

University of California
Berkeley
College of Engineering
Department of Electrical Engineering
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EECS140
[Analog Circuit Design](#)

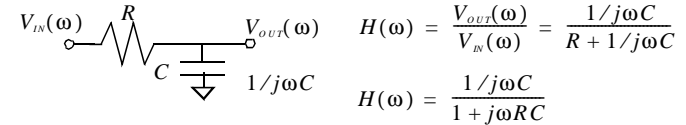
Lectures
on
FREQUENCY RESPONSE

Bode Plots

FR-1

Solve impedance network transfer function.

$$H(\omega) = \frac{V_{out}(\omega)}{V_{in}(\omega)} \quad (H(\omega), V_{out}(\omega) \text{ \& } V_{in}(\omega) \text{ are phasors})$$



Convert $H(\omega)$ to polar coordinates, $|H(\omega)| < \theta$

$$|H(\omega)| = [H(\omega)H^*(\omega)]^{1/2}$$

$$\theta = \text{atan} \left\{ \frac{I_m\{H(\omega)\}}{R_e\{H(\omega)\}} \right\}$$

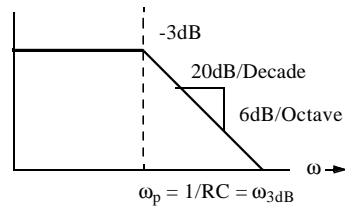
<p>IF $H(\omega) = \frac{N(\omega)}{D(\omega)}$ then</p> <p>$R_e\{H(\omega)\} = R_e\{N(\omega) \cdot D^*(\omega)\}$</p> <p>$I_m\{H(\omega)\} = I_m\{N(\omega) \cdot D^*(\omega)\}$</p>

Bode Plots (Cont.)

FR-2

$$\begin{aligned} (H(\omega) \cdot H^*(\omega))^{1/2} &= \left(\frac{1}{1 + j\omega RC} \right) \cdot \left(\frac{1}{1 - j\omega RC} \right) \\ &= \left(\frac{1}{1 + (\omega RC)^2} \right)^{1/2} = \left(\frac{1}{1 + \left(\frac{\omega}{\omega_{3dB}} \right)^2} \right)^{1/2} \end{aligned}$$

Bode Plot Magnitude $|H(\omega)|_{dB} = 20 \cdot \log |H(\omega)|$



$$\omega \gg \omega_{3dB} \quad |H(\omega)| \approx \left[\frac{1}{\left(\frac{\omega}{\omega_{3dB}} \right)^2} \right]^{1/2} = \frac{\omega_{3dB}}{\omega}$$

6dB/Octave - drops by 2 every time frequency doubles

Bode Plots (Cont.)

FR-3

$$\begin{aligned} H(\omega) &= \frac{1}{1 + j\frac{\omega}{\omega_{3dB}}} = |H(\omega)| \cdot \exp(j\theta(\omega)) \\ H(\omega) &= \frac{1}{1 + j\frac{\omega}{\omega_{3dB}}} \cdot \frac{\left(1 - j\frac{\omega}{\omega_{3dB}}\right)}{1 - j\frac{\omega}{\omega_{3dB}}} \\ &= \frac{1 - j\frac{\omega}{\omega_{3dB}}}{1 + \left(\frac{\omega}{\omega_{3dB}}\right)^2} \\ \theta(\omega) &= \text{atan} \left\{ \frac{I_m H(\omega)}{R_e H(\omega)} \right\} \end{aligned}$$

Bode Plots (Cont.)

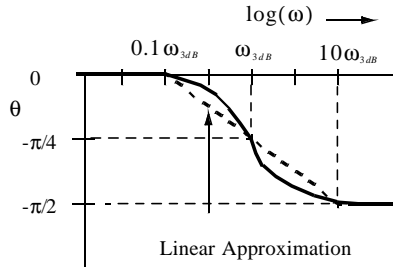
FR-4

$$Re\{H(\omega)\} = \frac{1}{1 + \left(\frac{\omega}{\omega_{3dB}}\right)^2}$$

$$Im\{H(\omega)\} = \frac{-\left(\frac{\omega}{\omega_{3dB}}\right)}{1 + \left(\frac{\omega}{\omega_{3dB}}\right)^2}$$

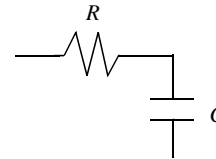
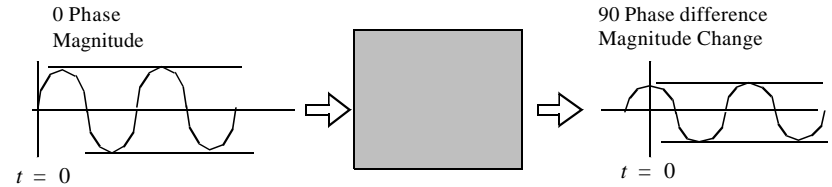
$$\theta(\omega) = \arctan\left\{-\frac{\omega}{\omega_{3dB}}\right\}$$

$$= -\arctan\left\{\frac{\omega}{\omega_{3dB}}\right\}$$



Bode Plots (Cont.)

FR-5



$$H(\omega) = \frac{1}{1 + j\omega RC}$$

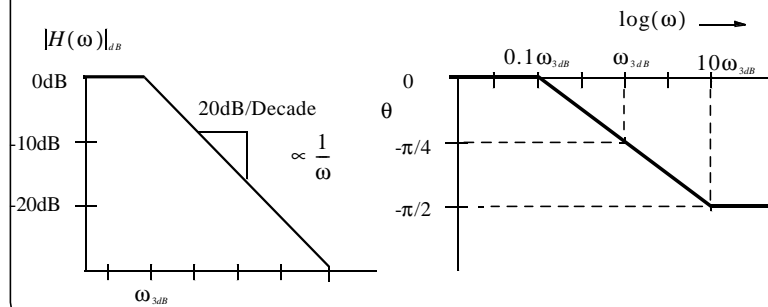
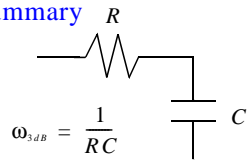
$$|H(\omega)| = (H(\omega) \cdot H^*(\omega))^{1/2}$$

Bode Plots (Cont.)

FR-6

1 Pole Summary

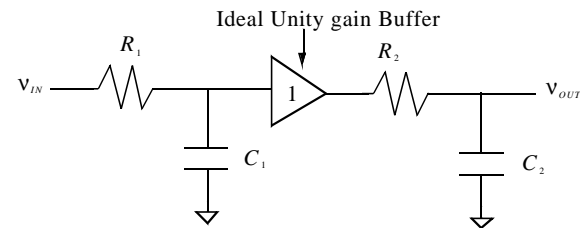
$$H(\omega) = \frac{1}{1 + j\frac{\omega}{\omega_{3dB}}}$$



Bode Plots (Cont.)

FR-7

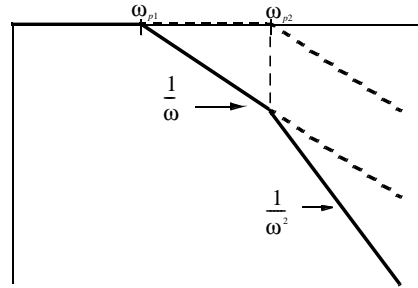
2 Poles



Two Poles: Bode Plots (Cont.) FR-8

$$H(\omega) = \left(\frac{1}{1 + j\frac{\omega}{\omega_{p1}}} \right) \cdot \left(\frac{1}{1 + j\frac{\omega}{\omega_{p2}}} \right) = H_{\omega_{p1}}(\omega) \cdot H_{\omega_{p2}}(\omega)$$

$$20 \cdot \log |H(\omega)| = 20 \cdot \log |H_{\omega_{p1}}(\omega)| + 20 \cdot \log |H_{\omega_{p2}}(\omega)|$$



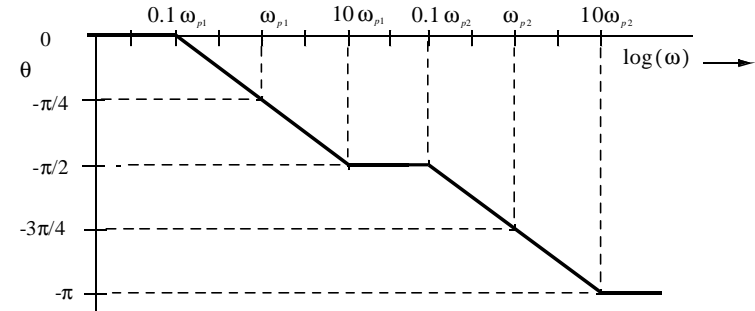
Bode Plots (Cont.) FR-9

$$H(\omega) = [|H_{\omega_{p1}}| \cdot \exp(j\theta_{\omega_{p1}})] \cdot [|H_{\omega_{p2}}| \cdot \exp(j\theta_{\omega_{p2}})]$$

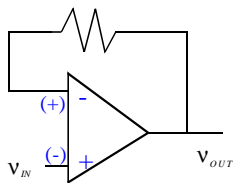
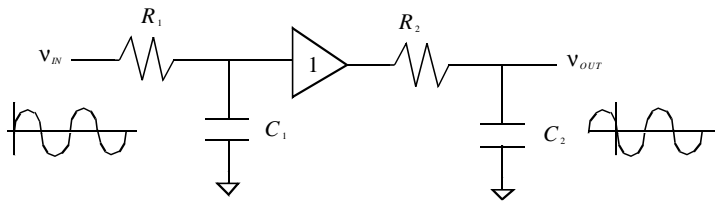
$$= |H_{\omega_{p1}}| \cdot |H_{\omega_{p2}}| \cdot \exp(j[\theta_{\omega_{p1}} + \theta_{\omega_{p2}}])$$

$$= |H(\omega)| \cdot \exp(j\theta(\omega))$$

$$\theta(\omega) = \theta_{\omega_{p1}} + \theta_{\omega_{p2}}$$

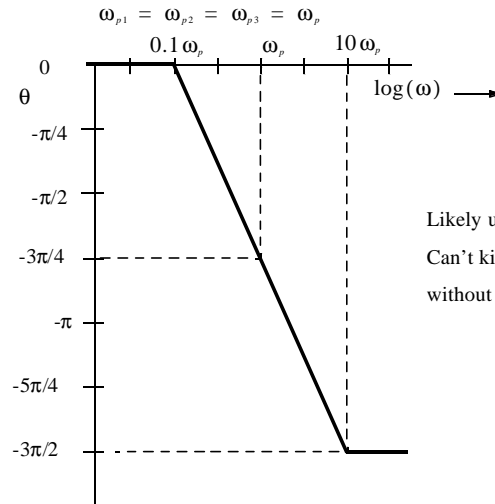


Bode Plots (Cont.) FR-10



If we have 180 degree phase shift we have a problem.
 The negative feedback will turn into positive feedback.
 Can't have positive feedback in a loop with gain > 1

Bode Plots (Cont.) FR-11



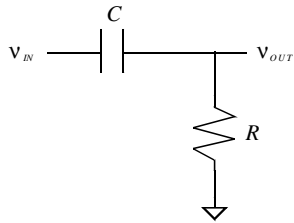
3 poles on top of each other

Likely unstable circuit
 Can't kill gain here
 without adding phase shift.

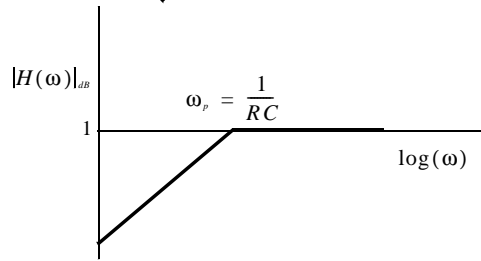
Bode Plots (Cont.)

FR-12

Zero at zero frequency, pole at $1/RC$

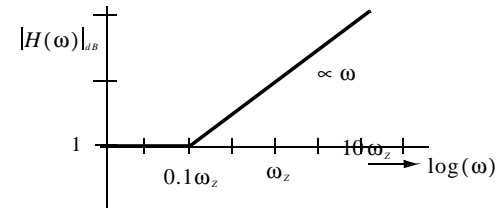
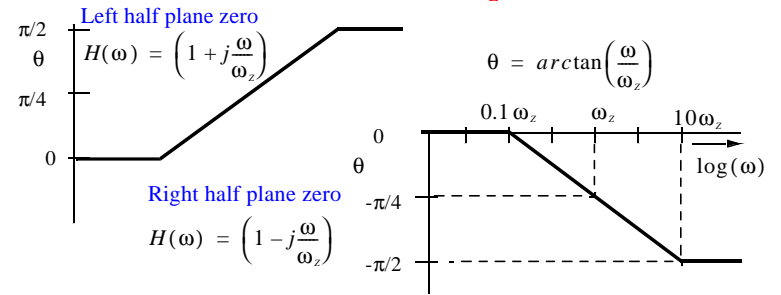


$$\frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC} = \frac{j\omega RC}{1 + j\frac{\omega}{\frac{1}{RC}}}$$



Single Zero at ω_z

FR-13

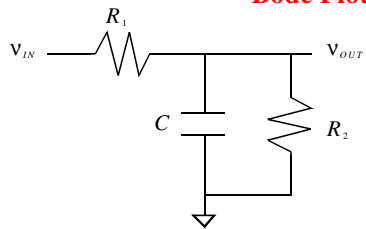


For both:

$$|H(\omega)| = \left(1 + \left(\frac{\omega}{\omega_z}\right)^2\right)^{1/2}$$

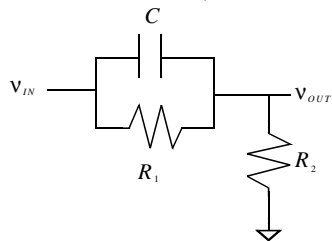
Bode Plots (Cont.)

FR-14



1 Pole

$$H(\omega) = \left(\frac{R_2}{R_1 + R_2}\right) \cdot \frac{1}{1 + j\omega(R_1 \parallel R_2)C}$$



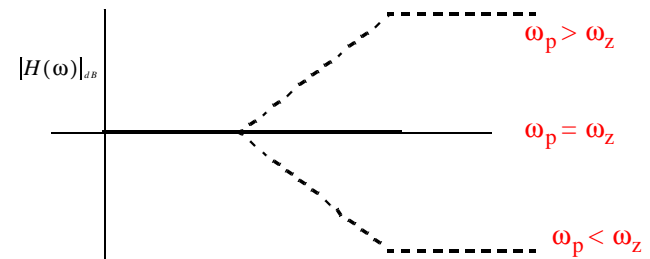
1 Pole, 1 Zero

$$H(\omega) = \left(\frac{R_2}{R_1 + R_2}\right) \cdot \frac{1 + j\omega R_1 C}{1 + j\omega(R_1 \parallel R_2)C}$$

Bode Plots (Cont.)

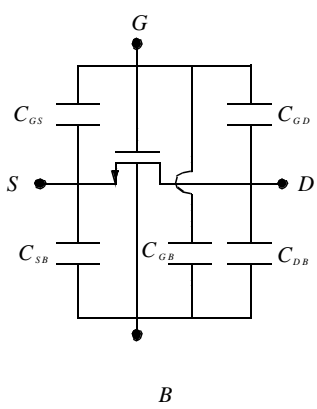
FR-15

1 Pole, 1 Zero



$$H(\omega) = \frac{1 + j\frac{\omega}{\omega_z}}{1 + j\frac{\omega}{\omega_p}}$$

Capacitances FR-16



$$C_{ox} = \frac{\epsilon_{SiO_2}}{t_{ox}} = 0.1 \frac{fF}{\mu^2} = 10^{-4} \frac{F}{m^2}$$

$$CGSO = 5 \times 10^{-10} \frac{F}{m}$$

$$CGDO = 5 \times 10^{-10} \frac{F}{m}$$

$$CGB0 = 4 \times 10^{-10} \frac{F}{m}$$

$$CJ = 10^{-4} \frac{F}{m^2}$$

$$PB = \phi_b = 0.8V$$

Capacitances (Cont.) FR-17

Sat :

$$C_{GS} = \frac{2}{3} \cdot C_{ox} \cdot L \cdot W + CGSO \cdot W$$

$$C_{GD} = CGDO \cdot W$$

$$C_{SB} = \frac{CJ \cdot AS}{\left(1 + \frac{V_{BS}}{PB}\right)^{MJ}} + \frac{CJSW \cdot PS}{\left(1 + \frac{V_{BS}}{PB}\right)^{MJSW}}$$

$$C_{GB} = CGBO \cdot L$$

Linear :

$$C_{GS} = \frac{C_{ox} \cdot L \cdot W}{2} + CGSO \cdot W$$

$$C_{GD} = \frac{C_{ox} \cdot L \cdot W}{2} + CGDO \cdot W$$

(similar for C_{DB})

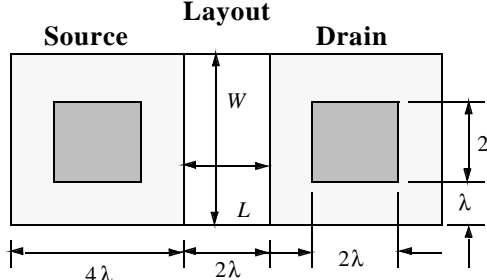
$$MJ = \frac{1}{2} \text{ (default)}$$

$$MJSW = 3 \text{ (default)}$$

PS = Perimeter of Source
 AS = Area of Source
 $CGBO$ = Capacitance of gate to bulk overlap

Capacitances (Cont.) FR-18

Layout



(Minimum size device, $W/L = 2$)

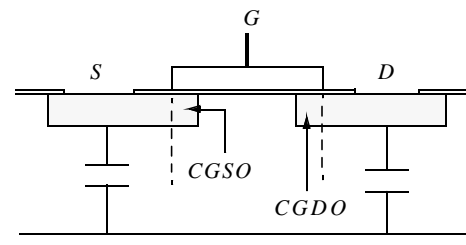
Area of Source = $AS = 4\lambda \cdot W$

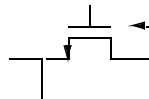
Area of Drain = $AD = AS$

Perimeter of Source = $PS = 8\lambda + W$

Capacitances (Cont.) FR-19

M1 1 2 3 4 NMOS L=2u W=2u
 + AS=4p AD=4p PS=6u PD=6u

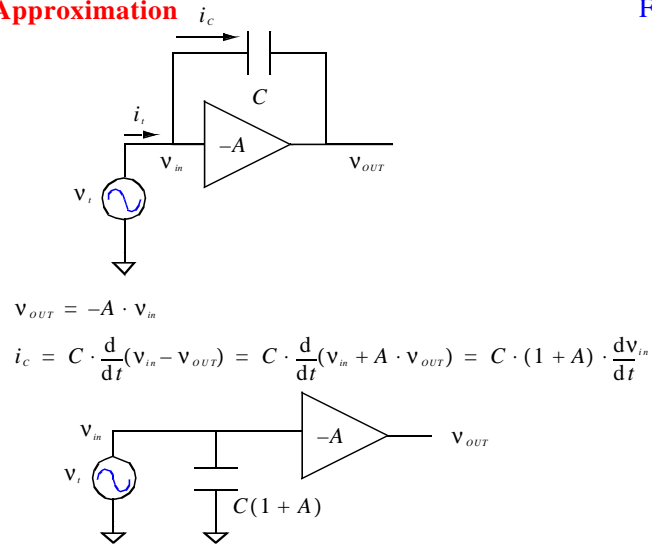




Capacitor (in linear)

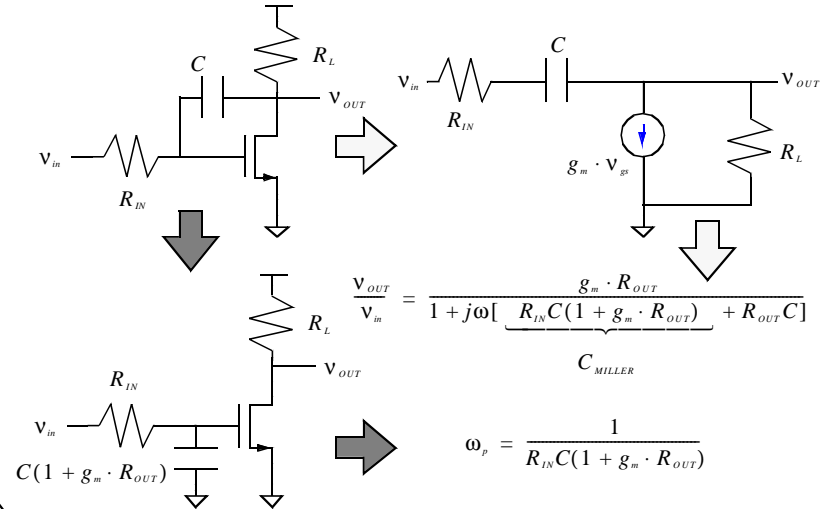
Miller Approximation

FR-20



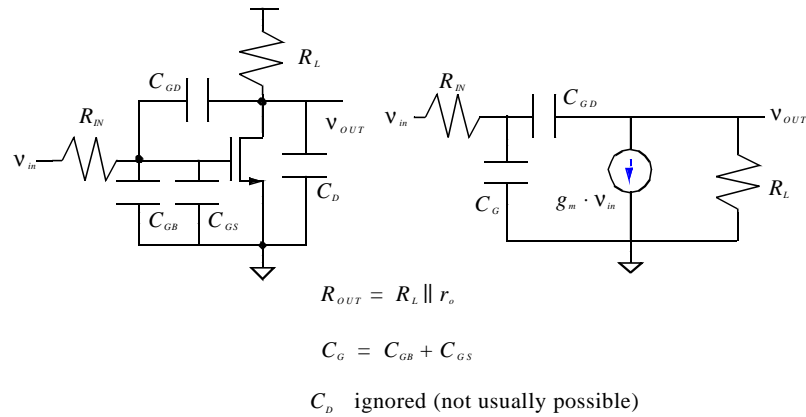
Miller Approximation (Cont.)

FR-21



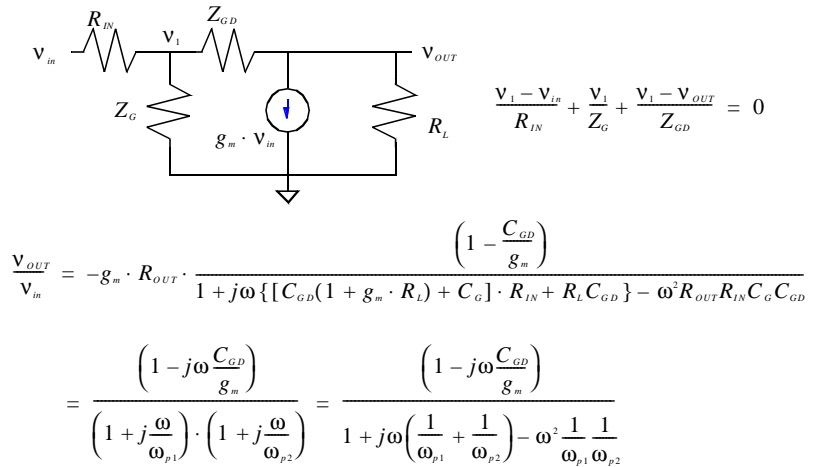
Inverter

FR-22



Inverter (Cont.)

FR-23



Inverter (Cont.)

FR-24

$$\omega_{p1} = -\frac{1}{R_{IN} \cdot [C_G + \underbrace{C_{GD}(1 + g_m R_{OUT})}_{C_{MILLER}}] + R_L C_{GD}}$$

$$\omega_{p2} = -\frac{1}{R_{OUT} C_{GD}} - \frac{1}{\left(R_{OUT} \parallel R_{IN} \parallel \frac{1}{g_m}\right) C_G}$$

$$\omega_z = \frac{g_m}{C_{GD}}$$

$$H(\omega) = \frac{\left(1 + j\frac{\omega}{\omega_z}\right)}{\left(1 + j\frac{\omega}{\omega_{p1}}\right) \cdot \left(1 + j\frac{\omega}{\omega_{p2}}\right)}$$

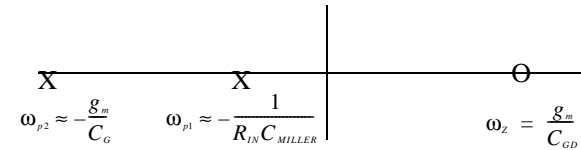
Inverter (Cont.)

FR-25

$$H(\omega) = \frac{\left(1 + j\frac{\omega}{\omega_z}\right)}{\left(1 + j\frac{\omega}{\omega_{p1}}\right) \cdot \left(1 + j\frac{\omega}{\omega_{p2}}\right)}$$

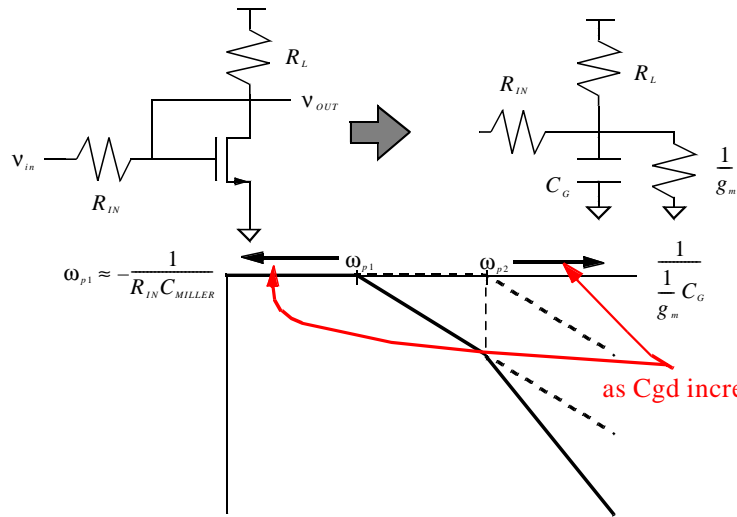
$$\begin{aligned} s_z &= -j\omega_z \\ s_{p1} &= -j\omega_{p1} \\ s_{p2} &= -j\omega_{p2} \end{aligned}$$

$$H(s) = \frac{\left(1 - \frac{s}{s_z}\right)}{\left(1 - \frac{s}{s_{p1}}\right) \cdot \left(1 - \frac{s}{s_{p2}}\right)}$$



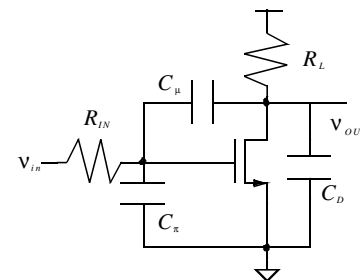
Inverter (Cont.)

FR-26



Inverter (Cont.)

FR-27



$$R_{OUT} = R_L \parallel r_o$$

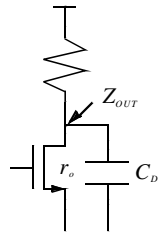
Case 1 (Miller Capacitance not important) :

$$R_{IN} C_{\pi} \gg R_{OUT} C_D \gg R_{IN} \cdot \underbrace{(1 + g_m R_{OUT}) C_{\mu}}_{C_{MILLER}}$$

$$\omega_{p1} = \frac{1}{R_{IN} C_{\pi}} \quad \omega_{p2} = \frac{1}{R_{OUT} C_D}$$

Inverter (Cont.)

FR-28



$$Z_{OUT} = R_L \parallel r_o \parallel \frac{1}{j\omega C_D}$$

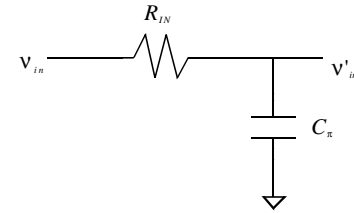
$$= R_{OUT} \parallel \frac{1}{j\omega C_D}$$

$$A_v = -g_m \cdot Z_{OUT} = -g_m \cdot \frac{\frac{R_{OUT}}{j\omega C_D}}{R_{OUT} + \frac{1}{j\omega C_D}} = -g_m \cdot \frac{R_{OUT}}{1 + j\omega R_{OUT} C_D}$$

$$\omega_p = \frac{1}{R_{OUT} C_D}$$

Inverter (Cont.)

FR-29



$$\frac{v'_{in}}{v_{in}} = \frac{\frac{1}{j\omega C_\pi}}{R_{IN} + \frac{1}{j\omega C_\pi}} = \frac{1}{1 + j\omega R_{IN} C_\pi}$$

$$\omega_p = \frac{1}{R_{IN} C_\pi}$$

Inverter (Cont.)

FR-30

Case 2 (Large CD) :

$$R_{OUT} C_D \gg R_{IN} (1 + g_m R_{OUT}) C_\mu, R_{IN} C_\pi$$

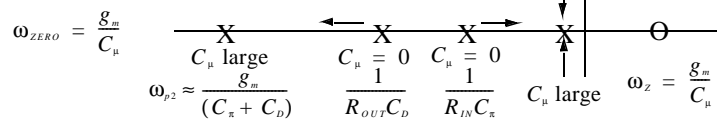
$$\omega_{p1} = \frac{1}{R_{OUT} C_D} \quad \omega_{p2} = \frac{1}{R_{IN} (C_\pi + C_\mu)}$$

Case 3 (Large Cμ) :

$$R_{IN} (1 + g_m R_{OUT}) C_\mu \gg R_{OUT} C_D, R_{IN} C_\pi$$

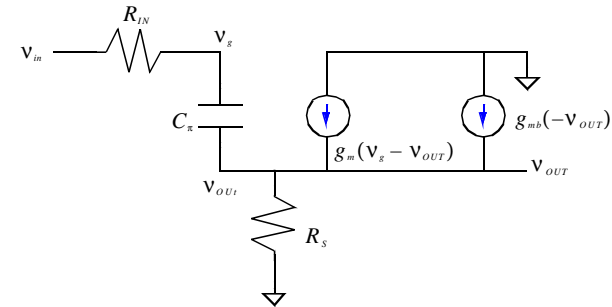
$$\omega_{p1} = \frac{1}{R_{IN} (1 + g_m R_{OUT}) C_\mu} \quad \omega_{p2} = \frac{1}{\frac{1}{g_m} (C_\pi + C_D)}$$

For case 2 and 3,



Source Follower

FR-31



$$v_g = \left(\frac{1}{1 + j\omega R_{IN} C_\pi} \right) \cdot (v_{in} - v_{OUT}) + v_{OUT}$$

$$\frac{v_{OUT}}{R_S} = \frac{v_g - v_{OUT}}{1} + g_m \cdot v_g - (1 + \chi) \cdot g_m \cdot v_{OUT}$$

Source Follower (Cont.)

FR-32

$$v_{OUT} \cdot \left(\frac{1}{R_S} + (1 + \chi) \cdot g_m + j\omega C_\pi \right) = (j\omega C_\pi + g_m) \cdot v_s$$

$$\frac{v_{OUT}}{v_{in}} = \frac{g_m R_S}{1 + (1 + \chi) \cdot g_m R_S} \cdot \left[\frac{(1 + j\omega \frac{C_\pi}{g_m})}{1 + j\omega R_{IN} \frac{C_\pi (1 + \chi \cdot g_m R_S)}{1 + (1 + \chi) g_m R_S}} \right]$$

$$\omega_z = \frac{g_m}{C_\pi}$$

$$\omega_p = \frac{1}{R_{IN} C_\pi (1 - A)}$$

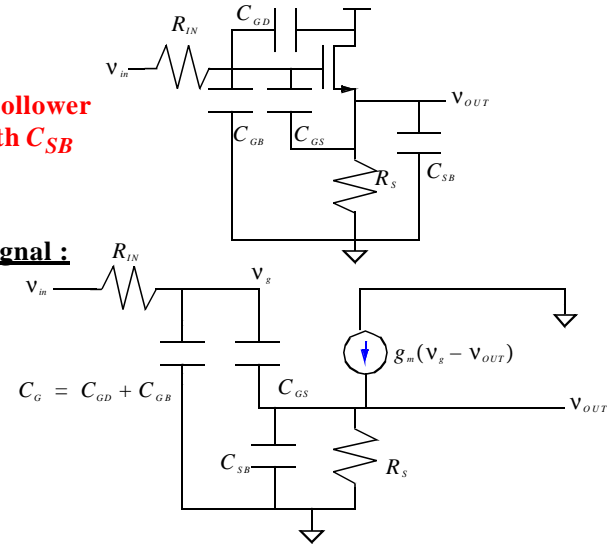
$$A = \frac{g_m R_S}{1 + (1 + \chi) g_m R_S}$$

$$A_v = \frac{g_m R_S}{1 + (1 + \chi) g_m R_S}$$

Source Follower again with C_{SB}

FR-33

Small Signal :



Source Follower (Cont.)

FR-34

$$\frac{v_{in} - v_G}{R_{IN}} = v_G \cdot j\omega C_G + (v_G - v_{OUT}) \cdot j\omega C_{GS}$$

$$(v_G - v_{OUT}) \cdot j\omega C_{GS} - g_m(v_G - v_{OUT}) = \frac{v_{OUT}}{R_S} + v_{OUT} \cdot j\omega C_{SB}$$

$$\frac{v_{OUT}}{v_{in}} = \frac{\frac{g_m R_S}{1 + g_m R_S} \cdot \left[1 + j\omega \frac{C_{GS}}{g_m} \right]}{1 + j\omega \left[R_{IN} C_G + \frac{R_{IN} C_{GS}}{1 + g_m R_S} + \frac{R_S (C_{GS} + C_{SB})}{1 + g_m R_S} \right] - \omega^2 R_S R_{IN} \left[\frac{C_{GS} C_G + C_{SB} (C_G + C_{GS})}{1 + g_m R_S} \right]}$$

let denominator = $\left(1 + j\frac{\omega}{p_1} \right) \cdot \left(1 + j\frac{\omega}{p_2} \right)$

$$= 1 + j\omega \left(\frac{1}{p_1} + \frac{1}{p_2} \right) - \frac{\omega^2}{p_1 p_2}$$

Source Follower (Cont.)

FR-35

for p_1 and p_2 widely separated, if we assume that p_1 is the dominant pole,

$$\frac{1}{p_1} \gg \frac{1}{p_2}$$

$$p_1 = \frac{1}{R_{IN} C_G + \frac{R_{IN} C_{GS}}{1 + g_m R_S} + \frac{R_S (C_{GS} + C_{SB})}{1 + g_m R_S}} = \frac{1}{R_{IN} C_G + \frac{R_{IN} C_{GS}}{1 + g_m R_S} + R_o (C_{GS} + C_{SB})}$$

where,

$$R_o = \frac{1}{g_m} \parallel R_S$$

thus,

$$p_2 = \frac{R_{IN} C_G + \frac{R_{IN} C_{GS}}{1 + g_m R_S} + R_o (C_{GS} + C_{SB})}{R_o R_{IN} [C_{GS} C_G + C_{SB} C_G + C_{SB} C_{GS}]}$$

Source Follower (Cont.)

FR-36

2 limiting cases,

Case 1:

$$R_{IN} \left(C_G + \frac{C_{GS}}{1 + g_m R_s} \right) \gg R_O (C_{GS} + C_{SB})$$

Miller cap = $C_{GS} \cdot (1 - A)$

$$A = \frac{g_m R_s}{1 + g_m R_s}$$

$$\omega_{p1} = \frac{1}{R_{IN} \left(C_G + \frac{C_{GS}}{1 + g_m R_s} \right)}$$

$$1 - A = 1 - \frac{g_m R_s}{1 + g_m R_s} = \frac{1}{1 + g_m R_s}$$

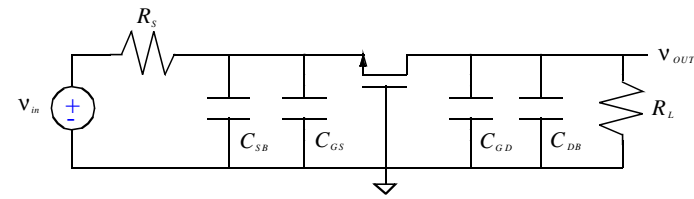
Case 2:

$$R_O (C_{GS} + C_{SB}) \gg R_{IN} \left(C_G + \frac{C_{GS}}{1 + g_m R_s} \right)$$

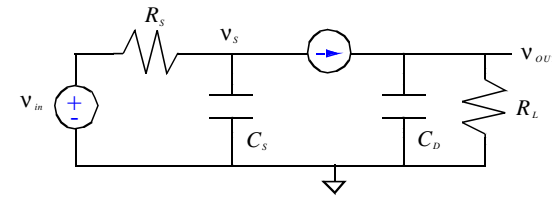
$$\omega_{p1} = \frac{1}{R_O (C_{GS} + C_{SB})}$$

Common Gate

FR-37



assume $r_o \rightarrow \infty$



Common Gate (Cont.)

FR-38

$$\text{KCL@ } v_s \quad \frac{v_{in} - v_s}{R_s} = v_s \cdot j\omega C_s + g_m v_s$$

$$\text{KCL@ } v_{OUT} \quad g_m v_s = v_{OUT} \cdot j\omega C_D + \frac{v_{OUT}}{R_L}$$

$$\frac{v_{OUT}}{v_{in}} = \frac{\frac{g_m R_L}{1 + g_m R_s}}{(1 + j\omega R_L C_D) \cdot \left(1 + j\omega \frac{R_s C_s}{1 + g_m R_s} \right)}$$

no zeros, poles @

$$p_1 = \frac{1}{R_L C_D}$$

$$p_2 = \frac{1}{\frac{R_s}{1 + g_m R_s} C_s} = \frac{1}{\left(R_s \parallel \frac{1}{g_m} \right) C_s}$$

Common Gate (Cont.)

FR-39

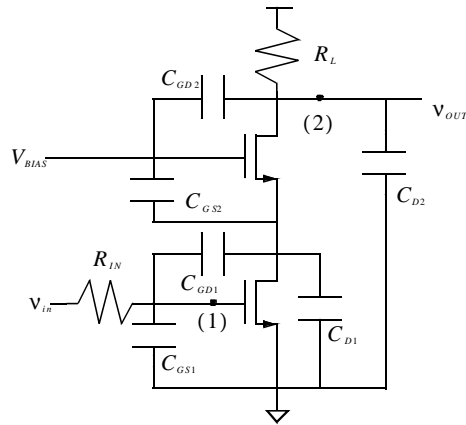
$$\text{@ } v_{OUT} \rightarrow R_{eq} = R_L \quad C_{eq} = C_D \quad \Rightarrow \quad p_1 = \frac{1}{R_{eq} C_{eq}} = \frac{1}{R_L C_D}$$

$$\text{@ } v_s \rightarrow R_{eq} = R_s \parallel \frac{1}{g_m} \quad C_{eq} = C_s \quad \Rightarrow \quad p_2 = \frac{1}{R_{eq} C_{eq}} = \frac{1}{\left(R_s \parallel \frac{1}{g_m} \right) C_s}$$

Since all caps go to ground, finding poles reduces to finding Req's and Ceq's at the nodes.

Cascode - reduces the Miller problem

FR-40



Cascode (Cont.)

FR-41

If we assume that R_s is very large, then,

At (1) :

$$R_{eq} = R_s$$

$$C_{eq} = C_{GS1} + C_{GD1} \cdot (1 - A)$$

$$A = -g_m \cdot \frac{1}{g_m} = -1$$

$$C_{eq} = C_{GS1} + 2 \cdot C_{GD1}$$

$$p_1 = \frac{1}{R_{eq} C_{eq}}$$

At (2) :

$$R_{eq} = R_L \parallel r_o \leftarrow \text{large}$$

$$C_{eq} = C_{GD2} + C_{D2}$$

$$p_2 = \frac{1}{R_{eq} C_{eq}}$$

No large Miller multiplication of a capacitance!

Zero Value Time Constant Analysis

FR-42

General case of dominant pole approximation. Use this technique for complex circuits where we can't identify node with large R_{eq} and C_{eq}

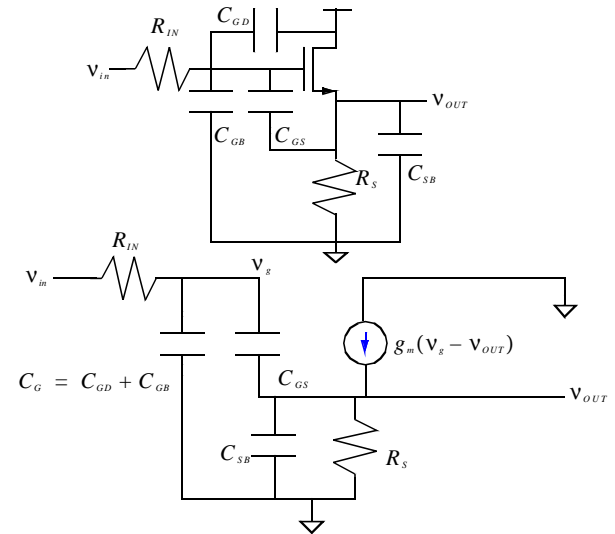
Strategy :

- 1) Set all caps $C_j=0$ except for C_i
- 2) Find resistance seen by C_i
- 3) Calculate $R_i C_i$ for all caps

$$4) \omega_{3dB} = \frac{1}{\sum R_i C_i}$$

ZVTC with Source Follower

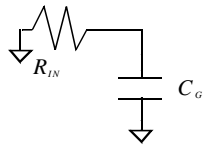
FR-43



ZVTC with Source Follower

FR-44

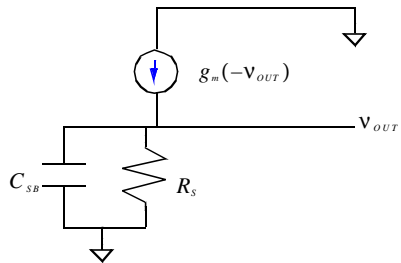
$C_1 :$



$$R_1 = R_{IN}$$

$$C_1 = C_G$$

$C_2 :$



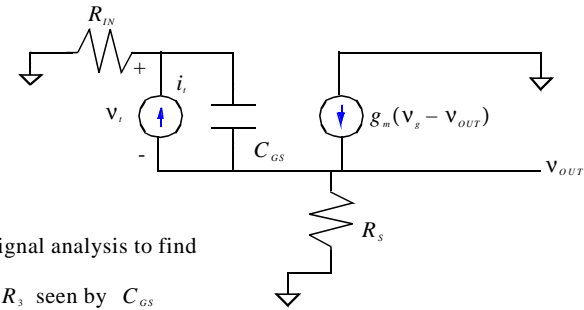
$$R_2 = R_S \parallel \frac{1}{g_m}$$

$$C_2 = C_{SB}$$

ZVTC with Source Follower (Cont.)

FR-45

$C_3 :$



do small signal analysis to find

$$\frac{v_i}{i_i} = R_3 \text{ seen by } C_{GS}$$

$$R_3 = R_o + \frac{R_{IN}}{1 + g_m R_S} \text{ where } R_o = R_S \parallel \frac{1}{g_m} \quad C_3 = C_{GS}$$

hence,

$$\omega_{s4B} = \frac{1}{R_1 C_1 + R_2 C_2 + R_3 C_3} = \frac{1}{R_{IN} C_G + R_o C_{SB} + \left(R_o + \frac{R_{IN}}{1 + g_m R_S} \right) C_{GS}}$$