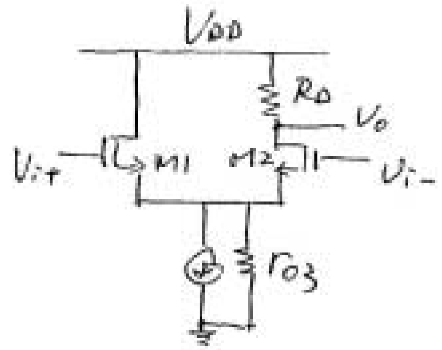
EE140 HW#4 Solution

1.

a)

Gm,dm calculation.

Since the circuit is asymmetric, we can not use the half circuit method. Instead of that, we can use linear superposition:



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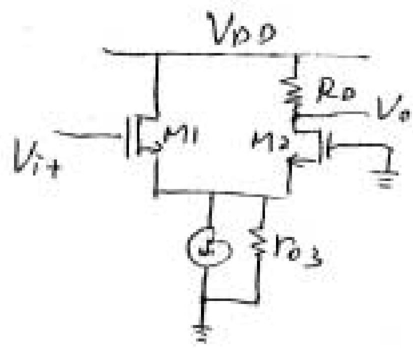

①

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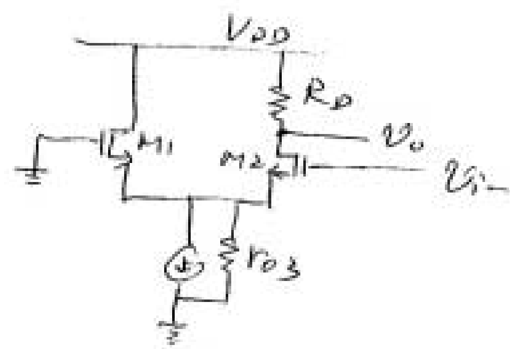
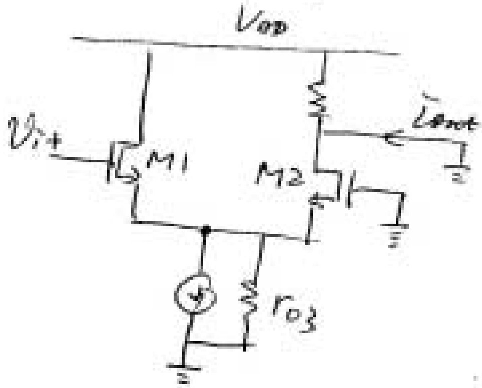
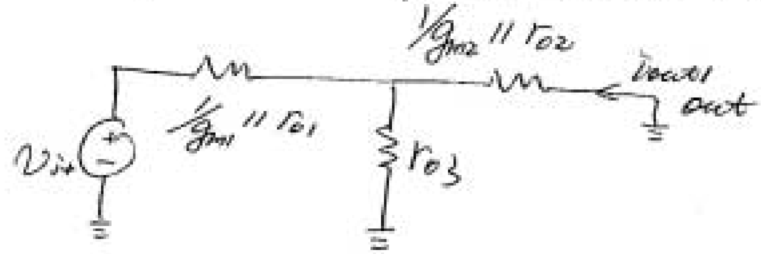


②

In circuit ⓑ, to calculate G_m, short output node to ground:

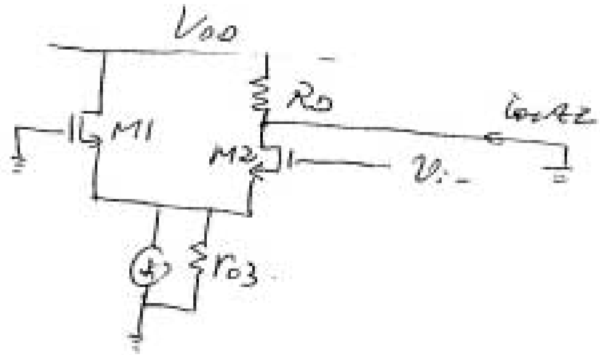


The eqv. small-signal model circuit:

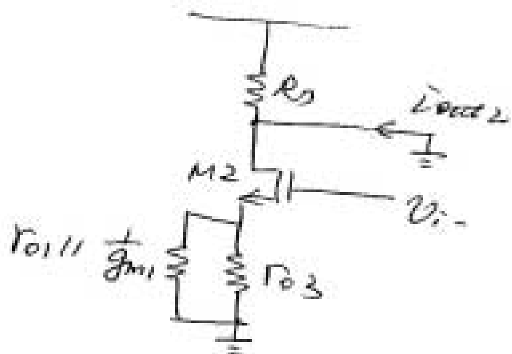


$$\therefore \frac{i_{out1}}{V_{i+}} = \frac{-1}{\frac{1}{g_{m1}} \| r_{o1} + r_{o3} \| r_{o2} \| \frac{1}{g_{m2}}} \times \frac{r_{o3}}{r_{o3} + \frac{1}{g_{m2}} \| r_{o2}}$$

In circuit ⓒ, to calculate G_m, short output node to ground:



The equ. small-signal model circuit:



$$\therefore \frac{i_{out2}}{V_{i-}} = \frac{g_{m2}}{1 + g_{m2} \cdot \left(r_{o3} \| r_{o1} \| \frac{1}{g_{m1}}\right)}.$$

Since $i_{out} = i_{out1} + i_{out2}$

$V_{i+} = \frac{V_{id}}{2}$ and $V_{i-} = \frac{-V_{id}}{2}$

$$\therefore i_{out} = -\frac{V_{id}}{2}\left[\frac{r_{o3}}{\frac{1}{g_{m1}} \| r_{o1} + r_{o3} \| r_{o2} \| \frac{1}{g_{m2}}} \cdot \frac{1}{r_{o3} + \frac{1}{g_{m2}} \| r_{o2}} + \frac{g_{m2}}{1 + g_{m2} \cdot \left(r_{o3} \| r_{o1} \| \frac{1}{g_{m1}}\right)}\right]$$

$$\therefore G_m = \frac{i_{out}}{v_{id}} = -\frac{1}{2}\left[\frac{r_{o3}}{\left(\frac{1}{g_{m1}}\|r_{o1} + r_{o3}\|r_{o2}\|\frac{1}{g_{m2}}\right)\left(r_{o3} + \frac{1}{g_{m2}}\|r_{o2}\right)} + \frac{g_{m2}}{1 + g_{m2}\cdot\left(r_{o3}\|r_{o1}\|\frac{1}{g_{m1}}\right)}\right]$$

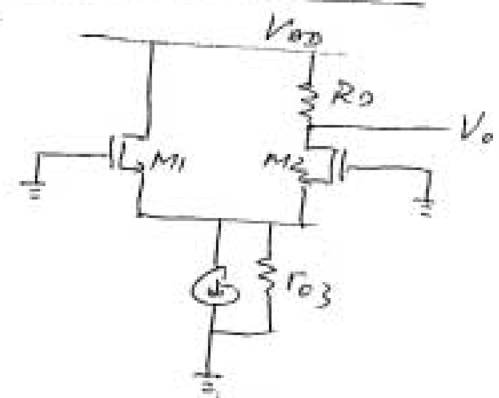
Assume $\frac{1}{g_m} \ll r_o$ and $g_{m1} = g_{m2} = g_{m1,2}$ (since V_{os} is small, we can ignore the channel length modulation in hand calculation and assume $i_{d1} = i_{d2}$)

$$\therefore G_{m_{dm}} \cong -\frac{1}{2} g_{m1,2} = -\frac{1}{2}\sqrt{2I_{D1,2}\cdot K'\cdot\frac{W}{L_{eff}}}$$

$K'\cdot\frac{W}{L_{eff}} = 16.6\,mA/V^2$ $\quad I_{D1,2} = \frac{1}{2}I_B = 300\,\mu A$

$\therefore \boxed{G_{m_{dm}} \cong -1.58\,mA/V}$ $\quad \boxed{-1.65\,ms \text{ from simulation}}$

Rout calculation:



The eqv. small signal model circuit is:

$$\therefore R_{out} = R_0 \,//\, \left[\frac{1}{g_{m1}} \,//\, r_{o3} + \left(1 + g_{m2} \cdot \left(r_{o3} \,//\, \frac{1}{g_{m1}}\right)\right) r_{o2} \right]$$

$$r_{o3} = \frac{1}{\lambda I_{o3}} = \frac{1}{0.2 \times 600\mu A} \approx 8.33\,k\Omega$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda I_{o1,2}} = \frac{1}{0.2 \times 300\mu A} \approx 16.67\,k\Omega.$$

$g_{m1,2} = 3.16\,mS$ $\quad R_0 = 1\,k\Omega$

$\therefore$ $\boxed{R_{out} = 970.6\,\Omega}$ (974.6 Ω from spice simulation)

$$A_{v-dm} = -G_{m,dm} \cdot R_{out}$$

$$= 1.58\,mA/V \times 970.6\,\Omega$$

$$= \boxed{1.53}$$ (1.61 from spice simulation).

$G_{m,cm}$ calculation

the common mode circuit is a common-source w/ degeneration circuit

$$\therefore G_{m,cm} = \frac{g_{m1,2}}{1 + g_{m1,2} \cdot 2\, r_{o3}} = \frac{3.16\,mS}{1 + 3.16\,mS \times 2 \times 8.33\,k\Omega}$$

$$= \boxed{0.059\,mS}$$ (0.050 mS from simulation)

$$\therefore A_{v-cm} = -G_{m,cm} \cdot R_{out}$$

$$= -0.059\,mS \cdot 970.6\,\Omega$$

$$= \boxed{-57.17m}$$ (−48.62m from simulation).

2.

a) $V_{GD5} = V_{GS6} = V_{t6} + V_{dsat6} > V_{t0}$ of NMOS

$\therefore$ There must exist channel around drain region of NMOS $\Rightarrow$ M5 in linear region

b) Since $V_{G2} = V_{GS6} + V_{ds5}$ (ignore body effect)

$$= V_{t0} + V_{dsat6} + V_{ds5}$$

Since $M1 \rightarrow M4$ have the same size

$\therefore I_{out} = I_{ref} \Rightarrow I_{DS6} = I_{DS2} = I_{DS1} = I_{ref}$

$\therefore V_{dsat6} = V_{dsat1}$

$\therefore V_{G2} = V_{t0} + 2V_{dsat1}$

In order to make M2 in the saturation region

$$V_{out(min)} = V_{G2} - V_{t0} = 2V_{dsat1}$$

c). Since M5 in linear region:

$$I_{D5} = K' m_5 \cdot \frac{W}{L}\left(V_{GS5} - V_{th5} - \frac{V_{DS5}}{2}\right) V_{DS5} \quad ①$$

define $m_5 = \dfrac{(\frac{W}{L})_5}{(\frac{W}{L})_6}$; $(\frac{W}{L})_6 = \frac{W}{L}$

$$I_{D6} = \frac{1}{2} K' \frac{W}{L} (V_{GS6} - V_{th6})^2 \quad ②$$

$I_{D5} = I_{D6}$ ③

$V_{GS5} = V_{GS6} + V_{DS5} = V_{GS6} + V_{dsat1}$ ④

$V_{GS6} = V_{th6} + V_{dsat6} = V_{th6} + V_{dsat1}$ ⑤

solve for $m_5 = \frac{1}{3}$

$\therefore (\frac{W}{L})_5 = \dfrac{\frac{1}{3} W}{L}$

3. In order to make a PMOS transistor in saturation region, It must satisfy:

$$V_{DG} \leq |V_{tp}|$$

For M1:

$$V_{DG1} = V_{D1} - V_{G1}$$
$$= V_{SG2} - I_{ref} \cdot R \quad (\text{From KVL})$$
$$= |V_{tp}| + V_{dsat2} - I_{ref} \cdot R$$

Since $V_{DG1} \leq |V_{tp}|$

$$\therefore |V_{tp}| + V_{dsat2} - I_{ref} \cdot R \leq |V_{tp}|$$

Solve for I_{ref}:

$$I_{ref} \geq \frac{V_{dsat2}}{R} = \frac{\sqrt{\frac{2 I_{ref}}{\mu_p C_{ox} \cdot \frac{W}{L}}}}{R}$$

$$\Rightarrow \boxed{I_{ref} \geq \frac{2}{\mu_p C_{ox} \cdot \left(\frac{W}{L}\right) \cdot R^2}}$$

For M2:

$$V_{DG2} = I_{ref} \cdot R \leq |V_{tp}|$$

$$\therefore \boxed{I_{ref} \leq \frac{|V_{tp}|}{R}}$$

SPICE deck for Prob.1

```
HW#4 Problem 1
.include 'model.m'

VDD    vdd    0     dc     1.2v
RD     vdd    vo    1k
IB     vdd    g4    dc     600u
Vid    vid    0     dc
Vic    vin    0     dc     0.8
Evi+   vi+    vin   vid    0     0.5
Evi-   vi-    vin   vid    0     -0.5

M1 vdd vi+ d3 d3 NMOS W=4u L=0.13u
M2 vo  vi- d3 d3 NMOS W=4u L=0.13u
M3 d3  g4  0  0  NMOS W=4u L=0.13u
M4 g4  g4  0  0  NMOS W=4u L=0.13u

.tf V(vo) vid
*.tf V(vo) vic

.op
.end
```

Output Result:

```
****    small-signal transfer characteristics
```
(Differential mode operation)

```
    v(vo)/vid                    =    1.6111
    input resistance at   vid    =  1.000e+20
    output resistance at v(vo)   =  974.5869
```

```
****    small-signal transfer characteristics
```
(Common mode operation)

```
    v(vo)/vic                    =  -48.6176m
    input resistance at   vic    =  1.000e+20
    output resistance at v(vo)   =  974.5869
```

Notes: Gm can be got from -Av/Rout.