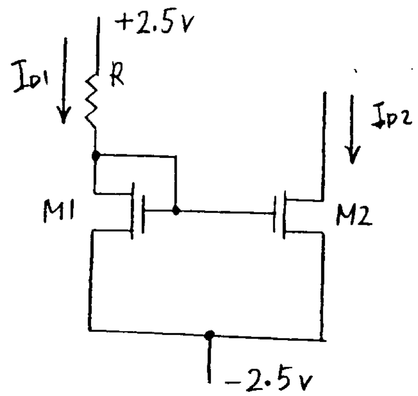
Chapter 20

Problem 20.1



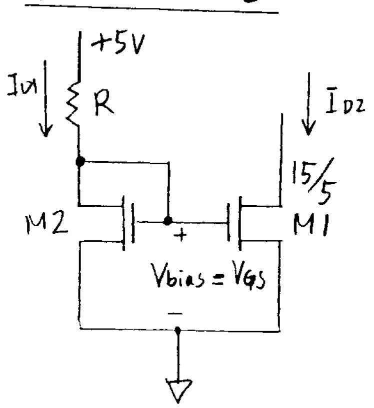
$$R = \frac{2.5 - V_{GS} - (-2.5)}{I_{D1}} = \frac{2.5 - 1 - (-2.5)}{1\,\mu A} = 4\,M\Omega$$

$$I_{D1} = I_{D2} = \frac{K_P}{2}\frac{W}{L}(V_{GS} - V_{THN})^2$$

$$\Rightarrow 1\,\mu A = \frac{50\,\mu A/V^2}{2} \times \frac{W}{L} \times (1V - 0.83V)^2$$

$$\Rightarrow \frac{W}{L} \simeq 1.38 \Rightarrow W_1 = W_2 = 7\,\mu m,\ L_1 = L_2 = 5\,\mu m$$

Problem 20.2



$$I_{D2} = 5\,\mu A = \frac{50\,\mu A/V^2}{2} \times \frac{15}{5} \times (V_{GS2} - V_{THN})^2$$

$$\Rightarrow V_{GS2} = V_{GS1} = V_{bias} \simeq 1.09\,(V)$$

We choose the size of M2 to be the same of that of M1,

$W_2 = 15\,\mu m,\ L_2 = 5\,\mu m$

$$I_{D1} = I_{D2} = 5\,\mu A = \frac{5 - 1.09}{R} \Rightarrow R = \underline{782K}$$

Problem 20.3

ANS: Let the drain current through M1 is IDS1, obviously,

VSG1 + IDS1*380k =5V

and IDS1 = (KPp/2)*(W1/L1)*(VSG1-Vthp)^2 where Vthp =0.916V

Solving the two eqations, ==> VSG1= 1.516V, ID1=9.169uA

Since VSG3 = VDD - ID2*380k = VDD - ID1*380 = VSG1, ==> ID3 = 3*ID1 = 27.507 uA

Problem 20.4

If the size of M2 increased to 30/5, $I_{D2} = I_{D3} = 20\mu A$.

V_{GS2} does not change, but V_{SG3} changes to about 1.32V.

Problem 20.5

ANS: The small signal output resistor of the right NMOS in figure P20.5 r0 is:

ro = 1/(λID) = 1/(0.06*5uA) = 10/3 megohms
it = vt/ro = 0.3 vt (uA) for vt's unit is volt

Problem 20.6

ANS: The schematic is shown below,

~~VSG1,3~~ VGS1,3 = Vthn + SQRT(2*ID/βn) = 0.83 + SQRT[2*1uA/(50*15/5)] = 0.95 V

Ignoring the body effect of M2 and M4, we have

VGS2,4 = 0.95 V

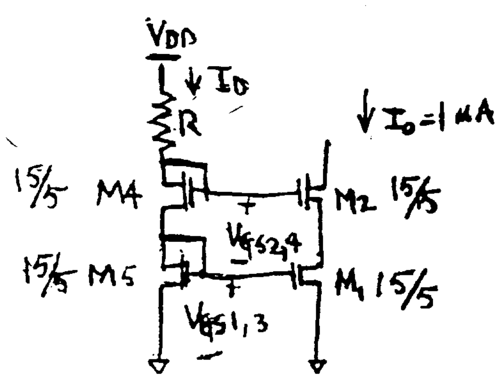
==> R=(VDD-VGS4-VGS3)/ID = (5-0.95-0.95)/1u = 3.1megohms

The small-signal resistance of M1 is ro = 1/(λID) = 1/(0.06*1uA) = 16.67 meg

The small-signal resistance of M2 is about = gm1*ro^2 = SQRT(2*β1*ID1)*ro2 = 17.32uA/V * (16.67)2 = =4.81 Gohms.

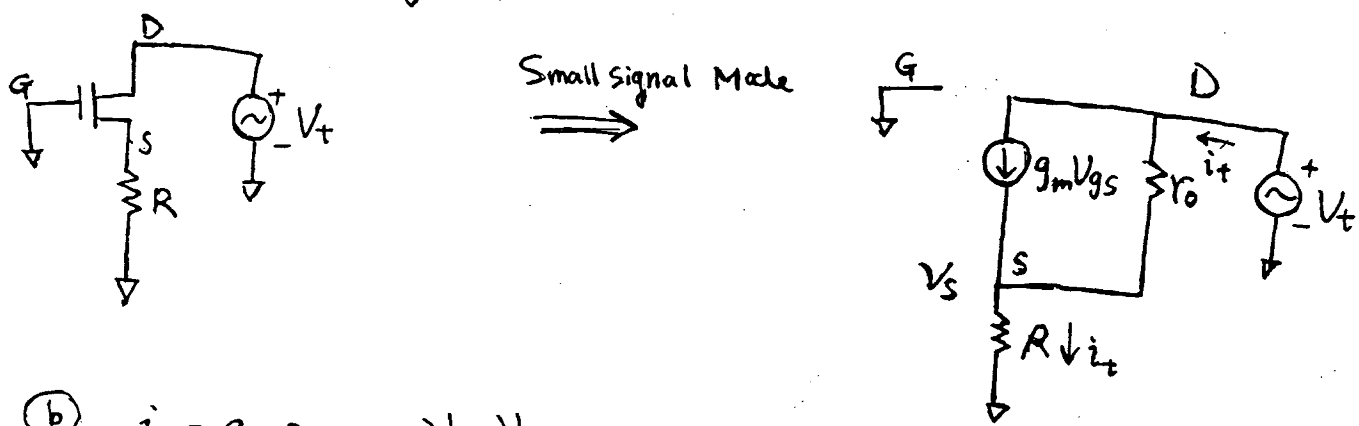
ΔV = VGS - Vthn = 0.95 - 0.83 = 0.12V

The minimum voltage across M1 and M2 = 2ΔV + Vthn = 2*0.12 + 0.83 = 1.07 V.



Problem 20.7

ANS. (a) add Test voltage V_T



(b) $$i_t = g_m v_{gs} + \frac{v_t - v_s}{r_o} \quad (1)$$

$$v_{gs} = 0 - v_s = -v_s = -(i_t R) = -i_t R \quad (2)$$

plugging $v_{gs} = -i_t R$ and $v_s = i_t R$ into (1)

$$\Rightarrow \quad i_t = g_m(-i_t R) + \frac{v_t - i_t R}{r_o}$$

$$\Rightarrow \quad i_t r_o + i_t R + i_t g_m R r_o = v_t$$

and $$R_o = \frac{v_t}{i_t} = \frac{i_t(r_o + R + g_m R r_o)}{i_t} = r_o + g_m R r_o + R$$

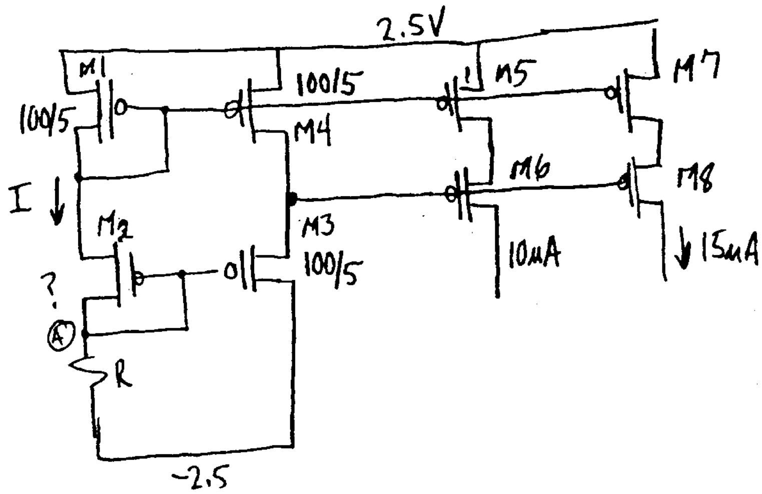
$$\Rightarrow \quad R_o = r_o(1 + g_m R) + R$$

Problem 20.9

ANS: $$S_{V_{SS}}^{I_o} = \frac{V_{SS}}{I_o}\frac{\partial I_o}{\partial V_{SS}} = \frac{V_{SS}}{I_o}\left(-\frac{1}{R}\right) = \frac{-2.5V}{10\mu A}\left(-\frac{1}{380k}\right) = 0.658$$

$$\frac{\Delta I_o}{I_o} = S_{V_{SS}}^{I_o}\frac{\Delta V_{SS}}{V_{SS}} = 0.658\,\frac{-2.6-(-2.4)}{-2.5} = 5.3\%$$

20.8.



Design this circuit.

This is the PMOS version of Figure 20.6.

In this case, $V_{SG} = \Delta V + |V_{THP}| \Rightarrow \Delta V = 1.2 - 0.91 = 0.29$

∴ The voltage at (A) is,

$V_{(A)} = 2.5 - (3\Delta V + 2|V_{THP}|)$ & the size of $M_3 = \frac{1}{4} M1$

$= 2.5 - (0.87 + 1.8) = \underline{-0.17\ V.}$

$I = \frac{17\mu A/V^2}{2}\left(\frac{100}{5}\right)(1.2 - 0.91)^2 = \underline{14.3\ \mu A}$

$\therefore\ R = \frac{-0.17 + 2.5}{14.3\mu A} = \underline{163\ k\Omega}$

$M_3 = \frac{1}{4} M1 = 25/5$

M5: $\frac{100/5}{(W/L)_5} = \frac{14.3\mu A}{10\mu A} \Rightarrow W_5 = \underline{70\ \mu m} = W_6$

M7: $\frac{100/5}{(W/L)_7} = \frac{14.3\mu A}{15\mu A} \Rightarrow W_7 = \underline{105\ \mu m} = W_8$

Estimate minimum voltages:

$V_{G6} = 2.5 - (2\Delta V + |V_{THP}|) = 1.01\ V$ ∴ $V_{D6(max)} = 1.01 + 0.91 = 1.92\ V$

∴ Minimum voltage across both sources is $2.5 - 1.92 = \underline{0.58 V.}$

20.8 (con't)

$$R_{out6} = g_{m6} r_o^2 = \sqrt{2(17\mu A/V^2)(70/5)(10\mu A)}\,(1.67M)^2 = 192.4\,M\Omega$$

$$R_{out8} \approx g_{m8} r_o^2 = \sqrt{2(17\mu A/V^2)(105/5)(15\mu A)}\,(1.1M)^2 = 125\,M\Omega$$

SPICE Analysis: see plots following.

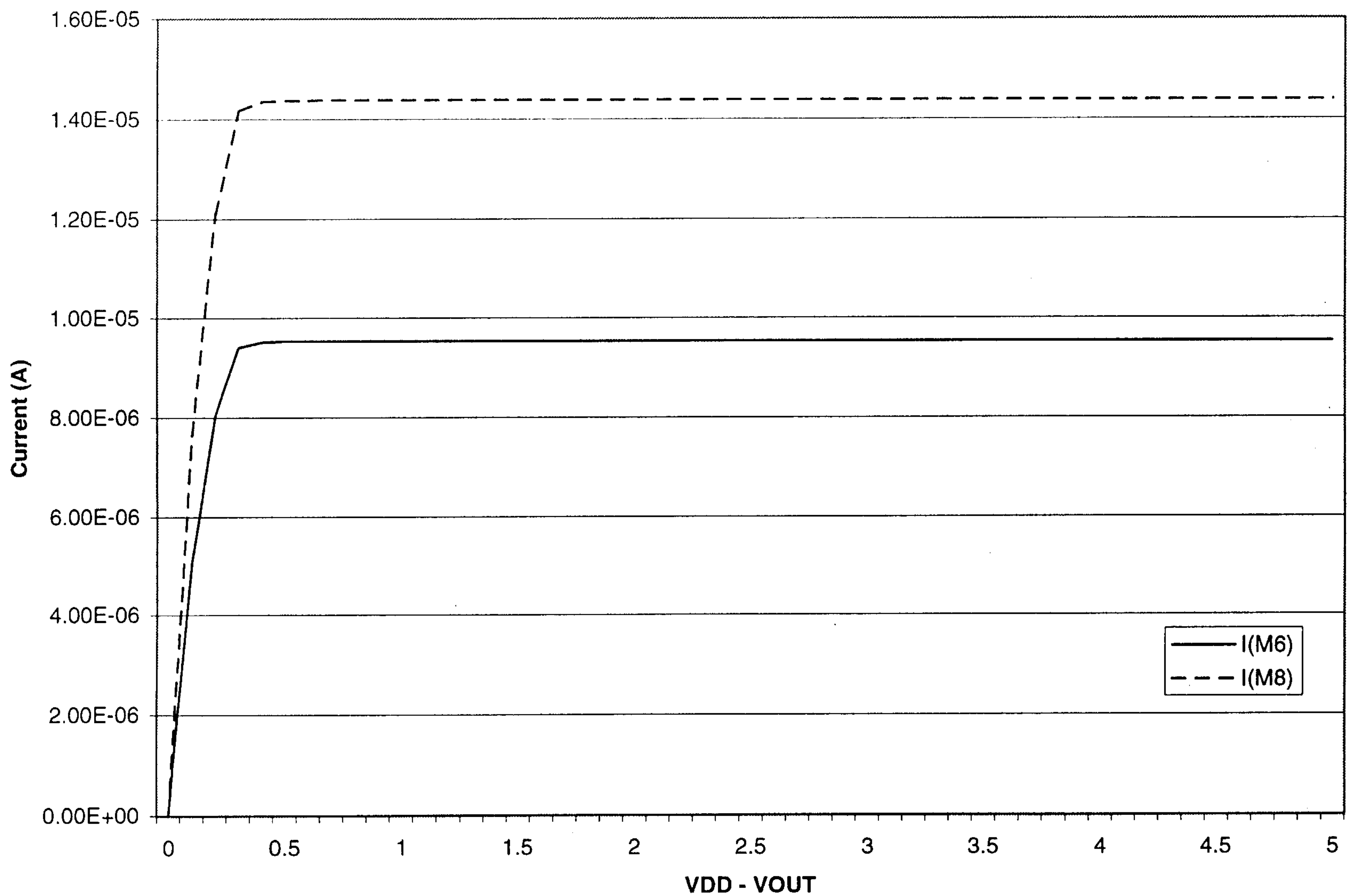
R_{out6}(HSPICE) = 770 MΩ

R_{out8}(HSPICE) = 502 MΩ

```
*problem 20.8 NOTE:  Bulk effects are ignored!

m1 g1 g1 vdd vdd cmospb w=100u l=5u
m2 g2 g2 g1 g1 cmospb w=25u l=5u
m3 vss g2 g6 g6 cmospb w=100u l=5u
m4 g6 g1 vdd vdd cmospb w=100u l=5u
m5 d5 g1 vdd vdd cmospb w=70u l=5u
m6 out1 g6 d5 d5 cmospb w=70u l=5u
m7 d7 g1 vdd vdd cmospb w=105u l=5u
m8 out2 g6 d7 d7 cmospb w=105u l=5u

r1 g2 vss 163k

vdd vdd 0 dc 2.5
vss vss 0 dc -2.5
vout1 out1 0 dc 0
vout2 out2 0 dc 0

.lib  ./../spice.models cmosnb
.lib  ./../spice.models cmospb

.op
.end
```

```
 ******  operating point information      tnom=  25.000 temp=  25.000
   node    =voltage       node     =voltage       node     =voltage

+0:d5        =   2.1756  0:d7       =   2.1755  0:g1       =   1.3037
+0:g2        =-201.3197m 0:g6       = 987.8969m 0:out1     =   0.
+0:out2      =   0.      0:vdd      =   2.5000  0:vss      =  -2.5000


**** mosfets

subckt
element  0:m1        0:m2        0:m3        0:m4        0:m5        0:m6
model    0:cmospb    0:cmospb    0:cmospb    0:cmospb    0:cmospb    0:cmospb
 id       -14.1023u  -14.1023u  -14.2386u  -14.2386u   -9.5349u   -9.5349u
 vgs       -1.1963    -1.5051    -1.1892    -1.1963    -1.1963    -1.1877
 vds       -1.1963    -1.5051    -3.4879    -1.5121  -324.4447m   -2.1756

subckt
element  0:m7        0:m8
model    0:cmospb    0:cmospb
 id       -14.3796u  -14.3796u
 vgs       -1.1963    -1.1876
 vds     -324.5369m   -2.1755
```

P20-8 Minimum Voltage Across Sources



Problem 20.9 (Simulation)

THe PSICE netlist is shown below

```
*** Top Level Netlist ***
M1        1 1 2 2 CMOSNB  L=5u W=15u
M2        3 1 2 2 CMOSNB  L=5u W=15u
R         1 6 380K TC1=0.002
V_AMM1    4 6 0V
VDD       4 0    DC 2.5 AC 0 0
VOUT 3 0 DC -1.3 AC 0 0
VSS       2 0    DC -2.5V AC 0 0

***** Spice models and macro models *****

.MODEL CMOSNB NMOS LEVEL=4
**BSIM MODEL PARAMETERS **

***** End of spice models and macro models *****

.DC VSS -2.4V -2.6V -0.01V
.SENS I(VOUT)
.PROBE
.end
```

The output sensitivity results are

DC SENSITIVITIES OF OUTPUT I(VOUT)

ELEMENT NAME	ELEMENT VALUE	ELEMENT SENSITIVITY (AMPS/UNIT)	NORMALIZED SENSITIVITY (AMPS/PERCENT)
R	3.800E+05	2.420E-11	9.197E-08
V_AMM1	0.000E+00	2.443E-06	0.000E+00
VDD	2.500E+00	-2.443E-06	-6.108E-08
VOUT	-1.300E+00	-8.494E-07	1.104E-08
VSS	-2.500E+00	3.293E-06	-8.231E-08

Problem 20.10

$$TC(I_o) = \frac{1}{I_o} \cdot \frac{\partial I_o}{\partial T} \qquad (1)$$

$$I_o = \frac{W_2 L_1}{W_1 L_2} \times \frac{V_{DD} - V_{GS} - V_{SS}}{R} \qquad (2)$$

Plugging (2) to (1), We get

$$TC(I_o) = \frac{1}{I_o} \times \frac{W_2 L_1}{W_1 L_2} \times \frac{-\frac{\partial V_{GS}}{\partial T} \times R - (V_{DD} - V_{GS} - V_{SS}) \frac{\partial R}{\partial T}}{R^2}$$

$$= -\frac{1}{I_o}\left[\frac{W_2 L_1}{W_1 L_2} \times \frac{1}{R} \times \frac{\partial V_{GS}}{\partial T} + \frac{I_o}{R} \frac{\partial R}{\partial T}\right]$$

Problem 20.11

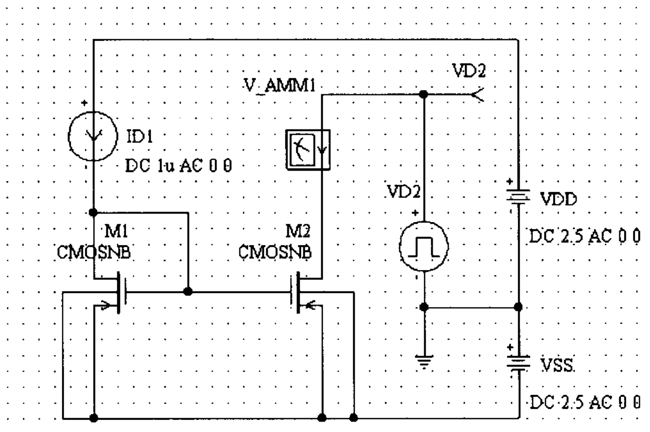
The temperature dependanceof the transconduction parameter KP(T) was expressed as below from Chp.9.

$KP(T) = KP(T0)*(T/T0)^{-1.5}$

$$\frac{\partial KP(T)}{\partial T} = KP(T_0) \cdot (-1.5) \cdot (\frac{T}{T_0})^{-2.5} \cdot \frac{1}{T_0}$$

$$\frac{1}{KP(T)} \frac{\partial KP(T)}{\partial T} = \frac{1}{KP(T_0) \cdot (T/T_0)^{-1.5}} KP(T_0) \cdot (-1.5) \cdot (\frac{T}{T_0})^{-2.5} \cdot \frac{1}{T_0} = \frac{-1.5}{T}$$

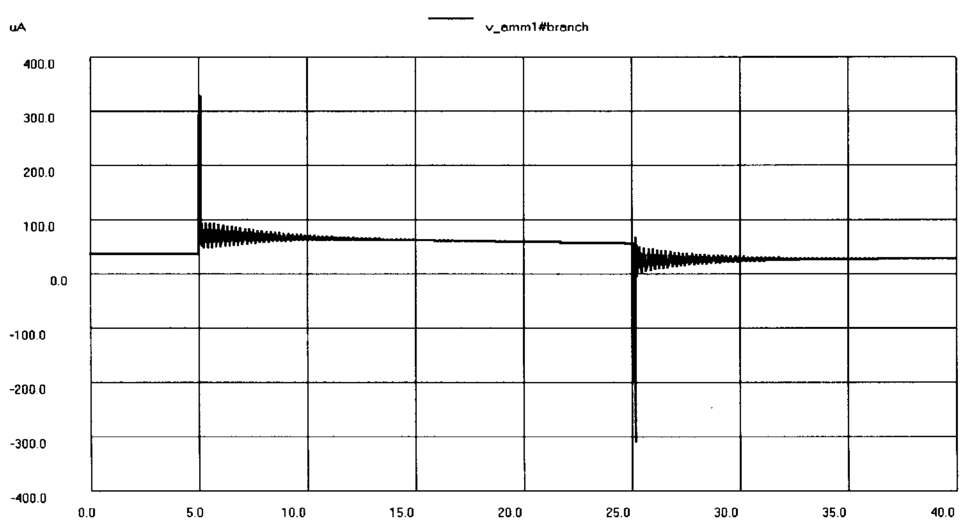
Problem 20.12



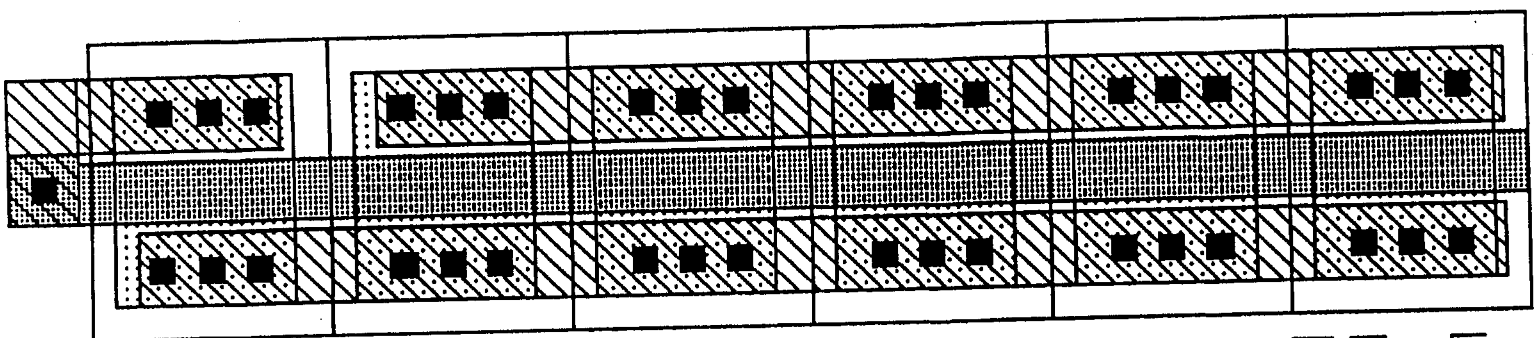
***** Top Level Netlist *****

```
ID1      5 1      DC 1u AC 0 0
M1       1 1 2 2 CMOSNB  L=5u W=5u
M2       3 1 2 2 CMOSNB  L=5u W=100u
V_AMM1          VD2 3 0V
VD2      VD2 0  DC 0 AC 0 0 PULSE(0 1 5n 0.1n 0.1n 20n 40n)
VDD      5 0      DC 2.5 AC 0 0
VSS      0 2      DC 2.5 AC 0 0

***** Spice models and macro models *****

.MODEL CMOSNB NMOS LEVEL=4

***** End of spice models and macro models *****

.OPTION ABSTOL=1u RELTOL=0.01 VNTOL=5mv
.tran 0.1n 50n 0 0.1n
.end
```

The SPICE simulation is shown below:



Prb 20.13



M1 15/5 M2 75/5

P20.13

ANS: See the Hardcopy of the Layout.

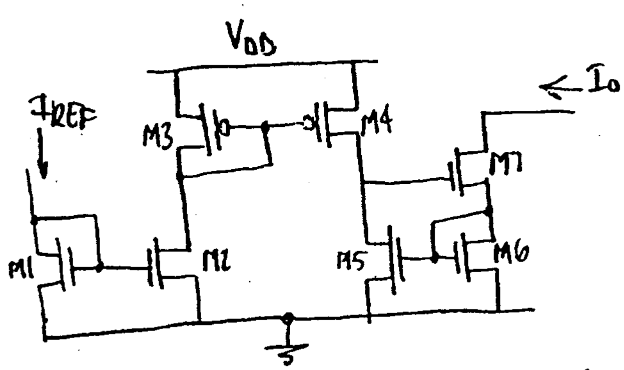
P20.14

ANS: a) $Io/ID1 = 1 - 2\Delta Vthn/(VGS-Vthn)^2$. The diffenence $2\Delta Vthn/(VGS-Vthn)^2$ will decrease as VGS increases.

b) Therefore, the matching would be better with large VGS.

c) The minimum voltage across the current mirror will increase if VGS is increased.

20.15



Design so that $I_O = 150\mu A$, $I_{REF} = 50\mu A$, $V_{DD} = 5V$, $V_{SS} = 0$

Philosophy: To minimize area, make $(W/L)_1 = (W/L)_2 = (W/L)_5$.
Make $(W/L)_3 = (W/L)_4$ & $(W/L)_6 = (W/L)_7 = 3(W/L)_5$.

Iteration 1: Assume $V_{GS} = V_{SG} = 1.2$ & $L = 5$

$\therefore$ M1, M2 & M5 $\Rightarrow$ $50\mu A = \frac{50\mu A/V^2}{2}(W/L)(1.2-0.83)^2$

$(W/L) = 14.6 \Rightarrow W_{1,2,5} = 73\mu m$

M3, M4 $\Rightarrow$ $50\mu A = \frac{17\mu A/V^2}{2}(W/L)(1.2-0.91)^2$

$(W/L) = 70 \Rightarrow W_{3,4} = 350\mu m$

Since there is no restriction on $V_{O(min)}$ we can shrink the design by sacrificing the $V_{GS} = V_{SG} = 1.2$ assumption.

Iteration 2: Increase $V_{GS} \simeq V_{SG} = 1.5V$. Same analysis as before:

$(W/L)_{1,2,5} = 4.5 \Rightarrow W_{1,2,5} = 22 \Rightarrow$ Let's make it $20\mu m$

$\therefore W_{3,4} \simeq 80\mu m$; & $W_{6,7} = 3(20) = 60\mu m$

See HSPICE info. following.

The second design will increase $V_{O(min)}$ by $\sim 0.25V$ over the 1st design, but at a greatly reduced area.

```
*problem 20.15 iteration 2
*

m1 g1 g1 vss vss cmosnb w=20u l=5u
m2 g3 g1 vss vss cmosnb w=20u l=5u
m3 g3 g3 vdd vdd cmospb w=80u l=5u
m4 g7 g3 vdd vdd cmospb w=80u l=5u
m5 g7 g6 vss vss cmosnb w=20u l=5u
m6 g6 g6 vss vss cmosnb w=60u l=5u
m7 out g7 g6 vss cmosnb w=60u l=5u

iref 0 g1 50u

vdd vdd 0 dc 5
vss vss 0 dc 0
vout out 0 dc 5

.lib  ./../spice.models cmosnb
.lib  ./../spice.models cmospb
.op
.dc vout 0 5 0.25
.print i(m7)

.end

**** mosfets
subckt
element   0:m1        0:m2        0:m3        0:m4        0:m5        0:m6
model     0:cmosnb    0:cmosnb    0:cmospb    0:cmospb    0:cmosnb    0:cmosnb
 id        50.0000u    55.7671u   -55.7671u   -55.5374u    55.5374u   153.7303u
 vgs        1.6097      1.6097     -1.5673     -1.5673      1.6059      1.6059
 vds        1.6097      3.4327     -1.5673     -1.4399      3.5601      1.6059

subckt
element   0:m7
model     0:cmosnb
 id       153.7303u
 vgs        1.9542
 vds        3.3941
```

20.16 see figure from problem 20.15

VDD = 5, VSS = 0, IREF = 10uA all W/L's = 10/10

a.) Calculate Rout, $r_o = \frac{1}{\lambda I_o} \approx \frac{1}{(0.06)(10\mu A)} = 1.67 M\Omega$

$R_{out} \approx 2r_o + g_m \frac{r_o^2}{2}$ (from eq. 20.47)

$g_{m2} = \sqrt{(50\mu A/V^2)(10/10)(2)(10\mu A)} = 3.16\times10^{-5}\ A/V$

$\therefore R_{out} \approx 2(1.67Meg) + (3.16\times10^{-5})\left(\frac{(1.67Meg)^2}{2}\right) = \underline{\underline{47.4 M\Omega}}$

b.) Verify w/SPICE. See input/output file that follows

$R_{out} = \frac{v_{oc}}{i_{oc}} = \frac{10mV}{53pA} = 188.7 M\Omega$! (HSPICE)

Hand analysis is off by a factor of 4.

c.) $V_{O(MIN)} = 2\sqrt{\frac{2I_oL}{KPW}} + V_{THN} = 2.095\ V$

d.) see Plot that follows

e) see Plot that follows - I_o won't go above ~20uA since my W/L's are so small! Eq. 20.47 doesn't take this into account.

f.) see layout that follows

```
*problem 20.16
 m1 g1 g1 vss vss cmosnb w=10u l=10u
 m2 g3 g1 vss vss cmosnb w=10u l=10u
 m3 g3 g3 vdd vdd cmospb w=10u l=10u
 m4 g7 g3 vdd vdd cmospb w=10u l=10u
 m5 g7 g6 vss vss cmosnb w=10u l=10u
 m6 g6 g6 vss vss cmosnb w=10u l=10u
 m7 out g7 g6 vss cmosnb w=10u l=10u

 iref 0 g1 10u

 vdd vdd 0 dc 5
 vss vss 0 dc 0
 vout out 0 ac 10m dc 5
```
← 10mV AC signal

```
* Insert models here
*operating point info to check DC values
.op
*AC analysis for checking Rout
.ac lin 10 100 10000
* Nested DC sweep for part (e)
.dc vout 0 5 0.25 iref 5u 50u 5u
.print dc i(m7)
.print ac i(vout)
.options ingold=2
.end
```

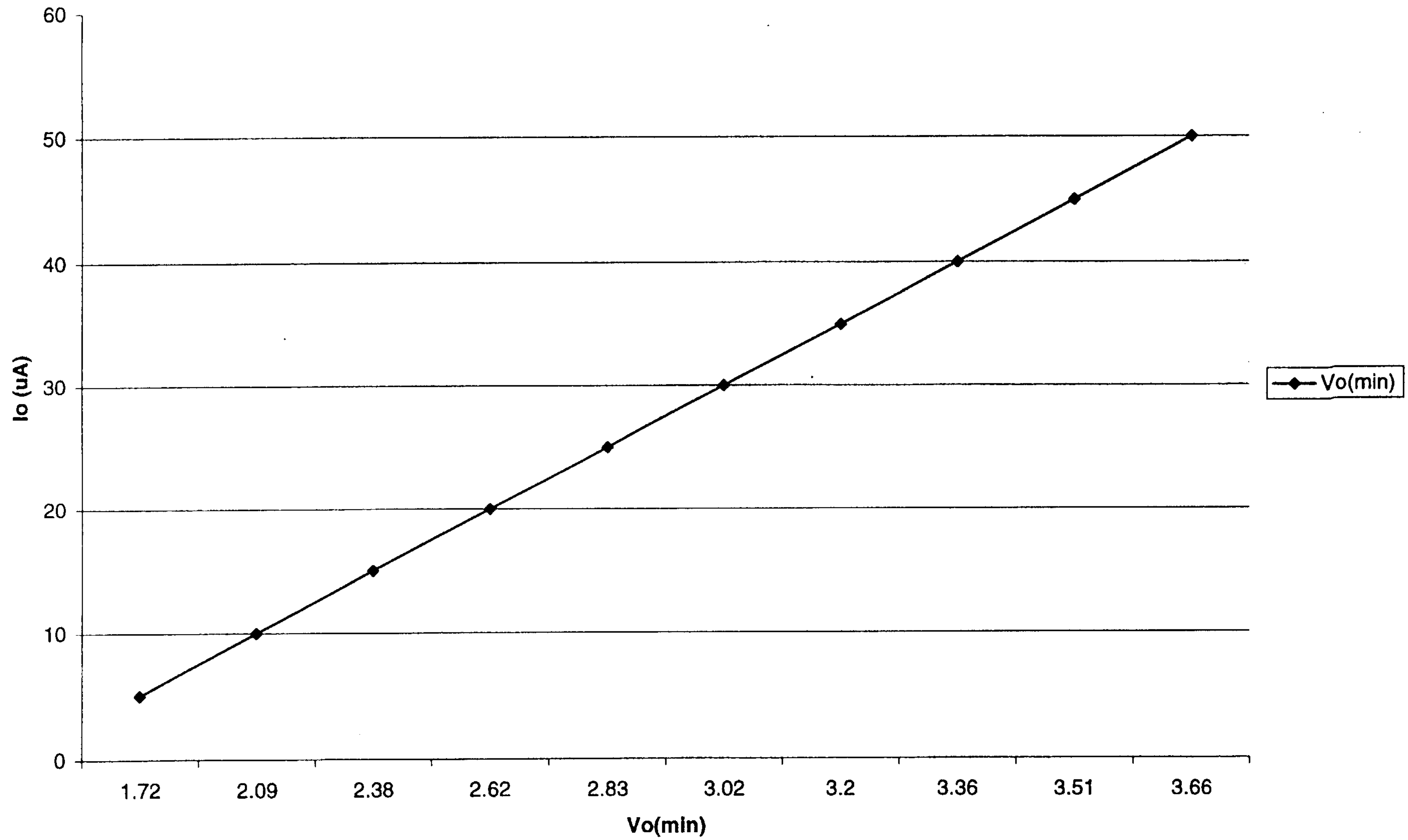
```
 *problem 20.16
  ******  ac analysis    tnom=  25.000 temp=  25.000
```

freq	current (vout)
1.0000e+02	5.304e-11
1.2000e+03	5.305e-11
2.3000e+03	5.305e-11
3.4000e+03	5.307e-11
4.5000e+03	5.309e-11
5.6000e+03	5.311e-11
6.7000e+03	5.314e-11
7.8000e+03	5.318e-11
8.9000e+03	5.322e-11
9.9999e+03	5.327e-11

← AC current through Vout

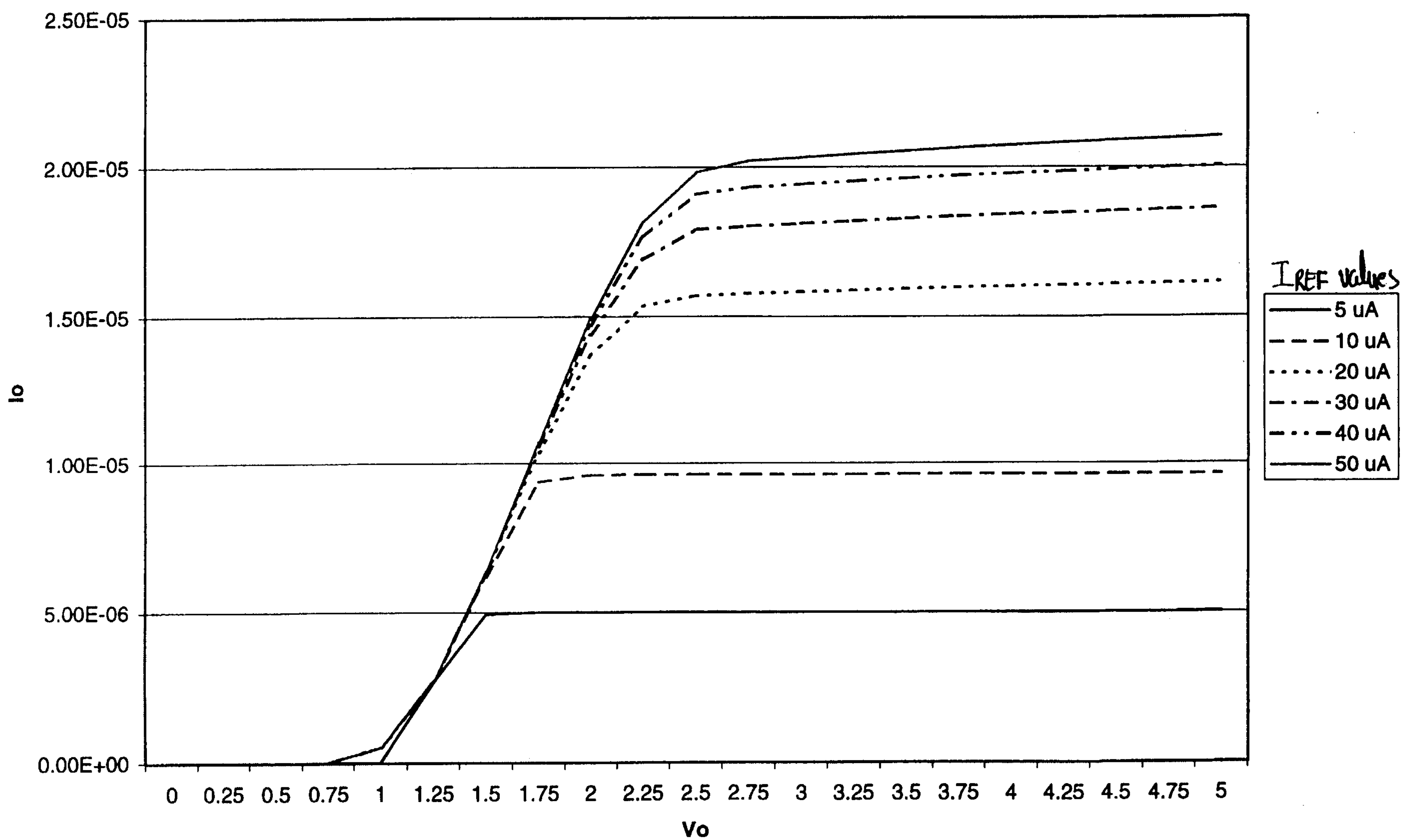
$$R_{out} = \frac{10\,mV}{53\,pA} = 188.7\,M\Omega$$

Prob. 20-16d



60
50
40
30
20
10
0
Io (uA)
1.72
2.09
2.38
2.62
2.83
3.02
3.2
3.36
3.51
3.66
Vo(min)
Vo(min)

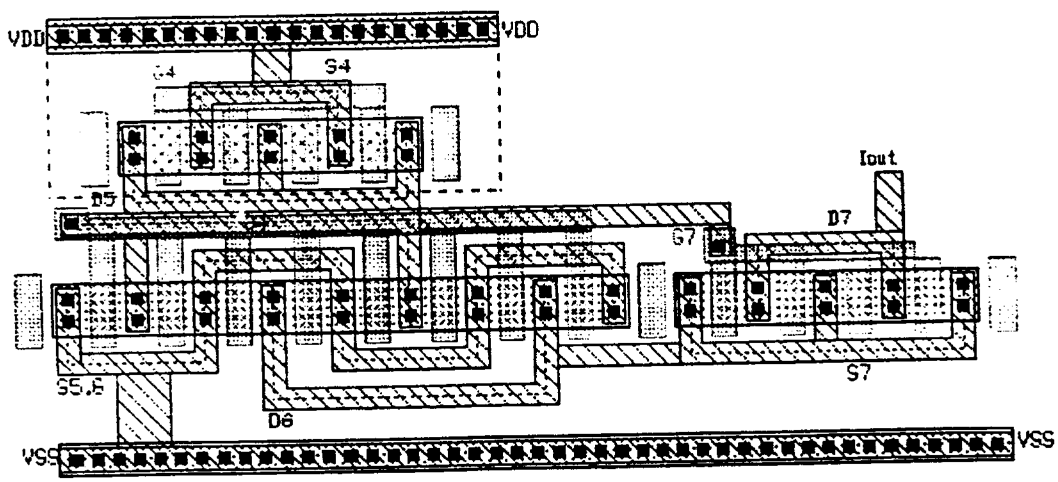
Prob. 20-16e



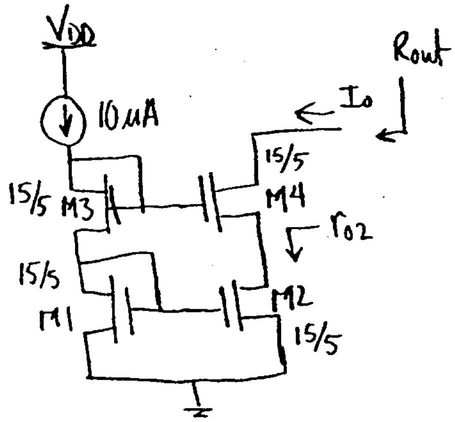
2.50E-05
2.00E-05
1.50E-05
1.00E-05
5.00E-06
0.00E+00
Io
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
2.75
3
3.25
3.5
3.75
4
4.25
4.5
4.75
5
Vo
IREF values
5 uA
10 uA
20 uA
30 uA
40 uA
50 uA

by Kevin McCall

Problem 20.16 f) Layout of devices M4 through M7, assuming that each device is 40/5 using an interditigated layout technique to match devices M5 and M6, using parallel devices for M4 and M7.



20.17



a.) Find Rout

$$R_{out} = r_{o2} + (1 + g_{m4} r_{o2}) r_{o4}$$

$$r_{o2} = r_{o4} = \frac{1}{\lambda I_D} = \frac{1}{(0.06)(10\mu A)} = 1.67 M\Omega$$

$$g_m = \sqrt{2(50\mu A/V^2)(15/5)(10\mu A)} = 5.48 \times 10^{-5}\ A/V$$

$$R_{out} = 1.67M + (1 + (5.48\times10^{-5})(1.67M))(1.67M) = \underline{\underline{156 M\Omega}}$$

b.) Verify w/ SPICE: (see page following)

c.) Calculate $V_{o(min)}$

$$V_{o(min)} = 2\overbrace{(\Delta V + V_{THN})}^{V_{G4}} - V_{THN}$$

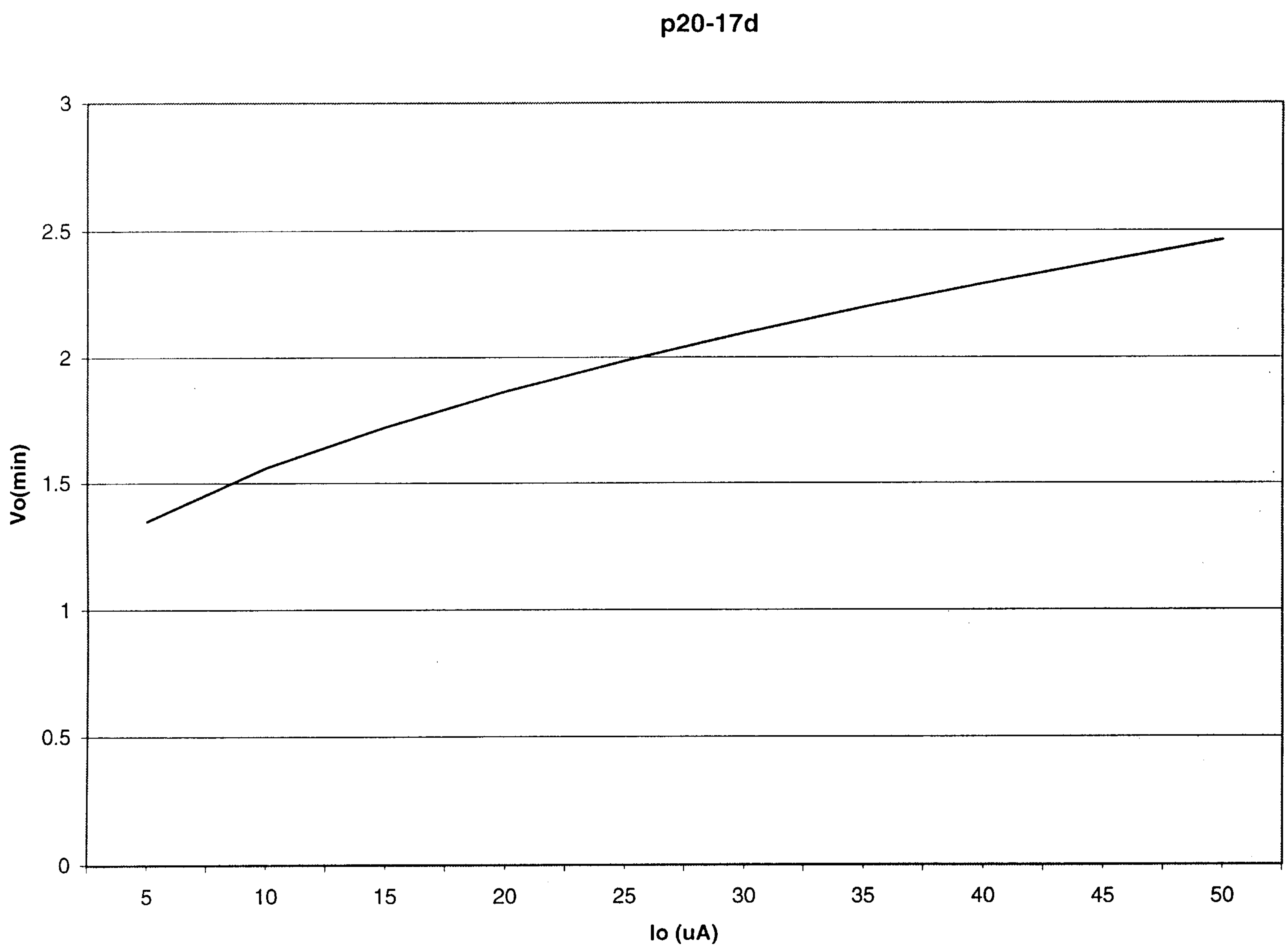
$$V_{GS1} = \sqrt{\frac{2(5)(10\mu A)}{(50\mu A/V)(15)}} + 0.83 \approx 1.2\,V$$

$$\therefore\ V_{o(min)} = 2(0.37) + 0.83 = \underline{\underline{1.57\,V}}$$

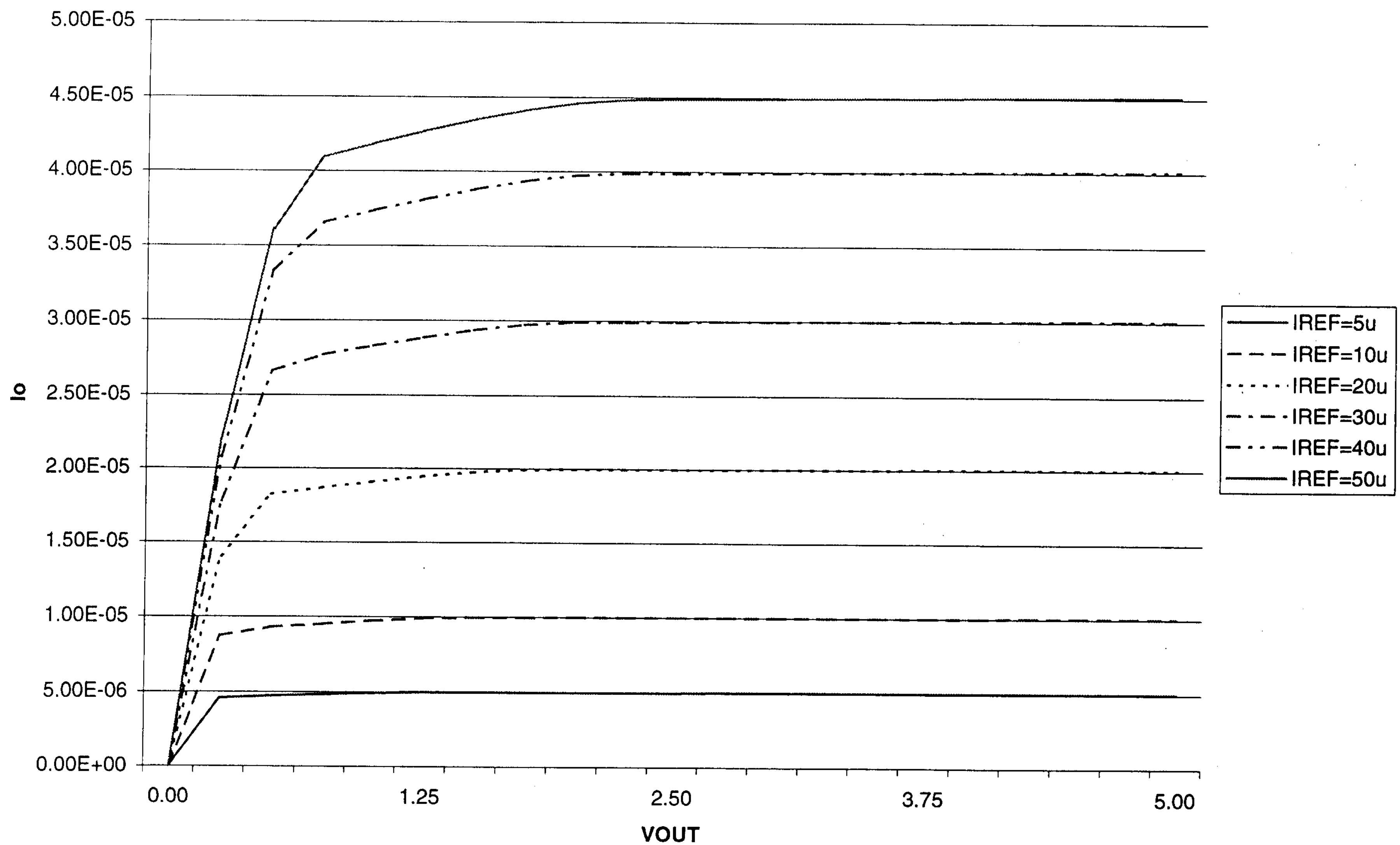
d.) Construct plot of I_o vs $V_{o(min)}$: See page following

e.) Generate I_o vs V_o curves w/ SPICE: see page following

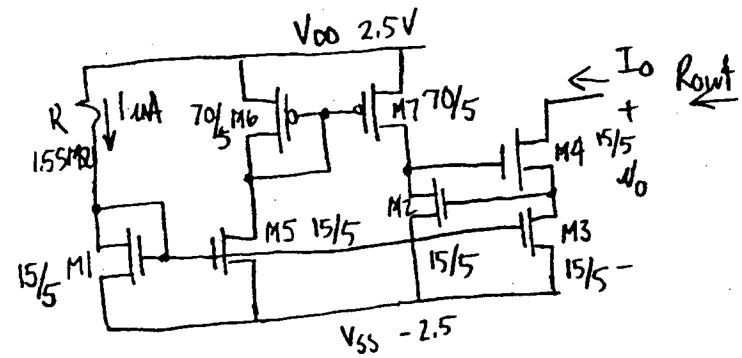
```
*problem 20.17  cascode current mirror
m1 g1 g1 vss vss cmosnb w=15u l=5u
m2 d2 g1 vss vss cmosnb w=15u l=5u
m3 g3 g3 g1 vss  cmosnb w=15u l=5u
m4 out g3 d2 vss cmosnb w=15u l=5u

iref 0 g3 10u

vss vss 0 dc 0
vout out 0 ac 10m dc 5

.lib  ./../spice.models cmosnb
.lib  ./../spice.models cmospb

.op
.dc vout 0 5 0.25 iref 5u 50u 5u
.ac lin 10 10 100
.print dc i(m4)
.print ac i(vout)
.options ingold=2

.end
**** mosfets

subckt
element  0:m1        0:m2        0:m3        0:m4
model    0:cmosnb    0:cmosnb    0:cmosnb    0:cmosnb
 id       1.000e-05   1.002e-05   1.000e-05   1.002e-05
 vgs      1.235e+00   1.235e+00   1.608e+00   1.579e+00
 vds      1.235e+00   1.263e+00   1.608e+00   3.736e+00

*problem 20.17  cascode current mirror

       freq     current
                   vout
 1.0000e+01    8.153e-11
 2.0000e+01    8.153e-11
 3.0000e+01    8.153e-11
 4.0000e+01    8.153e-11
 5.0000e+01    8.153e-11
 6.0000e+01    8.153e-11
 7.0000e+01    8.153e-11
 8.0000e+01    8.153e-11
 9.0000e+01    8.153e-11
 1.0000e+02    8.153e-11
```


p20-17d
Vo(min)
3
2.5
2
1.5
1
0.5
0
5
10
15
20
25
30
35
40
45
50
Io (uA)

p20-17e


5.00E-05
4.50E-05
4.00E-05
3.50E-05
3.00E-05
2.50E-05
2.00E-05
1.50E-05
1.00E-05
5.00E-06
0.00E+00
Io
0.00
1.25
2.50
3.75
5.00
VOUT
IREF=5u
IREF=10u
IREF=20u
IREF=30u
IREF=40u
IREF=50u

20.18



Design the regulated current mirror to sink 1uA.

Since no requirement is specified for $V_{o(min)}$, the design in Ex. 20.8 will work fine with difference being the value of R and the fact that V_{GS} for the NMOS devices will be $\simeq 0.95V$ & V_{SG} for all the PMOS devs will be $\simeq 1V$. see schematic above.

$$R = \frac{5 - 0.95}{1\mu A} = 4.05\ MEG$$

$$R_{out} \simeq \frac{g_m^2 r_o^3}{2} \Rightarrow g_m = \sqrt{2(50\times10^{-6} A/V^2)(15/5)(1\mu A)} = 1.732\times10^{-5}$$

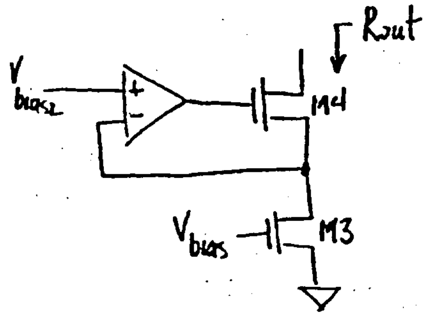
$$r_o \simeq \frac{1}{(0.06)(1\mu A)} = 16.67\ M\Omega$$

$$\underline{\underline{R_{out} = 6.94\times10^{11}\Omega}} \Rightarrow \underline{\underline{R_{out}\ (HSPICE) = 1.44\times10^{12}\Omega}}$$

See SPICE Plot on following page.

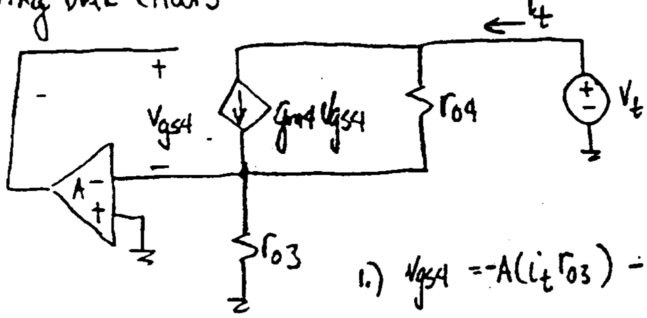
```
*problem 20.18  regulated cascode current mirror
m1 g1 g1 vss vss cmosnb w=15u l=5u
m2 g4 g2 vss vss cmosnb w=15u l=5u
m3 g2 g1 vss vss cmosnb w=15u l=5u
m4 out g4 g2 vss cmosnb w=15u l=5u
m5 g6 g1 vss vss cmosnb w=15u l=5u
m6 g6 g6 vdd vdd cmospb w=70u l=5u
m7 g4 g6 vdd vdd cmospb w=70u l=5u

r1 vdd g1 4.05meg

vdd vdd 0 dc 2.5
vss vss 0 dc -2.5

vout out 0 ac 10m dc 2.5

.lib  ./../spice.models cmosnb
.lib  ./../spice.models cmospb

.op
.ac lin 10 10 100
.print ac i(vout)
.options ingold=2
.end

**** mosfets

subckt
element  0:m1        0:m2        0:m3        0:m4        0:m5        0:m6
model    0:cmosnb    0:cmosnb    0:cmosnb    0:cmosnb    0:cmosnb    0:cmospb
 id       1.004e-06   1.126e-06   1.004e-06   1.004e-06   1.108e-06  -1.108e-06
 vgs      9.351e-01   9.401e-01   9.351e-01   1.243e+00   9.351e-01  -9.746e-01
 vds      9.351e-01   2.183e+00   9.401e-01   4.059e+00   4.025e+00  -9.746e-01

subckt
element  0:m7
model    0:cmospb
 id      -1.126e-06
 vgs     -9.746e-01
 vds     -2.816e+00
```

20.19



Find Rout.
Show that if the gain of A is $g_m(r_o/2)$, that Rout reduces to eq. 20.51.

ignoring bulk effects



1.) $V_{gs4} = -A(i_t r_{o3}) - i_t r_{o3} = -i_t r_{o3}(1+A)$

2.) $i_t r_{o3} + (i_t - g_m V_{gs4}) r_{o4} = V_t$

1.) → 2.)

$$i_t r_{o3} + (i_t + g_{m4} i_t r_{o3}(1+A)) r_{o4} = V_t$$

$$\therefore \frac{V_t}{i_t} = \underline{\underline{r_{o3} + (1 + g_{m4} r_{o3}(1+A)) r_{o4} = R_{out}}}$$

If $A = g_m r_o/2$, (and $r_{o3} = r_{o4} = r_o$), then

$$R_{out} = r_o + (1 + g_m r_o (1 + g_m r_o/2)) r_o$$

$$\simeq \underline{\underline{\frac{g_m^2 r_o^3}{2}}} \Rightarrow \text{same as eq. 20.51.}$$

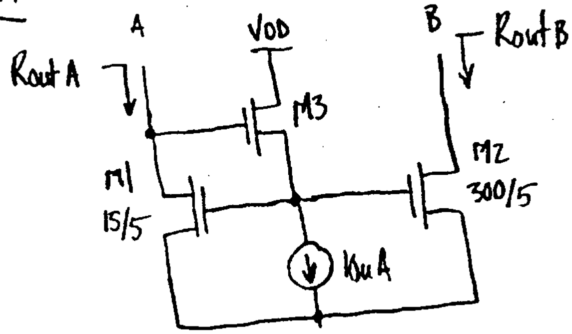
P20.20

ANS: a) VG5 = 2ΔV+Vthn. Plug it into Iref = (1/2) Kpn (W/4L) (VG5-Vthn)^2. ==> Iref = (1/2) Kpn (W/4L) (2ΔV+Vthn-Vthn)^2 = Iref = (1/2) Kpn (W/4L) (2ΔV)^2 ==>
Iref = (1/2) Kpn (W/L) (ΔV)^2 (1)

b) Same current goes through M3 and M4,
Iref = (1/2) Kpn (W/L) (VGS3,4 - Vthn)^2 (2)
Comparing equation (1) and (2) ==> VGS3,4 - Vthn = ΔV or VGS3,4 = ΔV+Vthn.

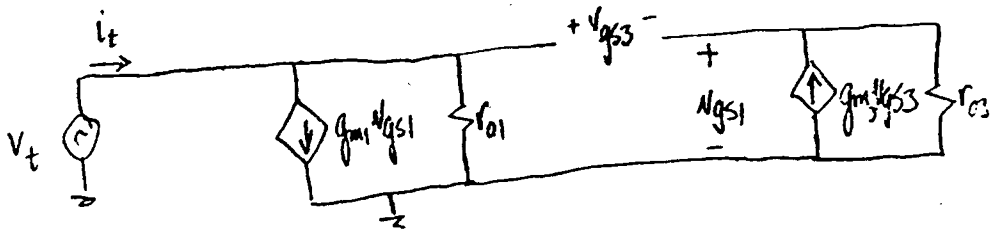
c) VD1,2 = VS3,4 = VG3,4 - VGS3,4 = VG5 - VGS3,4 = (2ΔV+Vthn) - (ΔV+Vthn) = ΔV

20.21



$\underline{\underline{Rout\ B = r_{o2}}}$

Rout A :



1.) $V_{gs1} = g_{m3} V_{gs3} r_{o3}$

2.) $V_{gs3} = V_t - g_{m3} V_{gs3} r_{o3} \Rightarrow V_{gs3}(1 + g_{m3} r_{o3}) = V_t$

3.) $V_t = (i_t - g_{m1} V_{gs1}) r_{o1}$

∴ 1.), 2) → 3.)

$$V_t = \left(i_t - g_{m1} g_{m3} r_{o3} \frac{V_t}{1+g_{m3} r_{o3}}\right) r_{o1}$$

$$V_t\left(1 + g_{m1} g_{m3} r_{o3}^2 \left(\frac{1}{1+g_{m3} r_{o3}}\right)\right) = i_t r_{o1}$$

$$\frac{V_t}{i_t} = \frac{r_{o1}}{\left(1 + g_{m1} g_{m3} r_{o3}^2 \left(\frac{1}{1+g_{m3} r_{o3}}\right)\right)}$$

$$\simeq \frac{r_{o1}}{1 + g_{m1} r_{o3}} = \frac{r_{o1}}{r_{o3}} \frac{1}{\frac{1}{r_{o3}} + g_{m1}} \simeq \underline{\underline{\frac{1}{g_{m1}}}} \quad (\text{if } r_{o1} \simeq r_{o3})$$

20.21 (con't)

If a resistor connects A to V_{DD} find an equation relating the resistor to both drain currents.

$$V_{GS1} = \sqrt{\frac{2I_{D1}}{\beta_1}} + V_{THN1} = \sqrt{\frac{2I_{D2}}{\beta_2}} + V_{THN2}$$

If $V_{THN1} = V_{THN2}$, then,

$$\frac{I_{D1}}{I_{D2}} = \frac{\beta_1}{\beta_2}$$

$$\frac{V_{DD} - (V_{GS1} + V_{GS3} + V_{SS})}{R} = I_{D1}$$

$$R = \frac{V_{DD} - V_{SS} - \left(\sqrt{\frac{2I_{D1}}{\beta_1}} + V_{THN1} + \sqrt{\frac{2(10\mu A)}{\beta_3}} + V_{THN3}\right)}{I_{D2}\,\beta_1/\beta_2}$$