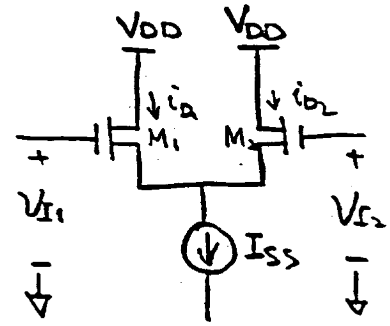
P24.1.

Solution: Large signal



① $i_{D1} + i_{D2} = I_{SS}$

② $v_{GS1} = V_{THN} + \sqrt{\frac{2 i_{D1}}{\beta_1}}$

$v_{GS2} = V_{THN} + \sqrt{\frac{2 i_{D2}}{\beta_2}}$

if, same size $\Rightarrow$ $\beta_1 = \beta_2 = \beta$

then $v_{I1} - v_{I2} = v_{GS1} - v_{GS2} = \sqrt{\frac{2 i_{D1}}{\beta}} - \sqrt{\frac{2 i_{D2}}{\beta}} = \sqrt{\frac{2}{\beta}}\left(\sqrt{i_{D1}} - \sqrt{i_{D2}}\right)$

③ Solving:

$$\begin{cases} i_{D1} + i_{D2} = I_{SS} & \text{(a)} \\ \sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{\frac{\beta}{2}}(v_{I1} - v_{I2}) & \text{(b)} \end{cases}$$

From (b), square both side

$$i_{D1} - 2\sqrt{i_{D1} i_{D2}} + i_{D2} = \frac{\beta}{2}(v_{I1} - v_{I2})^2$$

Note: $i_{D1} + i_{D2} = I_{SS}$

$$\Rightarrow \quad 2\sqrt{i_{D1} + i_{D2}} = I_{SS} - \frac{\beta}{2}(v_{I1} - v_{I2})^2 \qquad \text{(c)}$$

(a) + (c):

$$\Rightarrow \left(\sqrt{i_{D1}} + \sqrt{i_{D2}}\right)^2 = 2 I_{SS} - \frac{\beta}{2}(v_{I1} - v_{I2})^2$$

or $$\sqrt{i_{D1}} + \sqrt{i_{D2}} = \sqrt{2 I_{SS} - \frac{\beta}{2}(v_{I1} - v_{I2})^2} \qquad \text{(d)}$$

(b) + (d)

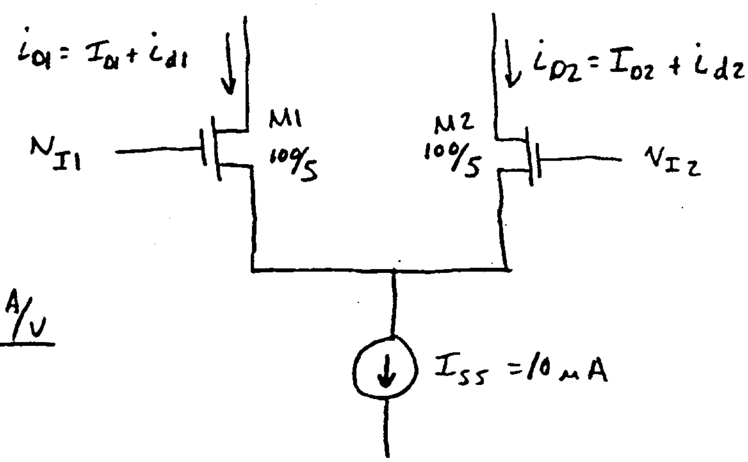
$$\Rightarrow 2\sqrt{i_{D1}} = \sqrt{\frac{\beta}{2}}(v_{I1} - v_{I2}) + \sqrt{2 I_{SS} - \frac{\beta}{2}(v_{I1} - v_{I2})^2}$$

or $$4 i_{D1} = \frac{\beta}{2}(v_{I1} - v_{I2})^2 + 2\sqrt{\frac{\beta}{2}}(v_{I1} - v_{I2})\sqrt{2 I_{SS} - \frac{\beta}{2}(v_{I1} - v_{I2})^2} + 2 I_{SS} - \frac{\beta}{2}(v_{I1} - v_{I2})^2$$

$$\Rightarrow \quad 4 i_{D1} = 2 I_{SS} + 2\sqrt{I_{SS}\,\beta(v_{I1} - v_{I2})^2 - \frac{\beta^2}{4}(v_{I1} - v_{I2})^4}$$

24.2

Repeat Ex. 24.1 if the widths of M1 & M2 are increased to 100 um. Determine the transconductance of the diff-amp. Write i_{d2} as a product of $g_m (= g_{m1} = g_{m2})$ and v_{I1} (with v_{I2} = AC ground), v_{I2} (with v_{I1} = AC ground), and $v_{I1} - v_{I2}$.



$$v_{DIMAX} = \sqrt{\frac{2 I_{SS}}{\beta}} = \sqrt{\frac{2 \cdot 10\mu}{50\mu \cdot \frac{100}{5}}} = \underline{141\,mV}$$

$$G_m = \frac{\sqrt{2\beta I_{SS}}}{4} = \frac{\sqrt{2 \cdot 50\mu(\frac{100}{5})(10\mu)}}{4} = \underline{35.3\,\mu A/V}$$

$$g_m = 4\,G_m = \underline{141.4\,\mu A/V}$$

- when $v_{I2} = 0$

$$i_{d2} = g_{m2} v_{gs2} = g_m\left(\frac{-v_{DI}}{2}\right) = \underline{-g_m \frac{v_{I1}}{2}}$$

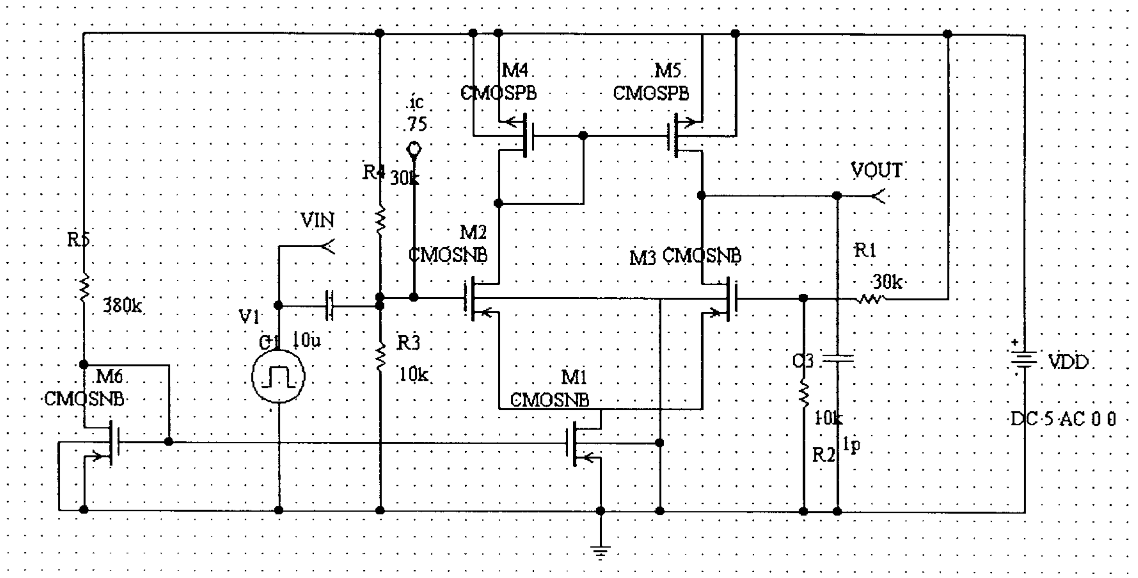
- when $v_{I1} = 0$

$$i_{d2} = g_{m2} v_{gs2} = g_m\left(\frac{v_{DI}}{2}\right) = \underline{g_m \frac{v_{I2}}{2}}$$

- In terms of $v_{I1} - v_{I2}$

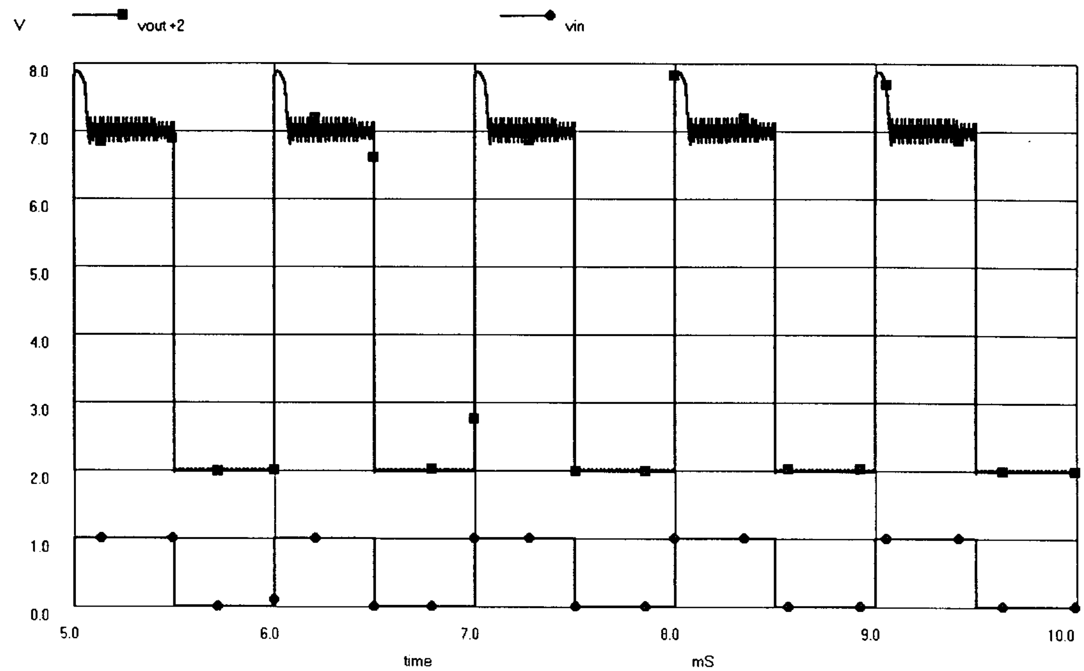
$$i_{d2} = g_{m2} v_{gs2} = g_m \frac{-v_{DI}}{2} = \underline{-g_m \frac{v_{I1} - v_{I2}}{2}}$$

Problem 24.3 and 24.4




```
*** Top Level Netlist ***
.ic     V(5)=.75
C1      VIN 5 10u
C3      0 VOUT 1p  IC=0
M1      1 2 0 0 CMOSNB  L=5u W=30u
M2      4 5 1 0 CMOSNB  L=5u W=15u
M3      VOUT 6 1 0 CMOSNB  L=5u W=15u
M4      4 4 7 7 CMOSPB  L=5u W=70u
M5      VOUT 4 7 7 CMOSPB  L=5u W=70u
M6      2 2 0 0 CMOSNB  L=5u W=15u
R1      6 7 30k
R2      0 6 10k
R3      5 0 10k
R4      7 5 30k
R5      7 2 380k
V1      VIN 0   DC 0 AC 0 0 PULSE(0 1 0 100n 100n 500u 1m)
VDD     7 0     DC 5 AC 0 0

***** Spice models and macro models *****

.MODEL CMOSNB NMOS LEVEL=4

.MODEL CMOSPB PMOS LEVEL=4

***** End of spice models and macro models *****

.OPTION ABSTOL=10u RELTOL=0.01 VNTOL=10mv
.tran 10u 10m 5m 10u uic
.end
```

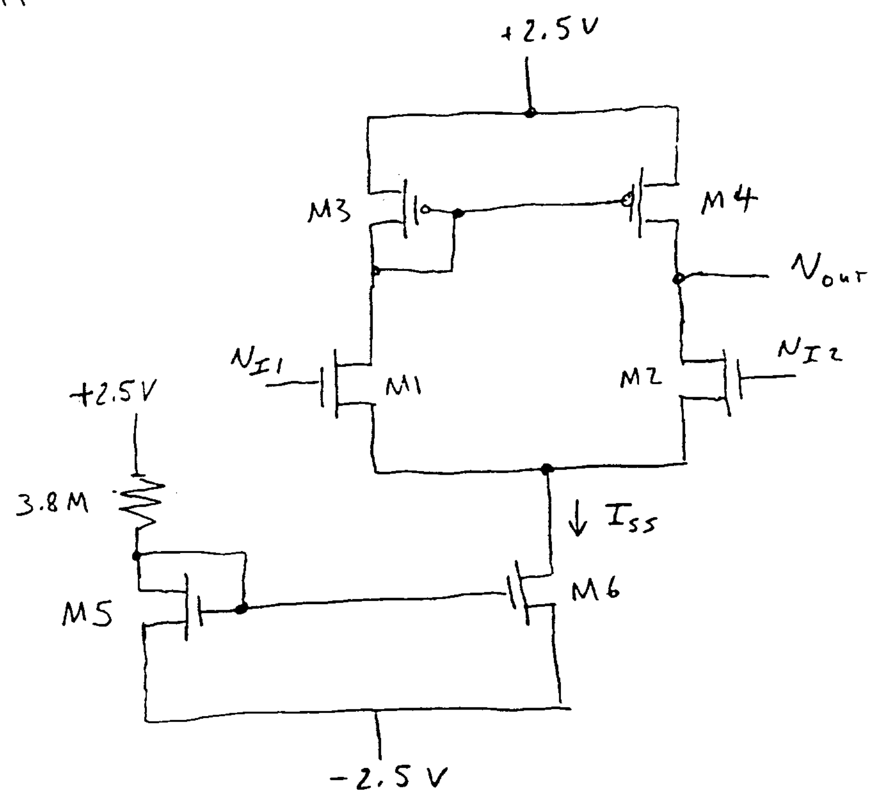
The SPICE simulation result is shown below:



24.5

Determine the small signal gain and the input common-mode-range (CMR) for the diff-amps show in Fig. P24.5. All n-channels are 15/5, all p-channels are 70/5, and the resistors are 3.8 MEG.

(a) First Circuit



- Find DC current I_{SS}

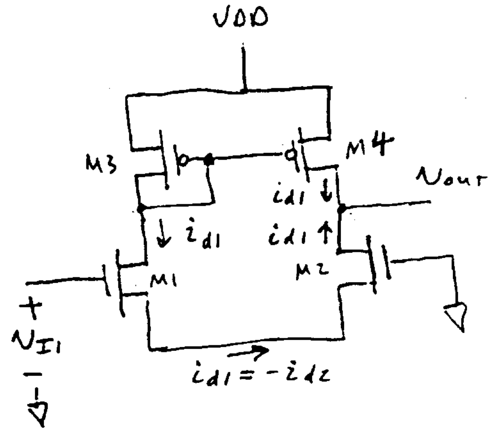
$$\frac{5.0 - V_{GS5}}{3.8(10)^6} = \frac{50\mu}{2}\left(\frac{15}{5}\right)\left(V_{GS5} - .83\right)^2$$

$$0 = V_{GS5}^2 - 1.6635\, V_{GS5} + .67136$$

$$V_{GS5} = .97478\ V$$

$$\therefore\ I_{SS} = \underline{1.06\ \mu A}$$

– Determine the small signal gain



AC currents

$$v_{out} = 2 i_{d1} (r_{o2} \| r_{o4})$$

$$v_{in} = v_{gs1} - v_{gs2}$$

$$= i_{d1} \frac{1}{g_{m1}} - i_{d2} \frac{1}{g_{m2}}$$

$$= 2 i_{d1} \frac{1}{g_{mn}}$$

$$A_v = \frac{v_{out}}{v_{in}} = \frac{\cancel{2 i_{d1}} (r_{o2} \| r_{o4})}{\cancel{2 i_{d1}} \frac{1}{g_{mn}}}$$

$$A_v = g_{mn} (r_{o2} \| r_{o4})$$

$$g_{mn} = \sqrt{2(50\mu)(3)\frac{1.06\mu}{2}} = 12.61\ \mu A/V$$

$$r_{o1} = r_{o2} = \frac{1}{.06\left(\frac{1.06\mu}{2}\right)} = 31.45\ M\Omega$$

$$\underline{A_v = 198\ V/V}$$

– Determine the CMR

The maximum v_I is limited by M1 + M2 going into the triode region of operation.

$$V_{IMAX} = V_{DD} - \left[\sqrt{\frac{I_{SS}}{B_3}} + V_{THP}\right] + V_{THN}$$

$$= 2.5 - \left[\sqrt{\frac{1.06\mu}{17\mu\left(\frac{70}{3}\right)}} + .91\right] + .83$$

$$= \underline{2.35\ V}$$

The minimum v_I is limited by M6 entering the triode region of operation

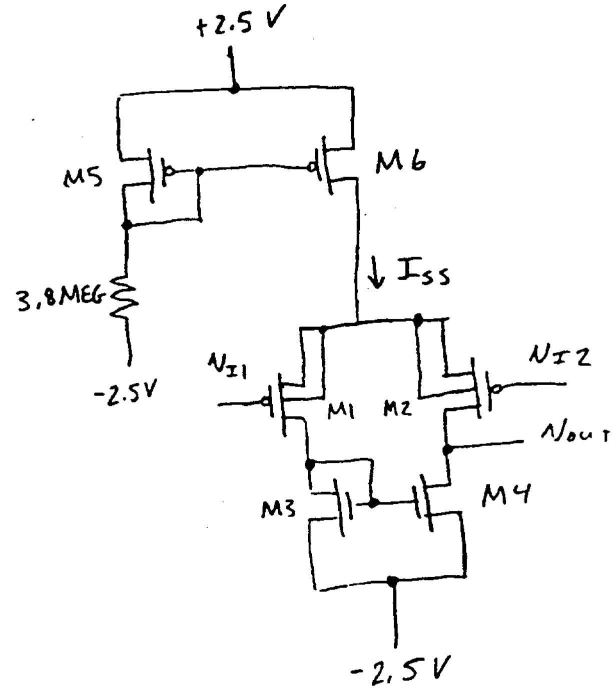
$$V_{IMIN} = V_{GS1} + V_{DS6} + V_{SS}$$

$$= \sqrt{\frac{I_{SS}}{B_1}} + V_{THN} + \sqrt{\frac{2 I_{SS}}{B_6}} + V_{SS}$$

$$= \sqrt{\frac{1.06\mu}{50\mu(3)}} + .83 + \sqrt{\frac{2(1.06\mu)}{50\mu(3)}} - 2.5$$

$$= -1.47\ V$$

(b) Second Circuit



- Determine the DC currents

$$\frac{5 - V_{SG3}}{3.8(10^6)} = \frac{17\mu}{2}\left(\frac{20}{5}\right)(V_{SG3} - .91)^2$$

$$0 = V_{SG}{}^3 - 1.818\, V_{SG3} + .81704$$

$$V_{SG3} = 1.004\ V$$

$$\underline{I_{SS} = 1.05\,\mu A}$$

- Determine the small signal gain

$$V_{OUT} = -2\, i_{d1}\,(r_{o2} \| r_{o4})$$

$$V_{IN} = -i_{d1} \cdot \frac{1}{g_{m1}} - i_{d2} \cdot \frac{1}{g_{m2}}$$

$$V_{IN} = -2\, i_{d1} \cdot \frac{1}{g_{mp}}$$

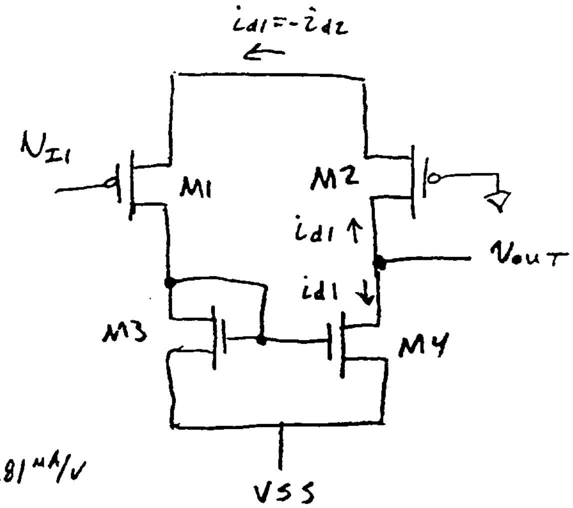
$$A_V = \frac{V_{OUT}}{V_{IN}} = \frac{\cancel{-2 i_{d1}}\,(r_{o2} \| r_{o4})}{\cancel{-2 i_{d1}} \cdot \frac{1}{g_{mp}}}$$

$$A_V = g_{mp}\,(r_{o2} \| r_{o4})$$

$$g_{mp} = \sqrt{2(17\mu)(14)\left(\frac{1.05\mu}{2}\right)} = 15.81\ \mu A/V$$

$$r_{o2} = r_{o4} = \frac{1}{(.06)\left(\frac{1.05\mu}{2}\right)} = 31.75\ MEG$$

$$\underline{A_V = 251\ V/V}$$



AC currents

- Determine the CMR

The maximum V_I is limited by M6 entering the triode region.

$$V_{IMAX} = -V_{SG1} - V_{SD6} + V_{DD}$$

$$= -\left(\sqrt{\frac{I_{SS}}{B_1}} + V_{THP}\right) - \sqrt{\frac{2I_{SS}}{B_6}} + V_{DD}$$

$$= -\left(\sqrt{\frac{1.05\mu}{17\mu(14)}} + .91\right) - \sqrt{\frac{2(1.05\mu)}{17\mu(14)}} + 2.5$$

$$= \underline{1.43\ V}$$

The minimum V_I is limited by M1 & M2 entering the triode region

$$V_{IMIN} = V_{D1} - V_{THP}$$

$$= \left[\sqrt{\frac{I_{SS}}{B_3}} + V_{THN}\right] + V_{SS} - V_{THP}$$

$$= \left[\sqrt{\frac{1.05\mu}{50\mu(3)}} + .83\right] - 2.5 - .91$$

$$= \underline{-2.50\ V}$$

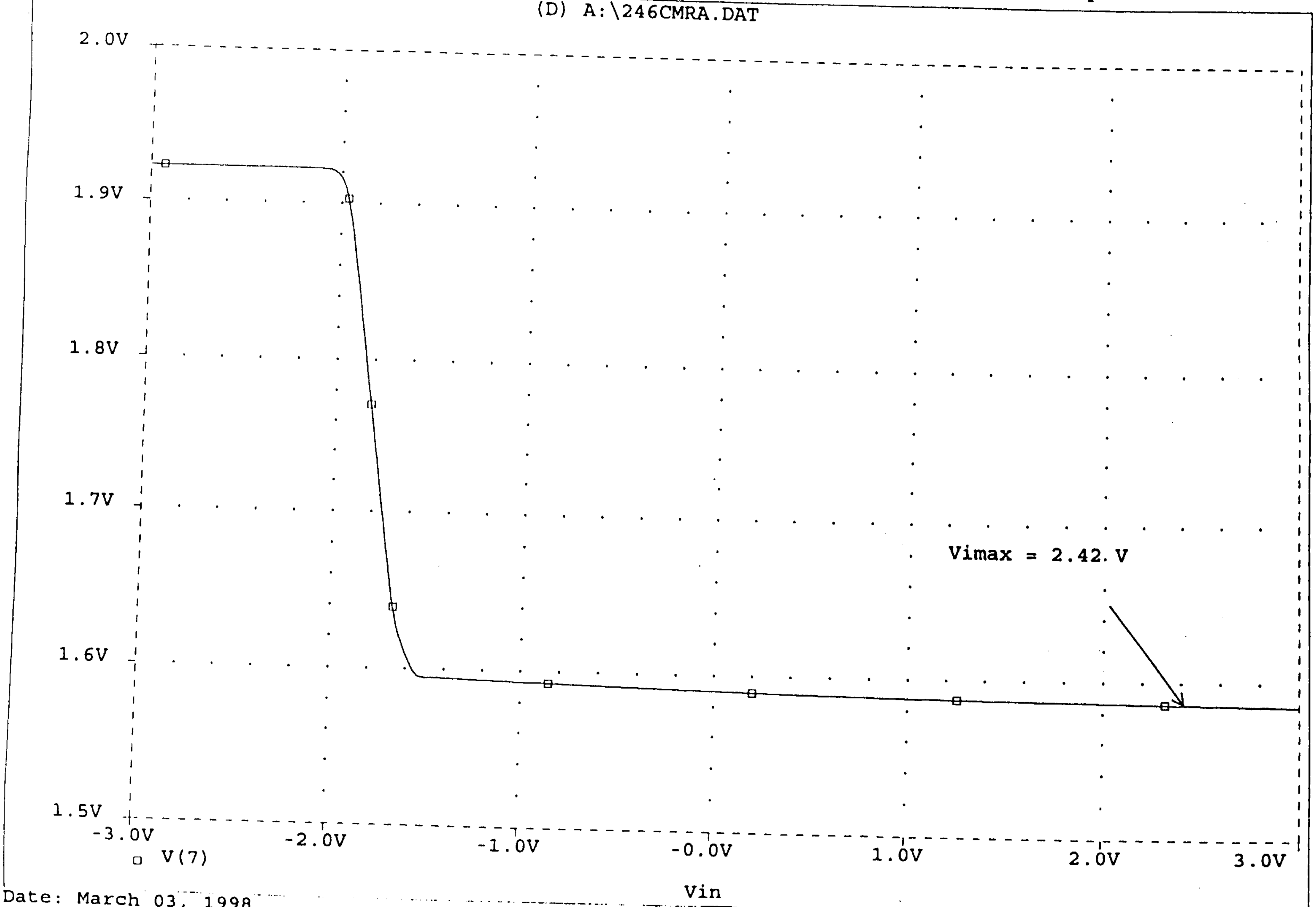
Problem 24.6

```
* Prob 24.6 (Gain simulation for circuit b)

R1 2 3  3.8E6
M1 6 5 4 4 CMOSPB L=5U W=70U
M2 8 0 4 4 CMOSPB L=5U W=70U
M3 6 6 3 3 CMOSNB L=5U W=15U
M4 8 6 3 3 CMOSNB L=5U W=15U
M5 2 2 1 1 CMOSPB L=5U W=70U
M6 4 2 1 1 CMOSPB L=5U W=70U
VDD 1 0 DC 2.5
VSS 3 0 DC -2.5
Vin 5 0 DC 0

* BSIM model for n-channel CN20
.MODEL CMOSNB NMOS LEVEL=4

* BSIM model for p-channel CN20
.MODEL CMOSPB PMOS LEVEL=4

.probe
.DC Vin -.2 .2 .001
.end
```

```
* Prob 24.6 (CMR simulation for circuit b)

R1 2 3  3.8E6
M1 6 5 4 4 CMOSPB L=5U W=70U
M2 8 5 4 4 CMOSPB L=5U W=70U
M3 6 6 3 3 CMOSNB L=5U W=15U
M4 8 6 3 3 CMOSNB L=5U W=15U
M5 2 2 1 1 CMOSPB L=5U W=70U
M6 4 2 1 1 CMOSPB L=5U W=70U
VDD 1 0 DC 2.5
VSS 3 0 DC -2.5
Vin 5 0 DC 0

* BSIM model for n-channel CN20
.MODEL CMOSNB NMOS LEVEL=4

* BSIM model for p-channel CN20
.MODEL CMOSPB PMOS LEVEL=4

.probe
.DC Vin -3 3 .01
.end
```

* Problem 24.6 (Simulation for CMR of circuit a)
Date/Time run: 03/03/98 17:25:28

Temperature: 27.0

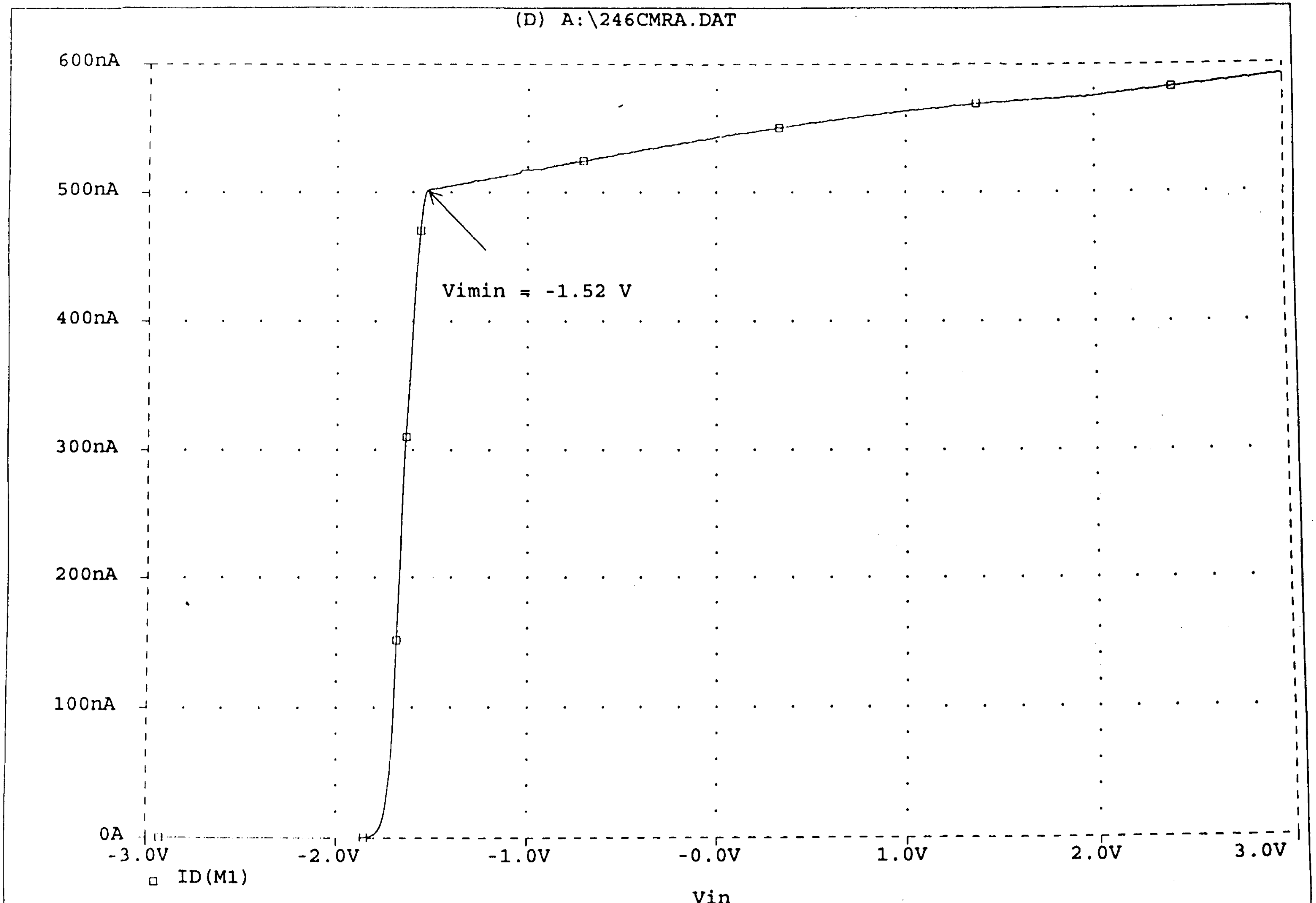


Date: March 03, 1998

Time: 17:31:08

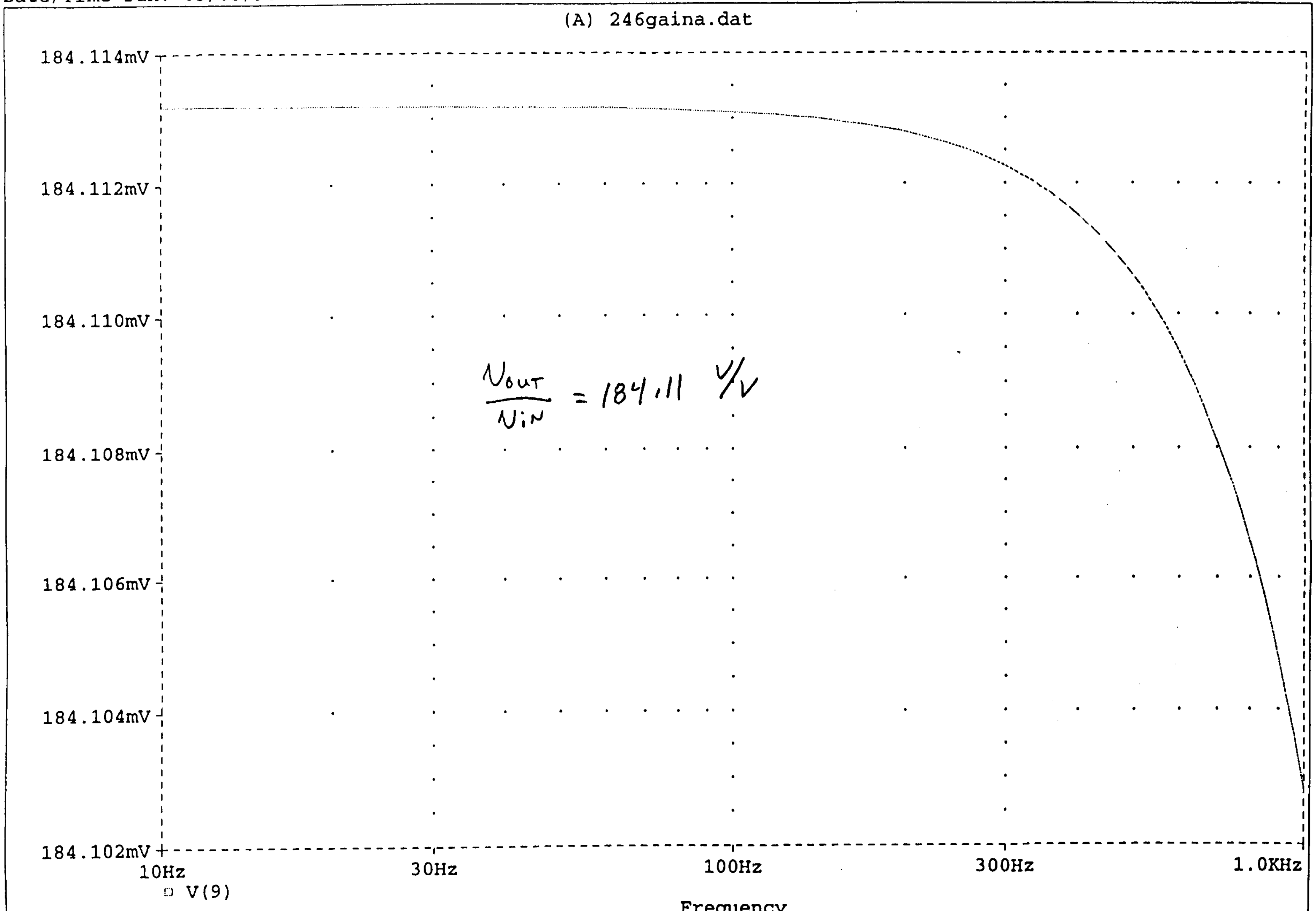
* Problem 24.6 (Simulation for CMR of circuit a)

Date/Time run: 03/03/98 17:25:28 Temperature: 27.0



* Problem 24.6 (Gain simulation for circuit a)

Date/Time run: 03/05/98 10:00:24 Temperature: 27.0



$\frac{V_{OUT}}{V_{IN}} = 184.11$ V/V

Date: March 05, 1998 Page 1 Time: 10:05:14

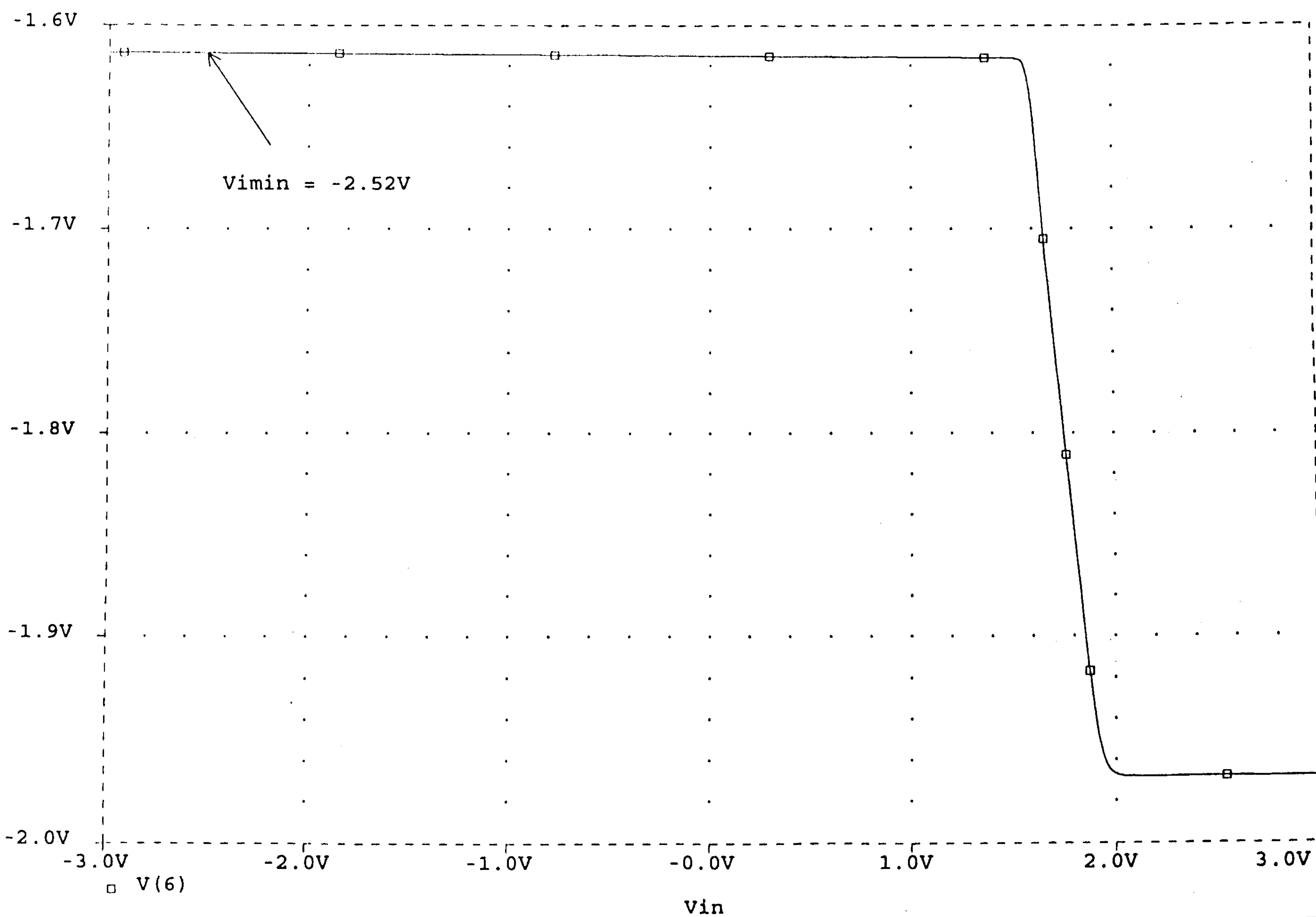
```
* Problem 24.6 (Gain simulation for circuit a)

R1 10 3  3.8E6
M5 3 3 4 4  CMOSNB L=5U W=15U
M6 5 3 4 4  CMOSNB L=5U W=15U
M1 7 6 5 4  CMOSNB L=5U W=15U
M2 9 0 5 4  CMOSNB L=5U W=15U
M3 7 7 10 10 CMOSPB L=5U W=70U
M4 9 7 10 10 CMOSPB L=5U W=70U
VDD 10 0 DC 2.5
VSS 4  0 DC -2.5
Vin 6 0 DC 0

* BSIM model for n-channel CN20
.MODEL CMOSNB NMOS LEVEL=4

* BSIM model for p-channel CN20
.MODEL CMOSPB PMOS LEVEL=4
.probe
.DC Vin -.2 .2 .001
.end
```

```
* Problem 24.6 (Simulation for CMR of circuit a)

R1 10 3  3.8E6
M5 3 3 4 4  CMOSNB L=5U W=15U
M6 5 3 4 4  CMOSNB L=5U W=15U
M1 7 6 5 4  CMOSNB L=5U W=15U
M2 9 6 5 4  CMOSNB L=5U W=15U
M3 7 7 10 10 CMOSPB L=5U W=70U
M4 9 7 10 10 CMOSPB L=5U W=70U
VDD 10 0 DC 2.5
VSS 4  0 DC -2.5
Vin 6 0 DC 0

* BSIM model for n-channel CN20
.MODEL CMOSNB NMOS LEVEL=4

* BSIM model for p-channel CN20
.MODEL CMOSPB PMOS LEVEL=4

.probe
.DC Vin -3 3 .01
.end
```

* Prob 24.6 (CMR simulation for circuit b)

Date/Time run: 03/03/98 16:58:54 Temperature: 27.0

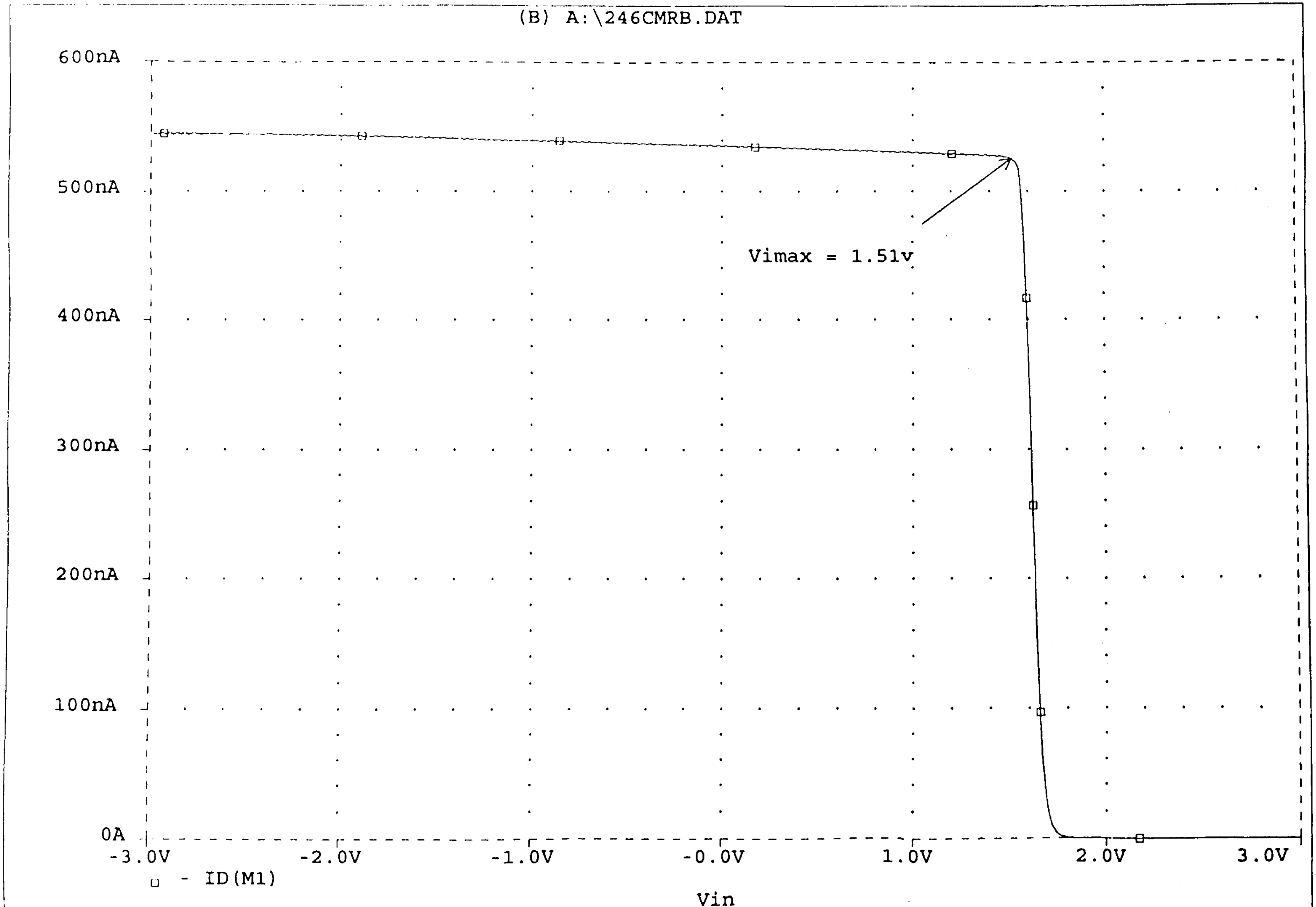
(B) A:\246CMRB.DAT



Date: March 03, 1998 Page 1 Time: 17:15:18

* Prob 24.6 (CMR simulation for circuit b)

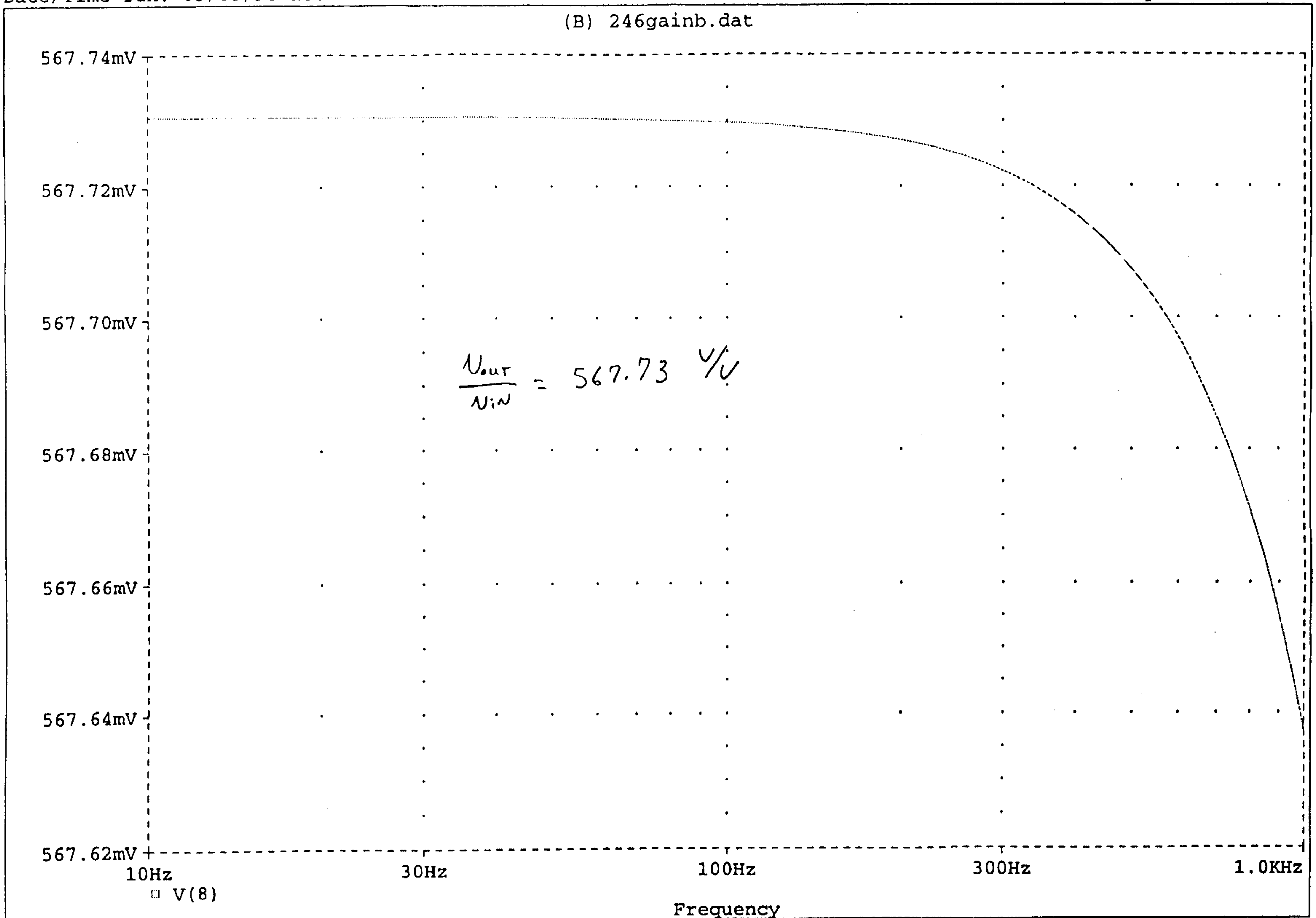
Date/Time run: 03/03/98 16:58:54 Temperature: 27.0



Date: March 03, 1998 Page 1 Time: 17:04:21

* Prob 24.6 (Gain simulation for circuit b)

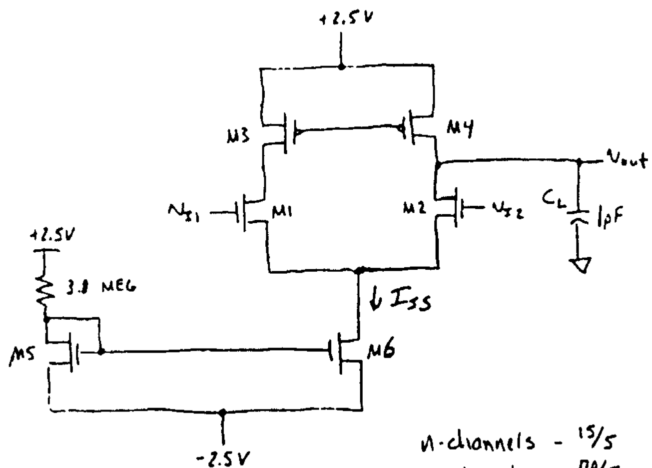
Date/Time run: 03/05/98 10:07:15 Temperature: 27.0



Date: March 05, 1998 Page 1 Time: 10:08:39

24.7

Estimate the slew-rate limitations in charging and discharging a 1pF capacitor tied to the outputs of the diff-amps shown in Fig. P24.5.



n-channels - 15/5
p-channels - 70/5

$$SR_{MAX} = \frac{dV}{dt} = \frac{I_{SS}}{C_L}$$

$$I_{SS} = \frac{5 - V_{GSS}}{3.8(10)^6} = \frac{50\mu}{2}\,\frac{15}{5}(V_{GSS} - .83)^2$$

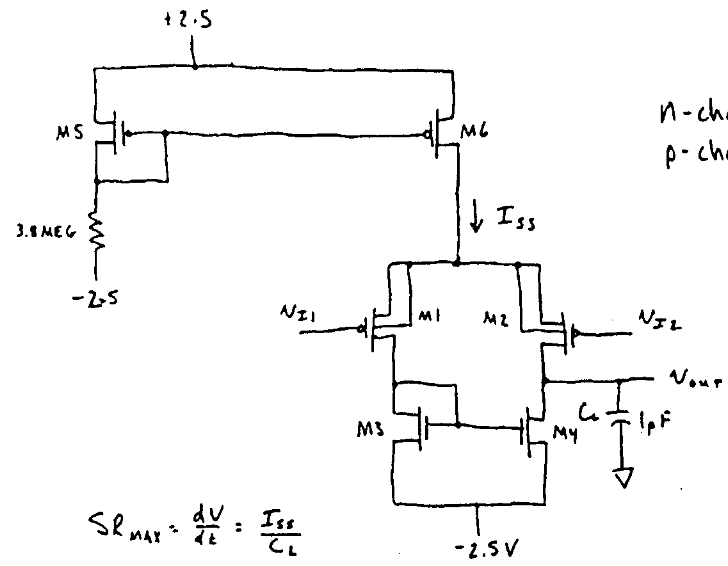
$$\frac{5 - V_{GSS}}{285} = V_{GSS}^2 - 1.66\,V_{GSS} + .6889$$

$$0 = V_{GSS}^2 - 1.6565\,V_{GSS} + .67136$$

$$V_{GSS} = 0.949 \text{ V}$$

$$I_{SS} = \frac{5 - .949}{3.8(10)^6} = 1.066\ \mu A$$

$$SR_{MAX} = \frac{1.066\mu}{1p} = \underline{1.066\ V/\mu s}$$



n-channels - 15/5
p-channels - 70/5

$$SR_{MAX} = \frac{dV}{dt} = \frac{I_{SS}}{C_L}$$

$$I_{SS} = \frac{5 - V_{SGS}}{3.8(10)^6} = \frac{17\mu}{2}\,\frac{70}{5}(V_{SGS} - .91)^2$$

$$\frac{5 - V_{SGS}}{452.2} = V_{SGS}^2 - 1.82\,V_{SGS} + .8281$$

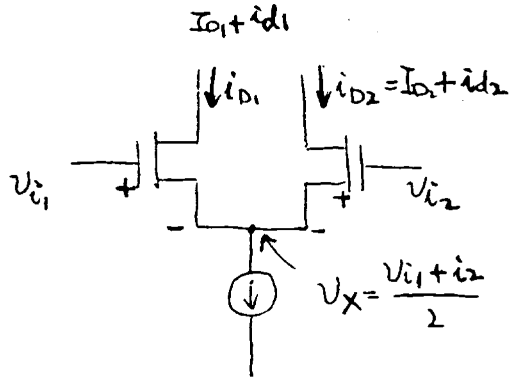
$$0 = V_{SGS}^2 - 1.8179\,V_{SGS} + .81704$$

$$V_{SGS} = 1.0046 \text{ V}$$

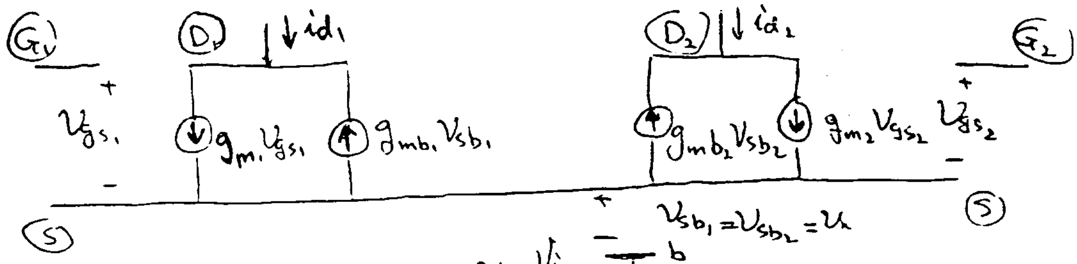
$$I_{SS} = \frac{5 - 1.0046}{3.8(10)^6} = 1.051\ \mu A$$

$$SR_{MAX} = \frac{1.051\mu}{1p} = \underline{1.051\ V/\mu s}$$

P4.8 ANS:



Small signal Model:



$$v_{gs1}=v_{i1}-v_x=v_{i1}-\frac{v_{i1}+v_{i2}}{2}=\frac{v_{i1}-v_{i2}}{2};\qquad v_{gs2}=v_{i2}-v_x=v_{i2}-\frac{v_{i1}+v_{i2}}{2}=-\frac{v_{i1}-v_{i2}}{2};$$

$$v_{sb1}=v_{sb2}=v_x=\frac{v_{i1}+v_{i2}}{2}.$$

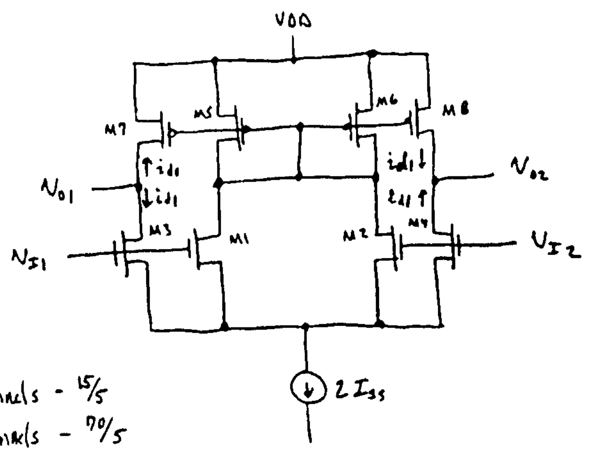
$$g_{m1}=g_{m2}=g_m,\qquad g_{mb1}=g_{mb2}=g_{mb}$$

$$i_{d1}=g_{m1}v_{gs1}-g_{mb1}v_{sb1}=g_m\frac{v_{i1}-v_{i2}}{2}-g_{mb}\frac{v_{i1}+v_{i2}}{2}=\boxed{\frac{g_m}{2}(v_{i1}-v_{i2})-\frac{g_{mb}}{2}(v_{i1}+v_{i2})}$$

$$i_{d2}=g_{m2}v_{gs2}-g_{mb2}v_{sb2}=g_m\left(-\frac{v_{i1}-v_{i2}}{2}\right)-g_{mb}\frac{v_{i1}+v_{i2}}{2}=\boxed{-\frac{g_m}{2}(v_{i1}-v_{i2})-\frac{g_{mb}}{2}(v_{i1}+v_{i2})}$$

24.9

The diff-amp configuration shown in Fig. P24.9 is useful in situations where a truly differential output is needed. Determine the following:



all n-channels - 15/5
all p-channels - 70/5

(a) The transconductance of the diff-amp.

$$G_m = \frac{g_{mn}}{4}$$

(b) The drain currents of all MOSFETs in terms of the input voltages and g_{mn}.

$$i_{d1} = g_{mn} v_{gs1} = g_{mn} \frac{v_{I1} - v_{I2}}{2}$$

$$i_{d2} = g_{mn} v_{gs2} = g_{mn} \frac{v_{I2} - v_{I1}}{2}$$

$$i_{d3} = g_{mn} v_{gs3} = g_{mn} \frac{v_{I1} - v_{I2}}{2}$$

$$i_{d4} = g_{mn} v_{gs4} = g_{mn} \frac{v_{I2} - v_{I1}}{2}$$

(c) The small-signal voltage gain, $(v_{O1} - v_{O2})/(v_{I1} - v_{I2})$

$$v_{O2} = (i_{d1} + i_{d1})(r_{on} \| r_{op})$$

$$v_{O1} = -(i_{d1} + i_{d1})(r_{on} \| r_{op})$$

$$i_{d1} = g_{mn} v_{gs1} = g_{mn} \frac{v_{I1} - v_{I2}}{2}$$

$$v_{O2} = g_{mn}(v_{I1} - v_{I2})(r_{on} \| r_{op})$$

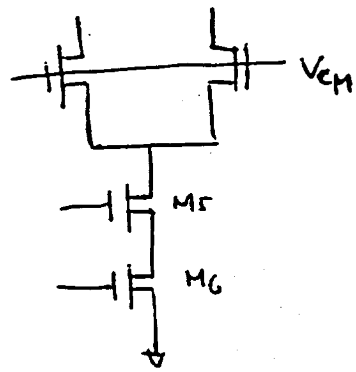
$$v_{O1} = -g_{mn}(v_{I1} - v_{I2})(r_{on} \| r_{op})$$

$$\frac{v_{O1} - v_{O2}}{v_{I1} - v_{I2}} = \underline{-2 g_{mn}(r_{on} \| r_{op})}$$

P24.10

Now. $R_{out} = r_{o5}(1+g_{m5} r_{o6})$

replace r_{o6} by R_{out} in equ. (24.36).



$$CMMR = 20 \log \left| 2 g_{m1} g_{m4} (r_{o2} \| r_{o4}) \times r_{o5}(1+g_{m5} r_{o6}) \right|$$

Problem 24.11

∵ Current source output impedance can be modeled by a simple capacitor,

Assume $R_{out} = \frac{1}{C_{out} \cdot S}$, where R_{out} is the output impedance of the current source.

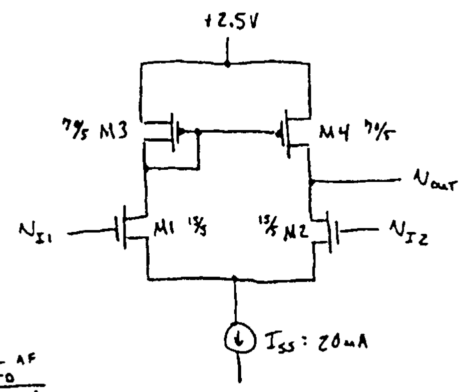
$$A_v = g_{m1} (r_{o2} // r_{o4}), \quad A_c = - \frac{1}{2 g_{m4} \frac{1}{C_{out} \cdot S}} = - \frac{C_{out} \cdot S}{2 g_{m4}}$$

$$\Longrightarrow CMRR = 20 \log \left| \frac{A_v}{A_c} \right| = 20 \log \left| \frac{2 g_{m1} g_{m4} (r_{o2} // r_{o4})}{C_{out} \cdot S} \right|$$

$$\Longrightarrow CMRR(j\omega) = 20 \log \left| \frac{2 g_{m1} g_{m4} (r_{o2} // r_{o4})}{C_{out} \cdot j\omega} \right|$$

24.12

Determine the RMS input noise voltage for the diff-amp of Ex. 24.2 over a bandwidth of 1 to 1 kHz. Assume $KF = 10^{-25}\ V^2F$ and $AF = 1.3$.



$$\overline{i_{n1}^2} = \overline{i^2_{therm}} + \overline{i^2_{1/f}}$$

$$= 6kT\sqrt{2\beta I_D} + \frac{KF \cdot I_D^{AF}}{f \cdot C'_{ox} \cdot L^2}$$

$k = 1.38 \times (10)^{-23}$

$T: 300\ ^oK$

$$= 6(1.38(10)^{-23})(300)\sqrt{2(50u)(\tfrac{15}{5})(10u)} + \frac{(10^{-25})(10u)^{1.3}}{f\ 800(10)^{-18} \cdot (5)^2}$$

$$= 1.3605 \cdot (10)^{-24} + \frac{1.5811(10)^{-18}}{f}$$

$$\overline{i_{n2}^2} = \overline{i_{n1}^2} = 1.3605(10)^{-24} + \frac{1.5811(10)^{-18}}{f}$$

$$\overline{i_{n3}^2} = \overline{i^2_{therm}} + \overline{i^2_{1/f}}$$

$$= 6(1.38 \cdot (10)^{-23})(300)\sqrt{2(50u)(\tfrac{70}{5})(10u)} + \frac{(10^{-25})(10u)^{1.3}}{f \cdot 800(10)^{-18} \cdot (5)^2}$$

$$= 2.939 \cdot (10)^{-24} + \frac{1.5811 \cdot (10)^{-18}}{f}$$

$$\overline{i_{n4}^2} = \overline{i_{n3}^2} = 2.939(10)^{-24} + \frac{1.5811 \cdot (10)^{-18}}{f}$$

$$\overline{v_{T,in}^2} = \int_{f_L}^{f_H} \frac{1}{g_{m1}^2}\left(\overline{i_{n1}^2} + \overline{i_{n2}^2} + \overline{i_{n3}^2} + \overline{i_{n4}^2}\right) df$$

$$g_{m1}^2 = 2\beta I_D = 2(50u)(\tfrac{15}{5})(10u) = 3 \cdot (10)^{-9}\ \frac{A^2}{V^2}$$

$$\overline{v_{T,in}^2} = \int_1^{1000} \frac{1}{3 \cdot 10^{-9}}\left(8.69 \cdot (10)^{-24} + \frac{6.324 \cdot (10)^{-18}}{f}\right) df$$

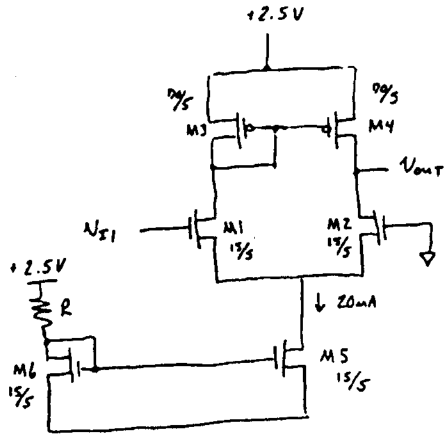
$$= 2.90 \cdot (10)^{-15}(1000-1) + 2.108(10)^{-9}\left[\ln(1000) - \ln(1)\right]\ V^2$$

$$= 14.57\ nV^2$$

$$\sqrt{\overline{v_{T,in}^2}} = .121\ mV$$

24.13

Verify the calculated noise voltage determined in Problem 24.12 with SPICE simulation results.



$$20\mu A = \frac{50\mu}{2}\frac{15}{5}\left(V_{GS5} - .83\right)^2$$

$$0.2667 = V_{GS5}^2 - 1.66\,V_{GS5} + .6889$$

$$0 = V_{GS5}^2 - 1.66\,V_{GS5} + .4222$$

$$V_{GS5} = 1.35\ V$$

$$20\mu = \frac{5 - 1.35\,V}{R}$$

$$R = 182.5\ k\Omega$$

```
* Problem 24.13 - Noise verification of Problem 24.12

.noise v(3,0) vin
.ac dec 100 1 1k
M1 2 4 5 7  CMOSNB L=5u W=15u
M2 3 0 5 7  CMOSNB L=5u W=15u
M3 2 2 1 1  CMOSPB L=5u W=70u
M4 3 2 1 1  CMOSPB L=5u W=70u
M5 5 6 7 7  CMOSNB L=5u W=15u
M6 6 6 7 7  CMOSNB L=5u W=15u
R1 1 6  182.5k
VDD 1 0 dc 2.5v
VSS 7 0 dc -2.5v
vin 4 0 ac 1v

* BSIM model for n-channel CN20
.MODEL CMOSNB NMOS LEVEL=4
AF=1.3 KF=1E-25

* BSIM model for p-channel CN20
.MODEL CMOSPB PMOS LEVEL=4
AF=1.3, KF=1E-25

.probe
.end
```

Problem 24.14

For both n and p channels are of size of $100\mu/5\mu$, we have

$$I_{SS} = 10\mu A = \frac{50\mu A/V^2 \times 100}{2 \times 5}(V_{GSN} - 0.83)^2$$

$$\Longrightarrow V_{GSN} = 0.97\,V$$

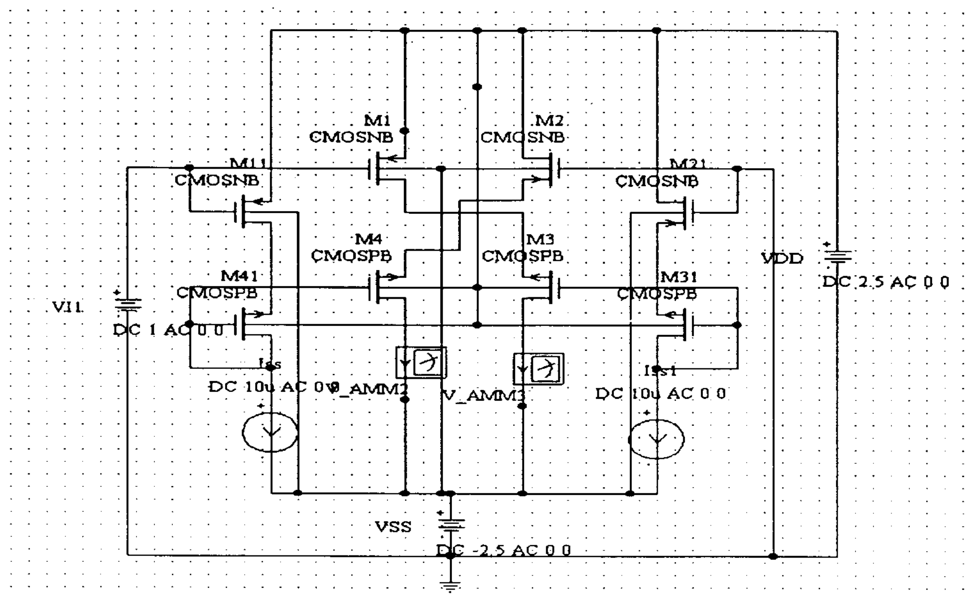
$$I_{SS} = 10\mu A = \frac{17\mu A/V^2 \times 100}{2 \times 5}(V_{SGP} - 0.91)^2$$

$$\Longrightarrow V_{SGP} = 1.15\,V$$

$$\therefore |V_{DMIN}| = -0.97\,V - 1.15\,V + 0.91\,V + 0.83\,V = \underline{\underline{-380\,mV}}.$$

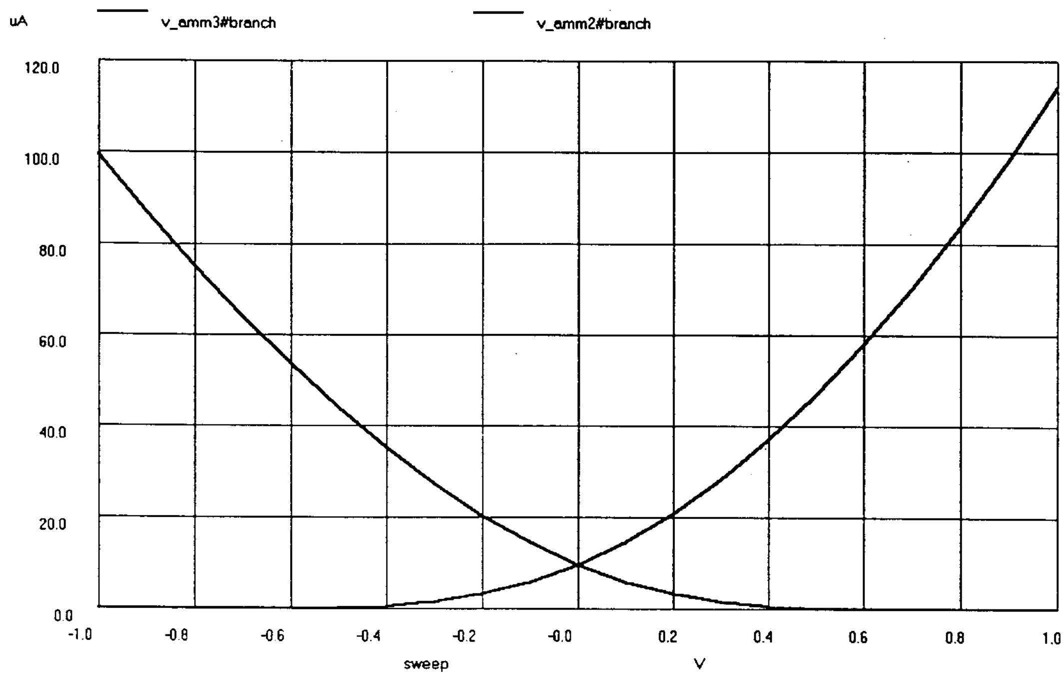
See next page for SPICE simulation.

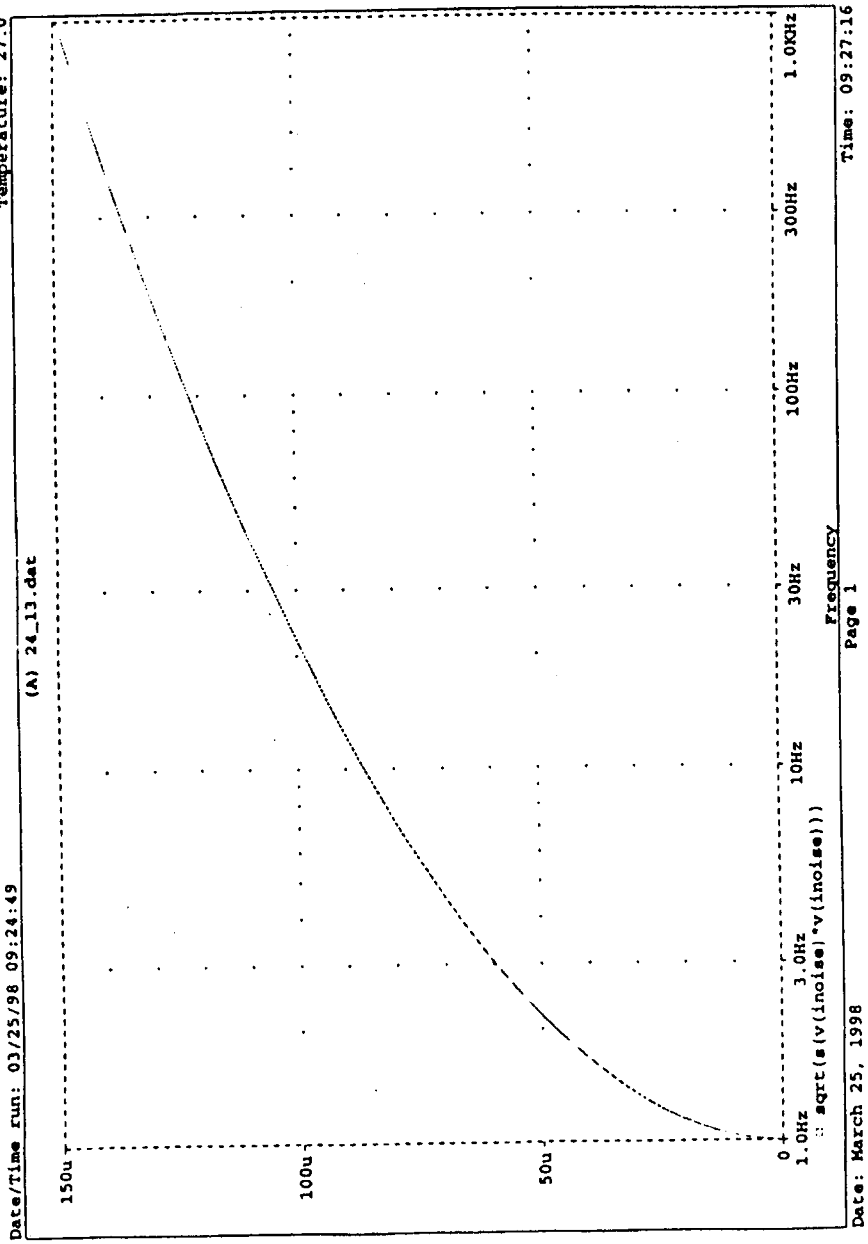
Problem 24.14 *(Simulation)*




```
*** Top Level Netlist ***
Iss      7 3       DC 10u AC 0 0
Iss1     6 3       DC 10u AC 0 0
M1       1 2 12 3 CMOSNB  L=5u W=100u
M11      8 2 12 3 CMOSNB  L=5u W=100u
M2       12 0 5 3 CMOSNB  L=5u W=100u
M21      12 0 9 3 CMOSNB  L=5u W=100u
M3       11 6 1 12 CMOSPB  L=5u W=100u
M31      6 6 9 12 CMOSPB  L=5u W=100u
M4       10 7 5 12 CMOSPB  L=5u W=100u
M41      7 7 8 12 CMOSPB  L=5u W=100u
V_AMM2           10 3 0V
V_AMM3           11 3 0V
VDD      12 0      DC 2.5 AC 0 0
VI1      2 0       DC 1 AC 0 0
VSS      3 0       DC -2.5 AC 0 0

***** Spice models and macro models *****

.MODEL CMOSNB NMOS LEVEL=4

.MODEL CMOSPB PMOS LEVEL=4

***** End of spice models and macro models *****

.DC VI1 -1 1 .1
.end
```

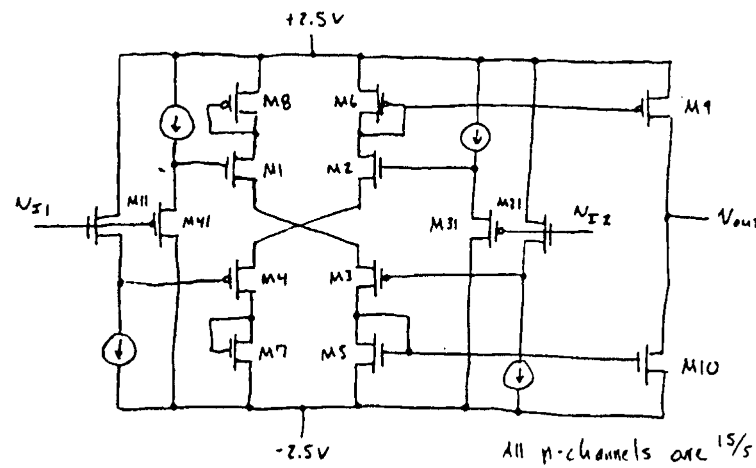
The SPICE simulation result is shown below:





24.15

Determine the small signal gain of the amplifier shown in Fig. P24.15. Also determine the input CMR of the amplifier if the minimum voltage across a current source/sink is 0.3V.



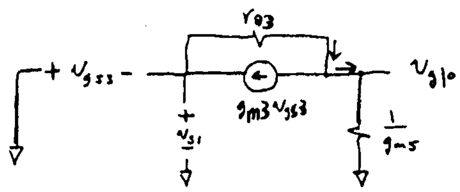
All n-channels are 15/5
All p-channels are 9/5
The current sources/sinks are 10 μA.

Use superposition to find the gain

If $v_{I2} = 0$, $\frac{v_{out}}{v_{I1}}$ for the 1st path is:

$$\frac{v_{g1}}{v_{I1}} \approx 1$$

$$\frac{v_{s1}}{v_{g1}} = \frac{g_{m1}\frac{1}{g_{m3}}}{1 + g_{m1}\frac{1}{g_{m3}}}$$



$$v_{s1} = \left(v_{g10}\, g_{m5} + g_{m3} v_{gs10}\right) r_{o3} + v_{g10}$$

$$v_{s1} = -v_{gs3}$$

$$v_{s1} = v_{g10}(1 + g_{m5} r_{o3}) - g_{m3} r_{o3} v_{s1}$$

$$\frac{v_{g10}}{v_{s1}} = \frac{1 + g_{m5} r_{o3}}{1 + g_{m3} r_{o3}} \simeq \frac{g_{m5}}{g_{m3}}$$

$$\frac{v_{out}}{v_{g10}} = -g_{m10}(r_{o9} \| r_{o10})$$

$\therefore$ When $v_{I2} = 0$, $\frac{v_{out}}{v_{I1}}$ for the 1st path is

$$\left(\frac{v_{out}}{v_{I1}}\right)_1 = \frac{v_{s1}}{v_{I1}} \cdot \frac{v_{s1}}{v_{g1}} \cdot \frac{v_{g10}}{v_{s1}} \cdot \frac{v_{out}}{v_{g10}}$$

$$= \left[\frac{g_{m1} \frac{1}{g_{m3}}}{1 + g_{m1} \frac{1}{g_{m3}}}\right] \cdot \left[\frac{g_{m5}}{g_{m3}}\right] \cdot \left[-g_{m10}(r_{o9} \| r_{o10})\right]$$

Find $\frac{v_{out}}{v_{I1}}$ when $v_{I2} = 0$. for the 2nd path:

$$\frac{v_{g4}}{v_{I1}} \approx 1$$

$$\frac{v_{s4}}{v_{g4}} = \frac{g_{m4} \frac{1}{g_{m2}}}{1 + g_{m4} \frac{1}{g_{m2}}}$$

$$\frac{v_{g9}}{v_{s4}} \simeq \frac{g_{m6}}{g_{m2}}$$

$$\frac{v_{out}}{v_{g9}} = -g_{m9}(r_{o9} \| r_{o10})$$

$\therefore$ When $v_{I2} = 0$, $\frac{v_{out}}{v_{I1}}$ for the 2nd path is

$$\left(\frac{v_{out}}{v_{I1}}\right)_2 = \frac{v_{g4}}{v_{I1}} \cdot \frac{v_{s4}}{v_{g4}} \cdot \frac{v_{g9}}{v_{s4}} \cdot \frac{v_{out}}{v_{g9}}$$

$$= \left[\frac{g_{m4} \frac{1}{g_{m2}}}{1 + g_{m4} \frac{1}{g_{m2}}}\right] \cdot \left[\frac{g_{m6}}{g_{m2}}\right] \cdot \left[-g_{m9}(r_{o9} \| r_{o10})\right]$$

Since $g_{mn} \approx g_{mp}$

$$\frac{v_{out}}{v_{I1}} = \left(\frac{v_{out}}{v_{I1}}\right)_1 + \left(\frac{v_{out}}{v_{I1}}\right)_2 = \underline{-g_m(r_{o9} \| r_{o10})}$$

First Find $\frac{v_{out}}{v_{I2}}$ when $v_{I1} = 0$ for the 1st path

$$\frac{v_{g3}}{v_{I2}} = \frac{g_{m11} r_{o21}}{1 + g_{m11} r_{o21}} \simeq 1$$

$$\frac{v_{g10}}{v_{g3}} = \frac{-g_{m3} \frac{1}{g_{m5}}}{1 + g_{m3} \frac{1}{g_{m1}}}$$

$$\frac{v_{out}}{v_{g10}} = -g_{m10}(r_{o9} \| r_{o10})$$

$\therefore$ When $v_{I1} = 0$, $\frac{v_{out}}{v_{I2}}$ for the 1st path is

$$\left(\frac{v_{out}}{v_{I2}}\right)_1 = \frac{v_{g3}}{v_{I2}} \cdot \frac{v_{g10}}{v_{g3}} \cdot \frac{v_{out}}{v_{g10}}$$

$$\left(\frac{V_{OUT}}{V_{I2}}\right)_1 = \left[\frac{-g_{m3}\left(\frac{1}{g_{m5}}\right)}{1 + g_{m3}\frac{1}{g_{m1}}}\right]\left[-g_{m10}(r_{o9} \| r_{o10})\right]$$

Find $\frac{V_{OUT}}{V_{I2}}$ when $V_{I1}=0$ for the 2nd path

$$\frac{V_{g2}}{V_{I2}} \simeq 1$$

$$\frac{V_{g9}}{V_{g2}} = \frac{-g_{m2}\frac{1}{g_{m6}}}{1+g_{m2}\frac{1}{g_{m4}}}$$

$$\frac{V_{OUT}}{V_{g9}} = -g_{m9}(r_{o9} \| r_{o10})$$

∴ When $V_{I1} = 0$, $\frac{V_{OUT}}{V_{I2}}$ for the 2nd path is

$$\left(\frac{V_{OUT}}{V_{I2}}\right)_2 = \frac{V_{g2}}{V_{I2}} \cdot \frac{V_{g9}}{V_{g2}} \cdot \frac{V_{OUT}}{V_{g9}}$$

$$= \left[\frac{-g_{m2}\frac{1}{g_{m6}}}{1+g_{m2}\frac{1}{g_{m4}}}\right]\left[-g_{m9}(r_{o9} \| r_{o10})\right]$$

so if $g_{mn} \approx g_{mp}$

$$\frac{V_{OUT}}{V_{I2}} \approx \left(\frac{V_{OUT}}{V_{I2}}\right)_1 + \left(\frac{V_{OUT}}{V_{I2}}\right)_2 = \underline{g_m(r_{o9} \| r_{o10})}$$

$$\frac{V_{OUT}}{V_{I1} - V_{I2}} \approx -\underline{g_m(r_{o9} \| r_{o10})}$$

$$\approx \underline{-46.5\ V/V}$$

$$g_{mn} = \sqrt{2(50\mu)\left(\tfrac{15}{5}\right)(10\mu)} = 54.8\ \mu A/V$$

$$r_o = \frac{1}{.06(10\mu)} = 1.7\ M\Omega$$

Determine the CMR

For M11 + M41 :

$$V_{IMIN} = V_{GS11} + .3 + VSS$$

$$= \sqrt{\frac{2I_D}{\beta_{11}}} + V_{THN} + .3 + VSS$$

$$= \sqrt{\frac{2(10\mu)}{50\mu\left(\frac{15}{5}\right)}} + .83 + .3 - 2.5$$

$$= -1.0\ V$$

$$V_{IMAX} = -V_{SG41} - .3 + VDD$$

$$= -\sqrt{\frac{2I_D}{\beta_4}} - V_{THN} - .3 + VDD$$

$$= -\sqrt{\frac{2(10\mu)}{17\mu(70/5)}} - .11 - .3 + 2.5$$

$$= 1.0\ V$$

P24.16

1) DC Biasing same as in P24.15

$I_{SS} = 10\,\mu A$ through all MOSFET.

2) AC gain expression are same

$$\frac{v_{out}}{v_{i1} - v_{i2}} = -\frac{2(R_{o9} \parallel R_{o10})}{\frac{1}{g_{m1}} + \frac{1}{g_{m3}}}$$

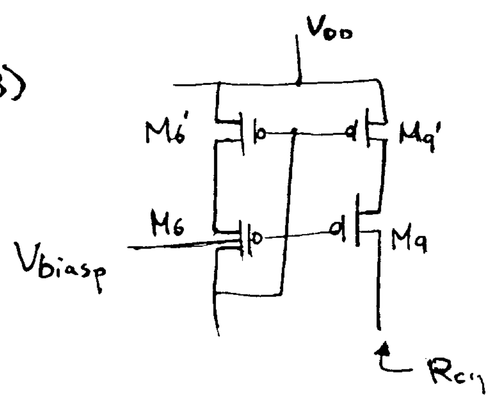
but $R_{o9} = r_{op}(1 + g_{mp} r_{op}) \approx g_{mp} r_{op}^2 = 68.99\mu \times 1.67M^2 = 192.4\ M\Omega$

$R_{10} = r_{on}(1 + g_{mn} r_{on}) \approx g_{mn} r_{on}^2 = 54.77 \times 1.67^2 = 152.8\ M\Omega$

$$\frac{v_{out}}{v_{i1} - v_{i2}} = -\frac{2(152.8 \parallel 192.4)}{\frac{1}{54.77} + \frac{1}{68.99}} \approx \boxed{-5200\ V/V}$$

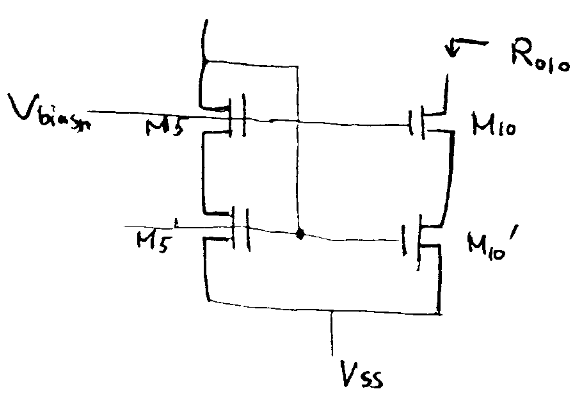
$\left(r_{on} = r_{op} = \frac{1}{0.06 \times 10\mu} = 1.67\ M \right)$

3)



$V_{bias,p} = V_{DD} - V_{DS9'sat} - V_{SG9} = 2.5 - (1.2 - 0.916) - 1.2 \cong 1V$

$V_{bias,n} = V_{GS10} + V_{DS10'sat} + V_{SS} = 1.2 + (1.2 - 0.83) - 2.5 = -0.93\ V$



CMR Same as $(-1V \sim 1V)$

Problem 24.17

$$\frac{I_{SS}}{2} = 1\mu A = \frac{\beta_{C1}}{2}(V_{GS1_{(C1)}} - V_{THN})^2 = \frac{50\mu A/V^2 \times \frac{15}{5}}{2} \times (V_{GS1_{(C1)}} - 0.83)^2$$

$$\Longrightarrow V_{GS1} = V_{GS2} \approx 0.95\ (V)$$

∴ The minimum voltage of V_{GSC6} is $V_{GSC1} + V_{DS,sat} \approx 0.95V + 0.12V$
$= 1.07\ V$

Design $I_{BIAS} = 1\mu A$, for MC6, we have

$$1\mu A = \frac{50\mu A/V^2 \times W_{C6}}{2 \times L_{C6}} \times (1.07 - 0.83)^2 \Longrightarrow \frac{W_{C6}}{L_{C6}} \approx 0.694$$

Set $\underline{L_{C6} = 10\mu m}$, $\underline{W_{C6} = 7\mu m}$.

Assume $1\mu A$ current flowing through M5 (15/5) ($R = 9.05M$)

We can set MB3 = MB1 (15/5) and M6 = 3 MB1 (45/5)

All the remaining MOSFETs still use the size of Fig 24.25.

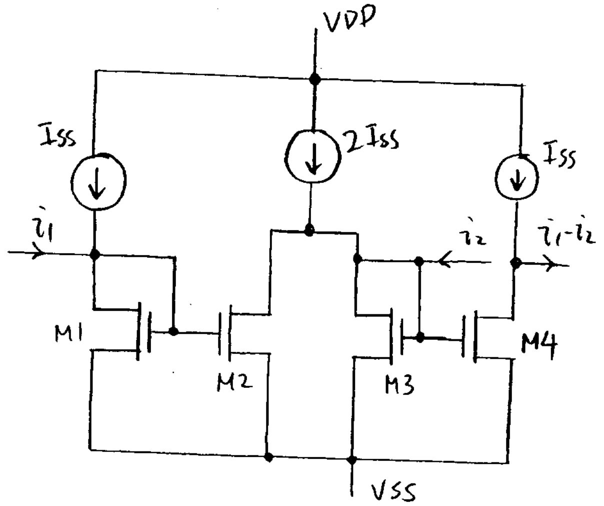
$$A_v = g_{m1}(g_{m2} r_{o2}^2 // g_{m4} r_{o4}^2) = 17.32\mu A/V \times \left(17.32\mu A/V \times (16.6M)^2 \;//\; 21.82\mu A/V \times (16.6M)^2\right)$$

$$\approx \underline{46473\ (V/V)}$$

$V_{INMIN} = 0.115V + 0.115V + 0.83V - 5V = \underline{-3.94(V)}$

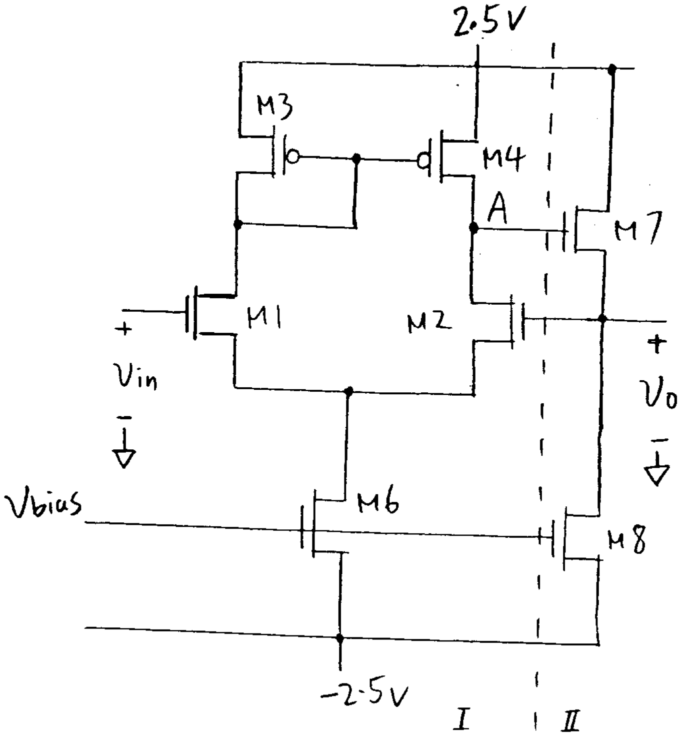
$V_{INMAX} = 5V - 0.115V - 0.092V - 0.092V + 0.4V - 2 \times 0.91V + 0.83V = \underline{4.11\ (V)}$.

Problem 24.18



When $i_1 = i_2 = -I_{SS}$, all the devices will be cut off. Therefore, $CMR(-) = -I_{SS} = -10\,\mu A$. The maximum common mode input is the maximum input current that will keep output MOSFETs of the current diff-amp operating in saturation region. The value of CMR(+) depends on the sizes of devices, including MOSFETs implementing the current sources in the circuit.

Problem 24.19



For the dif-amp stage,

$$v_{out}' = v_A = g_{m1,2}(r_{o2} \parallel r_{o4})(v_{in} - v$$

For the second stage, which is a voltag follower,

$$v_o \approx v_A.$$

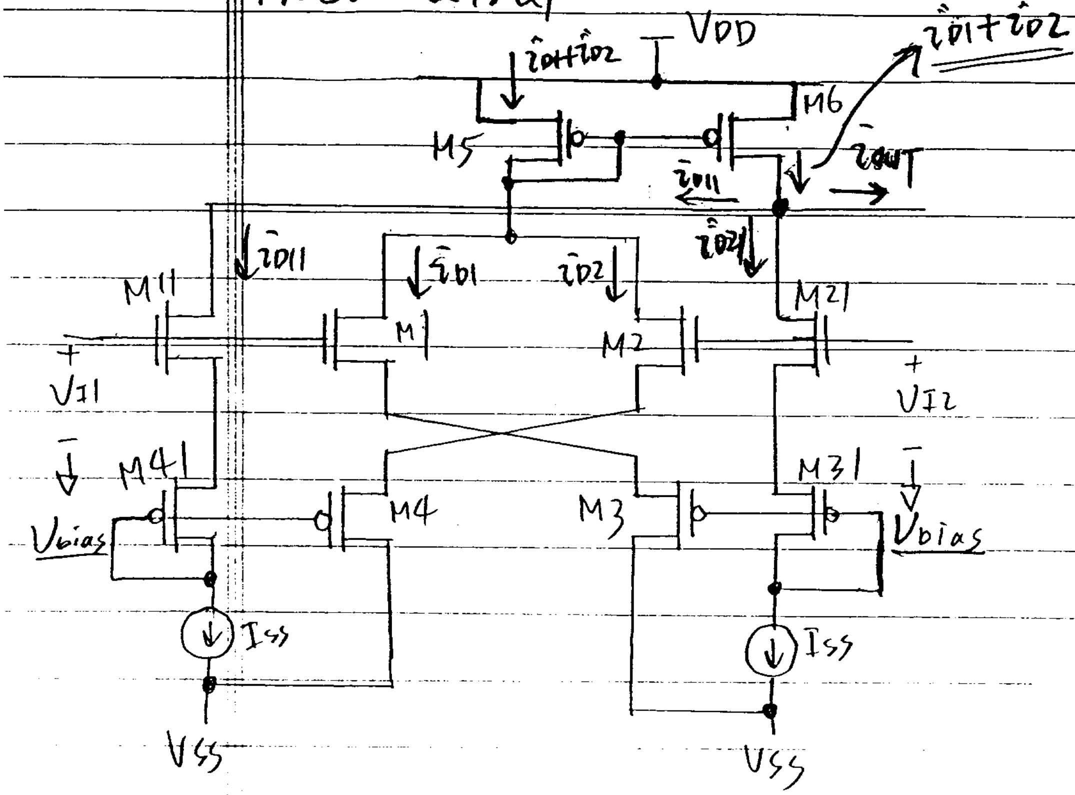
$$\therefore v_o = v_A = g_{m1,2}(r_{o2} \parallel r_{o4})(v_{in} - v_o)$$

$$\Rightarrow \frac{v_o}{v_{in}} = \frac{g_{m1,2}(r_{o2} \parallel r_{o4})}{1 + g_{m1,2}(r_{o2} \parallel r_{o4})} \approx 1.$$

Problem 24.20

If the common mode input voltage becomes large enough to shut the P-channel diff-pair off, the MOSFETs MS1 and MS2 are also off, causing the current in M5 to become I_o and current in M6 become $3I_o$. The circuit thus remains a constant g_m. Similarly, when n-channel diff-pair off, MS3 and MS4 are off, causing $3I_o$ flowing through M7 and keeping the transconductance a constance.

Problem 24.21



For the output node, using KCL, we have

$$i_{OUT} = (i_{D1} + i_{D2}) - (i_{D11} + i_{D21})$$

$(I_{D5} = I_{D6} = i_{D1} +$

$$\because \begin{cases} i_{D1} = \frac{1}{2}(V_{DI} - {}^{*}V_{bias} - V_{THP} - V_{THN})^2 \cdot \frac{\beta_1 \beta_3}{(\sqrt{\beta_1} + \sqrt{\beta_3})^2} & \cdots \\ i_{D2} = \frac{1}{2}(-V_{DI} - {}^{*}V_{bias} - V_{THP} - V_{THN})^2 \cdot \frac{\beta_2 \beta_4}{(\sqrt{\beta_2} + \sqrt{\beta_4})^2} & \cdots \\ V_{DI} = V_{I1} - V_{I2} & \cdots \end{cases}$$

Assume $\beta_1 = \beta_2$, $\beta_3 = \beta_4$, $(1) + (2) \Rightarrow$

$$i_{D1} + i_{D2} = \frac{\beta_1 \beta_3}{(\sqrt{\beta_1} + \sqrt{\beta_3})^2}\left[V_{DI}^2 + (V_{bias} + V_{THP} + V_{THN})^2\right.$$

$$i_{D11} + i_{D21} = I_{SS} + I_{SS} = 2I_{SS}$$

Now determin the I_{SS}:

Problem 24.21 (cont.)

determin I_{SS} :

When V_{I2} grounded ($V_{I2}=0$), we have

$$V_{I2} = 0 = V_{GS21} + V_{SG31} + V_{bias}$$

$$= \sqrt{\frac{2I_{SS}}{\beta_{21}}} + V_{THN} + \sqrt{\frac{2I_{SS}}{\beta_{31}}} + V_{THP} + V_{bias} \ldots$$

$$(4) \Rightarrow I_{SS} = \frac{(V_{THN}+V_{THP}+V_{bias})^2}{2\left(\frac{1}{\sqrt{\beta_{21}}}+\frac{1}{\sqrt{\beta_{31}}}\right)^2} = \frac{(V_{THN}+V_{THP}+V_{bias})^2 \beta}{2(\sqrt{\beta_{21}}+\sqrt{\beta_{31}})^2}$$

$$\therefore i_{OUT} = (i_{D1}+i_{D2}) - (i_{D11}+i_{D21}) = (i_{D1}+i_{D2}) - 2I_{SS}$$

$$= \frac{\beta_1\beta_3}{(\sqrt{\beta_1}+\sqrt{\beta_3})^2}\left[V_{DI}^2 + (V_{bias}+V_{THP}+V_{THN})^2\right] - \frac{\beta_{21}\beta_{31}}{(\sqrt{\beta_{21}}+\sqrt{\beta_{31}})^2}\left[(V_{THN}+V_{THP}+\right.$$

If $\beta_{21} = \beta_2 = \beta_1$; and $\beta_{31} = \beta_3 = \beta_4$ (all matched)

Then we have

$$i_{OUT} = \frac{\beta_1\beta_3}{(\sqrt{\beta_1}+\sqrt{\beta_3})^2} V_{DI}^2 = \frac{\beta_1\beta_3}{(\sqrt{\beta_1}+\sqrt{\beta_3})^2}(V_{I1} - V_{I2}$$