Frequency Response

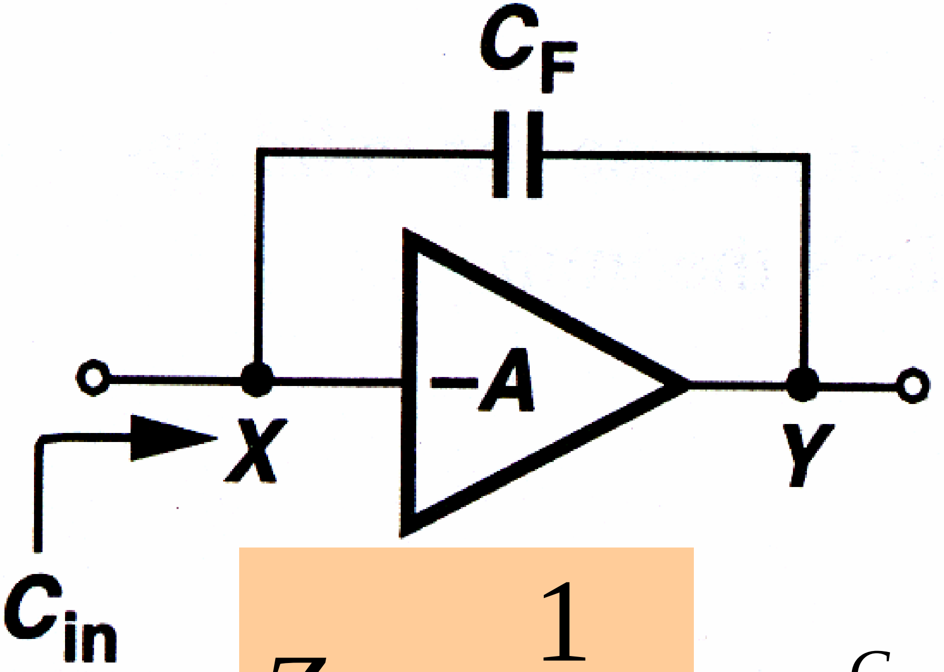
(1) Miller effect

(2) single TR amp (CS, CG, CD)

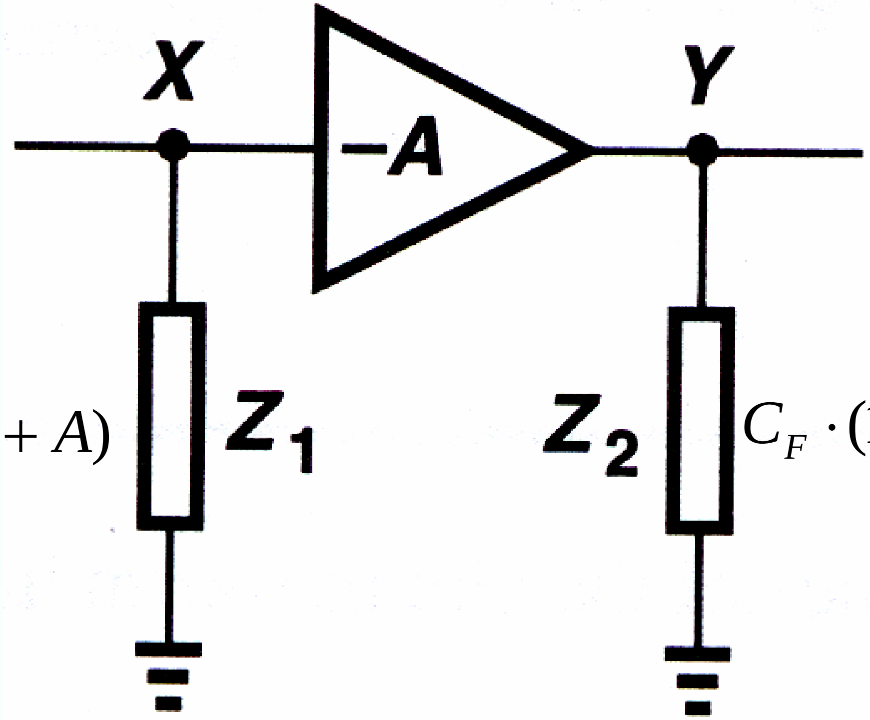
Consider Av(s), Zin(s), Zout(s)

Miller theorem

Voltage amp



$$Z = \frac{1}{sC_F}$$



$$C_F \cdot (1 + A)$$

$$C_F \cdot (1 + \frac{1}{A})$$

$$Z = Z_1 + Z_2$$

$$Z_1 = \frac{Z}{1 - \frac{V_Y}{V_X}}$$

$$Z_2 = \frac{Z}{1 - \frac{V_X}{V_Y}}$$

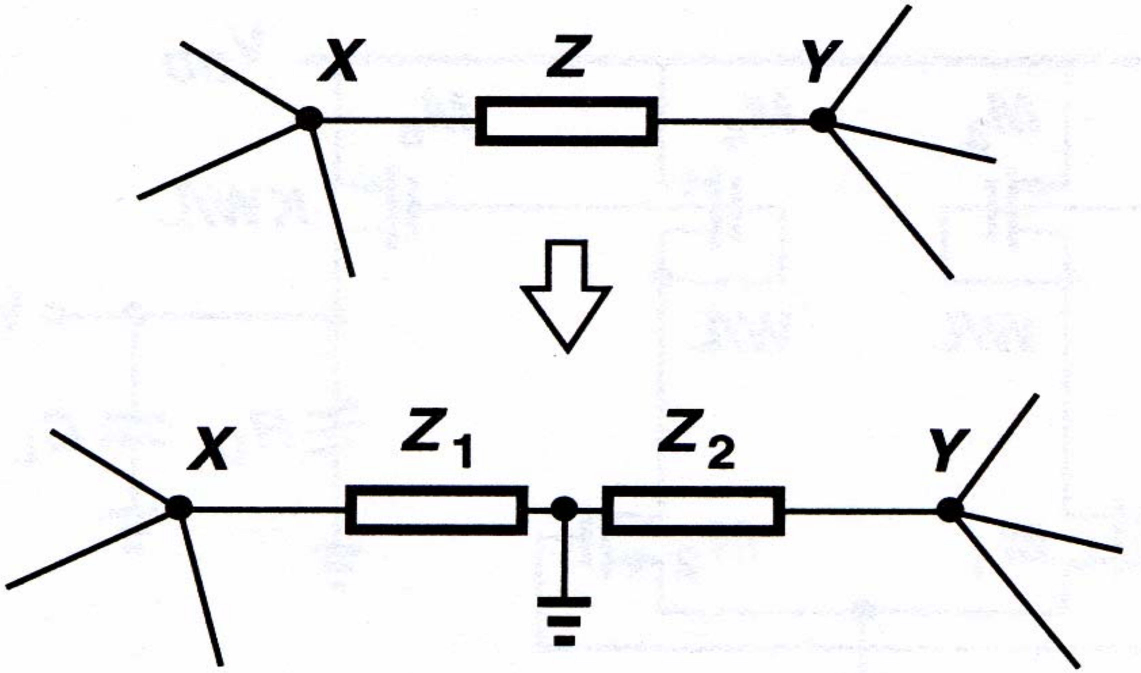
Razavi

The same current I flows through Z1 and Z2.

$$V_X = I \cdot Z_1 \qquad V_Y = -I \cdot Z_2 \qquad I = \frac{V_X - V_Y}{Z}$$

$$Z = Z_a + Z_b$$

No current through mid-ground



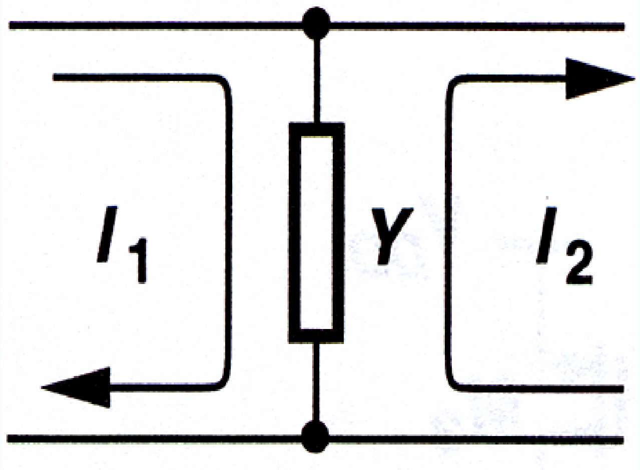
$$Z_a = \frac{Z}{1 - V_Y/V_X}$$

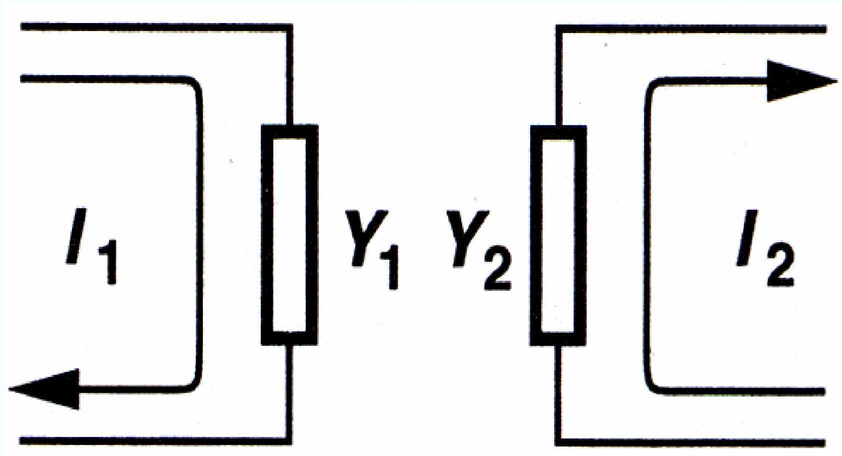
$$Z_b = \frac{Z}{1 - V_X/V_Y}$$

Razavi

Dual of Miller theorem

Current amp





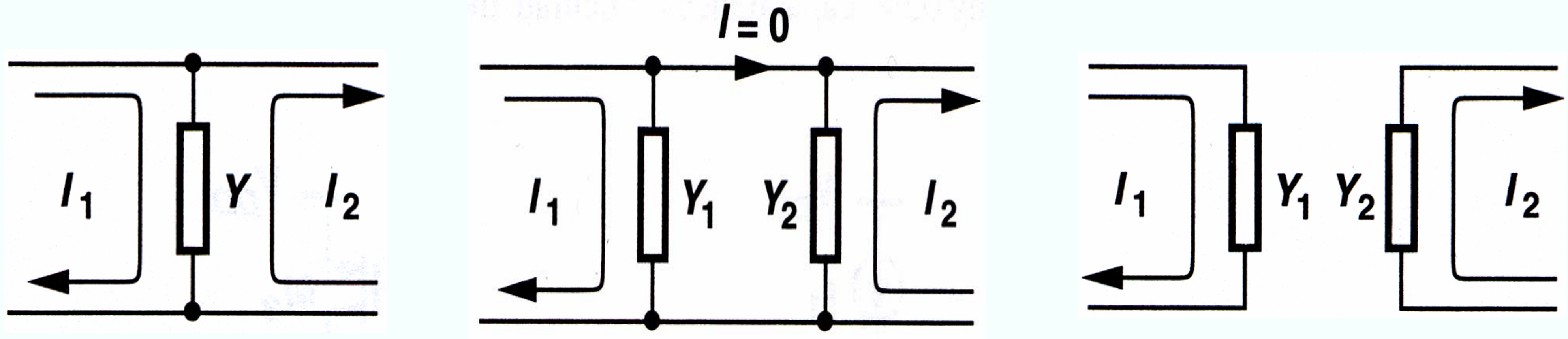
$$Y_1 = \frac{Y}{1 - I_2/I_1}$$

$$Y_2 = \frac{Y}{1 - I_1/I_2}$$

Razavi

Dual of Miller theorem

Current amp



The same voltage drop across $Y1$ *and* $Y2$.

$$\frac{I_1}{Y_1} = \frac{I_1 - I_2}{Y}$$

$$-\frac{I_2}{Y_2} = \frac{I_1 - I_2}{Y}$$

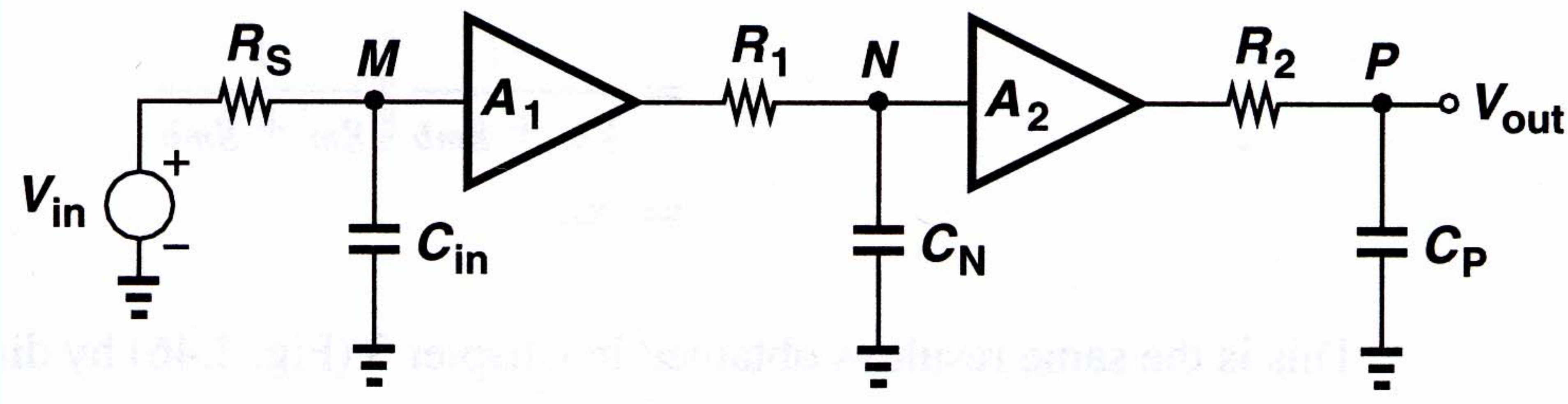
$$Y = Y_1 + Y_2$$

$$Y_1 = \frac{Y}{1 - I_2/I_1}$$

$$Y_2 = \frac{Y}{1 - I_1/I_2}$$

Razavi

Association of a pole with a node

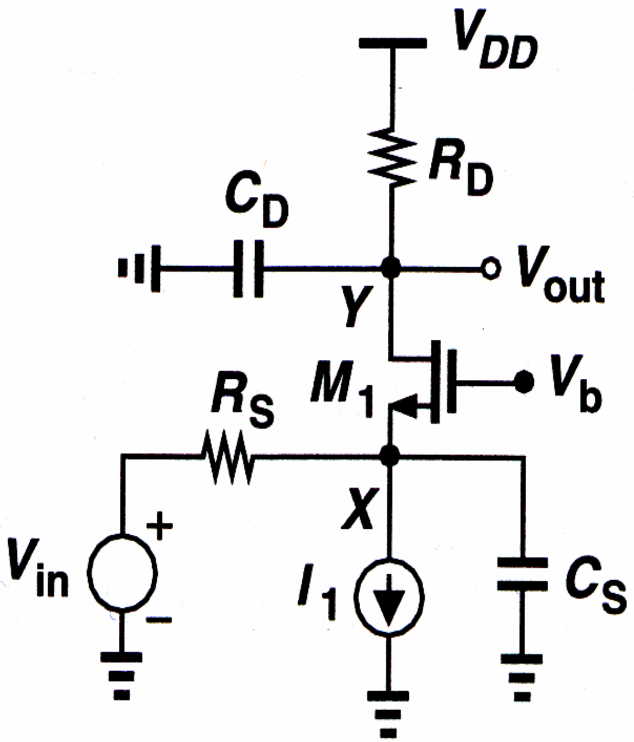


Every node has a pole

$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + R_S C_{in} s} \cdot \frac{A_2}{1 + R_1 C_N s} \cdot \frac{1}{1 + R_2 C_P s}$$

Razavi

Association of a pole with a node



Every node has a pole

CG amp (ro neglected)

$$\omega_{in} = \left[(C_{GS1} + C_{SB1})\left(R_S \middle\| \frac{1}{g_{m1} + g_{mb1}}\right)\right]^{-1}$$

$$\omega_{out} = [(C_{DG} + C_{DB})R_D]^{-1}$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \cdot \frac{1}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

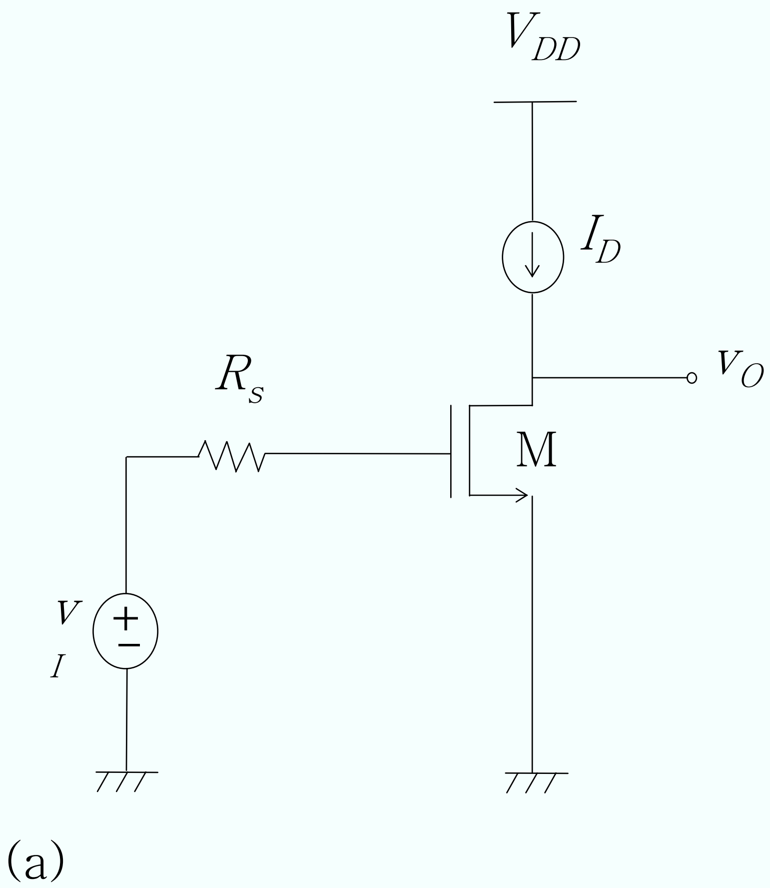
Razavi

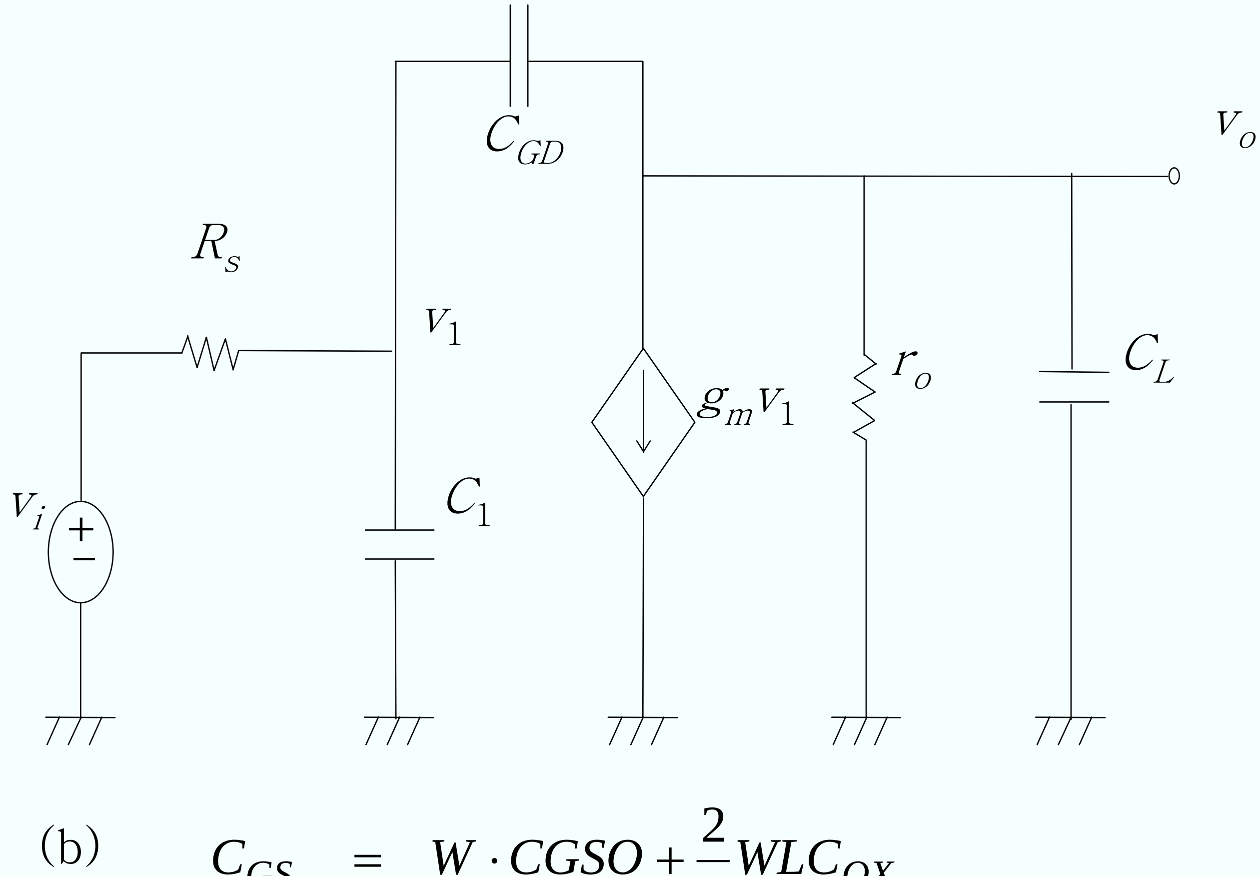
	CS amp	CG amp	CD amp
A_v'	$-g_m \cdot (r_o \parallel R_L)$	$+(g_m + g_{mb}) \cdot (r_o \parallel R_L)$	$+\dfrac{g_m}{g_m + g_{mb} + \dfrac{1}{r_o \parallel R_L}}$
R_i	∞	$\dfrac{1}{g_m + g_{mb} + \dfrac{1}{r_o}} \cdot \left(1 + \dfrac{R_L}{r_o}\right)$	∞
R_o	r_o	$r_o + R_s + (g_m + g_{mb}) \cdot r_o \cdot R_s$	$\dfrac{1}{g_m + g_{mb}}$
ω_{-3dB}	$\dfrac{1}{R_s \cdot \left\{ C_{GD} \cdot g_m (r_o \parallel R_L) + C_{GS} + C_{GB} + C_{GD} \right\}}$	$\dfrac{1}{(R_s \parallel R_i) \cdot (C_{GS} + C_{SB})}$	$\dfrac{1}{R_s \cdot \left\{ C_{GD} + C_{GB} + C_{GS} \cdot \dfrac{g_{mb}}{g_m + g_{mb}} \right\}}$
Linear input voltage range (distortion < 10%)	$\lvert v_i' \rvert < 0.2 (V_{GS} - V_{TH})$	$\lvert v_i' \rvert < 0.2 (V_{GS} - V_{TH})$	$\lvert v_i' \rvert < \left\{ \dfrac{g_m + g_{mb}}{g_{mb}} \right\} \cdot 0.2 (V_{GS} - V_{TH})$
Suitable amp type	**Voltage Input Current Output**	**Current Input Current Output**	**Voltage Input Voltage Output**

Freq response summary of CS, CG, CD amp

	−3dB Frequency	Zin(s)	Zout(s)
CS amp	Low	Capacitive	Capacitive
CG amp	High	Capacitive	Capacitive
CD amp	Very High	Capacitive	Capacitive If Rs < 1/gm Inductive If Rs > 1/gm

Frequency response of CS amp



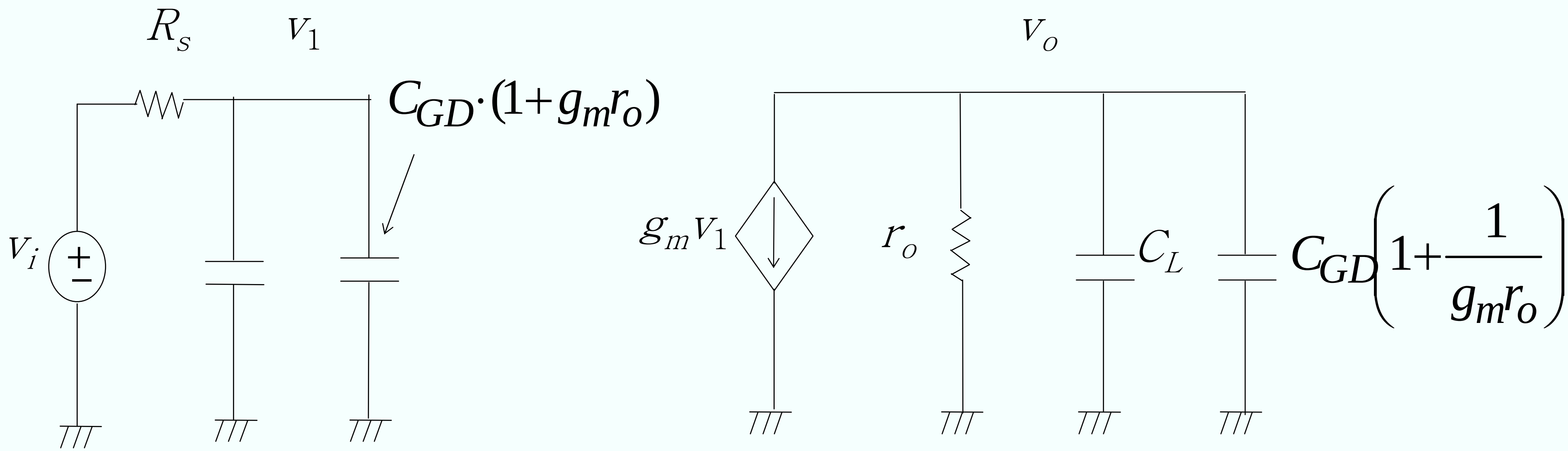


$$C_{GS} = W \cdot CGSO + \frac{2}{3} WLC_{OX}$$

$$C_{GD} = W \cdot CGDO$$

$$C_{GB} = L \cdot CGBO$$

Frequency response of CS amp (Miller effect)



$(C_1 = C_{GS} + C_{GB})$

$$\omega_{p1} = \frac{1}{R_s\{C_1 + C_{GD}(1+g_m r_o)\}}$$

$$A_v(s) = \frac{v_o(s)}{v_i(s)} = -\frac{g_m r_o}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

$$\omega_{p2} = \frac{1}{r_o\left(C_L + C_{GD}\left(1+\frac{1}{g_m r_o}\right)\right)}$$

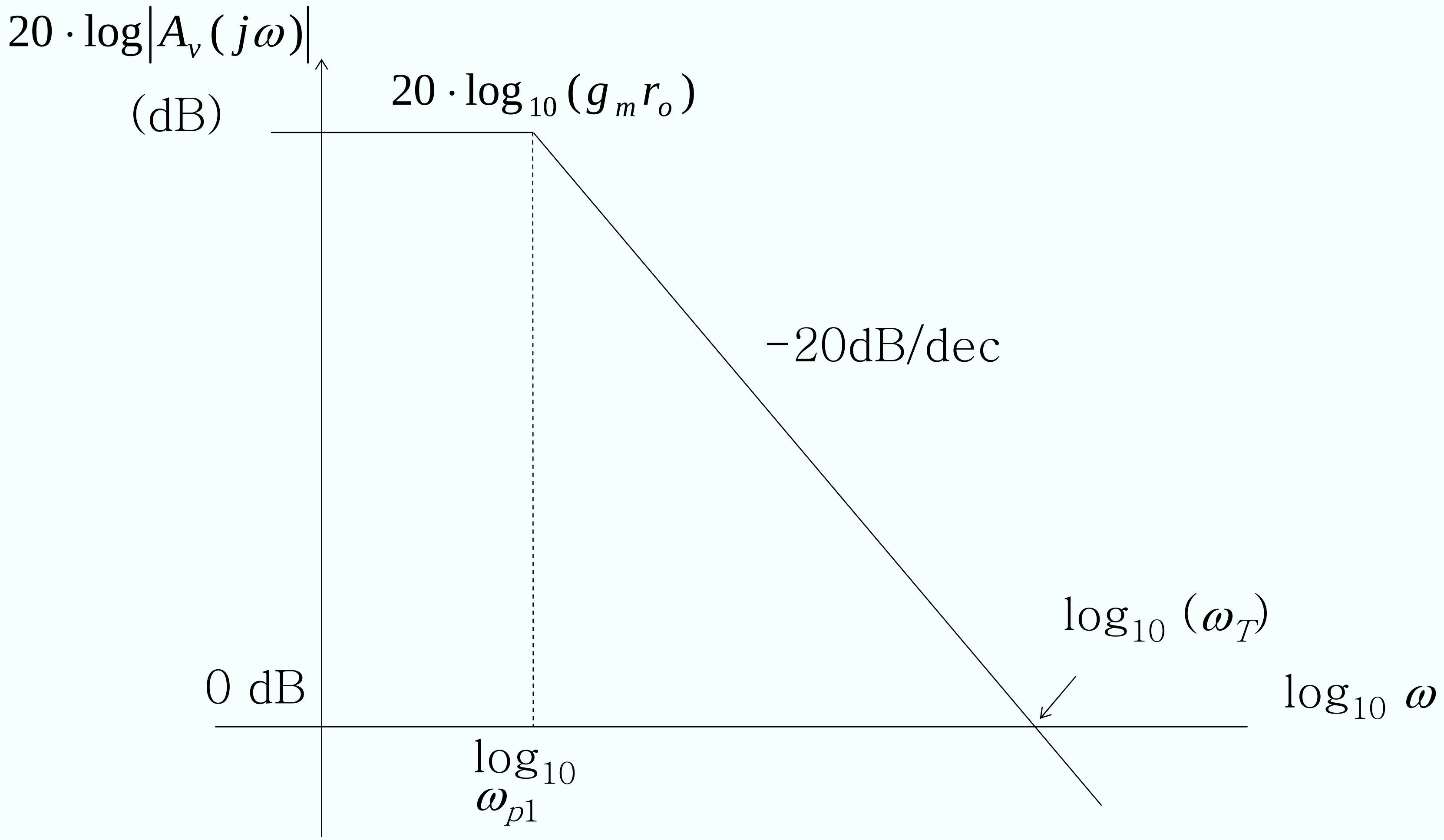


Fig 4.2.8 Frequency response of common source amplifier (amplitude response)

2-pole 1-zero

$$A_v(s) = -\frac{g_m r_o \cdot \left(1 - \dfrac{s}{(g_m/C_{GD})}\right)}{1+s\cdot\{g_m R_s r_o C_{GD} + R_s(C_1+C_{GD}) + r_o(C_L+C_{GD})\} + s^2\{R_s r_o\cdot(C_1 C_L + C_{GD}C_L + C_{GD}C_1)\}}$$

$$\left(1+\frac{s}{\omega_{p1}}\right)\cdot\left(1+\frac{s}{\omega_{p1}}\right) = 1 + s\cdot\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right) + s^2\cdot\frac{1}{\omega_{p1}\omega_{p2}} \approx 1 + s\cdot\frac{1}{\omega_{p1}} + s^2\cdot\frac{1}{\omega_{p1}\omega_{p2}}$$

$$\omega_{p1} = \frac{1}{(g_m r_o)\cdot(R_s C_{GD}) + R_s(C_1+C_{GD}) + r_o(C_L+C_{GD})}$$

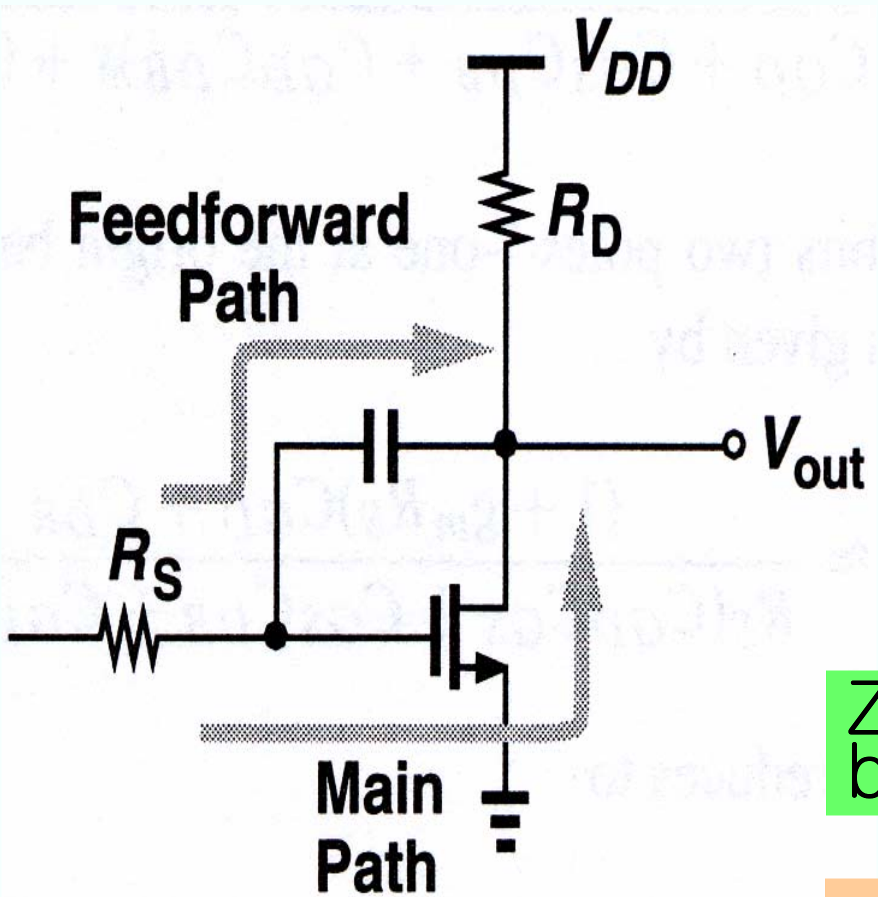
Dominant pole approx

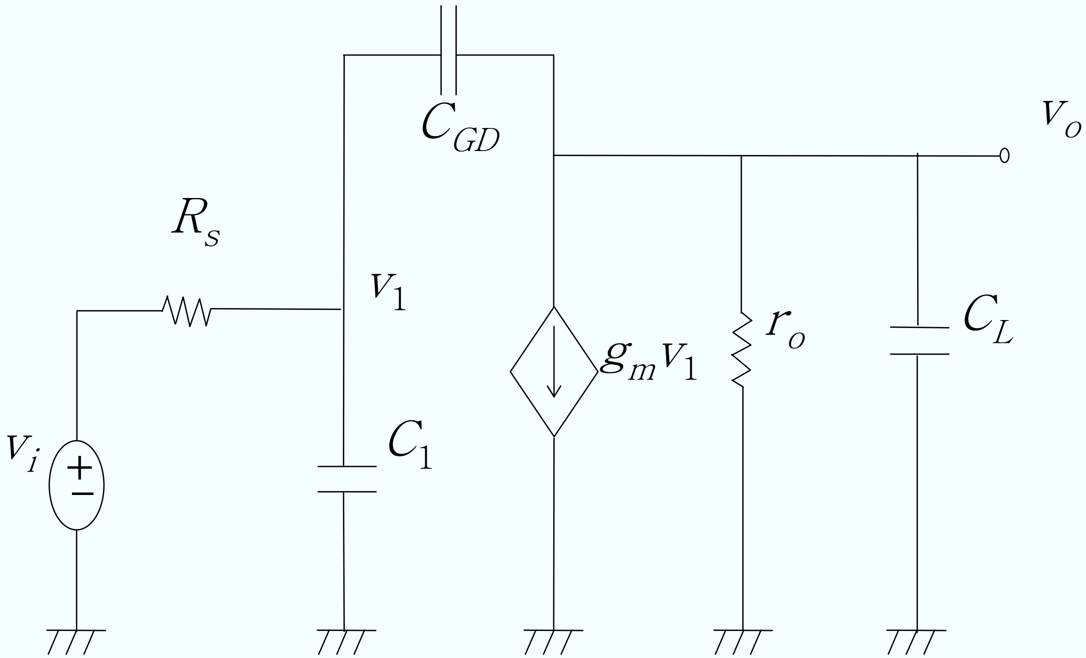
$$\omega_{p1} \approx \frac{1}{R_s\, C_{GD}\, g_m r_o}$$

Using Miller approx.

$$\omega_{p2} \approx \frac{g_m}{C_L\cdot\left(1+\dfrac{C_1}{C_{GD}}\right)} < \omega_z$$

Feed-forward capacitor between input & output nodes of inverting amp ➔
Positive real zero : + gm / CGD



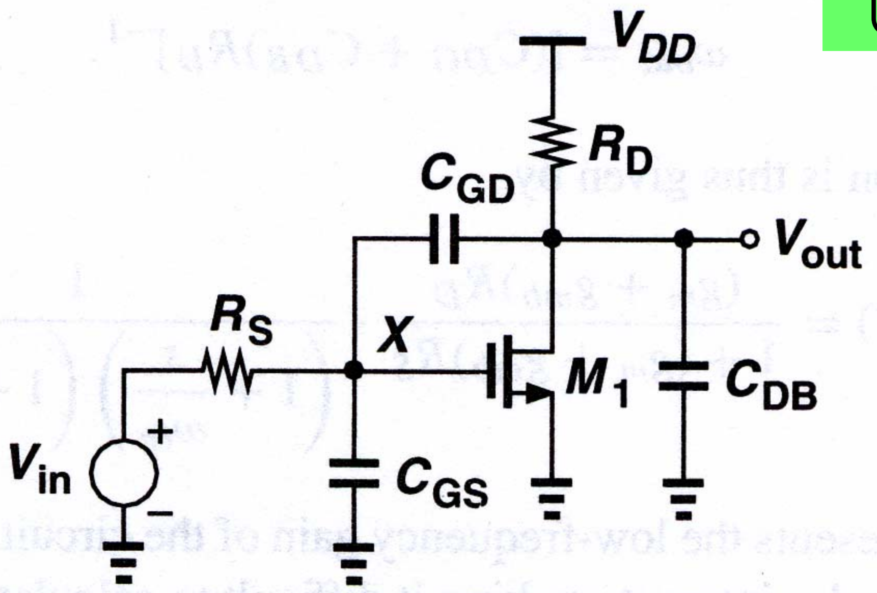


Zero: s value at which vo becomes 0 at non-zero input

$$s\,C_{GD} \cdot v_1 = g_m \cdot v_1$$

Input impedance of CS amp

Using Miller effect | ro neglected



$$\omega_{in} = \frac{1}{R_S[C_{GS} + (1 + g_m R_D)C_{GD}]}$$

$$\omega_{out} = \frac{1}{R_D(C_{DB} + C_{GD})}$$

Not very accurate

$$\frac{V_{out}}{V_{in}}(s) = \frac{-g_m R_D}{\left(1 + \frac{s}{\omega_{in}}\right)\left(1 + \frac{s}{\omega_{out}}\right)}$$

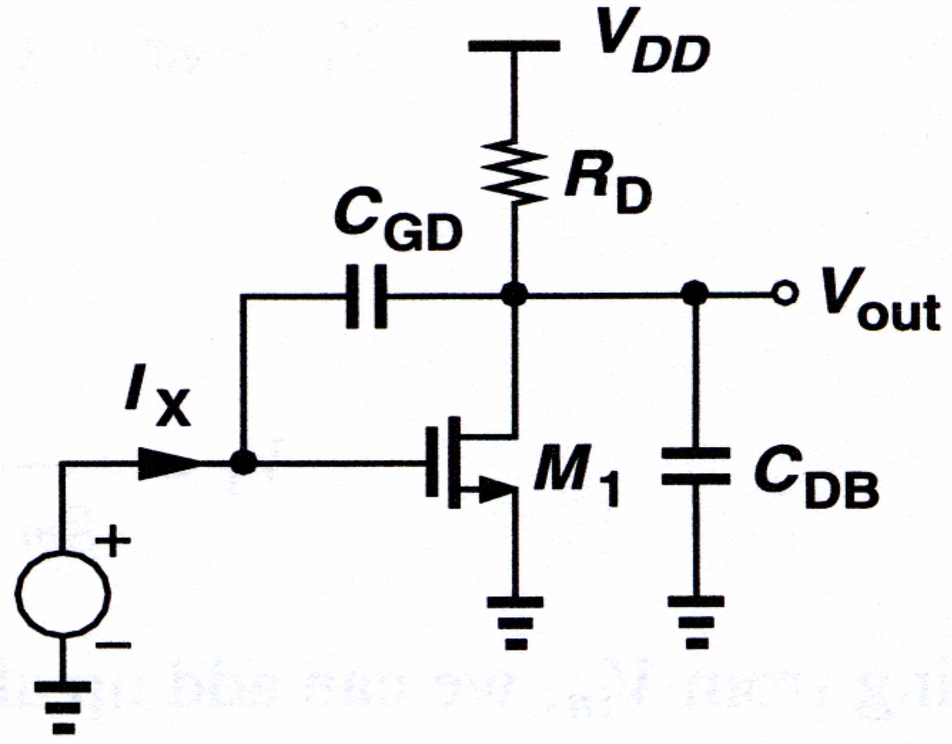
$$Z_{IN}(s) = R_s + \frac{1}{s\,(C_{GS} + (1 + g_m R_D)\,C_{GD})} \quad \text{at Low Freq}$$

Razavi

Input impedance Zin (Cgd neglected)

$$(I_X - g_m V_X)\frac{R_D}{1 + R_D C_{DB} s} + \frac{I_X}{C_{GD} s} = V_X$$

$$\frac{V_X}{I_X} = \frac{1 + R_D(C_{GD} + C_{DB})s}{C_{GD}s(1 + g_m R_D + R_D C_{DB} s)}$$



$$Z_{in} = \frac{1}{sC_{GD} \cdot (1 + g_m R_D)}$$

Low frequency Input impedance : Miller cap

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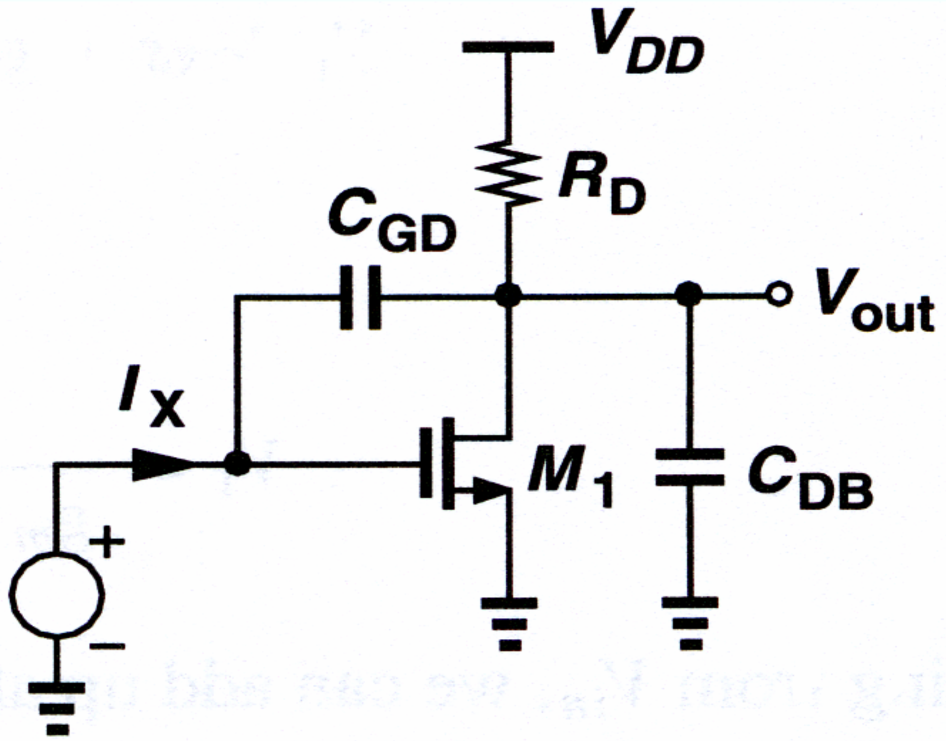
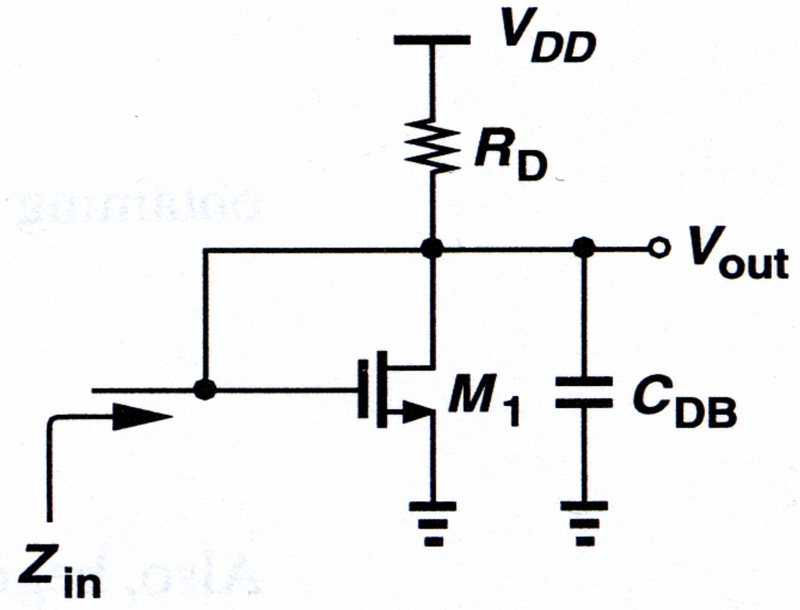
High freq Input impedance: CS amp

$$\frac{V_X}{I_X} = \frac{1 + R_D(C_{GD} + C_{DB})s}{C_{GD}s(1 + g_m R_D + R_D C_{DB}s)}$$

Capacitive at high frequency



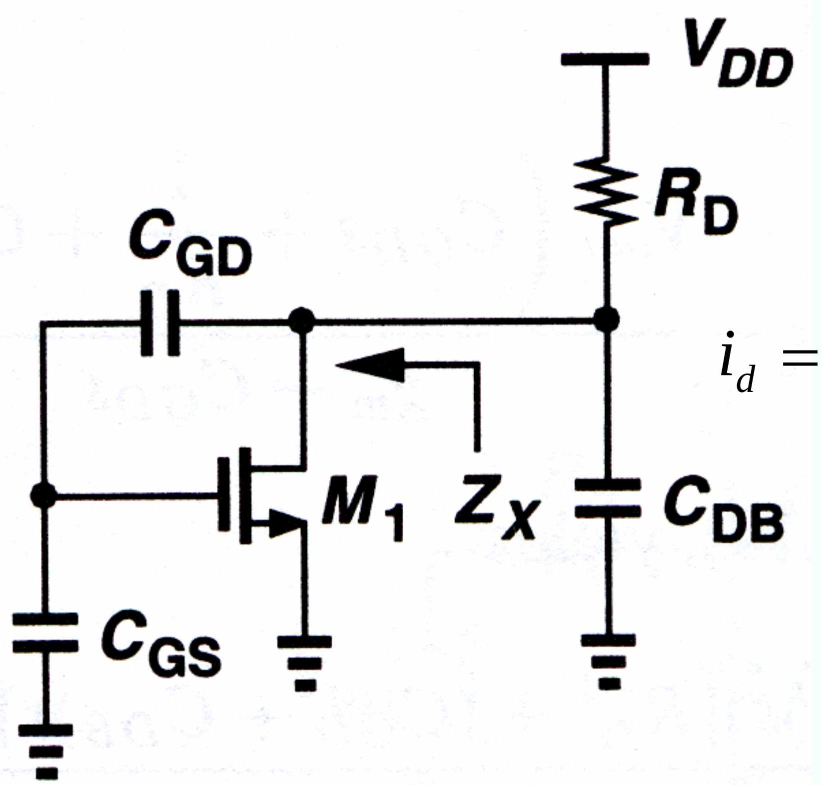


High frequency Input impedance

$$Z_{in} = \frac{1}{g_m} \| R_D \| \frac{1}{sC_{DB}}$$

$$= \frac{1}{\frac{1}{R_D} + g_m + sC_{DB}}$$

Razavi



Output impedance Zx at high frequency for high Rs

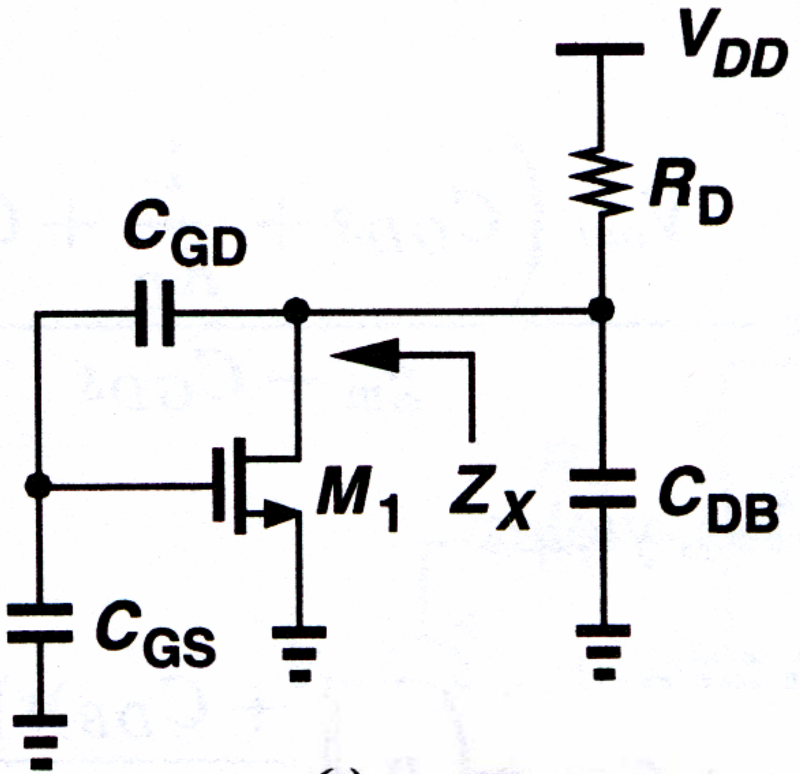
vx is applied at drain

$$i_d = g_m \cdot v_{gs} = g_m \cdot \frac{\frac{1}{C_{GS}} \cdot v_X}{\frac{1}{C_{GD}} + \frac{1}{C_{GS}}} = \frac{C_{GD}}{C_{GS} + C_{GD}} \cdot g_m v_X$$

$$C_{eq} = \frac{C_{GD} \cdot C_{GS}}{C_{GD} + C_{GS}}$$

$$Z_X = \frac{1}{C_{eq} s} \Bigg\| \left(\frac{C_{GD} + C_{GS}}{C_{GD}} \cdot \frac{1}{g_{m1}} \right)$$

High-frequency Output impedance: CS amp



$$Z_X = \frac{1}{C_{eq}s} \left\| \left(\frac{C_{GD} + C_{GS}}{C_{GD}} \cdot \frac{1}{g_{m1}} \right) \right.$$

Capacitive at high frequency

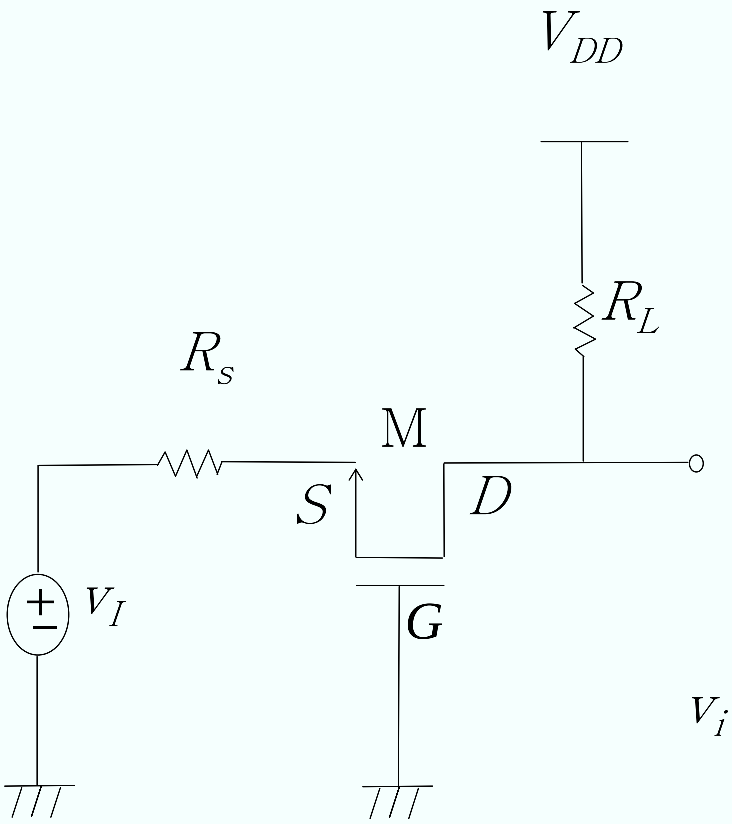
$$\omega_{out} = \frac{1}{\left[R_D \left\| \left(\frac{C_{GD} + C_{GS}}{C_{GD}} \cdot \frac{1}{g_{m1}} \right) \right. \right] (C_{eq} + C_{DB})}$$

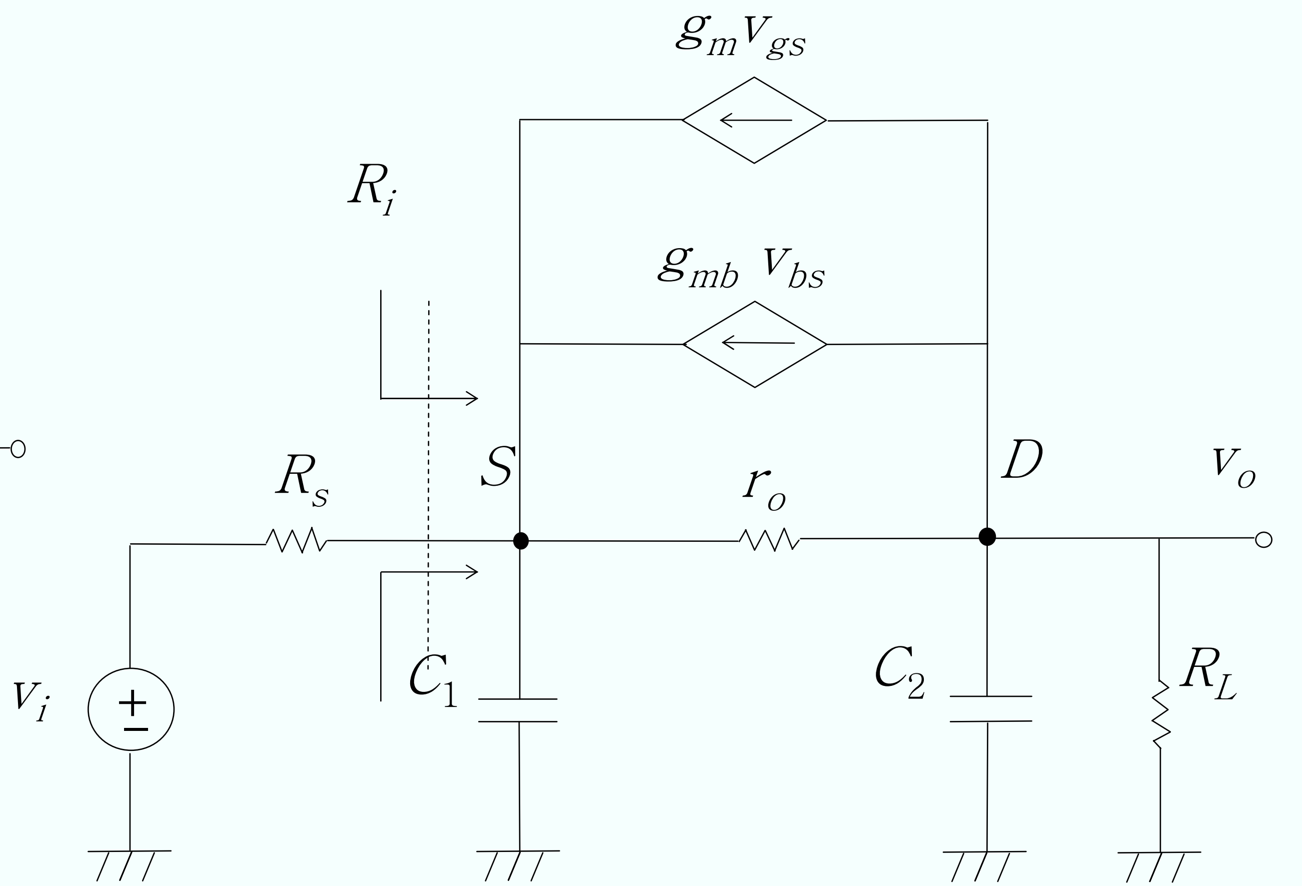
More accurate than

$$\omega_{out} = \frac{1}{R_D(C_{DB} + C_{GD})}$$

Razavi

Frequency response of CG amp



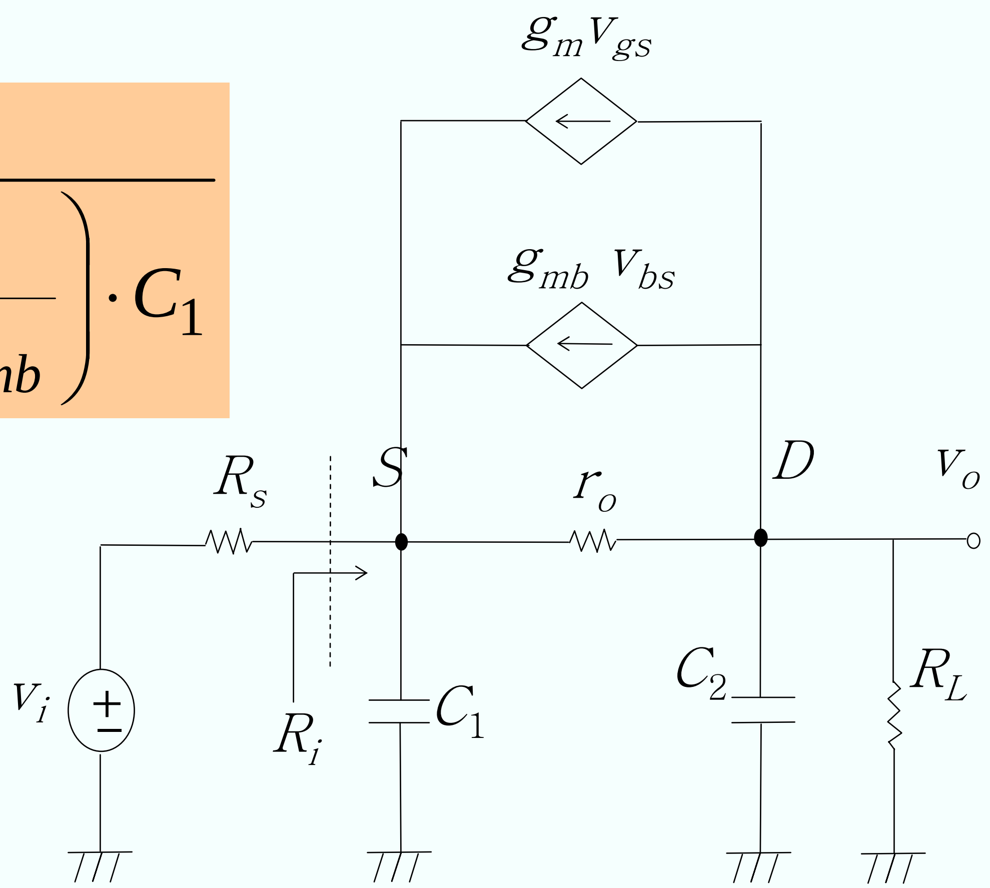


$$C_1 = C_{GS} + C_{BS} \qquad C_2 = C_{GD} + C_{BD}$$

No Capacitance across Input(S) and Output(D) => No Miller effect => high speed

$$\omega_p = \frac{1}{(R_s \parallel R_i)\cdot C_1}$$

$$\frac{1}{R_s C_1} \le \omega_p \le \frac{1}{\left(R_s \,\Big\|\, \dfrac{1}{g_m + g_{mb}}\right)\cdot C_1}$$



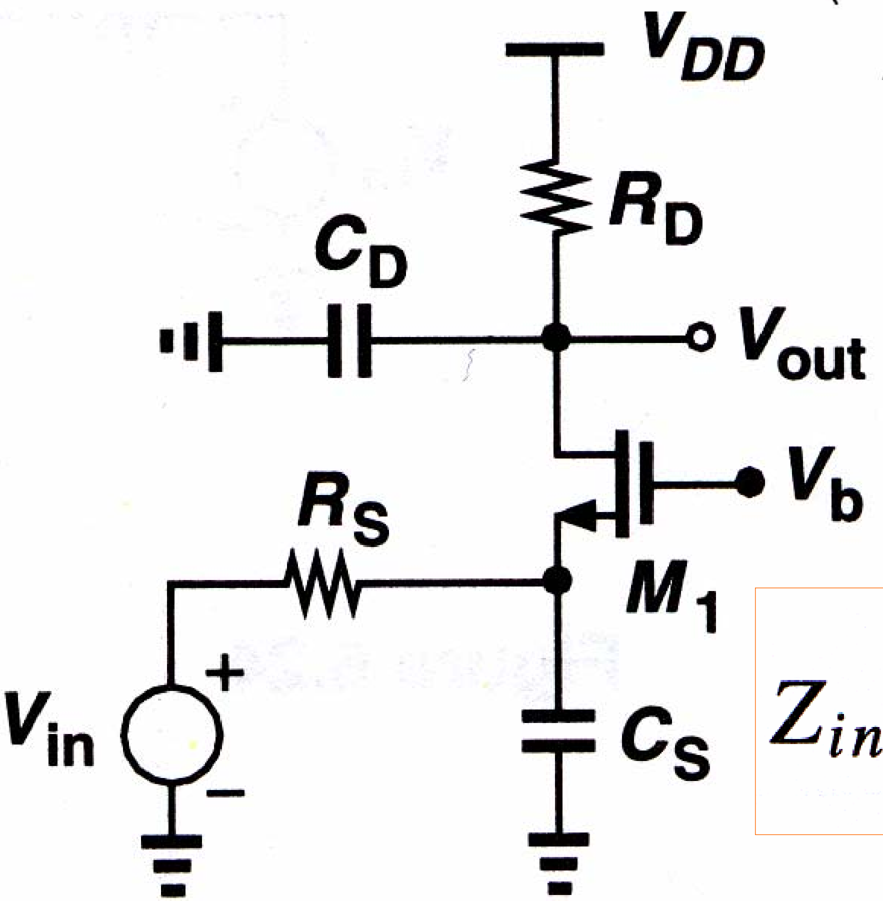
$$\frac{1}{R_s C_1} = \frac{1}{R_s\cdot\left\{\frac{2}{3}WLC_{ox} + W\cdot CGSO + CBS\right\}}$$

$$> \frac{1}{R_s\cdot\left\{W\cdot CGDO\cdot(g_m r_o + 1) + \frac{2}{3}\cdot WLC_{ox} + W\cdot CGSO + L\cdot CGBO\right\}}$$

CS amp

Frequency response of CG amp

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \frac{1}{\left(1 + \frac{C_S}{g_m + g_{mb} + R_S^{-1}} s\right)(1 + R_D C_D s)}$$



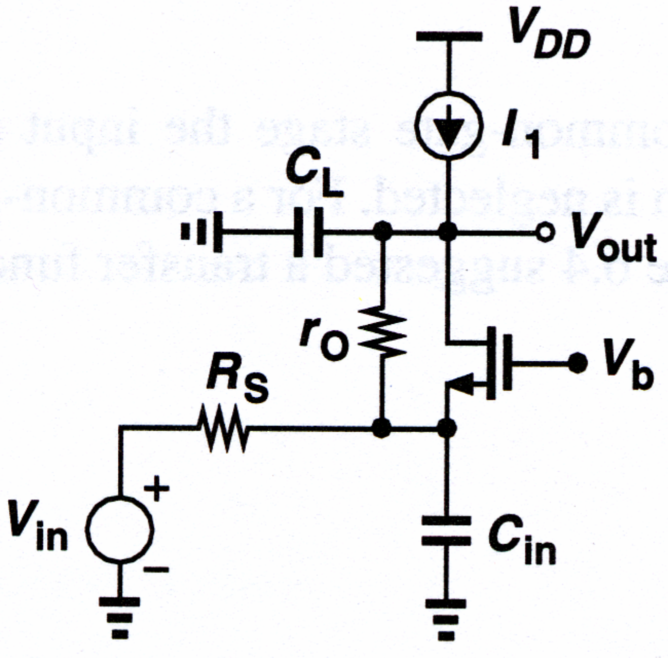
$$R_i \triangleq \frac{v_i}{i_i} = \frac{1}{g_m + g_{mb} + \frac{1}{r_o}} \cdot \left(1 + \frac{R_L}{r_o}\right)$$

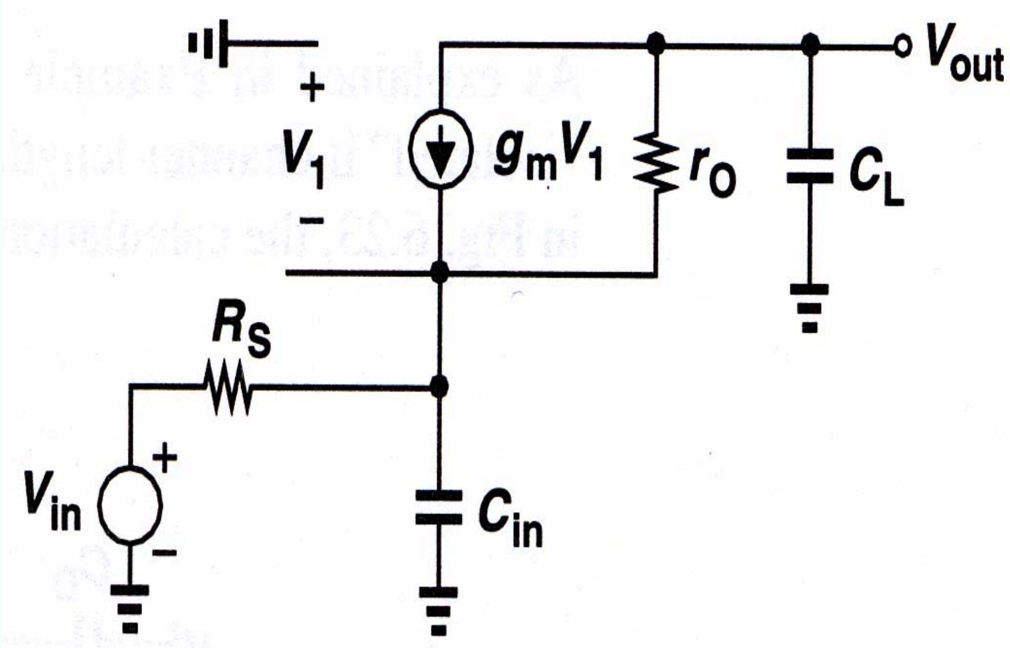
$$Z_{in} \approx \frac{Z_L}{(g_m + g_{mb}) r_O} + \frac{1}{g_m + g_{mb}}$$

Zin : capacitive

Razavi

CG amp with current source load





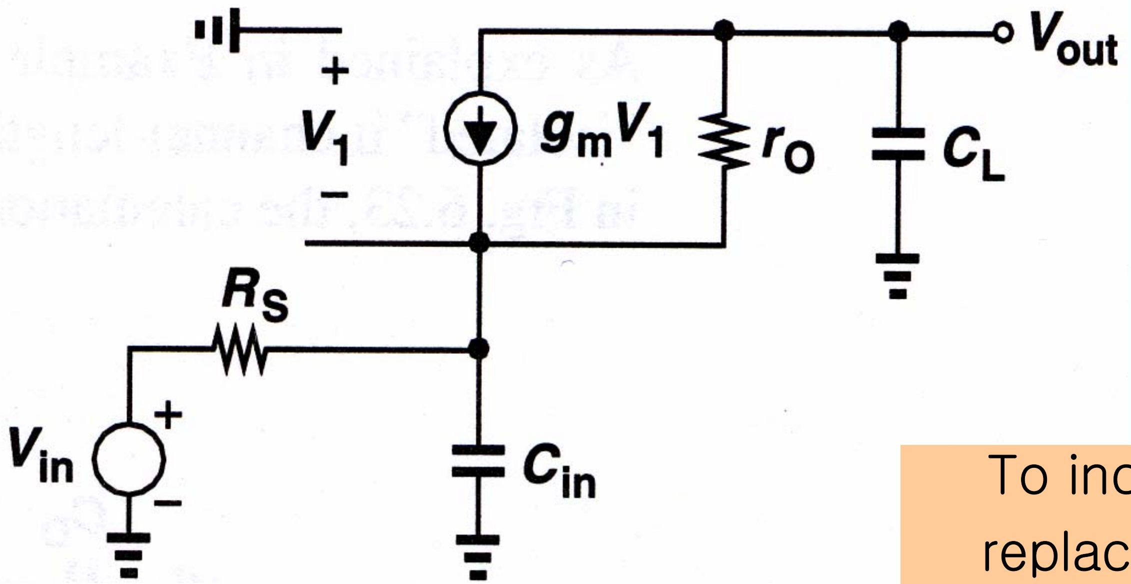
$$(-V_{out}C_L s + V_1 C_{in} s)R_S + V_{in} = -V_1$$

$$V_1 = -\frac{-V_{out}C_L s R_S + V_{in}}{1 + C_{in}R_S s}$$

$$r_O(-V_{out}C_L s - g_m V_1) - V_1 = V_{out}$$

Razavi

CG amp with current source load



To include body effect, replace gm by gm+gmb

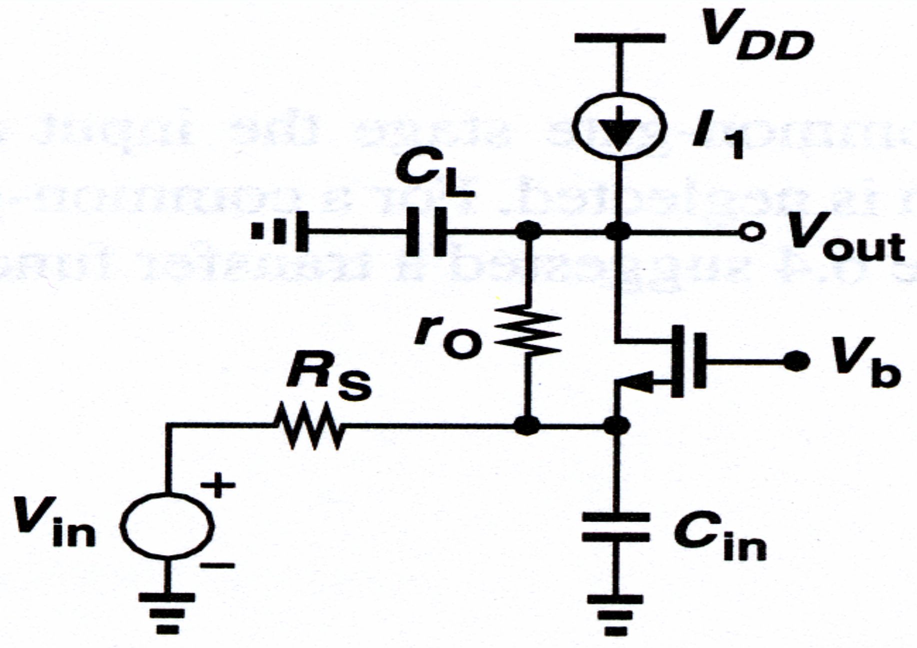
$$\frac{V_{out}}{V_{in}}(s) = \frac{1 + g_m r_O}{r_O C_L C_{in} R_S s^2 + [r_O C_L + C_{in} R_S + (1 + g_m r_O) C_L R_S] s + 1}$$

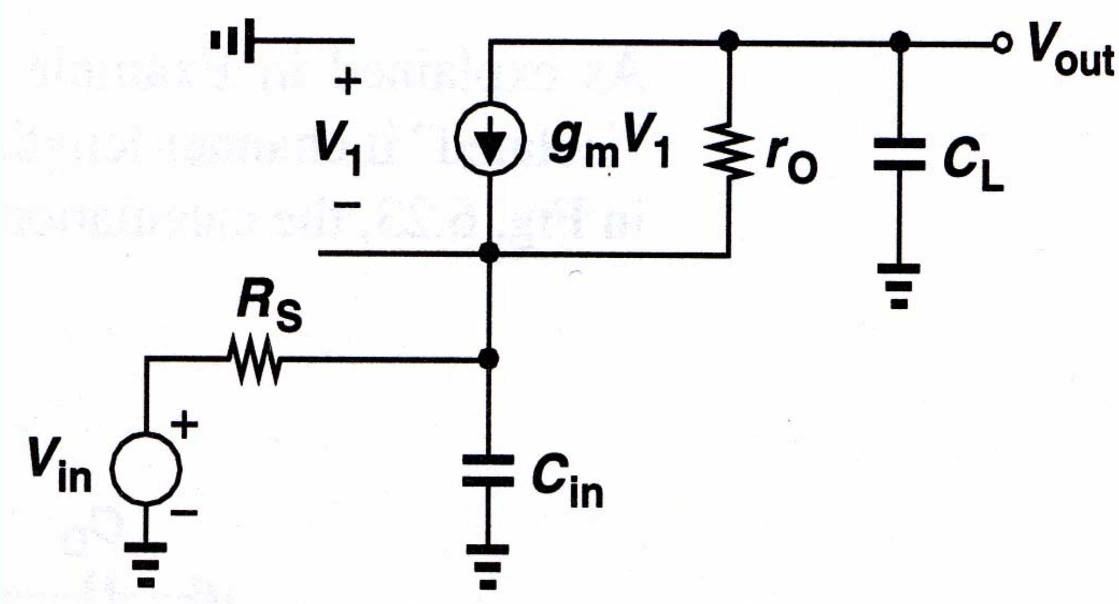
$$Z_{in} = \frac{1}{g_m + g_{mb}} + \frac{1}{C_L s} \cdot \frac{1}{(g_m + g_{mb}) r_O}$$

Zin(s): Capacitive

Razavi

CG amp with current source load





$$Z_{in} = \frac{1}{g_m + g_{mb}} + \frac{1}{C_L s} \cdot \frac{1}{(g_m + g_{mb}) r_O}$$

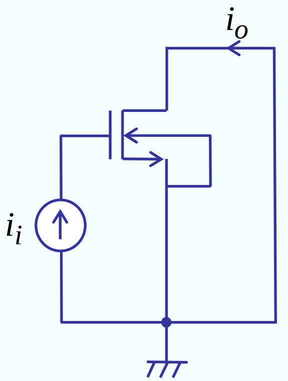
At high frequency, load shunted by CL ➔

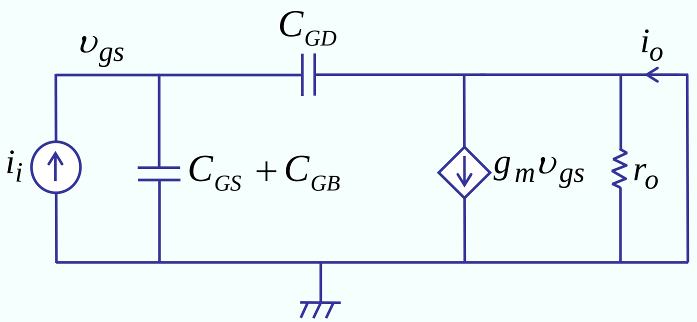
Lower output impedance ➔ HF Zin ~= 1/(gm+gmb)

$$\omega_{p,in} = \frac{1}{\left(R_S \middle\| \frac{1}{g_m + g_{mb}}\right) C_{in}}$$

Razavi

f_T : frequency where the magnitude of small-signal short circuit current gain becomes 1





$$i_o = g_m \upsilon_{gs} - j\omega C_{GD} v_{gs} \approx g_m v_{gs} \qquad for \quad \omega << \frac{g_m}{C_{GD}}$$

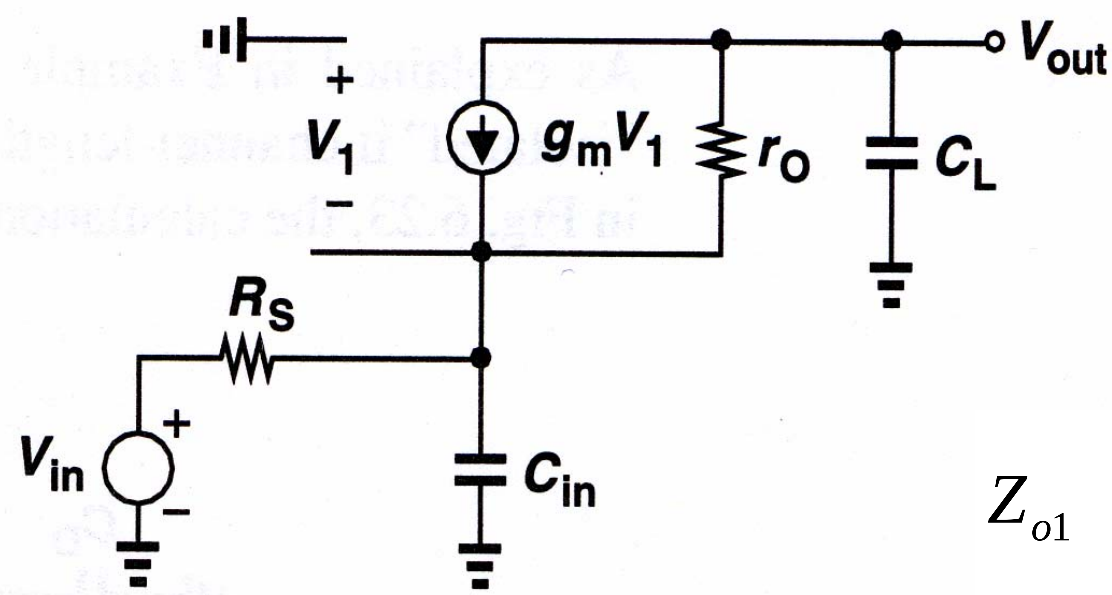
$$i_o = g_m \cdot \frac{i_i}{j\omega\left(C_{GS} + C_{GD} + C_{GB}\right)}$$

$$|A_i| = \left|\frac{i_o}{i_i}\right| = 1$$

$$\omega_T = 2\pi f_T = \frac{g_m}{C_{GS} + C_{GB} + C_{GD}}$$

Currently several tens of GHz

Output impedance of CG amp



$$Z_{o1} = (g_m r_o + 1)\cdot\frac{R_s}{1+sR_sC_{in}} + r_o$$

$$Z_{out}(s) = Z_{o1}(s) \,\|\, \frac{1}{sC_L}$$

$$= \frac{(g_m r_o + 1)R_s + r_o + s r_o R_s C_{in}}{1 + s\{R_sC_{in} + C_L(g_m r_o R_s + R_s + r_o)\} + s^2 r_o R_s C_{in} C_L}$$

Zout(s): capacitive

CD amp with DC current source load

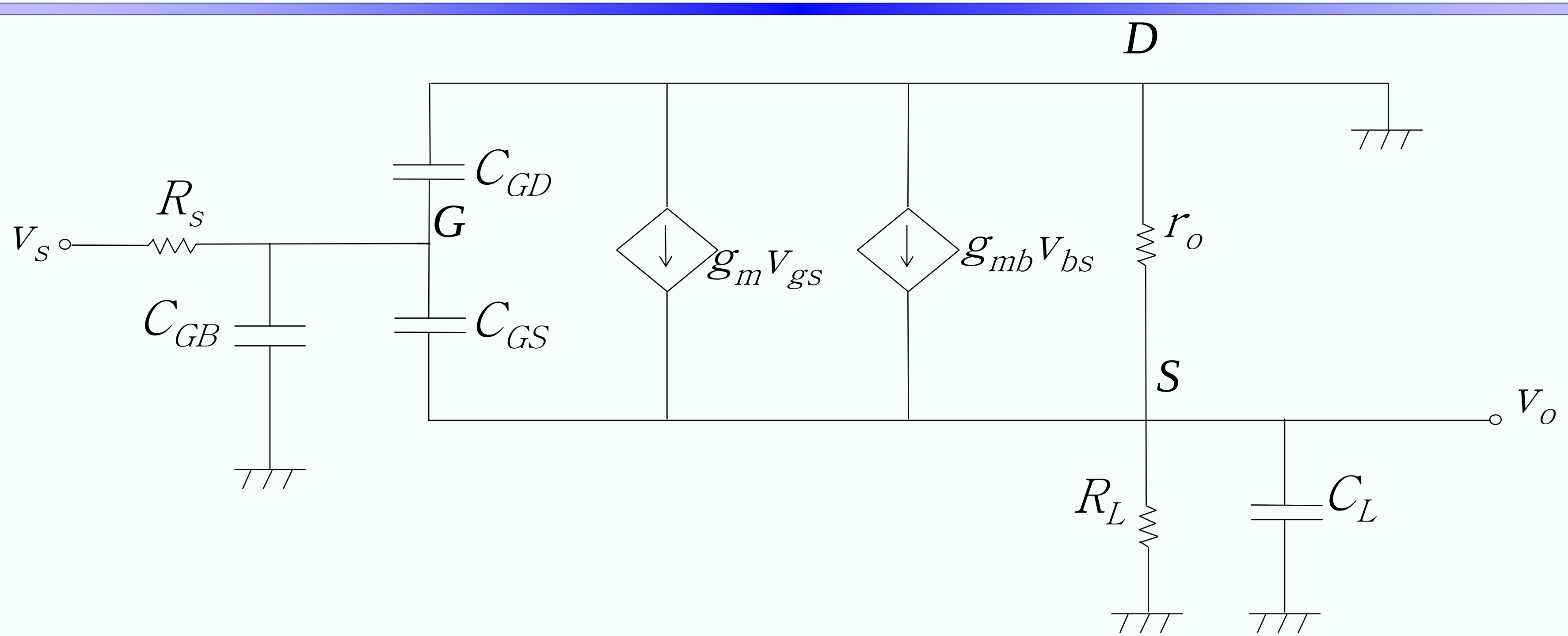


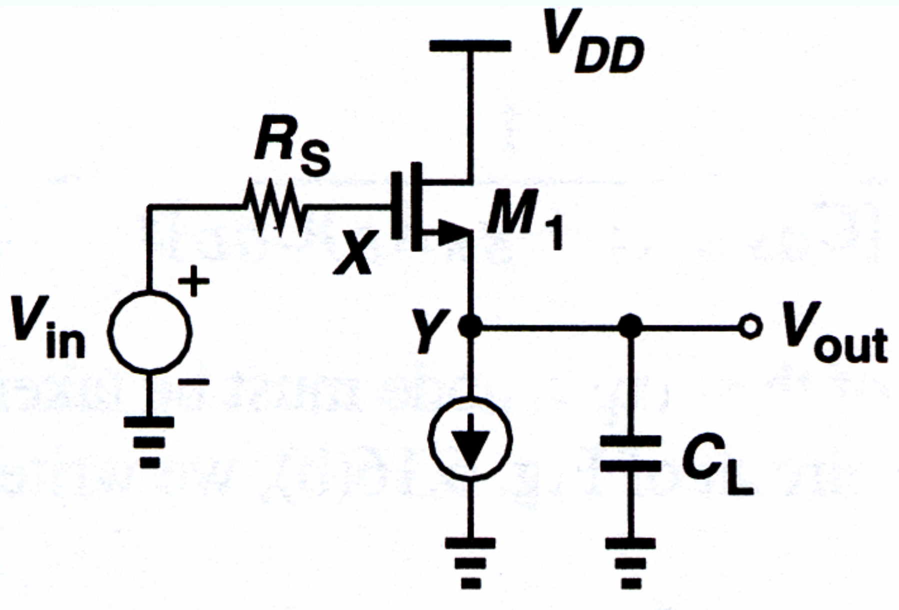
Fig 4.4.5 Small-signal equivalent circuit for frequency response

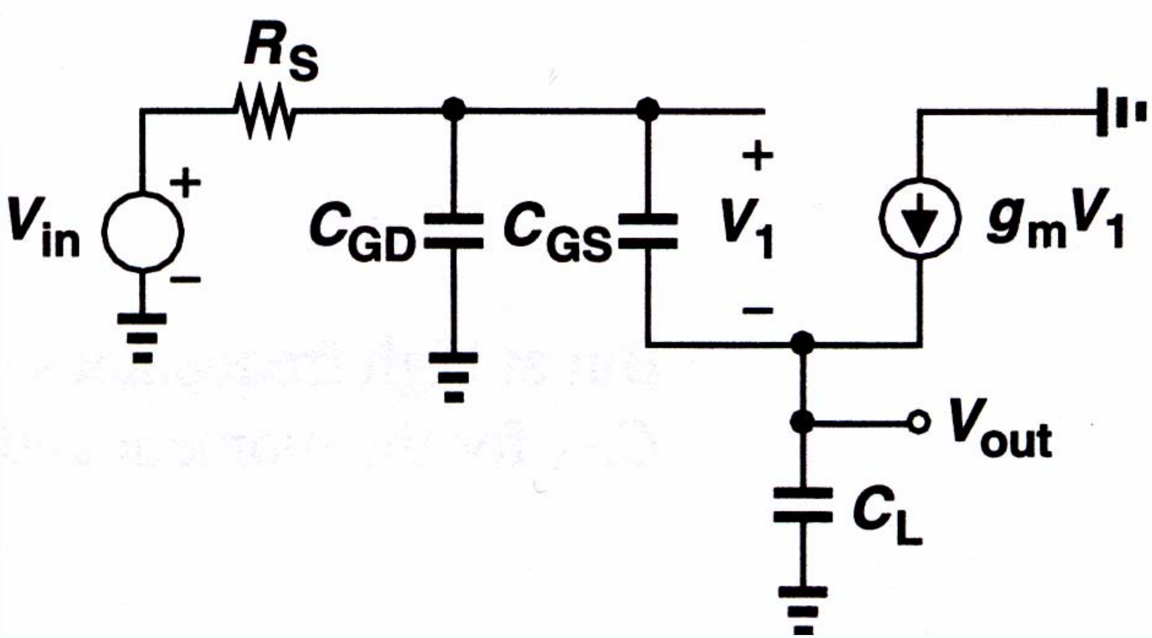
$$\omega_{-3dB(\text{input})} = \frac{1}{R_s \cdot \left\{ C_{GD} + C_{GB} + C_{GS} \cdot \left(\frac{g_{mb}}{g_m + g_{mb}} \right) \right\}}$$

$$\omega_{-3dB(\text{output})} \approx \frac{g_m + g_{mb}}{C_L}$$

Faster than CS, CG amp

CD amp with current source load



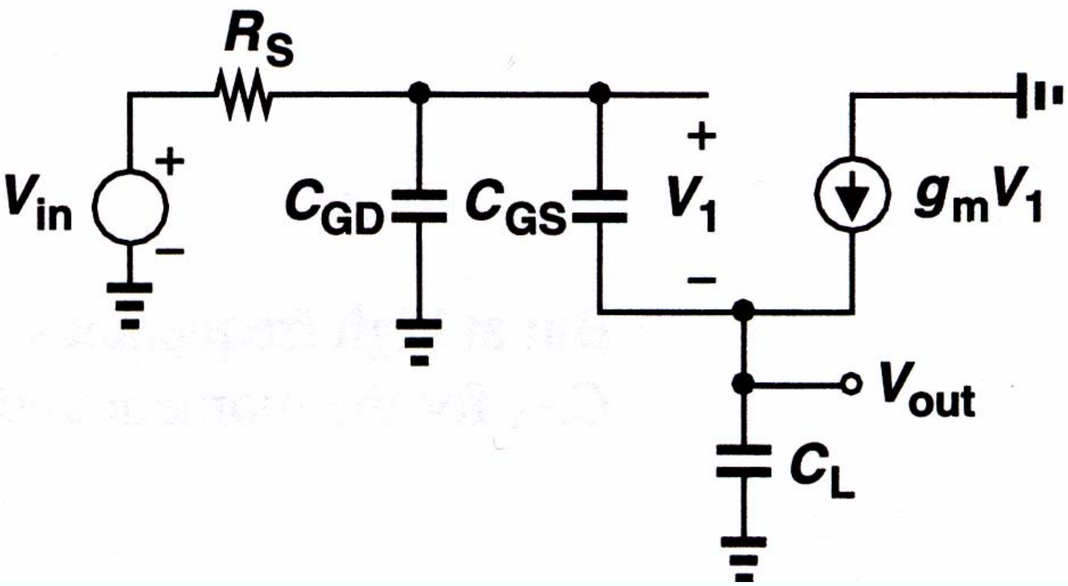


gmb, ro neglected

Razavi

CD amp with current source load

gmb, ro neglected



$$V_1 C_{GS} s + g_m V_1 = V_{out} C_L s$$

$$V_1 = \frac{C_L s}{g_m + C_{GS} s} V_{out}$$

$$V_{in} = R_S[V_1 C_{GS} s + (V_1 + V_{out}) C_{GD} s] + V_1 + V_{out}$$

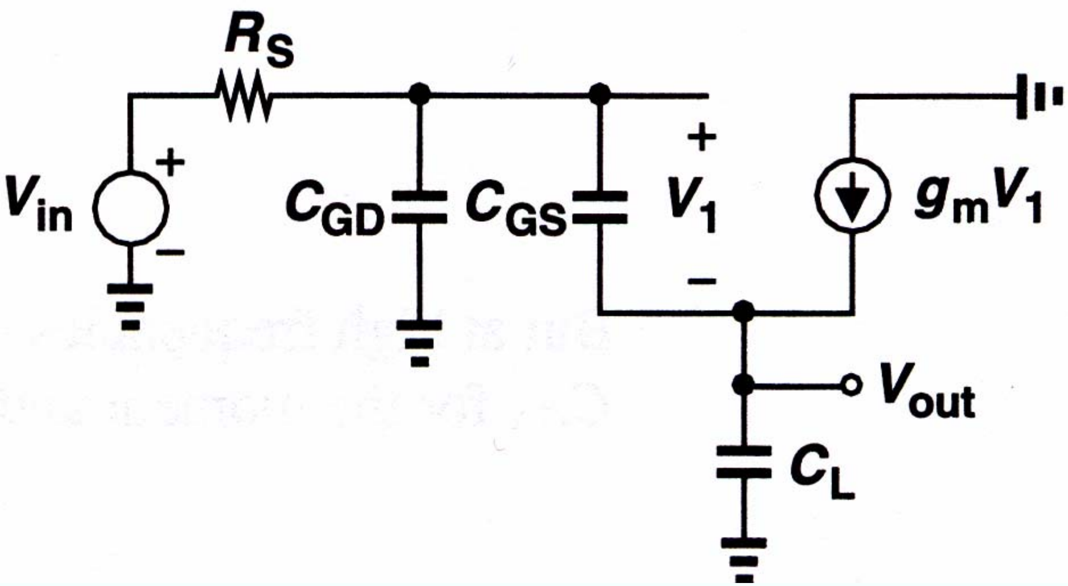
$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS} s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$

Razavi

CD amp with current source load

gmb neglected

$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS}s}{R_S(C_{GS}C_L + C_{GS}C_{GD} + C_{GD}C_L)s^2 + (g_m R_S C_{GD} + C_L + C_{GS})s + g_m}$$



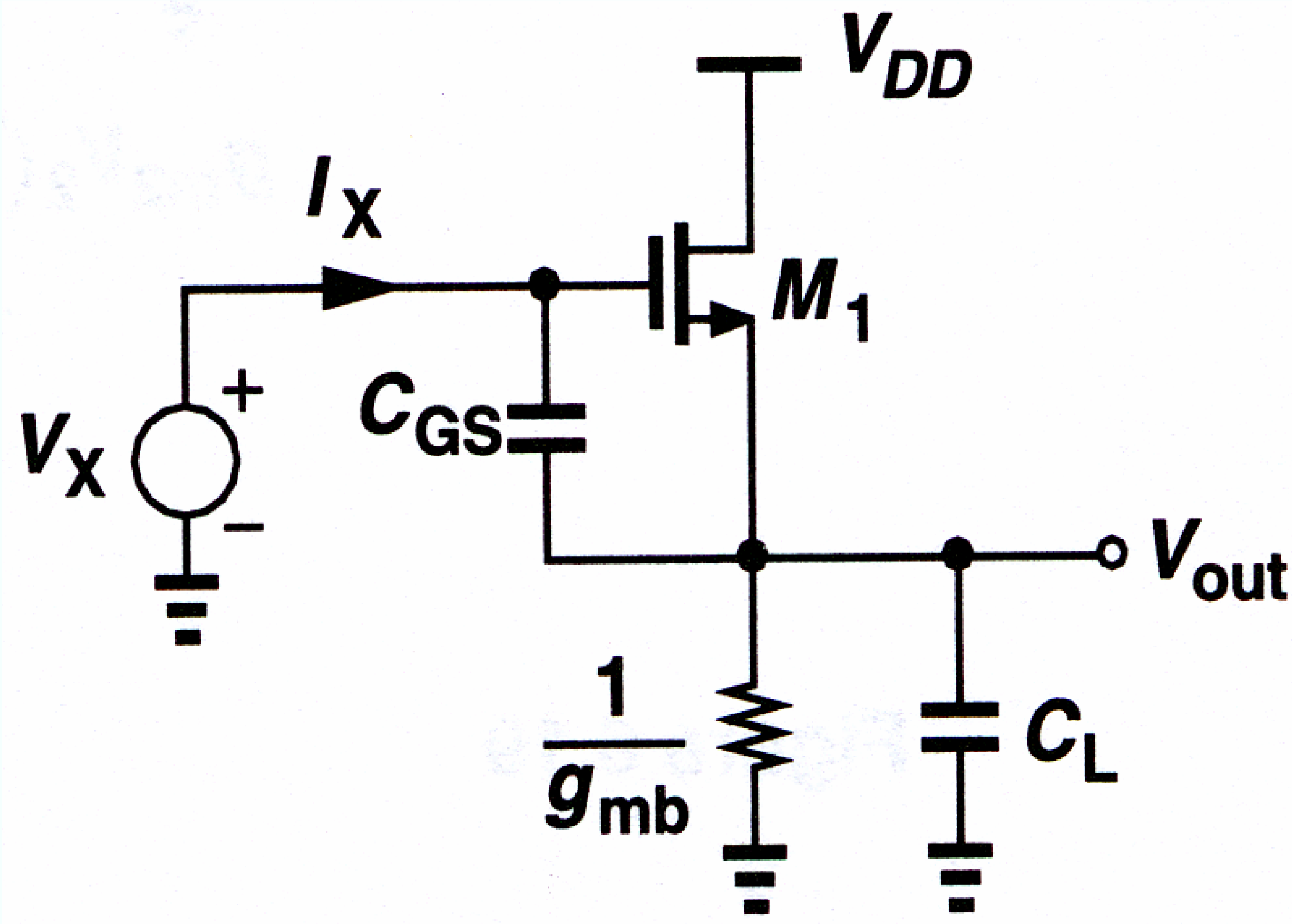
$$\omega_{p1} \approx \frac{g_m}{g_m R_S C_{GD} + C_L + C_{GS}} = \frac{1}{R_S C_{GD} + \dfrac{C_L + C_{GS}}{g_m}}$$

Dominant pole

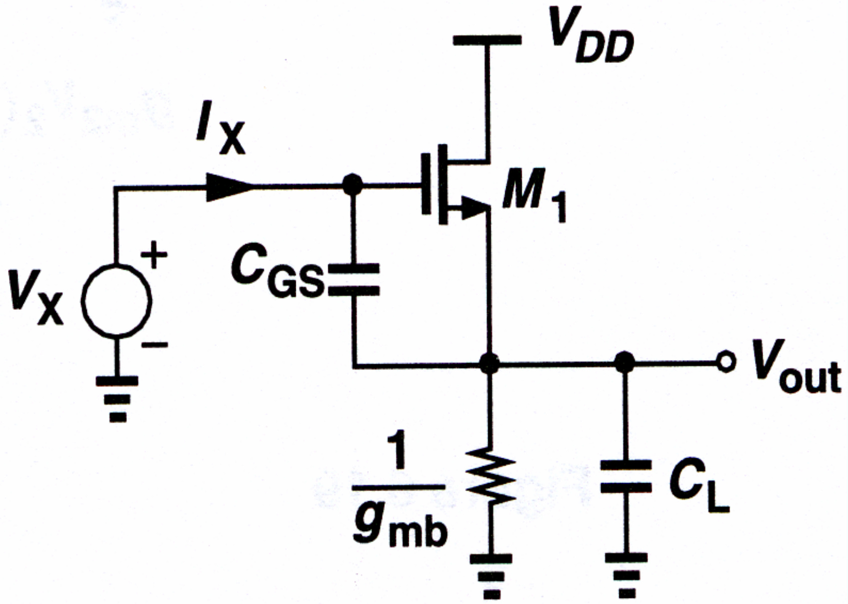
Negative real zero: −gm/Cgs :
due to feedforward through Cgs across non-inverting amp
(negative real zero: helpful for phase margin)

Razavi

Input impedance Zin



Razavi



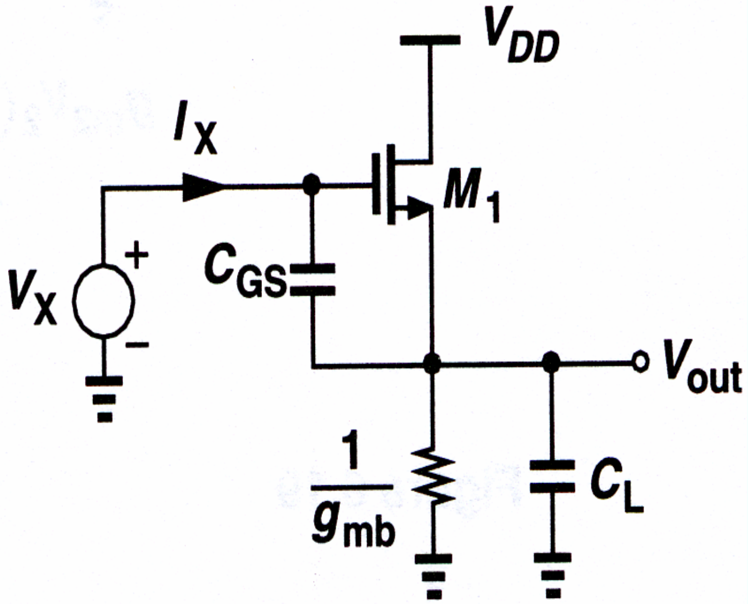
Input impedance Zin

$$V_X = \frac{I_X}{C_{GS}s} + \left(I_X + \frac{g_m I_X}{C_{GS}s}\right)\left(\frac{1}{g_{mb}} \Bigg\| \frac{1}{C_L s}\right)$$

$$Z_{in} = \frac{1}{C_{GS}s} + \left(1 + \frac{g_m}{C_{GS}s}\right)\frac{1}{g_{mb} + C_L s}$$

Razavi

$$Z_{in} = \frac{1}{C_{GS}s} + \left(1 + \frac{g_m}{C_{GS}s}\right)\frac{1}{g_{mb} + C_L s}$$



Input impedance Zin

$$Z_{in} \approx \frac{1}{C_{GS}s}\left(1 + \frac{g_m}{g_{mb}}\right) + \frac{1}{g_{mb}}$$

At LF, Zin=series Cgs*gmb/(gm+gmb) and 1/gmb

$$Z_{in} \approx \frac{1}{C_{GS}s} + \frac{1}{C_L s} + \frac{g_m}{C_{GS}C_L s^2}$$

At HF, Zin=series Cgs, CL and negative R

Zin(s): Capacitive

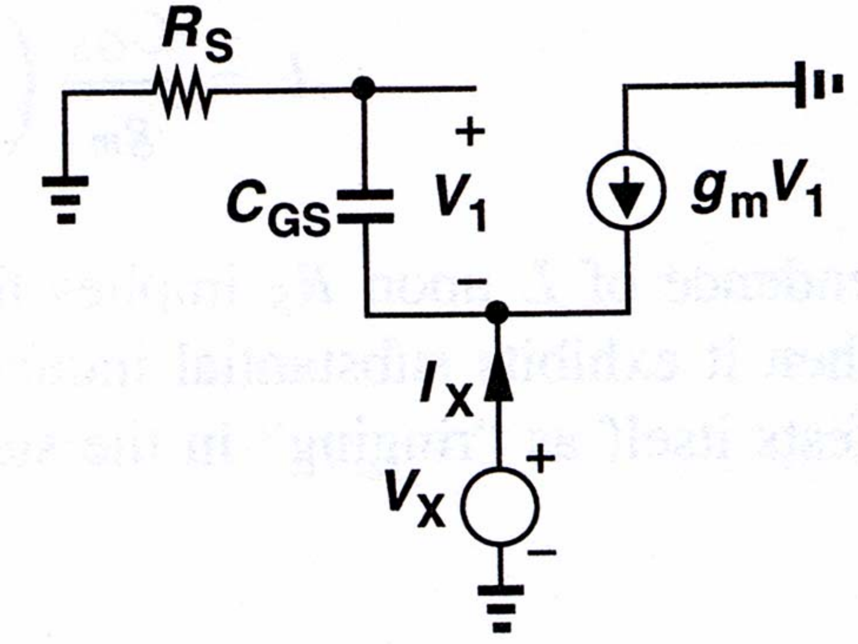
Razavi

Output impedance Zout

Gmb neglected

$$s C_{gs} v_1 + g_m v_1 = -i_x$$

$$s C_{gs} v_1 R_s + v_1 = -v_x$$



$$Z_{out} = \frac{V_X}{I_X}$$

$$= \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s}$$

Zout = 1/gm at LF

Zout = Rs at HF (Cgs shorts G and S at HF)

Razavi

CD amp with current source load

Output impedance Zout

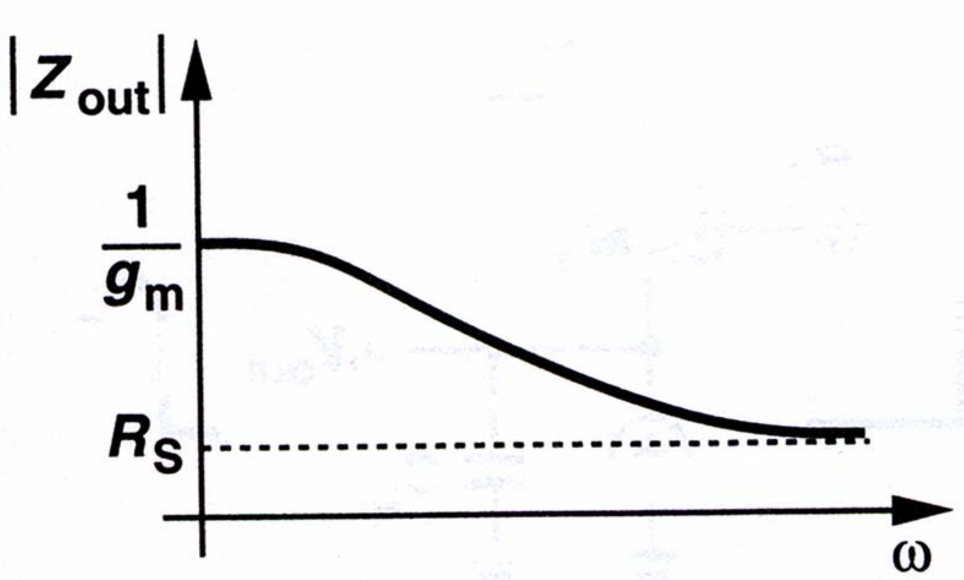
$$Z_{out} = \frac{V_X}{I_X}$$

$$= \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s}$$

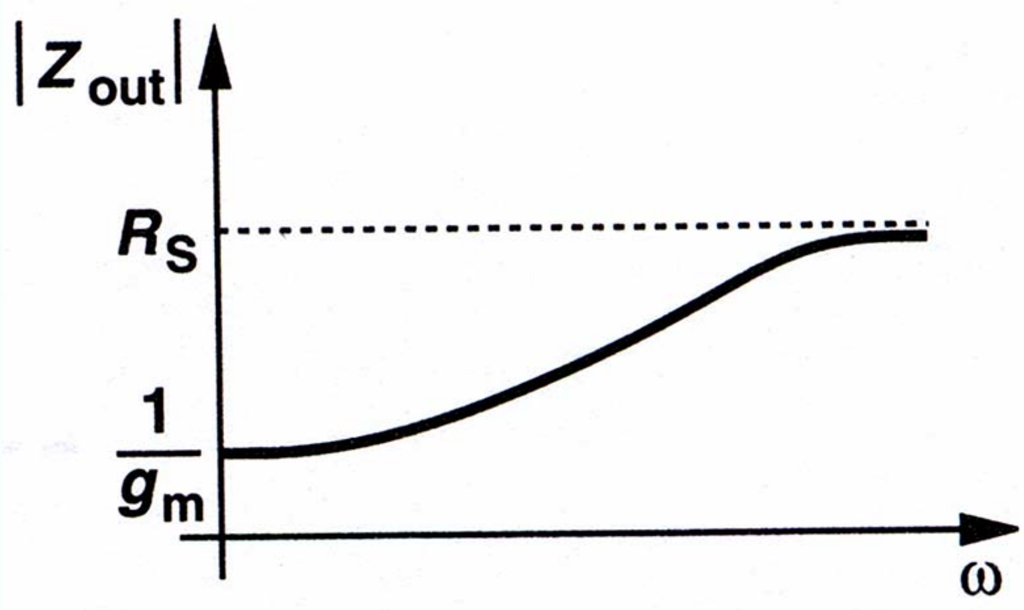
Gmb neglected

Zout = 1/gm at LF

Zout = Rs at HF (Cgs shorts G and S at HF)



Rs < 1/gm: Capacitive



Rs > 1/gm: Inductive (Troublesome)

Razavi

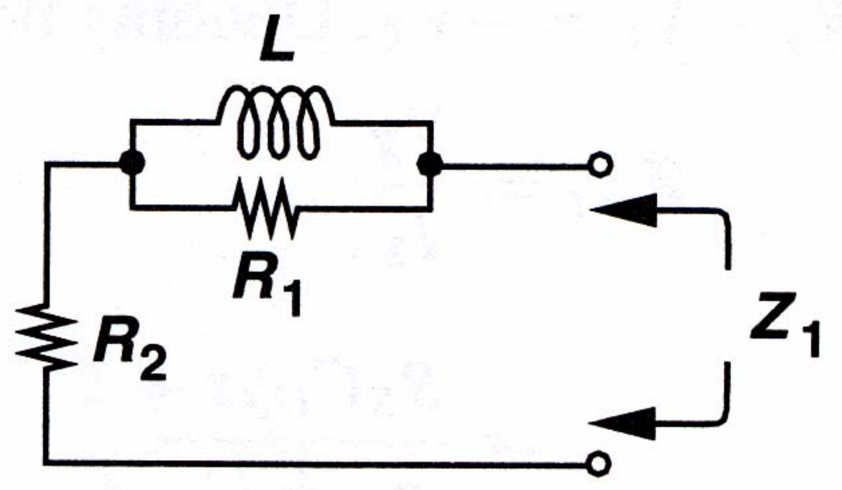
CD amp with current source load

$$Z_{out} = \frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{g_m + C_{GS} s}$$

gmb neglected

Output impedance Zout when Rs > 1/gm

$$\frac{1}{Z_{out} - \frac{1}{g_m}} = \frac{1}{R_S - \frac{1}{g_m}} + \frac{1}{\frac{C_{GS} s}{g_m}\left(R_S - \frac{1}{g_m}\right)}$$



R1 = Rs – (1/gm)

R2 = 1/gm

$$L = \frac{C_{GS}}{g_m}\left(R_S - \frac{1}{g_m}\right)$$

1/gmb & ro in parallel with Z1

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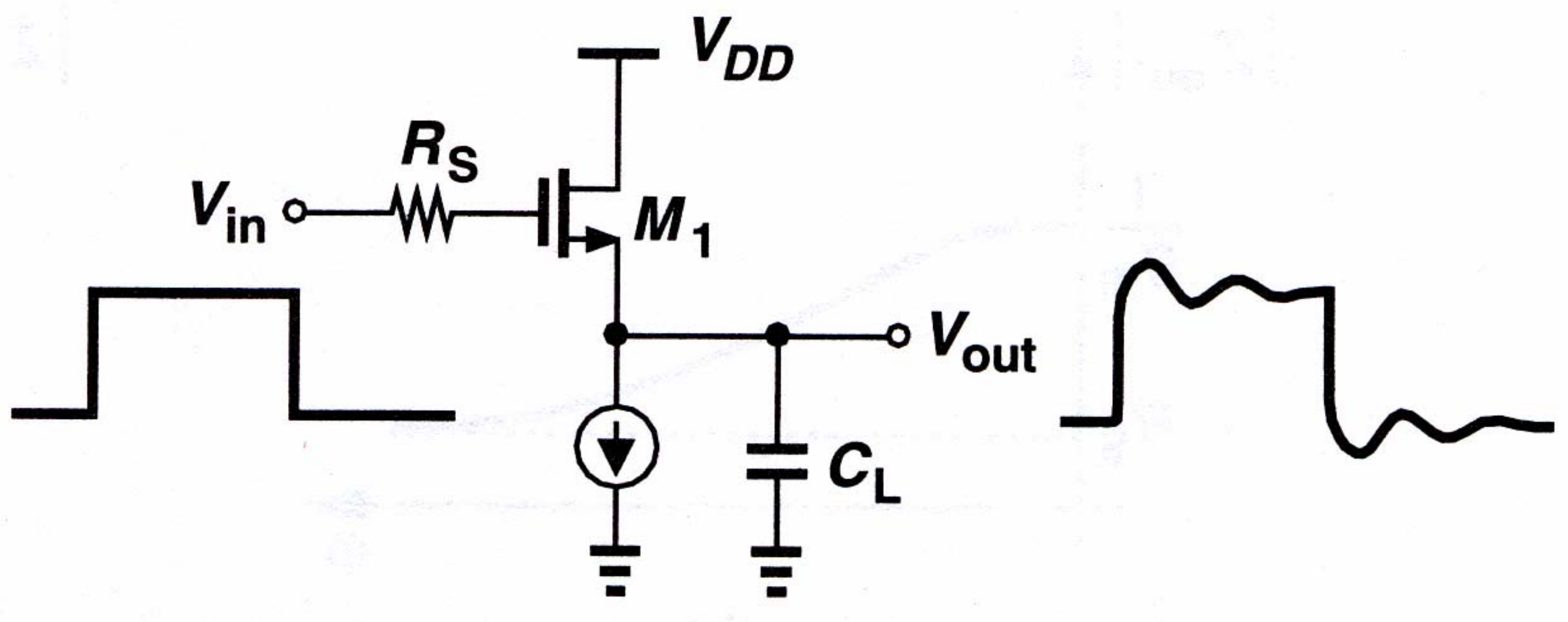
Output impedance Zout when Rs > 1/gm



Ringing due to inductive components of Zout

(when Rs > 1/gm)

LC resonance => ringing

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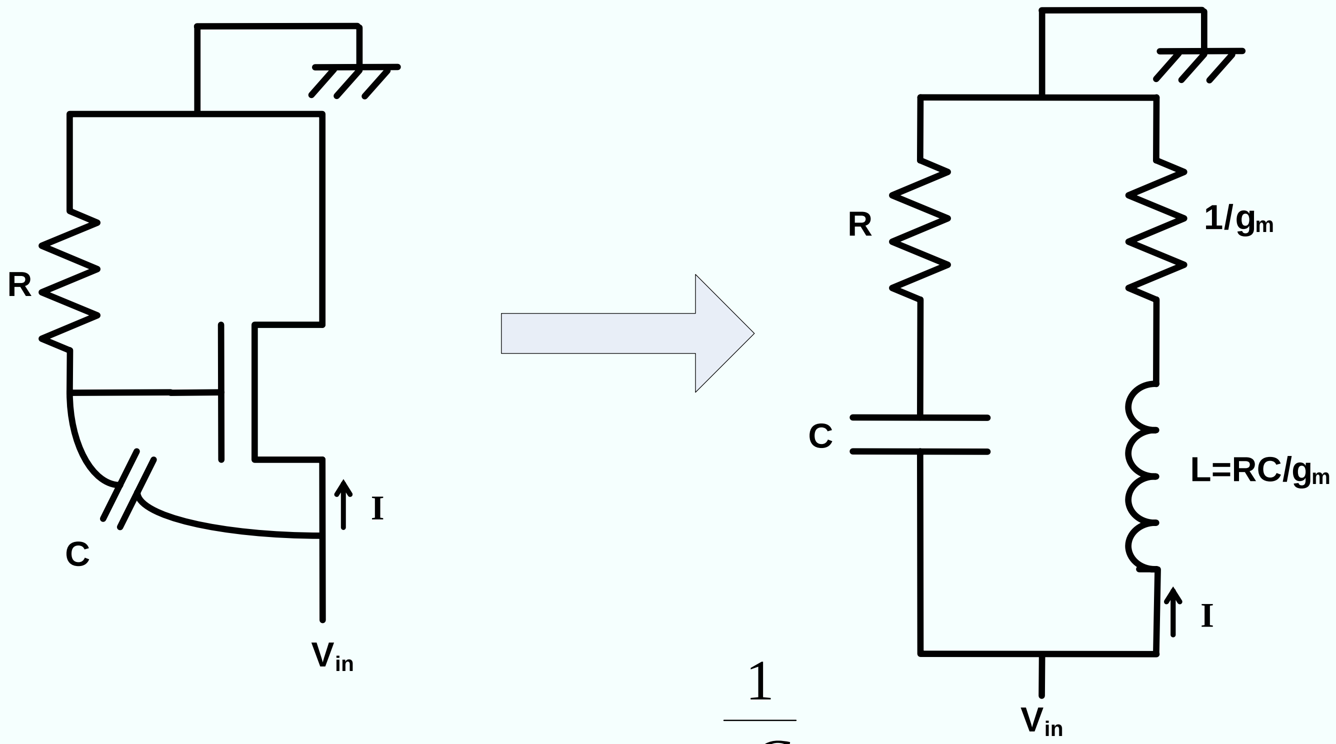
Active Inductor



$$I = -V_{gs} \cdot g_m, \quad V_{gs} = -V_{in} \cdot \frac{\frac{1}{sC}}{\frac{1}{sC} + R}$$

$$\therefore I = V_{in} \frac{\frac{1}{sC}}{\frac{1}{sC} + R} \cdot g_m, \quad \frac{V_{in}}{I} = \frac{1}{g_m} \cdot (1 + sCR) = \frac{1}{g_m} + sL$$

Freq response of CS, CG, CD amp

	−3dB Frequency	Zin(s)	Zout(s)
CS amp	Low	Capacitive	Capacitive
CG amp	High	Capacitive	Capacitive
CD amp	Very High	Capacitive	Capacitive If Rs < 1/gm Inductive If Rs > 1/gm