

University of California
Berkeley
College of Engineering
Department of Electrical Engineering
and Computer Science

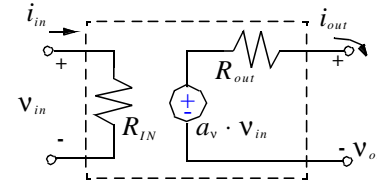
Robert W. Brodersen
EECS140

Analog Circuit Design

Lectures
on
SINGLE TRANSISTOR CIRCUITS

Two Port Analysis

S-1



Small Signal
Analysis

R_{IN}, R_{OUT}, a_v

Set independent sources to zero

$$R_{in} = \left. \frac{v_{in}}{i_{in}} \right|_{i_{out}=0}$$

$$R_{out} = \left. \frac{v_{out}}{(-i_{out})} \right|_{v_{in}=0}$$

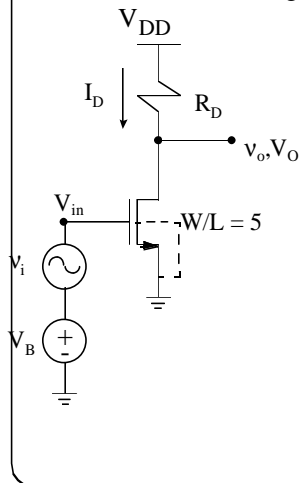
$$a_v = \left. \frac{v_{out}}{v_{in}} \right|_{i_{out}=0}$$

v_{in} at input

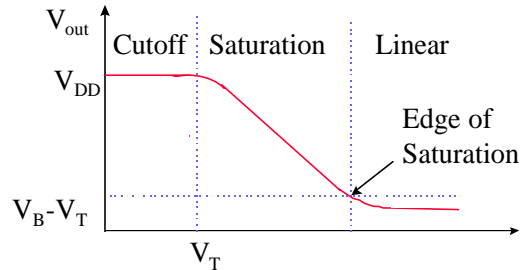
Common Source (Inverter, Gain Stage)

S-2

(High Gain, Moderate R_{out})



$V_T = 1V$ $V_{DD} = 5V$
 $k' = 60\mu A/V^2$ $R_D = 1k\Omega$
 $\lambda = 0.01$ $\gamma = 0.1$



Common Source (Cont.)

S-3

DC Analysis :

$$V_o = V_{DD} - I_D R_D$$

$$V_o = V_{DD} - \frac{k'}{2} \cdot \frac{W}{L} \cdot \left(\frac{V_{GS}}{V_B} - V_T \right)^2 \cdot R_D$$

$$V_o \geq V_{DSAT} = V_{GS} - V_T = V_B - V_T$$

$$V_{oEOS} = V_{DD} - \frac{k'}{2} \cdot \frac{W}{L} \cdot (V_B - V_T)^2 \cdot R_D$$

$$V_B = V_T + V_{DD} - \frac{k'}{2} \cdot \frac{W}{L} \cdot (V_B - V_T)^2 \cdot R_D$$

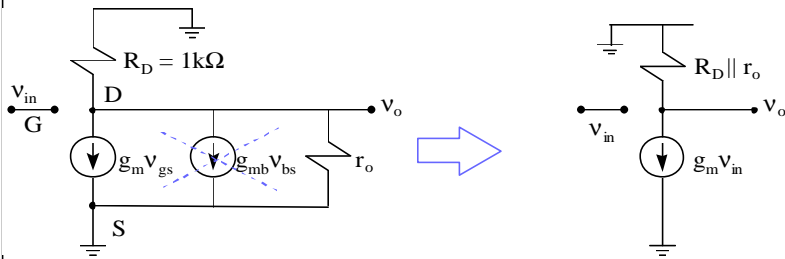
$$V_B^{(A)} = \left[\frac{V_{DD} - V_B^{(B)} + V_T}{\frac{k'}{2} \cdot \frac{W}{L} \cdot R_D} \right]^{\frac{1}{2}} + V_T$$

| $V_B^{(A)}$ | $V_B^{(B)}$ |
|-------------|-------------|
| 3.00 | 4.47 |
| 4.47 | 3.19 |
| 3.19 | 4.33 |
| : | : |
| | 4.12 |

Common Source (Cont.)

S-4

Small Signal Analysis :



Lets say $V_{out} = 2.5V$, then $I_D = 2.5mA$

$$v_o = -g_m \cdot (R_D \parallel r_o) \cdot v_{in} = -\left(2k' \cdot \frac{W}{L} \cdot I_{DS}\right)^{\frac{1}{2}} \cdot \underbrace{(R_D \parallel r_o)}_{1k\Omega} \cdot v_{in}$$

$$r_o = \frac{1}{\lambda \cdot I_D} = \frac{1}{0.01 \cdot 2.5 \cdot 10^{-3}} = 40k\Omega$$

$$\frac{v_o}{v_{in}} = -g_m \cdot (1k\Omega \parallel 40k\Omega) \approx -g_m \cdot (1k\Omega) = 1.2$$

Common Source (Cont.)

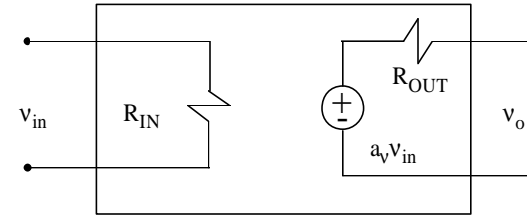
S-5

If we bias the output at $V_{DD}/2$, then the equation for gain is,

$$I_{DS} = \frac{V_{DD}}{2 \cdot R_D}$$

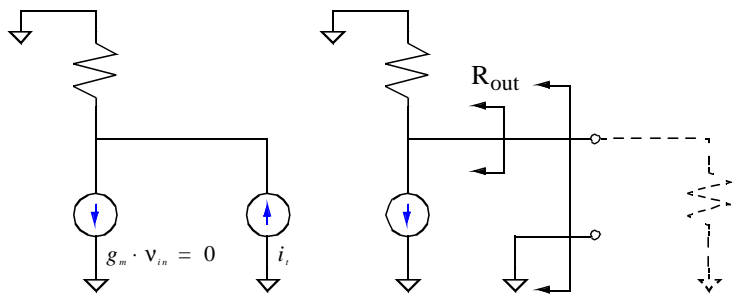
$$a_v = \frac{v_o}{v_{in}} = -g_m \cdot R_D = -\left(2k' \cdot \frac{W}{L} \cdot \frac{V_{DD}}{2}\right)^{\frac{1}{2}} \cdot R_D^{\frac{1}{2}}$$

$$R_{IN} = \infty \quad R_{OUT} = R_D \parallel r_o$$



Rout Calculation :

S-6



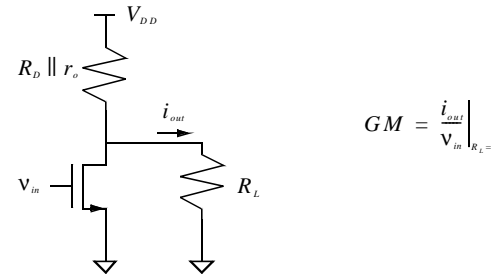
$$R_{out} = \left. \frac{v_t}{i_t} \right|_{v_{in}=0}$$

$$v_t = (R_D \parallel r_o) \cdot i_t$$

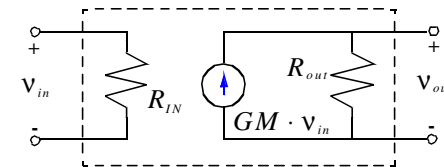
$$R_{out} = \frac{v_t}{i_t} = R_D \parallel r_o$$

Circuit Transconductance GM :

S-7

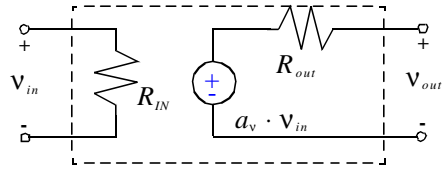


$$GM = \left. \frac{i_{out}}{v_{in}} \right|_{R_L=0}$$



$$v_{out} = GM \cdot v_{in} \cdot R_{out}$$

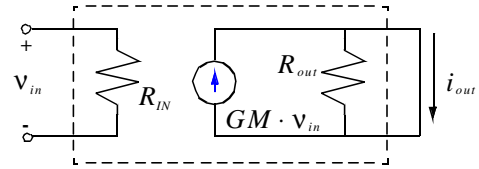
S-8



$$v_{out} = a_v \cdot v_{in}$$

$$v_{out} = GM \cdot v_{in} \cdot R_{out}$$

$$\frac{v_{out}}{v_{in}} = a_v = GM \cdot R_{out}$$

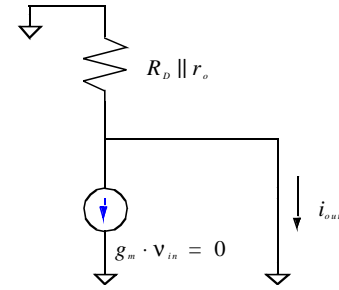


for $R_L = 0$,

$$i_{out} = GM \cdot v_{in}$$

$$GM = \frac{i_{out}}{v_{in}}$$

S-9



$$i_{out} = -g_m \cdot v_{in}$$

$$GM = \frac{i_{out}}{v_{in}} = -g_m$$

S-10

$$a_v = GM \cdot R_{out} \qquad a_v = -g_m \cdot (R_D \parallel r_o) \qquad r_o = \frac{1}{\lambda \cdot I_{DS}}$$

$$g_m \propto I_{DS}^{\frac{1}{2}} \qquad GM \propto I_{DS}^{\frac{1}{2}} \qquad R_D \gg r_o$$

$$a_{v,max} = -g_m \cdot r_o \propto \frac{I_{DS}^{\frac{1}{2}}}{I_{DS}} \sim \frac{1}{I_{DS}^{\frac{1}{2}}}$$

For Bipolar

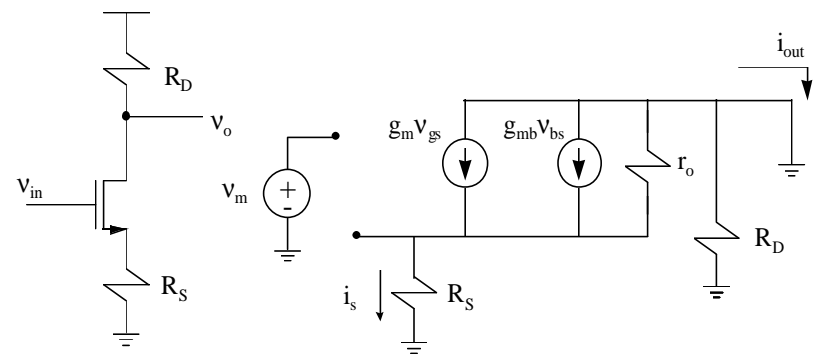
$$g_m = \frac{I_C}{V_{THERMAL}}$$

$$r_o = \frac{V_{EARLY}}{I_C}$$

$$(g_m \cdot r_o)_{MAX} = \frac{V_{EARLY}}{V_{THERMAL}}$$

S-11

Common Source with Source Degeneration



Common Source with Source Degeneration (Cont.)

S-12

Calculate circuit transconductance GM :

$$G_m = \left. \frac{i_{out}}{v_{in}} \right|_{R_L = 0\Omega} = -\frac{i_s}{v_{in}}$$

$$i_s = g_m \cdot (v_{in} - v_s) + g_{mb} \cdot (-v_s) - \frac{v_s}{r_o}$$

$$i_s = v_{in} \cdot g_m - v_s \left(g_m + g_{mb} + \frac{1}{r_o} \right)$$

small compared to g_m

$$v_s = i_s \cdot R_s$$

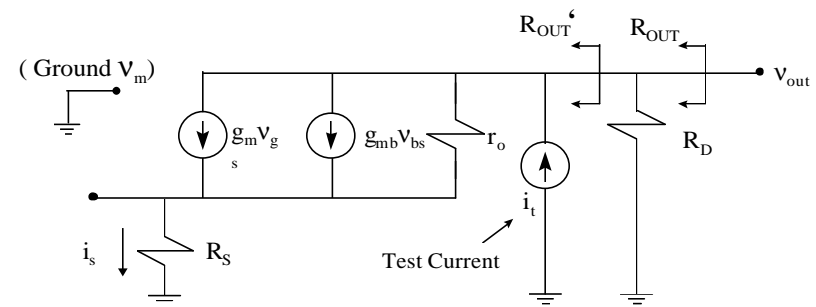
$$GM = \frac{i_s}{v_{in}} = \frac{-g_m}{1 + R_s \cdot \underbrace{(g_m + g_{mb})}_{\chi}} = \frac{-g_m}{1 + R_s \cdot g_m \cdot (1 + \chi)}$$

$\left(\chi = \frac{g_{mb}}{g_m} \right)$

Common Source with Source Degeneration (Cont.)

S-13

Rout Calculation :



Common Source with Source Degeneration (Cont.)

S-14

$$R_{OUT} = R'_{OUT} \parallel R_D$$

$$i_s = i_t \quad v_s = i_t \cdot R_s \quad v_g = v_b = 0$$

$$v_t = i_o \cdot r_o + v_s$$

$$= (i_t - g_m \cdot v_{gs} - g_{mb} \cdot v_{bs}) \cdot r_o + i_t \cdot R_s$$

$$= (i_t - (g_m + g_{mb}) \cdot v_s) \cdot r_o + i_t \cdot R_s$$

$$i_t \cdot R_s$$

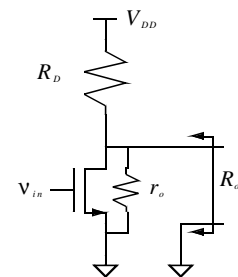
$$v_t = i_t \cdot [R_s + r_o \cdot \{1 + (g_m + g_{mb}) \cdot R_s\}]$$

$$R_{OUT} = \frac{v_t}{i_t} = \{R_s + r_o \cdot [1 + (g_m + g_{mb}) \cdot R_s]\} \parallel R_D$$

$$A_v = GM \cdot R_{OUT} = -\frac{g_m \cdot R_s}{1 + R_s \cdot (g_m + g_{mb})} - \underbrace{g_m \cdot r_o}_{\text{unchanged}} \quad \text{if } R_D \rightarrow \infty$$

$\approx \text{unchanged}$

S-15

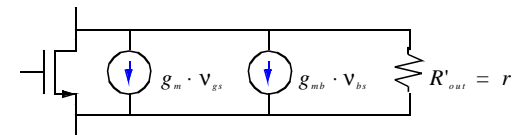


$$GM = \frac{i_{out}}{v_{in}} = -g_m$$

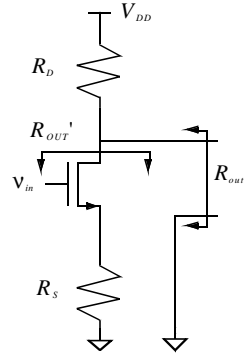
$$R_{OUT} = R_D \parallel r_o$$

$$R_{IN} = \infty$$

$$A_v = GM \cdot R_{OUT}$$



S-16



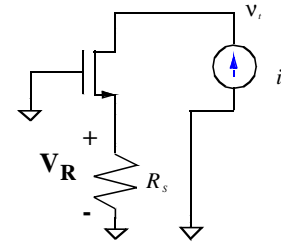
$$\chi = \frac{g_{mb}}{g_m}$$

$$R_{OUT}' = R_S + r_o \cdot \{1 + (1 + \chi) \cdot g_m \cdot R_S\}$$

$$R_{out} = R_D \parallel R_{OUT}'$$

$$g_m \cdot R_S \gg 1 \quad R_{OUT}' \approx r_o \cdot (1 + \chi) \cdot g_m \cdot R_S$$

S-17

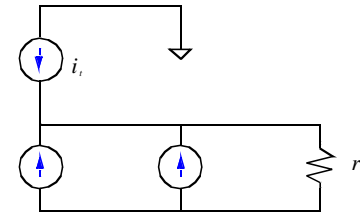


$$R_{out}' = \frac{v_o}{i_i}$$

$$V_R = R_S \cdot i_i$$

$$V_{GS} = V_G - V_S = -R_S \cdot i_i$$

$$V_{BS} = V_B - V_S = -R_S \cdot i_i$$



More current gets shoved into r_o because of polarity of V_{GS} and V_{BS} due to R_S

S-18

$$GM = \frac{-g_m}{1 + R_S \cdot g_m \cdot (1 + \chi)}$$

$$a_v = GM \cdot R_{OUT}$$

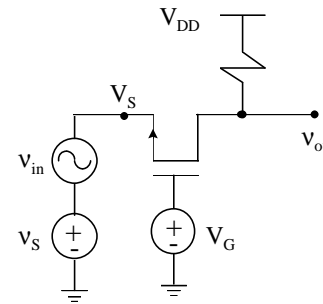
$$= \frac{-g_m}{1 + R_S \cdot g_m \cdot (1 + \chi)} \cdot \{r_o \cdot [1 + (g_m + g_{mb}) \cdot R_S]\} \parallel R_D$$

$$R_D \rightarrow \infty$$

$$a_v \rightarrow -g_m \cdot r_o$$

Common Gate (High Gain, Non-Inverting)

S-19



$$\chi = \frac{\gamma}{2(2\phi_f + V_{BS})}$$

$$V_T = V_{T0} + \gamma \cdot [(2 \cdot \phi_f + V_{BS})^{\frac{1}{2}} - (2 \cdot \phi_f)^{\frac{1}{2}}]$$

For Saturation :

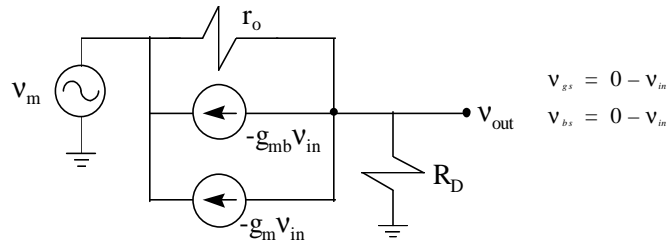
$$V_{OUT} > V_G - V_S - V_T$$

$$g_m = k' \cdot \frac{W}{L} \cdot (V_{GS} - V_T)$$

$$I_{DS} = \frac{k'}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2$$

Common Gate (Cont.)

S-20



$$v_{gs} = 0 - v_{in}$$

$$v_{bs} = 0 - v_{in}$$

$$v_o = (g_m + g_{mb}) \cdot R_D \cdot v_{in} + \frac{v_{in} - v_{OUT}}{r_o} \cdot R_D$$

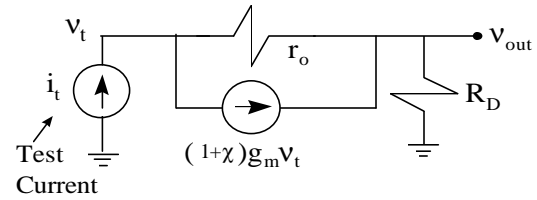
$$A_v = \frac{v_o}{v_{in}} = \frac{[1 + (g_m + g_{mb}) \cdot r_o] \cdot R_D}{r_o + R_D}$$

$$G_m = \frac{1}{r_o} + g_m \cdot (1 + \chi) \quad R_{OUT} = r_o \parallel R_D \quad A_v = G_m \cdot R_{OUT}$$

Common Gate (Cont.)

S-21

R_{in} for common gate :



$$i_t = -\frac{v_t - v_{OUT}}{r_o} - g_m \cdot (1 + \chi) \cdot v_t = 0$$

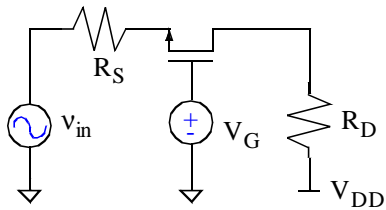
$$v_{OUT} = i_t \cdot R_D \quad \frac{v_t}{i_t} = R_{in} = \frac{r_o + R_D}{1 + (1 + \chi) \cdot g_m \cdot r_o}$$

$$R_D < r_o \quad r_o \rightarrow \infty \quad R_{in} \rightarrow \frac{1}{(1 + \chi) \cdot g_m}$$

$$R_D > r_o \quad R_D \rightarrow \infty \quad R_{in} \rightarrow \frac{R_D}{(1 + \chi) \cdot g_m \cdot r_o}$$

Common Gate with R_S

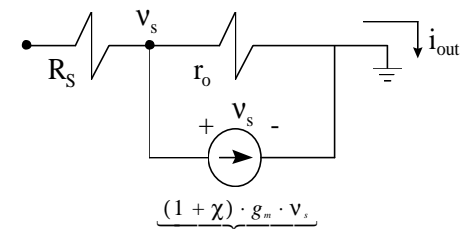
S-22



$$R_{OUT} = (R_S + r_o \cdot [1 + (g_m + g_{mb}) \cdot R_S]) \parallel R_D$$

Common Gate with R_S (Cont.)

S-23



$$(1 + \chi) \cdot g_m \cdot v_s$$

$$R_{eq} = \frac{1}{(1 + \chi) \cdot g_m}$$

Common Gate with R_S (Cont.)

S-24

$$i_{OUT} = \frac{v_{in}}{R_S + \frac{1}{(1 + \chi) \cdot g_m}}$$

$$G_m = \frac{i_{OUT}}{v_{in}} = \frac{1}{R_S + \frac{1}{(1 + \chi) \cdot g_m}}$$

$$= \frac{(1 + \chi) \cdot g_m}{1 + (1 + \chi) \cdot g_m \cdot R_S}$$

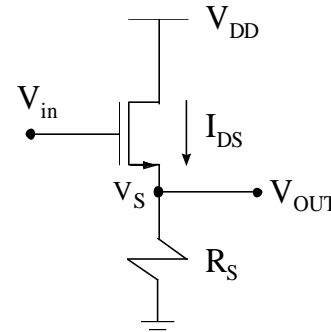
$A_v = GAIN = G_m \cdot R_{OUT}$ for $R_D \text{ LARGE} \gg r_o$

$A_v = g_m \cdot (1 + \chi) \cdot r_o$ $R_S \text{ typical} \ll r_o$

Source Follower

S-25

DC Analysis :



Good for Buffering & Impedance Transformation

Voltage GAIN ~ 1

High $R_{in} = \infty$

Low $R_{OUT} \sim 10 - 1k\Omega$

Source Follower (Cont.)

S-26

$$I_{DS} = \frac{k'}{2} \cdot \frac{W}{L} \cdot (V_{IN} - V_{OUT} - V_T)^2$$

$$V_{OUT} = I_{DS} \cdot R_S$$

$$V_{OUT} = \left(R_S \cdot \frac{k'}{2} \cdot \frac{W}{L} \right) \cdot (V_{IN} - V_{OUT} - V_T)^2$$

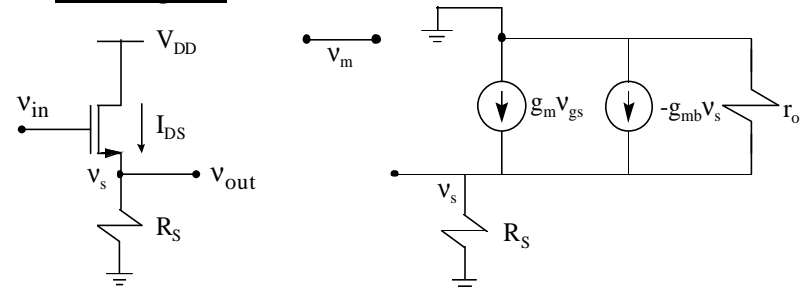
$$V_{OUT} + V_T + \left(\frac{V_{OUT} \cdot 2}{R_S \cdot k' \cdot \frac{W}{L}} \right)^{\frac{1}{2}} = V_{IN}$$

Solve iteratively !

Source Follower (Cont.)

S-27

Small Signal :



$$v_{gs} = v_{in} - v_s \quad v_{OUT} = v_s$$

$$v_s = (g_m \cdot v_{gs} - g_{mb} \cdot v_s) \cdot (r_o \parallel R_S)$$

$$= g_m \cdot R_S \cdot v_{in} - (1 + \chi) \cdot g_m \cdot R_S \quad r_o \gg R_S$$

$$v_s = \frac{g_m \cdot R_S \cdot v_{in}}{1 + (1 + \chi) \cdot g_m \cdot R_S} = v_{OUT}$$

Source Follower (Cont.) S-28

$v_{gs} = -v_s \mid v_{in}=0$

$R_{OUT} = \frac{1}{g_m} \parallel \frac{1}{g_{mb}} \parallel r_o \parallel R_S$

Single Transistor Circuits Summary S-29

Common Source :

$A_v = GM \cdot R_{out}$
 $GM = -g_m$
 $R_{OUT} = r_o \parallel R_D$
 $A_{v,max} = -\frac{2}{V_{DSAT}} \cdot \frac{1}{\lambda}; (R_D \gg r_o)$

Common Source with Source Degeneration :

$GM = \frac{-g_m}{1 + g_m \cdot (1 + \chi) \cdot R_S}$
 $R_{OUT} \approx r_o \cdot [1 + g_m \cdot (1 + \chi) \cdot R_S] \parallel R_D$

Single Transistor Circuits Summary S-30

Common Drain (Source Follower) :

$A_v = \frac{g_m \cdot R_S}{(1 + \chi) \cdot g_m \cdot R_S + 1}$
 $R_{OUT} = \frac{1}{(1 + \chi) \cdot g_m} \parallel R_S$
 $A_{v,max} = \frac{1}{1 + \chi}; (R_S \gg \frac{1}{g_m})$

Common Gate :

$A_v = g_m \cdot (1 + \chi) \cdot R_{OUT}$
 $R_{OUT} = R_D \parallel r_o$
 $R_{in} = \frac{r_o + R_D}{1 + (1 + \chi) \cdot g_m \cdot r_o}$

Common Source $A_{v,max}$ Calculation S-31

$$A_{v,MAX} = -g_m \cdot R_{OUT} \Big|_{R_D \gg r_o} = -g_m \cdot r_o$$

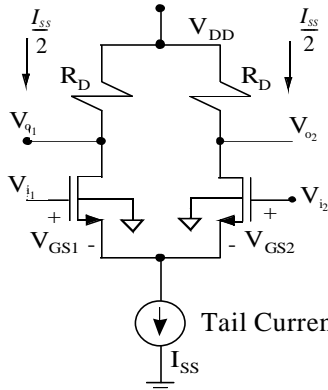
$$= k' \cdot \frac{W}{L} \cdot (V_{GS} - V_T) \cdot \frac{1}{\lambda \cdot I_{DS}} = \frac{k' \cdot \frac{W}{L} \cdot (V_{GS} - V_T)}{\lambda \cdot \frac{k'}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2}$$

$$= -\frac{2}{(V_{GS} - V_T)} \cdot \frac{1}{\lambda} = -\frac{2}{\lambda \cdot V_{DSAT}}$$

Differential Pair

S-32

DC Analysis :

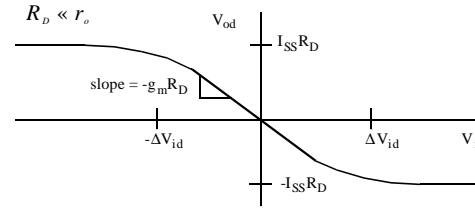


$$V_{GS} = V_T + \left(\frac{I_{SS}}{k' \cdot \frac{W}{L}} \right)^{\frac{1}{2}}$$

Tail Current Source

Differential Pair (Cont.)

S-33



$$V_{od} = V_{o1} - V_{o2}$$

$$V_{id} = V_{i1} - V_{i2}$$

Solve eqns : $V_{i1} - V_{GS1} + V_{GS2} - V_{i2} = 0$

$$I_{DS1} + I_{DS2} = I_{SS}$$

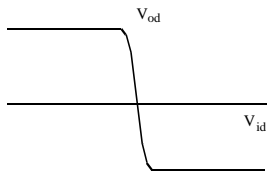
$$V_{GS1} = V_T + \left(\frac{2 \cdot I_{DS1}}{k' \cdot \frac{W}{L}} \right)^{\frac{1}{2}} \quad g_m = \left(2 \cdot k' \cdot \frac{W}{L} \cdot \tilde{I}_{DS} \right)^{\frac{1}{2}} = \frac{I_{SS}}{2 \cdot V_{GS1}}$$

$$\text{also } \Delta V_{GS} \approx \frac{I_{SS} \cdot R_D}{SLOPE} = \frac{I_{SS} \cdot R_D}{g_m \cdot R_D} = \left(\frac{I_{SS}}{k' \cdot \frac{W}{L}} \right)^{\frac{1}{2}}$$

Differential Pair (Cont.)

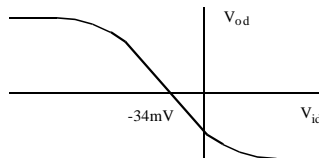
S-34

If we have a lot of gain, low current and a large R_D , then



This can quickly become unbiased

To make sure your output voltage is at zero when your input voltage is zero, do a DC sweep in spice.



Differential Pair (Cont.)

S-35

Define New Variables for Differential 2-Port :



Differential Mode :

$$V_{od} = V_{OUT1} - V_{OUT2}$$

$$V_{id} = V_{in1} - V_{in2}$$

Then the differential mode gain is

Common Mode :

$$V_{oc} = \frac{V_{OUT1} + V_{OUT2}}{2}$$

$$V_{ic} = \frac{V_{in1} + V_{in2}}{2}$$

and the common mode gain is

$$A_{DM} = \frac{V_{od}}{V_{id}}$$

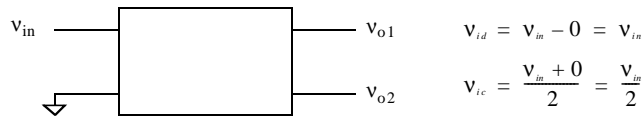
$$A_{CM} = \frac{V_{oc}}{V_{ic}}$$

Differential Pair (Cont.)

S-36

$$v_{o1} = v_{oc} + \frac{v_{od}}{2} \quad v_{i2} = v_{ic} - \frac{v_{id}}{2}$$

$$v_{o2} = v_{oc} - \frac{v_{od}}{2} \quad v_{i1} = v_{ic} + \frac{v_{id}}{2}$$



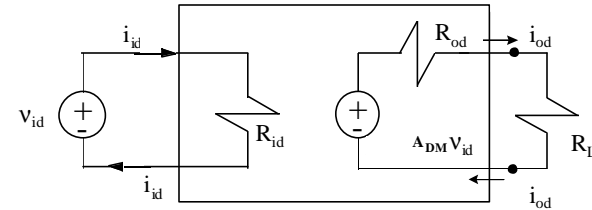
$$v_{o1} = v_{oc} + \frac{v_{od}}{2} = A_{CM} \cdot v_{in} + \frac{A_{DM}}{2} \cdot v_{in}$$

So for our example, $v_{o1} = A_{CM} \cdot \frac{v_{in}}{2} + \frac{A_{DM}}{2} \cdot v_{in} = v_{in} \cdot \left(\frac{A_{CM}}{2} + \frac{A_{DM}}{2} \right)$

Differential Pair (Cont.)

S-37

Differential Mode 2-Port :

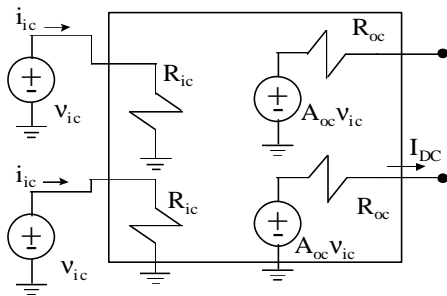


$$A_{DM} = \frac{v_{od}}{v_{id}} \quad R_{id} = \frac{v_{id}}{i_{id}} \quad R_{od} = \frac{v_{od}}{i_{od}}$$

Differential Pair (Cont.)

S-38

Common Mode 2-Port :

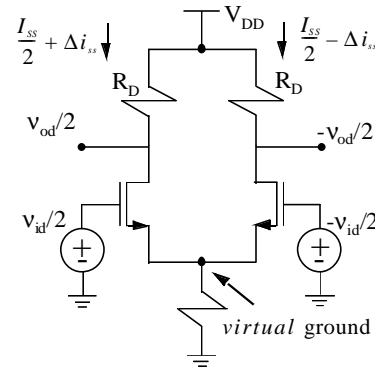


$$A_{CM} = \frac{v_{oc}}{v_{ic}} \quad R_{ic} = \frac{v_{in}}{i_{ic}} \quad R_{oc} = \frac{v_{in}}{i_{ic}}$$

Differential Pair - Half Circuits

S-39

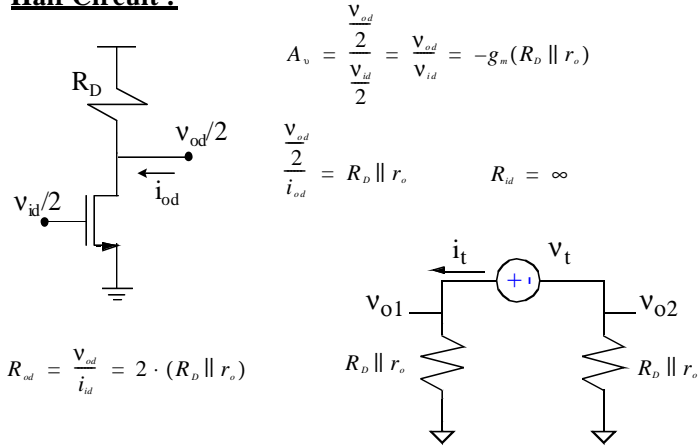
Differential Mode Operation :



Differential Pair - Half Circuits (Cont.)

S-40

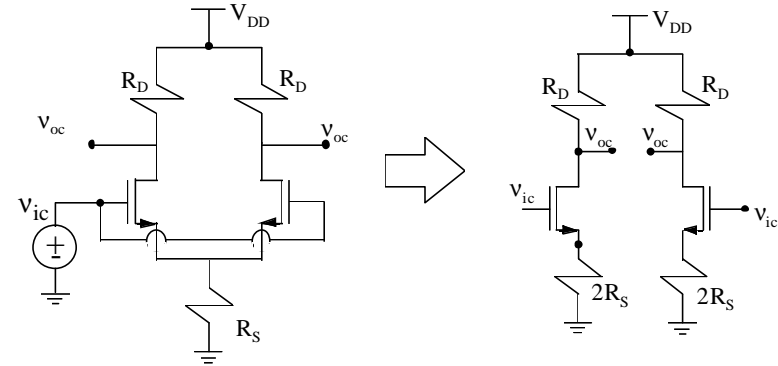
Half Circuit :



Differential Pair - Half Circuits (Cont.)

S-41

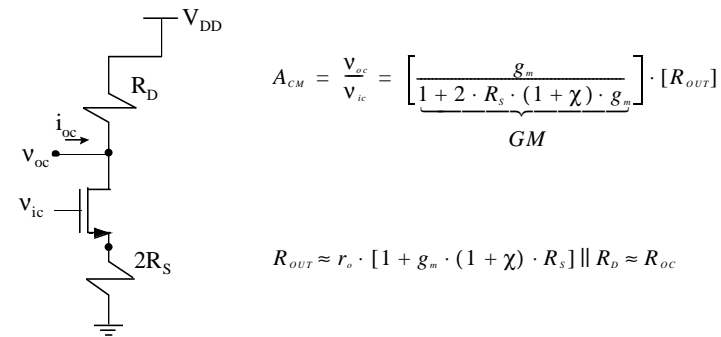
Common Mode Operation :



Differential Pair - Half Circuits (Cont.)

S-42

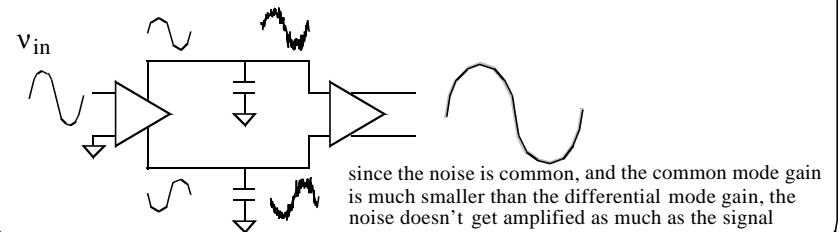
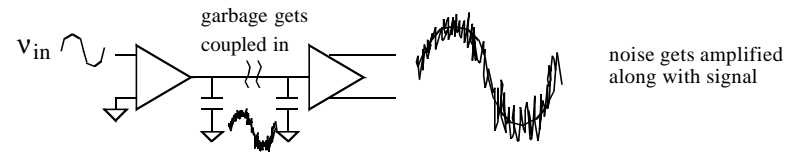
Half circuit :



CMRR and Single vs Double Ended Diff. Pairs

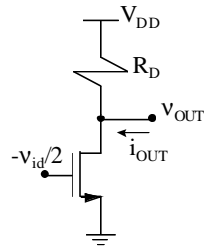
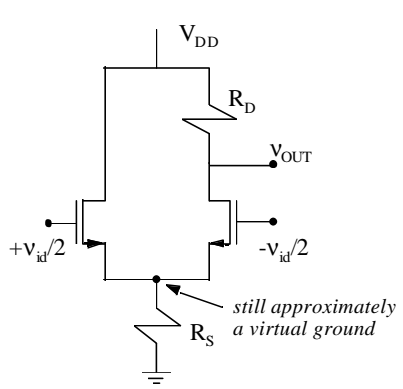
S-43

$$CMRR \equiv \text{Common Mode Rejection Ratio (dB)} = -20 \cdot \log \frac{A_{cm}}{A_{dm}}$$



Single Ended Differential Pair

S-44



$$\frac{v_{OUT}}{-\frac{v_{id}}{2}} = -g_m \cdot (R_D \parallel r_o)$$

$$\frac{v_{OUT}}{v_{id}} = \frac{g_m \cdot (R_D \parallel r_o)}{2}$$

$$R_{OUT} = \frac{v_{OUT}}{i_{OUT}} = R_D \parallel r_o$$