

hence

$$r_5 \left(\frac{V_{DS5}}{2} + V_{d2sat} + V_{TH2} - V_{TH1} \right) \cdot V_{DS5} = \frac{1}{2} \cdot r_2 \cdot (V_{d2sat})^2$$

$$\Delta V_{TH} = V_{TH2} - V_{TH1}$$

$$r = \frac{r_5}{r_2}$$

given

$$r \cdot (V_{DS5} + 2 \cdot V_{d2sat} + 2 \cdot \Delta V_{TH}) \cdot V_{DS5} = V_{d2sat}^2$$

$$\text{find}(V_{d2sat}) \rightarrow \left[r \cdot V_{DS5} + \left(r^2 \cdot V_{DS5}^2 + r \cdot V_{DS5}^2 + 2 \cdot r \cdot V_{DS5} \cdot \Delta V_{TH} \right)^{\left(\frac{1}{2}\right)} \cdot V_{DS5} - \left(r^2 \cdot V_{DS5}^2 + r \cdot V_{DS5}^2 + 2 \cdot r \cdot V_{DS5} \cdot \Delta V_{TH} \right)^{\left(\frac{1}{2}\right)} \right]$$

$$V_{d2sat} = r \cdot V_{DS5} \cdot \left(1 + \sqrt{1 + \frac{1}{r} + \frac{2 \cdot \Delta V_{TH}}{r \cdot V_{DS5}}} \right)$$

check

given

$$x \cdot \left(1 + \sqrt{1 + \frac{1}{x}} \right) = 1$$

$$\text{find}(x) \rightarrow \frac{1}{3} \quad \text{should be } 1/3$$

we need $V_{DS5} = K \cdot V_{d1sat}$

$$r_1 \cdot V_{d1sat}^2 = r_2 \cdot V_{d2sat}^2$$

hence

$$V_{d1sat} = V_{d2sat} \cdot \sqrt{\frac{r_2}{r_1}}$$

$$V_{d1sat} = V_{DS5} \cdot \left(\frac{r_5}{r_2} \right) \cdot \sqrt{\frac{r_2}{r_1}} \cdot \left(1 + \sqrt{1 + \frac{1}{r} + \frac{2 \cdot \Delta V_{TH}}{r \cdot V_{DS5}}} \right)$$

$$V_{d1sat} = V_{DS5} \cdot \frac{r_5}{\sqrt{r_1 \cdot r_2}} \cdot \left(1 + \sqrt{1 + \frac{r_2}{r_5} + \frac{2 \cdot \Delta V_{TH}}{r \cdot V_{DS5}}} \right)$$

ignoring body-effect:

$$V_{d1sat} = V_{DS5} \cdot \frac{r_5}{\sqrt{r_1 \cdot r_2}} \cdot \left(1 + \sqrt{1 + \frac{r_2}{r_5}} \right)$$

$$V_{d1sat} = V_{DS5} \cdot \frac{W_5}{L_5} \cdot \sqrt{\frac{L_1 \cdot L_2}{W_1 \cdot W_2}} \cdot \left(1 + \sqrt{1 + \frac{W_2 \cdot L_5}{L_2 \cdot W_5}} \right)$$

Problem: for $L_1 \neq L_2$ end up with ratios of channel length, no matter the choice of L_5

given

$$\frac{1}{K} = \frac{r_5}{\sqrt{r_1 \cdot r_2}} \cdot \left(1 + \sqrt{1 + \frac{r_2}{r_5}} \right)$$

$$\text{find}(r_5) \rightarrow \left[\frac{1}{2 \cdot (-4 \cdot r_1 + K^2 \cdot r_2)} \right] \cdot \left[2 \cdot r_2 \cdot K + 4 \cdot (r_1 \cdot r_2) \left(\frac{1}{2} \right) \right] \cdot \frac{r_1}{K} \left[\frac{1}{2 \cdot (-4 \cdot r_1 + K^2 \cdot r_2)} \right] \cdot \left[2 \cdot r_2 \cdot K - 4 \cdot (r_1 \cdot r_2) \left(\frac{1}{2} \right) \right] \cdot \frac{r_1}{K}$$

$$r_5(K, r_1, r_2) := r_1 \cdot \frac{K \cdot r_2 + 2 \cdot \sqrt{r_1 \cdot r_2}}{K \cdot (4 \cdot r_1 + K^2 \cdot r_2)} \quad (\text{ugly}) \quad V_{DS1} = K \cdot V_{d1sat}$$

Examples:

$L_2 := 0.25, 0.5..1$

$$r_5 \left(2, \frac{100}{1}, \frac{100}{L_2} \right) = r_5 \left(2, \frac{10}{1}, \frac{10}{L_2} \right) =$$

30
28.452
26.658
25

3
2.845
2.666
2.5

for large $K=2$, $(W/L)_5$ is fairly *insensitive* to L_2

$$r_5 \left(1.2, \frac{100}{1}, \frac{100}{L_2} \right) = r_5 \left(1.2, \frac{10}{1}, \frac{10}{L_2} \right) =$$

75.137
63.329
55.031
49.02

7.514
6.333
5.503
4.902

for small $K=1.2$, $(W/L)_5$ is fairly sensitive to L_2

Conclusion: large K for insensitivity to process variations and mismatch. Well, we knew that already!