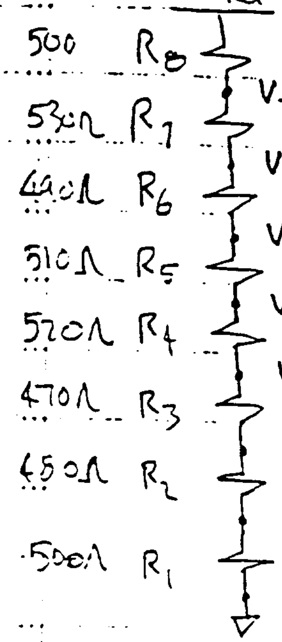
# Chapter 29

29.1

$V_{REF} = 5V$

$I = \frac{5V}{4k\Omega} = 1.25 mA$




$V_7 = 4.375V$

$V_6 = 3.7125V$

$V_5 = 3.1V$

$V_4 = 2.4625V$

$V_3 = 1.8125V$

$V_2 = 1.225V$

$V_1 = 0.625V$

$V_0 = 0V$

| Digital Input | Step Height (V) Actual | Ideal | DNL(V) | (LSB) | Voltage Output (V) Actual | Ideal | INL(V) | |
|---|---|---|---|---|---|---|---|---|
| 000 | | | | | 0 | 0 | 0 | |
| 001 | 0.625 | 0.625 | 0 | 0 | 0.625 | 0.625 | 0 | |
| 010 | 0.595 | " | -0.03 | -0.048 | 1.225 | 1.25 | -0.025 | - |
| 011 | 0.5925 | " | -0.0325 | -0.052 | 1.8125 | 1.875 | -0.0625 | - |
| 100 | 0.65 | " | 0.025 | 0.04 | 2.4625 | 2.5 | -0.0375 | - |
| 101 | 0.6375 | " | 0.0125 | 0.02 | 3.1 | 3.125 | -0.025 | - |
| 110 | 0.6125 | " | -0.0125 | -0.02 | 3.7125 | 3.75 | -0.0375 | - |
| 111 | 0.6625 | " | 0.0375 | 0.06 | 4.375 | 4.375 | 0 | |

$$|DNL|_{MAX} = \boxed{0.0375\,V = 0.06\ LSB}$$

$$|INL|_{max} = \boxed{0.0625\,V = 0.1\ LSB}$$

29.2

$$|DNL|_{max} = \left| \frac{V_{REF}}{2^N} \cdot \left(\frac{\Delta R}{R}\right)_{max} \right| \ (V)$$

$$= \left| \left(\frac{\Delta R}{R}\right) \right|_{max} \cdot (LSB)$$

$$= \boxed{0.01\ LSB}$$

$$|INL|_{max} = \left| \frac{V_{REF}}{2^N} \cdot 2^{N-1} \left(\frac{\Delta R}{R}\right)_{max} \right| \ (V)$$

$$= \left| 2^{N-1} \left(\frac{\Delta R}{R}\right)_{max} \right| \ LSB$$

$$= \boxed{1.28\ LSB}$$

The effective resolution is $\boxed{8-2=6}$

29.3

Digital Input Code

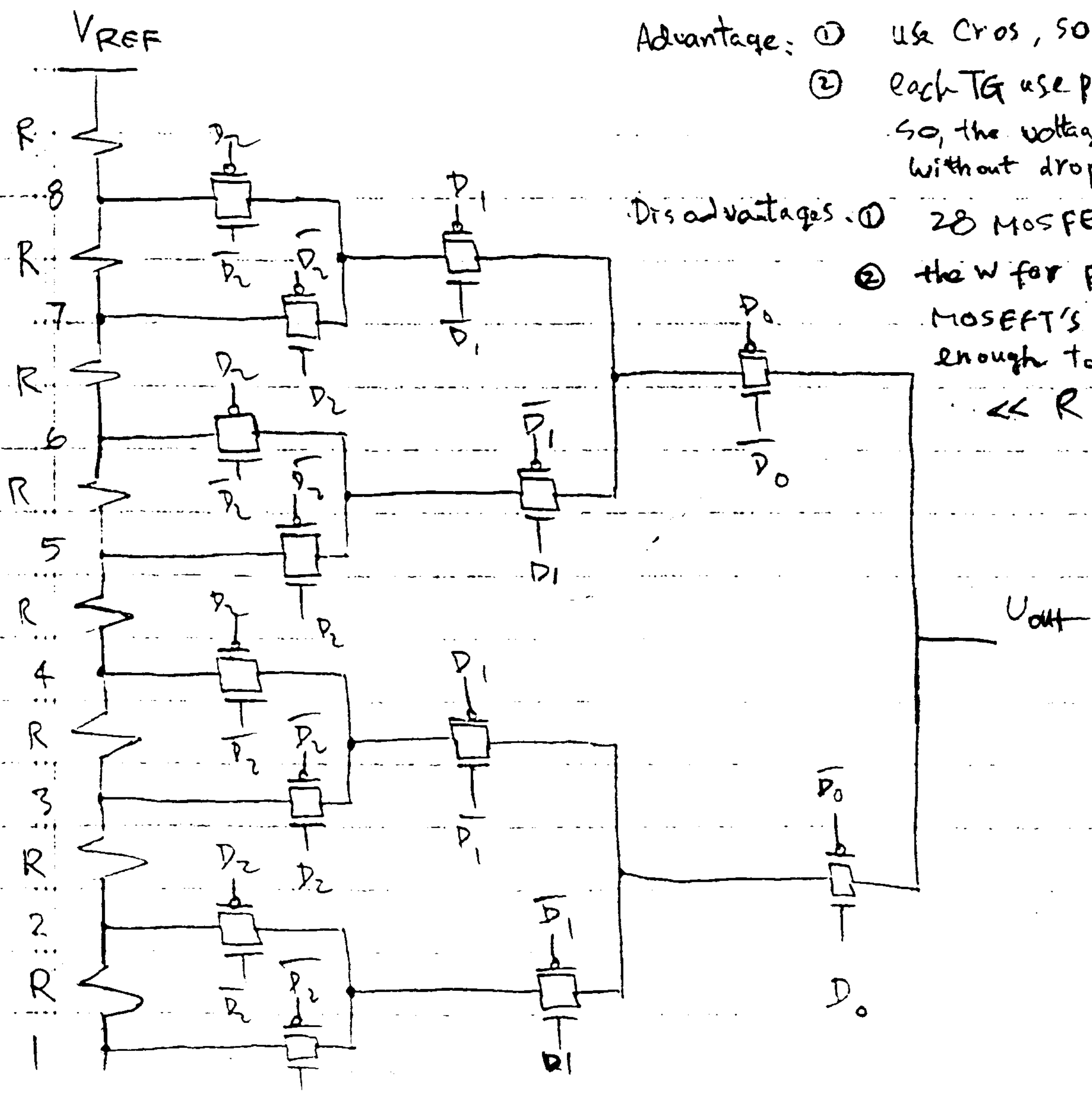
| | Resistor String DAC Figure 29.2(a) | | | | | | | | Resistor String DAC figure 29.2(b) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $D_7$ | $D_6$ | $D_5$ | $D_4$ | $D_3$ | $D_2$ | $D_1$ | $D_0$ | $D_2$ | $D_1$ | $D_0$ |
| 0 | | | | | | | | 1 | 0 | 0 | 0 |
| 1 | | 0 | | | | | 1 | | 0 | 0 | 1 |
| 2 | | | | | | 1 | | | 0 | 1 | 0 |
| 3 | | | | | 1 | | | | 0 | 1 | 1 |
| 4 | | | | 1 | | | | | 1 | 0 | 0 |
| 5 | | | 1 | | | | 0 | | 1 | 0 | 1 |
| 6 | | 1 | | | | | | | 1 | 1 | 0 |
| 7 | 1 | | | | | | | | 1 | 1 | 1 |

Advantage: ① use CMOS, so easy to fabrica

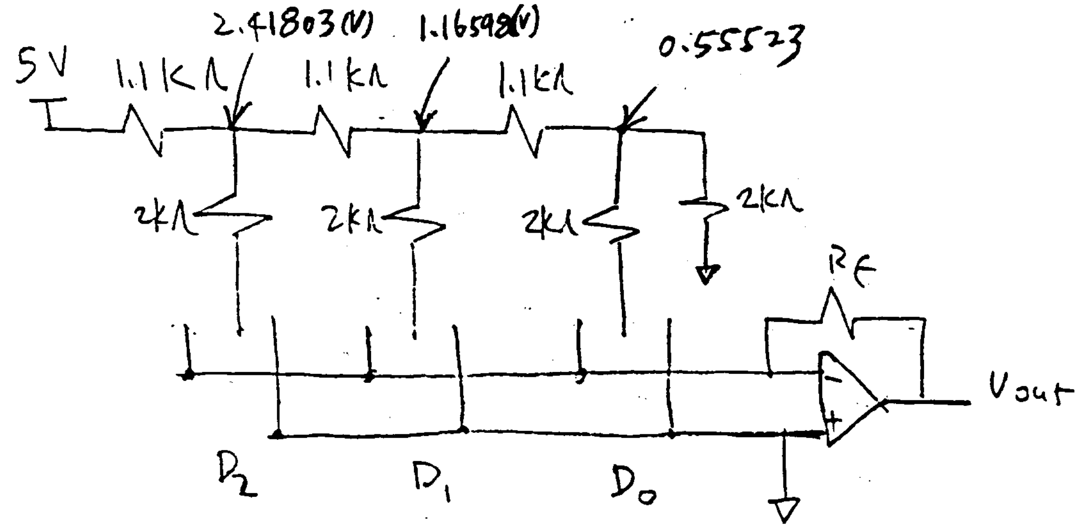
② each TG use p & n channel types so, the voltage can transfer without dropping

Disadvantages. ① 28 MOSFET's are used

② the W for p & n channel MOSEFT's should be big enough to let the TG resi $\ll R$.



29.4



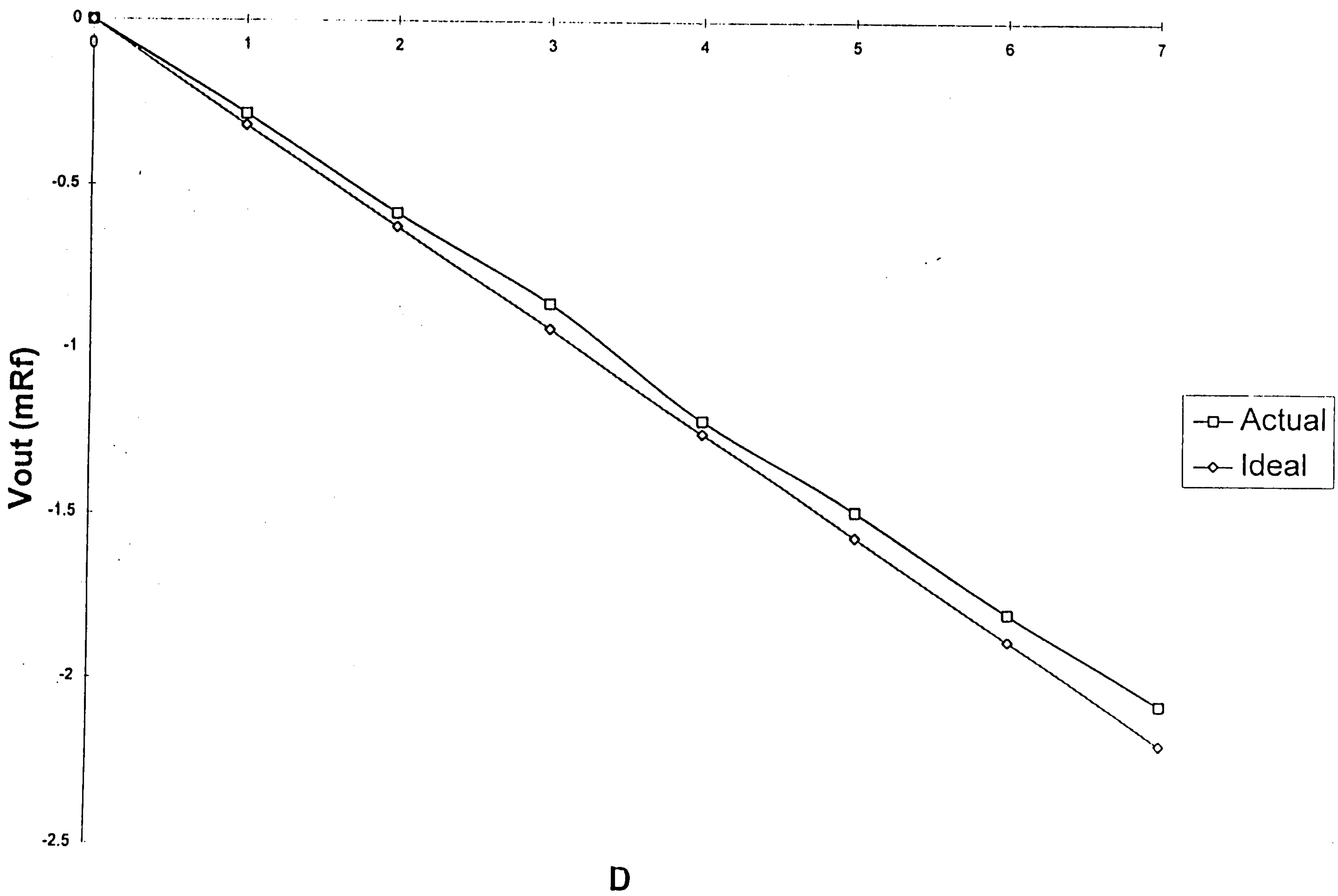

$V_{out} = -I_{TOT} \cdot R_F$

| Digital Input $D_2$ $D_1$ $D_0$ | Voltage Output (V) Actual | Voltage Output (V) Ideal | INL (LSB) | Step Height (V) Actual | Step Height (V) Ideal | DN |
|---|---|---|---|---|---|---|
| 0 0 0 | 0 | 0 | 0 | | | |
| 0 0 1 | $-0.2776\,mR_f$ | $-0.3125\,mR_f$ | -0.11 | $-0.2776\,mR_F$ | $-0.3125\,mR_F$ | -0.11 |
| 0 1 0 | $-0.5830\,mR_f$ | $-0.625\,mR_f$ | -0.13 | $-0.3054\,mR_f$ | " | -0.0 |
| 0 1 1 | $-0.8603\,mR_f$ | $-0.9375\,mR_f$ | -0.25 | $-0.2778\,mR_f$ | " | -0.11 |
| 1 0 0 | $-1.2090\,mR_f$ | $-1.25\,m\,R_f$ | -0.13 | $-0.3482\,mR_f$ | " | 0.11 |
| 1 0 1 | $-1.4866\,mR_f$ | $-1.5625\,mR_f$ | -0.24 | $-0.2776\,mR_f$ | " | -0.11 |
| 1 1 0 | $-1.7920\,mR_f$ | $-1.875\,m\,R_f$ | -0.27 | $-0.3054\,mR_f$ | " | -0.0 |
| 1 1 1 | $-2.0696\,mR_f$ | $-2.1875\,m\,R_f$ | -0.38 | $-0.2776\,mR_f$ | " | -0.11 |

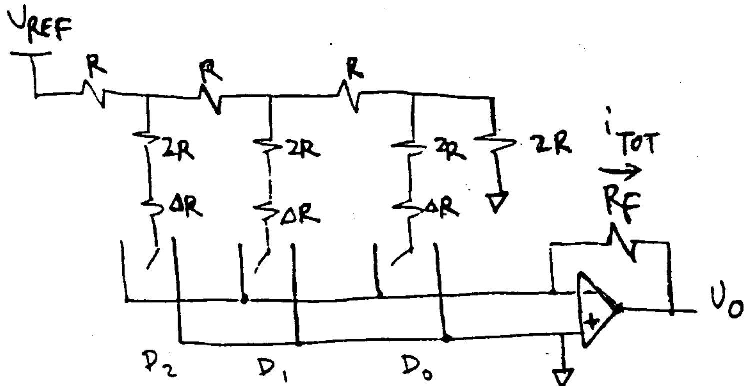
$$\frac{\text{Actual} - \text{Ideal}}{-0.3125\,m\,R_F}$$

$$-0.625\,\frac{R_F}{2K}$$

$$\frac{\text{Actual} - \text{Ideal}}{-0.3125\,m\,R_F}$$

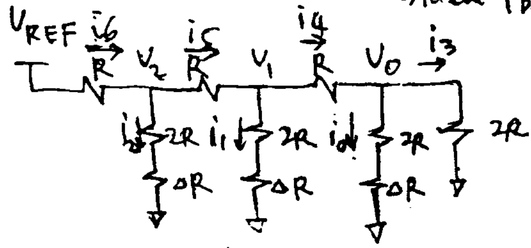

Vout (mRf)
0
-0.5
-1
-1.5
-2
-2.5
0
1
2
3
4
5
6
7
Actual
Ideal
D

Prob.29.5



$$INL = \frac{V_{o,actual} - V_{o,ideal}}{V_{o,ideal}\big|_{D_2D_1D_0=001}} = \frac{(-i_{TOT,actual}) - (-i_{TOT,ideal})}{-i_{TOT,ideal}\big|_{D_2D_1D_0=001}} \quad (LSB)$$

$$DNL = \frac{(V_{o,actual_k} - V_{o,actual_{k-1}}) - (V_{o,ideal_k} - V_{o,ideal_{k-1}})}{V_{o,ideal}\big|_{D_2D_1D_0=001}} = \frac{(-i_{TOT,actual_k} + i_{TOT,actual_{k-1}}) - i_{TOT,ideal}\big|_{D,C}}{-i_{TOT,ideal}\big|_{D_2D_1D_0=001}} \quad (LSB$$



$$i_6 = i_2 + i_5 : \quad \frac{V_{REF} - V_2}{R} = \frac{V_2 - V_1}{R} + \frac{V_2}{2R+\Delta R} \rightarrow \left(2 + \frac{1}{2+\Delta R/R}\right) V_2 = V_1 + V_{REF} \quad ①$$

$$i_5 = i_1 + i_4 : \quad \frac{V_2 - V_1}{R} = \frac{V_1 - V_0}{R} + \frac{V_1}{2R+\Delta R} \rightarrow \left(2 + \frac{1}{2+\Delta R/R}\right) V_1 = V_0 + V_2 \quad ②$$

$$i_4 = i_0 + i_3 : \quad \frac{V_1 - V_0}{R} = \frac{V_0}{2R} + \frac{V_0}{2R+\Delta R} \rightarrow \left(1.5 + \frac{1}{2+\Delta R/R}\right) V_0 = V_1 \quad ③$$

with MathCAD, increase $\Delta R$ to make INL or DNL $\doteq 0.5$ LSB get.

when $\Delta R = 305\,\Omega$ $\quad |INL|_{max} = |INL_7| = 0.496$ LSB.
$\quad |DNL|_{max} = |DNL_4| = 0.243$ LSB

So, $\boxed{\Delta R \doteq 300\,\Omega}$ is the upper limit to keep the DAC have 3 bit resolution

Vref := 5 DR := .305 R := 1

$$DRR := \frac{DR}{R}$$

$$alfa := 1.5 + \frac{1}{2 + DRR}$$

$$V0 := \frac{Vref}{((0.5 + alfa)\cdot((0.5 + alfa)\cdot alfa - 1) - alfa)} \qquad V0 = 0.705$$

$$V1 := alfa \cdot V0 \qquad V1 = 1.364$$

$D2 := 0 \quad D1 := 0 \quad D0 := 0$

$$iTAC0 := D0\cdot\frac{V0}{2\cdot R + DR} + D1\cdot\frac{V1}{2\cdot R + DR} + D2\cdot\frac{V2}{2\cdot R + DR} \qquad iTAC0 = 0$$

$iTID0 := 0$

$D2 := 0 \quad D1 := 0 \quad D0 := 1$

$$iTAC1 := D0\cdot\frac{V0}{2\cdot R + DR} + D1\cdot\frac{V1}{2\cdot R + DR} + D2\cdot\frac{V2}{2\cdot R + DR} \qquad iTAC1 = 0.306$$

$iTID1 := .3125 \quad iLSB := iTID1$

$D2 := 0 \quad D1 := 1 \quad D0 := 0$

$$iTAC2 := D0\cdot\frac{V0}{2\cdot R + DR} + D1\cdot\frac{V1}{2\cdot R + DR} + D2\cdot\frac{V2}{2\cdot R + DR} \qquad iTAC2 = 0.592$$

$iTID2 := .625$

$D2 := 0 \quad D1 := 1 \quad D0 := 1$

$$iTAC3 := D0\cdot\frac{V0}{2\cdot R + DR} + D1\cdot\frac{V1}{2\cdot R + DR} + D2\cdot\frac{V2}{2\cdot R + DR} \qquad iTAC3 = 0.898$$

$iTID3 := .9375$

$D2 := 1 \quad D1 := 0 \quad D0 := 0$

$$iTAC4 := D0\cdot\frac{V0}{2\cdot R + DR} + D1\cdot\frac{V1}{2\cdot R + DR} + D2\cdot\frac{V2}{2\cdot R + DR} \qquad iTAC4 = 1.134$$

$iTID4 := 1.25$

$D2 := 1 \quad D1 := 0 \quad D0 := 1$

$$iTAC5 := D0\cdot\frac{V0}{2\cdot R + DR} + D1\cdot\frac{V1}{2\cdot R + DR} + D2\cdot\frac{V2}{2\cdot R + DR} \qquad iTAC5 = 1.441$$

$iTID5 := 1.5625$

$D2 := 1 \quad D1 := 1 \quad D0 := 0$

$$iTAC6 := D0\cdot\frac{V0}{2\cdot R + DR} + D1\cdot\frac{V1}{2\cdot R + DR} + D2\cdot\frac{V2}{2\cdot R + DR} \qquad iTAC6 = 1.726$$

$iTID6 := 1.875$

$D2 := 1 \quad D1 := 1 \quad D0 := 1$

$$iTAC7 := D0\cdot\frac{V0}{2\cdot R + DR} + D1\cdot\frac{V1}{2\cdot R + DR} + D2\cdot\frac{V2}{2\cdot R + DR} \qquad iTAC7 = 2.032$$

## INL and DNL

$INL0 := \frac{iTAC0 - iTID0}{iLSB}$ $\quad$ $INL0 = 0$

$INL1 := \frac{iTAC1 - iTID1}{iLSB}$ $\quad$ $INL1 = -0.021$

$INL2 := \frac{iTAC2 - iTID2}{iLSB}$ $\quad$ $INL2 = -0.106$

$INL3 := \frac{iTAC3 - iTID3}{iLSB}$ $\quad$ $INL3 = -0.127$ $\quad$ (LSB)

$INL4 := \frac{iTAC4 - iTID4}{iLSB}$ $\quad$ $INL4 = -0.37$

$INL5 := \frac{iTAC5 - iTID5}{iLSB}$ $\quad$ $INL5 = -0.39$

$INL6 := \frac{iTAC6 - iTID6}{iLSB}$ $\quad$ $INL6 = -0.476$

$INL7 := \frac{iTAC7 - iTID7}{iLSB}$ $\quad$ $INL7 = -0.496$

$DNL1 := \frac{iTAC1 - iTAC0 - iLSB}{iLSB}$ $\quad$ $DNL1 = -0.021$

$DNL2 := \frac{iTAC2 - iTAC1 - iLSB}{iLSB}$ $\quad$ $DNL2 = -0.085$

$DNL3 := \frac{iTAC3 - iTAC2 - iLSB}{iLSB}$ $\quad$ $DNL3 = -0.021$

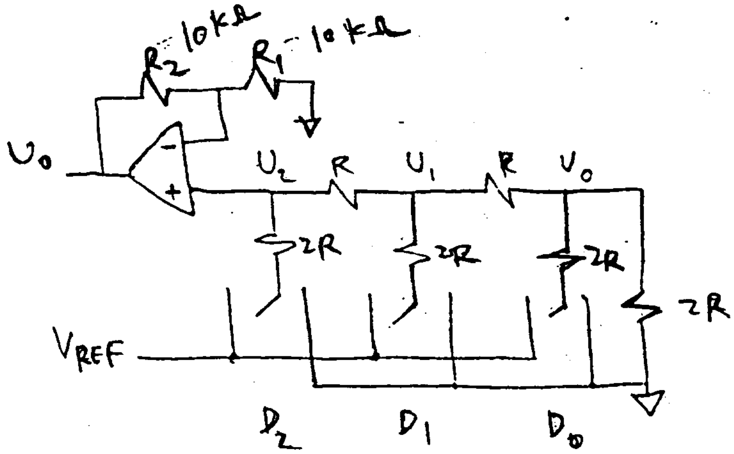
$DNL4 := \frac{iTAC4 - iTAC3 - iLSB}{iLSB}$ $\quad$ $DNL4 = -0.243$ $\quad$ (LSB)

$DNL5 := \frac{iTAC5 - iTAC4 - iLSB}{iLSB}$ $\quad$ $DNL5 = -0.021$

$DNL6 := \frac{iTAC6 - iTAC5 - iLSB}{iLSB}$ $\quad$ $DNL6 = -0.085$

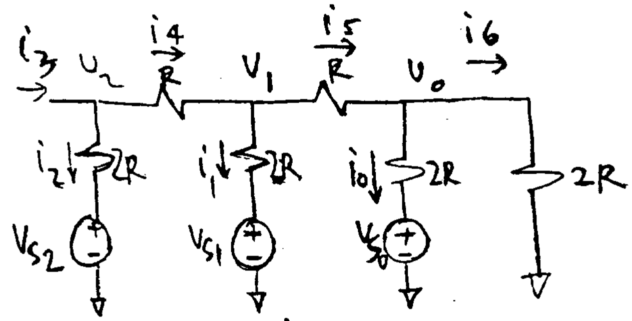
$DNL7 := \frac{iTAC7 - iTAC6 - iLSB}{}$ $\quad$ $DNL7 = -0.021$

29.6




$R = 1K\Omega$

$$V_o = \left(1+\frac{R_2}{R_1}\right) V_2 = 2V_2$$




ideal Op Amp

$$i_3 = i_2 + i_4 : \quad 0 = \frac{V_2 - V_1}{R} + \frac{V_2 - V_{S2}}{2R} \rightarrow 1.5 V_2 = V_1 + 0.5 V_{S2}$$

$$i_4 = i_1 + i_5 : \quad \frac{V_2 - V_1}{R} = \frac{V_1 - V_0}{R} + \frac{V_1 - V_{S1}}{2R} \rightarrow V_2 = 2.5 V_1 - V_0 - 0.5$$

$$i_5 = i_0 + i_6 : \quad \frac{V_1 - V_0}{R} = \frac{V_0}{2R} + \frac{V_0 - V_{S0}}{2R} \rightarrow V_1 = 2V_0 - 0.5 V_{S0}$$

$$\Rightarrow \begin{cases} V_0 = \frac{1}{4}(0.5 V_{S2} + 0.75 V_{S1} + 1.375 V_{S0}) \\ V_1 = 2V_0 - 0.5 V_{S0} \\ V_2 = 2.5 V_1 - V_0 - 0.5 V_{S1} \end{cases}$$

| $D_2D_1D_0$ | $V_{S2}$ | $V_{S1}$ | $V_{S0}$ (V) | $V_0$ | $V_1$ | $V_2$ (V) | $V_{out} = 2V_2$ (V) |
|---|---|---|---|---|---|---|---|
| 000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 001 | 0 | 0 | 5 | 1.719 | 0.938 | 0.625 | 1.25 |
| 010 | 0 | 5 | 0 | 0.938 | 1.875 | 1.25 | 2.5 |
| 011 | 0 | 5 | 5 | 2.656 | 2.813 | 1.85 | 3.75 |
| 100 | 5 | 0 | 0 | 0.625 | 1.25 | 2.5 | 5 |
| 101 | 5 | 0 | 5 | 2.344 | 2.188 | 3.125 | 6.25 |
| 110 | 5 | 5 | 0 | 1.563 | 3.125 | 3.75 | 7.5 |

Prob.29.6

D=000

$VS2 := 0 \quad VS1 := 0 \quad VS0 := 0$

$V0 := \frac{1}{4}\cdot(0.5\cdot VS2 + 0.75\cdot VS1 + 1.375\cdot VS0) \qquad V0 = 0$

$V1 := 2\cdot V0 - 0.5\cdot VS0 \qquad V1 = 0$

$V2 := 2.5\cdot V1 - V0 - .5\cdot VS1 \qquad V2 = 0$

D=001

$VS2 := 0 \quad VS1 := 0 \quad VS0 := 5$

$V0 := \frac{1}{4}\cdot(0.5\cdot VS2 + 0.75\cdot VS1 + 1.375\cdot VS0) \qquad V0 = 1.719$

$V1 := 2\cdot V0 - 0.5\cdot VS0 \qquad V1 = 0.938$

$V2 := 2.5\cdot V1 - V0 - .5\cdot VS1 \qquad V2 = 0.625$

D=010

$VS2 := 0 \quad VS1 := 5 \quad VS0 := 0$

$V0 := \frac{1}{4}\cdot(0.5\cdot VS2 + 0.75\cdot VS1 + 1.375\cdot VS0) \qquad V0 = 0.938$

$V1 := 2\cdot V0 - 0.5\cdot VS0 \qquad V1 = 1.875$

$V2 := 2.5\cdot V1 - V0 - .5\cdot VS1 \qquad V2 = 1.25$

D=011

$VS2 := 0 \quad VS1 := 5 \quad VS0 := 5$

$V0 := \frac{1}{4}\cdot(0.5\cdot VS2 + 0.75\cdot VS1 + 1.375\cdot VS0) \qquad V0 = 2.656$

$V1 := 2\cdot V0 - 0.5\cdot VS0 \qquad V1 = 2.813$

$V2 := 2.5\cdot V1 - V0 - .5\cdot VS1 \qquad V2 = 1.875$

D=100

$VS2 := 5 \quad VS1 := 0 \quad VS0 := 0$

$V0 := \frac{1}{4}\cdot(0.5\cdot VS2 + 0.75\cdot VS1 + 1.375\cdot VS0) \qquad V0 = 0.625$

$V1 := 2\cdot V0 - 0.5\cdot VS0 \qquad V1 = 1.25$

## D=101

$VS2 := 5 \qquad VS1 := 0 \qquad VS0 := 5$

$VO := \frac{1}{4}\cdot(0.5\cdot VS2 + 0.75\cdot VS1 + 1.375\cdot VS0) \qquad V0 = 2.344$

$V1 := 2\cdot V0 - 0.5\cdot VS0 \qquad V1 = 2.188$

$V2 := 2.5\cdot V1 - V0 - .5\cdot VS1 \qquad V2 = 3.125$

## D=110

$VS2 := 5 \qquad VS1 := 5 \qquad VS0 := 0$

$V0 := \frac{1}{4}\cdot(0.5\cdot VS2 + 0.75\cdot VS1 + 1.375\cdot VS0) \qquad V0 = 1.563$

$V1 := 2\cdot V0 - 0.5\cdot VS0 \qquad V1 = 3.125$

$V2 := 2.5\cdot V1 - V0 - .5\cdot VS1 \qquad V2 = 3.75$

## D=111

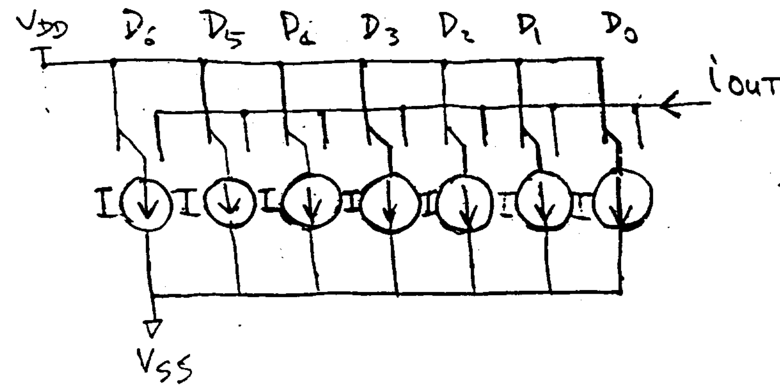
$VS2 := 5 \qquad VS1 := 5 \qquad VS0 := 5$

$V0 := \frac{1}{4}\cdot(0.5\cdot VS2 + 0.75\cdot VS1 + 1.375\cdot VS0) \qquad V0 = 3.281$

$V1 := 2\cdot V0 - 0.5\cdot VS0 \qquad V1 = 4.063$

$V2 := 2.5\cdot V1 - V0 - .5\cdot VS1 \qquad V2 = 4.375$

29.7




I=5mA

| | $D_6$ | $D_5$ | $D_4$ | $D_3$ | $D_2$ | $D_1$ | $D_0$ | $i_{out}$ |
|---|---|---|---|---|---|---|---|---|
| 000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 001 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | I |
| 010 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2I |
| 011 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 3I |
| 100 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 4I |
| 101 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 5I |
| 110 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 6I |
| 111 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 7I |

29.8 for nonbinary weight current steering DAC

$|DNL|_{max} = |\Delta I_{max}|\ (A) = |\Delta I_{max}|/I = 0.05\%$ (LSB) in our case

$|INL|_{max} = 2^{N-1}|\Delta I|_{max}\ (A) = 2^{N-1}|\Delta I|_{max}/I = 2^{N-1}\cdot 0.05\%$ (LSB) in our case

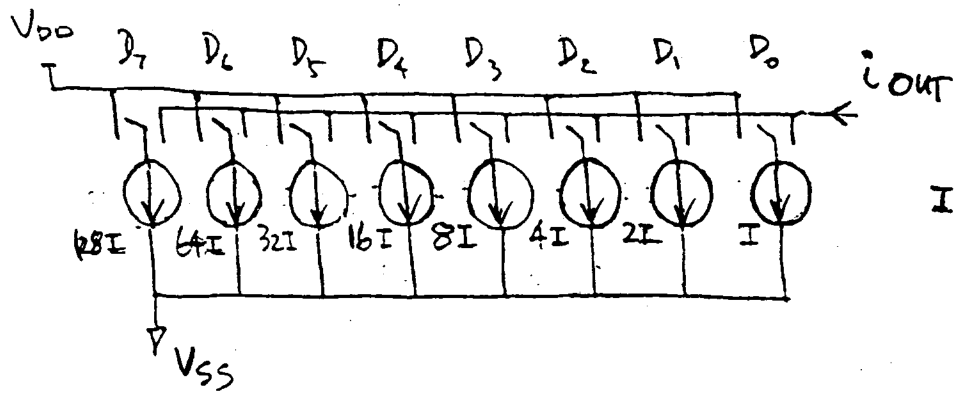
$$N = \frac{\lg|INL|_{max} - \lg(0.05\%)}{\lg 2} + 1$$

let $|INL|_{max} = 0.5$ LSB

$$N = \frac{\lg(0.5) - \lg(0.05\%)}{\lg 2} + 1 = 10.97$$

the maximum resolution the DAC can attain using the process is 10 bit

29.9




$I = 1\mu A$

The range of values that the current source corresponding to the MS can have :

① $|INL|_{max} \le \frac{1}{2}$ LSB

$$|\Delta I|_{max, INL} = \frac{|INL|_{max}}{2^{8-1}} = \frac{0.5}{2^7} = 3.906 \times 10^{-3}$$

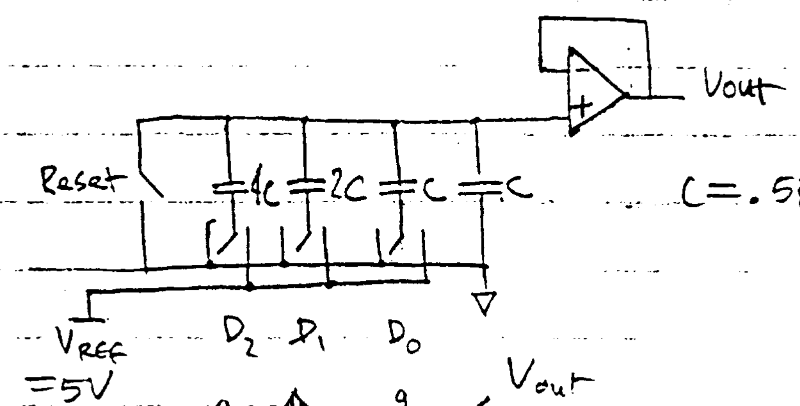
$$I_{MSB} = 128 \pm 128 \cdot |\Delta I|_{max, INL} = \boxed{128 \pm 0.5 \ \mu A} \quad (= 3.906 \ nA)$$

② $|DNL|_{max} \le \frac{1}{2}$ LSB

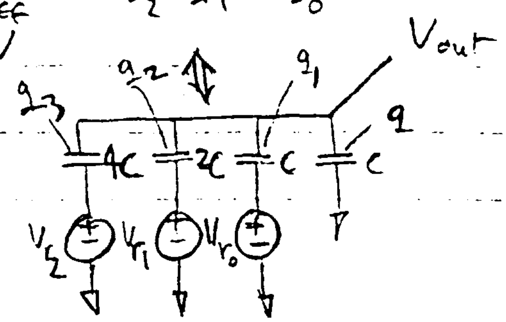
$$|\Delta I|_{max, DNL} = \frac{|DNL|_{max}}{2^8 - 1} = \frac{0.5}{2^8 - 1} = 1.961 \times 10^{-3} \text{ LSB} = 1.961 \ nA$$

$$I_{MSB} = 128 \pm 128 |\Delta I|_{max, DNL} = \boxed{128 \pm 0.25 \ \mu A}$$

29.10




$C = .5pF$




$q_0 + q_1 + q_2 + q = 0$

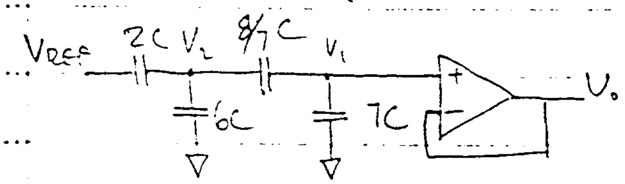
$$(1+2+4+1)\,V_{out} = V_{r0} + 2V_{r1} + 4V_{r2}$$

$$\boxed{V_{out} = \frac{1}{8}V_{r0} + \frac{1}{4}V_{r1} - \frac{1}{2}V_{r2}}$$

| $D_2D_1D_0$ | $V_{r2}$(V) | $V_{r1}$(V) | $V_{r0}$(V) | $V_{out}$(V) |
|---|---|---|---|---|
| 000 | 0 | 0 | 0 | 0 |
| 001 | 0 | 0 | 5 | 0.625 |
| 010 | 0 | 5 | 0 | 1.250 |
| 011 | 0 | 5 | 5 | 1.875 |
| 100 | 5 | 0 | 0 | 2.500 |
| 101 | 5 | 0 | 5 | 3.125 |
| 110 | 5 | 5 | 0 | 3.750 |
| 111 | 5 | 5 | 5 | 4.375 |

29.11 D = 000010

equivalent CKT:

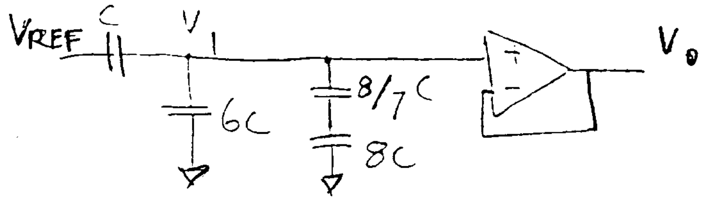



$$V_2 = \frac{2}{\left(6+\frac{8/7\cdot 7}{8/7+7}\right)+2}V_{REF} = \frac{2}{8+56/57}$$

$$V_0 = V_1 = \frac{8/7}{8/7+7}\cdot V_2 = \frac{8}{57}\cdot\frac{2}{8+56/57}V_{REF} = \boxed{\frac{V_{REF}}{32}}$$

D = 001000

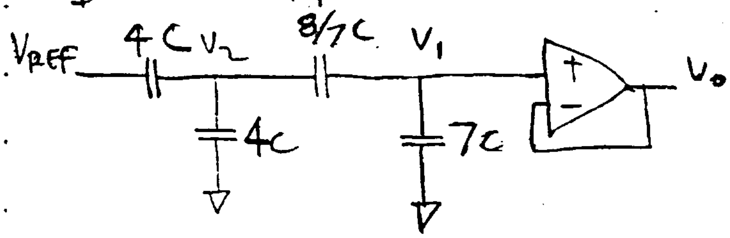
equivalent CKT



$$V_o = V_1 = \frac{1}{\frac{8/7\cdot 8}{8/7+8}+6+1} = \boxed{\frac{V_{REF}}{8}}$$

D = 000100

equivalent CKT

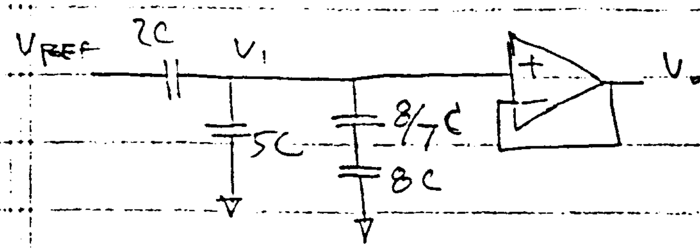



$$V_2 = \frac{4}{\left(\frac{8/7\cdot 7}{8/7+7}+4\right)+4} V_{REF} = \frac{4}{8+56/57} V_{REF}$$

$$V_0 = V_1 = \frac{8/7}{8/7+7}\cdot V_2 = \frac{8}{57}\cdot\frac{4}{8+56/57}V_{REF} = \boxed{\frac{V_{REF}}{16}}$$

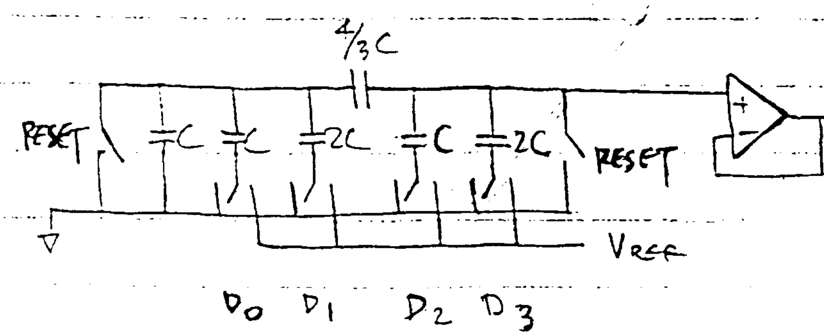
D = 0100000

equivalent CKT



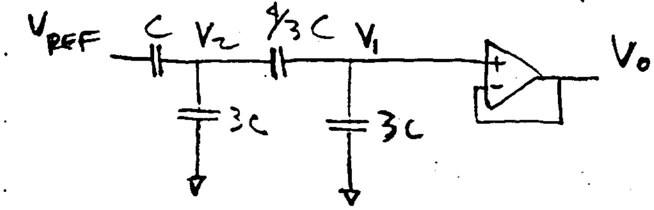

$$V_o = V_1 = \frac{2}{5+\frac{8/7\cdot 8}{8/7+8}+2} V_{REF} = \boxed{\frac{1}{4} V_{REF}}$$

29.12




C = .5pF

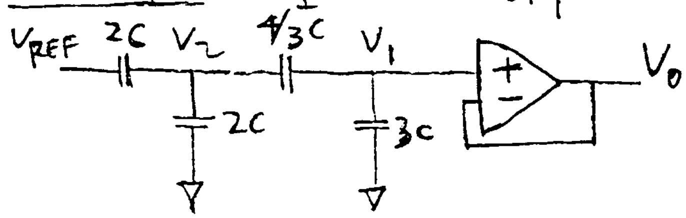
D = 0001 equivalent CKT




$$V_2 = \frac{1}{\left(\frac{4/3 \cdot 3}{4/3+3} + 3\right)+1} V_{REF} = \frac{1}{4+12/13} V_{REF}$$

$$V_o = V_1 = \frac{4/3}{4/3+3} \cdot V_2 = \frac{4/3}{4/3+3} \cdot \frac{1}{4+12/13} V_{REF} = \frac{5}{16} = \boxed{0.3125}$$

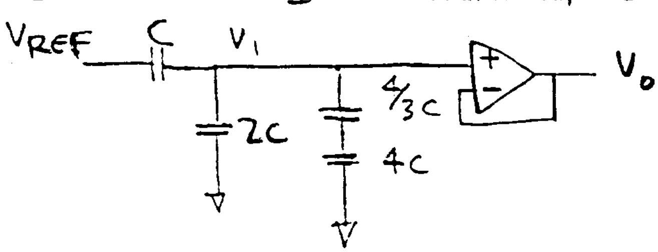
D = 0010 equivalent CKT




$$V_2 = \frac{2}{\left(\frac{4/3 \cdot 3}{4/3+3} + 2\right)+2} V_{REF} = \frac{2}{4+12/13} V_{REF}$$

$$V_o = V_1 = \frac{4/3}{4/3+3} \cdot \frac{2}{4+12/13} V_{REF} = \frac{5}{8} = \boxed{0.625V}$$

D = 0100 equivalent CKT

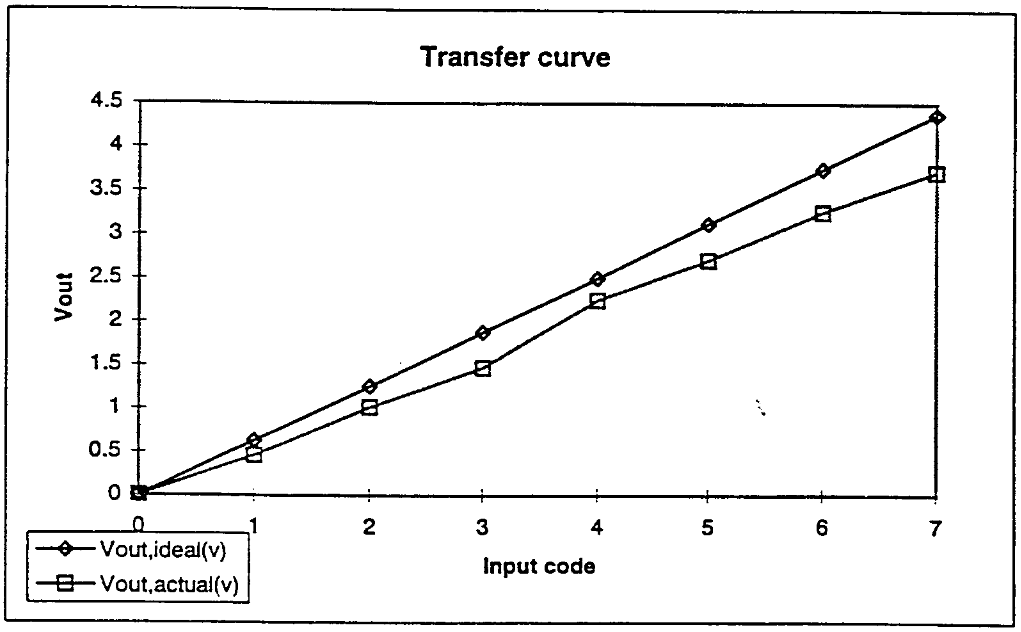



$$V_o = V_1 = \frac{1}{\left(\frac{4/3 \cdot 4}{4/3+4} + 2\right)+1} V_{REF} = \frac{5}{4} = \boxed{1.25V}$$

D = 1000 equivalent CKT

29.13.

| Input | D2 | D1 | D0 | Vout,ideal(v) | Vout,actual(v) | Vout,nogain(v) | INL(v) | INL(LSBs) | DNL(v) | DNL(LSBs) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0.625 | 0.455625 | 0.536119516 | -0.08888 | -0.142209 | -0.08888 | -0.1422088 |
| 2 | 0 | 1 | 0 | 1.25 | 1.0125 | 1.191376702 | -0.058623 | -0.093797 | 0.030257 | 0.0484115 |
| 3 | 0 | 1 | 1 | 1.875 | 1.468125 | 1.727496218 | -0.147504 | -0.236006 | -0.08888 | -0.1422088 |
| 4 | 1 | 0 | 0 | 2.5 | 2.25 | 2.647503782 | 0.147504 | 0.2360061 | 0.295008 | 0.4720121 |
| 5 | 1 | 0 | 1 | 3.125 | 2.705625 | 3.183623298 | 0.058623 | 0.0937973 | -0.08888 | -0.1422088 |
| 6 | 1 | 1 | 0 | 3.75 | 3.2625 | 3.838880484 | 0.08888 | 0.1422088 | 0.030257 | 0.0484115 |
| 7 | 1 | 1 | 1 | 4.375 | 3.718125 | 4.375 | 8.88E-16 | 1.421E-15 | -0.08888 | -0.1422088 |
| | | | | | MAXIMUM | | 0.147504 | 0.2360061 | ~~0.08888~~ | ~~0.1422088~~ |




$V_{out,actual} = V_{REF} \sum_{K=0}^{N-1} A^{N-K} \cdot D_K = V_{REF}\left[A \cdot D_2 + A^2 \cdot D_1 + A^1 \cdot D_0\right]$ $A = 0.45$

offset = 0

Gain error = Gain,ideal − Gain,actual

$= 1 - \frac{3.71825/0.625}{7} \doteq 1 - 0.85 = 0.15 \ \text{LSB}/\text{LSB}$

After removing offset, gain error

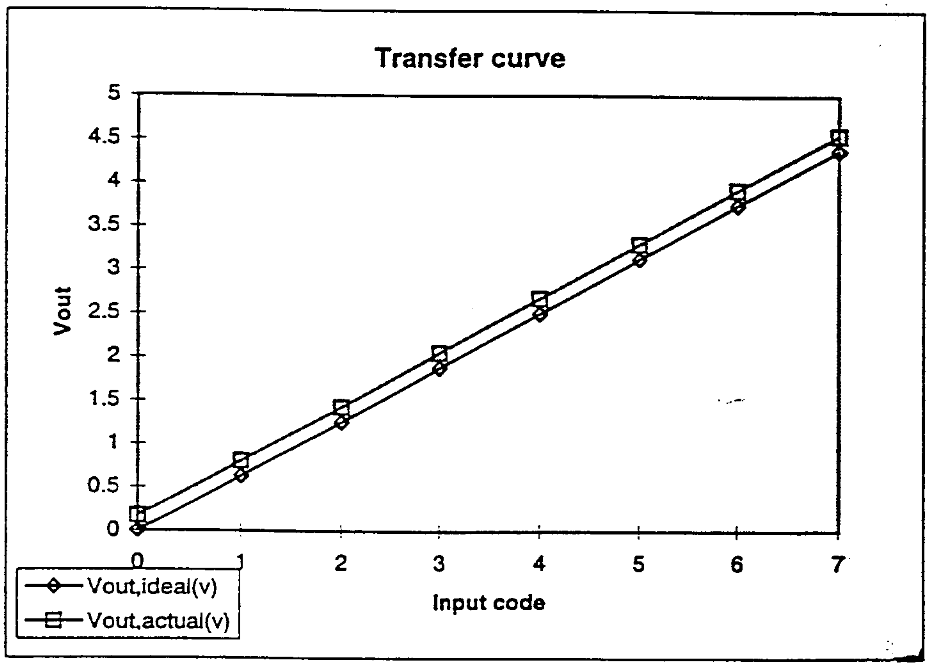
$|INL|_{max} = 0.236 \ \text{LSBs} = 0.1475\ V$

$|DNL|_{max} = 0.472 \ \text{LSBs} = 0.295\ V$

29.14

| Input | D2 | D1 | D0 | Vout,ideal(v) | Vout,actual(v) | Vout,nooff(v) | INL(v) | INL(LSBs) | DNL(v) | DNL(LSBs) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0.175 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0.625 | 0.8 | 0.625 | 0 | 0 | 0 | 0 |
| 2 | 0 | 1 | 0 | 1.25 | 1.425 | 1.25 | 0 | 0 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1.875 | 2.05 | 1.875 | 0 | 0 | 0 | 0 |
| 4 | 1 | 0 | 0 | 2.5 | 2.675 | 2.5 | 0 | 0 | 0 | 0 |
| 5 | 1 | 0 | 1 | 3.125 | 3.3 | 3.125 | 0 | 0 | 0 | 0 |
| 6 | 1 | 1 | 0 | 3.75 | 3.925 | 3.75 | 0 | 0 | 0 | 0 |
| 7 | 1 | 1 | 1 | 4.375 | 4.55 | 4.375 | 0 | 0 | 0 | 0 |
| | | | | | MAXIMUM | | 0 | 0 | 0 | 0 |

Offset=0.175v




Assume: $V_{off} = 0.2V$

$A = 0.5$

$V_{off} = 0.2V$

$$V_{out} = V_{ref} \sum_{K=0}^{N-1} A^{N-K} \cdot D_K + \sum_{K=0}^{N-1} A^{N-K} \cdot V_{off}$$

$$= V_{REF} \left[ A \cdot D_2 + A^2 \cdot D_1 + A^3 D_0 \right] + V_{off} \left[ A + A^2 + A^3 \right]$$

$$= V_{out,ideal} + 0.175V$$

offset $= 0.175V$

gain error $= 0$

After removing offset, gain error.

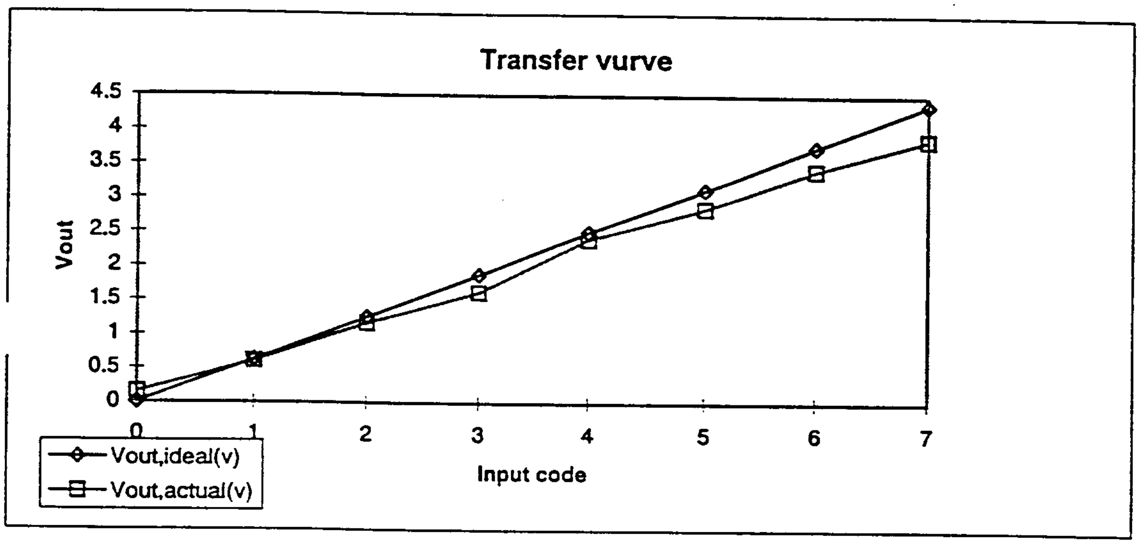
$|INL|_{max} = 0$

$|DNL|_{max} = 0$

# 29.15

| Input | D2 | D1 | D0 | Vout,ideal(v) | Vout,actual(v) | Vout,nooff(v) | Vout,nogain(v) | INL(v) | INL(LSBs) | DNL(v) | DNL(LSBs) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0.148725 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0.625 | 0.60435 | 0.455625 | 0.536119516 | -0.08888 | -0.14221 | -0.08888 | -0.14221 |
| 2 | 0 | 1 | 0 | 1.25 | 1.161225 | 1.0125 | 1.191376702 | -0.05862 | -0.0938 | 0.030257 | 0.048411 |
| 3 | 0 | 1 | 1 | 1.875 | 1.61685 | 1.468125 | 1.727496218 | -0.1475 | -0.23601 | -0.08888 | -0.14221 |
| 4 | 1 | 0 | 0 | 2.5 | 2.398725 | 2.25 | 2.647503782 | 0.147504 | 0.236006 | 0.295008 | 0.472012 |
| 5 | 1 | 0 | 1 | 3.125 | 2.85435 | 2.705625 | 3.183623298 | 0.058623 | 0.093797 | -0.08888 | -0.14221 |
| 6 | 1 | 1 | 0 | 3.75 | 3.411225 | 3.2625 | 3.838880484 | 0.08888 | 0.142209 | 0.030257 | 0.048411 |
| 7 | 1 | 1 | 1 | 4.375 | 3.86685 | 3.718125 | 4.375 | 0 | 0 | -0.08888 | -0.14221 |
| | | | | | MAXIMUM | | | 0.147504 | 0.236006 | ~~0.08888~~ 0.295008 | ~~0.142209~~ 0.472012 |

Offset=0.175v




$$V_{out} = V_{ref}\left[A\cdot D_2 + A^2\cdot D_1 + A^3\cdot D_0\right] + V_{off}\left[A + A^2 + A^2\right]$$

$A = 0.45$

$V_{off} = 0.2\,V$

offset $= 0.148725\ V = 0.23796$ LSBs

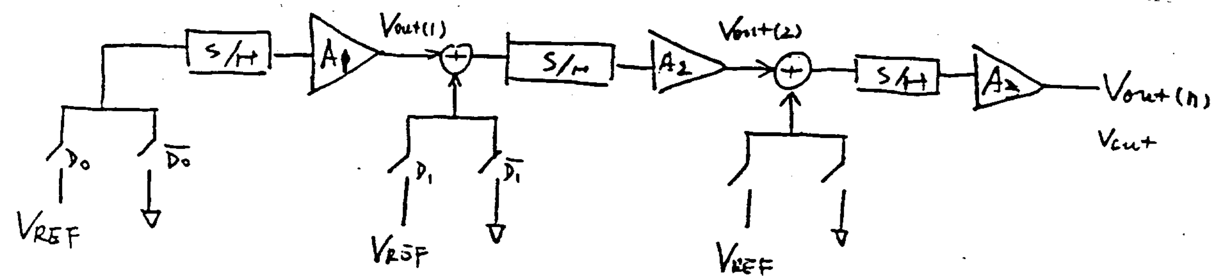
gain error $= 1 - \dfrac{3.718125/0.625}{7} = 1 - 0.85 = 0.15$ LSB/LSB

after removing offset, gain error.

$|INL|_{max} = 0.1475\ V = 0.236$ LSBs

$|DNL|_{max} = 0.295\ V = 0.472$ LSBs

29.16.




$A_{ideal} = 1/2$ , $V_{REF} = 5V$

a). $A_2 = A_3 = A_{ideal} = 1/2 = A$.

$V_{out}(1) = D_0 \cdot V_{REF} \cdot A_1$

$V_{out}(2) = (D_1 \cdot V_{REF} + D_0 V_{REF} \cdot A_1) A_2$

$V_{out}(3) = (D_2 V_{REF} + (D_1 \cdot V_{REF} + D_0 V_{REF} A_1) A_2) A_3$

$= V_{REF} ( D_0 \cdot A_1 A_2 A_3 + D_1 \cdot A_2 \cdot A_3 + D_2 \cdot A_3 )$

In this case. $V_{out,\,actual} = V_{REF} \left(D_2 \cdot \frac{1}{2} + D_1 \cdot \frac{1}{4} + D_0 \cdot \frac{A_1}{4}\right)$

| Input | Vout, actual | DNL (no offset) |
|---|---|---|
| 000 | 0 | 0 |
| 001 | $V_{REF}\left(\frac{A_1}{4}\right)$ | $x \cdot V_{REF}\left(\frac{A_1}{4}\right) - \frac{V_{REF}}{8}$ |
| 010 | $V_{REF}\left(\frac{1}{4}\right)$ | $x \cdot V_{REF}\left(\frac{1-A_1}{4}\right) - \frac{V_{REF}}{8}$ |
| 011 | $V_{REF}\left(\frac{1}{4}+\frac{A_1}{4}\right)$ | $x \cdot V_{REF}\left(\frac{A_1}{4}\right) - \frac{V_{REF}}{8}$ |
| 100 | $V_{REF}\left(\frac{1}{2}\right)$ | $x \cdot V_{REF}\left(\frac{1-A_1}{4}\right) - \frac{V_{REF}}{8}$ |
| 101 | $V_{REF}\left(\frac{1}{2}+\frac{A_1}{4}\right)$ | $x \cdot V_{REF}\left(\frac{A_1}{4}\right) - \frac{V_{REF}}{8}$ |
| 110 | $V_{REF}\left(\frac{1}{2}+\frac{1}{4}\right)$ | $x \cdot V_{REF}\left(\frac{1-A_1}{4}\right) - \frac{V_{REF}}{8}$ |
| 111 | $V_{REF}\left(\frac{1}{2}+\frac{1}{4}+\frac{A_1}{4}\right)$ | $x \cdot V_{REF}\left(\frac{A_1}{4}\right) - \frac{V_{REF}}{8}$ |

offset = 0

gain error factor = $x$

$$x = \frac{\frac{7}{8} V_{ref}}{V_{ref}\left(\frac{1}{2}+\frac{1}{4}+\frac{A_1}{4}\right)} = \frac{7}{2(3+A_1)}$$

Set $|DNL| \le \frac{1}{2} LSBs = \frac{1}{2} \cdot \frac{V_{REF}}{8} = \frac{1}{16} V_{REF}$

$$\left.\begin{array}{l} \left| \frac{7}{2\cdot(3+A_1)} \cdot \frac{A_1}{4} - \frac{1}{8} \right| \le \frac{1}{16} \Rightarrow \frac{3}{13} \le A_1 \le \frac{9}{11} \\ \left| \frac{7}{2(3+A_1)} \cdot \frac{1-A_1}{4} - \frac{1}{8} \right| \le \frac{1}{16} \Rightarrow \frac{5}{17} \le A_1 \le \frac{11}{15} \end{array}\right\} \Rightarrow \frac{5}{17} \le A_1 \le \frac{11}{15}$$

$0.294 \le A \le 0.733$

In order to let the DAC have $\le \pm\frac{1}{2}$ LSBs of DNL, the gain of first ct should be :

$$\boxed{0.294 < A < 0.733}$$

b) $V_{out,actual} = V_{REF}\left(D_2 \cdot \frac{1}{2} + D_1 \cdot \frac{A_2}{2} + D_0 \cdot \frac{A_2}{4}\right)$

| Input | $V_{out}$ actual | DNL |
|---|---|---|
| 000 | 0 | 0 |
| 001 | $V_{REF}\left(\frac{A_2}{4}\right)$ | $\alpha \cdot V_{REF}\left(\frac{A_2}{4}\right) - \frac{1}{8}V_{ref}$ |
| 010 | $V_{REF}\left(\frac{A_2}{2}\right)$ | $\alpha \cdot V_{REF}\left(\frac{A_2}{4}\right) - \frac{1}{8}V_{ref}$ |
| 011 | $V_{REF}\left(\frac{3A_2}{4}\right)$ | $\alpha \cdot V_{REF}\left(\frac{A_2}{4}\right) - \frac{1}{8}V_{ref}$ |
| 100 | $V_{REF}\left(\frac{1}{2}\right)$ | $\alpha \cdot V_{REF}\left(\frac{1}{2} - \frac{3A_2}{4}\right) - \frac{1}{8}V_{ref}$ |
| 101 | $V_{REF}\left(\frac{1}{2} + \frac{A_2}{4}\right)$ | $\alpha \cdot V_{REF}\left(\frac{A_2}{4}\right) - \frac{1}{8}V_{ref}$ |
| 110 | $V_{REF}\left(\frac{1}{2} + \frac{A_2}{2}\right)$ | $\alpha \cdot V_{REF}\left(\frac{A_2}{4}\right) - \frac{1}{8}V_{ref}$ |
| 111 | $V_{REF}\left(\frac{1}{2} + \frac{3A_2}{4}\right)$ | $\alpha \cdot V_{REF}\left(\frac{A_2}{4}\right) - \frac{1}{8}V_{ref}$ |

offset $= 0$

gain error factor $\alpha$

$$\alpha = \frac{\frac{7}{8}V_{ref}}{V_{ref}\left(\frac{1}{2} + \frac{3}{4}A_2\right)} = \frac{7}{2(2+3A_2)}$$

Set $|DNL| \le \frac{1}{16}V_{REF}$

$$\left|\frac{7}{2(2+3A_2)}\left(\frac{A_2}{4}\right) - \frac{1}{8}\right| \le \frac{1}{16} \Rightarrow \frac{2}{11} \le A_2 \le \frac{6}{5}$$

$$\left|\frac{7}{2(2+3A_2)}\left(\frac{1}{2} - \frac{3A_2}{4}\right) - \frac{1}{8}\right| \le \frac{1}{16} \Rightarrow \frac{22}{51} \le A_2 \le \frac{26}{45}$$

$$\Rightarrow \frac{22}{51} \le A_2 \le \frac{26}{45}$$

$$\boxed{0.4314 \le A_2 \le 0.5778}$$

c) $V_{out,actual} = V_{REF}\left(D_2 \cdot A_3 + D_1 \frac{A_3}{2} + D_0 \cdot \frac{A_3}{4}\right)$

| Input | $V_{out,actual}$ | DNL |
|---|---|---|
| 000 | 0 | 0 |
| 001 | $V_{REF}\left(\frac{A_3}{4}\right)$ | $\alpha V_{REF}\left(\frac{A_3}{4}\right) - \frac{1}{8}V_{ref}$ |
| 010 | $V_{REF}\left(\frac{A_3}{2}\right)$ | $\alpha \cdot V_{REF}\left(\frac{A_3}{4}\right) - \frac{1}{8}V_{ref}$ |
| 011 | $V_{REF}\left(\frac{3A_3}{4}\right)$ | $\alpha \cdot V_{REF}\left(\frac{A_3}{4}\right) - \frac{1}{8}V_{ref}$ |
| 100 | $V_{REF}(A_3)$ | $\alpha \cdot V_{REF}\left(\frac{3A_3}{4}\right) - \frac{1}{8}V_{ref}$ |
| 101 | $V_{REF}\left(\frac{5A_3}{4}\right)$ | $\alpha \cdot V_{REF}\left(\frac{1}{4}A_3\right) - \frac{1}{8}V_{ref}$ |
| 110 | $V_{REF}\left(\frac{6A_3}{4}\right)$ | $\alpha \cdot V_{REF}\left(\frac{A_3}{4}\right) - \frac{1}{8}V_{ref}$ |
| 111 | $V_{REF}\left(\frac{7}{4}A_3\right)$ | $\alpha \cdot V_{REF}\left(\frac{A_3}{4}\right) - \frac{1}{8}V_{ref}$ |

offset $= 0$

gain error factor $\alpha$

$$\alpha = \frac{\frac{7}{8}V_{ref}}{V_{ref}\left(\frac{7}{4}A_3\right)} = \frac{1}{2A_3}$$

Set $|DNL| \le \frac{1}{16}V_{REF}$

$$\left|\frac{1}{2A_3} \cdot \frac{A_3}{4} - \frac{1}{8}\right| = |0| \le \frac{1}{16}V_{ref}$$

In this case, $\boxed{\text{no restriction for } A_3}$ but there will be gain error

$V_{off} = 0.25 \qquad A_1 = A_2 = A_3 = A = \frac{1}{2}$

29.17 a) $V_{out1} = (D_0 \cdot V_{REF} + V_{off})\, A_1$

$V_{out2} = (D_1 \cdot V_{REF} + (D_0 \cdot V_{REF} + V_{off}) \cdot A_1)\, A_2$

$V_{out3} = (D_2 \cdot V_{REF} + (D_1 V_{REF} + (D_0 \cdot V_{REF} + V_{off}) A_1) A_2)\, A_3$

$= V_{REF}(D_2 \cdot A_3 + D_1 \cdot A_2 A_3 - D_0 \cdot A_1 A_2 A_3) + V_{off} \cdot A_1 A_2 A_3$

$= V_{REF} \cdot (D_2 A + D_1 A^2 + D_0 A^3) + 0.03125V = V_{i,ideal} + 0.5125V$

offset $= 0.03125\,V = 0.05$ LSBs

After removing offset. $\quad V_{out,\,no\ offset} = V_{i,\,ideal}$

$|INL|_{max} = 0$

$|DNL|_{max} = 0$

b) $V_{out} = V_{REF} \cdot (D_2 \cdot A + D_1 \cdot A^2 + D_0 A^3) + V_{off} \cdot A_2 \cdot A_3$

$= V_{i,ideal} + 0.0625V$

offset $= 0.0625\,V = 0.1$ LSBs

$|INL|_{max} = 0, \qquad |DNL|_{max} = 0$

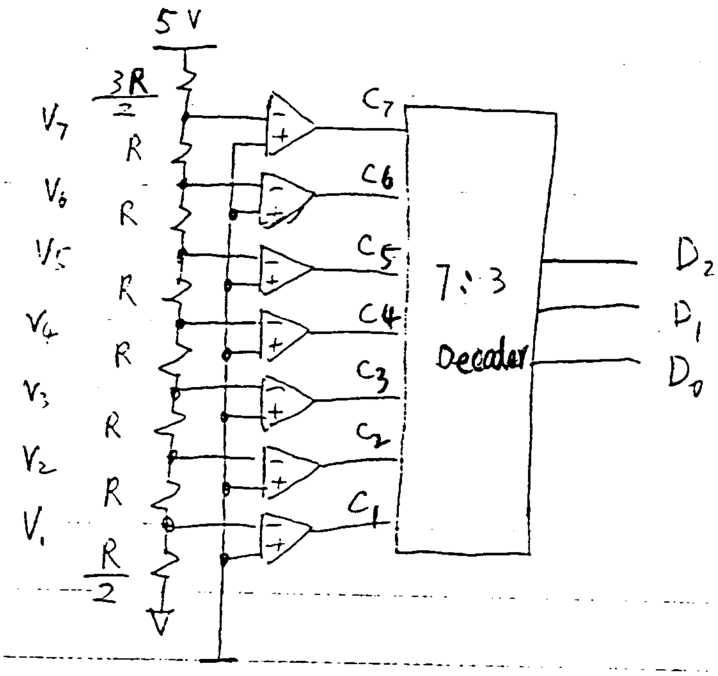
c) $V_{out} = V_{REF}(D_2 \cdot A + D_1 \cdot A^2 + D_0 A^3) + V_{off} \cdot A_3$

$= V_{i,ideal} + 0.125\,V$

offset $= 0.125V = 0.2$ LSBs

$|INL|_{max} = 0 \qquad |DNL|_{max} = 0$

29.18 3-bit flash ADC




| $V_{in}$ | $C_7C_6C_5C_4C_3C_2C_1$ | $D_2D_1D_0$ |
|---|---|---|
| $0 \le V_{in} < 0.3125$ | 0000000 | 000 |
| $0.3125 \le V_{in} < 0.9375$ | 0000001 | 001 |
| $0.9375 \le V_{in} < 1.5625$ | 0000011 | 010 |
| $1.5625 \le V_{in} < 2.1875$ | 0000111 | 011 |
| $2.1875 \le V_{in} < 2.8125$ | 0001111 | 100 |
| $2.8125 \le V_{in} < 3.4375$ | 0011111 | 101 |
| $3.4375 \le V_{in} < 4.0625$ | 0111111 | 110 |
| $4.0625 \le V_{in}$ | 1111111 | 111 |

$$|DNL|_{max} = \frac{5\text{V}}{8} \times 0.05 = 31.25\text{mV} = 0.05\text{LSB}$$

$$|INL|_{max} = \frac{5}{2} \times 0.05 = 125\text{mV} = 0.2\text{LSB}$$

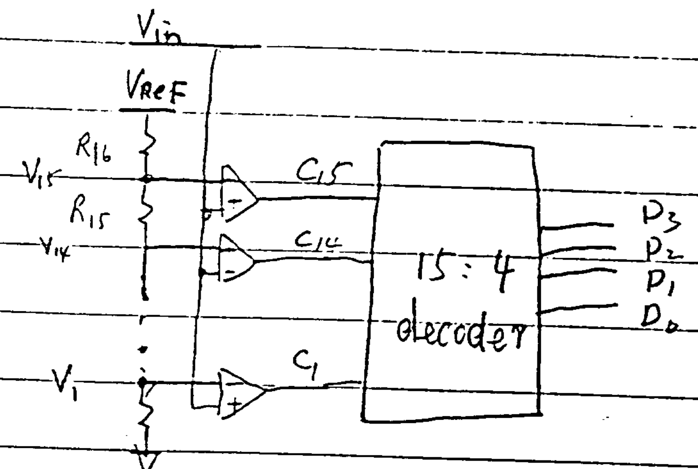
②

29.19 $\quad 1LSB = \frac{5}{8}V = 625\,mV$

$$|DNL|_{max} = \frac{V_{REF}}{2^N}\left|\frac{\Delta R}{R}\right|_{max} + 2\,|V_{os}|_{max} \le \frac{625}{2}mV$$

$$\therefore\ |V_{os}|_{max} \le \left(\frac{625}{2}mV - 31.25\,mV\right)/2 = \boxed{140.625\,mV}$$

29.20 $$\sum_{i=1}^{16}\left(\frac{\Delta R}{R}\right)_i = (2+1.5+0-1-0.5+1+1.5+3+2.5+1-0.5-1.5-2+0+1+1)\% = 8\% \ne 0$$




| n | R | $V_n$ (V) | $V_n$,ideal (V) | $V_{n,\text{no-of}}$ (V) | $V_n$, ~~nulled~~ (V) | INL (mV) | DNL (mV) |
|---|---|---|---|---|---|---|---|
| 1 | 1.02 | 0.317 | 0.3125 | 0.3125 | 0.31290 | 0.4 | 0.4 |
| 2 | 1.015 | 0.633 | 0.6250 | 0.6285 | 0.6293 | 4.3 | 3.9 |
| 3 | 1 | 0.944 | 0.9375 | 0.9395 | 0.9407 | 3.2 | 3.9 |
| 4 | 0.99 | 1.252 | 1.2500 | 1.2475 | 1.2491 | 0.9 | -1.1 |
| 5 | 0.995 | 1.561 | 1.5625 | 1.5565 | 1.5585 | -4.0 | -4.1 |
| 6 | 1.01 | 1.875 | 1.8750 | 1.8705 | 1.8729 | -2.1 | -3.1 |
| 7 | 1.015 | 2.191 | 2.1875 | 2.1865 | 2.1893 | -1.8 | -1.9 |
| 8 | 1.02 | 2.508 | 2.5000 | 2.5035 | 2.5067 | -6.7 | -2.5 |
| 9 | 1.025 | 2.826 | 2.8125 | 2.8215 | 2.8251 | 12.6 | 4.9 |
| 10 | 1.01 | 3.141 | 3.1250 | 3.1365 | 3.1405 | 15.5 | 4.9 |
| 11 | 0.995 | 3.450 | 3.4375 | 3.4455 | 3.4499 | 12.4 | 2.9 |
| 12 | 0.985 | 3.756 | 3.7500 | 3.7515 | 3.7563 | -6.3 | -3.1 |
| 13 | 0.98 | 4.061 | 4.0625 | 4.0565 | 4.0617 | -0.8 | -6.1 |
| 14 | 1 | 4.372 | 4.3725 | 4.3675 | 4.3731 | -0.6 | -7.1 |
| 15 | 1.01 | 4.686 | 4.6825 | 4.6815 | 4.6875 | 0 | -1.1 |
| 16 | | | | | | | 1.0 |

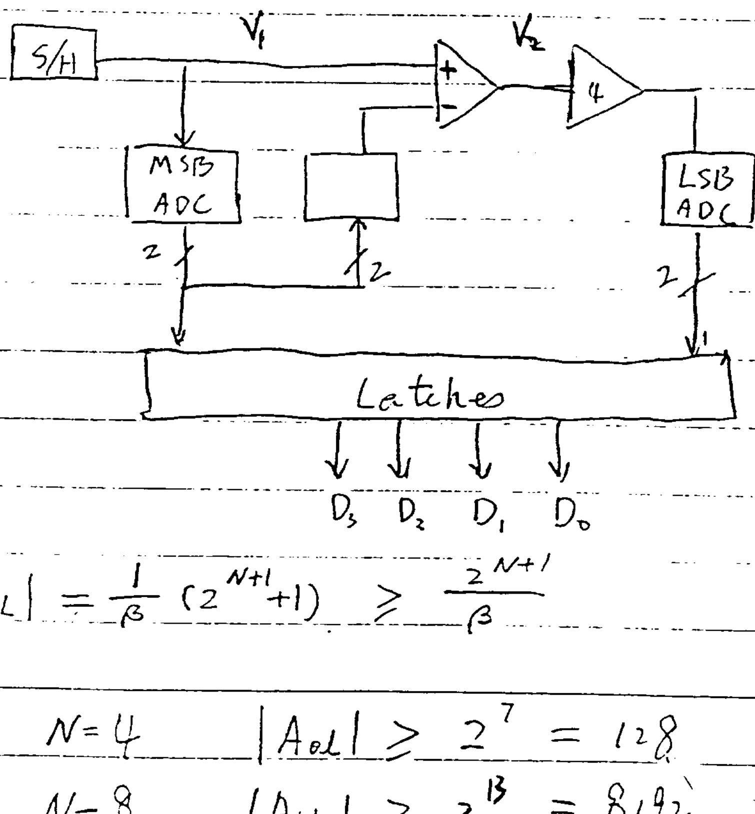
$$|INL|_{max} = 15.5\,mV \leq \frac{5}{2^{N+1}}$$

$$N = \log_2\left(\frac{5}{15.5\times10^{-3}}\right) - 1 = 7.333 \approx 7$$

This converter possesses 7-bit resolution

29.21

$$A_{CL} \cong \frac{1}{\beta} = 2^{N/2} \qquad |A_{OL}| \geq \frac{2^{N+1}}{\beta} = 2^{\frac{3N}{2}+1}$$




$$|A_{OL}| = \frac{1}{\beta}(2^{N+1}+1) \geq \frac{2^{N+1}}{\beta}$$

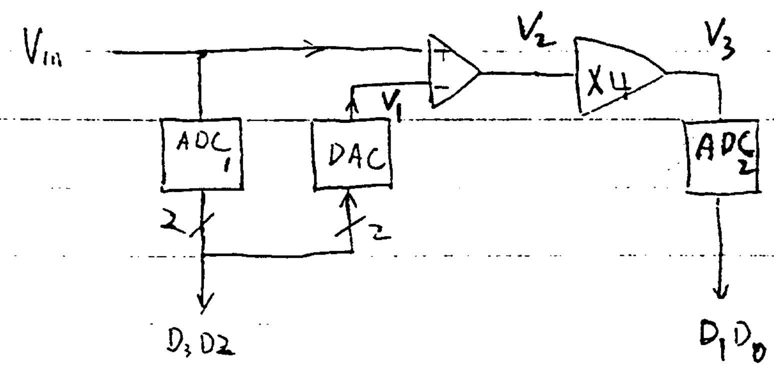
(a) For $N=4$ $\quad |A_{OL}| \geq 2^7 = 128$

(b) For $N=8$ $\quad |A_{OL}| \geq 2^{13} = 8192$

(c) For $N=10$ $\quad |A_{OL}| \geq 2^{16} = 65536$

29.22

4-bit, two step flash ADC




IF $ADC_2$ has an accuracy of $\frac{1}{2^{N_2}}$

$$\Delta V_3 = \frac{V_{REF}}{2^{N_2+1}} \qquad \Delta V_2 = \frac{\Delta V_3}{4} = \frac{V_{REF}}{2^{N_2+3}}$$

Assume ADC1 is ideal, then

$$\Delta V_{in} = \Delta V_2 = \frac{V_{REF}}{2^{N_2+3}} \leq \frac{V_{REF}}{2^{4+1}} \Rightarrow N_2 \geq 2$$

So $ADC_2$ has 2-bit accuracy at least.

If $ADC_1$ has an accuracy of $\frac{1}{2^{N_1}}$

$$\Delta V_{in} = \Delta V_1 = \frac{V_{REF}}{2^{N_1+1}} = \Delta V_2 \Rightarrow \Delta V_3 = \frac{4 V_{REF}}{2^{N_1+1}} \leq \frac{V_{REF}}{2^{2+1}}$$

$$\Rightarrow N_1 = 4.$$

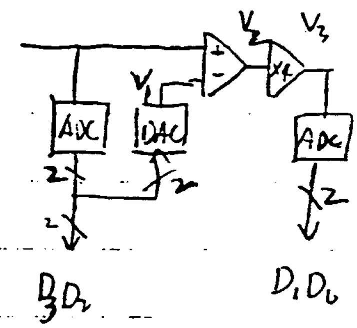
So $ADC_1$ has 4-bit accuracy at least.

∴ The first flash converter needs to be more accurate than the second one

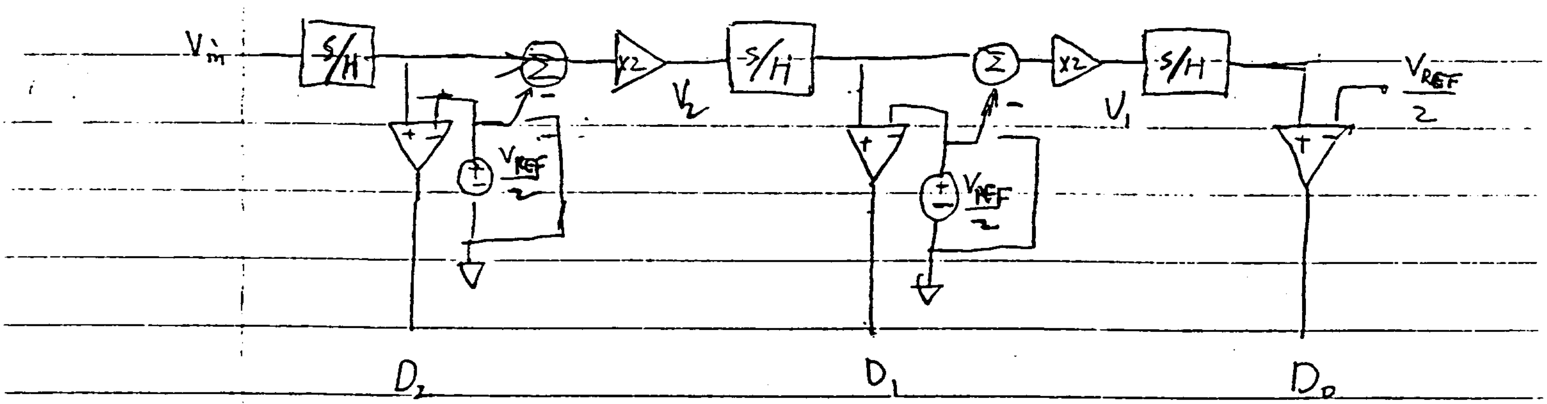
5

29.23. $V_{in} = 3, 5, 7.5, 14.75V$

| $V_{in}$ | $V_1$ | $V_2$ | $V_3$ | $D_3D_2D_1D_0$ |
|---|---|---|---|---|
| 3 | 0 | 3 | 12 | 0011 |
| 5 | 4 | 1 | 4 | 0101 |
| 7.5 | 4 | 3.5 | 14 | 0111 |
| 14.75 | 12 | 2.75 | 11 | 1110 |




29.24 $V_{REF} = 8V$




| $V_{in}$ | $V_2$ | $V_1$ | $D_2$ | $D_1$ | $D_0$ |
|---|---|---|---|---|---|
| 1 | 2 | 4 | 0 | 0 | 1 |
| 4 | 0 | 0 | 1 | 0 | 0 |
| 6 | 4 | 0 | 1 | 1 | 0 |
| 7 | 6 | 4 | 1 | 1 | 1 |

29.25 For 8 bits, it is best to write a program that generates the transfer curve, for increasing values of $V_{IN}$. INL & DNL can be determined from the resulting data.

29.26 Use eqs. 29.61, 29.62 & 29.63 to calculate the values of $V_{IN}$ that cause the digital output to change.

For a 3 bit pipeline, there are only two residue amplifiers.

when the 1st residue amp. has a gain of 2.2 the new switch points occur at:

$V_{IN,1} = 2.5V$

$$V_{p22} = 2.5V = \left[V_{IN} - \frac{1}{2} D_{N-1} \cdot V_{REF}\right] \underbrace{2.2}_{\text{gain of 1st residue amp.}}$$

$$\text{or } V_{IN} = \frac{2.5}{2.2} + \frac{1}{2} D_{N-1} \cdot V_{REF} \quad \text{where } D_{N-1} = \underbrace{D_2}_{MSB} \text{ \& can be either 0 or 1.}$$

$\therefore D_1$ switches when

$V_{IN} = 1.136\text{ V}, 3.6364$

$$\text{since } V_{p3} = 2.5 = \left[\left[V_{IN} - \frac{1}{2} \cdot D_2 \cdot V_{REF}\right] \underbrace{A_2}_{\text{gain of 1st residue amp}} - \left[\frac{1}{2} \cdot D_1 \cdot V_{REF}\right]\right] \underbrace{A_1}_{\text{gain of the 2nd residue amp.}}$$

$\therefore D_0$ switches when

$$V_{IN} = \frac{\frac{2.5}{2} + 2.5 D_1}{2.2} + 2.5 D_2 \quad \text{for } D_2 D_1 \text{ 4 combinations}$$

or $V_{IN} = 0.5682, 3.068, 1.7045, 4.2045$

29.26 con't.

∴ when $A_2$ (the 1st residue amp) has a gain of 2.2, $V_{IN}$ switches at:

| $A_2=2.2$ | | Ideal | DNL (V) | INL (V) |
|---|---|---|---|---|
| 0 | 000 | 0 | | |
| 0.5682 | 001 | 0.625 | -0.0568 | -0.0568 |
| 1.136 | 010 | 1.25 | -0.0572 | -0.114 |
| 1.7045 | 011 | 1.875 | -0.0565 | -0.1705 |
| 2.5 | 100 | 2.5 | 0 | 0 |
| 3.068 | 101 | 3.125 | -0.057 | -0.057 |
| 3.6364 | 110 | 3.75 | -0.0566 | -0.1136 |
| 4.2045 | 111 | 4.375 | -0.0569 | -0.1705 |

INL + DNL can be calculated by comparing the new switching pts to the ideal case, as see above.

Now perform same analysis using $A_1 = 2.2$, while $A_2 = 2.0$. This yields,

$$V_{IN} = \frac{\frac{2.5}{2.2} + 2.5 D_1}{2.0} + 2.5 D_2 \quad \text{or}$$

$$V_{IN} = 0.5682,\ 1.818,\ 3.0682,\ 4.3182$$

Note that since the 1st residue is now ideal (gain of 2), all the other switching points are ideal (eq. 29.63, 29.61)

$A_1=2.2$ yields the following switch points,

| $V_{IN}$ | DNL | INL |
|---|---|---|
| 0 | – | 0 |
| 0.5682 | -0.0568 | -0.0568 |
| 1.25 | 0.0568 | 0 |
| 1.818 | -0.057 | -0.057 |
| 2.5 | 0.057 | 0 |
| 3.0682 | -0.0568 | -0.0568 |
| 3.75 | 0.0568 | 0 |
| 4.3182 | -0.0568 | -0.0568 |

Compare these values to the previous case shows both DNL + INL are improved!

29.27

$t_{c(max)} \Rightarrow$ occurs when $V_{IN} = V_{FS}$

$\therefore$ eqs. (29.8) and (29.79)

$$t_c = \frac{V_{FS}}{V_{REF}} \cdot 2^N \cdot T_{CLK}$$

$$= \frac{2^N - 1}{\cancel{2^N}} \cdot \frac{\cancel{V_{REF}}}{\cancel{V_{REF}}} \cdot \cancel{2^N} \cdot T_{CLK}$$

$$= (255)(1\mu s) = 255\mu s.$$

eq. (29.80) $F_{max} = \frac{f_{sample}}{2} = \frac{V_{REF}}{V_{FS} \cdot 2^{N+1}} \cdot 1MHz = \frac{1MHz}{510} = \underline{\underline{1.96\ kHz}}$ !

$$V_{MAX} = V_{FS} = \frac{255}{256} \cdot V_{REF} = \underline{\underline{4.98\,V.}}$$

29.28

eq. (29.81) $\Rightarrow V_C = \frac{V_{REF} \cdot t_c}{RC}$

since the conversion time, $t_c$, is dependent on the value of $v_{IN}$, for the worst case of $V_{IN} = V_{FS} = V_C$

$$V_{C(ideal)} = \frac{V_{REF} \cdot (2^N - 1) T_{CLK}}{RC} = \frac{2^N - 1}{2^N} V_{REF}$$

$\therefore$ The RC time constant must be:

$$RC = 2^N T_{CLK} = \underline{\underline{256\mu s}}$$

Tolerance on clock (Assume $V_{IN} = 2.5V$ worst-case)

$V_{c,ideal} = 2.5V$

for ½ LSB INL, $V_{c,actual} - 2.5 = \frac{5}{2^9} = \pm 9.765\,mV$

or $V_{c,actual} = 2.5 \pm 9.765\,mV$ & $f_{CLK} = \frac{2^N \cdot 2.5}{V_{c,actual} \cdot RC}$ (29.82)

+ $996.11\,kHz \le f_{CLK} \le 1.0389\,MHz$

29.29 Max Conversion time:

1st integration takes $2^N$ clock cycles

2nd integration takes $2^N - 1$ maximum clock cycles. (when $V_{IN} = V_{FS}$)

∴ Max conversion time is,

$$t_{c(max)} = (2^N + 2^N - 1)\,1\mu s \text{ or } 511\ \mu s$$

Minimum conversion time still requires the

1st integration of $2^N$ clock cycles

If $V_{IN} = 0$, then $t_{c(min)} = 256\,\mu s$

For $V_{IN} = 2.5$ V,

$$t_c = \underbrace{2^N}_{\text{1st int.}} + \underbrace{\overbrace{\tfrac{1}{2}}^{\frac{V_{IN}}{V_{REF}}}(2^N)}_{\text{2nd}} = 256 + 128 = \underline{\underline{384\,\mu s}}$$

29.30 Advantages: Dual slope removes nonidealities of R, C, + $F_{CLK}$ so is the more accurate of two converters

Disadvantage: The first integration can be considered as a calibration phase which requires a hefty $2^N$ clock cycle overhead for each conversion.

29.31

$V_{REF} = 5\,V$, $V_{IN} = 1\,V$

| Step | $B_3 B_2 B_1 B_0$ | $D_3' D_2' D_1' D_0'$ | $V_{OUT}$ | Comp-Out | $D_3 D_2 D_1 D_0$ |
|---|---|---|---|---|---|
| 1 | 1000 | 1000 | $\frac{1}{2} V_{REF} = 2.5V$ | 1 | 0000 |
| 2 | 0100 | 0100 | $\frac{1}{4} V_{REF} = 1.25V$ | 1 | 0000 |
| 3 | 0010 | 0010 | $\frac{1}{8} V_{REF} = 0.625V$ | 0 | 0010 |
| 4 | 0001 | 0011 | $\frac{3}{16} V_{REF} = 0.9375V$ | 0 | 0011 |

$V_{IN} = 3\,V$

| Step | $B_3 B_2 B_1 B_0$ | $D_3' D_2' D_1' D_0'$ | $V_{OUT}$ | Comp-Out | $D_3 D_2 D_1 D_0$ |
|---|---|---|---|---|---|
| 1 | 1000 | 1000 | 2.5 V | 0 | 1000 |
| 2 | 0100 | 1100 | 3.75 V | 1 | 1000 |
| 3 | 0010 | 1010 | 3.125 V | 1 | 1000 |
| 4 | 0001 | 1001 | 2.8125 V | 0 | 1001 |

$V_{IN} = V_{FS} = 4.6875$

| Step | $B_3 B_2 B_1 B_0$ | $D_3' D_2' D_1' D_0'$ | $V_{OUT}$ | Comp-Out | $D_3 D_2 D_1 D_0$ |
|---|---|---|---|---|---|
| 1 | 1000 | 1000 | 2.5 V | 0 | 1000 |
| 2 | 0100 | 1100 | 3.75 V | 0 | 1100 |
| 3 | 0010 | 1110 | 4.375 V | 0 | 1110 |
| 4 | 0011 | 1111 | 4.6875 | 0 | 1111 |

29.32 Refer to Fig. 29.36 This problem refers to the 4bit ADC used in P29.31

$V_{IN} = 2.49$

Because of offset, Comp-out = 0 (wrong decision)

| Step | $D_3' D_2' D_1' D_0'$ | $V_{OUT}$ | Comp-OUT | $D_3 D_2 D_1 D_0$ |
|---|---|---|---|---|
| | | | | 1 0 0 0 |
| 1 | 1 0 0 0 | 2.5 V | 0 | 1 0 0 0 |
| 2 | 1 1 0 0 | 3.75 V | 1 | 1 0 0 0 |
| 3 | 1 0 1 0 | 3.125 | 1 | 1 0 0 0 |
| 4 | 1 0 0 1 | 2.8125 | 1 | |

The answer should be 0111

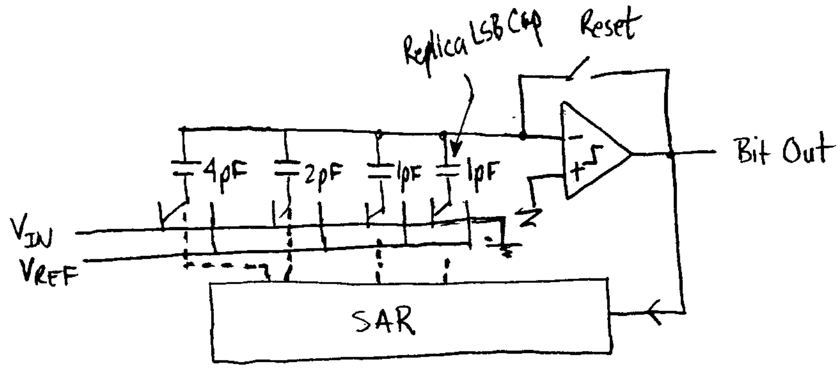
$V_{IN} = 0.3025$

| Step | $D_3' D_2' D_1' D_0'$ | $V_{OUT}$ | Comp-OUT | $D_3 D_2 D_1 D_0$ |
|---|---|---|---|---|
| 1 | 1 0 0 0 | 2.5 | 1 | 0000 |
| 2 | 0 1 0 0 | 1.25 | 1 | 0000 |
| 3 | 0 0 1 0 | 0.625 | 1 | 0000 |
| 4 | 0 0 0 1 | 0.3125 | 0 | 0001 |

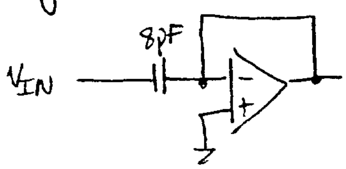
The answer should be 0000 since

$V_{IN} < 1\,LSB$!

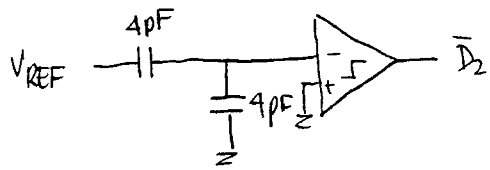
29.33



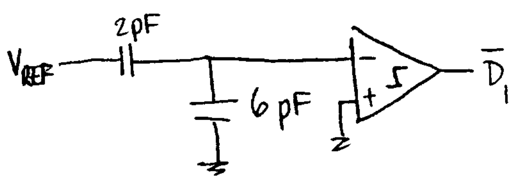

Sampling



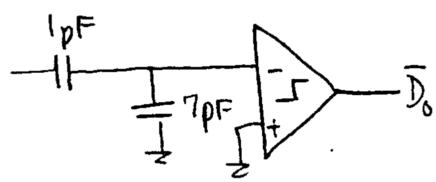

MSB ($D_2$ Comparison)




$D_1$ Comparison ( $D_2 = 0$ )




$D_0$ Comparison ($D_2 + D_1 = 0$)




Note that after sampling, the bottom plate of the replica LSB capacitor always stays on ground.

For $V_{IN} = 2$ V,

| Step | $D_2 D_1 D_0$ | $V_{TOP}$ |
|---|---|---|
| 1 | 0 0 0 | $-2\,V$ |
| 2 | 0 1 0 | $-2\,V + \frac{V_{REF}}{4} = -0.75\,V$ |
| 3 | 0 1 1 | $-2V + \frac{V_{REF}}{4} + \frac{V_{REF}}{8} = -0.125\,V$ |

See eq. (29.89)

29.33 (Con't)

For $V_{IN} = 3\,V$

| Step | $D_2\ D_1\ D_0$ | $V_{TOP}$ |
|---|---|---|
| 1 | 1 0 0 | $-3 + \frac{V_{REF}}{2} = -0.5$ |
| 2 | 1 0 0 | $-0.5\,V$ |
| 3 | 1 0 0 | $-0.5\,V$ |

For $V_{IN} = 4\,V$

| Step | $D_2 D_1 D_0$ | $V_{TOP}$ |
|---|---|---|
| 1 | 1 0 0 | $-3 + \frac{V_{REF}}{2} = -0.5$ |
| 2 | 1 1 0 | $-3 + \frac{V_{REF}}{2} + \frac{V_{REF}}{4} = -0.25\,V$ |
| 3 | 1 1 0 | $-0.25\,V$ |

Note that once $V_{TOP}$ is within $\pm 1$ LSB, that the converted digital output doesn't change.

29.34

eq. (29.91) $|INL|_{max} = 4 \cdot (0.01) = \underline{\underline{0.04}}$

eq. (29.93) $DNL_{max} = (7)(0.01) = \underline{\underline{0.07}}$

Note that the INL & DNL are independent of the absolute size of the capacitor.

29.35

See problem 29.33

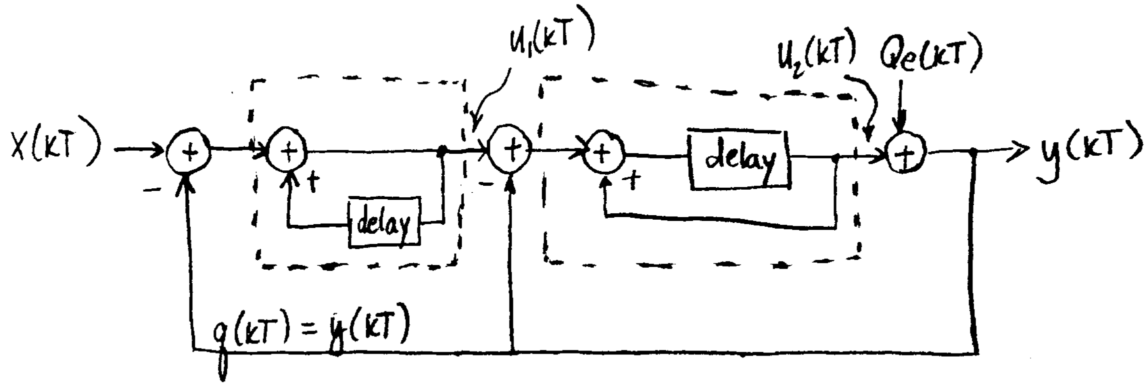
If $V_{OS} = 0.3$ Then, for $v_{IN} = 2.0V$

| Step | $D_2\ D_1\ D_0$ | $V_{TOP}$ |
|---|---|---|
| 1 | 0 0 0 | $-2 + 0.3 = -1.7V$ |
| 2 | 0 1 0 | $-2 + \frac{V_{REF}}{4} + 0.3 = -0.45$ |
| 3 | 0 1 1 | $-2 + \frac{V_{REF}}{4} + \frac{V_{REF}}{8} + 0.3 = 0.175$ |

Even though $V_{TOP}$ is now positive, because of the comparator's offset, the output stays the same. The comparator doesn't switch states until the plus input is 0.3V above ground.

29.36 See sec. 29.2.6

Problem 29.38




1.) $u_2(kT) = u_1(kT-T) - y(kT-T) + u_2(kT-T)$

2.) $u_1(kT) = x(kT) - y(kT) + u_1(kT-T)$

3) = 2) → 1.) $u_2(kT) = x(kT-T) - y(kT-T) + u_1(kT-2T) \overbrace{- y(kT-T) + u_2(kT-T)}^{-Q_e(kT-T)}$

4.) $y(kT) = Q_e(kT) + u_2(kT)$

plugging 3.) → 4.) $= x(kT-T) - y(kT-T) + u_1(kT-2T) + Q_e(kT) - Q_e(kT-T)$

from 1), we know

$$u_1(kT-2T) = u_2(kT-T) + y(kT-2T) - u_2(kT-2T)$$

∴ 4.) becomes,

$$y(kT) = x(kT-T) \overbrace{- y(kT-T) + u_2(kT-T)}^{-Q_e(kT-T)} + \overbrace{y(kT-2T) - u_2(kT-2T)}^{Q_e(kT-2T)}$$
$$+ Q_e(kT) - Q_e(kT-T)$$

$$\therefore\ y(kT) = x(kT-T) + Q_e(kT) - 2Q_e(kT-T) + Q_e(kT-2T)$$