EECS 140 Solution HW6

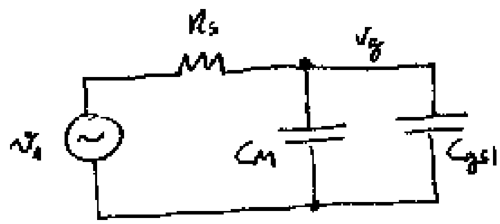
(1) (a)

$\frac{1}{g_{m2}}\left(1+\frac{R_D}{r_{o2}}\right) \cong \frac{R_D}{r_{o2}\, g_{m2}}$ for $R_D \gg r_{o2}$

r_o

$$\Rightarrow R_X = \frac{1}{\frac{1}{r_{o1}} + \frac{g_{m2} r_{o2}}{R_D}} \quad ; \quad A_{vx} = \frac{-g_{m1}}{\frac{1}{r_{o1}} + \frac{g_{m2} r_{o2}}{R_D}} \quad //$$

(b)



$C_M = C_{OL1}\left(1 - A_{vx}\right)$

$$\Rightarrow f_{p1} = \frac{1}{2\pi} \frac{1}{R_s\left[C_{gs1} + C_{OL1}\left(1 - A_{vx}\right)\right]} \quad //$$

(c)

$$g_{m2} \cong \sqrt{2 I_D k'\left(\tfrac{W}{L}\right)_2} = \sqrt{2m \cdot 1m} = 1.41\ mS$$

$$g_{m1} \cong \sqrt{2 I_D k'\left(\tfrac{W}{L}\right)_1} = \sqrt{2m \cdot 20m} = 6.32\ mS; \quad C_{gs1} = \tfrac{2}{3} \cdot 100.5 = 666\,fF$$

$$r_{o1} \cong r_{o2} \cong \frac{1}{\lambda I_D} = \frac{1}{0.1 \cdot 1mA} = 10\,k\Omega\ ; \quad C_{OL1} = 100 \cdot 0.5\,fF = 50\,fF$$

$$\Rightarrow R_X = 10k\Omega \,\|\, \left(\frac{1M\Omega}{1.41mS \cdot 10k\Omega}\right) = 8.76\,k\Omega$$

$$\Rightarrow A_{vx} = -g_{m1} \cdot R_X = 6.32mS \cdot 8.76k\Omega = \underline{\underline{-55.39}}$$

$$\Rightarrow f_{p1} = \frac{1}{2\pi} \frac{1}{500k\left(500fF + 50fF \cdot 56.39\right)} = \underline{\underline{95.89\,kHz}}$$

$$f_{p1}' = \frac{1}{2\pi} \frac{1}{500k\ 666\,fF} = 954.9\ kHz$$

$$\frac{f_{p1}}{f_{p1}'} = \frac{95.89}{636.6} = \underline{\underline{15.1\,\%}} \quad !$$

① d) → see attached plots

DC: found $V_{B1} = 1.3049\,V \Rightarrow V_o = 5.015\,V$

AC:

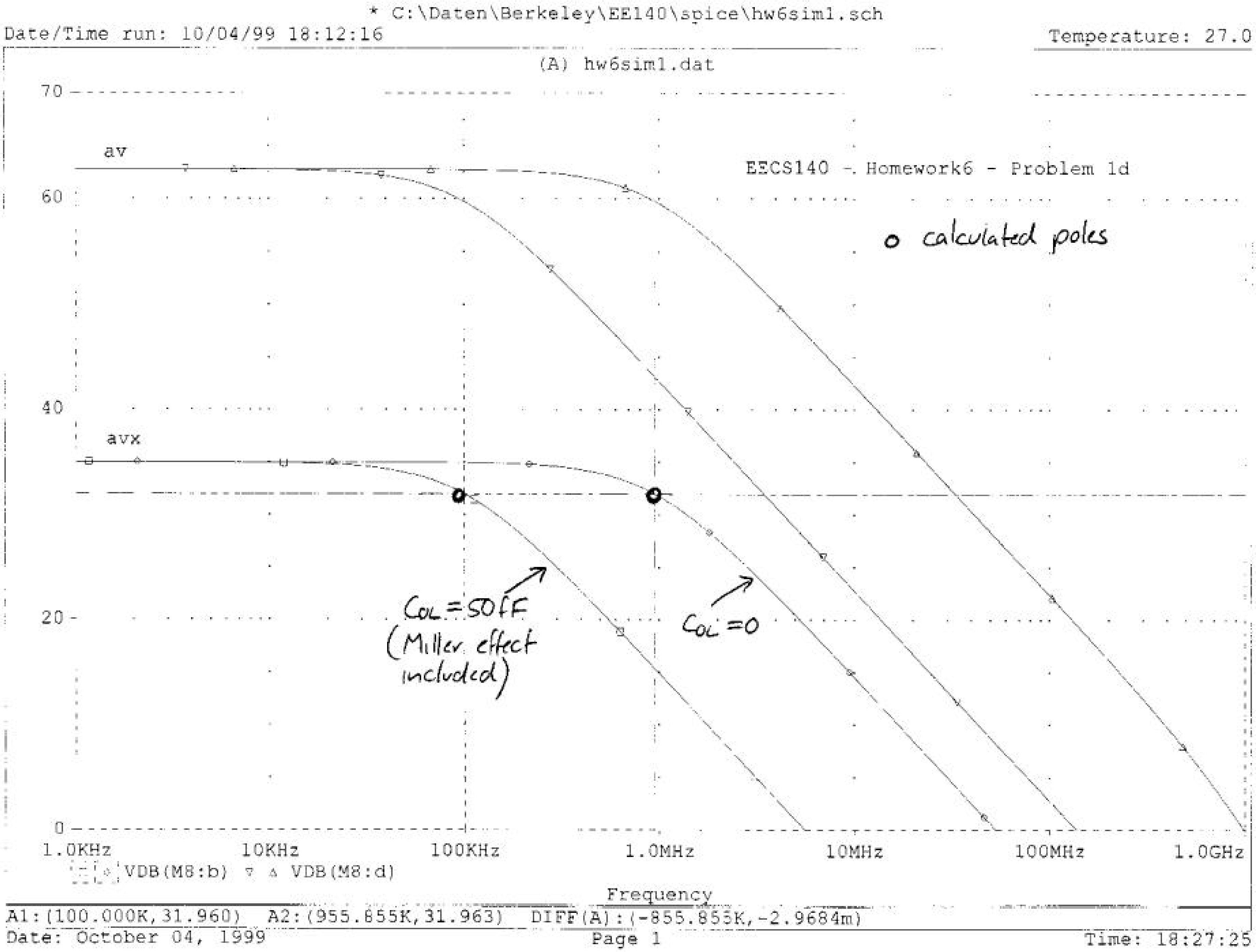
	SIMULATED	CALCULATED
A_{vxo}	34.97 dB $\hat{=}$ 56.0	55.39 $\hat{=}$ 34.86 dB
(C_{OL} = 0) f_{p1}	954.8 kHz	954.9 kHz
(C_{OL} = 50fF) f_{p1}	99 kHz	95.89 kHz

→ good agreement

e) → decrease R_D ⇒ "less" Miller effect, but also less gain a_{vo}!

→ increase g_{m2} by making $\left(\frac{W}{L}\right)_2$ larger

⇒ reduced Miller effect



* C:\Daten\Berkeley\EE140\spice\hw6sim1.sch
Date/Time run: 10/04/99 18:12:16
Temperature: 27.0
(A) hw6sim1.dat
EECS140 - Homework6 - Problem 1d
o calculated poles
av
avx
70
60
40
20
0
C_{OL} = 50fF (Miller effect included)
C_{OL} = 0
1.0KHz
10KHz
100KHz
1.0MHz
10MHz
100MHz
1.0GHz
VDB(M8:b)
VDB(M8:d)
Frequency
A1:(100.000K,31.960) A2:(955.855K,31.963) DIFF(A):(-855.855K,-2.9684m)
Date: October 04, 1999
Page 1
Time: 18:27:25

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(2) a)

$$a_{voa} = -g_{m1} r_{o1}$$

$$a_{vob} = -g_{m1}\left(r_{o2}\left[1 + g_{m2} r_{o1}\right]\right); \quad g_{m2} r_{o1} \gg 1$$

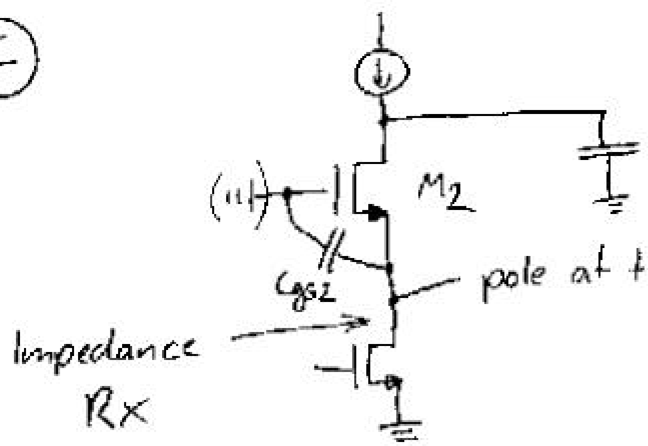
$$\Rightarrow a_{vob} \cong -g_{m1} r_{o1} r_{o2} g_{m2}$$

$$\Rightarrow \frac{a_{vob}}{a_{voa}} = \underline{\underline{g_{m2} r_{o2}}}$$

b)

$$\left.\begin{aligned} f_{p1a} &= \frac{1}{2\pi} \frac{1}{r_{o1} C_L} \\ f_{p1b} &= \frac{1}{2\pi} \frac{1}{g_{m2} r_{o2} r_{o1} C_L} \end{aligned}\right\} \frac{f_{p1a}}{f_{p1b}} = \underline{\underline{g_{m2} r_{o2}}}$$

c)



$C_L \rightarrow$ short for HF

pole at this node caused by C_{gs2}.

$$f_{p2} \cong \frac{1}{2\pi} \frac{1}{C_{gs} R_X}$$

$$R_X = \frac{1}{g_{m2}} \| r_{o1} \cong \frac{1}{g_{m2}}$$

$$\Rightarrow f_{p2} \cong \underline{\underline{\frac{1}{2\pi} \frac{g_{m2}}{C_{gs2}}}}$$

$\rightarrow$ this pole will typically occur at several GHz and is thus irrelevant!

d)

$$\left.\begin{aligned} f_{ua} &\cong a_{voa} \cdot f_{p1a} \\ f_{ub} &\cong a_{vob} \cdot f_{p1a} \end{aligned}\right\} \Rightarrow \underline{\underline{\frac{f_{ub}}{f_{ua}} = 1}} \quad !$$

(e) (a) $g_{m1} \cong g_{m2} \cong \sqrt{2 I_D k' \frac{W}{L}} = \sqrt{1m \cdot 4m} = 2\,mS = g_m$

$r_{o1} \cong r_{o2} \cong \frac{1}{\lambda I_D} = \frac{1}{0.1 \cdot 0.5mA} = 20k\Omega = r_o$

$\Rightarrow a_{voa} = -g_m r_o = \underline{\underline{40}}$

$a_{vob} = -g_m^2 r_o^2 = \underline{\underline{1600}}$ $\Bigg\} \quad \frac{a_{vob}}{a_{voa}} = 40$

(b) $f_{p1a} = \frac{1}{2\pi} \frac{1}{r_o C_L} = \frac{1}{2\pi\, 20k \cdot 0.5p} = \underline{\underline{15.92\,MHz}}$

$f_{p1b} = \frac{1}{2\pi} \frac{1}{g_m r_o^2 C_L} = \underline{\underline{397.89\,kHz}}$ $\Bigg\} \quad \frac{f_{p1a}}{f_{p1b}} = 40$

(c) $f_{p2b} \cong \frac{1}{2\pi} \frac{g_{m2}}{C_{gs2}}$; $C_{gs2} = \frac{2}{3} \cdot 20 \cdot 5fF = 66\,fF$

$\Rightarrow f_{p2b} \cong \frac{1}{2\pi} \frac{2mS}{66fF} = \underline{\underline{4.823\,GHz}}$

(d) $f_{u1a} \cong f_{u1b} \cong a_{voa} \cdot f_{p1a} = \underline{\underline{636.8\,MHz}}$

(f) $\rightarrow$ see attached plot

<u>DC:</u> found $V_{B1} = 1.465\,V \Rightarrow V_o = 4.901\,V$

<u>AC:</u>

	SIMULATED	CALCULATED
a_{voa}	37.34 dB	40 $\hat{=}$ 32 dB
a_{vob}	69.87 dB	1600 $\hat{=}$ 64 dB
f_{p1a}	10.6 MHz	15.9 MHz
f_{p1b}	218 kHz	397 kHz
f_{ua}	783 MHz	637 MHz
f_{ub}	665 MHz	637 MHz

② f) continued:

→ large discrepancies between simulation and hand analysis! The discrepancies are due to neglecting the finite λ in the g_m and r_o calculations!

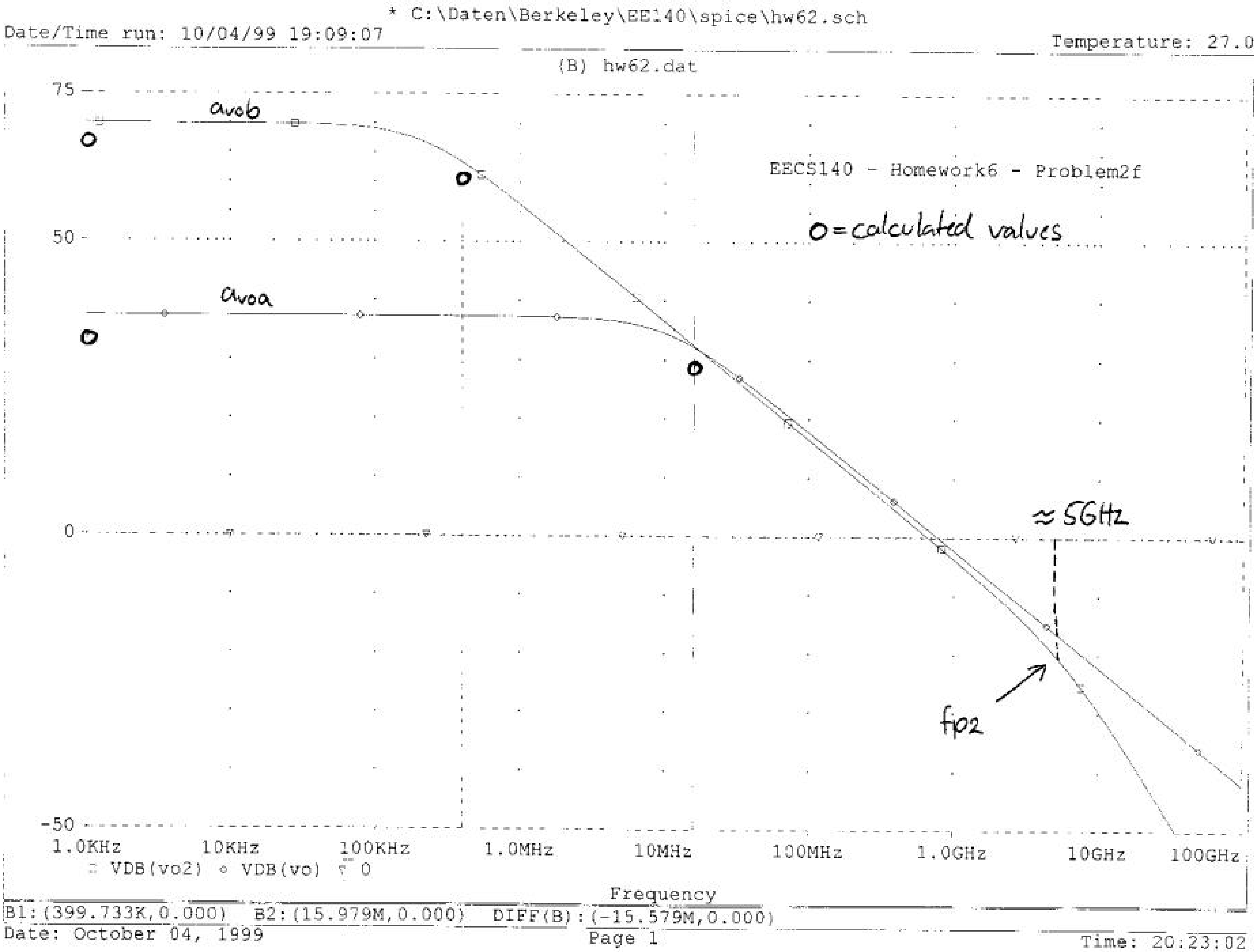
	Calculated	simulated (SPICE listing)	
g_m	2mS	M_{1b}: 2.15mS M_{2b}: 2.31mS	M_{1a}: 2.45mS
r_o	20kΩ	M_{1b}: 23.1kΩ M_{2b}: 26.7kΩ	M_{1a}: 30kΩ

!

→ this is a good example on how using the approximate equations $r_o \cong \frac{1}{\lambda I_D}$ and $g_m = \sqrt{2 I_D k' \frac{W}{L}}$

can introduce large discrepancies for "large" λ (short channel).

→ There is an exact agreement between hand calculations and simulation if the effect of λ is included into the hand calculations!


* C:\Daten\Berkeley\EE140\spice\hw62.sch
Date/Time run: 10/04/99 19:09:07
Temperature: 27.0
(B) hw62.dat
EECS140 - Homework6 - Problem2f
o = calculated values
avob
avoa
≈ 5GHz
fp2
75
50
0
-50
1.0KHz
10KHz
100KHz
1.0MHz
10MHz
100MHz
1.0GHz
10GHz
100GHz
VDB(vo2) ◇ VDB(vo) ▽ 0
Frequency
B1:(399.733K,0.000) B2:(15.979M,0.000) DIFF(B):(-15.579M,0.000)
Date: October 04, 1999
Page 1
Time: 20:23:02

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3 a) single device: $V_{min} = V_{dsat}$

$$C_{OUT} = AD_2 \frac{C_{db0}}{\sqrt{1+\frac{V_{db}}{\psi_0}}} \propto W_2$$

cascode $V_{min} = V_{d1a}^{sat} + V_{d1b}^{sat} = 2V_{d1sat}$

$$C_{OUT} = AD_1 \frac{C_{db0}}{\sqrt{1+\frac{V_{db}}{\psi_0}}} \propto W_1$$

$\Rightarrow \underline{\underline{W_2 = W_1}}$ for same C_{OUT}

$I_{D1} = I_{D2}$ and $2V_{d1}^{sat} = V_{d2}^{sat}$; $W_2 = W_1$

$$\Rightarrow \cancel{\tfrac{1}{2}k'}\left(\frac{W_1}{L_1}\right)\left(V_{d1}^{sat}\right)^2 = \cancel{\tfrac{1}{2}k'}\left(\frac{W_2}{L_2}\right)\left(2V_{d1}^{sat}\right)^2$$

$\Rightarrow \underline{\underline{L_2 = 4L_1}}$

b) $R_{o1} \cong g_{m1} r_{o1}^2$; $R_{o2} = 4 r_{o1}$ since $\lambda_2 = \frac{1}{4}\lambda_1$

$$\Rightarrow \frac{R_{o1}}{R_{o2}} = \frac{g_{m1} r_{o1}}{4}$$

c) $\left(\frac{W}{L}\right)_1 = \frac{10\mu}{2\mu} \rightarrow \left(\frac{W}{L}\right)_2 = \frac{10\mu}{8\mu}$

$$V_{min} = V_{dsat2} \cong \sqrt{\frac{2I_D}{k'\left(\frac{W}{L}\right)_2}} = \sqrt{\frac{200\mu A}{200\mu A/V^2 \cdot \frac{10}{8}}} = \underline{\underline{894 mV}}$$

$$g_{m1} \cong \sqrt{2I_D k'\left(\frac{W}{L}\right)_1} = \sqrt{2 \cdot 100\mu \cdot 1m} = 0.447 mS$$

$$R_{o2} = \frac{1}{\lambda_2 I_D} = \frac{1}{\frac{0.1}{8} \cdot 100\mu} = \underline{\underline{800 k\Omega}}$$

$$r_{o1} \cong \frac{1}{\lambda_2 I_D} = \frac{1}{\frac{0.1}{2} \cdot 100\mu} = 200 k\Omega$$

$$R_{o1} \cong g_{m1} r_{o1}^2 = \underline{\underline{17.88 M\Omega}}$$