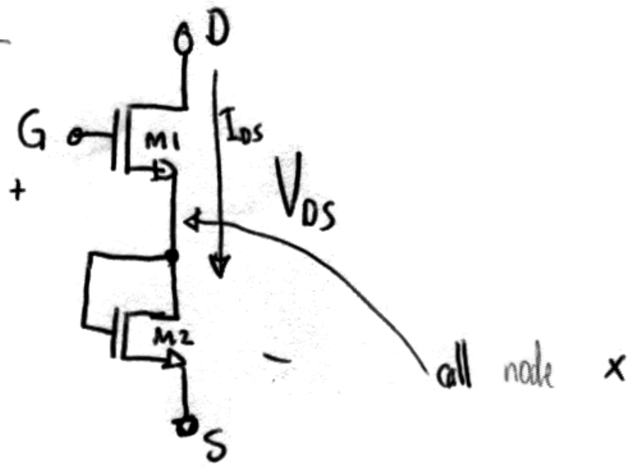
EE140 Homework #2, Solution Set

Problem 1



Cutoff mode

To be in cutoff either $V_{GS1} < V_T$ or $V_{GS2} < V_T$ (or bot).

V_{GS} $V_{GS1} + V_{GS2} \Rightarrow V_{GS} < 2 \cdot V_T$ ~~[illegible]~~ causes cutoff

Also, we know that $V_x < V_D \Rightarrow$ if $V_D < V_T$, M2 cutoff

$\Rightarrow$

> Cutoff happens when $V_{GS} < 2V_T$
> or $V_D < V_T$.
> $I_{CUTOFF} = \emptyset$

Saturation mode

For the "calsistor to be in saturation, BOTH M_1 & M_2 must in saturation $\Rightarrow V_{GS1} > V_T$ and $V_{GS2} > V_T$ and $V_{DS1} > V_{GS1} - V_T$

$\Rightarrow$

> conditions for saturation are:
>
> $V_{GS} > 2V_T$ and
>
> $V_{DS} > V_{GS} - V_T$

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saturation mode current:

Note that $I_{DS1} = I_{DS2} = I_{DS} \Rightarrow V_{DSAT1} = V_{DSAT2}$. Therefore

$V_{GS1} = V_T + V_{DSAT1}$ equals $V_{GS2} = V_T + V_{DSAT2}$

$$\left.\begin{aligned} I_{DS2} &= \tfrac{1}{2}k\tfrac{W}{L}(V_X - V_S - V_T)^2 \\ I_{DS1} &= \tfrac{1}{2}k\tfrac{W}{L}(V_G - V_X - V_T)^2 \end{aligned}\right\} \rightarrow I_{DS1} = I_{DS2} \Rightarrow (V_X - V_S - V_T) = (V_G - V_X - V_T)$$

So $V_X - V_S - V_T = V_G - V_X - V_T \Rightarrow V_X = \dfrac{V_G + V_S}{2}$

$$I_{DS} = I_{DS2} = \tfrac{1}{2}k\tfrac{W}{L}\left(\frac{V_G + V_S}{2} - V_S - V_T\right)^2 = \tfrac{1}{2}k\tfrac{W}{L}\left(\frac{V_G - V_S}{2} - V_T\right)^2$$

$$\boxed{I_{DS,SATURATION} = \tfrac{1}{2}k\tfrac{W}{L}\left(\frac{V_{GS}}{2} - V_T\right)^2}$$

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Linear mode: By process of elimination, linear mode occurs when

$$\boxed{\begin{array}{c} V_{GS} > 2V_T \\ V_T < V_{DS} < V_{GS} - V_T \end{array}}$$

In this mode: M1 is in linear and M2 is in saturation
(NOTE: M2 can NEVER be in linear mode as $V_{GS} = V_{DS}$.)

$$I_2 = \frac{k}{2}\frac{W}{L}\left(V_x - V_s - V_T\right)^2 \qquad \text{(eq 1)}$$

$$I_1 = k\frac{W}{L}\left(V_G - V_x - V_T - \frac{V_D - V_x}{2}\right)\left(V_D - V_x\right) \qquad \text{(eq 2)}$$

$$I_1 = I_2 \qquad \text{(eq 3)}$$

Plugging eq 1 & eq 2 into eq 3, we get

$$\frac{1}{2}\left(V_x - V_s - V_T\right)^2 = \left(V_G - V_T - \frac{V_D}{2} - \frac{V_x}{2}\right)\left(V_D - V_x\right)$$

without loss of generality, assume $V_s = 0$ and expand: (eq 4)

$$\frac{1}{2}\cancel{V_x^2} - V_x V_T + \frac{V_T^2}{2} = \left(V_G - V_T - \frac{V_D}{2}\right)V_D - \left(V_G - V_T - \frac{V_D}{2}\right)V_x - \frac{V_x V_D}{2} + \cancel{\frac{V_x^2}{2}}$$

Regrouping terms to solve for V_x:

$$\frac{V_T^2}{2} - \left(V_G - V_T - \frac{V_D}{2}\right)V_D = V_x\left(V_T - V_G + V_D + \frac{V_D}{2} - \frac{V_D}{2}\right) \qquad \text{(eq 5)}$$

or $$\frac{V_T^2}{2} - V_G - V_T - \frac{V_D}{2})V_D = -V_x\left(V_G - 2V_T\right. \qquad \text{(eq 6)}$$

Next page

Rearranging (eq 6):

$$V_x = \frac{\left(V_G - V_T - \frac{V_D}{2}\right)V_D - \frac{V_T^2}{2}}{V_G - 2V_T} \qquad \text{(eq -7)}$$

And, we know

$$I_{DS}\Big|_{V_S=0} = I_{DS1}\Big|_{V_S=0} = \frac{k}{2}\frac{W}{L}\left(V_x - V_T\right)^2\Big|_{V_S=0} \qquad \text{(eq 8)}$$

From (eq-7): $$V_x - V_T = \frac{-\frac{V_D^2}{2} + V_D V_G - V_D V_T - V_G V_T + \frac{3}{2}V_T^2}{V_G - V_T} \qquad \text{(eq 9)}$$

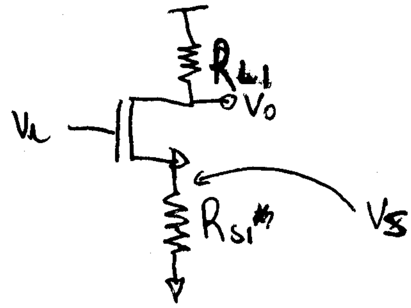
Plugging (9) into (8) we get:

$$I_{DS,Linear} = \frac{k}{2}\frac{W}{L}\left(\frac{-\frac{V_D^2}{2} + V_D V_G - V_D V_T - V_G V_T + \frac{3}{2}V_T^2}{V_G - V_T}\right)^2 \quad \text{if } V_S = 0$$

or more generally:

$$I_{DS,linear} = \frac{k}{2}\frac{W}{L}\left(\frac{-\frac{V_{DS}^2}{2} + V_{DS}V_{GS} - V_{DS}V_T - V_{GS}V_T + \frac{3}{2}V_T^2}{V_{GS} - V_T}\right)^2$$

P-2 1^{st} circuit:



(a) $V_o = 1.5V \Rightarrow I_{DS} = \frac{V_{DD} - V_o}{R_{L1}} = \frac{3V - 1.5V}{10k\Omega} = 150\mu A$

$V_S = I_{DS} \cdot R_{S1} = 150\mu A \cdot 2k\Omega = 300mV \Rightarrow V_{SB} = 300mV$

$V_T(V_{SB}) = V_{TO} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)$

$= 0.5V + 0.2\left(\sqrt{0.900V} - \sqrt{0.6V}\right) = 0.535V$

Now find required V_{GS}:

$\frac{1}{2}k\frac{W}{L}(V_{GS} - V_T)^2(1 + \lambda V_{DS}) = I_{DS}$ (assume saturation mode)

$\Rightarrow (V_{GS} - V_T)^2 = \frac{I_{DS}}{1 + \lambda \cdot V_{DS}} \cdot \frac{2}{k \cdot \frac{W}{L}}$

$\Rightarrow V_{GS} = V_T + \sqrt{\frac{2I_{DS}}{(1 + \lambda V_{DS}) \cdot k\frac{W}{L}}} = 0.535 + \sqrt{\frac{2 \cdot 150e\text{-}6}{[1 + 0.1 \cdot (1.5 - 0.3)] \cdot 8e\text{-}3}}$

$V_{GS} = 0.718V$

$V_{IN} = V_{GS} + V_S = 0.718V + 0.3V = \boxed{1.018V = V_{IN}}$

Check for validity of saturation mode:

$V_{GS} = 0.718V$

$V_{DS} = 1.2V$

Is $V_{GS} > V_T$? Yes!

Is $V_{DS} > V_{GS} - V_T$? Yes!

1st circuit, part b

$\boxed{I_{DS} = 150\mu A}$ (calculated in part a)

$\boxed{V_T = 0.535V}$ (calculated in part a)

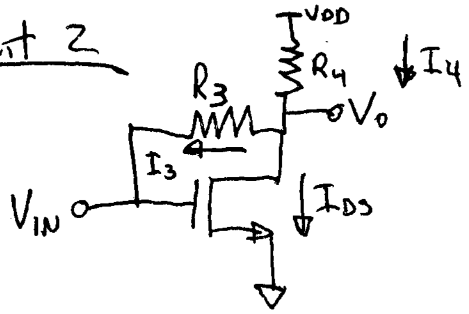
$V_{DSAT} = V_{GS} - V_T = 0.718V - 0.535V = \boxed{0.183V = V_{DSAT}}$

$g_m \triangleq k\frac{W}{L} V_{DSAT} = 8e\text{-}3\ A/V^2 \cdot 0.183V = \boxed{1.46e\text{-}3\ A/V = g_m}$

$g_{mbs} = g_m \cdot \frac{\gamma}{2\sqrt{2\phi_f + V_{SB}}} = 1.46e\text{-}3 \cdot \frac{0.2}{2\sqrt{0.6+0.3}} = \boxed{1.539e\text{-}4\ A/V = g_{mb}}$

$r_o = \frac{1}{\lambda I_D} = \frac{10V}{150\mu A} = 6.66e{+}4\ \frac{V}{A} = \boxed{66.66\ k\Omega = r_o}$

Circuit 2



(a) $V_o = 1.5 \Rightarrow I_4 = \dfrac{V_{DD} - V_o}{R_4} = \dfrac{1.5V}{10k\Omega} = 150\mu A$

$I_{DS} = \frac{1}{2} k \frac{W}{L} (V_{GS} - V_t)^2 (1 + \lambda V_{DS}) = 4 \, mA/V \cdot (V_{IN} - 0.5)^2 \cdot (1 + 0.1 \cdot 1.5)$

$= 4.6 \frac{mA}{V} \cdot (V_{IN} - 0.5)^2$

$I_3 = \dfrac{V_o - V_{IN}}{R_3} = \dfrac{\overset{1.5V}{\cancel{V_o}}}{10k} - \dfrac{V_{IN}}{10k}$

By KCL: $I_3 + I_{DS} = I_4$

$\Rightarrow \left(\dfrac{\overset{1.5V}{\cancel{V_o}}}{10k} - \dfrac{V_{IN}}{10k} \right) + 4.6e\text{-}3 \cdot (V_{IN} - 0.5)^2 = 150e\text{-}6$

Solving for $V_{IN} \Rightarrow \boxed{V_{IN} = 0.6156V}$ or ~~0.406V~~ less than V_T, hence invalid solution

Checking assumptions:

$I_{DS} = \frac{1}{2} k \frac{W}{L} (0.1156)^2 (1.15) = \boxed{61.5\mu A = I_{DS}}$

$I_3 = \dfrac{1.5 - 0.615}{10k\Omega} V = \boxed{88.5\mu A = I_3}$

$I_3 + I_{DS} = 61.5\mu A + 88.5\mu A = 150\mu A \overset{\checkmark}{=} I_4$

Saturation? $V_{GS} > V_T$? Yes

$V_{DS} > V_{GS} - V_T$? Yes

Circuit 2, part b

$$I_{DS} = \frac{1}{2} k' \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) = 4 mA/V^2 \cdot (0.1156V)^2 (1 + 0.1 \cdot 1.5V)$$

$$\boxed{I_{DS} = 61.5 \mu A}$$

$$\boxed{V_T = 0.5V}$$

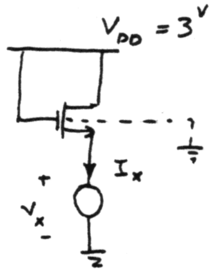
$$V_{DSAT} = V_{GS} - V_T = \boxed{0.1156V = V_{DSAT}}$$

$$g_m = \frac{2I_D}{V_{DSAT}} = \frac{123 \mu A}{0.1156V} = \boxed{1.064 \frac{mA}{V} = g_m}$$

$$g_{mbs} = g_m \cdot \frac{\gamma}{2\sqrt{2\phi_F}} = 1.064 \frac{mA}{V} \cdot \frac{0.2\sqrt{V}}{2\sqrt{0.6V}} = \boxed{0.1374 \frac{mA}{V} = g_{mbs}}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{10V}{61.5\mu A} = \boxed{162 k\Omega = r_o}$$

2.5) a)



$\lambda = 0.1$, $\gamma = 0.45$, $2\phi_F = 0.9$, $V_{TH0} = 0.7$

$V_{GS} = 3 - V_x$, $V_{DS} = 3 - V_x$, $V_{SB} = V_x$

$$V_{TH} = V_{TH0} + \gamma\left(\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F}\right)$$

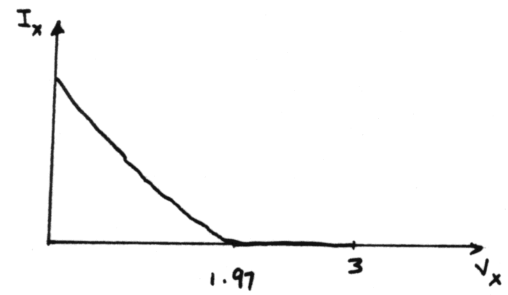
So, $$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(3 - V_x - 0.7 - 0.45\left(\sqrt{0.9 + V_x} - \sqrt{0.9}\right)\right)^2\left(1 + \lambda(3 - V_x)\right)$$

The above equation is valid for

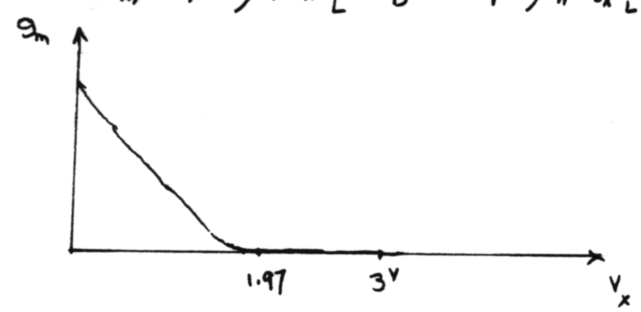
$3 - V_x - 0.7 - 0.45\left(\sqrt{0.9 + V_x} - \sqrt{0.9}\right) > 0$, i.e. $V_x < 1.97^V$

So, $$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(2.727 - V_x - 0.45\sqrt{0.9 + V_x}\right)^2(1.3 - 0.1V_x)$$

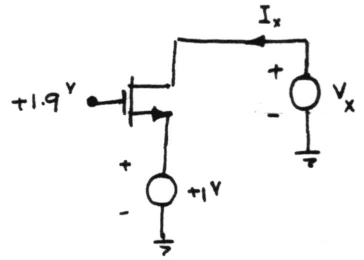
and $I_x = 0$ for $1.97 < V_x$



$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D} = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_x}$$



2.5) b)



$\lambda = \gamma = 0 \qquad V_{TH} = 0.7$

for $0 < V_x < 1$, S and D exchang their roles.

$V_{GS} = 1.9 - V_x \qquad V_{DS} = 1 - V_x \quad , V_{OD} = 1.2 - V_x$

$$I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[(1.2 - V_x)\times 2\times(1 - V_x) - (1 - V_x)^2\right]$$

$$I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(1 - V_x)(1.4 - V_x).$$

$$g_m = \mu_n C_{ox}\frac{W}{L}V_{DS} = \mu_n C_{ox}\frac{W}{L}(1 - V_x) \quad \text{(absolute Value)}$$

The above equations are valid for $V_x < 1$

Then the direction of current is reversed.

$V_{GS} = 1.9 - 1 = 0.9 \qquad V_{DS} = V_x - 1 \quad , V_{OD} = 0.9 - 0.7 = 0.2$

for $V_x < 1.2$, device operates in the triode region.

$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[2\times 0.2\times(V_x - 1) - (V_x - 1)^2\right]$$

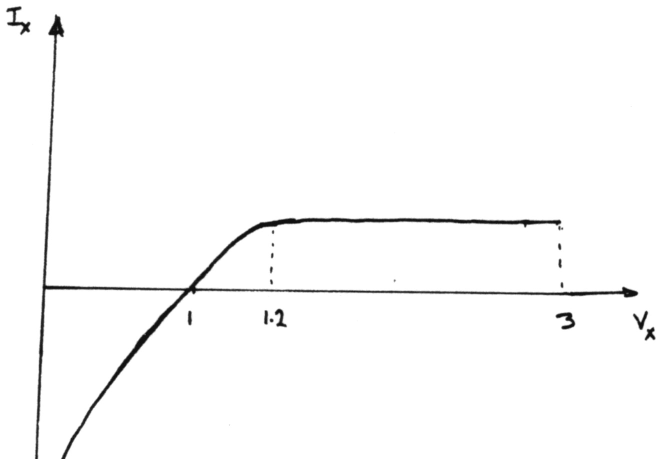
$$g_m = \mu_n C_{ox}\frac{W}{L}(V_x - 1)$$

for $V_x > 1.2$, Device goes into saturation region

2.5) b Cont

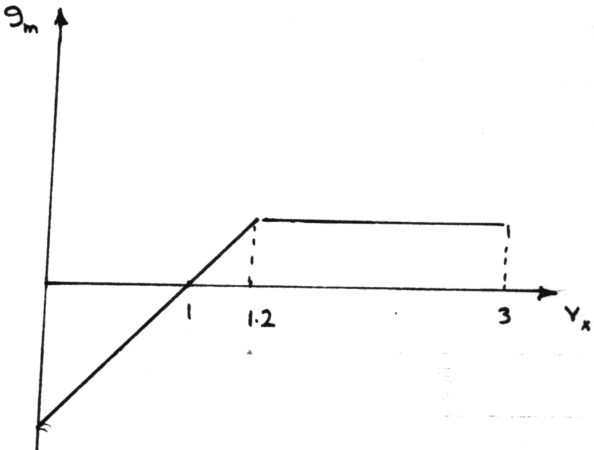
$$\text{So,} \quad I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.2)^2 ,$$

$$g_m = \mu_n C_{ox}\frac{W}{L}(0.2)$$



$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_x - 1)(1.4 - V_x) \; ; \; x < 1.2$$

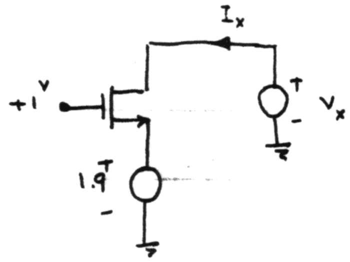
$$I_x = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.2)^2 \; ; \; x > 1.2$$



$$g_m = \mu_n C_{ox}\frac{W}{L}(V_x - 1) \; ; \; x < 1.2$$

$$g_m = \mu_n C_{ox}\frac{W}{L}(0.2) \; ; \; x > 1.2$$

2.5) c



$\lambda = \gamma = 0$ $\qquad V_{TH} = 0.7$

S and D exchange their roles.

$V_{GS} = 1 - V_x \qquad V_{DS} = 1.9 - V_x \qquad V_{OD} = V_{GS} - V_{TH} = 0.3 - V_x$

Device is in Saturation region, So, $I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.3 - V_x)^2$

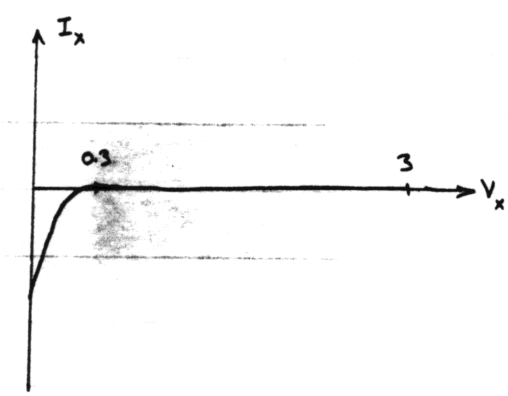
Device turns off when $V_x = 0.3$ and never turns on again.

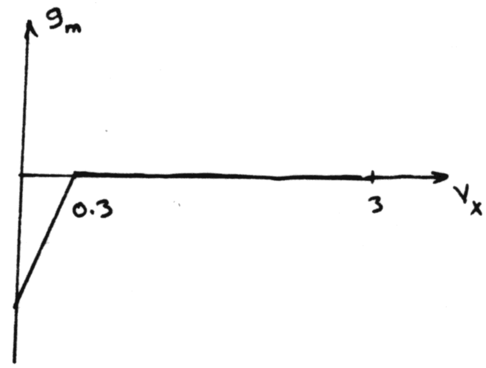
So, $\quad I_x = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(0.3 - V_x)^2 \quad ; x < 0.3$

$\qquad I_x = 0 \quad$; otherwise

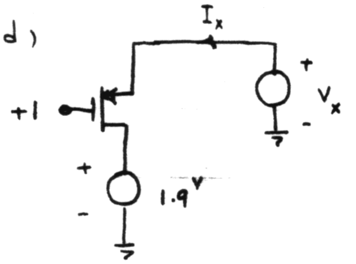
Then $\quad g_m = -\mu_n C_{ox}\frac{W}{L}(0.3 - V_x) \quad ; x < 0.3$

$\qquad g_m = 0 \quad$; Otherwise





2.5) d)



$V_{TH} = -0.8$ $\quad\gamma = 0$

D and S exchang their roles.

$V_{GS} = -0.9 \qquad V_{DS} = V_x - 1.9$

for $V_x < 1.8$:

$$I_x = -\frac{1}{2}\mu_p C_{ox}\frac{W}{L}(0.1)^2$$

$$g_m = -\mu_p C_{ox}\frac{W}{L}(0.1)$$

Device remains in the saturation region until

$V_x = 1.9 - 0.1 = 1.8$, then device goes into the triode

region.

for $1.8 < V_x < 1.9$:

$$I_x = -\mu_p C_{ox}\frac{W}{L}\left[(-0.1)(V_x - 1.9) - \frac{1}{2}(V_x - 1.9)^2\right]$$

$$g_m = +\mu_p C_{ox}\frac{W}{L}(V_x - 1.9)$$

for $V_x > 1.9$:

S and D exchange their roles again, when $V_x = 1.9$

for $V_x > 1.9$, Device operates in the triode region.

$V_{GS} = 1 - V_x \quad , \quad V_{DS} = 1.9 - V_x$

$$I_x = +\mu_p C_{ox}\frac{W}{L}\left[(1.8 - V_x)(1.9 - V_x) - \frac{1}{2}(1.9 - V_x)^2\right]$$

$$g_m = -\mu_p C_{ox}\frac{W}{L}(1.9 - V_x)$$

2.5) d so, $0 < V_x < 1.8$ $I_x = -\frac{1}{2}\mu_p C_{ox}\frac{W}{L}(0.1)^2$

$$g_m = -\mu_p C_{ox}\frac{W}{L}(0.1)$$

$1.8 < V_x < 3$ $I_x = +\mu_p C_{ox}\frac{W}{L}\times\frac{1}{2}(V_x-1.9)(V_x-1.7)$

$$g_m = \mu_p C_{ox}\frac{W}{L}(V_x-1.9)$$