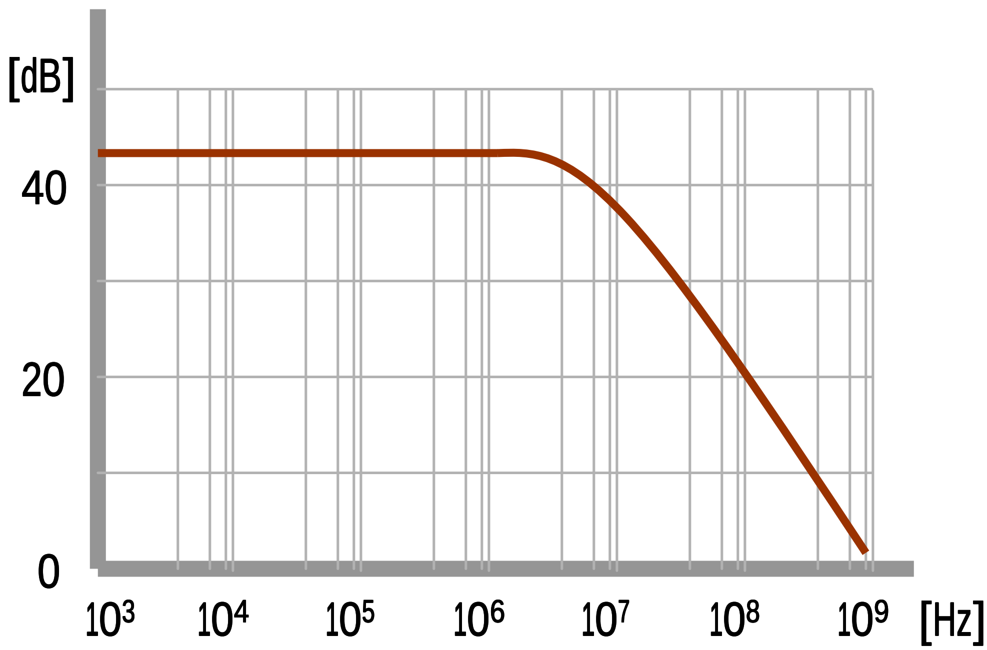
# VLSI Design II

*Frequency Response of Single Stage Amplifiers*




## Circuit Analysis

- *the precise way:* solving complex equations
- *the approximate way:* find the dominant pole
- *the handy way:* let Spice do it precisely

Goal: You are able to identify the dominant pole in a transistor circuit. You can approximately determine the contribution of each node in a circuit to the total frequency response.

# *Outline*

- Frequency response
  - common-source amplifier
  - source-follower amplifier
  - source-follower amplifier with compensation technique
  - cascode gain stage

- Johns&Martin
  - frequency response (chap 3.11)
- Gray&Meyer
  - estimation of dominant poles
  - zero-Value Time Constant Analysis (pp500 ff)

    (Analysis and Design of Analog Integrated Circuits, 3rd edition, Wiley and Sons, ISBN-0471-59984-0)

- Exercises
  - hand calculations
  - spice simulations

# *Frequency Response*
# *Dominant Pole Approximation*

- precise calculation of frequency response is a complex task and thus different approximation methods exist
- one method is the zero-value time constant analysis
- first some ideas about dominant-pole approximation are developed

transfer function by small-signal analysis

$$A(s) = \frac{N(s)}{D(s)} \frac{a_0 + a_1 s + a_2 s^2 + \cdots + a_m s^m}{1 + b_1 s + b_2 s^2 + \cdots + b_n s^n}$$

very often the zeros are unimportant, thus

$$A(s) = \frac{K}{\left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right)\cdots\left(1 - \frac{s}{p_n}\right)}$$

Where K is a constant and $p_1, p_2$ ... are poles of the transfer function,

thus $$b_1 = \sum_{i=1}^{n} \left(-\frac{1}{p_i}\right)$$

# *Dominant Pole Approximation (con't 2)*

$$b_1 = \sum_{i=1}^{n}\left(-\frac{1}{p_i}\right)$$

an important practical case occurs when one pole is dominant

$$|p_1| << |p_2|, |p_3|, \ldots \qquad \left|\frac{1}{p_1}\right| >> \left|\sum_{i=2}^{n}\left(-\frac{1}{p_i}\right)\right|$$

thus $b_1 \cong \left|\frac{1}{p_1}\right|$

the gain magnitute in the frequency domain is

$$A(j\omega) = \frac{K}{\sqrt{\left(1+\left(\frac{\omega}{p_1}\right)^2\right)\left(1+\left(\frac{\omega}{p_2}\right)^2\right)\cdots\left(1+\left(\frac{\omega}{p_n}\right)^2\right)}}$$

with a dominant pole we simply get

$$|A(j\omega)| \cong \frac{K}{\sqrt{\left(1+\left(\frac{\omega}{p_1}\right)^2\right)}}$$

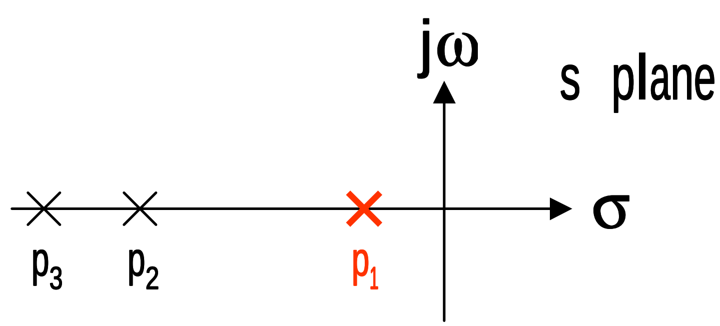
# *Dominant Pole Approximation (con't 3)*

this approximation will be quite accurate as long as $\omega \cong |p_1|$

thus for a dominant pole situation the -3dB frequency is

$$\omega_{-3dB} \cong |p_1| \qquad\qquad \omega_{-3dB} \cong \frac{1}{b_1}$$

pole plot for a circuit with a dominant pole



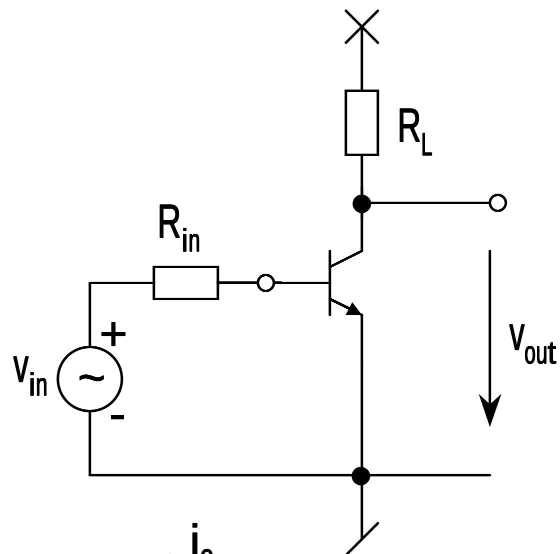
# *Zero-Value Time Constant*

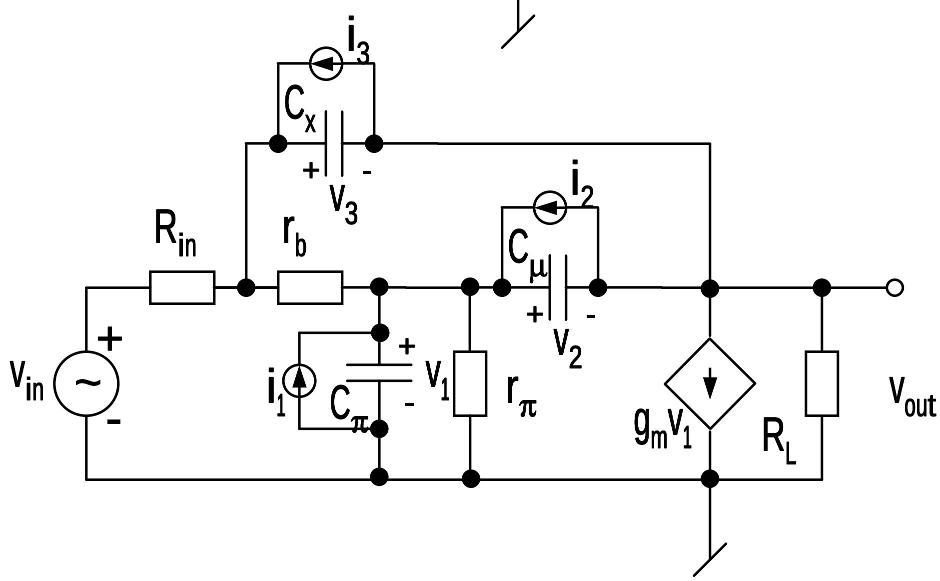
Method for finding the time constant associated with a capacitor in the small signal equivalent circuit

- replace the capacitor $C_x$ by a voltage source $V_x$
- set all independent sources to ground
- set all other network capacitors to zero
- find admittance $Y_x$ ($=1/R_x$) which is driven by a voltage source $V_x$
- the time constant $\tau_x$ is given by:

$$\tau_x = R_x C_x$$

# *Frequency Response*
# *Zero-Value Time Constant*




We can show that with this choice od variables the circuit equations are of the form:

$$i_1 = (g_{11} + sC_\pi)v_1 + g_{12}v_2 + g_{13}v_3$$

$$i_2 = g_{21}v_1 + (g_{22} + sC_\mu)v_2 + g_{23}v_3$$

$$i_3 = g_{31}v_1 + g_{32}v_2 + (g_{33} + sC_x)v_3$$

# *Zero-Value Time Constant (con't 1)*

The poles of the transfer function are the zeros of the determinant $\Delta$ of the circuit equations, which can be written in the form:

$$\Delta(s) = K_0 + K_1 s + K_2 s^2 + K_3 s^3$$

$$\Delta(s) = K_0\left(1 + b_1 s + b_2 s^2 + b_3 s^3\right)$$

If all capacitors are zero:

$$K_0 = \Delta\Big|_{C_\pi = C_\mu = C_x = 0} \equiv \Delta_0$$

Consider now the term $K_1 s$, this is the sum of the terms involving s that are obtained when the system determinant is evaluated. However it is apparent, that s only occurs when associated with a capacitance:

$$K_1 s = h_1 s C_\pi + h_2 s C_\mu + h_3 s C_x$$

The terms are constants. $h_1$ can be evaluated by expanding the determinant about the first row:

$$\Delta(s) = (g_{11} + sC_\pi)\Delta_{11} + g_{12}\Delta_{12} + g_{13}\Delta_{13}$$

With cofactors $\Delta_{xx}$ of the determinant. The term $sC_\pi$ is found by evaluating $\Delta_{11}$ with $C_\mu$ and $C_x$ equal zero

$$h_1 = \Delta_{11}\Big|_{C_\mu = C_x = 0}$$

# *Zero-Value Time Constant (con't 2)*

Now consider expansion of the determinant about the second row.

$$\Delta(s) = g_{21}\Delta_{21} + (g_{22} + sC_{\mu})\Delta_{22} + g_{23}\Delta_{23}$$

With cofactors $\Delta_{xx}$ of the determinant. The term $sC_{\mu}$ is found by evaluating $\Delta_{22}$ with $C_{\pi}$ and $C_x$ equal zero

$$h_2 = \Delta_{22}\Big|_{C_{\pi}=C_x=0}$$

similarly

$$h_3 = \Delta_{33}\Big|_{C_{\mu}=C\pi=0}$$

Combining these equations gives:

$$K_1 = \Delta_{11}\Big|_{C_{\mu}=C_x=0} C_{\pi} + \Delta_{22}\Big|_{C_{\pi}=C_x=0} C_{\mu} + \Delta_{33}\Big|_{C_{\mu}=C_{\pi}=0} C_x$$

and:

$$b_1 = \frac{K_1}{K_0} = \frac{\Delta_{11}\Big|_{C_{\mu}=C_x=0}}{\Delta_0} C_{\pi} + \frac{\Delta_{22}\Big|_{C_{\pi}=C_x=0}}{\Delta_0} C_{\mu} + \frac{\Delta_{33}\Big|_{C_{\mu}=C_{\pi}=0}}{\Delta_0} C_x$$

# *Zero-Value Time Constant (con't 3)*

Now consider putting $i_2 = i_3 = 0$ and solving for $v_1$

$$\frac{v_1}{i_1} = \frac{\Delta_{11}}{\Delta(s)}$$

The driving-point resistance at the $C_\pi$ node pair with all capacitors equal to zero:

$$\frac{\Delta_{11}\big|_{C_\mu = C_x = 0}}{\Delta_0} = \frac{\Delta_{11}}{\Delta}\bigg|_{C_\mu = C_\pi = C_x = 0}$$

We now define

$$R_{\pi 0} = \frac{\Delta_{11}}{\Delta_0}\bigg|_{C_\mu = C_x = 0}$$

We can write now:

$$b_1 = R_{\pi 0} C_\pi + R_{\mu 0} C_\mu + R_{x0} C_x$$

Thus:

$$\omega_{-3dB} \cong \frac{1}{b_1} \qquad \omega_{-3dB} \cong \frac{1}{\sum T_0}$$

Thus the sum of the zero-value time constants leads to the -3dB frequency

# *Summary: Frequency Analysis Methods*

## The precise way:

- Add the parasitic capacitors to the equivalent circuit. Use nodal analysis for evaluating the transfer function.

## The approximate way:

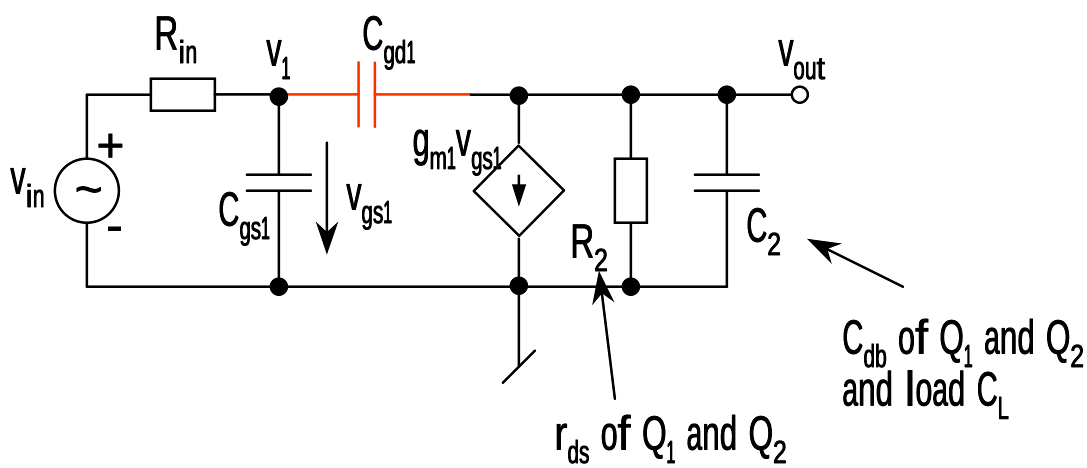
- if there exists a pole $p_1 << p_2, p_3, \ldots$, and the transfer function is already given be the transfer function $A(s)=N(s)/D(s)$
  with $$D(s)=1+b_1 s+b_2 s^2+\ldots+b_n s^n$$
  the pole $p_1$ is given by: $p_1 = 1/b_1$
- the dominant pole may be found directly in the circuit diagram by looking for the node with the largest impedance. Take care of the Miller Effect.
- The time constant (and its influence on the frequency response) associated with a single parasitic capacitor can be estimated with the zero value time constant method:
  - set all independent sources to zero
  - replace the interesting capacitor $C_x$ by a voltage source $V_x$
  - set all other capacitors to zero
  - evaluate the impedance $R_x$ seen by the voltage source $V_x$
  - the time constant is equal to $C_x R_x$

## The handy way:

- AC analysis with Spice

# *Frequency Response*
# *Common-Source Amplifier*

- precise calculation of frequency response is most often left to computer simulations
- much insight can be obtained by finding the dominant frequency effects (dominant poles, zeros)




nodel analysis ...

# *Frequency Analysis (con't)*

$$\frac{v_{out}}{v_{in}} = \frac{-g_{m1}R_2\left(1 - s\frac{C_{gd1}}{g_{m1}}\right)}{1 + sa + s^2 b}$$

at frequencies where gain has just started to decrease

$$a = R_{in}\left[C_{gs1} + C_{gd1}(1 + g_{m1}R_2)\right] + R_2(C_{gd1} + C_2)$$

$$b = R_{in}R_2(C_{gd1}C_{gs1} + C_{gs1}C_2 + C_{gd1}C_2)$$

$$\omega_{-3db} = \frac{1}{a}$$

$$\omega_{-3db} = \frac{1}{R_{in}\left[C_{gs1} + \underbrace{C_{gd1}(1 + g_{m1}R_2)}_{\text{Miller capacitance}}\right] + R_2(C_{gd1} + C_2)}$$

Miller capacitance

for $R_{in} >> R_2$

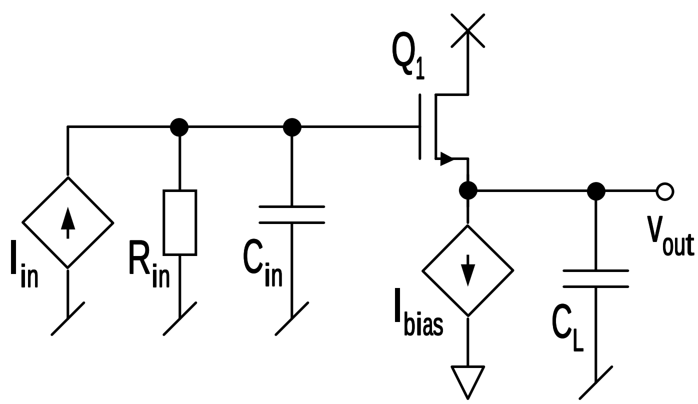
analysis for high frequencies for widely separated poles

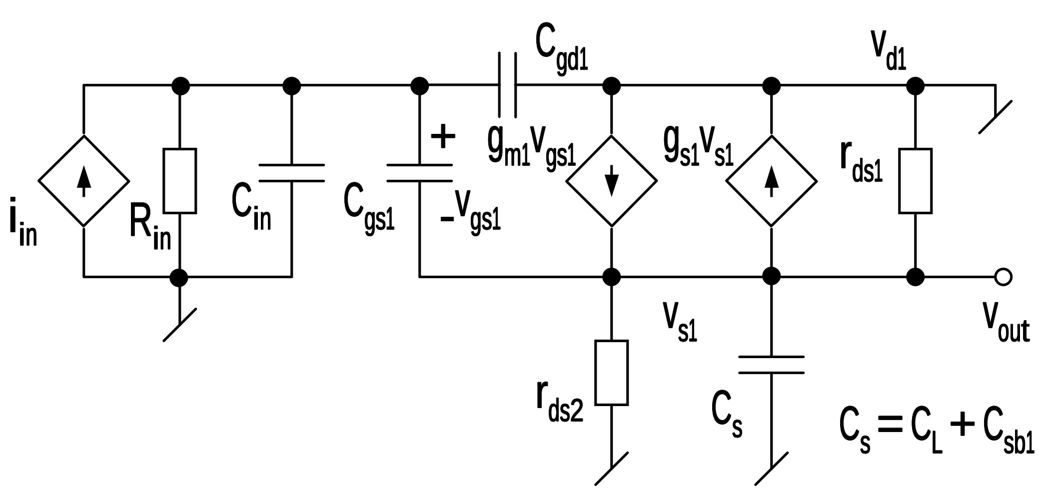
$$D(s) = \left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right) \cong 1 + \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

$$\omega_{p2} \cong \frac{g_{m1}C_{gd1}}{C_{gs1}C_{gd1} + C_{gs1}C_2 + C_{gd1}C_2}$$

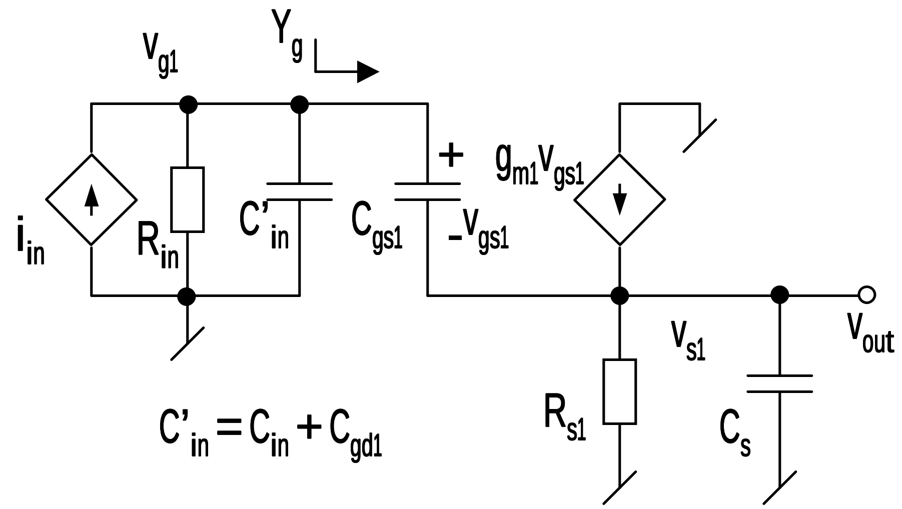
# *Frequency Response*
# *Source-Follower Amplifier*

- source followers can have complex poles and thus exhibit overshoot
- a compensation technique resulting in only real axis poles is shown, resulting in no overshooting



# *Source-Follower Amplifier (con't 1)*




$$R_{s1} = r_{ds1} \| r_{ds2} \| (1/g_{s1})$$

1. gain from $v_{g1}$ to $v_{out}$ is found
2. admittance $Y_g$ looking into gate of $Q_1$ without considering $C_{gd1}$ is found
3. Gain from $i_{in}$ to $v_{g1}$ is found
4. overall gain from $v_{in}$ to $v_{out}$ is found and results interpreted

1. gain from $v_{g1}$ to $v_{out}$ is found

$$v_{out}(sC_s + sC_{gs1} + G_{s1}) - v_{g1}sC_{gs1} - g_{m1}(v_{g1} - v_{out}) = 0$$

$$\frac{v_{out}}{v_{g1}} = \frac{sC_{gs1} + g_{m1}}{s(C_{gs1} + C_s) + g_{m1} + G_{s1}}$$

# *Source-Follower Amplifier (con't 2)*

1. gain from $v_{g1}$ to $v_{out}$ is found
2. admittance $Y_g$ looking into gate of $Q_1$ without considering $C_{gd1}$ is found
3. Gain from $i_{in}$ to $v_{g1}$ is found
4. overall gain from $v_{in}$ to $v_{out}$ is found and results interpreted

2. admittance $Y_g$ looking into gate of $Q_1$ without considering $C_{gd1}$ is found

$$Y_g = \frac{i_{g1t}}{v_{g1}} = \frac{sC_{gs1}\left(sC_s + G_{sq}\right)}{s\left(C_{gs1} + C_s\right) + g_{m1} + G_{s1}}$$

3. Gain from $i_{in}$ to $v_{g1}$ is found

$$\frac{v_{g1}}{i_{in}} = \frac{s\left(C_{gs1} + C_s\right) + g_{m1} + G_{s1}}{a + sb + s^2 c}$$

4. overall gain from $v_{in}$ to $v_{out}$ is found and results interpreted

$$A(s) = \frac{v_{out}}{i_{in}} = \frac{sC_{gs1} + g_{m1}}{a + sb + s^2 c}$$

# *Source-Follower Amplifier (con't 3)*

$\omega_0$ is the pole frequency
Q is the Q factor

$$A(s) = A(0)\frac{N(s)}{1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2}}$$

There is no peaking and the transfer functions maximum is at dc if:

$$Q < \sqrt{1/2} \cong 0.707$$

$\omega_0$ is the -3dB frequency if:

$$Q = \sqrt{1/2}$$

Step input function:

no peaking for $Q \le 0.5$

peaking for $Q > 0.5$
(complex conjugate poles)

$$\%\,\text{overshoot} = 100 e^{-\pi/\sqrt{4Q^2-1}}$$

For the source follower:

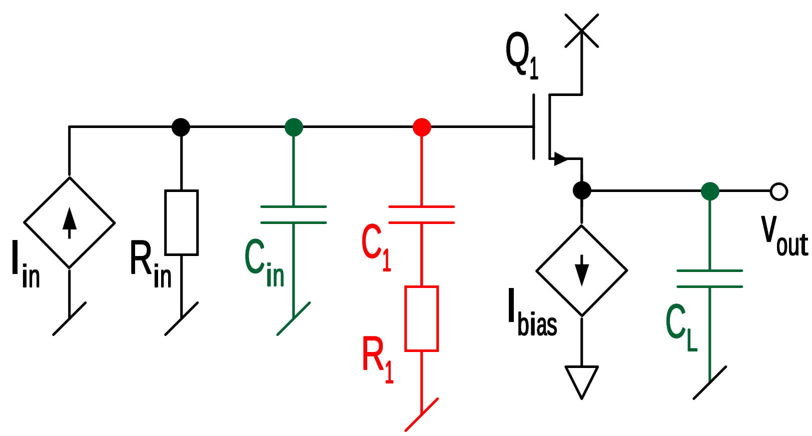
$$\omega_Z = \frac{-g_{m1}}{C_{gs1}} \qquad \omega_0 = \sqrt{\frac{G_{in}(g_{m1} + G_{s1})}{C_{gs1}C_s + C'_{in}(C_{gs1} + C_s)}}$$

$$Q = \frac{\sqrt{G_{in}(g_{m1} + G_{s1})\left[C_{gs1}C_s + C'_{in}(C_{gs1} + C_s)\right]}}{G_{in}C_s + C'_{in}(g_{m1} + G_{s1}) + C_{gs1}G_{s1}}$$

Source follower circuits can exhibit large amounts of overshoot under certain conditions. In practical uE circuits the parasitic capacitances and the output capacitance results in only moderate overshoot for worst-case conditions.

# *Source-Follower Amplifier Compensation Technique*

- source followers can have complex poles and thus exhibit overshoot
- overshooting may be reduced by:
  - increasing $C_{in}$ or $C_s$ or both
  - adding a compensation network




$$C_1 = \frac{C_{gs1}\left(C_s g_{m1} - C_{gs1} G_{s1}\right)}{\left(g_{m1} + G_{s1}\right)\left(C_{gs1} + C_s\right)} \cong \frac{g_{m1} C_{gs1} C_s}{\left(g_{m1} + G_{s1}\right)\left(C_{gs1} + C_s\right)}$$

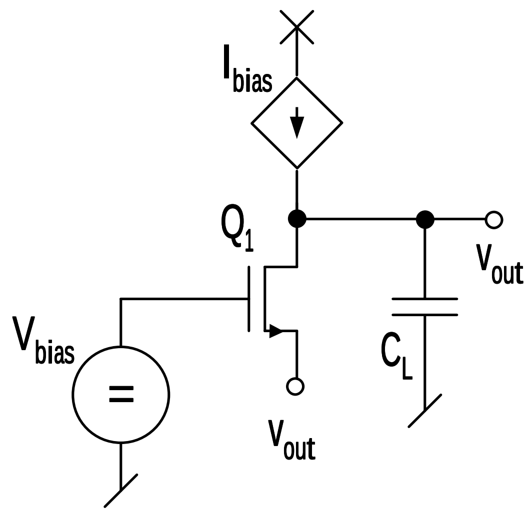
$$R_1 = \frac{\left(C_{gs1} + G_s\right)^2}{C_{gs1}\left(C_s g_{m1} - C_{gs1} G_{s1}\right)} \cong \frac{\left(C_{gs1} + G_s\right)^2}{C_{gs1} C_s g_{m1}}$$

$$C_2 = \frac{C_{gs1} C_s}{C_{gs1} + C_s}$$

(see Johns/Martin pp160-162)

# *Frequency Response Common-Gate Amplifier*

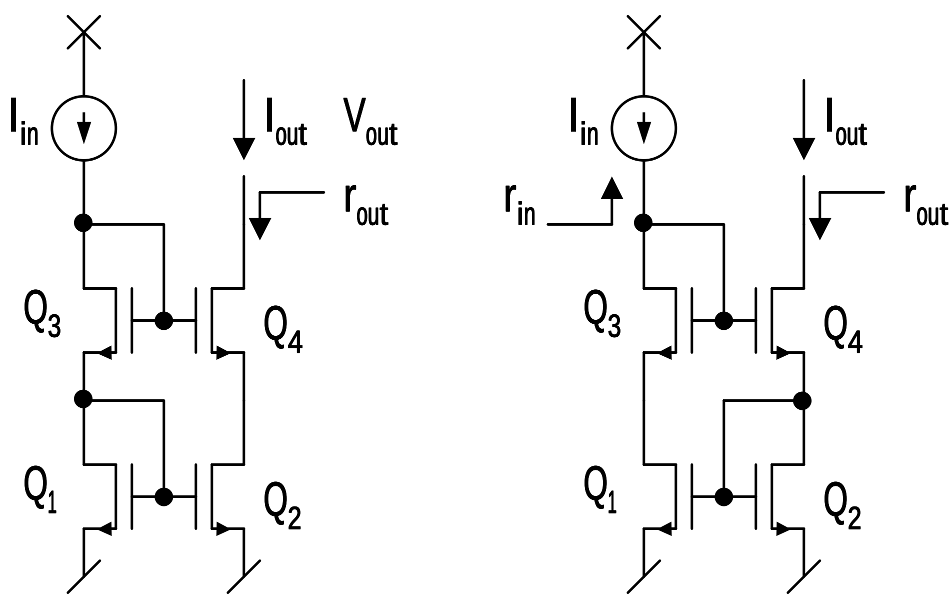
- The frequency response of the common-gate stage is usually superior to that of the common-source stage due to the low impedance, $r_{in}$, at the source node, assuming $G_L = (sC_L + g_{ds2})$ is not considerably smaller than $g_{ds1}$.




(see Johns/Martin pp160-162)

# *Frequency Response High-Ouput Impedance Mirrors*

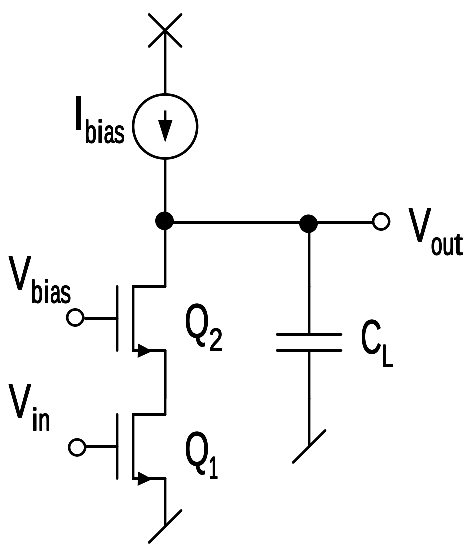
- Both the Wilson and the cascode current mirrors introduce high-frequency poles into the signal transfer function.
- The approximate time constant of these poles is $C_{gs}/g_m$, the roof of this statement can be found by doing high-frequency, small-signal analysis.




(see Johns/Martin pp163)

# *Frequency Response Cascode Gain Stage*

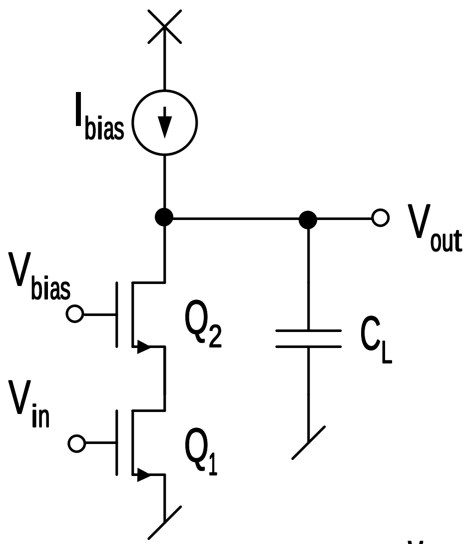
- The exact high-frequency analysis of a cascode gain stage is usually left to simulation on a computer.
- at high-frequencies, the time constant due to the output node almost always dominates since the impedance is so large at that node:
  - $C_{out} = (C_{gd2} + C_{db2}) + C_L + C_{bias}$
  - $C_L$ is normally the major contributor




$$\omega_{-3dB} \cong \frac{1}{R_{out} C_L} \cong \frac{2 g_{ds}^2}{g_m C_L}$$

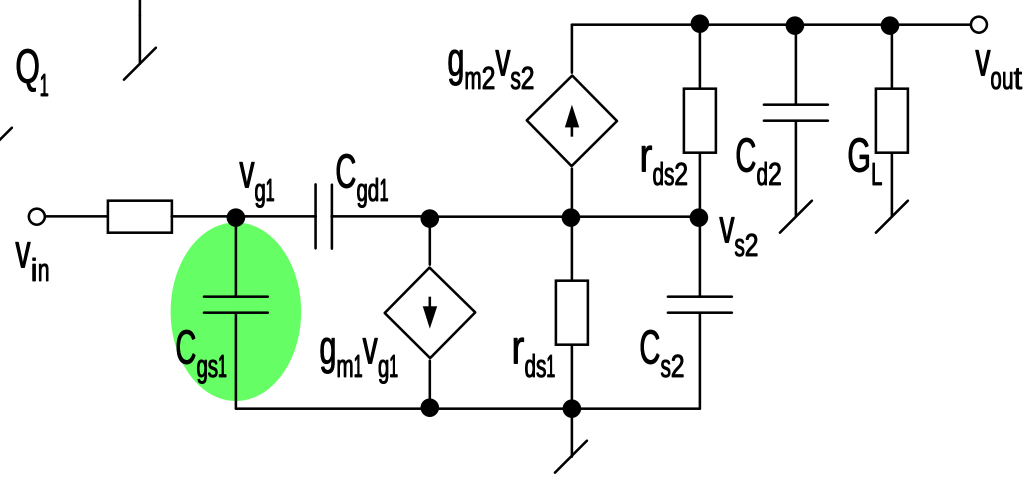
# *Cascode Gain Stage (con't 1)*

- Zero-value time constant analysis method used




$$C_{d2} = C_{gd2} + C_{db2} + C_L + C_{bias}$$

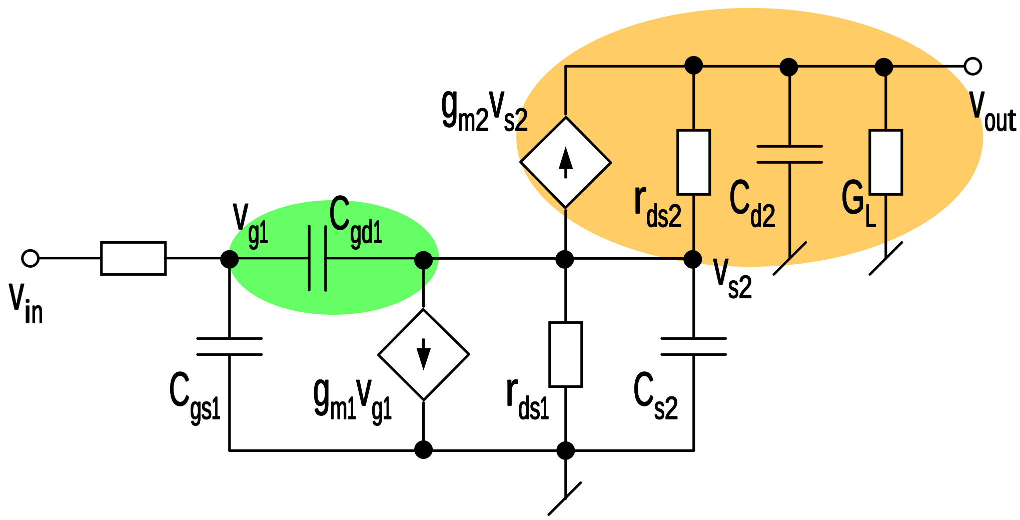
$$C_{s2} = C_{db1} + C_{sb2} + C_{gs2}$$



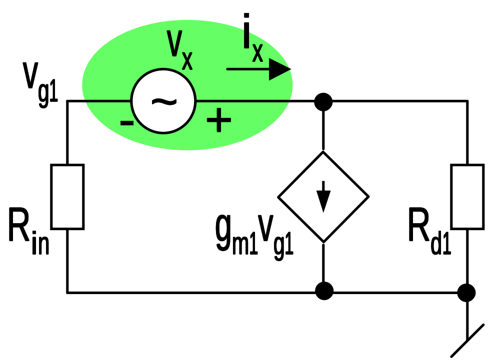

- All independent sources have to be set to zero ($v_{in} = 0$)

node $v_{g1}$ $\tau_{Cgs1} = C_{gs1} R_{in}$

# *Cascode Gain Stage (con't 2)*




nodes $v_{g1}$,$v_{s2}$ the capacitor $C_{gd1}$ is replaced by a voltage source $v_x$ in order to calculate the input resistance seen from that node.

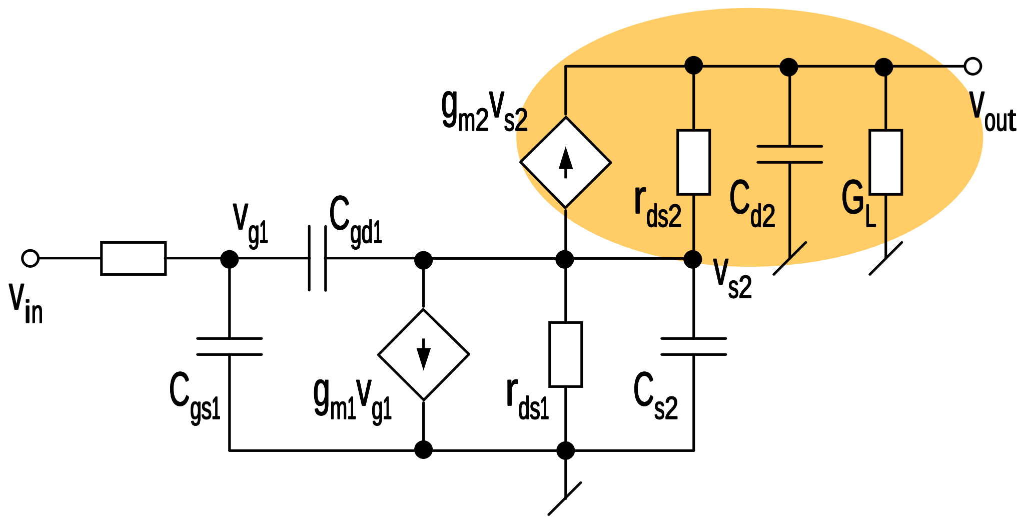



$$G_{d1} = g_{ds1} + Y_{s2}$$

admittance looking into the source of a cascode transistor is $Y_{s2}$

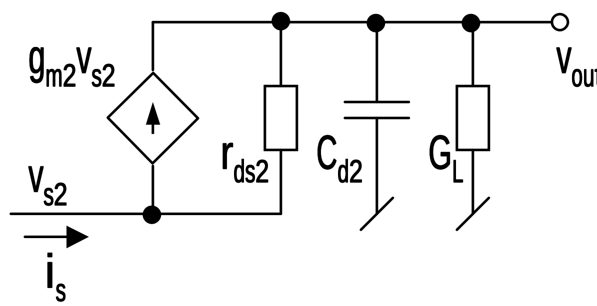
$$\tau_{Cgd1} = C_{gd1} R_{d1} \left(1 + R_{in} \left[G_{d1} + g_{m1}\right]\right)$$

# *Cascode Gain Stage (con't 3)*




$$G_{d1} = g_{ds1} + Y_{s2}$$

admittance looking into the source of a cascode transistor is $Y_{s2}$




$$Y_{s2} = i_s / v_{s2}$$

for

$g_{ds} << g_m$ $\quad R_L \cong g_m r_{ds}^2$

(see cascode current mirror impedance, pp137, vlsi-25/17)

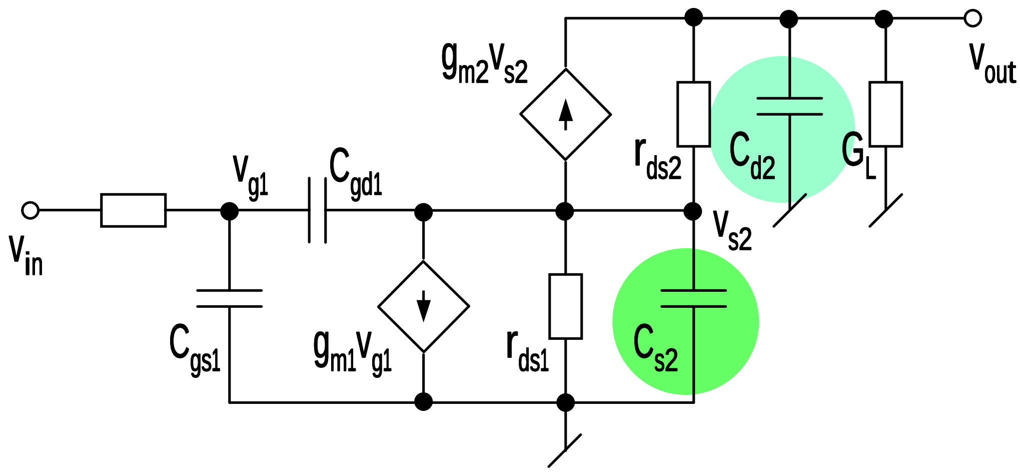
$$Y_{s2} \cong g_{ds}$$

$$\tau_{Cgd1} \cong C_{gd1} \frac{r_{ds}}{2} (1 + g_m R_{in})$$

$$\tau_{Cgd1} \cong C_{gd1} \frac{g_m r_{ds}^2}{2}$$

for $R_{in}$ is large and equal $r_{ds}$

# *Cascode Gain Stage (con't 4)*




**node $v_{s2}$**

the resistance seen by the capacitor $C_{s2}$ is $r_{ds1}$ in paralell with the impedance seen looking in the source of $Q_2$ which is approximately $r_{ds}$, thus:

$$\tau_{Cs2} \cong C_{s2} \frac{r_{ds}}{2}$$

**node $v_{out}$**

The resistance seen by $C_{d2}$ is the output impedance of the cascode amplifier, thus:

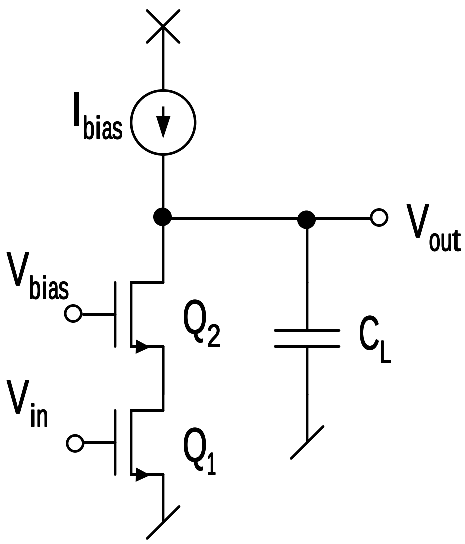
$$\tau_{Cd2} \cong C_{d2} \frac{g_m r_{ds}^2}{2}$$

$$\tau_{total} \cong \tau_{Cgs1} + \tau_{Cgd1} + \tau_{Cs1} + \tau_{Cd1}$$

$$\tau_{total} \cong C_{gs1}R_{in} + C_{gd1}\frac{g_m r_{ds}^2}{2} + C_{s2}\frac{r_{ds}}{2} + C_{d2}\frac{g_m r_{ds}^2}{2}$$

# *Cascode Gain Stage Comments*

- High frequencies considerations




one pole dominates, thus the gain is:

$$A(s) = \frac{A_v}{1 + s/\omega_{-3dB}}$$

at frequencies substantial larger than $\omega_{-3dB}$:

$$A(s) \cong \frac{A_v}{s/\omega_{-3dB}} \cong -\frac{g_{m1}}{sC_L}$$

- upper limit of the unity-gain frequency of an amplifier that uses a cascode gain stage is limited by source node of $Q_2$:

$$\omega_{p2} = \frac{1}{\tau_{s2}} > \frac{3\mu_p V_{eff2}}{2L_2^2}$$

# *Coming Up...*

- Next topic…

  Basic OpAmp design and compensation

- Readings for next time…

  *Johns&Martin*: Sections 3.11

- Exercises:

  Have a look at the exercises in *Johns&Martin*.

# *Exercises VLSI-26 #1*

Johns&Martin chap 3.11 pp156: 3.8 (difficulty: easy):

Consider the common-source amplifier shown on transparency vlsi-26/6 where $I_{in}=100\mu A$ and all transistors have $W=100\mu m$ and $L=1.6\mu m$. Given $R_{in}=180k\Omega$, $C_L=0.3pF$, $C_{gs1}=0.2pF$, $C_{gd1}=15fF$, $C_{db1}=20fF$, $C_{db2}=36fF$, $\mu_n C_{ox}=90\mu A/V^2$, $\mu_p C_{ox}=30\mu A/V^2$, and $r_{ds\text{-}n}=8000$ [L ($\mu$m)]/[ID (mA)], $r_{dsp}=12000$ [L ($\mu$m)]/[ID (mA)]. Estimate the 3db frequency response.

Result: $f_{-3db}=554kHz$

Johns&Martin chap 3.11 pp160: 3.9 (difficulty: easy):

Analyse the source follower and assume that $I_{bias}=100\mu A$ and all transistors have $W=100\mu m$ and $L=1.6\mu m$. Given $R_{in}=180k\Omega$, $C_L=10pF$, $C_{gs1}=0.2pF$, $C_{gd1}=15fF$, $C_{sb1}=40fF$, $C_{in}=30fF$, $\mu_n C_{ox}=90\mu A/V^2$, $\mu_p C_{ox}=30\mu A/V^2$, and $r_{ds\text{-}n}=8000$ [L ($\mu$m)]/[ID (mA)]. Find $\omega_0$, Q, and $\omega_z$ of the source follower.

Result: $\omega_0=52MHz$, $Q=0.8$, % overshoot = 8.1%, $\omega_z=5.3GHz$

# *Exercises VLSI-26 #2*

Johns&Martin chap 3.11 pp166:3.11 (difficulty: easy):

Assume that for the input transistors and the cascode transistors, $g_m = 1mA/V$, $r_{ds} = 100k\Omega$, $R_{in} = 180k\Omega$, $C_L = 5pF$, $C_{gs} = 0.2pF$, $C_{gd} = 15fF$, $C_{sb} = 40fF$, $C_{db} = 20fF$, $C_{bias} = 20fF$, Estimate the -dB frequency of the cascode amplifier (transparency 19).

Result: $\omega_{-3dB} = 2\pi\ 6.3MHz$

Johns&Martin chap 3.11 pp168: 3.12 (difficulty: easy):

Estimate the lower bound on the frequency of the second pole of a folded-cascode amplifier for a 0.8µm technology, where a typical value of 0.25V is chosen for $V_{eff2}$. $L_2 = 1.5L_{min}$, $\mu_p = 0.02m^2/Vs$.

Result: $\omega_{p2} = 2\pi\ 414MHz$