① a) $I_D = \frac{1}{2} k' \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$

$I_D = \frac{V_{DD} - V_0}{R_L} \qquad V_{GS} = V_0 - V_I \qquad V_{DS} = V_0$

$\Rightarrow V_I = V_0 - V_{TH} - \sqrt{\frac{2(V_{DD} - V_0)}{R_L (1 + \lambda V_0) k' \frac{W}{L}}}$

→ for the circuit of Fig. 1b, V_{TH} depends on V_0

$|V_{SB}| = V_{DD} - V_0 \quad , \quad V_{TH} = V_{T0} + \gamma \left(\sqrt{V_{SB} + 2\phi_f} - \sqrt{2\phi_f} \right)$

$V_0 = 5V: \quad V_{TH} = 1 + 0.5 \left(\sqrt{5 + 0.6} - \sqrt{0.6} \right) = 1.796\,V$

$V_0 = 9V: \quad V_{TH} = 1 + 0.5 \left(\sqrt{1 + 0.6} - \sqrt{0.6} \right) = 1.245\,V$

⇒ For circuit 1a:

R_L	V_0	V_I	$\frac{\Delta V_0}{\Delta V_I}$
1kΩ	5V 9V	2.174V 7.275V	$\frac{4}{5.101} = 0.784$
1MΩ	5V 9V	3.942V 7.977V	$\frac{4}{4.35} = 0.991$

⇒ For circuit 1b:

R_L	V_0	V_{TH}	V_I	$\frac{\Delta V_0}{\Delta V_I}$
1kΩ	5V 9V	1.796V 1.245V	1.378V 7.029V	$\frac{4}{5.651} = 0.708$
1MΩ	5V 9V	1.796V 1.245V	3.146 7.732	$\frac{4}{4.586} = 0.872$

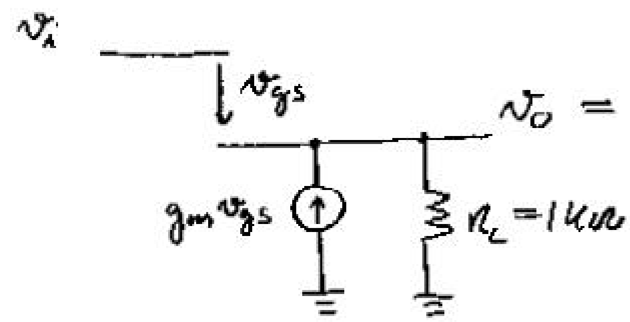
① a) ctd. Analysis:

→ $R_L \to \infty \Rightarrow \frac{\Delta V_O}{\Delta V_I} \to 1$

→ $\frac{\Delta V_O}{\Delta V_I}$ decreases for small R_L

→ Including the body effect (Fig. 1b):
$\frac{\Delta V_O}{\Delta V_I} \not\to 1$ even for large R_L, due to the change in V_{TH}! (True for large signal swing at the output)

b)



$v_O = g_m v_{gs} R_L$

$v_{gs} = v_i - v_o \Rightarrow v_o = (v_i - v_o) g_m R_L$

$$\Rightarrow a_v = \frac{g_m R_L}{1 + g_m R_L} \quad ; \quad g_m = \sqrt{2 I_D k' \frac{W}{L}}$$

bias: $I_D = \frac{V_{DD} - V_O}{R_L} \Rightarrow g_m = \sqrt{\frac{2(V_{DD} - V_O) k' \frac{W}{L}}{R_L}}$

$$\Rightarrow a_v = \frac{1}{1 + \frac{1}{\sqrt{2 R_L k' \frac{W}{L} (V_{DD} - V_O)}}} = f(R_L, V_O, \dots)$$

$$a_v(V_O = 5V) = \frac{1}{1 + \frac{1}{\sqrt{2 \cdot 1 \cdot 2 \cdot 5}}} = \underline{\underline{0.817}}$$

$$a_v(V_O = 5V) = \frac{1}{1 + \frac{1}{\sqrt{2 \cdot 1 \cdot 2 \cdot 1}}} = \underline{\underline{0.667}}$$

-18.4%

① ⓒ from DC sweep: ($\lambda = 0$)

$V_I(V_O = 5V) = 1.764V$

$V_I(V_O = 9V) = 7.000V$

→ .tf results (<u>see attached</u>):

$a_v(V_O = 5V) = 0.817$

$a_v(V_O = 9V) = 0.667$

→ -18.4%

→ Simulation results match hand analysis

EECS140 - Homework 5 - Problem 1c

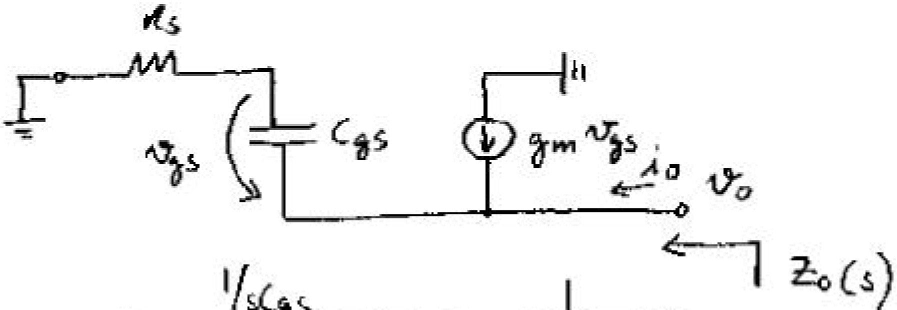
```
**** mosfet bias
element  0:m1
model    0:pmos1
 id         -4.9999m
 ibs          0.
 ibd         50.0006f
 vgs         -3.2361
 vds         -5.0001
 vbs          0.
 vth         -1.0000
 vdsat       -2.2361
 beta         2.0000m
 gam eff    500.0000m
 gm           4.4721m
 gds          0.
 gmb          1.4434m

****     small-signal transfer characteristics
     v(vo)/vi                                     =  817.2552m
     input resistance at          vi              =  1.000e+20
     output resistance at v(vo)                   =  182.7448
```

```
**** mosfet bias
element  0:m1
model    0:pmos1
 id         -1.0000m
 ibs          0.
 ibd         90.0000f
 vgs          2.0000
 vds         -9.0000
 vbs          0.
 vth          1.0000
 vdsat       -1.0000
 beta         2.0000m
 gam eff    500.0000m
 gm           2.0000m
 gds          0.
 gmb        645.4972u

****     small-signal transfer characteristics
     v(vo)/vi                                     =  666.6667m
     input resistance at          vi              =  1.000e+20
     output resistance at v(vo)                   =  333.3333
```

(2) (a)



$$v_{gs} = -v_o \frac{1/sC_{gs}}{1/sC_{gs} + R_s} = \frac{1}{1 + sR_sC_{gs}}$$

$$i_o = v_o\left(\frac{1}{R_s + 1/C_{gs}}\right) + g_m v_o\left(\frac{1}{1 + sR_sC_{gs}}\right)$$

$$\Rightarrow Y_o = \frac{i_o}{v_o} = \frac{g_m + sC_{gs}}{1 + sR_sC_{gs}} = g_m \frac{1 + s\frac{C_{gs}}{g_m}}{1 + sR_sC_{gs}}$$

$$\Rightarrow Z_o(s) = \frac{1}{g_m} \cdot \frac{1 + sR_sC_{gs}}{1 + s\frac{C_{gs}}{g_m}} \quad //$$

(b) $$Z_{RL(s)} = \frac{(R_1 + sL)R_2}{R_1 + R_2 + sL} \cong \frac{(R_1 + sL)R_2}{R_2 + sL} \quad \text{for } R_2 \gg R_1$$

$$= R_1 \frac{1 + s\frac{L}{R_1}}{1 + s\frac{L}{R_2}}$$

→ compare with $Z_o(s)$

$$\Rightarrow R_1 = \frac{1}{g_m}\ ; \quad R_2 = R_s\ ; \quad L = \frac{R_sC_{gs}}{g_m} \quad //$$

→ see also reader pp. 7T14.

(c) $$g_m = \sqrt{2I_D k' \frac{W}{L}} = 2.82\ \frac{mA}{V}\ ; \quad C_{gs} = \frac{2}{3}\, 20\mu m^2 \cdot 5fF/\mu m^2 = 66.$$

$$\Rightarrow R_1 = \frac{1}{g_m} = \underline{\underline{356.6\ \Omega}}$$
$$R_2 = R_s = \underline{\underline{1M\Omega}}$$

} ⇒ $R_s \gg \frac{1}{g_m}$ holds ✓

$$L = \frac{R_sC_{gs}}{g_m} = \underline{\underline{23.6\mu H}}$$

(2) d) → see attached plot

→ no discrepancies, exact match between model and real circuit!
(at least after setting capop = 0!)

e) → see attached plot

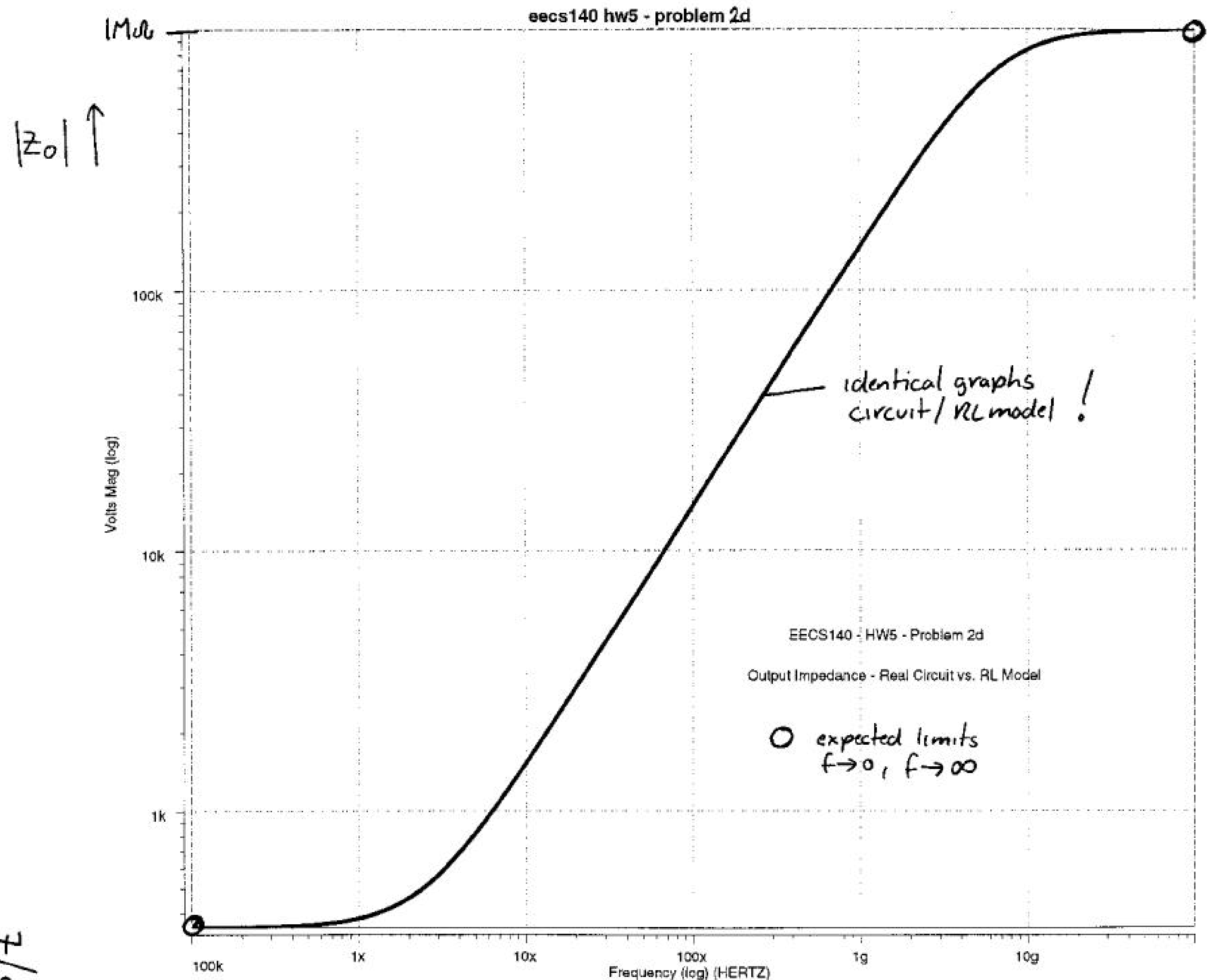
→ Extracted from plot:

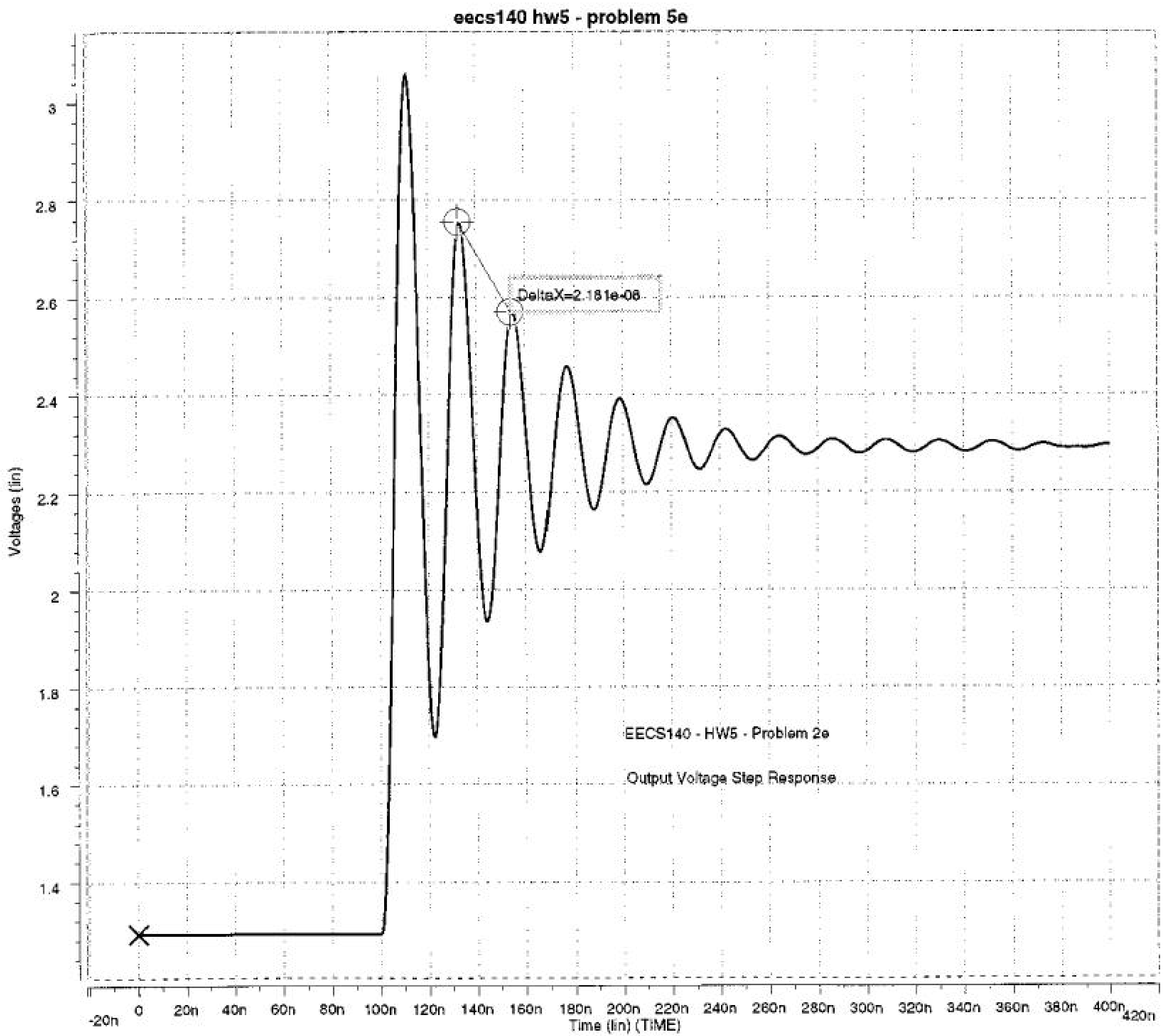
$T \approx 21.8\,ns \Rightarrow f \approx \underline{\underline{45.87\,MHz}}$ (ringing frequency)

→ Expected:

$$f_0 = \frac{1}{2\pi\sqrt{L C_L}} = \frac{1}{2\pi\sqrt{23.6\mu H \cdot 0.5pF}} = \underline{\underline{46.3\,MHz}}$$

⇒ Ringing frequency observed on plot is close to resonant frequency of an ideal LC resonator!


eecs140 hw5 - problem 2d
1MΩ
$|Z_0|$ ↑
100k
10k
1k
Volts Mag (log)
100k
1x
10x
100x
1g
10g
Frequency (log) (HERTZ)
identical graphs circuit / RL model !
EECS140 - HW5 - Problem 2d
Output Impedance - Real Circuit vs. RL Model
○ expected limits $f \to 0$, $f \to \infty$


eecs140 hw5 - problem 5e
DeltaX=2.181e-08
EECS140 - HW5 - Problem 2e
Output Voltage Step Response
Voltages (lin)
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
-20n
0
20n
40n
60n
80n
100n
120n
140n
160n
180n
200n
220n
240n
260n
280n
300n
320n
340n
360n
380n
400n
420n
Time (lin) (TIME)