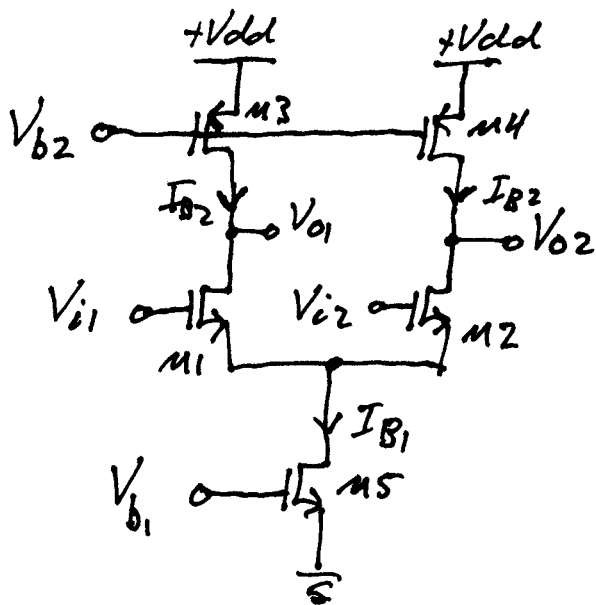


Single-Stage Diff. Amps

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- As seen in the diff. pair notes, active loading is required to achieve high gain, just as seen in the CS amp discussion.
- One option is to apply I-source loading (as in CS-amp)



• SSM same as before with

$$R_L = r_{ds_{3,4}}$$

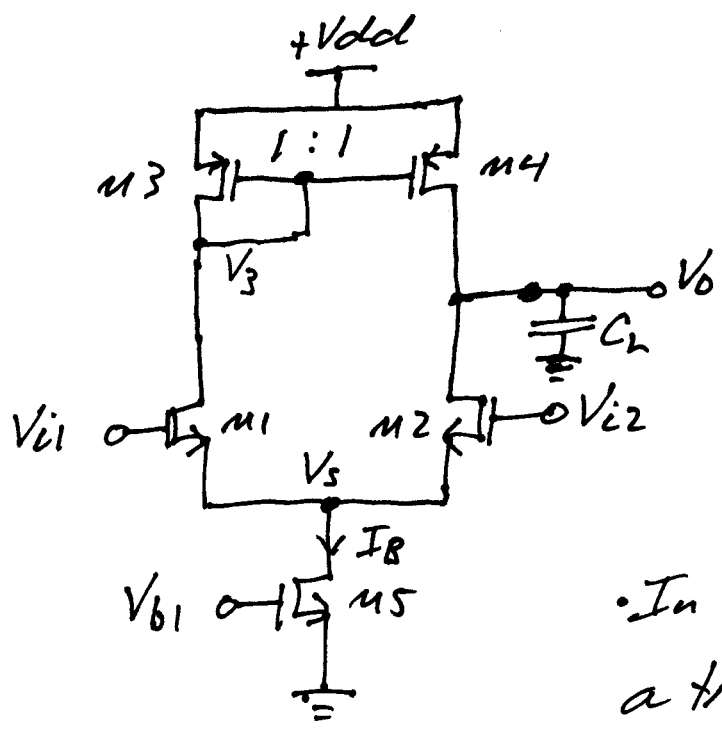
$$\Rightarrow A_{dm} = \frac{v_{od}}{v_{id}} = -g_{m_{1,2}} (r_{ds_{1,2}} \parallel r_{ds_{3,4}})$$

• Note: $v_{od} = v_{o1} - v_{o2}$; if only one output is used \Rightarrow half gain!

• However, there is a biasing problem: we must have: $I_{B1} = 2 \cdot I_{B2} \Rightarrow$ with such high imped. at V_o , even small differences due to I -mirror errors in generating V_{b1} & V_{b2} may cause V_{oc} to push $M_{3,4}$ or $M_{1,2}$ into triode.

• This circuit requires active feedback to control V_{b1} & V_{b2} : Common-Mode Feedback (CMFB).

- CMFB circuits will be introduced at the end of the semester.
- Another solution which avoids the need for CMFB & converts the diff. output into a single-ended output is a I-mirror load (similar to "self-biasing" in ref circuits):



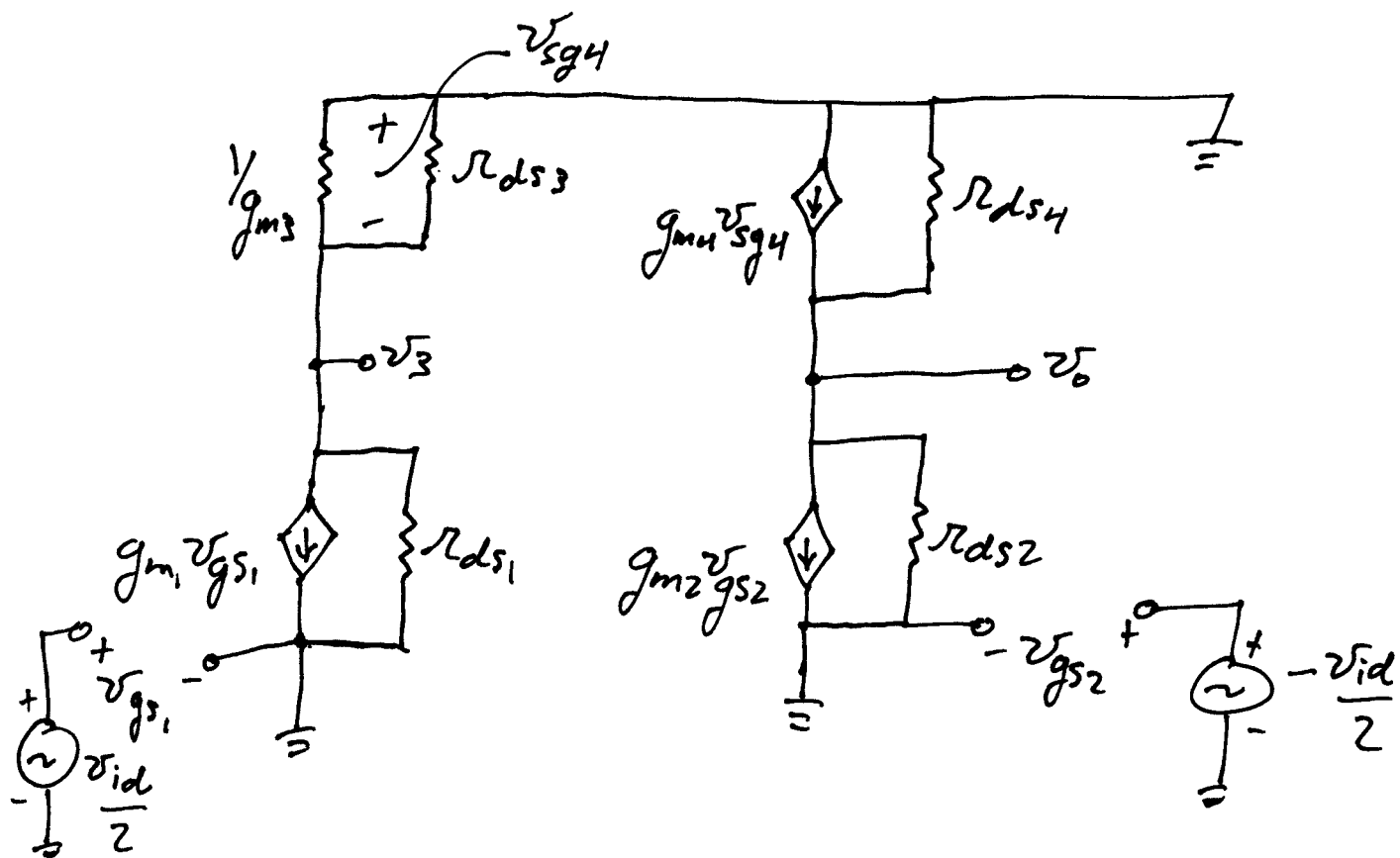
- Now only a single bias is used, I_B
- The mirror $M_{3,4}$ provides biasing & mirrors gain from M_1 to the single ended output, V_o .

- In the ssm, V_s is no longer a true small-signal gnd.
- However, it is still low imped. & the approx of $V_s = 0$ for diff inputs still gives good results & intuitive design-oriented insight.

* To solve ssm: { Diff input: $V_s \approx 0$.
 Same as w/ R_L loading. { Common input: split M_5 to two 2-roads & open circuit @ V_s .

• Also, use I-mirror of $M_{3,4}$ to transfer 3/16
 current gain of M_1 to V_o . We will show details below, but move directly to output node in the future († assume perfect I-mirror).

• Solve diff. gain: $A_{dm} = \frac{v_o}{v_{id}}$ ← single ended
 → set $v_s = 0$



$$\Rightarrow v_{gs1} = \frac{v_{id}}{2} = -v_{gs2} ; v_{sg4} = -v_3$$

$$v_3 = \left(\frac{g_{m1} v_{id}}{2} \right) \left(r_{ds1} \parallel r_{ds3} \parallel \frac{1}{g_{m3}} \right) \overset{\text{neglecting } r_{ds}}{\sim} - \left(\frac{g_{m1} v_{id}}{2} \right) \left(\frac{1}{g_{m3}} \right)$$

$$\Rightarrow \frac{v_{sg4}}{g_{m4}} = \left(\frac{g_{m1} v_{id}}{2} \right) \left(\frac{1}{g_{m3}} \right) g_{m4}$$

4/16

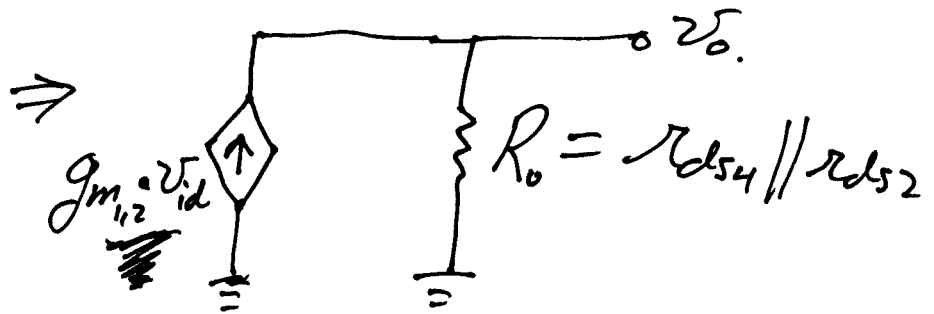
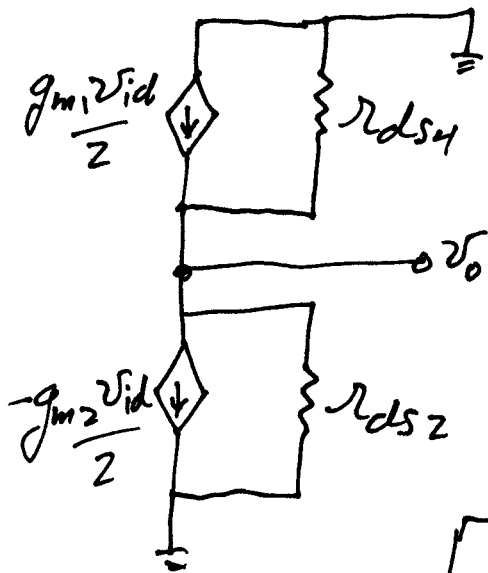
$$= \left(\frac{g_{m1} v_{id}}{2} \right) \Rightarrow \text{by neglecting } r_{ds1} \text{ \& } r_{ds3},$$

can assume i-source of $M4$ is equal to source of $M1$!

↑
Current from $M1$ device transconductance.

⇒ Right-half circuit is modeled as:

• for $g_{m3} = g_{m4}$; $g_{m1} = g_{m2}$



$$\Rightarrow A_{dm} = \frac{v_o}{v_{id}} = g_{m_{1,2}} \cdot R_o \quad \leftarrow \text{Same as CS-amp.}$$

Note: A_{dm} is non-inverting because of the definition of $v_{id} = v_{i1} - v_{i2} \Rightarrow (+)$ non-inverting input is v_{i1} & $(-)$ inverting input is v_{i2} .

In general, you can find the (+) & (-) 5/16 inputs by starting at a given input & counting the # of inv. & non-inv. stages from input to output. For pg. 2, from V_{IC} ,

two stages: $(-) \cdot (-) = (+) \Rightarrow$ non-inverting input.
 $\uparrow \quad \quad \uparrow$
 $M1, CS \quad M4, CS$

- From $M2$: $(-) = (-) \Rightarrow$ inverting input.
 \uparrow
 $M2, CS$

• The CMRR can be solved from the SSM as well. We will not show details here, but qualitatively it can be seen that variations

in $\uparrow V_{IC} \Rightarrow \uparrow V_S \Rightarrow \uparrow I_B \Rightarrow \uparrow I_{D1} \& \uparrow I_{D2}$
 $\uparrow \quad \quad \uparrow$
 $V_{GS, 2} \quad \text{due to}$
 $\neq \text{const.} \quad \lambda \text{ of } M5$

• Since $\uparrow I_{D1} = \uparrow I_{D2} = \frac{\uparrow I_B}{2}$, if I -mirror $M3$ & $M4$ is ideal $\Rightarrow \uparrow I_{D4} = \uparrow I_{D2}$ } two cancel & result in no change in $V_O \Rightarrow A_{cm} \approx 0$

• Thus $CMRR = \frac{A_{dcm}}{A_{cm}} \approx \underline{\underline{\infty}}$

6/16

• In reality M_3/M_4 I-mirror is not ideal &

CMRR depends on $\left[g_{m12} (2r_{ds}) \right] \neq \underbrace{g_{m3,4}}_{\text{quality of I-mirror}} \cdot R_o$
↑
quality of I_B

⇒ $CMRR \approx g_{m12} (2r_{ds}) \cdot g_{m3,4} \cdot R_o$

→ e.g. see [Gray/Meyer] 4.3.5.

• Solve for approx BW: ZVTC requires adding all τ_i 's ⇒ many caps even in simple circuit of pg 2. Also, now there are two ^{signal} paths to the output with different delays in each path → requires modification to ZVTC for exact solution.

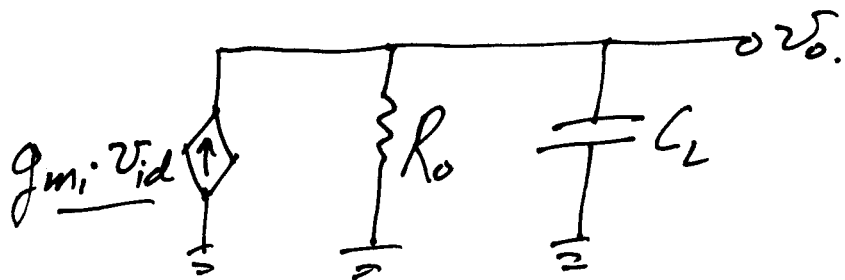
⇒ 1st: we will ~~search~~ ^{search} for a dominant τ_i

with large R_{oi} & C_i ⇒ if there is a clear

dominant τ_i that affects both signal paths,

then we can neglect smaller τ_i 's for BW estimate.

- For pg 2 circuit: only large impedance node v_o is $v_o \Rightarrow$ dominant τ_o , especially if $C_L = \text{large}$.
- Note: By neglecting smaller τ_i 's in Z_{UTL} , the method is no longer guaranteed to be conservative, and may overestimate the BW.
- Include C_L in simplified ssm of pg 4:



$$A_{dm} = g_{m_i} R_o$$

$$R_o = r_{ds1} \parallel r_{ds2}$$

$$BW \approx \frac{1}{2\pi R_o C_L}$$

- Next: Consider large-signal & biasing constraints on input & output signals for the A_{dm} , R_o & BW estimates to be valid.

\Rightarrow primary constraint: all devices active.

• Consider first DC biasing of input & output. 8/16

• We need to know V_o to determine op-modes.

• Generally, V_o must be forced to a desired point by applying V_{IO} (DC diff. input) for biasing.

• Define: $V_{OFFSET} = V_{off} =$ "offset voltage"

$V_{off} = V_{IO}$ required to force V_o to desired DC operating point.

• If $V_{IO} = 0$, find V_o in pg2 fig: assume 1st that I -mirror M_3/M_4 is ideal:

$$\Rightarrow I_{O1} = I_{O2} \Rightarrow \begin{cases} I_{O1} = K_{n2} (V_{IC} - V_s - V_t)^2 (1 + \lambda(V_3 - V_s)) \\ \text{for matched} \leftarrow \begin{cases} I_{O2} = K_{n2} (V_{IC} - V_s - V_t)^2 (1 + \lambda(V_o - V_s)) \end{cases} \end{cases}$$

devices

$$\Rightarrow \underline{V_3 = V_o}$$

* If this is desired $V_o \Rightarrow \underline{V_{off} = 0}$.

\Rightarrow In reality: M_3/M_4 is not perfect I -mirror & matching is not perfect $\Rightarrow V_{off} \neq 0$ ($100\text{ }\mu\text{V}$ to mV)

Let's neglect these errors: $V_{off}=0, V_{IO}=0$. 9/16

$$V_0 = V_3 \text{ (DC op)}$$

• Now, solve input common mode range (ICMR) to maintain active operation:

$$(V_{IC})_{max} \stackrel{\substack{\Rightarrow \\ \uparrow \\ M_{1,2} \text{ Triode}}}{=} V_{DD} - V_{SG3} + V_{t1} = V_{DD} - V_{OV3} - V_{t3} + V_{t1}$$

$\underbrace{-V_{t3} + V_{t1}}_{\text{may cancel or be small + or -}}$

$$\approx V_{DD} - V_{OV3} \Rightarrow \text{Can get close to } V_{DD}!$$

\uparrow
 $\sqrt{\frac{I_3}{K_3}}$

$$(V_{IC})_{min} \stackrel{\substack{\Rightarrow \\ \uparrow \\ M_5, \text{ Triode}}}{=} 0 + V_{OV5} + V_{GS_{1,2}} = \underbrace{V_{OV5} + V_{OV1} + V_{t1}}_{2V_{OV} + V_{t1}!}$$

\Rightarrow at least 1V above GND (or $-V_{SS}$).

Note: use p-mos diff pair \Rightarrow - close to GND. (within V_{OV})
 - $2V_{OV} + V_{t1}$ below V_{OV}

• Output voltage range:

10/16

$$(V_o)_{\max} = V_{DD} - V_{OV4} \leftarrow \text{close to } V_{DD}.$$

\uparrow
M4, triode

$$(V_o)_{\min} = V_{IC} - V_{t2} \leftarrow \text{Limited by } V_{IC} \Rightarrow \underline{\text{not good!}}$$

\uparrow
M2, triode

Note: Limits for I_{CMR} & (V_o) range are the extreme limits. At these ~~points~~ points, only infinitesimally small ac signals can be applied & maintain active devices. To amplify larger signals, V_{IC} or V_o will have to be appropriately biased away from these limits.

• The last large-signal limitation we will address is slew-rate (SR), which limits the amount of current that can be supplied to caps during transients.

• Since $i_c = C \frac{dv}{dt} \Rightarrow SR_c^+ = \left(\frac{dv}{dt} \right)_{\max^+} = \frac{I_c^+}{C}$ 11/16

$$SR_c^- = \left(\frac{dv}{dt} \right)_{\max^-} = \frac{I_c^-}{C}, \text{ where}$$

I_c^+ = max positive current that can be delivered to C with large V_{id} .

I_c^- = max negative current . . .

• For fig on pg. 2 \Rightarrow consider C_L :

$$I_{C_L}^+ = I_B \rightarrow \text{large } +V_{id} \Rightarrow M_2 \text{ cut off, } I_{D4} = I_B$$

$$I_{C_L}^- = I_B \rightarrow \text{large } -V_{id} \Rightarrow M_1, M_3, M_4 \text{ cut off}$$

$$I_{D2} = I_B$$

\Rightarrow SR limits are generally achieved when one or more devices enter cut-off.

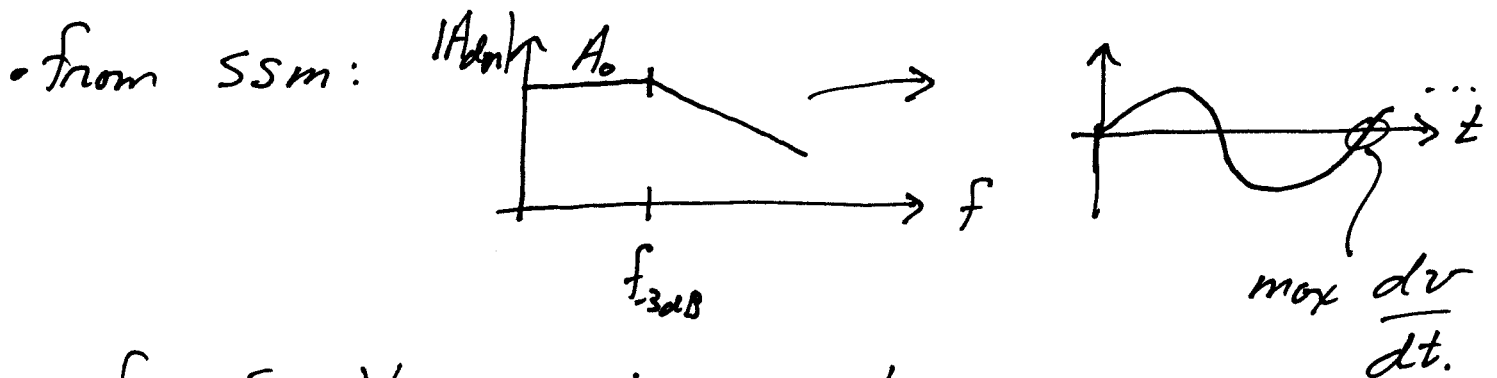
\Rightarrow For pg 2: $SR_{C_L} = \frac{I_B}{C}$ \leftarrow both pos & neg.
(~~symmetrical~~).
(symmetrical)

• Relationship between SR & BW:

12/16

→ BW: small-signal estimate of ~~max~~ freq. where signals are attenuated by -3dB (0.708), regardless of absolute amplitude.

SR: large-signal limitation on $\frac{dv}{dt}$ due to finite bias current \Rightarrow devices enter cut-off, directly dependant on signal amplitude.



• for $v_{in} = V_i \sin \omega_x t$, expect:

$$v_o = V_o \sin \omega_x t, \quad \frac{V_o}{V_i} = A_0 \text{ for } \omega_x < \omega_{-3dB},$$

$$\Rightarrow \left. \frac{dv_o}{dt} \right|_{t=0} = V_o \omega_x \cos \omega_x t \Big|_{t=0}$$

(neglecting -3dB attenuation + -45° phase shift @ f_{-3dB}).

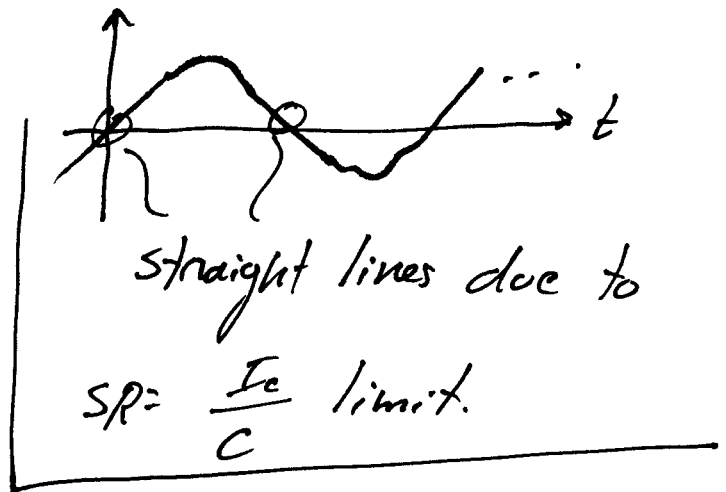
$$= \underline{\underline{V_o \omega_x}}$$

• Therefore, SSM predicts $\frac{dv}{dt}$ up to $V_0 \omega_c$ 13/16

with ω_c up to $\omega_{-3dB} \Rightarrow \frac{dv}{dt}$ up to $V_0 \omega_{-3dB}$

• If SR limit: $SR < V_0 \omega_{-3dB} \Rightarrow$ will have a large-signal SR limit near zero crossings.

leading to distortion:



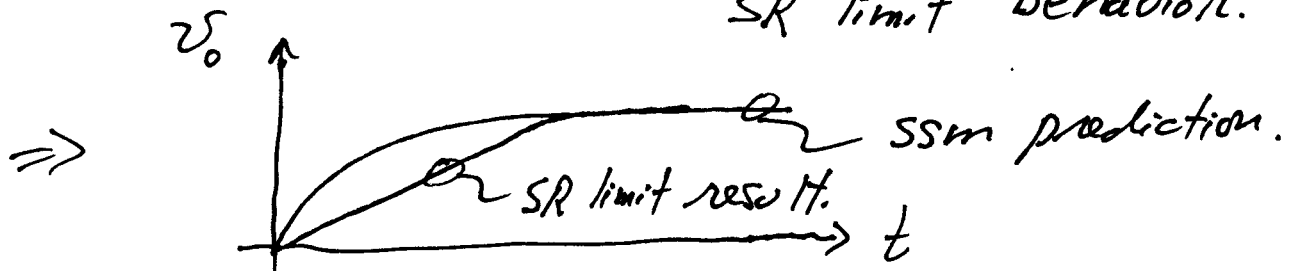
• For step response:

SSM w/ single pole @ f_{3dB} :

$$v_o = V_0 (1 - e^{-t/\tau})$$

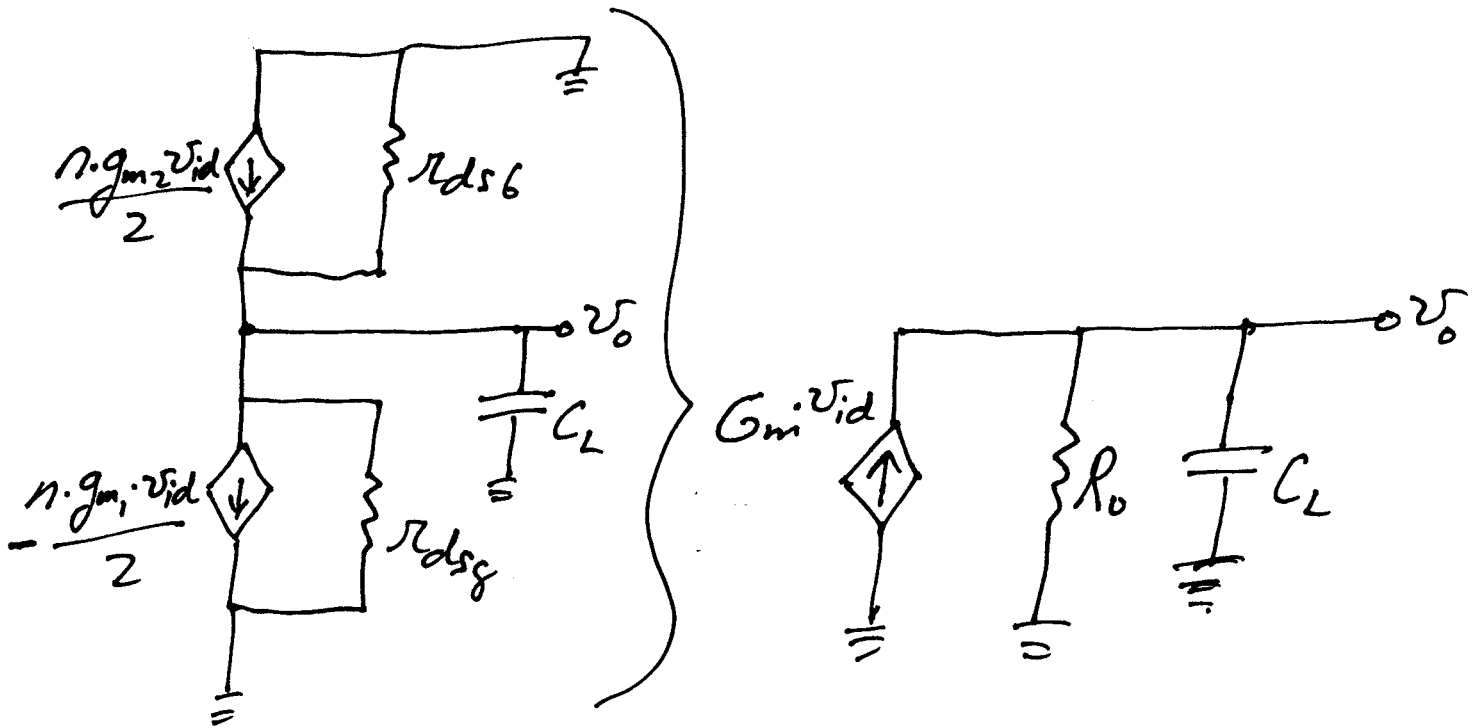
$$\Rightarrow \left. \frac{dv_o}{dt} \right|_{t=0} = \frac{V_0}{\tau}, \quad \tau = \text{time constant} \approx \frac{1}{\omega_{-3dB}}$$

$\Rightarrow \underline{V_0 \omega_{-3dB}}$; if $SR < V_0 \omega_{-3dB} \Rightarrow$ will have SR limit behavior.



⇒ SSm at output:

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$$G_m = n \cdot g_{m1,2} = n \cdot 2 \cdot \sqrt{k_{1,2} \frac{I_B}{2}}$$

$$R_o = r_{ds6} \parallel r_{ds8} = \frac{1}{\frac{n \cdot \lambda_B \cdot I_B}{2}} \parallel \frac{1}{\lambda_B \left(n \frac{I_B}{2} \right)}$$

$$BW \approx \frac{1}{2\pi R_o C_L}, \quad A_{dm} = G_m \cdot R_o.$$

$$\Rightarrow A_{dm} \cdot BW = \text{"gain \cdot BW product"} = \frac{G_m}{2\pi C_L}$$

↳ independant of R_o . ⇒ more on this later.

- DC limitations of SR essentially same as 16/16
Pg 2.

$$(V_{IC})_{max}^+ = V_{DD} - V_{S63} + V_{t1} = V_{DD} - V_{ov3} + V_{t1} - V_{t3}$$

$$(V_{IC})_{min} = V_{ov9} + V_{ov1} + V_{t1}$$

$$(V_o)_{max} = V_{DD} - V_{ov6} ; (V_o)_{min} = V_{ov8} \left. \vphantom{(V_o)_{max}} \right\} \text{improved!}$$

⇒ ~~near~~ near "rail-to-rail" output!

$$SR^+ = SR^- = \frac{n \cdot I_B}{C_L}$$

- Total DC power loss ("quiescent power")

$$\Rightarrow V_{DD} \cdot I_{TOT} = V_{DD} \left(\frac{n I_B}{2} + I_B + \frac{n I_D}{2} \right)$$

$$P_Q = V_{DD} \cdot I_Q = V_{DD} (I_B (n+1))$$

- Note: Pg 14 circuit is called "Symmetrical OTA"
Since n_1, n_2 loading is symmetrical, OTA = operational transconductance amplifier (V_{id} to current output).
high output imped.
- Next: design procedure given specs...