① a) $|V_{BE}| = V_t \ln \frac{I_{OUT}}{I_S} = 26mV \cdot \ln \frac{100\mu A}{1fA} = 658.5mV$

$R = \frac{|V_{BE}|}{I_{OUT}} = \frac{658.5mV}{100\mu A} = \underline{\underline{6.585k\Omega}}$

b) $I_{OUT} = \frac{V_t}{R} \ln \frac{I_{OUT}}{I_S}$; $S_{I_S}^{I_{OUT}} = \frac{I_S}{I_{OUT}} \frac{dI_{OUT}}{dI_S}$

→ need to find derrivative of implicit function

$f_{(I_{OUT}, I_S)} = I_{OUT} - \frac{V_t}{R} \ln \frac{I_{OUT}}{I_S} = 0$

$\frac{dI_{OUT}}{dI_S} = \frac{-\partial f / \partial I_S}{\partial f / \partial I_{OUT}} = \frac{\frac{V_t}{R} \cancel{\frac{I_S}{I_{OUT}}} \cdot \left(\cancel{\frac{-I_{OUT}}{I_S^2}}\right)}{1 - \frac{V_t}{R} \cancel{\frac{I_S}{I_{OUT}}} \cdot \frac{1}{I_S}}$

$\frac{dI_{OUT}}{dI_S} = \frac{V_t}{RI_S - \cancel{V_t \frac{I_S}{I_{OUT}}}}$ (small) $RI_S \simeq 6 \cdot 10^{-12}$, $V_t \frac{I_S}{I_{OUT}} \simeq 3 \cdot 10^{-13}$

$\Rightarrow S_{I_S}^{I_{OUT}} = \frac{\cancel{I_S}}{I_{OUT}} \cdot \frac{V_t}{R\cancel{I_S}} = \frac{V_t}{R\, I_{OUT}} = \frac{26mV}{6.585k \cdot 100\mu A} = \underline{\underline{3.95\%}}$

So, for a 10% in I_S:

$\left.\frac{\Delta I_{OUT}}{I_{OUT}}\right|_{\frac{\Delta I_S}{I_S} = 10\%} = 3.95\% \cdot 10\% = \underline{\underline{0.395\%}}$ (only!)

c) $TC_F = \frac{1}{I_{OUT}} \frac{\partial I_{OUT}}{\partial T} = \frac{1}{I_{OUT}} \frac{\partial}{\partial T}\left(\frac{V_{BE}}{R}\right)$

$= \frac{1}{I_{OUT}} \frac{\frac{\partial V_{BE}}{\partial T} \cdot R - V_{BE} \frac{\partial R}{\partial T}}{R^2}$

$= \frac{1}{100\mu A} \frac{-2mV/°C \cdot 6.585k - 685.5mV \cdot \left(1200 \cdot 10^{-6} \frac{1}{°C} \cdot 6.585k\right)}{(6.585k)^2}$

$TC_F = \underline{\underline{-0.424\%}}$

② a) $|V_{GS1}| = |V_{TH}| + \sqrt{\frac{2 I_{OUT}}{k' W/L}} = 1.1V + \sqrt{\frac{2 \cdot 100\mu A}{100 \frac{\mu A}{V^2} \cdot 200}} = 1.1V$

$$\Rightarrow R = \frac{|V_{GS1}|}{I_{OUT}} = \frac{1.1V}{100\mu A} = \underline{\underline{11k\Omega}}$$

b) $I_{OUT} = \frac{V_{TH}}{R} + \frac{1}{R}\sqrt{\frac{2 I_{OUT}}{k' W/L}}$; $S^{I_{OUT}}_{V_{TH}} = \frac{V_{TH}}{I_{OUT}} \cdot \frac{d I_{OUT}}{d V_{TH}}$

$$f(I_{OUT}, V_{TH}) = \frac{V_{TH}}{R} + \frac{1}{R}\sqrt{\frac{2 I_{OUT}}{k' W/L}} - I_{OUT} = 0$$

$$\frac{\partial f}{\partial I_{OUT}} = 0 + \frac{1}{R} \frac{\cancel{2}}{k' W/L} \frac{1}{\cancel{2}\sqrt{\frac{2 I_{OUT}}{k' W/L}}} - 1$$

$$\frac{\partial f}{\partial V_{TH}} = \frac{1}{R}$$

$$\Rightarrow \frac{d I_{OUT}}{d V_{TH}} = -\frac{\partial f / V_{TH}}{\partial f / I_{OUT}} = \frac{1}{R - \cancel{\frac{1}{\sqrt{2 I_{OUT} k' W/L}}}}$$

small ≈ 500Ω, R=11K

$$\Rightarrow S^{I_{OUT}}_{V_{TH}} = \frac{V_{TH}}{I_{OUT}} \cdot \frac{1}{R} = \frac{1V}{100\mu A \cdot 11k\Omega} = \underline{\underline{0.909}}$$

So, for a 100 mV change in V_{TH}:

$$\left.\frac{\Delta I_{OUT}}{I_{OUT}}\right|_{\Delta V_{TH}=0.1} = \frac{0.1}{1} \cdot 0.909 = \underline{\underline{9.09\%}}$$

which is large compared to ①(b)!

c) $TC_F = \frac{1}{I_{OUT}} \frac{\partial I_{OUT}}{\partial T} = \frac{1}{I_{OUT}} \frac{\partial}{\partial T}\left(\frac{V_{TH} + \cancel{\sqrt{\frac{2 I_{OUT}}{k' W/L}}}}{R}\right)$ → neglect

$$TC_F = \frac{1}{I_{OUT}} \frac{\frac{\partial V_{TH}}{\partial T} \cdot R - V_{TH} \frac{\partial R}{\partial T}}{R^2} = \frac{-2 \frac{mV}{°C} \cdot 11k - 1V \cdot 1200 \cdot 10^{-6}/°C \cdot 11k}{100\mu A \cdot (11k)^2}$$

$$TC_F = \underline{\underline{-0.291\%}}$$