EECS 140 Solution HW3

1. a) V_{GS}, V_{DS} and I_D are all negative.

b) TRIODE REGION: $I_D = -\mu C_{ox} \frac{W}{L}\left(V_{GS} - V_{TH} - \frac{1}{2}V_{DS}\right)V_{DS}$

FORWARD ACTIVE: $I_D = -\frac{1}{2}\mu C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH}\right)^2\left(1 - \lambda V_{DS}\right)$

c) (i) $|V_{DS}| = 3V > |V_{GS}| - |V_{TH}| = 3V - 1V = 2V \Rightarrow$ FORWARD ACTIVE

$$I_D = -\frac{1}{2}\, 100\,\frac{\mu A}{V^2}\cdot 10\left[-3-(-1)\right]^2\left[1 - 0.1\cdot(-3)\right]$$

or $$I_D = -\frac{1}{2}\, 100\,\frac{\mu A}{V^2}\cdot 10\,(3-1)^2\,(1+0.1\cdot 3) = \underline{\underline{-2.6\,mA}}$$

→ note that it's OK to use the n-channel expression. Just use absolute values for V_{GS}, V_{DS} and V_{TH} and then add a negative sign for I_D...

(ii) $|V_{DS}| = 0V < |V_{GS}| - |V_{TH}| = 5V - 1V = 4V \Rightarrow$ TRIODE

$I_D = \underline{\underline{0}}$ since $V_{DS} = 0$!

(iii) $|V_{DS}| = 2.5V > |V_{GS}| - |V_{TH}| = 1.5V - 1V = 0.5V \Rightarrow$ FORWARD ACTIVE

$$I_D = -\frac{1}{2}\, 100\,\frac{\mu A}{V^2}\cdot 10\cdot(0.5V)^2\,(1+2.5\cdot 0.1) = \underline{\underline{-156.25\,\mu A}}$$

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2. a) $$I_D = \mu C_{ox}\frac{W}{L}\left[\left(V_{GS} - V_{TH}\right)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$g_m' = \frac{dI_D}{dV_{GS}} = \underline{\underline{\mu C_{ox}\frac{W}{L}V_{DS}}}$$

$$\frac{1}{r_o'} = \frac{dI_D}{dV_{DS}} = \underline{\underline{\mu C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH} - V_{DS}\right)}}$$

or $$r_o' = \frac{1}{\mu C_{ox}\frac{W}{L}\left(V_{GS} - V_{TH} - V_{DS}\right)}$$

(2) b) assuming FORWARD ACTIVE leads to a contradiction: *

$$I_D = \frac{1}{2} k' \frac{W}{L} (V_{GS} - V_{TH})^2$$

$$I_D = \frac{1}{2} \frac{mA}{V^2} (2V)^2 = 2mA > I = 1mA \quad \rightarrow\leftarrow$$

→ so the device must be operating in the triode region:

$$I_D = k' \frac{W}{L} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$

$$10^{-3} = 10^{-3} \left(2 V_{DS} - \frac{1}{2} V_{DS}^2 \right)$$

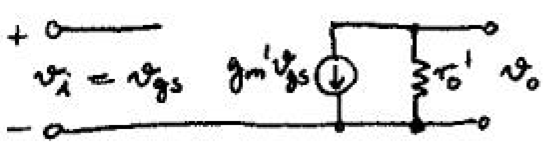
$$0 = V_{DS}^2 - 4 V_{DS} + 2$$

$$V_{DS1,2} = 2 \pm \sqrt{4-2} = 2 \pm 1.414$$

$$V_{DS1} = 3.414V \qquad V_{DS2} = 0.586V$$

→ V_{DS1} can be discarded, because of *

⇒ $V_{DS} = 0.586 V$



$$g_m' = k' \frac{W}{L} V_{DS} = 1 \frac{mA}{V^2} \cdot 0.586V = \underline{\underline{0.586 \frac{mA}{V}}}$$

$$r_o' = 1 / k' \frac{W}{L} (V_{GS} - V_{TH} - V_{DS}) = 1 / 1 \frac{mA}{V^2} (3-1-0.586)V = \underline{\underline{707 \Omega}} \ !$$

$$a_{vo}' = -g_m' r_o' = 0.586 \cdot 10^{-3} \cdot 707 = \underline{\underline{-0.414}} \ !$$

(2) c) $g_m \cong \sqrt{2 k' \frac{W}{L} I_D} = \sqrt{2 \cdot 1 \frac{mA}{V^2} \cdot 1mA} = \underline{\underline{1.414 \ \frac{mA}{V}}}$

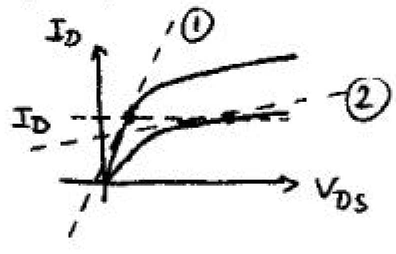
$r_o \cong \frac{1}{\lambda I_D} = \frac{1}{0.05 \cdot 1mA} = \underline{\underline{20 \ k\Omega}}$

$a_{vo} = -g_m r_o = \underline{\underline{-28.28}}$

d)

	TRIODE	FORWARD ACTIVE
g_m	$0.586 \ \frac{mA}{V}$	$1.414 \ \frac{mA}{V}$
r_o	$707 \ \Omega$	$20 \ k\Omega$
a_{v_o}	-0.414	-28.28

→ For the same drain current bias, g_m and r_o are significantly lower in the triode region of operation yielding a poor gain performance. Therefore, devices are usually operated in the forward active region.



① slope $\frac{dI_D}{dV_{DS}}$ triode ≫
② slope " forward active
and $r_o \propto 1 / \frac{dI_D}{dV_{DS}}$

e) TRIODE: $g_m' r_o' = \frac{\cancel{k' \frac{W}{L}} V_{DS}}{\cancel{k' \frac{W}{L}} (V_{GS} - V_{TH} - V_{DS})} = \underline{\underline{\frac{V_{DS}}{V_{GS} - V_{TH} - V_{DS}}}}$

FORWARD ACTIVE: $g_m r_o = \frac{2 \cancel{I_D}}{(V_{GS} - V_{TH}) \lambda \cancel{I_D}} = \underline{\underline{\frac{2}{\lambda (V_{GS} - V_{TH})}}}$

② c) ctd.

TRIODE

$$|a_v| = \frac{V_{DS}}{V_{GS} - V_{TH} - V_{DS}}$$

$$= \frac{1}{\frac{V_{dsat}}{V_{DS}} - \cancel{1}} \quad \text{small}$$

$$\cong \frac{V_{DS}}{V_{Dsat}}$$

→ $V_{DS} \ll V_{DSAT}$ in TRIODE REGION

⇒ low gain!

FORWARD ACTIVE

$$|a_v| = \frac{2}{\lambda\,(V_{GS} - V_{TH})}$$

$$\cong \frac{2V_A}{V_{Dsat}}$$

→ $2V_A \gg V_{Dsat}$ in FORWARD ACTIVE REGION

⇒ high gain!

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③ a) (i) $\frac{W}{L} = \frac{10\mu}{1\mu}$; $I_D = 0.2\,mA$; $\lambda = 0.1$

$$I_D = \frac{1}{2} k' \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$0.2 = (V_{GS} - 1)^2 (1 + 0.1 \cdot 3)$$

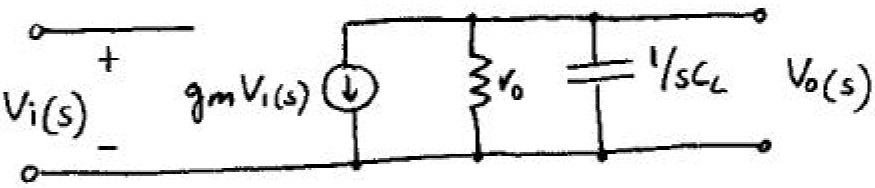
$$\Rightarrow V_{GS} = 0.3922 \pm 1V \Rightarrow V_{GS} = V_A = \underline{\underline{1.3922\,V}}$$

$$g_m = \frac{2 I_D}{V_{GS} - V_{TH}} = \frac{400\mu A}{0.3922V} = \underline{\underline{1020\,\frac{\mu A}{V}}}$$

$$r_o = \frac{2}{k' \frac{W}{L} (V_{GS} - V_{TH})^2 \lambda} = \frac{2}{1\,\frac{mA}{V^2}\,(0.3922V)^2 \cdot 0.1} = \underline{\underline{65.01\,k\Omega}}$$

$$a_{vo} = -g_m r_o = -66.31 \hat{=} \underline{\underline{36.43\,dB}}$$

③ ⓐ (i) ctd.



$$H_{(s)} = \frac{V_o(s)}{V_i(s)} = -\frac{g_m}{\frac{1}{r_o} + sC_L}$$

steady state: $s \rightarrow j\omega$; define $\omega_o = \frac{1}{r_o C_L}$, $a_{vo} = -g_m r_o$

$$\Rightarrow H_{(\omega)} = \frac{a_{vo}}{1 + j\frac{\omega}{\omega_o}} \quad ; \quad |H_{(\omega)}| = \frac{|a_{vo}|}{\sqrt{1+\left(\frac{\omega}{\omega_o}\right)^2}}$$

$$f_o = \frac{1}{2\pi r_o C_L} = \frac{1}{2\pi\, 65.01\text{k} \cdot 1\text{pF}} = \underline{\underline{2.448\ \text{MHz}}}$$

$$|H_{(\omega)}| = 1 \Rightarrow f_u = f_o\sqrt{a_{vo}^2 - 1} = \underline{\underline{163.04\ \text{MHz}}}$$

(ii) found $V_A = 1.3922\text{V}$ @ $V_{DS} = 3\text{V}$

→ exact match. Need to use 5 digits because of high gain!

(iii) from SPICE listing:

$V_{DS} = 3.002\ \text{V}$

$g_m = 1020\ \frac{\mu A}{V}$

$r_o = 65.0\ \text{k}\Omega$

→ device is in forward active region ✓

→ exact match to hand analysis

(IV) see attached plot. No discrepancies!

(3) (b) $\frac{W}{L} = \frac{20\mu}{2\mu}$; $I_D = 0.2\,mA$; $\lambda = 0.05$

→ hand analysis and simulation agree, obtain the following values:

$V_A = 1.4170\,V$ ($\Rightarrow V_{DS} = 3.003\,V$ in SPICE)

$g_m = 959\ \frac{\mu A}{V}$

$r_o = 115\,k\Omega$

$|a_{vo}| = 40.9\,dB$

$f_o = 1.38\,MHz$

$f_v = 153\,MHz$

(c) $\frac{W}{L} = \frac{20\mu}{1\mu}$; $I_D = 0.4\,mA$; $\lambda = 0.1$

→ hand analysis and simulation agree, obtain the following values:

$V_A = 1.3922\,V$ ($\Rightarrow V_{DS} = 3.002\,V$ in SPICE)

$g_m = 2040\ \frac{\mu A}{V}$

$r_o = 32.5\,k\Omega$

$|a_{vo}| = 36.4\,dB$

$f_o = 4.89\,MHz$

$f_v = 324\,MHz$

(3) (d) → see attached plot

Results:

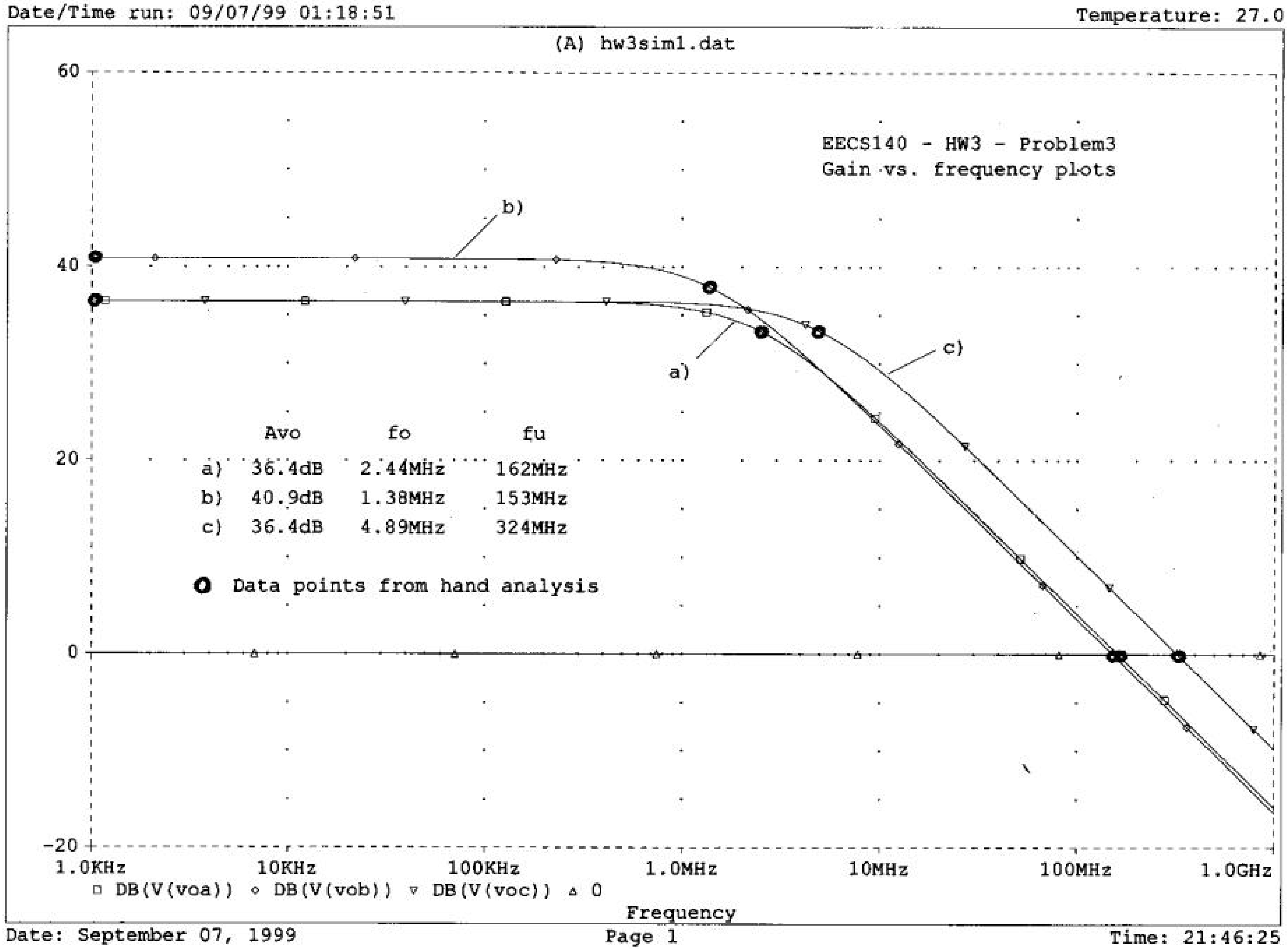
	GAIN / dB	BANDWIDTH / MHz
(a) (nominal)	36.4	162
(b) ($W \uparrow 2x$, $L \uparrow 2x$)	40.9	153
(c) ($I_D \uparrow 2x$, $W \uparrow 2x$)	36.4	324

"small change"

→ Looking at the SPICE plot, note that gain and bandwidth can be controlled <u>independently</u> of each other (... at least to first order)

$(a) \xrightarrow{W\uparrow,\ L\uparrow} (b)$ constant bandwidth, increased gain due to smaller λ

$(a) \xrightarrow{W\uparrow,\ I_D\uparrow} (c)$ constant gain, increased bandwidth due to smaller r_o. Gain constant since g_m increases to yield the same $g_m \cdot r_o$ product


Date/Time run: 09/07/99 01:18:51
Temperature: 27.0
(A) hw3sim1.dat
EECS140 - HW3 - Problem3
Gain vs. frequency plots
b)
a)
c)
Avo fo fu
a) 36.4dB 2.44MHz 162MHz
b) 40.9dB 1.38MHz 153MHz
c) 36.4dB 4.89MHz 324MHz
Data points from hand analysis
60
40
20
0
-20
1.0KHz
10KHz
100KHz
1.0MHz
10MHz
100MHz
1.0GHz
DB(V(voa)) DB(V(vob)) DB(V(voc)) 0
Frequency
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Page 1
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