

EE140 HW#1 Solution

①

1.

(a) As $V_{SB} = 0V$, $V_T = V_{T0} = 0.6V$

1) When $V_{GS} = 0.5V \Rightarrow$ cut off $\Rightarrow I_{DS} = 0$

2) when $V_{GS} = 1.5V$, $V_{GS} - V_T = 1.5V - 0.6V = 0.9V$.

$$\begin{aligned} I_{DS} (V_{DS} = 0.9V) &= \frac{1}{2} K' \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \\ &= \frac{1}{2} \times 194 \mu A/V^2 \times 10 \times 0.9V^2 \times \\ &\quad (1 + 0.024V^{-1} \times 0.9V) \\ &= 803 \mu A \end{aligned}$$

$$\begin{aligned} I_{DS} (V_{DS} = 3V) &= \frac{10}{2} \times 194 \mu A/V^2 \times (0.9V)^2 \times (1 + \\ &\quad 0.024V^{-1} \times 0.9V) \\ &= 842 \mu A \end{aligned}$$

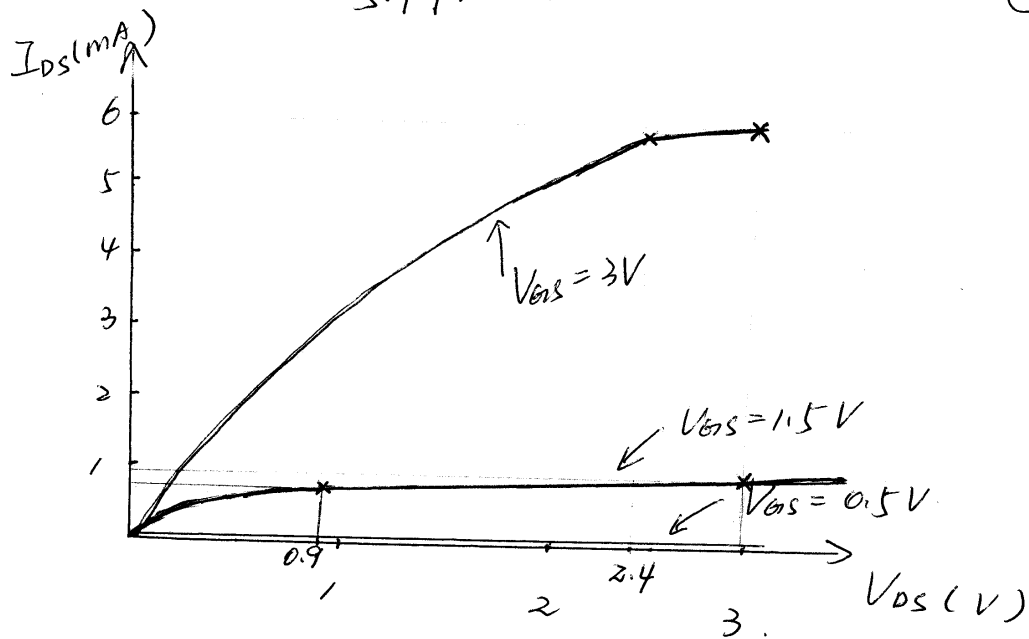
3) when $V_{GS} = 3V$, $V_{GS} - V_T = 3V - 0.6V = 2.4V$

$$\begin{aligned} I_{DS} (V_{DS} = 2.4V) &= \frac{1}{2} \times 10 \times 194 \mu A/V^2 \times (2.4V)^2 \\ &\quad \times (1 + 0.024V^{-1} \times 2.4V) \\ &= 5.91 mA. \end{aligned}$$

$$I_{DS} (V_{DS} = 3V) = \frac{1}{2} \times 10 \times 194 \mu A/V^2 \times (2.4V)^2 \times (1 + 0.024V^{-1} \times 3V)$$

$$= 5.99 \text{ mA}$$

(2)



$$\begin{aligned}
 (b) \quad V_T &= V_{T0} + \frac{1}{C_{ox}} \sqrt{2q\epsilon N_A} \left(\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right) \\
 &= 0.6 + \left(\frac{3.9 \times 8.854 \times 10^{-14}}{80 \times 10^{-8}} \right)^{-1} \cdot \sqrt{2 \times 1.6 \times 10^{-19} \times 11.7 \times 8.85 \times 10^{-14} \times 5 \times 10^{15}} \\
 &\quad \cdot \left(\sqrt{0.6 \text{ V} + V_{SB}} - \sqrt{0.6 \text{ V}} \right)
 \end{aligned}$$

$$\therefore V_T (V_{SB} = 0 \text{ V}) = 0.6 \text{ V}$$

$$V_T (V_{SB} = 0.5 \text{ V}) = 0.626 \text{ V}$$

$$V_T (V_{SB} = 1.0 \text{ V}) = 0.646 \text{ V}$$

Since $V_{DS} = 2V > V_{GS} - V_T \Rightarrow$ always in saturation. (3)

$$I_{DS} = \frac{1}{2} \frac{\mu C'}{L} \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

When $V_{GS} = 2V$, $V_{SB} = 0V$

$$I_{DS} = \frac{1}{2} \times 194 \mu \times 10 \times (2 - 0.6)^2 \times (1 + 0.024 \times 2)$$

$$= 1.99 \text{ mA}$$

$V_{SB} = 0.5V$:

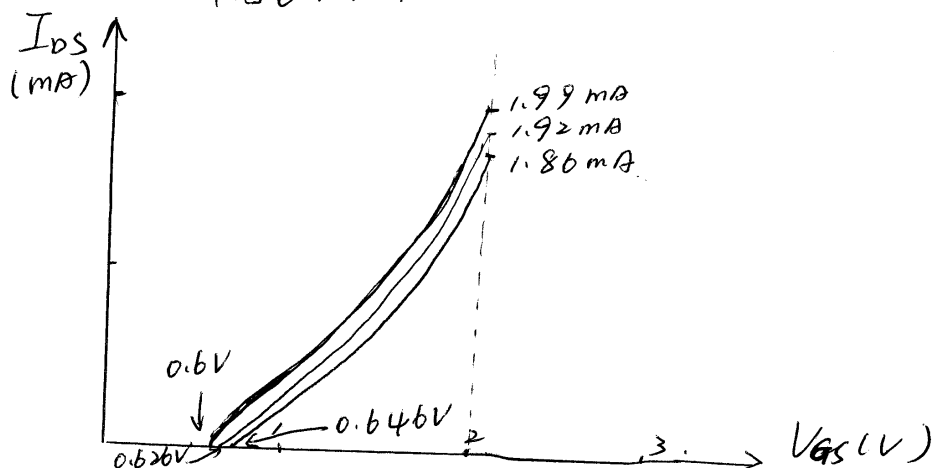
$$I_{DS} = \frac{1}{2} \times 194 \mu \times 10 \times (2 - 0.626)^2 \times (1 + 0.024 \times 2)$$

$$= 1.92 \text{ mA}$$

$V_{SB} = 1V$:

$$I_{DS} = \frac{1}{2} \times 194 \mu \times 10 \times (2 - 0.646)^2 \times (1 + 0.024 \times 2)$$

$$= 1.86 \text{ mA}$$



(4)

(2)

V_{T0} : for $V_{DS} = 1.5V$, $V_{BS} = 0$, find I_{DS} for two different V_{GS} and take the ratio

$$\frac{I_{DS}(V_{GS} = 1.5V)}{I_{DS}(V_{GS} = 1.2V)} \Bigg|_{\substack{V_{SB} = 0 \\ V_{DS} = 1.5V}} = \frac{10.3 \text{ mA}}{5.5 \text{ mA}}$$

$$= \left(\frac{1.5 - V_{T0}}{1.2 - V_{T0}} \right)^2 \Rightarrow \boxed{V_{T0} \cong 0.4V}$$

γ : for $V_{DS} = 1.5V$, $V_{GS} = 1.2V$. Find I_{DS} for two different V_{BS} and take the ratio

$$\frac{I_{DS}(V_{SB} = 0V)}{I_{DS}(V_{SB} = 0.3V)} = \frac{5.5 \text{ mA}}{5 \text{ mA}} = \left(\frac{1.2V - V_{T0}}{1.2V - V_T} \right)^2$$

$$\Rightarrow V_T - V_{T0} = 37 \text{ mV} = \gamma (\sqrt{0.6V + 0.3V} - \sqrt{0.6V})$$

$$\therefore \boxed{\gamma \cong 0.21 \text{ V}^{-\frac{1}{2}}}$$

(5)

λ : for $V_{GS} = 1.2V$, $V_{SB} = 0V$. find I_{DS} for two different V_{DS} and take the ratio.

$$\frac{I_{DS}(V_{GS} = 1.5V)}{I_{DS}(V_{GS} = 1V)} = \frac{5.5mA}{5.26mA} = \frac{1 + \lambda \cdot 1.5V}{1 + \lambda \cdot 1V}$$

$$\Rightarrow \boxed{\lambda \cong 0.1V^{-1}}$$

k' : plug into the equation for saturation current.

$$10.3mA = \frac{k'}{2} \cdot 50 \cdot (1.5V - 0.4V)^2 (1 + 0.1 \cdot 1.5)$$

$$\Rightarrow \boxed{k' = 296 \mu A/V^2}$$

point A: $I_{DS} = 10.5mA$. $V_{GS} = 1.5V$

$$\therefore g_m = \frac{2I_{DS}}{V_{GS} - V_{t0}} = \frac{2 \times 10.5mA}{1.5V - 0.4V} = \underline{\underline{19.1ms}}$$

$$r_o = \frac{1}{\lambda I_{DS}} = \frac{1}{0.1 \times 10.5} = \underline{\underline{952 \Omega}}$$

$$g_{mb} = g_m \cdot \frac{\gamma}{2\sqrt{2\phi_f + V_{GS}}} = \underline{\underline{2.59ms}}$$

$$\text{point B: } g_m = \frac{2I_{DQ}}{V_{GS} - V_{t0}} = \frac{2 \times 5.3 \text{ mA}}{1.2 \text{ V} - 0.4 \text{ V}} \quad (6)$$

$$= \underline{\underline{13.25 \text{ mS}}}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{0.1 \text{ V}^{-1} \times 5.3 \text{ mA}} = \underline{\underline{1.89 \text{ k}\Omega}}$$

$$g_{mbs} = g_m \cdot \frac{\beta}{2\sqrt{2\phi_f + V_{SB}}} \approx \underline{\underline{1.80 \text{ mS}}}$$

$$\text{point C: } g_m = \frac{2I_{DQ}}{V_{GS} - V_t} = \frac{2 \times 5 \text{ mA}}{1.2 \text{ V} - 0.4 \text{ V}}$$

$$= \underline{\underline{13.16 \text{ mS}}}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{0.1 \text{ V}^{-1} \times 5 \text{ mA}} = \underline{\underline{2 \text{ k}\Omega}}$$

$$g_{mbs} = g_m \cdot \frac{\beta}{2\sqrt{2\phi_f + V_{SB}}} = \underline{\underline{1.46 \text{ mS}}}$$