UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
and Computer Sciences

Midterm 1

- closed book, 5.5"x 8.5" summary ok -

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EECS 140
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Device Parameters (except where otherwise noted):

Parameter	Unit	NMOS	PMOS
Threshold voltage, V_{TH}	V	1	-1
Gate capacitance, C_{ox}	fF/μm²	5	5
Transconductance coefficient, $k_p=\mu C_{ox}$	μA/V²	200	100
Channel length modulation, λ(L), L in μm	V⁻¹	0.1/L	0.1/L
Body effect coefficient, γ	$\sqrt{V}$	0.5	0.5
Surface potential, $2\phi_F$	V	0.6	0.6
Overlap Capacitances	$C_{OL} = W \cdot 0.5fF/\mu m$		
Drain and source area	$AS = AD = W \cdot 1\mu m$		
Zero bias junction capacitance	$C_{sb0} = C_{db0} = 1fF/\mu m^2$		
Built-in junction potential	$\psi_0 = 0.5V$		
Fractional temperature coefficient of resistors	$TCF = \frac{1}{R} \cdot \frac{\partial R}{\partial T} = -1000\,ppm/°C$		
Base-Emitter voltage temperature dependence	$\frac{dV_{BE}}{dT} = -1.5mV/°C$		

- Show your calculations on exam sheets (no credit for numerical result only) -

Name: Solution (5 points)

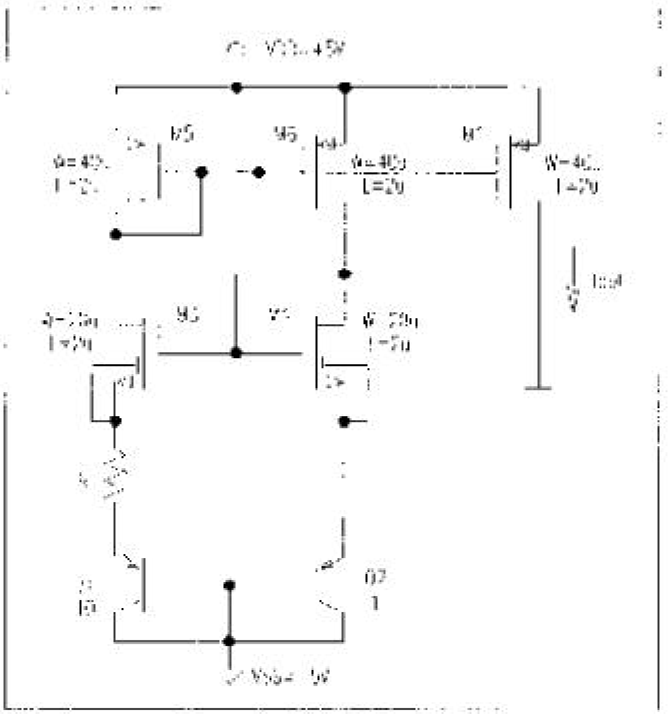
Result summary: (5 points)

Problem 1 a) R = 239.5 Ω
b) T_{CF} = 4333 ppm/°C
Problem 2 I_{D1} = 4.675 μA
Problem 3 a) a_{v0} = −42.16
b) f_p = 2.39 MHz
Problem 4 a) C_x = 51.7 fF
b) f_{p1} = 711.7 MHz
Problem 5 a) V_T = 1.979 V
b) C_L = 46.3 fF

- Each Problem is worth 18 points-

Problem 1

a) Find the value of R such that $I_{out}=250\mu A$ at room temperature. Assume $\lambda=0$ and $\beta \to \infty$.
b) Find TC_F of I_{out} for $R=1k\Omega$ (this is not the correct answer for (a)!)



ⓐ $$I_{OUT} = \frac{V_t}{R}\ln n$$

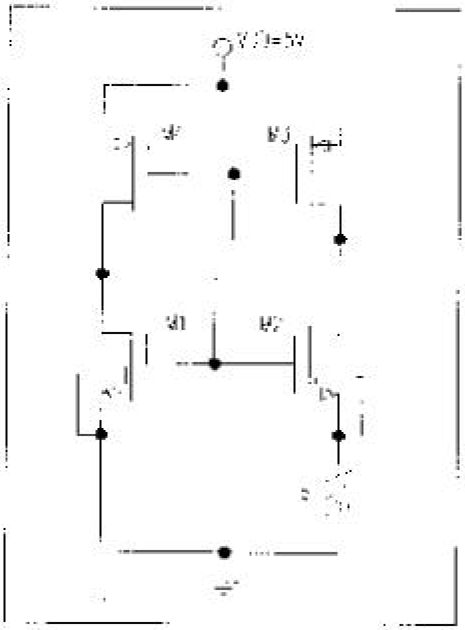
$$R = \frac{V_t}{I_{OUT}}\ln n = \frac{26mV}{0.25mA}\cdot \ln 10 = \underline{\underline{239.5\,\Omega}}$$

ⓑ $$TC_F = \frac{1}{I_{OUT}}\frac{\partial I_{OUT}}{\partial T} = \frac{1}{T} - \frac{1}{R}\frac{\partial R}{\partial T}$$

$$TC_F = 3333\ ppm/°C + 1000\ ppm/°C = \underline{\underline{4333\ ppm/°C}}$$

Problem 2

Find the drain current I_{D1} in M1. Assume that all devices operate in the forward active region and $\lambda=0$. $(W/L)_1= 10$, $(W/L)_2= 100$, $(W/L)_3=(W/L)_4$, $R=10k\Omega$.



$$R I_{D1} + \sqrt{\frac{2I_{D1}}{k'\left(\frac{W}{L}\right)_2}} - \sqrt{\frac{2I_{D1}}{k'\left(\frac{W}{L}\right)_1}} = 0$$

$$R\sqrt{I_{D1}}\left(\sqrt{I_{D1}} + \frac{1}{R}\left[\sqrt{\frac{2}{k'\left(\frac{W}{L}\right)_2}} - \sqrt{\frac{2}{k'\left(\frac{W}{L}\right)_1}}\right]\right) = 0 \quad ; \quad \sqrt{I_{D1}} \neq 0$$

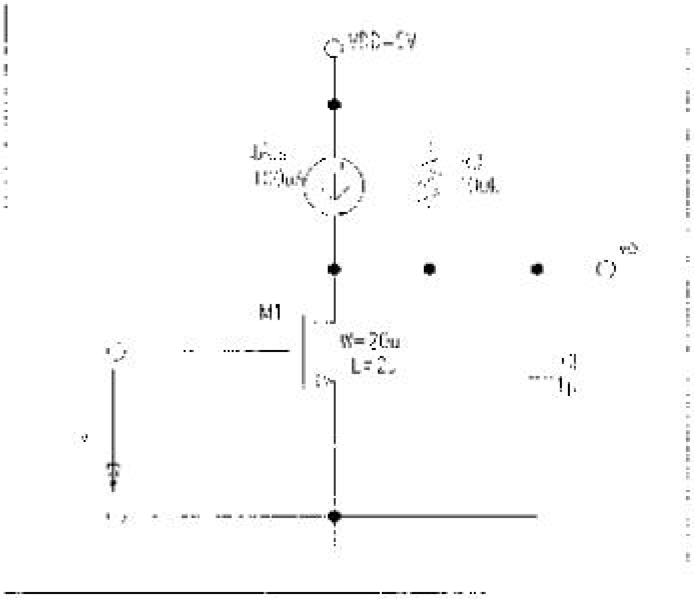
$$\Rightarrow \quad \sqrt{I_{D1}} = \frac{1}{R}\left(\sqrt{\frac{2}{k'\left(\frac{W}{L}\right)_1}} - \sqrt{\frac{2}{k'\left(\frac{W}{L}\right)_2}}\right)$$

$$\sqrt{I_{D1}} = \frac{1}{10k}\left(\sqrt{\frac{2}{2mA/V^2}} - \sqrt{\frac{2}{20mA/V^2}}\right) \Rightarrow I_{D1} = \underline{\underline{4.675\,\mu A}}$$

Problem 3

In the circuit below, the DC value of V_i is adjusted such that V_o=2V

a) What is the low frequency small signal voltage gain $A_{v0}=v_o/v_i$?
b) Find the pole frequency f_p of $A_{v(s)} = V_{o(s)}/V_{i(s)}$.



ⓐ $r_{o1} = \frac{1}{\lambda I_D} = \frac{1}{0.05 \cdot 0.1m} = 200k\Omega$

$g_{m1} = \sqrt{2 I_D k' \frac{W}{L}} = \sqrt{0.2m \cdot 2m} = 632.5\, mS$

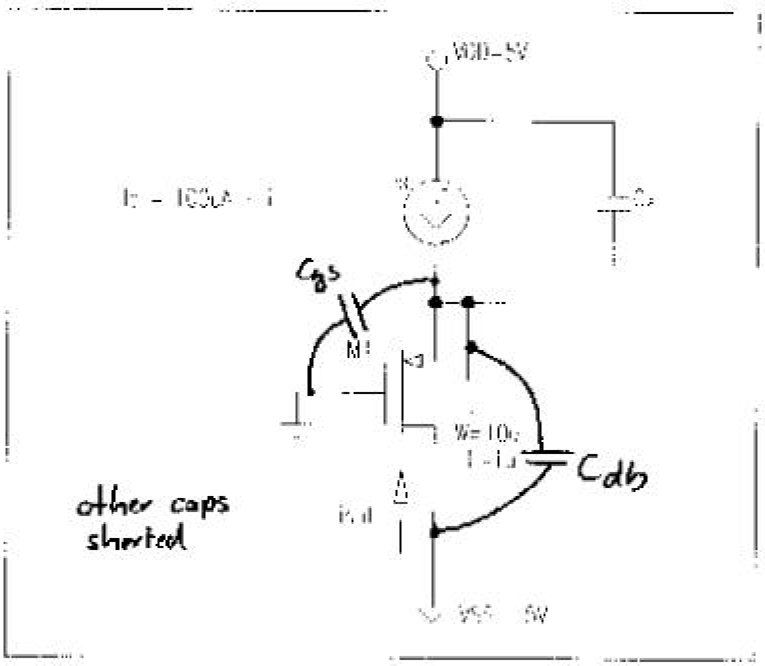
$A_{vo} = -g_m \left(r_{o1} \| R_D \right) = -632.5mS \cdot 66k\Omega = \underline{\underline{-42.16}}$

ⓑ $f_p = \frac{1}{2\pi} \frac{1}{(r_{o1} \| R_D) C_L} = \frac{1}{66k\Omega \cdot 1pF} \cdot \frac{1}{2\pi} = \underline{\underline{2.39\, MHz}}$

Problem 4

a) In the circuit below, C_x represents the intrinsic and extrinsic capacitances associated with M1. Find the value of C_x. Hint: Draw the small signal model for the circuit. **Use AS=AD= W·5µm.**

b) Assuming C_x=100fF (not the correct answer for (a)!), find the pole frequency f_{p1} of the small signal current gain $Ai(s)=i_{out(s)}/i_{s(s)}$.



ⓐ $C_x = C_{gs} + C_{db}$; $C_{gs} = \frac{2}{3} WL\, C_{ox} + C_{OL} = 33.3fF + 5fF = 38.3fF$

$V_{db} = 5V + V_{GS}$; $V_{GS} \cong 1V + \sqrt{\frac{2 \cdot 100\mu}{1m}} = 1.447V$

$\Rightarrow V_{db} = 5V + 1.447V = 6.447\,V$ $\quad C_{db} = AD \frac{C_{db0}}{\sqrt{1 + \frac{V_{db}}{\varphi_o}}}$

$\Rightarrow C_{db} = 50\mu m^2 \frac{1fF/\mu m^2}{\sqrt{1 + \frac{6.447}{0.5}}} = 13.41\ fF$

$\Rightarrow C_x = 38.3fF + 13.41\ fF = \underline{\underline{51.7\ fF}}$

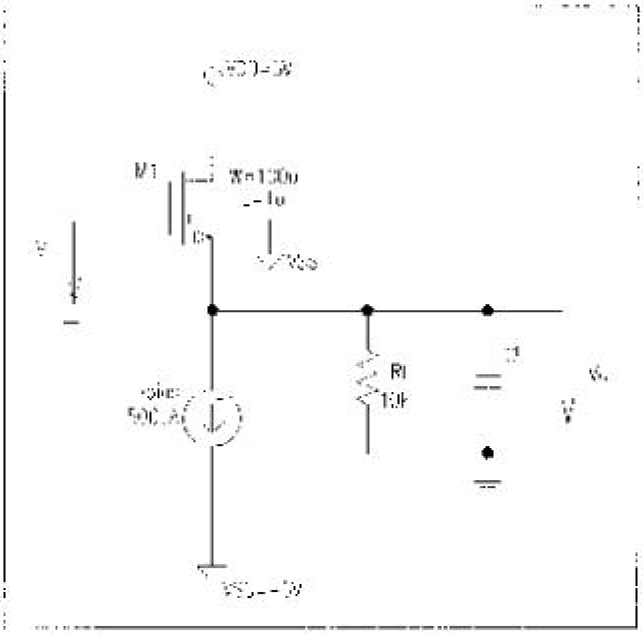
ⓑ $f_{p1} = \frac{1}{2\pi} \frac{g_m}{C_x}$; $g_m = \sqrt{2 \cdot 100\mu \cdot 1m} = 0.447\ mS$

$f_{p1} = \frac{1}{2\pi} \frac{0.447\ mS}{100\ fF} = \underline{\underline{711.7\ MHz}}$

Problem 5

a) Find the large signal value of V_i for $V_o = 0V$. Beware: M1 has body effect!

b) Find the value of C_L for which the bandwidth of the circuit goes to infinity (i.e. all poles and zeros cancel).



ⓐ $$V_{TH} = V_{T0} + \gamma\left(\sqrt{V_{SB} + 2\phi_f} - \sqrt{2\phi_f}\right)$$

$$V_{TH} = 1V + 0.5V\left(\sqrt{5+0.6} - \sqrt{0.6}\right) = 1.796V$$

$$I_D = \frac{1}{2}\mu' \frac{W}{L}\left(V_I - V_{TH}\right)^2\left(1 + \lambda V_{DS}\right)$$

$$0.5 = 10\left(V_I - 1.796\right)^2\left(1 + 0.5\right) \Rightarrow \underline{\underline{V_I = 1.979V}}$$

ⓑ $$z = \frac{g_m}{C_{gs}} \; ; \; p = \frac{g_m + \frac{1}{R_L'}}{(C_{gs} + C_L)} \qquad R_L' = R_L \,||\, \frac{1}{g_{mb}} \,||\, r_o$$

$$g_m = \sqrt{2 \cdot 500\mu \cdot 20m} = 4.472\,mS \; ; \quad C_{gs} = \frac{2}{3}WL \cdot 5fF/\mu m^2 = 333fF$$

$$g_{mb} = g_m \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} = 4.472mS \frac{0.5}{2\sqrt{0.6+5}} = 0.472\,mS = \frac{1}{2.117k}$$

$$r_o = \frac{1}{\lambda I_D} = 20k\Omega$$

$$z = p \Rightarrow C_L = \frac{1}{g_m R_L'} C_{gs} = \frac{1}{4.472mS \cdot (10k \,||\, 2.117k \,||\, 20k)} \cdot 333fF$$

$$\underline{\underline{C_L = 46.3\,fF}}$$