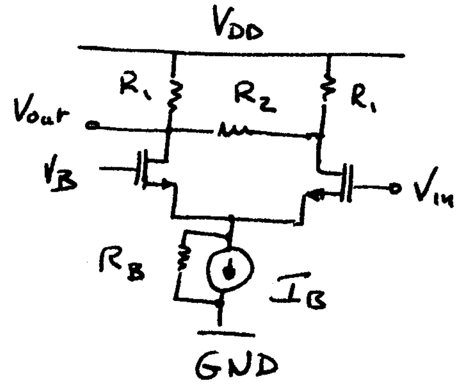
Homework # 4 Solution

1)

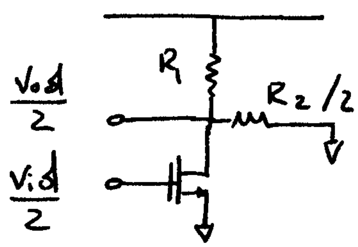


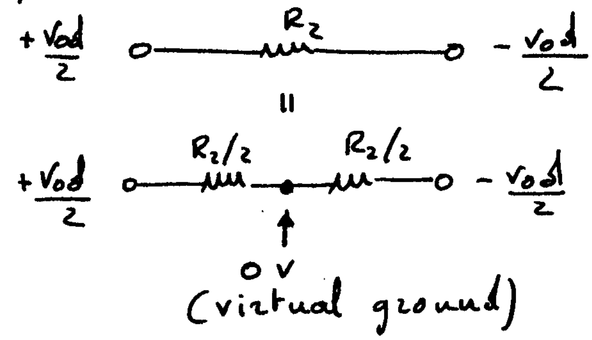
We will decompose the circuit into a differential mode half-circuit and a common mode half-circuit.

Let the subscripts 1 refer to the left side of the circuit and the subscripts 2 to the right side of the circuit.

$$\begin{cases} V_{i_1} = 0 \\ V_{i_2} = V_{in} \end{cases} \Rightarrow \begin{cases} V_{ic} = (V_{i_1} + V_{i_2})/2 = V_{in}/2 \\ V_{id} = V_{i_1} - V_{i_2} = -V_{in} \end{cases}$$

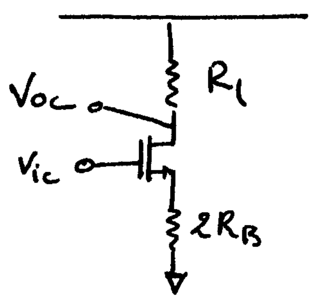
differential-mode half-circuit

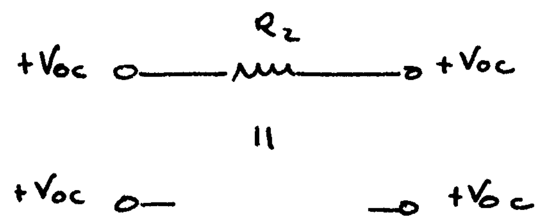




$$A_{dm} = -g_m \left(r_o /\!/ R_1 /\!/ \frac{R_2}{2}\right)$$

common-mode half-circuit





$$A_{cm} = -\frac{g_m (r_o /\!/ R_1)}{1 + g_m 2 R_B}$$

total gain

$$
\begin{aligned}
V_{out} &= V_{o_1} \\
&= V_{oc} + V_{od}/2 \\
&= A_{cm} V_{ic} + A_{dm} V_{id}/2 \\
&= -\frac{g_m (r_o \| R_1)}{1 + 2 g_m R_B} \frac{V_{in}}{2} \\
&\quad - g_m \left(r_o \| R_1 \| \frac{R_2}{2}\right) (-V_{in})/2 \\
&= \frac{g_m}{2} \left[r_o \| R_1 \| \frac{R_2}{2} - \frac{r_o \| R_1}{1 + 2 g_m R_B} \right]
\end{aligned}
$$

$$A = \frac{V_{out}}{V_{in}}$$

$$\Rightarrow \boxed{A = \frac{g_m}{2} \left[r_o \| R_1 \| \frac{R_2}{2} - \frac{r_o \| R_1}{1 + 2 g_m R_B} \right]}$$

2) $k' = \mu_0 C_{ox} = \mu_0 \dfrac{\varepsilon_0 \varepsilon_{r,SiO_2}}{t_{ox}}$

$$k'_{NMOS} = 250 \frac{cm^2}{Vs} \frac{8.85 \cdot 10^{-12} F/m \cdot 3.9}{2.6\ nm}$$

$$= 250 \frac{cm^2}{Vs} \quad 13.3\ fF/\mu m^2$$

$$= 332\ \mu A/V^2$$

$$k'_{NMOS} \frac{W}{L_{eff}} = k'_{NMOS} \frac{W}{L - 2L_D} = 332 \frac{\mu A}{V^2} \frac{4\mu m}{0.13\mu m - 0.05\mu m}$$

$$= 16.6\ mA/V^2$$

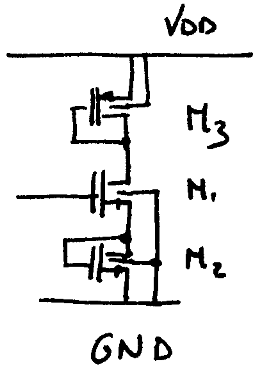
$$k'_{PMOS} = 100 \frac{cm^2}{Vs} \quad 13.3\ fF/\mu m^2$$

$$= 133\ \mu A/V^2$$

$$k'_{PMOS} \frac{W}{L_{eff}} = k'_{PMOS} \frac{W}{L - 2L_D} = 133 \frac{\mu A}{V^2} \frac{4\mu m}{0.13\mu m - 0.05\mu m}$$

$$= 6.64\ mA/V^2$$

Large-signal analysis



- $I_{DS_1} = I_{DS_2} = |I_{DS_3}| = 150\mu A$
- Assume that M_1 is in saturation

- $$I_{DS_2} = \frac{k'}{2}\frac{W}{L_{eff}}(V_{GS_2} - V_{T0})^2(1+\lambda V_{GS_2})$$

(since $V_{GS_2} = V_{DS_2}$)

We will solve this equation iteratively:

$$V_{GS_2} = V_{T0} + \sqrt{\frac{I_{DS_2}}{\frac{k'}{2}\frac{W}{L_{eff}}(1+\lambda V_{GS_2})}}$$

We start with an initial guess for V_{GS_2}, fill this in in the right hand side of the equation and we get another value for V_{GS_2}; and so on... This iterative process should converge to an accurate enough estimate after a couple of iterations

$V_{GS_2} = 0 \longrightarrow V_{GS_2} = 434\,mV \longrightarrow V_{GS_2} = 429\,mV$

$\longrightarrow \boxed{V_{GS_2} = 429\,mV}$ (two iterations would have been enough)

- $$|V_{GS_3} - V_{T0}| = \sqrt{\frac{|I_{DS_3}|}{\frac{k'}{2}\frac{W}{L_{eff}}(1+\lambda|V_{GS_3}|)}}$$

$|V_{GS_3}| = 0\,V \longrightarrow |V_{GS_3}| = 513\ mV \longrightarrow$ $\boxed{|V_{GS_3}| = 505\ mV}$

- $V_{SB_1} = V_{GS_2} = 429\ mV$

$\hookrightarrow V_{T_1} = V_{T0} + \gamma\left(\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right)$

$= 348\ mV$

$$\begin{aligned} V_{DS_1} &= V_{DD} - V_{GS_2} - |V_{GS_3}| \\ &= 1.2V - 429\,mV - 505\,mV \\ &= 266\,mV \end{aligned}$$

$$V_{GS_1} = V_{T_1} + \sqrt{\frac{I_{DS_1}}{\frac{k'}{2}\frac{W}{L_{eff}}(1+\lambda V_{DS_1}}} = 348\,mV + 131\,mV = 479\,mV$$

(notice that $V_{GS_1} - V_{T_1} = 131\,mV < V_{DS_1} = 266\,mV$
so that M_1 is in fact in saturation
and our assumptions are self-consistent

$V_{IN} = V_{GS_2} + V_{GS_1} = 429\,mV + 479\,mV$

$\Rightarrow \boxed{V_{IN} = 908\,mV}$

Small signal analysis

ⓐ $R_{out} = \frac{1}{g_{m_3}} \,//\, \left[\frac{1}{g_{m_2}} + r_{o_1} + r_{o_1}\frac{g_{m_1} + g_{mb_1}}{g_{m_2}}\right] \,//\, r_{o_3}$

$\approx \frac{1}{g_{m_3}} \,//\, 2r_{o_1} \,//\, r_{o_3}$

$\approx \frac{1}{g_{m_3}}$

$\frac{1}{g_{m_3}} = \frac{V_{Dsat_3}}{2I_{DS_3}} = \frac{205\,mV}{300\,\mu A} = 683\,\Omega$

$\Rightarrow \boxed{R_{out} \approx 683\,\Omega}$ SPICE : $667\,\Omega$

$\left(\frac{1}{g_{m_3}} \,//\, 2r_{o_1} \,//\, r_{o_3} = 666\,\Omega\right)$

$a_v = \frac{-g_{m_1}}{1 + \frac{g_{m_1}(1 + \chi_1)}{g_{m_2}}} \cdot \frac{1}{g_{m_3}}$ $(= +G_m \cdot R_{out})$

$\frac{g_{m_1}(1 + \chi_1)}{g_{m_2}} \approx 1$

$\Rightarrow a_v = -\frac{V_{Dsat_3}/V_{Dsat_1}}{2}$

$\Rightarrow \boxed{a_v = -0.782}$ SPICE : -0.734

(b)

$$R_{out} = \frac{1}{g_{m_2}} \; // \; \frac{r_{o_1} + 1/g_{m_3}}{1 + g_{m_1}(1+\chi_1) r_{o_1}}$$

$$\approx \frac{1}{g_{m_2}} \; // \; \frac{1}{g_{m_1}(1+\chi_1)}$$

$$\chi_1 = \frac{\gamma}{2\sqrt{2\phi_f + V_{SB_1}}} = \frac{0.2\, V^{1/2}}{2\sqrt{600mV + 429mV}} = 0.1$$

$$\Rightarrow \boxed{R_{out} \approx 206\,\Omega}$$

SPICE : 206 Ω

$$a_v = G_m R_{out} = g_{m_1} \left(\frac{1}{g_{m_2}} \; // \; \frac{1}{g_{m_1}(1+\chi_1)} \right)$$

$$= \frac{1}{1+\chi_1} \; // \; \frac{g_{m_1}}{g_{m_2}}$$

$$= \frac{1}{1+\chi_1} \; // \; \frac{V_{Dsat_2}}{V_{Dsat_1}}$$

$$\approx \frac{1}{2+\chi_1}$$

$$\Rightarrow \boxed{a_v \approx 0.476}$$

SPICE : 0.463

SPICE deck for problem 2a:

```
.title Homework 4 - Problem 2a

.option nomod post
.include ../models.sp

m1 out1 in out2 0 nmos w=4u l=0.13u
m2 out2 out2 0 0 nmos w=4u l=0.13u
m3 out1 out1 dd dd pmos w=4u l=0.13u

vdd dd 0 dc=1.2
vin in 0 dc=0.908 ac=1

.op
.tf v(out1) vin
.end
```

SPICE result for problem 2a:

```
***** operating point status is all       simulation time is     0.
    node    =voltage      node    =voltage      node    =voltage

 +0:dd      =   1.2000  0:in      = 908.0000m 0:out1    = 695.0375m
 +0:out2    = 429.0277m



 **** mosfets


 subckt
 element  0:m1       0:m2       0:m3
 model    0:nmos     0:nmos     0:pmos
 region     Saturati   Saturati   Saturati
  id       150.0512u  150.0512u -150.0512u
  ibs       -4.2903f    0.         0.
  ibd       -6.9504f   -4.2903f    5.0496f
  vgs      478.9723m  429.0277m -504.9625m
  vds      266.0098m  429.0277m -504.9625m
  vbs     -429.0277m    0.         0.
  vth      347.9627m  300.0000m -300.0000m
  vdsat    131.0096m  129.0277m -204.9625m
  beta      17.4849m   18.0262m    7.1437m
  gam eff  200.0000m  200.0000m  200.0000m
  gm         2.2907m    2.3259m    1.4642m
  gds       28.4943u   27.6387u   20.9229u
  gmb      225.8149u  300.2692u  189.0250u



 ****     small-signal transfer characteristics

      v(out1)/vin                            = -734.3052m
      input resistance at         vin        =  1.000e+20
      output resistance at v(out1)           =  667.2029
```

SPICE deck for problem 2b:

```
.title Homework 4 - Problem 2b

.option nomod post
.include ../models.sp

m1 out1 in out2 0 nmos w=4u l=0.13u
m2 out2 out2 0 0 nmos w=4u l=0.13u
m3 out1 out1 dd dd pmos w=4u l=0.13u

vdd dd 0 dc=1.2
vin in 0 dc=0.908 ac=1

.op
.tf v(out2) vin
.end
```

SPICE result for problem 2b:

```
***** operating point status is all       simulation time is     0.
    node    =voltage      node    =voltage      node    =voltage

 +0:dd      =   1.2000  0:in      = 908.0000m 0:out1    = 695.0375m
 +0:out2    = 429.0277m



**** mosfets


 subckt
 element  0:m1       0:m2       0:m3
 model    0:nmos     0:nmos     0:pmos
 region     Saturati   Saturati   Saturati
  id       150.0512u  150.0512u -150.0512u
  ibs       -4.2903f    0.         0.
  ibd       -6.9504f   -4.2903f    5.0496f
  vgs      478.9723m  429.0277m -504.9625m
  vds      266.0098m  429.0277m -504.9625m
  vbs     -429.0277m    0.         0.
  vth      347.9627m  300.0000m -300.0000m
  vdsat    131.0096m  129.0277m -204.9625m
  beta      17.4849m   18.0262m    7.1437m
  gam eff  200.0000m  200.0000m  200.0000m
  gm         2.2907m    2.3259m    1.4642m
  gds       28.4943u   27.6387u   20.9229u
  gmb      225.8149u  300.2692u  189.0250u



 ****     small-signal transfer characteristics

      v(out2)/vin                             =  463.3582m
      input resistance at         vin         =  1.000e+20
      output resistance at v(out2)            =  206.1600
```