - g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM}$$

Common-mode to differential-mode conversion

Due to transistor mismatch

$$V_X - V_Y = -\frac{g_{m1} - g_{m2}}{(g_{m1} + g_{m2})R_{SS} + 1} R_D V_{in,CM}$$

$$V_X - V_Y = V_{in,CM} \cdot \frac{g_m \Delta R_D}{1 + 2g_m R_{SS}}$$



$$A_{vc} = \frac{v_{oc}}{v_{ic}} = -\frac{R_L}{r_{s1} + 2r_{o3}} = -\frac{g_{m1}R_L}{1 + 2g_{m1}r_{o3}}$$

$$A_{CM-DM} = -\frac{\Delta g_m R_D}{(g_{m1} + g_{m2})R_{SS} + 1}$$

Add gm x DEL RD in numerator

$$CMRR = \left| \frac{A_{vd}}{A_{vc}} \right| = 1 + 2g_{m1}r_{o3} \approx 2g_{m1}r_{o3} \quad \text{Conv def: Very large}$$

More practical CMRR definition

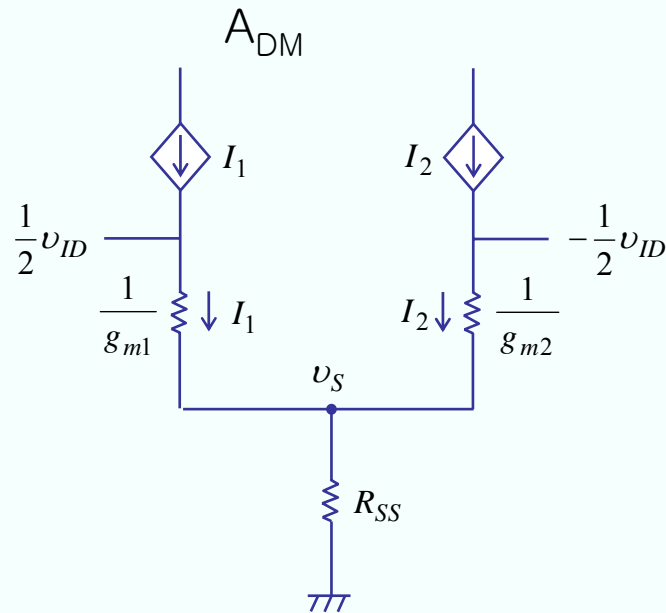
$$|A_{DM}| = \frac{R_D}{2} \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{1 + (g_{m1} + g_{m2})R_{SS}}$$

$$A_{CM-DM} = -\frac{\Delta g_m R_D}{(g_{m1} + g_{m2})R_{SS} + 1}$$

Add gm x DEL RD in numerator

$$CMRR = \left| \frac{A_{DM}}{A_{CM-DM}} \right| = \frac{g_{m1} + g_{m2} + 4g_{m1}g_{m2}R_{SS}}{2\Delta g_m}$$

$$\approx \frac{g_m}{\Delta g_m} (1 + 2g_m R_{SS})$$



Apply KCL at V_S node

$$g_{m1} \cdot \left(v_S - \frac{1}{2} v_{ID} \right) + g_{m2} \cdot \left(v_S + \frac{1}{2} v_{ID} \right) + \frac{1}{R_{SS}} \cdot v_S = 0$$

$$v_S = \frac{g_{m1} - g_{m2}}{g_{m1} + g_{m2} + \frac{1}{R_{SS}}} \times \frac{1}{2} v_{ID} = \frac{(g_{m1} - g_{m2}) \cdot R_{SS}}{1 + (g_{m1} + g_{m2}) \cdot R_{SS}} \times \frac{1}{2} v_{ID}$$

$$I_1 = g_{m1} \cdot \left(\frac{1}{2} v_{ID} - v_S \right) = \frac{g_{m1} + 2 g_{m1} g_{m2} R_{SS}}{1 + (g_{m1} + g_{m2}) R_{SS}} \times \frac{1}{2} v_{ID}$$

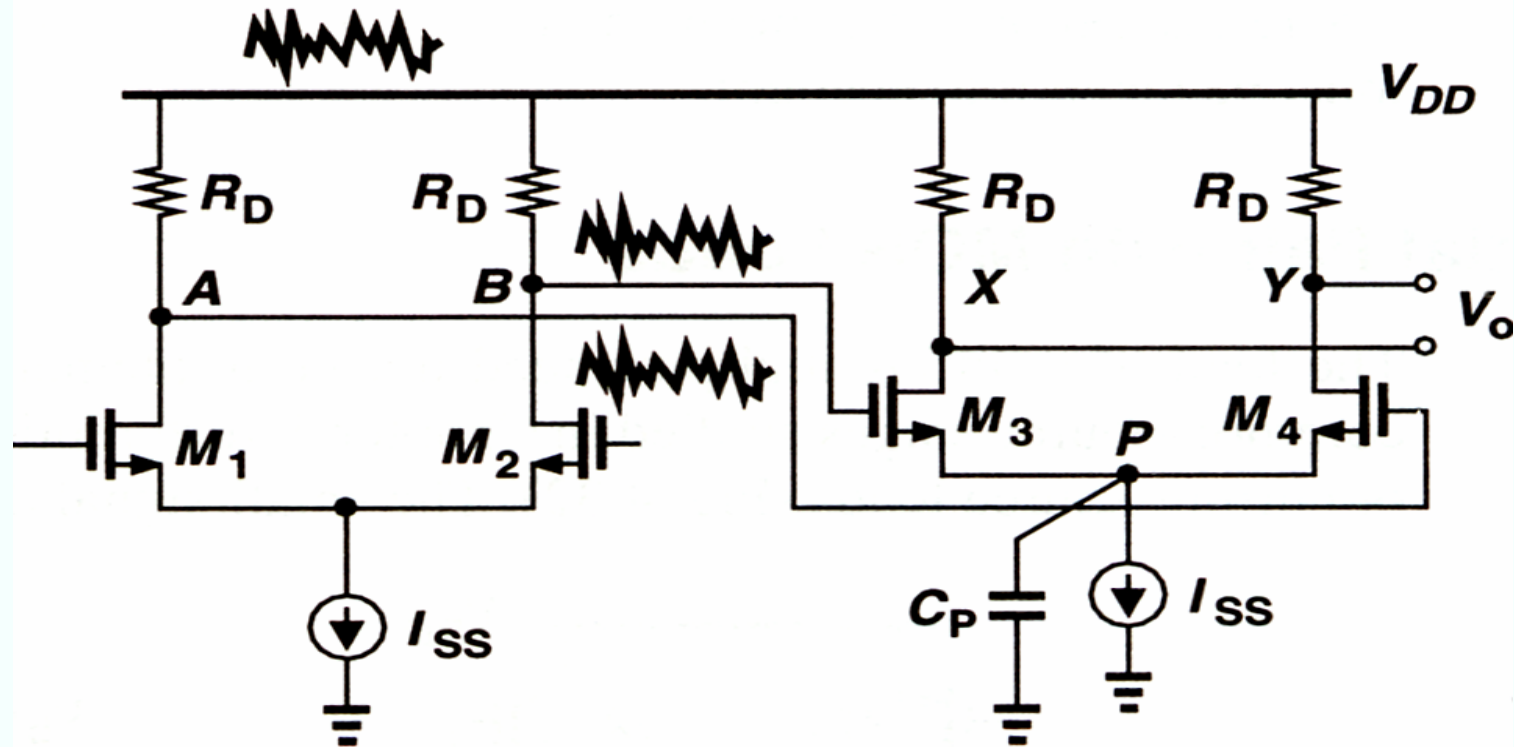
$$I_2 = g_{m2} \cdot \left(-\frac{1}{2} v_{ID} - v_S \right) = \frac{(-g_{m2} - 2 g_{m1} g_{m2} R_{SS})}{1 + (g_{m1} + g_{m2}) R_{SS}} \times \frac{1}{2} v_{ID}$$

$$v_{OD} = (-I_1 + I_2) \cdot R_D = -\frac{g_{m1} + g_{m2} + 4 g_{m1} g_{m2} R_{SS}}{1 + (g_{m1} + g_{m2}) \cdot R_{SS}} \times \frac{1}{2} R_D v_{ID}$$

$$A_{DM} = \frac{v_{OD}}{v_{ID}} = -\frac{1}{2} R_D \cdot \frac{g_{m1} + g_{m2} + 4 g_{m1} g_{m2} R_{SS}}{1 + (g_{m1} + g_{m2}) \cdot R_{SS}}$$

$$A_{DM} = -g_m R_D$$

if $g_{m1} = g_{m2} = g_m$



$$|A_{CM-DM}| = \frac{\Delta g_m R_D}{\sqrt{1 + (g_{m3} + g_{m4})^2 \left| \frac{1}{C_P \omega} \right|^2}}$$

$R_{ss} \Rightarrow 1/(s C_p)$

More significant at high frequency