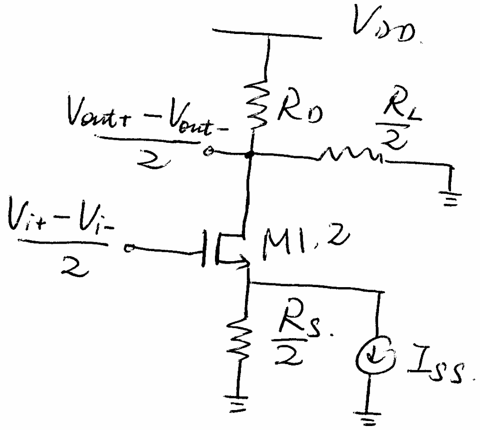
EE140 HW#3 Solution

1.

Differential half circuit:



It is a common source amplifier with source degeneration $R = \frac{R_S}{2}$.

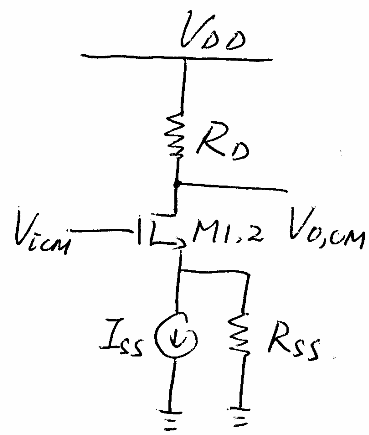
$g_{m1} = g_{m2} = g_{m1,2}$

$$\therefore A_{v\text{-}diff} = \frac{V_{out+} - V_{out-}}{V_{i+} - V_{i-}} \cong \frac{R_D // \frac{R_L}{2}}{\frac{1}{g_{m1,2}} + \frac{R_S}{2}}$$

$\left[\text{Assume } \left(R_D // \frac{R_L}{2}\right) << \frac{1}{2} g_m R_S \cdot r_o\right]$

$$= \frac{g_{m1,2} \cdot \left(R_D // \frac{R_L}{2}\right)}{1 + g_{m1,2} \cdot \frac{R_S}{2}}$$

Common mode half circuit:



$$\therefore A_{v-cm} = \frac{V_{O,CM}}{V_{i,CM}}$$

$$\cong \frac{R_D}{\frac{1}{g_{m1,2}} + R_{SS}}$$

[Assume $R_D << g_m \cdot R_{SS} \cdot r_o$ and R_{SS} is the output resistance of the current source]

$$= \frac{g_{m1,2} R_D}{1 + g_{m1,2} \cdot R_{SS}}$$

Since in this problem, we assume the current source is ideal $R_{SS} \to \infty$. $\therefore A_{v-cm} = 0$.

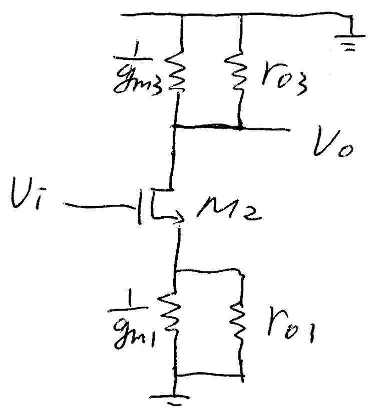
$$\boxed{CMRR = \frac{A_{v\text{-}diff}}{A_{v-cm}} = \infty}$$

* If the current source is not ideal.

$$CMRR = \frac{A_{v\text{-}diff}}{A_{v-cm}} = \frac{R_D // \frac{R_L}{2}}{R_D} \cdot \frac{1 + g_{m1,2} \cdot R_{SS}}{1 + g_{m1,2} \cdot \frac{R_S}{2}}$$

2.

(a). The small-signal circuit is shown below. M2 is CS w/ degeneration:



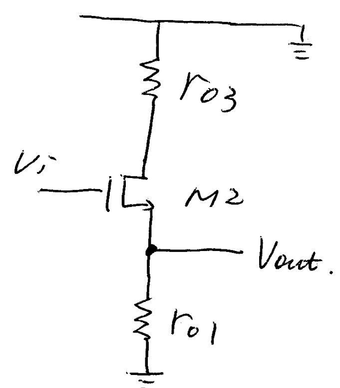
Assume $\frac{1}{g_m} \ll r_o$, then

$$\frac{1}{g_m} // r_o \cong \frac{1}{g_m}.$$

$$A_v \cong \frac{-\frac{1}{g_{m3}}}{\frac{1}{g_{m1}} + \frac{1}{g_{m2}}}$$

$$= \boxed{\frac{-g_{m1} // g_{m2}}{g_{m3}}}$$

(b) The small-signal circuit is shown below: M2 is source follower (CD) connection:



$$A_v = -G_m \cdot R_{out}$$

$$= \boxed{1/2\, g_{m2} \cdot \Big(\underbrace{r_{o1}}_{\substack{\text{resistor} \\ \text{looking down}}} // \underbrace{\frac{r_{o3} + r_{o2}}{1 + g_{m2} r_{o2}}}_{\substack{\text{resistor} \\ \text{looking up}}} \Big)}$$

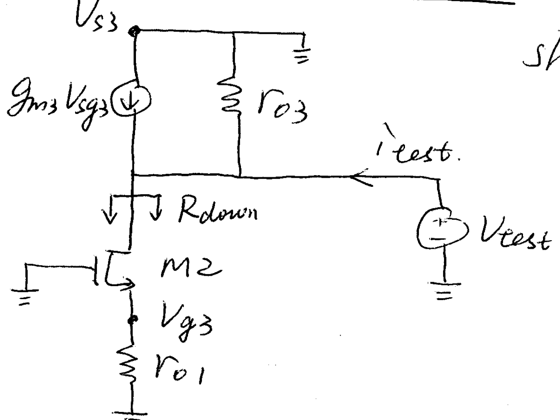
* Notes: If we assume $g_{m2} r_{o2} >> 1$, and $r_{o3} = r_{o2}$ (M2 & M3 have the same channel length,

the $A_v \cong 1/2 g_{m2} \cdot (r_{o1} // \frac{2}{g_{m2}})$

$\cong 1/2 g_{m2} \cdot \frac{2}{g_{m2}} = 1$

(c) $A_v = -G_m \cdot R_{out}$.

First calculate R_{out}: Apply V_{test} at output short input node to ground.



$R_{out} = \frac{V_{test}}{i_{test}}$

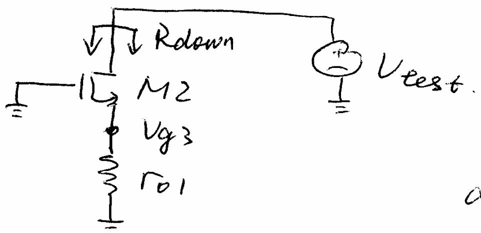
$$i_{test} = \frac{V_{test}}{r_{o3}} - g_{m3} \cdot V_{sg3} + \frac{V_{test}}{R_{down}} \quad (1)$$

R_{down} is the output resistance of a common source stage (M2) with degeneration resistor $= r_{o1}$

$$\therefore R_{down} = r_{o1} + (1 + g_{m2} \cdot r_{o1}) \cdot r_{o2} \quad (2)$$

$$V_{sg3} = V_{s3} - V_{g3} = 0 - V_{g3} = -V_{g3}$$

To calculate V_{g3}:



In order to calculate V_{g3}, we can use the concept of voltage divider:

$$V_{g3} = V_{test} \cdot \frac{r_{o1}}{R_{down}} \qquad (3)$$

Substitute (3), (4) into (1)

$$i_{test} = \frac{V_{test}}{r_{o3}} + g_{m3} \cdot V_{test} \cdot \frac{r_{o1}}{R_{down}} + \frac{V_{test}}{R_{down}}$$

$$\therefore R_{out} = \frac{V_{test}}{i_{test}} = r_{o3} // R_{down} // \frac{R_{down}}{g_{m3} \cdot r_{o1}} \qquad (5)$$

Substitute (2) into (5) and assume $g_m \cdot r_o \gg 1$.

$$R_{out} \cong r_{o3} // \frac{g_{m2} \cdot r_{o1} \cdot r_{o2}}{g_{m3} \cdot r_{o1}}$$

$$= \boxed{r_{o3} // \frac{g_{m2} \cdot r_{o2}}{g_{m3}}}$$

Calculation of G_m:

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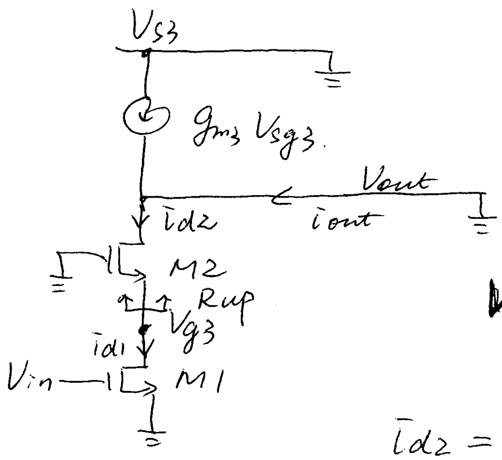
In order to calculate G_m, we need to short output to ground. apply v_{in}, then

$G_m = \dfrac{i_{out}}{v_{in}}$. the equivalent ckt is:



KCL @ output:

$$i_{out} = -g_{m3} \cdot V_{sg3} + i_d. \quad (1)$$

$$V_{sg3} = V_{S3} - V_{g3} = 0 - V_{g3} = -V_{g3} \quad (2)$$

$$i_{d2} = i_{d1} = g_{m1} \cdot v_{in} \quad (3)$$

If we define $A_{v1} = \dfrac{v_{g3}}{v_{in}}$, the A_{v1} is the vol. gain of a common source (M1) amplifier.

$$A_{v1} = -g_{m1} \cdot (R_{up} // r_{o1}) = -g_{m1} \cdot \left(\frac{1}{g_{m2}} // r_{o1}\right) \approx -g_{m1}/g_{m2}$$

$$\therefore v_{g3} = v_{in} A_{v1} = -v_{in} \cdot \frac{g_{m1}}{g_{m2}} \quad (4)$$

substitute (2), (3), (4) in (1)

$$i_{out} = g_{m3} \cdot v_{g3} + g_{m1} \cdot v_{in}$$
$$= \left(g_{m1} - g_{m3} \cdot \frac{g_{m1}}{g_{m2}}\right) v_{in}.$$

$$G_m = \frac{i_{out}}{v_{in}} = \boxed{g_{m1} - g_{m3} \cdot \frac{g_{m1}}{g_{m2}}}$$

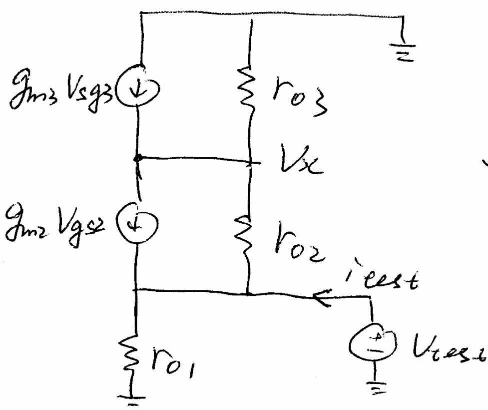
$$\therefore A_v = -G_m \cdot R_{out}$$

$$= \boxed{-\left(g_{m1} - g_{m3} \cdot \frac{g_{m1}}{g_{m2}}\right) \cdot \left(r_{o3} \,\|\, \frac{g_{m2} \cdot r_{o2}}{g_{m3}}\right)}$$

(d) $A_v = -G_m \cdot R_{out}$.

$$\boxed{G_m = g_{m1}}$$

<u>Calculation of Rout</u>



i_{test} has two parts, one is flowing down through r_{o1}; another part flows up we define it as i_{up}

According to the KCL @ output & node v_x

$$i_{up} = \frac{V_{test} - V_x}{r_{o2}} - g_{m2} \cdot V_{gs2} = \frac{V_x}{r_{o3}} - g_{m3} \cdot V_{sg3}($$

$$v_{gs2} = v_{g2} - v_{s2} = 0 - v_{test} = -v_{test}$$

$$v_{sg3} = v_{s3} - v_{g3} = 0 - v_{test} = -v_{test}.$$

substitute in ①, solve for v_x:

$$v_x = v_{test}\left[(r_{o2} // r_{o3})\left(\frac{1}{r_{o2}} + g_{m2} - g_{m3}\right)\right] \quad ②$$

substitute in ①, solve for i_{up}:

$$i_{up} = \frac{v_x}{r_{o3}} + g_{m3} \cdot v_{test}.$$

$$= \left[\frac{r_{o2} // r_{o3}}{r_{o3}}\left(\frac{1}{r_{o2}} + g_{m2} - g_{m3}\right) + g_{m3}\right] \cdot v_{test}$$

$$\therefore R_{up} = \frac{v_{test}}{i_{up}} = \frac{1}{\frac{r_{o2} // r_{o3}}{r_{o3}}\left(\frac{1}{r_{o2}} + g_{m2} - g_{m3}\right) + g_{m3}}$$

if g_{m2} is close to g_{m3}, $g_m \cdot r_o >> 1$

then $R_{up} \approx \frac{1}{g_{m3}}$.

Total $R_{out} = R_{up} // r_{o1} \cong \frac{1}{g_{m3}} // r_{o1} \cong \frac{1}{g_{m3}}$

$$\therefore A_v = -G_m \cdot R_{out}$$

$$\cong -\frac{g_{m1}}{g_{m3}}.$$