EE140 HW#2 Solution

1.

$V_{SG2} = V_{R1}$

$V_{SD2} = V_{R1} + V_{R2}$.

$\therefore V_{SG2} < V_{SD2} \Rightarrow$ M2 either in cut-off or saturation region

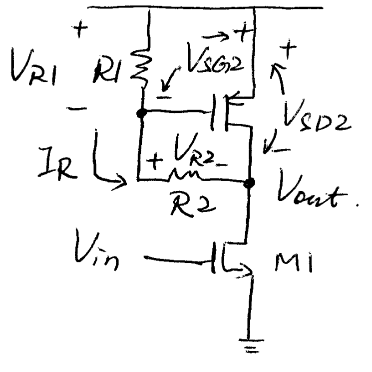
when $V_{in} < V_{to(NMOS)} = 0.5V$. M1 is cut-off

$I_{DS1} = I_{DS2} = I_{R1} = I_{R2} = 0 \quad \therefore V_{out} = V_{DD}$.

When $V_{R1} = V_{R2} \geq |V_{to(pmos)}| = 0.6V$. M2 is in the saturation region

When $V_{R1} = V_{R2} = 0.6V \quad V_{out} = V_{DD} - V_{R1} - V_{R2}$

$= 1.8V$



Assume M1 is in the saturation region:

KCL @ output

$I_{SD2} + I_R = I_{DS1}$ ①

$I_{SD2} = \frac{1}{2} K_p'(\frac{W}{L})_p (V_{SG2} - |V_{tp}|)^2$ ②

$I_R = \frac{0.6V}{R_2} = \frac{0.6V}{10K\Omega} = 60\mu A.$ ③

$I_{DS1} = \frac{1}{2} K_n'(\frac{W}{L})_n (V_{GS1} - |V_{tn}|)^2$ $V_{GS1} = V_{in}$

$\therefore I_{DS1} = \frac{1}{2} K_n'(\frac{W}{L})_n (V_{in} - |V_{tn}|)^2$ ④ $V_{SG2} = 0.6V$

Substitute ②, ③, ④ in ①.

$\frac{1}{2} \times 3mA/V^2 \times (0.6V - 0.6V)^2 + 60\mu A =$

$\frac{1}{2} \times 8mA/V^2 \times (V_{in} - 0.5)^2$

Solve $\boxed{V_{in} = 0.62V.}$

* Check the operation region of M1:

$V_{GS1} = V_{in} = 0.62V.$

$V_{DS1} = V_{out} = 1.8V$ $\Big\}$ $V_{DS1} > V_{GS1} - V_{t1}$

M1 is in the sat. region. Assumption is correct.

Another breaking point is when M1 goes into linear region. (boundry point).

$I_{DS1} = K_n'(\frac{W}{L})_n [(V_{GS1} - V_t) \cdot V_{DS1} - \frac{V_{DS1}^2}{2}]$ ⑤

$V_{GS1} = V_{in}$ $V_{DS1} = V_{out}$

$I_D = (V_{DD} - V_{out})/$ ⑥

$$V_{SG2} = \frac{V_{DD} - V_{out}}{2} \qquad ⑦.$$

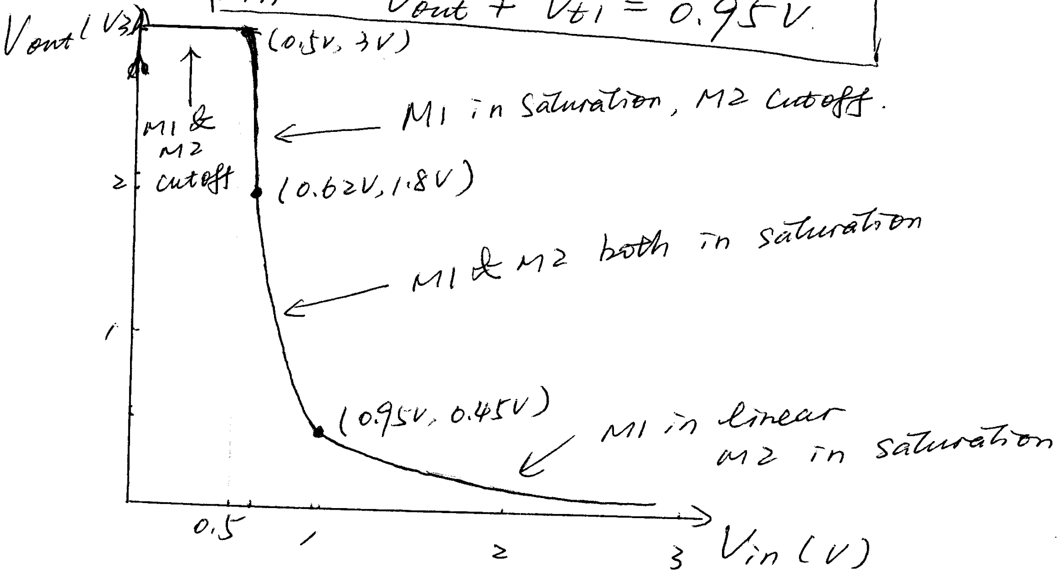
$$V_{in} - V_{t1} = V_{DS1} \qquad ⑧$$

Substitute ②⑤, ⑥, ⑦,⑧ into ①.

$$\frac{1}{2} \times 3\,mA/V^2 \times \left(\frac{3V - V_{out}}{2} - 0.6\right)^2 + \frac{3V - V_{out}}{20k\Omega} = 8\,mA/V^2 \times \left[V_{out}^2 - \frac{V_{out}^2}{2}\right]$$

Solve

$$\boxed{\begin{aligned} V_{out} &= 0.45V \\ V_{in} &= V_{out} + V_{t1} = 0.95V \end{aligned}}$$



2.

a) <u>First circuit</u> :

$V_{out} = 1.5V$.

$$I_{DS} = \frac{V_{DD} - V_{out}}{R_{L1}} = \frac{3V - 1.5V}{10k\Omega}$$

$$= \boxed{150\mu A.}$$

$V_D = 1.5V$, $V_G = 1.2V$ $\Rightarrow$ Transistor must be in sat. region.

$\therefore I_{DS} = \frac{1}{2}K'\frac{W}{L}(V_{GS} - V_t)^2(1 + \lambda V_{DS})$ ①.

$V_{GS} = V_G - V_S = 1.2V - V_S$ ②.

$V_{DS} = V_D - V_S = 1.5V - V_S$ ③.

$V_t = V_{t0} + \gamma(\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f})$ ④.

$V_{SB} = V_S - V_B = V_S$ ⑤.

Substitute ②, ③ into ① and ⑤ into ④.

we got:

$150\mu A = \frac{1}{2} \times 8m \times (1.2V - V_S - V_t)^2[1 + 0.1(1.5 - V_S)]$ ⑥

$V_t = 0.5 + 0.2(\sqrt{0.6 + V_S} - \sqrt{0.6})$ ⑦.

Use iteration to solve ⑥ & ⑦:

first guess $V_s \cong 0.5V$:

$$\begin{tikzcd}
V_s & V_t \\
0.5V \arrow[dr, "\text{use ⑦}"] & \\
& 0.55V \arrow[dl, "\text{use⑥}"'] \\
0.462 \arrow[dr, "\text{use ⑦}"] & \\
& 0.551V.
\end{tikzcd}$$

The V_t difference between the first & second iteration is only 0.001V, enough accuracy.

So $V_s \cong 0.462V$. $\boxed{V_t \cong 0.551V.}$

$$V_{in} = V_s - R_s \cdot I_{os} = 0.462V - 0.2k\Omega \times 0.15mA$$

$$= \underline{\underline{0.432V.}}$$

Second circuit :

$V_{out} = 1.5V$

$$\therefore I_{os} = \frac{V_{out}}{R_{L2}} = \frac{1.5V}{10k\Omega} = 0.15mA$$

$V_{in,max} = 3V$.

$\therefore V_{GS,max} = 3V - 1.5V = 1.5V$.

$V_{DS} = V_{DD} - V_{out} = 1.5V$

Therefore, transistor must be in saturation region.

$I_{DS} = \frac{1}{2} K' \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS})$ ⑧.

$V_{GS} = V_G - V_S = V_{in} - 1.5V$ ⑨

$V_{DS} = V_{DD} - V_{out} = 1.5V$. ⑩

$V_t = V_{t0} + \gamma(\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f})$

$V_{SB} = V_S - V_B = 1.5V$

$\therefore V_t = 0.5V + 0.2 \times (\sqrt{0.6V + 1.5V} - \sqrt{0.6V})$

$= \boxed{0.635V}$ ⑪

Sub. ⑨, ⑩, ⑪ into ⑧.

$0.15mA = \frac{1}{2} \times 8m \times (V_{in} - 1.5 - 0.635)^2 (1 + 0.1 \times 1.5)$

$\therefore \boxed{V_{in} = 2.32V}$

b) First circuit.

$I_{DS} = 150\mu A.$

$V_T = 0.551\,V$

$V_{dsat} = V_{GS} - V_t$

$= 1.2V - 0.462V - 0.551V$

$= 0.187\,V.$

$$g_m = \frac{2I_D}{V_{dsat}}$$

$$= \frac{2\times 150\mu A}{0.187V} \simeq 1.6\,mS$$

$$r_o = \frac{1}{\lambda I_{DS}}$$

$$= \frac{1}{0.1\times 150\mu A} = 66.7\,K\Omega$$

$$g_{mbs} = g_m \cdot \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}}$$

$$= 1.6ms \cdot \frac{0.2}{2\sqrt{0.6+0.462}}$$

$= 0.155\,ms$

Second circuit.

$I_{DS} = 150\mu A$

$V_T = 0.635V$

$V_{dsat} = V_{GS} - V_t$

$= 2.32V - 1.5V$

$- 0.635V$

$= 0.185V$

$$g_m = \frac{2I_D}{V_{dsat}}$$

$$= \frac{2\times 150\mu A}{0.185V}$$

$\simeq 1.6\,mS.$

$$r_o = \frac{1}{\lambda I_{DS}}$$

$$= \frac{1}{0.1\times 150\mu A}$$

$= 66.7\,K\Omega$

$$g_{mbs} = g_m \cdot \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}}$$

$$= 1.6ms \cdot \frac{0.2}{2\sqrt{0.6+1.5}}$$

$= 0.11\,mS$