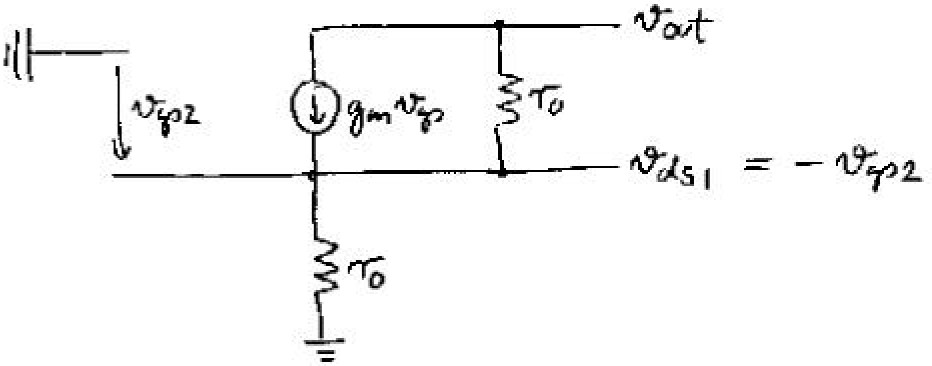
Solution HW7

① ⓐ



$$0 = -v_{ds1}\left(\frac{1}{r_o}+\frac{1}{r_o}+g_m\right)+\frac{1}{r_o}v_{out}$$

$$\Rightarrow \frac{v_{ds1}}{v_{out}} = \frac{1}{r_o\left(\frac{1}{r_o}+\frac{1}{r_o}+g_m\right)} \cong \frac{1}{g_m r_o}$$

$$\varepsilon = \lambda\left(V_{DS1}-V_{DS3}\right)$$

small signal: $\varepsilon = \lambda\left(v_{ds1} - \overset{0}{\cancel{v_{ds3}}}\right) = \lambda\frac{v_{out}}{g_m r_o}$ $\quad v_{in} = 0$

$$\varepsilon = \frac{\lambda(v_{out}-v_{in})}{g_m r_o} \xrightarrow{\approx} \frac{\lambda\left(V_{OUT}-V_{IN}\right)}{g_m r_o}$$

$$\underline{\underline{\varepsilon = \frac{\lambda\Delta V}{g_m r_o}}}$$

ⓑ simple mirror: $\varepsilon_s = \lambda\Delta V$

cascode: $\varepsilon_c = \frac{\lambda\Delta V}{g_m r_o}$ $\Big\}$ $\frac{\varepsilon_s}{\varepsilon_c} = \frac{1}{g_m r_o}$

ⓒ $\lambda = 0.05$; $\quad g_m = \sqrt{0.4m\cdot 1m} = 0.632\,mS$

$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.05\cdot 0.2m} = 100\,k\Omega$

simple mirror: $\varepsilon = \lambda\Delta V = 0.05\cdot 2V = \underline{\underline{10\%}}$

cascode: $\varepsilon = \frac{10\%}{0.632\cdot 100} = \underline{\underline{0.16\%}}$

②

$$I_{OUT} R_S = \sqrt{\frac{2I_{IN}}{k'(\frac{W}{L})}} - \sqrt{\frac{2I_{OUT}}{k'(\frac{W}{L})}}$$

let $x = \sqrt{I_{OUT}}$

$K = \sqrt{\frac{2}{k'\frac{W}{L}}}$

$$0 = x^2 + \frac{K}{R_S} x - \frac{K}{R_S}\sqrt{I_{IN}}$$

$$x_{1,2} = -\frac{K}{2R_S} \overset{+}{(-)} \sqrt{\frac{K^2}{4R_S^2} + \frac{K}{R_S}\sqrt{I_{IN}}}$$

$$\Rightarrow \sqrt{I_{OUT}} = \sqrt{\frac{K^2}{4R_S^2} + \frac{K}{R_S}\sqrt{I_{IN}}} - \frac{K}{2R_S}$$

$$\frac{K}{R_S} = \frac{\sqrt{\frac{2}{20m}}}{100}$$

$$\frac{K}{R_S} = 0.1$$

$$\sqrt{I_{OUT}} = \sqrt{\frac{1}{4}\cdot 0.01 + 0.1\sqrt{0.2m}} - 0.05$$

$$\sqrt{I_{OUT}} = 12.564 \quad \sqrt{A} \Rightarrow I_{OUT} = 157.8\mu A$$

$$\Rightarrow \varepsilon = \frac{157.8}{200} - 1 = \underline{\underline{-21.1\%}}$$

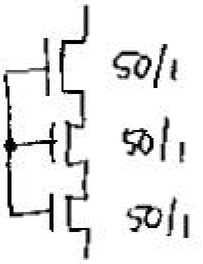
$$\Delta V_{GS} = I_{OUT} R_S = \sqrt{\frac{0.4}{20}} - \sqrt{\frac{0.3156}{20}} = \underline{\underline{15.8mV}} \quad !$$

③ ⓐ $V_{min} = 2V_{dsat} = 400mV \Rightarrow V_{dsat} = 200mV$

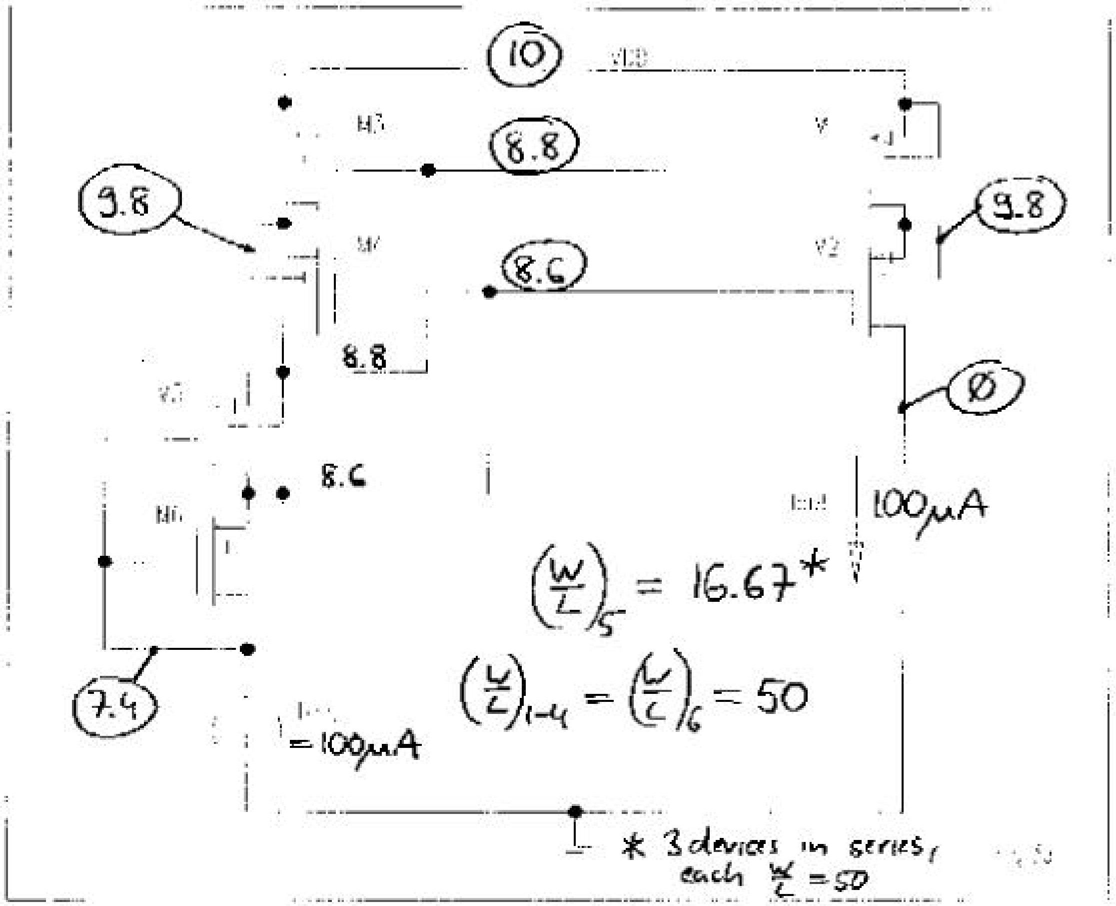
$$V_{dsat} = \sqrt{\frac{2I_D}{k'\frac{W}{L}}} = \left(\frac{W}{L}\right) = \frac{2I_D}{V_{dsat}^2 k'} = \frac{0.2m}{0.04m \cdot 0.1m} = \underline{\underline{50}}$$

Can choose all $\left(\frac{W}{L}\right) = 50$, except $\left(\frac{W}{L}\right)_5 = \frac{1}{3}\frac{W}{L} = \underline{\underline{16.67}}$ *

* in practice, you would use 3 devices in series:



③ ⓑ



$\left(\frac{W}{L}\right)_5 = 16.67^*$

$\left(\frac{W}{L}\right)_{1-4} = \left(\frac{W}{L}\right)_6 = 50$

* 3 devices in series, each $\frac{W}{L} = 50$

check ROP for M4: $V_{DS} = 9.8 - 8.8 = 1V \geq V_{dsat} = 0.2V$ ✓

$\Rightarrow$ SAT

ⓒ • $\mathcal{E} = 0$ with M4! (Since $V_{DS3} = V_{DS1}$)

• without M4: $V_{DS3} = V_{TH} + V_{dsat}$

$V_{DS4} = V_{dsat}$

$$\Rightarrow \underline{\underline{\mathcal{E} = -\lambda V_{TH}}}$$

So, say I used $L = 1\mu m$ in my design $\Rightarrow \mathcal{E} = -10\%$!

$L = 2\mu m \Rightarrow \mathcal{E} = 5\%$

$L = 3\mu m \Rightarrow \mathcal{E} = 3.3\%$

⋮

→ better $\mathcal{E}$ for longer channels!

④ ⓐ → see text pp 37.36

$$\Delta I_D = \frac{1}{2} k' \frac{W}{L} V_{id} \sqrt{\frac{4I_{SS}}{k'\frac{W}{L}} - V_{id}^2} \qquad (3.162)$$

from Fig. 4: $V_{od} = R_D \Delta I_D$; $\Delta I_D = I_{D1} - I_{D2}$

$$\Rightarrow V_{od} = \frac{1}{2} k' \frac{W}{L} R_D V_{id} \sqrt{\frac{4I_{SS}}{k'\frac{W}{L}} - V_{id}^2}$$

$$\frac{dV_{od}}{dV_{id}} = \frac{1}{2} k' \frac{W}{L} R_D \left(\sqrt{\frac{4I_{SS}}{k'\frac{W}{L}} - V_{id}^2} + \frac{-2V_{id}^2}{2\sqrt{\frac{4I_{SS}}{k'\frac{W}{L}} - V_{id}^2}} \right)$$

$$\therefore \underline{\underline{A_{dm0} = \frac{1}{2} k' \frac{W}{L} R_D \sqrt{\frac{4I_{SS}}{k'\frac{W}{L}} - V_{id}^3} \left(1 - \frac{V_{id}^2}{\frac{4I_{SS}}{k'\frac{W}{L}} - V_{id}^2} \right)}}$$

ⓑ for $V_{id} = 0$: $A_{dm0} = \frac{1}{2} k' \frac{W}{L} R_D \sqrt{\frac{4I_{SS}}{k'\frac{W}{L}}}$

$A_{dm0} = R_D \cdot \sqrt{I_{SS} k' \frac{W}{L}}$; $I_{SS} = 2I_D$

$A_{dm0} = g_m R_D$!! $\quad g_m = \sqrt{0.2m \cdot 10m} = 1.41mS$

$\Rightarrow A_{dm0} = 1.41 \cdot 30 = \underline{\underline{42.42}}$

for $V_{id} = 100mV$:

$$A_{dm0} = 150 \sqrt{\frac{0.8m}{10m} - 0.01} \left(1 - \frac{0.01}{\frac{0.8m}{10m} - 0.01} \right) = \underline{\underline{34.01}}$$

ⓒ see attached plot:

(i) for $V_{id} = 100mV \rightarrow V_{od} = \underline{\underline{3.97V}}$

(ii) max at $V_{id} = 0 \rightarrow A_{dm0} = \underline{\underline{42.4}}$ MAX

④ ⓒ (ii) ctd.

<u>min</u>: @ $V_{od} = 5.5 \rightarrow V_{id} = 155mV \rightarrow A_{dm} = 20.3$

@ $V_{od} = -5.5 \rightarrow V_{id} = -155mV \rightarrow A_{dm} = 20.3$

$\Rightarrow$ minimum small signal gain $\underline{\underline{20.3}}$! MIN

$$\frac{A_{dm\phi}(max) - A_{dm\phi}(min)}{A_{dm\phi}(max)} = \frac{42.4 - 20.3}{20.3} = \underline{\underline{52\%}} \; !$$

→ The gain changes about 50% over a reasonable output swing ! In practice, if the amplifiers output swings from -5.5 to +5.5V, the <u>lowest</u> gain in this operating range should be quoted in a specification !

ⓓ → see text p. 3T.44

$$A_{cm} \cong \frac{g_m R_D}{1 + 2 g_m R_X} = \frac{R_D}{\frac{1}{g_m} + 2R_X}$$

$$\underline{\underline{A_{cm} \cong \frac{1}{2}\frac{R_D}{R_X}}}$$

ⓔ $g_m = 1.41mS = \frac{1}{700\Omega}$ O.K.

$\underline{R_X = 100k\Omega}$: $A_{cm} = \frac{1}{2}\frac{30}{100} = \underline{\underline{0.15}} \mathrel{\hat{=}} -16.5dB$

$\underline{R_X = 10M\Omega}$: $A_{cm} = \frac{1}{2}\frac{30}{10\,000} = \underline{\underline{1.5 \cdot 10^{-3}}} \mathrel{\hat{=}} -56.5dB$

eecs140 hw7 - problem 4