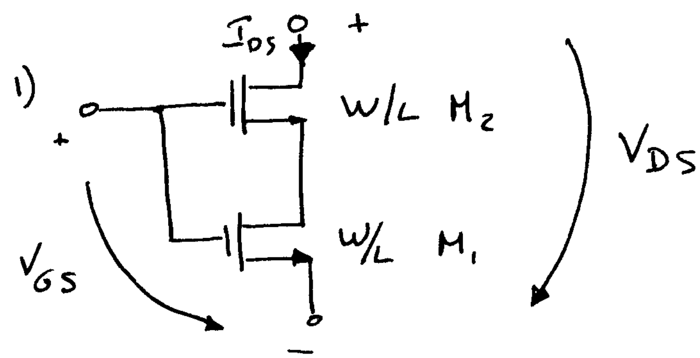
1)



- From inspection of the circuit:

$$I_{DS_1} = I_{DS_2} = I_{DS} \qquad (1)$$
$$V_{DS} = V_{DS_1} + V_{DS_2} \qquad (2)$$
$$V_{GS} = V_{GS_1} \qquad (3)$$
$$= V_{GS_2} + V_{DS_1} \qquad (4)$$

- For $V_{GS} < V_T$: $I_{DS_1} = 0 \Rightarrow I_{DS} = 0$

Assume now that $V_{GS} > V_T \Rightarrow I_{DS_1} = I_{DS_2} = I_{DS} > 0$

From (3) and (4): $V_{GS_2} - V_T = V_{GS_1} - V_T - V_{DS_1}$
$= V_{Dsat_1} - V_{DS_1}$

Since $I_{DS_2} > 0$: $V_{GS_2} - V_T > 0$

$\Rightarrow V_{DS_1} < V_{Dsat_1}$

$\boxed{M_1 \text{ is in the linear region}}$

- Assume M_2 is in the linear region also:

$$I_{DS_1} = k' \frac{W}{L}\left((V_{GS_1} - V_T)V_{DS_1} - V_{DS_1}^2/2\right)$$

$$= k' \frac{W}{L}\left((V_{GS} - V_T)V_{DS_1} - V_{DS_1}^2/2\right)$$

$$I_{DS_2} = k' \frac{W}{L}\left((V_{GS_2} - V_T)V_{DS_2} - V_{DS_2}^2/2\right)$$

$$= k' \frac{W}{L}\left((V_{GS} - V_T)V_{DS_2} - V_{DS_1}V_{DS_2} - V_{DS_2}^2/2\right)$$

(using (4))

$$I_{DS} = \frac{I_{DS} + I_{DS}}{2} = \frac{I_{DS_1} + I_{DS_2}}{2}$$

$$= k' \frac{W}{2L}\left((V_{GS} - V_T)(V_{DS_1} + V_{DS_2}) - \frac{V_{DS_1}^2 + 2V_{DS_1}V_{DS_2} + V_{DS_2}^2}{2}\right)$$

$$\boxed{I_{DS} = k' \frac{W}{2L}\left((V_{GS} - V_T)V_{DS} - V_{DS}^2/2\right)}$$

The structure is equivalent to a single transistor with ratio $W/2L$

- Assume M_2 is in the saturation region:

From (2) and (4): $V_{GS_2} - V_{DS_2} = V_{GS} - V_{DS}$

$\Leftrightarrow \quad V_{Dsat_2} - V_{DS_2} = V_{Dsat} - V_{DS}$

where $V_{Dsat} = V_{GS} - V_T$

$\Rightarrow \quad V_{DS_2} > V_{Dsat_2} \Leftrightarrow V_{DS} > V_{Dsat}$

$$I_{DS_1} = k'\frac{W}{L}\left((V_{GS} - V_T)V_{DS_1} - V_{DS_1}^2/2\right)$$

$$I_{DS_2} = \frac{k'}{2}\frac{W}{L}\left(V_{GS_2} - V_T\right)^2$$

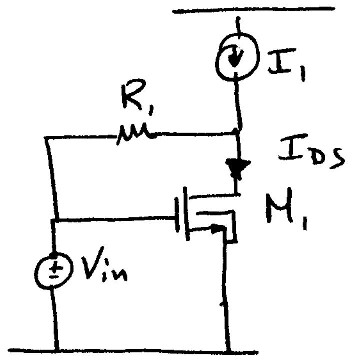
$$= k'\frac{W}{L}\frac{(V_{GS} - V_T - V_{DS_1})^2}{2}$$

$$= k'\frac{W}{L}\left[(V_{GS}-V_T)\left(\frac{V_{GS}-V_T}{2} - V_{DS_1}\right) + V_{DS_1}^2/2\right]$$

$$I_{DS} = \frac{I_{DS} + I_{DS}}{2} = \frac{I_{DS_1} + I_{DS_2}}{2}$$

$$\boxed{I_{DS} = \frac{k'}{2}\frac{W}{2L}(V_{GS} - V_T)^2}$$

2)



• For $V_{in} \le V_{T0} = 0.5V$, M_1 is in cut-off and $I_{DS} = 0$

• $V_D = V_G + R_1 (I_1 - I_{DS})$

$V_{DS} = V_{Dsat} + V_T + R_1 (I_1 - I_{DS})$

$\Rightarrow$ M_1 is in saturation when $V_T + R_1(I_1 - I_{DS}) \geq 0$

$$\Leftrightarrow V_{GS} - V_T \le \sqrt{\frac{R_1 I_1 + V_T}{R_1 \frac{k'}{2} \frac{W}{L}}} = 1.06V$$

For $0.5\,V \le V_{in} \le 1.56V$, M_1 is in saturation

$$I_{DS} = \frac{4mA}{V^2} (V_{GS} - V_T)^2$$

$$I_{DS}\Big|_{V_{GS} = 1.56V} = 4.5mA \quad \left(= \frac{R_1 I_1 + V_T}{R_1} \right)$$

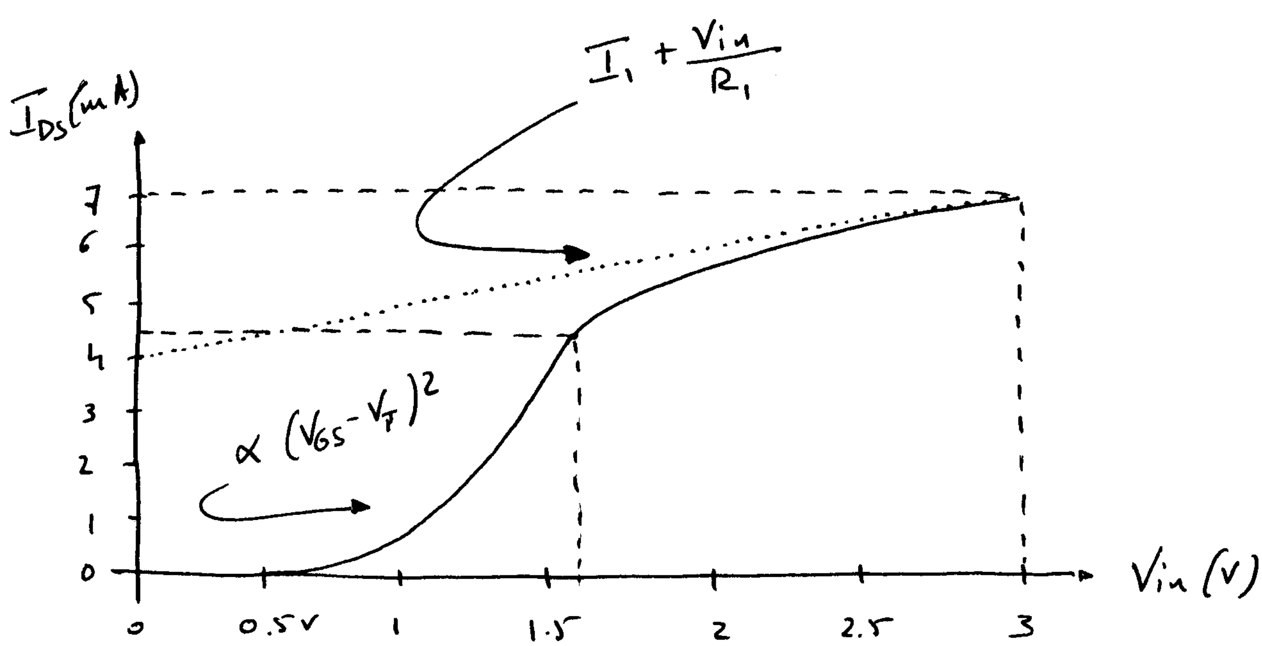
$\boxed{\text{For } V_{in} \gtrsim 1.56\,V,\ M_1 \text{ is in the linear region}}$

The exact behavior of $I_{DS}(V_{in})$ in the linear region is hard to calculate, but we can easily calculate the asymptote for $V_{in} \gg 1.56\,V$

For $V_{in} \gg 1.56\,V$, $V_{DS} \ll V_{in}$

$$\Rightarrow \quad I_{DS} \approx I_1 + \frac{V_{in}}{R_1}$$

$\boxed{\text{Sketch of } I_{DS}(V_{in})}$



3) $\boxed{a}$

$$V_{OUT} = 1.5\,V \Rightarrow V_{R_1} = 1.5\,V$$

$$\Rightarrow I_{R_1} = V_{R_1}/R_1 = 1.5\,mA$$

$$= I_{DS_1} = \frac{k'}{2}\frac{W}{L}(V_{GS_1} - V_T)^2(1 + \lambda V_{DS_1})$$

(Assuming M_1 is in saturation)

$$\Rightarrow V_{GS_1} - V_T = \sqrt{\frac{I_{DS_1}}{\frac{k'}{2}\frac{W}{L}(1 + \lambda V_{DS_1})}}$$

$$= \sqrt{\frac{1.5\,mA}{\frac{2\,mA}{V^2}(1 + 0.375)}}$$

$$= 0.74\ V$$

$$V_{IN} = V_{GS} + V_T$$

$$\boxed{V_{IN} = 1.24\ V}$$

$$\boxed{\begin{aligned} I_{DS} &= 1.5\,mA \\ V_T &= 0.5\,V \\ V_{Dsat} &= 0.74\,V \end{aligned}}$$

$$\boxed{\begin{aligned} g_m &= 4.06\,mS \\ g_{mb} &= 1.3\,mS \\ r_o &\approx 2.7\,k\Omega \end{aligned}}$$

$= 2I_{DS}/V_{GS} - V_T$

$= \chi g_m \qquad \chi = \gamma / 2\sqrt{2\phi_F} = 0.32$

$r_o \approx 1/\lambda I_{DS}$

1b

$V_{OUT} = 1.5V \rightarrow V_{R_1} = 1.5V$

$\Rightarrow I_{R_1} = 1.5 mA$

$= I_{DS_1} = \frac{k'}{2} \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS})$

First, we need to calculate V_{SB}:

$V_{SB} = R_2 I_{DS_1} = 0.45V$

$\Rightarrow V_T = V_{T0} + \gamma \left(\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f} \right)$

$= 0.625 V$

$V_{DS} = V_{OUT} - V_{SB} = 1.05V$

$\rightarrow V_{GS} - V_T = \sqrt{\dfrac{I_{DS}}{\frac{k'}{2} \frac{W}{L} (1 + \lambda V_{DS})}} = 0.77V$

$V_{IN} = (V_{GS} - V_T) + V_T + V_{SB}$

$\boxed{V_{IN} = 1.85 V}$

$I_{DS} = 1.5 mA$

$V_t = 0.625 V$

$V_{Dsat} = 0.77V$

$g_m = 3.89 mS$ $= 2 I_{DS} / V_{GS} - V_T$

$g_{mb} = 0.95 mS$ $= \chi g_m$ $\chi = \gamma / 2\sqrt{2\phi_f}$

$r_o \approx 2.7 k\Omega$ $r_o \approx 1/\lambda I_{DS}$