Chapter 3

STABILITY AND COMPENSATION

3.1 Stability

For a feedback amplifier shown in Fig. 3.1, the closed-loop transfer function is given by

$$A_f(s) = \frac{A(s)}{1 + A(s)f(s)} \tag{3.1}$$

Set $s = j\omega$ to get

$$A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)f(j\omega)} \tag{3.2}$$

Consider the loop gain

$$T(j\omega) = A(j\omega)f(j\omega) = |A(j\omega)f(j\omega)| \cdot e^{j\phi(\omega)} \tag{3.3}$$

where $\phi(\omega)$ is the phase of the loop gain $T(j\omega)$ which is dependent on the feedback factor $f(j\omega)$ and the frequency ω.

Typically, the magnitude and the phase of both the open-loop gain $A(j\omega)$ and the loop gain $T(j\omega)$ decrease as the frequency increases. Consequently, at some frequency high enough, the phase of the loop gain will eventually reach -180 degrees, the loop gain will change sign, and the original *negative* feedback will become *positive* feedback.

Define ω_{180} as the frequency at which the phase of the loop gain $\phi(\omega)$ is -180^o and ω_o as the unity-gain frequency at which the magnitude of the loop gain $|T(j\omega)|$ is 1, that is

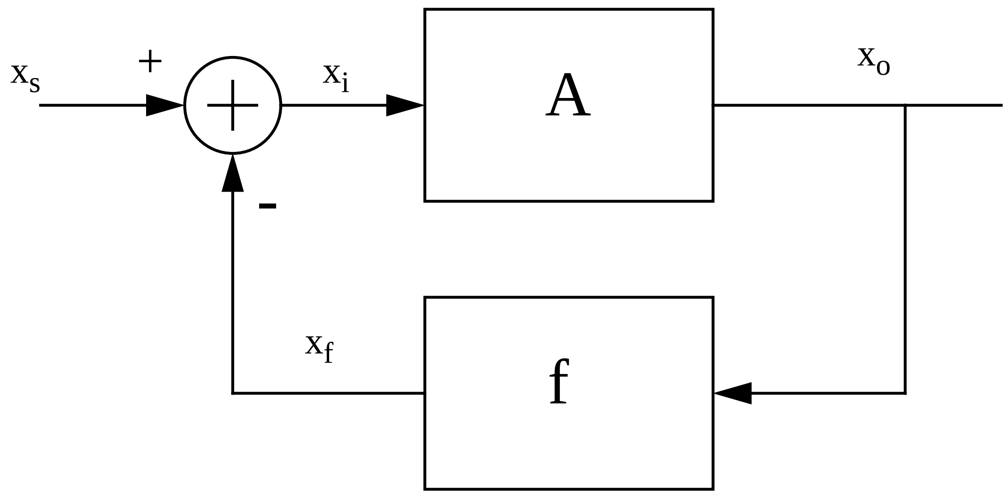


Fig. 3.1 Block diagram of a feedback system

$$\phi(\omega_{180}) = -180^\circ \Rightarrow e^{j\phi(\omega_{180})} = -1 \tag{3.4}$$

$$|T(\omega_o)| = |A(j\omega_o)f(j\omega_o)| = 1 \tag{3.5}$$

1. If $|T(j\omega_{180})| > 1$, we have a positive feedback with the loop gain > 1, ie. an unstable system.

2. If $|T(j\omega_{180})| = 1$, $A_f = \infty$ .It follows that a zero input can introduce and sustain a non-zero output, ie. an oscillating system. Conceptually, any disturbance in the feedback loop that consists of ω_{180} component will be sustained. To see this, set $x_s = 0$. The output at the frequency ω_{180} can be calculated as follows:

$$x_f(\omega_{180}) = T(j\omega_{180})(x_s - x_f) = (-x_f) = x_i \tag{3.6}$$

That is, although the input is zero, there will exist a sinusoidal signal at the input and output of the amplifier oscillating at the frequency ω_{180}.

3. If $|T(j\omega_{180})| < 1$, we have a positive feedback system with loop gain < 1, ie. a stable system. (See Appendix).

Note: For real and typical amplifier systems, both the phase and magnitude of $|T(j\omega_{180})|$ decrease monotonically as the frequency increase. As a result,

i) $|T(j\omega_{180})| > 1$ is equivalent to $\phi\{T(j\omega_o)\} < -180^o$ (larger in size)

ii) $|T(j\omega_{180})| < 1$ is equivalent to $\phi\{T(j\omega_o)\} > -180^o$ (smaller in size)

As examples, the systems whose Bode's plots for T(jω) are as shown in Figs. 3.2(a) and 3.2(b) are stable and unstable because $|T(j\omega_{180})| < 1$ and $|T(j\omega_{180})| > 1$, respectively.

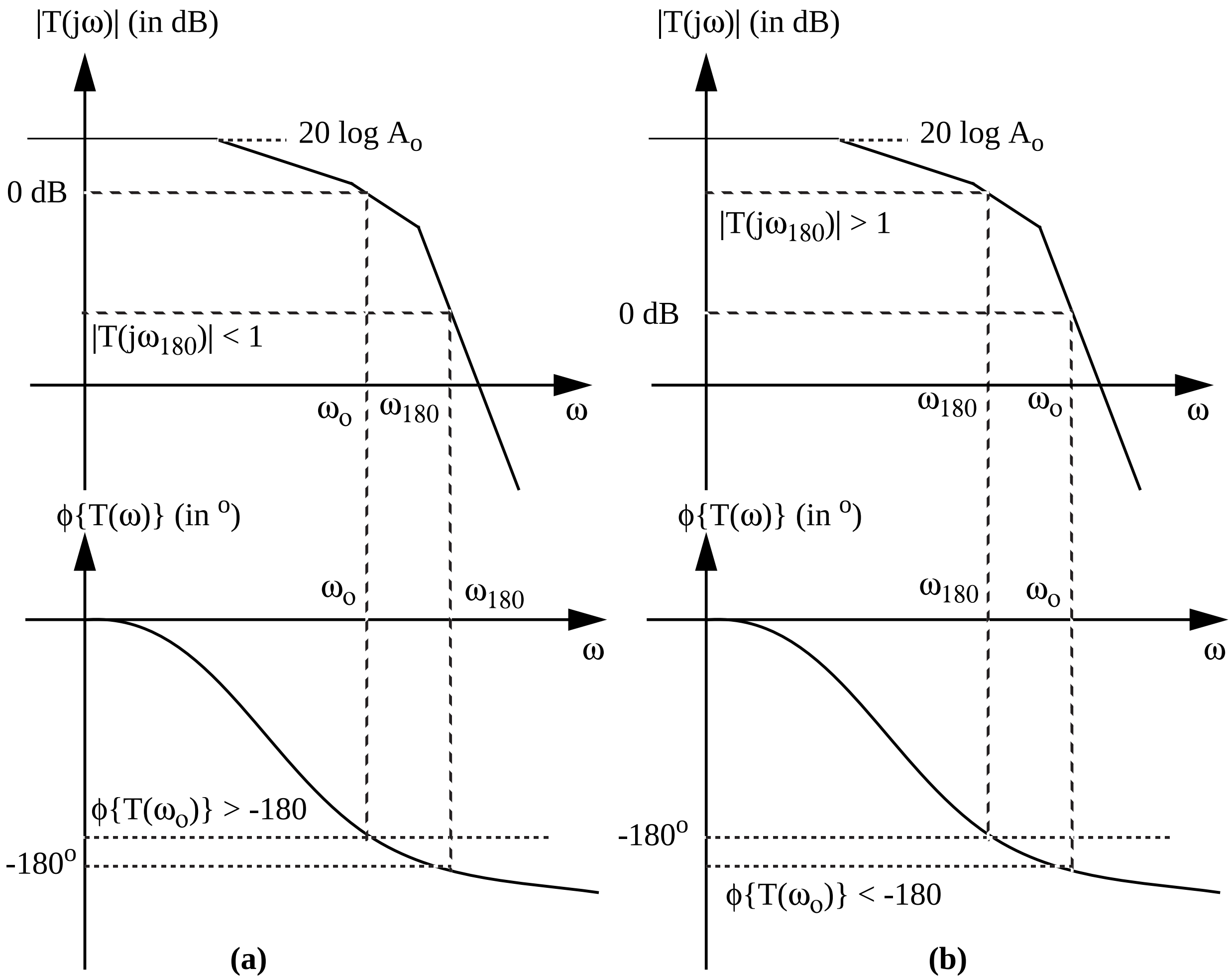


Fig. 3.2 Example of Bode's plots for T(jω) of (a) stable and (b) unstable systems

3.2 Nyquist Plot

A Nyquist plot is a polar plot of the loop gain T(jω) = A(jω) f(jω) with the frequency ω being the variable parameter. Figures 3.3 and 3.4 show examples of such Nyquist plots. Note that the dash curves are for negative frequencies!

3.3 Nyquist Criteria

If the Nyquist plot *does not encircle* the point (-1,0), the magnitude of the loop gain at the frequency ω_{180}, $|T(j\omega_{180})|$, is *smaller* than unity. As a result, it is a positive feedback with a loop gain *smaller* than 1, and, as mentioned earlier, the amplifier is *stable*! An example is shown in Fig. 3.3.

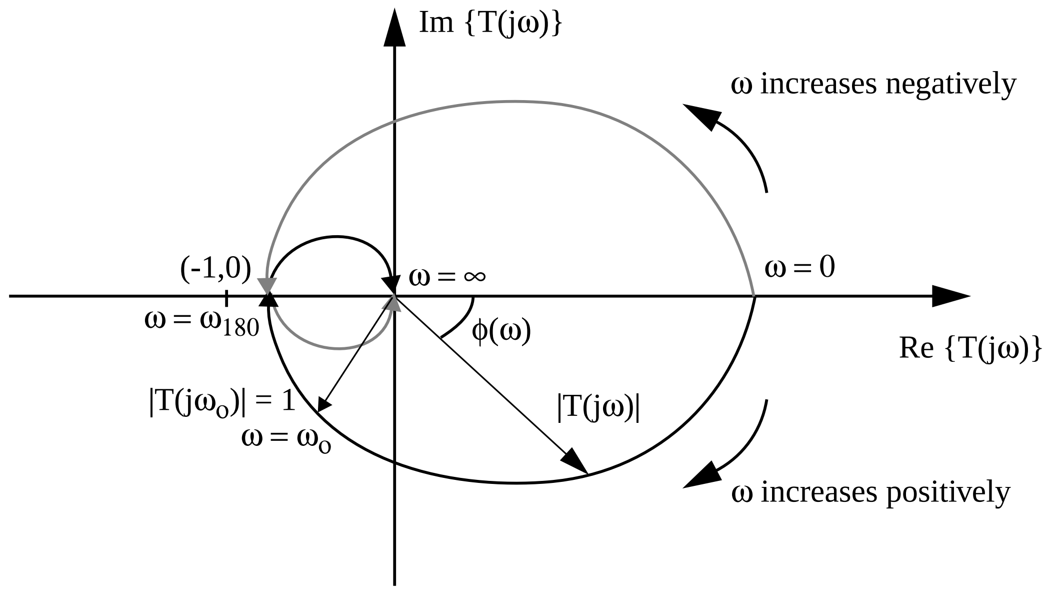


Fig. 3.3 Example of a Nyquist plot of a *stable* system

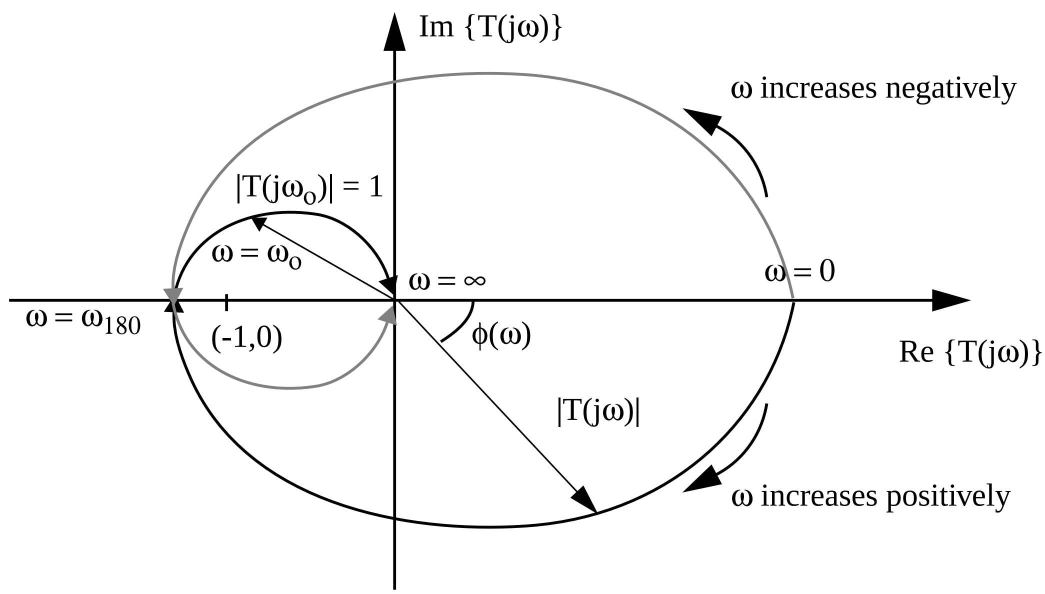


Fig. 3.4 Example of a Nyquist plot of an *unstable* system

On the contrary, if the Nyquist plot *encircles* the point (-1,0), the magnitude of the loop gain at the frequency ω_{180}, $|T(j\omega_{180})|$, is *larger* than unity. It is a positive feedback with a loop gain *more* than 1, and as a consequence, the amplifier is *unstable*! Figure 3.4 illustrates such an example.

Interpretation:

a) If the magnitude of the loop gain at the frequency ω_{180} where the phase is -180^o, $|T(j\omega_{180})|$, is larger than 1, the amplifier is unstable!

b) Alternatively, if the phase of the loop gain at the unity-gain frequency ω_o, $\phi(\omega_o)$, is larger than -180^o, the amplifier is stable!

3.4 Bode's Plot

Bode's plots of the *open-loop gain*, A(jω), can also be used to determine the stability of an amplifier with a feedback network. Consider the Bode's plots of the open-loop gain A(jω) in Fig. 3.5.

At low frequency, if the loop gain is larger than unity, ie. $T = A_o f >> 1$,

$$A_f = \frac{A}{1 + Af} \approx \frac{1}{f} \tag{3.7}$$

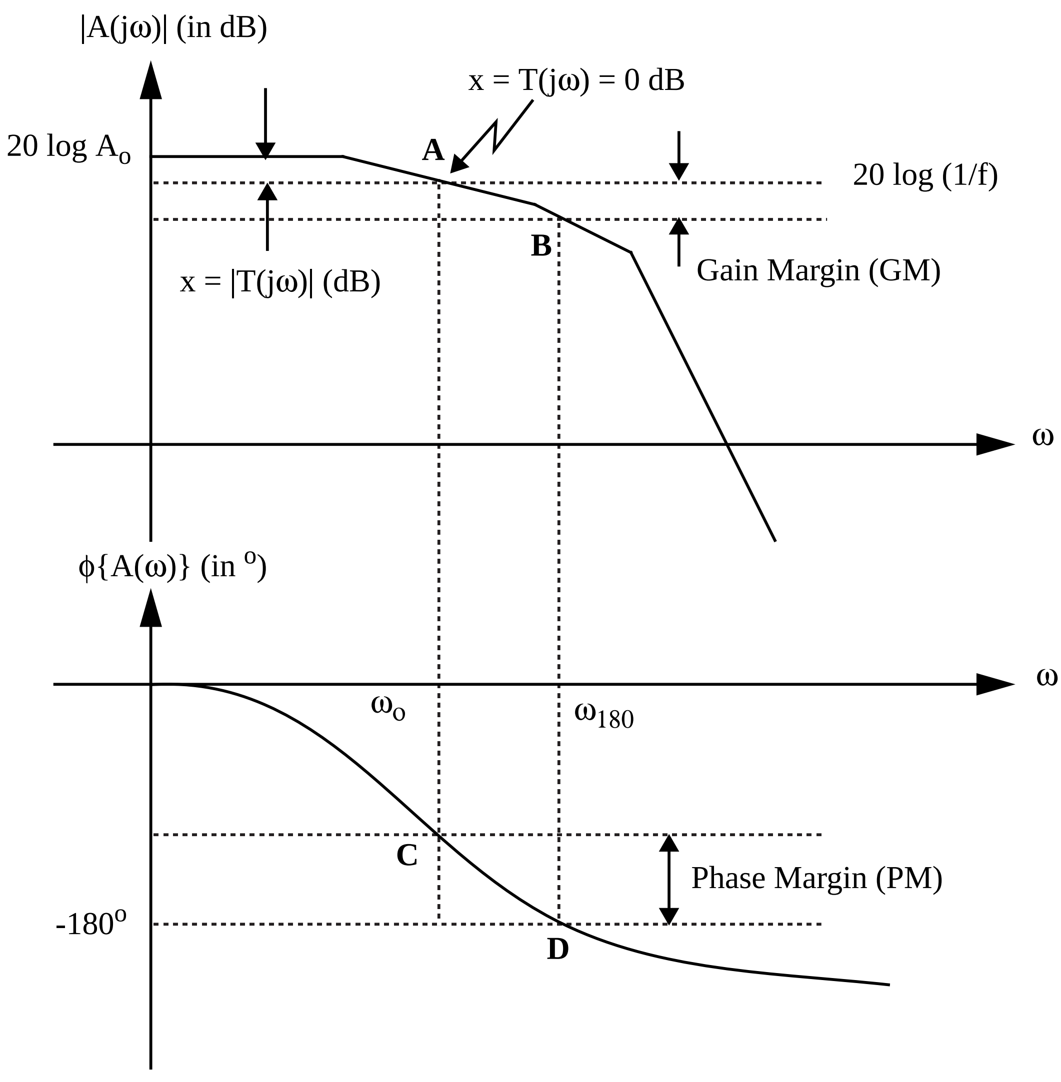


Fig. 3.5 Bode plots for the open-loop gain A(jω)

At the intersecting point of the horizontal line (20 log 1/f) and the magnitude plot $|A(j\omega)|$, labelled A,

$$20\log|A(j\omega)| = 20\log\frac{1}{f} \Rightarrow |T(j\omega)| = |A(j\omega)| \cdot f = 1 \qquad (3.8)$$

This goes to show that the loop gain at Point A is in fact unity, and therefore the frequency at Points A and C, as defined above, is the unity-gain frequency ω_o. Also, as indicated on the magnitude plot,

$$x = 20\log|A(j\omega)| - 20\log\frac{1}{f} = 20\log(|A(j\omega)| \cdot f) = 20\log(|T(j\omega)|) \qquad (3.9)$$

which is nothing but the loop gain in dB.

Since the phase of the loop gain at Point D is -180^o, the frequency at Points B and D is ω_{180}. According to the Nyquist criteria, if B is lower than A, ie. the loop gain at ω_{180} is less than 1, the amplifier is stable.

Alternatively, if the phase of the loop gain at Point C, whose frequency is the unity-gain frequency, is larger than -180^o (smaller in magnitude), the amplifier is stable.

Note: For a passive feedback network, the feedback factor f is real, and therefore the phase of the open-loop gain is the same as the phase of the loop gain, ie. $\phi\{A(j\omega)\} = \phi\{T(j\omega)\}$.

3.5 Phase Margin & Gain Margin

Both phase margin and gain margin can be used to indicate and measure the degree of stability of a feedback amplifier.

3.5.1 Phase Margin

The phase margin, PM, is defined as the difference (in degrees) between the phase of the loop gain at the unity-gain frequency and -180^o, that is:

$$PM \equiv 180° + \phi\{T(j\omega_o)\} \qquad |T(j\omega_o)| = 1 \qquad (3.10)$$

As shown in the Bode's plots in Fig. 3.5, the PM is the distance from Point C to Point D. According to the Nyquist criteria, the amplifier is stable if and only if PM is positive.

3.5.2 Gain Margin

The gain margin, GM, is defined as the loop gain (in dB) at the frequency ω_{180} where the phase of the loop gain is -180^o. That is,

$$GM \equiv |T(j\omega_{180})| \qquad \phi\{T(j\omega_{180})\} = -180° \tag{3.11}$$

In Fig. 3.5, GM is measured by the distance between Point A and Point B. Again, based on the Nyquist criteria, as long as the gain margin is less than 1, the feedback amplifier is stable.

3.6 Stability & Pole Location

3.6.1 Single-Pole Basic Amplifier

For amplifiers with only a single pole, since the phase is larger or equal -90^o at all frequencies, the phase margin PM is greater than 90^o, and the amplifier is *unconditionally stable!*

3.6.2 Multiple-Pole Basic Amplifier

Consider an amplifier with a pair of complex conjugate poles, ie.

$$p_{1,2} = \sigma_o \pm j\omega_n \tag{3.12}$$

Recall that

$$\mathfrak{I}^{-1}\left[\frac{1}{s+\alpha}\right] = e^{-\alpha t}u(t) \tag{3.13}$$

As a result, the transient response for the amplifier is given by

$$v(t) \propto \exp(p_1 t) + \exp(p_2 t) = \exp(\sigma_o t)\{\exp(j\omega_n t) + \exp(-j\omega_n t)\} = 2e^{\sigma_o t}\cos(\omega_n t) \tag{3.14}$$

a) If $\sigma_o > 0$, the response is exponentially growing, and the amplifier is thus unstable. (See Fig. 3.6a)

b) If $\sigma_o = 0$, the response is purely sinusoidal, and the amplifier oscillates. (See Fig. 3.6b)

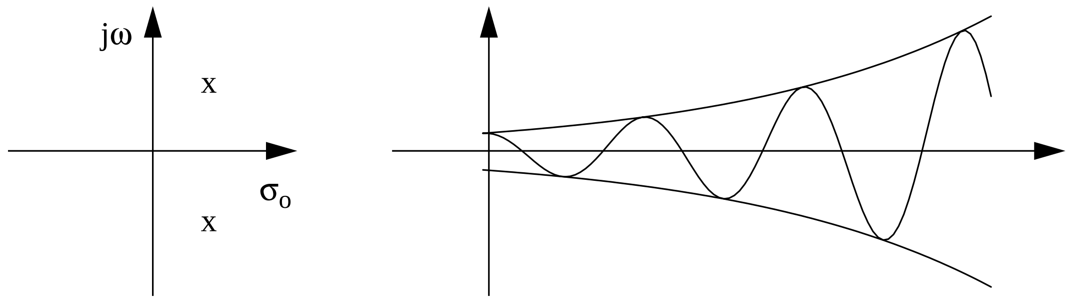
c) If $\sigma_o < 0$, the response is exponentially decaying, and the amplifier is stable! (See Fig. 3.6c)

3.7 Compensation

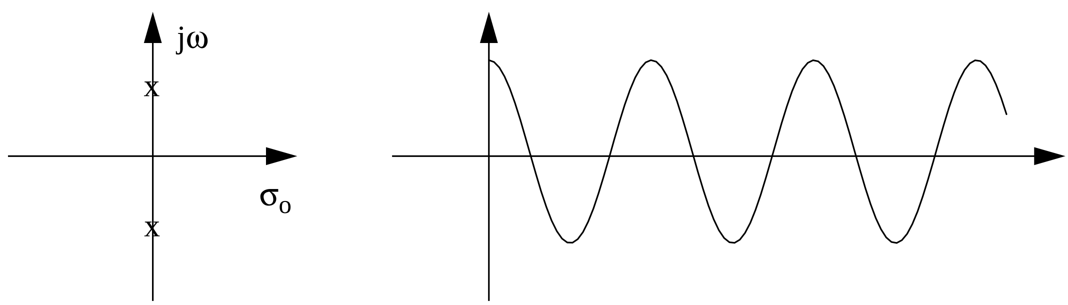
3.7.1 Motivation

Recall that, for stability, it is required that the phase margin should be positive, PM > 0, or the gain margin should be less than unity, GM < 1. Consider an amplifier with the Bode's plots for the open-loop gain being as shown in Fig. 3.7.

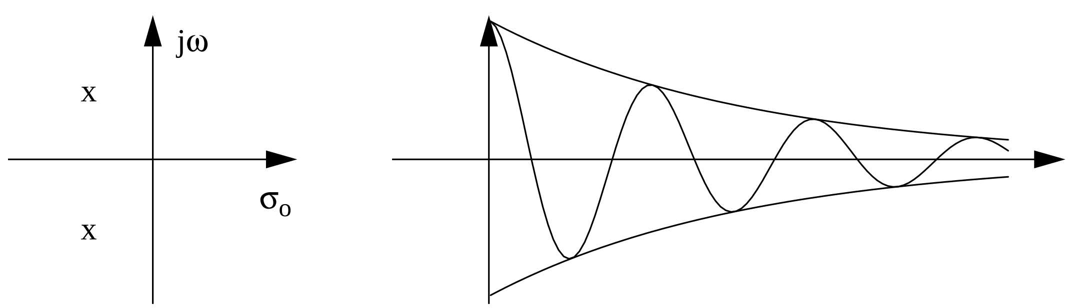
With feedback factor f_1,



a) $\sigma_o > 0$: unstable



b) $\sigma_o = 0$: oscillate



c) $\sigma_o < 0$: stable

Fig. 3.6 Relationship between the pole location and stability

$$\omega_o = \omega_{180} \tag{3.15}$$

Therefor, the phase margin is zero, and the system would be *marginally stable!*

As the feedback factor is increased, the phase margin will become more and more negative, and the circuit becomes more and more unstable. As illustrated in Fig. 3.7, for *any* feedback factor $f_2 > f_1$, the phase margin is negative, and the circuit is unstable.

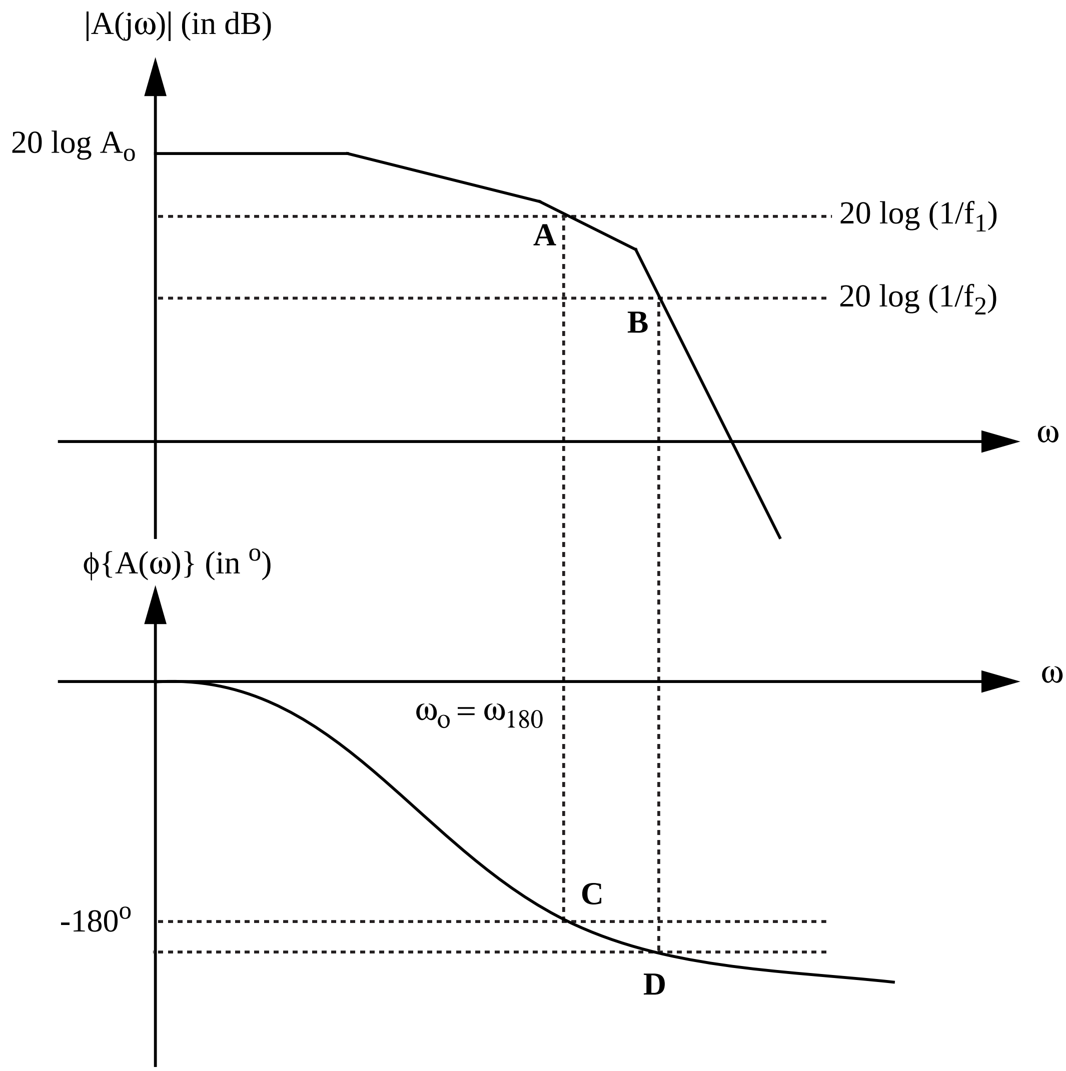


Fig. 3.7 Compensation is needed for any feedback with $f > f_1$

3.7.2 Narrowbanding

<u>Idea</u>: A dominant pole is *intentionally* inserted into the amplifier to force the phase margin to be positive to achieve stability. In the Fig. 3.8, before compensation, the first two dominant poles are p_1 and p_2, and for the feedback factor f as shown, the phase margin is negative, PM < 0, and thus the feedback circuit would be unstable.

To achieve stability, a dominant pole p_D is introduced so that the loop gain $|T(j\omega)|$ is unity at the original first pole p_1. That is, the open-loop gain $|A(j\omega)|$ is equal to the closed-loop gain (1/f) at p_1.

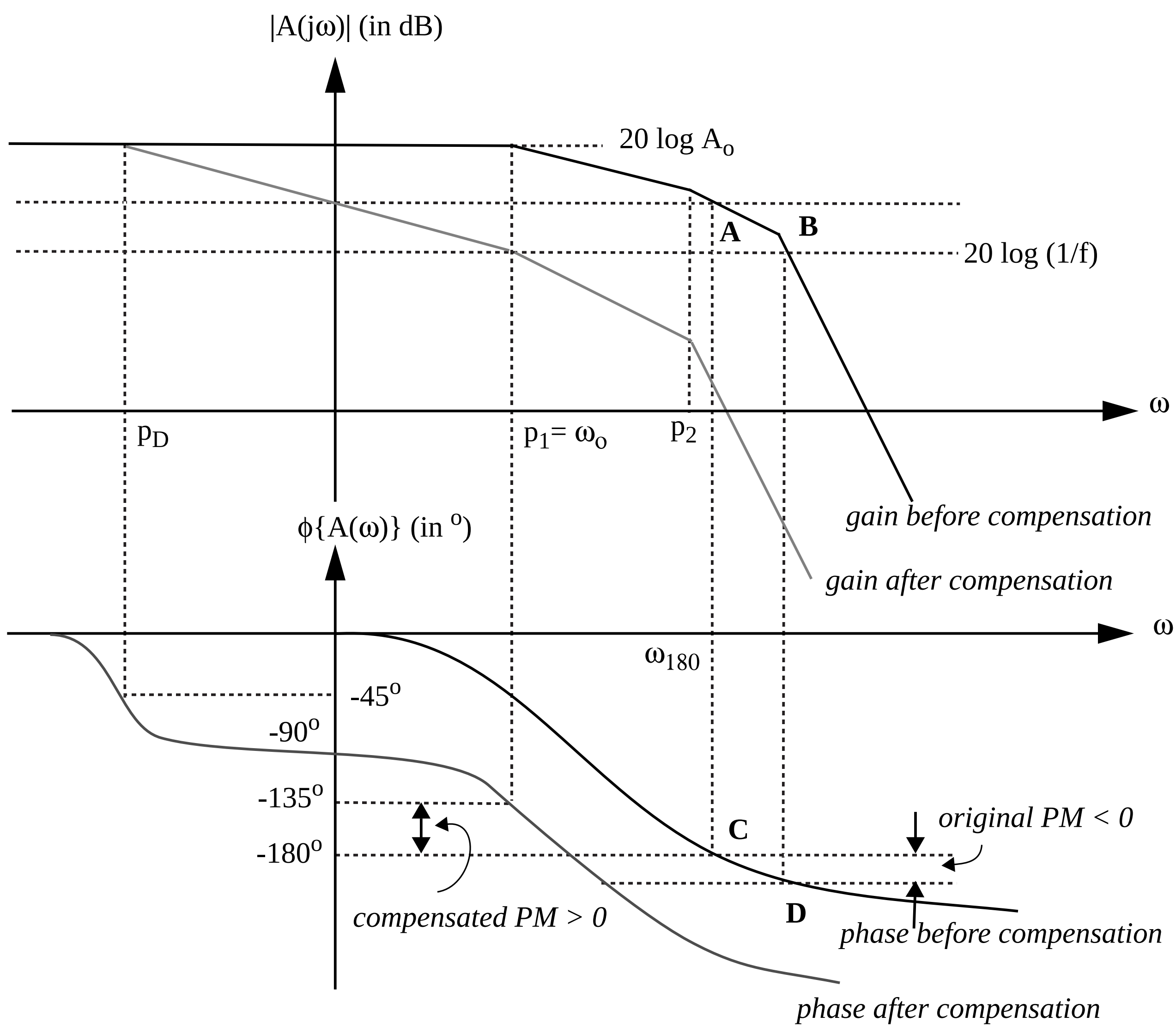


Fig. 3.8 Bodes' plots showing narrow-banding compensation technique

$$|A(j\omega)|\big|_{\omega = p_1} = |A(jp_1)| = |A_f(jp_1)| = \frac{1}{f} \tag{3.16}$$

With such a compensation, as shown in the Bode's plots, the phase margin becomes positive, PM = $45^{o} > 0$, and as a consequence, the feedback circuit gets stabilized!

Since the 3dB bandwidth is effectively much reduced by the compensation, the technique is referred to as *narrowbanding*.

<u>Disadvantage</u>:

a) The unity-gain bandwidth ω_o and the 3dB bandwidth are reduced to p_1 and p_D, respectively.

b) With feedback, the loop-gain starts to decrease as $f > p_D$. As a result, the gain is no longer constant, and the effect of feedback diminishes.

3.7.3 Improved Compensation Technique

Instead of introducing a new dominant pole p_D and making p_1 the second dominant pole as in narrowbanding technique, compensation can be done more effectively by *pushing p_1 to p_D and making p_2 the unity-gain frequency.*

Let's look again at the same amplifier considered in Fig. 3.9. As discussed above, the amplifier is unstable without compensation since the phase margin is negative. As shown in Fig. 3.9, the circuit can be stabilized without sacrificing too much bandwidth by somehow *replacing* p_1 with a smaller pole p_D and at the same time making the original second pole p_2 the unity-gain frequency.

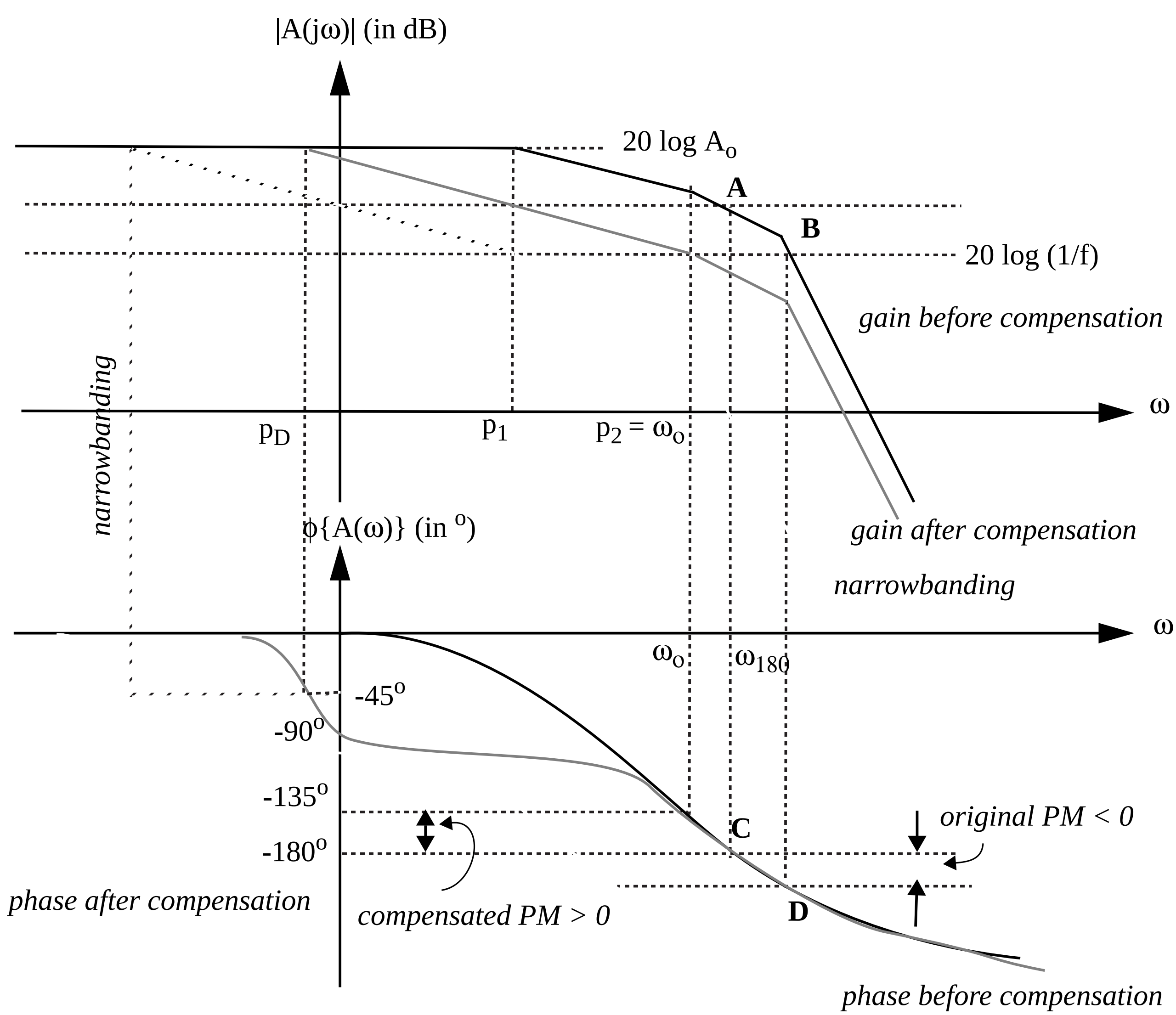


Fig. 3.9 Comparison of narrow-banding compensation and pole-splitting technique

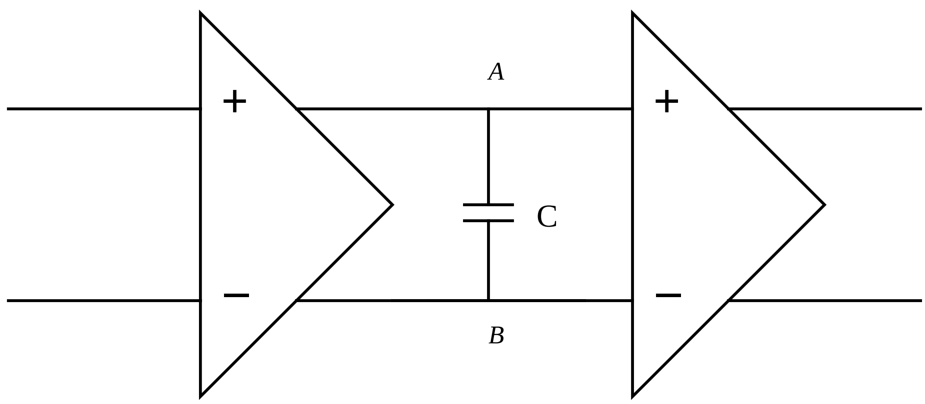
As can be seen from Fig. 3.9, this technique is much better compared to the narrowbanding technique since both the bandwidth and the unity-gain frequency of the compensated circuit are much larger.

3.8 Compensation Implementation

The most straight-forward way to do compensation is to add a large capacitor at a high capacitive and resistive node so that a desired first dominant pole is achieved.

As an example, Fig. 3.10a shows a circuit in which a capacitor C is placed at the output of the first stage for compensation. In Fig. 3.10b is the equivalent circuit, from which the first dominant pole p_D can be estimated to be:

(a)



(b)

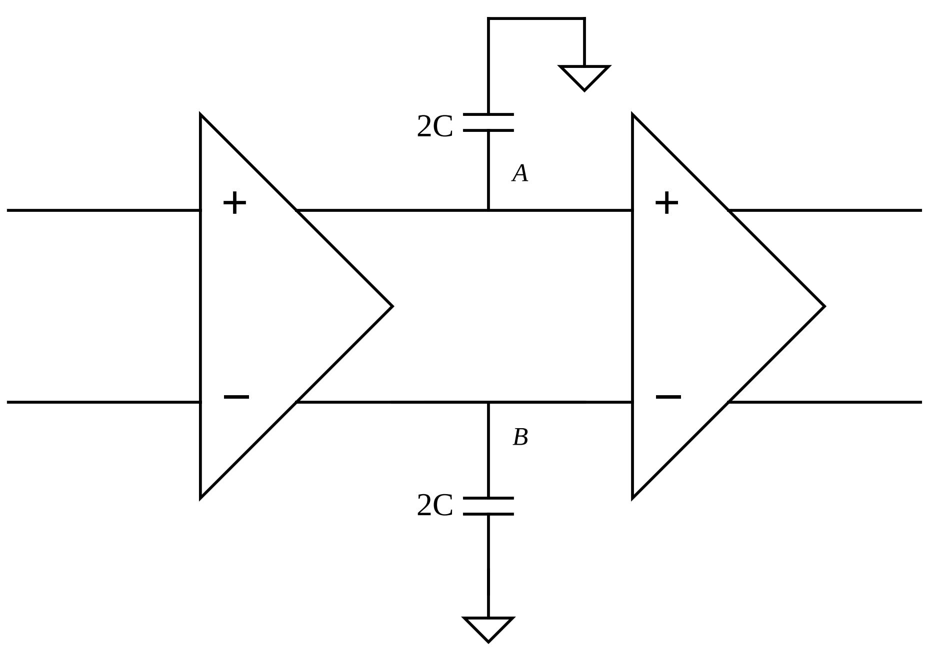


Fig. 3.10 Practical implementation of narrow-banding compensation

$$|p_D| = \frac{1}{2CR} \tag{3.17}$$

where R is the equivalent resistance at output nodes A and B. The main problem with such a compensation method is a large capacitor required (~ 1nF!).

3.9 Miller Compensation

To significantly reduce the capacitor value required for compensation, Miller effect can be used as in the real implementation of the op amp 741 and as shown in Fig. 3.11a.

By connecting the compensation capacitor C from the input to the output of the gain stage, the capacitor is effectively multiplied by the Miller-effect factor (1+A) where A is the voltage gain of the gain stage. As illustrated in Fig. 3.11b and, the first dominant pole p_D can be estimated to be:

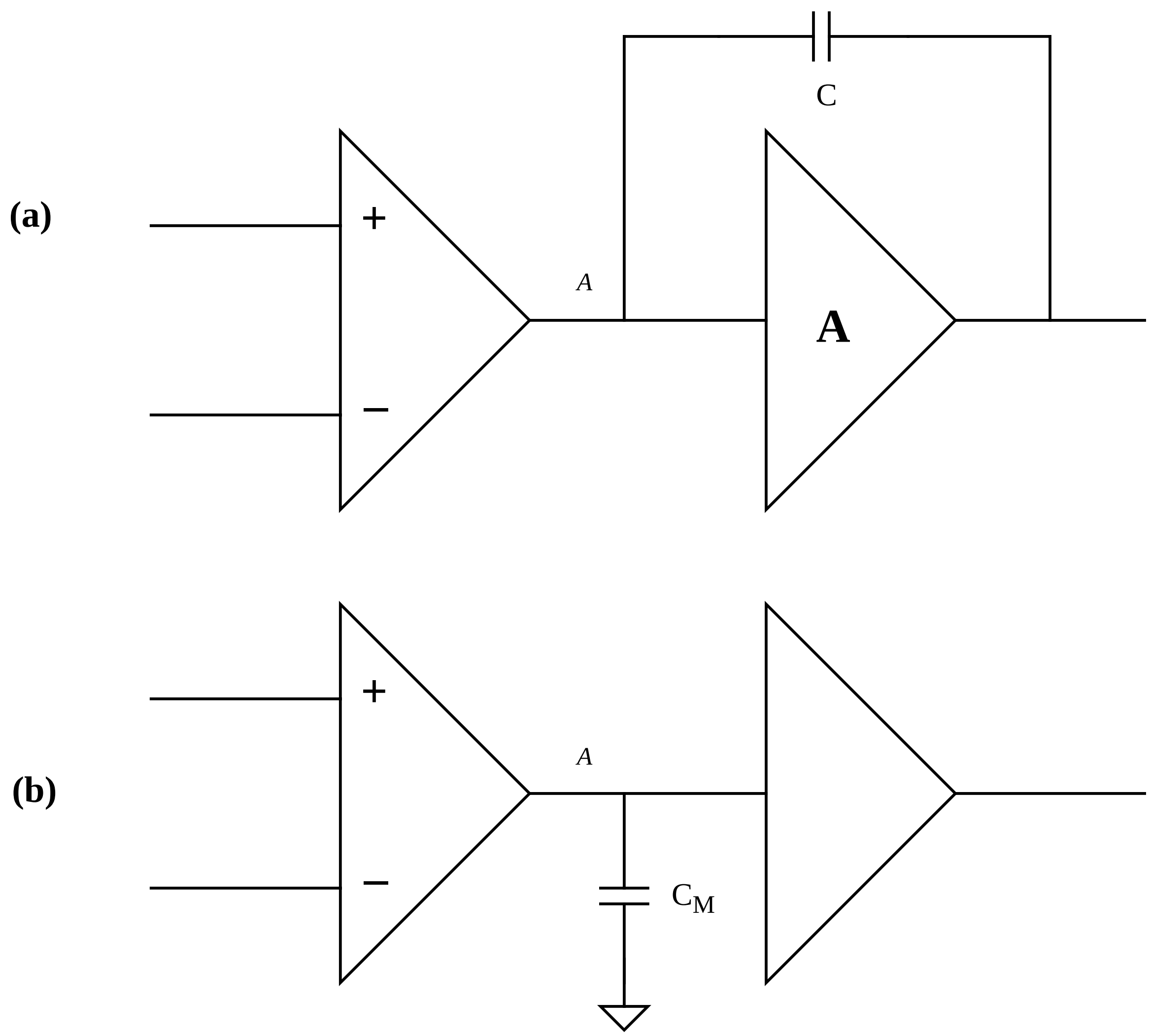


Fig. 3.11 Practical implementation of Miller (pole-splitting) compensation

$$|p_D| = \frac{1}{C_M R} = \frac{1}{C(1+A)R} \tag{3.18}$$

where R is the equivalent resistance at node A. For the same desired dominant pole, the capacitor is reduced by a factor of (1+A) compared to the narrowbanding compensation technique.

3.9.1 Pole Splitting

Another advantage of the Miller compensation technique is pole-splitting, ie. introducing a dominant pole by decreasing the first dominant pole but *at the same time* increasing the second dominant pole. Effectively, a much higher unity-gain frequency is achieved.

Consider the simple common-emitter amplifier and its small-signal equivalent circuit shown in Fig. 3.12, where the capacitor C_C is used for compensation.

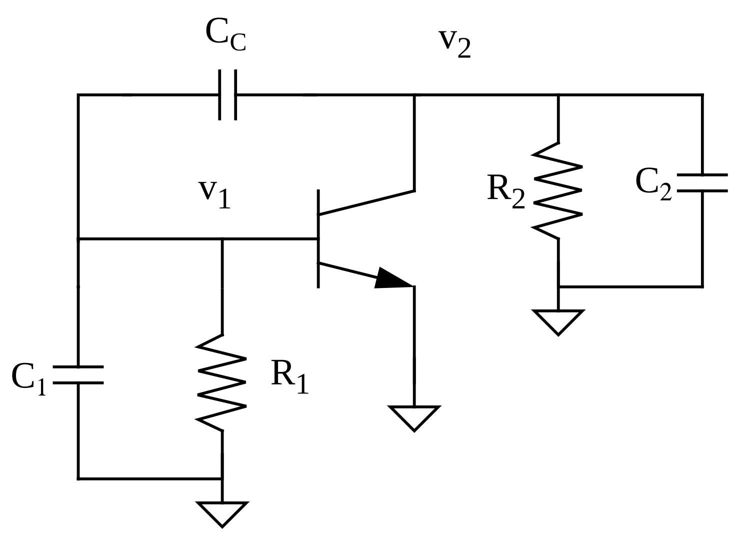


Fig. 3.12 Circuit schematic to demonstrate the pole-splitting phenomenon

Without the compensation capacitor C_C, the pole locations can be found to be:

$$|p_1| = \frac{1}{C_1 R_1} \tag{3.19}$$

$$|p_2| = \frac{1}{C_2 R_2} \tag{3.20}$$

With the compensation capacitor C_C,

$$\frac{v_o}{i_S} = \frac{(g_m - C_C s)R_1R_2}{1 + s[(C_2 + C_C)R_2 + (C_1 + C_C)R_1 + g_m(R_1R_2C_C)] + s^2R_1R_2(C_2C_1 + C_1C_C + C_2C_C)} \qquad (3.21)$$

The denominator can be expressed as:

$$D(s) = \left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right) = 1 - s\left(\frac{1}{p_1} + \frac{1}{p_2}\right) + \frac{s^2}{p_1p_2} \qquad (3.22)$$

Assume that p_1 is the dominant pole and C_C is large. Equate the coefficients for the denominator to get:

$$p_1 = -\frac{1}{(C_2 + C_C)R_2 + (C_1 + C_C)R_1 + g_m(R_1R_2C_C)} \approx -\frac{1}{g_m(R_1R_2C_C)} \qquad (3.23)$$

$$p_2 = -\frac{g_mC_C}{C_2C_1 + C_1C_C + C_2C_C} \qquad (3.24)$$

This goes to show that as C_C is increased, not only does $|p_1|$ decreases but $|p_2|$ also increases, implying that the poles are indeed splitting!

Note that if the big capacitor C_C were connected from the base to ground, ie. “No-Miller” compensation, there would be *NO* pole splitting. In this case, the pole p_1 does decreases,

$$|p_1| = \frac{1}{(C_1 + C_C)R_1} \qquad (3.25)$$

But the pole p_2 remains unchanged!

$$|p_2| = \frac{1}{C_2R_2} \qquad (3.26)$$

Table 1 summarizes the pole locations before and after compensation with Miller compensation and with No-Miller compensation.

As an illustration, Figs. 3.13a and 3.13b show graphically the pole locations of the op amp 741 before and after compensation, respectively. Before compensation, there are two dominant poles at 18.9 kHz and 328 kHz. By introducing the compensation capacitor at the critical node, the first dominant is moved back to 5 Hz whereas the second dominant pole is pushed to 10 MHz range. That is, the compensation is achieved with a much higher unity-gain frequency.

	$\lvert p_1 \rvert$	$\lvert p_2 \rvert$	C_C
Before Compensation	18.9 KHz	328 KHz	0
Miller Compensation	5 Hz	> 10 MHz	30 pF
No-Miller Compensation	0.27 Hz	294 KHz	300 nF

Table 1 Pole locations of 741 op amp before and after compensation

3.9.2 Right Half-Plane Zero

As derived in Eq. 3.21, with Miller compensation, a right half-plane (RHP) zero is introduced with the magnitude given by:

$$|z| = \frac{g_m}{C_C} \tag{3.27}$$

This RHP zero increases the gain but at the same time contributes a -90^o phase shift and thus makes the phase margin smaller.

For BJT transistors, due to their high g_m's, this RHP zero is far away from the two dominant poles p_1 and p_2 and thus would have no effect on the phase margin and stability. However, for MOS devices, due to their low g_m's, the zero can be located very close to the second dominant pole p_2 and would hence reduce the phase margin and affect the stability.

<u>Solution</u>: One effective and popular solution is to insert a nulling resistor to cancel the RHP zero as indicated in Fig. 3.14. With the nulling resistor R_Z being connected in series with the compensation capacitor C_C, the zero becomes:

$$|z| = \frac{1}{C_C\left(\frac{1}{g_m} - R_Z\right)} \tag{3.28}$$

If the nulling resistor R_Z is chosen to be:

$$R_Z = \frac{1}{g_m} \tag{3.29}$$

the zero would effectively eliminated! In practice, the resistor can be realized by using a MOS

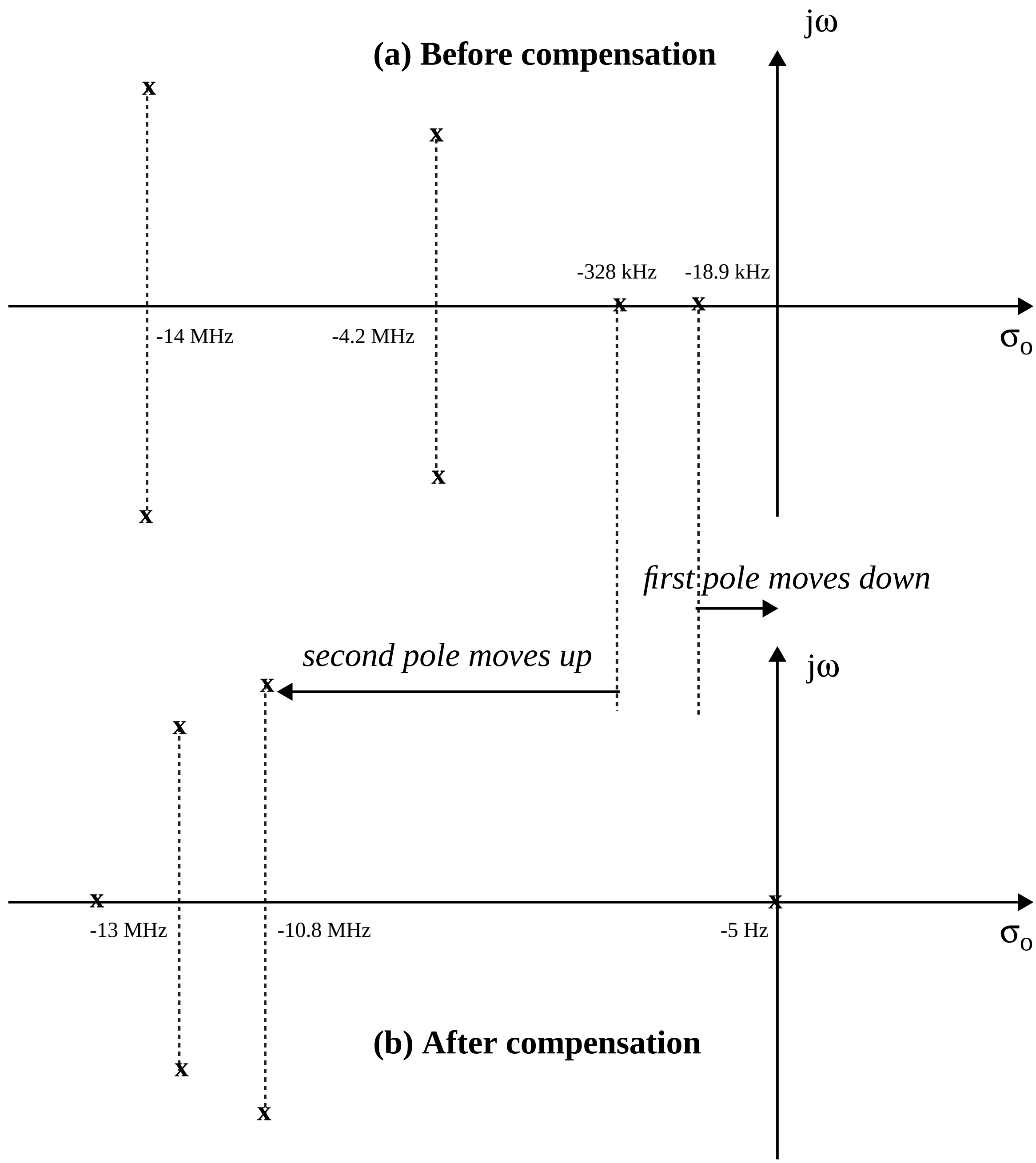


Fig. 3.13 Pole locations of 741 op amp before and after compensation

transistor operated in linear region.

3.10 Phase Margin & Frequency Response:

Consider a feedback amplifier with the closed-loop transfer function given by:

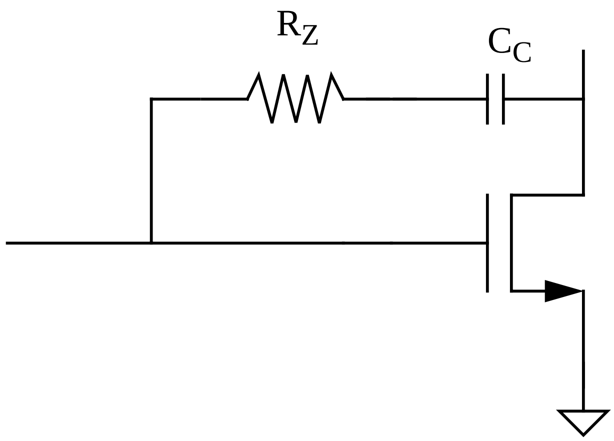


Fig. 3.14 Schematic showing a series resistor to null out the RHP zero

$$A_f(j\omega) = \frac{A(j\omega)}{1 + A(j\omega)f} = \frac{A(j\omega)}{1 + T(j\omega)} \quad (3.30)$$

where

$$T(j\omega) = A(j\omega)f = |T(j\omega)|e^{j\phi(j\omega)} \quad (3.31)$$

and $\phi(j\omega)$ denotes the phase of the loop gain.

At the unity-gain frequency, ω_o, the closed-loop gain is given by:

$$|A_f(j\omega_o)| = \left|\frac{A(j\omega_o)}{1 + |T(j\omega_o)|e^{j\phi(j\omega_o)}}\right| = \frac{1}{f}\cdot\left|\frac{1}{1 + e^{j\phi(j\omega_o)}}\right| \quad (3.32)$$

since by the definition of the unity-gain frequency,

$$|T(j\omega_o)| = |A(j\omega_o)f| = 1 \quad (3.33)$$

Combining with the definition of the phase margin,

$$PM = 180° + \phi(j\omega_o) \quad (3.34)$$

the closed-loop gain as a function of the phase margin at the unity-gain frequency is obtained to be:

$$|A_f(j\omega_o)| = \frac{1}{f}\cdot\left|\frac{1}{1 + e^{j[PM - 180]}}\right| \quad (3.35)$$

a) PM = 45°:

$$|A_f(j\omega_o)| = \frac{1}{f} \cdot \left|\frac{1}{1 + e^{j(-135)}}\right| = \frac{1.3}{f} \tag{3.36}$$

That is, the frequency response is peaking at the unity-gain frequency, 2.4dB above the low-frequency gain 1/f.

b) PM = 60°:

$$|A_f(j\omega_o)| = \frac{1}{f} \cdot \left|\frac{1}{1 + e^{j(-120)}}\right| = \frac{1}{f} \tag{3.37}$$

There is *neither* peaking *nor* gain reduction at the unity-gain frequency.

c) PM = 90°:

$$|A_f(j\omega_o)| = \frac{1}{f} \cdot \left|\frac{1}{1 + e^{j(-90)}}\right| = \frac{0.7}{f} \tag{3.38}$$

There is a gain reduction of 3dB (below the low-frequency gain 1/f) at the unity-gain frequency.

The results are summarized in Fig. 3.15, where it can be concluded that as the phase margin PM decreases, the gain peak increases. When the phase margin reaches zero, the feedback amplifier becomes oscillating.

In addition, from the figure, the best compensation is for the phase margin of 60° for which the maximum bandwidth can be achieved without peaking or gain reduction.

3.11 Slew Rate:

Consider an amplifier connected in a unity-gain buffer (voltage follower) configuration with a large step input signal as shown in Figs. 3.16a and 3.16b.

Assume that the amplifier has a single dominant pole and the transfer function is given by:

$$\frac{v_o}{v_{in}}(s) = \frac{A}{1 + \tau s} \tag{3.39}$$

Since the amplifier is connected as a unity-gain buffer, the low-frequency gain is unity, and:

$$\frac{v_o}{v_{in}}(s = 0) = A = 1 \tag{3.40}$$

As a result,

$$\frac{v_o}{v_{in}}(s) = \frac{1}{1 + \tau s} \tag{3.41}$$

For a step input with an amplitude of V_a,

$$v_{in}(t) = V_a u(t) \qquad v_{in}(s) = \frac{V_a}{s} \tag{3.42}$$

the output voltage can be obtained to be:

$$v_o(s) = \frac{V_a}{s} \cdot \frac{1}{1 + \tau s} = \frac{V_a}{s} - \frac{V_a}{s + 1/\tau} \tag{3.43}$$

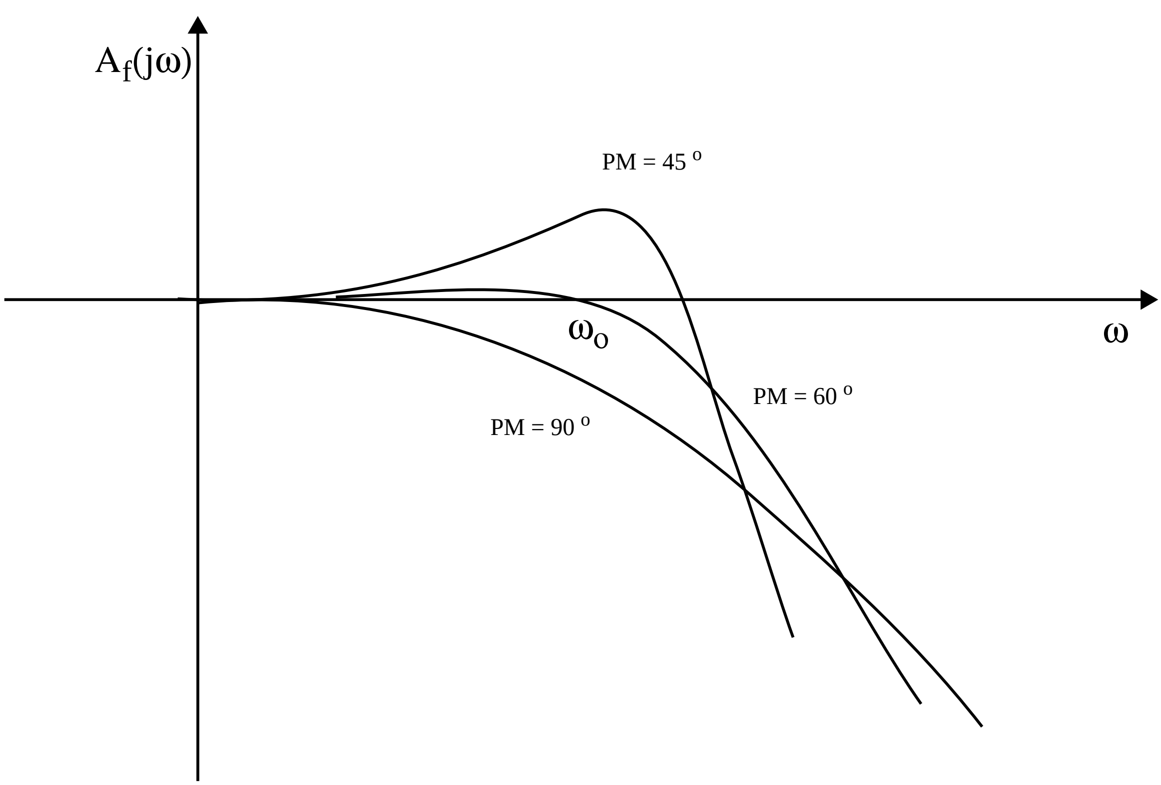


Fig. 3.15 Bode's s showing relationship between frequency response and phase margins

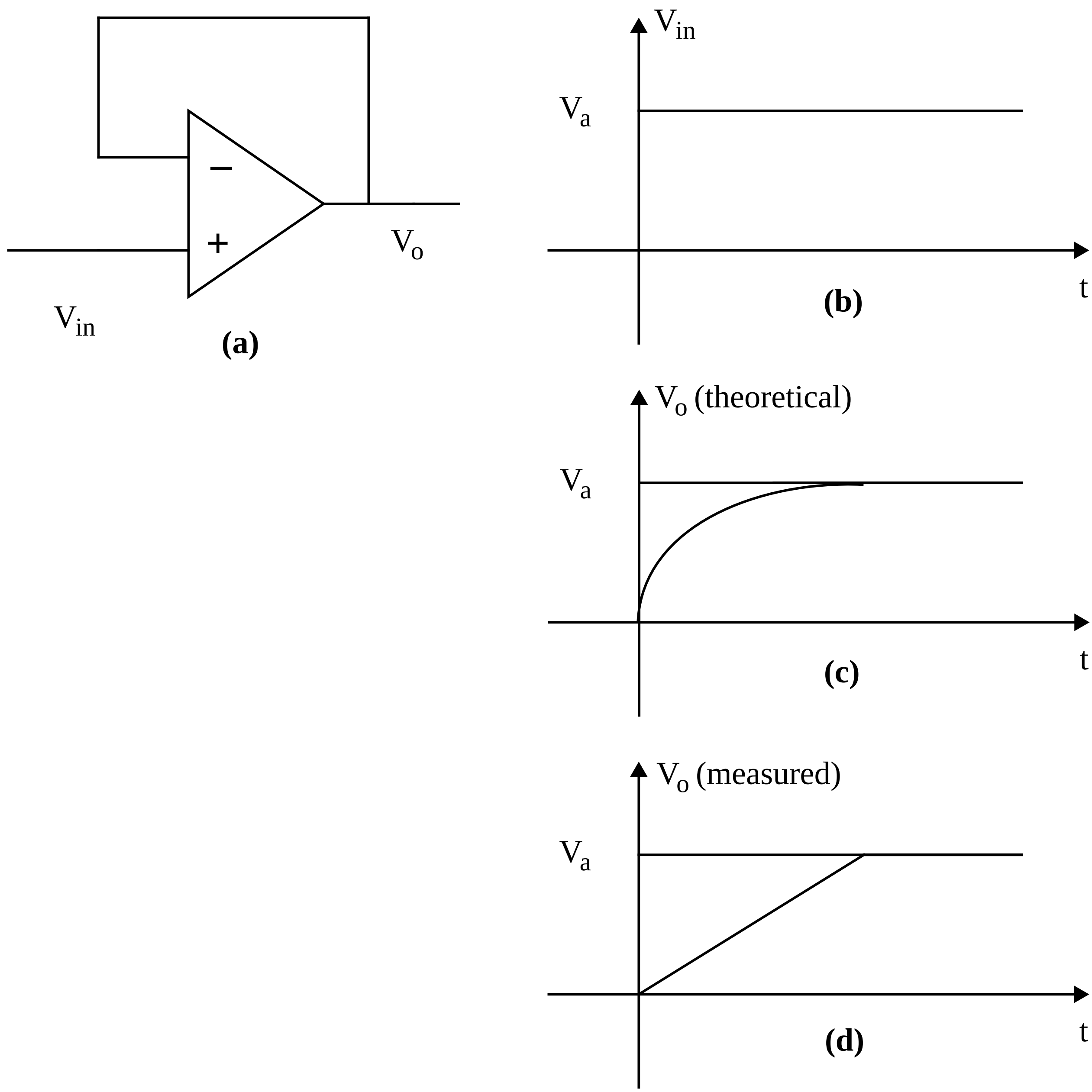


Fig. 3.16 Comparison of output voltage of an op amp with and without slew-limiting

$$v_o(t) = V_a(1 - e^{-t/\tau}) \tag{3.44}$$

Although it is expected that the output would follow the input and step up, the output waveform would rise exponentially as plotted in Fig. 3.16c. This is due to the finite bandwidth of the amplifier according to Eq. 3.44. However, in practice, the measured output waveform looks like that shown in Fig. 3.16d, where the output increases linearly with time. The slope of this output waveform, dV_o/dt, is referred to as the *slew rate* of the amplifier.

3.11.1 Explanation

At t = 0, the input changes abruptly from 0 to V_a. Because of the unity feedback configuration, the output tries to follow. However, due to a limited current available to charge the load capacitor, typically the compensation capacitor, the output voltage cannot change instantaneously. As a consequence, a differential voltage with a magnitude of V_a is applied at the input and drives the amplifier out of its linear range of operation.

As illustrated in Fig. 3.17,

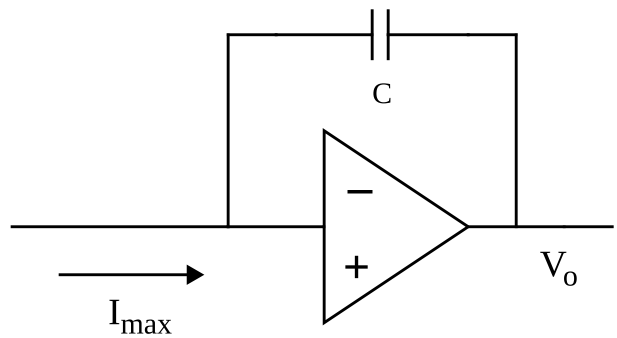


Fig. 3.17 Schematic showing compensation capacitor and maximum input current

$$v_o(t) = \frac{1}{C}\int I_{max}dt \qquad (3.45)$$

$$\frac{d}{dt}v_o(t) = \frac{I_{max}}{C} \qquad (3.46)$$

As an example, consider the op amp 741, for which I_{max} = 24 µA, C = 30 pF,

$$\frac{d}{dt}v_o(t) = \frac{I_{max}}{C} = 0.8V/\mu s \qquad (3.47)$$

which is very close to experimental result!

3.11.2 Relationship of Slew Rate and Unity-Gain Frequency

Shown in Fig. 3.18 is a typical op amp configuration in which a compensation capacitor connected across the gain stage is driven by an input stage with transconductance g_m.

$$\frac{\Delta V_o}{\Delta I_x} = \frac{1}{sC} \qquad (3.48)$$

But

$$\Delta I_x = g_m \Delta V_{in} \qquad (3.49)$$

Therefore,

$$\frac{\Delta V_o}{\Delta V_{in}}(j\omega) = \frac{g_m}{j\omega C} \tag{3.50}$$

Assume that the compensation is done for unity-gain operation with a phase margin of 45^o,

$$\frac{\Delta V_o}{\Delta V_{in}}(j\omega_2) = 1 \tag{3.51}$$

where ω_2 is the unity-gain frequency is compensated as the second dominant pole, we obtain:

$$\left|\frac{g_m}{j\omega_2 C}\right| = 1 \tag{3.52}$$

or

$$\frac{1}{C} = \frac{\omega_2}{g_m} \tag{3.53}$$

Put all together, the slew rate as a function of the unity-gain frequency is given by:

$$SR \equiv \frac{d}{dt}v_o(t) = \frac{I_{max}}{C} = \frac{\omega_2}{g_m}I_{max} \tag{3.54}$$

From Eq. 3.54, in order to increase the slew rate, it is necessary to:

- For a given unity-gain frequency, increase the ratio I_{max}/g_m.
- For a given I_{bias}, minimize the input transconductance g_m, which can be achieved by using emitter degeneration or using MOS devices.

3.11.3 Effect of Slew Rate on Amplifiers with Large Sinusoidal Input

For sinusoidal input signal,

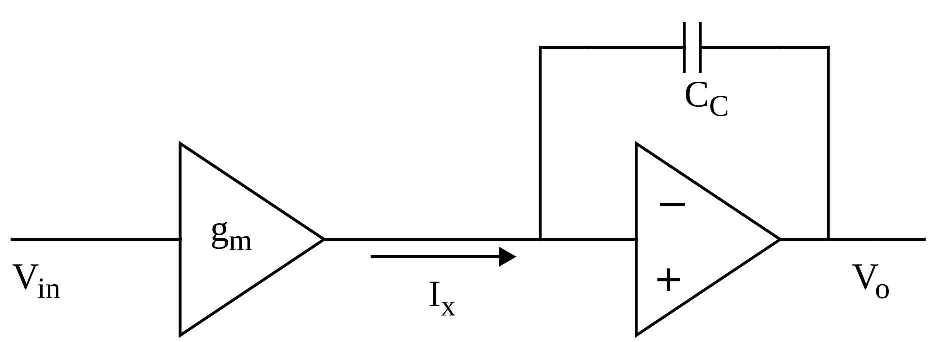


Fig. 3.18 Typical op amp configuration with a g_m-stage driving a compensation capacitor

$$v_{in}(t) = V_a \sin(\omega t) \tag{3.55}$$

$$\left.\frac{d}{dt}v_{in}(t)\right|_{max} = \omega V_a \cos(\omega t)|_{max} = \omega V_a \tag{3.56}$$

It follows that, if the slew rate SR > ωV_a, the output will follow the input closely, as illustrated in Fig. 3.19(a). Vice versa, if SR < ωV_a, the amplifier is *slew-limited*, and there will be a lot of distortion at the output as shown in Fig. 3.19(b)!

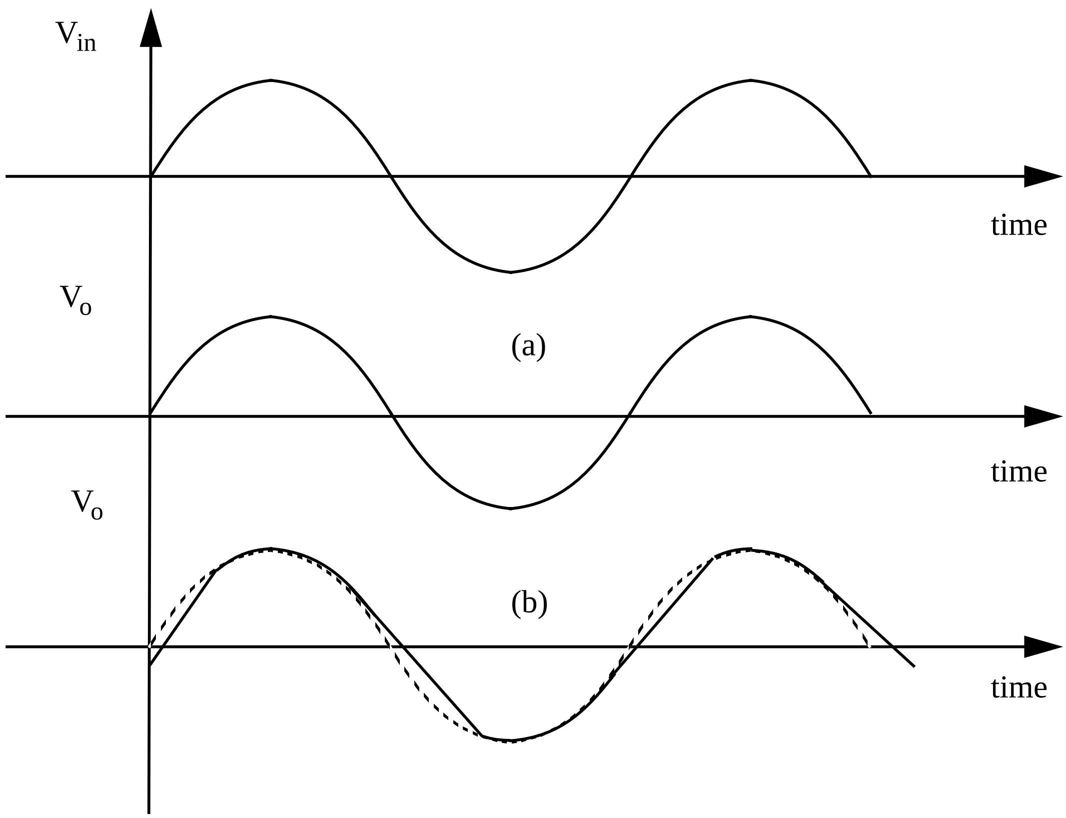


Fig. 3.19 Output waveform (a) without slew-limiting and (b) with slew-limiting

3.11.4 Full-Power Bandwidth

For a sinusoidal input signal with a given amplitude V_a, as the frequency is increased high enough, the slew limiting will occur. The full-power bandwidth ω_{FP} is defined as the frequency at which this happens. From Eq. 3.56, it is obvious to obtain:

$$\omega_{FP} = \frac{SR}{V_a} \tag{3.57}$$

3.12 Appendix on Positive Feedback

This appendix is to prove that a system with a positive feedback is still stable as long as the loop gain is less than unity. Consider the feedback system shown in Fig. 3.20. For the feedback to be positive, it is necessary that the loop gain $T = Af > 0$.

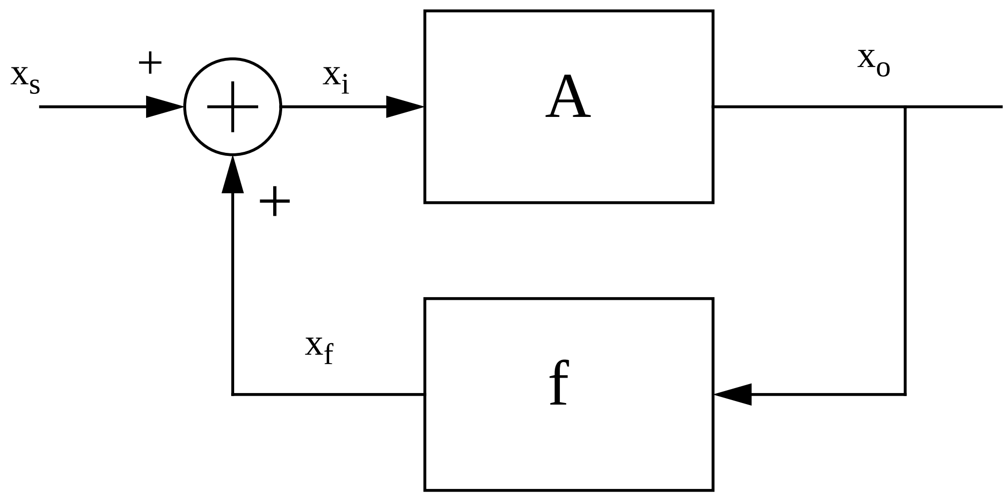


Fig. 3.20 Block diagram of a *positive* feedback system

Denote x_{in} as the value of x_i after going through the loop n times, we have:

$$x_{i_o} = x_S \tag{3.58}$$

$$x_{i_1} = x_S + T \cdot x_{i_o} = (1+T)x_S \tag{3.59}$$

$$x_{i_2} = x_S + T \cdot x_{i_1} = (1+T+T^2)x_S \tag{3.60}$$

By induction,

$$x_{i_n} = x_S + T \cdot x_{i_{(n-1)}} = (1+T+T^2+\ldots+T^n)x_S = \frac{1-T^{(n+1)}}{1-T}x_S \tag{3.61}$$

As a result, for $T > 1$, the system blows up, ie. the signal approaches infinity and the system becomes unstable. However, for $T < 1$,

$$x_{i_n} = \frac{1}{1-T}x_S \tag{3.62}$$

which apparently is finite as long as x_s is finite. In other words, even if a system has a positive feedback, it can still be stable as long as the loop gain is less than 1.

Note that the above derivation does not hold for *negative feedback* systems. The main reason is that for a negative feedback, the feedback signal x_f is always driven in the *opposite* direction of the input signal x_s. It follows that the output signal x_o is also driven from one polarity to another, and as a result, at some point the output surely crosses the point where the feedback signal is equal to the input signal. At this point, the error signal x_i becomes zero, the output signal stops changing, and the system becomes stable.