UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
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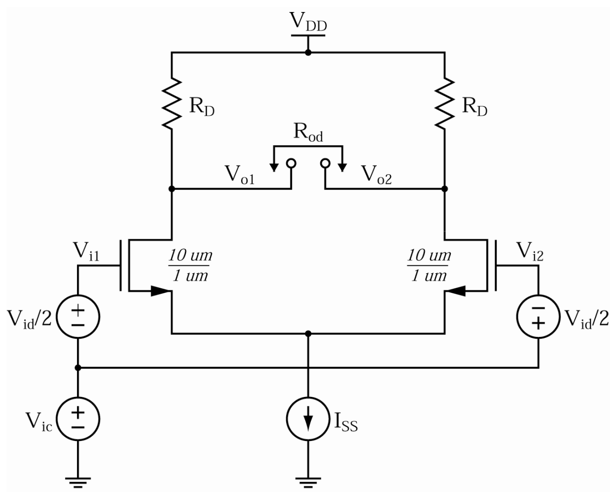
EECS 140
Fall 2004

PROBLEM SET #3
(SOLUTION)



3) In the above circuit, use V_{DD}=1.8 V, I_{SS}=14 µA, R_D=100 kΩ, W=10 µm, L=1 µm.
a) Calculate V_{DS} of the two transistors by hand, with V_{ic}=0.9 V, V_{id}=0 V, and verify with SPICE.

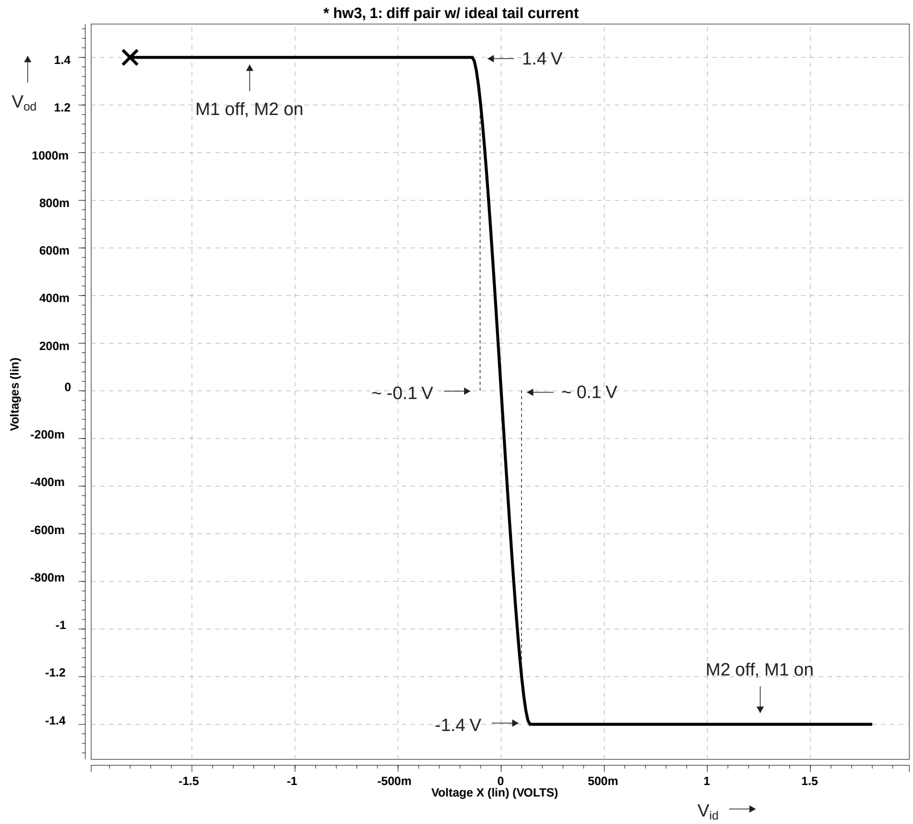
$$V_{o1} = V_{o2} = V_{DD} - \frac{I_{SS}}{2} R_D = 1.1\,\text{V}$$

$$V_{DSAT} = \sqrt{\frac{2I_{DS}}{k'\frac{W}{L}}} = \sqrt{\frac{I_{SS}}{k'\frac{W}{L}}} = 0.1\,\text{V}$$

$$\begin{aligned} V_S &= V_{ic} - V_T - V_{DSAT} \\ &= V_{ic} - \left(V_{T0} + \gamma\left(\sqrt{2\phi + V_S} - \sqrt{2\phi}\right)\right) - V_{DSAT} \\ &= 0.9 - \left(0.5 - 0.5\left(\sqrt{0.6 + V_S} - \sqrt{0.6}\right)\right) - 0.1 \\ &= 0.3 - 0.5\left(\sqrt{0.6 + V_S} - \sqrt{0.6}\right) \end{aligned}$$

Solving iteratively, we find $V_S = 0.231\,\text{V}$, and therefore, $V_{DS} = 0.87\,\text{V}$.

b) Plot $V_{od}=(V_{o1}-V_{o2})$ vs. V_{id}, with $V_{ic}=0.9$ V over the range $-1.8 < V_{id} < +1.8$ V.



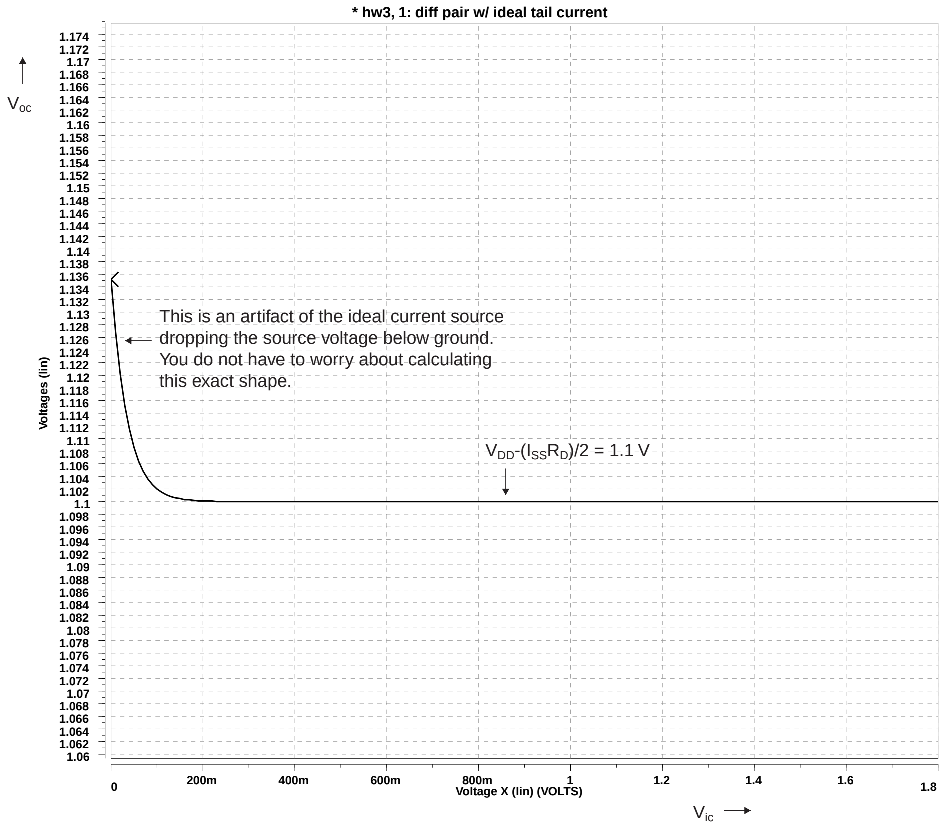
The slope around $V_{id}=0$ V is $A_{dm}=-g_m R_{out}\approx -g_m R_D = -\left(\sqrt{2k'\frac{W}{L}I_{DS}}\right)R_D = -14$.

The curves limit when all of the current is switched to one side of the differential pair. When this happens, the differential voltage is $I_{SS}R_D = 1.4\text{ V}$.

The voltage that this occurs at can be approximated by $\Delta V_{id} = \dfrac{I_{SS}R_D}{g_m R_D} = 0.1\text{ V}$.

It is also correct to find the exact point where one transistor turns off and the other takes all of the current. This occurs for $\Delta V_{id} = \sqrt{\dfrac{2I_{SS}}{k'\frac{W}{L}}} = 0.14\text{ V}$.

c) Plot $V_{oc}=(V_{o1}+V_{o2})/2$ vs. V_{ic}, with V_{id}=0 V over the range 0 < V_{ic} < +1.8 V.



There are no breakpoints in this curve. The strange behavior near V_{oc}=0 V is only an artifact of the ideal current source pulling the source voltage below ground. When V_S<0 V, the source-bulk junction diode begins to be forward biased and starts to conduct current. This will not happen with any real implementation, and you do not have to worry about calculating this.

d) Calculate A_{dm} with V_{ic}=0.9 V, V_{id}=0 V. Over what range of V_{id} will the gain remain high? Why does the gain drop off?

$$A_{dm} = -g_m(R_D \| r_o) \approx -g_m R_D$$

$$= -\sqrt{2(140\ \mu\text{A}/\text{V}^2)(10)(70\ \mu\text{A})}(100\ \text{k})$$

$$= -14$$

As calculated in part (b), the gain is high over the range of –0.1 V < V_{id} < 0.1 V. The gain drops off because one of the transistors goes into cutoff.

e) Calculate A_{cm} with V_{ic}=0.9 V, V_{id}=0 V.

$$A_{cm} \approx -\frac{g_m R_D}{1+(1+\chi)g_m(2r_o)}$$

For an ideal current source, the output resistance is $r_o = \infty$, and therefore the common-mode gain is, $A_{cm} = 0$.

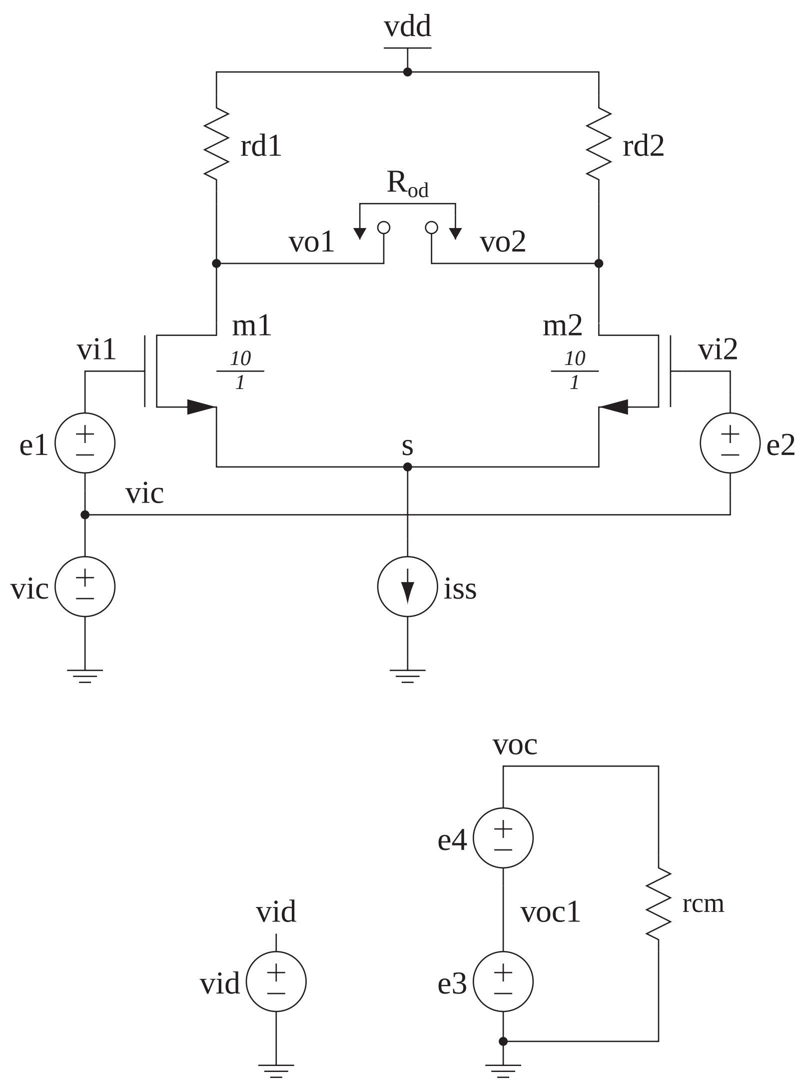
f) Calculate R_{od} with V_{ic}=0.9 V, V_{id}=0 V.

$$R_{od} = 2(R_D \| r_o) = 2(100\text{ k}\|1.4\text{ M})$$
$$= 187\text{ k}\Omega$$

Verify (d)-(f) with SPICE using the .TF analysis option.

```
* hw3, 3: diff pair w/ ideal tail current

.model nch nmos level=1 tox=25 vto=0.5 kp=140e-6 lambda=0.1 gamma=0.5 phi=0.6

vdd    vdd 0 1.8

* set up common mode and differential inputs
vic    vic 0 0.9
vid    vid 0 0
e1     vi1 vic vid 0 0.5
e2     vi2 vic vid 0 -0.5

* set up to measure output common mode
e3     voc1 0 vo1 0 0.5
e4     voc voc1 vo2 0 0.5
rcm    voc 0 1

rd1    vdd vo1 100k
rd2    vdd vo2 100k
m1     vo1 vi1 s 0 nch w=10u l=1u
m2     vo2 vi2 s 0 nch w=10u l=1u
iss    s 0 14u

.options post=2 nomod

.probe dc vod=v(vo1,vo2) voc=v(voc)

.op
.dc    vid -1.8 1.8 0.01
.dc    vic 0 1.8 0.01

.tf    v(vo1,vo2) vid

.alter
.tf    v(voc) vic

.end


**** mosfets

subckt
element  0:m1       0:m2
model    0:nch      0:nch
 id         7.0000u    7.0000u
 vgs      665.4148m  665.4148m
 vds      865.4147m  865.4147m                              Vds
 vth      569.4798m  569.4798m
 vdsat     95.9349m   95.9349m


****     small-signal transfer characteristics

     v(vo1,vo2)/vid                          =  -13.7100          Adm
     input resistance at         vid         =  1.000e+20
     output resistance at v(vo1,vo2)         =  187.8949k         Rod


****     small-signal transfer characteristics

     v(voc)/vic                              =  -79.0254n         Acm
     input resistance at         vic         =  1.000e+20
     output resistance at v(voc)             =    0.
```

The schematic for the circuit used in Problem is:



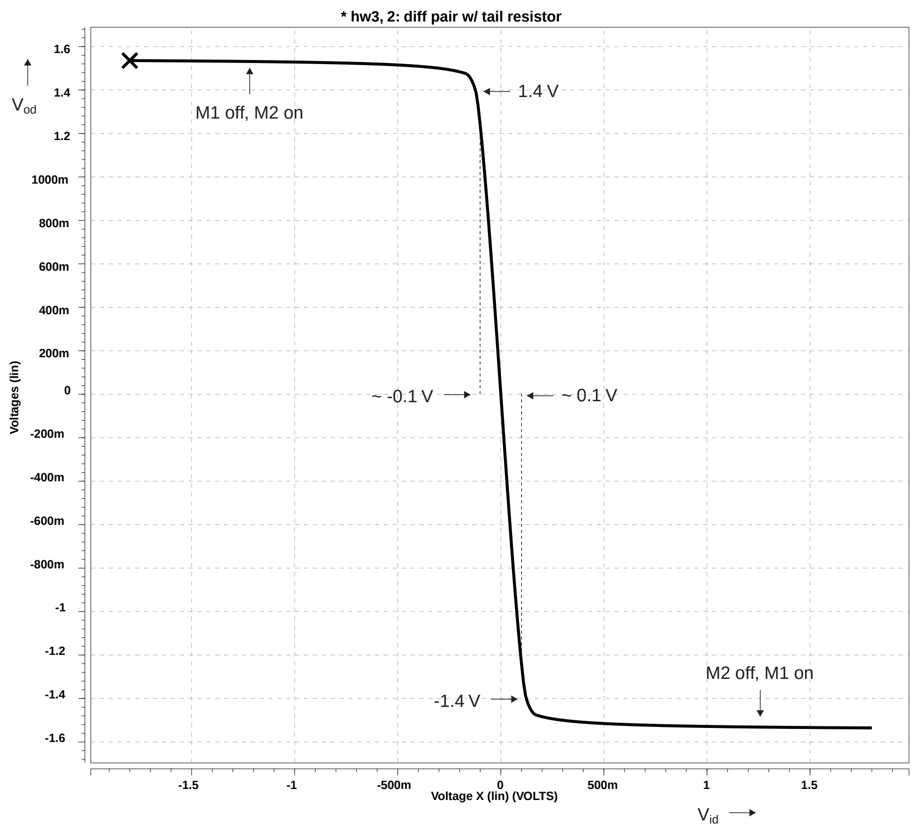
replace the I_{SS} current source with a resistor which results in the same I_{DS} currents when V_{ic}=0.9 V and V_{id}=0 V.

a) To get the same current, $R_{SS} = \frac{V_S}{I_{SS}} = \frac{0.231\,\text{V}}{14\,\mu\text{A}} = 16.5\,\text{k}\Omega$

Since the current and source voltage are the same, V_{DS} is also the same.

$$V_{DS} = 0.87\text{ V}$$

b) Plot V_{od} vs. V_{id} with V_{ic}=0.9 V over the range –1.8 < V_{id} < 1.8 V.

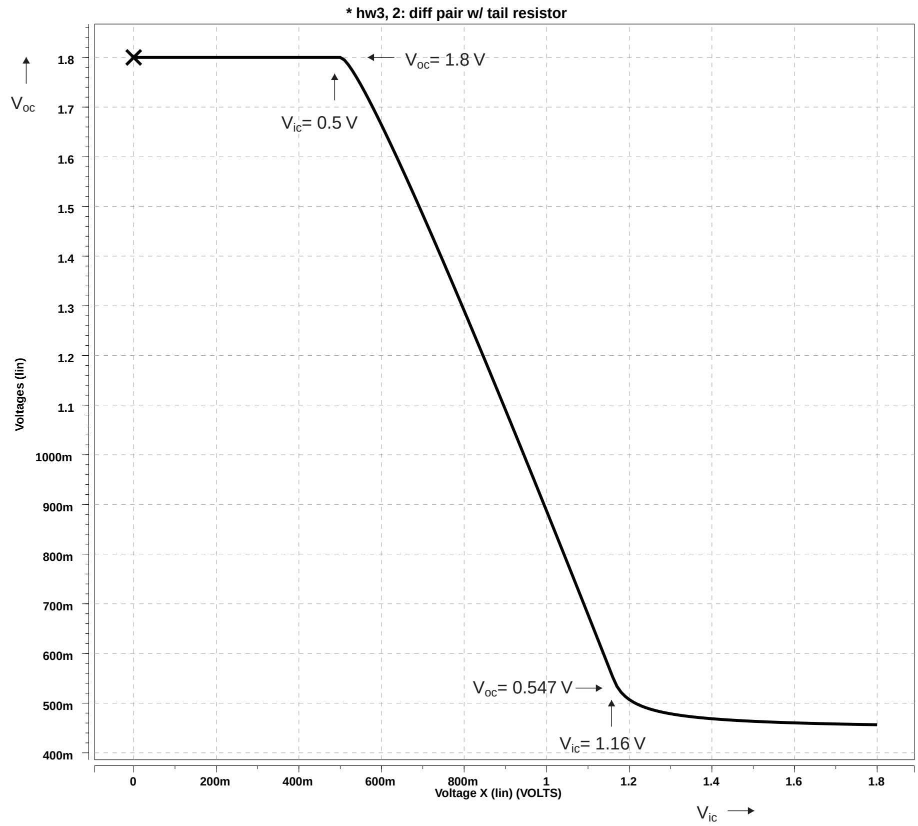


Differentially, this circuit behaves the same way as the circuit with an ideal current source. Therefore the breakpoints are the same as before:

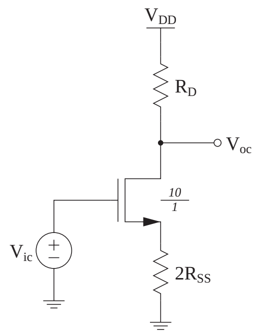
$$\Delta V_{id} = \pm 0.1\,\text{V}, \;\; \Delta V_{od} = \pm 1.4\,\text{V}$$

When V_{id} is increased beyond this range, the current increases causing a small slope in the transfer function. This can be approximated by a flat line and the exact shape is not important.

c) Plot V_{oc} vs. V_{ic} with V_{id}=0 V over the range $0 < V_{ic} < +1.8$ V.



The equivalent half-circuit is



When $V_{ic} < V_T$, the transistor is operating in cutoff and the output is at V_{DD}=1.8 V.

When $V_{ic} > V_T$, the transistor operates as a common-source with source-degeneration amplifier until it goes into triode. We need to find the point where this occurs. At the edge of saturation,

$$V_{DS} = V_{DSAT}$$

$$V_{DD} - I_{DS}(R_D + 2R_{SS}) = \sqrt{\frac{2I_{DS}}{k'\frac{W}{L}}}$$

$$(R_D + 2R_{SS})I_{DS} + \left(\sqrt{\frac{2}{k'\frac{W}{L}}}\right)\sqrt{I_{DS}} - V_{DD} = 0$$

Solving the quadratic equation, $I_{DS} = 12.5\,\mu\text{A}$, and $V_{DSAT} = 0.134\text{ V}$.

Therefore, $V_S = I_{DS}(2R_{SS}) = 0.413\text{ V}$, and $V_T = 0.616\text{ V}$. So, the transistor goes from saturation to triode at the point,

$$V_{ic} = V_S + V_T + V_{DSAT} = 1.16\text{ V}$$

$$V_{oc} = V_S + V_{DSAT} = 0.547\text{ V}$$

d) Since the differential half-circuit is exactly the same as with the ideal tail current source, this is the same as in (3d).

$$A_{dm} = -g_m(R_D \| r_o) = -14$$

and the gain is large over the range –0.1 V < V_{id} < 0.1 V. The gain drops off because one of the transistors goes into cutoff.

e) The common-mode gain is slightly different due to the source resistor.

$$A_{cm} = -\frac{g_m R_D}{1 + (1+\chi)g_m(2R_{SS})} = -2$$

where

$$g_m = 140\,\mu\text{S}$$

$$\chi = \frac{\gamma}{2(2\phi + V_{SB})^{1/2}} = 0.274$$

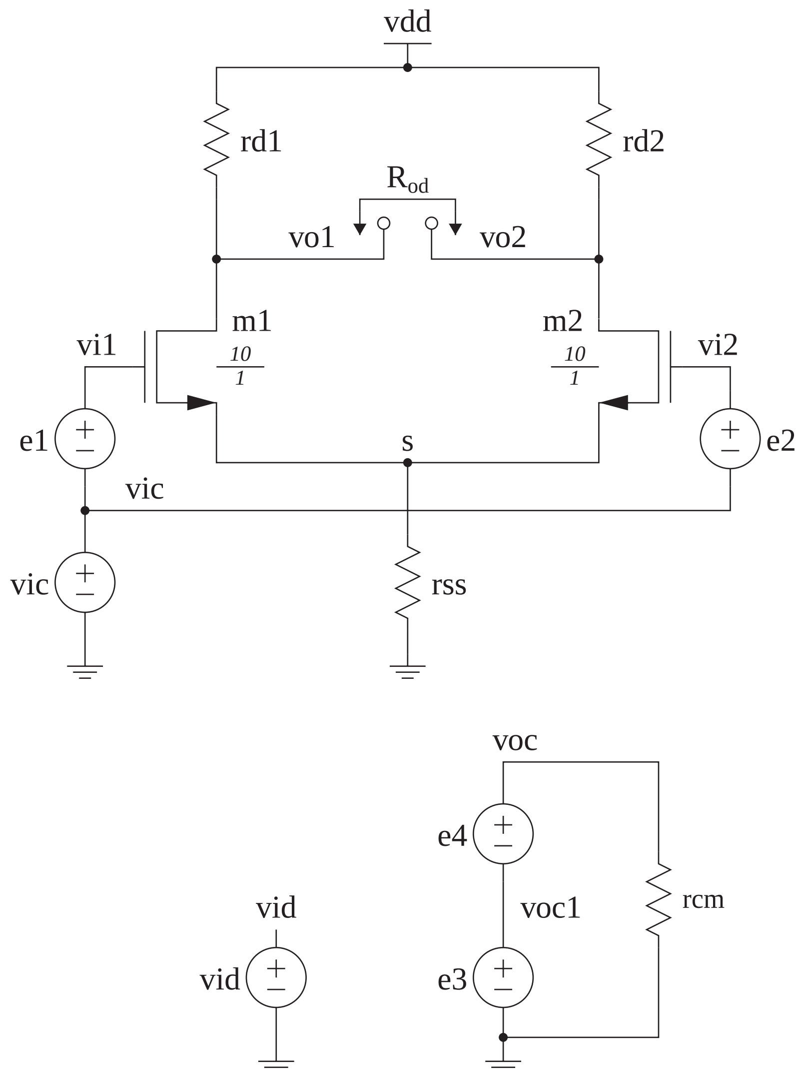
f) The differential output resistance is the same as in (4f),

$$R_{od} = 2(R_D \| r_o) = 187\text{ k}\Omega$$

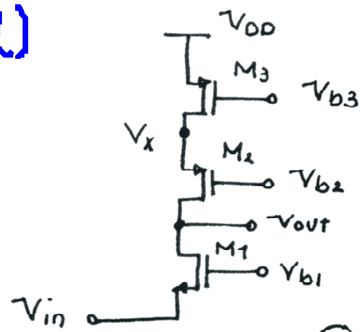
```
* hw3, 4: diff pair w/ tail resistor

.model nch nmos level=1 tox=25 vto=0.5 kp=140e-6 lambda=0.1 gamma=0.5 phi=0.6

vdd    vdd 0 1.8

* set up common mode and differential inputs
vic    vic 0 0.9
vid    vid 0 0
e1     vi1 vic vid 0 0.5
e2     vi2 vic vid 0 -0.5

* set up to measure output common mode
e3     voc1 0 vo1 0 0.5
e4     voc voc1 vo2 0 0.5
rcm    voc 0 1

rd1    vdd vo1 100k
rd2    vdd vo2 100k
m1     vo1 vi1 s 0 nch w=10u l=1u
m2     vo2 vi2 s 0 nch w=10u l=1u
rss    s 0 16.5k

.options post=2 nomod

.probe dc vod=v(vo1,vo2) voc=v(voc)

.op
.dc    vid -1.8 1.8 0.01
.dc    vic 0 1.8 0.01

.tf    v(vo1,vo2) vid

.alter
.tf    v(voc) vic

.end
```

```
**** mosfets

subckt
element  0:m1       0:m2
model    0:nch      0:nch
 id         7.0927u    7.0927u
 vgs      665.9422m  665.9422m
 vds      856.6759m  856.6759m                                    Vds
 vth      569.3355m  569.3355m
 vdsat     96.6067m   96.6067m


****     small-signal transfer characteristics

     v(vo1,vo2)/vid                          =  -13.7831          Adm
     input resistance at         vid         =  1.000e+20
     output resistance at v(vo1,vo2)         =  187.7352k         Rod


****     small-signal transfer characteristics

     v(voc)/vic                              =   -2.0228          Acm
     input resistance at         vic         =  1.000e+20
     output resistance at v(voc)             =    0.
```

The schematic for the circuit used is:



2)



(a)

① ② ③

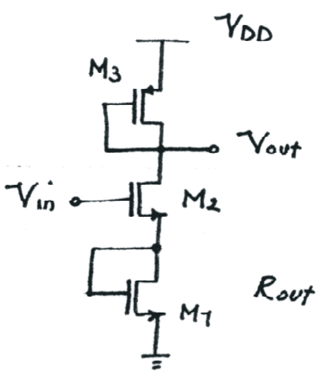
$$-\frac{V_x}{r_{o3}} = g_{m2}V_x + \frac{V_x - V_{out}}{r_{o2}} = \frac{V_{out} - V_{in}}{r_{o1}} - g_{m1}V_{in}$$

①, ② → $$V_x = \frac{V_{out}}{1 + r_{o2}\left(g_{m2} + \frac{1}{r_{o3}}\right)}$$

①, ③ → $$\frac{V_{out} - V_{in}}{r_{o1}} - g_{m1}V_{in} = -\frac{V_{out}}{r_{o3} + r_{o2}(1 + g_{m2}\cdot r_{o3})}$$

$$V_{out}\left[\frac{1}{r_{o1}} + \frac{1}{r_{o3} + r_{o2}(1 + g_{m2}\cdot r_{o3})}\right] = \left(g_{m1} + \frac{1}{r_{o1}}\right)V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{(1 + g_{m1}\cdot r_{o1})\left[r_{o3} + r_{o2}(1 + g_{m2}\cdot r_{o3})\right]}{r_{o1} + r_{o3} + r_{o2}(1 + g_{m2}r_{o3})}$$

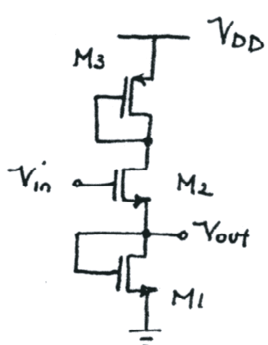


(b)

$$G_m = \frac{g_{m2}\cdot r_{o2}}{\left(\frac{1}{g_{m1}} \| r_{o1}\right) + \left[1 + g_{m2}\left(\frac{1}{g_{m1}} \| r_{o1}\right)\right]r_{o2}}$$

$$R_{out} = \left(\frac{1}{g_{m3}} \| r_{o3}\right) \| \left\{\left[1 + g_{m2}\left(\frac{1}{g_{m1}} \| r_{o1}\right)\right]r_{o2} + \left(\frac{1}{g_{m1}} \| r_{o1}\right)\right\}$$

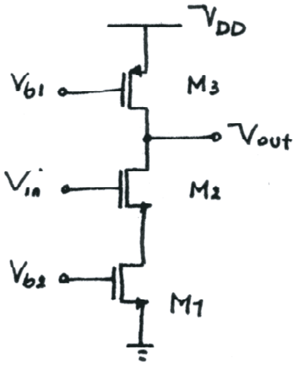
$$A_V = -G_m\cdot R_{out} = -\frac{g_{m2}r_{o2}\left(\frac{1}{g_{m3}} \| r_{o3}\right)}{\left(\frac{1}{g_{m3}} \| r_{o3}\right) + \left\{\left[1 + g_{m2}\left(\frac{1}{g_{m1}} \| r_{o1}\right)\right]r_{o2} + \left(\frac{1}{g_{m1}} \| r_{o1}\right)\right\}}$$



(c)

resistance seen looking up at the source of M_2, $$\frac{\left(\frac{1}{g_{m3}} \| r_{o3}\right) + r_{o2}}{1 + g_{m2}\cdot r_{o2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{\left(\frac{1}{g_{m1}} \| r_{o1}\right)}{\left(\frac{1}{g_{m1}} \| r_{o1}\right) + \frac{\left(\frac{1}{g_{m3}} \| r_{o3}\right) + r_{o2}}{1 + g_{m2}\cdot r_{o2}}}$$

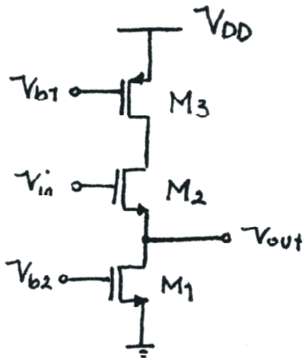


(d)

$$G_m = \frac{g_{m2} \cdot r_{o2}}{r_{o1} + \left[1 + g_{m2} \cdot r_{o1}\right] r_{o2}}$$

$$R_{out} = r_{o3} \parallel \left[(1 + g_{m2} \cdot r_{o1}) r_{o2} + r_{o1}\right]$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m2} \cdot r_{o2} \cdot r_{o3}}{r_{o3} + (1 + g_{m2} \cdot r_{o1}) r_{o2} + r_{o1}}$$

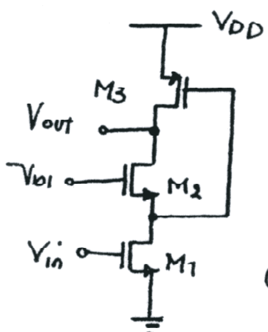


(e)

resistance seen looking up at the source of M_2

$$R_{in} = \frac{r_{o3} + r_{o2}}{1 + g_{m2} \cdot r_{o2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{r_{o1}}{r_{o1} + \dfrac{r_{o3} + r_{o2}}{1 + g_{m2} r_{o2}}} = \frac{r_{o1}(1 + g_{m2} \cdot r_{o2})}{r_{o1}(1 + g_{m2} \cdot r_{o2}) + r_{o2} + r_{o3}}$$



$$\overset{①}{-\left(\frac{V_{out}}{r_{o3}} + g_{m3} V_X\right)} = \overset{②}{\left(\frac{V_{out} - V_X}{r_{o2}} - g_{m2} V_X\right)} = \overset{③}{\frac{V_X}{r_{o1}} + g_{m1} V_{in}}$$

$$①, ② \rightarrow \frac{V_X}{r_{o2}} + g_{m2} V_X - g_{m3} V_X = \frac{V_{out}}{r_{o2}} + \frac{V_{out}}{r_{o3}} \rightarrow V_X = \frac{\dfrac{1}{r_{o2}} + \dfrac{1}{r_{o3}}}{\dfrac{1}{r_{o2}} + g_{m2} - g_{m3}} V_{out}$$

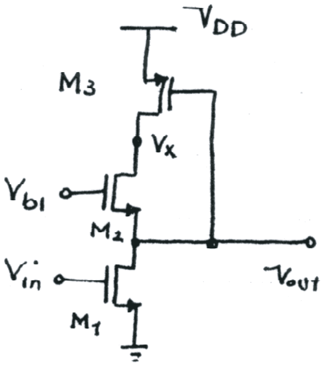
$$①, ③ \quad -\frac{V_{out}}{r_{o3}} - g_{m3} V_X = \frac{V_X}{r_{o1}} + g_{m1} V_{in}$$

$$-\frac{V_{out}}{r_{o3}} - \left(g_{m3} + \frac{1}{r_{o1}}\right) \frac{\dfrac{1}{r_{o2}} + \dfrac{1}{r_{o3}}}{\dfrac{1}{r_{o2}} + g_{m2} - g_{m3}} V_{out} = g_{m1} \cdot V_{in}$$

$$-V_{out}\left[\frac{1}{r_{o3}} + \frac{\left(g_{m3} + \frac{1}{r_{o1}}\right)\left(\frac{1}{r_{o2}} + \frac{1}{r_{o3}}\right)}{\dfrac{1}{r_{o2}} + g_{m2} - g_{m3}}\right] = g_{m1} \cdot V_{in}$$

$$-V_{out}\left[\frac{1}{r_{o3}} + \frac{(1+g_{m3}r_{o1})(r_{o3}+r_{o2})}{r_{o1}r_{o3}\left[1+(g_{m2}-g_{m3})r_{o2}\right]}\right] = g_{m1}\cdot V_{in}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}r_{o1}r_{o3}\left[1+(g_{m2}-g_{m3})r_{o2}\right]}{r_{o1}\left[1+(g_{m2}-g_{m3})r_{o2}\right]+(1+g_{m3}\cdot r_{o1})(r_{o3}+r_{o2})}$$



(g)

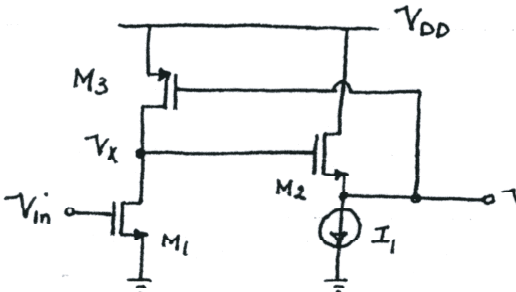
$$V_X = \frac{\frac{1}{r_{o2}}+g_{m2}-g_{m3}}{\frac{1}{r_{o2}}+\frac{1}{r_{o3}}}\cdot V_{out}$$

$$-\frac{V_X}{r_{o3}} - g_{m3}V_{out} = \frac{V_{out}}{r_{o1}} + g_{m1}\cdot V_{in}$$

$$-\frac{\frac{1}{r_{o2}}+g_{m2}-g_{m3}}{\frac{r_{o3}}{r_{o2}}+1}V_{out} - g_{m3}V_{out} = \frac{V_{out}}{r_{o1}} + g_{m1}V_{in}$$

$$-V_{out}\left(\frac{1+(g_{m2}-g_{m3})r_{o2}}{r_{o3}+r_{o2}} + g_{m3} + \frac{1}{r_{o1}}\right) = g_{m1}V_{in}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}r_{o1}(r_{o2}+r_{o3})}{r_{o1}\left[1+(g_{m2}-g_{m3})r_{o2}\right]+(r_{o2}+r_{o3})(1+g_{m3}\cdot r_{o1})}$$



(h)

(1) (2)

$$-\left(\frac{V_X}{r_{o3}} + g_{m3}V_{out}\right) = g_{m1}V_{in} + \frac{V_X}{r_{o1}}$$

(3)

$$-\frac{V_{out}}{r_{o2}} + g_{m2}(V_X - V_{out}) = 0 \quad \text{@ output node}$$

(3) → $$V_X = \frac{\frac{1}{r_{o2}}+g_{m2}}{g_{m2}}\cdot V_{out} = \frac{1+g_{m2}r_{o2}}{g_{m2}r_{o2}}V_{out}$$

(1), (2) → $$-\left[\left(\frac{1}{r_{o3}}+\frac{1}{r_{o1}}\right)\frac{1+g_{m2}\cdot r_{o2}}{g_{m2}\cdot r_{o2}} + g_{m3}\right]V_{out} = g_{m1}\cdot V_{in}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m1}\cdot g_{m2}r_{o1}r_{o2}r_{o3}}{(r_{o1}+r_{o3})(1+g_{m2}\cdot r_{o2}) + g_{m2}g_{m3}r_{o1}r_{o2}r_{o3}}$$

1. $I_{D1} = I_{D2} \Rightarrow V_{in} = 1.254 \Rightarrow I_D = 2.316$ mA

$g_{m1} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{THn}) = 1.34 \times 10^{-4} \times 100 \times (1.254 - 0.7) = 7.43\ m℧$

$g_{m2} = \mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{THP}) = 3.84 \times 10^{-5} \times 100\ (3 - 1.254 - 0.8) = 3.63\ m℧$

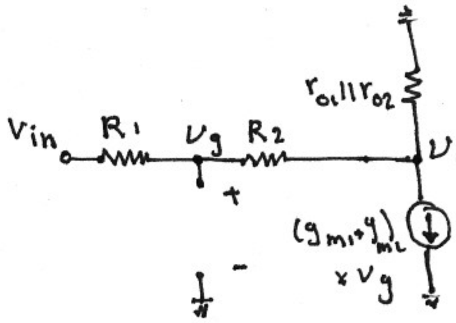
$r_{o1} = \frac{1}{\lambda_1 I_1} = 4.317\ K$ $\qquad r_{o2} = \frac{1}{\lambda_2 I_2} = 2.159\ K$

(a)

a) $A_V = -(g_{m1} + g_{m2})(r_{o1} \| r_{o2}) = 15.91$

$R_{out} = r_{o1} \| r_{o2} = 1439\ \Omega$

b) $v_g = v_{in} \times \frac{R_2}{R_1 + R_2} + V_o \times \frac{R_1}{R_1 + R_2}$



$$\frac{v_o - v_{in}}{R_1 + R_2} + \frac{v_o}{(r_{o1} \| r_{o2})} + (g_{m1} + g_{m2})\left[V_{in}\frac{R_2}{R_1 + R_2} + v_o \frac{R_1}{R_1 + R_2}\right] = 0$$

$$\Rightarrow A_v = \frac{v_o}{v_{in}} = -\frac{(g_{m1} + g_{m2})R_2 - 1}{1 + (g_{m1} + g_{m2})R_1 + \frac{R_1 + R_2}{r_{o1} \| r_{o2}}} = 5.57$$

to calculate Rout :

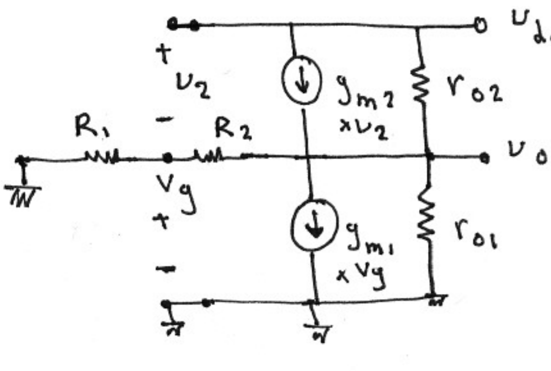
$v_{in} = 0$

$$i_o = \frac{v_o}{r_{o1} \| r_{o2}} + \frac{v_o}{R_1 + R_2} + (g_{m1} + g_{m2}) \times \left[V_2 \times \frac{R_1}{R_1 + R_2}\right]$$

$$= v_o \left[\frac{1}{r_{o1} \| r_{o2}} + \frac{1 + (g_{m1} + g_{m2})R_1}{R_1 + R_2}\right]$$

$$\Rightarrow R_{out} = \frac{v_o}{i_o} = 557\ \Omega$$

(b) We figure out sensitivity for (b),
(a) is a special case where
$R_1 = 0 \qquad R_2 = \infty$



$$V_g = V_o \times \frac{R_1}{R_1 + R_2}$$

$$\frac{V_o - V_{dd}}{r_{o2}} + \frac{V_o}{r_{o1}} + \frac{V_o}{R_1 + R_2} + g_{m1} \times V_o \times \frac{R_1}{R_1 + R_2}$$

$$+ g_{m2}\left(V_o \times \frac{R_1}{R_1 + R_2} - V_{dd}\right) = 0$$

$$\Rightarrow \frac{g_{m2} + \frac{1}{r_{o2}}}{\frac{1}{r_{o1}} + \frac{1}{r_{o2}} + \frac{1}{R_1 + R_2} + \frac{(g_{m1} + g_{m2}) R_1}{R_1 + R_2}} = 2.28 = A_v$$

if $R_1 = 0$ and $R_2 = \infty$

$$A_v = \left[g_{m2} + \frac{1}{r_{o2}} \right] (r_{o1} \parallel r_{o2}) = [3.63 + 0.46] \times 1.439$$

$$= 5.84$$