

CORRECTIONS TO SOLUTIONS MANUAL

In the new edition, some chapter problems have been reordered and equations and figure references have changed. The solutions manual is based on the preview edition and therefore must be corrected to apply to the new edition. Below is a list reflecting those changes.

The “NEW” column contains the problem numbers in the new edition. If that problem was originally under another number in the preview edition, that number will be listed in the “PREVIEW” column on the same line. In addition, if a reference used in that problem has changed, that change will be noted under the problem number in quotes. Chapters and problems not listed are unchanged.

For example:

NEW	PREVIEW
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4.18	4.5
“Fig. 4.38”	“Fig. 4.35”
“Fig. 4.39”	“Fig. 4.36”

The above means that problem 4.18 in the new edition was problem 4.5 in the preview edition. To find its solution, look up problem 4.5 in the solutions manual. Also, the problem 4.5 solution referred to “Fig. 4.35” and “Fig. 4.36” and should now be “Fig. 4.38” and “Fig. 4.39,” respectively.

CHAPTER 3

NEW	PREVIEW
-----	-----
3.1	3.8
3.2	3.9
3.3	3.11
3.4	3.12
3.5	3.13
3.6	3.14
3.7	3.15
“From 3.6”	“From 3.14”
3.8	3.16
3.9	3.17
3.10	3.18
3.11	3.19
3.12	3.20
3.13	3.21
3.14	3.22
3.15	3.1

3.16	3.2
3.17	3.2'
3.18	3.3
3.19	3.4
3.20	3.5
3.21	3.6
3.22	3.7
3.23	3.10
3.24	3.23
3.25	3.24
3.26	3.25
3.27	3.26
3.28	3.27
3.29	3.28

CHAPTER 4

NEW	PREVIEW
-----	-----
4.1	4.12
4.2	4.13
4.3	4.14
4.4	4.15
4.5	4.16
4.6	4.17
4.7	4.18
“p. 4.6”	“p. 4.17”
4.8	4.19
4.9	4.20
4.10	4.21
4.11	4.22
4.12	4.23
4.13	4.24
“p. 4.9”	“p. 4.20”
4.14	4.1
“(4.52)”	“(4.51)”
“(4.53)”	“(4.52)”
4.15	4.2
4.16	4.3
4.17	4.4
4.18	4.5
“Fig. 4.38”	“Fig. 4.35”
“Fig. 4.39”	“Fig. 4.36”
4.19	4.6
“Fig 4.39(c)”	“Fig 4.36(c)”

4.20	4.7
4.21	4.8
4.22	4.9
4.23	4.10
4.24	4.11
4.25	4.25
4.26	4.26
“p. 4.9”	“p. 4.20”

CHAPTER 5

NEW	PREVIEW
-----	-----
5.1	5.16
5.2	5.17
5.3	5.18
5.4	5.19
5.5	5.20
5.6	5.21
5.7	5.22
5.8	5.23
5.9	5.1
5.10	5.2
5.11	5.3
5.12	5.4
5.13	5.5
5.14	5.6
5.15	5.7
5.16	5.8
5.17	5.9
5.18	5.10
“Similar to 5.18(a)”	“Similar to 5.10(a)”
5.19	5.11
5.20	5.12
5.21	5.13
5.22	5.14
5.23	5.15

CHAPTER 6

NEW	PREVIEW
-----	-----
6.1	6.7
6.2	6.8

6.3	6.9
“from eq(6.23)”	“from eq(6.20)”
6.4	6.10
6.5	6.11
“eq (6.52)”	“eq (6.49)”
6.6	6.1
6.7	6.2
6.8	6.3
6.9	6.4
6.10	6.5
6.11	6.6
6.13	6.13
“eq (6.56)”	“eq (6.53)”
“problem 3”	“problem 9”
6.16	6.16
“to (6.23) & (6.80)”	“to (6.20) & (6.76)”
6.17	6.17
“equation (6.23)”	“equation (6.20)”

CHAPTER 7

NEW	PREVIEW
-----	-----
7.2	7.2
“eqn. (7.59)”	“eqn. (7.57)”
7.17	7.17
“eqn. (7.59)”	“eqn. (7.57)”
7.19	7.19
“eqns 7.66 and 7.67”	“eqns 7.60 and 7.61”
7.21	7.21
“eqn. 7.66”	“eqn. 7.60”
7.22	7.22
“eqns 7.70 and 7.71”	“eqns. 7.64 and 7.65”
7.23	7.23
“eqn. 7.71”	“eqn. 7.65”
7.24	7.24
“eqn 7.79”	“eqn 7.73”

CHAPTER 8

NEW	PREVIEW
-----	-----
8.1	8.5
8.2	8.6

8.3	8.7
8.4	8.8
8.5	8.9
8.6	8.10
8.7	8.11
8.8	8.1
8.9	8.2
8.10	8.3
8.11	8.4
8.13	8.13
“problem 8.5”	“problem 8.9”

CHAPTER 13

NEW	PREVIEW
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3.17	3.17
“Eq. (3.123)”	“Eq. (3.119)”

CHAPTER 14 - New Chapter, “Oscillators”

CHAPTER 15 - New Chapter, “Phase-Locked Loops”

CHAPTER 16 - Was Chapter 14 in Preview Ed.

Change all chapter references in solutions manual from 14 to 16.

CHAPTER 17 - Was Chapter 15 in Preview Ed.

Change all chapter references in solutions manual from 15 to 17.

CHAPTER 18 - Was Chapter 16 in Preview Ed.

NEW	PREVIEW
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18.3	16.3
“Fig. 18.12(c)”	“Fig. 16.13(c)”
18.8	16.8
“Fig. 18.33(a,b,c,d)”	“Fig. 16.34(a,b,c,d)”

Also, change all chapter references from 16 to 18.

14.1 Open-Loop Transfer Function:

$$H(s) = \frac{-(g_m R_D)^2}{\left(1 + \frac{s}{\omega_0}\right)^2}, \quad \omega_0 = \frac{1}{R_D C_L}$$

The gain drops to unity at $\frac{g_m R_D}{\left(1 + \frac{\omega_u^2}{\omega_0^2}\right)^{1/2}} = 1$, which for $g_m R_D \gg 1$, yields, $\omega_u \gg \omega_0$ and $\omega_u \cong \omega_0 \cdot g_m R_D = \frac{g_m}{C_L}$. The phase changes from -180° at $\omega \approx \omega_0$ to $-2 \tan^{-1} \frac{\omega_u}{\omega_0} - 180^\circ$ at ω_u ; i.e., the phase change at ω_u is $-2 \tan^{-1}(g_m R_D)$ and the phase margin is equal to $180^\circ - 2 \tan^{-1}(g_m R_D)$.

14.2 (a) $g_m R_D \cong 2 \Rightarrow R_D \geq 400 \Omega$.

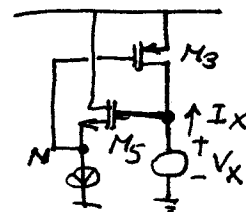
$$(b) \begin{cases} \omega_{osc} = \sqrt{3} \omega_0 = \sqrt{3} / (R_D C_L) \\ \text{Total Gain} = (g_m R_D)^3 = 16 \Rightarrow R_D = 504 \Omega \end{cases} \Rightarrow C_L = 0.547 \text{ pF}$$

14.3 Each stage must provide a small-signal gain of 2. That is, $g_{m1} R_1 = 2$. With small swings, each transistor carries half of the tail current. For square-law devices, therefore, we have

$$g_{m1} R_1 = 2 = \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} R_1 = 2 \Rightarrow I_{SS} \geq \frac{4}{\mu_n C_{ox} \frac{W}{L} R_1^2}$$

14.4 Neglecting body effect of M_5 , we have

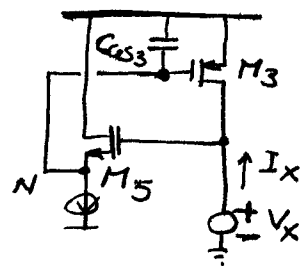
$V_N \approx V_X$. Thus, the gate and drain of M_3 experience equal voltage variations. That is, M_3 operates as a diode-connected device, providing an impedance of $1/g_{m3}$.



$$14.5 \quad \frac{V_N}{V_X} = \frac{\frac{1}{C_{GS3}S}}{\frac{1}{C_{GS3}S} + \frac{1}{g_{m5}}} \quad (r = \lambda = 0)$$

$$= \frac{g_{m5}}{g_{m5} + C_{GS3}S} \quad \Rightarrow \quad \frac{I_X}{V_X} = \frac{g_{m3} g_{m5}}{g_{m5} + C_{GS3}S}$$

$$\Rightarrow \frac{V_X}{I_X} = \frac{1}{g_{m3}} + \frac{C_{GS3}}{g_{m3} g_{m5}} S \quad \Rightarrow \text{The impedance is always inductive.}$$



$$14.6 \quad \text{To avoid latchup, } g_m R_S < 1 \Rightarrow R_S < \frac{1}{g_m}.$$

14.7 The drain currents saturate near I_{SS} and 0 for a short while, creating a "suarish" waveform. The output voltages are the result of injecting the currents into the tanks. Since the tanks provide suppression at higher harmonics, V_X and V_Y are filtered versions of I_{D1} and I_{D2} .

14.8 For the circuit to oscillate, the loop gain must exceed unity: $g_m R_p > 1 \Rightarrow$

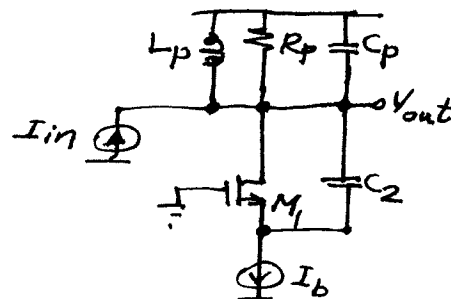
$$g_m > \frac{1}{R_p}. \text{ For square-law devices, } \sqrt{\mu_n C_{ox} \frac{W}{L} I_{SS}} > \frac{1}{R_p}. \text{ Thus,}$$

$$I_{SS} > \frac{1}{\mu_n C_{ox} \frac{W}{L} R_p^2}.$$

For M_1 and M_2 not to enter the triode region, the maximum value of V_X and the minimum value of V_Y must differ by no more than V_{TH} . That is, the peak-to-peak swing at X or Y must be less than V_{TH} . Since the peak-to-peak swing is $\approx I_{SS} R_p$, we must have $I_{SS} R_p < V_{TH}$.

14.9 Since the total current flowing thru M_1 and C_2 is equal to I_b , a constant value.

$$\text{Thus, } \frac{V_{out}}{I_{in}} = (L_p S) \parallel R_p \parallel \frac{1}{C_p S},$$



14.10 Replace R_p with $R_p \parallel \frac{1}{C_p s} = \frac{R_p}{R_p C_p s + 1}$ in Eq. (14.40). The

denominator then reduces to:

$$R_p C_1 C_2 L_p S^3 + (C_1 + C_2) L_p R_p C_p S^3 + (C_1 + C_2) L_p S^2 + [g_m L_p R_p C_p S + g_m L_p + R_p (C_1 + C_2)] S + g_m R_p$$

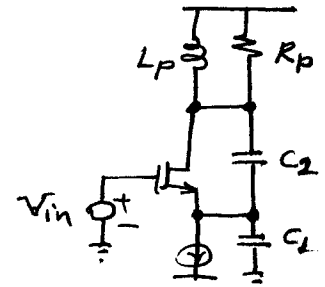
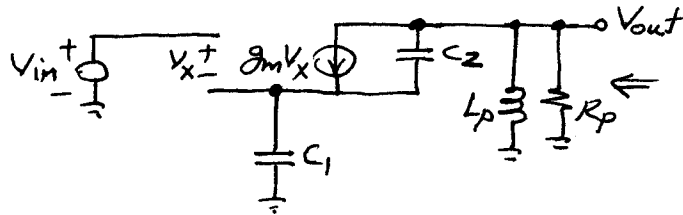
Grouping the imaginary terms and equating their sum to zero, we have

$$-R_p L_p \omega^3 [C_1 C_2 + (C_1 + C_2) C_p] + [g_m L_p + R_p (C_1 + C_2)] \omega = 0$$

Assuming $g_m L_p \ll R_p (C_1 + C_2)$, we obtain

$$\omega^2 = \frac{1}{L_p \left(\frac{C_1 C_2}{C_1 + C_2} + C_p \right)}$$

14.11



The current thru $R_p \parallel (L_p s)$ is equal to $V_{out} \left(\frac{1}{R_p} + \frac{1}{L_p s} \right)$. The negative of this current flows thru C_1 , generating a voltage $-V_{out} \left(\frac{1}{R_p} + \frac{1}{L_p s} \right) \frac{1}{C_1 s}$ across it. Thus, $V_x = V_{in} + V_{out} \left(\frac{1}{R_p} + \frac{1}{L_p s} \right) \frac{1}{C_1 s}$. Also, the current thru C_2 is equal to $\left[V_{out} + V_{out} \left(\frac{1}{R_p} + \frac{1}{L_p s} \right) \frac{1}{C_1 s} \right] C_2 s$.

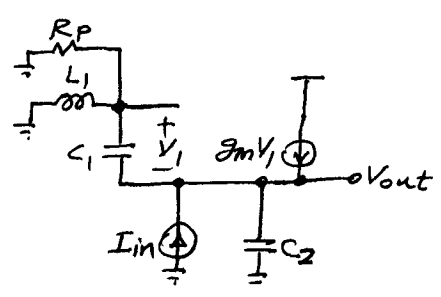
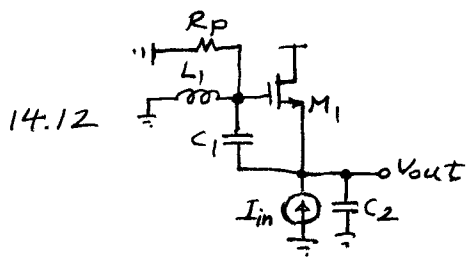
Adding $g_m V_x$ and the current thru C_2 and equating the result to $-V_{out} \left(\frac{1}{R_p} + \frac{1}{L_p s} \right)$, we have

$$\left[V_{in} + V_{out} \left(\frac{1}{R_p} + \frac{1}{L_p s} \right) \frac{1}{C_1 s} \right] g_m + \left[V_{out} + V_{out} \left(\frac{1}{R_p} + \frac{1}{L_p s} \right) \frac{1}{C_1 s} \right] C_2 s = -V_{out} \left(\frac{1}{R_p} + \frac{1}{L_p s} \right)$$

It follows that

$$\frac{V_{out}}{V_{in}} = \frac{-g_m L_p R_p C_1 S^2}{R_p L_p C_2 C_1 S^3 + L_p (C_1 + C_2) S^2 + [g_m L_p + R_p (C_1 + C_2)] S + g_m R_p}$$

Note that the denominator is the same as in Eq. (14.40).



$$V_1 = -(I_{in} - V_{out} C_2 s + g_m V_1) / C_1 s \Rightarrow V_1 (1 + g_m / C_1 s) = \frac{-I_{in} + V_{out} C_2 s}{C_1 s}$$

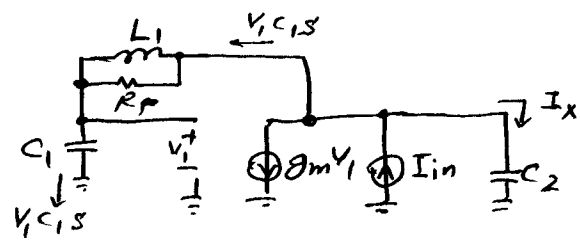
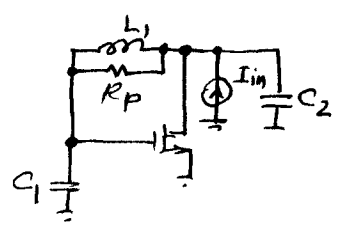
$$\Rightarrow V_1 = \frac{-I_{in} + V_{out} C_2 s}{g_m + C_1 s}$$

writing a KVL, we have $-V_1 C_1 s \frac{R_p L_1 s}{R_p + L_1 s} = V_1 + V_{out}$.

It follows that

$$V_{out} = - \frac{I_{in} + V_{out} C_2 s}{g_m + C_1 s} \left[1 + \frac{C_1 s R_p L_1 s}{R_p + L_1 s} \right]$$

Simplifying and calculating the denominator of V_{out}/I_{in} , we have $R_p L_1 C_1 C_2 s^3 + L_1 (C_1 + C_2) s^2 + [R_p (C_1 + C_2) + g_m L_1] s + g_m R_p$, which is the same as Eq. (14.40). Thus, the oscillation conditions are the same as those of Colpitts oscillator.



We can consider V_1 as the output because for oscillation to begin the gain from I_{in} to V_1 must be infinite as well. First, assume $R_p \rightarrow \infty$:

$$I_x = +V_1 C_1 s (L_1 s + \frac{1}{C_1 s}) C_2 s = -g_m V_1 + I_{in} - V_1 C_1 s$$

$$\Rightarrow V_1 \left[C_1 C_2 s^2 (L_1 s + \frac{1}{C_1 s}) + g_m + C_1 s \right] = I_{in}$$

Now, include R_p : $V_1 \left[C_1 C_2 s^2 \left(\frac{R_p L_1 s}{R_p + L_1 s} + \frac{1}{C_1 s} \right) + g_m + C_1 s \right] = I_{in}$

$$\Rightarrow V_1 \left[\frac{C_1 C_2 s^2 (R_p C_1 L_1 s^2 + R_p + L_1 s) + (g_m + C_1 s) (C_1 s) (R_p + L_1 s)}{C_1 s (R_p + L_1 s)} \right] = I_{in}$$

\Rightarrow denominator of V_1/I_{in} is

($C_1 s$ is factored from numerator & denominator.)

$$R_p C_1 C_2 L_1 s^3 + R_p C_2 s + L_1 C_2 s^2 + g_m R_p + g_m L_1 s + C_1 R_p s + C_1 L_1 s^2$$

$$= R_p C_1 C_2 L_1 s^3 + L_1 (C_1 + C_2) s^2 + [R_p (C_1 + C_2) + g_m L_1] s + g_m R_p,$$

the same as that in Eq. (14.40).

14.13 $I_T = 1 \text{ mA}, (\frac{W}{L})_{1,2} = 50/0.5$

(a) For a three-stage ring, the minimum gain per stage at low freqs must be 2. Thus, $g_{m1,2} R_{1,2} = 2$ (when no current flows thru M_3 and M_4). $\Rightarrow R_{1,2} = 2/g_{m1,2}$. ($g_{m1,2} = \sqrt{\mu_n C_{ox} (\frac{W}{L})_{1,2} I_T}$.)

(b) $g_{m3,4} R = 0.5$ with $I_{D3,4} = 0.5 \text{ mA}$.

$$g_{m3,4} = \sqrt{\mu_n C_{ox} (\frac{W}{L})_{3,4} I_T} = g_{m1,2} \sqrt{\frac{(\frac{W}{L})_{3,4}}{(\frac{W}{L})_{1,2}}} = \frac{2}{R}$$

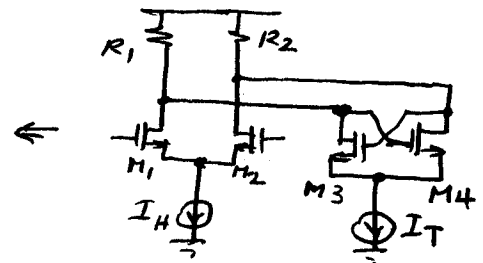
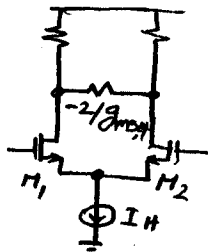
$$\Rightarrow \frac{2}{R} \sqrt{\frac{(\frac{W}{L})_{3,4}}{(\frac{W}{L})_{1,2}}} R = 0.5$$

$$\Rightarrow (\frac{W}{L})_{3,4} = 0.25^2 (\frac{W}{L})_{1,2}$$

(c) The voltage gain must be equal to 2 with a diff pair tail current of I_H while M_3 and M_4 carry all of I_T .

$$|A_v| = g_{m1,2} (R_{1,2} \parallel \frac{-1}{g_{m3,4}})$$

$$= g_{m1,2} \frac{R_{1,2}}{1 - g_{m3,4} R_{1,2}}$$



If $g_{m3,4} R_{1,2} < 1$ (to avoid latch-up), then

$$g_{m1,2} R_{1,2} > 2(1 - g_{m3,4} R_{1,2})$$

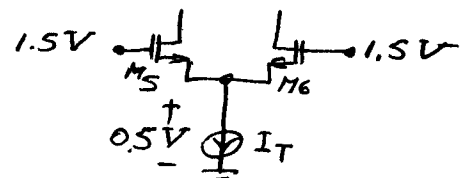
$$\Rightarrow \sqrt{2 \frac{I_H}{2} \mu_n C_{ox} (\frac{W}{L})_{1,2}} R_{1,2} > 2(1 - \sqrt{2 \frac{I_T}{2} \mu_n C_{ox} (\frac{W}{L})_{3,4}} R_{1,2})$$

Thus, I_H can be determined.

(d) Neglecting body effect for simplicity, we have

$$\frac{I_T}{2} = \frac{1}{2} \mu_n C_{ox} (\frac{W}{L})_{5,6} (V_{G5,6} - V_{TH5,6})^2$$

$$\Rightarrow (\frac{W}{L})_{5,6} = \frac{I_T}{\mu_n C_{ox} (V_{G5,6} - V_{TH5,6})^2} \text{ and } V_{G5,6} + 0.5 \text{ V} = 1.5 \text{ V}$$



14.14 If each inductor contributes a cap of C_1 , then

$$f_{osc, \min} = \frac{1}{2\pi\sqrt{L(C_0 + C_1)}} \quad , \quad f_{osc, \max} = \frac{1}{2\pi\sqrt{L(0.62C_0 + C_1)}}$$

Thus, the tuning range is given by $\frac{f_{osc, \max}}{f_{osc, \min}} = \sqrt{\frac{C_0 + C_1}{0.62C_0 + C_1}}$,

which is less than 27%. For example, if $C_1 = 0.2C_0$, then,

$$f_{osc, \max} / f_{osc, \min} \approx 1.21.$$

14.15 (a) $L_p = 5 \text{ nH}$, $C_x = 0.5 \text{ pF}$ $f_{osc} = 1 \text{ GHz} = \frac{1}{2\pi\sqrt{5 \text{ nH} \times (C_x + C_D)}}$

$$\Rightarrow C_D = 4.566 \text{ pF}.$$

(b) $Q = \frac{\omega L}{R_p} = 4 \Rightarrow R_p = 125.7 \Omega \Rightarrow$

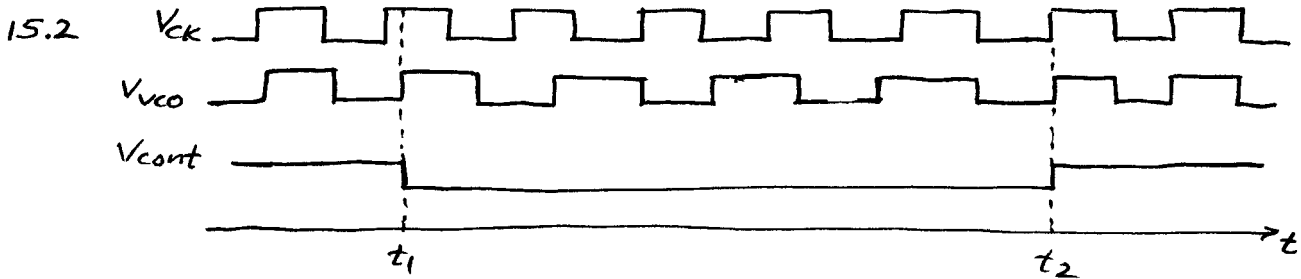
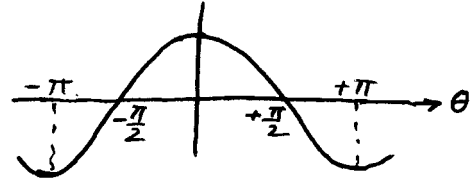
With a 1-mA tail current, the peak-to-peak swing on each side is approximately equal to 126 mV.

Chapter 15
Phase-Locked Loops

15.1

15.1 With two signals $V_1 \cos \omega t$ and $V_2 \cos(\omega t + \theta)$, the product is $V_{out} = \frac{1}{2} V_1 V_2 [\cos(2\omega t + \theta) + \cos \theta]$. If the high-freq. component is filtered out, $\overline{V_{out}} \propto \cos \theta$.

The phase detector is linear only for a small neighborhood around $\theta = \pm \frac{\pi}{2}$.



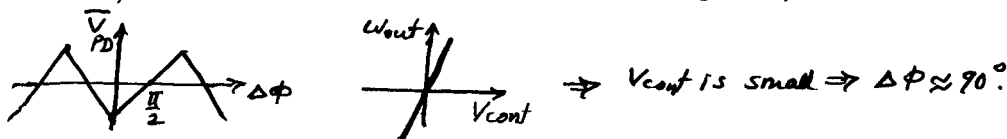
The difference between the two frequencies is integrated between t_1 and t_2 to accumulate a difference of ϕ_0 :

$$(f_H - f_L)(t_2 - t_1) = \frac{\phi_0}{2\pi}$$

$$\Rightarrow t_2 - t_1 = \frac{\phi_0}{2\pi(f_H - f_L)}$$

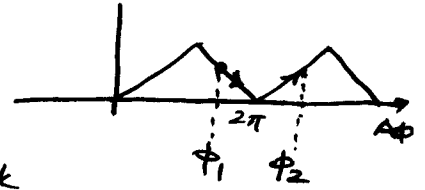
15.3 The VCO still requires a dc voltage that defines the frequency of operation. A high-pass filter would not provide the dc component.

15.4 The loop must lock such that the phase difference is away from zero because the PD gain drops to zero at $\Delta\phi = 0$. With a large loop gain, the PD output settled around half of its full scale. This point can be better seen in a fully-differential implementation:

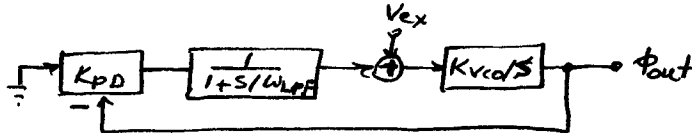


15.5 Suppose the loop begins with $\Delta\phi = \phi_1$.

If the feedback is positive, the loop accumulated so much phase to drive the PD toward ϕ_2 , where the feedback is negative and the loop can settle.



15.6 Note: ϕ_{ex} should be changed to V_{ex} .



$$(-\phi_{out} \cdot K_{PD} \cdot \frac{1}{1 + \frac{s}{W_{LPF}}} + V_{ex}) \frac{K_{VCO}}{s} = \phi_{out}$$

$$\Rightarrow \phi_{out} \left(1 + \frac{K_{PD} K_{VCO}}{s(1 + \frac{s}{W_{LPF}})} \right) = V_{ex} \frac{K_{VCO}}{s} \Rightarrow$$

$$\frac{\phi_{out}}{V_{ex}} = \frac{K_{VCO}}{s + \frac{K_{PD} K_{VCO}}{1 + \frac{s}{W_{LPF}}}} = \frac{K_{VCO} (1 + \frac{s}{W_{LPF}})}{\frac{s^2}{W_{LPF}} + s + K_{PD} K_{VCO}}$$

15.7

$$\zeta = \frac{1}{2} \sqrt{\frac{W_{LPF}}{K_{PD} K_{VCO}}} \quad \sqrt{\frac{K_{VCO1}}{K_{VCO2}}} = 1.5$$

$$\Rightarrow \frac{K_{VCO1}}{K_{VCO2}} = 2.25$$

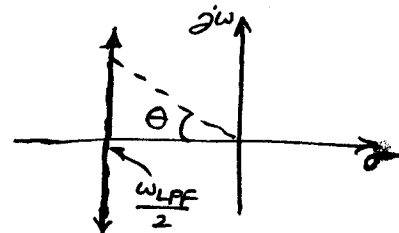
The slope can vary by a factor of 2.25.



15.8

$$\tan \varphi = \frac{\text{Im}(\text{pole})}{-\text{Re}(\text{pole})} = \frac{\sqrt{1 - \zeta^2}}{\zeta}$$

This is indeed as if $\zeta = \cos \varphi$ and $\sqrt{1 - \zeta^2} = \sin \varphi$.



15.9

$$K_{VCO} = 100 \text{ MHz/V}, \quad K_{PD} = 1 \text{ V/rad}, \quad W_{LPF} = 2\pi(1 \text{ MHz})$$

$$\Rightarrow \zeta = \frac{1}{2} \sqrt{\frac{1 \text{ MHz}}{(1 \text{ V/rad})(100 \text{ MHz/V})}} = 0.05$$

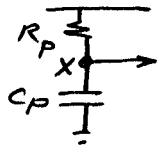
$$\frac{W_n}{2\pi} = \sqrt{(1 \text{ MHz})(1 \text{ V/rad})(100 \text{ MHz/V})} = 10 \text{ MHz}$$

The loop is heavily underdamped.

$$\tau = 318 \text{ ns}$$

$$\text{Step response} \approx [1 - e^{-t/318 \text{ ns}} \sin(2\pi \times 10 \text{ MHz} \times t + \theta)] u(t), \quad \theta \leq 90^\circ$$

15.10

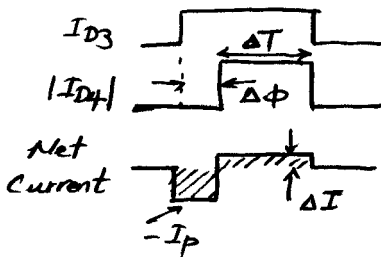


If the control voltage is sensed at node X, then R_p appears in series with the current sources in the charge pump, failing to provide a zero.

15.11 From (15.40), $\frac{I_{out}(s)}{\Delta\phi} = \frac{I_p}{2\pi}$. Since I_{out} is multiplied by the series combination of R_p and C_p :

$$\frac{V_{out}(s)}{\Delta\phi} = \frac{I_p}{2\pi} (R_p + \frac{1}{C_p s}).$$

15.12



$\Delta\phi$ must be such that the net current is zero. If the current mismatch equals ΔI and the width of $|I_{D4}|$ pulses is ΔT , then
 $(\frac{\Delta\phi}{2\pi} \cdot T_p) I_p = \Delta T \cdot \Delta I$, where T_p is the period.
 $\Rightarrow \Delta\phi = 2\pi \frac{\Delta T}{T_p} \cdot \frac{\Delta I}{I_p}$

15.13 $\omega_{out} = \omega_0 + K_{vco} V_{cont}$, $V_{cont} = V_m \cos \omega_m t$. The VCO output is

$$V_{out} = V_0 \cos \left[\int \omega_{out} dt \right] = V_0 \cos \left[\omega_0 t + K_{vco} V_m \int \cos \omega_m t dt \right]$$

$$= V_0 \cos \omega_0 t \cos \left(K_{vco} \frac{V_m}{\omega_m} \sin \omega_m t \right) - V_0 \sin \omega_0 t \sin \left(K_{vco} \frac{V_m}{\omega_m} \sin \omega_m t \right).$$

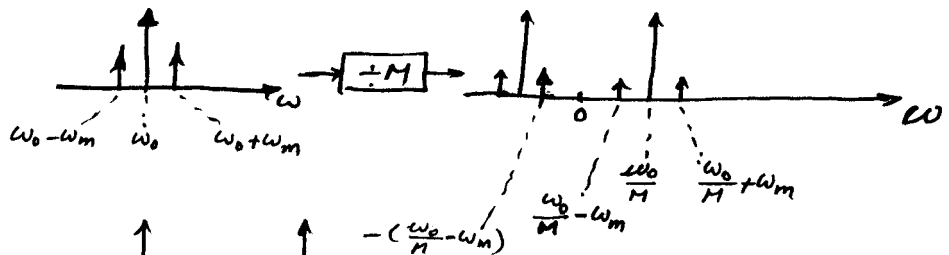
For small V_m , $V_{out}(t) \approx V_0 \cos \omega_0 t - \frac{K_{vco} V_m V_0}{2 \omega_m} [\cos(\omega_0 - \omega_m)t - \cos(\omega_0 + \omega_m)t]$.

The divider output is expressed as

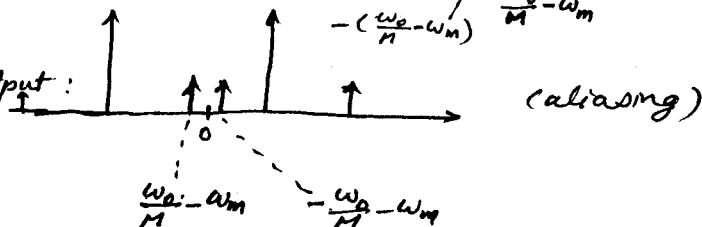
$$V_{out,M} = V_0 \cos \left[\frac{\omega_0 t}{M} + \frac{K_{vco} V_m}{M} \int \cos \omega_m t dt \right]$$

$$\approx V_0 \cos \frac{\omega_0}{M} t - \frac{K_{vco} V_m V_0}{2 M \omega_m} \left[\cos \left(\frac{\omega_0}{M} - \omega_m \right) t - \cos \left(\frac{\omega_0}{M} + \omega_m \right) t \right].$$

If $\frac{\omega_0}{M} > \omega_m$,



If $\frac{\omega_0}{M} < \omega_m$, output:



$$15.14 \quad S_{1,2} = -\xi \omega_n \pm \omega_n \sqrt{\xi^2 - 1} \quad \begin{array}{l} \xi \propto \sqrt{I_p K_{vco}} \\ \omega_n \propto \sqrt{I_p K_{vco}} \end{array}$$

As $I_p K_{vco}$ starts from small values, $S_{1,2}$ are complex:

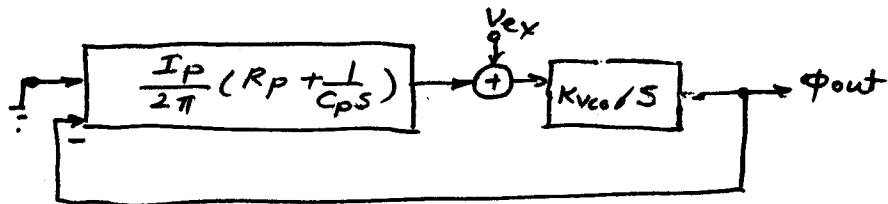
$$\operatorname{Re}\{S_{1,2}\} = -\xi \omega_n \quad \operatorname{Im}\{S_{1,2}\} = \pm \omega_n \sqrt{1 - \xi^2}$$

Noting that $\omega_n = \frac{2\xi}{R_p C_p}$, we can write $\omega_n^2 - \frac{2\xi \omega_n}{R_p C_p} = 0$

Adding $(\frac{1}{R_p C_p})^2$ to both sides and subtracting and adding $-\xi^2 \omega_n^2$, we obtain $(-\xi \omega_n + \frac{1}{R_p C_p})^2 + \omega_n^2(1 - \xi^2) = (\frac{1}{R_p C_p})^2$, which is a circle centered at $-\frac{1}{R_p C_p}$ with a radius equal to $\frac{1}{R_p C_p}$.

For $\xi \geq 1$, the poles become real and move away from each other: $-\xi \omega_n + \omega_n \sqrt{\xi^2 - 1}$ and $-\xi \omega_n - \omega_n \sqrt{\xi^2 - 1}$. If $\xi \rightarrow \infty$, then $-\xi \omega_n + \omega_n \sqrt{\xi^2 - 1} = \omega_n(-\xi + \sqrt{\xi^2 - 1}) = \omega_n \xi(-1 + \sqrt{1 - \frac{1}{\xi^2}}) \approx \omega_n \xi(-1 + (1 - \frac{1}{2\xi^2})) \approx \frac{-\omega_n}{2\xi} = \frac{-1}{R_p C_p}$.

15.15 Note: ϕ_{ex} should be changed to V_{ex} .

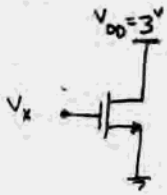


$$\begin{aligned} & \left[-\phi_{out} \cdot \frac{I_p}{2\pi} \left(\frac{R_p C_p s + 1}{C_p s} \right) + V_{ex} \right] \frac{K_{vco}}{s} = \phi_{out} \\ \Rightarrow \phi_{out} \left[1 + \frac{I_p K_{vco} (R_p C_p s + 1)}{2\pi C_p s^2} \right] &= V_{ex} \frac{K_{vco}}{s} \Rightarrow \\ \frac{\phi_{out}}{V_{ex}} &= \frac{K_{vco} (2\pi C_p s^2)}{2\pi C_p s^2 + I_p K_{vco} R_p C_p s + 1} \end{aligned}$$

15.16 When the VCO frequency is far from the input frequency, the PFD operates as a frequency detector, comparing the VCO and input frequencies. Thus, the VCO transfer function must relate the output frequency to the control voltage:

$\Delta \omega_{\text{out}} = K_{\text{VCO}} \Delta V_{\text{cont}} \Rightarrow$ the order of the system falls by one (compared to when the VCO phase is of interest: K_{VCO}/s .)

2.1) a) NMOS :

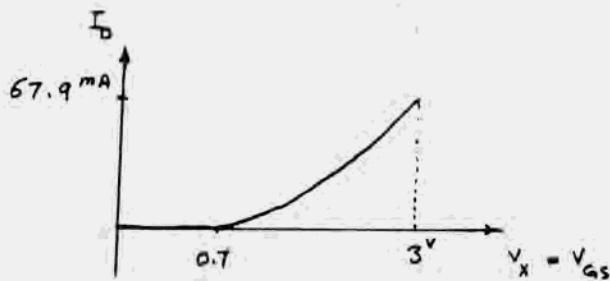


for $V_x < V_{TH} (= 0.7)$ device is off, $I_D \approx 0$

for $V_x \geq 0.7$

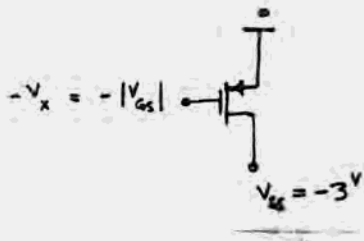
$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_x - 0.7)^2 (1 + \lambda \cdot 3V) \quad (L_{eff} = 0.5^{\mu} - 2L_0)$$

$$I_D = 12.8 \left(\frac{mA}{V^2} \right) \cdot (V_x - 0.7)^2$$



b) PMOS :

Solution is the same

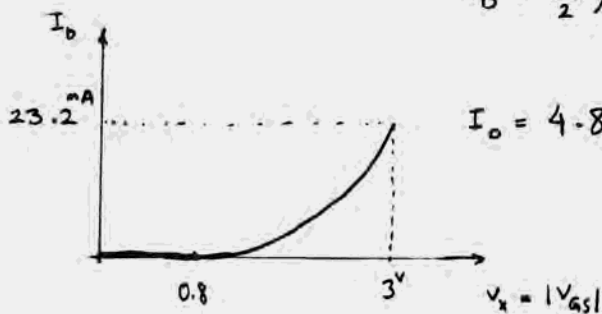


for $|V_{GS}| < V_{TH} (= 0.8)$ $I_D \approx 0$

for $|V_{GS}| \geq 0.8$

$$I_D = \frac{1}{2} \mu_p C_{ox} \frac{W}{L_{eff}} (V_x - 0.8)^2 (1 + \lambda \cdot 3V)$$

$$I_D = 4.8 \left(\frac{mA}{V^2} \right) \cdot (V_x - 0.8)^2$$



2.2) a) Nmos

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 3.66 \frac{mA}{V} \quad (\text{Neglecting } L_D)$$

$$r_o = \frac{1}{\lambda I_D} = 20 \text{ k}\Omega$$

$$\text{Intrinsic gain} = g_m r_o = 733 \frac{V}{V}$$

b) PMOS

$$g_m = \sqrt{2\mu_p C_{ox} \frac{W}{L} I_D} = 1.96 \frac{mA}{V}$$

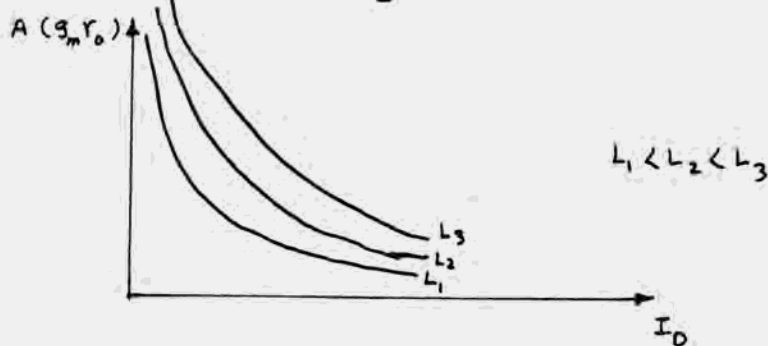
$$r_o = \frac{1}{\lambda I} = \frac{1}{0.2 \cdot 0.5 \text{ mA}} = 10 \text{ k}\Omega$$

$$g_m r_o = 19.6 \frac{V}{V}$$

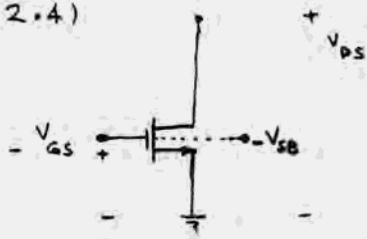
$$2.3) \quad g_m = \sqrt{2\mu C_{ox} \frac{W}{L} I_D} \quad r_o = \frac{1}{\lambda I_D} \quad \text{Assume } \lambda = \frac{k}{L}$$

$$A = g_m r_o = \sqrt{2\mu C_{ox} \frac{W}{L} I_D} \cdot \frac{L}{\alpha I_D}$$

$$A = K \cdot \sqrt{\frac{WL}{I_D}} \quad (K; \text{Constant})$$



2.4)


 I_D versus V_{GS} : (for NMOS)

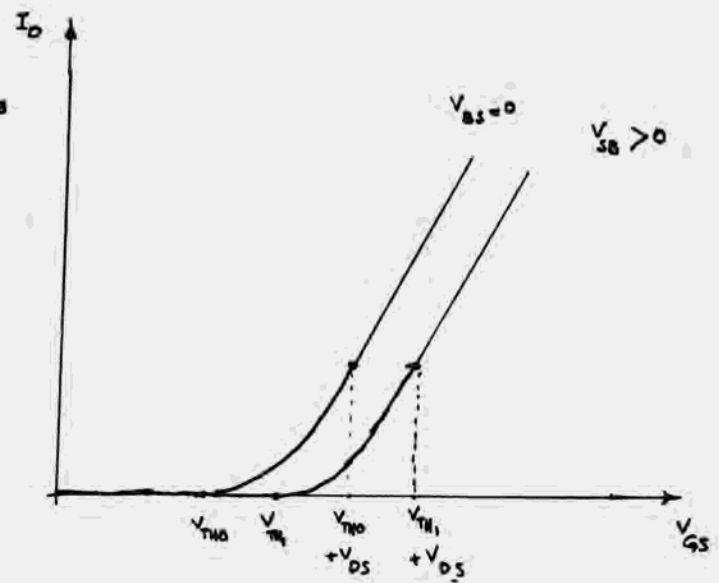
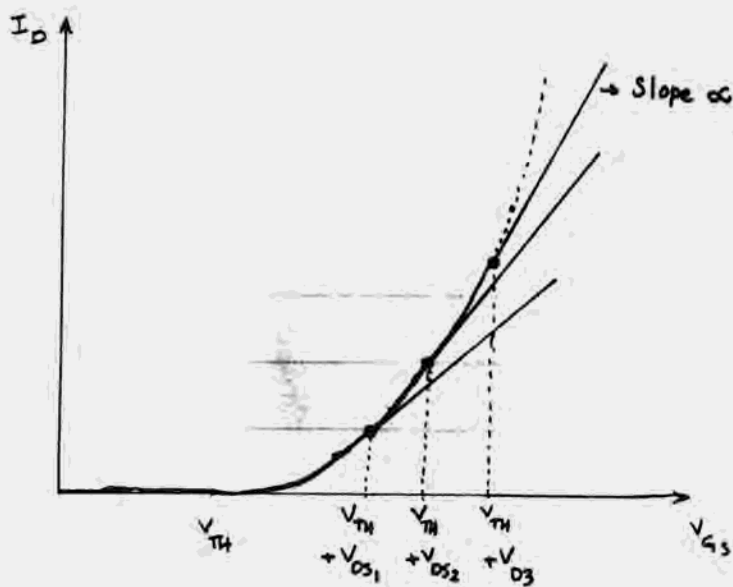
 I) for $V_{GS} < V_{TH}$, $I_D \approx 0$

 II) for $V_{TH} < V_{GS} < V_{TH} + V_{DS} \Rightarrow$ Device is in the Saturation region

$$I_D \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$$

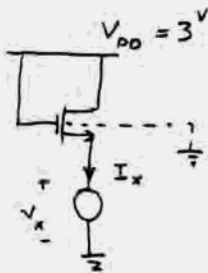
 III) for $V_{GS} > V_{TH} + V_{DS} \Rightarrow$ Device operates in the triode region

$$I_D \approx \mu_n C_{ox} \frac{W}{L} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right]$$



Changing V_{SB} just shifts the curve to the right for $V_{SB} > 0$ or to the left for $V_{SB} < 0$.

2.5) a)



$$\lambda = 0.1, \quad \gamma = 0.45, \quad 2\phi_F = 0.9, \quad V_{TH0} = 0.7$$

$$V_{GS} = 3 - V_x, \quad V_{DS} = 3 - V_x, \quad V_{SB} = V_x$$

$$V_{TH} = V_{TH0} + \gamma (\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F})$$

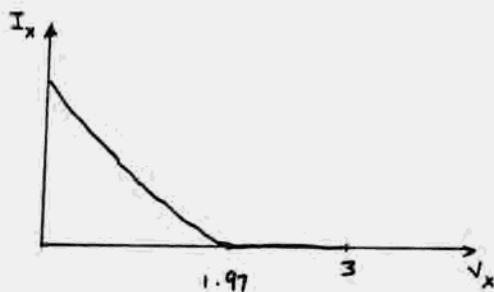
$$\text{So, } I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (3 - V_x - 0.7 - 0.45(\sqrt{0.9 + V_x} - \sqrt{0.9}))^2 (1 + \lambda(3 - V_x))$$

The above equation is valid for

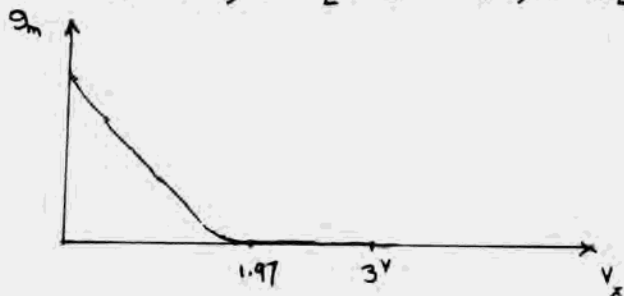
$$3 - V_x - 0.7 - 0.45(\sqrt{0.9 + V_x} - \sqrt{0.9}) > 0, \quad \text{i.e. } V_x < 1.97 \text{ V}$$

$$\text{So, } I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2.727 - V_x - 0.45\sqrt{0.9 + V_x})^2 (1.3 - 0.1V_x)$$

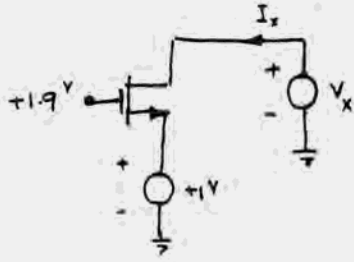
$$\text{and } I_x = 0 \quad \text{for } 1.97 < V_x$$



$$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_x}$$



2.5) b)



$$\lambda = \gamma = 0 \quad V_{TH} = 0.7$$

for $0 < V_x < 1$, S and D exchange their roles.

$$V_{GS} = 1.9 - V_x \quad V_{DS} = 1 - V_x, \quad V_{DD} = 1.2 - V_x$$

$$I_x = -\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[(1.2 - V_x) \cdot 2 \cdot (1 - V_x) - (1 - V_x)^2 \right]$$

$$I_x = -\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (1 - V_x) (1.4 - V_x)$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{DS} = \mu_n C_{ox} \frac{W}{L} (1 - V_x) \quad (\text{absolute value})$$

The above equations are valid for $V_x < 1$

Then the direction of current is reversed.

$$V_{GS} = 1.9 - 1 = 0.9 \quad V_{DS} = V_x - 1, \quad V_{DD} = 0.9 - 0.7 = 0.2$$

for $V_x < 1.2$, device operates in the triode region.

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 \times 0.2 \times (V_x - 1) - (V_x - 1)^2 \right]$$

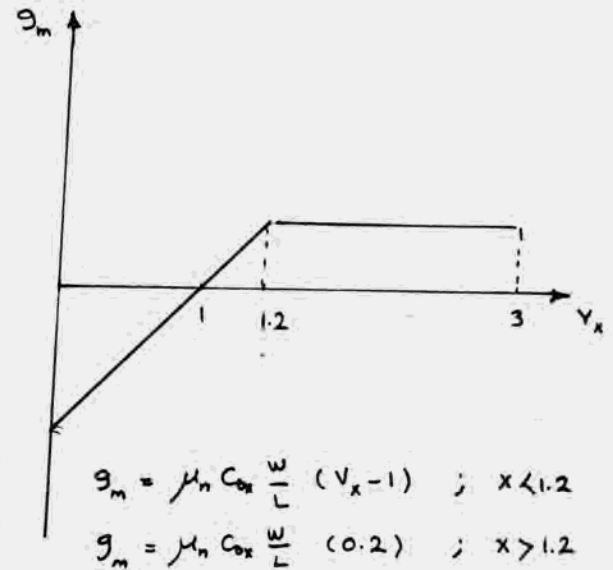
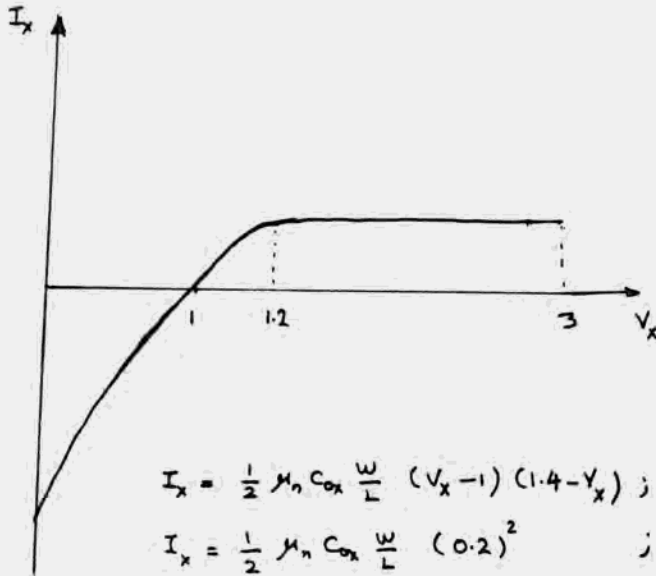
$$g_m = \mu_n C_{ox} \frac{W}{L} (V_x - 1)$$

for $V_x > 1.2$, Device goes into saturation region

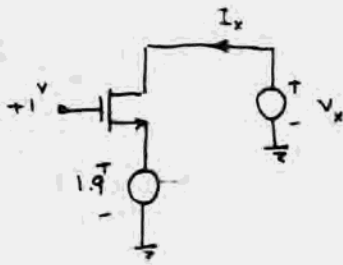
2.5) b Cont

$$\text{So, } I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.2)^2$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (0.2)$$



2.5) c



$$\lambda = \gamma = 0$$

$$V_{TH} = 0.7$$

S and D exchange their roles.

$$V_{GS} = 1 - V_x \quad V_{DS} = 1.9 - V_x \quad V_{GD} = V_{GS} - V_{TH} = 0.3 - V_x$$

Device is in saturation region, so, $I_x = -\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.3 - V_x)^2$

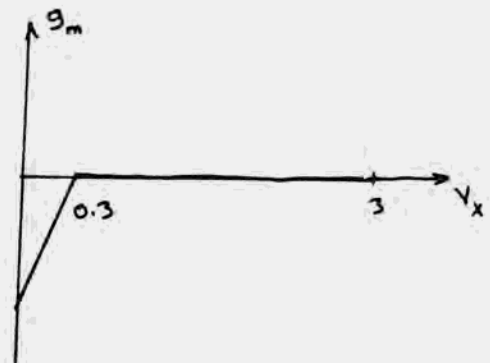
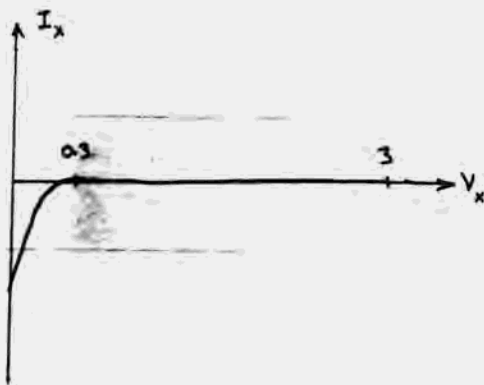
Device turns off when $V_x = 0.3$ and never turns on again.

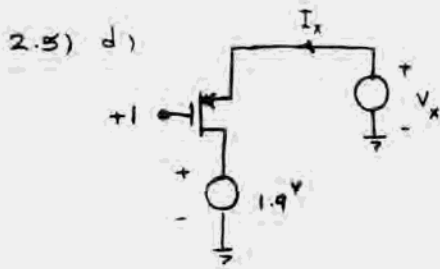
$$\text{So, } I_x = -\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.3 - V_x)^2 \quad ; x < 0.3$$

$$I_x = 0 \quad ; \text{otherwise}$$

$$\text{Then } g_m = -\mu_n C_{ox} \frac{W}{L} (0.3 - V_x) \quad ; x < 0.3$$

$$g_m = 0 \quad ; \text{otherwise}$$





$$V_{TH} = -0.8 \quad \gamma = 0$$

D and S exchange their roles.

$$V_{GS} = -0.9 \quad V_{DS} = V_x - 1.9$$

for $V_x < 1.8$:

$$I_x = -\frac{1}{2} \mu_p C_{ox} \frac{W}{L} (0.1)^2$$

$$g_m = -\mu_p C_{ox} \frac{W}{L} (0.1)$$

Device remains in the saturation region until

$V_x = 1.9 - 0.1 = 1.8$, then device goes into the triode

region.

for $1.8 < V_x < 1.9$:

$$I_x = -\mu_p C_{ox} \frac{W}{L} \left[(-0.1)(V_x - 1.9) - \frac{1}{2} (V_x - 1.9)^2 \right]$$

$$g_m = +\mu_p C_{ox} \frac{W}{L} (V_x - 1.9)$$

for $V_x > 1.9$:

S and D exchange their roles again, when $V_x = 1.9$

for $V_x > 1.9$, Device operates in the triode region.

$$V_{GS} = 1 - V_x \quad , \quad V_{DS} = 1.9 - V_x$$

$$I_x = +\mu_p C_{ox} \frac{W}{L} \left[(1.8 - V_x)(1.9 - V_x) - \frac{1}{2} (1.9 - V_x)^2 \right]$$

$$g_m = -\mu_p C_{ox} \frac{W}{L} (1.9 - V_x)$$

$$2.5)d \quad 50, \quad 0 < V_x < 1.8$$

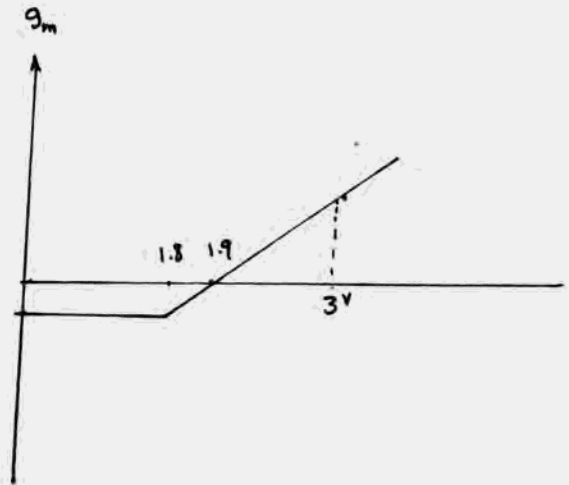
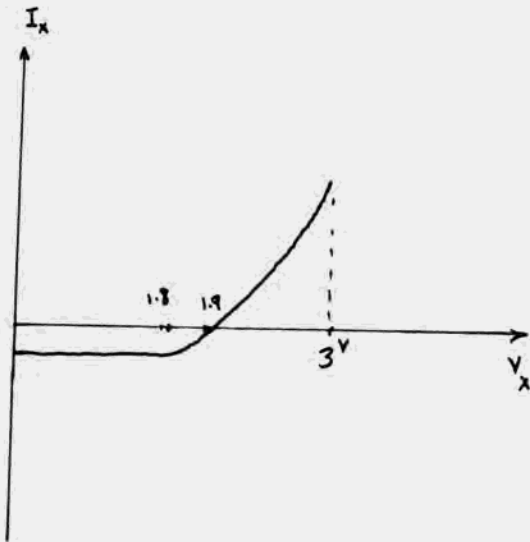
$$I_x = -\frac{1}{2} \mu_p C_{ox} \frac{W}{L} (0.1)^2$$

$$g_m = -\mu_p C_{ox} \frac{W}{L} (0.1)$$

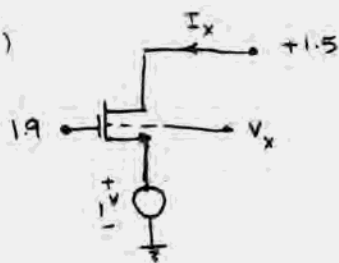
$$1.8 < V_x < 3$$

$$I_x = +\mu_p C_{ox} \frac{W}{L} \times \frac{1}{2} (V_x - 1.9)(V_x - 1.7)$$

$$g_m = \mu_p C_{ox} \frac{W}{L} (V_x - 1.9)$$



2.5) e)



$$V_{TH0} = 0.7 \quad \gamma = 0.45 \quad 2\phi_f = 0.9 \quad , \lambda = 0$$

$$V_{SB} = 1 - V_x$$

$$V_{TH} = 0.7 + 0.45 (\sqrt{0.9 + 1 - V_x} - \sqrt{0.9})$$

$$V_{GS} = 0.9$$

$$V_{DS} = 0.5$$

for $V_x = 0$, $V_{TH} = 0.893$ So device is in saturation region.

$$\text{So } I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.2 - 0.45 (\sqrt{1.9 - V_x} - \sqrt{0.9}))^2$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (0.2 - 0.45 (\sqrt{1.9 - V_x} - \sqrt{0.9}))$$

These equations are valid upto the edge of triode region, i.e.

$$0.2 - 0.45 (\sqrt{1.9 - V_x} - \sqrt{0.9}) = 0.5 \quad \Rightarrow \quad V_x = 1.82$$

Above $V_x = 1.82$, device is in the triode region.

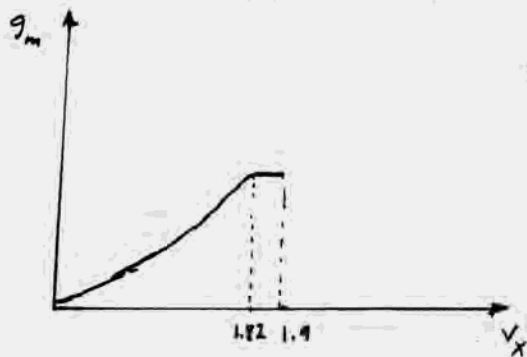
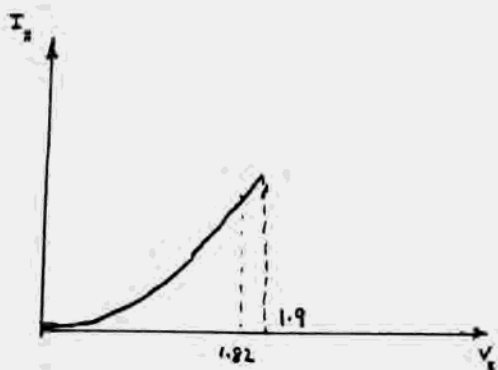
$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2 \times 0.5 \times (0.2 - 0.45 (\sqrt{1.9 - V_x} - \sqrt{0.9})) - 0.5^2]$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (0.5)$$

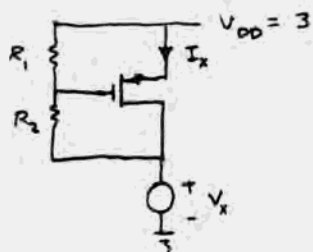
; This problem has been considered

only for $0 < V_x < 1.9$ in which

Schichman-Hodges Eq. is valid for V_{TH} .



2.4) a)



$$\gamma = 0$$

$$V_{SG} = (V_{DD} - V_x) \frac{R_1}{R_1 + R_2}$$

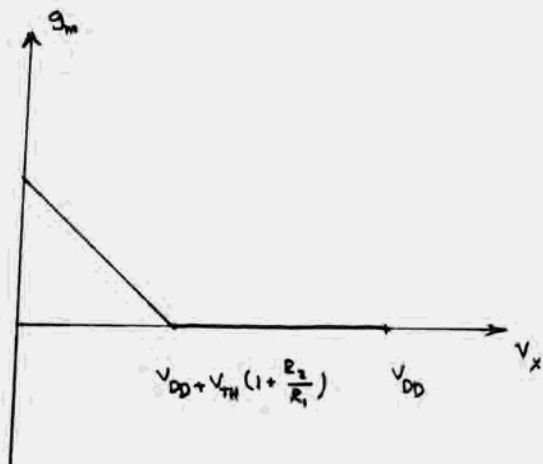
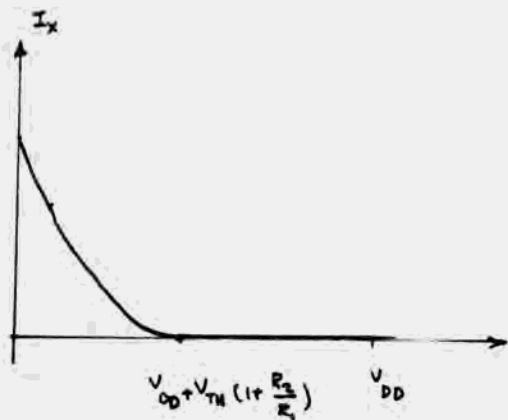
$$V_{SD} = V_{DD} - V_x$$

for $V_{SG} > |V_{TH}|$ Device is in the Saturation Region (Device is

off; otherwise) $(V_{DD} - V_x) \frac{R_1}{R_1 + R_2} > -V_{TH}$

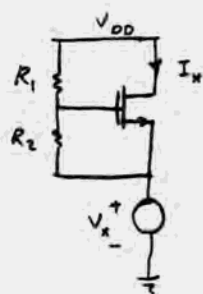
$$V_x < V_{DD} + V_{TH} \left(1 + \frac{R_2}{R_1}\right) \Rightarrow I_x = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} \left[(V_{DD} - V_x) \frac{R_1}{R_1 + R_2} + V_{TH} \right]^2$$

$$g_m = \mu_p C_{ox} \frac{W}{L} \left[(V_{DD} - V_x) \frac{R_1}{R_1 + R_2} + V_{TH} \right]$$



If $V_{DD} + V_{TH} \left(1 + \frac{R_2}{R_1}\right) < 0$ (e.g. for small value of R_1), device never turns on!

2.6) b)



$$\gamma = 0$$

$$V_{GS} = (V_{DD} - V_x) \frac{R_2}{R_1 + R_2}$$

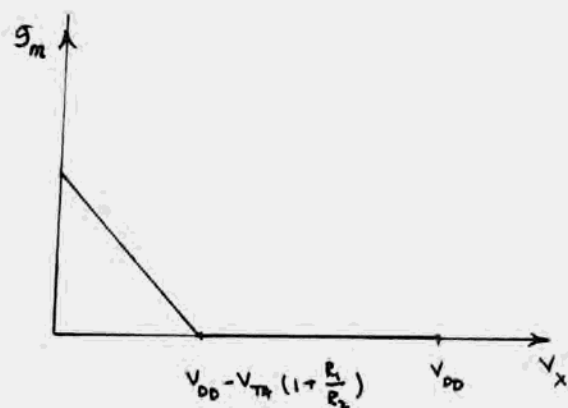
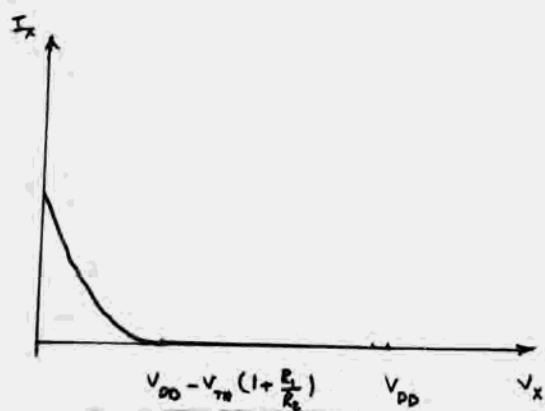
$$V_{DS} = V_{DD} - V_x$$

for $V_{GS} > V_{TH}$, Device is in the saturation region and

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[(V_{DD} - V_x) \frac{R_2}{R_1 + R_2} - V_{TH} \right]^2$$

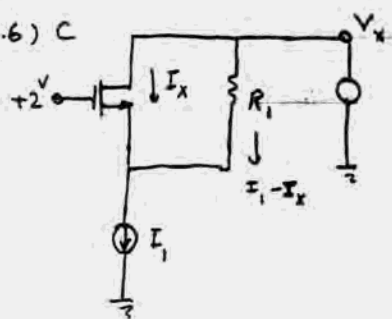
$$g_m = \mu_n C_{ox} \frac{W}{L} \left[(V_{DD} - V_x) \frac{R_2}{R_1 + R_2} - V_{TH} \right]$$

for $V_x < V_{DD} - V_{TH} \left(1 + \frac{R_1}{R_2}\right)$ (i.e. $V_{GS} > V_{TH}$)



If $V_{DD} - V_{TH} \left(1 + \frac{R_1}{R_2}\right) < 0$ device doesn't turn on.

2.6) C



I_x and $I_R = I_1 - I_x$ have the same polarity

So, $0 \leq I_x \leq I_1$

for $0 < V_x < 2 - V_{TH}$ (1.3) Device is in the triode.

$$V_{GS} = 2 - V_x + R_1 (I_1 - I_x) \quad , \quad V_{DS} = R_1 (I_1 - I_x)$$

$$I_x = I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2(V_{GS} - V_{TH}) - V_{DS}] V_{DS}$$

$$\Rightarrow (*) \quad I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [R_1 (I_1 - I_x) + 2(2 - V_{TH} - V_x)] \{R_1 (I_1 - I_x)\}$$

The above equation presents $I_x - V_x$ characteristics in this region.

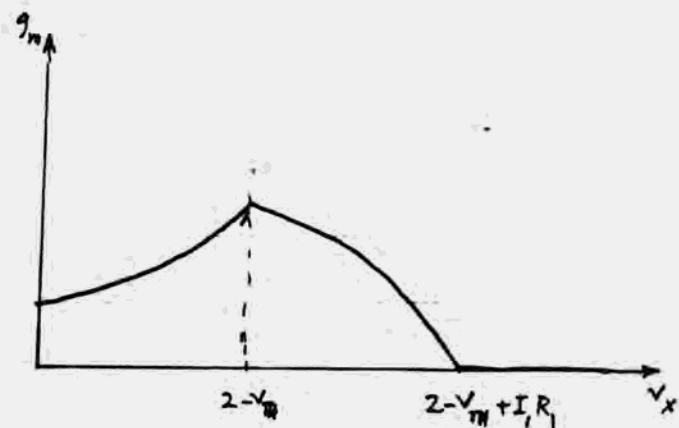
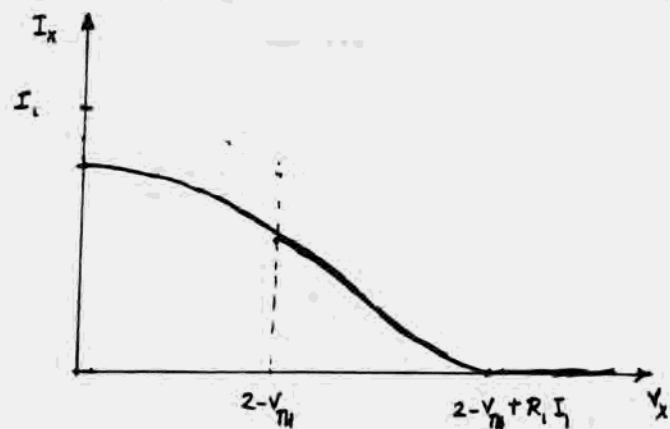
In this region $g_m = \mu_n C_{ox} V_{DS} = \mu_n C_{ox} R_1 (I_1 - I_x)$

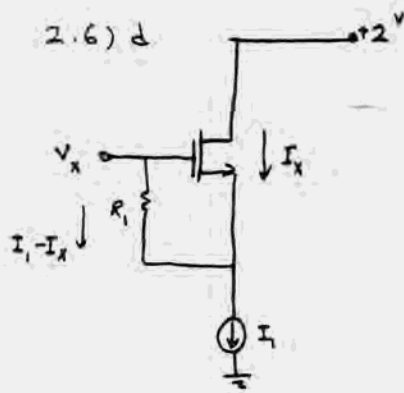
Then device enters the saturation region; $V_{GS} = 2 - V_x + R_1 (I_1 - I_x)$

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2 - V_x + R_1 (I_1 - I_x) - V_{TH}]^2$$

$$g_m = \mu_n C_{ox} \frac{W}{L} [2 - V_x + R_1 (I_1 - I_x) - V_{TH}]$$

Then device turns off when $V_x = 2 - V_{TH} + R_1 I_1$





Assumption: $R_1 I_1 > V_{TH}$

for $0 < V_x < 2 + V_{TH}$: Device is in the saturation region

$$V_{GS} = R_1 (I_1 - I_x)$$

$$I_D = I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [R_1 (I_1 - I_x) - V_{TH}]^2$$

I_x is a constant that can be derived by solving the above equation.

Then device enters the triode region for $V_x > 2 + V_{TH}$

In this case $V_{GS} = R_1 (I_1 - I_x)$ $V_{DS} = 2 - [V_x - R_1 (I_1 - I_x)] = 2 - V_x + R_1 (I_1 - I_x)$

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2 (V_{GS} - V_{TH}) V_{DS} - V_{DS}^2] = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2 [R_1 (I_1 - I_x) - V_{TH}] - 2 + V_x - R_1 (I_1 - I_x)] \times (2 - V_x + R_1 (I_1 - I_x))$$

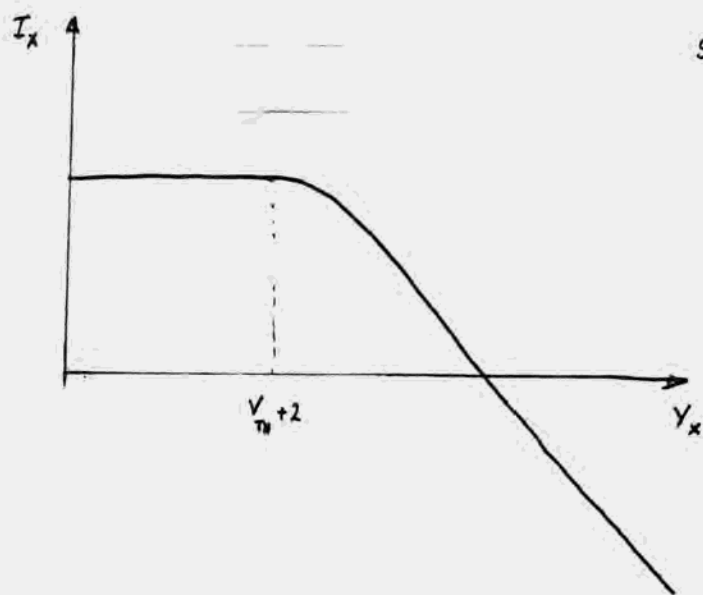
$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [(R_1 (I_1 - I_x) - V_{TH}) + (V_x - 2 - V_{TH})] [(R_1 (I_1 - I_x) - V_{TH}) - (V_x - 2 - V_{TH})]$$

$$(*) \quad I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [(R_1 (I_1 - I_x) - V_{TH})^2 - (V_x - 2 - V_{TH})^2]$$

The second term shows that I_x decreases when we increase V_x

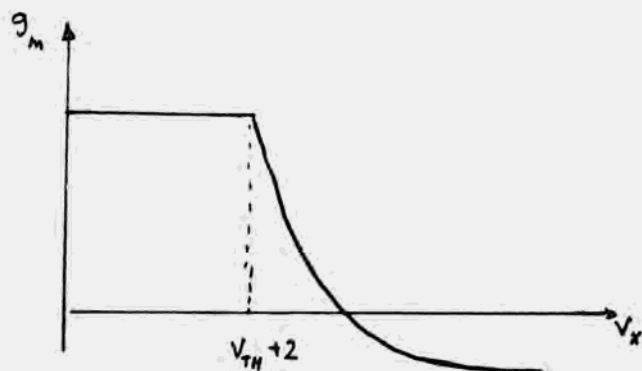
The polarity of I_x changes for higher V_x (Device still is in triode)

(*) presents $I_x - V_x$ relationship in this region.

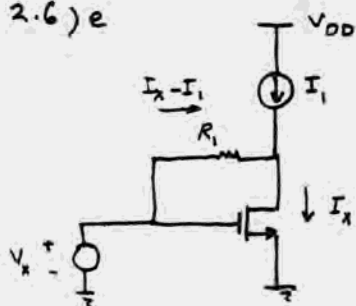


$$g_m = \mu_n C_{ox} \frac{W}{L} [R_1 (I_1 - I_x) - V_{TH}] \quad ; V_x < 2 + V_{TH}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{DS} = \mu_n C_{ox} \frac{W}{L} [R_1 (I_1 - I_x) + 2 - V_x] \quad ; V_x > 2 + V_{TH}$$



2.6) e

for $0 < V_x < V_{TH}$

Device is off

$$I_x = 0 \quad g_m = 0$$

Then device turns on (in the saturation region)

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_x - V_{TH})^2$$

Transistor is in the saturation until

$$V_{GD} = R_1 (I_x - I_1) = V_{TH} \quad , \text{ Then device}$$

enters the triode region. (When $I_x = I_1 + \frac{V_{TH}}{R_1}$, i.e. $V_x = V_{TH} + \sqrt{\frac{2I_1 + 2V_{TH}/R_1}{\mu_n C_{ox} \frac{W}{L}}}$)

$$\text{So, } V_{TH} < V_x < V_{TH} + \sqrt{\frac{2I_1 + 2V_{TH}/R_1}{\mu_n C_{ox} \frac{W}{L}}}$$

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_x - V_{TH})^2$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_x - V_{TH})$$

2.6) e Cont.

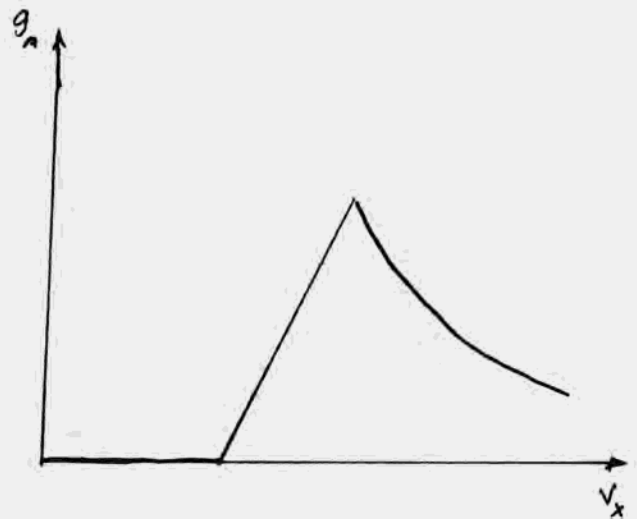
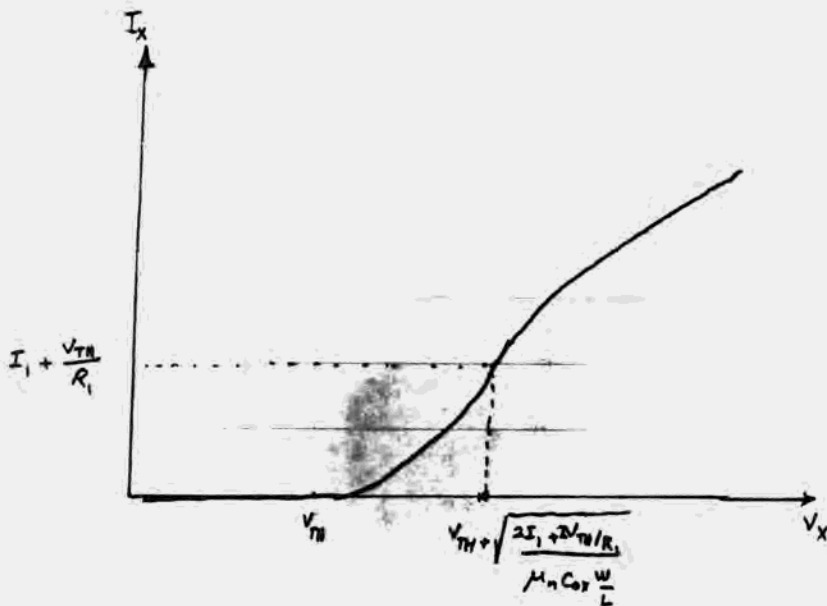
Then device enters the triode region. $V_{GS} = V_X$ $V_{DS} = V_X - R_1(I_X - I_1)$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH}) - V_{DS} \right] V_{DS} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_X - V_{TH}) - V_X + R_1(I_X - I_1) \right] \times (V_X - R_1(I_X - I_1))$$

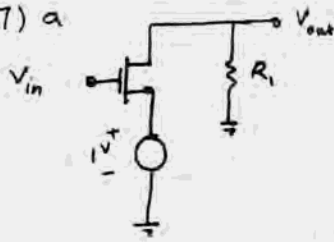
$$(*) I_X = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_X + R_1(I_X - I_1) - 2V_{TH})(V_X - R_1(I_X - I_1))$$

The above equation presents $I_X - V_X$ relationship in triode region.

In this region, $g_m = \mu_n C_{ox} \frac{W}{L} V_{DS} = \mu_n C_{ox} \frac{W}{L} (V_X - R_1(I_X - I_1))$



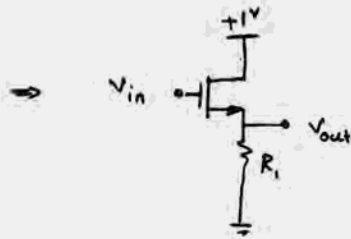
2.7) a



$$\lambda = \gamma = 0$$

$$V_{TH} = 0.7$$

Drain and source exchange their roles.



for $0 < V_{in} < 0.7$ device is off $V_{out} = 0$

for $0.7 < V_{in} < 1.7$ device is in the saturation region

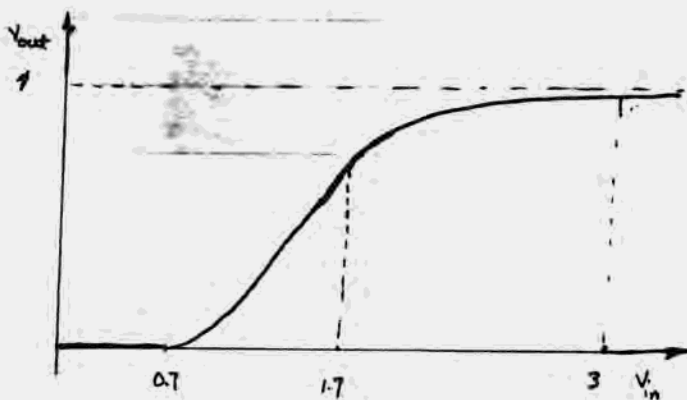
$$(*) \quad I_D = \frac{V_{out}}{R_1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{out} - 0.7)^2 \Rightarrow \text{Input-output relationship}$$

for $1.7 < V_{in} < 3$ device is in the triode region

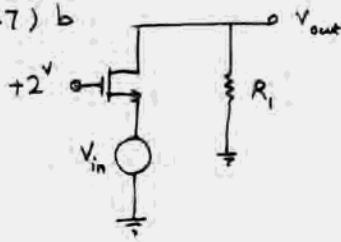
$$V_{GS} = V_{in} - V_{out} \quad V_{DS} = 1 - V_{out}$$

$$(*) \quad I_D = \frac{V_{out}}{R_1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{in} - V_{out} - 0.7)(1 - V_{out}) - (1 - V_{out})^2 \right]$$

\Rightarrow Input-output relationship

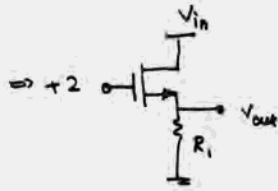


2.7) b



$$\gamma = \lambda = 0 \quad V_{TH} = 0.7$$

Drain and source exchange their roles!



for $0 < V_{in} < 1.3$ device is in triode

$$V_{GS} = 2 - V_{out} \quad V_{DS} = V_{in} - V_{out}$$

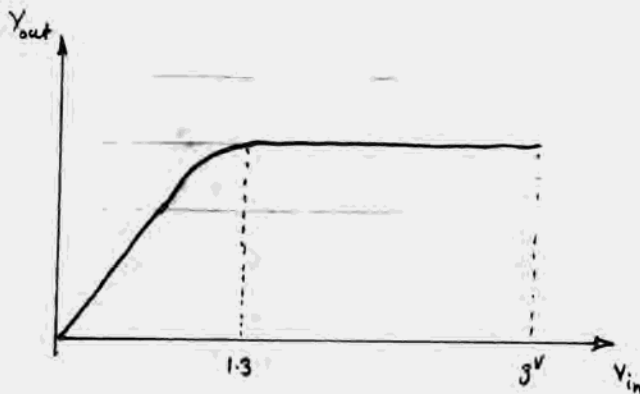
$$(*) \quad I_D = \frac{V_{out}}{R_L} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(2 - V_{out} - 0.7)(V_{in} - V_{out}) - (V_{in} - V_{out})^2 \right]$$

Input output relationship is presented by the above equation.

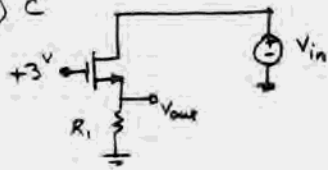
for $1.3 < V_{in} < 3$ device is in the saturation region

$$I_D = \frac{V_{out}}{R_L} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2 - V_{out} - 0.7)^2$$

V_{out} doesn't depend on V_{in} and it is constant for $V_{in} > 1.3$



2.7) c



$$\gamma = \lambda = 0 \quad V_{TH} = 0.7$$

for $0 < V_{in} < 2.3$ device is in triode

$$V_{GS} = 3 - V_{out} \quad V_{DS} = V_{in} - V_{out}$$

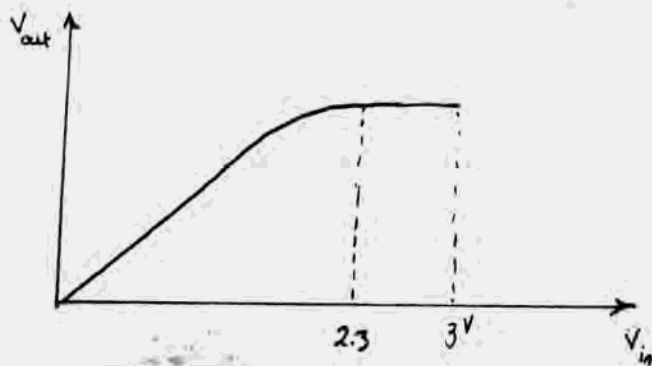
$$(*) \quad I_D = \frac{V_{out}}{R_i} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(3 - V_{out} - 0.7)(V_{in} - V_{out}) - (V_{in} - V_{out})^2 \right]$$

Input-output relationship is presented by the above equation.

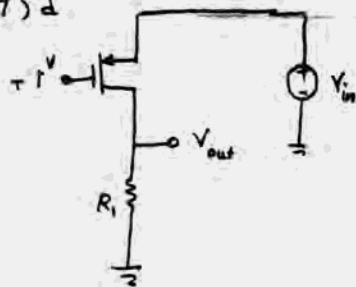
for $2.3 < V_{in} < 3$ device is in the saturation region

$$I_D = \frac{V_{out}}{R_i} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (3 - V_{out} - 0.7)^2$$

V_{out} is constant for $V_{in} > 2.3$ (It doesn't depend on V_{in})



2.7) d



$$|V_{TH}| = 0.8 \quad \gamma = \lambda = 0$$

for $0 < V_{in} < 1.8$ device is off $\Rightarrow V_{out} = 0$

Then device turns on (in sat.) and V_{out} goes up

until $V_{out} = 1.8$, then device enters the triode region

for $V_{in} > 1.8$ and $V_{out} < 1.8$

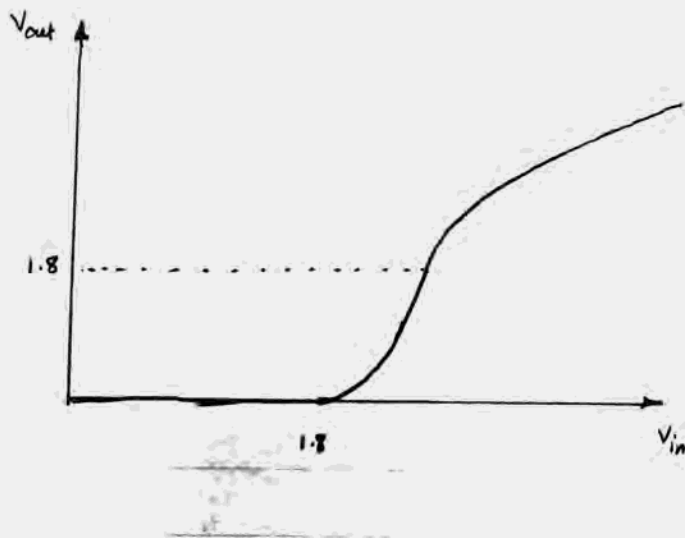
$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} (V_{in} - 1.8)^2 \Rightarrow V_{out} = \frac{1}{2} \mu_p C_{ox} R_1 \frac{W}{L} (V_{in} - 1.8)^2$$

This is good for $1.8 < V_{in} < 1.8 + \sqrt{\frac{2 \times 1.8 V}{\mu_p C_{ox} \frac{W}{L} R_1}}$

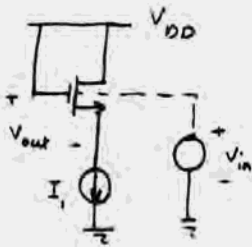
for $V_{in} > 1.8 + \sqrt{\frac{2 \times 1.8 V}{\mu_p C_{ox} \frac{W}{L} R_1}}$

$$I_D = \frac{V_{out}}{R_1} = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} \left[2 (V_{in} - 1.8) (V_{in} - V_{out}) - (V_{in} - V_{out})^2 \right]$$

Input-output relationship is presented by the above equation.



2.8) a



$$V_S = V_{DD} - V_{out} \quad V_B = V_{in} \quad V_{SB} = V_{DD} - V_{out} - V_{in}$$

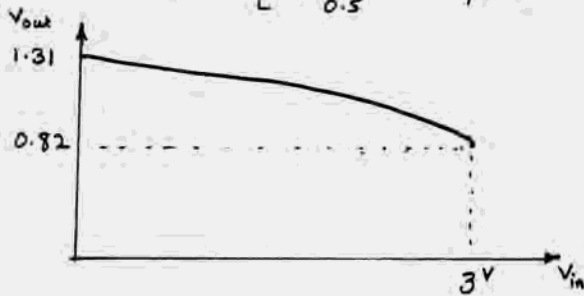
$$I_D = I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{out} - V_{TH})^2$$

$$V_{TH} = V_{TH0} + \gamma (\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F})$$

$$\Rightarrow I_1 = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{out} - V_{TH0} - \gamma (\sqrt{2\phi_F + V_{DD} - V_{out} - V_{in}} - \sqrt{2\phi_F}))^2$$

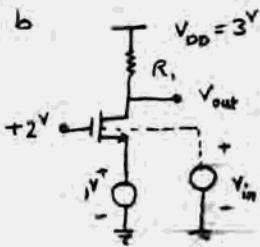
for each V_{in} , the above equation should be solved to obtain V_{out}

$$\text{for } \frac{W}{L} = \frac{50}{0.5} \quad I_1 = 1 \mu A$$



$$\text{Assumption: } 2\phi_F + V_{DD} - V_{out} - V_{in} > 0$$

2.8) b



$$V_{SB} = 1 - V_{in}$$

$$V_{GS} = 1$$

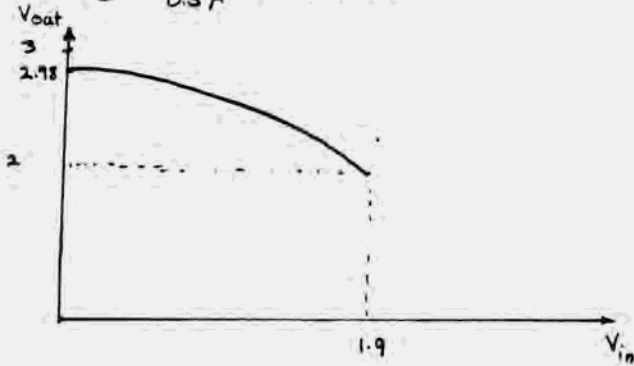
$$V_{TH} = V_{TH0} + \gamma (\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F})$$

$$V_{TH} = 0.7 + 0.45 (\sqrt{1.9 - V_{in}} - \sqrt{0.9})$$

Assumption: V_{in} varies from 0 to 1.9 and R_1 is small enough to guarantee that the device remains in the saturation region.

$$V_{out} = 3 - R_1 \cdot \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (0.3 - 0.45 (\sqrt{1.9 - V_{in}} - \sqrt{0.9}))^2$$

for $\frac{W}{L} = \frac{50 \mu\text{m}}{0.5 \mu\text{m}}$, $R = 0.2 \text{ k}$



2.8) C

Drain and Source exchange their roles

 $V_{TH0} = 0.7$ $\gamma = 0.45$ $2\phi_F = 0.9$ 

Assumption: $V_{SB} > -2\phi_F$ ($V_{out} - V_{in} > -2\phi_F$) \Rightarrow Device is in the Saturation

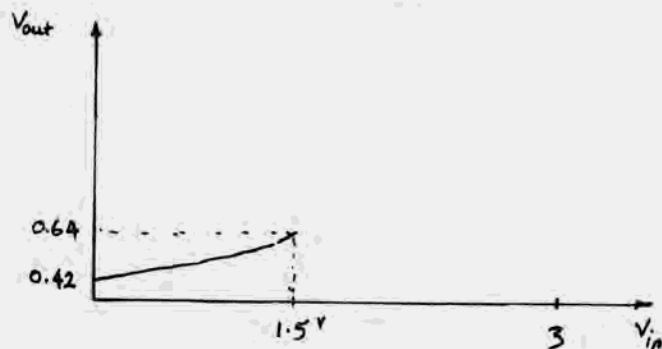
$$V_{TH} = 0.7 + 0.45 (\sqrt{0.9 + V_{out} - V_{in}} - \sqrt{0.9}) \quad V_{GS} = 2 - V_{out}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2 - V_{out} - 0.7 - 0.45 (\sqrt{0.9 + V_{out} - V_{in}} - \sqrt{0.9}))^2$$

$$I_D = \frac{V_{out}}{R_i}$$

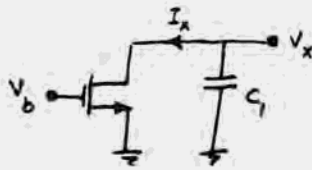
$$(*) \quad \frac{V_{out}}{R_i} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2 - V_{out} - 0.7 - 0.45 (\sqrt{0.9 + V_{out} - V_{in}} - \sqrt{0.9}))^2$$

Input-output relationship is presented by the above equation.



$\frac{W}{L} = \frac{50}{0.5}$ $R = 100 \Omega$

2.9) a



$$\beta = \lambda = 0$$

$$V_{TH} = 0.7$$

for $V_b - 0.7 < V_x < 3$ device is in saturation

Assume $V_b > V_{TH}$

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{TH})^2$$

$$V_x = -\frac{1}{C_1} \int I_x dt + 3^V = 3 - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{TH})^2 t$$

Then device goes into triode, for $0 < V_x < V_b - 0.7$

$$I_x = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2(V_b - 0.7)V_x - V_x^2] = -\frac{dV_x}{dt} \times C_1$$

$$\Rightarrow -dt \underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \times \frac{1}{C_1}}_{\alpha} = \frac{dV_x}{V_x [2(V_b - 0.7) - V_x]}$$

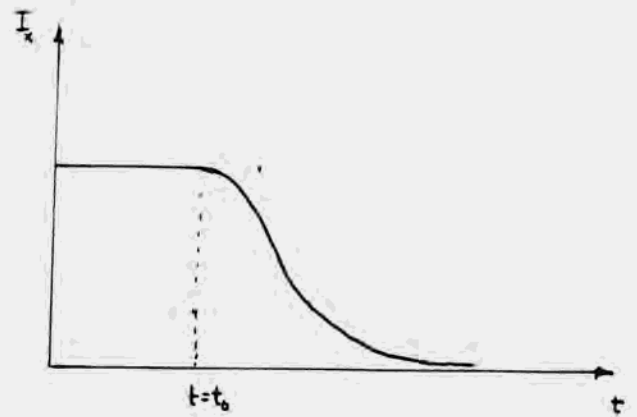
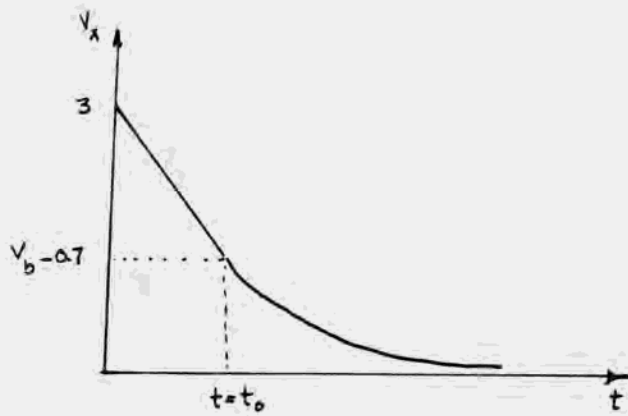
$$-\alpha dt = \left[\frac{1}{V_x} + \frac{1}{2(V_b - 0.7) - V_x} \right] \times \frac{1}{2(V_b - 0.7)}$$

$$\Rightarrow -\alpha(t - t_0) = \left[\ln \frac{V_x}{2(V_b - 0.7) - V_x} \right] \frac{1}{2(V_b - 0.7)} \quad @t=t_0, V_x = V_b - 0.7$$

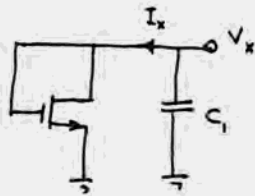
$$\Rightarrow \frac{2(V_b - 0.7) - V_x}{V_x} = e^{2\alpha(V_b - 0.7)(t - t_0)}$$

$$\Rightarrow V_x = \frac{2(V_b - 0.7)}{1 + e^{2\alpha(V_b - 0.7)(t - t_0)}}$$

$$I_x = -C_1 \frac{dV_x}{dt} = \frac{4\alpha C_1 (V_b - 0.7)^2 e^{2\alpha(V_b - 0.7)(t - t_0)}}{\left(1 + e^{2\alpha(V_b - 0.7)(t - t_0)}\right)^2}$$



2.9) b



Device is always in the saturation region.

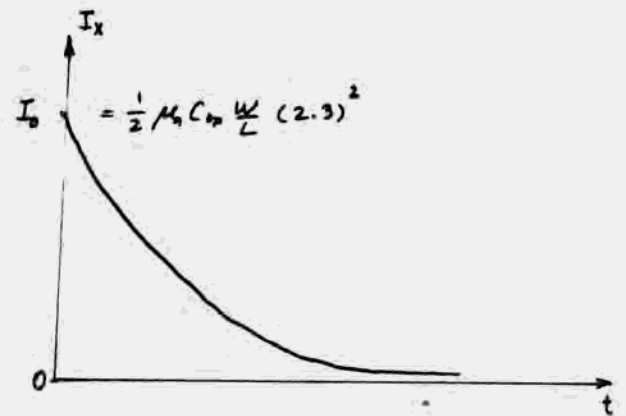
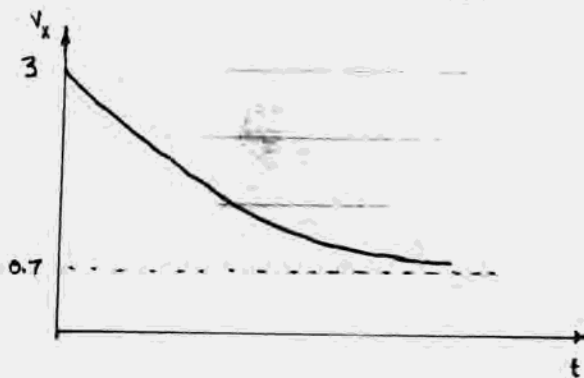
$$I_x = -C_1 \frac{dV_x}{dt} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_x - 0.7)^2$$

$$\Rightarrow \underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L}}_{\alpha} \frac{1}{C_1} dt = - \frac{dV_x}{(V_x - 0.7)^2}$$

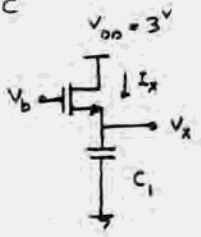
$$\Rightarrow \alpha t = \frac{1}{V_x - 0.7} + K$$

$$@ t=0, V_x=3 \Rightarrow \alpha t = \frac{1}{V_x - 0.7} - \frac{1}{2.3} \Rightarrow V_x = 0.7 + \frac{1}{\alpha t + 1/2.3}$$

$$I_x = -C_1 \frac{dV_x}{dt} = \frac{\alpha C_1}{(\alpha t + \frac{1}{2.3})^2}$$

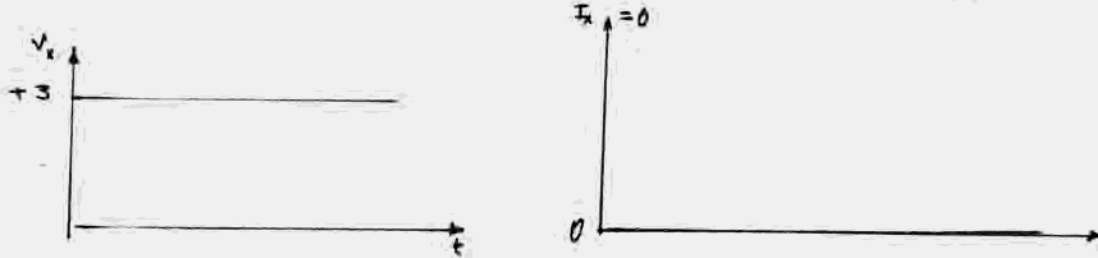


2.9) c

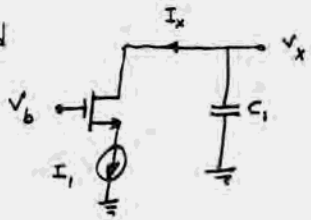


@ $t=0$ $V_x = 3$, $V_{op} = 3^V \Rightarrow V_{os} = 0 \Rightarrow I_x = 0$

And the circuit remains in this state



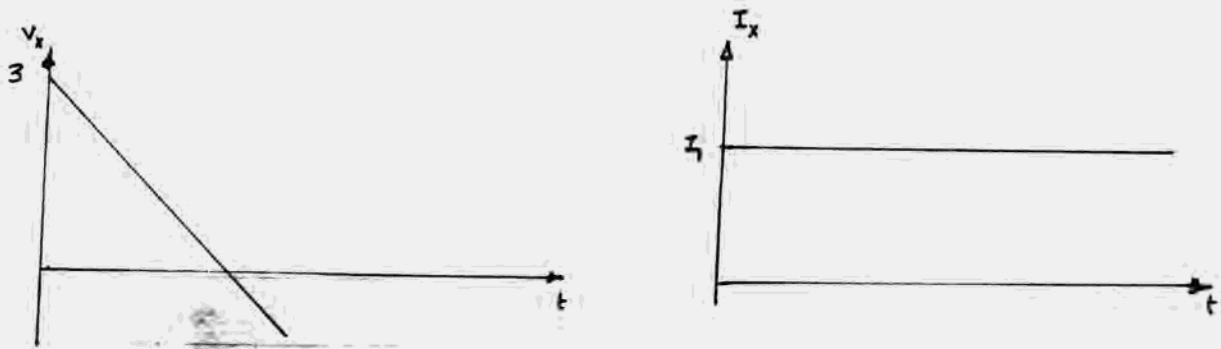
2.9) d



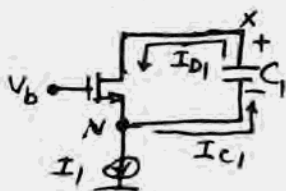
$I_x = I_1$

$-C_1 \frac{dV_x}{dt} = I_1 \Rightarrow V_x = 3 - \frac{I_1}{C} t$

In fact these Equations are valid until I_1 is no longer an ideal current source.



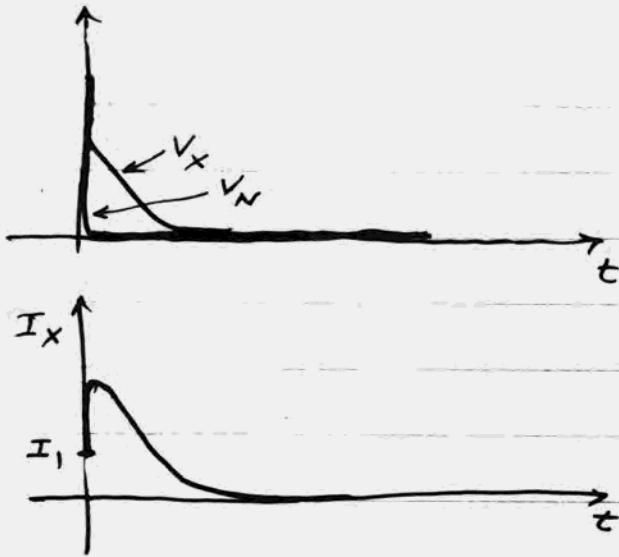
2.9) e Initially, the current thru $M_1 = I_1 \Rightarrow$ certain V_{gs} is developed and $V_x = V_b - V_{gs1} + 3V$ and $I_x = I_1$. However, at $t=0^+$, the drain current of M_1 flows from C_1 : $I_{D1} - I_{C1} = I_1$. But,

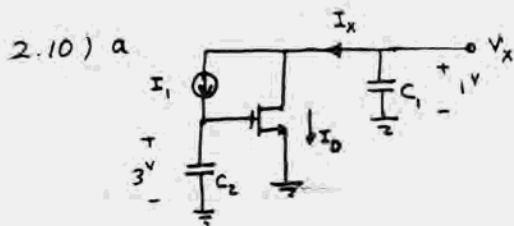


$I_{D1} = I_{C1} \Rightarrow I_1 = 0$. If the current source is ideal, V_x jumps to $-\infty$ (actually about 0.6V below 0, where the S-B diode turns on)

If I_1 is not ideal, V_x jumps to zero and C_1 discharges

2.9) e (cont'd)

through M_1 :

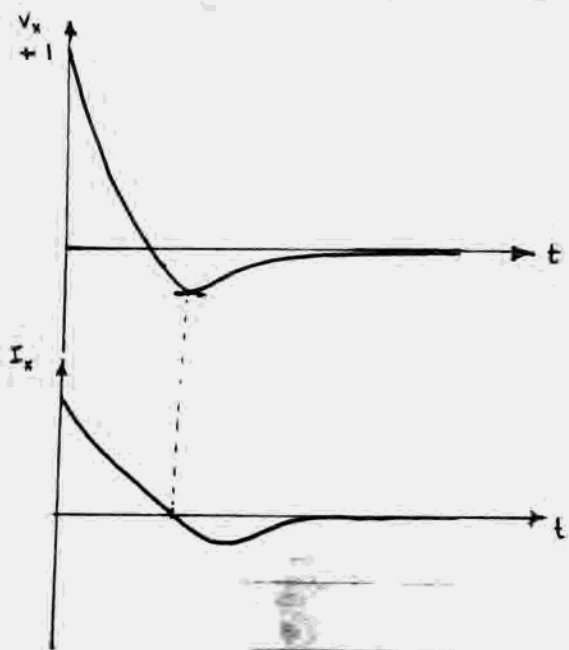


$$V_G = 3 + \frac{I_1 t}{C}$$

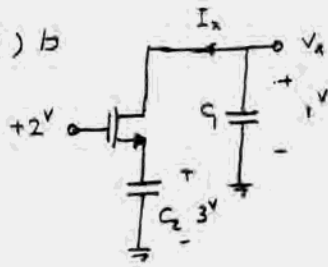
This circuit settles at $t = \infty$, when $V_G = \infty$
 $I_D = -I_1$, $V_{DS} = 0$ (Actually, Drain and Source exchange their roles after a specific time at which $I_x = I_1$, and afterward V_x becomes negative) However, transistor always operates in the triode region.

$$I_x = I_1 + \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 \left(3 + \frac{I_1}{C_2} t - 0.7 \right) V_x - V_x^2 \right] = -C \frac{dV_x}{dt}$$

The values of V_x can be obtained by numerical methods



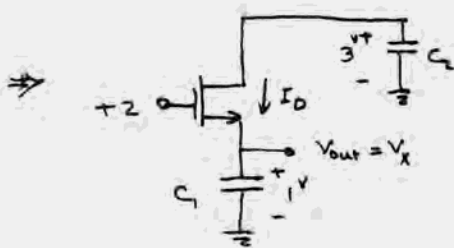
2.10) b



Drain and source exchange their roles.

$$(\gamma = \lambda = 0) \quad V_{TH} = 0.7$$

$$\int I_D dt = q \quad V_x = 1 + \frac{q}{C_1} \quad , \quad V_0 = V_{C_2} = 3 - \frac{q}{C_2}$$

 V_x goes up until transistor turns off when $V_x = 1.3$


Assumption: Transistor is in saturation.

This assumption is correct if: $V_0 = 3 - \frac{q}{C_2} > 1.3 \quad (2 - 0.7)$

$$V_x(\infty) = 1 + \frac{q(\infty)}{C_1} = 1.3$$

$$V_0(\infty) = 3 - \frac{q(\infty)}{C_2} = 3 - 0.3 \frac{C_1}{C_2} > 1.3$$

$$0.3 \frac{C_1}{C_2} < 1.7$$

$$C_1 < 5.67 C_2$$

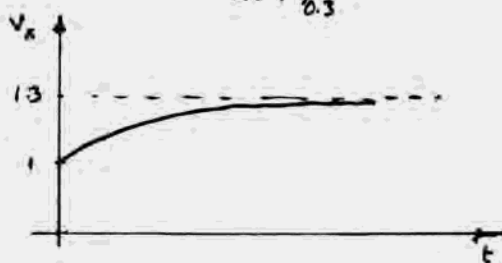
$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(2 - 1 - \frac{q}{C_1} - 0.7 \right)^2 = \frac{dq}{dt}$$

$$\Rightarrow \underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{1}{C_1}}_{\alpha} dt = \frac{dq/C_1}{\left(0.3 - \frac{q}{C_1} \right)^2} \quad \Rightarrow \alpha t = \frac{1}{0.3 - \frac{q}{C_1}} + K \quad (t=0, q=0)$$

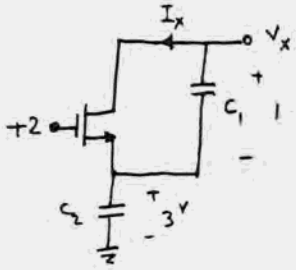
$$\Rightarrow \alpha t = \frac{1}{0.3 - \frac{q}{C_1}} - \frac{1}{0.3} \quad \Rightarrow \frac{q}{C_1} = 0.3 - \frac{1}{\alpha t + \frac{1}{0.3}} \quad V_x = 1 + \frac{q}{C_1}$$

$$\Rightarrow V_x = 1.3 - \frac{1}{\alpha t + \frac{1}{0.3}}$$

$$I_x = -C_1 \frac{dV_x}{dt} = \frac{-\alpha C_1}{\left(\alpha t + \frac{1}{0.3} \right)^2}$$



2.10) C

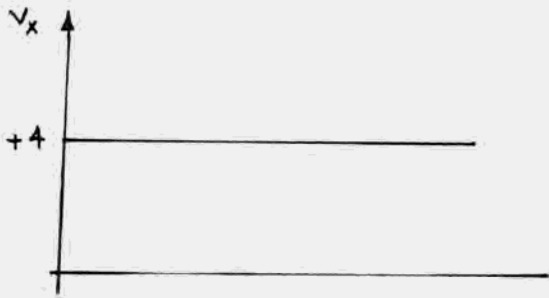


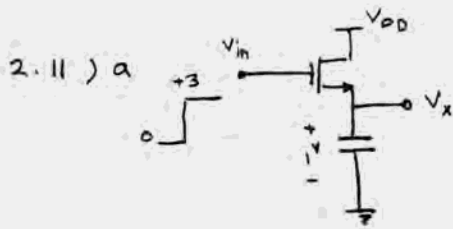
$$\text{At } t=0 \quad V_G = 2 \quad V_S = 3 \quad V_D = 4$$

Device is off and doesn't turn on.

The Circuit remains in this state.

$$\text{So, } V_x = 4 \quad I_x = 0$$





$$\gamma = \lambda = 0$$

$$V_{TH} = 0.7$$

At $t=0^+$, device turns on (in sat) and starts charging the capacitor, until device turns off

when; $V_x = V_{in} - V_{TH} = 3 - 0.7 = 2.3$

$$I_c = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2.3 - V_x)^2$$

$$; V_{GS} = 3 - V_x - 0.7$$

$$I_c = C_i \frac{dV_x}{dt}$$

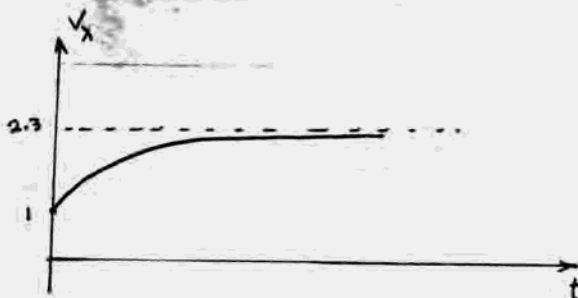
$$\Rightarrow \underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \times \frac{1}{C_i}}_{\alpha} (2.3 - V_x)^2 = \frac{dV_x}{dt}$$

$$\Rightarrow \alpha dt = \frac{dV_x}{(2.3 - V_x)^2} \quad \Rightarrow \alpha t + K_0 = \frac{1}{2.3 - V_x}$$

$$(t=0, V_x=1) \quad \alpha \cdot 0 + K_0 = \frac{1}{2.3 - 1} \quad \Rightarrow K_0 = \frac{1}{1.3}$$

$$\Rightarrow \frac{1}{1.3} + \alpha t = \frac{1}{2.3 - V_x} \quad \Rightarrow 2.3 - V_x = \frac{1}{\alpha t + \frac{1}{1.3}}$$

$$\Rightarrow V_x = 2.3 - \frac{1}{\alpha t + \frac{1}{1.3}}$$



2.11) b



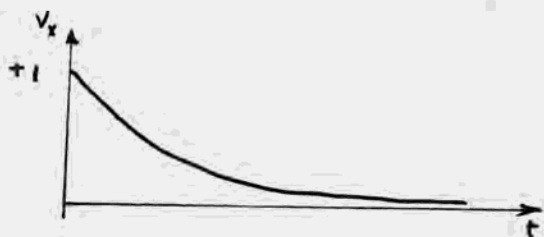
Transistor turns on at $t=0$, and discharges c_1
 until $V_x=0$, (device always operates in triode)

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2(3-0.7)V_x - V_x^2] = -c_1 \frac{dV_x}{dt}$$

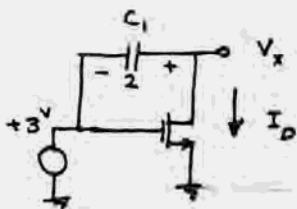
$$\underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \times \frac{1}{c_1}}_{\alpha} [4.6 V_x - V_x^2] = -\frac{dV_x}{dt} \Rightarrow -\alpha dt = \frac{dV_x}{V_x(4.6 - V_x)}$$

$$\Rightarrow -\alpha t = \left(\frac{1}{V_x} + \frac{1}{4.6 - V_x} \right) \frac{1}{4.6} + K, \quad @ t=0, V_x=1$$

$$\frac{1}{3.6} e^{-\alpha t} = \frac{V_x}{4.6 - V_x} \Rightarrow V_x = \frac{4.6}{1 + 3.6 e^{4.6 \alpha t}}$$



2.11) c



At $t=0^+$, $V_x=5$, device is in Saturation region

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (3-0.7)^2, \quad V_x \text{ decreases until}$$

$V_x = 2.3$ at $t=t_0$, then device enters triode region

$$\text{for } t < t_0 \quad (V_x > 2.3) \quad V_x = 5 - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2.3)^2 t / c_1$$

$$\text{for } t > t_0 \quad I_D = -c_1 \frac{dV_x}{dt} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2(3-0.7)V_x - V_x^2]$$

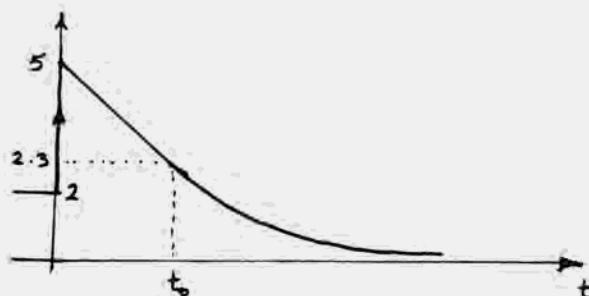
$$\Rightarrow \frac{dV_x}{V_x(4.6 - V_x)} = -\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot \frac{1}{c_1} dt$$

2.11) c, cont.

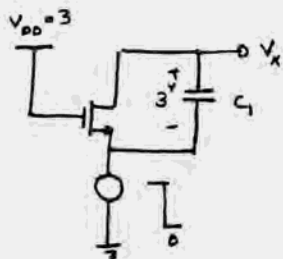
$$-\alpha(t-t_0) = \left[\ln \frac{V_x}{4.6 - V_x} \right] \cdot \frac{1}{4.6}$$

$$t=t_0, V_x=2.3$$

$$\Rightarrow V_x = \frac{4.6}{1 + e^{4.6\alpha(t-t_0)}}$$



2.11) d

At $t=0^+$, $V_x = 3$ device is in saturation

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (3 - 0.7)^2, V_x \text{ decreases until}$$

 $V_x = 2.3$ at $t=t_0$, then device enters triode region.for $t < t_0$

$$V_x = 3 - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (2.3)^2 \frac{t}{C_1}; \quad 2.3 < V_x < 3$$

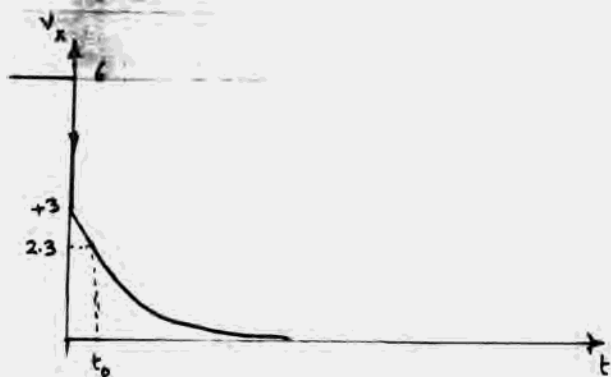
for $t > t_0$

$$I_D = -C_1 \frac{dV_x}{dt} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} [2(3-0.7)V_x - V_x^2]$$

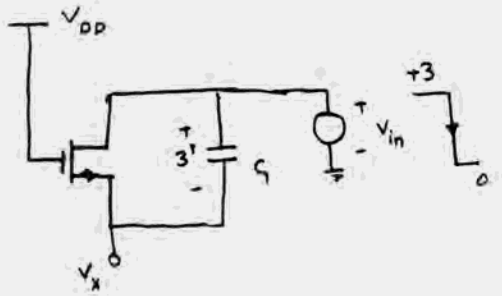
$$\frac{dV_x}{V_x(4.6 - V_x)} = -\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{1}{C_1} dt \quad , (t=t_0, V_x=2.3)$$

$$-\alpha(t-t_0) = \left[\ln \frac{V_x}{4.6 - V_x} \right] \frac{1}{4.6}$$

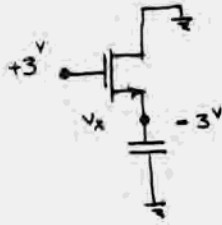
$$\Rightarrow V_x = \frac{4.6}{1 + e^{4.6\alpha(t-t_0)}}$$



2.12) a)



Device is in the triode region.

 $t \gg 0^+$ 

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(2.3 - V_x)(-V_x) - V_x^2 \right]$$

$$I_D = C_1 \frac{dV_x}{dt}$$

$$\begin{cases} V_{GS} = 3 - V_x \\ V_{DS} = -V_x \end{cases}$$

$$\Rightarrow \underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \times \frac{1}{C_1}}_{\alpha} \left[V_x^2 - 4.6 V_x \right] = \frac{dV_x}{dt}$$

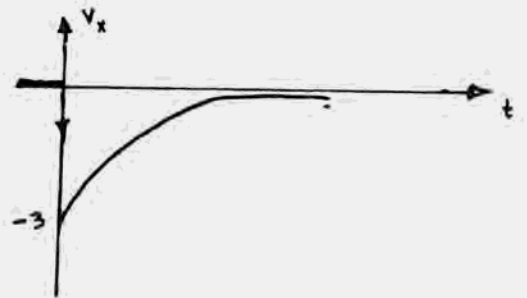
$$\Rightarrow \alpha dt = \frac{dV_x}{V_x^2 - 4.6 V_x} = dV_x \left(\frac{1}{V_x - 4.6} + \frac{-1}{V_x} \right) \times \frac{1}{4.6}$$

$$\Rightarrow -4.6 dt + K_0 = \ln \left(\frac{V_x - 4.6}{V_x} \right) \quad ; \quad V_x(0^+) = -3$$

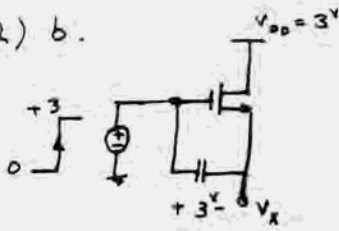
$$\Rightarrow K_0 = \ln \frac{7.6}{3} \quad \Rightarrow \frac{V_x - 4.6}{V_x} = \frac{7.6}{3} e^{4.6 dt}$$

$$\Rightarrow \frac{4.6}{V_x} = 1 - \frac{7.6}{3} e^{4.6 dt}$$

$$\Rightarrow V_x = \frac{-4.6}{\frac{7.6}{3} e^{4.6 dt} - 1}$$

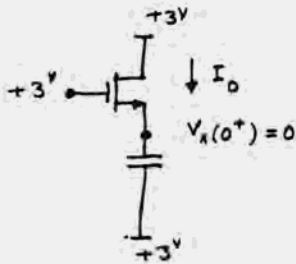


2.12) b.



Device is in saturation region.

$t = 0^+$

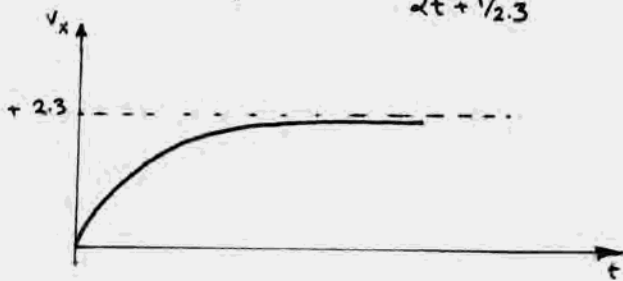


$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (3 - V_x - 0.7)^2 = C_1 \frac{dV_x}{dt}$$

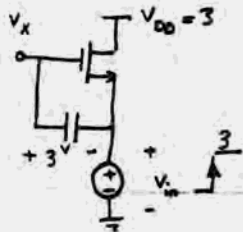
$$\frac{dV_x}{(2.3 - V_x)^2} = \underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot \frac{1}{C_1}}_{\alpha} dt$$

$$\Rightarrow \frac{1}{2.3 - V_x} = \alpha t + K \quad (t=0, V_x=0) \Rightarrow \frac{1}{2.3 - V_x} - \frac{1}{2.3} = \alpha t$$

$$\Rightarrow V_x = 2.3 - \frac{1}{\alpha t + 1/2.3}$$



2.12) c

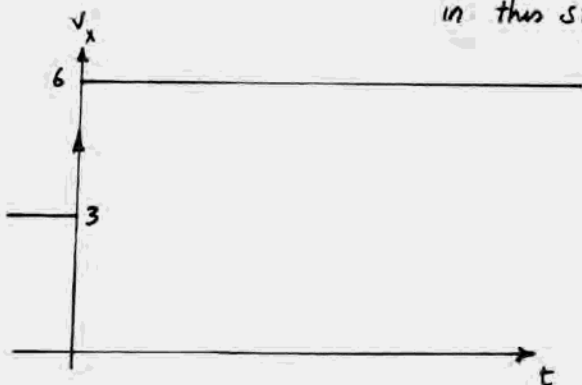


At $t = 0^+$ $V_D = 3$ $V_S = 3$ $V_G = 6$

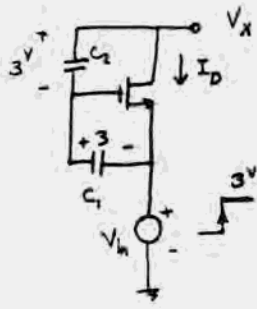
So, $V_{GS} = 0$ and $I_C = I_D = 0$ And circuit remains

in this state.

$$V_x(0^-) = 3, \quad V_x(t) = 6$$



2.12) d



Assume that the device remains in the saturation region until it turns off when $V_{gs} = 0.7$

$$V_{C1} = V_{gs} = 3 - \frac{1}{C_1} \int I_D dt \quad V_{C2} = V_{dg} = 3 - \frac{1}{C_2} \int I_D dt$$

This assumption is correct if $V_{dg} > -0.7$ when $V_{gs} = 0.7$

$$\int I_D dt = q(t) \quad V_{gs} = 3 - \frac{q}{C_1} = 0.7 \Rightarrow \frac{q}{C_1} = 2.3 \quad V_{dg} = 3 - \frac{q}{C_2} > -0.7$$

$$\Rightarrow \frac{q}{C_2} < 3.7 \quad 2.3 \frac{C_1}{C_2} < 3.7 \Rightarrow C_1 < 1.61 C_2$$

With this assumption,

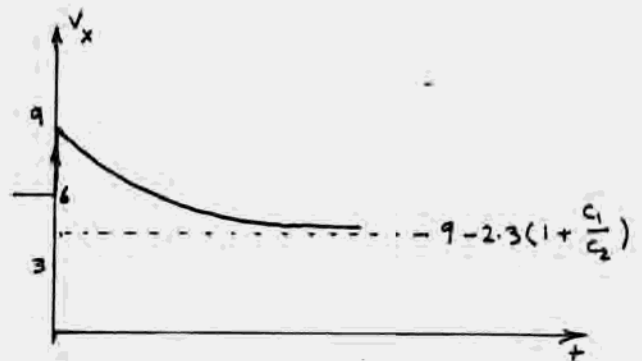
$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(3 - \frac{q}{C_1} - 0.7 \right)^2 = \frac{dq}{dt}$$

$$\Rightarrow \underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot \frac{1}{C_1}}_{\alpha} dt = \frac{dq/C_1}{\left(3 - \frac{q}{C_1} - 0.7 \right)^2} \Rightarrow \alpha t = \frac{1}{3 - \frac{q}{C_1} - 0.7} + K \quad (t=0, q=0)$$

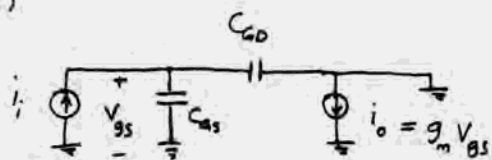
$$\Rightarrow \alpha t = \frac{1}{2.3 - \frac{q}{C_1}} - \frac{1}{2.3} \Rightarrow \frac{q}{C_1} = 2.3 - \frac{1}{\alpha t + \frac{1}{2.3}}$$

$$V_x = 3 + 3 - \frac{q}{C_1} + 3 - \frac{q}{C_2} = 9 - \frac{q}{C_1} \left(1 + \frac{C_1}{C_2} \right)$$

$$V_x(t) = 9 - \left(1 + \frac{C_1}{C_2} \right) \frac{2.3 \alpha t}{\alpha t + \frac{1}{2.3}}$$



2.13) a)



$$i_i = (C_{gs} + C_{gd}) s V_{gs}$$

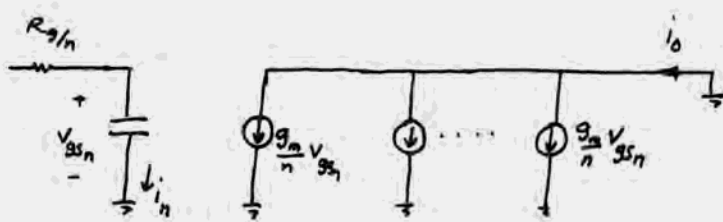
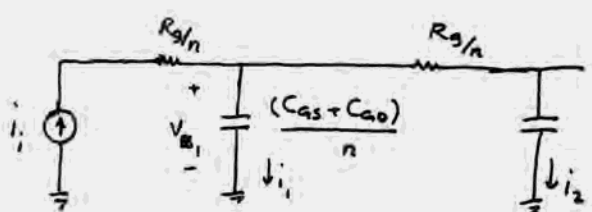
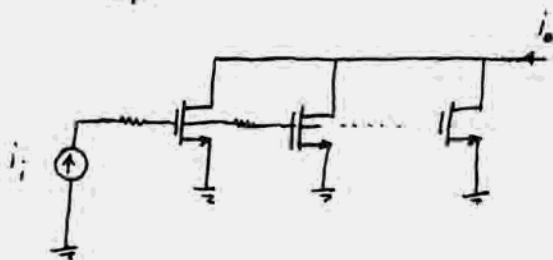
$$i_o = g_m V_{gs}$$

$$\beta = \frac{i_o}{i_i} = \frac{g_m}{(C_{gs} + C_{gd}) s} ; |\beta| = 1 \Rightarrow \frac{g_m}{(C_{gs} + C_{gd}) \omega_T} = 1$$

$$\Rightarrow \omega_T = \frac{g_m}{(C_{gs} + C_{gd})} \Rightarrow f_T = \frac{\omega_T}{2\pi} = \frac{g_m}{2\pi (C_{gs} + C_{gd})}$$

Approximation: $g_m V_{gs}$ is the output current.

b)



$$i_k = \frac{1}{n} (C_{gs} + C_{gd}) s V_{gsk} \quad k = 1 \dots n$$

$$(*) \quad i_i = i_1 + i_2 + \dots + i_n = \frac{1}{n} (C_{gs} + C_{gd}) s (V_{gs1} + V_{gs2} + \dots + V_{gsn})$$

$$(**) \quad i_o = \frac{g_m}{n} V_{gs1} + \dots + \frac{g_m}{n} V_{gsn} = \frac{g_m}{n} (V_{gs1} + V_{gs2} + \dots + V_{gsn})$$

$$(*), (**) \Rightarrow \beta = \frac{i_o}{i_i} = \frac{g_m}{(C_{gd} + C_{gs}) s} ; |\beta| = 1 \Rightarrow f_T = \frac{\omega_T}{2\pi} = \frac{g_m}{2\pi (C_{gs} + C_{gd})}$$

$$c) \quad f_T = \frac{g_m}{2\pi (C_{GS} + C_{GD})}$$

$$g_m = \mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$C_{GS} + C_{GD} \approx C_{ox} WL$$

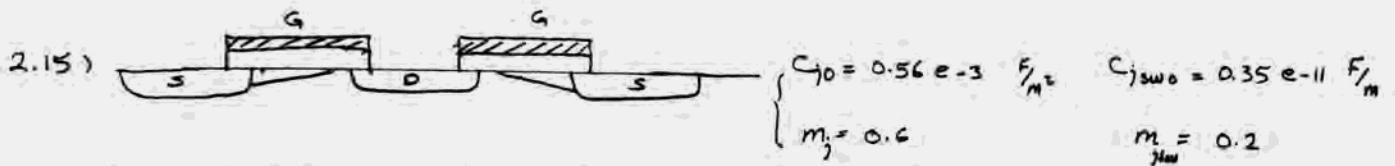
$$\Rightarrow f_T = \frac{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}{2\pi C_{ox} WL} \approx \frac{\mu}{2\pi} \frac{(V_{GS} - V_{TH})}{L^2}$$

2.1A)

$$f_T = \frac{g_m}{2\pi (C_{GS} + C_{GD})} ; \quad g_m = \frac{I_D}{5V_T}$$

In the subthreshold $C_{GS} = C_{GD} = WC_{ov}$ (Fig 2.33)

$$\text{So, } f_T = \frac{I_D / 5V_T}{4\pi WC_{ov}} = \frac{I_D}{4\pi 5V_T WL C_{ox}}$$



$$C_{DB} = \frac{W}{2} E C_j + 2 \left(\frac{W}{2} + E \right) C_{jsw}$$

$$C_j = \frac{C_{j0}}{\left(1 + \frac{V_A}{2\phi_F}\right)^m}$$

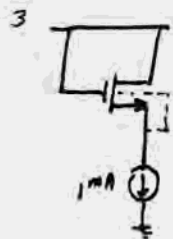
$$C_{SB} = 2 \left[\frac{W}{2} E C'_j + 2 \left(\frac{W}{2} + E \right) C'_{jsw} \right]$$

$$C_{GD} = 2 \left(\frac{W}{2} C_{ov} \right) \quad C_{ov} = L_D C_{ox}$$

$$C_{GS} = \frac{2WL C_{ox}}{3} + WC_{ov}$$

$$C_{GB} = (WL C_{ox}) C_d / (WL C_{ox} + C_d) ; C_d = NL \sqrt{q \epsilon_{si} N_{sub} / 2 \phi_F}$$

$$W = 50 \mu \quad L = 0.5 \mu \quad E = 1.5 \mu$$



$$I = \frac{1}{2} \mu_n C_{ox} \frac{W}{L - 2L_D} (V_{GS} - V_{TH})^2, \quad I^{mA} = \frac{1}{2} \times 0.13429 \times \frac{50}{0.5 - 0.16} (V_{GS} - 0.7)^2$$

$$V_{GS} = 1.0182 \quad g_m = \frac{2I_D}{V_{GS} - V_{TH}} = 6.285 \text{ mA/V}, \quad V_{DS} = 1.0182$$

$$\lambda = 0, \quad L_D = 0.08 \mu\text{m}$$

$$\frac{W}{L} = \frac{50 \mu\text{m}}{0.5 \mu\text{m}}, \quad V_{TH} = 0.7$$

$$C_{GD} = 15.4 \text{ fF}$$

$$C_{GS} = 79.36 \text{ fF}$$

$$\mu_n C_{ox} = 134.29 \text{ mA/V}^2$$

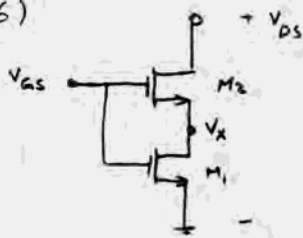
$$C_{ox} = 3.84 \times 10^{-3} \text{ F/m}^2$$

$$C_{SB} = 42.4 \text{ fF}$$

$$C_{DB} = 13.5 \text{ fF}$$

$$f_T = \frac{g_m}{2\pi(C_{GD} + C_{GS})} = 10.6 \text{ GHz}$$

2.16)

CASE I, M_1 : Triode M_2 : Triode

$$V_{GS1} = V_{GS} - V_{TH}$$

$$V_{GS2} = V_{GS} - V_X - V_{TH}$$

$$V_{DS1} = V_X$$

$$V_{DS2} = V_{DS} - V_X$$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH}) V_X - V_X^2 \right] \quad (*)$$

$$I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH} - V_X)(V_{DS} - V_X) - (V_{DS} - V_X)^2 \right]$$

$$I_{D1} = I_{D2} \Rightarrow 2(V_{GS} - V_{TH}) V_X - V_X^2 = 2(V_{GS} - V_{TH}) V_{DS} + 2V_X^2 - 2V_X(V_{GS} - V_{TH}) - 2V_X V_{DS} - V_{DS}^2 - V_X^2 + 2V_X V_{DS}$$

$$\Rightarrow 2 \left[2(V_{GS} - V_{TH}) V_X - V_X^2 \right] = 2(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \quad (**)$$

$$(*), (**) \Rightarrow I_{D1} = I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \times \frac{1}{2} \left[2(V_{GS} - V_{TH}) V_{DS} - V_{DS}^2 \right] \left(\frac{W}{2L} \text{ in Triode} \right)$$

CASE II, M_1 : Triode, M_2 : Sat

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH}) V_X - V_X^2 \right] \quad (**)$$

$$I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_X - V_{TH})^2$$

$$I_{D1} = I_{D2} \Rightarrow V_X^2 - 2V_X(V_{GS} - V_{TH}) + (V_{GS} - V_{TH})^2 = 2(V_{GS} - V_{TH}) V_X - V_X^2$$

$$\Rightarrow (V_{GS} - V_{TH})^2 = 2 \left[2(V_{GS} - V_{TH}) V_X - V_X^2 \right] \quad (***)$$

2.16) Cont. $(*) , (**) \Rightarrow I_{D1} = I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \times \frac{1}{2} (V_{GS} - V_{TH})^2 \left(\frac{W}{2L} \text{ in Sat} \right)$

Note that M_1 is always in triode, because V_{OD2} is always positive

i.e. $V_{GS2} - V_{TH} > 0 \Rightarrow V_{GS} - V_X - V_{TH} > 0 \Rightarrow V_{GS} - V_{TH} > V_X$

$\Rightarrow V_{GS1} - V_{TH} > V_{DS1} \Rightarrow M_1$ is in the triode region.

Saturation - triode transition edge of M_2 :

We show that the transition point the saturation and triode region of the equivalent transistor is the same as that of M_2 .

$$V_{OD2} = V_{GS} - V_X - V_{TH} \quad V_{DS2} = V_{DS} - V_X$$

for $V_{OD2} > V_{DS2}$, M_2 is in the triode region, i.e. $V_{GS} - V_{TH} > V_{DS}$

It means that when M_2 is in the saturation, then the equivalent

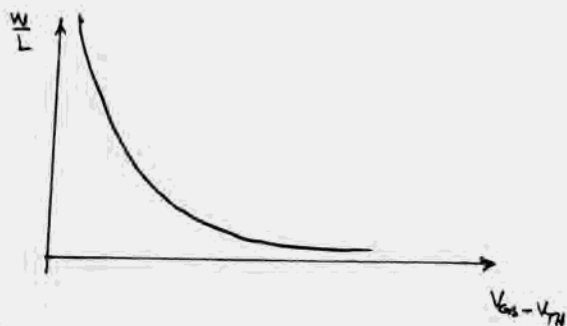
transistor is in the saturation, and vice versa.

2.17)

In Saturation region,

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2$$

$$\Rightarrow \frac{W}{L} = \frac{2 I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^2}$$

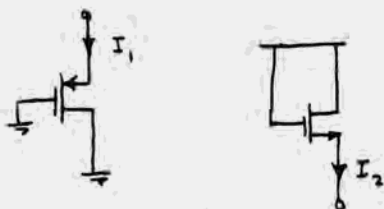


$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$\Rightarrow \frac{W}{L} = \frac{g_m}{\mu_n C_{ox} (V_{GS} - V_{TH})}$$



2.18)



These structures cannot operate as current sources, because

their currents strongly depend on source voltages, but

an ideal current source should provide a constant current,

independent of its voltage.

2.19) From Eq. (2.1) we know that $V_{TH} = \phi_{MS} + 2\phi_F + \frac{Q_{dep}}{C_{ox}}$, where

ϕ_{MS} and ϕ_F are constant values, so any changes in V_{TH}

come from the third term, in fact $\Delta V_{TH} = \frac{\Delta Q_{dep}}{C_{ox}}$ and

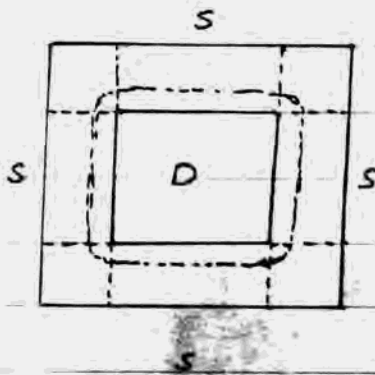
From Eq. (2.22), we have $\Delta V_{TH} = \gamma (\sqrt{2\phi_F + V_{SB}} - \sqrt{2\phi_F})$ (in fact,

this is definition of γ). From pn junction theory we know

that Q_{dep} is proportional to $\sqrt{N_{sub}}$, so γ is directly

proportional to $\sqrt{N_{sub}}$ and inversely proportional to C_{ox} .

2.20)



This structure operates as a traditional device does, in fact if we neglect edges

we have four MOSFETs in parallel,

where the aspect ratio of each is $\frac{W}{L}$.

So the overall aspect ratio is almost $\frac{4W}{L}$.

Drain junction capacitance: $C_{DB} = W^2 C_j + 4WC_{jsw}$

Drain junction capacitance of devices shown in Fig. 2.32 a, b for the aspect ratio of $\frac{4W}{L}$

$$C_{DB(a)} = 4WE C_j + (8W + 2E) C_{jsw}$$

$$C_{DB(b)} = 2WE C_j + (4W + 2E) C_{jsw}$$

The value of side wall capacitance in the ring structure is less than that in folded and traditional structures, but the bottom capacitance of ring structure

is higher than that of the other two structures. (for $w > 4E$)

2.21) We first check the terminals of the device with a multimeter

in order to find BS or BD junctions. There are 12 experiments

in total of which two lead to conduction and remaining ones show

no conduction. If we find one of those two conduction then we

are done. Finding B and S (or D), we need to do one other

experiment between B (Cathode of junction) and one of the two

remaining terminals; In case of no connection, the terminal under

test is G, otherwise it is D (or S). In worst case with a maximum

of 8 experiments, each terminal can be specified. It is as follows:

Assume, the two selected terminals do not conduct in both

directions and this is the case for the other two terminals.

Up to this point, four experiments have been done while not yet

encountering any conduction. It is clear that one group consists of

G and B and the other comprises from D and S, Because at least one conduction should be observed if B were in the same group with one of the source or Drain. In the next step, we pick up one terminal from each group to undergo the conductivity test. Assume, no conduction happens in either direction (Worst case). It means that we had chosen G from (GB) group. Thus far, we have done six experiments. We change both of terminals and now we have chosen B for sure. and in worst case, we will find a connection in 8th experiment. Now, we know B and S (D), Bulk's groupmate is Gate and Source's (Drain's) groupmate is Drain (Source).

2.22) If we don't know the type of device, In eight experiment we cannot distinguish between B and S (D) and we should perform another experiment, which is exchanging one of

2.22) Cont. terminals with its groupmate. If we still had the

Conduction then the exchanged terminal and its groupmate

are source and Drain, otherwise the exchanged terminal

is Bulk.

2.23) a) NO, Because in DC model equations of MOSFET, we

always have the product of $\mu_n C_{ox}$ and $\frac{W}{L}$.

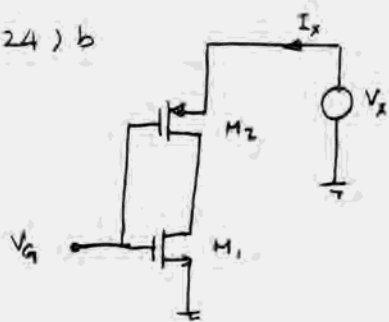
b) No, Because we cannot obtain as many independent

equations as the unknown quantities. But if the

difference between the aspect ratios is known, then $\mu_n C_{ox}$

and both $\frac{W}{L}$, are attainable.

2.24) b



CASE I : $V_G < V_{THN} \Rightarrow M_1$: off $I_x = 0$

$$g_m = 0$$

CASE II : $V_G > V_{THN}$

for $0 < V_x < V_G + |V_{THP}| \Rightarrow I_x = 0$ (M_2 : off) $g_m = \frac{\partial I_x}{\partial V_G} = 0$

Then M_2 turns on (in sat), M_1 still is in triode region

$$I_x = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)^2$$

This is correct until M_1 goes into saturation, when

$$\frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)^2 = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_G - V_{THN})^2$$

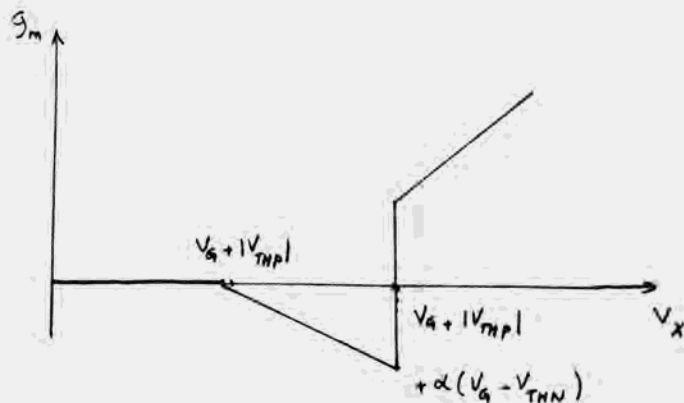
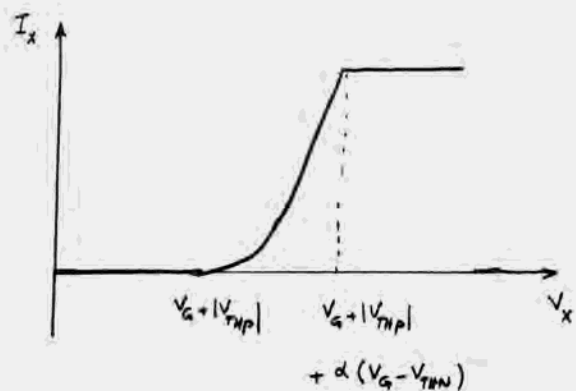
$$\text{i.e. } V_x = V_G + |V_{THP}| + \sqrt{\frac{\mu_n (W/L)_n}{\mu_p (W/L)_p}} (V_G - V_{THN})$$

And afterward, M_2 goes into triode region and $I_x = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_G - V_{THN})^2$

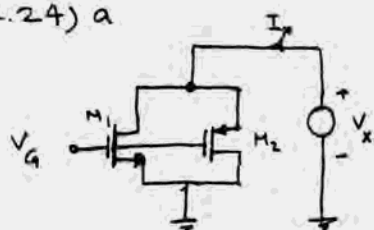
So, $0 < V_x < V_G + |V_{THP}| \Rightarrow I_x = 0$ $g_m = \frac{\partial I_x}{\partial V_G} = 0$

$$V_G + |V_{THP}| < V_x < V_G + |V_{THP}| + \alpha (V_G - V_{THN}) \quad I_x = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)^2 \quad g_m = \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_G + |V_{THP}| - V_x)$$

$$V_G + |V_{THP}| + \alpha (V_G - V_{THN}) < V_x \quad I_x = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_G - V_{THN})^2 \quad g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_G - V_{THN})$$



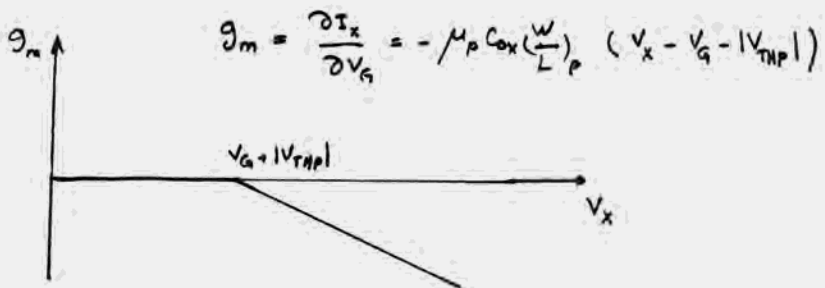
2.24) a



CASE I : $V_G < V_{TNN}$ M_1 : off

for $0 < V_x < V_G + |V_{THP}|$ $I_x = 0$, $g_m = \frac{\partial I_x}{\partial V_G} = 0$

for $V_G + |V_{THP}| < V_x \Rightarrow I_x = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)^2$



CASE II : $V_G > V_{TNN}$

for $0 < V_x < V_G - V_{TNN}$ (M_2 : off M_1 : triode)

$I_x = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n [2(V_G - V_{TNN})V_x - V_x^2]$ $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_n V_x$

for $V_G - V_{TNN} < V_x < V_G + |V_{THP}|$ (M_2 : off M_1 : sat)

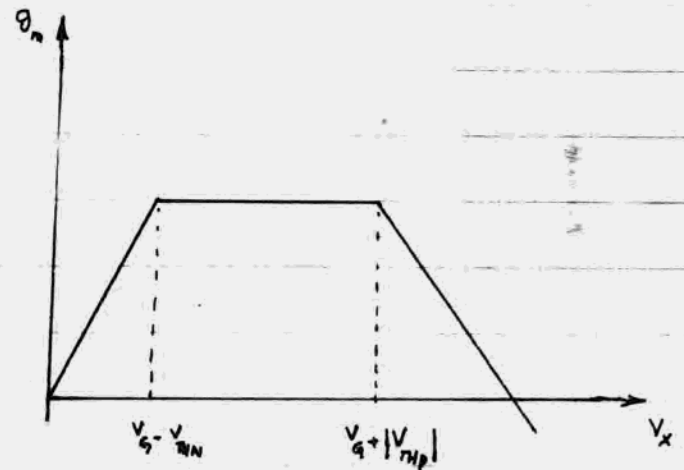
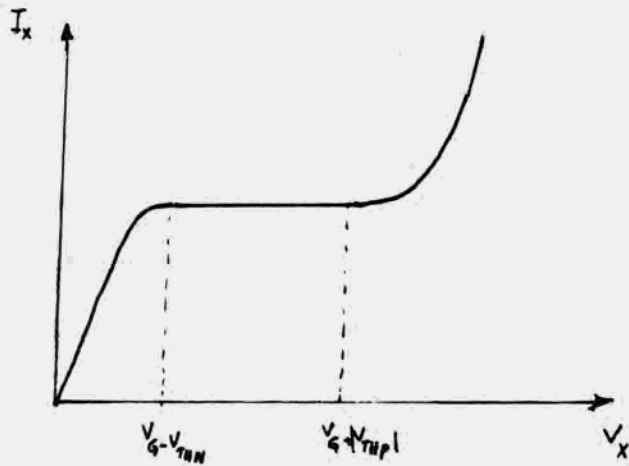
$I_x = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_G - V_{TNN})^2$ $g_m = \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_G - V_{TNN})$

for $V_G + |V_{THP}| < V_x$ (M_2 : sat M_1 : sat)

2.24) a Cont.

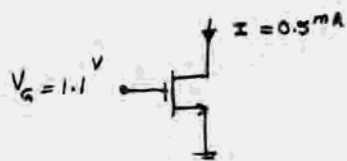
$$I_x = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_G - V_{THN})^2 + \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)^2$$

$$g_m = \frac{\partial I_x}{\partial V_G} = \mu_n C_{ox} \left(\frac{W}{L}\right)_n (V_G - V_{THN}) - \mu_p C_{ox} \left(\frac{W}{L}\right)_p (V_x - V_G - |V_{THP}|)$$



2.25)

$$V_{TH} = 0.7 \quad \lambda = 0.1 \quad (\text{for } L = 0.5 \mu\text{m})$$



$$\text{for } L = 0.5 \mu\text{m} \quad \lambda = 0.1 \rightarrow r_o = \frac{1}{\lambda I_D} = 20 \text{ k}\Omega$$

$$V_{D0} = V_{GS} - V_{TH} = 0.4 \Rightarrow V_{GS} = 1.1 \text{ V}$$

Calculating W ,

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2$$

$$0.5 \text{ mA} = \frac{1}{2} \times 0.1343 \frac{\text{mA}}{\text{V}^2} \times \frac{W}{0.5 \mu\text{m} - 0.16 \mu\text{m}} \times (0.4)^2$$

$$\frac{W}{L_{eff}} \approx 47$$

 \Rightarrow

$$W = 15.82 \mu\text{m}$$

$$C_{gs} = \frac{2}{3} WL C_{ox} + WC_{ov} = 25 \text{ fF}$$

$$C_{gd} = WC_{ov} = 4.85 \text{ fF}$$

$$C_{DB} = \frac{W}{2} \epsilon C_j + 2 \left(\frac{W}{2} + E \right) C_{jsw} \quad (@ V_D = 0.4) = 10.7 \text{ fF}$$

(for folded structure)

$$\left(C_j = \frac{C_{j0}}{\left(1 + \frac{V_{DB}}{2\phi_F}\right)^{m_j}} = 0.449 \times 10^{-3} \frac{\text{F}}{\text{m}^2}, \quad C_{jsw} = \frac{C_{jsw0}}{\left(1 + \frac{V_{DB}}{2\phi_F}\right)^{m_{jsw}}} = 0.325 \times 10^{-11} \frac{\text{F}}{\text{m}} \right)$$

$$C_{ox} = 3.84 \times 10^{-3} \frac{\text{F}}{\text{m}}$$

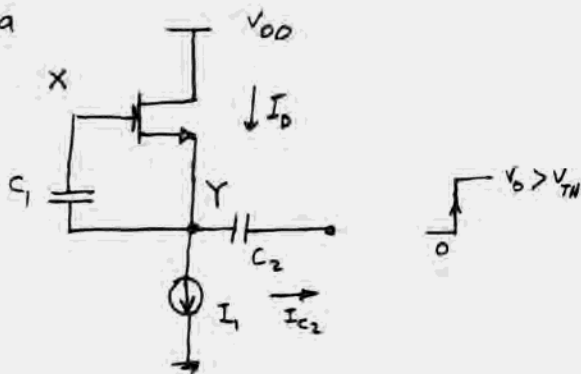
$$C_{j0} = 0.56 \times 10^{-3}$$

$$m_j = 0.6$$

$$C_{jsw0} = 0.35 \times 10^{-11}$$

$$m_{jsw0} = 0.2$$

2.26) a



Before applying the pulse

$$X(0^-) = V_{DD}$$

$$Y(0^-) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}}$$

After Applying the Pulse

$$X(0^+) = V_{DD} + V_0$$

$$Y(0^+) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} + V_0$$

for $t > 0$

$$\begin{cases} X(t) = V_{DD} + \alpha(t) \\ Y(t) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} + \alpha(t) \end{cases}$$

 $\alpha(0^+) = V_0$, Device is in triode

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2 \right] = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2 \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} - \left(V_{TH} + \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} - \alpha(t) \right) \right. \\ \left. \left(V_{TH} + \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} - \alpha(t) \right) \right]$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}} - (\alpha(t) - V_{TH})^2 \right] = I_1 - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (\alpha(t) - V_{TH})^2$$

$$I_{C2} = I_D - I_1 = -\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (\alpha(t) - V_{TH})^2 = C_2 \frac{dV_{C2}}{dt} = C_2 \frac{d\alpha(t)}{dt}$$

$$\underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot \frac{1}{C_2}}_K dt = \frac{-d\alpha}{(\alpha - V_{TH})^2} \Rightarrow Kt = \frac{1}{\alpha - V_{TH}} - \frac{1}{V_0 - V_{TH}}$$

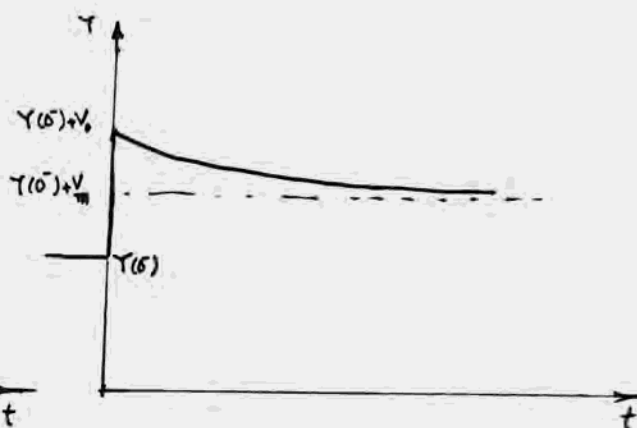
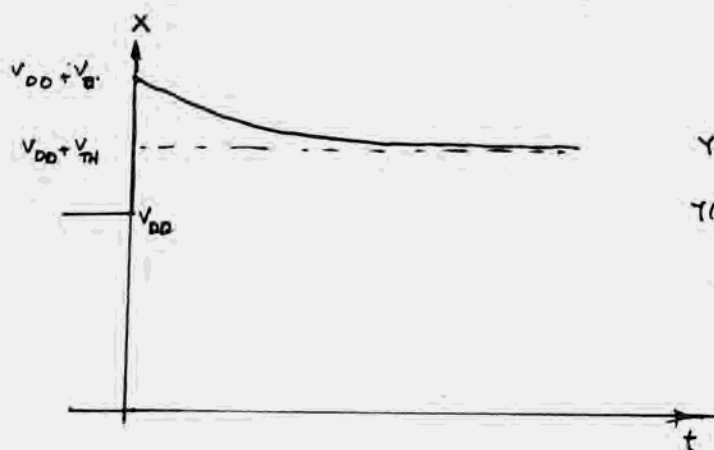
$$\Rightarrow \alpha(t) = V_{TH} + \frac{1}{Kt + \frac{1}{V_0 - V_{TH}}}$$

$$\alpha(\infty) = V_{TH}$$

2.26) a Cont.

$$X(\infty) = V_{DD} + V_{TH}$$

$$Y(\infty) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} + V_{TH} = V_{DD} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}}$$



2.26) b

Before applying the pulse.

$$X(0^-) = V_{DD}$$

$$Y(0^-) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}}$$

After applying the pulse

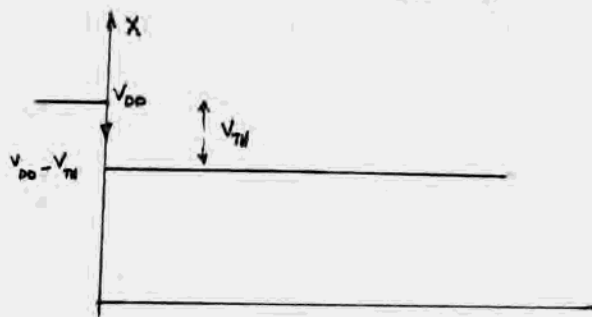
$$X(0^+) = V_{DD} - V_{TH}$$

$$Y(0^+) = V_{DD} - V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}} - V_{TH}$$

After applying the pulse, device remains in the saturation region, and its current doesn't change, so, $I_{C1} = I_{C2} = 0$

Therefore, the circuit keeps its state.

$$X(t) = X(0^+) = V_{DD} - V_{TH}$$



$$Y(t) = Y(0^+) = V_{DD} - 2V_{TH} - \sqrt{\frac{2I_1}{\mu_n C_{ox} \frac{W}{L}}}$$



2.27)

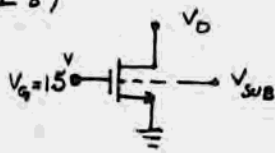
$$I_D = I_0 \exp \frac{V_{GS}}{\xi V_T}$$

$$\frac{I_{D2}}{I_{D1}} = \exp \frac{V_{GS2} - V_{GS1}}{\xi V_T} \quad \frac{I_{D2}}{I_{D1}} = 10 \quad \Rightarrow \quad \Delta V_{GS} = \xi V_T \ln 10$$

$$\Delta V_{GS} = 1.5 \times \ln 10 \times 26 \text{ mV} = 89.8 \text{ mV}$$

$$g_m = \frac{I_D}{\xi V_T} = \frac{10 \mu\text{A}}{1.5 \times 26 \text{ mV}} = 0.26 \text{ mA/V}$$

2.28)



- a) If we decrease V_D below zero, source and drain exchange their roles and device operates in the triode region.

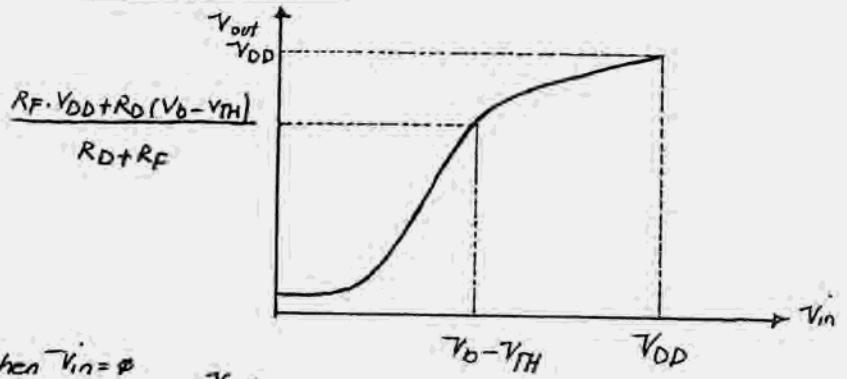
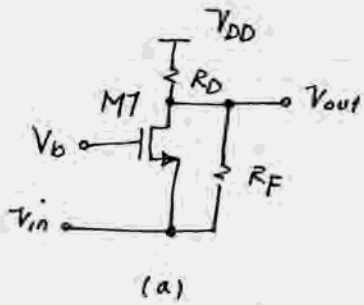
- b) If we increase V_B , V_{TH} decreases, because

$$\Delta V_{TH} = \gamma (\sqrt{2\phi_F - V_B} - \sqrt{2\phi_F}) \text{ is negative.}$$

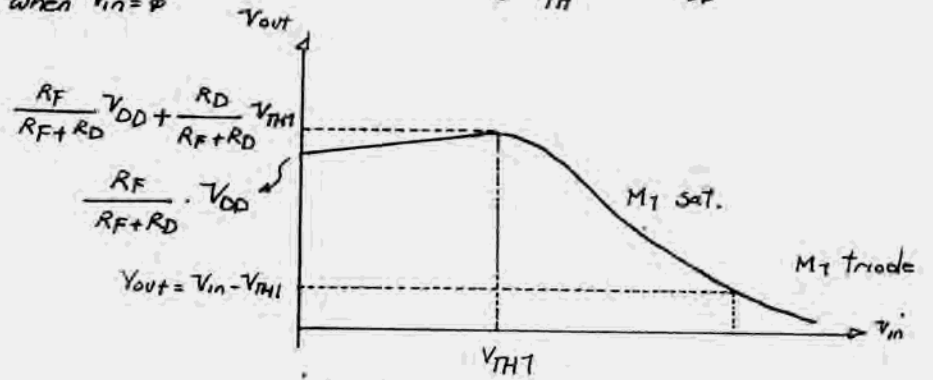
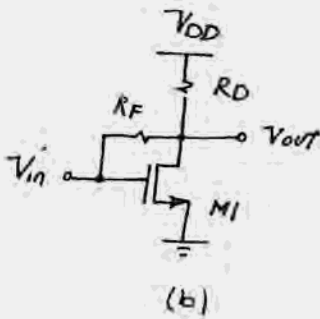
Therefore, I_D increases.

Chapter 3

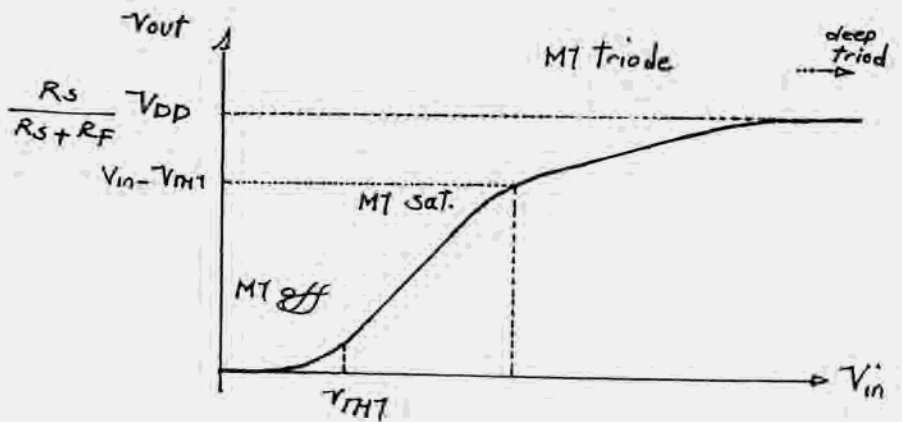
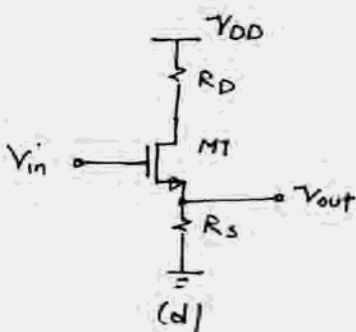
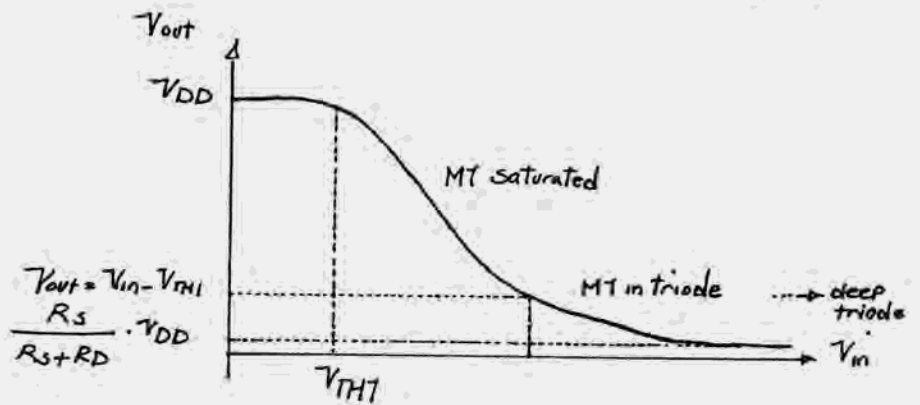
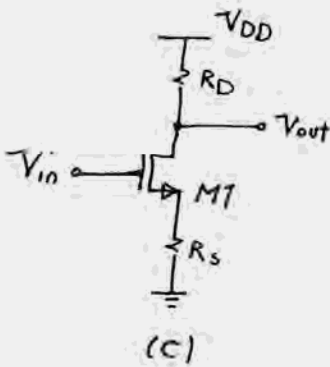
3.1.

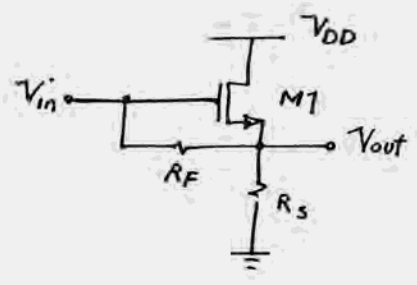


We assume that M_1 is saturated when $V_{in} = \phi$

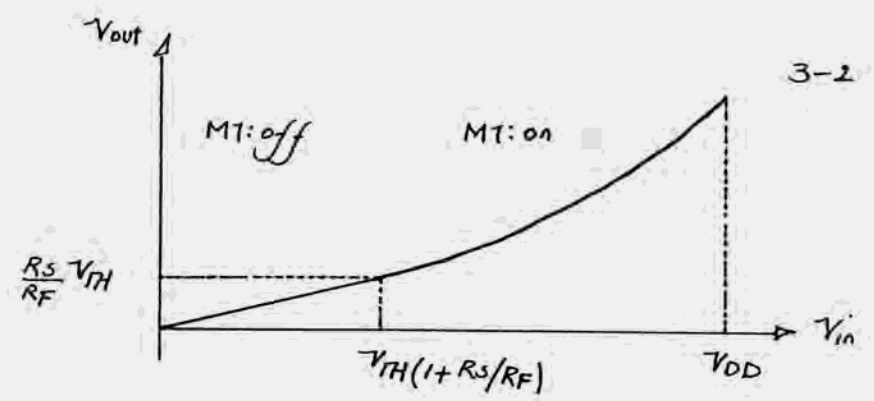


If $V_{in} < V_{TH1} \rightarrow V_{out} = \frac{R_F}{R_F + R_D} V_{DD} + \frac{R_D}{R_F + R_D} V_{in}$



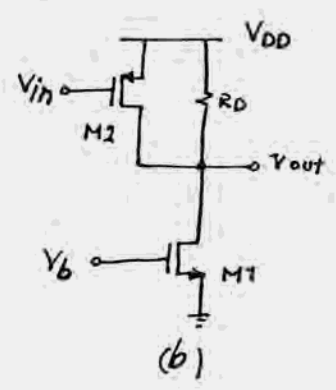


(c)

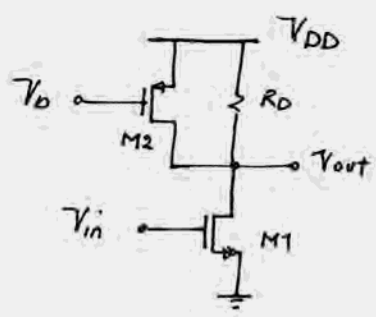
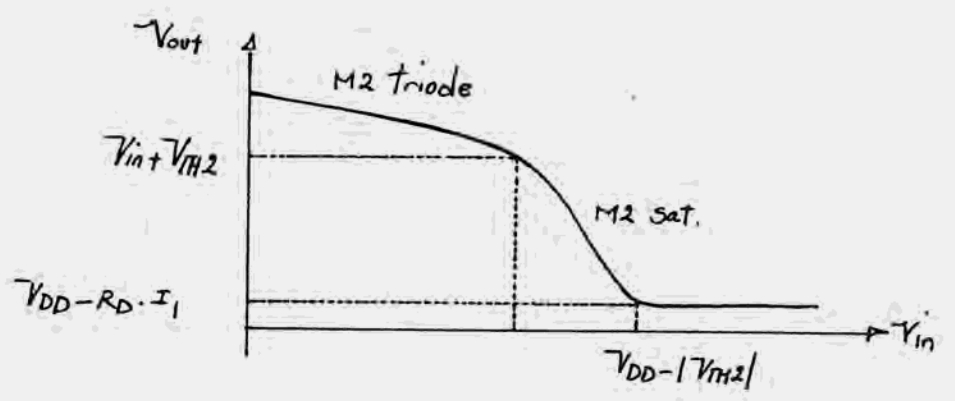


$$\text{if } V_{in} < (1 + R_S/R_F) V_{TH} \rightarrow V_o = \frac{R_S}{R_S + R_F} V_{in}$$

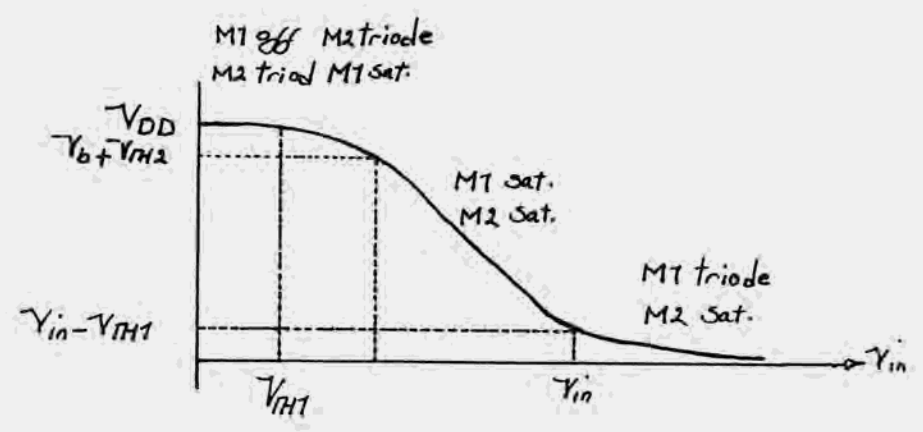
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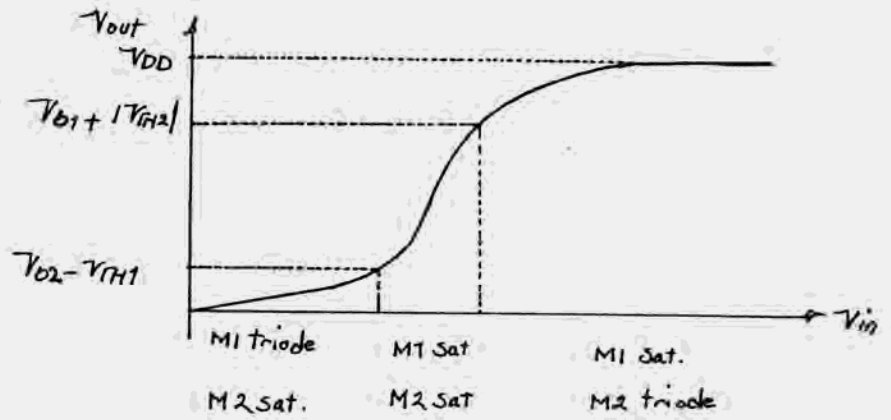
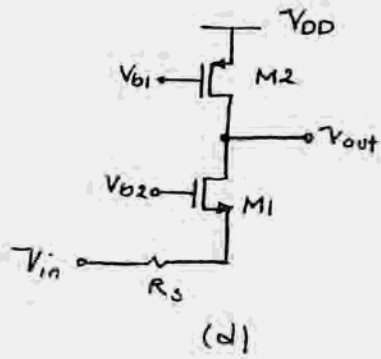
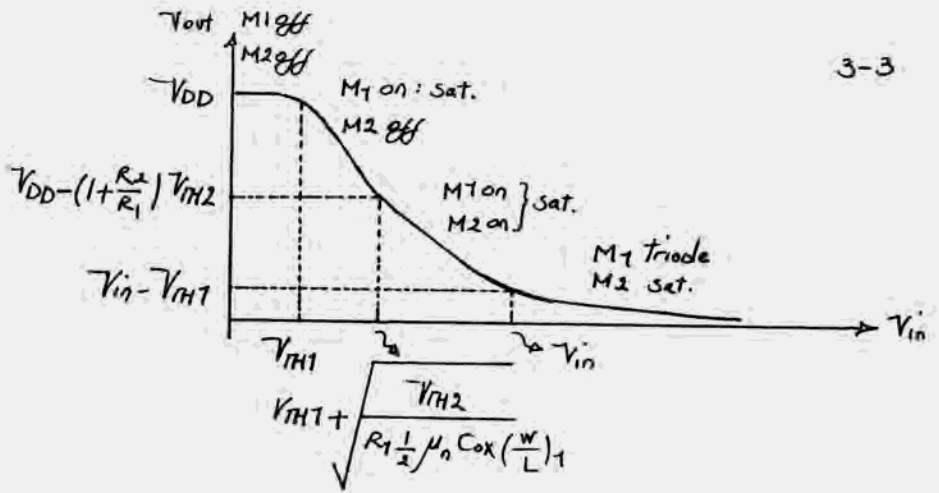
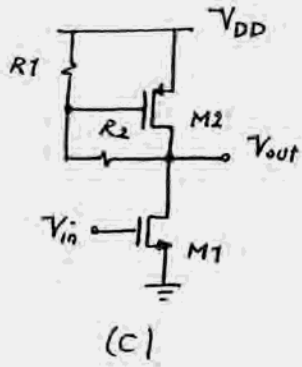


(b)

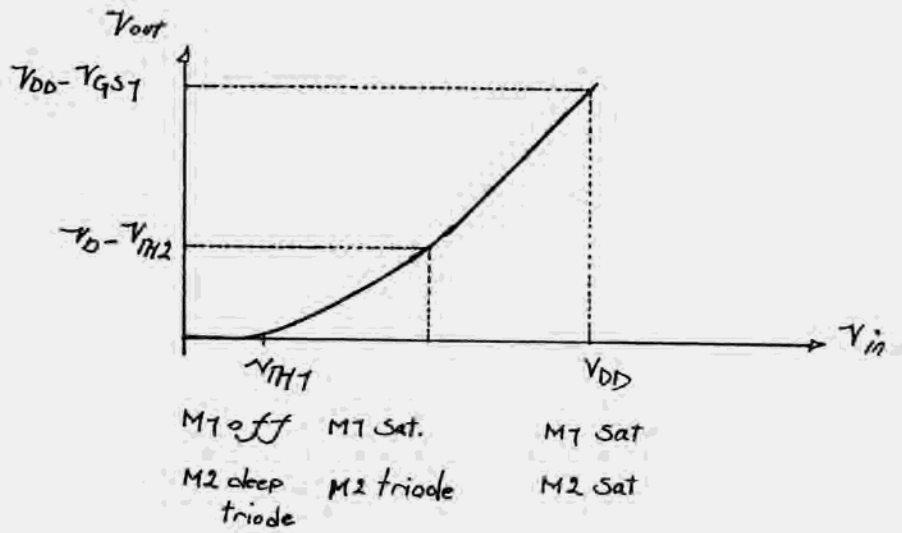
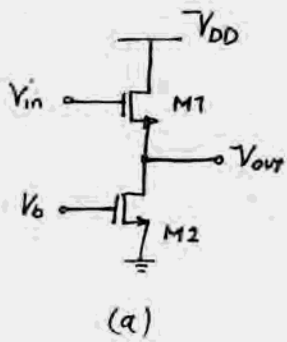


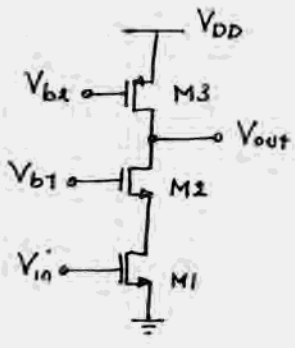
(a)



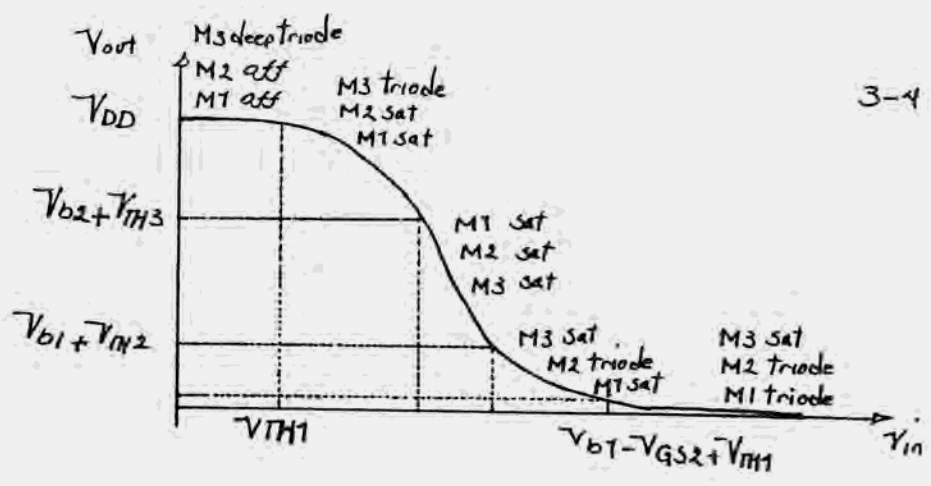


3.2.

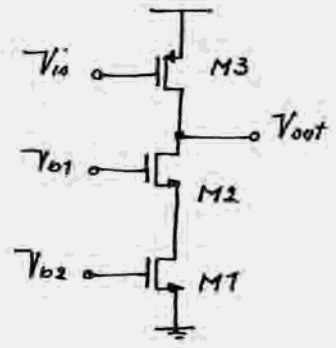




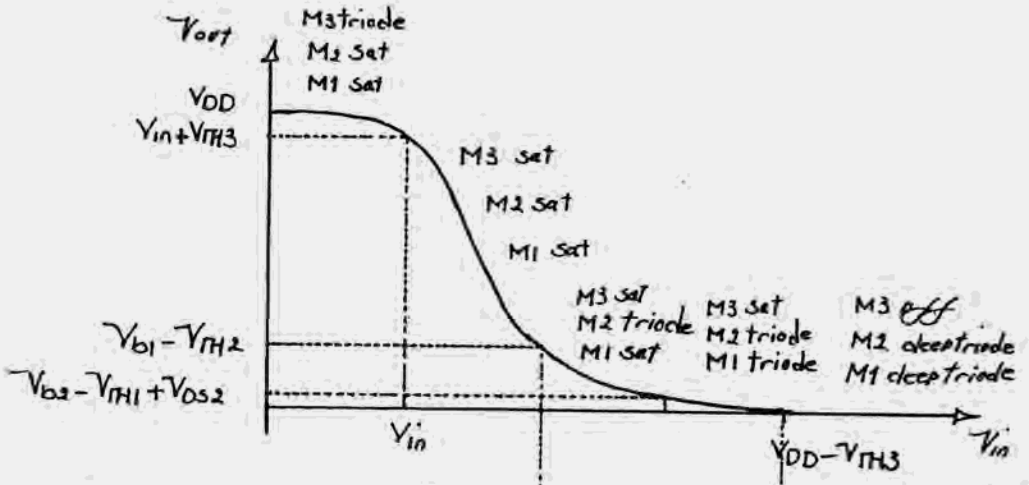
(b)



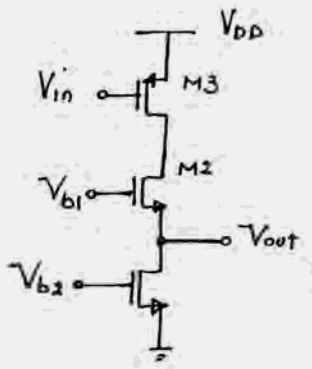
3-4



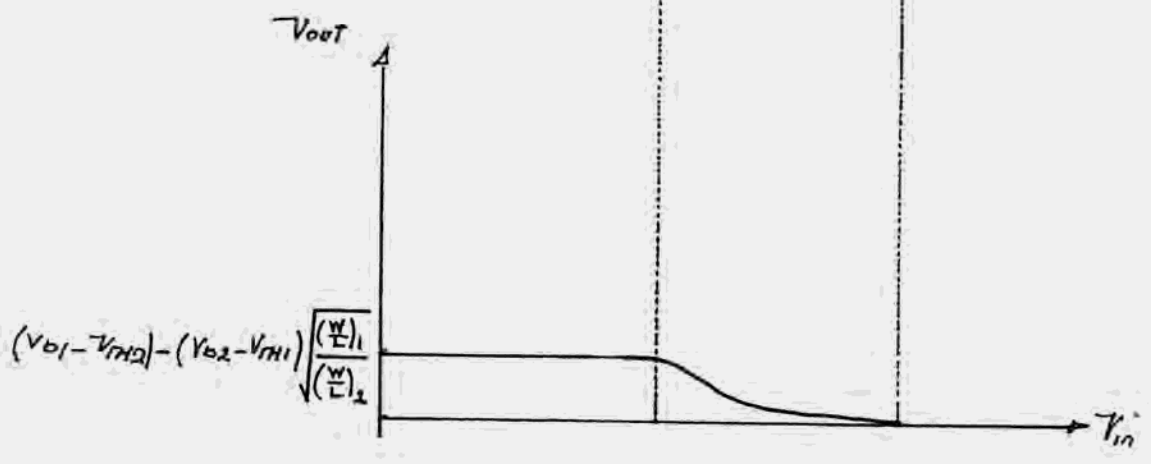
(c)



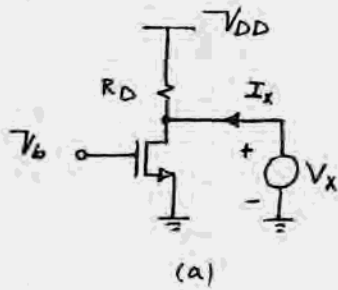
V_{DS2} is obtained from $\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{b2} - V_{TH1})^2 = \mu_n C_{ox} \left(\frac{W}{L}\right)_2 \left[(V_{b1} - V_{b2} + V_{TH1} - V_{TH2}) V_{DS2} - \frac{V_{DS2}^2}{2} \right]$



(d)

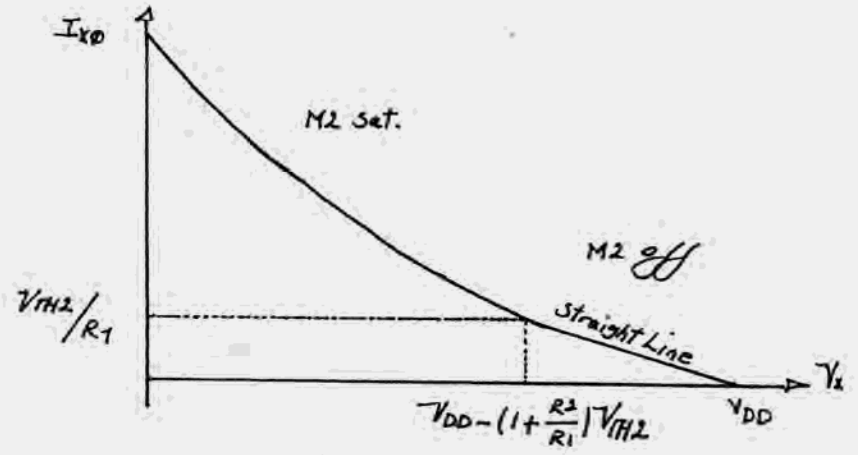
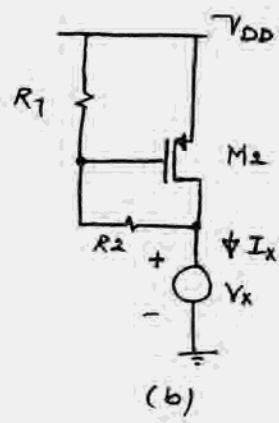
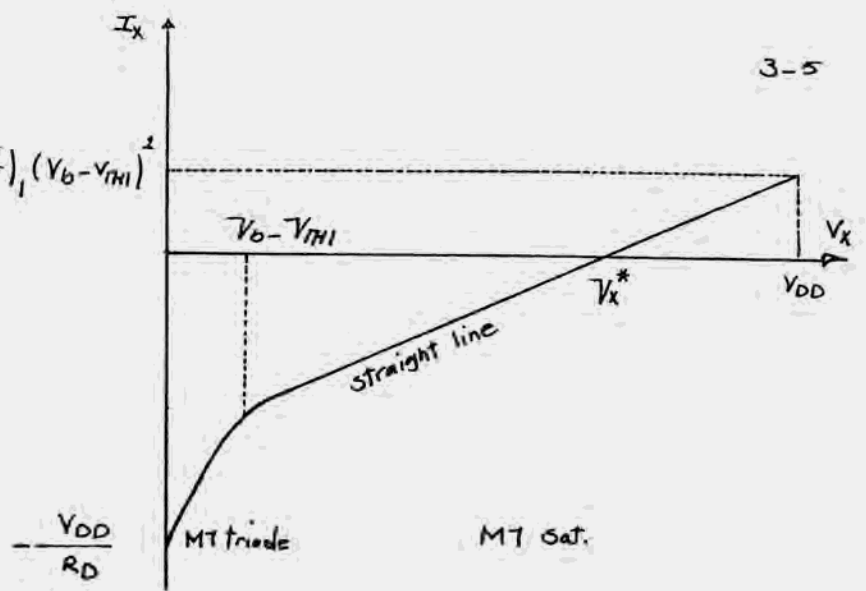


3.3.

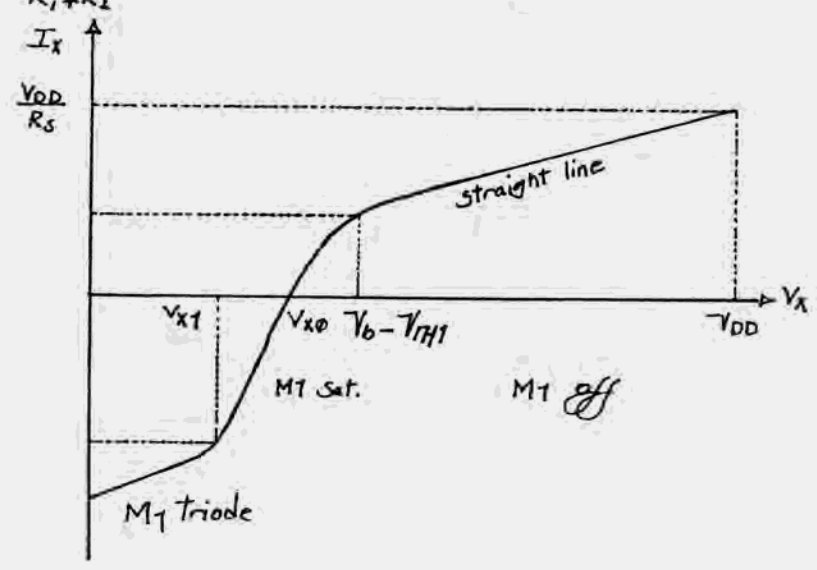
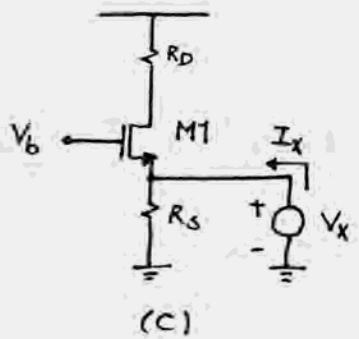


$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_b - V_{TH1})^2$$

$$V_x^* = V_{DD} - R_D \left[\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_b - V_{TH1})^2 \right]$$



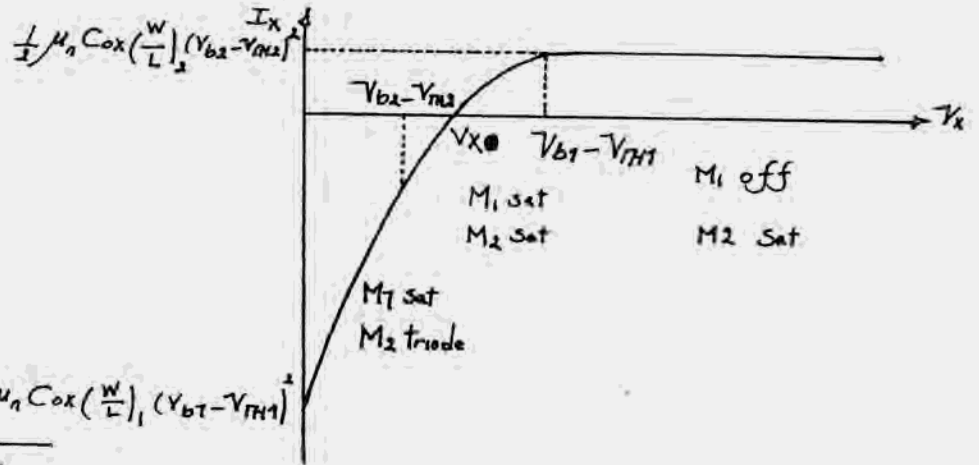
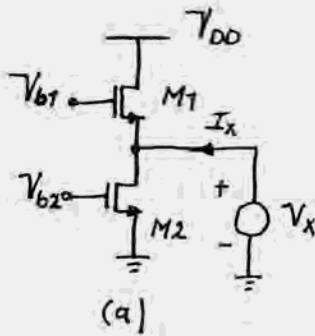
$\text{If } V_x < V_{DD} - (1 + \frac{R_2}{R_1}) V_{TH2} \quad I_x = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 \left[\frac{(V_{DD} - V_x)}{R_1 + R_2} \cdot R_1 - V_{TH2} \right]^2 + \frac{V_{DD} - V_x}{R_1 + R_2}$
 $\text{If } V_x > V_{DD} - (1 + \frac{R_2}{R_1}) V_{TH2} \quad I_x = \frac{V_{DD} - V_x}{R_1 + R_2}$



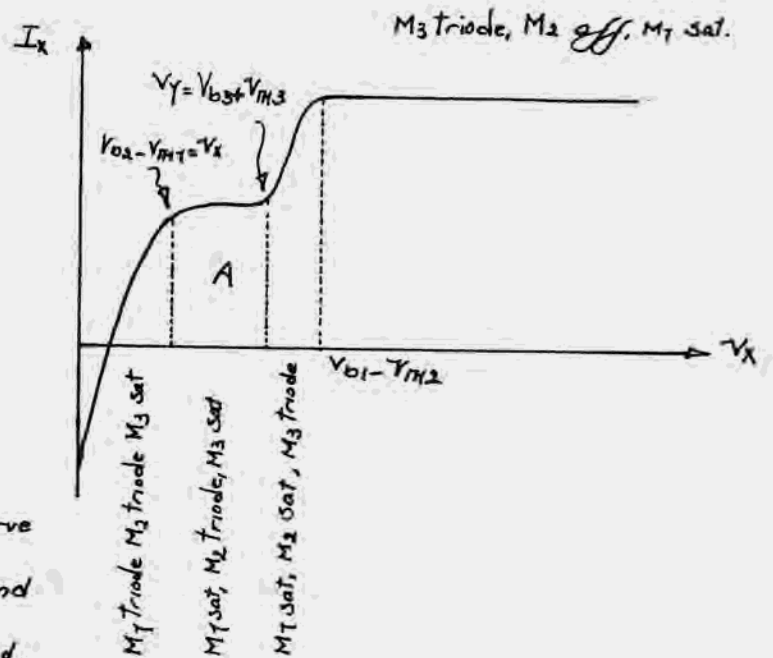
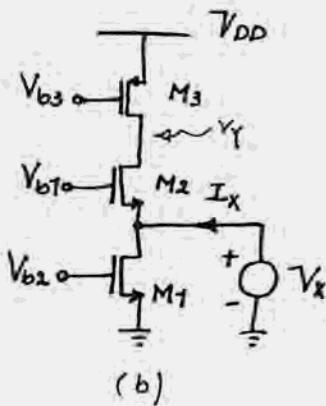
$$V_{X0} = V_{DD} - \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_b - V_{TH1})^2 \cdot R_D$$

$$V_{XT} = V_b - V_{TH1} - \left(\frac{2(V_{DD} - V_b + V_{TH1})}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \cdot R_D} \right)^{1/2}$$

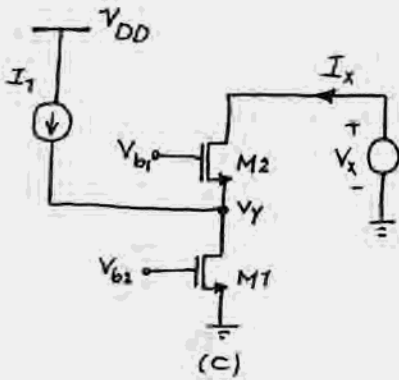
3.4.



$$V_{X0} = V_{b1} - V_{TH1} - \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}} (V_{b2} - V_{TH2})$$

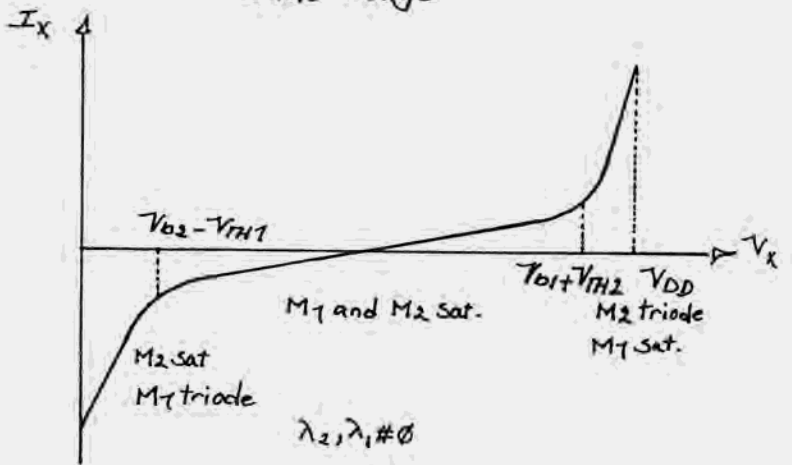
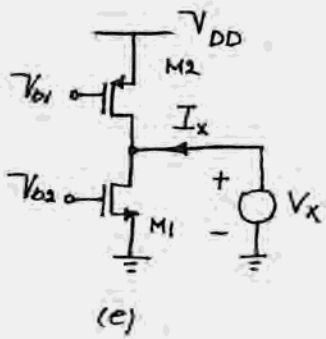
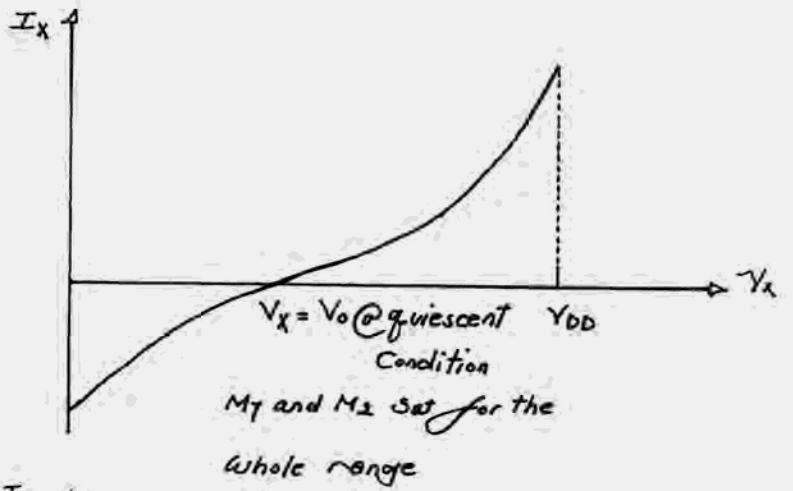
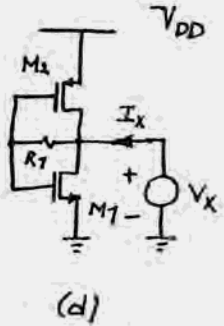
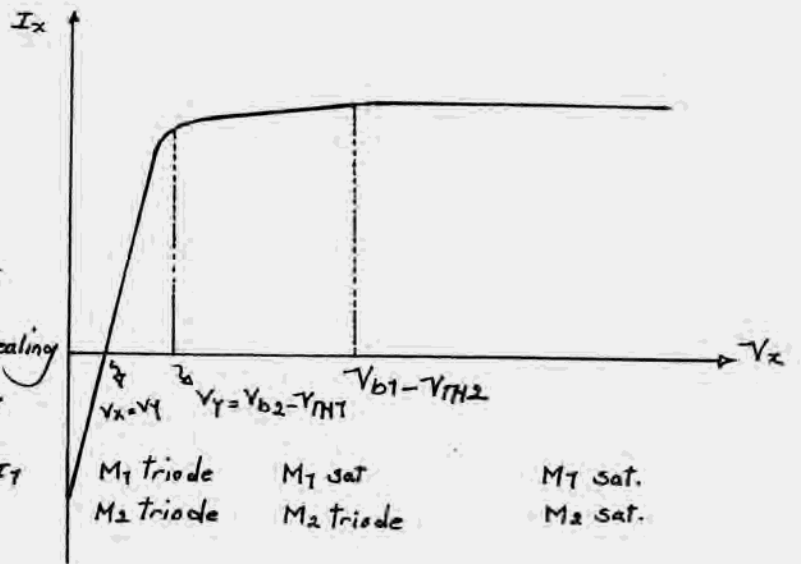


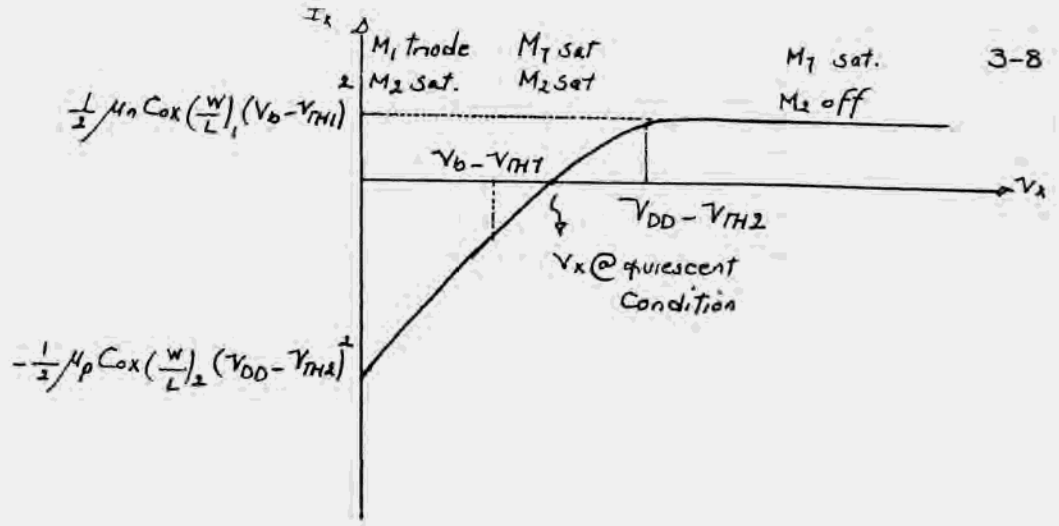
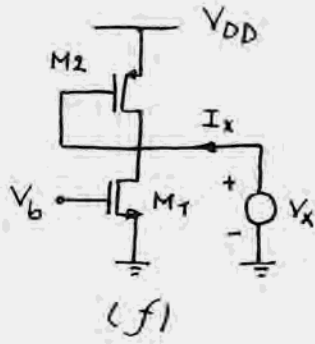
It's worth mentioning that the I_x/V_x Curve varies with the value of bias voltages and aspect ratios, therefore, some region(s), based on the aforementioned parameters, gets wider or narrower, especially the region called "A" in the above figure.



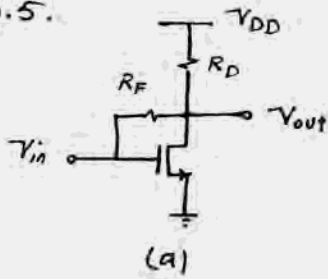
we assume $V_{b1} > V_{b2}$ and both M_1 and M_2 operate in saturation region if $V_x = V_{DD}$

Below V_x , for which $V_x = V_Y$, drain current of M_2 flows in opposite direction, revealing the fact the drain and source terminals of M_2 are reversed. As expected, most of I_1 flow through M_2 when $V_x = 0$, because we assume that $V_{b1} > V_{b2}$.



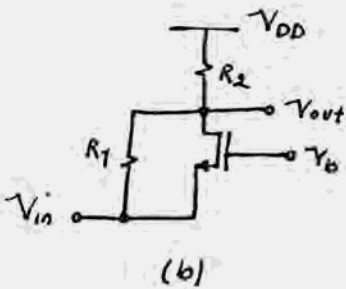


3.5.



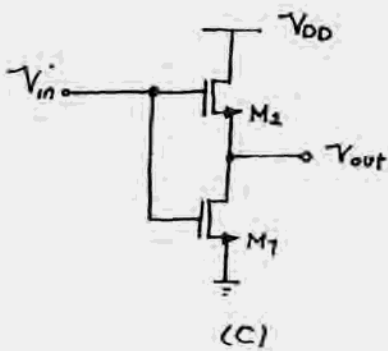
$$\frac{V_o - V_{in}}{R_F} + g_{m1} V_{in} + \frac{V_o}{r_{o1}} + \frac{V_o}{R_D} = 0$$

$$A_V = \frac{V_o}{V_{in}} = - \frac{g_{m1} - 1/R_F}{\frac{1}{R_F} + \frac{1}{r_{o1}} + \frac{1}{R_D}}$$



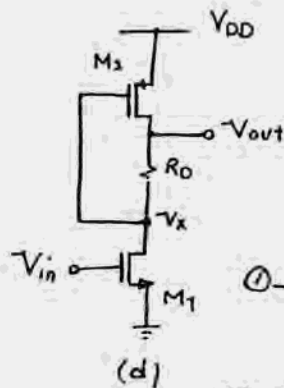
$$\frac{V_o}{R_2} + (V_o - V_{in}) \left(\frac{1}{R_1} + \frac{1}{r_{o1}} \right) - g_{m1} V_{in} = 0$$

$$\frac{V_o}{V_{in}} = \frac{g_{m1} + \frac{1}{R_1} + \frac{1}{r_{o1}}}{\frac{1}{R_2} + \frac{1}{R_1} + \frac{1}{r_{o1}}}$$



$$g_{m2} (V_{in} - V_{out}) + \frac{-V_{out}}{r_{o2}} = g_{m1} V_{in} + \frac{V_{out}}{r_{o1}}$$

$$\frac{V_{out}}{V_{in}} = - \frac{g_{m1} - g_{m2}}{g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}}}$$



$$\textcircled{1} (g_{m1} V_{in} + \frac{V_x}{r_{o1}}) R_D + V_x = V_{out}, \quad \textcircled{2} - (g_{m2} V_x + \frac{V_{out}}{r_{o2}}) = g_{m1} V_{in} + \frac{V_x}{r_{o1}}$$

$$\textcircled{2} \rightarrow V_x (-g_{m2} - \frac{1}{r_{o1}}) = g_{m1} V_{in} + \frac{V_{out}}{r_{o2}} \quad \textcircled{2} \rightarrow V_x = - \frac{g_{m1} V_{in} + V_{out}/r_{o2}}{g_{m2} + \frac{1}{r_{o1}}} \quad \textcircled{3}$$

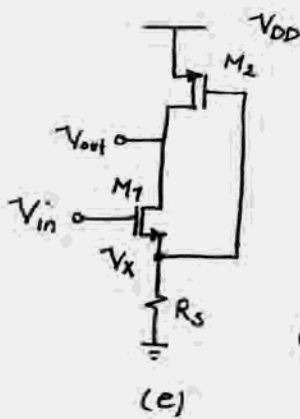
$$\textcircled{1} \rightarrow g_{m1} R_D V_{in} + (1 + \frac{R_D}{r_{o1}}) V_x = V_o \quad \textcircled{4}$$

$$\textcircled{3}, \textcircled{4} \rightarrow g_{m1} R_D V_{in} - \frac{(1 + \frac{R_D}{r_{o1}})(g_{m1} V_{in} + \frac{V_{out}}{r_{o2}})}{g_{m2} + \frac{1}{r_{o1}}} = V_{out}$$

$$\left[g_{m1} R_D - \frac{g_{m1} (1 + R_D/r_{o1})}{g_{m2} + \frac{1}{r_{o1}}} \right] V_{in} = \left[1 + \frac{\frac{1}{r_{o2}} (1 + R_D/r_{o1})}{g_{m2} + \frac{1}{r_{o1}}} \right] V_{out}$$

$$\left[g_{m1} R_D (g_{m2} + \frac{1}{r_{o1}}) - g_{m1} (1 + \frac{R_D}{r_{o1}}) \right] V_{in} = \left[g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}} (1 + \frac{R_D}{r_{o1}}) \right] V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1} (g_{m2} R_D - 1)}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}} (1 + \frac{R_D}{r_{o1}})}$$



$$- \left(\frac{V_{out}}{r_{o2}} + g_{m2} V_x \right) = \frac{V_{out} - V_x}{r_{o1}} + g_{m1} (V_{in} - V_x) = \frac{V_x}{R_S} \quad \textcircled{1} \quad \textcircled{2} \quad \textcircled{3}$$

$$\textcircled{1}, \textcircled{3} \rightarrow V_x = - \frac{V_{out}}{r_{o2} (g_{m2} + \frac{1}{R_S})}$$

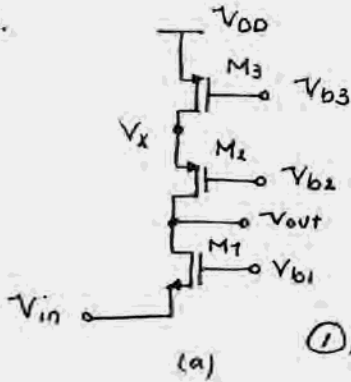
$$\textcircled{2}, \textcircled{3} \rightarrow \frac{V_{out}}{r_{o1}} + g_{m1} V_{in} = - \frac{V_{out}}{r_{o2} (g_{m2} + \frac{1}{R_S})} \left(\frac{1}{R_S} + g_{m1} + \frac{1}{r_{o1}} \right)$$

(e)

$$\frac{V_{out}}{r_{o1}} \cdot r_{o2} (g_{m2} + \frac{1}{R_S}) + g_{m1} V_{in} \cdot r_{o2} (g_{m2} + \frac{1}{R_S}) = -V_{out} \left(\frac{1}{R_S} + g_{m1} + \frac{1}{r_{o1}} \right)$$

$$\frac{V_{out}}{V_{in}} = - \frac{g_{m1} (g_{m2} + 1/R_S) r_{o2}}{g_{m1} + \frac{1}{R_S} + \frac{1}{r_{o1}} \left[1 + r_{o2} (g_{m2} + \frac{1}{R_S}) \right]}$$

3.6.



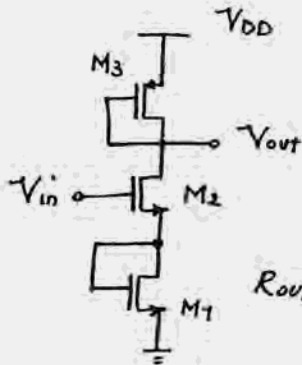
$$\textcircled{1} \quad -\frac{V_x}{r_{o3}} = g_{m2} V_x + \frac{V_x - V_{out}}{r_{o2}} = \frac{V_{out} - V_{in}}{r_{o1}} - g_{m1} V_{in}$$

$$\textcircled{1}, \textcircled{2} \rightarrow V_x = \frac{V_{out}}{1 + r_{o2} (g_{m2} + \frac{1}{r_{o3}})}$$

$$\textcircled{1}, \textcircled{3} \rightarrow \frac{V_{out} - V_{in}}{r_{o1}} - g_{m1} V_{in} = -\frac{V_{out}}{r_{o3} + r_{o2} (1 + g_{m2} \cdot r_{o3})}$$

$$V_{out} \left[\frac{1}{r_{o1}} + \frac{1}{r_{o3} + r_{o2} (1 + g_{m2} \cdot r_{o3})} \right] = (g_{m1} + \frac{1}{r_{o1}}) V_{in}$$

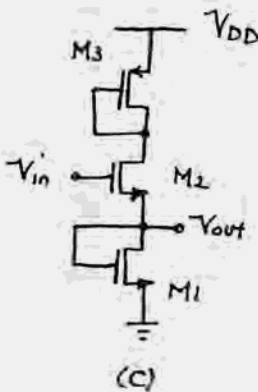
$$\frac{V_{out}}{V_{in}} = \frac{(1 + g_{m1} \cdot r_{o1}) [r_{o3} + r_{o2} (1 + g_{m2} \cdot r_{o3})]}{r_{o1} + r_{o3} + r_{o2} (1 + g_{m2} \cdot r_{o3})}$$



$$G_m = \frac{g_{m2} \cdot r_{o2}}{(\frac{1}{g_{m1}} \parallel r_{o1}) + \left[1 + g_{m2} \left(\frac{1}{g_{m1}} \parallel r_{o1} \right) \right] r_{o2}}$$

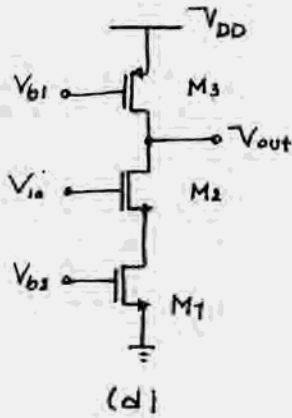
$$R_{out} = \left(\frac{1}{g_{m3}} \parallel r_{o3} \right) \parallel \left\{ \left[1 + g_{m2} \left(\frac{1}{g_{m1}} \parallel r_{o1} \right) \right] r_{o2} + \left(\frac{1}{g_{m1}} \parallel r_{o1} \right) \right\}$$

$$(b) \quad A_v = -G_m \cdot R_{out} = -\frac{g_{m2} r_{o2} \left(\frac{1}{g_{m3}} \parallel r_{o3} \right)}{\left(\frac{1}{g_{m3}} \parallel r_{o3} \right) + \left\{ \left[1 + g_{m2} \left(\frac{1}{g_{m1}} \parallel r_{o1} \right) \right] r_{o2} + \left(\frac{1}{g_{m1}} \parallel r_{o1} \right) \right\}}$$



resistance seen looking up at the source of M2, $\frac{(\frac{1}{g_{m3}} \parallel r_{o3}) + r_{o2}}{1 + g_{m2} \cdot r_{o2}}$

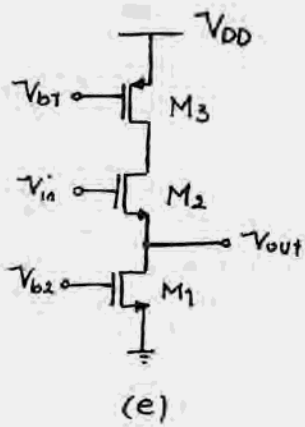
$$\frac{V_{out}}{V_{in}} = \frac{\left(\frac{1}{g_{m1}} \parallel r_{o1} \right)}{\left(\frac{1}{g_{m1}} \parallel r_{o1} \right) + \frac{\left(\frac{1}{g_{m3}} \parallel r_{o3} \right) + r_{o2}}{1 + g_{m2} \cdot r_{o2}}}$$



$$G_m = \frac{g_{m2} \cdot r_{o2}}{r_{o1} + (1 + g_{m2} \cdot r_{o1}) r_{o2}}$$

$$R_{out} = r_{o3} \parallel \left[(1 + g_{m2} \cdot r_{o1}) r_{o2} + r_{o1} \right]$$

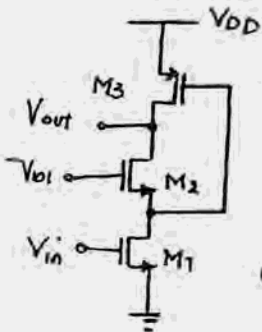
$$\frac{V_{out}}{V_{in}} = \frac{g_{m2} \cdot r_{o2} \cdot r_{o3}}{r_{o3} + (1 + g_{m2} \cdot r_{o1}) r_{o2} + r_{o1}}$$



resistance seen looking up at the source of M2

$$R_{in} = \frac{r_{o3} + r_{o2}}{1 + g_{m2} \cdot r_{o2}}$$

$$\frac{V_{out}}{V_{in}} = \frac{r_{o1}}{r_{o1} + \frac{r_{o3} + r_{o2}}{1 + g_{m2} \cdot r_{o2}}} = \frac{r_{o1} (1 + g_{m2} \cdot r_{o2})}{r_{o1} (1 + g_{m2} \cdot r_{o2}) + r_{o2} + r_{o3}}$$



$$\textcircled{1} \quad - \left(\frac{V_{out}}{r_{o3}} + g_{m3} V_X \right) = \textcircled{2} \quad \left(\frac{V_{out} - V_X}{r_{o2}} - g_{m2} V_X \right) = \textcircled{3} \quad \frac{V_X}{r_{o1}} + g_{m1} V_{in}$$

$$\textcircled{1}, \textcircled{2} \rightarrow \frac{V_X}{r_{o2}} + g_{m2} V_X - g_{m3} V_X = \frac{V_{out}}{r_{o2}} + \frac{V_{out}}{r_{o3}} \rightarrow V_X = \frac{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}}{\frac{1}{r_{o2}} + g_{m2} - g_{m3}} V_{out}$$

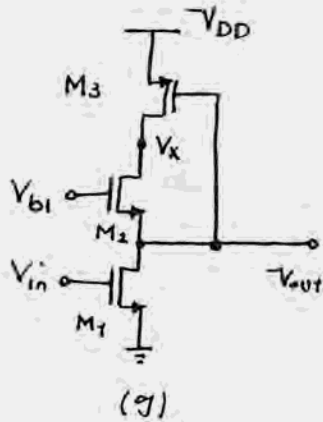
$$\textcircled{1}, \textcircled{3} \rightarrow -\frac{V_{out}}{r_{o3}} - g_{m3} V_X = \frac{V_X}{r_{o1}} + g_{m1} V_{in}$$

$$-\frac{V_{out}}{r_{o3}} - \left(g_{m3} + \frac{1}{r_{o1}} \right) \frac{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}}{\frac{1}{r_{o2}} + g_{m2} - g_{m3}} V_{out} = g_{m1} V_{in}$$

$$-V_{out} \left[\frac{1}{r_{o3}} + \frac{\left(g_{m3} + \frac{1}{r_{o1}} \right) \left(\frac{1}{r_{o2}} + \frac{1}{r_{o3}} \right)}{\frac{1}{r_{o2}} + g_{m2} - g_{m3}} \right] = g_{m1} V_{in}$$

$$-V_{out} \left[\frac{1}{r_{o3}} + \frac{(1+g_{m3}r_{o1})(r_{o3}+r_{o2})}{r_{o1}r_{o3}[1+(g_{m2}-g_{m3})r_{o2}]} \right] = g_{m1} \cdot V_{in}$$

$$\frac{V_{out}}{V_{in}} = - \frac{g_{m1} r_{o1} r_{o3} [1+(g_{m2}-g_{m3})r_{o2}]}{r_{o1} [1+(g_{m2}-g_{m3})r_{o2}] + (1+g_{m3} \cdot r_{o1})(r_{o3}+r_{o2})}$$



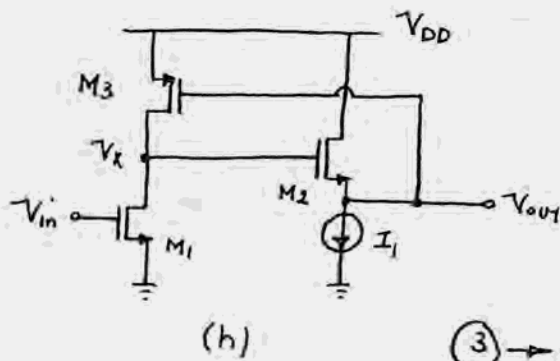
$$V_x = \frac{\frac{1}{r_{o2}} + g_{m2} - g_{m3}}{\frac{1}{r_{o2}} + \frac{1}{r_{o3}}} \cdot V_{out}$$

$$-\frac{V_x}{r_{o3}} - g_{m3} V_{out} = \frac{V_{out}}{r_{o1}} + g_{m1} \cdot V_{in}$$

$$-\frac{\frac{1}{r_{o2}} + g_{m2} - g_{m3}}{\frac{r_{o3}}{r_{o2}} + 1} V_{out} - g_{m3} V_{out} = \frac{V_{out}}{r_{o1}} + g_{m1} V_{in}$$

$$-V_{out} \left[\frac{1+(g_{m2}-g_{m3})r_{o2}}{r_{o3}+r_{o2}} + g_{m3} + \frac{1}{r_{o1}} \right] = g_{m1} V_{in}$$

$$\frac{V_{out}}{V_{in}} = - \frac{g_{m1} r_{o1} (r_{o2}+r_{o3})}{r_{o1} [1+(g_{m2}-g_{m3})r_{o2}] + (r_{o2}+r_{o3})(1+g_{m3} \cdot r_{o1})}$$



$$-\left(\frac{V_x}{r_{o3}} + g_{m3} V_{out} \right) = g_{m1} V_{in} + \frac{V_x}{r_{o1}} \quad \text{①}$$

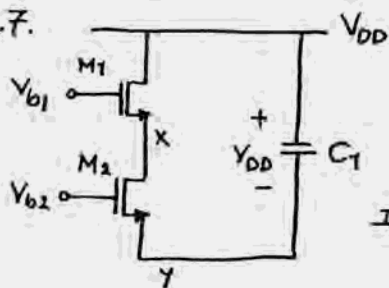
$$-\frac{V_{out}}{r_{o2}} + g_{m2} (V_x - V_{out}) = 0 \quad \text{② @ output node}$$

$$\text{③} \rightarrow V_x = \frac{\frac{1}{r_{o2}} + g_{m2}}{g_{m2}} \cdot V_{out} = \frac{1+g_{m2}r_{o2}}{g_{m2}r_{o2}} V_{out}$$

$$\text{①, ②} \rightarrow - \left[\left(\frac{1}{r_{o3}} + \frac{1}{r_{o1}} \right) \frac{1+g_{m2} \cdot r_{o2}}{g_{m2} \cdot r_{o2}} + g_{m3} \right] V_{out} = g_{m1} \cdot V_{in}$$

$$\frac{V_{out}}{V_{in}} = - \frac{g_{m1} \cdot g_{m2} r_{o1} r_{o2} r_{o3}}{(r_{o1}+r_{o3})(1+g_{m2} \cdot r_{o2}) + g_{m2} g_{m3} r_{o1} r_{o2} r_{o3}}$$

3.7.



$$V_Y(t=0) = -V_{C1} + V_{DD} = -V_{DD} + V_{DD} = 0$$

$$I_{D1} = I_{D2} \rightarrow \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[V_{b1} - V_X(t=0) - V_{TH1} \right]^2 = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2$$

$$(a) \quad V_X(t=0) = V_{b1} - V_{TH1} - \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}} (V_{b2} - V_{TH2})$$

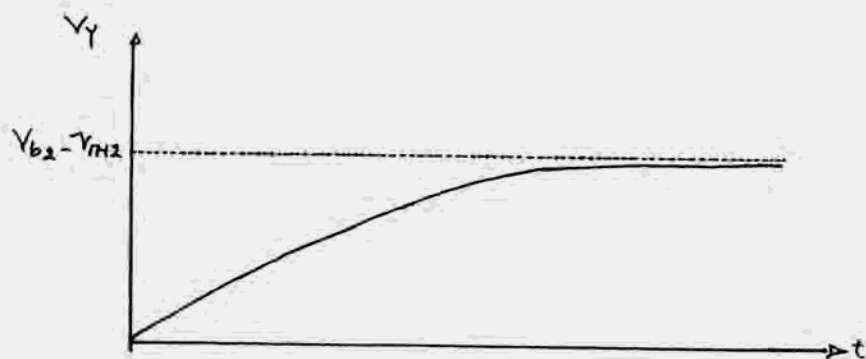
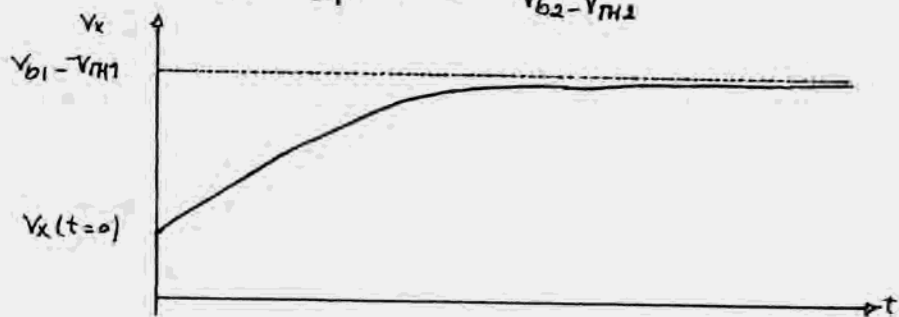
We assume that $V_X(t=0) > V_{b2} - V_{TH2}$, therefore, M_2 is always saturated.

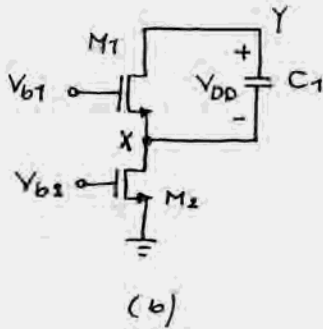
$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{b1} - V_X - V_{TH1})^2 = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{b2} - V_Y - V_{TH2})^2 = C_1 \frac{dV_Y}{dt} \quad (1) \quad (2) \quad (3)$$

$$(2), (3) \rightarrow \frac{dV_Y}{(V_{b2} - V_Y - V_{TH2})^2} = \frac{1}{2} \mu_n \frac{C_{ox}}{C_1} \left(\frac{W}{L}\right)_2 dt$$

$$\frac{1}{V_{b2} - V_Y - V_{TH2}} = \frac{1}{2} \mu_n \frac{C_{ox}}{C_1} \left(\frac{W}{L}\right)_2 t + K, \quad K = \frac{1}{V_{b2} - V_{TH2}} \text{ because } V_Y(t=0) = 0$$

$$V_Y = V_{b2} - V_{TH2} - \frac{1}{\frac{1}{2} \mu_n \frac{C_{ox}}{C_1} \left(\frac{W}{L}\right)_2 t + \frac{1}{V_{b2} - V_{TH2}}}, \quad V_X = V_{b1} - V_{TH1} - (V_{b2} - V_Y - V_{TH2}) \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}} \leftarrow (2), (1)$$





3-14

The drain current of M_2 is zero, therefore, M_2 operates in deep triode region, pulling down V_X to zero potential.

$$V_X = 0 \text{ for } 0 < t < \infty$$

$V_Y(t=0) = V_{DD} \rightarrow M_1$ starts in saturation.

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2 = -C_1 \frac{dV_{C1}}{dt} = -C_1 \frac{dV_Y}{dt}$$

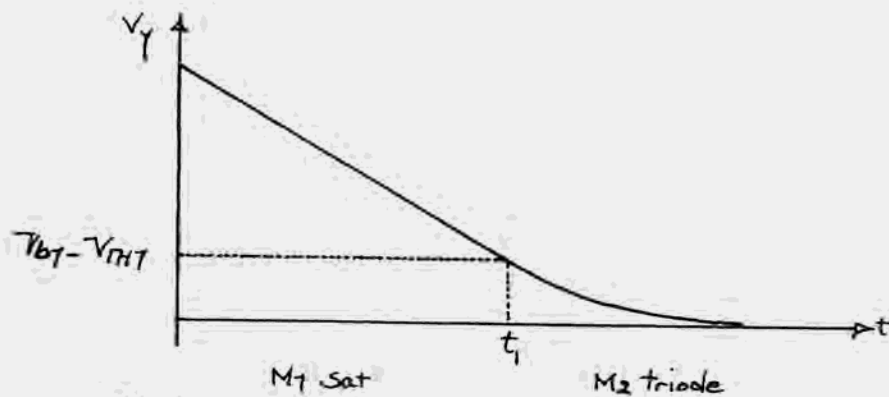
$$\textcircled{1} V_Y = V_{C1} = V_{DD} - \frac{1}{2} \mu_n \frac{C_{ox}}{C_1} \left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2 t$$

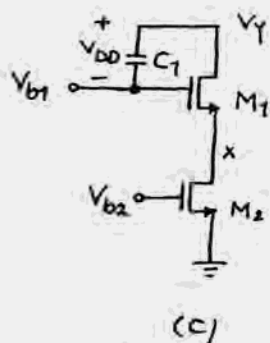
When $V_Y = V_{b1} - V_{TH1}$ $\textcircled{2}$, M_1 enters triode region.

Substituting $\textcircled{2}$ in $\textcircled{1}$, we calculate the time when M_1 is at the edge of triode region.

$$t_1 = \frac{V_{DD} - V_{b1} + V_{TH1}}{\frac{1}{2} \mu_n \frac{C_{ox}}{C_1} \left(\frac{W}{L}\right)_1 (V_{b1} - V_{TH1})^2}$$

$$\text{for } t > t_1: \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[(V_{b1} - V_{TH1}) V_Y - \frac{V_Y^2}{2} \right] = -C_1 \frac{dV_Y}{dt} \rightarrow V_Y = \dots$$





$V_Y(t=0) = V_{DD} + V_{b1}$, both transistors are saturated.

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{b1} - V_x - V_{TH1})^2$$

$$V_x = V_{b1} - V_{TH1} - (V_{b2} - V_{TH2}) \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}}$$

$$C_1 \frac{dV_{C1}}{dt} = -\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 \rightarrow V_{C1} = V_{DD} - \frac{1}{2} \mu_n \frac{C_{ox}}{C_1} \left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 t$$

$$V_Y = V_{C1} + V_{b1} = V_{DD} + V_{b1} - \frac{1}{2} \mu_n \frac{C_{ox}}{C_1} \left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 t$$

@ $t = t_1$, we have $V_Y = V_{b1} - V_{TH1}$, polarity of voltage across C_1 has already changed.

$$V_{DD} + V_{b1} - \frac{1}{2} \mu_n \frac{C_{ox}}{C_1} \left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2 t_1 = V_{b1} - V_{TH1}$$

$$t_1 = \frac{2(V_{DD} + V_{TH1}) C_1}{\mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2}$$

For $t > t_1$, M_1 enters triode region. We assume that still M_2 is saturated.

$$V_Y = V_{DD} + V_{b1} - \frac{1}{C_1} I_{D2} t \quad \text{where } I_{D2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH2})^2$$

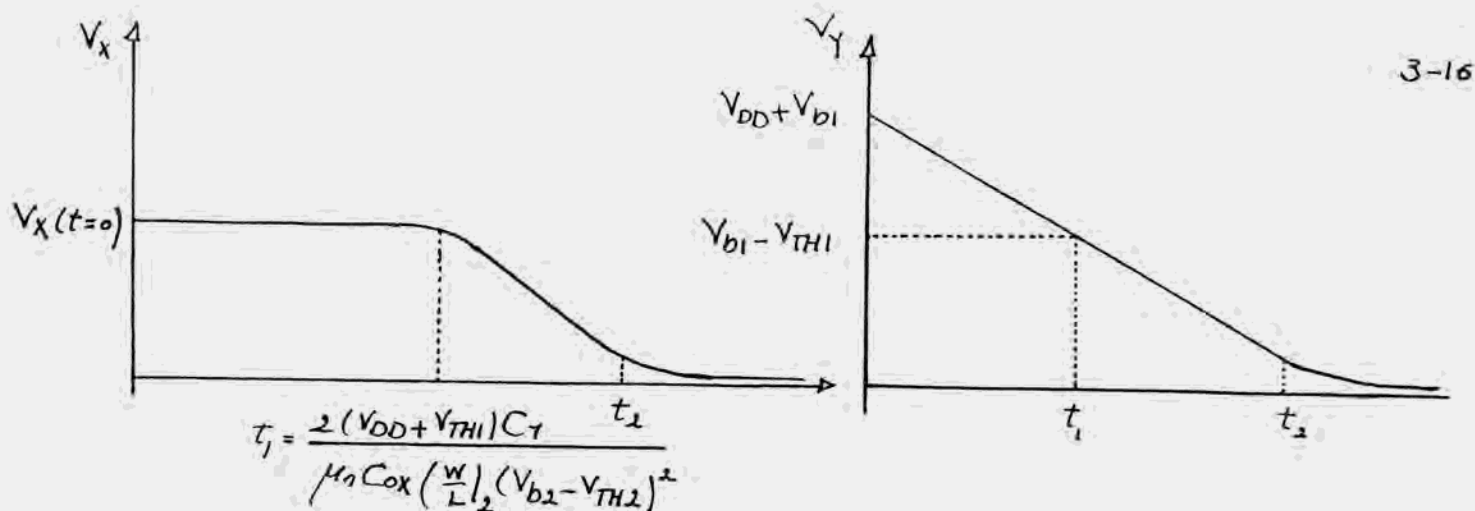
$$\text{and } I_{D2} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[(V_{b1} - V_x) \left(V_{DD} + V_{b1} - \frac{1}{C_1} I_{D2} t - V_x \right) - \frac{\left(V_{DD} + V_{b1} - \frac{1}{C_1} I_{D2} t - V_x \right)^2}{2} \right]$$

$\rightarrow V_x$ is obtained

When $V_x = V_{b2} - V_{TH2}$, M_2 enters the triode region, too.

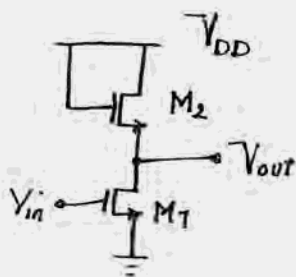
$$\mu_n C_{ox} \left(\frac{W}{L}\right)_2 \left[(V_{b2} - V_{TH2}) V_x - \frac{V_x^2}{2} \right] = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[(V_{b1} - V_x - V_{TH1}) (V_Y - V_x) - \frac{(V_Y - V_x)^2}{2} \right] = -C_1 \frac{dV_Y}{dt}$$

V_x and V_Y are obtained. This regime continues until V_x and V_Y drop to zero, and C_1 charges up to $-V_{b1}$.



for $0 < t < t_1$ M_1 Sat, M_2 Sat, for $t_1 < t < t_2$ M_1 triode, M_2 Sat.
for $t_2 < t$ M_1 triode, M_2 triode

3.8



$$\left(\frac{W}{L}\right)_1 = \frac{50}{0.5}, \quad \left(\frac{W}{L}\right)_2 = \frac{10}{0.5}, \quad I_{D1} = I_{D2} = 0.5 \text{ mA}$$

$$\mu_n C_{ox} = \frac{350 \text{ Cn}^2}{\text{V}\cdot\text{s}} \times \frac{8.85 \times 10^{-14} \times 3.9 \text{ Farad/Cn}}{9 \times 10^{-7} \text{ Cn}} =$$

$$1.34225 \times 10^{-4} \text{ A/V}^2$$

$$\mu_p C_{ox} = \frac{100 \text{ Cn}^2}{\text{V}\cdot\text{s}} \times \frac{8.85 \times 10^{-14} \times 3.9 \text{ Farad/Cn}}{9 \times 10^{-7} \text{ Cn}} =$$

$$3.835 \times 10^{-5} \text{ A/V}^2$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = 20 \text{ k}, \quad I_{D2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TH2})^2 (1 + \lambda_n V_{DS2})$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \times 20 \left[3 - V_o - 0.7 - 0.45 (\sqrt{0.9 + V_o} - \sqrt{0.9}) \right]^2 [1 + 0.1(3 - V_o)]$$

$$2.3 - 0.45 (\sqrt{0.9 + V_o} - \sqrt{0.9}) - \frac{1}{2.6845 (1.3 - 0.1V_o)} = V_o \rightarrow V_o = 1.466 \text{ V}$$

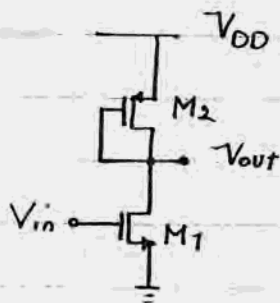
$$g_{m1} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 0.5 \times 10^{-3}} = 3.66 \times 10^{-3} \text{ A/V}$$

$$g_{m2} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 20 \times 0.5 \times 10^{-3}} = 1.63 \times 10^{-3} \text{ A/V}$$

$$g_{mb2} = \frac{\lambda \cdot g_{m2}}{2\sqrt{2\phi_F + V_{SB}}} = \frac{0.45}{2\sqrt{0.9 + 1.466}} \times 1.63 \times 10^{-3} = 2.3843 \times 10^{-4} \text{ A/V}$$

$$R_{out} = \frac{1}{g_{m2} + g_{mb2} + r_{o2}^{-1}} \parallel r_{o1} = \frac{1}{1.63 \times 10^{-3} + 2.3843 \times 10^{-4} + (20 \times 10^3)^{-1}} \parallel 20 \times 10^3 \quad 3.17$$

$$R_{out} = 508 \Omega \quad A_V = -g_{m1} \cdot R_{out} = -3.66 \times 10^{-3} \times 508 = -1.85$$



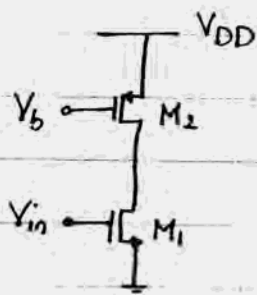
$$g_{m2} = \sqrt{2 \times 3.835 \times 10^{-5} \times 20 \times 0.5 \times 10^{-3}} = 8.7578 \times 10^{-4}$$

$$r_{o2} = \frac{1}{\lambda_p I} = \frac{1}{0.2 \times 0.5 \times 10^{-3}} = 10K$$

$$R_{out} = \frac{1}{g_{m2} + r_{o2}^{-1}} \parallel r_{o1} = 974.8628 \Omega$$

$$A_V = -g_{m1} \cdot R_{out} = -0.8537$$

3.9.



$$\left(\frac{W}{L}\right)_1 = 50/0.5, \quad \left(\frac{W}{L}\right)_2 = 50/2, \quad I_{D1} = I_{D2} = 0.5 \text{ mA}$$

$$r_{o1} = \frac{1}{\lambda_n I} = \frac{1}{0.1 \times 0.5 \times 10^{-3}} = 20K, \quad r_{o2} = \frac{1}{\lambda_p I} = \frac{1}{0.2 \times 0.5 \times 10^{-3}} = 40K$$

$$g_{m1} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 0.5 \times 10^{-3}} = 3.6636 \times 10^{-3}$$

$$A_V = -g_{m1} (r_{o1} \parallel r_{o2}) = -48.84$$

If we assume that M_1 is in the edge of the triode region, then, we have:

$$V_{GS} - V_{TH1} = V_{DS1} = V_{out}, \quad I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2 (1 + \lambda_n V_{DS})$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \times 100 V_{DS}^2 (1 + 0.1 V_{DS}) \rightarrow \sqrt{\frac{1}{13.4225 (1 + 0.1 V_{DS})}} = V_{DS}$$

$$V_{DSmin} = V_{omin} = 0.2693$$

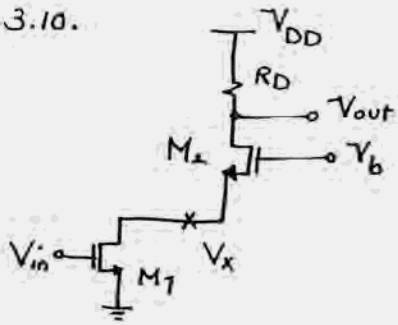
If we assume that M_2 is in the edge of the triode region, then, we have:

$$I_D = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{SG} - |V_{TH2}|)^2 (1 + \lambda_p V_{SD}), \quad 0.5 \times 10^{-3} = \frac{1}{2} \times 3.835 \times 10^{-5} \times 25 V_{SD}^2 (1 + \lambda_p V_{SD})$$

$$\sqrt{\frac{1}{0.95875 (1 + 0.05 V_{SD})}} = V_{SD} \rightarrow V_{SDmin} = 0.9967V, \quad V_{omax} = V_{DD} - V_{SDmin},$$

$$V_{omax} = 2V$$

3.10.



$$\left(\frac{W}{L}\right)_1 = 50/0.5, \quad \left(\frac{W}{L}\right)_2 = 10/0.5 \quad I_{D1} = I_{D2} = 0.5 \text{ mA} \quad 3-18$$

$$R_D = 1 \text{ k}\Omega$$

$$V_{DS, \text{sat}1} = V_{GS1} - V_{TH1} = \left(\frac{2I_{D1}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}\right)^{1/2} = \left(\frac{2 \times 0.5 \times 10^{-3}}{1.34225 \times 10^{-4} \times 100}\right)^{1/2}$$

$$V_{DS, \text{sat}1} = 0.2729 \text{ V}$$

$$V_{X, \text{Bias}} = 0.2729 + 50 \times 10^{-3} = 0.3229 \text{ V}$$

$$V_{TH2} = V_{TH0} + \delta \left(\sqrt{2\beta_F + V_{SB}} - \sqrt{2\beta_F} \right) = 0.7 + 0.15 \left(\sqrt{0.9 + 0.3229} - \sqrt{0.9} \right)$$

$$V_{TH2} = 0.77073 \text{ V}$$

$$V_{GS2} = V_{TH2} + \left(\frac{2I_{D2}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_2}\right)^{1/2} = 0.77073 + \left(\frac{2 \times 0.5 \times 10^{-3}}{1.34225 \times 10^{-4} \times 20}\right)^{1/2} = 1.38107 \text{ V}$$

$$V_b = V_{GS2} + V_X$$

$$V_b = 1.38107 + 0.3229 = 1.7 \text{ V}, \quad g_{m1} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 0.5 \times 10^{-3}} = 3.6636 \times 10^{-3} \text{ A/V}$$

$$g_{m2} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 20 \times 0.5 \times 10^{-3}} = 1.6384 \times 10^{-3} \text{ A/V}$$

$$g_{mb2} = \frac{0.45}{2\sqrt{0.9 + 0.3229}} \times 1.6384 \times 10^{-3} = 3.3336 \times 10^{-4} \text{ A/V}, \quad r_{o1} = r_{o2} = \frac{1}{\lambda_n I} = \frac{1}{0.1 \times 0.5 \times 10^{-3}} = 20 \text{ k}\Omega$$

$$R_{out} = R_D \parallel \left\{ \left[1 + (g_{m2} + g_{mb2}) r_{o2} \right] r_{o1} + r_{o2} \right\} = 10 \text{ k}\Omega \parallel \left\{ \left[1 + (1.6384 \times 10^{-3} + 3.3336 \times 10^{-4}) 20 \times 10^3 \right] 20 \times 10^3 + 20 \times 10^3 \right\}$$

$$R_{out} = 998.7947 \Omega, \quad G_m = \frac{g_{m1} \cdot r_{o1} \left[r_{o2} (g_{m2} + g_{mb2}) + 1 \right]}{r_{o1} \cdot r_{o2} (g_{m2} + g_{mb2}) + r_{o1} + r_{o2}}$$

$$G_m = \frac{3.6636 \times 10^{-3} \times 20 \times 10^3 \left[20 \times 10^3 (1.6384 \times 10^{-3} + 3.3336 \times 10^{-4}) + 1 \right]}{(20 \times 10^3)^2 (1.6384 \times 10^{-3} + 3.3336 \times 10^{-4}) + 2 \times 20 \times 10^3} = 3.5751 \times 10^{-3} \text{ A/V}$$

$$A_v = -G_m R_{out} = -3.57$$

We obtain the small signal voltage gain from input to node x.

$$R_{out@x} = r_{o1} \parallel \frac{R_D + r_{o2}}{1 + (g_{m2} + g_{mb2}) r_{o2}} = 20 \times 10^3 \parallel \frac{10 + 20 \times 10^3}{1 + (1.6384 \times 10^{-3} + 3.3336 \times 10^{-4}) 20 \times 10^3}$$

$$R_{out@x} = 506.2 \Omega$$

$$A_{Vx} = -g_{m1} \cdot R_{out@x} = -1.8545$$

$$\text{If } V_x = V_{x \text{ min}} = V_{DS, \text{sat}1}, \quad \Delta V_x = -50 \text{ mV} \rightarrow \Delta V_{in} = \frac{-50 \times 10^{-3}}{-1.8545} = 26.96 \times 10^{-3}$$

$$\Delta V_{out} = 26.96 \times 10^{-3} \times (-3.57) = -96.25 \times 10^{-3}$$

3-19

$$V_{out, min} = V_{DD} - R_D I_D + \Delta V_o = 3 - 1 \times 0.5 - 96.25 \times 10^{-3} = 2.4V$$

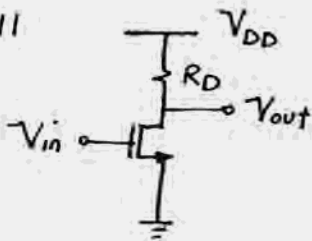
$$V_{out, max} = 3V, \Delta V_o = 3 - 2.5 = 0.5V, \Delta V_{in} = \frac{0.5}{-3.57} = -0.14V$$

$$\Delta V_x = -1.8545 (-0.14) = 0.2597$$

$$V_{x, max} = V_{x, Bias} + 0.2597 = 0.3229 + 0.2597 = 0.5826V$$

If we take $V_{out, min} = V_b - V_{TH2} = 1.7 - 0.77073 = 0.92927V$, $\Delta V_o = -1.57$ which translates into a huge negative swing at x that makes the final voltage at node x negative. Therefore, M_1 limits the negative going output swing because the voltage gain from input to node x is quite large.

3.11



$$\left(\frac{W}{L}\right)_1 = 50/0.5, R_D = 2k\Omega, \lambda = \emptyset$$

$$r_{o1} = \frac{1}{\lambda n I_D} = \frac{1}{0.1 \times 10^{-3}} = 10k$$

$$R_{out} = r_{o1} \parallel R_D = 10k \parallel 2k = \frac{5000}{3} \Omega$$

$$g_{m1} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 10^{-3}} = 5.1812 \times 10^{-3}$$

$$A_v = -g_{m1} R_{out} = -5.1812 \times 10^{-3} \times \frac{5000}{3} = -8.6353$$

At the edge of the triode region: $V_{out} = V_{GS} - V_{TH} = V_{GS} - 0.7$, $I_{D1} = \frac{V_{DD} - V_{out}}{R_D} =$

$$\frac{3 - V_{GS} + 0.7}{2 \times 10^3} = \frac{3.7 - V_{GS}}{2 \times 10^3}, I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2$$

$$\frac{3.7 - V_{GS}}{2 \times 10^3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \times 100 (V_{GS} - 0.7)^2$$

$$13.4225 V_{GS}^2 - 17.7915 V_{GS} - 10.277025 = 0 \rightarrow V_{GS} = 1.137V$$

$$I_D @ \text{the edge of the triode} = \frac{1}{2} \times 1.34225 \times 10^{-4} \times 100 (1.137 - 0.7)^2 = 1.28157 \times 10^{-3}$$

$$g_{m1} @ \text{the edge of the triode} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 1.28157 \times 10^{-3}} = 5.8653 \times 10^{-3}$$

$$r_{o1} = \frac{1}{0.1 \times 1.28157 \times 10^{-3}} = 7.8 \times 10^3$$

$$A_v @ \text{the edge of the triode} = -g_{m1} (r_{o1} \parallel R_D) = -5.8653 \times 10^{-3} (7.8 \times 10^3 \parallel 2 \times 10^3)$$

$$A_v = -9.3374$$

$$V_o @ \text{the edge of the triode} = V_{DD} - R_D \times I_D = 3 - 2 \times 1.2815 \times 10^{-3} = 0.4369 \text{ V} \quad 3-20$$

$$V_{DS} = V_{DS, \text{sat1}} - 50 \times 10^{-3} = 0.4369 - 50 \times 10^{-3} = 0.3869 \text{ V}$$

$$I_D = \frac{V_{DD} - V_{DS}}{R_D} = \frac{3 - 0.3869}{2 \times 10^3} = 1.3065 \times 10^{-3}$$

$$I_D = \mu_n C_{ox} \left(\frac{W}{L} \right)_1 \left[(V_{GS} - V_{TH1}) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

$$1.3065 \times 10^{-3} = 1.34225 \times 10^{-4} \times 100 \left[(V_{GS} - 0.7) \cdot 0.3869 - \frac{(0.3869)^2}{2} \right] \Rightarrow V_{GS} = 1.145$$

$$g_{m1} = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \left(\frac{W}{L} \right)_1 \cdot V_{DS}$$

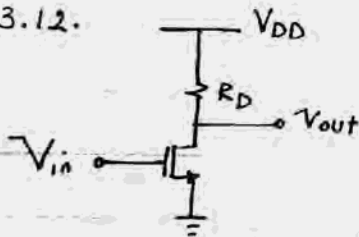
$$g_{m1} @ \text{the point where } 50 \text{ mV into the triode} = 1.34225 \times 10^{-4} \times 100 \times 0.3869 = 5.1942 \times 10^{-3}$$

$$R_o^{-1} = \frac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox} \left(\frac{W}{L} \right)_1 (V_{GS} - V_{TH1} - V_{DS}) \rightarrow R_o = \frac{1}{1.34225 \times 10^{-4} \times 100 (1.145 - 0.7 - 0.3869)}$$

$$R_o = 1.2835 \times 10^3 \Omega$$

$$A_V @ 50 \text{ mV into the triode region} = -5.1942 \times 10^{-3} (1.2835 \times 10^3 || 2 \times 10^3) = -4$$

3.12.



$$\left(\frac{W}{L} \right)_1 = 50/0.5 \quad R_D = 2 \text{ k}, \quad \lambda = 0$$

$$I_{D1} @ V_{out} = 7 \text{ V} = \frac{V_{DD} - V_o}{R_D} = \frac{3 - 1}{2 \times 10^3} = 10^{-3} \text{ A}$$

$$V_{in} = V_{TH1} + \left(\frac{2 I_{D1}}{\mu_n C_{ox} \left(\frac{W}{L} \right)_1} \right)^{1/2} = 0.7 + \left(\frac{2 \times 10^{-3}}{1.34225 \times 10^{-4} \times 100} \right)^{1/2} \rightarrow V_{in} @ V_{out} = 7 \text{ V} = 1.086 \text{ V}$$

$$I_{D1} @ V_{out} = 2.5 \text{ V} = \frac{3 - 2.5}{2 \times 10^3} = 2.5 \times 10^{-4} \text{ A} \quad V_{in} @ V_{out} = 2.5 \text{ V} = 0.7 + \left(\frac{2 \times 2.5 \times 10^{-4}}{1.34225 \times 10^{-4} \times 100} \right)^{1/2} = 0.893 \text{ V}$$

$$g_{m1} @ V_{out} = 7 \text{ V} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 10^{-3}} = 5.1812 \times 10^{-3}$$

$$g_{m1} @ V_{out} = 2.5 \text{ V} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 2.5 \times 10^{-4}} = 2.59 \times 10^{-3}$$

$$r_{o1} @ V_{out} = 7V = \frac{1}{0.1 \times 10^{-3}} = 10K, R_{out} = r_{o1} \parallel R_D = 10000 \parallel 2000 = \frac{5000}{3} \Omega$$

3-21

$$A_v @ V_{out} = 7V = -g_m \cdot R_{out} = -5.1812 \times 10^{-3} \times \frac{5000}{3} = -8.6353$$

$$r_{o1} @ V_{out} = 2.5V = \frac{1}{0.1 \times 2.5 \times 10^{-4}} = 40K, R_{out} = r_{o1} \parallel R_D = 40000 \parallel 2000 = 1.7 \times 10^3$$

$$A_v @ V_{out} = 2.5V = -g_m \cdot R_{out} = -2.59 \times 10^{-3} \times 1.7 \times 10^3 = -4.9221$$

$$3.13. \quad \left(\frac{W}{L}\right) = 50/0.5 \quad |I_D| = 0.5 \text{ mA}$$

$$\uparrow 100/1$$

$$\text{For NMOS device with } \left(\frac{W}{L}\right) = 50/0.5, r_o = \frac{1}{\lambda n I_D} = \frac{1}{0.1 \times 0.5 \times 10^{-3}} = 20K$$

$$g_m = \sqrt{2 \times 1.34225 \times 10^{-4} \times 100 \times 0.5 \times 10^{-3}} = 3.6636 \times 10^{-3}$$

$$g_m r_o = 73.27$$

$$\text{For PMOS device with } \left(\frac{W}{L}\right) = 50/0.5, r_o = \frac{1}{\lambda p I_D} = \frac{1}{0.2 \times 0.5 \times 10^{-3}} = 10K$$

$$g_m = \sqrt{2 \times 3.835 \times 10^{-5} \times 100 \times 0.5 \times 10^{-3}} = 1.9583 \times 10^{-3}$$

$$g_m r_o = 19.5831$$

$$\text{For NMOS device with } \left(\frac{W}{L}\right) = 100/1, r_o = \frac{1}{\lambda n I_D} = \frac{1}{0.1 \times 0.5 \times 10^{-3}} = 40K$$

$$g_m = 3.6636 \times 10^{-3}, g_m r_o = 146.5469$$

$$\text{For PMOS device with } \left(\frac{W}{L}\right) = 100/1, r_o = \frac{1}{\lambda p I_D} = \frac{1}{0.2 \times 0.5 \times 10^{-3}} = 20K$$

$$g_m = 1.9583 \times 10^{-3}, g_m r_o = 39.1663$$

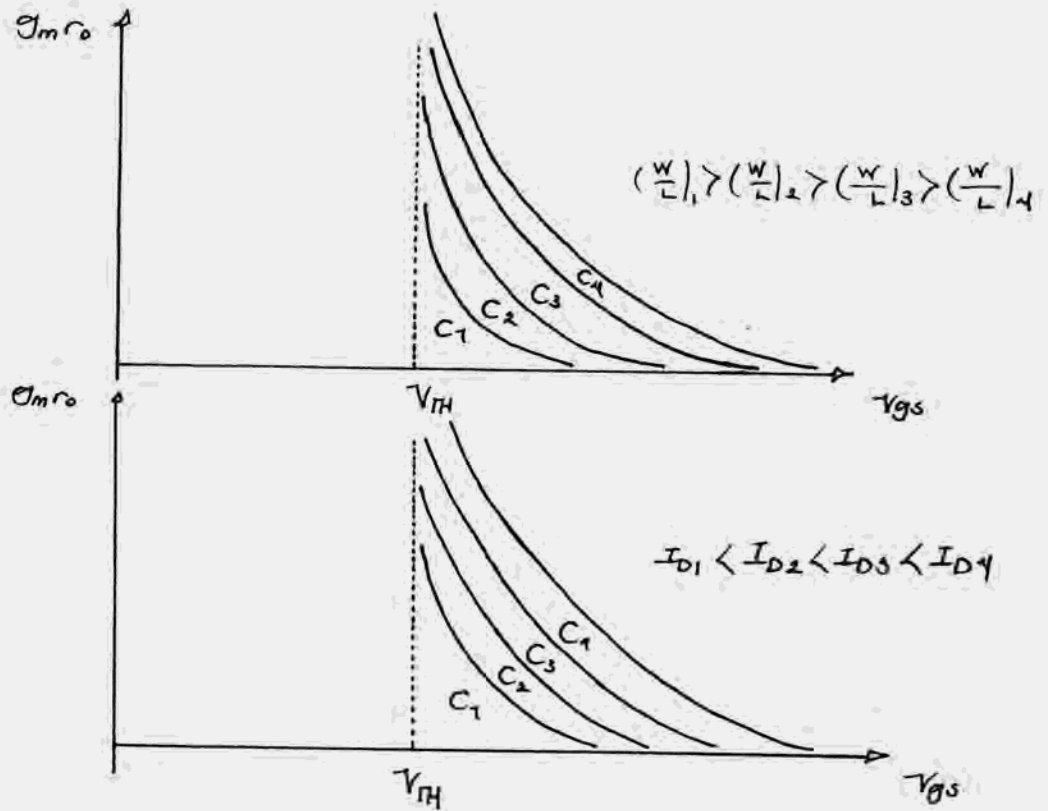
$$3.14. \quad I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS}) \quad (1)$$

$$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH}) (1 + \lambda V_{DS}) \quad (2)$$

Substituting $(1 + \lambda V_{DS})$ from (1) in (2), we have.

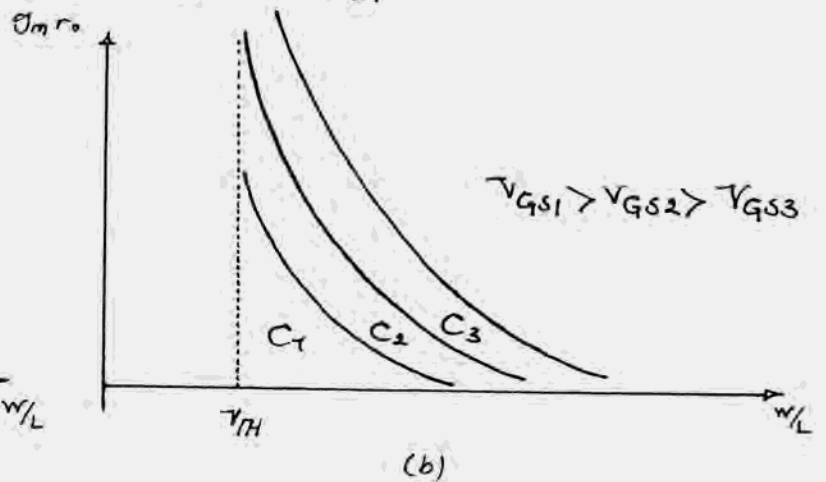
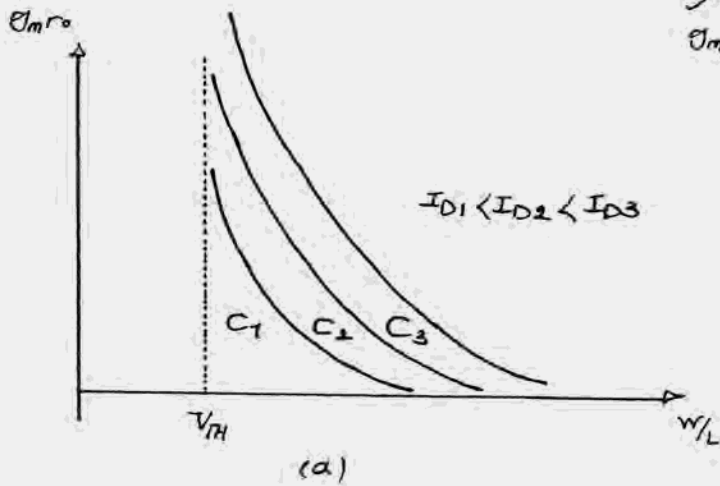
$$g_m = \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH}) \frac{I_D}{\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2} = \frac{2I_D}{V_{GS} - V_{TH}}$$

$$g_{mro} = \frac{2I_D}{V_{GS} - V_{TH}} \frac{1 + \lambda V_{DS}}{\lambda I_D} = \frac{2(1 + \lambda V_{DS})}{\lambda (V_{GS} - V_{TH})} = \frac{4I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^3 \lambda \left(\frac{W}{L}\right)}$$



3.15. From 3.14. we have:

$$g_{mro} = \frac{4I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^3 \lambda \left(\frac{W}{L}\right)}$$



$$3.16. \quad \frac{W}{L} = 50/0.5 \quad V_G = +1.2V \quad V_S = 0 \quad 0 < V_D < 3 \quad V_{bulk} = 0$$

$$V_{Dsat} = V_{GS} - V_{TH} = 1.2 - 0.7 = 0.5V, \quad \text{for a saturated device } g_{mro} = \frac{2(1 + \lambda V_{DS})}{\lambda(V_{GS} - V_{TH})}$$

$$\text{@ the edge of the triode region } g_{mro} = \frac{2(1 + 0.5 \times 0.1)}{0.1(1.2 - 0.7)} = 42 \quad \lambda(V_{GS} - V_{TH})$$

We cannot neglect the channel-length modulation in the triode region, because it would lead to a discontinuity at the transition point between the saturation and the triode region.

@ triode region

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \left(\frac{W}{L}\right) V_{DS} (1 + \lambda V_{DS})$$

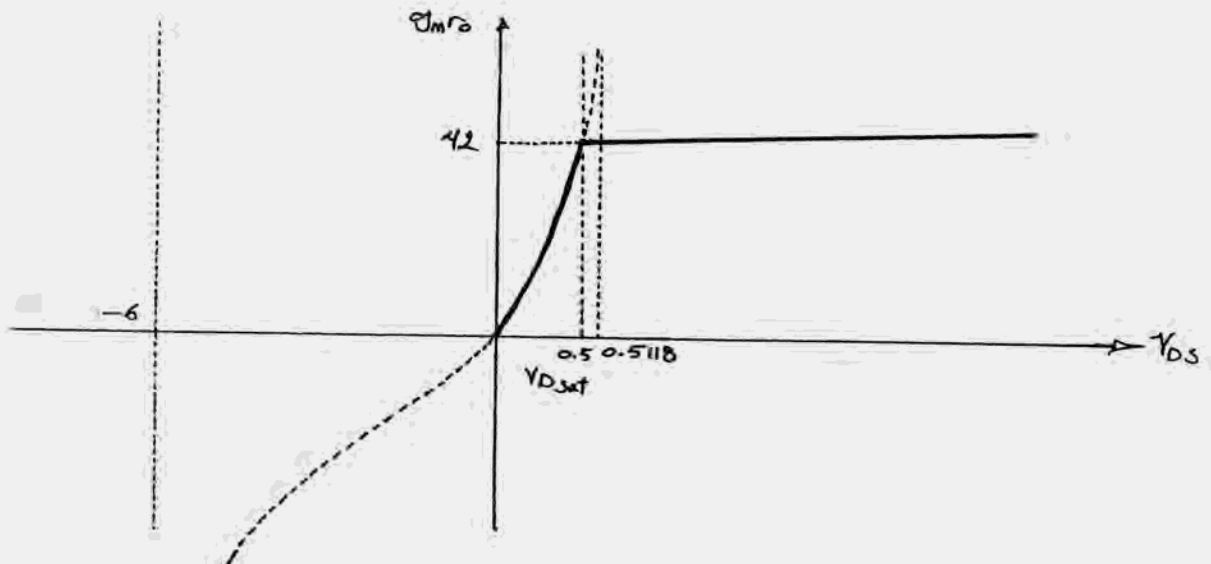
$$g_o = \frac{\partial I_D}{\partial V_{DS}} = \mu_n C_{ox} \left(\frac{W}{L}\right) \left\{ (V_{GS} - V_{TH} - V_{DS})(1 + \lambda V_{DS}) + \lambda \left[(V_{GS} - V_{TH}) V_{DS} - \frac{V_{DS}^2}{2} \right] \right\}$$

$$\text{in the triode region } g_{mro} = \frac{(1 + \lambda V_{DS}) V_{DS}}{(V_{GS} - V_{TH} - V_{DS})(1 + \lambda V_{DS}) + \lambda \left[(V_{GS} - V_{TH}) V_{DS} - \frac{V_{DS}^2}{2} \right]}$$

$$\text{In Saturation } g_{mro} = \frac{2(1 + 0.1 V_{DS})}{0.1(1.2 - 0.7)} = 40 + 4 V_{DS} \quad V_{DS} > 0.5V$$

$$\text{In triode } g_{mro} = \frac{(1 + 0.1 V_{DS}) V_{DS}}{(0.5 - V_{DS})(1 + 0.1 V_{DS}) + 0.1 \times 0.5 V_{DS} (1 - V_{DS})}$$

$$g_{mro} = \frac{0.1 V_{DS}^2 + V_{DS}}{-0.15 V_{DS}^2 - 0.9 V_{DS} + 0.5}$$



$$V_{\text{bulk}} = -1V, V_{\text{SB}} = +1V$$

$$V_{\text{TH}} = V_{\text{TH0}} + \gamma \left(\sqrt{2|\phi_F| + V_{\text{SB}}} - \sqrt{2|\phi_F|} \right) = 0.7 + 0.45 \left(\sqrt{0.9+1} - \sqrt{0.9} \right) = 0.8933V$$

$$\text{In Saturation } g_m r_o = \frac{2(1+0.1V_{\text{DS}})}{0.1(1.2-0.8933)} = 65.2262 + 6.5226 V_{\text{DS}}$$

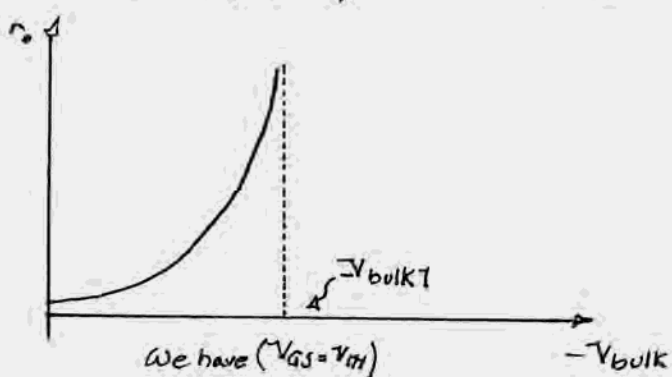
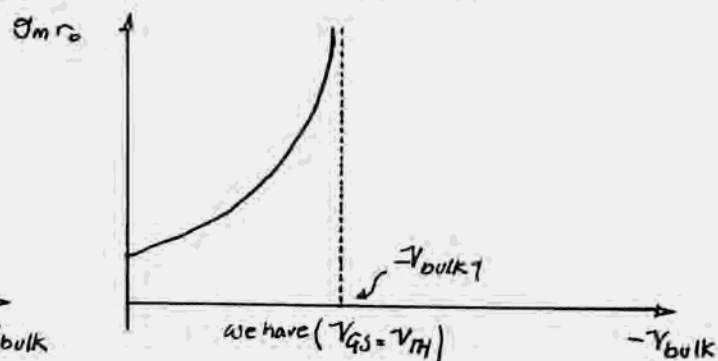
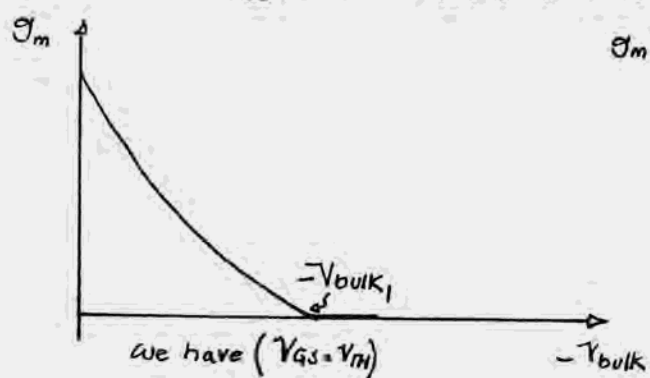
$$V_{\text{DSsat}} = V_{\text{GS}} - V_{\text{TH}} = 1.2 - 0.8933 = 0.3066V, \text{ @ the edge of the triode } g_m r_o = 67.2262$$

$$g_m r_o = \frac{(1+0.1V_{\text{DS}})V_{\text{DS}}}{(1.2-0.8933-V_{\text{DS}})(1+0.1V_{\text{DS}}) + 0.1 \left[(1.2-0.8933)V_{\text{DS}} - 0.5V_{\text{DS}}^2 \right]}$$

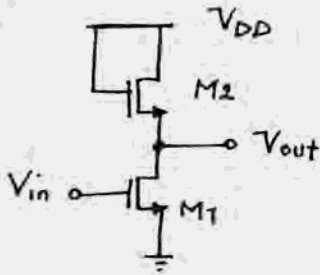
$$g_m r_o = \frac{(1+0.1V_{\text{DS}})V_{\text{DS}}}{-0.15V_{\text{DS}}^2 - 0.9386V_{\text{DS}} + 0.3066}$$

$$3.17. \quad g_m = \mu_n C_{\text{ox}} \left(\frac{W}{L} \right) \left[V_{\text{GS}} - V_{\text{TH0}} - \gamma \left(\sqrt{2|\phi_F| + V_{\text{SB}}} - \sqrt{2|\phi_F|} \right) \right] (1 + \lambda V_{\text{DS}})$$

$$r_o = \frac{1}{\frac{1}{2} \mu_n C_{\text{ox}} \left(\frac{W}{L} \right) (V_{\text{GS}} - V_{\text{TH}})^2 \lambda}, \quad g_m r_o = \frac{2(1 + \lambda V_{\text{DS}})}{\lambda (V_{\text{GS}} - V_{\text{TH}})}$$



3.18.



$$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad \left(\frac{W}{L}\right)_2 = 10/0.5, \quad \lambda = \delta = 0$$

M_1 at the edge of the triode region $\rightarrow V_{out} = V_{in} - V_{TH1}$

$$I_{D1} = I_{D2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} + V_{TH1} - V_{TH2})^2$$

$$\left(\frac{W}{L}\right)_1^{1/2} (V_{in} - V_{TH1}) = \left(\frac{W}{L}\right)_2^{1/2} (V_{DD} - V_{in}) \rightarrow (V_{in} - V_{TH1}) = \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}} (V_{DD} - V_{in})$$

$$V_{in} = \left(\sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}} V_{DD} + V_{TH1} \right) / \left(1 + \sqrt{\frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1}} \right) = \left[\left(\frac{10}{50} \right)^{1/2} \times 3 + 0.7 \right] / \left[1 + \left(\frac{10}{50} \right)^{1/2} \right] = 1.41 \text{ V}$$

$$A_V = - \sqrt{\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2}} = - \sqrt{\frac{50}{10}} = -2.236$$

at the edge of the triode region $V_{out} = 1.41 - 0.7 = 0.71 \text{ V}$

50 mV into the triode region $V_{out} = 0.71 - 50 \times 10^{-3} = 0.66 \text{ V}$

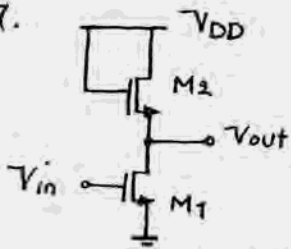
$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})^2 = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[(V_{in} - V_{TH1}) V_{out} - \frac{V_{out}^2}{2} \right]$$

$$V_{in} = \frac{\left(\frac{W}{L}\right)_2}{\left(\frac{W}{L}\right)_1} \frac{(V_{DD} - V_{out} - V_{TH2})^2}{V_{out}} + \frac{V_{out}}{2} + V_{TH1} = \frac{10}{50} \frac{(3 - 0.66 - 0.7)^2}{0.66} + \frac{0.66}{2} + 0.7$$

$$V_{in} = 1.8437, \quad I_D = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[(V_{in} - V_{TH1}) V_{out} - \frac{V_{out}^2}{2} \right], \quad \frac{\partial I_D}{\partial V_{in}} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \cdot V_{out}$$

$$A_V = - \frac{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \cdot V_{out}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})} = - \frac{\frac{50}{0.5} \times 0.66}{\frac{10}{0.5} \times (3 - 0.66 - 0.7)} = -2.015$$

3.19.



$$\left(\frac{W}{L}\right)_1 = 50/0.5 \quad \left(\frac{W}{L}\right)_2 = 10/0.5 \quad \lambda = 0$$

$$V_{out} = V_{in} - V_{TH1} \quad V_{TH2} = V_{TH2,0} + \gamma \left(\sqrt{2|f_F| + V_{SB}} - \sqrt{2|f_F|} \right)$$

$$I_{D1} = I_{D2} \Rightarrow \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_{in} + V_{TH1} - V_{TH2,0} - 0.45(\sqrt{0.9 + V_{out}} - \sqrt{0.9}))^2$$

$$\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \left(\frac{W}{L}\right)_2 \left[V_{DD} - V_{in} - 0.45(\sqrt{0.9 + V_{in} - 0.7} - \sqrt{0.9}) \right]^2$$

$$V_{in} = \sqrt{\frac{1}{5}} \left[3 - V_{in} - 0.45(\sqrt{0.2 + V_{in}} - \sqrt{0.9}) \right] + 0.7 \rightarrow \text{After enough iterations} \rightarrow$$

$$V_{in} = 1.3685, \quad V_{out} = 0.6685, \quad \eta = \frac{\gamma}{2 \sqrt{2(2|f_F| + V_{SB})}^{1/2}} = \frac{0.45}{2(0.9 + 0.6685)^{1/2}} = 0.1796$$

$$A_V = -\frac{g_{m1}}{g_{m2}(1 + \eta_2)} = -\frac{\sqrt{\left(\frac{W}{L}\right)_1} \cdot 1}{\sqrt{\left(\frac{W}{L}\right)_2} (1 + \eta_2)} = -\frac{\sqrt{50}}{\sqrt{10} (1 + 0.1796)} = -1.8955$$

$$V_{out} = 0.6685 - 50 \times 10^{-3} = 0.6185$$

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})^2 = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 \left[(V_{in} - V_{TH1}) V_{out} - \frac{V_{out}^2}{2} \right]$$

$$V_{TH2} = V_{TH2,0} + \gamma \left(\sqrt{2|f_F| + V_{SB}} - \sqrt{2|f_F|} \right) = 0.7 + 0.45(\sqrt{0.9 + 0.6185} - \sqrt{0.9}) = 0.8276$$

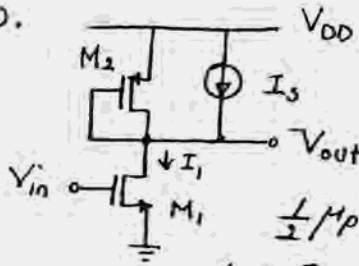
$$24.1453 = \frac{50}{0.5} \left[(V_{in} - 0.7) 0.6185 - \frac{0.6185^2}{2} \right] \rightarrow V_{in} = 1.3996$$

$$\eta = \frac{0.45}{2(0.9 + 0.6185)^{1/2}} = 0.1825$$

$$A_V = -\frac{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 \cdot V_{out}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TH2})(1 + \eta)} =$$

$$\frac{\left(\frac{W}{L}\right)_1 V_{out}}{\left(\frac{W}{L}\right)_2 (V_{DD} - V_{out} - V_{TH2})(1 + \eta_2)} = \frac{50 \times 0.6185}{10(3 - 0.6185 - 0.8276)(1 + 0.1825)} = -1.6829$$

3.20.



$(\frac{W}{L})_1 = 20/0.5, I_1 = 1\text{mA}, I_S = 0.75\text{mA}, \lambda = 0$ 3.27
 M_1 at the edge of the triode region $V_{out} = V_{in} - V_{TH1}$

$$\frac{1}{2} \mu_p C_{ox} (\frac{W}{L})_2 (V_{DD} - V_{out} - |V_{TH2}|)^2 + I_S = \frac{1}{2} \mu_n C_{ox} (V_{in} - V_{TH1})^2 = 10^{-3}$$

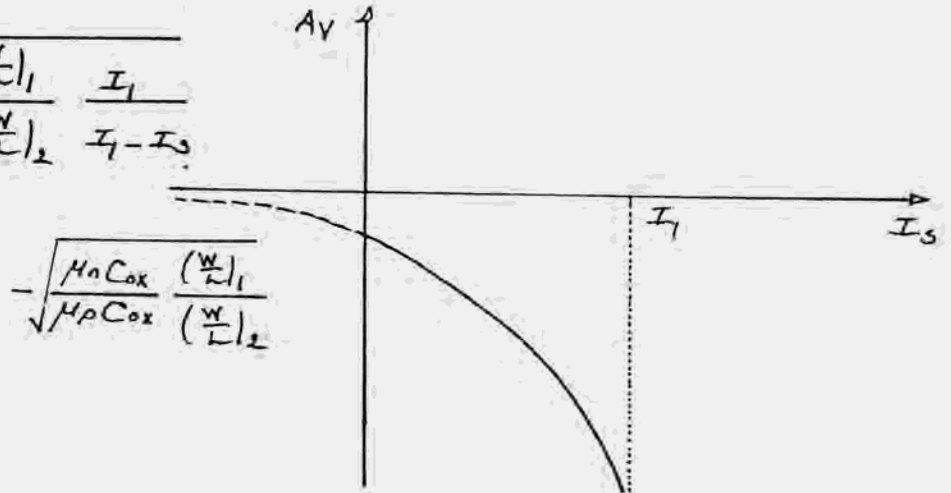
$$\frac{1}{2} \mu_p C_{ox} (\frac{W}{L})_2 (V_{DD} - V_{in} + V_{TH1} - |V_{TH2}|)^2 + I_S = \frac{1}{2} \mu_n C_{ox} (V_{in} - V_{TH1})^2 = 10^{-3}$$

$$(V_{in} - V_{TH1})^2 = \frac{2I_1}{\mu_n C_{ox} (\frac{W}{L})_1} \rightarrow V_{in} = \sqrt{\frac{2I_1}{\mu_n C_{ox} (\frac{W}{L})_1}} + V_{TH1} = 0.7 + \sqrt{\frac{2 \times 10^{-3}}{1.34225 \times 10^{-4} \times 20/0.5}} = 1.31$$

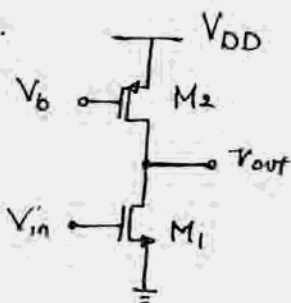
$$\frac{1}{2} \times 3.835 \times 10^{-5} (\frac{W}{L})_2 (3 - 1.31 + 0.7 - 0.8)^2 + 0.75 \times 10^{-3} = 10^{-3}, (\frac{W}{L})_2 = 5.159$$

$$A_V = - \frac{g_{m1}}{g_{m2}} = - \sqrt{\frac{\mu_n C_{ox} (\frac{W}{L})_1 I_1}{\mu_p C_{ox} (\frac{W}{L})_2 I_2}} = - \sqrt{\frac{1.34225 \times 10^{-4} \times 20/0.5 \times 10^{-3}}{3.835 \times 10^{-5} \times 5.159 \times 2.5 \times 10^{-4}}} = -10.418$$

3.21. $A_V = - \sqrt{\frac{\mu_n C_{ox} (\frac{W}{L})_1 I_1}{\mu_p C_{ox} (\frac{W}{L})_2 I_1 - I_S}}$



3.22.



output voltage swing = 2.2

$$I_{D1} = I_{D2} = 1\text{mA}$$

$$A_V = 100$$

$$V_{out, min} = \left(\frac{2I_{D1}}{\mu_n C_{ox} (\frac{W}{L})_1} \right)^{1/2}, V_{out, max} = V_{DD} - \left(\frac{2I_{D2}}{\mu_p C_{ox} (\frac{W}{L})_2} \right)^{1/2}$$

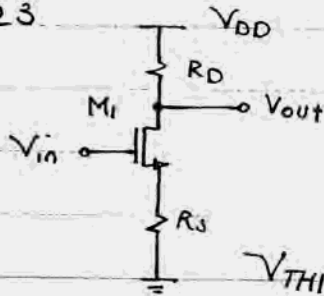
$$V_{DD} - \left(\frac{2I_D}{\mu_p C_{ox} \left(\frac{W}{L} \right)_2} \right)^{1/2} - \left(\frac{2I_D}{\mu_n C_{ox} \left(\frac{W}{L} \right)_1} \right)^{1/2} = 2.2, \quad r_{o1} = \frac{1}{\lambda_1 I_D} = \frac{1}{0.1 \times 10^{-3}} = 10K$$

$$r_{o2} = \frac{1}{\lambda_2 I_D} = \frac{1}{0.2 \times 10^{-3}} = 5K, \quad r_{o1} \parallel r_{o2} = \frac{10^4}{3}, \quad \beta_{M1} (r_{o1} \parallel r_{o2}) = 100 \rightarrow \beta_{M1} = \frac{100 \times 3}{10^4} = 0.03$$

$$2\mu_n C_{ox} \left(\frac{W}{L} \right)_1 \times 10^{-3} = 9 \times 10^{-4} \rightarrow \left(\frac{W}{L} \right)_1 = \frac{9 \times 10^{-4}}{2 \times 1.34225 \times 10^{-4} \times 10^{-3}} = 3352.5796$$

$$3 - \left(\frac{2 \times 10^{-3}}{1.34225 \times 10^{-4} \times 3352.5796} \right)^{1/2} - 2.2 = \left(\frac{2 \times 10^{-3}}{3.835 \times 10^{-5} \left(\frac{W}{L} \right)_2} \right)^{1/2} \rightarrow \left(\frac{W}{L} \right)_2 = 96.97$$

3.23



$$\left(\frac{W}{L} \right)_1 = 50/0.5 \quad R_D = 2K \quad R_S = 200\Omega$$

$$r_{o1} = \frac{1}{\lambda I_D} = \frac{1}{0.1 \times 0.5 \times 10^{-3}} = 20K, \quad V_S = R_S I_D = 200 \times 0.5 \times 10^{-3} = 0.1$$

$$V_{TH1} = V_{TH1,0} + \delta \left(\sqrt{2I_D f_1 + V_{SB}} - \sqrt{2I_D f_1} \right) = 0.7 + 0.45 \left(\sqrt{0.9 + 0.1} - \sqrt{0.9} \right)$$

$$V_{TH1} = 0.723, \quad V_{out} = V_{DD} - R_D I_D = 3 - 2 \times 10^3 \times 0.5 \times 10^{-3} = 2$$

$$V_{DS} = 2 - 0.1 = 1.9$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right)_1 (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$g_m = \mu_n C_{ox} \left(\frac{W}{L} \right)_1 (V_{GS} - V_{TH}) (1 + \lambda V_{DS}) = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L} \right)_1 (1 + \lambda V_{DS}) I_D}$$

$$g_{m1} = \sqrt{2 \times 1.34225 \times 10^{-4} \times \frac{50}{0.5} (1 + 0.1 \times 0.9 / 0.5 \times 10^{-3})} = 3.8249 \times 10^{-3}$$

$$\eta = \frac{0.45}{2(0.1 + 0.9)^{1/2}} = 0.225 \quad G_m = \frac{g_{m1} r_{o1}}{R_S + [1 + (1 + \eta) g_{m1} R_S] r_{o1}}$$

$$G_m = \frac{3.8249 \times 10^{-3} \times 20 \times 10^3}{200 + [1 + (1 + 0.225) 3.8249 \times 10^{-3} \times 200] 20 \times 10^3} = 1.9644 \times 10^{-3}$$

$$R_{out} = [1 + (g_m + g_{m0}) r_o] R_S + r_o$$

Seen looking down at the drain of M1

$$R_{out} = \left[1 + (1 + 0.225) 3.8249 \times 10^{-3} \right] 200 + 20 \times 10^3 = 20.2 \times 10^3 \quad 3-29$$

$$R_{out, total} = R_{out} \parallel R_D = 1819.8274, \quad A_V = -G_m \cdot R_{out, total} = -1.96 \times 10^{-3} \times 1819.8$$

$$A_V = -3.57$$

$V_{out} = V_{in} - V_{TH1}$ @ the edge of the triode region

$$V_{in} = V_{GS1} + R_S I_D$$

$$V_{DD} - R_D I_D = V_{out}, \quad V_{DD} - R_D I_D = V_{GS1} + R_S I_D - V_{TH1}, \quad V_{DD} - (R_S + R_D) I_D = V_{GS1} - V_{TH1}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right)_1 (V_{GS1} - V_{TH1})^2 = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L} \right)_1 \left[V_{DD} - (R_S + R_D) I_D \right]^2 =$$

$$\frac{1}{2} \times 1.34225 \times 10^{-4} \times \frac{50}{0.5} \left[3 - (2000 + 200) I_D \right]^2$$

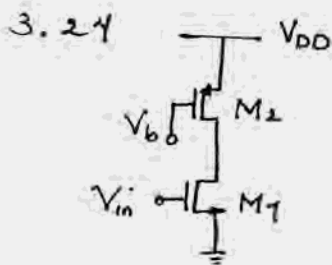
$$I_D = 6.71125 \times 10^{-3} (3 - 2200 I_D)^2 \rightarrow 32482.45 I_D^2 - 89.5885 I_D + 60.40125 \times 10^{-3} = 0$$

$$I_{D1} = 1.5844 \times 10^{-3} \text{ (not acceptable)}, \quad I_{D2} = 1.17355 \times 10^{-3} \text{ (acceptable!)}$$

$$V_{in} = V_{DD} - R_D I_D + V_{TH1} = 3 - 2000 \times 1.17355 \times 10^{-3} + 0.7 = 1.35280 \text{ V}$$

$$g_{m1} = \sqrt{2 \times 1.34225 \times 10^{-4} \times \frac{50}{0.5} \times 1.17355 \times 10^{-3}} = 5.6128 \times 10^{-3}$$

$$G_m = \frac{g_{m1}}{1 + g_{m1} R_S} = 2.6443 \times 10^{-3} \quad A_V = -G_m R_D = -2.6443 \times 10^{-3} \times 2000 = -5.2887$$



$$A_V = -5, \quad \left(\frac{W}{L} \right)_1 = 20/0.5, \quad I_{D1} = 0.5 \text{ mA}, \quad V_b = 0$$

$$g_{m1} = 2 \mu_n C_{ox} \left(\frac{W}{L} \right)_1 (1 + \lambda V_{DS}) I_D$$

$$g_{m1} = \sqrt{2 \times 1.34225 \times 10^{-4} \times \frac{20}{0.5} (1 + 0.1 V_{DS}) \times 0.5 \times 10^{-3}}$$

$$r_{o1} = \frac{1}{\lambda n I} = \frac{1}{0.1 \times 0.5 \times 10^{-3}} = 20 \text{ K}, \quad r_{o2} = \frac{1}{\lambda p I} = \frac{1}{0.2 \times 0.5 \times 10^{-3}} = 10 \text{ K}$$

The key point here is that the channel length modulation effect in M_1 cannot be neglected because its drain-source voltage is quite large. We take this effect into account with a few iterations.

First we let $V_{DS1} = 0$, then, we have, $g_{m1} = 2.31711 \times 10^{-3}$ (as $A_v = -5$) 3-30

$$R_{out, total} = 2157.86 \Omega$$

$$r_{o2} = \frac{1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{SG} - |V_{TH2}| - V_{SD})} = 2418.8356$$

$$0.5 \times 10^{-3} = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 \left[(V_{SG} - |V_{TH2}|) V_{SD} - \frac{V_{SD}^2}{2} \right], \text{ by dividing these two relations together.}$$

$$1.2094 = \frac{(3 - 0.8) V_{SD} - 0.5 V_{SD}^2}{3 - 0.8 - V_{SD}} = \frac{4.4 V_{SD} - V_{SD}^2}{4.4 - 2 V_{SD}}, \quad V_{SD}^2 - 6.8188 V_{SD} + 5.3214 = 0$$

$V_{SD} = 0.8987$, now second iteration starts, with the aid of the value we obtain for V_{SD} (or V_{DS}) from the first iteration, we have:

$$g_{m1} = 2.5489 \times 10^{-3}, \quad R_{out} = 1961.6020 \Omega$$

$$r_{o2} = 2174.9182, \quad 1.087459 = \frac{4.4 V_{SD} - V_{SD}^2}{4.4 - 2 V_{SD}}, \quad V_{SD}^2 - 6.5749 V_{SD} + 4.7848 = 0$$

$$V_{SD} = 0.8336 V$$

Third iteration starts now:

By substituting the value of V_{SD} from the second iteration in the relation for g_{m1} , we get:

$$g_{m1} = 2.5558 \times 10^{-3}, \quad R_{out} = 1956.3119, \quad r_{o2} = 2168.4169 \Omega$$

$$1.0842 = \frac{4.4 V_{SD} - V_{SD}^2}{4.4 - 2 V_{SD}}, \quad V_{SD}^2 - 6.5684 V_{SD} + 4.77051 = 0$$

$$V_{SD} = 0.8315$$

By doing the fourth iteration:

$$g_{m1} = 2.5560 \times 10^{-3}$$

$$R_{out} = 1956.1662, \quad r_{o2} = 2168.2379, \quad 1.0841189 = \frac{4.4 V_{SD} - V_{SD}^2}{4.4 - 2 V_{SD}}$$

$$V_{SD}^2 - 6.5682 V_{SD} + 4.77012 = 0$$

$$V_{SD} = 0.8315$$

$$I = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 \left[(V_{GS} - |V_{TH2}|) V_{SD} - \frac{V_{SD}^2}{2} \right], \left(\frac{W}{L}\right)_2 = \frac{0.5 \times 10^{-3}}{3.835 \times 10^{-5} \left[(3-0.8)0.8315 - \frac{0.8315^2}{2} \right]}$$

$$\left(\frac{W}{L}\right)_2 = 8.7878$$

If M_1 is at the edge of the triode region: $V_{out} = V_{in} - V_{TH1} = V_{in} - 0.7$

$$I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1}) = I_{D2} = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 \left[(V_{DD} - |V_{TH2}|)(V_{DD} - V_o) - \frac{(V_{DD} - V_o)^2}{2} \right] \times$$

$$V_{out} = \sqrt{\frac{2 \times 3.835 \times 10^{-5}}{1.34225 \times 10^{-4}} \frac{8.7878}{40} \left[2.2(3 - V_o) - \frac{(3 - V_o)^2}{2} \right] (1.6 - 0.2V_o)} \quad (1 + 0.2(V_{DD} - V_o))$$

$$V_o = 0.6663, V_{in} = 1.3663, g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1}) = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{out} = 1.34225 \times 10^{-4} \times \frac{20}{0.5} \times 0.6663 = 3.5773 \times 10^{-3}$$

However, M_2 is no longer in triode region because $V_o = 0.66 < V_b + |V_{TH2}| = 0.8$

Therefore, we should recalculate V_o with the assumption that M_2 is saturated

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{out}^2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_b - |V_{TH2}|)^2 \left[1 + \lambda_p (V_{DD} - V_o) \right]$$

$$536.9 V_o^2 + 32.623 V_o - 260.9845 = 0, V_{out} = 0.6674, V_{in} = 1.3674$$

$$g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1}) = 3.5837 \times 10^{-3}, I_{D1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 V_{out}^2 = 1.196 \times 10^{-3}$$

$$r_{out} = r_{o1} \parallel r_{o2} = \frac{1}{(\lambda_p + \lambda_n) I} = 2786.962 \Omega$$

$$A_v = -g_{m1} \cdot r_{out} = -9.9877$$

$$V_{out} = 0.8, \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - |V_{TH2}|)^2 \left[1 + \lambda_p (V_{DD} - V_o) \right]$$

$$1.34225 \times 10^{-4} \times 40 \times (V_{GS} - 0.7)^2 = 3.835 \times 10^{-5} \times 8.7878 (3 - 0.8)^2 \left[1 + 0.2(3 - 0.8) \right]$$

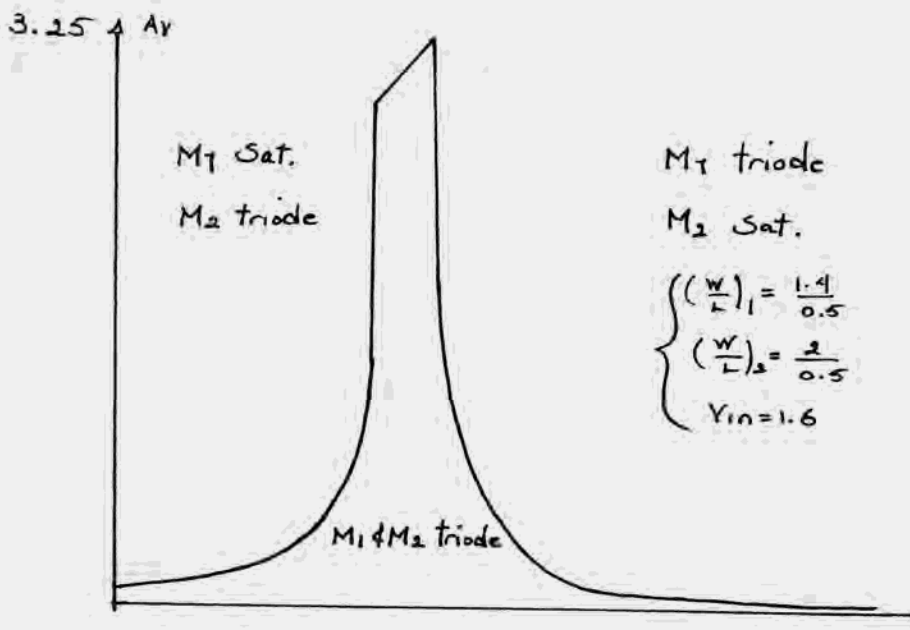
$$V_{in} = 1.3614$$

$$g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1}) = 3.5512 \times 10^{-3}$$

$$I = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH1})^2 = 1.1744 \times 10^{-3}$$

$$r_{out} = \frac{1}{(\lambda_P + \lambda_N)I} = 2838.2553$$

$$A_V = -g_{m1} \cdot r_{out} = -10.08$$



For M_1 to enter the triode region before M_2 is saturated, the overdrive voltage of M_1 must be increased.

Comparing the two curves, we observe that at $V_b = 0$ small signal voltage gain in (a) is higher than that in (b). That is because

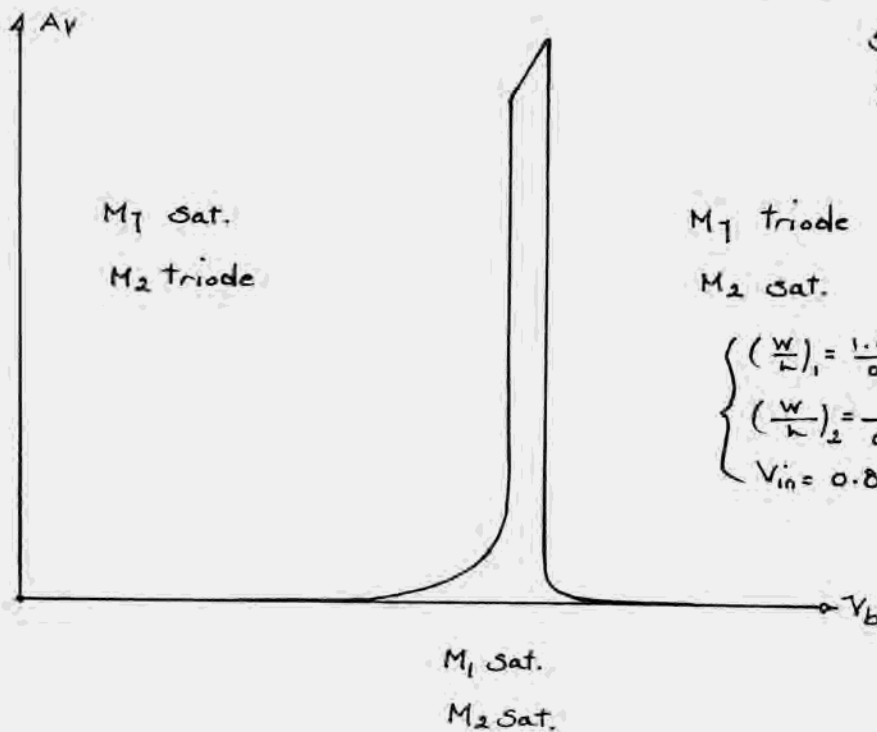
g_{m1} in (a) is higher than that in (b). However,

generally, small signal

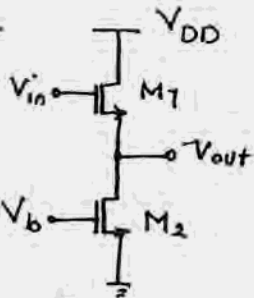
M_1 triode voltage gain in (a) is less than that in (b),

M_2 Sat. because when v_b sweeps all the way from 0 to V_{DD} , nowhere

are both devices simultaneously in the saturation region.



3.26.



$$V_{in} - V_{out} = 1V, I_{D1} = I_{D2} = 0.5 \text{ mA}, V_{GS2} - V_{GS1} = 0.5$$

$$\lambda = \gamma = 0$$

3.39

$$I_{D1} = I_{D2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - V_{out} - V_{TH1})^2 = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_b - V_{TH2})^2$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \left(\frac{W}{L}\right)_1 (1 - 0.7)^2$$

$$\left(\frac{W}{L}\right)_1 = 82.77$$

$$V_{GS2} = 0.5 + 1 = 1.5$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \left(\frac{W}{L}\right)_2 (1.5 - 0.7)^2 \rightarrow \left(\frac{W}{L}\right)_2 = 11.64$$

$$\gamma = 0.45 \text{ V}^{-1}, V_{in} = 2.5 \text{ V}, V_{in} - V_{out} = 1, I_{D1} = I_{D2} = 0.5 \text{ mA}, V_{GS2} - V_{GS1} = 0.5$$

$$V_{out} = V_{in} - 1 = 2.5 - 1 = 1.5, V_{GS2} = 0.5 + (2.5 - 1.5) = 1.5 \text{ V}$$

$$V_{TH1} = V_{TH0} + \gamma (\sqrt{2|f_f| + V_0} - \sqrt{2|f_f|}) = 0.7 + 0.45 (\sqrt{0.9 + 1.5} - \sqrt{0.9}) = 0.97022$$

$$I_{D1} = I_{D2} = 0.5 \times 10^{-3} = \frac{1}{2} \mu_n C_{ox} S_1 (V_{GS1} - V_{TH1})^2 = \frac{1}{2} \mu_n C_{ox} S_2 (V_{GS2} - V_{TH2})^2$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} S_1 (1 - 0.97)^2 = \frac{1}{2} \times 1.34225 \times 10^{-4} S_2 (1.5 - 0.7)^2$$

$$S_1 = \left(\frac{W}{L}\right)_1 = 8278$$

$$S_2 = \left(\frac{W}{L}\right)_2 = 11.64$$

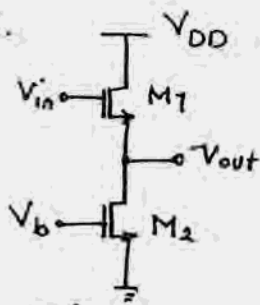
$$V_{out} = V_b - V_{TH2} = 1.5 - 0.7 = 0.8$$

$$V_{TH1} = 0.7 + 0.45 (\sqrt{0.9 + 0.8} - \sqrt{0.9}) = 0.8598$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \times 8278 (V_{in} - 0.8 - 0.8598)^2$$

$$V_{in} = 1.6897$$

3.26.



$$V_{in} - V_{out} = 1V, I_{D1} = I_{D2} = 0.5 \text{ mA}, V_{GS2} - V_{GS1} = 0.5$$

$$\lambda = \gamma = 0$$

$$I_{D1} = I_{D2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - V_{out} - V_{TH1})^2 = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_b - V_{TH2})^2$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \left(\frac{W}{L}\right)_1 (1 - 0.7)^2$$

$$\left(\frac{W}{L}\right)_1 = 82.77$$

$$V_{GS2} = 0.5 + 1 = 1.5$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \left(\frac{W}{L}\right)_2 (1.5 - 0.7)^2 \rightarrow \left(\frac{W}{L}\right)_2 = 11.64$$

$$\gamma = 0.45 \text{ V}^{-1}, V_{in} = 2.5 \text{ V}, V_{in} - V_{out} = 1, I_{D1} = I_{D2} = 0.5 \text{ mA}, V_{GS2} - V_{GS1} = 0.5$$

$$V_{out} = V_{in} - 1 = 2.5 - 1 = 1.5, V_{GS2} = 0.5 + (2.5 - 1.5) = 1.5 \text{ V}$$

$$V_{TH1} = V_{TH0} + \gamma (\sqrt{2|I_{D1}|} + V_0 - \sqrt{2|I_{D2}|}) = 0.7 + 0.45 (\sqrt{0.9 + 1.5} - \sqrt{0.9}) = 0.97022$$

$$I_{D1} = I_{D2} = 0.5 \times 10^{-3} = \frac{1}{2} \mu_n C_{ox} S_1 (V_{GS1} - V_{TH1})^2 = \frac{1}{2} \mu_n C_{ox} S_2 (V_{GS2} - V_{TH2})^2$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} S_1 (1 - 0.97)^2 = \frac{1}{2} \times 1.34225 \times 10^{-4} S_2 (1.5 - 0.7)^2$$

$$S_1 = \left(\frac{W}{L}\right)_1 = 8278$$

$$S_2 = \left(\frac{W}{L}\right)_2 = 11.64$$

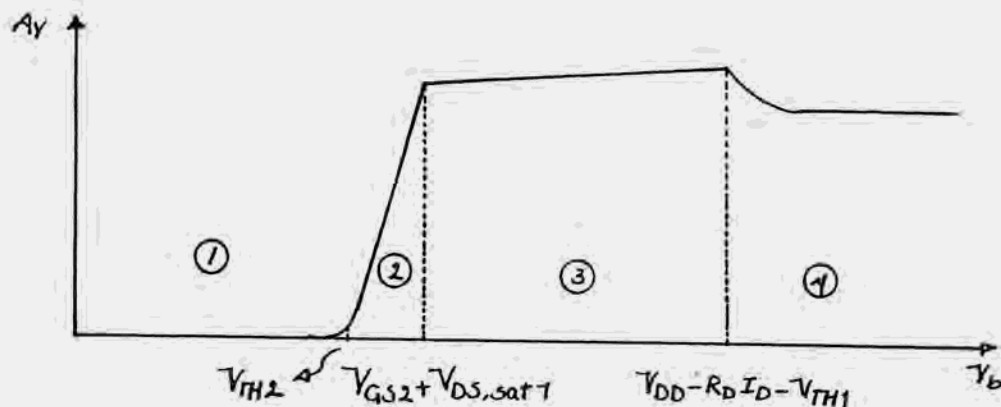
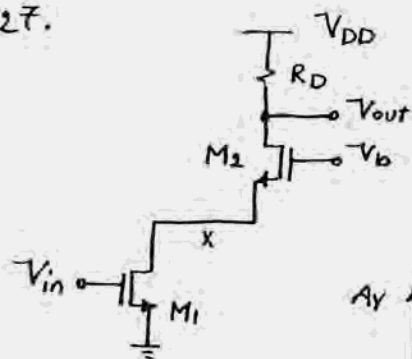
$$V_{out} = V_b - V_{TH2} = 1.5 - 0.7 = 0.8$$

$$V_{TH1} = 0.7 + 0.45 (\sqrt{0.9 + 0.8} - \sqrt{0.9}) = 0.8598$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \times 8278 (V_{in} - 0.8 - 0.8598)^2$$

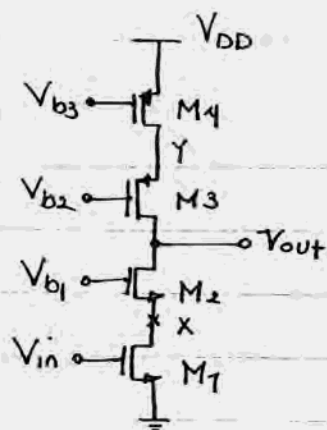
$$V_{in} = 1.6897$$

3.27.



- ① In this region V_b is less than V_{TH2} , so M_1 and M_2 are off. It is worth mentioning that M_2 is saturated-off and M_1 is off in triode region.
- ② V_b is increasing above V_{TH2} , as a result, a current establishes in circuit. M_1 operates in triode region and M_2 does in saturation. The higher V_b , the higher the drain-source voltage of M_1 , increasing the output impedance of M_1 which, in turn, causes the small signal voltage gain of the circuit increases.
- ③ Both devices are in saturation region and the maximum gain is attainable in this region. The slight increase in A_v is because of increasing the transconductance of M_1 with increasing V_x (or V_b).
- ④ M_2 enters the triode region, as a result, the total output impedance decreases down to the limit of $r_{o11} || R_D$. Consequently, the small signal voltage gain experiences a similar change.

3.28



$$\text{output swing} = 1.9 \text{ V}$$

$$I_{\text{bias}} = 0.5 \text{ mA}$$

$$\gamma = 0 \quad \left(\frac{W}{L}\right)_{1-4} = \left(\frac{W}{L}\right)$$

$$V_{b1} - V_{TH1} < V_{out} < V_{b2} + V_{TH3}$$

$$V_{b2} + V_{TH3} - (V_{b1} - V_{TH1}) = 1.9$$

$$V_{b2} + 0.8 - V_{b1} + 0.7 = 1.9, \quad V_{b2} - V_{b1} = 0.4$$

$$0.5 \times 10^{-3} = \frac{1}{2} \mu_n C_{ox} S (V_{in} - V_{TH1})^2 = \frac{1}{2} \mu_n C_{ox} S (V_{b1} - V_X - V_{TH2})^2 = \frac{1}{2} \mu_p C_{ox} S (V_Y - V_{b2} - |V_{TH3}|)^2 = \frac{1}{2} \mu_p C_{ox} S (V_{DD} - V_{b3} - |V_{TH4}|)^2$$

$$V_{DD} - V_{SD_{min,4}} - V_{SD_{min,3}} - V_{DS_{min,1}} - V_{DS_{min,2}} = 1.9$$

$$1.1 = \left(\frac{2I_D}{\mu_p C_{ox} S}\right)^{1/2} + \left(\frac{2I_D}{\mu_p C_{ox} S}\right)^{1/2} + \left(\frac{2I_D}{\mu_n C_{ox} S}\right)^{1/2} + \left(\frac{2I_D}{\mu_n C_{ox} S}\right)^{1/2}$$

$$1.1 = 2\sqrt{2I_D} \left(\frac{1}{\sqrt{\mu_p C_{ox} S}} + \frac{1}{\sqrt{\mu_n C_{ox} S}}\right) \frac{1}{\sqrt{S}} \rightarrow S = \frac{8I_D \left(\sqrt{\frac{1}{\mu_p C_{ox}}} + \sqrt{\frac{1}{\mu_n C_{ox}}}\right)^2}{1.1^2}$$

$$S = \frac{8 \times 0.5 \times 10^{-3} \left(\frac{1}{\sqrt{1.34225 \times 10^{-4}}} + \frac{1}{\sqrt{3.835 \times 10^{-5}}}\right)^2}{1.1^2} = 202.98 \rightarrow S = 203$$

$$V_{DS_{min,1}} = \left(\frac{2I_D}{\mu_n C_{ox} S}\right)^{1/2} = \left(\frac{2 \times 0.5 \times 10^{-3}}{1.34225 \times 10^{-4} \times 203}\right)^{1/2} = 0.1915$$

$$V_{SD_{min,4}} = \left(\frac{2 \times 0.5 \times 10^{-3}}{3.835 \times 10^{-5} \times 203}\right)^{1/2} = 0.3584$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 1.34225 \times 10^{-4} \times 203 (V_{b1} - V_X - 0.7)^2$$

$$V_{b1} - V_X = 0.8915$$

$$0.5 \times 10^{-3} = \frac{1}{2} \times 3.835 \times 10^{-5} \times 203 (V_Y - V_{b2} - 0.8)^2$$

3.35

$$V_Y - V_{b2} = 1.1584$$

$$V_{b2} - V_{b1} = 0.4$$

$$\text{If } V_X = 0.1915 \rightarrow V_{b1} = 1.083, V_{b2} = 1.483, V_Y = 2.6414$$

$V_{SD4} = V_{DD} - V_Y = 0.3586$, as a result, M_1 and M_2 are at the edge of the triode region.

$$g_{m1} = \sqrt{2\mu_n C_{ox} S (1 + \lambda V_{DS}) I_D} = \sqrt{2 \times 1.34225 \times 10^{-4} \times 203 \times 0.5 \times 10^{-3}}$$

$$g_{m1} = g_{m2} = 5.2199 \times 10^{-3}$$

$$r_{o1} = r_{o2} = \frac{1}{0.1 \times 0.5 \times 10^{-3}} = 20K \quad r_{o3} = r_{o4} = \frac{1}{0.2 \times 0.5 \times 10^{-3}} = 10K$$

$$G_m = \frac{g_{m1} \cdot r_{o1} \cdot (1 + g_{m2} \cdot r_{o2})}{r_{o1} \cdot r_{o2} \cdot g_{m2} + r_{o1} + r_{o2}} = \frac{5.2199 \times 10^{-3} \times 20 \times 10^3 (20 \times 10^3 \times 5.2199 \times 10^{-3} + 1)}{(20 \times 10^3)^2 \times 5.2199 \times 10^{-3} + 2 \times 20 \times 10^3}$$

$$G_m = 5.17 \times 10^{-3}, \text{ neglecting the body effect.}$$

$$R_{out} = \left[(1 + g_{m2} r_{o2}) r_{o1} + r_{o2} \right] \parallel \left[(1 + g_{m3} r_{o3}) r_{o1} + r_{o3} \right]$$

$$R_{out} = \left[(1 + 5.2199 \times 10^{-3} \times 20 \times 10^3) 20 \times 10^3 + 20 \times 10^3 \right] \parallel \left[(1 + 2.79 \times 10^{-3} \times 10 \times 10^3) 10 \times 10^3 + 10 \times 10^3 \right]$$

$$R_{out} = 262.1766 \times 10^3, \quad A_V = -G_m R_{out} = -5.17 \times 10^{-3} \times 262.1766 \times 10^3$$

$$A_V = -1355.45 \quad g_{m3} = g_{m4} = \sqrt{2 \times 3.835 \times 10^{-5} \times 203 \times 0.5 \times 10^{-3}} = 2.79 \times 10^{-3}$$

Chapter 4: Differential Amplifiers

4.1

$$(a) \quad A_v \cong - \frac{g_{mN}}{g_{mP}} = - \sqrt{\frac{\mu_n (W/L)_N}{\mu_p (W/L)_P}} \quad (4.52)$$

$$A_v = - \sqrt{\frac{350}{100} \times \frac{50/0.5}{50/1}} = - \sqrt{7} = \underline{\underline{-2.65}}$$

$$(b) \quad A_v = - g_{mN} (r_{oN} \parallel r_{oP}) \quad (4.53)$$

$$I_D = \frac{I_{SS}}{2} = 0.5 \text{ mA} \quad \mu_n C_{ox} = 350 \mu\text{A/V}^2 \times \frac{8.85 \times 10^{-14} \times 3.9}{9 \times 10^{-7}} = 0.134 \text{ mA/V}^2$$

$$g_{mN} = \sqrt{2 I_D \mu_n C_{ox} \frac{W}{L}} = \sqrt{2 \times 0.5 \text{ mA} \times 0.134 \text{ mA/V}^2 \times 100} = 3.66 \text{ mA/V}$$

$$L_N = 0.5 \mu\text{m} \Rightarrow \lambda_n = 0.1 \Rightarrow r_{oN} = \frac{1}{\lambda_n I_D} = \frac{1}{0.1 \times 0.5 \text{ mA}} = 20 \text{ k}\Omega$$

$$L_P = 1 \mu\text{m}; \lambda_p = 0.2 \text{ for } L = 0.5 \mu\text{m}; \lambda \propto \frac{1}{L} \Rightarrow \lambda_p = 0.1$$

$$r_{oP} = \frac{1}{\lambda_p I_D} = \frac{1}{0.1 \times 0.5 \text{ mA}} = 20 \text{ k}\Omega$$

$$A_v = - g_{mN} (r_{oN} \parallel r_{oP}) = - 3.66 \text{ mA/V} (20 \text{ k}\Omega \parallel 20 \text{ k}\Omega) = \underline{\underline{-36.6}}$$

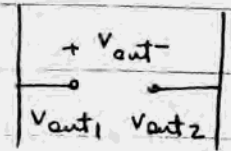
$$(V_{in,cm})_{min} = 0.4 + V_{GS1} \quad \text{for both circuits}$$

$$V_{GS1} = V_{TH} + \sqrt{\frac{2 I_D}{\mu_n C_{ox} (W/L)_N}} = 0.7 + \sqrt{\frac{2 \times 0.5 \text{ mA}}{0.134 \text{ mA/V}^2 \times 100}} = 0.7 + 0.27 = 0.97 \text{ V}$$

$$\rightarrow (V_{in,CM})_{\min} = 0.4 + 0.97 = 1.37 \text{ V}$$

max output voltage swing:

$$(a) (V_{out,1,2})_{\max} = V_{DD} - |V_{TH,P}| = 3 - 0.8 = 2.2 \text{ V}$$



There are two constraints for $(V_{out,1,2})_{\min}$:

$$1) M_1 \text{ enters triode: } (V_{out,1,2})_{\min} = 0.4 + V_{GS1} - V_{TH,N} \\ = 0.4 + 0.97 - 0.7 = 0.67 \text{ V}$$

2) all of I_{SS} goes through M_3 :

$$(V_{out,1,2})_{\min} = V_{DD} - |V_{GS3}|_{I_D = I_{SS}} = V_{DD} - |V_{TH,P}| + \sqrt{\frac{2 I_{SS}}{\mu_p C_{ox} (\frac{W}{L})_3}} \\ = 3 - 0.8 - \sqrt{\frac{2 \times 1 \text{ m}}{38.3 \mu\text{A/V}^2 \times 50}} = 3 - 0.8 - 1.02 = 1.18 \text{ V}$$

$$\mu_p C_{ox} = 100 \times \frac{8.85 \times 10^{-14} \times 3.9}{9 \times 10^{-7}} = 38.3 \mu\text{A/V}^2 \Rightarrow (V_{out,1,2})_{\min} = 1.18 \text{ V}$$

$$\text{Max swing of } V_{out,1,2} = 2.2 - 1.18 = 1.02 \text{ V}$$

$$\text{Max swing of } V_{out} = 2 \times 1.02 = 2.04 \text{ V}$$

$$(b) (V_{out,1,2})_{\max} = V_{DD} - |V_{GS3} - V_{TH,P}| = 3 - 0.72 = 2.28 \text{ V}$$

$$(V_{out,1,2})_{\min} = 0.4 + V_{GS1} - V_{TH,N} = 0.67 \text{ V}$$

$$\text{Max swing of } V_{out} = 2(2.28 - 0.67) = 3.22 \text{ V}$$

$$4.2 \quad I_{SS} = 1 \text{ mA}$$

(a)

$$A_v = -g_{m1} \left(\frac{1}{g_{m3}} \parallel r_{o1} \parallel r_{o3} \parallel r_{o5} \right) \approx -\frac{g_{m1}}{g_{m3}} = \sqrt{\frac{\mu_n}{\mu_p} \times \frac{I_{D1}}{I_{D3}}}$$

$$= \sqrt{\frac{350}{100} \times \frac{\frac{1}{2} I_{SS}}{0.2 \frac{I_{SS}}{2}}} = -4.18$$

$$(b) \quad I_{D5} = I_{D6} = 0.8 \left(\frac{I_{SS}}{2} \right) = 0.4 \text{ mA}$$

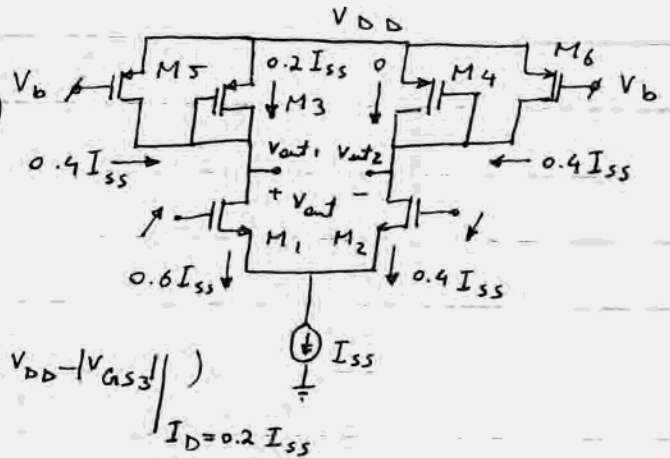
$$|V_{GS5}| = V_{DD} - V_b \Rightarrow V_b = V_{DD} - |V_{GS5}| = V_{DD} - |V_{TH,P}| - \sqrt{\frac{2I_{D5}}{\mu_p C_{ox} \frac{W}{L}}}$$

$$V_b = 3 - 0.8 - \sqrt{\frac{2 \times 0.4 \text{ mA}}{38.3 \mu\text{A/V}^2 \times 100}} = 1.74$$

(c)

$$(V_{out1,2})_{\max} = \min(V_b + |V_{TH,P}|, V_{DD} - |V_{TH,P}|)$$

$$= (1.74 + 0.8, 3 - 0.8) = 2.2 \text{ V}$$



$$(V_{out1,2})_{\min} = \max(V_{GS1} \Big|_{I_D=0.6 I_{SS}} + V_{GS3} \Big|_{I_D=0.2 I_{SS}} - V_{TH,n}, V_{DD} - |V_{GS3}| \Big|_{I_D=0.2 I_{SS}})$$

$$V_{GS1} \Big|_{I_D=0.6 I_{SS}} = V_{TH,n} + \sqrt{\frac{2 \times 0.6 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = V_{TH,n} + 0.299 \text{ V}$$

$$|V_{GS3}| \Big|_{I_D=0.2 I_{SS}} = |V_{TH,P}| + \sqrt{\frac{2 \times 0.2 I_{SS}}{\mu_p C_{ox} \frac{W}{L}}} = 0.8 + 0.323 \text{ V} = 1.12 \text{ V}$$

$$(V_{out1,2})_{\min} = \max(0.4 + 0.299, 3 - 1.12) = 1.88 \text{ V}$$

$$\text{Max swing of } V_{out} = 2(2.2 - 1.88) = 0.64 \text{ V}$$

$$4.2 \quad I_{SS} = 1 \text{ mA}$$

(a)

$$A_v = -g_{m1} \left(\frac{1}{g_{m3}} \parallel r_{o1} \parallel r_{o3} \parallel r_{o5} \right) \approx -\frac{g_{m1}}{g_{m3}} = \sqrt{\frac{\mu_n}{\mu_p} \times \frac{I_{D1}}{I_{D3}}}$$

$$= \sqrt{\frac{350}{100} \times \frac{\frac{1}{2} I_{SS}}{0.2 \frac{I_{SS}}{2}}} = -4.18$$

$$(b) \quad I_{D5} = I_{D6} = 0.8 \left(\frac{I_{SS}}{2} \right) = 0.4 \text{ mA}$$

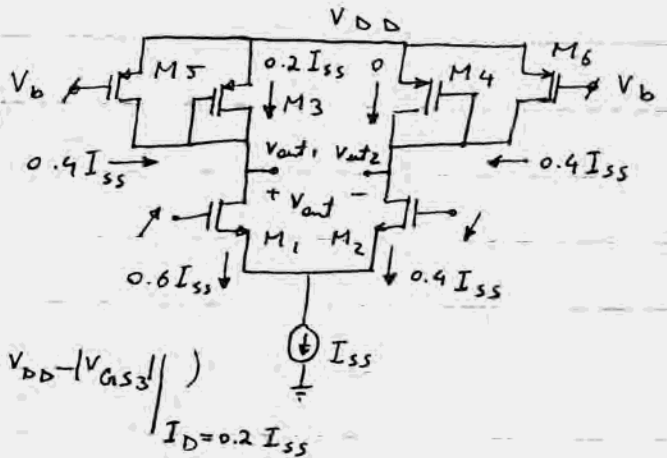
$$|V_{GS5}| = V_{DD} - V_b \Rightarrow V_b = V_{DD} - |V_{GS5}| = V_{DD} - |V_{TH,P}| - \sqrt{\frac{2I_{D5}}{\mu_p C_{ox} \frac{W}{L}}}$$

$$V_b = 3 - 0.8 - \sqrt{\frac{2 \times 0.4 \text{ m}}{38.3 \mu \times 100}} = 1.74$$

(c)

$$(V_{out1,2})_{\max} = \min(V_b + |V_{TH,P}|, V_{DD} - |V_{TH,P}|)$$

$$= (1.74 + 0.8, 3 - 0.8) = 2.2 \text{ V}$$



$$(V_{out1,2})_{\min} = \max \left(V_{GS1} \Big|_{I_D=0.6 I_{SS}} + V_{TH,N}, V_{DD} - |V_{GS3}| \Big|_{I_D=0.2 I_{SS}} \right)$$

$$V_{GS1} \Big|_{I_D=0.6 I_{SS}} = V_{TH,N} + \sqrt{\frac{2 \times 0.6 I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} = V_{TH,N} + 0.299 \text{ V}$$

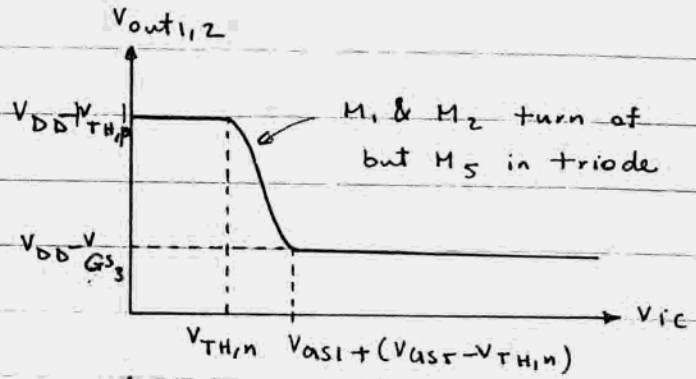
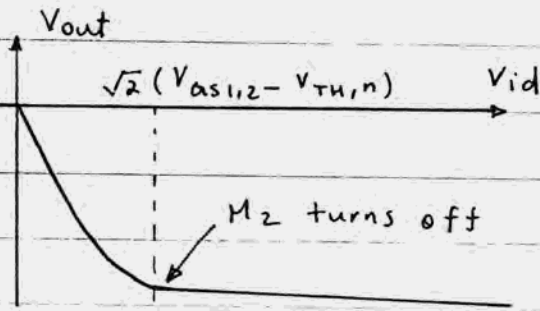
$$|V_{GS3}| \Big|_{I_D=0.2 I_{SS}} = |V_{TH,P}| + \sqrt{\frac{2 \times 0.2 I_{SS}}{\mu_p C_{ox} \frac{W}{L}}} = 0.8 + 0.323 \text{ V} = 1.12 \text{ V}$$

$$(V_{out1,2})_{\min} = \max(0.4 + 0.299, 3 - 1.12) = 1.88 \text{ V}$$

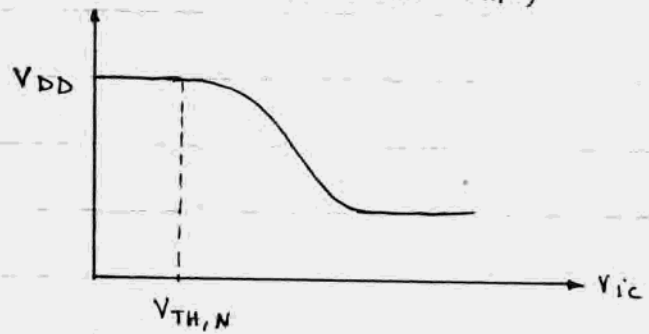
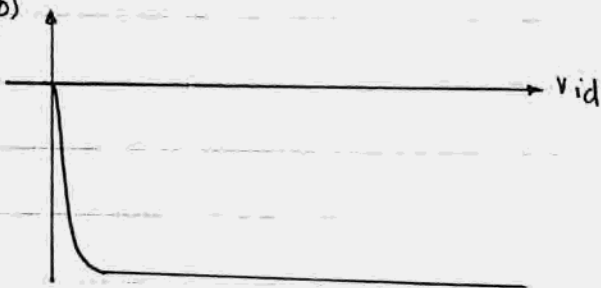
$$\text{Max swing of } V_{out} = 2(2.2 - 1.88) = 0.64 \text{ V}$$

4.3

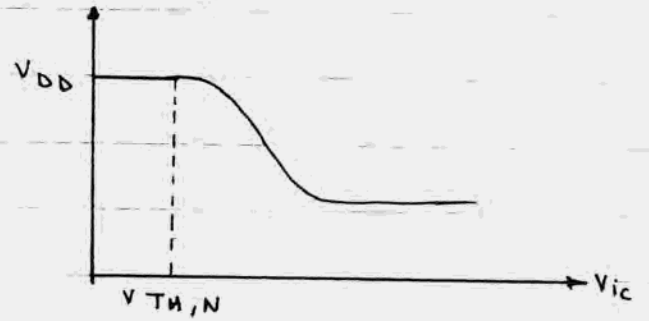
(a)



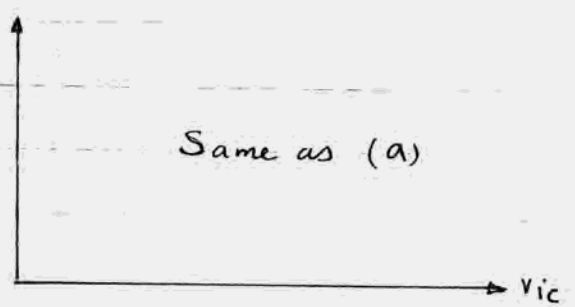
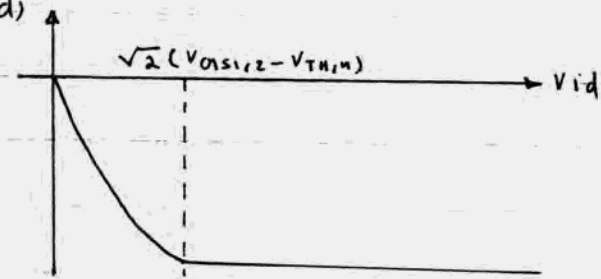
(b)



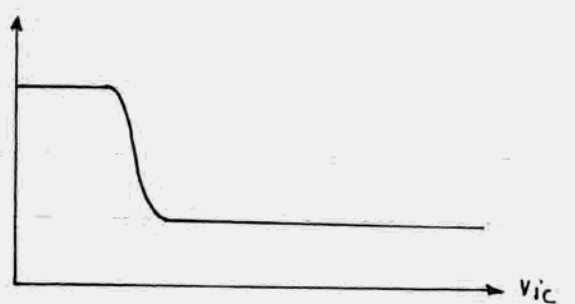
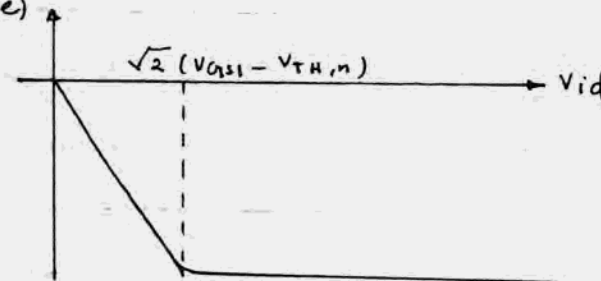
(c)



(d)

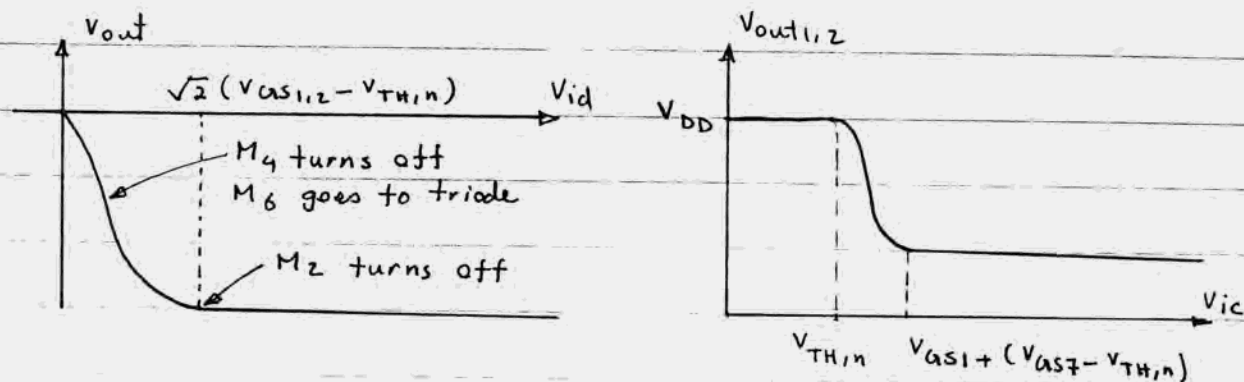


(e)

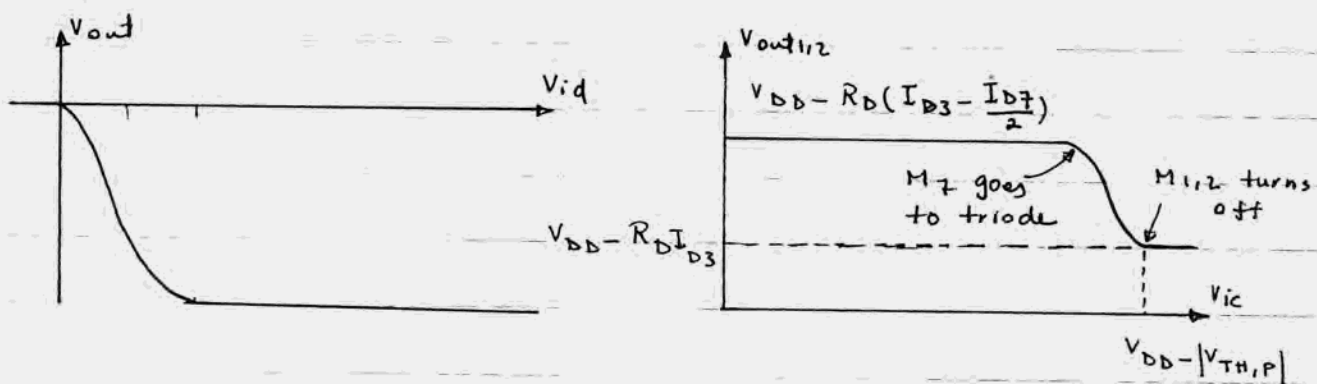


4.4

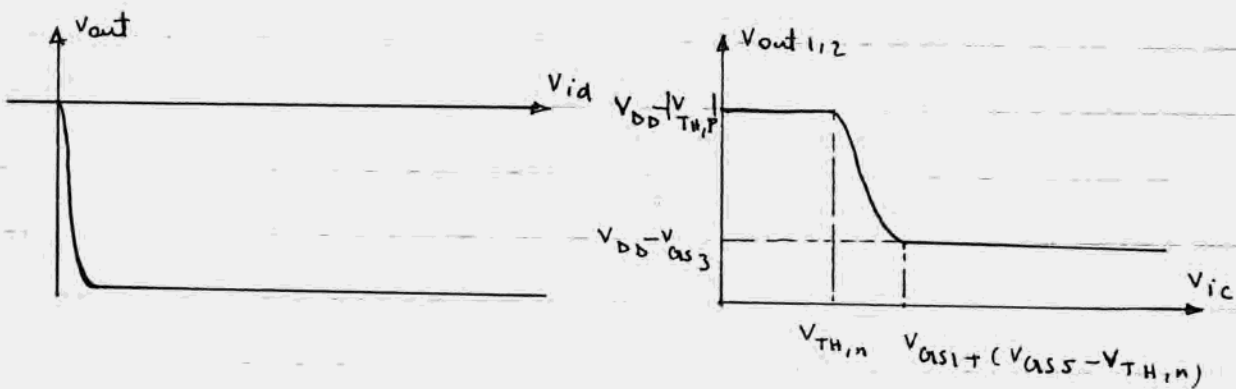
(a)



(b)



(c)



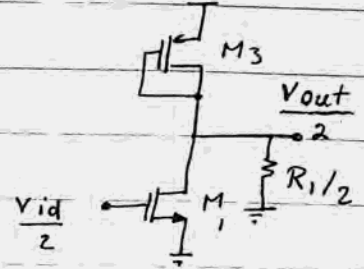
4.5

Fig. 4.35 Using half circuit we have:

(a) we define $V_{id} = v_{in1} - v_{in2}$

$$A_v = \frac{V_{out}}{V_{id}} = -g_{m1} \left(\frac{1}{g_{m3}} \parallel \frac{R_1}{2} \parallel r_{o1} \parallel r_{o3} \right)$$

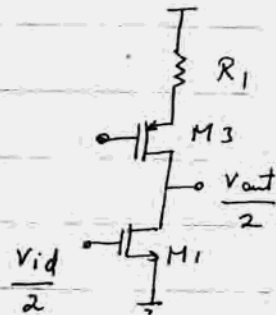
$$\approx -g_{m1} \left(\frac{1}{g_{m3}} \parallel \frac{R_1}{2} \right) = -\frac{g_{m1} R_1}{2 + g_{m3} R_1}$$



(b)

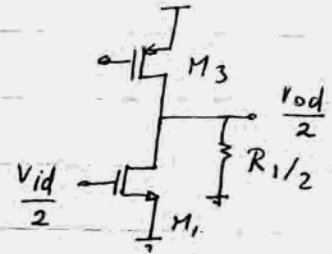
$$A_v = -g_{m1} \left[r_{o1} \parallel (R_1 g_{m3} r_{o3} + R_1 + r_{o3}) \right]$$

$$\approx -g_{m1} (r_{o1} \parallel R_1 g_{m3} r_{o3})$$



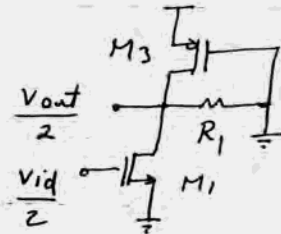
(c)

$$A_v = -g_{m1} \left(r_{o1} \parallel r_{o3} \parallel \frac{R_1}{2} \right)$$

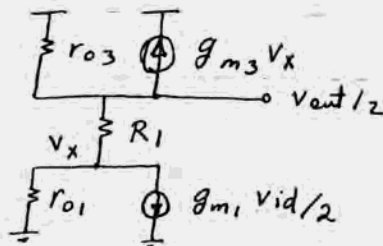
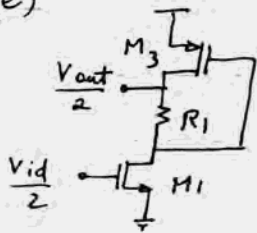


(d)

$$A_v = -g_{m1} (r_{o1} \parallel r_{o3} \parallel R_1)$$



(e)



$$\text{KCL: } \frac{v_x}{r_{o1}} + \frac{v_x - v_{out}/2}{R_1} + g_{m1} \frac{v_{id}}{2} = 0$$

$$\frac{v_{out}/2}{r_{o3}} + \frac{v_{out}/2 - v_x}{R_1} + g_{m3} v_x = 0 \Rightarrow v_x = \frac{(r_{o3} + R_1) v_{out}}{2 r_{o3} (1 - R_1 g_{m3})}$$

$$\left(\frac{R_1 + r_{o1}}{R_1 r_{o1}} \times \frac{r_{o3} + R_1}{r_{o3} (1 - R_1 g_{m3})} - \frac{1}{R_1} \right) v_{out} + g_{m1} v_{id} = 0$$

$$\frac{(R_1 + r_{o1})(R_1 + r_{o3}) - r_{o1} r_{o3} (1 - R_1 g_{m3})}{R_1 r_{o1} r_{o3} (1 - R_1 g_{m3})} v_{out} + g_{m1} v_{id} = 0$$

$$\frac{R_1 + r_{o1} + r_{o2} + r_{o1} r_{o3} g_{m3}}{r_{o1} r_{o3} (1 - R_1 g_{m3})} v_{out} + g_{m1} v_{id} = 0$$

$$\Rightarrow A_v = \frac{v_{out}}{v_{id}} = - \frac{g_{m1} r_{o1} r_{o3} (1 - R_1 g_{m3})}{R_1 + r_{o1} + r_{o3} + r_{o1} r_{o3} g_{m3}} \approx - \frac{g_{m1}}{g_{m3}} (1 - R_1 g_{m3})$$

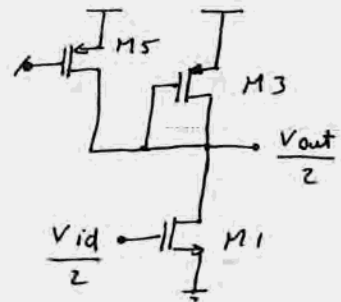
$$R_1 g_{m3} < 1$$

Fig. 4.36

(a)

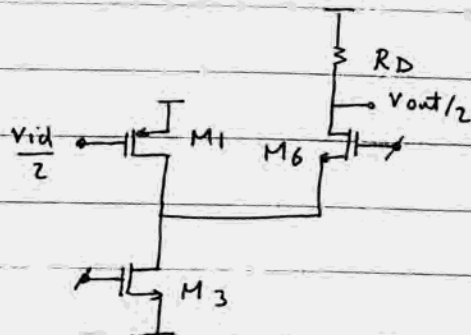
$$A_v = - g_{m1} \left(\frac{1}{g_{m3}} \parallel r_{o1} \parallel r_{o3} \parallel r_{o5} \right)$$

$$\approx - \frac{g_{m1}}{g_{m3}}$$



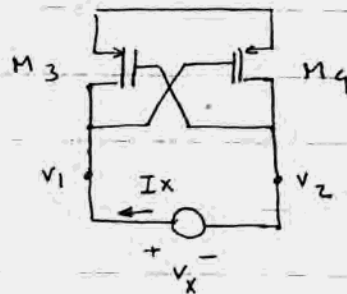
(b)

$$A_v = -g_{m1} (R_D \parallel [r_{o6} g_{m6} (r_{o1} \parallel r_{o3})])$$



(c)

if we neglect r_{o3} & r_{o4} at the moment,
we have:



$$I_x = g_{m3} v_2 \quad g_{m3} = g_{m4} = g_{m3,4}$$

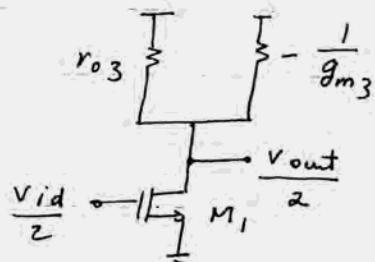
$$I_x = -g_{m4} v_1$$

$$2I_x = -g_{m3,4} (v_1 - v_2) = -g_{m3,4} v_x$$

$$\rightarrow \frac{v_x}{I_x} = -\frac{2}{g_{m3,4}}$$

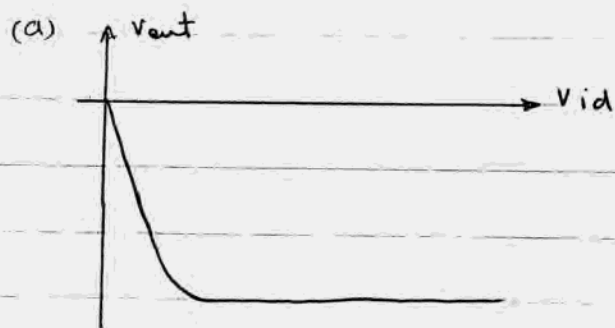
$$A_v = -g_{m1} (r_{o1} \parallel r_{o3} \parallel \frac{-1}{g_{m3}})$$

$$A_v = -\frac{g_{m1}}{\frac{1}{r_{o1}} + \frac{1}{r_{o3}} - g_{m3}} \quad \left(\frac{1}{r_{o1}} + \frac{1}{r_{o3}} > g_{m3} \right)$$



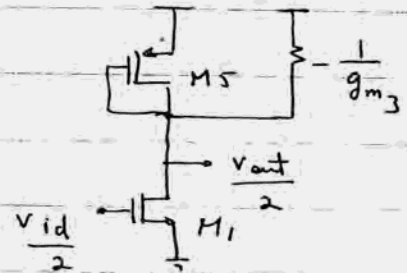
if $g_{m3} \geq \frac{1}{r_{o1}} + \frac{1}{r_{o3}}$ then the circuit is not stable and small signal model is not valid.

4.6



(b)

Similar to what we had in the previous problem (Fig 4.36 (c)), M_3 gives a negative resistance at the output.



$$A_v = -g_{m1} \left(\frac{1}{g_{m5}} \parallel \frac{-1}{g_{m3}} \right) \quad (\lambda = \infty)$$

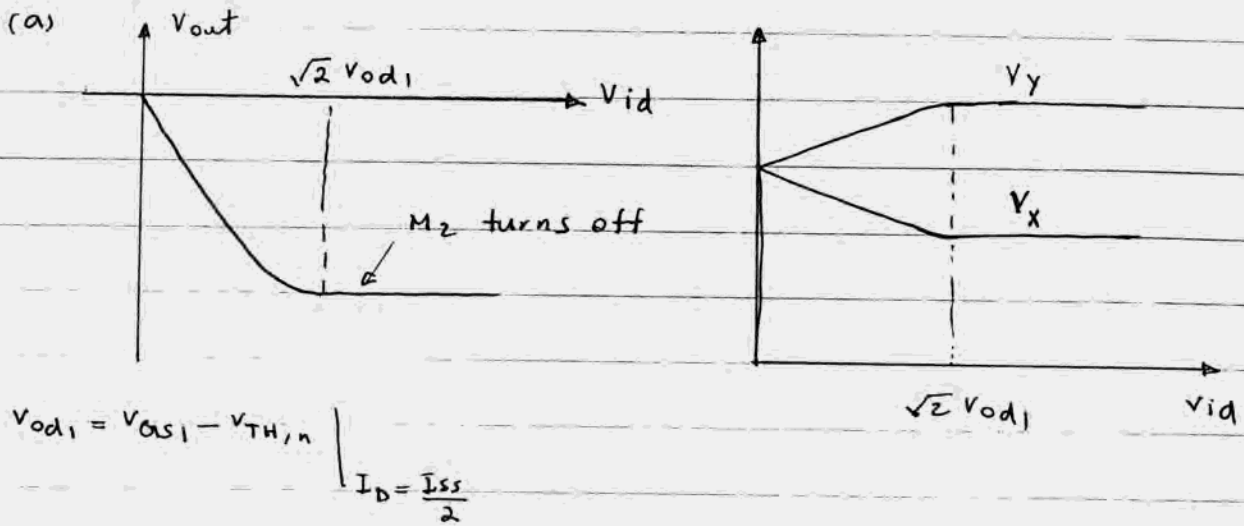
$$A_v = - \frac{g_{m1}}{g_{m5} - g_{m3}} \quad (g_{m3} \text{ must be less than } g_{m5})$$

$$g_m = \mu_p C_{ox} \frac{W}{L} (V_{GS} - V_T) \quad V_{GS\ 3,4} = V_{GS\ 5,6}$$

$$\Rightarrow \frac{g_{m3,4}}{g_{m5,6}} = \frac{(W/L)_{3,4}}{(W/L)_{5,6}} = 0.8$$

$$\Rightarrow A_v = - \frac{g_{m1}}{g_{m5} - 0.8 g_{m5}} = - \frac{5 g_{m1}}{g_{m5}}$$

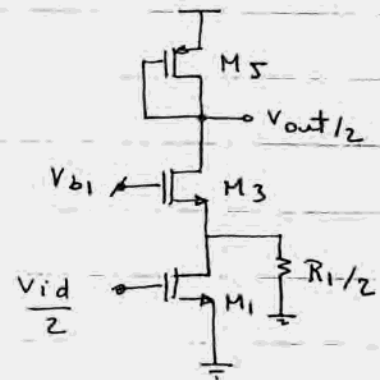
4.7



(b)

$$A_v = \frac{V_{out}}{V_{id}} = -g_{m1} \left(\frac{R_1}{2} \parallel \frac{1}{g_{m3}} \right) g_{m3} g_{m5}^{-1}$$

$$= - \frac{g_{m1}}{g_{m5} \left(1 + \frac{2}{R_1 g_{m3}} \right)}$$



4.8

By using superposition, we have:

$$G_{m1} = \frac{I_1}{V_{in1}} \Bigg|_{V_{in2}=0} = \frac{g_{m1}}{1 + (g_{m1} + g_{mb1}) \underbrace{\left(R_{S1} + R_{S2} + \frac{1}{g_{m2} + g_{mb2}} \right)}_{R_{eq1}}}$$

(Transconductance of c_s)

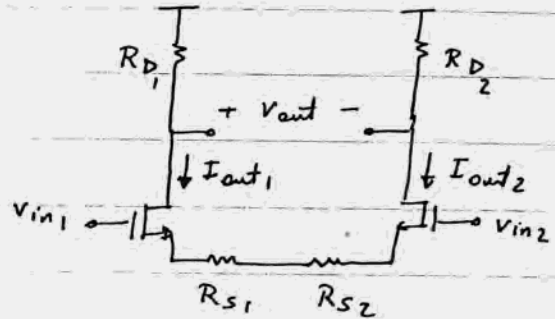
$$R_{eq1} = R_{S1} + R_{S2} + \frac{1}{g_{m2} + g_{mb2}}$$

Similarly:

$$G_{m2} = \frac{I_2}{V_{in2}} \Big|_{V_{in1}=0} = \frac{g_{m2}}{1 + (g_{m2} + g_{mb2})(R_{S1} + R_{S2}) + g_{m1} + g_{mb1}}$$

$$I_{out1} = I_1 - I_2$$

$$I_{out2} = I_2 - I_1$$



$$V_{out} = V_{out1} - V_{out2} = -R_{D1} I_{out1} + R_{D2} I_{out2}$$

$$V_{out} = -R_{D1} (I_1 - I_2) - R_{D2} (I_1 - I_2)$$

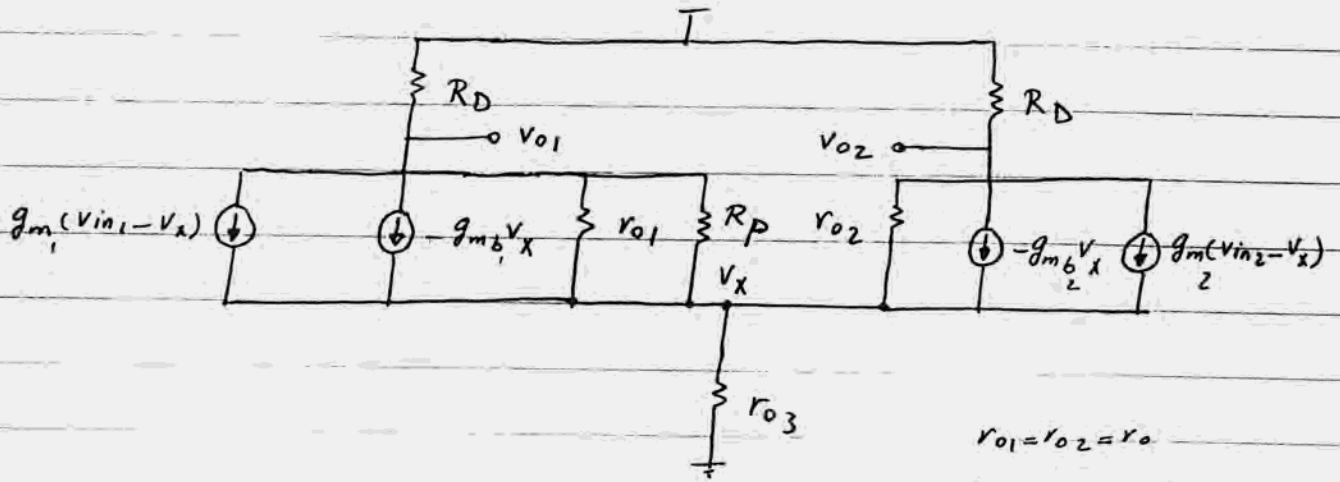
$$V_{out} = -(R_{D1} + R_{D2})(I_1 - I_2)$$

$$V_{out} = -(R_{D1} + R_{D2})(G_{m1} V_{in1} - G_{m2} V_{in2})$$

or equivalently:

$$V_{out} = -(R_{D1} + R_{D2}) \left[(G_{m1} + G_{m2}) \frac{V_{in1} - V_{in2}}{2} + (G_{m1} - G_{m2}) \frac{V_{in1} + V_{in2}}{2} \right]$$

4.9



$$r_{01} = r_{02} = r_o$$

$$g_{m1} = g_{m2} = g_m$$

$$g_{mb1} = g_{mb2} = g_{mb}$$

KCL:

$$\textcircled{1} \quad \frac{V_{01}}{R_D} + \frac{V_{01} - V_X}{r_o} + g_m (v_{in1} - V_X) - g_{mb} V_X + \frac{V_{01} - V_X}{R_P} = 0$$

$$\textcircled{2} \quad \frac{V_{02}}{R_D} + \frac{V_{02} - V_X}{r_o} + g_m (v_{in2} - V_X) - g_{mb} V_X = 0$$

$$\frac{V_{01}}{R_D} + \frac{V_{02}}{R_D} + \frac{V_X}{r_{03}} = 0 \Rightarrow V_X = -r_{03} \frac{V_{01} + V_{02}}{R_D}$$

we define:
$$\begin{cases} V_{od} = V_{01} - V_{02} \\ V_{oc} = \frac{V_{01} + V_{02}}{2} \end{cases} \Rightarrow \begin{cases} V_{01} = V_{oc} + \frac{V_{od}}{2} \\ V_{02} = V_{oc} - \frac{V_{od}}{2} \end{cases}$$

Now by substituting V_{01} , V_{02} and V_X in $\textcircled{1}$ and $\textcircled{2}$ we have:

$$\begin{cases} (V_{oc} + \frac{V_{od}}{2}) (\frac{1}{R_D} + \frac{1}{R_P} + \frac{1}{r_o}) + g_m v_{in1} - (\frac{1}{R_P} + g_m + g_{mb} + \frac{1}{r_o}) (-\frac{2r_{03}}{R_D} V_{oc}) = 0 \\ (V_{oc} - \frac{V_{od}}{2}) (\frac{1}{R_D} + \frac{1}{r_o}) + g_m v_{in2} - (g_m + g_{mb} + \frac{1}{r_o}) (-\frac{2r_{03}}{R_D} V_{oc}) = 0 \end{cases}$$

or:

$$\left[\frac{1}{R_D} + \frac{1}{R_P} + \frac{1}{r_o} + (\frac{1}{R_D} + \frac{1}{r_o} + g_m + g_{mb}) (\frac{2r_{03}}{R_D}) \right] V_{oc}$$

$$+ \frac{1}{2} (\frac{1}{R_D} + \frac{1}{R_P} + \frac{1}{r_o}) V_{od} + g_m v_{in1} = 0 \quad \textcircled{3}$$

$$\left[\frac{1}{R_D} + \frac{1}{r_o} + (g_m + g_{mb} + \frac{1}{r_o}) \left(\frac{2r_{o3}}{R_D} \right) \right] V_{oc} - \frac{1}{2} \left(\frac{1}{R_D} + \frac{1}{r_o} \right) V_{od}$$

$$+ g_m V_{in2} = 0 \quad (4)$$

From equation (3) and (4) V_{od} and V_{oc} can be solved in terms of V_{in1} and V_{in2} .

Now if $\lambda = \delta = 0$, we have:

$$(5) \quad \frac{V_{o1}}{R_D} + g_m (V_{in1} - V_x) + \frac{V_{o1} - V_x}{R_P} = 0$$

$$\frac{V_{o2}}{R_D} + g_m (V_{in2} - V_x) = 0 \Rightarrow V_x = V_{in2} + \frac{V_{o2}}{g_m R_D}$$

$$V_{o1} + V_{o2} = 0, \quad V_{out} = V_{o1} - V_{o2} \Rightarrow V_{o1} = \frac{V_{out}}{2}, \quad V_{o2} = -\frac{V_{out}}{2}$$

$$(5) \quad \frac{V_{out}}{2R_D} + g_m V_{in1} - g_m V_{in2} + \frac{V_{out}}{2R_D} + \frac{V_{out}}{2R_P} - \frac{V_{in2}}{R_P} + \frac{V_{out}}{2R_P g_m R_D} = 0$$

$$V_{out} \left(\frac{1}{R_D} + \frac{1}{2R_P} + \frac{1}{2g_m R_P R_D} \right) = -g_m (V_{in1} - V_{in2}) + \frac{V_{in2}}{R_P}$$

$$V_{in2} = \frac{V_{in1} + V_{in2}}{2} - \frac{V_{in1} - V_{in2}}{2}$$

$$\Rightarrow V_{out} \left(\frac{1}{R_D} + \frac{1}{2R_P} + \frac{1}{2g_m R_P R_D} \right) = - \left(g_m + \frac{1}{2R_P} \right) (V_{in1} - V_{in2}) + \frac{(V_{in1} + V_{in2})}{2R_P}$$

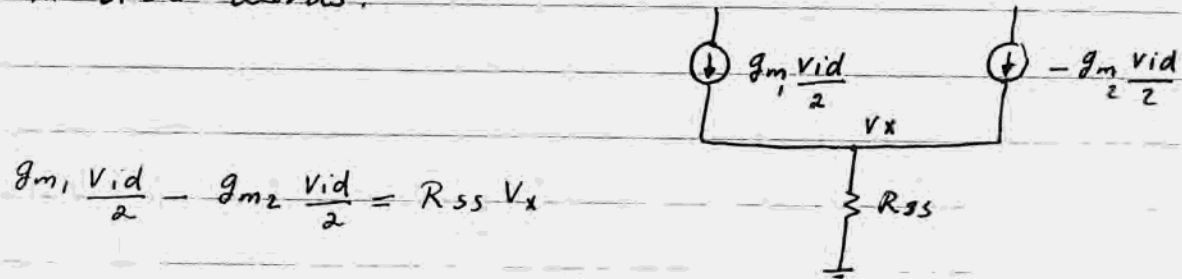
$$V_{out} = \left[- \left(g_m + \frac{1}{2R_P} \right) (V_{in1} - V_{in2}) + \frac{V_{in1} + V_{in2}}{2R_P} \right] (R_D \parallel 2R_P \parallel 2g_m R_P R_D)$$

$$CMRR = \frac{g_m + \frac{1}{2R_P}}{\frac{1}{R_P}} = \frac{2R_P g_m + 1}{2}$$

$$A_{dm-dm} = - \left(g_m + \frac{1}{2R_P} \right) (R_D \parallel 2R_P \parallel 2g_m R_P R_D) \quad A_{cm-dm} = - \frac{R_D \parallel 2R_P \parallel 2g_m R_P R_D}{R_P}$$

4.10

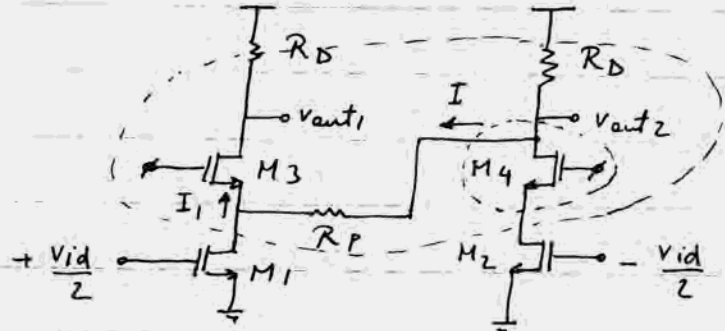
$\lambda = 0$, so for a differential input, symmetry in the input is enough to for the tail node to be grounded.
in other words:



$$g_{m1} \frac{v_{id}}{2} - g_{m2} \frac{v_{id}}{2} = R_{SS} v_x$$

$$g_{m1} = g_{m2} \Rightarrow v_x = 0$$

$$(v_{gs1} = v_{gs2} \Rightarrow g_{m1} = g_{m2} \\ \text{but } g_{m3} \neq g_{m4})$$



For the cutsets shown:

$$\frac{v_{out1}}{R_D} + \frac{v_{out2}}{R_D} + \frac{v_{id}}{2} g_{m1} - \frac{v_{id}}{2} g_{m2} = 0 \Rightarrow v_{out1} + v_{out2} = 0 \quad (1)$$

$$\text{KCL: } \frac{v_{out2}}{R_D} + I - \frac{v_{id}}{2} g_{m2} = 0 \quad g_{m1} = g_{m2} \Rightarrow I = \frac{v_{id} g_{m1}}{2} - \frac{v_{out2}}{R_D} \quad (2)$$

$$\text{KVL: } v_{out2} = I R_P + \frac{I_1}{g_{m3}}, \quad \text{but } I_1 = \frac{v_{out1}}{R_D}$$

$$\Rightarrow v_{out2} = I R_P + \frac{v_{out1}}{R_D g_{m3}} \quad (3)$$

$$(2), (3) \Rightarrow v_{out2} = \left(\frac{v_{id} g_{m1}}{2} - \frac{v_{out2}}{R_D} \right) R_P + \frac{v_{out1}}{R_D g_{m3}}$$

$$v_{out1} = -v_{out2} \Rightarrow -v_{out1} \left(1 + \frac{R_P}{R_D} + \frac{1}{R_D g_{m3}} \right) = \frac{g_{m1} R_P}{2} v_{id}$$

4.15

$$A_{vd} = \frac{V_{out1} - V_{out2}}{V_{id}} = 2 \frac{V_{out1}}{V_{id}} = \frac{g_{m1} R_P}{1 + \frac{R_P}{R_D} + \frac{1}{R_D g_{m3}}}$$

$$= \frac{g_{m1} R_D}{1 + \frac{R_D}{R_P} + \frac{1}{R_P g_{m3}}} = \frac{g_{m1} R_D}{1 + \frac{1}{R_P} (R_D + \frac{1}{g_{m3}})}$$

Since $\lambda = 0 \Rightarrow r_{o5} = \infty \Rightarrow A_{cm} = 0 \Rightarrow CMRR = \infty$

4.11

$$I_D = \frac{1}{2} \mu_p C_{ox} \frac{W}{L} [2(V_{GS} - V_{TH,P})V_{DS} - V_{DS}^2] \approx \mu_p C_{ox} \frac{W}{L} (V_{GS} - V_{TH,P})V_{DS}$$

$$R_{on} = \frac{|V_{DS}|}{I_D} = \frac{1}{\mu_p C_{ox} (V_{GS} - V_{TH,P})}$$

$$\text{if } R_{on} = 2 \text{ k}\Omega \Rightarrow |V_{GS3} - V_{TH1,P}| = \frac{1}{2 \text{ k} \times 38.3 \mu \text{A/V}} = 0.131 \text{ V}$$

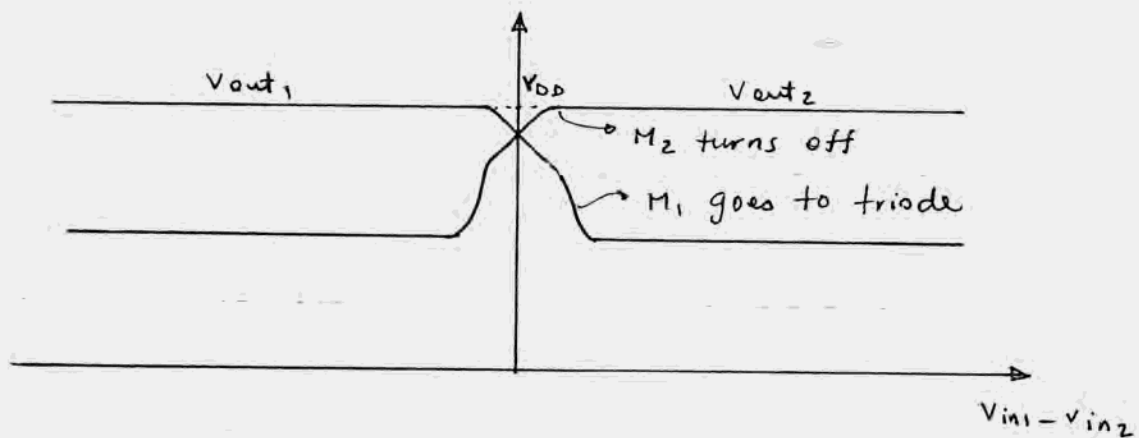
$$I_{SS} = 20 \mu\text{A} \Rightarrow V_{GS1} = V_{TH1,N} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} = 0.7 + \sqrt{\frac{2 \times 10^{-6}}{0.134 \text{ m} \times 100}} = 0.739 \text{ V}$$

$$V_{DD} = |V_{GS3}| - V_{GS1} + V_{in,cm}$$

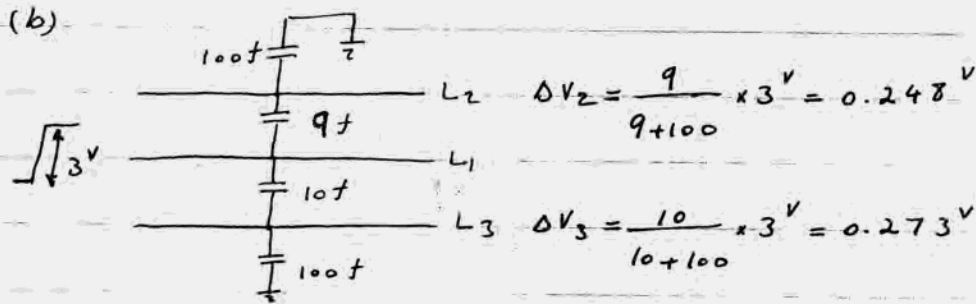
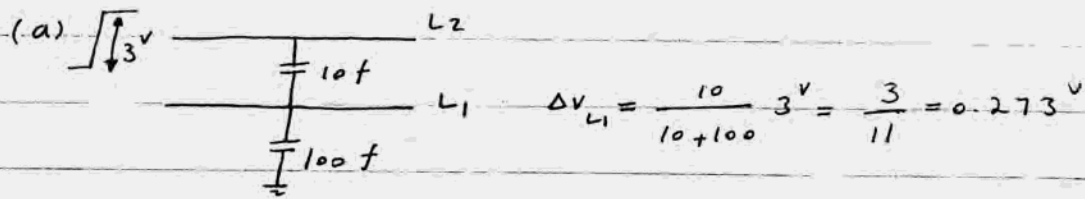
$$\Rightarrow V_{in,cm} = 3 - (0.131 + 0.8) + 0.739 = 2.81 \text{ V}$$

$$|V_{DS3}| = R \frac{I_{SS}}{2} = 2 \text{ k} \times 10 \mu\text{A} = 20 \text{ mV} \Rightarrow |V_{DS3}| < |V_{GS3} - V_{TH1,P}| \Rightarrow M_3 \text{ \& } M_4 \text{ are in triode}$$

$$V_{D1} - V_{G1} = (3 - 20 \text{ mV}) - 2.81 = 0.17 \text{ V} > -V_{TH1,N} \Rightarrow M_1 \text{ \& } M_2 \text{ are in Sat.}$$



4.12

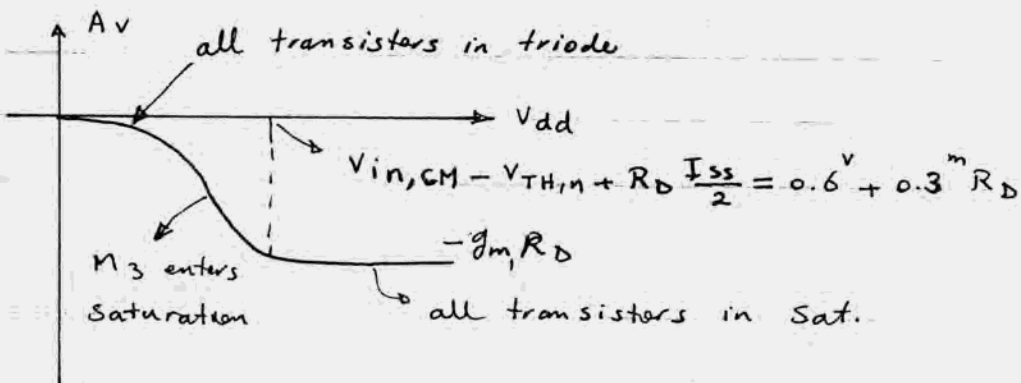


$$\Rightarrow \Delta V_{23} = 0.273 - 0.248 = 25 \text{ mV}$$

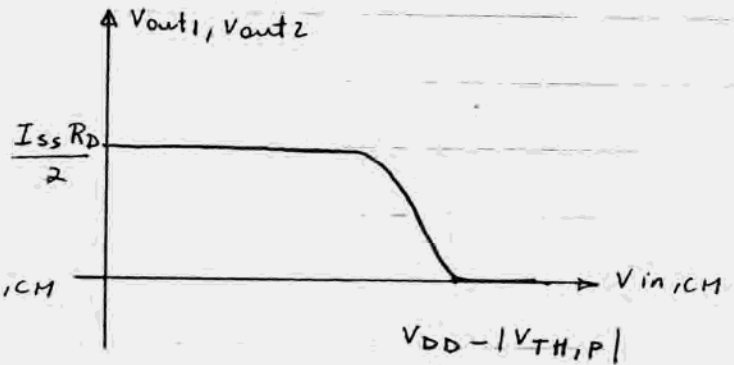
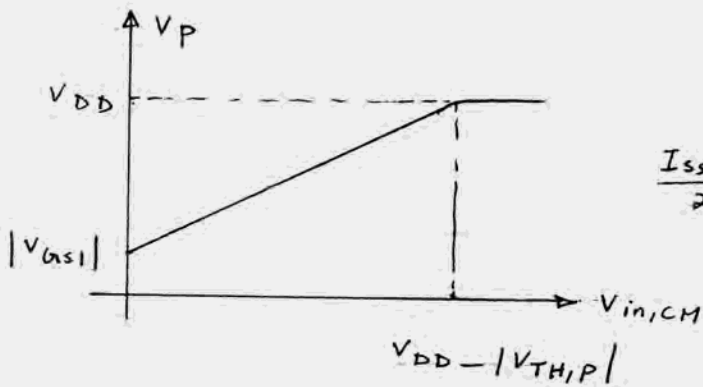
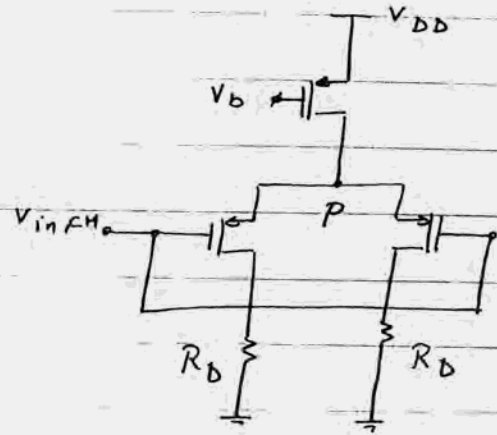
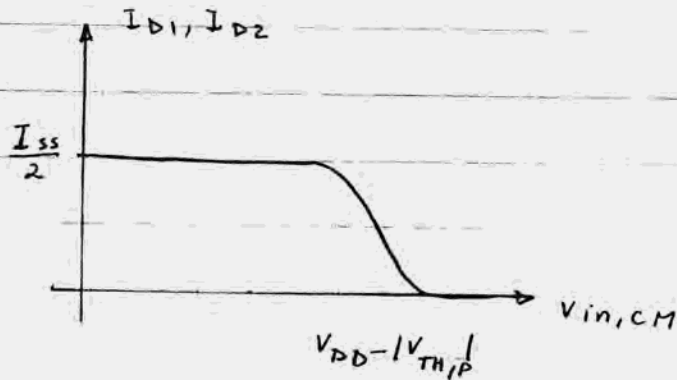
4.13

Fig. 4.8 (a) $I_{SS} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_3 (V_{GS3} - V_{TH,n})^2$

$$V_{GS3} = V_b = 1V \Rightarrow I_{SS} = 0.5 \times 0.134^m \times 100 (1 - 0.7)^2 = 0.603 \text{ mA}$$



4.14



$$I_{D1,2} = \frac{V_{out1,2}}{R_D}$$

4.15 (a) $(V_{out1,2})_{max} = V_{DD} = 3V$

$$(V_{out1,2})_{min} = V_{in,CM} - V_{TH,N} = 1.2 - 0.7 = 0.5V$$

Max swing at $V_{out} = 2(3 - 0.5) = 5V$

(b) $A_v = -g_m R_D$ $g_m = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L}\right) I_D} = \sqrt{2 \times 0.134 \times 100 \times \frac{0.5}{2}} = 2.59 \text{ m}$

to get max swing: $R_D \frac{I_{SS}}{2} = \frac{(V_{out1,2})_{max} - (V_{out1,2})_{min}}{2} = 1.25$

$\Rightarrow R_D = 5 \text{ k}\Omega$ $\Rightarrow A_v = -2.59 \text{ m} \times 5 \text{ k} = -13$

4.16

$$(a) \quad V_{GS} - V_{TH} = \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} = \sqrt{\frac{2 \times 0.5 \text{ mA}}{0.134 \text{ mA/V}^2 \times 100}} = 0.273 \text{ V}$$

$$(b) \quad \Delta I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{in2}) \sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - (V_{in1} - V_{in2})^2}$$

(4.9)

$$\Delta I_D = \frac{1}{2} \times 0.134 \text{ mA/V}^2 \times \frac{50}{0.5} \times 50 \text{ m} \sqrt{\frac{4 \times 1 \text{ mA}}{0.134 \text{ mA/V}^2 \times \frac{50}{0.5}} - (50 \text{ m})^2}$$

$$\Delta I_D = 182 \text{ } \mu\text{A} \Rightarrow \begin{cases} I_{D1} = 0.5 \text{ mA} + \frac{0.182 \text{ mA}}{2} = 0.591 \text{ mA} \\ I_{D2} = 0.5 \text{ mA} - \frac{0.182 \text{ mA}}{2} = 0.409 \text{ mA} \end{cases}$$

(c)

$$G_m = \frac{\partial I_D}{\partial \Delta V_{in}} = \frac{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - 2 \Delta V_{in}}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - \Delta V_{in}^2}} \quad (4.10)$$

$$\Delta V_{in} = 50 \text{ mV} \Rightarrow G_m = 3.61 \text{ m}\Omega^{-1}$$

$$(d) \quad \Delta V_{in} = 0 \Rightarrow G_{m0} = 3.66 \text{ m}\Omega^{-1}$$

$$G_m = \frac{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} \frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - 2 \Delta V_{in}}{\sqrt{\frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}} - \Delta V_{in}^2}}$$

$$\text{if we define } A = \frac{4I_{SS}}{\mu_n C_{ox} \frac{W}{L}}, \quad B = \left(\frac{G_m}{\frac{1}{2} \mu_n C_{ox} \frac{W}{L}} \right)^2$$

$$\Rightarrow B = \frac{(A - 2\Delta V_{in})^2}{A - \Delta V_{in}^2} \quad 4\Delta V_{in}^4 - (4A - B)\Delta V_{in}^2 + A^2 - AB = 0$$

$$\Delta V_{in}^2 = \frac{4A - B \pm \sqrt{(4A - B)^2 - 16(A^2 - AB)}}{8}$$

taking the smaller value: $\Delta V_{in}^2 = \frac{4A - B - \sqrt{8AB + B^2}}{8}$

So for different value of G_m , B can be calculated and then ΔV_{in} is found.

$$G_{m_{10\%}} = 0.9 \times 3.66^m = 3.294 \text{ m}\Omega^{-1} \Rightarrow |\Delta V_{in}| = 139 \text{ mV} \quad 10\% \text{ drop}$$

$$G_{m_{90\%}} = 0.1 \times 3.66^m = 0.366 \text{ m}\Omega^{-1} \Rightarrow |\Delta V_{in}| = 372 \text{ mV} \quad 90\% \text{ drop}$$

4.17

(a) $V_{od} = V_{GS} - V_{TH} = 0.386 \text{ V}$

(b) $\Delta I_D = 0.129 \text{ mA} \Rightarrow I_{D1} = 0.565 \text{ mA}$
 $I_{D2} = 0.435 \text{ mA}$

(c) $G_m = 2.57 \text{ m}\Omega^{-1}$

(d) $\Delta V_{in} = 0 \Rightarrow G_{m0} = 2.59 \text{ m}\Omega^{-1}$

$$G_{m_{10\%}} = 0.9 \times G_{m0} = 0.9 \times 2.59^m \Rightarrow \Delta V_{in} = 197 \text{ mV}$$

$$G_{m_{90\%}} = 0.1 \times G_{m0} = 0.1 \times 2.59^m \Rightarrow \Delta V_{in} = 526 \text{ mV}$$

For a given current, by reducing $\frac{W}{L}$, overdrive voltage

increases while G_m decreases. In this case for a fixed ΔV_{in} , I_D and G_m change less so the circuit has a wider linear range.

4.18

$$(a) \quad V_{od} = 0.386 \text{ V}$$

$$(b) \quad \Delta I_D = 0.258 \text{ mA} \quad \left\{ \begin{array}{l} I_{D1} = 1.13 \text{ mA} \\ I_{D2} = 0.87 \text{ mA} \end{array} \right.$$

$$(c) \quad G_m = 5.14 \text{ m}\Omega^{-1}$$

$$(d) \quad \Delta V_{in} = 0 \rightarrow G_{m0} = 5.18 \text{ m}\Omega^{-1}$$

$$G_{m_{10\%}} = 0.9 \times 5.18 \text{ m} \Rightarrow \Delta V_{in} = 197 \text{ mV}$$

$$G_{m_{90\%}} = 0.1 \times 5.18 \text{ m} \Rightarrow \Delta V_{in} = 526 \text{ mV}$$

In this case v_{od} and G_m have increased but

the linearity range of G_m is same as (p. 4.17)

$$4.19 \quad I_{D1} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH,n})^2$$

$$I_{D2} = \frac{1}{2} \mu_n C_{ox} \frac{2W}{L} (V_{in2} - V_{TH,n})^2$$

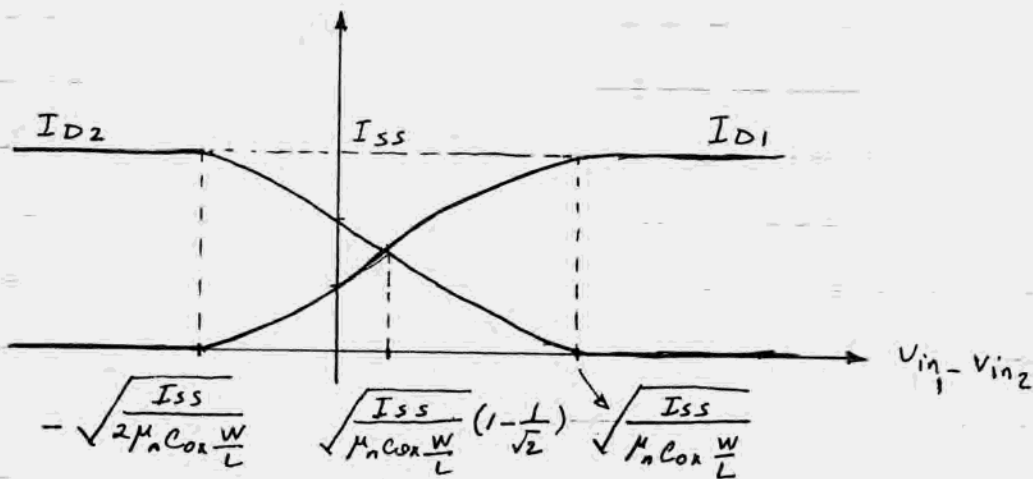
$$I_{D1} + I_{D2} = I_{SS}$$

if $I_{D1} = I_{D2}$ then:

$$I_{D1} = \frac{I_{SS}}{2} \Rightarrow v_{in1} = v_{TH,n} + \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}}$$

$$I_{D2} = \frac{I_{SS}}{2} \Rightarrow v_{in2} = v_{TH,n} + \sqrt{\frac{I_{SS}}{2\mu_n C_{ox} \frac{W}{L}}}$$

$$v_{in1} - v_{in2} = \sqrt{\frac{I_{SS}}{\mu_n C_{ox} \frac{W}{L}}} \left(1 - \frac{1}{\sqrt{2}}\right)$$



$$\text{if } v_{in1} = v_{in2} \Rightarrow I_{D1} = \frac{I_{SS}}{3}, \quad I_{D2} = \frac{2I_{SS}}{3}$$

4.20

$$A_{dm-dm} = -g_m R_D$$

$$A_{cm-dm} = \frac{g_m}{1 + 2g_m R_{SS}} R_D - \frac{g_m}{1 + 2g_m R_{SS}} (R_D + DR_D)$$

$$A_{cm-dm} = - \frac{g_m}{1 + 2g_m R_{SS}} \Delta R_D$$

$$SNR = \frac{(A_{dm-dm} \cdot V_{in,dm})^2}{(A_{cm-dm} \cdot V_{in,cm})^2} = \left(\frac{g_m R_D \times 10^m}{\frac{g_m}{1 + 2g_m R_{SS}} \Delta R_D \times 100^m} \right)^2$$

$$SNR = \frac{(1 + 2g_m R_{SS})^2}{\left(\frac{\Delta R}{R}\right)^2} \cdot \left(\frac{1}{10}\right)^2$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = 3.66 \text{ mS}^{-1} \quad (I_D = 0.5 \text{ m})$$

$$L_{SS} = 0.5 \mu\text{m} \Rightarrow \lambda = 0.1 \text{ V}^{-1} \Rightarrow R_{SS} = \frac{1}{\lambda I_D} = \frac{1}{0.1 \times 1^{\text{m}}} = 10 \text{ k}\Omega$$

$$\Rightarrow SNR = \left(\frac{1 + 2 \times 3.66^{\text{m}} \times 10^{\text{k}}}{0.05} \right)^2 \left(\frac{1}{10} \right)^2 = 22000$$

$$\text{or } SNR = 10 \log 22000 = 43.4 \text{ dB}$$

$$CMRR = \left| \frac{A_{dm-dm}}{A_{cm-dm}} \right| = \frac{g_m R_D}{\frac{g_m \Delta R_D}{1 + 2g_m R_{SS}}} = \frac{1 + 2g_m R_{SS}}{\Delta R_D / R_D}$$

$$CMRR = 1484 \quad \text{or} \quad CMRR = 20 \log 1484 = 63.4 \text{ dB}$$

$$4.21 \quad A_{cm-dm} = - \frac{\Delta g_m R_D}{(g_{m1} + g_{m2}) R_{SS} + 1} \quad g_{m1} + g_{m2} = 2g_m$$

$$SNR = \left(\frac{A_{dm-dm} \cdot V_{in-dm}}{A_{cm-dm} \cdot V_{in-cm}} \right)^2 = \left(\frac{g_m R_D}{\frac{\Delta g_m R_D}{(g_{m1} + g_{m2}) R_{SS} + 1}} \right)^2 \left(\frac{10^{\text{m}}}{100^{\text{m}}} \right)^2$$

$$SNR = \left(\frac{2g_m R_{SS} + 1}{\frac{\Delta g_m}{g_m}} \right)^2 \times \left(\frac{1}{10} \right)^2$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \Rightarrow \Delta g_m = -\mu_n C_{ox} \frac{W}{L} \Delta V_{TH}$$

$$|\Delta g_m| = 0.134^m \times 100 \times 1^{mv} = 13.4 \mu A^{-1}$$

$$\Rightarrow SNR = \frac{(2 \times 3.66^m \times 10^k + 1)^2}{\left(\frac{13.4 \mu}{3.66^m} \right)^2} \times \left(\frac{1}{10} \right)^2 = 4.1 \times 10^6$$

$$\text{or } SNR = 10 \log 4.1 \times 10^6 = 66.1 \text{ dB}$$

$$CMRR = \left| \frac{A_{dm-dm}}{A_{cm-dm}} \right| = \frac{1 + 2g_m R_{SS}}{\Delta g_m / g_m} = 20300$$

$$\text{or } CMRR = 20 \log 20300 = 86.1 \text{ dB}$$

4.22 (a)

$$\left(\frac{W}{L} \right)_{SS} = \frac{50}{0.5}, I_{SS} = 0.5 \text{ mA}$$

$$V_{od_{SS}} = V_{GS3} - V_{TH} = \sqrt{\frac{2 I_{SS}}{\mu_n C_{ox} \left(\frac{W}{L} \right)_{SS}}} = 0.273 \text{ V}$$

$$I_{D1} = \frac{I_{SS}}{2} = 0.25 \text{ mA} \Rightarrow V_{od1} = \sqrt{\frac{2 I_{D1}}{\mu_n C_{ox} \frac{W}{L}}} = 0.193 \text{ V}$$

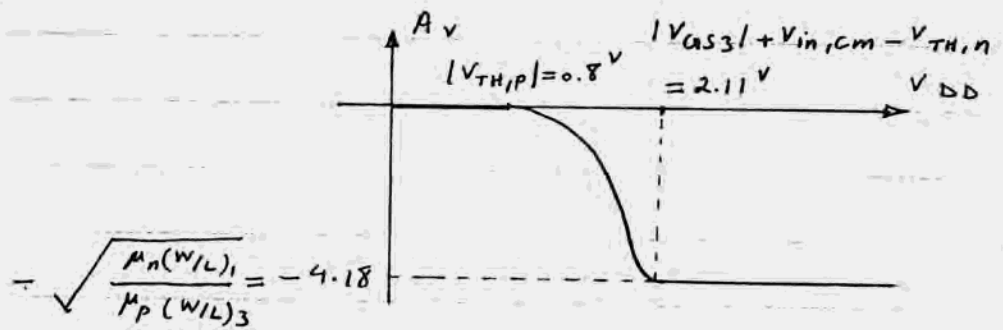
$$(V_{in,cm})_{min} = V_{GS1} + V_{od_{SS}} = 0.7 + 0.193 + 0.273 = 1.17 \text{ V}$$

$$(V_{in,cm})_{max} = V_{DD} - |V_{GS3}| + V_{TH,N}$$

$$|V_{GS3}| = |V_{TH,P}| + \sqrt{\frac{2I_{D3}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}} = 1.61 \text{ V}$$

$$(V_{in,cm})_{max} = 3 - 1.61 + 0.7 = 2.09 \text{ V}$$

$$(b) \quad V_{in,cm} = 1.2 \text{ V}$$



4.23

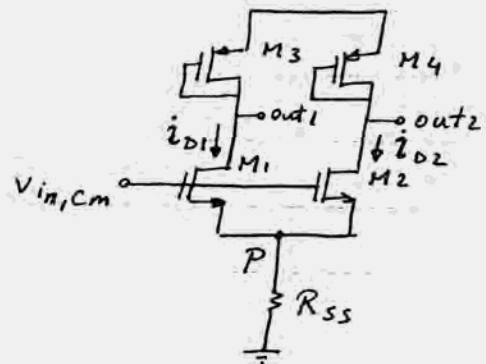
This mismatch in V_{TH} of M_1 and M_2 makes I_{D1} and I_{D2} unequal. Therefore $g_{m1} \neq g_{m2}$, $g_{m3} \neq g_{m4}$. Using the equivalent circuit below to calculate A_{cm-dm} , we have:

$$i_{D1} = g_{m1} (V_{in,cm} - V_P)$$

$$i_{D2} = g_{m2} (V_{in,cm} - V_P)$$

$$V_{out1} = -\frac{i_{D1}}{g_{m3}} = -\frac{g_{m1}}{g_{m3}} (V_{in,cm} - V_P)$$

$$V_{out2} = -\frac{i_{D2}}{g_{m4}} = -\frac{g_{m2}}{g_{m4}} (V_{in,cm} - V_P)$$



$$\frac{g_{m1}}{g_{m3}} = \frac{\sqrt{2I_{D1}\mu_n C_{ox} \left(\frac{W}{L}\right)_{1,2}}}{\sqrt{2I_{D3}\mu_p C_{ox} \left(\frac{W}{L}\right)_{3,4}}} = \sqrt{\frac{\mu_n \left(\frac{W}{L}\right)_{1,2}}{\mu_p \left(\frac{W}{L}\right)_{3,4}}}, \quad \text{Similarly} \quad \frac{g_{m2}}{g_{m4}} = \sqrt{\frac{\mu_n \left(\frac{W}{L}\right)_{1,2}}{\mu_p \left(\frac{W}{L}\right)_{3,4}}}$$

$$\Rightarrow V_{out1} = V_{out2} \Rightarrow A_{cm-dm} = 0, \quad CMRR = \infty$$

4.24

$$P \quad 4.20 \quad CMRR = \frac{1 + 2g_{m1}R_{SS}}{\Delta R_D / R_D}$$

$$R_{D1} = \frac{1}{g_{m3}}, \quad R_{D2} = \frac{1}{g_{m4}}$$

$$\frac{\Delta R_D}{R_D} = \frac{R_{D1} - R_{D2}}{R_{D1}} = 1 - \frac{R_{D2}}{R_{D1}} = 1 - \frac{g_{m3}}{g_{m4}} = 1 - \frac{\sqrt{2\mu_p C_{ox}(\frac{W}{L})_3 I_D}}{\sqrt{2\mu_p C_{ox}(\frac{W}{L})_4 I_D}}$$

$$\frac{\Delta R_D}{R_D} = 1 - \sqrt{\frac{10}{11}} = 0.0465$$

$$\Rightarrow CMRR = 2248 \quad \text{or} \quad CMRR = 20 \log 2248 = 67 \text{ dB}$$

4.25

$$(a) \quad A_V = -g_{m1}(r_{o1} \parallel r_{o3})$$

$$g_{m1} = 3.66 \text{ mS}^{-1} \quad r_{o1} = \frac{1}{\lambda I_D} = \frac{1}{0.1 \times 0.5 \text{ m}} = 20 \text{ k}\Omega$$

$$r_{o3} = \frac{1}{\lambda I_D} = \frac{1}{0.2 \times 0.5 \text{ m}} = 10 \text{ k}\Omega \quad \Rightarrow \quad A_V = -3.66 \text{ m} (10 \text{ k} \parallel 20 \text{ k}) = -24.4$$

$$(b) \quad (V_{out,1,2})_{\min} = 1.5 - V_{TH,n} = 1.5 - 0.7 = 0.8 \text{ V}$$

$$(V_{out,1,2})_{\max} = V_{DD} - (|V_{DS3}| - |V_{TH,p}|)$$

$$= V_{DD} - \sqrt{\frac{2I_D}{\mu_p C_{ox}(\frac{W}{L})_3}} = 3 - \sqrt{\frac{2 \times 0.5 \text{ m}}{38.3 \mu\text{A/V}^2}} = 2.49 \text{ V}$$

$$\text{Max swing of } V_{out} = 2(2.49 - 0.8) = 3.38 \text{ V}$$

4.26

$$P \ 4.20 : \quad CMRR = \frac{1 + 2g_{m1} R_{SS}}{\Delta R_D / R_D}$$

$$R_D = \frac{1}{g_{m3} \parallel r_{o1} \parallel r_{o3} \parallel r_{o5}} \approx \frac{1}{g_{m3}}$$

$$I_{D3} + I_{D5} = I_{D4} + I_{D6} = \frac{I_{SS}}{2} \Rightarrow \Delta I_{D3} = -\Delta I_{D5} \quad (1)$$

$$I_{D5} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{GS5} - V_{TH1P})^2 \Rightarrow \frac{\partial I_{D5}}{\partial V_{TH1P}} = -\mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{GS5} - V_{TH1P})$$

$$\Delta I_{D5} \approx \mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{DD} - V_b - |V_{TH1P}|) \Delta V_{TH1P} \quad (2)$$

$$g_{m3} = \sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right)_3 I_{D3}} \quad \frac{\partial g_{m3}}{\partial I_{D3}} = \sqrt{\frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}{2 I_{D3}}}$$

$$\Rightarrow \Delta g_{m3} \approx \sqrt{\frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}{2 I_{D3}}} \Delta I_{D3} \quad (3)$$

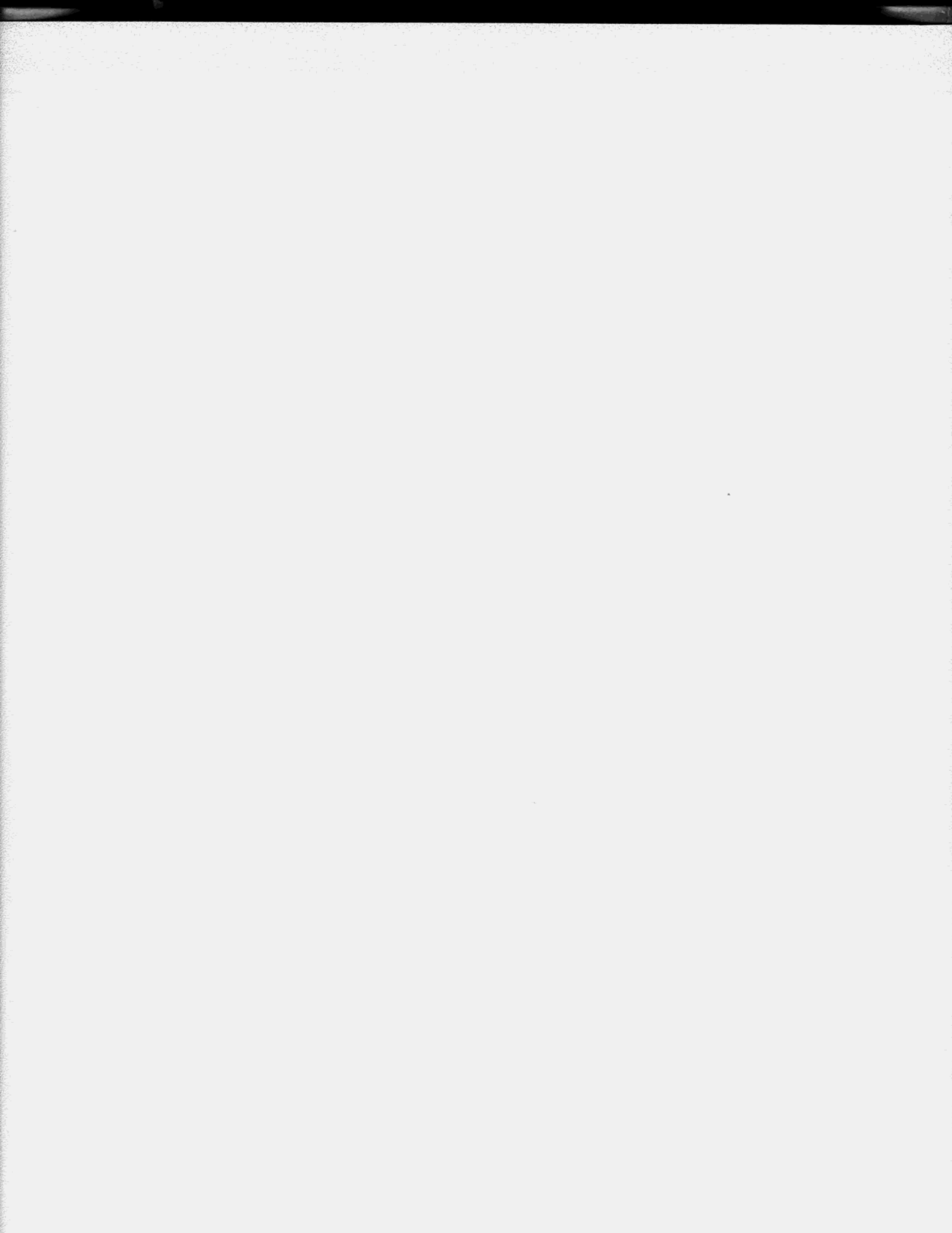
$$R_D = \frac{1}{g_{m3}} \quad \frac{\partial R_D}{\partial g_{m3}} = -\frac{1}{g_{m3}^2} \Rightarrow \frac{\Delta R_D}{R_D} \approx -\frac{\Delta g_{m3}}{g_{m3}}$$

$$\begin{aligned} (1), (2), (3) \Rightarrow \frac{\Delta R_D}{R_D} &= -\frac{\Delta g_{m3}}{g_{m3}} = -\frac{\sqrt{\frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_3}{2 I_{D3}}} \Delta I_{D3}}{\sqrt{2 \mu_p C_{ox} \left(\frac{W}{L}\right)_3 I_{D3}}} = -\frac{\Delta I_{D3}}{2 I_{D3}} \\ &= \frac{\Delta I_{D5}}{2 I_{D3}} = \frac{\mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{DD} - V_b - |V_{TH1P}|) \Delta V_{TH1P}}{2 I_{D3}} = \frac{I_{D5}}{I_{D3}} \frac{\Delta V_{TH1P}}{V_{DD} - V_b - |V_{TH1P}|} \end{aligned}$$

$$\text{or } \frac{\Delta R}{R} = \frac{I_{D5}}{I_{D3}} \frac{V_{TH1P}}{V_{DD} - V_b - |V_{TH1P}|} \cdot \frac{\Delta V_{TH1P}}{V_{TH1P}}$$

$$\text{but } \frac{I_{D5}}{I_{D3}} = 4$$

$$\Rightarrow \frac{\Delta R_D}{R_D} = 4 \cdot \frac{V_{TH1P}}{V_{DD} - V_b - |V_{TH1P}|} \cdot \frac{\Delta V_{TH1P}}{V_{TH1P}}$$



S.1 (a) M1 and M2 are off when $V_{DD} < V_{TH1,2}$.

At this time, $V_{x,y} = V_{DD}$ because $I_{1,2} = 0$.

Once $V_{DD} \geq V_{TH1,2}$, M1 and M2 turn on.

Since M1 and M2 are symmetric and the

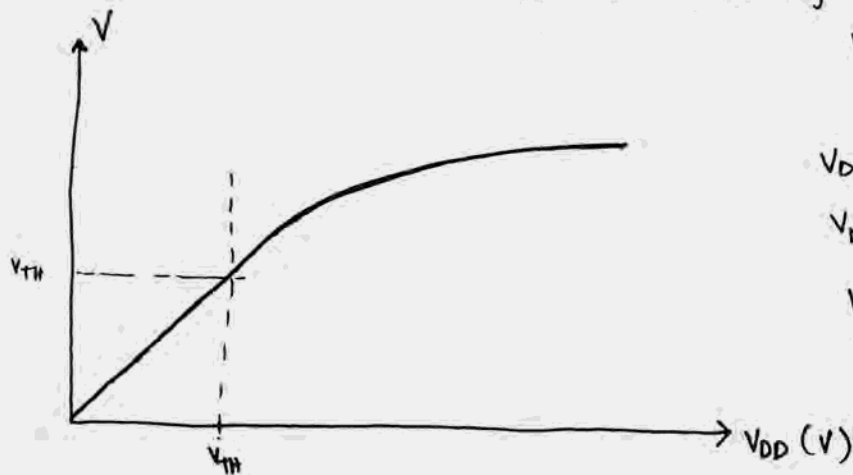
V_{gs} for both are the same $I_1 = I_2$ and

$V_x = V_y = V_{gs2,1}$. V_{gs} is a quadratic solution as a function of V_{DD} and follows the below relationship:

$$V_{gs2} = V_{DD} - I_2 R$$

$$= V_{DD} - \frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{gs2} - V_{th})^2 \cdot R$$

$$K = \frac{1}{2} \mu C_{ox} \frac{W}{L}$$

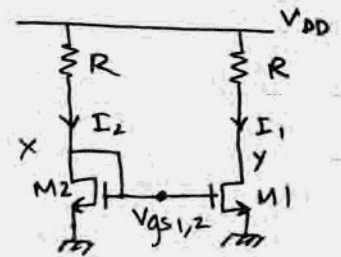


$$V_{DD} < V_{TH}: V_x = V_y = V_{DD}$$

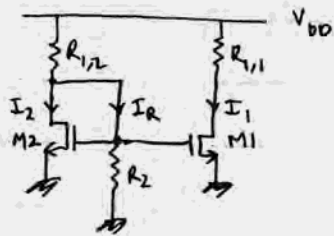
$$V_{DD} \geq V_{TH}:$$

$$V_{gs} = \frac{2V_{th} - \frac{1}{KR} + \sqrt{(2V_{th} - \frac{1}{KR})^2 - 4(\frac{V_{th}^2 - V_{DD}}{KR})}}{2}$$

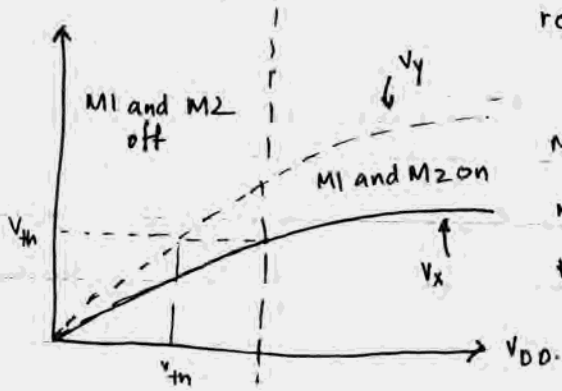
(b) We have the same solution as S.1(a) because if M1, M2 are symmetric and $V_{gs1} = V_{gs2}$ then $V_x = V_y$ and $I_1 = I_2$, in this case, no current ever flows through R_2 .



5.1 (c)



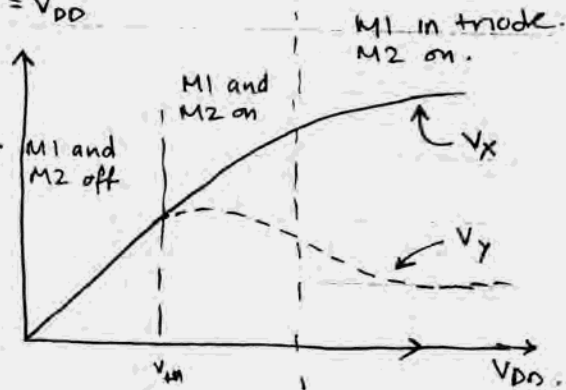
When $V_{DD} < V_{th,1,2}$ no current flows through M1 and M2. the only current that flows is I_R through $R_{1,2}$ and R_2 . V_x is set by the resistor divider $V_x = V_{DD} \cdot \frac{R_2}{R_{1,2} + R_2}$ and $V_y = V_{DD}$



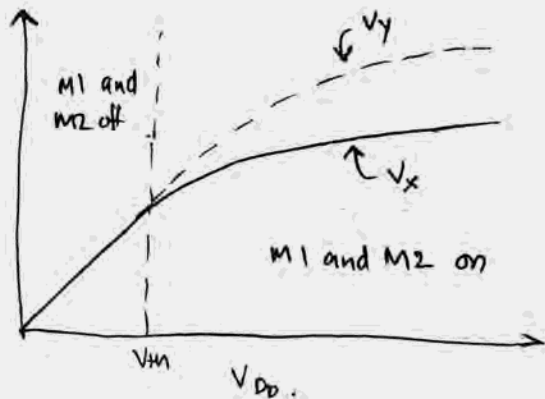
M1 and M2 turn on only when $V_x \geq V_{th,1,2}$. Once M1 and M2 are on, I_1 and I_2 are equal because they have the same V_{gs} . But $V_y > V_x$ because the current through $R_{1,2}$ consist of I_2 plus I_R which is greater than the current through $R_{1,1}$.

(d) When $V_{DD} < V_{th,1,2}$, $I_1 = I_2 = 0$ and $V_x = V_y = V_{DD}$

Once $V_{DD} \geq V_{th,1,2}$, M1 and M2 turn on. the R_2 at the source of M2 causes $V_{gs2} < V_{gs1}$. Thus $I_2 < I_1$ and $V_x > V_y$. At some point, V_{gs1} becomes so large that M1 goes into triode as seen in the graph.

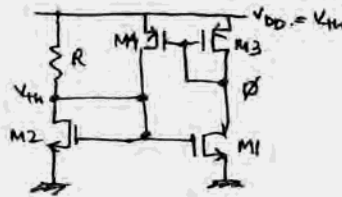


(e) Here when $V_{DD} > V_{th,1,2}$, $V_{gs2} > V_{gs1}$ and $I_1 < I_2$. Thus $V_x < V_y$, but because M2 is diode connected, M2 never goes into triode.

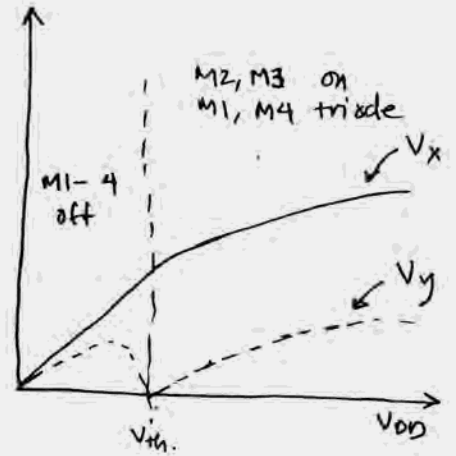


5.2 (a) When $V_{DD} < V_{th1,2}$, all transistors are off. $V_x = V_{DD}$ and V_y is floating between GND and V_{DD} since it is isolated. From either node.

Once $V_{DD} = V_{th1,2}$ all transistors turn on and the voltages at nodes are as follows:

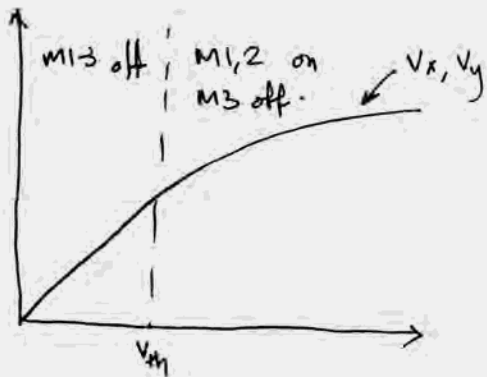


$V_x = V_{DD} = V_{th2}$ and $V_y = V_{DD} - V_{th3} \approx 0$ if we assume $V_{th2} = V_{th3}$. M1 and M4 are in triode and stay in triode always as $V_{DD} > V_{th2}$.

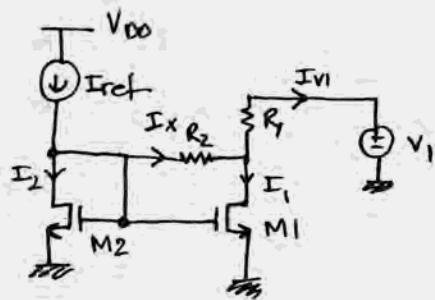


As $V_{DD} > V_{th2}$, $V_x = V_{gs2}$ and $V_y = V_{DD} - V_{gs3} \approx V_{DD} - V_{gs2}$. $V_x > V_y$.

(b) When $V_{DD} < V_{th1,2}$, $I_1, I_2 = 0$ and $V_x = V_y = V_{DD}$.
 When $V_{DD} \geq V_{th1,2}$, $I_1 = I_2$ because $V_{gs1} = V_{gs2}$ and therefore $V_x = V_y$. Since $V_x \approx V_y$, $V_{gs3} = 0$ and M3 is always off.



5.3 (a)



for V_1 : $0 \leq V_1 \leq V_{DD}$ M_1 and M_2 are always on: $I_1 = I_2$
 V_y and V_x are related as a function of R_1 , R_2 and V_1 .

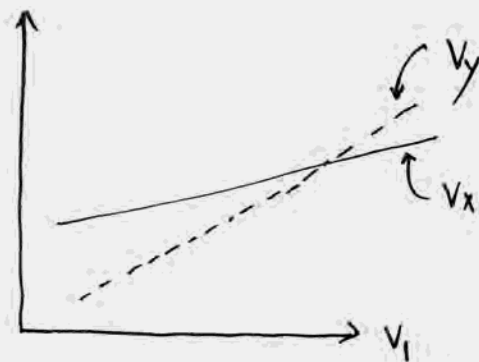
$$(1) \quad 2I_1 = I_{ref} - I_{V1} = I_{ref} - \frac{V_y - V_1}{R_1}$$

$$(2) \quad I_1 = \frac{V_x - V_y}{R_2} - \frac{V_y - V_1}{R_1}$$

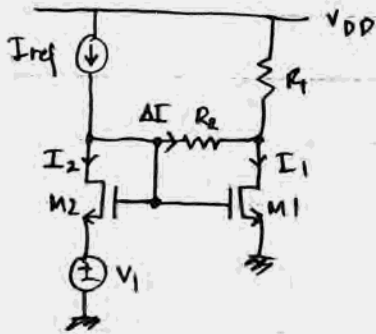
Solving these 2 equations for V_y , we get

$$V_y = \frac{R_2 V_1 + 2R_2 V_x - I_{ref}}{R_1 + R_2}$$

as V_1 increase, $I_{1,2}$ increase and therefore V_x and V_y increase.
 The slope of V_y is greater because it is a linear combination of $V_x + V_1$, but V_y starts off less than V_x because of the constant subtraction of $I_{ref}/(R_1 + R_2)$.

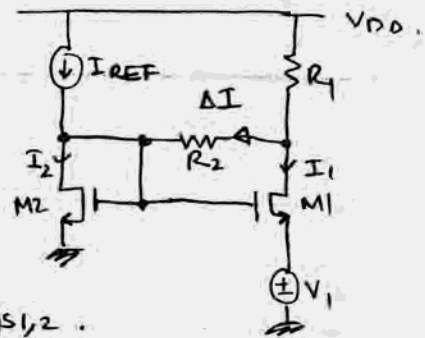
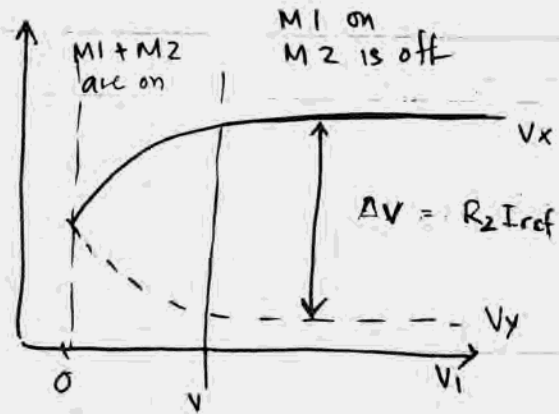


5.3 (b) When $V_1 = 0$, $I_1 = I_2 = I_{ref}$ and $V_x \approx V_y$, $\Delta I = 0$



as V_1 increases, I_2 gradually decreases and part of I_{ref} flows through R_2 . V_x increases and $V_y = V_x - R_2 \Delta I$, decreases.

Finally when V_1 is large enough such that M_2 turns off $V_y = V_x - R_2 I_{ref}$ and both V_x and V_y are set at a constant voltage.



(c) When $V_1 = 0$, $I_1 = I_2$, $V_x \approx V_y$. Here

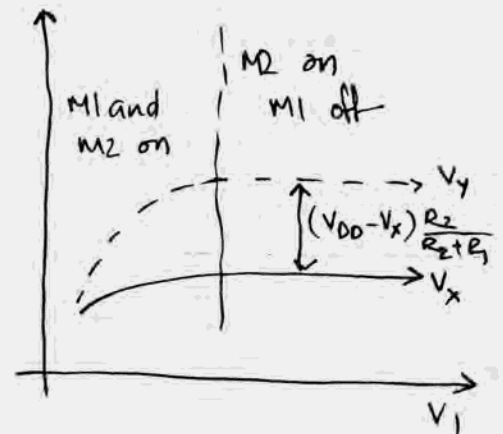
maybe small variations if $V_{DD} - R_1 I_1 \neq V_{gs1,2}$.

as V_1 increases, I_1 decreases and the extra current flows through M_2 . I_2 increases.

$$V_y = V_x + \Delta I R_2$$

Once V_1 gets large enough, M_1 shuts off.

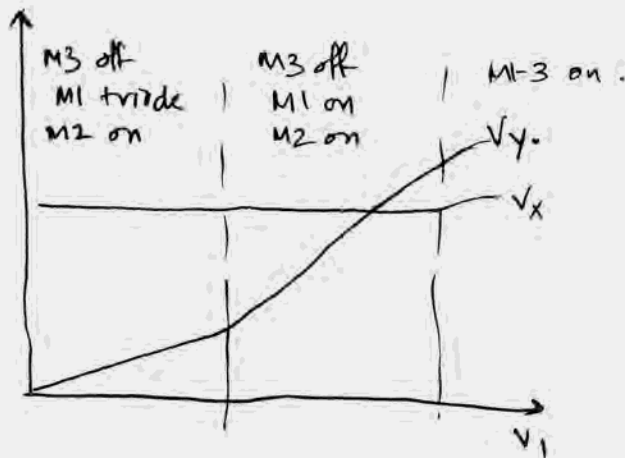
$$V_y = \frac{(V_{DD} - V_x) R_2}{R_1 + R_2} + V_x$$



5.4 (a) V_x is constant until V_i gets high enough that $V_y - V_x$ is greater than V_{th3} .

Initially $M1$ is in triode with $V_y = V_i \frac{1}{1 + g_{m1} R_2}$ until $M1$ is in saturation.

When $M1$ is sat., $V_y = V_i - I_{REF} R_1$.



(b). When $V_i = 0$, $M1$ and $M2$ are off and $M3$ is on, but in a flipped position, source and drain switch as show below.

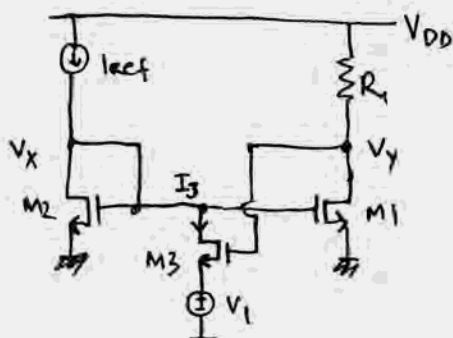
$V_y = V_{DD}$ and $V_x = V_i + V_{DSS}$ where $V_{DSS} = I_{REF} / g_{m3}$

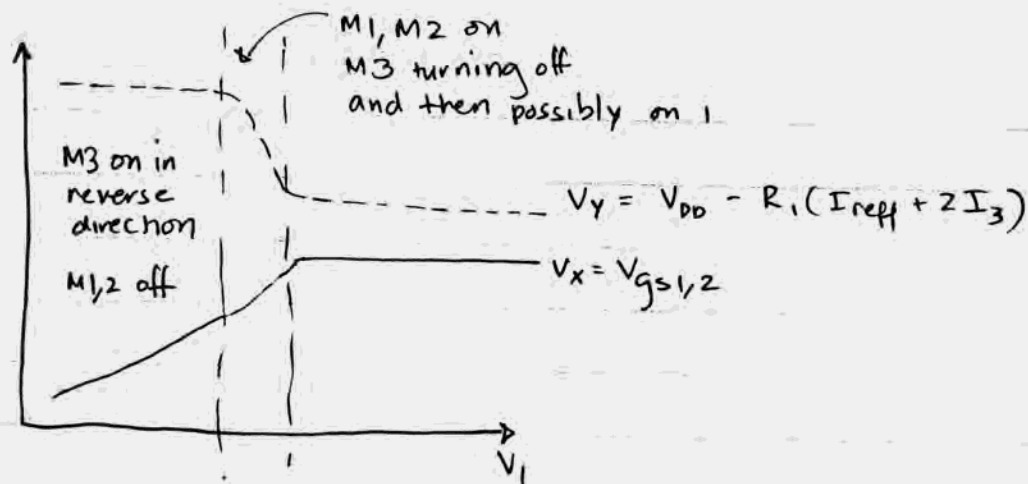
As V_i increase, V_x increases in the same amount until $V_x = V_{th1,2}$.

Now that $M1$ and $M2$ turn on, V_y drops down due to $I_1 R_1$, until $M3$ turns off. Once $M3$ is off, $V_x = V_{GS1,2}$ and $V_y = V_{DD} - R_1 I_{REF}$.

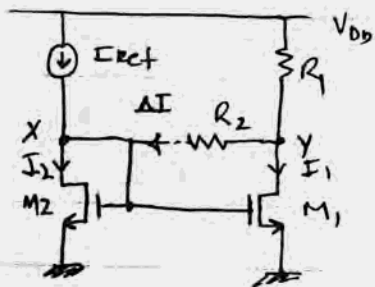
If at this point, $V_{th3} < V_y - V_x$, $M3$ will turn on to increase the current through $M2$ and hence $M1$ so

$$V_y = V_{DD} - R_1 (I_{REF} + 2 I_3)$$





S.5



When $I_{ref} = 0$, current I_1 and I_2 are supplied by V_{DD} through R_1

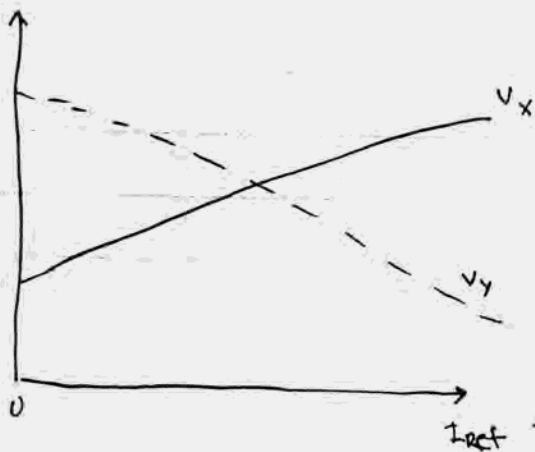
The initial points can be solved with

$$I_{1,2} = \frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{gs1,2} - V_{th})^2 \quad (1)$$

$$V_{gs2,1} = V_{DD} - (2R_1 + R_2) I_{1,2} \quad (2)$$

where $V_x = V_{gs2,1}$

$$V_y = V_x + \frac{V_{DD} - V_x}{2R_1 + R_2} R_2$$



As I_{ref} increases, I_2 increases and hence $V_{gs2} = V_{gs1} = V_x$ increases.

Following KCL, we can find V_y as a function of V_x, I_{ref}

$$V_y = V_{DD} \frac{R_2}{2R_1 + R_2} - \frac{I_{ref} R_1 R_2}{2R_1 + R_2} + \frac{2R_1 V_x}{2R_1 + R_2}$$

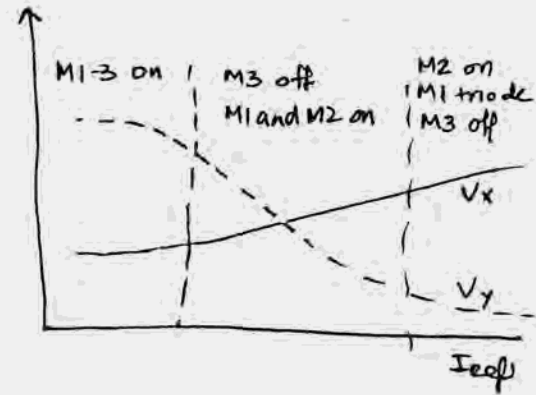
5.5(b)

Initially when $I_{ref} = 0$, this ckt is on with the follow condition
 $I_1 = I_2 = I_3$ since all transistors are assumed equal and all on.

$$I_{1-3} = \frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{gs1-3} - V_{th})^2 \quad (1)$$

$$2V_{gs1-3} = V_{DD} - R_1 I_{1-3} \quad (2)$$

where $V_x = V_{gs1-3}$ and $V_y = 2V_{gs1-3}$.



Once I_{ref} increase, $V_{gs1,2}$ goes up and
 V_y drops down, decreasing V_{gs3} until
 $M3$ turns off. Then $M1$ and $M2$ act as

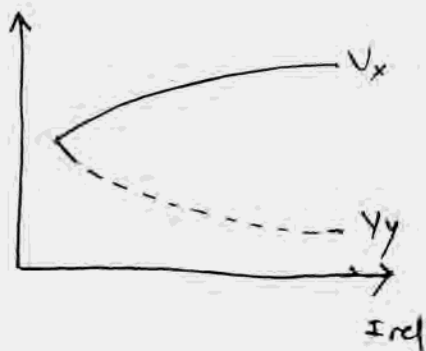
a typical current mirror for I_{ref} . Then when I_{ref} get so large,
 the $I_{ref} R_1$ drop increases to a point when $M1$ goes into triode and
 can't sustain the I_{ref} current.

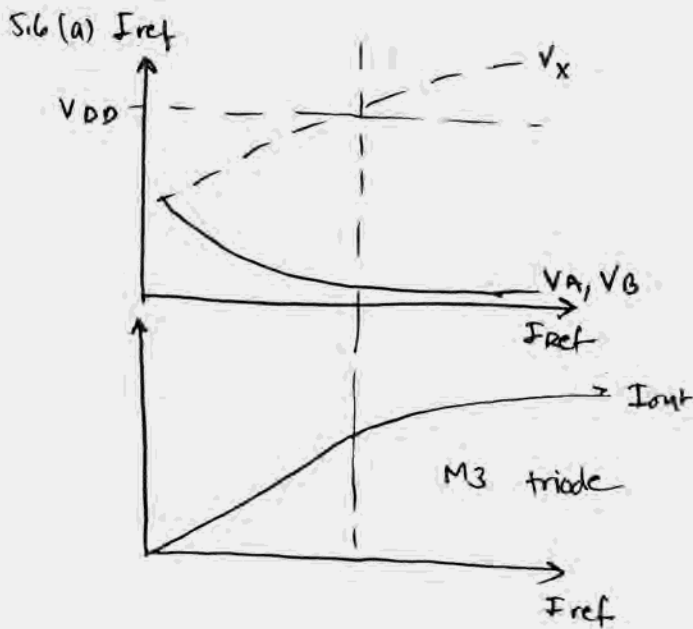
(c)

At $I_{ref} = 0$, all transistors are in saturation mode with $V_x = V_y$.

Once I_{ref} turns on, $M1$ goes into triode and then $M4$ goes
 into triode also

$$V_x \approx V_{gs2} \quad \text{and} \quad V_y = V_{DD} - V_{gs3} \approx V_{DD} - V_{gs2}$$





I_{out} follows I_{ref} for all I_{ref} until $V_x > V_{DD}$. Even if M_1 and M_2 go into triode, they still generate similar currents since V_A and V_B match. As I_{ref} increase, V_A, B decrease since $V_{gs4,3}$ increases and V_x increases since $V_{gs1,2}$ also increase.

Once $V_x > V_{DD}$, M_3 goes into triode and reduces I_{out} w.r.t I_{ref} .

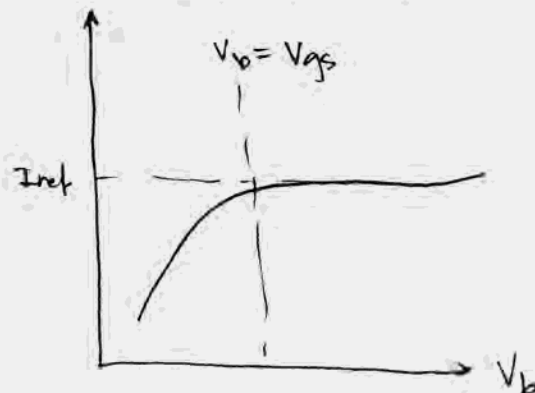
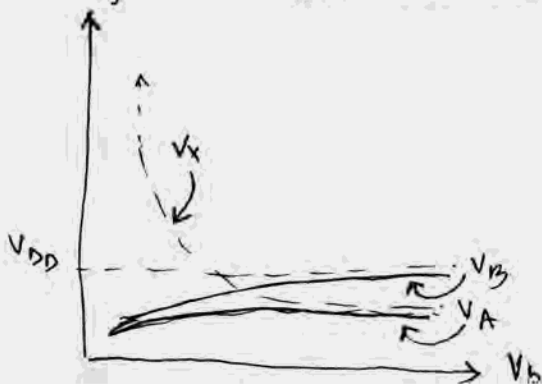
(b) If V_b is less than $1V_{gs}$ for I_{REF} current, V_x goes up to a very large voltage to allow for I_{REF} to flow through M_2 and M_4 if we assume channel resistance.

$$I = \frac{1}{2} \mu C_{ox} \frac{W}{L} (V_{gs} - V_{th}) (1 + \lambda V_{ds})$$

Once $V_b = 1V_{gs}$, V_A and V_B are $\emptyset V$ and $I_{out} = I_{ref}$.

As V_b increases, V_A and V_B increase to turn M_1 and M_2 on.

V_x is then equal to $1V_{gs}$. As V_b increase further, M_3 and M_4 go into triode while M_1 and M_2 are still in sat.



5.7 (a) $\gamma = 0$

$$K_0 = \frac{1}{2} \mu C_{ox}$$

$$I_{REF} = K_0 \frac{W_0}{L_0'} (V_{gs1} - V_{th})^2 (1 + \lambda V_{ds1})$$

$$I_{out} = K_0 \frac{W_0}{L_0'} (V_{gs1} - V_{th})^2 (1 + \lambda V_{ds2})$$

where $V_{ds1} = V_{gs1}$ and $V_{ds2} = 2V_{gs1} - V_{gs4} - V_{gs3}$

if we assume $I_{out} \approx I_{REF}$, $V_{gs3} \approx V_{gs1}$,

$$\text{so } V_{ds2} = V_{gs1} - V_{gs4}$$

$$V_{gs1} = V_{th} + \sqrt{\frac{I_{REF} L_0'}{K_0 W_0}} \quad V_{gs4} = V_{th} + \sqrt{\frac{I_1 L_4'}{K_0 W_4}} \quad L' = L - 2L_D$$

$$\frac{I_{out}}{I_{REF}} = \frac{1 + \lambda V_{ds2}}{1 + \lambda V_{ds1}} = \frac{1 + \lambda \left(\sqrt{\frac{I_{REF} L_0'}{K_0 W_0}} - \sqrt{\frac{I_1 L_4'}{K_0 W_4}} \right)}{1 + \lambda \left(V_{th} + \sqrt{\frac{I_{REF} L_0'}{K_0 W_0}} \right)}$$

(b) $\gamma \neq 0$

$$V_{th} = V_{th0} + \gamma \left(\sqrt{2\phi_f + V_{sb}} - \sqrt{2\phi_f} \right) \quad \phi_f \approx 45 \text{ v (work func.)}$$

$$V_{sb} \equiv \text{source - substrate volt}$$

Find V_{gs1} , V_{gs2} , V_{gs4} and V_{gs3}

$$V_{gs1} = V_{th0} + \sqrt{\frac{I_{REF} L_0'}{K_0 W_0}}$$

$$V_{gs0} = V_{th0} + \sqrt{\frac{I_{REF} L_0'}{K_0 W_0}} + \gamma \left(\sqrt{2\phi_f + V_{gs1}} - \sqrt{2\phi_f} \right)$$

$$V_{gs3} = V_{th0} + \sqrt{\frac{I_{out} L_0'}{K_0 W_0}} + \gamma \left(\sqrt{2\phi_f + V_{ds2}} - \sqrt{2\phi_f} \right)$$

$$\text{if we assume } I_{out} \approx I_{REF} \text{ and } V_{ds2} \approx \sqrt{\frac{I_{REF} L_0'}{K_0 W_0}} - \sqrt{\frac{I_1 L_4'}{K_0 W_4}}$$

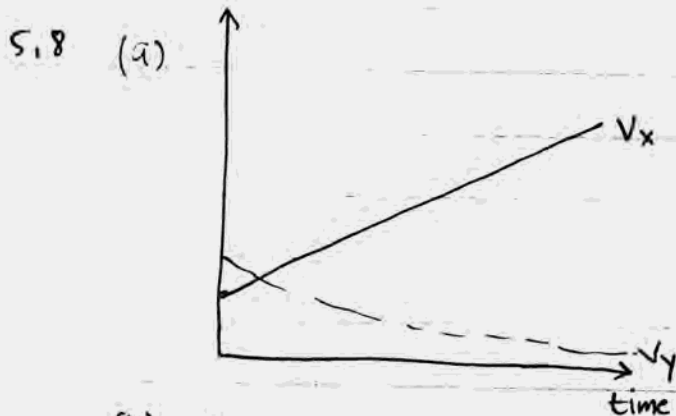
we can estimate V_{gs3}

$$V_{gs3} \approx V_{th0} + \sqrt{\frac{I_{REF} L_0'}{K_0 W_0}} + \gamma \left(\sqrt{2\phi_f + \sqrt{\frac{I_{REF} L_0'}{K_0 W_0}} - \sqrt{\frac{I_1 L_4'}{K_0 W_4}}} - \sqrt{2\phi_f} \right)$$

$$5.7 \quad V_{gs4} = V_{th0} + \sqrt{\frac{I_1}{k_0} \frac{L_4}{W_4}} + \gamma \left(\sqrt{2\phi_f + V_{gs3} + V_{ds2}} - \sqrt{2\phi_f} \right)$$

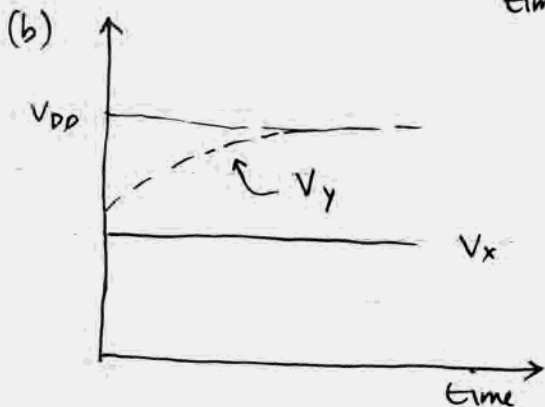
Now we can plug everything in to the final solution

$$\frac{I_{out}}{I_{ref}} \approx \frac{1 + \lambda (V_{gs1} + V_{gs0} - V_{gs4} - V_{gs3})}{1 + \lambda (V_{gs1})}$$



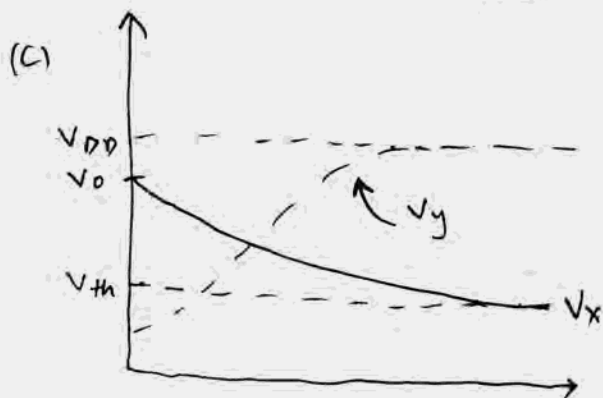
C_1 is continuously charged w/
 I_{ref} so V_c and V_x increase with
 time $V_c = \int_0^t \frac{I_{ref}}{C_1} dt$
 $V_x = V_c + V_{gs2}$

$V_{gs} = V_x$ and M_1 goes into triode



M_2 is on with fixed $V_x = V_{gs2}$
 C_1 is charged with current I_1
 until M_1 turns off.

$V_y = V_{DD} - I_1 R_1$ where
 I_1 goes from I_{ref} to \emptyset .



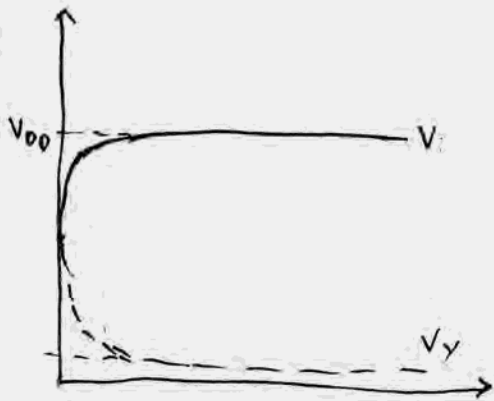
With C_1 initially charged w/ V_0 ,
 M_2 is on and discharges C_1
 until $V_x = V_{th}$ and $I_2 = 0$.
 $V_y = V_{DD} - R_1 I_2$ and as
 I_2 goes to \emptyset , V_y goes to V_{DD} .

5.8 (d)



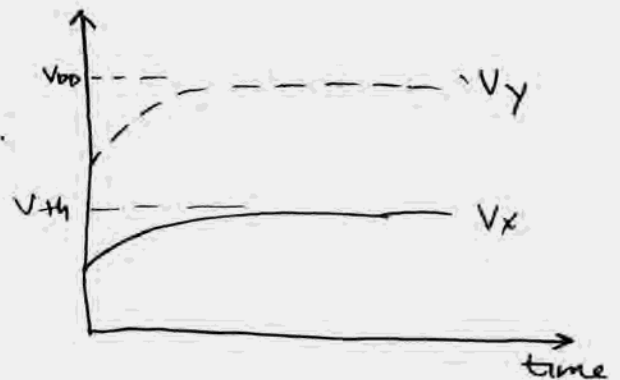
M2 and M1 are initially both on, M1 discharges all the charge in C_1 such that $V_y \rightarrow 0$ and M1 turns off. Since current through M1 reduces, current through M2 increases and V_x increases slightly.

(c)

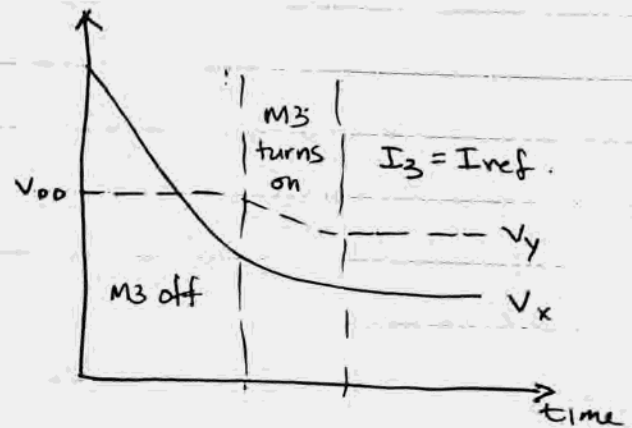


Since M2 can sustain no current, V_x goes up to V_{DD} . This pushes M1 into triode and drops V_y to very small voltage. The voltage across C_1 drops to V_{th} to sustain $I_2 = 0$.

5.9 (a) At $t=0$, when C_1 is charged to 0V, $V_x = V_B - V_{gs3}$ where the $I_3 = I_{ref}$ and $V_y = V_{DD} - R_1 I_{ref}$. As the current flows into C_1 , the cap charges up and shuts off M1. V_x is charged up to $V_B - V_{th}$ to sustain $I_{in} = 0$ and $V_y = V_{DD}$.



5.9 (b) At $t=0$, V_x is so high that M_3 is off. Current for M_1 and M_2 is generated by I_{ref} and as I_1 flows through M_1 , C_1 discharges enough to allow current flow through M_3 . Once $I_3 = I_{REF}$, C_1 no longer discharges and has a constant set voltage.



(c) At $t=0$, all transistors are on.

$V_y = V_{gs2} + V_{DD}$ and V_x is

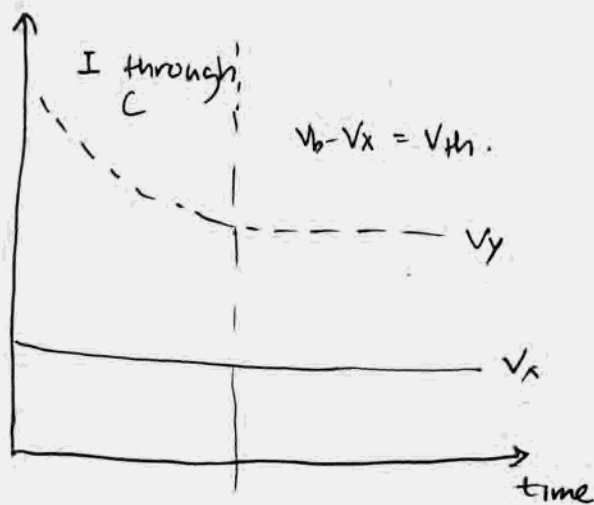
$$\text{such that } I_3 = k_0 \frac{W_3}{L_3} (V_b - V_x - V_{th})^2 (1 + \lambda V_{DS3}),$$

as C_1 discharges, V_{DS3} decreases and

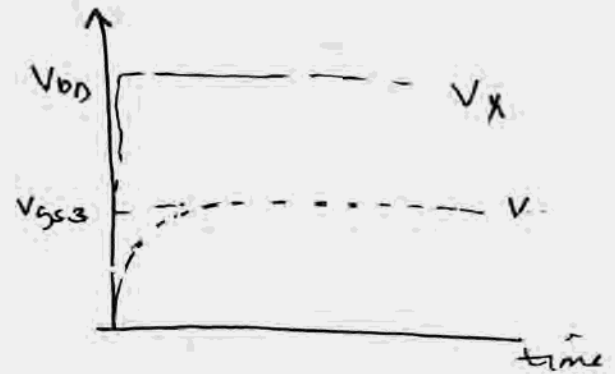
V_{gs3} increases, lowering V_x . At the

point where $V_b - V_x = V_{th}$ and $I_3 = I_1 = 0$,

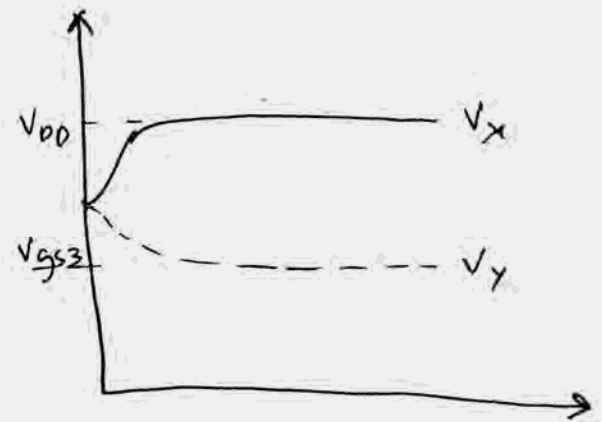
V_y stops decreasing.



5.10 (a) at $t=0$, $V_{gs2} = 3V$ and forces current through M2. since the source of M2 is attached to the gate of M1, no current can flow and $V_x = V_{DD}$. C_1 charges up such that $I_3 = I_1$ and $V_y = V_{DD} - V_{gs2}$



(b) Similar to 5.10(a), but since current can flow through C_1 to charge capacitor. V_x does not instantaneously reach V_{DD} , but slowly charges to V_{DD} . $V_y = V_{DD} - V_{gs2}$ also.



5.11 small signal model.



$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{in}}{g_{m2}V_1}$$

$$= - \frac{C_1 s + g_{m3}}{g_{m2}g_{m1}}$$

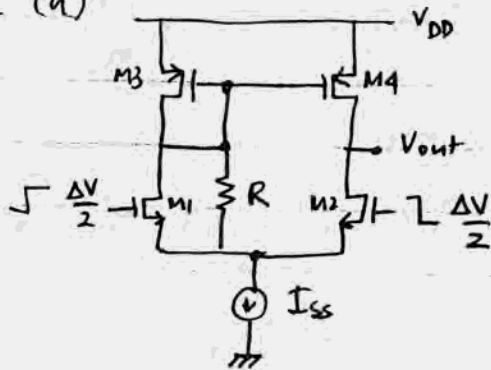
$$V_1 = -g_{m1} \left(\frac{1}{C_1 s + g_{m3}} \right) V_{in}$$

if all transistors are equal
 $g_{m1} = g_{m2} = g_{m3}$

$$Z_{in} = - \frac{C_1 s}{g_{m^2}} - \frac{1}{g_m}$$

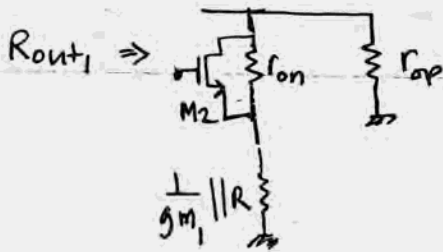
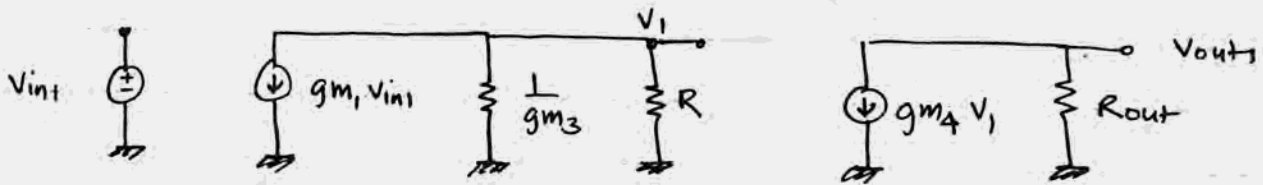
negative $C = -\frac{C_1}{g_{m^2}}$ and negative $R = \frac{1}{g_m}$.

5.12 (a)



Solve for gain by superposition

$$v_{in} = \Delta V = \frac{\Delta V}{2} - \left(-\frac{\Delta V}{2}\right) = v_{in1} + v_{in2}$$

1. for $+\frac{\Delta V}{2}$, ground M2 gate and find gain.

$$R_{out} = R_{op} \parallel \left(\frac{1}{g_{m1}} + r_{on} + \frac{g_{m2} r_{on}}{g_{m1}} \right) \\ \approx R_{op} \parallel 2r_{on} \approx \frac{2}{3} r_o = \frac{2}{3} \frac{1}{\lambda I_0} = \frac{4}{3} \frac{1}{\lambda I_{SS}}$$

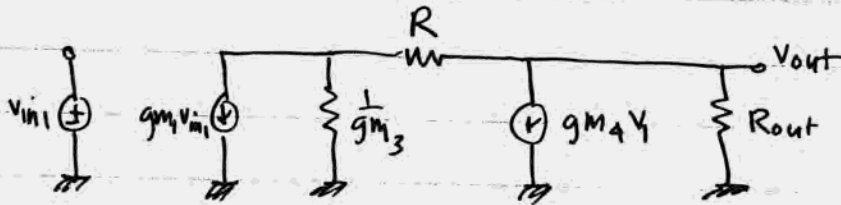
$$\frac{V_{out1}}{v_{in1}} = +g_{m1} \left(\frac{1}{g_{m3}} \parallel R_o \right) g_{m4} R_{out}$$

2. for $-\frac{\Delta V}{2}$

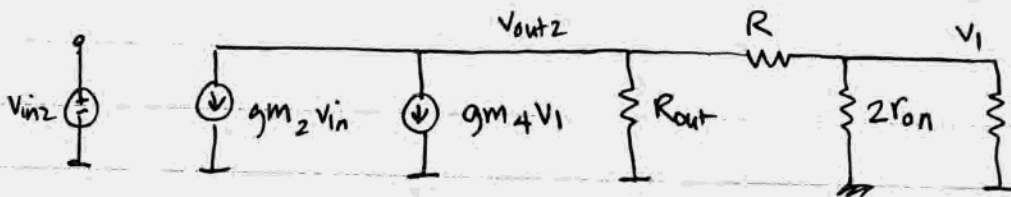
$$V_{out} = -g_{m2} v_{in2} R_{out}$$

assume $g_{m1} = g_{m2}$ and $g_{m3} = g_{m4}$

$$\text{Gain} = \frac{V_{out1} + V_{out2}}{\Delta V} = \frac{g_{m1}}{2} R_{out} \left[1 + \frac{R g_{m3}}{1 + R g_{m3}} \right]$$

(b) for $v_{in1} = +\frac{\Delta V}{2}$ 

$$\frac{v_{out1}}{v_{in1}} = g_{m1} R_{out} \left[\frac{R g_{m3} - 1}{R g_{m3} + 2 R_{out} + 1} \right]$$

for $v_{in2} = -\frac{\Delta V}{2}$ 

$$v_1 = v_{out2} \cdot \frac{2r_{on} \parallel \frac{1}{g_{m3}}}{R + 2r_{on} \parallel \frac{1}{g_{m3}}} = v_{out2} \cdot \frac{2r_{on}}{R + 2r_{on} + 2g_{m3}r_{on}R}$$

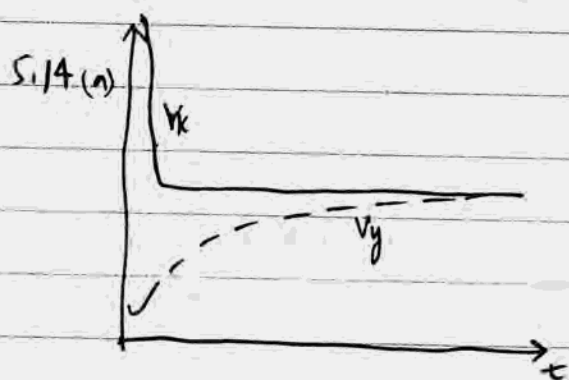
$$v_{out2} = \left(g_{m2} v_{in2} - g_{m4} v_{out2} \frac{2r_{on}}{R + 2r_{on} + 2g_{m3}r_{on}R} \right) \left[R_{out} \parallel \left(R + 2r_{on} \parallel \frac{1}{g_{m3}} \right) \right]$$

$$\frac{v_{out2}}{v_{in2}} = \frac{-g_{m2} \left[R_{out} \parallel \left(R + 2r_{on} \parallel \frac{1}{g_{m3}} \right) \right]}{1 + \frac{g_{m4} 2r_{on}}{R + r_{on} + 2g_{m3}r_{on}R} \cdot \left[R_{out} \parallel \left(R + 2r_{on} \parallel \frac{1}{g_{m3}} \right) \right]}$$

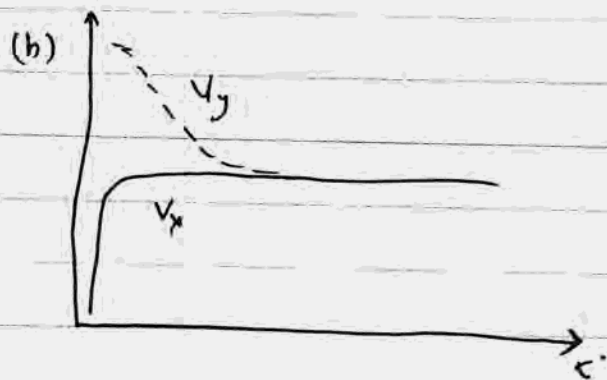
$$\text{Gain} = \frac{v_{out1} + v_{out2}}{\Delta V}$$

$$S.13 \quad V_{min} = V_p + V_{Dsat,2} \quad V_p = V_{CMI,2} - V_{GS1,2}$$

$$= V_{CMI,2} - V_{GS1,2} + V_{Dsat,2}$$

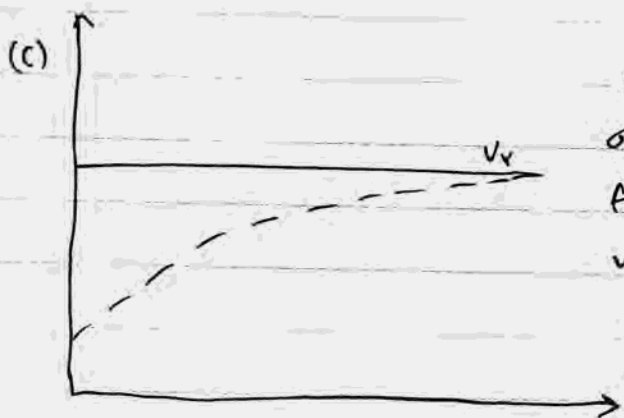


M3 and M4 are initially off. M1 is on and M2 is in triode. The current through M1 initially comes from the C_i charge until V_x drops in voltage and M3 and M4 turn on. M2 is still in triode until the current from M4 charges up V_y to the same voltage as V_x .



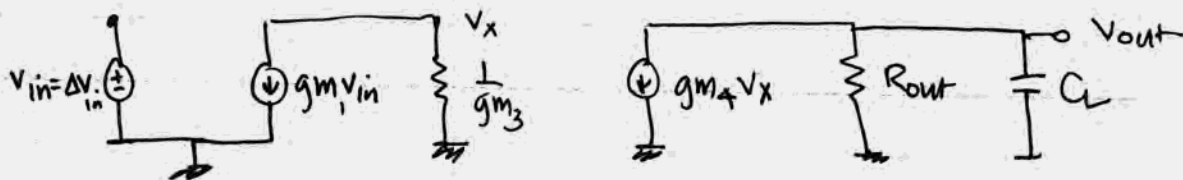
V_x starts at $1.5V - V_{GS1,2}$ where M1 is in triode and M3 is on strong. M4 and M2 can't sustain that high current w/ tail current = I_{SS} so V_y goes up enough to put M4 in triode and reduce current. Current through M3 charges C_i and V_y reduced its voltage w/ discharge in parasitics

M3



Initial short between source & drain of M2 puts M2 in triode w/ minimal current flow. Current from M4 is used to charge up C_i . As C_i charges up, some current starts flowing through M2 until V_{GS2} is high enough that M2 is in sat and all current is diverted to M2.

S. 15 Initial value $\Rightarrow V_{in} = V_1$ $V_{out} = V_x$

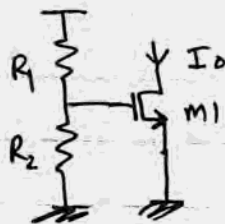


$$\Delta V_{out} = g_{m1} \Delta V_{in} \frac{1}{g_{m3}} g_{m4} R_{out} \quad (\text{final})$$

final value $\Rightarrow V_{in} = V_1 - \Delta V_{in}$ $V_{out} = V_x - \Delta V_{out}$

$$\tau_c = R_{out} C_L$$

S. 16



$$\frac{W}{L} = \frac{50 \mu\text{m}}{15 \mu\text{m}}; \lambda = 0; I = 0.5 \text{ mA}; K_p = \mu C_{ox} = 137 \times 10^{-4} \frac{\text{A}}{\text{V}^2}; L_D = 90 \text{ nm}$$

a) R_2/R_1

$$V_{gs1} = V_{DD} \frac{R_2}{R_1 + R_2} = \sqrt{\frac{2I}{K_p \frac{W}{L}}} + V_{th} \quad L' = L - 2L_D$$

$$\text{Let } R_x = R_2/R_1$$

$$R_x = \frac{\sqrt{2I/K_p \frac{W}{L}} + V_{th}}{V_{DD} - (\sqrt{2I/K_p \frac{W}{L}} + V_{th})} = 0.4386$$

$$b) I_D = \frac{1}{2} \mu C_{ox} \frac{W}{L} \left(V_{DD} \frac{R_x}{1+R_x} - V_{th} \right)^2$$

$$\frac{\left(\frac{\partial I_D}{\partial V_{DD}} \right)}{I_D} = \frac{\mu C_{ox} \frac{W}{L} \left(V_{DD} \frac{R_x}{1+R_x} - V_{th} \right) \frac{R_x}{1+R_x}}{\frac{1}{2} \mu C_{ox} \frac{W}{L} \left(V_{DD} \frac{R_x}{1+R_x} - V_{th} \right)^2}$$

$$= \frac{2}{V_{DD} - V_{th} \left(1 + \frac{1}{R_x} \right)} = 2.84$$

$$5.16 \quad (c) \quad \frac{\partial I_0}{\partial V_{th}} = -\mu C_{ox} \frac{W}{L} \left(V_{DD} \frac{R_x}{1+R_x} - V_{th} \right)$$

$$\Delta I_0 \approx -\mu C_{ox} \frac{W}{L} \left(V_{DD} \frac{R_x}{1+R_x} - V_{th} \right) \Delta V_{th} = -233 \mu A$$

$$\Delta I_0 = I_0(V_{th} = .75) - I_0(V_{th} = .7) = -205 \mu A$$

$$(d) \quad \frac{\partial I_0}{\partial T} = -\frac{3}{2} \left(\frac{T}{T_0} \right)^{-3/2} \cdot \frac{1}{T} \cdot I_0 \quad T = T_0 + \Delta T$$

$$\Delta I_0 \approx -\frac{3}{2} \left(\frac{T}{T_0} \right)^{-3/2} \frac{1}{T} \cdot I_0 \Delta T = -103 \mu A \quad *$$

$$\Delta I_0 = I_0(T=370K) - I_0(T=300K) = -135 \mu A \quad *$$

$$(e) \quad \Delta I_{\text{worst-case}} = I_{\text{worst-case}} - I_0$$

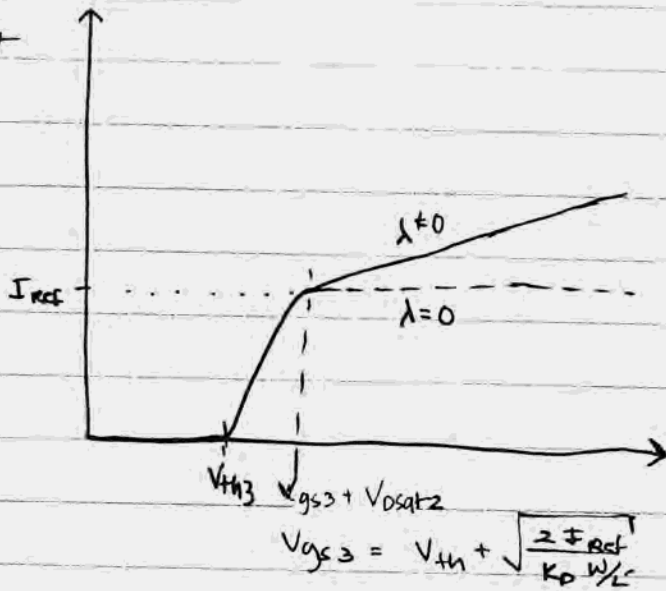
$$I_{\text{worst-case}} = \frac{1}{2} \mu_0 \left(\frac{T_0 + \Delta T}{T_0} \right)^{-3/2} \left((V_{DD} - \Delta V_{DD}) \frac{R_x}{1+R_x} - (V_{th} + \Delta V_{th}) \right)$$

$$= 43 \mu A$$

$$\Delta I_{\text{worst-case}} = -457 \mu A$$

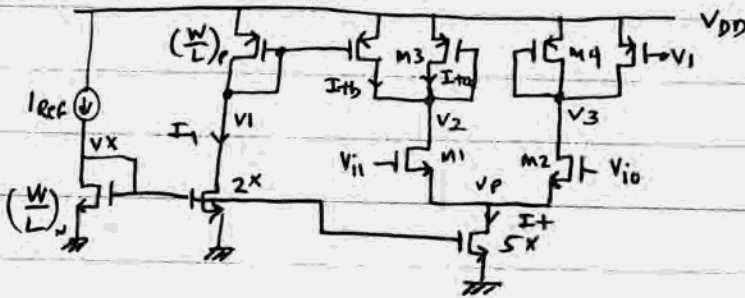
* Note as temperature changes, so does V_{th} . In this calculation, we do not include the temperature effects on threshold voltage.

S.17



$I_{out} = 0$ until $V_{DD} = V_{th3,4}$ when M3 and M4 are off, M2 in triode and M1 on. Once $V_{DD} > V_{th3,4}$, there is current flow but very little until M2 is in saturation
 At $V_{DD} = V_{gs3} + V_{osat2}$, $I_{out} = I_{ref}$
 At $V_{DD} = V_{gs1} + V_{gs}$, $I_{out} = I_{ref}$ exactly and grows as V_{os2} increases!

S.18



$(W/L)_N = \frac{10}{.5}$ $(W/L)_{p2}$
 $(W/L)_P = \frac{10}{.5}$ arbitrary
 $I_{ref} = 100 \mu A$
 $K_p = 38 \frac{\mu A}{V^2}$ $K_n = 139 \frac{\mu A}{V^2}$

(a) $\lambda = 0$

$$I = \frac{1}{2} K_p \frac{W}{L-2L_0} (V_{gs} - V_t)^2$$

$$V_1 = V_{DD} - V_{gs(P)} = V_{DD} - V_{tp} - \sqrt{\frac{2 \cdot 2 \cdot I_{ref}}{K_p \frac{W}{L-2L_0}}} = 1.619 V$$

$$V_2 = V_3 = V_{DD} - V_{gs(S)} = V_{DD} - V_{tp} - \sqrt{\frac{I_{ref}}{K_p \frac{W}{L-2L_0}}} = 1.909 V$$

$$V_p = V_{cm} - V_{gs(I)} = 1.3 - V_{th} - \sqrt{\frac{2.5 \cdot I_{ref}}{K_n W}} = .3747 V \quad (\gamma = 0)$$

V_p for $\gamma = .45$ is found iteratively by finding V_{th} iteratively also.

$$V_{th}(V_p = .3747) = V_{t0} + \gamma \sqrt{2\phi + V_{SB}} - \gamma \sqrt{2\phi} \quad \gamma = .45, \phi = .45$$

$$= .78$$

$$V_p(V_{th} = .78) = .29 \quad \text{until} \quad V_{th}(\text{final}) = .767$$

$$V_p(\text{final}) = .307$$

S.18 (b) $\lambda = 2$

$$V_{GS}(0) = V_{tn} + \sqrt{\frac{2I_{ref}}{K_n \cdot \frac{W}{L-2L_D}}} = 0.9253$$

for I_+ , initially assume $V_p = .307$ from part (a)

iterate with

$$I_+ = I_{ref} \cdot 5 \cdot \frac{1 + \lambda V_p}{1 + \lambda V_{GS}(0)}$$

and

$$V_p = V_{cm,1,2} - V_{GS}(1,2) \quad \text{with} \quad V_{tn,1,2} = V_{t0} + \gamma \sqrt{2\phi + V_p} - \gamma \sqrt{2\phi}$$

$$I_+ = 448 \mu A \quad \text{and} \quad V_p = 0.317$$

for V_1 , iterate also

$$I_1 = I_{ref} \cdot 2 \cdot \frac{1 + \lambda V_1}{1 + \lambda V_{GS}(0)}$$

$$V_1 = V_{DD} - |V_{GS,p}| = V_{DD} - V_{tp} - \sqrt{\frac{2 \cdot I_1}{K_p \cdot \frac{W}{L-2L_D}}}$$

$$\text{Final } I_1 = 222 \mu A, \quad V_1 = 1.59 V$$

for V_2, V_3 , iterate also.

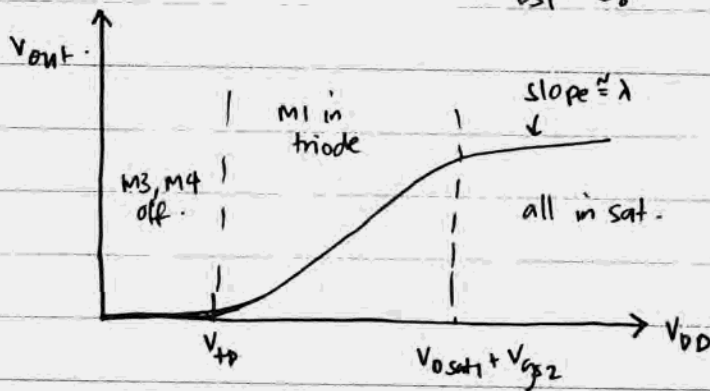
$$\begin{cases} I_{ta} = I_{+2} - I_{tb} \\ V_{2,3} = V_{DD} - |V_{GS3,6}| = V_{DD} - V_{tp} - \sqrt{\frac{2 \cdot I_{ta}}{K_p \cdot \frac{W}{L-2L_D}}} \\ I_{tb} = I_1 \cdot \frac{1 + \lambda |V_{GS3,6}|}{1 + \lambda |V_{GS,p}|} \end{cases}$$

$$I_{ta} = 17.3 \mu A; \quad I_{tb} = 207 \mu A; \quad V_{2,3} = 2.029 V$$

5.19 $V_{DD} < V_{TP}$: M2 and M3 off, $I_3 = 0$, $V_{out} = 0$

$V_{TP} \leq V_{DD} < V_{DSat1} + V_{GS2}$: M1 in triode, linearly approaching saturation. M2 and M3 are on with V_{out} increasing linearly.

$V_{DD} > V_{DSat1} + V_{GS2}$: All transistors are on and as V_{DD} increases, V_{DS1} increases so current increases as $\lambda V_{DS1} \cdot I_0$.



5.20 $\gamma = 0$ $\left(\frac{W}{L}\right)_{1-3} = \frac{40}{5}$ $I_{ref} = .3 \text{ mA}$ $K_n = 138 \frac{\mu\text{A}}{\text{V}^2}$ $L_D = 80 \text{ nm}$

$$(a) \quad V_b = 2V_{GS1} \quad V_{GS1} = V_{tn} + \sqrt{\frac{2I_{ref}}{K_n \frac{W}{L-2L_D}}} = .892 \text{ V}$$

$$V_D = 1.78 \text{ V}$$

$$(b) \quad I_{out} = I_{ref} \cdot \frac{(1 + \lambda(V_{GS1} + \Delta V_b))}{1 + \lambda V_{GS1}} = 295 \mu\text{A} \quad \Delta V_b = -.100 \text{ V}$$

$$\Delta I_{out} = \frac{I_{ref} \cdot \lambda \Delta V_b}{1 + \lambda V_{GS1}}$$

5.20 (c) V_p increases by 1V

V_y increases by ΔX

solve for 2 unknowns and 2 equations

$$\left\{ \begin{array}{l} I_{out} = I_{ref} \left(\frac{1 + \lambda(V_{gs1} - \Delta X)}{1 + \lambda V_{gs1}} \right) \quad (M2) \\ \end{array} \right. \quad \text{eq 1.}$$

$$\left\{ \begin{array}{l} I_{out} = \frac{1}{2} k_n \frac{W}{L-2L_D} (V_{gs1} - \Delta X)^2 \left(\frac{1 + \lambda(V_{gs1} + 1V - \Delta X)}{1 + \lambda V_{gs1}} \right) \quad (M3) \end{array} \right. \quad \text{eq 2}$$

$$\Delta X = 13 \text{ mV}$$

$$V_y = V_{gs1} + \Delta X = .905 \text{ V}$$

5.21 (a) $V_x = V_{gs1} = V_{th} + V_{Dsat1} = .863$

$$V_{Dsat1} = \sqrt{\frac{2I_{ref}}{k_n \frac{W}{L-2L_D}}} = .163 \quad V_{th} = .7 \text{ V}$$

$$V_b \geq V_{Dsat1} + V_{gs2}$$

$$\begin{aligned} V_{gs2} &= V_{th0} + \sigma \sqrt{2\phi + V_{Dsat1}} - \sigma \sqrt{2\phi} + \sqrt{\frac{2I_{ref}}{k_n \frac{W}{L-2L_D}}} \\ &= .7 + .037 + .163 = .900 \text{ V} \end{aligned}$$

As V_b increases, M2 and M4 go into triode and $V_A \sim V_x$ and $V_B \sim V_{M4, \text{drain}}$. As long as $V_{M4, \text{drain}}$ does not drop below V_{Dsat1} , I_{out} will reasonably follow I_{ref} .

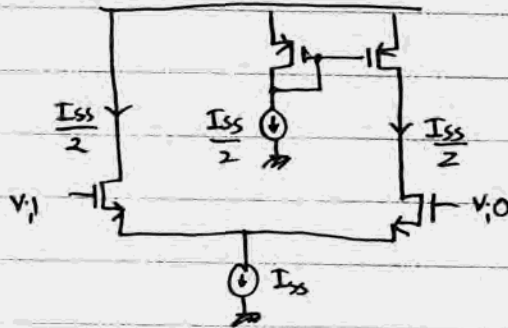
b) $V_{M4, \text{drain}} \uparrow 1V$, $V_B \uparrow \Delta X$

use eq 1 and 2 from 5.20 to solve for change in current.

$$I_{out} = 301.1 \mu\text{A} \quad \Delta X = 21 \text{ mV}$$

S.22 Assume $\lambda=0$ for bias purpose and $\lambda=.2$ for small signal analysis.

(a) DC Bias



$$g_{m_{1,2}} = \frac{I_{SS}}{V_{dsat_{1,2}}} = 3.18 \text{ mS}$$

$$V_{dsat_{1,2}} = \sqrt{\frac{I_{SS}}{K_n \frac{W}{L} \cdot 2}} = .157$$

$$r_{on} = r_{op} = \frac{1}{\lambda \frac{I_{SS}}{2}} = 20 \text{ K}$$

small signal: $\text{Gain} = \frac{v_{out}}{v_{i1} - v_{i0}} = \frac{1}{2} g_{m_{1,2}} R_{out} = 21.2$

$$R_{out} = r_{op} \parallel \left(2r_{on} + \frac{1}{g_{m_1}} \right) \approx \frac{2}{3} r_o = 13.33 \text{ K}$$

(b) Maximum output voltage swing.

$$\begin{aligned} V_{out(\min)} &= V_{cm} - V_{gs_{1,2}} \left(I = \frac{I_{SS}}{2} \right) + V_{dsat_{1,2}} \\ &= V_{cm} - V_{tn_{1,2}} \\ &= 1.3 - .778 = .522 \text{ V} \end{aligned}$$

$$V_{out(\max)} = V_{dd} \quad (\text{M4 in triode})$$

$$\begin{aligned} V_{out(\max)} - V_{out(\min)} &= V_{DD} - V_{cm} + V_{tn_{1,2}} \\ &= 3.0 \text{ V} - 1.3 \text{ V} + .778 = 2.48 \text{ V} \end{aligned}$$

$$V_{tn} = V_{t0} + \sqrt{2\phi + .36} - \sqrt{2\phi} = .778$$

5.23. Assume $I_3 = I_4$ though $V_{th3} \neq V_{th4}$

$$(a) \quad I_3 = \frac{1}{2} K_p \frac{W}{L-2L_D} (V_{gs3} - V_{th3})^2 (1 + \lambda |V_{gs3}|)$$

$$I_4 = \frac{1}{2} K_p \frac{W}{L-2L_D} (V_{gs} - V_{th4})^2 (1 + \lambda (|V_{gs3}| - \Delta x))$$

$$K' = \frac{1}{2} K_p \frac{W}{L-2L_D}$$

$$K' (V_{dsat3})^2 (1 + \lambda |V_{gs3}|) = K' (|V_{dsat3} - 1mV|)^2 (1 + \lambda |V_{gs3}| + \lambda \Delta x)$$

$$\Delta x \approx \frac{1mV \cdot (1 + \lambda |V_{gs3}|)}{\lambda |V_{dsat3}|}$$

$$V_F = V_{gs3} - \Delta x.$$

$$(b) \quad CMRR \approx \left| \frac{A_{dm}}{A_{cm}} \right|$$

$$A_{dm} = g_{m_{1,2}} (r_{o_{3,4}} \parallel r_{o_{1,2}})$$

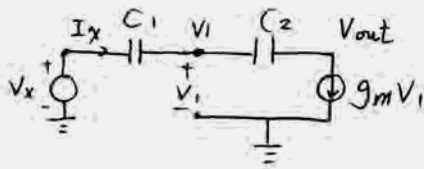
$$\approx g_{m_{1,2}} \left(\frac{1}{2} r_o \right)$$

$$A_{cm} \approx \frac{-1}{1 + 2g_{m_{1,2}} r_o} \cdot \frac{g_{m_{1,2}}}{g_{m_{3,4}}}$$

$$CMRR = (1 + 2g_{m_{1,2}} r_o) g_{m_{3,4}} (r_{o_{1,2}} \parallel r_{o_{3,4}})$$

Chapter 6

6.1(a)



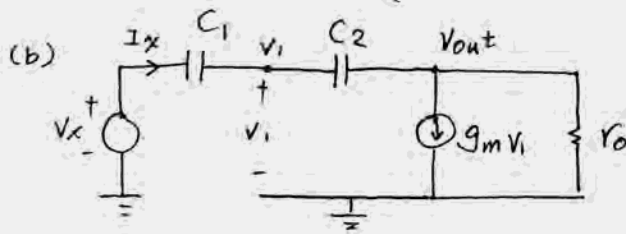
g_m : transconductance of M_1

$$I_x = sC_1(V_x - V_1) = sC_2(V_1 - V_{out}) = g_m V_1$$

$$\therefore sC_1 V_x = (g_m + sC_1)V_1 \Rightarrow V_1 = \left[\frac{sC_1}{g_m + sC_1} \right] V_x$$

$$\Rightarrow I_x = g_m V_1 = \left[\frac{g_m sC_1}{g_m + sC_1} \right] V_x$$

$$\Rightarrow Z_{in} = \frac{V_x}{I_x} = \frac{g_m + sC_1}{g_m sC_1} *$$



$$g_m = g_{m1} + g_{m2}$$

$$r_o = r_{o1} \parallel r_{o2}$$

g_{m1}, g_{m2} : transconductance for M_1, M_2

r_{o1}, r_{o2} : output resistance for M_1, M_2

$$\therefore I_x = sC_1(V_x - V_1) = \underbrace{sC_2(V_1 - V_{out})}_{\textcircled{1}} = \underbrace{g_m V_1 + \frac{V_{out}}{r_o}}_{\textcircled{2}}$$

from $\textcircled{1}$:

$$\frac{V_{out}}{V_1} = \frac{sC_2 - g_m}{sC_2 + \frac{1}{r_o}}$$

from $\textcircled{2}$:

$$(sC_1 + sC_2)V_1 = sC_1 V_x + sC_2 V_{out}$$

$$= sC_1 V_x + \frac{s^2 C_2^2 - g_m sC_2}{sC_2 + \frac{1}{r_o}} V_1$$

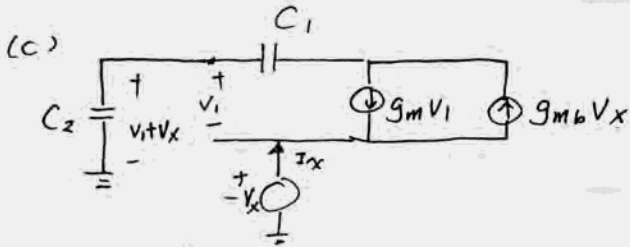
$$\Rightarrow \left[sC_1 + sC_2 - \frac{s^2 C_2^2 - g_m sC_2}{sC_2 + \frac{1}{r_o}} \right] V_1 = sC_1 V_x$$

$$\Rightarrow \left[\frac{s^2 C_1 C_2 + \frac{s C_1}{r_o} + \frac{s C_2}{r_o} + g_m s C_2}{s C_2 + \frac{1}{r_o}} \right] V_1 = s C_1 V_x$$

$$\therefore V_1 = \left[\frac{s C_1 C_2 + \frac{C_1}{r_o}}{s C_1 C_2 + \frac{C_1}{r_o} + \frac{C_2}{r_o} + g_m C_2} \right] V_x$$

$$I_x = s C_1 (V_x - V_1) = s C_1 \cdot \left[\frac{\frac{C_2}{r_o} + g_m C_2}{s C_1 C_2 + \frac{C_1}{r_o} + \frac{C_2}{r_o} + g_m C_2} \right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s C_1 C_2 + \frac{C_1}{r_o} + \frac{C_2}{r_o} + g_m C_2}{s C_1 C_2 \left(\frac{1}{r_o} + g_m \right)}$$



$$s C_2 (V_1 + V_x) + g_m V_1 = g_{mb} V_x$$

$$\Rightarrow (s C_2 + g_m) V_1 = (g_{mb} - s C_2) V_x$$

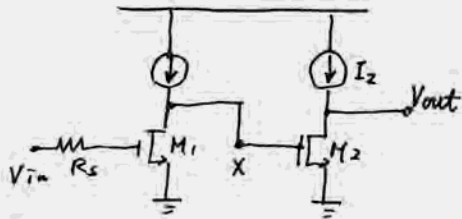
$$\frac{V_x}{V_1} = \frac{s C_2 + g_m}{g_{mb} - s C_2}$$

$$I_x = -g_m V_1 + g_{mb} V_x$$

$$= \left[-g_m \cdot \frac{g_{mb} - s C_2}{s C_2 + g_m} + g_{mb} \right] V_x = \left[\frac{(g_m + g_{mb}) s C_2}{s C_2 + g_m} \right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s C_2 + g_m}{s C_2 (g_m + g_{mb})}$$

6.2 (a)



There are three poles associated with this circuit.

The first pole @ V_{out}

$$\omega_{pout} = \frac{1}{r_{o2} \cdot (C_{gd2} + C_{db2})}$$

The pole @ the input

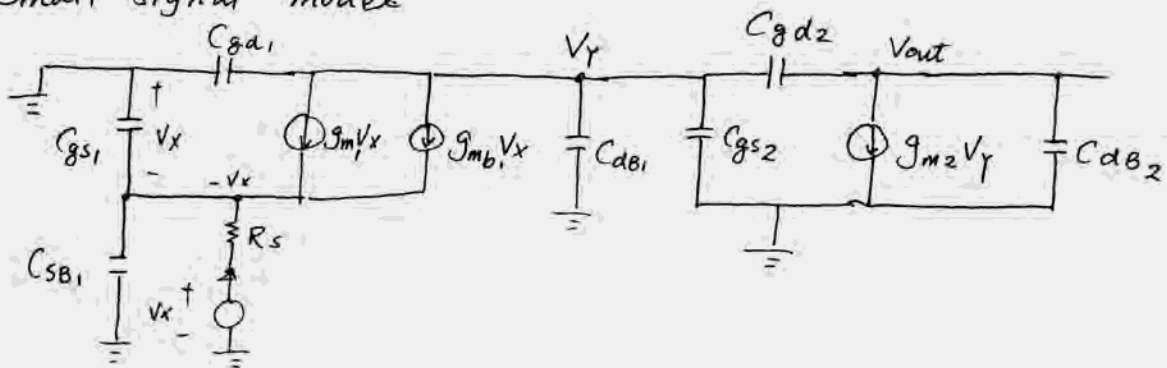
$$\omega_{p,in} = \frac{1}{R_s \cdot [(1 + g_{m1} r_{o1}) C_{gd1} + C_{gs1}]}$$

The pole @ node X

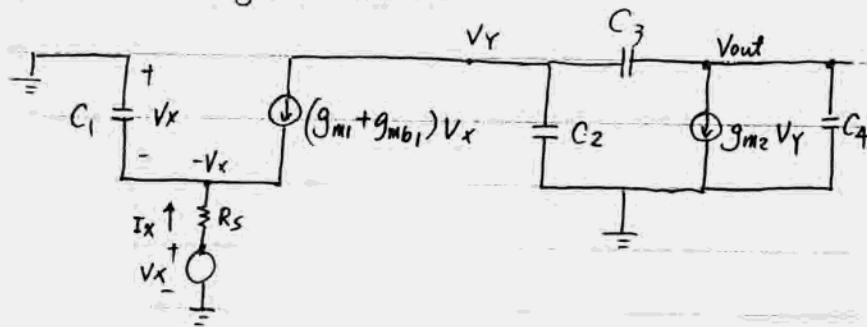
$$\omega_{p,X} = \frac{1}{r_{o1} \cdot [(C_{gd1} + C_{db1} + C_{gs2}) + (1 + g_{m2} r_{o2}) \cdot C_{gd2}]}$$

Please note that the above approximation is based on Miller effect. In order to get more accuracy approximation, transfer function has to be derived.

(b) Small signal model



Redraw small signal model



where,

$$C_1 = C_{gs1} + C_{SB1}$$

$$C_2 = C_{gs2} + C_{dB1} + C_{gd1}$$

$$C_3 = C_{gd2}$$

$$C_4 = C_{dB2}$$

$$\text{KCL @ } V_{out} : S C_3 (V_Y - V_{out}) = g_{m2} V_Y + S C_4 V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_Y} = \frac{-g_{m2} + S C_3}{S (C_3 + C_4)}$$

$$\text{KCL @ } V_Y : (g_{m1} + g_{mb1}) V_X + S C_2 V_Y + S C_3 (V_Y - V_{out}) = 0$$

$$(g_{m1} + g_{mb1}) V_X = -V_Y \left(S C_2 + \frac{S^2 C_3 C_4 + S C_3 g_{m2}}{S (C_3 + C_4)} \right)$$

$$\frac{V_Y}{V_X} = - \frac{g_{m1} + g_{mb1}}{\left[S (C_2 C_3 + C_2 C_4 + C_3 C_4) + C_3 g_{m2} \right] / (C_3 + C_4)}$$

$$\text{KCL @ } V_X : \frac{V_{in} + V_X}{R_s} + S C_1 V_X + (g_{m1} + g_{mb1}) V_X = 0$$

$$\frac{V_X}{V_{in}} = - \frac{1}{S C_1 R_s + (1 + (g_{m1} + g_{mb1}) \cdot R_s)}$$

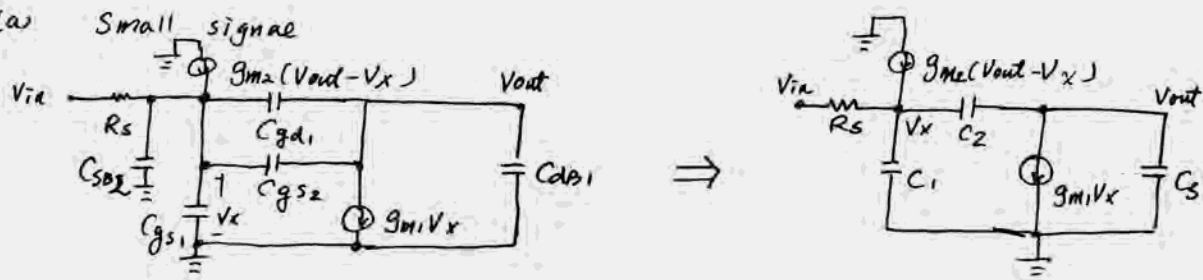
Thus, there are three poles

$$\omega_{p0} = 0$$

$$\omega_{p1} = \frac{-C_3 g_{m2}}{C_2 C_3 + C_2 C_4 + C_3 C_4} \quad *$$

$$\omega_{p2} = \frac{-(1 + (g_{m1} + g_{mb1}) \cdot R_s)}{C_1 R_s} \quad *$$

6.3 (a)



$$C_1 = C_{gs1} + C_{s02}$$

$$C_2 = C_{gd1} + C_{gs2}$$

$$C_3 = C_{db1}$$

KCL @ \$V_{out}\$:

$$sC_2(V_x - V_{out}) = g_{m1}V_x + sC_3V_{out}$$

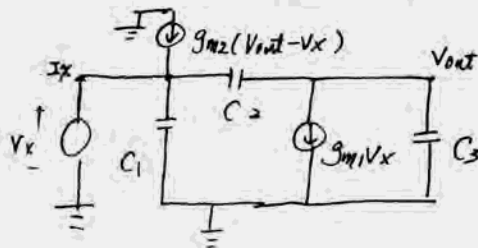
$$\therefore \frac{V_{out}}{V_x} = \frac{sC_2 - g_{m1}}{s(C_2 + C_3)} \quad \text{--- } \textcircled{D}$$

$$\text{KCL @ } V_x: \frac{V_{in} - V_x}{R_s} + g_{m2}(V_{out} - V_x) = sC_1V_x + sC_2(V_x - V_{out})$$

$$\Rightarrow \frac{V_x}{R_s} = V_x \left(\frac{1}{R_s} + g_{m2} + sC_1 + sC_2 \right) - (g_{m2} + sC_2) \cdot \left[\frac{sC_2 - g_{m1}}{s(C_2 + C_3)} \right] V_x$$

$$= V_x \cdot \frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s \left(\frac{1}{R_s}(C_2 + C_3) + g_{m1}C_2 + g_{m2}C_2 \right) + g_{m1}g_{m2}}{s(C_2 + C_3)}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_x} \cdot \frac{V_x}{V_{in}} = \frac{\frac{1}{R_s}(sC_2 - g_{m1})}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s \left[\frac{1}{R_s}(C_2 + C_3) + g_{m1}C_2 + g_{m2}C_3 \right] + g_{m1}g_{m2}} \quad \star$$



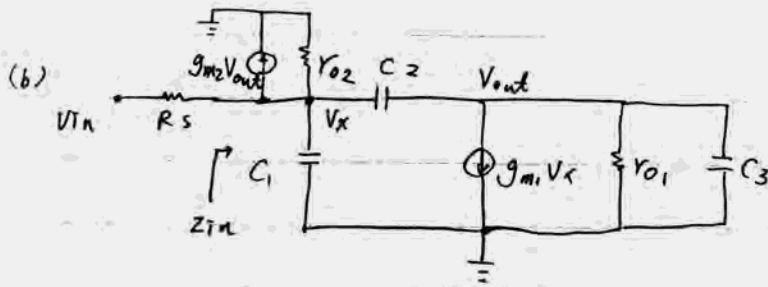
$$I_x = sC_1V_x + sC_2(V_x - V_{out}) + g_{m2}(V_x - V_{out})$$

$$\text{from } \textcircled{D}: V_x - V_{out} = \left(\frac{sC_3 + g_{m1}}{s(C_2 + C_3)} \right) V_x$$

$$\therefore I_x = \left[sC_1 + \frac{s^2C_2C_3 + g_{m1}sC_2 + g_{m2}sC_3 + g_{m1}g_{m2}}{s(C_2 + C_3)} \right] V_x$$

$$= \left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m1}C_2 + g_{m2}C_3) + g_{m1}g_{m2}}{s(C_2 + C_3)} \right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s(C_2 + C_3)}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m1}C_2 + g_{m2}C_3) + g_{m1}g_{m2}} \quad \star$$



$$C_1 = C_{gs1} + C_{db2}$$

$$C_2 = C_{gd1} + C_{gd2}$$

$$C_3 = C_{db1} + C_{gs2}$$

$$\text{KCL @ } V_{out} : sC_2(V_x - V_{out}) = g_{m1}V_x + V_{out}\left(\frac{1}{r_{o1}} + sC_3\right)$$

$$\frac{V_{out}}{V_x} = \frac{sC_2 - g_{m1}}{s(C_2 + C_3) + \frac{1}{r_{o1}}}$$

$$\text{KCL @ } V_x : \frac{V_{in} - V_x}{R_S} = g_{m2}V_{out} + V_x\left(\frac{1}{r_{o2}} + sC_1\right) + sC_2(V_x - V_{out})$$

$$\frac{V_{in}}{R_S} = \left(sC_1 + sC_2 + \frac{1}{r_{o2}} + \frac{1}{R_S}\right)V_x - \frac{(g_{m2} + sC_2)(sC_2 - g_{m1})}{s(C_2 + C_3) + \frac{1}{r_{o1}}} \cdot V_x$$

$$= \frac{s^2(C_1C_2 + C_1C_3 + C_2C_3) + s\left[\left(\frac{1}{r_{o2}} + \frac{1}{R_S}\right)(C_2 + C_3) + \frac{1}{r_{o1}}(C_1 + C_2) + g_{m1}C_2 + g_{m2}C_2\right] - g_{m1}g_{m2} + \frac{1}{r_{o1}}\left(\frac{1}{r_{o2}} + \frac{1}{R_S}\right)}{s(C_2 + C_3) + \frac{1}{r_{o1}}}$$

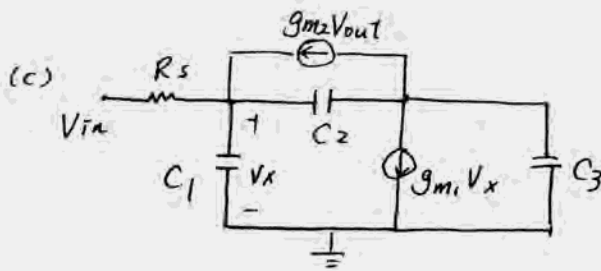
$$\therefore \frac{V_{out}}{V_{in}} = \frac{-\frac{1}{R_S}(sC_2 - g_{m1})}{s^2(C_1C_2 + C_1C_3 + C_2C_3) + s\left[\left(\frac{1}{r_{o2}} + \frac{1}{R_S}\right)(C_2 + C_3) + \frac{1}{r_{o1}}(C_1 + C_2) + g_{m1}C_2 + g_{m2}C_2\right] - g_{m1}g_{m2} + \frac{1}{r_{o1}}\left(\frac{1}{r_{o2}} + \frac{1}{R_S}\right)}$$

For Z_{in}

$$I_x = g_{m2}V_{out} + V_x\left(\frac{1}{r_{o2}} + sC_1\right) + sC_2(V_x - V_{out})$$

$$= \left[\frac{s^2(C_1C_2 + C_1C_3 + C_2C_3) + s\left[\frac{1}{r_{o2}}(C_2 + C_3) + \frac{1}{r_{o1}}(C_1 + C_2) + g_{m1}C_2 + g_{m2}C_2\right] + \left(-g_{m1}g_{m2} + \frac{1}{r_{o1}}\frac{1}{r_{o2}}\right)}{s(C_2 + C_3) + \frac{1}{r_{o1}}} \right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s(C_2 + C_3) + \frac{1}{r_{o1}}}{s^2(C_1C_2 + C_1C_3 + C_2C_3) + s\left[\frac{1}{r_{o2}}(C_2 + C_3) + \frac{1}{r_{o1}}(C_1 + C_2) + g_{m1}C_2 - g_{m2}C_2\right] + \left(-g_{m1}g_{m2} + \frac{1}{r_{o1}}\frac{1}{r_{o2}}\right)}$$



$$C_1 = C_{gs1} + C_{db2} + C_{gd2}$$

$$C_2 = C_{gd1}$$

$$C_3 = C_{db1} + C_{sb2} + C_{gs2}$$

$$\text{KCL@ } V_{out} : sC_2(V_x - V_{out}) = g_{m1}V_x + sC_3V_{out} + g_{m2}V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_x} = \frac{sC_2 - g_{m1}}{s(C_2 + C_3) + g_{m2}}$$

$$\text{KCL@ } V_x : \frac{V_{in} - V_x}{R_s} + g_{m2}V_{out} = sC_1V_x + sC_2(V_x - V_{out})$$

$$\frac{V_{in}}{R_s} = \left(\frac{1}{R_s} + sC_1 + sC_2\right)V_x - (g_{m2} + sC_2)V_{out} = \left(\frac{1}{R_s} + sC_1 + sC_2\right)V_x - \frac{(g_{m2} + sC_2)(sC_2 - g_{m1})}{s(C_2 + C_3) + g_{m2}}$$

$$\frac{V}{V} = \frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[\frac{1}{R_s}(C_2 + C_3) + g_{m2}C_1 + g_{m1}C_2\right] + \left[\frac{g_{m2}}{R_s} + g_{m1}g_{m2}\right]}{s(C_2 + C_3) + g_{m2}}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_x} \cdot \frac{V_x}{V_{in}}$$

$$= \frac{\frac{1}{R_s} [sC_2 - g_{m1}]}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s\left[\frac{1}{R_s}(C_2 + C_3) + g_{m2}C_1 + g_{m1}C_2\right] + \left(\frac{g_{m2}}{R_s} + g_{m1}g_{m2}\right)}$$

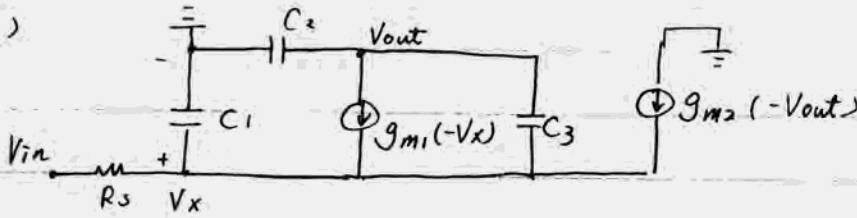
For Z_{in}

$$I_x = sC_1V_x - g_{m2}V_{out} + sC_2(V_x - V_{out})$$

$$= \left[\frac{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m2}C_1 + g_{m1}C_2) + g_{m1}g_{m2}}{s(C_2 + C_3) + g_{m2}} \right] V_x$$

$$Z_{in} = \frac{s(C_2 + C_3) + g_{m2}}{s^2(C_1C_2 + C_2C_3 + C_1C_3) + s(g_{m2}C_1 + g_{m1}C_2) + g_{m1}g_{m2}}$$

(d)



$$C_1 = C_{gs1} + C_{sb1} + C_{DB2}$$

$$C_2 = C_{gd1} + C_{gs2} + C_{db1}$$

$$C_3 = C_{gd2}$$

$$\text{KCL @ } V_{out} : -sC_2 V_{out} = -g_{m1} V_x + sC_3 (V_{out} - V_x)$$

$$\frac{V_{out}}{V_x} = \frac{sC_3 + g_{m1}}{s(C_2 + C_3)}$$

$$\text{KCL @ } V_x : \frac{V_{in} - V_x}{R_s} = sC_1 V_x + g_{m1} V_x + sC_3 (V_x - V_{out}) + g_{m2} V_{out}$$

$$\begin{aligned} \frac{V_{in}}{R_s} &= \left[\frac{1}{R_s} + s(C_1 + C_3) + g_{m1} \right] V_x + \frac{(sC_3 + g_{m1})(g_{m2} - sC_3)}{s(C_2 + C_3)} V_x \\ &= V_x \left[\frac{s^2(C_1 C_2 + C_2 C_3 + C_1 C_3) + s \left[\frac{C_1}{R_s} + \frac{C_3}{R_s} + g_{m1} C_2 + g_{m2} C_3 \right] + g_{m1} g_{m2}}{s(C_2 + C_3)} \right] \end{aligned}$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{sC_3 + g_{m1}}{s^2 R_s (C_1 C_2 + C_2 C_3 + C_1 C_3) + s [C_2 + C_3 + R_s (g_{m1} C_2 + g_{m2} C_3)] + g_{m1} g_{m2} R_s}$$

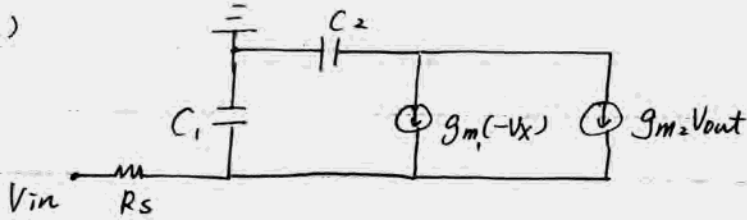
For Z_{in}

$$I_x = sC_1 V_x + g_{m1} V_x + sC_3 (V_x - V_{out}) - g_{m2} V_{out}$$

$$= \left[\frac{s^2(C_1 C_2 + C_2 C_3 + C_1 C_3) + s(g_{m1} C_2 + g_{m2} C_3) + g_{m1} g_{m2}}{s(C_2 + C_3)} \right] V_x$$

$$\therefore Z_{in} = \frac{s(C_2 + C_3)}{s^2(C_1 C_2 + C_2 C_3 + C_1 C_3) + s(g_{m1} C_2 + g_{m2} C_3) + g_{m1} g_{m2}}$$

(e)



$$C_1 = C_{gs1} + C_{sb1} + C_{db2} + C_{gd2}$$

$$C_2 = C_{gd1} + C_{sb2} + C_{gs2} + C_{db1}$$

$$\text{KCL @ } V_{out} \Rightarrow -sC_2 V_{out} = -g_{m1} V_x + g_{m2} V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_x} = \frac{g_{m1}}{sC_2 + g_{m2}}$$

$$\text{KCL @ } V_x \Rightarrow \frac{V_{in} - V_x}{R_s} = sC_1 V_x + g_{m1} V_x - g_{m2} V_{out}$$

$$\Rightarrow \frac{V_{in}}{R_s} = \left[\frac{1}{R_s} + sC_1 + g_{m1} \right] V_x - \frac{g_{m1} g_{m2}}{(sC_2 + g_{m2})} V_x$$

$$= \frac{s^2 C_1 C_2 + s \left[\left(\frac{1}{R_s} + g_{m1} \right) C_2 + g_{m2} C_1 \right] + \frac{g_{m2}}{R_s}}{sC_2 + g_{m2}}$$

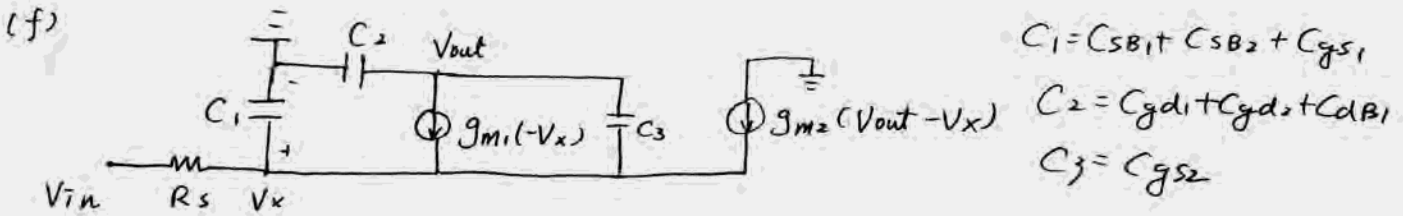
$$\therefore \frac{V_{out}}{V_{in}} = \frac{g_{m1}}{s^2 R_s C_1 C_2 + s \left[(1 + g_{m1} R_s) C_2 + g_{m2} R_s C_1 \right] + g_{m2}}$$

For Z_{in}

$$I_x = sC_1 V_x + g_{m1} V_x - g_{m2} V_{out}$$

$$= \left[\frac{s^2 C_1 C_2 + s \left[g_{m1} C_2 + g_{m2} C_1 \right]}{sC_2 + g_{m2}} \right]$$

$$\therefore Z_{in} = \frac{sC_2 + g_{m2}}{s^2 C_1 C_2 + s \left(g_{m1} C_2 + g_{m2} C_1 \right)}$$



$$\text{KCL @ } V_{out} : sC_2(-V_{out}) = g_{m1}(-V_x) + sC_3(V_{out} - V_x)$$

$$\Rightarrow \frac{V_{out}}{V_x} = \frac{g_{m1} + sC_3}{s(C_2 + C_3)}$$

$$\text{KCL @ } V_x : \frac{V_{in} - V_x}{R_s} + g_{m1}(-V_x) + g_{m2}(V_{out} - V_x) = sC_1 V_x + sC_3(V_x - V_{out})$$

$$\frac{V_{in}}{R_s} = V_x \left(\frac{1}{R_s} + g_{m1} + g_{m2} + sC_1 + sC_3 \right) - \frac{(g_{m2} + sC_3)(g_{m1} + sC_3)}{s(C_2 + C_3)} \cdot V_x$$

$$= \frac{s^2(C_1 C_2 + C_2 C_3 + C_1 C_3) + s \left[\frac{C_2}{R_s} + \frac{C_3}{R_s} + g_{m1} C_2 + g_{m2} C_2 \right] - g_{m1} g_{m2}}{s(C_2 + C_3)}$$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1} + sC_3}{s^2 R_s (C_1 C_2 + C_2 C_3 + C_1 C_3) + s [C_2 + C_3 + R_s (g_{m1} C_2 + g_{m2} C_2)] - g_{m1} g_{m2} R_s}$$

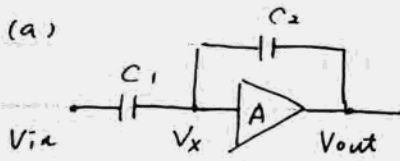
For Z_{in}

$$I_x = g_{m1} V_x + g_{m2} (V_x - V_{out}) + sC_1 V_x + sC_3 (V_x - V_{out})$$

$$= \left[\frac{s^2(C_1 C_2 + C_2 C_3 + C_1 C_3) + s [g_{m1} C_2 + g_{m2} C_2] - g_{m1} g_{m2}}{s(C_2 + C_3)} \right] V_x$$

$$Z_{in} = \frac{V_x}{I_x} = \frac{s(C_2 + C_3)}{s^2(C_1 C_2 + C_2 C_3 + C_1 C_3) + s(g_{m1} C_2 + g_{m2} C_2) - g_{m1} g_{m2}}$$

6.4 (a)



$$A = -\infty$$

(i) At low frequency, V_x is like virtual ground

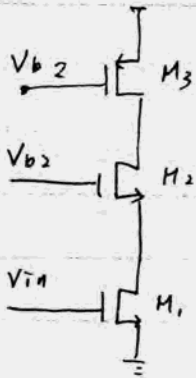
$$sC_1 V_{in} = -sC_2 V_{out}$$

$$\frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2}$$

(ii) At high frequency, C_1, C_2 is like a short circuit

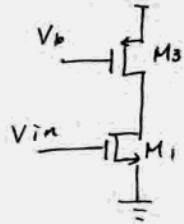
$$\frac{V_{out}}{V_{in}} = 1$$

(b) At low frequency, the equivalent circuit is shown as



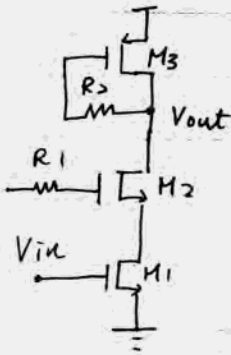
$$A_v \approx -g_{m1} r_{o3} \rightarrow \infty, \text{ if } \lambda = 0$$

(ii) At high frequency, the equivalent circuit

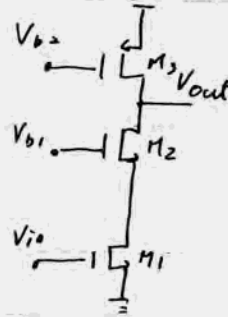


$$A_v = -g_{m1}(r_{o1} || r_{o3}) \rightarrow \infty \text{ if } \lambda = 0$$

(c) (i) At low frequency, the equivalent circuit



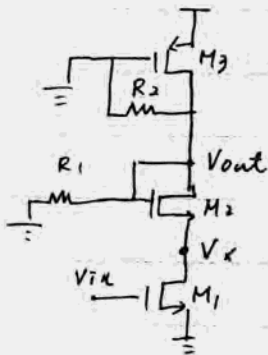
R_1, R_2 can be ignored
 \Rightarrow



The impedance @ $V_{out} = \frac{1}{g_{m3}}$

$$A_v \cong -g_{m1} \cdot \frac{1}{g_{m3}} = -\frac{g_{m1}}{g_{m3}}$$

(ii) At high frequency,



$$\frac{V_x}{V_{in}} = -g_{m1} \cdot \frac{1}{g_{m2}}$$

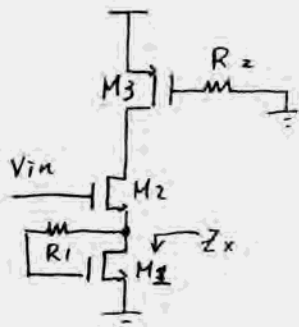
↳ the impedance looking into V_x

The impedance @ $V_{out} = R_1 || R_2$

$$\therefore A_v = \left(-g_{m1} \cdot \frac{1}{g_{m2}} \right) \cdot g_{m2} \cdot (R_1 || R_2) = -g_{m1} (R_1 || R_2)$$

#

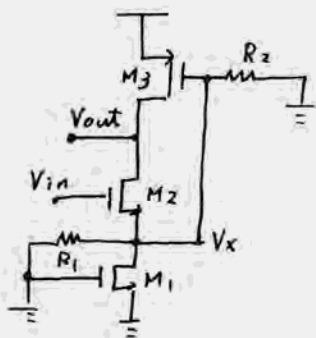
(d) (i) At low frequency, the equivalent circuit is



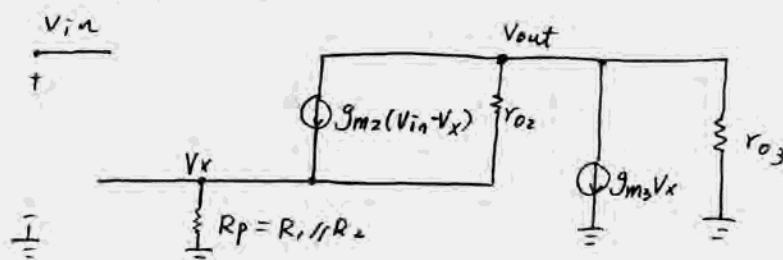
$$\frac{V_{out}}{V_{in}} = \frac{g_{m2}(r_{o3} \parallel (1 + g_{m2}r_{o2})Z_x)}{1 + g_{m2}Z_x} \approx \frac{g_{m2}(r_{o3} \parallel r_{o2})}{1 + \frac{g_{m2}}{g_{m1}}} \rightarrow \infty \text{ if } \lambda = 0$$

$$Z_x = \frac{1}{g_{m1}}$$

(ii) At high frequency



Small-signal model



$$\text{KCL @ } V_x, V_{out} : \frac{V_x}{R_p} = g_{m2}(V_{in} - V_x) + \frac{V_{out} - V_x}{r_{o2}} = -\left(g_{m3}V_x + \frac{V_{out}}{r_{o3}}\right)$$

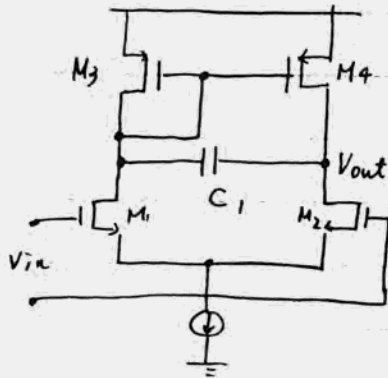
$$\frac{V_x}{R_p} = -\left(g_{m3}V_x + \frac{V_{out}}{r_{o3}}\right) \Rightarrow \frac{V_{out}}{V_x} = -r_{o3}\left(g_{m3} + \frac{1}{R_p}\right)$$

$$g_{m2}(V_{in} - V_x) + \frac{V_{out} - V_x}{r_{o2}} = \frac{V_x}{R_p} \Rightarrow g_{m2}V_{in} = \left(\frac{1}{R_p} + \frac{1}{r_{o2}} + g_{m2}\right)V_x + \frac{V_{out}}{r_{o2}}$$

$$\Rightarrow g_{m2}V_{in} = \left[-\left(\frac{1}{R_p} + \frac{1}{r_{o2}} + g_{m2}\right) \frac{1}{r_{o3}\left(g_{m3} + \frac{1}{R_p}\right)} + \frac{1}{r_{o2}}\right] V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{-g_{m2}r_{o3}\left(g_{m3} + \frac{1}{R_p}\right)}{\left(\frac{1}{R_p} + \frac{1}{r_{o2}} + g_{m2}\right) - \frac{r_{o3}\left(g_{m3} + \frac{1}{R_p}\right)}{r_{o2}}} \rightarrow \infty \text{ if } \lambda = 0$$

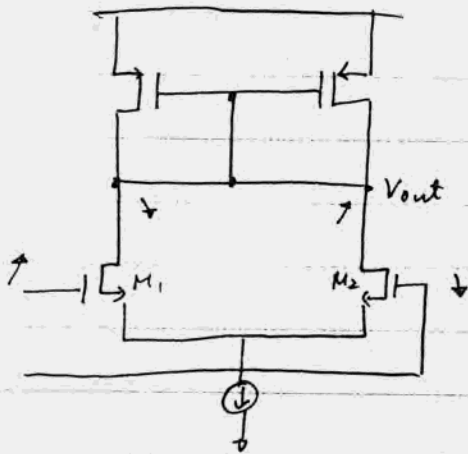
6.5(a) (i) At low frequency



C_1 is like an open circuit @ very low frequency

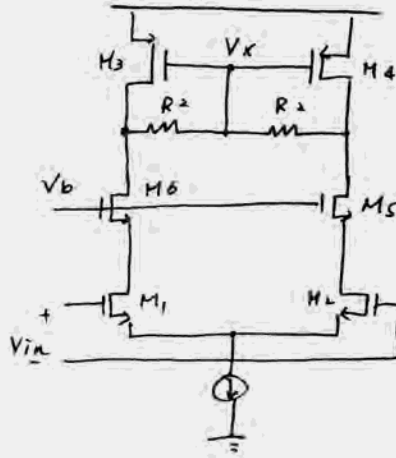
$$\Rightarrow \frac{V_{out}}{V_{in}} = -g_{m1}(r_{o2} \parallel r_{o4}) \rightarrow \infty \text{ if } \lambda = 0$$

(ii) At very high frequency, C_1 is like a short circuit

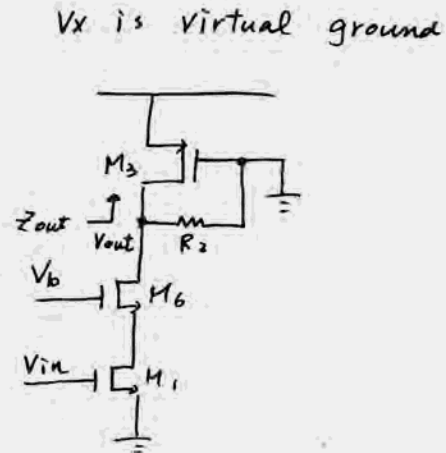


Gain = 0

(b) (i) At low frequency, the equivalent circuit is



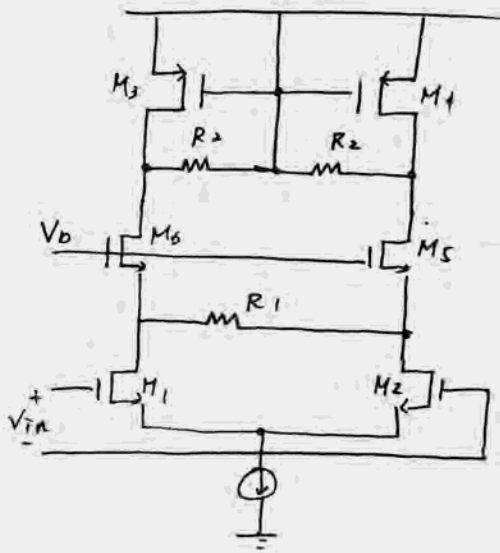
half circuit



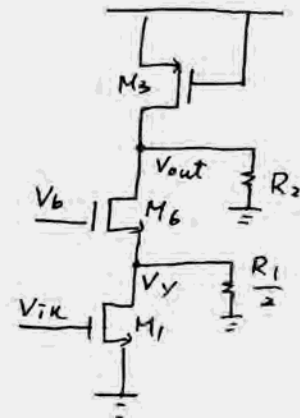
$$Z_{out} \approx r_{o3} \parallel R_2 \approx R_2$$

$$A_v = -g_{m1} \cdot (R_2 \parallel r_{o3}) \approx -g_{m1} R_2$$

(ii) At high frequency



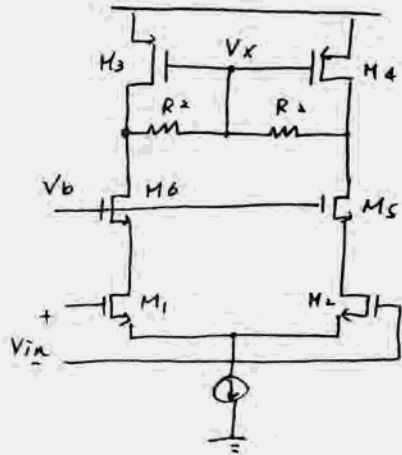
half circuit



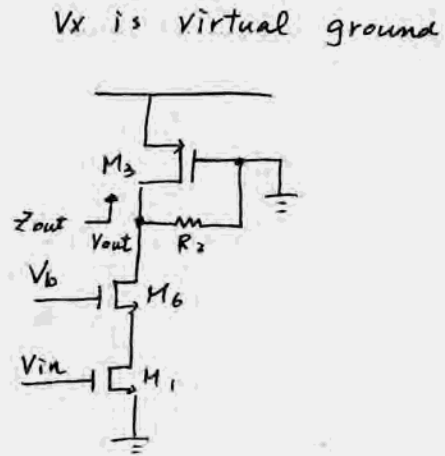
$$\frac{V_y}{V_{in}} = -g_{m1} \left(\frac{1}{g_{m6}} \parallel \frac{R_1}{2} \right), \quad \frac{V_{out}}{V_y} \approx +g_{m6} \cdot R_2$$

$$\frac{V_{out}}{V_{in}} = - \frac{g_{m1} g_{m6} R_1 R_2}{(2 + g_{m6} \cdot R_1)}$$

(b) (i) At low frequency, the equivalent circuit is



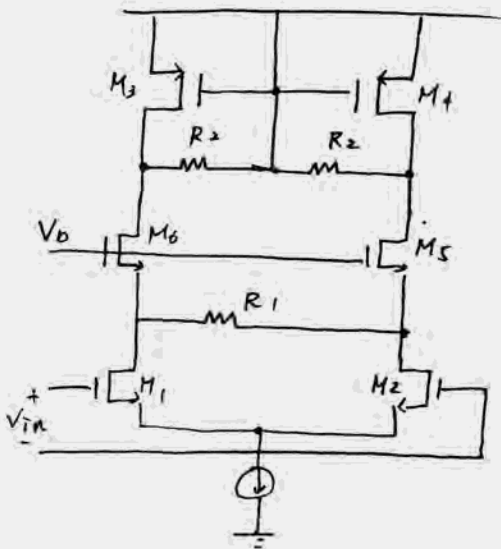
half circuit



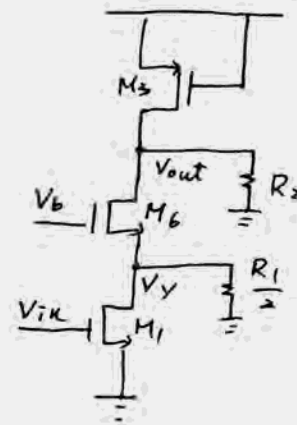
$$Z_{out} \cong r_{o3} \parallel R_2 \cong R_2$$

$$A_v = -g_{m1} \cdot (R_2 \parallel r_{o3}) \cong -g_{m1} R_2 \quad \#$$

(ii) At high frequency



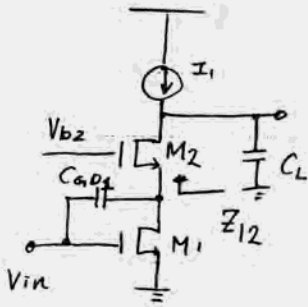
half circuit



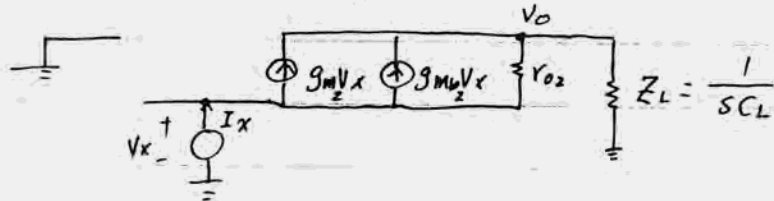
$$\frac{V_y}{V_{in}} = -g_{m1} \left(\frac{1}{g_{m6}} \parallel \frac{R_1}{2} \right), \quad \frac{V_{out}}{V_y} \cong +g_{m6} \cdot R_2$$

$$\frac{V_{out}}{V_{in}} = - \frac{g_{m1} g_{m6} R_1 R_2}{(2 + g_{m6} \cdot R_1)} \quad \#$$

6.6



The impedance Z_{12} can be derived from the following small signal model



$$\text{KCL @ } V_o: \frac{V_o}{Z_L} + \frac{V_o - V_x}{r_{o2}} = (g_{m2} + g_{mb2})V_x \Rightarrow \left(\frac{1}{Z_L} + \frac{1}{r_{o2}}\right)V_o = \left(g_{m2} + g_{mb2} + \frac{1}{r_{o2}}\right)V_x$$

$$\Rightarrow V_o = \left(\frac{g_{m2} + g_{mb2} + \frac{1}{r_{o2}}}{1 + \frac{Z_L}{r_{o2}}}\right)V_x$$

$$\Rightarrow I_x = \frac{V_o}{Z_L} = \left[\frac{g_{m2} + g_{mb2} + \frac{1}{r_{o2}}}{1 + \frac{Z_L}{r_{o2}}}\right]V_x \Rightarrow \frac{V_x}{I_x} = Z_{12} = \frac{1 + \frac{Z_L}{r_{o2}}}{g_{m2} + g_{mb2} + \frac{1}{r_{o2}}}$$

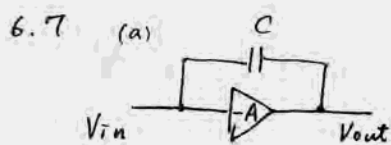
$$\Rightarrow Z_{12} = \frac{r_{o2} + Z_L}{1 + (g_{m2} + g_{mb2})r_{o2}}$$

The miller multiplication for $C_{GD1} = 1 + g_{m1}Z_{12}$

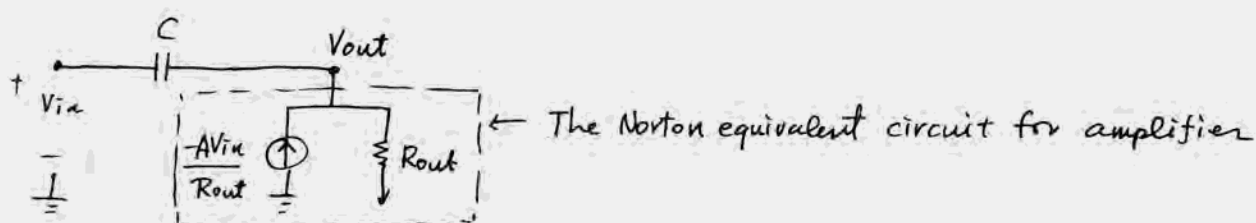
$$= 1 + \frac{g_{m1}(r_{o2} + Z_L)}{1 + (g_{m2} + g_{mb2})r_{o2}} \quad \text{--- } \textcircled{1}$$

If C_L is relatively large $\Rightarrow \left|\frac{1}{sC_L}\right| \ll r_{o2}$

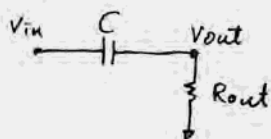
$$\text{eg } \textcircled{1} \text{ can be approximated as } \cong 1 + \frac{g_{m1}r_{o2}}{1 + (g_{m2} + g_{mb2})r_{o2}} \cong 1 + \frac{g_{m1}}{g_{m2} + g_{mb2}}$$



Assume the amplifier output resistance R_{out}
The small signal model is as follows



As we can see, the above circuit forms a high pass network



Thus, when there is a step ΔV at the input, output will follow input, a step ΔV , first. Then, it will settle down to $-AV_{in}$ as the steady state

(b) KCL @ V_{out} :

$$-\frac{AV_{in}}{R_{out}} = \frac{V_{out}}{R_{out}} + sC(V_{out} - V_{in})$$

$$\Rightarrow \left(sC - \frac{A}{R_{out}}\right)V_{in} = \left(\frac{1}{R_{out}} + sC\right)V_{out} \Rightarrow \frac{V_{out}}{V_{in}} = \frac{sCR_{out} - A}{1 + sCR_{out}} \quad *$$

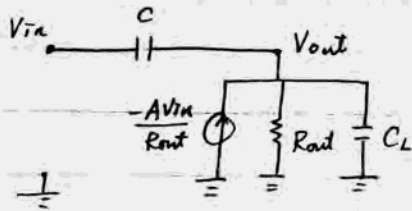
for the step response, $x(t) = u(t)$, $t \geq 0 \rightarrow X(s) = \frac{1}{s}$

$$Y(s) = \frac{1}{s} \cdot \frac{V_{out}}{V_{in}}(s) = \frac{sCR_{out} - A}{s(1 + sCR_{out})} = \frac{-A}{s} + \frac{(A+1) \cdot R_{out}C}{1 + sCR_{out}}$$

$$\Rightarrow y(t) = -Au(t) + (A+1)e^{-\frac{t}{R_{out}C}}, t \geq 0$$

For a ΔV input step, output = $-A \cdot \Delta V + (A+1) \cdot \Delta V \cdot e^{-\frac{t}{R_{out}C}}$ *

6.8 (a) Small-signal circuit model



When input has ΔV jump, V_{out} will follow
and the output jump = $\left(\frac{C}{C_L + C}\right) \Delta V$ *

(b) The transfer function $H(s)$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} \quad \text{KCL @ } V_{out}$$

$$\Rightarrow -\frac{A V_{in}}{R_{out}} = \frac{V_{out}}{R_{out}} + sC(V_{out} - V_{in}) + sC_L V_{out}$$

$$\Rightarrow V_{in} \left(sC - \frac{A}{R_{out}} \right) = V_{out} \left(\frac{1}{R_{out}} + sC + sC_L \right)$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{sC R_{out} - A}{1 + sR_{out}(C + C_L)} \quad *$$

step response

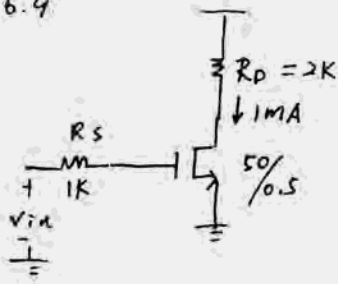
$$Y(s) = \frac{1}{s} \cdot \frac{sC R_{out} - A}{1 + sR_{out}(C + C_L)} = \frac{-A}{s} + \frac{[(A+1)C + AC_L] R_{out}}{1 + sR_{out}(C + C_L)} = \frac{-A}{s} + \frac{\frac{(A+1)C + AC_L}{C + C_L}}{s + \frac{1}{R_{out}(C + C_L)}}$$

$$y(t) = -A u(t) + \frac{(A+1)C + AC_L}{C + C_L} e^{-\frac{t}{R_{out}(C + C_L)}} u(t)$$

For a step ΔV @ the input

$$\text{output} = -A \Delta V + \left[\frac{(A+1)C + AC_L}{C + C_L} \right] \Delta V \cdot e^{-\frac{t}{R_{out}(C + C_L)}} \quad *$$

6.9



$$\lambda = 0.1$$

$$C_{ox} = \frac{\epsilon_{SiO_2}}{t_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14}}{9 \times 10^{-7}} = 3.835 \times 10^{-7}$$

$$\mu_n = 350$$

$$I_D = 10^{-3} = \frac{1}{2} \times 350 \times 3.835 \times 10^{-7} \times \left(\frac{50}{0.5 - 2 \times 0.08} \right) (V_{GS} - V_T)^2 (1 + 0.1 \times V_{DS})$$

$$= 67.113 \times 10^{-6} \times \frac{50}{0.34} \times 1.1 \times (V_{in} - 0.7)^2$$

$$\Rightarrow V_{in} = 1.0035$$

$$g_m = \frac{2I_D}{(V_{GS} - V_T)} = 6.59 \times 10^{-3}$$

$$C_{gs} = \frac{2}{3} C_{ox} w L + C_{ov} \cdot w$$

$$= \frac{2}{3} \times 3.835 \times 10^{-7} \times 50 \times (0.5 - 0.08 \times 2) \times 10^{-8} + 3.835 \times 10^{-7} \times 0.08 \times 10^{-4} \times 50 \times 10^{-4}$$

$$= 53.7 \times 10^{-15}$$

$$C_{gd} = C_{gd0} \cdot w = 0.4 \times 10^{-11} \times 50 \times 10^{-6} = 2 \times 10^{-16}$$

$$C_{db} = \frac{0.56 \times 10^{-3} \times 75 \times 10^{-12}}{\left(1 + \frac{1}{0.9}\right)^{0.6}} + \frac{0.35 \times 10^{-11} \times 103 \times 10^{-6}}{\left(1 + \frac{1}{0.9}\right)^{0.2}} = 2.714 \times 10^{-14}$$

According to eq (20)

$$zero = \frac{g_m}{C_{gd}} = \frac{6.59 \times 10^{-3}}{2 \times 10^{-16}} = 3.3 \times 10^{13} \text{ rad/sec} *$$

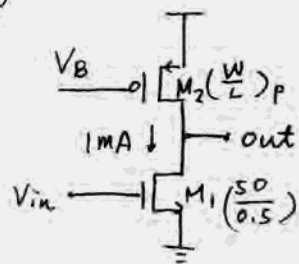
pde is the root of $R_s R_D (C_{gs} C_{gd} + C_{gs} C_{db} + C_{gd} C_{db}) s^2 + [R_s (1 + g_m R_D) C_{gd} + R_s C_{gs} + R_D (C_{gd} + C_{db})] s + 1 \dots$ from eq (6.20)

$$\Rightarrow 2.95 \times 10^{-21} s^2 + 1.112 \times 10^{-10} s + 1 = 0$$

$$\omega_{p1} = -14.82 \times 10^9 \text{ rad/sec} *$$

$$\omega_{p2} = -22.88 \times 10^9 \text{ rad/sec} *$$

6.10



(a) The maximum output level = 2.6 V

→ V_B can be as low as $2.6 - |V_{THP}| = 2.6 - 0.8 = 1.8V$

Let's choose output DC bias @ 1.5V, such that

 M_1, M_2 are both in saturation region

$$\text{Thus, } I_{D1} = 10^{-3} = \frac{1}{2} \times 350 \times 3.835 \times 10^{-7} \times \left(\frac{50}{0.5 - 2 \times 0.08}\right) (V_{in} - 0.7)^2 (1 + 0.1 \times 1.5)$$

$$\Rightarrow V_{in} = 0.997 \approx 1V$$

$$\text{Also, } I_{D2} = 10^{-3} = \frac{1}{2} \times 100 \times 3.835 \times 10^{-7} \times \left(\frac{W_P}{0.5 - 2 \times 0.09}\right) (3 - 1.8 - 0.8)^2 (1 + 0.2 \times 1.5)$$

$$W_P \approx 80.5 \mu m = 81 \mu m$$

Therefore, we can choose $\left(\frac{W}{L}\right)_p = \left(\frac{81 \mu m}{0.5 \mu m}\right)$ with gate bias 1.8Vso that M_1, M_2 are both in saturation region and $V_{out} \approx 1.5V$

$$V_{out, low} = V_{in} - |V_{THN}| = 0.3V$$

Thus, the maximum output peak-to-peak swing = $2.6 - 0.3 = 1.3V$ *

(b) This problem is similar to problem 6.9 except

$$R_D \rightarrow (r_{o1} \parallel r_{o2})$$

$$C_{DB} = C_{DB1} + C_{DB2} + C_{gd2}$$

$$\therefore g_{m1} = \frac{2I_D}{V_{GS} - V_{t1}} = \frac{2 \times 10^{-3}}{0.297} = 6.73 \times 10^{-3}$$

$$r_{op} = \frac{1}{\left(\frac{\lambda_P I_D}{1 + \lambda_P V_{DS}}\right)} \approx 6.5K$$

$$r_{oN} = \frac{1}{\left(\frac{\lambda_N I_D}{1 + \lambda_N V_{DS}}\right)} = 11.5K$$

$$\therefore R_D = r_{op} \parallel r_{oN} = 4.1K$$

$$R_S = 1K$$

$$C_{gs1} = \frac{2}{3} C_{ox} W_1 L_1 + C_{ox} W_1 \cdot \Delta L = 58.8 \times 10^{-15} F$$

$$C_{gd1} = 2 \times 10^{-16}$$

$$C_{db1} = \frac{0.56 \times 10^{-3} \times 75 \times 10^{-12}}{\left(1 + \frac{1.5}{0.9}\right)^{0.6}} + \frac{0.35 \times 10^{-11} \times 107 \times 10^{-6}}{\left(1 + \frac{1.5}{0.9}\right)^{0.2}} = 23.38 \times 10^{-15} \text{ F}$$

$$C_{db2} = \frac{0.94 \times 10^{-3} \times 121.5 \times 10^{-12}}{\left(1 + \frac{1.5}{0.9}\right)^{0.5}} + \frac{0.32 \times 10^{-11} \times 165 \times 10^{-6}}{\left(1 + \frac{1.5}{0.9}\right)^{0.3}} = 70.33 \times 10^{-15} \text{ F}$$

$$C_{gd2} = 81 \times 0.3 \times 10^{-11} \times 10^{-6} = 0.243 \times 10^{-15} \text{ F}$$

$$\omega_3 = -\frac{g_{m1}}{C_{gd1}} = -\frac{6.59 \times 10^{-3}}{2 \times 10^{-16}} = -3.3 \times 10^{13} \text{ rad/sec}$$

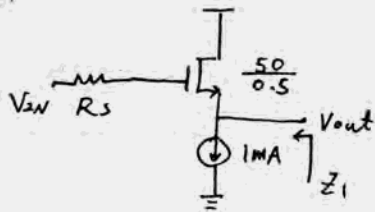
ω_{p1}, ω_{p2} is the root of the equation

$$R_s R_D (C_{gs}, C_{gd1} + C_{gs}, C_{db} + C_{gd}, C_{db}) + [R_s (1 + g_{m1} R_D) C_{gd1} + R_s C_{gs} + R_D (C_{gd1} + C_{db})] + 1$$

$$\Rightarrow \omega_{p1} = -2.2 \times 10^9 \text{ rad/sec}$$

$$\omega_{p2} = -17.36 \times 10^9 \text{ rad/sec}$$

6.11

Assume $\sigma = 0$

$$g_m = \frac{2I_D}{(V_{GS} - V_{t})}$$

$$= \frac{2 \times 10^{-3}}{0.3} = 6.67 \times 10^{-3}$$

From eq (6.49) $Z_1 = \frac{R_S C_{gs} s + 1}{g_m + C_{gs} s}$

Since $\frac{1}{g_m} < R_S$

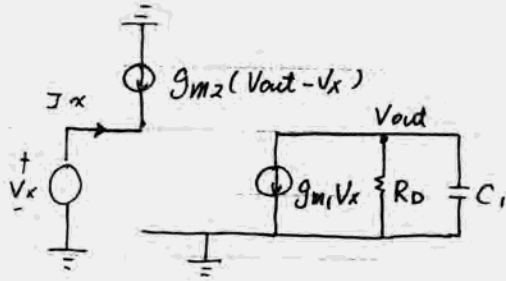
, thus Z_1 is inductive and the equivalent inductance is

$$= \frac{C_{gs}}{g_m} \left(R_S - \frac{1}{g_m} \right)$$

$$C_{gs} = 58 \times 10^{-15} \text{ F}$$

$$\therefore L = 8.56 \times 10^{-8} \text{ H}$$

6.12 (a)



$$V_{out} = -g_{m1} V_x (R_D \parallel \frac{1}{sC_1})$$

$$I_x = -g_{m2} (V_{out} - V_x) = -g_{m2} V_{out} + g_{m2} V_x = [g_{m2} g_{m1} (R_D \parallel \frac{1}{sC_1}) + g_{m2}] V_x$$

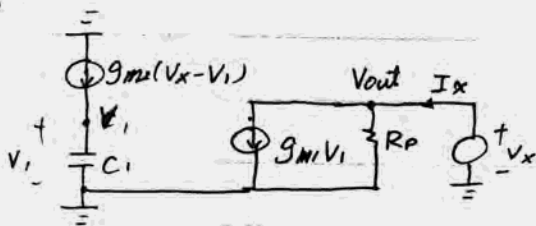
$$Z_x = \frac{V_x}{I_x} = \frac{1}{g_{m2} g_{m1} \frac{R_D / sC_1}{R_D + 1/sC_1} + g_{m2}} = \frac{1}{g_{m2} \left[\left(\frac{g_{m1} R_D}{1 + sR_D C_1} \right) + 1 \right]}$$

Thus, $Z_x(s \rightarrow 0) = \frac{1}{g_{m2}(1 + g_{m1} R_D)}$

$Z_x(s \rightarrow \infty) = \frac{1}{g_{m2}}$



(b)



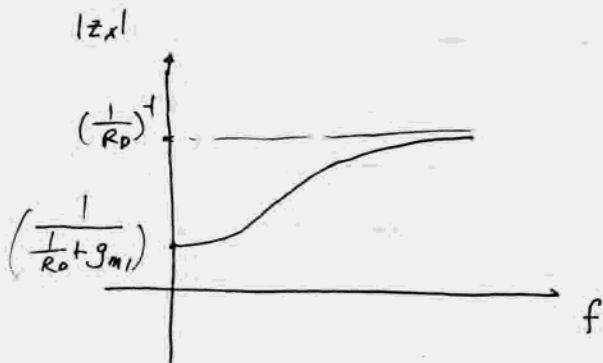
KCL @ V_i : $g_{m2}(V_x - V_i) = sC_1 V_i$

$$\Rightarrow V_i = \left(\frac{g_{m2}}{sC_1 + g_{m2}} \right) V_x$$

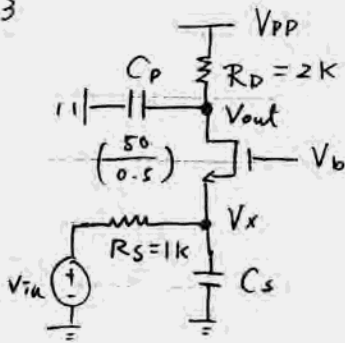
KCL @ V_{out} : $I_x = \frac{V_x}{R_D} + g_{m1} V_i$

$$= \frac{V_x}{R_D} + \frac{g_{m1} g_{m2}}{sC_1 + g_{m2}} V_x$$

$$\Rightarrow Z_x = \frac{1}{\frac{1}{R_D} + \frac{g_{m1} g_{m2}}{sC_1 + g_{m2}}}$$



6.13



from eq (6.53)

$$\frac{V_{out}}{V_{in}}(s) = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} \frac{1}{\left(1 + \frac{C_S}{g_m + g_{mb} + R_S^{-1}} \cdot s\right) (1 + R_D C_D)}$$

Assume V_b is chosen appropriately such that $V_x \approx 0$ (no body effect)

$$g_m \sim 6.59 \times 10^{-3}$$

$$g_{mb} = \left[\frac{\sigma}{2\sqrt{2\phi_f}} \right] g_m = 1.563 \times 10^{-3}$$

$$\left. \begin{aligned} C_S &= C_{SB} + C_{gs} = 42.4 \times 10^{-15} + 58.8 \times 10^{-15} \\ C_D &= C_{DB} = 27.14 \times 10^{-15} \text{ F} \end{aligned} \right\} \text{from problem 9}$$

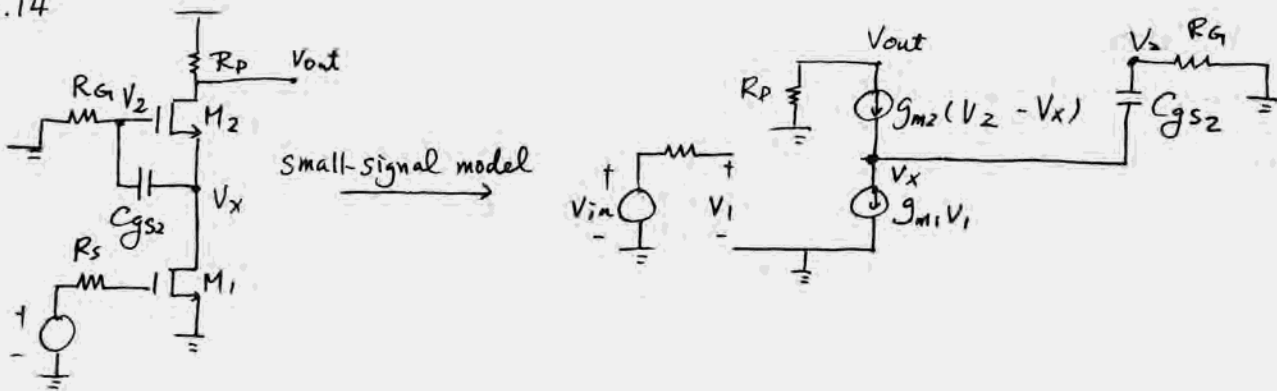
$$A_v(\text{low frequency}) = \frac{(g_m + g_{mb})R_D}{1 + (g_m + g_{mb})R_S} = 1.44$$

$$\omega_{p1} = - \frac{g_m + g_{mb} + R_S^{-1}}{C_S} = - \frac{6.59 \times 10^{-3} + 1.563 \times 10^{-3} + 10^{-3}}{42.4 \times 10^{-15} + 58.8 \times 10^{-15}} = -9.044 \times 10^{10} \text{ rad/s}$$

$$\omega_{p2} = - \frac{1}{R_D C_D} = - \frac{1}{2 \times 10^3 \times 27.14 \times 10^{-15}} = -1.84 \times 10^{10} \text{ rad/s}$$

Compared with the pole locations in problem 9, the poles for common-gate configuration are much larger because there is no Miller-effect for C_{gd} in this case.

6.14



$$\text{KCL @ } V_2 : \frac{V_2}{R_G} = sC_{gs2}(V_x - V_2)$$

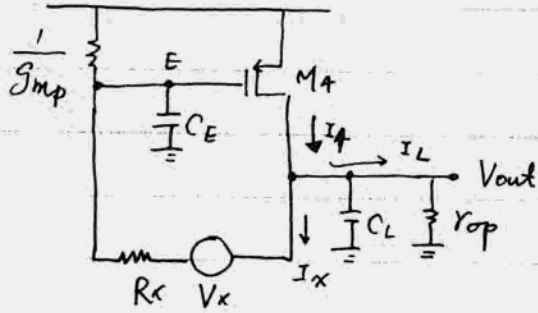
$$\Rightarrow \frac{V_2}{V_x} = \frac{sC_{gs2}}{\frac{1}{R_G} + sC_{gs2}} = \frac{sR_G C_{gs2}}{1 + sR_G C_{gs2}} \quad \text{--- ①}$$

$$\text{KCL @ } V_{out} : V_{out} = -g_{m2}(V_2 - V_x)R_D = \left[\frac{g_{m2}R_D}{1 + sR_G C_{gs2}} \right] V_x \quad \text{--- ②}$$

$$\begin{aligned} \text{KCL @ } V_x : \quad g_{m1}V_1 &= g_{m1}V_{in} = (g_{m2} + sC_{gs2})(V_2 - V_x) \\ &= \frac{-(g_{m2} + sC_{gs2})}{1 + sR_G C_{gs2}} V_x \quad \text{--- ③} \end{aligned}$$

$$\text{From ②, ③, } \frac{V_{out}}{V_{in}} = \frac{-g_{m1}g_{m2}R_D}{g_{m2} + sC_{gs2}} \quad *$$

6.15



For zero frequency, $I_L = 0$ & $I_4 = I_x$

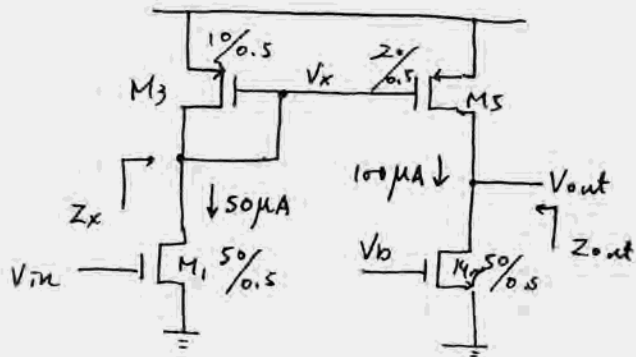
$$I_4 = -g_{mp} V_E$$

$$I_x = V_E (g_{mp} + sC_E)$$

$$\therefore I_4 = I_x \Rightarrow -g_{mp} = g_{mp} + sC_E$$

$$s_z = \frac{-2g_{mp}}{C_E} *$$

6.15 Half circuit can be drawn as follows



Since $R_s = 0$. According to (6.20) & (6.76)

$$\frac{V_{out}}{V_{in}}(s) = - \frac{(C_{gd1} s - g_{m1}) \cdot \frac{1}{g_{m3}} \cdot g_{m5} \cdot (r_{o5} \parallel r_{o7})}{\frac{1}{g_{m3}} (C_{gd1} + C_x) s + 1} \cdot \frac{1}{1 + (r_{o5} \parallel r_{o7}) \cdot C_L \cdot s}$$

where $C_L = C_{db7} + C_{gd7} + C_{db5}$

$$C_x = C_{gs3} + C_{db1} + C_{gs5} + C_{db3} + C_{gd5} (1 + g_{m5} (r_{o5} \parallel r_{o7}))$$

$$Z_{out} = (r_{o5} \parallel r_{o7})$$

$$Z_x = \frac{1}{g_{m3}}$$

First of all, let's calculate V_x operating point

$$I_{d3} = 50 \times 10^{-6} = \frac{1}{2} \times 100 \times 3.835 \times 10^{-7} \times \frac{10}{0.5 - 0.09 \times 2} (3 - V_x - 0.8)^2 (1 + 0.2(3 - V_x))$$

$$V_x \approx 1.94 \text{ V}$$

For V_{in} operating point

$$I_{d1} = 50 \times 10^{-6} = \frac{1}{2} \times 250 \times 3.835 \times 10^{-7} \times \frac{50}{0.5 - 0.08 \times 2} (V_{gs1} - 0.7)^2 (1 + 0.1 \cdot 1.94)$$

$$V_{gs1} \approx 0.765 \text{ V}$$

$$\Rightarrow g_{m1} = \frac{2 I_{d1}}{(V_{gs1} - V_t)} \approx 1.54 \times 10^{-3}$$

$$g_{m3} = \frac{2 I_{d1}}{(3 - 1.94 - 0.8)} \approx 3.73 \times 10^{-4} \Rightarrow g_{m5} = 2 \cdot g_{m3} = 7.46 \times 10^{-4}$$

$$g_{dss} = 2 \times 50 \times 10^{-6} \cdot \lambda / (1 + \lambda \cdot 1.06) = 10^{-4} \cdot 0.2 / 1.212 \approx 1.649 \times 10^{-5}$$

$$g_{ds7} = 10^{-4} \times 0.1 / (1 + 0.196) \approx 8.36 \times 10^{-6}$$

$$r_{os} // r_{o7} = 40290$$

$$C_L = C_{DB5} + C_{DB7} + C_{gd7} = \left[\frac{0.94 \times 10^{-3} \times 30 \times 10^{-12}}{\left(1 + \frac{1.06}{0.9}\right)^{0.5}} + \frac{0.32 \times 10^{-11} \times 43 \times 10^{-6}}{\left(1 + \frac{1.06}{0.9}\right)^{0.3}} \right]$$

$$+ \left[\frac{0.56 \times 10^{-3} \times 75 \times 10^{-12}}{\left(1 + \frac{1.94}{0.9}\right)^{0.6}} + \frac{0.35 \times 10^{-11} \times 103 \times 10^{-6}}{\left(1 + \frac{1.94}{0.9}\right)^{0.2}} \right] + 50 \times 0.4 \times 10^{-17}$$

$$= 19.22 \times 10^{-15} + 21.36 \times 10^{-15} + 2 \times 10^{-16} = 40.78 \times 10^{-15}$$

$$C_{gs3} = \frac{2}{3} \times 3.835 \times 10^{-9} \times 10 \times 0.32 + 3.835 \times 10^{-9} \times 10 \times 0.09 = 11.633 \times 10^{-15}$$

$$C_{AB1} \approx C_{AB7} \approx 21.36 \times 10^{-15}$$

$$C_{gs5} = 2 \cdot C_{gs3} = 23.266 \times 10^{-15}$$

$$C_{AB3} = \frac{1}{2} \cdot C_{DB5} = 9.61 \times 10^{-15}$$

$$C_{gd5} = 6.3 \times 10^{-11} \times 20 \times 10^{-6} = 6 \times 10^{-17}$$

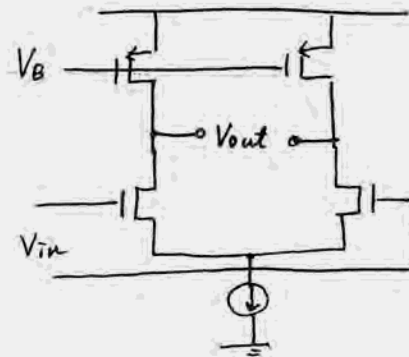
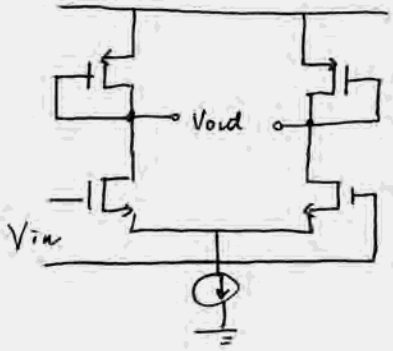
$$C_x = [11.633 + 21.36 + 23.27 + 9.61 + 0.06 (1 + g_{ms} \cdot (r_{os} // r_{o7}))] \times 10^{-15} = 67.732 \times 10^{-15}$$

$$\therefore \omega_z = \frac{g_{m1}}{C_{gd1}} = 7.7 \times 10^{12} \text{ rad/sec}$$

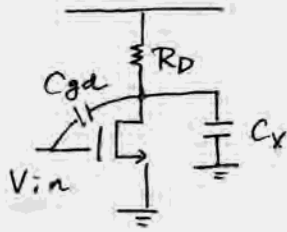
$$\omega_{p1} = - \frac{1}{C_L \cdot (r_{os} // r_{o7})} = - \frac{1}{40290 \times 40.78 \times 10^{-15}} = 6.08 \times 10^8 \text{ rad/sec}$$

$$\omega_{p2} = - \frac{g_{m3}}{(C_{gd1} + C_x)} = \frac{-3.73 \times 10^{-4}}{2 \times 10^{-16} + 67.732 \times 10^{-15}} = 5.5 \times 10^9 \text{ rad/sec}$$

6.17 (a)



Both of these two differential pair can be simplified as common-source amplifier with different load resistance and capacitance



Since $R_s = 0$, equation (6.20) can be simplified as

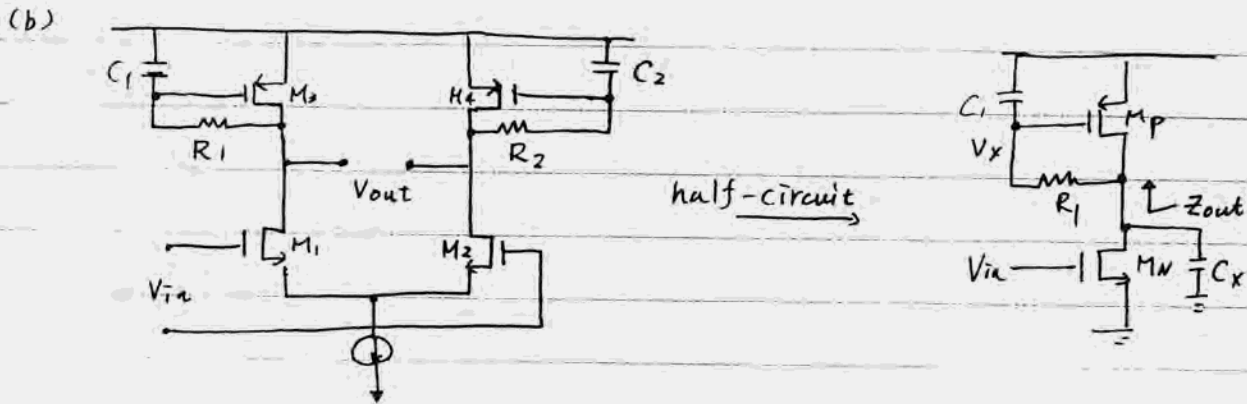
$$\frac{V_{out}}{V_{in}}(s) = \frac{(C_{gd}s - g_m)R_D}{s[R_D(C_{gd} + C_x)] + 1}$$

where $\left\{ \begin{array}{l} R_D = \frac{1}{g_{mp}} \text{ for diode connected load} \\ C_x = C_{dbN} + C_{dbP} + C_{gsP} \end{array} \right.$

$\left\{ \begin{array}{l} R_p \equiv (r_{oN} \parallel r_{oP}) \text{ for current mirror load} \\ C_x = C_{dbN} + C_{dbP} + C_{gdP} \end{array} \right.$

Although there is right-half-plane zero, however this zero is much larger than the dominant pole.

Therefore, the maximum phase shift it can achieve is $\sim 90^\circ$ before the gain is down to unity.



(i) At low frequency, C_1 is open circuit,

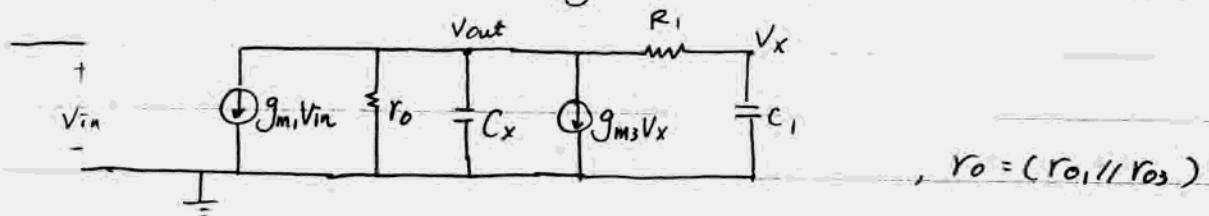
M_p is like a diode-connected device $\rightarrow Z_{out} \sim \frac{1}{g_{mp}}$

(ii) At high frequency, C_1 is short circuit.

M_p is like a current source device $\rightarrow Z_{out} \sim (r_{on} \parallel r_{op})$

Since $(r_{on} \parallel r_{op}) \gg \frac{1}{g_{mp}}$, Z_{out} exhibits an inductive behavior.

For transfer function, small-signal model



$$\text{KCL @ } V_x : \frac{V_{out} - V_x}{R_1} = sC_1 V_x \Rightarrow V_{out} = (1 + sR_1C_1)V_x$$

$$\Rightarrow V_x = \frac{V_{out}}{1 + sR_1C_1}$$

$$\text{KCL @ } V_{out} : -g_{m1}V_{in} = V_{out} \left(\frac{1}{r_o} + sC_x \right) + \frac{1}{R_1}(V_{out} - V_x) + g_{m3}V_x$$

$$= V_{out} \left(\frac{1}{r_o} + sC_x + \frac{1}{R_1} \right) + \left(g_{m3} - \frac{1}{R_1} \right) \cdot \frac{V_{out}}{1 + sR_1C_1}$$

$$-g_m V_{in} = V_{out} \left(\frac{\frac{1}{r_o} + \frac{1}{R_1} + sC_x + \left(\frac{R_1}{r_o} + 1\right) sC_1 + s^2 R_1 C_1 C_x + g_{m3} \frac{1}{R_1}}{1 + sR_1 C_1} \right)$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_m (1 + sR_1 C_1)}{s^2 R_1 C_1 C_x + s \left(C_1 + \frac{R_1 C_1}{r_o} + C_x \right) + \left(g_{m3} + \frac{1}{r_o} \right)}$$

From the above transfer function,

$$\omega_z = -\frac{1}{R_1 C_1}$$

$$\text{the sum of two poles} = -\frac{1}{R_1 C_1 C_x} \left(C_1 + \frac{R_1 C_1}{r_o} + C_x \right) = -\frac{1}{R_1 C_1} \left(1 + \frac{C_1}{C_x} \left(1 + \frac{R_1}{r_o} \right) \right)$$

usually $C_1 > C_x$, C_1 at least = C_{gs3}

Thus, the sum of two poles $> -\frac{2}{R_1 C_1}$, which means that at least one of the poles are larger than zero \Rightarrow It's quite impossible to produce 135° phase shift

Thus, this circuit still can't produce 135° phase shift.

However, it's more likely for it to generate 90° phase shift

@ unity-gain frequency.

CHAPTER 7: NOISE

(7.1)

$$|A_v| = g_m R_D$$

$$\overline{V_{n,out}^2} = \left(4KT \frac{2}{3} g_m + \frac{4KT}{R_D} \right) R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{A_v^2} = 4KT \left(\frac{2}{3} \frac{1}{g_m} + \frac{1}{g_m^2 R_D} \right)$$

$$\overline{V_{n,in_{TOT}}} = \sqrt{\overline{V_{n,in}^2} \cdot BW}$$

$$g_m = \sqrt{2I_D \mu_n C_{ox} \left(\frac{W}{L} \right)} = \sqrt{2(1mA)(134.28 \frac{\mu A}{V^2}) \left(\frac{50\mu m}{0.34\mu m} \right)} \approx 6.28 \frac{mA}{V}$$

$$4KT = 1.656 \times 10^{-20} \text{ V}^2, \quad R_D = 2k\Omega, \quad BW = 100 \text{ MHz}$$

$$\therefore \overline{V_{n,in_{TOT}}} = \sqrt{(1.966 \times 10^{-18} \frac{V^2}{Hz})(100 \text{ MHz})} \approx 14 \mu\text{V rms} //$$

(7.2)

using eqn. (7.57)

$$\overline{V_{n,in}^2} = 4KT \left(\frac{2}{3} \frac{1}{g_{m1}} + \frac{2}{3} \frac{g_{m2}}{g_{m1}^2} \right)$$

$$\overline{V_{n,in}} = \sqrt{4KT \frac{2}{3}} \sqrt{\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}}$$

$$\frac{g_{m2}}{g_{m1}^2} = \left(\frac{1}{5} \right)^2 \frac{1}{g_{m2}} \Rightarrow g_{m2} = \left(\frac{1}{5} \right)^2 g_{m1}$$

$$g_m = \frac{2I_D}{V_{GS} - V_T} \Rightarrow V_{GS} - V_T = \frac{2I_D}{g_m}$$

$$\therefore \text{Output Swing} = V_{DD} - (V_{GS1} - V_{T1}) - |V_{GS2} - V_{T2}|$$

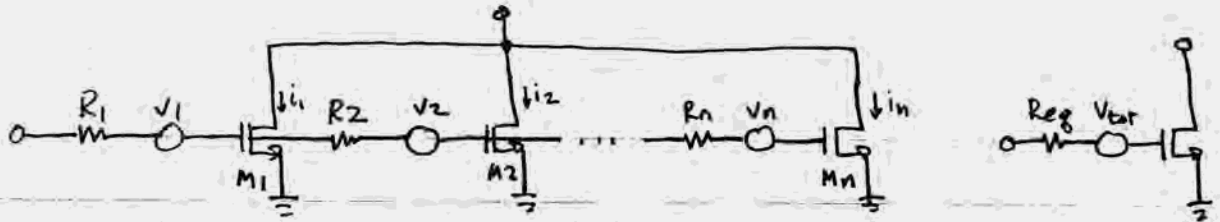
$$= V_{DD} - 2I_D \left(\frac{1}{g_{m1}} + \frac{1}{g_{m2}} \right)$$

$$= V_{DD} - 2I_D \left(\frac{1}{g_{m1}} \right) (1 + 5^2)$$

$$g_{m1} = \sqrt{2I_D \mu_n C_{ox} \left(\frac{W}{L} \right)} = \sqrt{2(61mA)(134.28 \frac{\mu A}{V^2}) \left(\frac{50\mu m}{0.34\mu m} \right)} \approx 1.986 \frac{mA}{V}$$

$$\therefore \text{Output Swing} = 3V - 2(61mA) \left(\frac{1}{1.986 \text{ mA/V}} \right) (26) \approx 0.38 \text{ V} //$$

(7.3)



The drain noise current of M_1 resulting from the gate resistance is

$$i_1 = g_{m1} V_1 \quad \text{where } V_1 \text{ is the noise voltage of } R_1.$$

Similarly, $i_2 = g_{m2} (V_1 + V_2)$

Thus, for transistor M_j , $i_j = g_{mj} (V_1 + V_2 + \dots + V_j)$

The total drain noise current is,

$$\begin{aligned} i_{tot} &= i_1 + i_2 + \dots + i_n \\ &= g_{m1} V_1 + g_{m2} (V_1 + V_2) + \dots + g_{mn} (V_1 + V_2 + \dots + V_n) \end{aligned}$$

If $g_{m1} = g_{m2} = \dots = g_{mn} = \frac{g_m}{n}$ then

$$i_{tot} = \frac{g_m}{n} [nV_1 + (n-1)V_2 + \dots + V_n]$$

Assuming v_1, \dots, v_n are uncorrelated,

$$\overline{i_{tot}^2} = \frac{g_m^2}{n^2} [n^2 \overline{v_1^2} + (n-1)^2 \overline{v_2^2} + \dots + \overline{v_n^2}]$$

If $R_1 = R_2 = \dots = R_n = R_g$ then $\overline{v_1^2} = \overline{v_2^2} = \dots = \overline{v_n^2} = 4kTB \frac{R_g}{n}$

$$\overline{i_{tot}^2} = \frac{g_m^2}{n^2} \frac{4kTB R_g}{n} [n^2 + (n-1)^2 + \dots + 1]$$

$$= g_m^2 (4kTB) R_g \frac{n(n+1)(2n+1)}{6n^3}$$

As $n \rightarrow \infty$

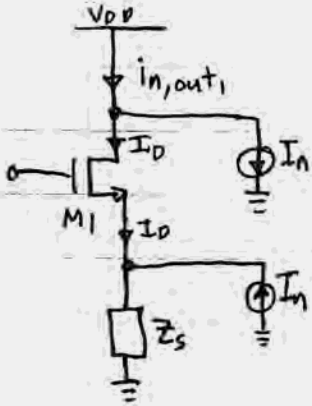
$$\overline{i_{tot}^2} = g_m^2 (4kTB \frac{R_g}{3})$$

which can be referred to the input as

$$\overline{v_{tot}^2} = \frac{\overline{i_{tot}^2}}{g_m^2} = 4kTB \left(\frac{R_g}{3} \right)$$

$$\Rightarrow \text{lumped resistance} = \frac{R_g}{3} //$$

(7.4)



$$I_{n,out,1} = I_D + I_n, \text{ KCL @ drain}$$

$$I_D = \frac{-z_s}{(\frac{1}{g_m} \parallel r_o) + z_s} I_n, \text{ current divider @ source}$$

$$\therefore I_{n,out,1} = \left(\frac{-z_s}{(\frac{1}{g_m} \parallel r_o) + z_s} + 1 \right) I_n$$

$$\therefore I_{n,out,1} = \frac{I_n}{z_s(g_m + \frac{1}{r_o}) + 1} //$$

(7.5)

$$|A_v| = (g_{m1} + g_{m2}) (r_{o1} \parallel r_{o2})$$

$$\overline{V_{n,out}^2} = 4kT \frac{2}{3} (g_{m1} + g_{m2}) (r_{o1} \parallel r_{o2})^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \frac{2}{3} \left(\frac{1}{g_{m1} + g_{m2}} \right)$$

$$\text{eqn (7.57)} \quad \overline{V_{n,in}^2} = 4kT \frac{2}{3} \left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2} \right)$$

increasing g_{m2} increases $\overline{V_{n,in}^2}$ in eqn. (7.57)

but reduces $\overline{V_{n,in}^2}$ for amplifier in figure 7.49.

(7.6)(a)

$$|A_v| = \frac{g_m R_D}{1 + g_m R_S}$$

$$\overline{V_{n,out}^2} = 4kT R_D + 4kT \frac{2}{3} \frac{1}{g_m} \left(\frac{g_m R_D}{1 + g_m R_S} \right)^2 + 4kT \frac{1}{R_S} \left(\frac{R_S}{\frac{1}{g_m} + R_S} \right)^2 R_D^2$$

$$= 4kT R_D + 4kT \frac{2}{3} \frac{1}{g_m} \left(\frac{g_m R_D}{1 + g_m R_S} \right)^2 + 4kT R_S \left(\frac{g_m R_D}{1 + g_m R_S} \right)^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \frac{2}{3} \frac{1}{g_m} + 4kT R_S + 4kT R_D \left(\frac{1 + g_m R_S}{g_m R_D} \right)^2 //$$

(7.6)(b)

$$|A_v| = g_m \left(\frac{1}{g_m} \parallel R_S \right)$$

$$\overline{V_{n,out}^2} = (4kT \frac{2}{3} g_m + 4kT \frac{1}{R_S}) \left(\frac{1}{g_m} \parallel R_S \right)^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \frac{2}{3} \frac{1}{g_m} + 4kT \frac{1}{g_m^2 R_S} //$$

$$(7.6)(c) \quad |A_v| = \frac{g_m}{1 + (g_m + \frac{1}{R_F})R_S} \cdot R_{out}$$

$$R_{out} = R_S + (1 + g_m R_S)R_F$$

$$\overline{V_{n,out}^2} = \left(\frac{4KT \frac{2}{3} g_m}{(1 + (g_m + \frac{1}{R_F})R_S)^2} + \frac{4KT \frac{1}{R_F}}{(1 + (g_m + \frac{1}{R_F})R_S)^2} + \frac{4KT \frac{1}{R_S} \cdot R_S^2}{(R_S + \frac{1}{g_m} \parallel R_F)^2} \right) R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4KT \left(\frac{2}{3} \frac{1}{g_m} + \frac{1}{g_m^2 R_F} + R_S \left(1 + \frac{1}{g_m R_F}\right)^2 \right) //$$

$$(7.6)(d) \quad |A_v| = \frac{g_{m1}}{1 + g_{m1} R_S} \cdot R_{out}$$

$$R_{out} = \frac{1}{g_{m2}}$$

$$\overline{V_{n,out}^2} = 4KT \frac{2}{3} g_{m2} R_{out}^2 + 4KT \frac{2}{3} \frac{1}{g_{m1}} |A_v|^2 + 4KT \frac{1}{R_S} \left(\frac{R_S}{\frac{1}{g_{m1}} + R_S} \right)^2 R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4KT \left[\frac{2}{3} \frac{1}{g_{m1}} + R_S + \frac{2}{3} g_{m2} \left(\frac{1 + g_{m1} R_S}{g_{m1}} \right)^2 \right] //$$

$$(7.6)(e) \quad |A_v| = g_{m1} R_D$$

$$\overline{V_{n,out}^2} = (4KT \frac{2}{3} g_{m1} + 4KT \frac{1}{R_D}) R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4KT \left(\frac{2}{3} \frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} \right) //$$

$M_2 + R_F$ do not contribute noise because $r_{o1} = \infty$

$$(7.6)(f) \quad |A_v| = g_{m1} \left(\frac{g_{m2} R_S}{1 + g_{m2} R_S} \right) R_D$$

$$\overline{V_{n,out}^2} = \left[4KT \frac{1}{R_D} + 4KT \frac{2}{3} \frac{1}{g_{m2}} \left(\frac{g_{m2}}{1 + g_{m2} R_S} \right)^2 + 4KT \frac{2}{3} \frac{1}{g_{m1}} \left(\frac{g_{m1} R_S}{\frac{1}{g_{m2}} + R_S} \right)^2 + 4KT \frac{1}{R_S} \left(\frac{R_S}{\frac{1}{g_{m2}} + R_S} \right)^2 \right] \cdot R_D^2$$

$$\text{note: } \frac{R_S}{\frac{1}{g_{m2}} + R_S} = \frac{g_{m2} R_S}{1 + g_{m2} R_S}$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4KT \left[\frac{2}{3} \frac{1}{g_{m1}} + \frac{2}{3} \frac{1}{g_{m2} (g_{m1} R_S)^2} + \frac{1}{g_{m1}^2 R_S} + \frac{1}{g_{m1}^2 R_D} \left(\frac{1 + g_{m2} R_S}{g_{m2} R_S} \right)^2 \right] //$$

$$(7.7)(a) \quad |A_v| = \frac{g_{m1} R_{out}}{1 + (g_{m1} + \frac{1}{R_F}) R_S}$$

$$R_{out} = R_S + (1 + g_{m1} R_S) R_F$$

$$\overline{V_{n,out}^2} = \left[\frac{4kT \frac{2}{3} g_{m1}}{(1 + (g_{m1} + \frac{1}{R_F}) R_S)^2} + \frac{4kT \frac{1}{R_F}}{(1 + (g_{m1} + \frac{1}{R_F}) R_S)^2} + \frac{4kT \frac{1}{R_S} \cdot R_S^2}{(R_S + \frac{1}{g_{m1}} \parallel R_F)^2} + 4kT \frac{2}{3} g_{m2} \right] R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left[\frac{2}{3} \frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_F} + R_S \left(1 + \frac{1}{g_{m1} R_F}\right)^2 + \frac{2}{3} g_{m2} \left(\frac{1 + (g_{m1} + \frac{1}{R_F}) R_S}{g_{m1}}\right)^2 \right] //$$

$$(7.7)(b) \quad |A_v| = \left(g_{m2} + \frac{g_{m1}}{1 + (g_{m1} + \frac{1}{R_F}) R_S} \right) \cdot R_{out}$$

$$R_{out} = R_S + (1 + g_{m1} R_S) R_F$$

$$\overline{V_{n,out}^2} = \left[\frac{4kT \frac{2}{3} g_{m1}}{(1 + (g_{m1} + \frac{1}{R_F}) R_S)^2} + \frac{4kT \frac{1}{R_F}}{(1 + (g_{m1} + \frac{1}{R_F}) R_S)^2} + \frac{4kT \frac{1}{R_S} \cdot R_S^2}{(R_S + \frac{1}{g_{m1}} \parallel R_F)^2} + 4kT \frac{2}{3} g_{m2} \right] R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left(\frac{1}{g_{m1} + g_{m2} (1 + (g_{m1} + \frac{1}{R_F}) R_S)} \right)^2 \left[\frac{2}{3} g_{m1} + \frac{1}{R_F} + R_S (g_{m1} + \frac{1}{R_F})^2 + \frac{2}{3} g_{m2} (1 + (g_{m1} + \frac{1}{R_F}) R_S)^2 \right] //$$

$$(7.7)(c) \quad |A_v| = \left(\frac{g_{m1}}{1 + g_{m1} R_S} \right) (1 + g_{m2} R_S) (R_D)$$

$$\overline{V_{n,out}^2} = 4kT R_D + 4kT \frac{2}{3} g_{m2} R_D^2 + 4kT \frac{2}{3} \frac{1}{g_{m1}} |A_v|^2 + 4kT \frac{1}{R_S} \left[\frac{R_S}{\frac{1}{g_{m1}} + R_S} - \frac{\frac{1}{g_{m1}} R_S}{\frac{1}{g_{m1}} + R_S} g_{m2} \right]^2 R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left[\frac{2}{3} \frac{1}{g_{m1}} + \left(\frac{1}{R_D} + \frac{2}{3} g_{m2} \right) \left(\frac{1 + g_{m1} R_S}{g_{m1} (1 + g_{m2} R_S)} \right)^2 + R_S (g_{m1} - g_{m2})^2 \left(\frac{1}{g_{m1} (1 + g_{m2} R_S)} \right)^2 \right] //$$

$$(7.7)(d) \quad |A_v| = g_{m1} R_D$$

$$\overline{V_{n,out}^2} = \left[4kT \frac{1}{R_D} + 4kT \frac{2}{3} g_{m3} + 4kT \frac{2}{3} g_{m1} \right] R_D^2$$

M2 does not contribute any noise because r_{o1} and $r_{o3} = \infty$.

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left[\frac{2}{3} \frac{1}{g_{m1}} + \frac{2}{3} \frac{g_{m3}}{g_{m1}^2} + \frac{1}{g_{m1}^2 R_D} \right] //$$

$$(7.8)(a) \quad |A_v| = g_{m1} R_D$$

$$\overline{V_{n,out}^2} = \left[4kT \frac{1}{R_D} + 4kT \frac{2}{3} g_{m1} + 4kT R_G g_{m1}^2 \right] R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left(\frac{2}{3} \frac{1}{g_{m1}} + R_G + \frac{1}{g_{m1}^2 R_D} \right) //$$

$$(7.8)(b) \quad |A_v| = \left(g_{m1} + \frac{1}{R_1} \right) (R_1 // R_D)$$

$$\overline{V_{n,out}^2} = \left[4kT \frac{2}{3} g_{m1} + 4kT \frac{1}{R_1} + 4kT \frac{1}{R_D} \right] (R_1 // R_D)^2$$

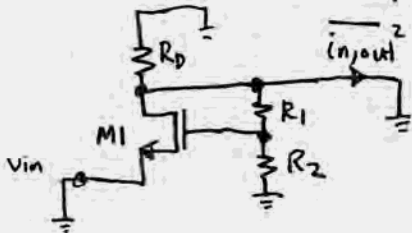
$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left(\frac{1}{g_{m1} + \frac{1}{R_1}} \right)^2 \left[\frac{2}{3} g_{m1} + \frac{1}{R_1} + \frac{1}{R_D} \right] //$$

$$(7.8)(c) \quad A_v = \left(-g_{m1} + \frac{1}{R_F} \right) (R_F // R_D)$$

$$\overline{V_{n,out}^2} = 4kT \left(\frac{1}{R_F} + \frac{1}{R_D} + \frac{2}{3} g_{m1} \right) (R_F // R_D)^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left(\frac{1}{-g_{m1} + \frac{1}{R_F}} \right)^2 \left[\frac{2}{3} g_{m1} + \frac{1}{R_F} + \frac{1}{R_D} \right] //$$

(7.8)(d) Find short circuit output noise current $\overline{i_{n,out}^2}$

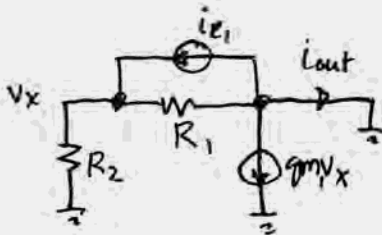


$$\overline{i_{n,out}^2} = 4kT \frac{1}{R_D} + 4kT \frac{2}{3} g_{m1} + \overline{i_{noise,R_1}^2} + \overline{i_{noise,R_2}^2}$$

$$\overline{i_{noise,R_1}^2} = 4kT \frac{1}{R_1} |A_{I,R_1}|^2$$

$$\overline{i_{noise,R_2}^2} = 4kT \frac{1}{R_2} |A_{I,R_2}|^2$$

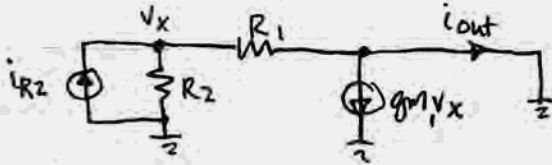
• small signal model used to find A_{I,R_1}



$$A_{I,R_1} = \frac{i_{out}}{i_{R_1}} = - \left(1 + (R_1 // R_2) \left(g_{m1} - \frac{1}{R_1} \right) \right)$$

(7.8) (d) cont.

- small signal model used to find $A_{I,R2}$



$$A_{I,R2} = \frac{i_{out}}{i_{R2}} = \left[\frac{R_2}{R_1 + R_2} - g_{m1} (R_1 \parallel R_2) \right]$$

$$\therefore \overline{i_{n,out}^2} = 4kT \frac{1}{R_D} + 4kT \frac{2}{3} g_{m1} + 4kT \frac{1}{R_1} \left(1 + (R_1 \parallel R_2) \left(g_{m1} - \frac{1}{R_1} \right) \right)^2 + 4kT \frac{1}{R_2} \left[\frac{R_2}{R_1 + R_2} - g_{m1} (R_1 \parallel R_2) \right]^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{i_{n,out}^2}}{g_{m1}^2} = 4kT \left[\frac{2}{3} \frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} + \frac{1}{g_{m1}^2 R_1} \left(1 + (R_1 \parallel R_2) \left(g_{m1} - \frac{1}{R_1} \right) \right)^2 + \frac{1}{g_{m1}^2 R_2} \left(\frac{R_2}{R_1 + R_2} - g_{m1} (R_1 \parallel R_2) \right)^2 \right] //$$

$$(7.9) (a) \quad |A_v| = g_{m1} R_D$$

$$\overline{V_{n,out}^2} = 4kT \left(\frac{1}{R_D} + \frac{2}{3} g_{m1} \right) R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left(\frac{2}{3} \frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} \right) //$$

$$(7.9) (b) \quad |A_v| = g_{m1} (R_D \parallel \frac{1}{g_{m2}})$$

$$\overline{V_{n,out}^2} = 4kT \left(\frac{1}{R_D} + \frac{2}{3} g_{m1} + \frac{2}{3} g_{m2} \right) (R_D \parallel \frac{1}{g_{m2}})^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left(\frac{2}{3} \frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} + \frac{2}{3} \frac{g_{m2}}{g_{m1}^2} \right) //$$

$$(7.9) (c) \quad |A_v| = g_{m1} R_D$$

$$\overline{V_{n,out}^2} = 4kT \left(\frac{2}{3} g_{m1} + \frac{1}{R_D} \right) R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left(\frac{2}{3} \frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} \right) //$$

$$(7.9) (d) \quad |A_v| = \frac{g_{m1}}{g_{m1} + g_{m2}}$$

$$\overline{V_{n,out}^2} = \left(4kT \frac{2}{3} g_{m1} + 4kT \frac{2}{3} g_{m2} \right) \left(\frac{1}{g_{m1} + g_{m2}} \right)^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left(\frac{2}{3} \frac{1}{g_{m1}} + \frac{2}{3} \frac{g_{m2}}{g_{m1}^2} \right) //$$

$$(7.9) (e) \quad |A_v| = 1, \quad \overline{V_{n,in}^2} = \overline{V_{n,out}^2} = 4kT \frac{2}{3} \frac{1}{g_{m1}} //$$

(7.10) • With the input shunted to ground,

$$\overline{V_{n,out}^2} = \frac{1}{C_{ox} f} \left[\frac{g_{m1}^2 k_n}{(WL)_1} + \frac{g_{m3}^2 k_p}{(WL)_3} + \frac{g_{m3}^2 k_p}{(WL)_4} \right] (r_{o1} \parallel r_{o3})^2$$

$$|A_v| = (g_{m1} + g_{mb1}) (r_{o1} \parallel r_{o3})$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = \frac{1}{C_{ox} f} \left[\frac{g_{m1}^2 k_n}{(WL)_1} + \frac{g_{m3}^2 k_p}{(WL)_3} + \frac{g_{m3}^2 k_p}{(WL)_4} \right] \frac{1}{(g_{m1} + g_{mb1})^2} //$$

• With the input open,

$$\overline{V_{n,out}^2} = \frac{1}{C_{ox} f} \left[g_{m2}^2 k_n \left(\frac{1}{(WL)_0} + \frac{1}{(WL)_2} \right) + g_{m3}^2 k_p \left(\frac{1}{(WL)_3} + \frac{1}{(WL)_4} \right) \right] R_{out}^2$$

$$R_{out} \approx r_{o3} \parallel (g_{m1}, r_{o1}, r_{o2})$$

$$\overline{I_{n,in}^2} = \frac{1}{C_{ox} f} \left[g_{m2}^2 k_n \left(\frac{1}{(WL)_0} + \frac{1}{(WL)_2} \right) + g_{m3}^2 k_p \left(\frac{1}{(WL)_3} + \frac{1}{(WL)_4} \right) \right] //$$

$$(7.11) \quad \overline{V_{n,out}^2} = \frac{k}{C_{ox} (WL)_1} \cdot \frac{1}{f} (g_{m1} R_{out})^2 + \frac{k}{C_{ox} (WL)_2} \cdot \frac{1}{f} (g_{m2} R_{out})^2$$

$$R_{out} = \left(\frac{1}{g_{m1}} \parallel \frac{1}{g_{mb1}} \parallel r_{o1} \parallel r_{o2} \right)$$

$$|A_v| = g_{m1} R_{out}$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = \frac{k}{C_{ox} f} \left[\frac{1}{(WL)_1} + \frac{g_{m2}^2}{(WL)_2 g_{m1}^2} \right] //$$

$$(7.12)(a) \quad \overline{V_{n,out}^2} = 4KT \frac{2}{3} (g_{m1} + g_{m2} + g_{m3} + g_{m4} + g_{m5} + g_{m6}) R_{out}^2$$

$$|A_v| = g_{m1} R_{out}$$

$$g_{m1} = g_{m2}, \quad g_{m3,4} = 0.5 g_{m5,6}$$

$$\overline{V_{n,in}^2} = 4KT \frac{2}{3} \left[\frac{2}{g_{m1}} + \frac{3 g_{m5}}{g_{m1}^2} \right] //$$

$$(7.12)(b) \quad |A_v| = g_{m1} (r_{o2} \parallel r_{o4})$$

$$\overline{V_{n,out}^2} = 4KT \frac{2}{3} [g_{m1} + g_{m2} + g_{m3} + g_{m4}]$$

$$g_{m1} = g_{m2}, \quad g_{m3} = g_{m4}$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4KT \left(\frac{2}{3} \right) \left[\frac{2}{g_{m1}} + \frac{2 g_{m3}}{g_{m1}^2} \right] //$$

$$(7.13)(a) \quad |A_v| = \frac{g_{m1} R_D}{1 + g_{m1} R_S}$$

$$\overline{V_{n,out}^2} = 4KT R_D + 4KT \frac{2}{3} \frac{1}{g_{m1}} |A_v|^2 + 4KT \frac{1}{R_S} \left(\frac{R_S}{R_S + \frac{1}{g_{m1}}} \right)^2 R_D^2$$

$$\overline{V_{n,in}^2} = 4KT \left[\frac{2}{3} \frac{1}{g_{m1}} + R_S + \frac{1}{R_D} \left(\frac{1 + g_{m1} R_S}{g_{m1}} \right)^2 \right]$$

$$(7.13)(b) \quad I R_S = V_{GS} - V_T \Rightarrow R_S = \frac{V_{GS} - V_T}{I}$$

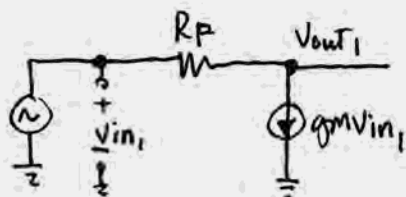
$$g_{m1} = \frac{2I}{V_{GS} - V_T} = \frac{2}{R_S}$$

$$\therefore 4KT \left[\frac{2}{3} g_m \right] = 4KT \frac{R_S}{3} \quad \leftarrow \text{Thermal noise of M1}$$

$$4KT R_S \quad \leftarrow \text{Thermal noise of } R_S$$

$\therefore R_S$ contributes 3x more noise power ($\overline{V_{n,in}^2}$) than M1 when $I R_S = V_{GS} - V_T$.

(7.14) Consider the following ckt with noise only due to resistor R_F :

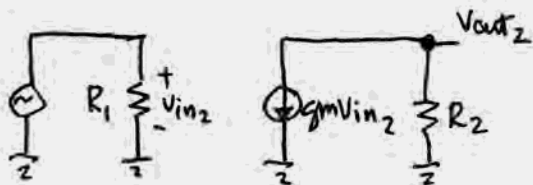


$$A_{v1} = \frac{V_{out1}}{V_{in1}} = -g_m R_F + 1$$

$$\overline{V_{n,out1}^2} = 4KT R_F$$

$$\overline{V_{n,in1}^2} = \frac{\overline{V_{n,out1}^2}}{|A_{v1}|^2} = 4KT R_F \left(\frac{1}{-g_m R_F + 1} \right)^2$$

using the Miller effect, we have the following ckt:



$$A_{v2} = \frac{V_{out2}}{V_{in2}} = -g_m R_2 = \frac{-g_m R_F A_{v1}}{A_{v1} - 1} = (-g_m R_F + 1) = A_{v1}$$

$$\overline{V_{n,out2}^2} = 4KT R_2$$

$$\overline{V_{n,in2}^2} = \frac{\overline{V_{n,out2}^2}}{|A_{v2}|^2} = 4KT \frac{1}{g_m^2 R_2} = 4KT \frac{1}{g_{m1}} \left(\frac{-1}{-g_m R_F + 1} \right)$$

$$R_1 = \frac{R_F}{1 - A_{v1}} \quad R_2 = \frac{R_F}{1 - \frac{1}{A_{v1}}}$$

notice: $\overline{V_{n,in1}^2} \neq \overline{V_{n,in2}^2} \Rightarrow$ cannot use Miller's Theorem //

(7.15) Using equation (7.26)

$$\begin{aligned} \overline{V_{n,out}^2} &= 4kT \left(\frac{2}{3} g_m\right) r_o^2 \\ &= (1.656 \times 10^{-20} \text{ V}\cdot\text{C}) \left(\frac{2}{3}\right) \left(4.44 \frac{\text{mA}}{\text{V}}\right) (20 \text{ k}\Omega)^2 \\ &= 19.6 \times 10^{-15} \frac{\text{V}^2}{\text{Hz}} \end{aligned}$$

$$\overline{V_{n,out,TOT}} = \sqrt{(19.6 \times 10^{-15} \frac{\text{V}^2}{\text{Hz}})(50 \text{ MHz})} \cong 990 \mu\text{V rms} //$$

(7.16) $|A_v|^2 = \frac{[g_m(R_D || r_o)]^2}{1 + (2\pi(R_D || r_o)C_L f)^2}$

$$\begin{aligned} \overline{V_{n,out}^2} &= 4kT \frac{(R_D || r_o)^2}{R_D} \frac{1}{1 + (2\pi(R_D || r_o)C_L f)^2} + 4kT \frac{2}{3} g_m (R_D || r_o)^2 \frac{1}{1 + (2\pi(R_D || r_o)C_L f)^2} \\ &\quad + \frac{k}{C_{ox} W L} g_m^2 (R_D || r_o)^2 \frac{1}{1 + (2\pi(R_D || r_o)C_L f)^2} \end{aligned}$$

$$\begin{aligned} \overline{V_{n,out,TOT}^2} &= 4kT (R_D || r_o) \left[\frac{(R_D || r_o)}{R_D} + \frac{2}{3} g_m (R_D || r_o) \right] \int_{f_L}^{f_H} \frac{df}{1 + (2\pi(R_D || r_o)C_L f)^2} \\ &\quad + \frac{k g_m^2 (R_D || r_o)^2}{C_{ox} W L} \int_{f_L}^{f_H} \frac{df}{1 + (2\pi(R_D || r_o)C_L f)^2} \end{aligned}$$

$$\begin{aligned} \overline{V_{n,out,TOT}^2} &= \frac{2kT}{\pi C_L} \left[\frac{(R_D || r_o)}{R_D} + \frac{2}{3} g_m (R_D || r_o) \right] \left[\tan^{-1}(2\pi(R_D || r_o)C_L f_H) - \tan^{-1}(2\pi(R_D || r_o)C_L f_L) \right] \\ &\quad + \frac{k g_m^2 (R_D || r_o)^2}{C_{ox} W L} \int_{f_L}^{f_H} \frac{df}{1 + (2\pi(R_D || r_o)C_L f)^2} // \end{aligned}$$

(7.17) Using equation number (7.57)

$$\overline{V_{n,in}^2} = 4kT \left(\frac{2}{3}\right) \left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right)$$

$$g_{m1} = \sqrt{2(0.5 \text{ mA})(134.29 \frac{\text{mA}}{\text{V}} \times \frac{50}{34})} = 4.44 \frac{\text{mA}}{\text{V}}$$

$$g_{m2} = \sqrt{2(0.5 \text{ mA})(38.37 \frac{\text{mA}}{\text{V}} \times \frac{50}{34})} = 2.36 \frac{\text{mA}}{\text{V}}$$

$$\begin{aligned} \therefore \overline{V_{n,in}^2} &= (1.656 \times 10^{-20} \text{ V}\cdot\text{C}) \left(\frac{2}{3}\right) (225.23 \Omega + 119.71 \Omega) \\ &\cong 3.81 \times 10^{-18} \frac{\text{V}^2}{\text{Hz}} \end{aligned}$$

$$\overline{V_{n,in}} \cong 1.95 \frac{\text{nV}}{\sqrt{\text{Hz}}} //$$

$$(7.18)(a) |A_v| = g_{m1} R_{out}$$

$$R_{out} = r_{o1} \parallel (R_S + (1 + g_{m2} R_S) r_{o2})$$

$$\overline{V_{n,out}^2} = \left[4kT \frac{1}{R_S} \left(\frac{R_S}{R_S + \frac{1}{g_{m2}}} \right)^2 + 4kT \frac{2}{3} \frac{1}{g_{m2}} \left(\frac{g_{m2}}{1 + g_{m2} R_S} \right)^2 + 4kT \frac{2}{3} g_{m1} \right] R_{out}^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left[\frac{2}{3} \frac{1}{g_{m1}} + R_S \left(\frac{g_{m2}}{1 + g_{m2} R_S} \right)^2 \frac{1}{g_{m1}^2} + \frac{2}{3} \frac{g_{m2}}{g_{m1}^2} \left(\frac{1}{1 + g_{m2} R_S} \right)^2 \right]$$

(b) R_S large

(7.19) Neglecting body effect and using eqns 7.60 and 7.61

$$\overline{V_{n,in}^2} = 4kT \left(\frac{2}{3} \frac{1}{g_m^2} + \frac{1}{g_m^2 R_D} \right)$$

$$\overline{I_{n,in}^2} = \frac{4kT}{R_D}$$

$$g_m = \sqrt{2(1mA)(134.29 \frac{\mu A}{V^2}) \left(\frac{50}{.94} \right)} = 6.28 \frac{mA}{V}$$

$$\therefore \overline{V_{n,in}^2} = (1.656 \times 10^{-20} V \cdot C) \left(\frac{2}{3} 159.24 \Omega + 25.3 \Omega \right) \cong 2.18 \times 10^{-18} \frac{V^2}{Hz} //$$

$$\overline{I_{n,in}^2} = \frac{(1.656 \times 10^{-20} V \cdot C)}{1k\Omega} = 16.56 \times 10^{-24} \frac{A^2}{Hz} //$$

$$(7.20)(a) \overline{I_{n,in}^2} = 4kT \frac{1}{R_D} + 4kT \frac{2}{3} g_{m2}$$

$$\overline{I_{n,in}^2} = \sqrt{4kT \left(\frac{1}{R_D} + \frac{2}{3} g_{m2} \right)}$$

$$\therefore \frac{2}{3} g_{m2} = \left(\frac{1}{5} \right)^2 \left(\frac{1}{R_D} \right)$$

$$\Rightarrow g_{m2} = \left(\frac{1}{25} \right) \left(\frac{1}{1000} \right) \left(\frac{3}{2} \right) = 60 \frac{\mu A}{V}$$

$$\left(\frac{V}{C} \right)_2 = \frac{g_{m2}^2}{2I_D \mu_n C_{ox}} = \frac{(60 \frac{\mu A}{V})^2}{2(0.05mA)(134.29 \frac{\mu A}{V^2})} \cong 0.268 //$$

$$(b) g_{m2} = \frac{2I_D}{(V_{GS} - V_T)_2} \Rightarrow (V_{GS} - V_T)_2 = \frac{2I_D}{g_m} = \frac{2(0.05mA)}{60 \frac{\mu A}{V}} \cong 1.67 V$$

$$(V_{GS} - V_T)_1 = \sqrt{\frac{2I_D}{\mu_n C_{ox} \left(\frac{W}{L} \right)}} = \sqrt{\frac{2(0.05mA)}{134.29 \frac{\mu A}{V^2} \left(\frac{50}{.94} \right)}} \cong 71.2 mV$$

neglecting body effect

$$V_b = (V_{GS} - V_T)_2 + V_{GS1} = 1.67 + 0.0712 + 0.7 = 2.4412 V //$$

$$\text{Output Swing} = V_{CC} - (V_{GS} - V_T)_1 - (V_{GS} - V_T)_2 = 3 - 1.67 - 0.0712 = 1.2588 V //$$

Note: output swing is not symmetric.

(7.21) Neglecting body effect and using the result of eqn. 7.60

$$\overline{V_{n,in}} = \sqrt{4kT \left(\frac{2}{3} \frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} \right)} = 3 \frac{mV}{\sqrt{Hz}}$$

$$\frac{2}{3} \frac{1}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} = 543.4 \Omega$$

note:

$$\frac{1}{g_{m1}} = \frac{(V_{GS} - V_T)}{2I_D} \cong \frac{\Delta V}{2I_D}$$

$$\text{also define } R_N = 543.4 \Omega$$

$$\Rightarrow \Delta V^2 + \frac{4}{3} I_D R_D \Delta V - 4 I_D^2 R_D R_N = 0 \quad ; \quad I_D = 0.5 \text{ mA}$$

One possible answer assuming $(\frac{W}{L})_1 = (\frac{W}{L})_2$ and a 3V supply

$$\Delta V_1 = \Delta V_2 = 562 \text{ mV} \quad ; \quad R_D = 1875 \Omega$$

$$\Rightarrow \text{Output swing} = 2 \cdot I_D R_D = 1.875 \text{ V} //$$

$$g_{m1} = g_{m2} = \frac{2I_D}{\Delta V} = 1.78 \frac{\text{mA}}{\text{V}}$$

$$\left(\frac{W}{L} \right) = \frac{g_m^2}{2I_D \mu_n C_{ox}} = \frac{(1.78 \text{ mA/V})^2}{2(0.5 \text{ mA})(134.27 \frac{\mu\text{A}}{\text{V}^2})} \cong 23.6 //$$

$$V_b = \Delta V_2 + V_{GS2} \cong 0.562 \text{ V} + 0.562 \text{ V} + 0.7 \text{ V} = 1.824 \text{ V} //$$

(7.22) Neglecting body effect and using eqns 7.64 + 7.65

$$\overline{V_{n,in}}^2 = 4kT \frac{2}{3} \left(\frac{1}{g_{m1}} + \frac{g_{m3}}{g_{m1}^2} \right)$$

$$\overline{I_{n,in}}^2 = 4kT \frac{2}{3} (g_{m2} + g_{m3})$$

$$g_{m1} = g_{m2} = \sqrt{2(0.5 \text{ mA})(134.27 \frac{\mu\text{A}}{\text{V}^2})(\frac{50}{.34})} \cong 4.44 \frac{\text{mA}}{\text{V}}$$

$$g_{m3} = \sqrt{2(0.5 \text{ mA})(38.37 \frac{\mu\text{A}}{\text{V}^2})(\frac{50}{.34})} \cong 2.38 \frac{\text{mA}}{\text{V}}$$

$$\therefore \overline{V_{n,in}}^2 = (1.656 \times 10^{-20} \text{ V}\cdot\text{C}) \left(\frac{2}{3} \right) (225.23 \Omega + 120.73 \Omega) \cong 3.82 \times 10^{-18} \frac{\text{V}^2}{\text{Hz}} //$$

$$\overline{I_{n,in}}^2 = (1.656 \times 10^{-20} \text{ V}\cdot\text{C}) \left(\frac{2}{3} \right) \left(4.44 \frac{\text{mA}}{\text{V}} + 2.38 \frac{\text{mA}}{\text{V}} \right) \cong 75.3 \times 10^{-24} \frac{\text{A}^2}{\text{Hz}} //$$

(7.23) Neglect body effect and use eqn. 7.65

$$\overline{I_{n,in}^2} = 4kT \frac{2}{3} (gm_2 + gm_3)$$

$$gm_1 = \sqrt{2(0.5mA)(134.29 \frac{\mu A}{V^2})(\frac{50}{34})} = 4.44 \frac{mA}{V}$$

$$(V_{gs} - V_t)_1 = \frac{2I_D}{gm_1} = \frac{2(0.5mA)}{4.44 mA/V} \approx 225.23 \text{ mV}$$

$$\text{define } \Delta V_x = |(V_{gs} - V_t)|_x$$

$$\therefore \text{Output Swing} = V_{CC} - (\Delta V_1 + \Delta V_2 + \Delta V_3) = 2V \quad ; \quad V_{CC} = 3V$$

$$\Rightarrow \Delta V_2 + \Delta V_3 = 774.77 \text{ mV}$$

note: $gm = \frac{2I_D}{\Delta V}$

$$\therefore \overline{I_{n,in}^2} = 4kT \left(\frac{2}{3}\right) 2I_D \left(\frac{\Delta V_2 + \Delta V_3}{\Delta V_2 \Delta V_3}\right)$$

to minimize $\overline{I_{n,in}^2}$ let $\Delta V_2 = \Delta V_3 \Rightarrow gm_2 = gm_3 = \frac{2(0.5mA)}{0.387V} = 2.58 \frac{mA}{V}$

$$\left(\frac{W}{L}\right)_2 = \frac{(gm_2)^2}{2I_{Dn} \mu C_{ox}} = \frac{(2.58 \frac{mA}{V})^2}{2(0.5mA)(134.29 \frac{\mu A}{V^2})} \approx 49.7 //$$

$$\left(\frac{W}{L}\right)_3 = \frac{(gm_3)^2}{2I_{Dp} \mu C_{ox}} = \frac{(2.58 \frac{mA}{V})^2}{2(0.5mA)(38.37 \frac{\mu A}{V^2})} \approx 174 //$$

(7.24) (a) Neglecting body effect and r_{o1}, r_{o2}

$$R_{out} = \frac{1}{g_{m1}} = \frac{1}{\sqrt{2I_D \mu_n C_{ox} \left(\frac{W}{L}\right)_1}} = 100 \Omega$$

$$\left(\frac{W}{L}\right)_1 = \frac{1}{2I_D \mu_n C_{ox} R_{out}^2} = \frac{1}{2(61 \mu A)(134.29 \frac{A^2}{V^2})(100 \Omega)^2} \approx 3723 //$$

(b) Using the result from eqn 7.73

$$\overline{V_{n,in}} = \sqrt{4kT \left(\frac{2}{3}\right) \left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right)}$$

$$\frac{g_{m2}}{g_{m1}^2} = \left(\frac{1}{5}\right)^2 \frac{1}{g_{m1}} \Rightarrow g_{m2} = \left(\frac{1}{5}\right)^2 g_{m1} = \frac{1}{2500 \Omega}$$

$$(V_{GS} - V_T)_2 = \frac{2I_D}{g_{m2}} = 0.5 V, \quad \left(\frac{W}{L}\right)_2 = \frac{(g_{m2})^2}{2I_D \mu_n C_{ox}} = \frac{(400 \mu A/V)^2}{2(61 \mu A)(134.29 \frac{A^2}{V^2})} \approx 596 //$$

$$(V_{GS} - V_T)_1 = \sqrt{\frac{2I_D}{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}} = \sqrt{\frac{2(0.1 \text{ mA})}{(134.29 \frac{A^2}{V^2})(3723)}} \approx 20 \text{ mV}$$

neglecting body effect

$$V_{GS1} \approx 0.7 + 0.02 = 0.72$$

$$\therefore \text{Output swing} \approx V_{CC} - V_{GS1} - (V_{GS} - V_T)_2 = 3 - 0.72 - 0.5 = 1.78 \text{ V}$$

$$(7.25) \quad |A_v|^2 = g_{m1}^2 \left(\frac{g_{m2}}{\omega C_x} \right)^2 \left(\frac{1}{1 + \left(\frac{g_{m2}}{\omega C_x} \right)^2} \right) R_D^2 \quad \omega = 2\pi f$$

$$\overline{V_{n,out}^2} = \left[4kT \frac{1}{R_D} + 4kT \frac{2}{3} \frac{1}{g_{m2}} \frac{g_{m2}^2}{1 + \left(\frac{g_{m2}}{\omega C_x} \right)^2} + 4kT \frac{2}{3} g_{m1} \left(\frac{g_{m2}}{\omega C_x} \right)^2 \left(\frac{1}{1 + \left(\frac{g_{m2}}{\omega C_x} \right)^2} \right) \right] R_D^2$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{|A_v|^2} = 4kT \left[\frac{2}{3} \frac{1}{g_{m1}} + \frac{2}{3} \frac{1}{g_{m2}} \left(\frac{\omega C_x}{g_{m1}} \right)^2 + \frac{1}{R_D} \left(\frac{g_{m2}^2 + (\omega C_x)^2}{g_{m1}^2 g_{m2}^2} \right) \right] //$$

(7.26) (a) $M_1 = M_2$, M_3 does not contribute differential noise

$$|A_v| = \frac{g_{m1} R_D}{1 + g_{m1} R_S}$$

$$\overline{V_{n,out_a}^2} = \left[2(4kT) \frac{1}{R_D} + 2(4kT) \frac{2}{3} \frac{1}{g_{m1}} \left(\frac{g_{m1}}{1 + g_{m1} R_S} \right)^2 + 2(4kT) \frac{1}{R_S} \left(\frac{R_S}{g_{m1} + R_S} \right)^2 \right] R_D^2$$

$$\overline{V_{n,in_a}^2} = \frac{\overline{V_{n,out_a}^2}}{|A_v|^2} = 2(4kT) \left[\frac{2}{3} \frac{1}{g_{m1}} + R_S + \frac{1}{R_D} \left(\frac{1 + g_{m1} R_S}{g_{m1}} \right)^2 \right] //$$

(b) $M_1 = M_2$, $M_3 = M_4$

$$\overline{V_{n,out_b}^2} = \overline{V_{n,out_a}^2} + 2(4kT) \left(\frac{2}{3} \right) g_{m3} \left(\frac{R_S}{g_{m1} + R_S} \right)^2 R_D^2$$

$$\overline{V_{n,in_b}^2} = \frac{\overline{V_{n,out_b}^2}}{|A_v|^2} = 2(4kT) \left[\frac{2}{3} \frac{1}{g_{m1}} + R_S + \frac{1}{R_D} \left(\frac{1 + g_{m1} R_S}{g_{m1}} \right)^2 + \frac{2}{3} g_{m3} R_S^2 \right] //$$

$$\therefore \overline{V_{n,in_b}^2} = \overline{V_{n,in_a}^2} + 2(4kT) \left(\frac{2}{3} g_{m3} R_S^2 \right)$$

$M_3 + M_4$
contribute differential
noise in (b)

8.1

$$V_{in} = I_{in} (Z_{in} + G_{22}) + G_{21} V_{out}$$

$$\frac{V_{out} - A_o I_{in} Z_{in}}{Z_{out}} = -G_{11} V_{out} - G_{12} I_{in}$$

$$\Rightarrow I_{in} \left(\frac{A_o Z_{in}}{Z_{out}} - G_{12} \right) = V_{out} \left(\frac{1}{Z_{out}} + G_{11} \right)$$

$$V_{in} = V_{out} \left[G_{21} + \frac{(Z_{in} + G_{22}) \left(\frac{1}{Z_{out}} + G_{11} \right)}{\frac{A_o Z_{in}}{Z_{out}} - G_{12}} \right]$$

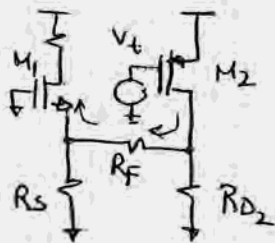
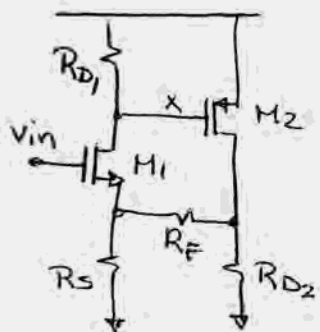
$$\frac{V_{out}}{V_{in}} = \frac{\frac{A_o Z_{in} - G_{12} Z_{out}}{Z_{out}}}{G_{21} \left(\frac{A_o Z_{in}}{Z_{out}} - G_{12} \right) + (Z_{in} + G_{22}) \left(\frac{1}{Z_{out}} + G_{11} \right)}$$

$$A_{v, \text{open loop}} = \frac{1}{Z_{out}} (A_o Z_{in} - G_{12} Z_{out}) \frac{1}{Z_{in} + G_{22}} \frac{1}{\frac{1}{Z_{out}} + G_{11}}$$

$$= \frac{G_{11}^{-1}}{G_{11}^{-1} + Z_{out}} \frac{1}{Z_{in} + G_{22}} (A_o Z_{in} - G_{12} Z_{out})$$

if $G_{12} \ll A_o Z_{in} / Z_{out}$ then the second term can be neglected

8.2



The current through R_S :

$$= \frac{g_{m2} \cdot v_t \cdot R_{D2}}{R_{D2} + R_F + (R_S \parallel \frac{1}{g_{m2}})}$$

The current through M_1 :

$$= \frac{g_{m2} v_t R_{D2}}{R_{D2} + R_F + (R_S \parallel \frac{1}{g_{m2}})} \cdot \frac{R_S}{R_S + \frac{1}{g_{m2}}}$$

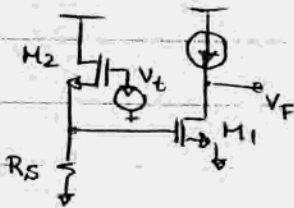
This current is multiplied by R_{D1} to produce v_f

loop gain:
$$\frac{g_{m2} R_{D2} R_S R_{D1}}{(R_{D2} + R_F) \left(R_S + \frac{1}{g_{m2}} \right) + R_S \cdot \frac{1}{g_{m2}}}$$

This result is accurate, whereas $G_{21} A_{v,open}$ is approximate because it neglects the signal propagating thru the feedback network from the input to the output.

8.3

voltage-current



loop gain:
$$\frac{R_S}{R_S + \frac{1}{g_{m2}}} \cdot g_{m1} r_{o1}$$

$$\frac{V_{in}}{R_S} \times \left(R_S \parallel \frac{1}{g_{m2}} \right) g_{m1} r_{o1} = V$$

$$\Rightarrow \left. \frac{V_{out}}{\frac{V_{in}}{R_S}} \right|_{open} = g_{m1} r_{o1} \left(R_S \parallel \frac{1}{g_{m2}} \right)$$

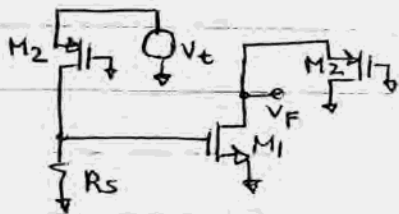
$$Z_{in,open} = \frac{1}{g_{m2}}$$

$$Z_{out,open} = r_{o1}$$

$$\left. \frac{V_{out}}{\frac{V_{in}}{R_S}} \right|_{closed} = \frac{1}{g_{m2}} \Rightarrow A_{v,closed} = \frac{1}{g_{m2} R_S}$$

$$Z_{in,closed} = 0$$

$$Z_{out,closed} = \frac{R_S + \frac{1}{g_{m2}}}{g_{m1} R_S} = \frac{1}{g_{m1}} + \frac{1}{g_{m1} g_{m2} R_S}$$



$$\frac{V_F}{V_t} = g_{m2} R_S \frac{g_{m1}}{g_{m2}} = g_{m1} R_S$$

$$Z_{in,open} = r_{o2}$$

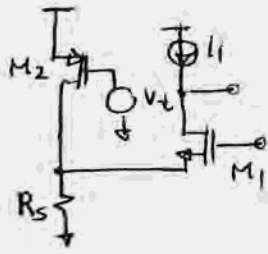
$$Z_{out,open} = \frac{1}{g_{m2}}$$

$$\left. \frac{V_{out}}{\frac{V_{in}}{R_S}} \right|_{open} = R_S \cdot \frac{g_{m1}}{g_{m2}}$$

$$A_{v,closed} = \frac{g_{m1}/g_{m2}}{1 + g_{m1} R_S}$$

$$Z_{in,closed} = \infty$$

$$Z_{out,closed} = \frac{1}{g_{m2} (1 + g_{m1} R_S)}$$



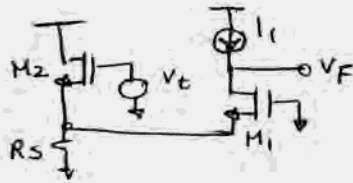
$$R_S g_{m2} V_t \frac{r_{o1}}{R_S + \frac{1}{g_{m1}}} = V_F \Rightarrow \text{loop gain: } \frac{R_S g_{m2} r_{o1}}{R_S + \frac{1}{g_{m1}}}$$

$$R_{out,open} = g_{m1} r_{o1} R_S + r_{o1} + R_S \approx g_{m1} r_{o1} R_S$$

$$R_{in,open} = \frac{1}{g_{m1}}$$

$$\left. \frac{V_{out}}{V_{in}} \right|_{open} = \frac{r_{o1}}{R_S + \frac{1}{g_{m1}}}$$

$$r_{o1} \rightarrow \infty \quad A_V = \frac{1}{R_S g_{m2}} \quad R_{in} = 0 \quad R_{out} = \frac{g_{m1} (R_S + \frac{1}{g_{m1}})}{g_{m2}}$$



$$\text{loop gain: } V_t \cdot \frac{R_S \parallel \frac{1}{g_{m1}}}{\frac{1}{g_{m2}} + R_S \parallel \frac{1}{g_{m1}}} \cdot g_{m1} r_{o1} = V_F$$

$$R_{in,open} = \frac{1}{g_{m1} + g_{m2}}$$

$$R_{out,open} = g_{m1} r_{o1} (R_S \parallel \frac{1}{g_{m2}})$$

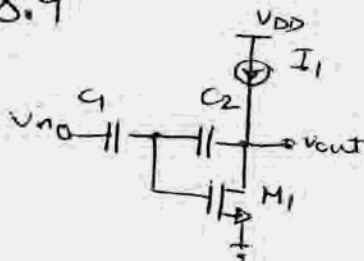
$$A_{V,open} \Rightarrow \frac{V_{in}}{R_S} (R_S \parallel \frac{1}{g_{m2}}) \times g_{m1} r_{o1} = V_{out} \Rightarrow A_{r,open} = \frac{g_{m1} r_{o1} (R_S \parallel \frac{1}{g_{m2}})}{R_S}$$

$$r_o \rightarrow \infty \quad A_{v,closed} = \frac{(R_S \parallel \frac{1}{g_{m2}}) (\frac{1}{g_{m2}} + R_S \parallel \frac{1}{g_{m1}})}{R_S (R_S \parallel \frac{1}{g_{m1}})}$$

$$R_{in,closed} = 0$$

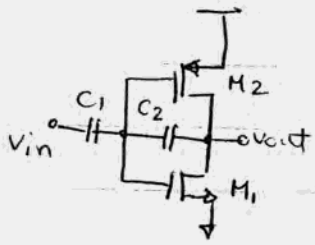
$$R_{out,closed} = \frac{(R_S \parallel \frac{1}{g_{m2}}) (\frac{1}{g_{m2}} + R_S \parallel \frac{1}{g_{m1}})}{R_S \parallel \frac{1}{g_{m1}}}$$

8.4



$$R_{in} = \frac{1}{C_1 S} + \frac{1}{g_m}$$

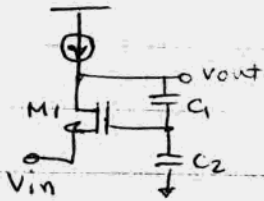
$$R_{out} = \frac{r_o}{1 + \frac{C_2}{C_1 + C_2} g_{m1} r_o} = \frac{C_1 + C_2}{g_{m1} C_2}$$



using the results in part (a)

$$R_{in} = \frac{1}{C_1 s} + \frac{1}{g_{m1} + g_{m2}}$$

$$R_{out} = \frac{C_1 + C_2}{(g_{m1} + g_{m2}) C_2}$$



$$\text{loop gain} = g_{m1} r_o \frac{C_1}{C_1 + C_2}$$

$$R_{in \text{ closed}} = \frac{1}{g_{m1}} + \frac{1}{C_2 s}$$

$$R_{out \text{ closed}} = \frac{r_o}{1 + g_{m1} r_o \frac{C_1}{C_1 + C_2}} = \frac{C_1 + C_2}{g_{m1} C_1}$$

8.5

$$- \frac{1}{\left(1 + \frac{1}{g_{m1} r_{o1}}\right) \frac{C_2}{C_1} + \frac{1}{g_{m1} r_{o1}}} = -0.95 \frac{C_1}{C_2} \Rightarrow \frac{C_1}{C_2} = 1.63$$

$g_{m1} r_{o1} = 50$

Open loop output impedance: r_o

$$\text{loop gain: } \frac{C_2}{C_1 + C_2} g_m r_o$$

$$\text{closed loop } R_{out} = \frac{r_o}{1 + \frac{C_2}{C_1 + C_2} g_m r_o} = 0.49 r_o$$

8.6

$$R_{in \text{ closed}} = \frac{1}{g_{m1}} \cdot \frac{1}{1 + g_{m2} R_D \frac{C_1}{C_1 + C_2}} \quad I_1 = I_2 \Rightarrow g_{m2} = \sqrt{2} g_{m1}$$

$$= \frac{1}{g_{m1}} \cdot \frac{1}{1 + 1000 \sqrt{2} g_{m1}} = 50 \Rightarrow g_{m1} = 3.42 \text{ mS}$$

$$g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L}} I_D \Rightarrow I_D = \frac{(3.42 \times 10^{-3})^2}{2 \times 1.342 \times 10^{-4} \times 100} = 435 \mu\text{A}$$

8.7

$$\frac{V_x}{I_x} = \frac{R_D}{1 + \frac{g_{m2} R_S (g_{m1} + g_{mb1}) R_D}{(g_{m1} + g_{mb1}) R_S + 1}} \cdot \frac{C_1}{C_1 + C_2} \stackrel{R_D \rightarrow \infty}{=} \frac{(g_{m1} + g_{mb1}) R_S + 1}{g_{m2} R_S (g_{m1} + g_{mb1})} \cdot \frac{C_1 + C_2}{C_1}$$

$$\text{if } (g_{m1} + g_{mb1}) R_S \gg 1 \Rightarrow \frac{V_x}{I_x} = \frac{1}{g_{m2}} \cdot \frac{C_1 + C_2}{C_1}$$

8.8

If f_{-3dB} of each stage is ω_0

$$\left| \frac{1}{\left(1 + \frac{s}{\omega_0}\right)^n} \right| = \frac{1}{\sqrt{2}} \Rightarrow \left(1 + \left(\frac{\omega}{\omega_0}\right)^2\right)^n = 2$$

if we indicate the Gain f_{-3dB} as $k = \text{const}$

$$\Rightarrow \left(\frac{K}{\omega_0}\right)^n = 500$$

$$\Rightarrow \frac{\ln 2}{\ln\left(1 + \left(\frac{\omega}{\omega_0}\right)^2\right)} = \frac{\ln 500}{\ln\left(\frac{K}{\omega_0}\right)} \Rightarrow 1 + \left(\frac{\omega}{\omega_0}\right)^2 = \left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}}$$

$$\Rightarrow \omega = \omega_0 \sqrt{\left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}} - 1}$$

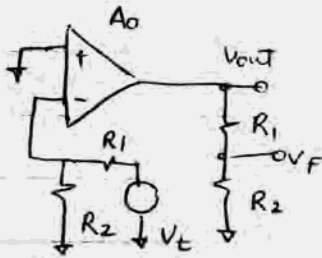
$$\frac{d\omega}{d\omega_0} = 0 \Rightarrow \sqrt{\left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}} - 1} + \frac{\omega_0}{2} \cdot \frac{1}{\sqrt{\left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}} - 1}} \cdot \left(-\frac{\ln 2}{\ln 500} \cdot \frac{1}{\omega_0} \cdot \left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}}\right)$$

$$\Rightarrow \left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}} - 1 - \frac{1}{2} \frac{\ln 2}{\ln 500} \left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}} = 0 \Rightarrow \frac{K}{\omega_0} = 1.67$$

\Rightarrow Gain per stage = 1.67

Stage BW = 598 MHz

8.9



$$A_{\text{open}} = A_0 \frac{R_2}{R_1 + R_2 + R_0}$$

$$R_{\text{out open}} = R_0 \parallel (R_1 + R_2)$$

Loop gain =

$$\left(\frac{R_2}{R_1 + R_2}\right) A_0 \left(\frac{R_2}{R_0 + R_1 + R_2}\right)$$

$$A_{\text{closed}} = \frac{A_0 \frac{R_2}{R_0 + R_1 + R_2}}{1 + \left(\frac{R_2}{R_1 + R_2}\right) A_0 \left(\frac{R_2}{R_0 + R_1 + R_2}\right)}$$

$$R_{\text{out closed}} = \frac{R_0 \parallel (R_1 + R_2)}{1 + \left(\frac{R_2}{R_1 + R_2}\right) A_0 \left(\frac{R_2}{R_0 + R_1 + R_2}\right)}$$

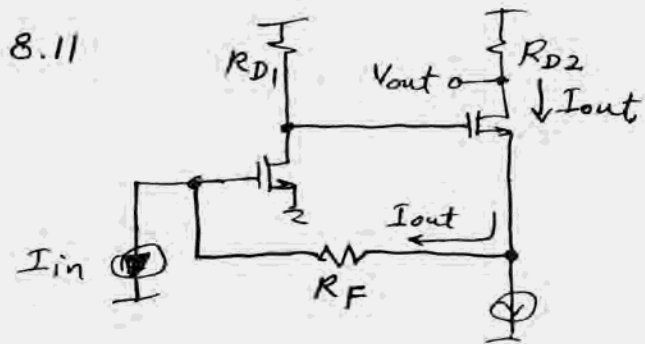
B.10

$$\frac{1 + \frac{C_2}{C_1}}{\left(1 + \frac{C_2}{C_1}\right) \frac{1}{g_{m1}(r_{o2} \parallel r_{o4})} + 1} = 0.95 \left(1 + \frac{C_2}{C_1}\right)$$

↑
5% gain error

$$g_{m1}(r_{o2} \parallel r_{o4}) \approx 24.4$$

$$\Rightarrow 1 + \frac{C_2}{C_1} \leq 1.28$$



I_{out} fully flows through $R_F \Rightarrow I_{out} = I_{in}$
and $V_{out} = I_{out} \cdot R_{D2}$.

Thus, the transimpedance is equal to R_{D2} .
(continued on next page)

8.11 (cn'td)

$$-\underbrace{(I_{out} R_S + V_{nRS} + V_{nRF} + V_{n1})}_{V_Y} g_{m1} R_D = V_X + V_{nRD} + V_{n2} = V_X$$

$$(V_X - V_Y) g_{m2} = I_{out}$$

$$\Rightarrow g_{m2} \left[(I_{out} R_S + V_{nRS} + V_{nRF} + V_{n1}) (-g_{m1} R_D) + V_{nRD} + V_{n2} - (I_{out} R_S + V_{nRS}) \right] = I_{out}$$

$$\Rightarrow I_{out} \left[1 + g_{m2} R_S (g_{m1} R_D + 1) \right] = g_{m2} \left[-g_{m1} R_D - 1 \right] V_{nRS} - g_{m1} R_D V_{nRF} - g_{m1} R_D V_{n1} + V_{nRD} + V_{n2}$$

$$\Rightarrow I_{out} = \frac{g_{m2} \left[-(1 + g_{m1} R_D) V_{nRS} - g_{m1} R_D V_{nRF} - g_{m1} R_D V_{n1} + V_{nRD} + V_{n2} \right]}{1 + g_{m2} R_S (1 + g_{m1} R_D)}$$

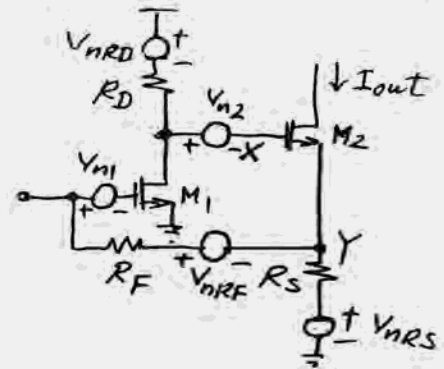
If we apply a current of I_{in} to the input, the resulting output current is obtained as :

$$\left\{ \underbrace{[(I_{in} + I_{out}) R_S + I_{in} R_F]}_{V_X} (-g_{m1} R_D) - \underbrace{(I_{in} + I_{out}) R_S}_{V_Y} \right\} g_{m2} = I_{out}$$

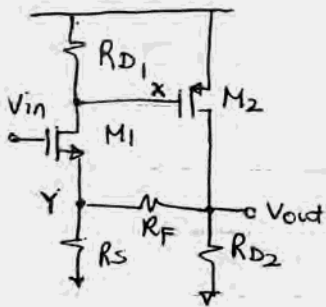
$$\frac{I_{out}}{I_{in}} = \frac{[-g_{m1} R_D (R_S + R_F) - R_S] g_{m2}}{1 + g_{m2} R_S (1 + g_{m1} R_D)}$$

Dividing the output noise current by the gain yields the input-referred noise current:

$$I_{n,in} = \frac{-(1 + g_{m1} R_D) V_{nRS} - g_{m1} R_D V_{nRF} - g_{m1} R_D V_{n1} + V_{nRD} + V_{n2}}{-g_{m1} R_D (R_S + R_F) - R_S}$$



8.12



$$I_{D2} = 1 \text{ mA} \Rightarrow \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{DD} - V_X - |V_{TP}|)^2 = 1 \text{ mA}$$

$$\Rightarrow V_X = 1.478$$

$$I_{D1} = 761 \mu\text{A}$$

$$\left\{ \begin{array}{l} \frac{V_Y}{2\text{k}} + \frac{V_Y - V_{out}}{2\text{k}} = 761 \mu\text{A} \Rightarrow V_{out} = 1.841 \\ \frac{V_{out}}{2\text{k}} + \frac{V_{out} - V_Y}{2\text{k}} = 1 \text{ mA} \Rightarrow V_Y = 1.681 \end{array} \right.$$

$$\left. \begin{array}{l} \frac{V_Y}{2\text{k}} + \frac{V_Y - V_{out}}{2\text{k}} = 761 \mu\text{A} \\ \frac{V_{out}}{2\text{k}} + \frac{V_{out} - V_Y}{2\text{k}} = 1 \text{ mA} \end{array} \right\} \Rightarrow V_Y = 1.681$$

$$I_{D1} = 0.761 \text{ mA} \Rightarrow \frac{1}{2} \times 1.342 \times 10^{-4} \times 100 (V_{in} - V_Y - V_{TN})^2 = 0.761 \text{ mA}$$

$$V_{in} - V_Y = 1.037 \Rightarrow V_{in} = 2.717 \text{ V}$$

$$G_{21} = 0.5$$

$$\left\{ \begin{array}{l} g_{m1} = \frac{2 \times 0.761 \text{ mA}}{1.037 - 0.7} = 4.52 \text{ mS} \\ g_{m2} = \frac{2 \times 1 \text{ mA}}{1.522 - 0.8} = 2.77 \text{ mS} \end{array} \right.$$

$$A_{V_{open}} = \frac{-2\text{k}}{1\text{k} + \frac{1}{4.52 \text{ mS}}} \left[-2.77 \left[2\text{k} \parallel 4\text{k} \right] \right] = 6.048$$

$$A_{V_{closed}} = \frac{6.048}{1 + 3.024} = 1.503$$

8.13

Problem 8.9

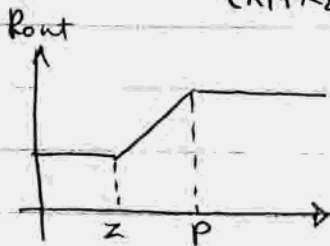
$$R_{out} = \frac{R_o (R_1 + R_2)}{R_o + R_1 + R_2 + \frac{R_2^2}{R_1 + R_2} \cdot \frac{A_o}{1 + \frac{s}{\omega_o}}}$$

Zero: ω_o

$$\text{pole: } \omega_o + \frac{A_o R_2^2}{(R_1 + R_2)(R_o + R_1 + R_2)} \omega_o$$

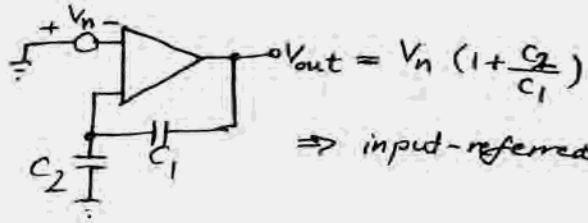
$$\text{DC value: } \frac{R_o (R_1 + R_2)}{R_o + R_1 + R_2 + A_o R_2^2 / (R_1 + R_2)}$$

$$\text{final value: } R_o \parallel (R_1 + R_2)$$

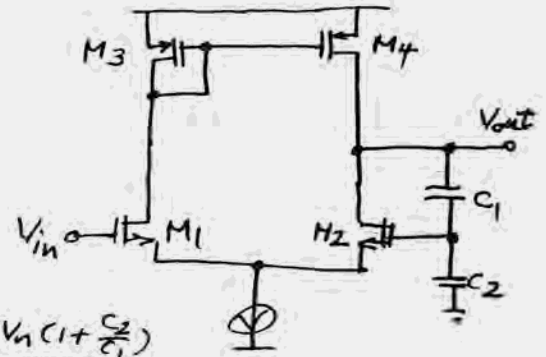


The output impedance is less reduced, as the loop gain gets smaller.

8.14 The input-referred noise voltage of the circuit is the same as that of the open-loop circuit:



$$\Rightarrow \text{input-referred noise} = \frac{V_n(1 + \frac{C_2}{C_1})}{1 + \frac{C_2}{C_1}} = V_n$$



The noise produced by $M_1 - M_4$ referred to the input is:

$$\overline{V_n^2} = 4kT \left(\frac{2}{3g_{m1,2}} + 2 \frac{2g_{m3,4}}{3g_{m1,2}^2} \right) + 2 \frac{K_N}{(WL)_{1,2} C_{ox} f} + 2 \frac{K_P}{(WL)_{3,4} C_{ox} f} \times \frac{g_{m3,4}^2}{g_{m1,2}^2}$$

8.15

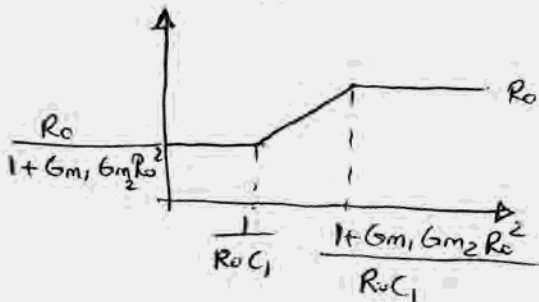
a) $Z_{in\ open} = R_o$

$$Z_{in\ closed} = \frac{R_o}{1 + G_{m1}G_{m2}R_o \frac{R_o}{1 + R_oC_1s}} = \frac{R_o(1 + R_oC_1s)}{R_oC_1s + 1 + G_{m1}G_{m2}R_o^2}$$

Zero: $\frac{1}{R_oC_1}$

pole: $\frac{1 + G_{m1}G_{m2}R_o^2}{R_oC_1}$

DC: $\frac{R_o}{1 + G_{m1}G_{m2}R_o^2}$ final: R_o



b) Heavy feedback at lower frequency. As frequency increases, feedback weakens since the output impedance of the feedforward amplifier reduces

8.15 (c) For input-referred noise voltage, we short the input,

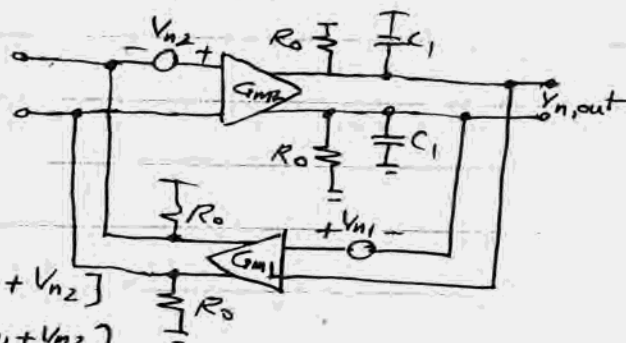
$$\text{hence } \overline{V_{n,out}^2} = 4kT \times 2 \left(\frac{2}{3}g_m + \frac{1}{R_0} \right) \left(R_0 \parallel \frac{1}{C_1 s} \right).$$

Dividing this by the voltage gain, $g_m^2 (R_0 \parallel \frac{1}{C_1 s})^2$, we have

$$\overline{V_{n,in}^2} = 8kT \left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_0} \right).$$

For the input noise current, we leave the input open.

Here, V_{n1} and V_{n2} represent the input noise of each differential pair (including the noise of resistors).



$$-V_{n,out} = G_{m2} (R_0 \parallel \frac{1}{C_1 s}) [(V_{n,out} + V_{n1}) G_{m1} R_0 + V_{n2}]$$

$$\Rightarrow V_{n,out} = - \frac{G_{m2} (R_0 \parallel \frac{1}{C_1 s}) (G_{m1} R_0 V_{n1} + V_{n2})}{1 + G_{m2} (R_0 \parallel \frac{1}{C_1 s}) G_{m1} R_0}$$

If we apply current between the two input terminals with value I_{in} , the output voltage is obtained as:

$$-V_{out} = (V_{out} G_{m1} + I_{in}) R_0 \cdot G_{m2} (R_0 \parallel \frac{1}{C_1 s})$$

$$\Rightarrow \frac{V_{out}}{I_{in}} = - \frac{G_{m2} R_0 (R_0 \parallel \frac{1}{C_1 s})}{1 + G_{m1} G_{m2} R_0 (R_0 \parallel \frac{1}{C_1 s})}$$

Dividing $V_{n,out}$ by this gain gives the input-referred noise

$$\text{current: } I_{n,in} = \frac{G_{m1} R_0 V_{n1} + V_{n2}}{R_0} \Rightarrow \overline{I_{n,in}^2} = G_{m1}^2 \overline{V_{n1}^2} + \frac{\overline{V_{n2}^2}}{R_0^2}$$

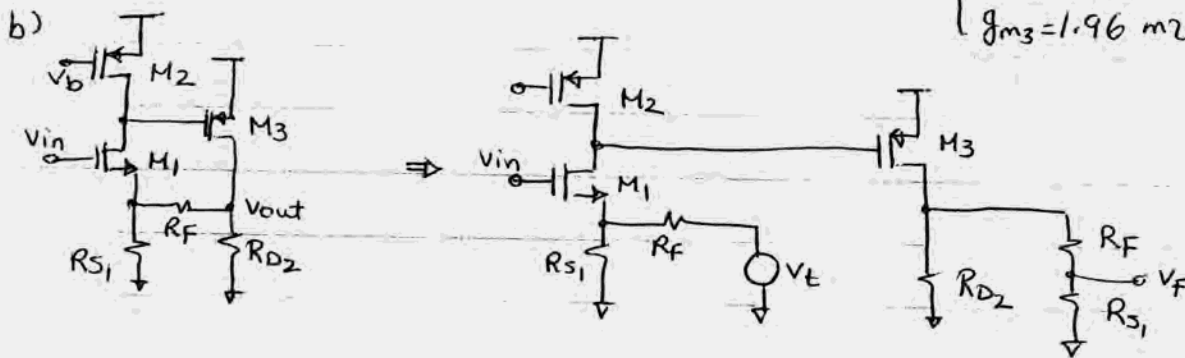
8.16

a) Due to symmetry of the π network, no current flows through R_F .

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - R_{S1} \cdot I_{D1} - V_{TN})^2 = 0.5 \text{ mA}$$

$$(V_{in} - 2.2)^2 = \frac{2 \times 0.5 \times 10^{-3}}{1.342 \times 10^{-4} \times 100} \Rightarrow V_{in} = 2.473 \text{ V}$$

$$\left. \begin{array}{l} g_{m1} = 3.66 \text{ mS} \\ g_{m2} = 1.96 \text{ mS} \\ g_{m3} = 1.96 \text{ mS} \end{array} \right\} \begin{array}{l} r_{o1} = 20 \text{ k}\Omega \\ r_{o2} = 10 \text{ k}\Omega \\ r_{o3} = 10 \text{ k}\Omega \end{array}$$



$$A_{v_{open}} = \frac{r_{o2}}{R_{S1} \parallel R_F + \frac{1}{g_{m1}}} \times g_{m3} [r_{o3} \parallel R_{D2} \parallel (R_F + R_{S1})] = 18.42$$

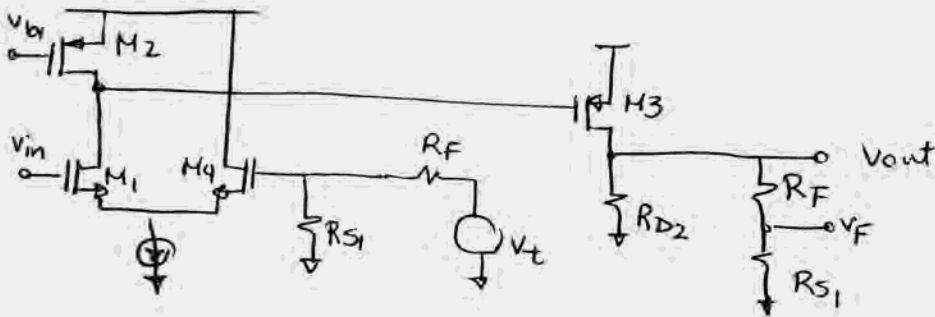
$$R_{out_{open}} = r_{o3} \parallel R_{D2} \parallel (R_F + R_{S1}) = 1.667 \text{ k}\Omega$$

$$\begin{aligned} \text{Loop gain: } \frac{V_t}{R_F} \times (R_F \parallel R_{S1}) \times \frac{r_{o2}}{R_{S1} \parallel R_F + \frac{1}{g_{m1}}} \times g_{m3} [r_{o3} \parallel R_{D2} \parallel (R_F + R_{S1})] \times \frac{R_{S1}}{R_F + R_{S1}} \\ = V_F \Rightarrow \text{loop gain} = 4.605 \end{aligned}$$

$$A_v = \frac{18.42}{1 + 4.605} = 3.286$$

$$R_{out} = \frac{1.667 \text{ k}\Omega}{1 + 4.605} = 297 \Omega$$

8.17



$$\begin{aligned} r_{o1} &= r_{o4} = 20k \\ r_{o3} &= 10k \quad r_{o2} = 10k \\ g_{m1} &= g_{m4} = 3.66 \text{ mS} \\ g_{m2} &= g_{m3} = 1.96 \text{ mS} \end{aligned}$$

$$A_{v_{open}} = \frac{1}{2} g_{m1} (r_{o1} \parallel r_{o2}) g_{m3} (r_{o3} \parallel R_{D2} \parallel (R_F + R_{S1})) = 39.85$$

$$R_{out} = r_{o3} \parallel R_{D2} \parallel (R_F + R_{S1}) = 1.667k$$

$$V_t \frac{R_{S1}}{R_{S1} + R_F} \times \frac{g_{m1}}{2} \times (r_{o1} \parallel r_{o2}) g_{m3} (r_{o3} \parallel R_{D2} \parallel (R_F + R_{S1})) \frac{R_{S1}}{R_{S1} + R_F} = V_F$$

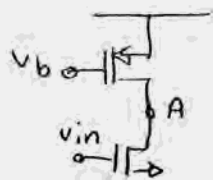
$$\text{loop gain} = 9.96$$

$$A_v = \frac{39.85}{1 + 9.96} = 3.63$$

$$R_{out} = \frac{1.667k}{1 + 9.96} = 153\Omega$$

Smaller output impedance compared to 8.16.

8.18



a) $I_{Dn} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GSn} - V_{THn})^2 \Rightarrow V_{GSn} = 0.973V$

$V_{GSp} = 1.311 \Rightarrow 3 - V_b = 1.311 \Rightarrow V_b = 1.69$

$V_{in} = R_1 \times I + V_{GSn}$

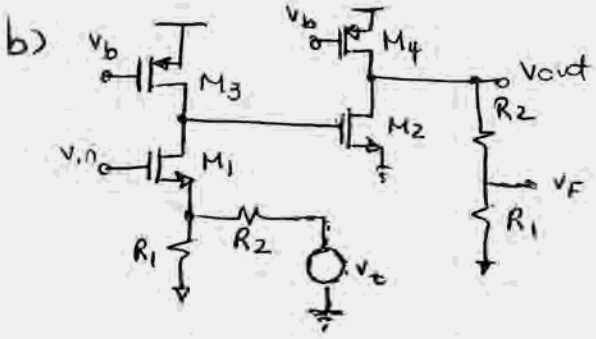
M₃: saturation $\Rightarrow -V_A + V_b > |V_{THp}| \Rightarrow V_A < 1.689 - 0.8 = 0.889$

M₄: saturation $\Rightarrow -V_{out} + V_b > V_{THp} \Rightarrow V_{out} < 0.889 \Rightarrow R_1 I < 0.889 \Rightarrow R_1 < 177$

M₁: saturation $\Rightarrow V_A > R_1 I + (V_{GS1} - V_{THn}) \Rightarrow 0.273 + R \times 0.5m < 0.889$
 $\Rightarrow R_1 < 1232$

M₂: saturation: $V_{out} = R_1 I > V_A - V_{tn} \Rightarrow R_1 > \frac{0.889 - 0.7}{0.5 \times 10^{-3}} = 378 \Omega$

$378 \leq R_1 \leq 1232 \Rightarrow 1.162 \leq V_{in} \leq 1.589$



- $R_1 = 805 \Omega$
- $r_{o1} = r_{o2} = 20k$
- $g_{m1} = 3.66 mS$
- $r_{o3} = r_{o4} = 10k$
- $g_{m2} = 3.66 mS$

open loop gain = $\frac{r_{o3}}{R_1 \parallel R_2 + \frac{1}{g_{m1}}} \times g_{m2} (r_{o4} \parallel (R_1 + R_2) \parallel r_{o2}) = 97.6$

Output impedance = $r_{o4} \parallel (R_1 + R_2) \parallel r_{o2} = 2422 \Omega$

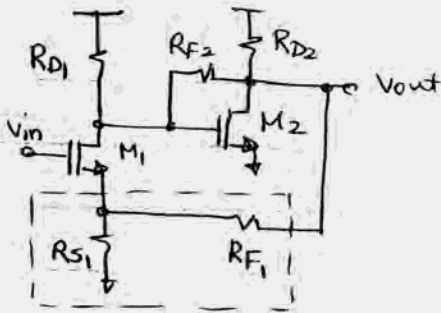
loop gain: $\frac{V_t}{R_2} \times (R_1 \parallel R_2) \times \frac{r_{o3}}{R_1 \parallel R_2 + \frac{1}{g_{m1}}} \times g_{m2} (r_{o4} \parallel (R_1 + R_2) \parallel r_{o2}) \frac{R_1}{R_1 + R_2} = V_F$

\Rightarrow loop gain = $\frac{1}{3000} \times 635 \times 97.6 \times \frac{805}{3805} = 4.37$

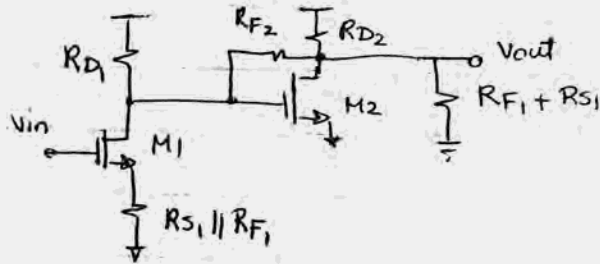
$A_v = \frac{97.6}{1 + 4.37} = 18.17$

$R_{out} = \frac{2422}{1 + 4.37} = 451 \Omega$

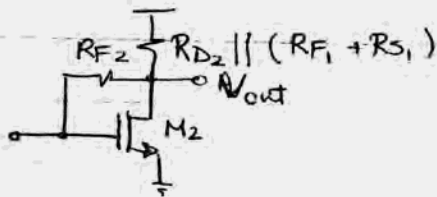
8.19



Voltage - Voltage



next we consider



$$R_{in2} = \frac{R_{F2}}{1 + g_{m2} [R_{D2} \parallel (R_{F1} + R_{S1})]} = 261 \Omega$$

$$R_{out2} = \frac{R_{D2} \parallel (R_{F1} + R_{S1})}{1 + g_{m2} [R_{D2} \parallel (R_{F1} + R_{S1})]} = 174 \Omega$$

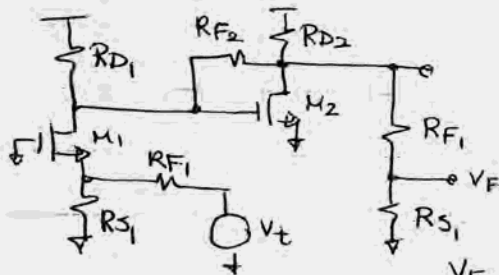
$$A_{v2} = \frac{R_D}{R_D + R_{F2}} (-g_{m2} R_{F2} + 1) = -3.6$$

$$R_D = R_{D2} \parallel (R_{F1} + R_{S1}) = 1333$$

$$\text{Open loop gain} = - \frac{R_{D1} \parallel R_{in2}}{R_{S1} \parallel R_{F1} + \frac{1}{g_{m1}}} \cdot A_{v2} = \frac{231}{1000 + 200} \times 3.6 = 0.69$$

Open loop output impedance = $R_{out2} = 174 \Omega$

loop gain:



$$\frac{V_t}{R_{F1}} \times (R_{S1} \parallel R_{F1}) \times \frac{R_{D1}}{R_{S1} \parallel R_{F1} + \frac{1}{g_{m1}}} \times A_{v2} \times \frac{R_{S1}}{R_{S1} + R_{F1}} = V_f$$

$$\frac{V_f}{V_t} = \frac{1}{2000} \times 1000 \times \frac{1000}{1200} \times 3.6 \times \frac{1}{2} = 0.75$$

$$A_{v \text{ closed}} = \frac{0.69}{1 + 0.75} = 0.394$$

$$R_{out \text{ closed}} = \frac{174}{1 + 0.75} = 99.5 \Omega$$

8.20

$$I_{D1} = I_{D2} \Rightarrow V_{in} = 1.2538 \Rightarrow I_D = 2.316 \text{ mA}$$

$$g_{m1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TH1}) = 1.342 \times 10^{-4} \times 100 \times (1.2538 - 0.7) = 7.432 \text{ mS}$$

$$g_{m2} = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TH2}) = 3.835 \times 10^{-5} \times 100 \times (3 - 1.2538 - 0.8) = 3.628 \text{ mS}$$

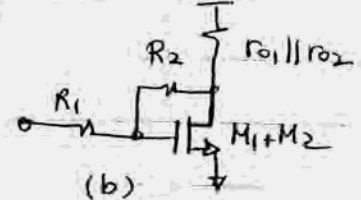
$$r_{o1} = \frac{1}{\lambda_1 I_1} = \frac{1}{0.1 \times 2.316 \times 10^{-3}} = 4.317 \text{ k}\Omega$$

$$r_{o2} = \frac{1}{\lambda_2 I_2} = \frac{1}{0.2 \times 2.316 \times 10^{-3}} = 2.159 \text{ k}\Omega$$

(a)

$$a) A_v = -(g_{m1} + g_{m2})(r_{o1} \parallel r_{o2}) = -(7.432 + 3.628) \times 1.439 = -15.91$$

$$R_{out} = r_{o1} \parallel r_{o2} = 1439 \Omega$$



(b)

$$b) A_v = \frac{1}{R_1} \cdot \frac{-(R_1 \parallel R_2)(g_{m1} + g_{m2})(R_2 \parallel r_{o1} \parallel r_{o2})}{1 + (g_{m1} + g_{m2})(R_2 \parallel r_{o1} \parallel r_{o2}) \frac{R_1}{R_1 + R_2}}$$

(eq. 8.70)

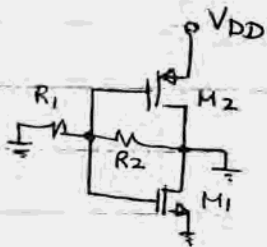
$$= \frac{1}{1} \times \frac{0.909 (7.432 + 3.628) \times 1.258}{1 + (7.432 + 3.628) \times 1.258 \times \frac{1}{11}} = 5.58$$

$$R_{out} = \frac{R_2 \parallel r_{o1} \parallel r_{o2}}{1 + (g_{m1} + g_{m2})(R_2 \parallel r_{o1} \parallel r_{o2}) \frac{R_1}{R_1 + R_2}} = \frac{1.258 \text{ k}\Omega}{2.26} = 556 \Omega$$

$$R_{out} = 1.258 \text{ k}\Omega / 2.26 = 556 \Omega$$

(b) We figure out sensitivity for (b), (a) is a special case where

$$R_1 = 0 \quad R_2 = \infty$$



$$G_m = g_{m2}$$

$$R_{out} = \text{same as before}$$

$$A_v = \frac{g_{m2} (R_2 \parallel r_{o1} \parallel r_{o2})}{1 + (g_{m1} + g_{m2})(R_2 \parallel r_{o1} \parallel r_{o2}) \frac{R_1}{R_1 + R_2}} = \frac{3.628 \times 1.258}{1 + 13.913 \times \frac{1.258}{11}} = 1.76$$

if $R_1 = 0$ and $R_2 = \infty$

$$A_v = g_{m2} (r_{o1} \parallel r_{o2}) = 3.628 \times 1.439 = 5.217$$

8.21

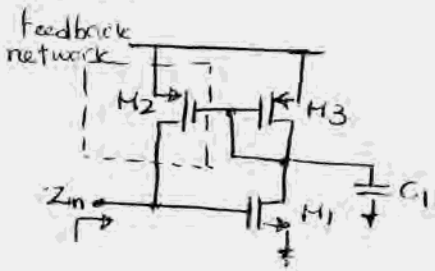
$$a) \overline{V_{out}^2} = 4KT \frac{2}{3} (g_{m1} + g_{m2}) (r_{o1} || r_{o2})^2$$

$$\overline{V_{n,in}^2} = \frac{4KT \frac{2}{3}}{g_{m1} + g_{m2}} =$$

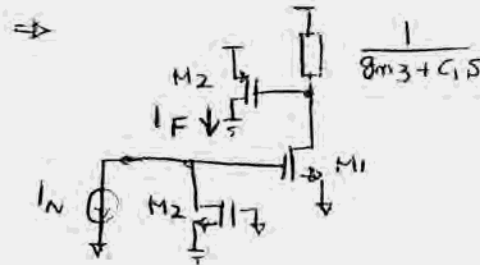
$$b) \overline{V_{n,in}^2} = 4KT R_1 + \frac{\left[\frac{4KT}{R_2} + 4KT \frac{2}{3} (g_{m1} + g_{m2}) \right] R_o^2}{A_v^2} = 4KT R_1 + \frac{\frac{4KT}{R_2} + 4KT \frac{2}{3} (g_{m1} + g_{m2})}{\left(\frac{R_2}{R_1 + R_2} \right)^2 (g_{m1} + g_{m2})}$$

$$= 4KT R_1 + \frac{4KT \frac{2}{3}}{g_{m1} + g_{m2}} + \frac{4KT/R_2}{\left(\frac{R_2}{R_1 + R_2} \right)^2 (g_{m1} + g_{m2})^2}$$

8.22



Voltage - Current



$$\left| \frac{I/F}{I_N} \right| = r_{o2} \cdot g_{m1} \cdot \frac{1}{g_{m3} + C_1 s} \cdot g_{m2}$$

$Z_{in, open} = r_{o2}$

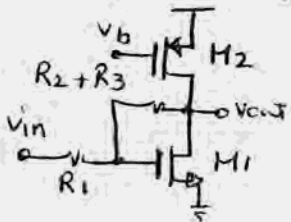
$$Z_{in, closed} = \frac{r_{o2}}{1 - \frac{r_{o2} g_{m1} g_{m2}}{g_{m3} + C_1 s}} = - \frac{g_{m3} + C_1 s}{g_{m1} g_{m2}}$$

$$= - \frac{g_{m3}}{g_{m1} g_{m2}} - \frac{C_1}{g_{m1} g_{m2}} s$$

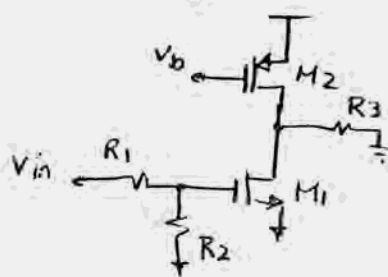
$r_{o2} \rightarrow \infty$

8.23

Very low freq.



Very high freq.



eq. (8.70)

$$A_v = \frac{1}{R_1} \cdot \frac{-(R_1 \parallel (R_2 + R_3)) g_{m1} (R_2 + R_3)}{1 + g_{m1} (R_2 + R_3) \frac{R_1}{R_1 + R_2 + R_3}}$$

$$A_v = \frac{R_2}{R_1 + R_2} (-g_{m1} R_3)$$

$$= \frac{-1}{2k} \frac{(2 \parallel 4) \frac{1}{200} \times 4k}{1 + \frac{1}{200} \times 4k \times \frac{1}{3}}$$

$$= \frac{1}{2} \left(-\frac{1}{200} \times 2000 \right) = -5$$

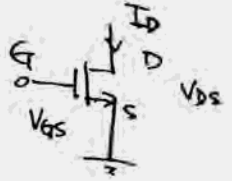
$$= -1.739$$

Chapter 9

Problem 9.1

(a) For a MOSFET in triode region,

$$I_D = \mu_n C_{ox} \left(\frac{W}{L} \right) (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \quad (*)$$



Transconductance, $g_m = \frac{\partial I_D}{\partial V_{GS}}$ (from definition)

$$\frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L} V_{DS}$$

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{DS}$$

Output resistance, $r_o = \frac{\partial V_{DS}}{\partial I_D}$

Take derivative of (*) on both sides

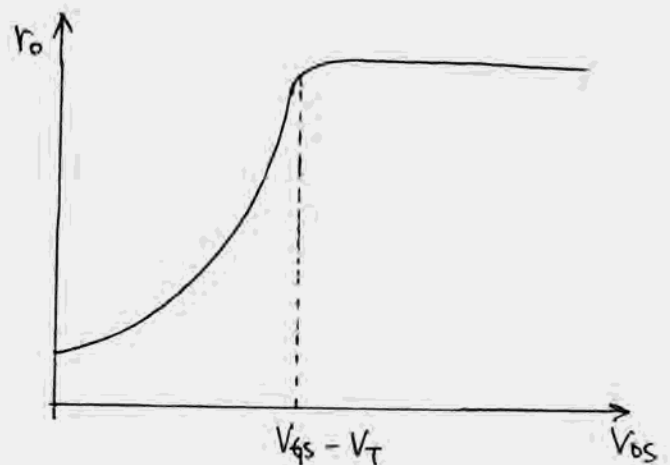
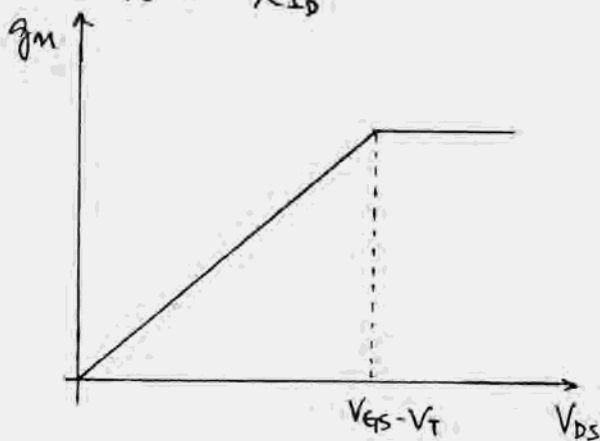
$$1 = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T - V_{DS}) \frac{\partial V_{DS}}{\partial I_D}$$

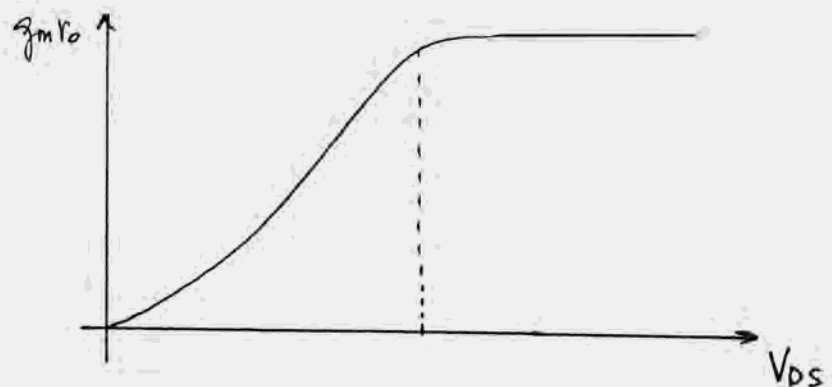
$$r_o = \frac{\partial V_{DS}}{\partial I_D} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T - V_{DS})}$$

we know in saturation,

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T) \quad \&$$

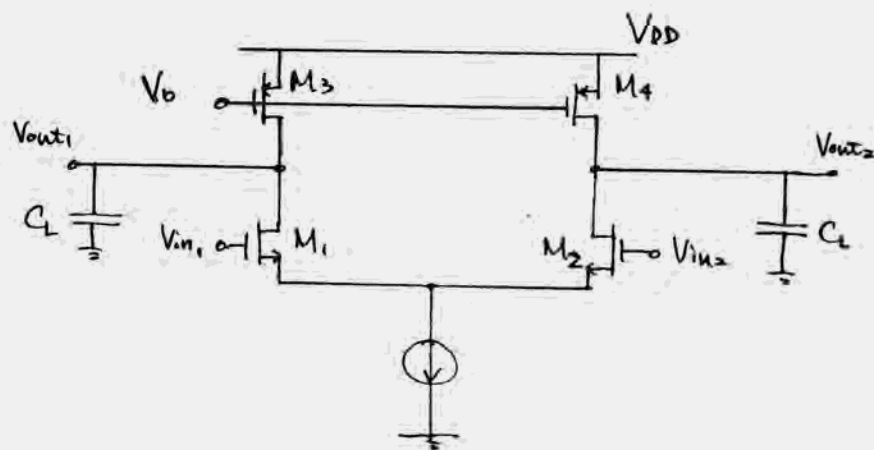
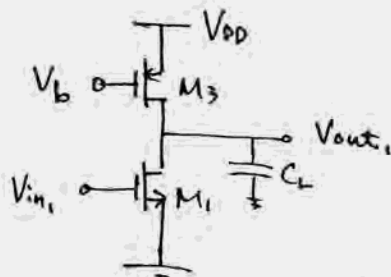
$$r_o = \frac{1}{\lambda I_D}$$





9.1 (b)

Use half-circuit concept:



$$A_v = g_{m1} (r_{o1} \parallel r_{o2})$$

$$g_{m1} = \sqrt{2\mu_n C_{ox} \frac{W}{L_{eff}} I_D} \quad \text{where } I_D = \frac{I_{SS}}{2} = 0.5 \text{ mA}$$

$$C_{ox} (@ t_{ox} = 400 \text{ \AA}) = 0.863 \text{ fF}/\mu\text{m}^2 = 86.3 \times 10^{-9} \text{ F/cm}^2$$

$$C_{ox} (@ t_{ox} = 9 \times 10^{-9} \text{ m}) = \frac{86.3 \times 10^{-9}}{9 \times 10^{-9}} \times 400 \times 10^{-10} = 383.56 \text{ nF/cm}^2$$

$$g_{m1} = \left[2 \times (350 \text{ cm}^2/\text{V}\cdot\text{sec}) (383.56 \times 10^{-9} \text{ F/cm}^2) \left(\frac{50}{0.5 - 0.08 \times 2} \right) (0.5 \times 10^{-3} \text{ A}) \right]^{\frac{1}{2}}$$

$$= 4.443 \text{ m}\Omega^{-1}$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.1)(1\text{V})(0.5 \text{ mA})}$$

$$= 20 \text{ k}\Omega$$

$$r_{o3} = \frac{1}{\lambda_3 I_{D3}} = \frac{1}{(0.2)(4\text{V})(0.5 \text{ mA})}$$

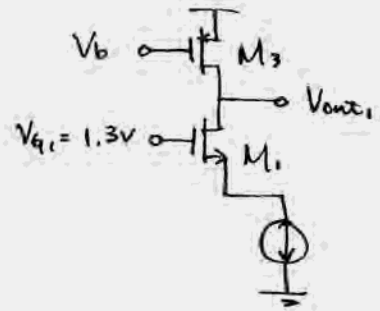
$$= 10 \text{ k}\Omega$$

$$A_v = g_{m1} (r_{o1} \parallel r_{o3})$$

$$= (4.443 \times 10^{-3} \Omega^{-1}) \left[\frac{20 \text{ k} \cdot 10 \text{ k}}{20 \text{ k} + 10 \text{ k}} \right] (\Omega)$$

$$= \boxed{29.6 = A_v}$$

To find maximum output swing
If we require both transistors are in saturation,



$$V_{DS1} \geq V_{GS1} - V_T$$

$$V_{D1} - V_{S1} \geq V_{G1} - V_{S1} - V_T$$

$$V_{D1} \geq V_{G1} - V_T = 1.3 - 0.7 = 0.6 \text{ V.}$$

$$\Rightarrow V_{out1 \text{ min}} = 0.6 \text{ V}$$

For M3: $I_{D3} = \frac{1}{2} \mu_p C_{ox} \frac{W}{L_3} (V_{GS3} - V_{TP})^2 (1 + \lambda V_{DS})$

For simplicity, assume channel length modulation is negligible, $\lambda \rightarrow 0$.

$$k_p' = \mu_p C_{ox}$$

$$= (100 \text{ cm}^2/\text{V}\cdot\text{sec}) (3.936 \times 10^{-7} \text{ F/cm}^2)$$

$$= 3.836 \times 10^{-5} \text{ A/V}^2$$

$$I_{D3} = 0.5 \text{ mA} = \frac{1}{2} k_p' \left(\frac{W}{L_{eff}} \right) (V_{GS3} - V_{TP})^2$$

$$V_{GS3} - V_{TP} = \left[\frac{(0.5 \text{ mA})(2)}{(3.836 \times 10^{-5} \text{ A/V}^2) \left(\frac{50}{0.5 - 0.09 \times 2} \right)} \right]^{\frac{1}{2}} = 0.408$$

$$V_{DS3} \geq V_{GS3} - V_{TP} = 0.408 \text{ V.}$$

$$V_{out1 \text{ max}} = V_{DD} - V_{DS3} = 3 \text{ V} - 0.408 \text{ V}$$

$$= 2.59 \text{ V.}$$

$$0.6 \text{ V} \leq V_{out1} \leq 2.59 \text{ V}$$

\therefore One sided output swing = $2.59 - 0.6 = 1.99 \text{ V}$.

$$\boxed{\begin{aligned} \text{Differential output swing} &= (1.99 \times 2) \text{ V} \\ &= 3.98 \text{ V} \end{aligned}}$$

9.1 (c). From part (b), M_3 will enter the triode region when $V_{DS3} < V_{GS3} - V_{TP} = 0.408 \text{ V}$.

At the peak of the output swing $V_{DS3} = 0.408 - 50 \text{ mV}$
 $= 0.358 \text{ V}$

$$r_{o3} = \frac{1}{\mu_p C_{ox} \frac{W}{L_{eff}} (V_{GS3} - V_{TP} - V_{DS3})} \quad (\text{from part a})$$

$$= \left[(3.836 \times 10^{-5} \text{ A/V}^2) \left(\frac{50}{0.32} \right) (0.408 - 0.358) \right]^{-1}$$

$$= 3.337 \text{ k}\Omega$$

$$A_v = g_{m1} (r_{o1} \parallel r_{o3})$$

$$= (4.443 \times 10^{-3} \text{ }\Omega^{-1}) \left(\frac{20 \times 3.337}{20 + 3.337} \text{ k}\Omega \right)$$

$$A_v = 12.7$$

Problem 9.2

(a) $V_b = 1.4V$ $I_{SS} = 1mA$ $\left(\frac{W}{L}\right)_{1-4} = \frac{100}{0.5}$

To keep M_3 in saturation,

$$V_A > V_b - V_{THn} = 1.4V - 0.7V = 0.7V.$$

Assume $M_5 - M_8$ are identical,

So $V_{GS5} = V_{GS7}$

$$V_{GS5} = \frac{V_{DD} - V_A}{2} = \frac{3 - 0.7}{2} = 1.15V.$$

$$I_{D5} = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_5 (V_{GS5} - V_{THp})^2 (1 + \lambda V_{DS5})$$

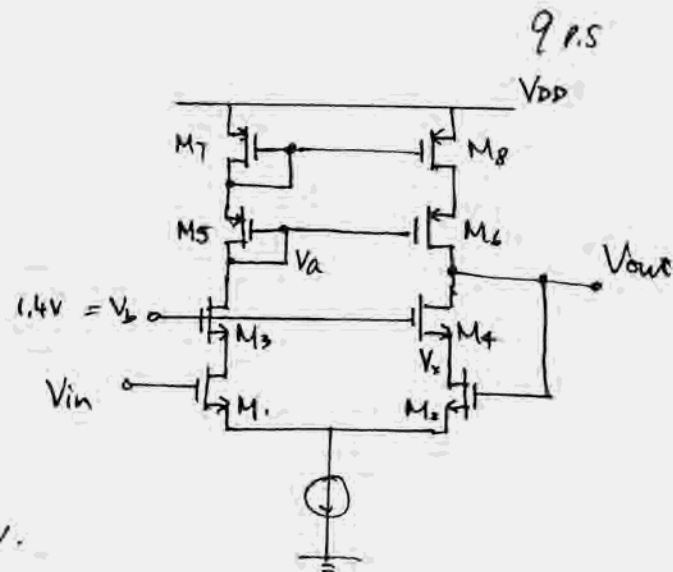
$$\left(\frac{W}{L}\right)_{5, \text{eff}} = \frac{2I_{D5}}{\mu_p C_{ox} (V_{GS5} - V_{THp})^2 (1 + \lambda V_{DS5})}$$

$$= \frac{2(0.5mA)}{(100)(3.836 \times 10^{-7})(1.15 - 0.8)^2 (1 + 0.2(1.15))} = 173.$$

$$W_5 = 173 \times L_{\text{eff}} = 173 \times (0.5 - 0.09 \times 2).$$

$$W_5 \approx 56 \mu m$$

$$W_{5-8} = 56 \mu m$$



(b) Max. output swing = $V_{TH4} - (V_{GS4} - V_{TH2}) = V_{TH4} + V_{TH2} - V_{GS4}$

$$I_{D4} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_4 (V_{GS4} - V_{TH4})^2 (1 + \lambda V_{DS4}) \quad \text{assume } \lambda \rightarrow 0 \text{ for simplicity.}$$

$$V_{GS4} - V_{TH4} \approx \left[\frac{2I_D}{\mu_n C_{ox} \left(\frac{W}{L}\right)_4} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2(0.5mA)}{(350)(3.836 \times 10^{-7}) \left(\frac{100}{0.34}\right)} \right]^{\frac{1}{2}}$$

$$= 0.159V$$

$$V_{GS4} = V_{TH4} + 0.159 = 0.859V$$

$$\text{Max. Output swing} = 0.7 + 0.7 - 0.859$$

$$= 0.541V$$

9.2 (c). $A_{v \text{ open loop}} = g_{m1} (g_{m4} r_{o4} r_{o2} \parallel g_{m6} r_{o6} r_{o8})$

$$g_{m1} = \sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right)_{eff} I_D}$$

$$= \sqrt{2 (350) (383.6 \times 10^{-9}) \left(\frac{100}{0.34}\right) (0.5 \text{ mA})}$$

$$= 6.28 \text{ m}\Omega^{-1}$$

$$g_{m4} = g_{m1} = 6.28 \text{ m}\Omega^{-1}$$

$$g_{m6} = \sqrt{2 \mu_n C_{ox} \left(\frac{W}{L}\right)_{eff} I_D}$$

$$= \sqrt{2 (100) (383.6 \times 10^{-9}) \left(\frac{56}{0.5 - 0.09 \times 2}\right) (0.5 \text{ mA})}$$

$$= 2.59 \text{ m}\Omega^{-1}$$

$$r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.5 \text{ mA})} = 20 \text{ k}\Omega$$

$$r_{o4} = r_{o2} = 20 \text{ k}\Omega$$

$$r_{o6} = r_{o8} = \frac{1}{(0.2)(0.5 \text{ mA})} = 10 \text{ k}\Omega$$

$$A_v = (6.28 \text{ m}) [(6.28 \text{ m} \times 20 \text{ k} \times 20 \text{ k}) \parallel (2.59 \text{ m} \times 10 \text{ k} \times 10 \text{ k})]$$

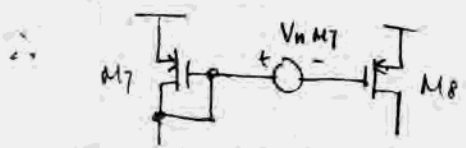
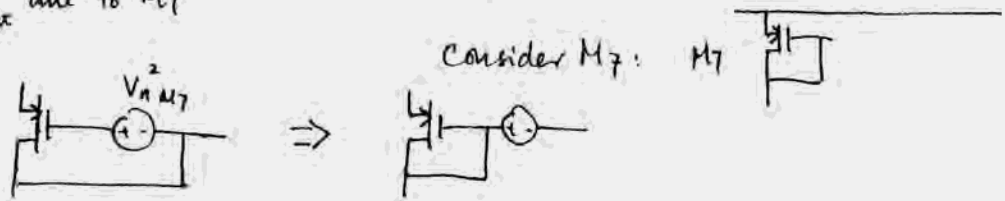
$$A_v = 1474$$

(d) Since this is a cascode configuration, the noise due to $M_{3,4,5,6}$ can be neglected.

$$\overline{V_{n \text{ input}}^2} \text{ due to } M_{1,2} = 4kTY \frac{1}{g_{m1,2}}$$

$$\overline{V_{n \text{ input}}^2} \text{ due to } M_8 = 4kTY \frac{g_{m8}}{g_{m1,2}^2} \text{ same as in cascode.}$$

$$\overline{V_{n \text{ input}}^2} \text{ due to } M_7$$



M_7 will induce the same noise as M_8 .

\therefore input-referred noise voltage

$$\overline{V_n^2} = \left[4kTY \frac{1}{g_{m1,2}} + 4kTY \frac{g_{m7,8}^2}{g_{m1,2}^2} \right] \times 2 \quad \text{where } \gamma = \frac{2}{3}$$

9.2 (d) cont.

$$\overline{V_n^2} = 4 \times 1.38 \times 10^{-23} \times 300 \times \frac{2}{3} \times 2 \left[\frac{1}{6.28 \text{ m}} + \frac{2.59 \text{ m}}{(6.28 \text{ m})^2} \right]$$

$$\overline{V_n^2} = 4.966 \times 10^{-18} \text{ V}^2/\text{Hz}$$

$$\text{or} = 2.23 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}}$$

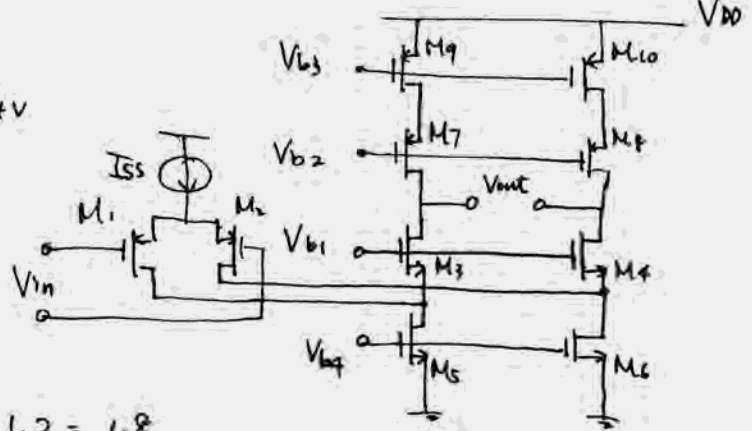
Problem 9.3

Requirements: Max. diff. swing 2.4V

$$P_{total} = 6mW$$

Max diff swing =

$$= 2[V_{DD} - (V_{D03} + V_{D05} + |V_{D07}| + |V_{D09}|)] = 2.4V$$



$$V_{D03} + V_{D05} + |V_{D07}| + |V_{D09}| = V_{DD} - 1.2 = 1.8$$

In general, assign $|V_{D07}|$ & $|V_{D09}|$ to be large than V_{D03} as PMOS has a smaller μ_p . Also $M5$ need large V_{D05} as I_{D5} is larger.

Let the followings:

$$V_{D03} = 0.3V, \quad V_{D05} = 0.44V \quad |V_{D07}| = |V_{D09}| = 0.53V \quad |V_{D01}| = 0.53V$$

$$P_{total} = 6mW = V_{DD} \times (I_{D5} + I_{D6})$$

$$I_{D5} = I_{D6} = \frac{6mW}{(3)(2)} = 1mA$$

$$I_D = \frac{1}{2} \mu C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2 (1 + \lambda V_{D0})$$

$$\left(\frac{W}{L}\right) = \frac{2I_D}{\mu C_{ox} (V_{GS} - V_{TH})^2 (1 + \lambda V_{D0})}$$

$$\left(\frac{W}{L}\right)_{5,6} = \frac{2(1mA)}{2(1mA)}$$

$$\frac{(350)(383.6 \times 10^{-9})(0.44)^2 (1 + (0.1)(0.44))}{2(1mA)} = 74$$

(Assume $V_{D0} \approx V_{GS} - V_{TH}$,

Since λ is small, the result will not be affected too much.)

$$W_{5,6} = (74)(0.34\mu m) = 25.16\mu m$$

Let $I_{D1,2} = 0.5mA$, $I_{D3,4} = 0.5mA$

$$\left(\frac{W}{L}\right)_{1,2} = \frac{2(0.5mA)}{(100)(383.6 \times 10^{-9})(0.53)^2 (1 + (0.2)(0.53))} = 84$$

$$W_{1,2} = 84(0.32\mu m) = 26.88\mu m$$

$$\left(\frac{W}{L}\right)_{3,4} = \frac{2(0.5mA)}{(350)(383.6 \times 10^{-9})(0.3)^2 (1 + (0.1)(0.3))} = 80$$

$$W_{3,4} = 80 \cdot 0.34\mu m = 27.2\mu m$$

$$\left(\frac{W}{L}\right)_{7,9} = \frac{2(0.5mA)}{(100)(383.6 \times 10^{-9})(0.53)^2 (1 + (0.2)(0.53))} = 84$$

9.3 cont.

$$W_{7,9} = 84 \times 0.32 \mu\text{m} = 26.88 \mu\text{m}$$

$$V_{b4} = V_{GS5} = V_{DD5} + V_{TH5} = 0.44 + 0.7$$

$$V_{b4} = 1.14 \text{ V}$$

$$\begin{aligned} V_{b1} &= V_{DD5} + V_{GS3} = V_{DD5} + V_{DD3} + V_{TH3} = V_{DD5} + V_{DD3} + V_{TH0} + Y(\sqrt{|-2\phi_F + V_{SB1}|} - \sqrt{|-2\phi_F|}) \\ &= 0.44 + 0.3 + 0.7 + 0.45(\sqrt{0.9 + 0.44} - \sqrt{0.9}) \\ &= 1.53 \text{ V} \end{aligned}$$

$$V_{b3} = V_{DD} - |V_{GS9}| = V_{DD} - [|V_{DD9}| + |V_{TH9}|] = 3 - 0.53 - 0.8$$

$$V_{b3} = 1.67 \text{ V}$$

$$\begin{aligned} V_{b2} &= V_{DD} - |V_{DD9}| - |V_{GS7}| = V_{DD} - |V_{DD9}| - |V_{DD7}| - |V_{TH7}| \\ &= 3 - 0.53 - 0.53 - [0.8 + 0.4(\sqrt{0.8 + 0.53} - \sqrt{0.8})] \end{aligned}$$

$$V_{b2} = 1.04 \text{ V}$$

$$\begin{aligned} V_{in, \text{ common mode}} &\leq V_{DD} - V_{ISS} - V_{GS1} = 3 - 0.3 - 0.8 - 0.53 = 1.37 \text{ V} \\ &\geq V_{DD5} - V_{TH1} = 0.44 - 0.8 = -0.36 \text{ V} \end{aligned}$$

$V_{in, \text{ common mode}}$
 can be zero
 ($V_{in} = 0$)
 CM

$$A_V = g_{m1} [(g_{m7} r_{o7} r_{o9}) // (g_{m3} r_{o3} (r_{o1} // r_{o5}))]$$

$$g_{m1} = \frac{2I_{D1}}{V_{GS} - V_{TH}} = \frac{2(0.5 \text{ mA})}{0.53} = 1.89 \text{ m}\Omega^{-1}$$

$$g_{m7} = g_{m1} = 1.89 \text{ m}\Omega^{-1}$$

$$g_{m3} = \frac{2I_{D3}}{V_{GS} - V_{TH}} = \frac{2(0.5 \text{ mA})}{0.3} = 3.33 \text{ m}\Omega^{-1}$$

$$r_{o7} = r_{o9} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(0.5 \text{ mA})} = 10 \text{ k}\Omega = r_{o1}$$

$$r_{o3} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.5 \text{ mA})} = 20 \text{ k}\Omega$$

$$r_{o5} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(1 \text{ mA})} = 10 \text{ k}\Omega$$

$$\begin{aligned} A_V &= (1.89 \text{ m}) [(1.89 \text{ m} (10 \text{ k})^2) // (3.33 \text{ m} (20 \text{ k}) (10 \text{ k} // 2))] \\ &= 228 \end{aligned}$$

$$W_{1,2} = 26.88 \mu\text{m}$$

$$V_{b1} = 1.53 \text{ V}$$

$$A_V = 228$$

$$W_{3,4} = 27.2 \mu\text{m}$$

$$V_{b2} = 1.04 \text{ V}$$

$$W_{5,6} = 25.16 \mu\text{m}$$

$$V_{b3} = 1.67 \text{ V}$$

$$W_{7,8,9,10} = 26.88 \mu\text{m}$$

$$V_{b4} = 1.14 \text{ V}$$

$$-0.36 \leq V_{in, \text{ CM}} \leq 1.37 \text{ V}$$

Problem 9.4

$$\left(\frac{W}{L}\right)_{1-8} = \frac{100}{0.5}, \quad I_{SS} = 1\text{mA}, \quad V_{b1} = 1.7\text{V}, \quad \gamma = 0$$

$$(a) \quad V_{in,CM,max} = V_{ISS} + V_{GS1} \\ = V_{ISS} + V_{TH1} + V_{OD1}$$

where V_{ISS} is the voltage across I_{SS} .

$$V_{in,CM,max} = V_Y + V_{TH1} ;$$

$$V_Y = V_{b1} - V_{GS3} = V_{b1} - V_{TH3} - V_{OD3}$$

$$V_{in,CM,max} = V_{b1} - V_{TH3} - V_{OD3} + V_{TH1} \quad \text{Assume } V_{TH3} = V_{TH1}$$

$$\boxed{V_{in,CM,max} = V_{b1} - V_{OD3}}$$

To calculate V_{OD3} , $I_{D3} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_3 (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$ Assume $\lambda \rightarrow 0$

$$V_{OD3} = V_{GS3} - V_{TH} = \left[\frac{2I_{D3}}{\mu_n C_{ox} \left(\frac{W}{L}\right)_3} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2(0.5\text{mA})}{350(383.6 \times 10^{-9}) \left(\frac{100}{0.34}\right)} \right]^{\frac{1}{2}} = 0.159\text{V}$$

$$\therefore \boxed{V_{in,CM,max} = 1.7 - 0.159 = 1.541\text{V}}$$

(b) $V_x = ?$ To find V_x , we can find V_{GS7}

$$V_{GS7} - V_{TH7} = \left[\frac{2I_{D7}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_7} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2(0.5\text{mA})}{(100)(383.6 \times 10^{-9}) \left(\frac{100}{0.32}\right)} \right]^{\frac{1}{2}} = 0.289$$

$$V_{GS7} = 0.289 + V_{TH7} = 1.089\text{V}$$

$$V_x = V_{DD} - V_{GS7} = 3 - 1.089\text{V}$$

$$\boxed{V_x = 1.911\text{V}}$$

(c) For details, please see page 284 (chapter 9).

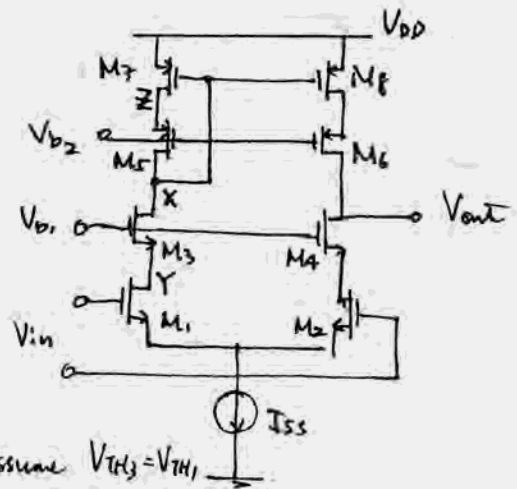
$$\text{Max. output swing} = V_{TH4} - (V_{GS4} - V_{TH4})$$

$$V_{GS3} = V_{GS4} \text{ by symmetry, } V_{GS3} - V_{TH3} = V_{GS4} - V_{TH4} = 0.159\text{V}$$

$$V_{GS4} = 0.7 + 0.159 = 0.859\text{V}$$

$$\text{Max output swing} = 0.7 - (0.859 - 0.7) =$$

$$\boxed{\text{Max output swing} = 0.541\text{V}}$$



9.4 cont.

1d). We know $V_x = 1.911V$, $V_{GS5} = V_{GS7} = 1.089V$

To keep M_7 in saturation,

$$V_Z < V_x + V_{THP} \quad ; \quad V_{b2} = V_Z - |V_{GS5}|$$

$$V_{b2} < V_x + V_{THP} - |V_{GS5}| = 1.911V + 0.8 - 1.089$$

$$V_{b2} < 1.622V$$

$$V_{b2} > V_x - V_{TH5} = 1.911 - 0.8 \Rightarrow V_{b2} > 1.111V$$

$$\therefore \boxed{1.111V < V_{b2} < 1.622V}$$

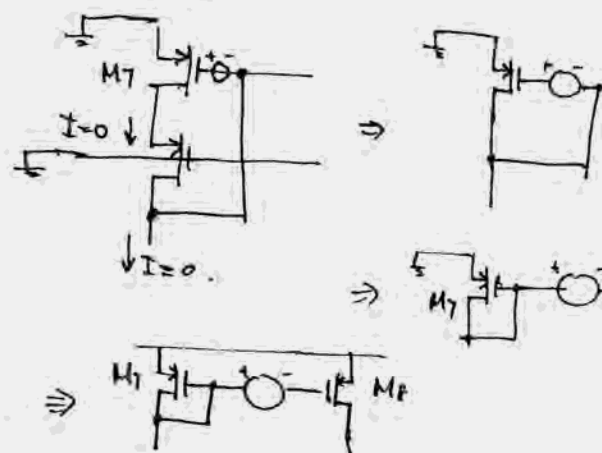
e). As this is a cascode configuration, M_3, M_4, M_5, M_6 have negligible.

Input referred noise voltage due to M_1, M_2

$$\overline{V_n^2}_{input} = 4kTY \frac{1}{g_{m1,2}}$$

$$\overline{V_n^2}_{input M_8} = 4kTY \frac{g_{m8}}{g_{m1,2}^2}$$

$$\overline{V_n^2}_{input M_7} = \overline{V_n^2}_{input \text{ due to } M_8} = 4kTY \frac{g_{m7}}{g_{m1,2}^2}$$



\therefore Input referred noise voltage

$$= \left[4kTY \left(\frac{1}{g_{m1,2}} + \frac{g_{m7,8}}{g_{m1,2}^2} \right) \right] \times 2$$

$$g_{m1,2} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L_{eff}} \right) I_D} = \left[2(350)(383.6 \times 10^{-9}) \left(\frac{100}{0.34} \right) (0.5mA) \right]^{\frac{1}{2}} = 6.28 \times 10^{-3} \Omega^{-1}$$

$$g_{m7,8} = \left[2(100)(383.6 \times 10^{-9}) \left(\frac{100}{0.32} \right) (0.5mA) \right]^{\frac{1}{2}} = 3.46 m\Omega^{-1}$$

$$\text{Input referred noise voltage} = \left[4 \times (1.38 \times 10^{-23}) \times 300 \times \frac{2}{3} \times \left(\frac{1}{6.28m} + \frac{3.46m}{6.28m^2} \right) \right] \times 2$$

$$= 5.45 \times 10^{-18} V^2/Hz$$

$$v_n = 2.34 \times 10^{-9} V/\sqrt{Hz}$$

Problem 9.5.

Requirement: Max. diff. swing = 2.4V
 Power_{max} = 6mW.

$V_{DD} \cdot I_{SS} = \text{Power}_{\text{max}} = 6\text{mW}$.

$I_{SS} = \frac{6\text{mW}}{3\text{V}} = 2\text{mA}$

$|V_{GS7}| - |V_{THP}| = \left[\frac{2I_{D7}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_7} \right]^{\frac{1}{2}}$ Assume $W_{7,8} = 100\mu\text{m}$
 $= \left[\frac{2(1\text{mA})}{(100)(383.6 \times 10^{-9}) \left(\frac{100}{0.32}\right)} \right]^{\frac{1}{2}} = 0.408\text{V}$

$|V_{GS7}| = 0.408 + |V_{THP}| = 1.208\text{V}$

$V_x = V_{DD} - |V_{GS7}| = 3 - 1.208 = 1.79\text{V}$

$V_x - V_{THS} < V_{b2} < V_x + V_{THP} - |V_{GS5}|$ Assume $W_{5,6} = 100\mu\text{m}$
 $0.99\text{V} < V_{b2} < 1.382\text{V}$

For larger output swing, choose $V_{b2} = 1.3\text{V}$.

$V_{out\text{max}} = V_{b2} + V_{THP} = 1.3 + 0.8 = 2.1\text{V}$

We need one sided output swing = 1.2V, so $V_{out\text{min}} = 0.9\text{V}$.

$|V_{OD1}| = |V_{OD3}| = 0.3 = V_{ISS}$

$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_{\text{eff}} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$ Assume $V_{DS} \approx V_{GS} - V_{TH}$
 $\left(\frac{W}{L}\right)_{\text{eff}} = \frac{2I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})} = \frac{2(1\text{mA})}{(350)(383.6 \times 10^{-9})(0.3)^2 (1 + 0.03)}$

$\left(\frac{W}{L}\right)_{\text{eff}} = 160 \Rightarrow W_{1,2,3,4} = (160)(L_{\text{eff}}) = (160)(0.34\mu\text{m})$ let $L = 0.5\mu\text{m}$

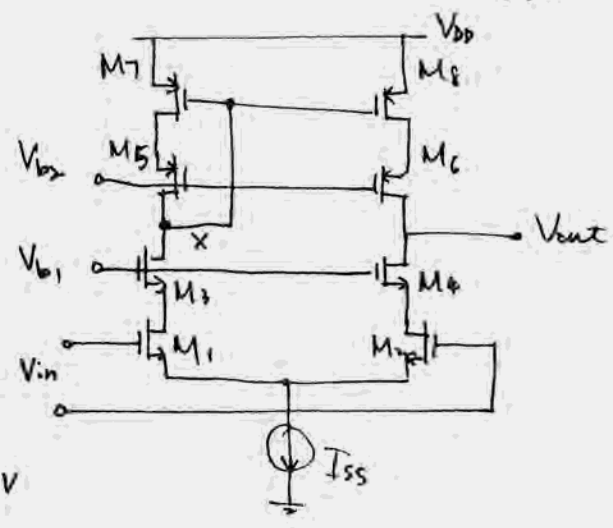
$W_{1-4} = 55\mu\text{m}$

$V_{in,CM} = V_{ISS} + V_{GS6,2} = V_{ISS} + V_{THn} + V_{OD1} = 0.3 + 0.7 + 0.3\text{V}$
 $= 1.3\text{V}$

$V_{b1} = V_{in,CM} + V_{OD1} = 1.3 + 0.3\text{V} = 1.6\text{V}$

Summary

$\left(\frac{W}{L}\right)_{1-4} = \frac{55}{0.5}$	$V_{in,CM} = 1.3\text{V}$	$I_{SS} = 2\text{mA}$
$\left(\frac{W}{L}\right)_{5-8} = \frac{100}{0.5}$	$V_{b1} = 1.6\text{V}$	
	$V_{b2} = 1.3\text{V}$	



Problem 9.6

(A) Given: $(\frac{W}{L})_{1-8} = 0.5$ $I_{SS} = 1\text{mA}$

$$I_{D5,6} = \frac{1}{2} \mu_p C_{ox} (\frac{W}{L})_{5,6} (V_{GS5,6} - V_{THP})^2 (1 + \lambda V_{DS5,6})$$

$$V_{GS5,6} - V_{THP} = \left[\frac{2I_{D5,6}}{\mu_p C_{ox} (\frac{W}{L})_{5,6}} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2(1\text{mA})}{(100)(383.6 \times 10^{-9})(\frac{100}{0.32})} \right]^{\frac{1}{2}} = 0.408\text{V}$$

$$V_{GS5,6} = 0.408 + 0.8 = 1.208\text{V}$$

$$V_{X,Y} = V_{DD} - V_{GS5,6} = 3 - 1.208\text{V}$$

$$V_{X,Y} = 1.792\text{V}$$

In order to keep M_1, M_2 in saturation,

$$V_{in,CM} < V_{X,Y} + V_{TH} = 1.792 + 0.7 = 2.492\text{V}$$

$$\therefore V_{in,CM,max} = 2.49\text{V}$$

(b) A_v of 1st stage = $g_{m1} (r_{o1} \parallel r_{o3})$

A_v of 2nd stage = $g_{m5} (r_{o5} \parallel r_{o7})$

$$A_{v,tot} = g_{m1} (r_{o1} \parallel r_{o3}) g_{m5} (r_{o5} \parallel r_{o7})$$

$$g_{m1} = \sqrt{2 \mu_n C_{ox} (\frac{W}{L})_{1,2} (I_{D1})} = \left[2(350)(383.6 \times 10^{-9})(\frac{100}{0.32})(0.5\text{mA}) \right]^{\frac{1}{2}}$$

$$= 6.28\text{ m}\Omega^{-1}$$

$$g_{m5} = \frac{2I_D}{V_{GS} - V_{TH}} = \frac{2(1\text{mA})}{0.408}$$

$$g_{m5} = 4.90\text{ m}\Omega^{-1}$$

$$r_{o1} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.5\text{mA})} = 20\text{k}\Omega$$

$$r_{o3} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(1\text{mA})} = 10\text{k}\Omega$$

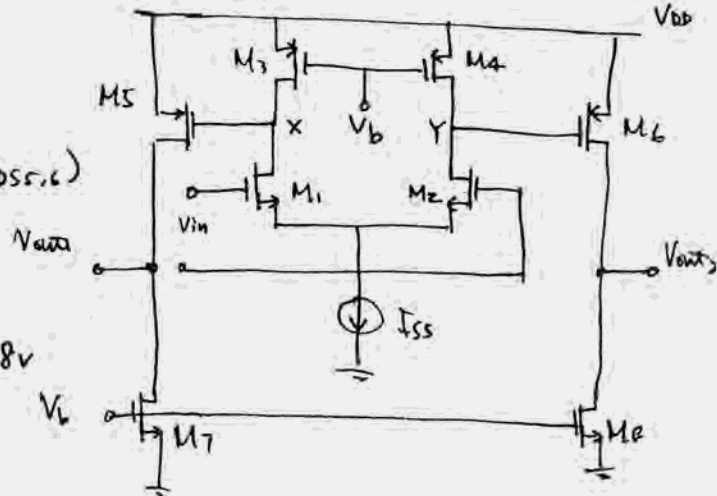
$$r_{o5} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(1\text{mA})} = 5\text{k}\Omega$$

$$r_{o7} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(1\text{mA})} = 10\text{k}\Omega$$

$$A_v = (6.28\text{m}) [20\text{k} \parallel 10\text{k}] (4.90\text{m}) [5\text{k} \parallel 10\text{k}]$$

$$A_v = 684$$

$$\text{Max output swing} = 2(V_{DD} - |V_{DS5}| - V_{DS7})$$



9 p.14

$$|V_{DS}| = |V_{GS}| - |V_{THP}| = 0.408V$$

$$V_{GS7} = V_{THN} = \left[\frac{2 I_D}{(M_n) C_{ox} \left(\frac{W}{L}\right)_7} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2 (mA)}{(350)(383.6 \times 10^{-9}) \left(\frac{100}{0.34}\right)} \right]^{\frac{1}{2}} = 0.225$$

$$\boxed{\begin{aligned} \text{Max output swing} &= 2(3 - 0.408 - 0.225) \\ &= 4.734V. \end{aligned}}$$

Problem 9.7

Design the op amp of fig. 9.21

Max diff. swing = 4V

total Power = 6mW $I_{SS} = 0.5mA$

Total current driven by $V_{OD} = \frac{6mW}{3V} = 2mA$

$I_{D5} + I_{D6} = 2mA - I_{SS} = 1.5mA \Rightarrow I_{D5} = I_{D6} = 0.75mA$

Max diff swing = $2[V_{DD} - |V_{OD5}| - V_{OD7}] = 4V$

$\Rightarrow |V_{OD5}| + |V_{OD7}| = 1$ choose $V_{OD5} = 0.6V, V_{OD7} = 0.4V$

$$I_D = \frac{1}{2} \mu C_{ox} \left(\frac{W}{L}\right)_{eff} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$\left(\frac{W}{L}\right)_5 = \frac{2I_D}{\mu C_{ox} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})} = \frac{2(0.75mA)}{(100)(383.6 \times 10^{-9})(0.6)^2 (1 + 0.2 \times 0.6)}$$

$$\left(\frac{W}{L}\right)_5 = 97 \Rightarrow W_{5,6} = 97 \times 0.32\mu = 31\mu m$$

$$\left(\frac{W}{L}\right)_7 = \frac{2(0.75mA)}{(350)(383.6 \times 10^{-9})(0.4)^2 (1 + 0.1(0.4))}$$

$$= 67 \Rightarrow W_{7,8} = 67 \times 0.34\mu = 23\mu m$$

We are generally not worried about the swing of 1st stage,

assume $|V_{OD3}| = 1V, V_{OD1} = 1V.$

$$\left(\frac{W}{L}\right)_3 = \frac{2(0.25mA)}{(100)(383.6 \times 10^{-9})(1)^2 (1 + 0.2(1))} = 10.86$$

$$W_{3,4} = 3.5\mu m$$

$$\left(\frac{W}{L}\right)_1 = \frac{2(0.25mA)}{(350)(383.6 \times 10^{-9})(1)^2 (1 + 0.1)} = 3.4$$

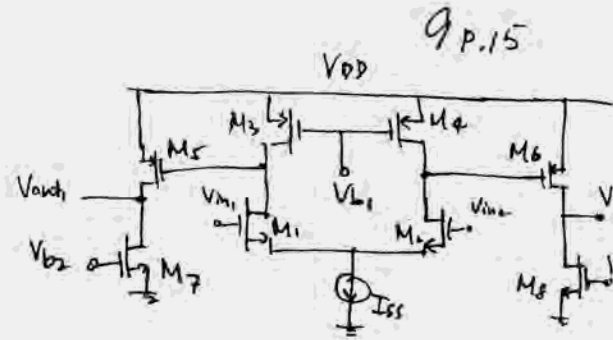
$$W_{1,2} = 1.2\mu m$$

$$V_{b1} = V_{DD} - |V_{OD3}| - V_{TH3} = 3 - 1 - 0.8 = 1.2V.$$

$$V_{in,CM} = V_{ISS} + V_{TH1} + V_{OD1} = 0.3 + 0.7 + 1.0 = 2V.$$

$$V_{b2} = V_{TH7} + V_{OD7} = 0.7 + 0.4 = 1.1V.$$

Summary	$L = 0.5\mu m$	
$W_{1,2} = 1.2\mu m$		$V_{b1} = 1.2V$
$W_{3,4} = 3.5\mu m$		$V_{b2} = 1.1V$
$W_{5,6} = 31\mu m$		$V_{in,CM} = 2V$
$W_{7,8} = 23\mu m$		



Problem 9.8

Given $I_{SS} = 1\text{mA}$, $I_{D9} - I_{D12} = 0.5\text{mA}$

$$\left(\frac{W}{L}\right)_{9-12} = \frac{100}{0.5}$$

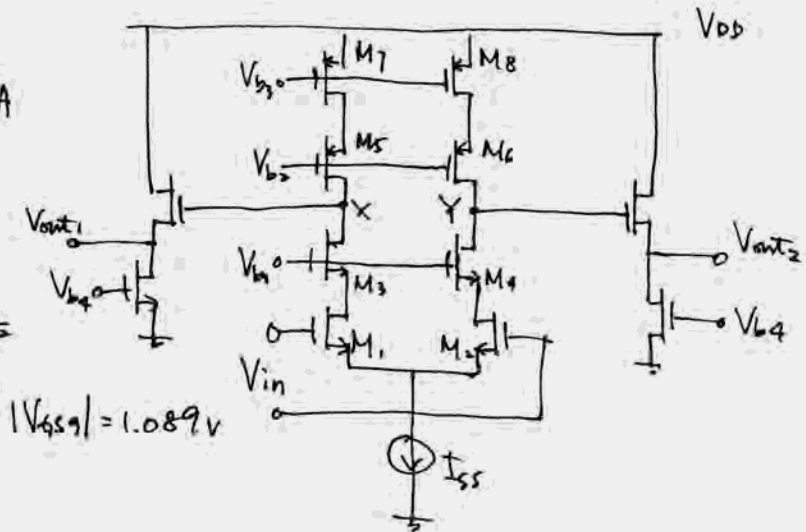
a. $V_{x,r, CM} = ?$

$$|V_{GS9}| - |V_{THP}| = \left[\frac{2I_{D9}}{\mu_p C_{ox} \left(\frac{W}{L}\right)_9} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2(0.5\text{mA})}{(100)(383.6\text{n}) \left(\frac{100}{0.32}\right)} \right]^{\frac{1}{2}}$$

$$= 0.289\text{V}, \rightarrow$$

$$V_{x,r, CM} = V_{DD} - |V_{GS9}| = 1.911\text{V}$$



$$|V_{GS9}| = 1.089\text{V}$$

b. $V_x \text{ swing} = 0.2\text{V}$, $V_{x, CM} = 1.911\text{V}$

$$V_{x, \max} = 2.011\text{V}$$

$$V_{x, \min} = 1.811\text{V}$$

$$V_{DD7} = V_{DD5} = \frac{V_{DD} - V_{x, \max}}{2} = \frac{3 - 2.011}{2} = 0.495$$

$$V_{DD1} = V_{DD3} = \frac{V_{x, \min} - V_{ISS}}{2} = \frac{1.811 - 0.4}{2} = 0.7055$$

$$\left(\frac{W}{L}\right)_{5-8} = \frac{2I_D}{\mu_p C_{ox} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})}$$

$$= \frac{2(0.5\text{mA})}{(100)(383.6\text{n})(0.495)(1 + 0.2)(0.495)}$$

$$= 97.02$$

$$W_{5-8} = 97.02 \times L_{\text{eff}} = 31.05\ \mu\text{m} \approx 31.1\ \mu\text{m}$$

$$\left(\frac{W}{L}\right)_{1-4} = \frac{2I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})}$$

$$= \frac{2(0.5\text{mA})}{(350)(383.6\text{n})(0.7055)(1 + 0.1)(0.7055)}$$

$$= 14$$

$$W_{1-4} \approx 4.8\ \mu\text{m}$$

$$1c) \quad A_v = g_{m1} (g_{m3} r_{o3} r_{o1} \parallel g_{m5} r_{o5} r_{o7}) g_{m9} (r_{o9} \parallel r_{o11})$$

$$g_{m1} = \frac{2 I_D}{V_{GS1} - V_{TH}} = \frac{2(0.5 \text{ mA})}{0.7055} = 1.417 \text{ m}\Omega^{-1}$$

$$g_{m3} = g_{m1} = 1.417 \text{ m}\Omega^{-1}$$

$$g_{m5} = \frac{2 I_D}{V_{GS5} - V_{TH5}} = \frac{2(0.5 \text{ mA})}{0.495} = 2.022 \text{ m}\Omega^{-1}$$

$$g_{m9} = \frac{2(0.5 \text{ mA})}{0.289} = 3.46 \text{ m}\Omega^{-1}$$

$$r_{oN} = r_{o1} = r_{o3} = r_{o11} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.5 \text{ mA})}$$

$$= 20 \text{ k}\Omega$$

$$r_{oP} = r_{o5} = r_{o7} = r_{o9} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(0.5 \text{ mA})}$$

$$= 10 \text{ k}\Omega$$

$$A_v = (1.417 \text{ m}) [1.417 \text{ m} \times 20 \text{ k} \times 20 \text{ k} \parallel 2.022 \text{ m} \times 10 \text{ k} \times 10 \text{ k}] \times 3.46 \text{ m} \times (10 \text{ k} \parallel 20 \text{ k})$$

$$A_v = 4871$$

Problem 9.9

$$\overline{V_n^2, \text{input}} |_{M_1} = 4kT\gamma \frac{1}{g_{m1}}$$

$$\overline{V_n^2, \text{input}} |_{M_2} \approx 0$$

$$\overline{V_n^2, \text{out}} |_{M_5} = 4kT\gamma g_{m5} R_{out}^2$$

$$\overline{V_n^2, \text{out}} |_{M_5} = \frac{4kT\gamma g_{m5} R_{out}^2}{(g_{m1} R_{out})^2} = 4kT\gamma \frac{g_{m5}}{g_{m1}^2}$$

Noise due to M_3, M_4 :

$$I_{n,34} = 4kT\gamma (g_{m3} + g_{m4})$$

$$R_{034} = r_{03} \parallel r_{04}$$

$$V_x = r_{01} \left(-\frac{V_{out}}{r_{05}} \right)$$

$$I_{03} = g_{m3} V_x = -\frac{g_{m3} r_{01} V_{out}}{r_{05}}$$

$$V_Y = R_{034} \left(I_{n,34} + \frac{g_{m3} r_{01}}{r_{05}} V_{out} \right)$$

neglect the r_{02} to approximate the result,

$$V_{out} = \frac{-r_{05}}{\frac{1}{g_{m2}} + r_{01}} V_Y = \frac{-r_{05}}{\frac{1}{g_{m2}} + r_{01}} R_{034} \left(I_{n,34} + \frac{g_{m3} r_{01}}{r_{05}} V_{out} \right)$$

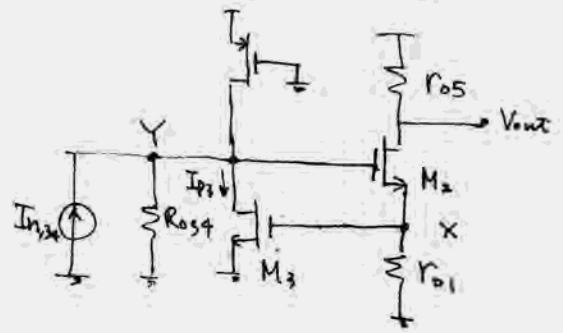
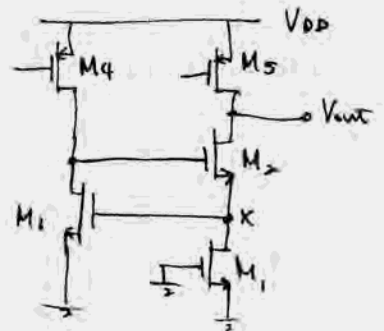
$$V_{out} \left(\frac{\frac{1}{g_{m2}} + r_{02}}{+ r_{05} R_{034}} + \frac{g_{m3} r_{01}}{r_{05}} \right) = -I_n$$

$$V_{out} = \frac{-I_n r_{05} R_{034}}{\frac{1}{g_{m2}} + r_{02} + g_{m3} r_{01} R_{034}} \approx \frac{-I_n r_{05} R_{034}}{g_{m3} r_{01} R_{034}} = \frac{-r_{05}}{r_{01} g_{m3}} I_n$$

$$\overline{V_n^2, \text{input}} |_{M_3, M_4} = \frac{\left(\frac{r_{05}}{r_{01} g_{m3}} \right)^2 I_n^2}{g_{m1}^2 (r_{05} \parallel [g_{m2} r_{02} r_{01} g_{m3} (r_{05} \parallel r_{04})])^2} \approx \frac{I_n \left(\frac{r_{05}}{g_{m3} r_{01}} \right)^2}{g_{m1}^2 r_{05}^2} = 4kT\gamma (g_{m3} + g_{m4}) \left[\frac{1}{g_{m1}^2 g_{m3}^2 r_{01}^2} \right]$$

This is negligible compared with the noise due to M_1, M_5

$$\boxed{\therefore \overline{V_n^2, \text{in total}} = 4kT\gamma \left[\frac{1}{g_{m1}} + \frac{g_{m5}}{g_{m1}^2} \right]}$$



Problem 9.10

1a) $I_1 = 100 \mu A, I_2 = 0.5 mA, (\frac{W}{L})_{1-3} = \frac{100}{0.5}$

$(\frac{W}{L})_p = \frac{50}{0.5}$

$I_{D3} = I_1 = \frac{1}{2} \mu_n C_{ox} (\frac{W}{L})_3 (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$

$V_{GS3} - V_{TH} = \left[\frac{2I_1}{\mu_n C_{ox} (\frac{W}{L})_3} \right]^{\frac{1}{2}}$
 $= \left[\frac{2(100 \mu A)}{350 \times (388.6 n) (\frac{100}{0.34})} \right]^{\frac{1}{2}} = 0.0712$

$V_{GS3} = 0.7712 V = V_{GS} = V_x$

$V_{GS2} = V_{THn} = \left[\frac{2I_2}{\mu_n C_{ox} (\frac{W}{L})_2} \right]^{\frac{1}{2}}$

$= \left[\frac{2(0.5 mA)}{(350)(388.6 n) (\frac{100}{0.34})} \right]^{\frac{1}{2}} = 0.159 V$

$V_{GS2} = 0.859 V$

$V_{G2} = V_{GS2} + V_x = 1.630 V$

1b) Max output swing:

$V_{out max} = V_{DD} - |V_{DS}|$

$V_{out min} = V_x + V_{DS2} = V_{GS3} + V_{DS2}$

Max output swing = $V_{DD} - |V_{DS}| - V_{DS2} - V_{GS3}$

$|V_{GS1} - V_{TH}| = \left[\frac{2I_D}{\mu_p C_{ox} (\frac{W}{L})_5} \right]^{\frac{1}{2}} = \left[\frac{2(0.5 mA)}{(100)(388.6 n) (\frac{50}{0.32})} \right]^{\frac{1}{2}}$
 $= 0.408 V = |V_{DS5}|$

Max output swing = $3 - 0.408 - 0.159 = 0.7712$
 $= 1.6618 V$

1c) $A_v = g_{m1} [r_{o5} || (g_{m2} r_{o2} r_{o1} g_{m3} (r_{o3} || r_{o4}))]$

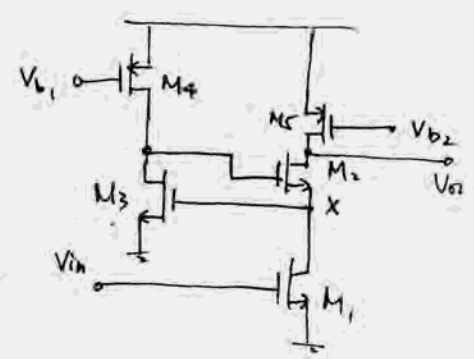
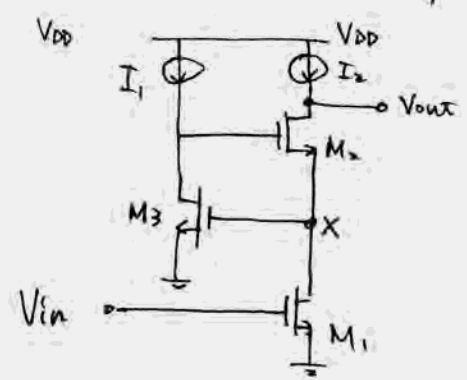
Note: r_{o5} is limiting the gain.

$\approx g_{m1} r_{o5}$

$g_{m1} = \sqrt{2 \mu_n C_{ox} (\frac{W}{L})_1 I_{D1}} = \sqrt{2(350)(388.6 n) (\frac{100}{0.34})(0.5 mA)}$
 $= 6.28 mS^{-1}$

$r_{o5} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(0.5 mA)} = 10 k\Omega$

$A_v = 62.8$



9.10 cont.

$$(c) \overline{V_{n,in}} = 4kTY \left[\frac{1}{g_{m1}} + \frac{g_{m5}}{g_{m1}^2} \right] + 4kTY \left(\frac{1}{g_{m3}} + \frac{g_{m4}}{g_{m3}^2} \right) \left[\frac{g_{m2} r_{o2} g_{m3} (r_{o3} \parallel r_{o4})}{g_{m1} (r_{o1} + r_{o2} + r_{o5})} \right]^2 \quad (\text{see})$$

$$g_{m1} = g_{m2} = 6.28 \text{ m}\Omega^{-1}$$

$$g_{m5} = \left[2(100)(383.6 \text{ n}) \left(\frac{50}{0.32} \right) (0.5 \text{ m}) \right]^{\frac{1}{2}} = 2.45 \text{ m}\Omega^{-1}$$

$$g_{m3} = \left[2(350)(383.6 \text{ n}) \left(\frac{100}{.34} \right) (100 \mu) \right]^{\frac{1}{2}} = 2.81 \text{ m}\Omega^{-1}$$

$$g_{m4} = \left[2(100)(383.6 \text{ n}) \left(\frac{50}{.32} \right) (100 \mu) \right]^{\frac{1}{2}} = 1.09 \text{ m}\Omega^{-1}$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.5 \text{ m})} = 20 \text{ k}\Omega$$

$$r_{o3} = \frac{1}{(0.1)(100 \mu)} = 100 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{(0.2)(100 \mu)} = 50 \text{ k}\Omega$$

$$r_{o5} = \frac{1}{(0.2)(0.5 \text{ m})} = 10 \text{ k}\Omega$$

$$\overline{V_{n,in}}^2 = 4(1.38 \times 10^{-23})(300) \left(\frac{2}{3} \right) \left[\frac{1}{6.28 \text{ m}} + \frac{2.45 \text{ m}}{6.28 \text{ m}^2} \right]$$

$$= 2.444 \times 10^{-18} \text{ V}^2/\text{Hz}$$

$$\overline{V_{n,in}} = 1.56 \times 10^{-9} \text{ V}/\sqrt{\text{Hz}}$$

Problem 9.11

$V_p = 100 \text{ mV}$

$V_{out, CM} = 1.5 \text{ V}, I_{D7,8} = 0.5 \text{ mA}$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_7 \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]$$

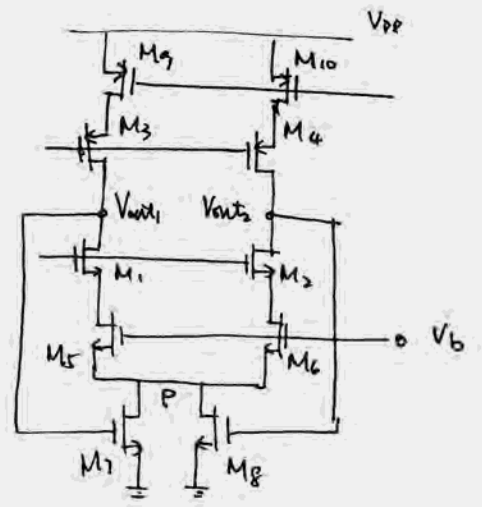
$$\left(\frac{W}{L}\right)_7 = \frac{2 I_D}{\mu_n C_{ox} \left[(V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2} \right]}$$

$$= \frac{2 (0.5 \text{ mA})}{(350)(383.6 \text{ n}) \left[(1.5 - 0.7)(0.1) - \frac{0.1^2}{2} \right]}$$

$$= 99.3$$

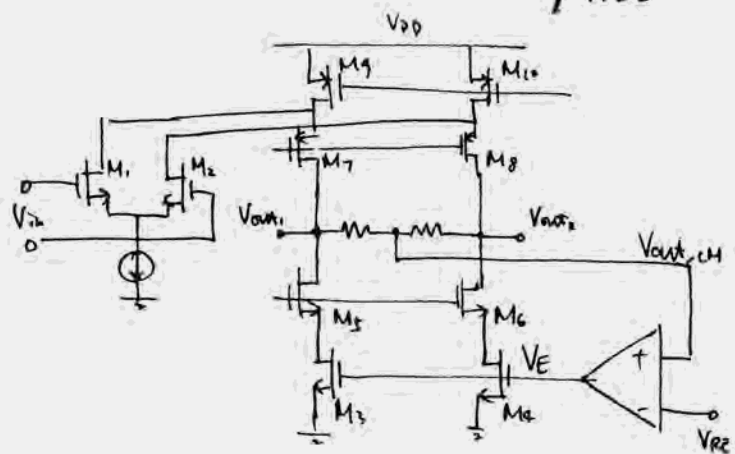
$W_{7,8} = 99.3 \times 0.34 \mu\text{m} = 33.762 \mu\text{m} \approx 34 \mu\text{m}$

$\left(\frac{W}{L}\right)_{7,8} = \frac{34}{0.5}$

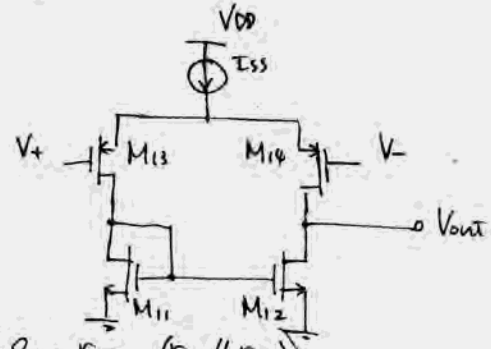


Problem 9.12

(A) PMOS devices should be used. Since $V_{out,CM}$ is in the middle voltage range, (around 1.5V), and $V_{GS,4}$ are in low voltage range, (around 0.7 - 0.8V), we should use PMOS to bring down the voltage.



=



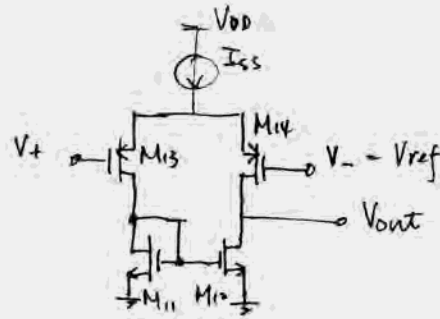
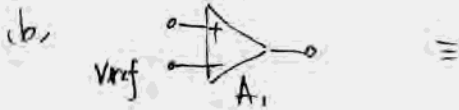
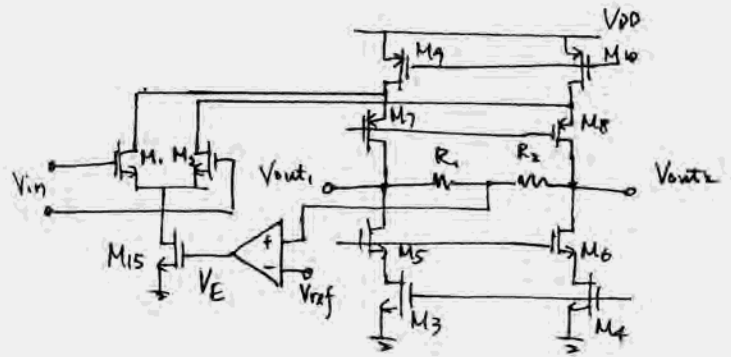
$$A_i = g_{m13} (r_{o12} \parallel r_{o14})$$

$$\frac{V_{out,CM}}{V_E} = -g_{m3,4} (g_{m5} r_{o5} r_{o3} \parallel g_{m7} r_{o7} (r_{o1} \parallel r_{o9}))$$

$$\text{Loop gain} = -g_{m3,4} \left[(g_{m5} r_{o5} r_{o3}) \parallel (g_{m7} r_{o7} (r_{o1} \parallel r_{o9})) \right] g_{m13} (r_{o12} \parallel r_{o14})$$

Problem 9.13

(a) Since we need to bring down $V_{out,cm}$ to fit the bias voltage of NMOS, which is relatively low, we should use PMOS for the input pair of amplifier.



$$A_1 = g_{m3} (r_{o12} \parallel r_{o14})$$

$$\frac{V_{out,cm}}{V_E} = -g_{m15} \left[(g_{m5} r_{o5} r_{o3}) \parallel (g_{m7} r_{o7} (r_{o9} \parallel g_{m1} r_{o1} r_{o15})) \right]$$

$$\text{loop gain} = -g_{m15} \left[(g_{m5} r_{o5} r_{o3}) \parallel (g_{m7} r_{o7} (r_{o9} \parallel g_{m1} r_{o1} r_{o15})) \right] g_{m13} (r_{o12} \parallel r_{o14})$$

Problem 9.14

(a) $(\frac{W}{L})_{1,4} = \frac{100}{0.5}$, $C_1 = C_2 = 0.5 \text{ pF}$, $I_{SS} = 1 \text{ mA}$

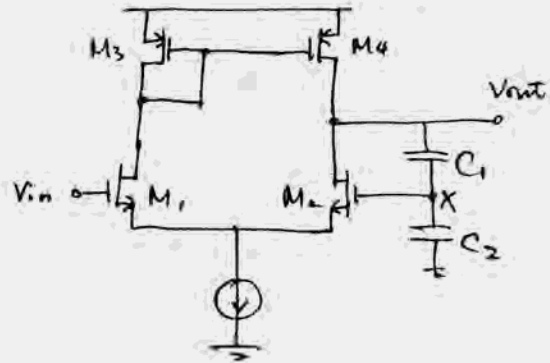
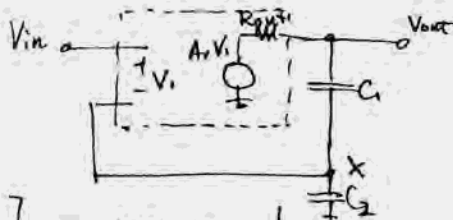
$A_v = g_{m1} (r_{o2} \parallel r_{o4})$

$R_{out} = r_{o2} \parallel r_{o4}$

$V_{in} = V_i + V_x$

$V_x = V_{out} \frac{C_1}{C_1 + C_2}$

$V_{out} = A_v V_i \left[\frac{\frac{C_1 C_2 s}{R_{out} + \frac{C_1 C_2 s}{A_v V_i}}}{1 + \frac{C_1 C_2 s}{A_v V_i}} \right] = A_v V_i \frac{1}{(C_1 \parallel C_2) R_{out} s + 1}$
 $= \frac{A_v}{1 + (C_1 \parallel C_2) R_{out} s} (V_{in} - V_{out} \frac{C_1}{C_1 + C_2})$



$(1 + (C_1 \parallel C_2) R_{out} s + A_v \frac{C_1}{C_1 + C_2}) V_{out} = A_v V_{in}$

$\frac{V_{out}}{V_{in}} = \frac{A_v}{1 + A_v \frac{C_1}{C_1 + C_2} + (C_1 \parallel C_2) R_{out} s} = \frac{A_v}{1 + \frac{A_v C_1}{C_1 + C_2} + \frac{(C_1 \parallel C_2) R_{out} s}{1 + A_v \frac{C_1}{C_1 + C_2}}}$

$\tau = \frac{\frac{C_1 C_2}{C_1 + C_2} R_{out}}{1 + A_v \frac{C_1}{C_1 + C_2}}$

(b) $I_{D2} = 0.1 I_{SS}$

Since I_{D2} is still small, we can solve this problem by assuming the current through C_1 & C_2 roughly equal to I_{SS}

$V_x(t) - V_x(0) = \frac{I_{SS}}{C_2} t$

At $t = 0^-$ $I_{D1} = I_{D2} = 0.5 \text{ mA}$

$V_{GS1,2} - V_{TH} = \left[\frac{2 I_D}{\mu_n C_{ox} (\frac{W}{L})_{eff}} \right]^{\frac{1}{2}} = \left[\frac{2(0.5 \text{ mA})}{(350)(303.6 \mu\text{m}) (\frac{100}{0.34})} \right]^{\frac{1}{2}} = 0.159 \text{ V}$

At time = t , when $I_{D2} = 0.1 I_{SS} \Rightarrow I_{D1} = 0.9 I_{SS}$

$I_D = \frac{1}{2} k_n C_{ox} (\frac{W}{L}) (V_{GS} - V_T)^2$

$\frac{I_{D1}}{I_{D2}} = \frac{0.9 I_{SS}}{0.1 I_{SS}} = \frac{(V_{GS1} - V_T)^2}{(V_{GS2} - V_T)^2} \Rightarrow \frac{(V_{GS1} - V_T)}{(V_{GS2} - V_T)} = 3$

$V_{GS1}(t) - V_T = 3(V_{GS2} - V_T)$

$(V_{GS1}(t) - V_T) = (V_{GS1}(0) - V_T) + 1 \text{ V} = 0.159 + 1 \text{ V} = 1.159 \text{ V} = 3(V_{GS2} - V_T)$

$V_{GS2}(t) - V_T = 0.386 \text{ V}$

Q.14 cont.

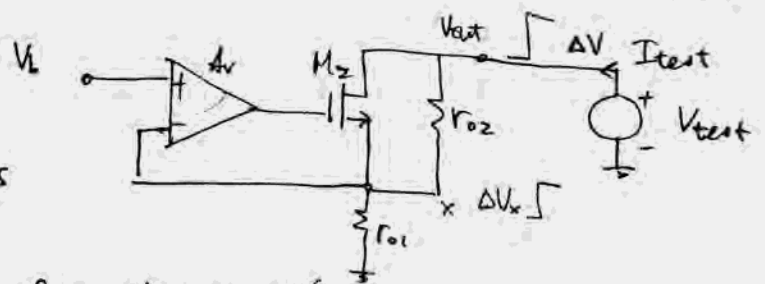
$$[V_{GS_2}(t) - V_T] - [V_{GS_2}(0) - V_T] = V_G(t) - V_G(0) = \frac{I_{SS}}{C_G} t$$

$$0.386 - 0.159 = 0.227 = \frac{1 \mu A}{0.5 \text{ pF}} (t)$$

$$\boxed{t = 113.5 \text{ ps.}}$$

Problem 9.15

The mistake is made when we say the current from V_{test} is equal to $\Delta V / r_{o2}$.



We can see it when we start from the amplifier.

If we ~~to~~ assume current from V^- is very small or negligible, the current through r_{o1} is equal to I_{test} , the current driven from V_{test} . The current through r_{o1} is $\frac{\Delta V_x}{r_{o1}}$, which is a much smaller value than $\Delta V / r_{o2}$.

The mistake is made because the current through r_{o2} is actually equal to $\Delta V / r_{o2}$ or $\approx \frac{\Delta V + \Delta V_x}{r_{o2}}$. This current is larger ~~since~~ than I_{test} since some extra current from M_2 makes the current through r_{o2} larger. As a result, $\Delta V_{r_{o2}}$ (Δ voltage across r_{o2}) increases by about ΔV , but the current from V_{test} ~~only~~ increases only by $\frac{\Delta V_x}{r_{o1}}$.

Problem 9.16

$$CMRR = \frac{\text{diff. gain}}{CM \text{ gain}}$$

$$\text{diff. gain} = g_{m1} (r_{o2} \parallel r_{o4})$$

CM gain: let R is the resistance of current source

$$\Delta V_{in1} = \Delta V_{in2} = V_{CM}$$

$$\frac{V_{out}}{\Delta V_{in1}} = \frac{-\frac{1}{g_{m2}}}{\frac{R}{2} + \frac{1}{g_{m1}}} g_{m4} (R_{out})$$

$$\frac{V_{out}}{\Delta V_{in2}} = \frac{-R_{out}}{\frac{R}{2} + \frac{1}{g_{m2}}}$$

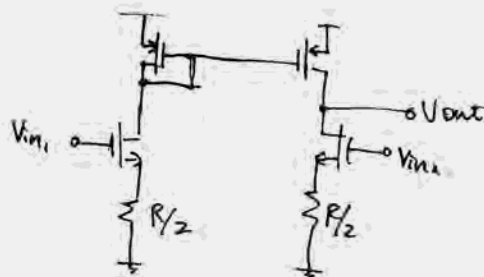
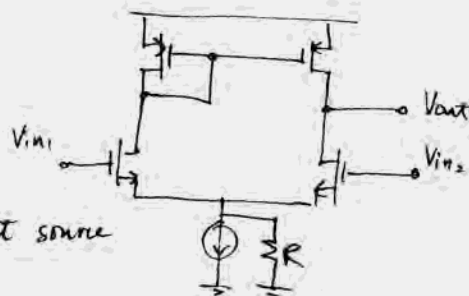
$$\left| \frac{V_{out}}{V_{CM}} \right| = \frac{2 R_{out}}{\frac{R}{2} + \frac{1}{g_{m1}}} \approx \frac{4 R_{out}}{R}$$

where $R_{out} = r_{o4} \parallel r_{o2}$

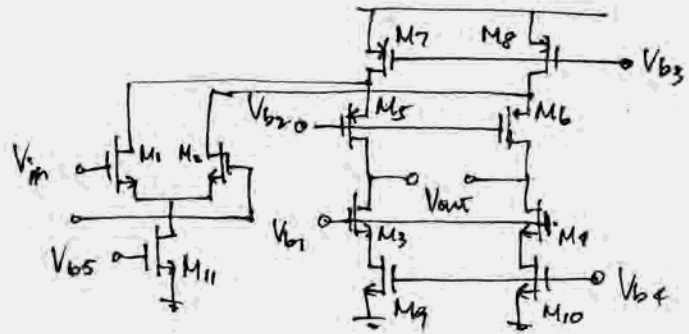
$$CM \text{ gain} = \frac{4(r_{o4} \parallel r_{o2})}{R}$$

$$CMRR = \frac{g_{m1} (r_{o2} \parallel r_{o4})}{\frac{4(r_{o2} \parallel r_{o4})}{R}}$$

$$\boxed{\frac{g_{m1} R}{4} = CMRR}$$



Problem 9.17



Neglect the noise due to $M_{11}, M_3, M_4, M_5, M_6$.

Input-referred flicker noise due to $M_{7,8} = \frac{\overline{V_{n,7,8}^2} \cdot g_{m7,8}^2 R_{out}^2}{A_v^2}$

where $A_v = g_{m1} (R_{out})$, $\overline{V_{n,7,8}^2} = \frac{K_p}{C_{ox} (WL)_{7,8}} \cdot \frac{1}{f}$

$\overline{V_{n, input}^2} |_{M_{9,10}} = \frac{\overline{V_{n,9,10}^2} \cdot g_{m9,10}^2 R_{out}^2}{g_{m1}^2 R_{out}^2} = \left[\overline{V_{n,9,10}^2} \frac{g_{m9,10}^2}{g_{m1,2}^2} \right]$

Total Input-referred flicker noise

$$= \frac{2K_n}{C_{ox} f} \left[\frac{1}{(WL)_{1,2}} + \frac{1}{(WL)_{9,10}} \frac{g_{m9,10}^2}{g_{m1,2}^2} \right] + \frac{2K_p}{C_{ox} f} \frac{1}{(WL)_{7,8}} \frac{g_{m7,8}^2}{g_{m1,2}^2}$$

Problem 9.18

$$P = 6mW, \text{ output swing} = 2.5V$$

$$L_{eff} = 0.5\mu m$$

$$(a) I_{D5,6} = 1mA. V_{D5} \approx V_{D6} = \frac{V_{DD} - \text{Output Swing}}{2} = \frac{3 - 2.5}{2} = 0.25V$$

$$I_D = \frac{1}{2} \mu C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

$$\left(\frac{W}{L}\right)_{eff} = \frac{2I_D}{\mu C_{ox} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})}$$

$$\left(\frac{W}{L}\right)_5 = \frac{2(1mA)}{(350)(383.6n)(0.25)^2(1 + 0.1)(0.25)}$$

$$I_{D5} = I_{D6} = 1mA$$

$$= 233$$

$$\left(\frac{W}{L}\right)_6 = \frac{2(1mA)}{(100)(383.6n)(0.25)^2(1 + 0.2)(0.25)}$$

$$= 795$$

$$\boxed{\left(\frac{W}{L}\right)_5 = 233 \quad \left(\frac{W}{L}\right)_6 = 795}$$

$$b. A_v \text{ of 1st stage} = g_{m5} (r_{o2} \parallel r_{o4})$$

$$A_v \text{ of 2nd stage} = g_{m5} (r_{o5} \parallel r_{o6})$$

$$g_{m5} = \frac{2I_D}{V_{GS} - V_{TH}} = \frac{2(1mA)}{0.25} = 8m\Omega^{-1}$$

$$r_{o5} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(1mA)} = 10k\Omega$$

$$r_{o6} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(1mA)} = 5k\Omega$$

$$\boxed{A_v \text{ of output stage} = (8m)(10k \parallel 5k) = 26.67}$$

$$(c) I_{D7} = 1mA \Rightarrow I_{D3} = I_{D4} = 0.5mA$$

$$V_{GS5} - V_{TH} = 0.25V \Rightarrow V_{GS5} = 0.25 + V_{TH} = 0.95V$$

$$V_{GS3} - V_{TH} = 0.25$$

$$\left(\frac{W}{L}\right)_{3,4} = \frac{2(0.5mA)}{(350)(383.6n)(0.25)^2(1 + 0.1)(0.25)}$$

$$\boxed{\left(\frac{W}{L}\right)_{3,4} = 116}$$

9.18

$$(a) \quad A_{v \text{ tot}} = g_{m1} (r_{o2} \parallel r_{o4}) g_{m5} (r_{o5} \parallel r_{o6})$$

$$r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(0.5 \text{ mA})} = 10 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{(0.1)(0.5 \text{ mA})} = 20 \text{ k}\Omega$$

$$r_{o2} \parallel r_{o4} = 6.67 \text{ k}\Omega$$

$$A_{v \text{ tot}} = g_{m1} (6.67 \text{ k}\Omega)(26.7) = 500$$

$$g_{m1} = 2.81 \text{ mS}^{-1}$$

$$g_{m1} = \sqrt{2\mu_p C_{ox} \left(\frac{W}{L}\right) I_D} \Rightarrow \left(\frac{W}{L}\right) = \frac{g_{m1}^2}{2\mu_p C_{ox} I_D}$$

$$\left(\frac{W}{L}\right)_{1,2} = \frac{(2.81 \text{ m})^2}{2(100)(383.6 \text{ n})(0.5 \text{ mA})}$$

$$\boxed{\left(\frac{W}{L}\right)_{1,2} = 206}$$

Problem 9.19

$$A_v \text{ of 2nd stage} = 20 \quad I_{D5,6} = 1 \text{ mA}$$

(a), $V_{DS5} = V_{DS6}$

$$A_v \text{ of 2nd stage} = g_{m5} (r_{o5} \parallel r_{o6})$$

$$= \frac{2I_D}{V_{DS5} - V_{TH}} \cdot \left[\frac{1}{\lambda I_{D5}} \parallel \frac{1}{\lambda I_{D6}} \right] = 20.$$

$$r_{o5} = \frac{1}{(0.1)(1 \text{ mA})} = 10 \text{ k}\Omega \quad r_{o6} = \frac{1}{(0.2)(1 \text{ mA})} = 5 \text{ k}\Omega \quad r_{o5} \parallel r_{o6} = 3.33 \text{ k}\Omega$$

$$V_{DS5} - V_{TH} = \frac{2I_D (r_{o5} \parallel r_{o6})}{A_v} = \frac{2(1 \text{ mA})(3.33 \text{ k}\Omega)}{20} = 0.333 \text{ V}$$

$$\left(\frac{W}{L}\right)_5 = \frac{2(1 \text{ mA})}{(350)(383.6 \text{ n})(0.33)^2 (1 + 0.1 \times 0.33)} = 132 = (W/L)_5$$

$$\left(\frac{W}{L}\right)_6 = \frac{2(1 \text{ mA})}{(100)(383.6 \text{ n})(0.33)^2 (1 + 0.2 \times 0.33)} = 449 = (W/L)_6$$

b), $r_{o6} = \frac{1}{\mu_p C_{ox} \left(\frac{W}{L}\right) (V_{DS6} - V_{THP} - V_{DS})}$

$$= \frac{1}{(100)(383.6 \text{ n})(449)(50 \text{ m})} = 1.16 \text{ k}\Omega$$

$$V_{DS6} - V_{THP} - V_{DS} = 50 \text{ mV}$$

$$A_v \text{ of 2nd stage} = \left[\frac{2(1 \text{ mA})}{0.333} \right] [10 \text{ k} \parallel 1.16 \text{ k}]$$

$$= 6.24 = A_v$$

Problem 9.20

$$(a) |V_{GS7} - V_{TH7}| = 0.4 \text{ V} = |V_{OD7}|$$

$$V_{in \text{ max}} = V_{DD} - |V_{OD7}| - |V_{OD1}| - |V_{TH1}|$$

$$\text{In part (d), Prob 9.18, } g_{m1} = 2.81 \text{ mS}^{-1} = \frac{2I_D}{V_{GS} - V_{TH}} = \frac{2(0.5 \text{ m})}{|V_{OD1}|}$$

$$|V_{OD1}| = 0.356 \text{ V}$$

$$V_{in \text{ max}} = 3 - 0.4 \text{ V} - 0.356 \text{ V} - 0.8 = 1.444 \text{ V}$$

$$V_{in \text{ min}} = |V_{OD3}| = 0.25 \text{ from Prob 9.18 (c)}$$

$$\text{Allowable input voltage range: } \boxed{0.25 \leq V_{in} \leq 1.44 \text{ V}}$$

(b) At $V_{in} = V_{out}$, $V_{in1} = V_{in2}$ since V_{in2} is connect to V_{out}

$$\text{Since } V_{in1} = V_{in2}, I_{D1} = I_{D2} \Rightarrow V_x = V_y.$$

$$\Rightarrow I_{D5} = 1 \text{ mA} \Rightarrow I_{D1} = I_{D2} = 0.5 \text{ mA}.$$

$$I_D = \frac{1}{2} \mu C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2 (1 + \lambda V_{GS})$$

$$V_{GS1} - V_{TH} = \sqrt{\frac{2I_D}{\mu C_{ox} \left(\frac{W}{L}\right) (1 + \lambda V_{GS})}}$$

$$= \left[\frac{2(0.5 \text{ m})}{(100)(393.6 \text{ n})(206)(1 + 0.2 \times 1.3)} \right]^{\frac{1}{2}}$$

$$= 0.317 \text{ V}$$

$$V_{GS1} = 0.317 + 0.7 = 1.017 \text{ V}$$

$$V_{in} = V_{GS3} + V_{GS1} = 0.95 + 1.017 \text{ V}$$

$$V_{in} = 1.97 \text{ V}$$

$$V_{GS3} - V_{TH} = 0.25 \Rightarrow V_{GS3} = 0.95$$

$$\Rightarrow V_{GS} \approx V_{DD} - V_{GS7} - V_{GS3} \approx 3 - 0.7 - 0.95 \approx 1.3 \text{ V}$$

Problem 9.21

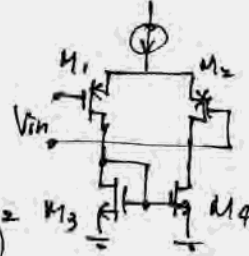
Noise due to $M7$ is negligible since induce common mode gain, which is very small.

Consider 1st stage:

$$\overline{V_{n, \text{input}}^2} \text{ 1st stage} = \left[4kT \left(\frac{1}{g_{m1,2}} + \frac{g_{m3,4}}{g_{m1,2}^2} \right) \right] \times 2$$

Consider 2nd stage

$$\overline{V_{n, \text{output}}^2} \text{ 2nd stage} = [4kT \gamma (g_{m5} + g_{m6})] (r_{o5} \parallel r_{o6})^2$$



Overall:

$$\overline{V_{n, \text{input}}^2} = \left[4kT \gamma \left(\frac{1}{g_{m1,2}} + \frac{g_{m3,4}}{g_{m1,2}^2} \right) \right] \times 2 + \frac{[4kT \gamma \left(\frac{1}{g_{m5}} + \frac{g_{m6}}{g_{m5}^2} \right)]}{[g_{m1} (r_{o2} \parallel r_{o4})]^2}$$

From Prob. 9.18

$$g_{m1,2} = 2.81 \text{ mS}^{-1}, \quad g_{m3,4} = \frac{2I_D}{V_{DS3} - V_{TH}} = \frac{2(0.5 \text{ m})}{0.25} = 4 \text{ mS}^{-1}$$

$$g_{m5} = 8 \text{ mS}^{-1}, \quad g_{m6} = 8 \text{ mS}^{-1}, \quad r_{o2} = 10 \text{ k}\Omega, \quad r_{o4} = 2 \text{ k}\Omega, \quad r_{o2} \parallel r_{o4} = 6.67 \text{ k}\Omega$$

$$\overline{V_{n, \text{input}}^2} = \left[4 \times 1.38 \times 10^{-23} \times 300 \times \frac{2}{3} \times \left(\frac{1}{2.81 \text{ m}} + \frac{4 \text{ m}}{2.81 \text{ m}^2} \right) \right] \times 2 + \frac{4 \times 1.38 \times 10^{-23} \times 300 \times \frac{2}{3} \times \left(\frac{1}{8 \text{ m}} + \frac{1}{8 \text{ m}} \right)}{[2.81 \text{ m} (6.67 \text{ k})]^2}$$

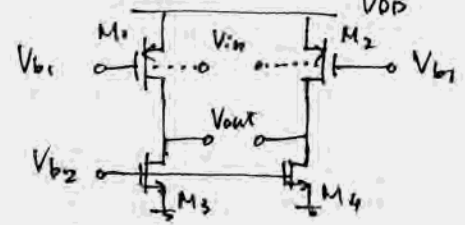
$$= 1.905 \times 10^{-17} \text{ V}^2/\text{Hz}$$

$$\overline{V_{n, \text{input}}^2} = 4.36 \times 10^{-9} \text{ V} / \sqrt{\text{Hz}}$$

9, 22.

$$(a) A_v = g_{mb1,2} (R_{o1} \parallel R_{o3})$$

b) $V_{in} > V_{DD} - V_{D1}$ where V_{D1} is the diode junction voltage of the diode between source and body.



$$(c) g_{mb} = g_m \frac{\gamma}{2\sqrt{|\phi_F| + |V_{SB}|}}$$

As $V_{in,cm}$ decreases, $|V_{SB}| \uparrow$, g_{mb} decreases.

More accurately, $g_{mb} \propto \frac{1}{\sqrt{|\phi_F| + |V_{SB}|}}$

As a result, A_v decreases.

(d)

$$\overline{V_{n,out}^2} = [4kT\gamma (g_{m1} + g_{m3}) R_{out}^2] \times 2$$

$$\overline{V_{n,in}^2} = \frac{4kT\gamma (g_{m1} + g_{m3}) R_{out}^2 \times 2}{[g_{mb1,2} (R_{out})]^2}$$

$$= \left[4kT\gamma \frac{g_{m1} + g_{m3}}{(g_{mb1,2})^2} \right] \times 2$$

Problem 9.23

a. Av of 1st stage = $g_{m1,2} (r_{o1} \parallel r_{o3})$

Av of 2nd stage = $[g_{m5,9} (r_{o7} \parallel r_{o5})] \times 2$

Av-tot = $g_{m1,2} (r_{o1} \parallel r_{o3}) g_{m5,9} (r_{o5} \parallel r_{o7}) \times 2$

b. 1st major pole:

$$W_1 = \frac{1}{(r_{o9} \parallel r_{o11}) [C_{DG9} + C_{DB9} + C_{GS11} + C_{DB11} + C_{GS7} + C_{GD7} (1 + g_{m7} (r_{o5} \parallel r_{o7}))]}$$

2nd major pole: node X, Y

$$W_X = \frac{1}{(r_{o1} \parallel r_{o3}) \left[C_{DG1} + C_{DB1} + C_{DG3} + C_{DB3} + C_{GS10} + C_{GD10} \left(1 + \frac{g_{m10}}{g_{m12}} \right) + C_{GS5} \right. \\ \left. + C_{GD5} (1 + g_{m5} (r_{o5} \parallel r_{o7})) \right]}$$

3rd major pole: node output

$$W_{out} = \frac{1}{(r_{o5} \parallel r_{o7}) (C_{GD5} + C_{DB5} + C_{GD7} + C_{DB7})}$$

Prob. 9.24.

$$A_v \text{ of fast path: } g_{m1}' (r_{o5} \parallel r_{o7})$$

$$A_v \text{ of slow path: } g_{m1} (r_{o1} \parallel r_{o3}) g_{m5} (r_{o5} \parallel r_{o7}).$$

$$\text{Overall gain } A_{v\text{-tot}} = \left[\frac{g_{m1}' + g_{m1} g_{m5} (r_{o1} \parallel r_{o3})}{2} \right] (r_{o5} \parallel r_{o7})$$

The output swing is usually limited by $M_5 - 8$, i.e.
 $V_{DS} - |V_{O07}| - V_{O05}$.

Problem 9.25

Noise due to $M_{1,2}$

$$\overline{V_{n, \text{input}}^2} | M_{1,2} = 4kTY \left(\frac{1}{g_{m_{1,2}}} \right) \times 2$$

$$\overline{V_{n, \text{input}}^2} | M_{1,2} = 4kTY \left(\frac{1}{g_{m_{1,2}}} \right) \times 2$$

$$\overline{V_{n, \text{input}}^2} | M_{3,4} = 4kTY \left(\frac{g_{m_{3,4}}}{g_{m_{1,2}}^2} \right) \times 2$$

$$\overline{V_{n, \text{output}}^2} | M_{5,6} = 4kTY g_{m_{5,6}} R_{\text{out}} \times 2$$

$$\overline{V_{n, \text{output}}^2} | M_{7,8} = 4kTY g_{m_{7,8}} R_{\text{out}} \times 2$$

$$\overline{V_{n, \text{input}}^2} | M_{5-8} = \frac{4kTY (g_{m_{5,6}} + g_{m_{7,8}}) \times 2}{\left(\frac{g_{m_1} + g_{m_1} g_{m_5} (R_{o1} || R_{o3})}{2} \right)^2}$$

$$\overline{V_{n, \text{input}}^2} |_{\text{tot}} = 2 \left[4kTY \left(\frac{1}{g_{m_{1,2}}} + \frac{1}{g_{m_{1,2}}} + \frac{g_{m_{3,4}}}{g_{m_{1,2}}^2} + \frac{4(g_{m_{5,6}} + g_{m_{7,8}})}{(g_{m_1} + g_{m_1} g_{m_5} (R_{o1} || R_{o3}))^2} \right) \right]$$

10.1 Two poles $\omega_{p1} = 10 \text{ MHz}$ $\omega_{p2} = 500 \text{ MHz}$

First find ω_i ($= G \times$) that gives phase $= -120^\circ$ (P.M. of 60°)

$$-120^\circ = -\tan^{-1} \frac{\omega_i}{\omega_{p1}} - \tan^{-1} \frac{\omega_i}{\omega_{p2}} \rightarrow \omega_i \cong 311 \text{ MHz}$$

$$A_o = (\log \frac{\omega_i}{\omega_{p1}}) (20 \text{ dB/dec}) = (\log \frac{311}{10}) (20) = \underline{\underline{29.9 \text{ dB}}}$$

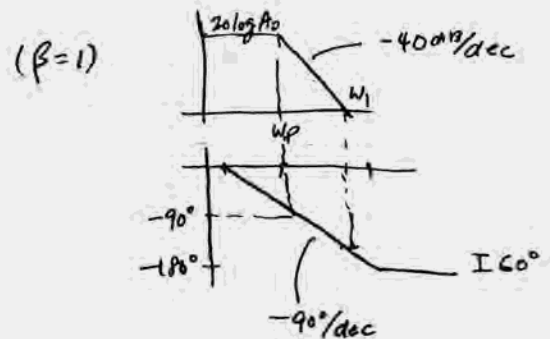
10.2 $\omega_{p1} = \omega_{p2} = \omega_p$

a) $60^\circ \cdot \frac{1}{90^\circ/\text{dec}} = 0.67 \text{ decade}$

$$\log \frac{10\omega_p}{\omega_i} = 0.67 \text{ dec} \quad (\omega_i \text{ is } G \times)$$

$$\Rightarrow \omega_i = 2.14 \omega_p$$

$$A_o = (\log \frac{2.14\omega_p}{\omega_p}) (40 \text{ dB/dec}) = \underline{\underline{13.2 \text{ dB}}}$$



b) For closed-loop gain = 4 $\Rightarrow \beta \cong \frac{1}{4}$

Thus A_o can increase by a factor of 4 to maintain 60° P.M.

$$\Rightarrow A'_o = 13.2 \text{ dB} + 20 \log 4 = \underline{\underline{25.2 \text{ dB}}}$$

10.3 $A_o = 1000$ $\omega_{p1} = 1 \text{ MHz}$

a) $= 60 \text{ dB} - (\log \frac{2 \text{ MHz}}{1 \text{ MHz}}) (20 \frac{\text{dB}}{\text{dec}}) = \underline{\underline{54 \text{ dB}}}$

$$\log \frac{\omega_i}{2 \text{ MHz}} = 54 \text{ dB} \frac{1}{40 \text{ dB/dec}} = 1.35 \text{ dec}$$

$$\omega_i = \underline{\underline{44.8 \text{ MHz}}}$$

$$\angle H(j\omega_i) = -\tan^{-1} \frac{\omega_i}{1 \text{ MHz}} - \tan^{-1} \frac{\omega_i}{2 \text{ MHz}} = -176.2^\circ \Rightarrow \text{P.M.} = 180^\circ - 176.2^\circ = \underline{\underline{3.8^\circ}}$$

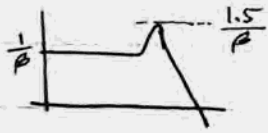
b) $\omega'_{p2} = 4 \text{ MHz}$

$$\log \frac{\omega'_i}{4 \text{ MHz}} = [60 \text{ dB} - (\log \frac{4 \text{ MHz}}{1 \text{ MHz}}) (20 \text{ dB/dec})] \frac{1}{40 \text{ dB/dec}} = 1.199 \text{ dec}$$

$$\Rightarrow \omega'_i = 63.2 \text{ MHz}$$

$$\angle H(j\omega'_i) = -175.5^\circ \Rightarrow \underline{\underline{\text{P.M.} = 4.5^\circ}}$$

10.4



$$\beta = 1$$

$$\text{At } \omega_X, H(j\omega_1) = 1 \cdot e^{j\theta_1}$$

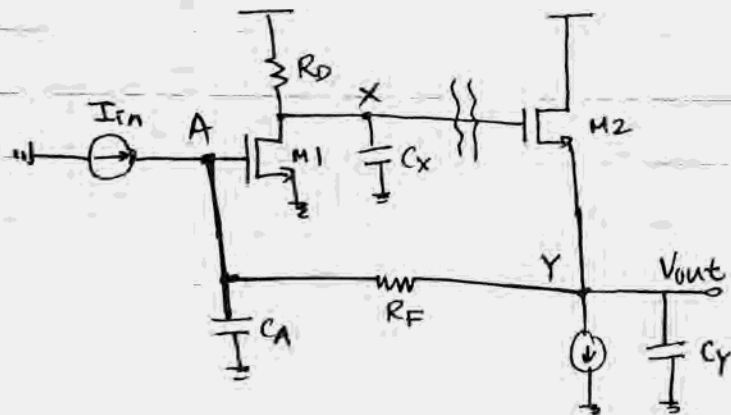
$$\text{Closed loop: } \left| \frac{Y}{X}(j\omega_1) \right| = \left| \frac{H(j\omega_1)}{1 + H(j\omega_1)} \right| = 1.5$$

$$\left| \frac{1}{1 + e^{j\theta_1}} \right| = 1.5$$

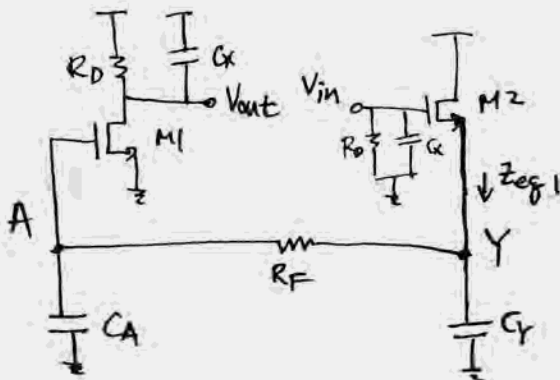
$$\rightarrow \frac{1}{\sqrt{1 + 2\cos\theta_1 + 1}} \rightarrow \frac{1}{2 + 2\cos\theta_1} = 1.5^2 \rightarrow \theta_1 = -14.1^\circ$$

$$\underline{\underline{P.M. = 38.9^\circ}}$$

10.5



Breaking the loop at node X as shown by $\{\}$ and replacing each end by the impedance each sees, we get the next circuit =



Next, calculate the loop gain

$$\frac{V_{out}}{V_{in}}(s) = \frac{V_Y}{V_{in}} \cdot \frac{V_A}{V_Y} \cdot \frac{V_{out}}{V_A} = A_{v1} \cdot A_{v2} \cdot A_{v3}$$

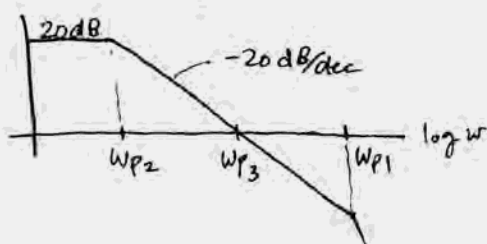
$$Z_{eg1} \approx \frac{1}{sC_Y} \text{ since } R_F = 10k\Omega \gg \frac{1}{g_{m2}}$$

$$A_{v1} = \frac{g_{m2} Z_{eg1}}{1 + g_{m2} Z_{eg1}} \approx \frac{g_{m2} \frac{1}{sC_Y}}{1 + g_{m2} \frac{1}{sC_Y}} = \frac{1}{s\left(\frac{C_Y}{g_{m2}}\right) + 1}$$

$$A_{v2} = \frac{\frac{1}{sC_A}}{R_F + \frac{1}{sC_A}} = \frac{1}{sC_A R_F + 1}$$

$$A_{v3} = -g_{m1} (R_D \parallel \frac{1}{sC_X}) = \frac{-g_{m1} R_D}{1 + sC_X R_D}$$

Hence $\omega_{p1} = \frac{g_{m2}}{C_Y} = 1 \times 10^{11} \text{ rad/s}$, $\omega_{p2} = \frac{1}{C_A R_F} = 1 \times 10^9 \text{ rad/s}$, $\omega_{p3} = \frac{1}{C_X R_D} = 1 \times 10^{10} \text{ rad/s}$
and $g_{m1} R_D = 10 \rightarrow 20 \text{ dB}$



$$\Rightarrow \underline{\underline{P.M. = 45^\circ}}$$

$$\boxed{10.6} \quad R_D' = 2 \text{ k}\Omega$$

$$\rightarrow g_{m1} R_D' = 20 \Rightarrow \underline{26.0 \text{ dB}}, \quad \omega_{P3}' = \frac{1}{C_x R_D'} = \underline{5 \times 10^9 \text{ rad/s}}$$

$$\omega_i' \Rightarrow 26 \text{ dB} - \left(\log \frac{\omega_{P3}'}{\omega_{P2}'} \right) (20 \text{ dB/dec}) - \left(\log \frac{\omega_i'}{\omega_{P3}'} \right) (40 \text{ dB/dec}) = 0 \text{ dB}$$

$$\underline{\omega_i' = 9.99 \times 10^9 \text{ rad/s}}$$

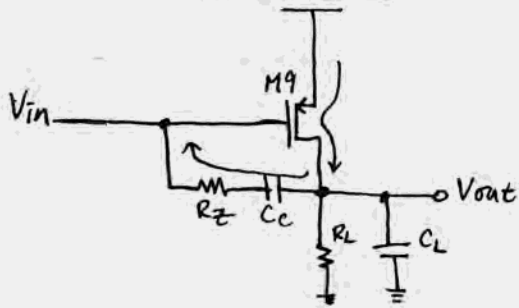
$$\phi = -\tan^{-1} \frac{\omega_i'}{1 \times 10^9} - \tan^{-1} \frac{\omega_i'}{5 \times 10^9} - \tan^{-1} \frac{\omega_i'}{1 \times 10^{11}} = -153.4^\circ$$

$$\underline{\underline{\text{P.M.} = 26.6^\circ}}$$

$$\boxed{10.7} \quad \text{From 10.5} \quad \omega_{P1} = \frac{g_{m2}}{C_Y} \quad \omega_{P2} = \frac{1}{C_A R_F} \quad \omega_{P3} = \frac{1}{C_x R_D}$$

- a) Increasing C_Y causes ω_{P1} to move towards ω_{P3} and will be less than 1 decade from ω_{P3} . This will reduce the already 45° -phase margin. Hence $C_{Y \text{ max}} = 100 \text{ fF}$.
- b) Increasing C_A will increase phase margin.
Hence $C_{A \text{ max}} = 100 \text{ fF}$.
- c) $C_{x \text{ max}} = 100 \text{ fF}$ since increasing C_x will reduce phase margin.

10.8 The approximation can be derived from the ideal case in which the circuit looks like the following:



At the zero, $V_{out} = 0$. and

$$\frac{-V_{in}}{R_Z + \frac{1}{s_z C_c}} = -g_{m9} V_{in}$$

$$\left(\frac{1}{g_{m9}} - R_Z \right)^{-1} = s_z C_c$$

$$\therefore s_z = \frac{1}{C_c (g_{m9}^{-1} - R_Z)}$$

$$\boxed{10.9} \quad \left(\frac{W}{L}\right)_{1-4} = \frac{50}{0.5} \quad I_{SS} = I_1 = 0.5 \text{ mA} \quad C_X = C_Y = 0.5 \text{ pF}$$

a)

$$\omega_{Px} \approx \frac{1}{C_X (r_{op3} \parallel r_{op2})}$$

$$\omega_{Py} \approx \frac{1}{C_Y \cdot (g_{m4}^{-1})}$$

In saturation :

$$r_o = \frac{1}{\lambda I_D}$$

$$\lambda_p = 0.2 \quad \lambda_n = 0.1 \quad \text{from Table 2.1}$$

$$r_{op3} = \frac{1}{0.2 (0.25 \text{ mA})} = 20 \text{ k}\Omega$$

$$r_{on2} = 2 r_{op3} = 40 \text{ k}\Omega$$

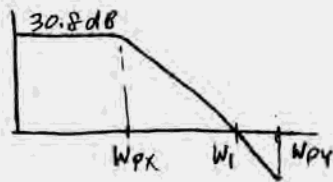
$$g_{m4} = \sqrt{2 I_D \mu_n C_{ox} \frac{W}{L}} = \sqrt{2 (0.5 \text{ mA}) (1.34 \times 10^{-4}) \left(\frac{50}{0.5}\right)} = \frac{1}{273} \text{ A/V}$$

$$g_{m4}^{-1} = 273.0 \Omega$$

$$\Rightarrow \omega_{Px} = 150 \times 10^6 \text{ rad/s}, \quad \omega_{Py} = 7.33 \times 10^9 \text{ rad/s} //$$

$$|\text{Low frequency gain}| \approx g_{m2} (r_{on2} \parallel r_{op2}) \cdot (1) \quad \left(g_{m2} = \frac{g_{m4}}{\sqrt{2}}\right)$$

$$\approx 34.5 \text{ V/V} \Rightarrow 30.8 \text{ dB} //$$



$$30.8 \text{ dB} - \left[\log \left(\frac{7.33 \times 10^9}{150 \times 10^6} \right) \right] (20 \text{ dB/dec}) = -3.78 \text{ dB}$$

$$\left(\log \frac{\omega_1}{150 \times 10^6} \right) (20 \text{ dB/dec}) = 30.8 \text{ dB}$$

$$\Rightarrow \omega_1 = 5.20 \times 10^9 \text{ rad/s} //$$

$$\phi = -\tan^{-1} \frac{\omega_1}{\omega_{Px}} - \tan^{-1} \frac{\omega_1}{\omega_{Py}} = -123.7^\circ$$

$$\Rightarrow \underline{\underline{\text{P.M.} = 56.3^\circ}}$$

b)

$$\phi = -\tan^{-1} \frac{\omega_1}{150 \times 10^6} - \tan^{-1} \frac{\omega_1}{\omega_{Py}} = -120^\circ$$

$$\text{If } \omega_1 \text{ is same as (a), then } \omega_{Py} = 8.43 \times 10^9 \text{ rad/s}$$

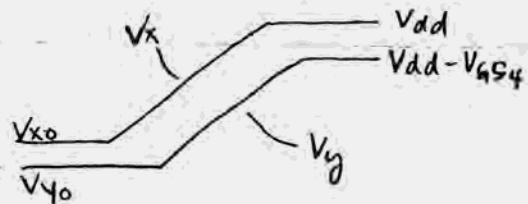
$$= \frac{1}{C_{Y \max} g_{m4}^{-1}}$$

$$\underline{\underline{C_{Y \max} = 434 \text{ fF}}}$$

10.10

For large positive step in V_{in} :

M_2 turns off. M_3 charges C_x and M_4 charges C_y .



(M_4 also provides I_1 .)

The slew rates of V_x and V_y (or V_{out}) must be exactly the same — regardless of C_x or C_y .

Hence slew rate due to positive step input $\cong \frac{I_{p3}}{C_x}$ for both parts (a) and (b) of 10.9.

$$\text{slew rate} \cong \frac{I_{p3}}{C_x} \cong \frac{0.25 \text{ mA}}{0.5 \text{ pF}} = 5.00 \times 10^8 \text{ V/s} //$$

For large negative step in V_{in} :

Again, V_{out} tracks V_x — as V_x drops, V_{out} drops at the same rate.

$$\text{slew rate} \cong -\frac{I_{cx}}{C_x}, \quad I_{p3} \cong 0.25 \text{ mA}, \quad I_{cx} \cong 0.5 \text{ mA} - I_{p3} = 0.25 \text{ mA}$$

$$\text{For both } C_y\text{'s}, \quad \text{slew rate} \cong -\frac{0.25 \text{ mA}}{0.5 \text{ pF}} = -5.00 \times 10^8 \text{ V/s} //$$

$$\boxed{10.11} \quad \left(\frac{W}{L}\right)_{5,6} = \frac{60}{0.5} \quad I_{SS} = 0.25 \text{ mA}$$

a) CM level $V_x = V_y = V_{DD} - V_{GS5} = V_{DD} - V_{GS6}$

$$I = 1 \text{ mA} = \frac{1}{2} \mu_p C_{ox} \left(\frac{60}{0.5}\right) (|V_{GS6}| - |V_{thp}|)^2$$

$$\underline{V_{GS6} = 1.46 \text{ V}} \rightarrow \underline{V_x = V_y = 3 - 1.46 = 1.54 \text{ V}}$$

b) Max. output swing:

$$V_{out, \max} = V_{DD} - V_{overdrive6} = 3 - (1.46 - 0.8) = \underline{2.34 \text{ V}}$$

$$V_{out, \min} = V_{overdrive8} = V_b - V_{thn} = \underline{0.39 \text{ V}}$$

$$(\text{since } V_b = 1.09 \text{ V from } 1 \text{ mA} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - 0.7)^2)$$

$$\text{Total max. swing} = 2.34 - 0.39 = \underline{1.95 \text{ V}}$$

$$c) \left. \begin{aligned} r_{on2} &= \frac{1}{(0.1)(0.125 \text{ mA})} = 80 \text{ k}\Omega \\ r_{op4} &= 40 \text{ k}\Omega \end{aligned} \right\} \rightarrow r_{on2} \parallel r_{op4} = \underline{26.67 \text{ k}\Omega}$$

$$r_{on8} = 10 \text{ k}\Omega, r_{op6} = 5 \text{ k}\Omega \rightarrow r_{on8} \parallel r_{op6} = \underline{3.33 \text{ k}\Omega}$$

$$g_{m2} = \sqrt{2(0.125 \text{ mA})(1.34 \times 10^{-4})\left(\frac{50}{0.5}\right)} = \underline{1.83 \times 10^{-3} \text{ A/V}}$$

$$g_{m6} = \mu_p C_{ox} \frac{W}{L} (V_{GS6} - V_{th}) = (3.83 \times 10^{-5}) \left(\frac{60}{0.5}\right) (1.46 - 0.8) = \underline{3.03 \times 10^{-3} \text{ A/V}}$$

$$A_{v2} = g_{m6} (3.33 \text{ k}\Omega) = \underline{-10.09 \text{ V/V}}$$

$$A_{v1} = g_{m2} (26.67 \text{ k}\Omega) = \underline{-48.8 \text{ V/V}} \rightarrow A_{v1}, A_{v2} = 492.4 \rightarrow \underline{53.8 \text{ dB}}$$

$$C_{out} = C_L + [C_{db6} + (1 + \frac{1}{|A_{v1}|}) C_{gd6}] + (C_{db8} + C_{gd8})$$

$$\cong 1 \text{ pF} + 52.1 + (1 + \frac{1}{10.09}) 0.18 + 23.4 + 0.2 = \underline{1.076 \text{ pF}}$$

$$C_Y = C_{gd4} + C_{db4} + C_{gd2} + C_{db2} + C_{gs6} + C_{gd6} (1 + |A_{v2}|)$$

$$\cong 0.15 + 43.8 + 0.2 + 23.4 + 76.6 + 0.18(11.09) = \underline{146.1 \text{ fF}}$$

Using:

$$\text{Overlap} = C_{GD0} \cdot W, \quad C_{db} = \frac{(CJ)(W \cdot 1.5 \mu\text{m})}{\left[1 - \frac{V_D}{\phi_B}\right]^{MS}} + \frac{CJSW(2W + 3\mu\text{m})}{\left[1 - \frac{V_D}{\phi_B}\right]^{MSW}}$$

(V_D = reverse bias junction voltage.)

10.11 c) cont. $C_{gs} = \frac{2}{3} C_{ox} W \cdot L$, Values from Table 2.1.

Before compensation:

$$\text{Dominant Pole: } \omega_p = \frac{1}{C_y R_y} = \frac{1}{(146.1 \text{ pF})(26.67 \text{ k}\Omega)} = \underline{2.57 \times 10^8 \text{ rad/s}}$$

$$\text{2nd Pole: } \omega_{out} = \frac{1}{C_{out} R_{out}} = \frac{1}{(1.076 \text{ pF})(3.33 \text{ k}\Omega)} = \underline{2.79 \times 10^8 \text{ rad/s}}$$

★ After compensation:

$$\text{2nd Pole: } \omega_{out}' \cong \frac{g_{m6}}{C_y + C_{out}} = \frac{3.03 \times 10^{-3}}{146.1 \text{ pF} + 1.076 \text{ pF}} = \underline{2.48 \times 10^9 \text{ rad/s}}$$

$$\text{Dominant Pole: } \omega_{p}' = \frac{1}{[C_y + (1 + |A_{v2}|) C_c] R_y}$$

(For 60° P.M.) $90^\circ + \tan^{-1} \frac{\omega_{p}'}{\omega_{out}'} = 120^\circ \rightarrow \omega_{p}' = \omega_{out}' \tan 30^\circ = \underline{1.43 \times 10^9 \text{ rad/s}}$

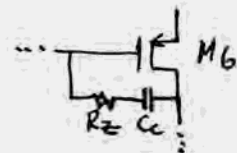
$$\log \frac{\omega_{p}'}{\omega_{p}} = \frac{53.8 \text{ dB}}{20 \text{ dB/dec}} \rightarrow \omega_{p}' = \frac{1}{10^{53.8/20}} \cdot \omega_p = \underline{2.91 \times 10^6 \text{ rad/s}}$$

$$C_c = \frac{[(2.91 \times 10^6)(26.67 \text{ k}\Omega)]^{-1} - 146.1 \text{ pF}}{1 + 10.08} = \underline{1.15 \text{ pF}}$$

$$\text{Zero: } \omega_z' = \frac{g_{m6}}{C_c + C_{gd6}} = \frac{3.03 \times 10^{-3}}{1.15 \text{ pF} + 0.18 \text{ pF}} = \underline{2.63 \times 10^9 \text{ rad/s}} \quad (\text{So } C_c \gg C_y) \quad (> \omega_{p}', \omega_{out}')$$

$$d) \omega_z = \frac{1}{C_c (\frac{1}{g_{m6}} + R_z)} = -|\omega_{out}'|$$

$$\rightarrow R_z = \frac{1}{g_{m6}} + \frac{1}{|\omega_{out}'| \cdot C_c} = \underline{680.7 \Omega}$$



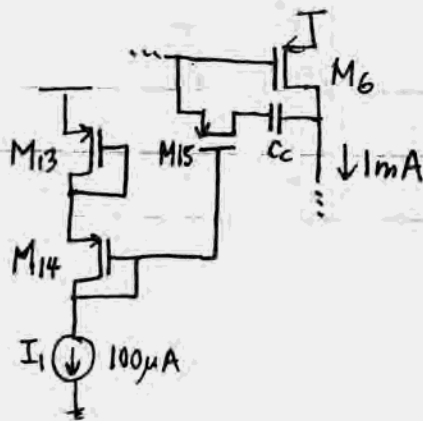
e) Slew rate: Symmetrical for large positive V_{in} or large negative V_{in} .

Large + V_{in} :

$$\text{Slew rate of } V_{out2} \cong -\frac{I_{D4}}{C_c} \cong -\frac{0.125 \text{ mA}}{1.15 \text{ pF}} = \underline{1.09 \times 10^8 \text{ V/s}}$$

$$\text{Slew rate of } V_{out1} = -(\text{slew rate of } V_{out2}) = \underline{1.09 \times 10^8 \text{ V/s}}$$

10.12



Want $|V_{gs13}| = |V_{gs6}| = 1.46V$ (from 10.11a)

$$100\mu A = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_{13} (|V_{gs6}| - |V_{tp}|)^2$$

$$\hookrightarrow \left(\frac{W}{L}\right)_{13} = \frac{2(100\mu A)}{(3.93 \times 10^{-5})(1.46 - 0.8)^2} = \underline{\underline{1200}} \quad \text{e.g. } \left(\frac{W}{L}\right)_{15} = \frac{6}{0.5}$$

Allowing 0.5V across I_1 and maximizing $V_{gs14} = V_{gs15}$,

We get $V_{gs14} = V_{gs15} = V_{DD} - 0.5 = 1.54 - 0.5 = \underline{\underline{1.04V}}$

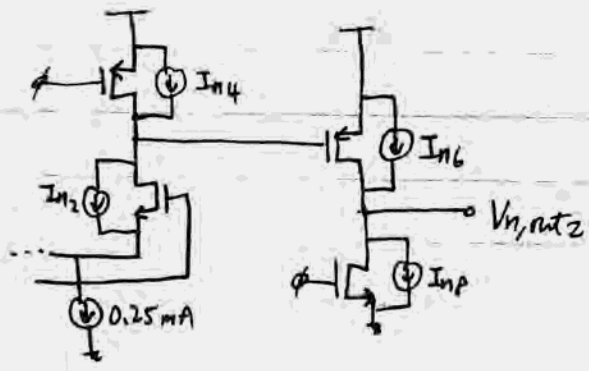
$$R_{on15} = \frac{1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_{15} (1.04 - 0.8)} = 680.7 \Omega$$

$$\hookrightarrow \left(\frac{W}{L}\right)_{15} = \underline{\underline{384}} \quad \text{e.g. } \frac{192}{0.5}$$

$$I_{D14} = 100\mu A = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_{14} (1.04 - 0.8)^2$$

$$\hookrightarrow \left(\frac{W}{L}\right)_{14} = \underline{\underline{90.7}} \quad \text{e.g. } \frac{45.5}{0.5}$$

10.13



$$\overline{I_n^2} = 4kT \frac{2}{3} g_m$$

$$V_{n,out2} = (I_{n2} + I_{n4})(r_{o2} || r_{o4}) \cdot A_{v2} + (I_{n6} + I_{n8})(r_{o6} || r_{o8})$$

$$\overline{V_{n,out2}^2} = (\overline{I_{n2}^2} + \overline{I_{n4}^2}) [(r_{o2} || r_{o4}) A_{v2}]^2 + (\overline{I_{n6}^2} + \overline{I_{n8}^2}) (r_{o6} || r_{o8})^2$$

$$\overline{V_{n,out1}^2} = (\overline{I_{n1}^2} + \overline{I_{n3}^2}) [(r_{o2} || r_{o4}) A_{v2}]^2 + (\overline{I_{n5}^2} + \overline{I_{n7}^2}) (r_{o6} || r_{o8})^2$$

$$\overline{V_{n,out}^2} = \overline{V_{n,out2}^2} + \overline{V_{n,out1}^2}$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{(A_{v1} A_{v2})^2}$$

$$= \left(\frac{1}{A_{v1} A_{v2}} \right)^2 \left\{ [(r_{o2} || r_{o4}) A_{v2}]^2 [\overline{I_{n1}^2} + \overline{I_{n2}^2} + \overline{I_{n3}^2} + \overline{I_{n4}^2}] + (r_{o6} || r_{o8})^2 (\overline{I_{n5}^2} + \overline{I_{n6}^2} + \overline{I_{n7}^2} + \overline{I_{n8}^2}) \right\}$$

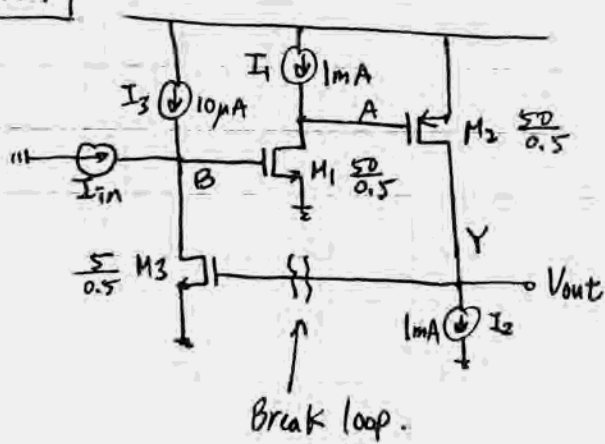
Aside

$$\left\{ \begin{aligned} g_{m1} &= g_{m2} = 1.83 \times 10^{-3} \text{ A/V} \\ g_{mP1} &= g_{m4} = \sqrt{2 I_{pp} \mu_n \frac{W}{L}} = \sqrt{2 (0.125 \text{ mA}) (3.83 \times 10^{-5}) \left(\frac{50}{0.5} \right)} = 9.79 \times 10^{-4} \text{ A/V} \\ g_{mN2} &= g_{m8} = 5.18 \times 10^{-3} \text{ A/V} ; & A_{v1} &= 48.8, & A_{v2} &= 10.09 \\ g_{mP2} &= g_{m6} = 3.03 \times 10^{-3} \text{ A/V} ; & r_{o2} || r_{o4} &= 26.67 \text{ k}\Omega, & r_{o6} || r_{o8} &= 3.33 \text{ k}\Omega \end{aligned} \right.$$

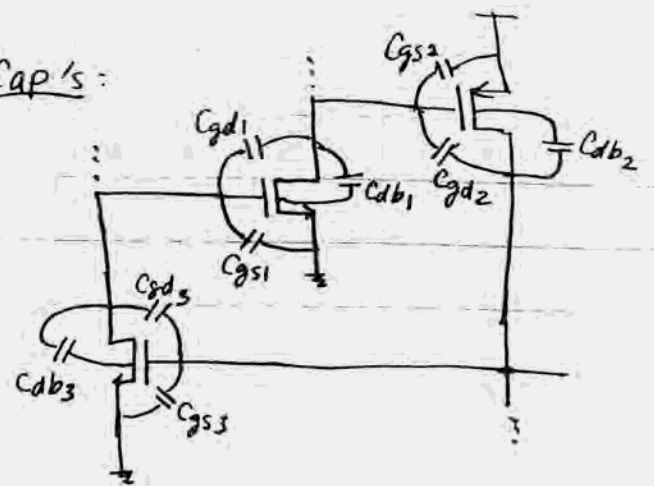
$$\overline{V_{n,in}^2} = 4kT \frac{2}{3} [1678.1 + 0.7511] = 4kT \frac{2}{3} [1678.85] \quad (4kT = 1.658 \times 10^{-20})$$

$$= \underline{\underline{1.86 \times 10^{-17} \text{ V}^2/\text{Hz}}}$$

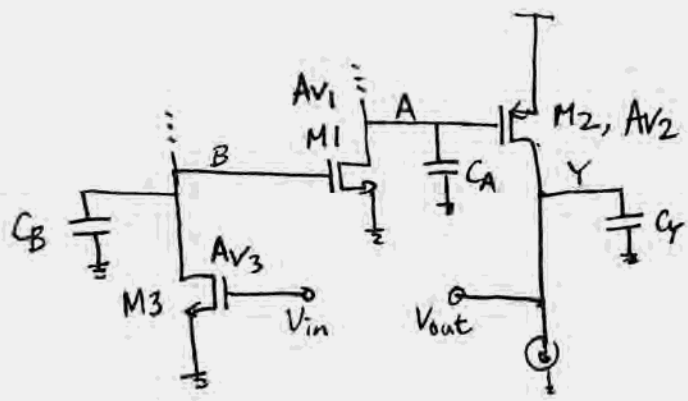
10.14



Cap's:



a)



$$C_B = C_{db3} + (1 + \frac{1}{|A_{v3}|}) C_{gd3} + C_{gs1} + (1 + |A_{v1}|) C_{gd1}$$

$$C_A = (1 + \frac{1}{|A_{v1}|}) C_{gd1} + C_{gs2} + (1 + |A_{v2}|) C_{gd2} + C_{db1}$$

$$C_Y = C_{db2} + (1 + \frac{1}{|A_{v2}|}) C_{gd2} + C_{gs3} + (1 + |A_{v3}|) C_{gd3}$$

$$r_{o1} = \frac{1}{(0.1)(1mA)} = 10 \text{ k}\Omega$$

$$r_{o2} = 5 \text{ k}\Omega$$

$$r_{o3} = 1 \text{ M}\Omega$$

Using capacitance formulas of (10.11), we get:

$$C_A = 80.4 \text{ fF}$$

$$C_B = 77.1 \text{ fF}$$

$$C_Y = 105.6 \text{ fF}$$

$$V_A = 1.48 \text{ V}$$

$$V_B = 1.09 \text{ V}$$

$$V_Y = 0.822 \text{ V}$$

Also,

$$g_{m1} = \sqrt{2(1mA)(1.34 \times 10^{-4})(\frac{50}{0.5})} = 5.18 \times 10^{-3} \text{ A/V}$$

$$g_{m2} = 2.77 \times 10^{-3} \text{ A/V}$$

$$g_{m3} = 1.64 \times 10^{-4} \text{ A/V}$$

$$A_{v1} = g_{m1} r_{o1} = -51.8 \text{ V/V}$$

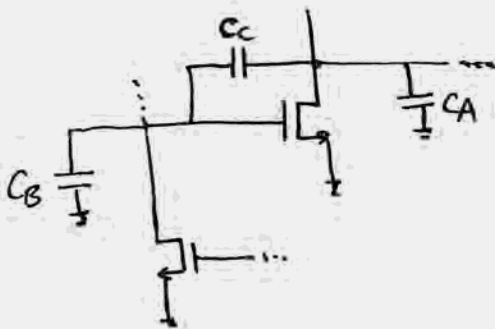
$$A_{v2} = g_{m2} r_{o2} = -13.85 \text{ V/V}$$

$$A_{v3} = g_{m3} r_{o3} = -164.0 \text{ V/V}$$

$$\left. \begin{array}{l} A_{v1} \\ A_{v2} \\ A_{v3} \end{array} \right\} \rightarrow |A_{v1} A_{v2} A_{v3}| = 1.18 \times 10^5 \rightarrow \underline{\underline{101.4 \text{ dB}}}$$

10.14 a) cont.

$$\begin{aligned} \omega_A &= \frac{1}{C_A r_{o1}} = 1.24 \times 10^9 \text{ rad/s} && \leftarrow \text{2nd} \\ \omega_B &= \frac{1}{C_B r_{o3}} = 1.30 \times 10^7 \text{ rad/s} && \leftarrow \text{Dominant} \\ \omega_Y &= \frac{1}{C_Y r_{o2}} = 1.89 \times 10^9 \text{ rad/s} && \leftarrow \text{3rd} \end{aligned}$$

(Phase Margin is $-78.3^\circ \rightarrow$ unstable system.)b) Compensate by adding C_c across G and D of M_1 .

$$\rightarrow \omega_A' \approx \frac{g_{m1}}{C_B + C_A} = \frac{5.18 \times 10^{-3}}{77.1 \text{ fF} + 80.4 \text{ fF}} = \underline{\underline{3.3 \times 10^{10} \text{ rad/s}}}$$

$$\rightarrow \omega_Y \text{ unchanged} = \underline{\underline{1.89 \times 10^9 \text{ rad/s}}}$$

$$90^\circ + \tan^{-1} \frac{\omega_1}{\omega_Y} = 120^\circ \quad (\text{For } 60^\circ \text{ P.M.})$$

$$\tan^{-1} \frac{\omega_1}{\omega_Y} = 30^\circ$$

$$\omega_1 = \underline{\underline{1.09 \times 10^9 \text{ rad/s}}}$$

$$\left(\log \frac{\omega_1}{\omega_B'} \right) \frac{20 \text{ dB}}{\text{dec}} = 101.4 \text{ dB}$$

$$\rightarrow \omega_B' = \underline{\underline{9.28 \times 10^3 \text{ rad/s}}}$$

Dominant = ω_B' , 2nd: ω_Y , 3rd: ω_A'

$$\omega_B' = \frac{1}{[C_B + (1 + |A_{v1}|) C_c] r_{o3}} \rightarrow C_c = \frac{1}{\omega_B' (1 \times 10^6) - 77.1 \text{ fF}} = \underline{\underline{2.04 \text{ pF}}}$$

$$\omega_Z' \approx \frac{g_{m1}}{C_c} = \frac{5.18 \times 10^{-3}}{2.04 \text{ pF}} = \underline{\underline{2.54 \times 10^9 \text{ rad/s}}} \quad (> \omega_Y)$$

10.14 c)

$$W_z = \frac{1}{C_c (g_{m1}^{-1} - R_z)} = -|W_y| = -1.89 \times 10^9$$

$$\hookrightarrow R_z = \frac{1}{g_{m1}} + \frac{1}{|W_y| C_c} = \underline{\underline{452.4 \Omega}}$$

10.15 a) Before compensation:

$$C_A = 80.4 \text{ fF}$$

$$C_B = 77.1 \text{ fF}$$

$$C_Y = 105.6 \text{ fF} + 0.5 \text{ pF} = 605.6 \text{ fF}$$

$$W_A = 1.24 \times 10^9 \text{ rad/s}$$

$$W_B = 1.30 \times 10^7 \text{ rad/s}$$

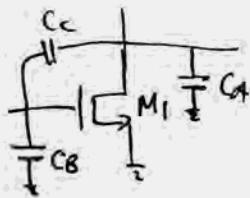
$$W_Y = \frac{1}{(605.6 \text{ fF})(5 \times 10^3)} = 3.30 \times 10^8 \text{ rad/s}$$

← 3rd

← Dominant

← 2nd

b) Two choices = We can put C_c across G & D of M_1 OR just add a C from G of M_1 to ground. Cannot take advantage of splitting 1st and 2nd pole here since 1st pole is at B and 2nd pole is at Y and the gain between those two nodes is > 0 .
Choose to put C_c across M_1 = splits 1st and 3rd poles, 2nd pole unchanged.



$$\text{3rd: } W'_A \cong \frac{g_{m1}}{C_A + C_B} = 3.30 \times 10^{10} \text{ rad/s}$$

$$\text{2nd: } W_Y = 3.30 \times 10^8 \text{ rad/s (unchanged)}$$

$$W'_1 = W_Y \cdot \tan 30^\circ = 1.91 \times 10^8 \text{ rad/s}$$

$$\text{Dominant: } W'_B = \frac{1}{10^{(101.4/20)}} \cdot W'_1 = 1.63 \times 10^3 \text{ rad/s}$$

$$C_c = \frac{1}{(1.63 \times 10^3)(1 \times 10^6)} - 77.1 \text{ fF} = \frac{52.8}{52.8} = \underline{\underline{11.6 \text{ pF}}}$$

10.15c)

$$\omega_z = \frac{1}{C_c(g_{m1}^{-1} - R_z)} = -|\omega_z| = -3.30 \times 10^8 \text{ rad/s}$$

$$\hookrightarrow R_z = \frac{1}{g_{m1}} + \frac{1}{|\omega_z| \cdot C_c}$$

$$R_z = \frac{1}{5.18 \times 10^{-3}} + \frac{1}{(3.3 \times 10^8)(11.6 \text{ pF})}$$

$$\underline{\underline{R_z = 454.3 \Omega}}$$

10.16

If M_1 turns off momentarily, I_1 causes a positive jump in voltage at node A. This causes M_2 to shut off momentarily so the slew rate is determined by I_2 and C_y .

$$i) \text{ Slew rate} = -\frac{I_2}{C_y} = -\frac{1 \text{ mA}}{105.6 \text{ fF}} = \underline{\underline{-9.47 \times 10^9 \text{ V/s}}}$$

(If unloaded)

$$ii) \text{ Slew rate} = -\frac{I_2}{C_y + C_L} = -\frac{1 \text{ mA}}{105.6 \text{ fF} + 0.5 \text{ pF}} = \underline{\underline{-1.65 \times 10^9 \text{ V/s}}}$$

(loaded.)

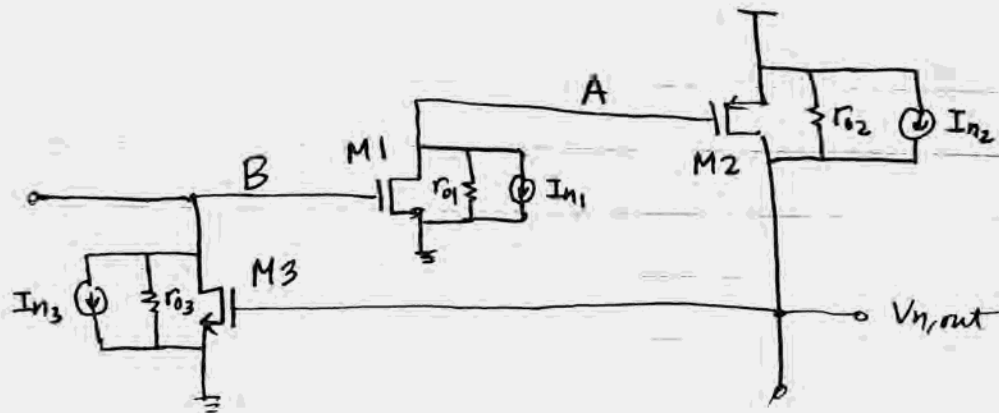
10.17

For problem 10.14, C_c should not be placed "across" M_2 or M_3 because of the location of the poles. Since the dominant pole was at node B, the 2nd pole at node A, and the 3rd pole at node Y, we need to split the 1st two poles by placing C_c across M_1 .

Putting C_c "across" M_2 only splits the 2nd and 3rd pole keeping the dominant pole unchanged. It moves the 2nd pole toward the dominant pole and the 3rd pole away. That cannot give a 60° phase margin.

Putting C_c "across" M_3 only affects the dominant pole and we cannot take advantage of pole-splitting to widen the bandwidth.

10.18



$$V_B = -r_{o3} (g_{m3} V_{n,out} + I_{n3})$$

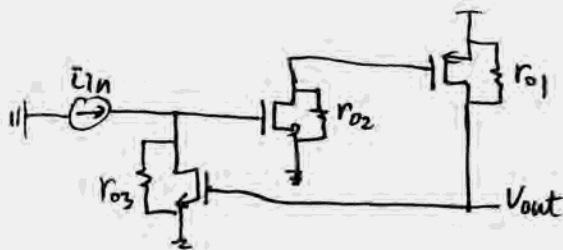
$$V_A = -r_{o1} (g_{m1} V_B + I_{n1}) = -r_{o1} (-g_{m1} g_{m3} r_{o3} V_{n,out} - g_{m1} r_{o3} I_{n3} + I_{n1})$$

$$V_{n,out} = -r_{o2} (g_{m2} V_A + I_{n2})$$

$$= -r_{o2} [I_{n2} + g_{m1} g_{m2} g_{m3} r_{o1} r_{o3} V_{n,out} + g_{m1} g_{m2} r_{o1} r_{o3} I_{n3} - g_{m2} r_{o1} I_{n1}]$$

$$\overline{V_{n,out}^2} = \frac{\overline{I_{n2}^2} + (g_{m1} g_{m2} r_{o1} r_{o3})^2 \overline{I_{n3}^2} + (g_{m2} r_{o1})^2 \overline{I_{n1}^2}}{(\frac{1}{r_{o2}} + g_{m1} g_{m2} g_{m3} r_{o1} r_{o3})^2}$$

Trans resistance of the circuit is :



$$\frac{V_{out}}{i_{in}} = \frac{1}{g_{m3} + [g_{m1} g_{m2} r_{o1} r_{o2} r_{o3}]^{-1}}$$

$$\star \text{ Input referred noise current: } \underline{\underline{\overline{i_{n,i}^2} = \frac{\overline{V_{n,out}^2}}{(\frac{V_{out}}{i_{in}})^2} = 7.25 \times 10^{-24} \text{ A}^2/\text{Hz}}}}$$

Aside { Since from (10.14) : $g_{m1} = 5.18 \times 10^{-3} \text{ A/V}$, $g_{m2} = 2.77 \times 10^{-3} \text{ A/V}$, $g_{m3} = 1.64 \times 10^{-4} \text{ A/V}$.
 $r_{o1} = 10 \text{ k}\Omega$, $r_{o2} = 5 \text{ k}\Omega$, $r_{o3} = 1 \text{ M}\Omega$.
 $\overline{I_{n2}^2} = 4kT \frac{2}{3} g_{m2} = 3.062 \times 10^{-23} \text{ A}^2/\text{Hz}$, $\overline{I_{n3}^2} = 1.813 \times 10^{-24} \text{ A}^2/\text{Hz}$,
 $\overline{I_{n1}^2} = 5.726 \times 10^{-23} \text{ A}^2/\text{Hz}$.

$$\boxed{10.19} \quad H_{open}(s) = \frac{A_0 \left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right)} \quad \omega_z \approx \omega_{p2}$$

a)

$$\begin{aligned} H_{closed}(s) &= \frac{A}{1 + A\beta} = \frac{A_0 \left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right) + A_0 \left(1 + \frac{s}{\omega_z}\right)} \\ &= \frac{A_0 \left(1 + \frac{s}{\omega_z}\right)}{\frac{s^2}{\omega_{p1}\omega_{p2}} + s \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \frac{A_0}{\omega_z}\right) + A_0 + 1} \quad \text{Q.E.D.} \end{aligned}$$

$$b) \quad D(s) = \left(1 + \frac{s}{\omega_{pA}}\right) \left(1 + \frac{s}{\omega_{pB}}\right) \approx 1 + \frac{s}{\omega_{pB}} + \frac{s^2}{\omega_{pA}\omega_{pB}} \quad (\omega_{pB} \ll \omega_{pA})$$

$$\omega_{pB} = \frac{A_0 + 1}{\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \frac{A_0}{\omega_z}} \quad //$$

$$\omega_{pA} = (1 + A_0) \omega_{p1} \omega_{p2} \cdot \frac{1}{\omega_{pB}} = \omega_{p2} + \omega_{p1} + \frac{A_0}{\omega_z} \omega_{p1} \omega_{p2} //$$

c) Using $\omega_z \approx \omega_{p2}$, $\omega_{p2} \ll (1 + A_0) \omega_{p1}$ or $\frac{1}{\omega_{p1}} \ll \frac{A_0 + 1}{\omega_{p2}}$

$$\omega_{pB} = \frac{A_0 + 1}{\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \frac{A_0}{\omega_z}} \approx \frac{A_0 + 1}{\frac{1}{\omega_{p1}} + \frac{A_0 + 1}{\omega_{p2}}} \approx \underline{\underline{\omega_{p2}}}$$

$$\omega_{pA} = \omega_{p2} + \omega_{p1} + \frac{A_0}{\omega_z} \omega_{p1} \omega_{p2} \approx \omega_{p2} + (A_0 + 1) \omega_{p1} \approx \underline{\underline{(A_0 + 1) \omega_{p1}}}$$

$$H_{closed}(s) \approx \frac{A_0 \left(1 + \frac{s}{\omega_z}\right)}{\left(1 + \frac{s}{(A_0 + 1) \omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right)} //$$

10.19 d) Step response: $Y(s)$

$$Y(s) = \frac{1}{s} \frac{A(1 + \frac{s}{w_z})}{(1 + \frac{s}{w_{PA}})(1 + \frac{s}{w_{PB}})}$$

$$A = \frac{A_0}{1+A_0}, \quad w_{PA} = (1+A_0)w_{P1}, \quad w_{PB} = w_{P2}$$

$$= \frac{K_1}{s} + \frac{K_2}{(1 + \frac{s}{w_{PA}})} + \frac{K_3}{(1 + \frac{s}{w_{PB}})}$$

$$= \frac{s^2 \left(\frac{K_1}{w_{PA} w_{PB}} + \frac{K_2}{w_{PB}} + \frac{K_3}{w_{PA}} \right) + s \left(\frac{K_1}{w_{PA}} + \frac{K_1}{w_{PB}} + K_2 + K_3 \right) + K_1}{s \left(1 + \frac{s}{w_{PA}} \right) \left(1 + \frac{s}{w_{PB}} \right)}$$

$$\boxed{K_1 = A}$$

$$\frac{A}{w_{PA}} + \frac{A}{w_{PB}} + K_2 + K_3 = \frac{A}{w_z} \longrightarrow K_2 + K_3 = A \left(\frac{1}{w_z} - \frac{1}{w_{PA}} - \frac{1}{w_{PB}} \right) \quad \dots (1)$$

$$\frac{A}{w_{PA} w_{PB}} + \frac{K_2}{w_{PB}} + \frac{K_3}{w_{PA}} = 0 \longrightarrow w_{PA} K_2 + w_{PB} K_3 = -A \quad \dots (2)$$

$$- (1) \times w_{PA} + (2) \longrightarrow - \left[w_{PA} K_2 + w_{PA} K_3 = A \left(\frac{w_{PA}}{w_z} - 1 - \frac{w_{PA}}{w_{PB}} \right) \right]$$

$$+ w_{PA} K_2 + w_{PB} K_3 = -A$$

$$\hline K_3 (w_{PB} - w_{PA}) = A \left(\frac{w_{PA}}{w_{PB}} - \frac{w_{PA}}{w_z} \right)$$

$$K_3 = \frac{A \left(\frac{w_{PA}}{w_{PB}} - \frac{w_{PA}}{w_z} \right)}{w_{PB} - w_{PA}}$$

Plug back into (2) to get K_2 :

$$w_{PA} K_2 + w_{PB} \left[\frac{A w_{PA} \left(\frac{1}{w_{PB}} - \frac{1}{w_z} \right)}{w_{PB} - w_{PA}} \right] = -A$$

$$w_{PA} K_2 = -A \left[1 + \frac{w_{PA} - \frac{w_{PA} w_{PB}}{w_z}}{w_{PB} - w_{PA}} \right] = -A \cdot \frac{w_{PB} - \frac{w_{PA} w_{PB}}{w_z}}{w_{PB} - w_{PA}}$$

$$K_2 = -A \frac{w_{PB} \left(1 - \frac{w_{PA}}{w_z} \right)}{w_{PA} (w_{PB} - w_{PA})}$$

10.19 d) (cont)

can simplify:

$$K_3 = \frac{A W_{PA} \left(\frac{1}{W_{PB}} - \frac{1}{W_Z} \right)}{W_{PB} - W_{PA}} \cong \frac{\frac{A_0}{A_0+1} (A_0+1) W_{P1} \left(\frac{1}{W_{P2}} - \frac{1}{W_Z} \right)}{W_{P2} - (A_0+1) W_{P1}}$$

$$\cong \underline{\underline{-\frac{A_0}{A_0+1} \left(\frac{1}{W_{P2}} - \frac{1}{W_Z} \right) = K_3}}$$

$$K_2 = -A \frac{W_{P2} \left(1 - \frac{(A_0+1) W_{P1}}{W_Z} \right)}{(A_0+1) W_{P1} (W_{P2} - (A_0+1) W_{P1})} \cong \frac{-A_0}{A_0+1} \frac{-(A_0+1) W_{P1}}{-(A_0+1)^2 W_{P1}^2}$$

$$\cong \underline{\underline{\frac{-A_0}{(A_0+1)^2 W_{P1}} = K_2}}$$

$$Y(s) = \frac{A}{s} + \frac{\frac{-A_0}{(A_0+1)^2 W_{P1}} \cdot W_{PA}}{W_{PA} + s} + \frac{\frac{-A_0}{A_0+1} \left(\frac{1}{W_{P2}} - \frac{1}{W_Z} \right) \cdot W_{PB}}{s + W_{PB}}$$

$$\cong \frac{A}{s} + \frac{\frac{-A_0}{A_0+1}}{s + (A_0+1) W_{P1}} + \frac{\frac{-A_0}{A_0+1} \left(1 - \frac{W_{P2}}{W_Z} \right)}{s + W_{P2}}$$

$$\star y(t) = \frac{A_0}{1+A_0} \left[1 - \left(1 - \frac{W_{P2}}{W_Z} \right) e^{-W_{P2}t} - e^{-(A_0+1)W_{P1}t} \right] u(t)$$

= small signal step response.

$$\star y(t) \cong \frac{A_0}{1+A_0} \left[1 - \left(1 - \frac{W_{P2}}{W_Z} \right) e^{-W_{P2}t} \right] u(t) \text{ since } (1+A_0)W_{P1} \gg W_{P2}.$$

Hence if W_Z and W_{P2} do not exactly cancel, there is an exponential term $\left(1 - \frac{W_{P2}}{W_Z} \right) e^{-W_{P2}t}$ with a time constant $\frac{1}{W_{P2}} \approx \frac{1}{W_Z}$. Q.E.D.

10.20

a) Perfect pole-zero cancellation.

Then

$$y(t) \cong \frac{A_0}{1+A_0} [1 - 0 - e^{-\omega_{p1}(A_0+1)t}] u(t)$$

$$\cong \frac{A_0}{1+A_0} u(t)$$

\Rightarrow Step. 

b) 10% mismatch.

$$y(t) \cong \frac{A_0}{1+A_0} [1 - 0.9 e^{-\omega_{p2}t}] u(t)$$

\Rightarrow  $\tau = \frac{1}{\omega_{p2}}$

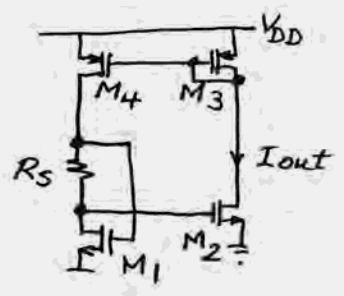
Chapter 11

11.1 Assuming all transistors are in saturation, we have

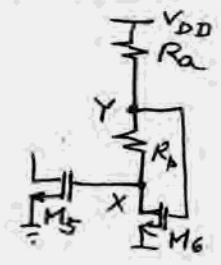
$$I_{out} R_s + \sqrt{\frac{2 I_{out}}{\mu_n C_{ox} (\frac{W}{L})_2}} + V_{TH2} = \sqrt{\frac{2 I_{out}}{\mu_n C_{ox} (\frac{W}{L})_1}} + V_{TH1}$$

where we have assumed $(\frac{W}{L})_4 = (\frac{W}{L})_3$ and $\lambda = 0$.

Thus,
$$I_{out} = \frac{1}{\mu_n C_{ox} R_s^2} \left(\sqrt{(\frac{L}{W})_1} - \sqrt{(\frac{L}{W})_2} \right)^2$$



11.2 When the circuit turns on, initially both M5 and M6 are off and V_x and V_y rise together, i.e., $V_x = V_y$. When V_y reaches V_{TH6} , V_x is also near V_{TH5} . Thus, M6 and M5 turn on almost simultaneously.



The surge in the drain current of M5 turns the rest of the circuit on. As V_y increases further, V_x begins to drop if M6 is turned on sufficiently because the voltage gain of M6 and R_b exceeds unity. For high values of V_y , V_x can be lower than V_{TH5} .

Since $(V_{DD} - I_{D6} \cdot R_a - V_{TH})^2 \cdot \mu_n C_{ox} (\frac{W}{L})_6 = I_{D6}$, we solve the quadratic equation:

$$R_a^2 I_{D6}^2 - I_{D6} \left(2 R_a \frac{(V_{DD} - V_{TH})}{\mu_n C_{ox} (\frac{W}{L})_6} \right) + (V_{DD} - V_{TH})^2 = 0$$

$$\Rightarrow I_{D6} = \frac{2 R_a (V_{DD} - V_{TH}) + \frac{1}{\mu_n C_{ox} (\frac{W}{L})_6} + \sqrt{\left[2 R_a (V_{DD} - V_{TH}) + \frac{1}{\mu_n C_{ox} (\frac{W}{L})_6} \right]^2 - 4 R_a^2 (V_{DD} - V_{TH})^2}}{2 R_a^2}$$

This value is substituted in the other condition:

$$V_{DD} - I_{D6} (R_a + R_b) \leq V_{TH5}$$

to give the condition for turning off M5.

11.3 (a) Since the output voltage is near 2.5V whereas $V_X \approx 2V_{BE}$,

$$\begin{aligned} \text{we write } \frac{I_{D1}}{I_{D2}} &\approx \frac{1 + \lambda (V_{DD} - 2V_{BE})}{1 + \lambda (V_{DD} - 2.5V)} \\ &\approx 1 + \lambda (2.5V - 2V_{BE}) \end{aligned}$$

$$\begin{aligned} \Rightarrow V_{BE2} - V_{BE4} &= V_T \ln n + V_T \ln \frac{I_{D1}}{I_{D2}} \quad \ln(1 + \epsilon) \approx \epsilon \\ &= V_T \ln n + V_T \lambda (2.5V - 2V_{BE}) \end{aligned}$$

The error $V_T \lambda (2.5V - 2V_{BE})$ directly appears in V_{out} .

This error is also divided by R_1 and multiplied by R_2 , giving another error component at the output. So the overall

error is equal to $(1 + \frac{R_2}{R_1}) V_T \lambda (2.5V - 2V_{BE})$.

$$\begin{aligned} (b) \quad \frac{I_{D3}}{I_{D4}} &\approx \frac{1 + \lambda (V_{DD} - V_{BE1})}{1 + \lambda (V_{DD} - V_{BE1} + V_T \ln n)} \\ &\approx 1 - \lambda V_T \ln n \end{aligned}$$

$$\begin{aligned} \text{The output error is then equal to } &V_T \ln(1 - \lambda V_T \ln n) \\ &\approx -V_T^2 \lambda \ln n. \end{aligned}$$

$$(c) \quad V_{TH1} = V_{TH}, \quad V_{TH2} = V_{TH} + \Delta V_{TH}$$

For small V_{TH} , we have $I_{D2} = I_{D1} + g_m \Delta V_{TH}$, where g_m is the mean transconductance of M_1 and M_2 . Thus,

$$\frac{I_{D1}}{I_{D2}} = 1 - \frac{g_m \Delta V_{TH}}{I_{D2}} = 1 - \frac{2 \Delta V_{TH}}{|V_{GS} - V_{TH}|_2}$$

$$\text{part (a), we have: output error} = (1 + \frac{R_2}{R_1}) (-V_T) \frac{2 \Delta V_{TH}}{|V_{GS} - V_{TH}|_2}$$

$$(d) \quad \frac{I_{D3}}{I_{D4}} = 1 - \frac{2 \Delta V_{TH}}{|V_{GS} - V_{TH}|_4} \Rightarrow \text{output error} = -V_T \cdot \frac{2 \Delta V_{TH}}{|V_{GS} - V_{TH}|_4}$$

11.4 $-V_{xy} \cdot A_1 = V_{DD} - |V_{GS2}|$ $|V_{GS2}| = \sqrt{\frac{2(V_T \ln n) / R_1}{\mu_n C_{ox} (\frac{W}{L})_2}} + |V_{TH2}|$

$A_1 \geq \left[V_{DD} - \sqrt{\frac{2(V_T \ln n) / R_1}{\mu_n C_{ox} (\frac{W}{L})_2}} - |V_{TH2}| \right] / (-V_e)$

11.5 The collector current of Q_4 is less than its emitter current.

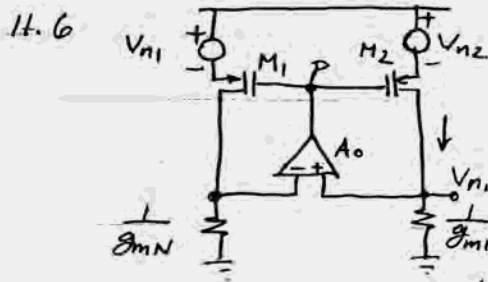
Thus, the current thru R_1 and R_2 is given by

$\frac{(V_T \ln n)}{R_1} \times \frac{\beta+1}{\beta}$, and hence the output has an

error equal to $\frac{1}{\beta} \frac{V_T \ln n}{R_1} R_2$.

Another source of error is the flow of base currents of Q_2 and Q_4 from M_3 and M_4 , respectively. That is, $|V_{BE1}|$ and $|V_{BE3}|$ are slightly less than the predicted value.

error = $V_T \ln \frac{\beta}{\beta+1}$.



For the noise due to M_1 :

$\frac{V_{n,out}}{R_1 + g_{mN}^{-1}} \cdot \frac{1}{g_{mp}} = V_p$, $(\frac{V_p}{A_o} + V_{n,out}) g_{mN} = |I_{D1}|$

$(\frac{V_{n,out}}{R_1 + g_{mN}^{-1}} \cdot \frac{1}{g_{mp}} \cdot \frac{1}{A_o} + V_{n,out}) g_{mN} (\frac{1}{g_{mp}}) = \frac{V_{n,out}}{R_1 + g_{mN}^{-1}} \cdot \frac{1}{g_{mp}} + V_{n1}$

$\Rightarrow V_{n,out} = V_{n1} \frac{1}{(\frac{1}{R_1 + g_{mN}^{-1}} \cdot \frac{1}{g_{mp}} \cdot \frac{1}{A_o} + 1) \frac{g_{mN}}{g_{mp}} - \frac{1}{(R_1 + g_{mN}^{-1})} \cdot \frac{1}{g_{mp}}}$

where $\overline{V_{n1}^2} = 4kT (\frac{2}{3g_{mp}}) + \frac{K_{FJP}}{WLC_{ox} f}$

For the noise due to M_2 :

$$\begin{cases} \frac{V_{n,out}}{R_1 + g_{mN}^{-1}} \cdot \frac{1}{g_{mP}} + V_{n2} = V_p \\ \left(\frac{V_p}{A_o} + V_{n,out} \right) g_{mN} = |I_{D1}| \end{cases}$$

$$\Rightarrow \left[\left(\frac{V_{n,out}}{R_1 + g_{mN}^{-1}} \cdot \frac{1}{g_{mP}} + V_{n2} \right) \frac{1}{A_o} + V_{n,out} \right] g_{mN} \left(\frac{1}{g_{mP}} \right) = \frac{V_{n,out}}{R_1 + g_{mN}^{-1}} \cdot \frac{1}{g_{mP}} + V_{n2}$$

$$\Rightarrow V_{n,out} \left[\frac{1}{(R_1 + g_{mN}^{-1}) g_{mP} A_o} + 1 \right] \times \frac{g_{mN}}{g_{mP}} \frac{1}{R_1 + g_{mN}^{-1}} \cdot \frac{1}{g_{mP}} = V_{n2} \left[1 - \frac{g_{mN}}{A_o g_{mP}} \right]$$

$$\Rightarrow V_{n,out} = V_{n2} \frac{1 - \frac{g_{mN}}{g_{mP}}}{\frac{g_{mN}/g_{mP}}{(R_1 + g_{mN}^{-1}) g_{mP} A_o} + \frac{g_{mN}}{g_{mP}} - \frac{1}{R_1 + g_{mN}^{-1}} \cdot \frac{1}{g_{mP}}}$$

where $V_{n2}^2 = 4kT \left(\frac{2}{3g_{mP}} \right) + \frac{K_{F,P}}{WL C_{ox}} \cdot \frac{1}{f}$

The overall noise is obtained by adding the noise powers.

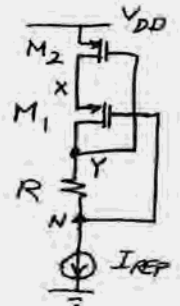
11.7 For M_1 to be in saturation, $R I_{REF} \leq |V_{TH1}|$.

For M_2 to be in saturation,

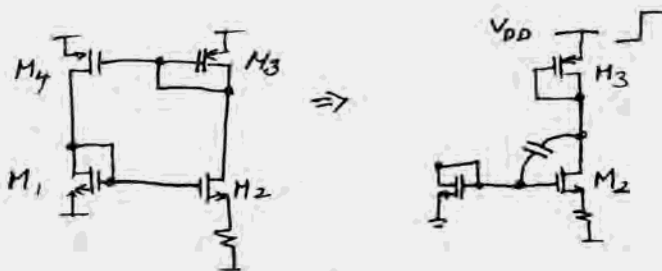
$$V_N + |V_{GS1}| \leq \underbrace{V_{DD} - |V_{GS2}|}_{V_Y = V_N + R I_{REF}} + |V_{TH2}|$$

$$\Rightarrow |V_{GS1}| \leq R I_{REF} + |V_{TH2}|$$

$$\Rightarrow |V_{GS1}| - |V_{TH2}| \leq R I_{REF} \leq |V_{TH1}|$$



11.8



When V_{DD} rises, M_3 turns on because the gate-drain overlap capacitance of M_2 must charge. The current flowing thru this capacitance may increase the gate voltage of M_2 sufficiently, turning this transistor on as well. When M_3 turns on, M_4 also turns on.

$$11.9 \quad \frac{\partial V_{BE}}{\partial T} = \frac{V_{BE} - E_g/q}{T} - (4+m) \frac{k}{q}$$

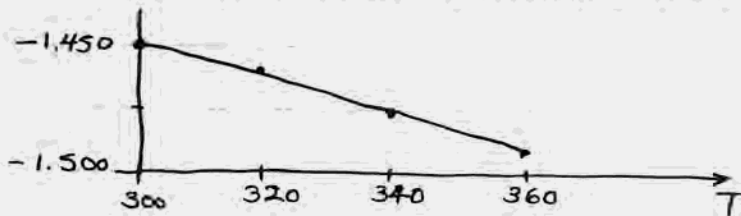
As T increased, V_{BE} drops. Thus, the TC becomes more negative. We can sketch the behavior by a piecewise linear approximation.

$$T = 300^\circ\text{K}, V_{BE} \approx 750 \text{ mV} \Rightarrow TC = -1.45 \text{ mV}/^\circ\text{K}$$

$$T = 320^\circ\text{K}, V_{BE} \approx 750 - 20(1.45) = 721 \text{ mV} \Rightarrow TC = -1.46 \text{ mV}/^\circ\text{K}$$

$$T = 340^\circ\text{K}, V_{BE} \approx 721 - 20(1.46) = 692 \text{ mV} \Rightarrow TC = -1.476 \text{ mV}/^\circ\text{K}$$

$$T = 360^\circ\text{K}, V_{BE} \approx 692 - 20(1.476) = 662 \text{ mV} \Rightarrow TC = -1.489 \text{ mV}/^\circ\text{K}$$



$$11.10 \quad \frac{\partial^2 V_{BE}}{\partial T^2} = \frac{(\partial V_{BE}/\partial T)T - (V_{BE} - \frac{E_g}{q})}{T^2} = \frac{1}{T} \frac{\partial V_{BE}}{\partial T} - \frac{1}{T^2} (V_{BE} - \frac{E_g}{q})$$

$$= \frac{V_{BE} - (4+m)V_T - E_g/q}{T^2} - \frac{1}{T^2} V_{BE} + \frac{1}{T^2} \frac{E_g}{q}$$

$$= -(4+m) \frac{V_T}{T^2} = -(4+m) \frac{k}{q} \cdot \frac{1}{T}$$



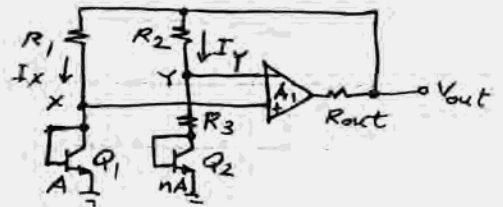
$$11.11 \quad V_Y - V_X = R_3 I_Y - V_T \ln n$$

$$\Rightarrow -A_1 (R_3 I_Y - V_T \ln n) - R_{out} (2 I_Y) = V_{out}$$

Assumed $I_X \approx I_Y$ here.

$$\text{We also note that: } I_Y = \frac{V_{out} - V_{BE2}}{R_2 + R_3}$$

$$\text{Thus, } V_{out} = \frac{(R_2 + R_3) (V_T \ln n) A_1 + (A_1 R_3 + 2R_{out}) V_{BE2}}{R_2 + 2R_{out} + A_1 R_3 + R_3}$$



Dividing the numerator and denominator by $A_1 R_3$ and assuming $\frac{R_3 + R_2 + 2R_{out}}{A_1 R_3} \ll 1$, we have:

$$V_{out} \approx \left[\left(1 + \frac{R_2}{R_3}\right) V_T \ln n + V_{BE} + \left(\frac{2R_{out}}{A_1 R_3} V_{BE}\right) \right] \left(1 - \frac{R_3 + R_2 + 2R_{out}}{A_1 R_3}\right)$$

The error is then equal to:

$$\frac{2R_{out}}{A_1 R_3} V_{BE} - \frac{R_3 + R_2 + 2R_{out}}{A_1 R_3} \left[\left(1 + \frac{R_2}{R_3}\right) V_T \ln n + V_{BE} \right]$$

11-12 $R_3 = 1 \text{ k}\Omega$ $I_{R_3} = 50 \mu\text{A}$ $R_1 = R_2$

$$V_{out} = V_{BE2} + (V_T \ln n) \left(1 + \frac{R_2}{R_3}\right) = 1.25 \text{ V}, \quad V_{BE2} \approx 750 \text{ mV}$$

$$I_{R_3} = \frac{V_{out} - V_{BE2}}{R_2 + R_3} = \frac{(V_T \ln n) \left(1 + \frac{R_2}{R_3}\right)}{R_2 + R_3} = 50 \mu\text{A}$$

$$\left. \begin{aligned} (\ln n) \left(1 + \frac{R_2}{R_3}\right) &\approx 17.2 \\ \Rightarrow R_2 &= 7.944 \text{ k}\Omega \\ \Rightarrow n &\approx 6.84 \end{aligned} \right\}$$

Some iteration is usually necessary to arrive at an integer n . (Of course, the current thru R_3 will be slightly different from $50 \mu\text{A}$.)

11-13 $I_{C1} = I_{C2} = 100 \mu\text{A}$ $I_{C3} = I_{C4} = 50 \mu\text{A}$ $R_1 = 1 \text{ k}\Omega$.

V_{DD} must be equal to 3 V.

Since $V_{out} \approx 2.5 \text{ V}$, M_2 and hence M_1 must be sized such that they remain in saturation.

$$\begin{cases} V_{BE3} + V_{BE4} + (1 + \frac{R_2}{R_1}) (2V_T) \ln \left(\frac{I_{C2}}{I_{C3}} n\right) \approx 2.5 \text{ V} \\ V_{out} - (V_{BE4} + V_{BE3}) = (50 \mu\text{A})(R_1 + R_2) \end{cases}$$

The two unknowns here are R_2 and n . Since Q_3 and Q_4 are biased at a relatively low current, we assume

$$V_{BE3} = V_{BE4} \approx 700 \text{ mV} \Rightarrow \left(1 + \frac{R_2}{R_1}\right) (2V_T) \ln(nn) \approx 1.8 \text{ V}$$

From the second equation, $R_1 + R_2 \approx 36 \text{ k}\Omega$, $\Rightarrow R_2 = 35 \text{ k}\Omega$.

Dividing the numerator and denominator by $A_1 R_3$ and assuming $\frac{R_3 + R_2 + 2R_{out}}{A_1 R_3} \ll 1$, we have:

$$V_{out} \approx \left[\left(1 + \frac{R_2}{R_3}\right) V_T \ln n + V_{BE} + \left(\frac{2R_{out}}{A_1 R_3} V_{BE}\right) \right] \left(1 - \frac{R_3 + R_2 + 2R_{out}}{A_1 R_3}\right)$$

The error is then equal to:

$$\frac{2R_{out}}{A_1 R_3} V_{BE} - \frac{R_3 + R_2 + 2R_{out}}{A_1 R_3} \left[\left(1 + \frac{R_2}{R_3}\right) V_T \ln n + V_{BE} \right]$$

11.12 $R_3 = 1 \text{ k}\Omega$ $I_{R_3} = 50 \mu\text{A}$ $R_1 = R_2$

$$V_{out} = V_{BE2} + (V_T \ln n) \left(1 + \frac{R_2}{R_3}\right) = 1.25 \text{ V}, \quad V_{BE2} \approx 750 \text{ mV}$$

$$I_{R_3} = \frac{V_{out} - V_{BE2}}{R_2 + R_3} = \frac{(V_T \ln n) \left(1 + \frac{R_2}{R_3}\right)}{R_2 + R_3} = 50 \mu\text{A}$$

$$\left. \begin{aligned} (\ln n) \left(1 + \frac{R_2}{R_3}\right) &\approx 17.2 \\ \Rightarrow R_2 &= 7.944 \text{ k}\Omega \\ \Rightarrow n &\approx 6.84. \end{aligned} \right\}$$

Some iteration is usually necessary to arrive at an integer

n . (Of course, the current thru R_3 will be slightly different from $50 \mu\text{A}$.)

11.13 $I_{C1} = I_{C2} = 100 \mu\text{A}$ $I_{C3} = I_{C4} = 50 \mu\text{A}$ $R_1 = 1 \text{ k}\Omega$.

V_{DD} must be equal to 3 V .

Since $V_{out} \approx 2.5 \text{ V}$, M_2 and hence M_1 must be sized such that they remain in saturation.

$$\begin{cases} V_{BE3} + V_{BE4} + \left(1 + \frac{R_2}{R_1}\right) (2V_T) \ln \left(\frac{I_{C2}}{I_{C3}} n\right) \approx 2.5 \text{ V} \\ V_{out} - (V_{BE4} + V_{BE3}) = (50 \mu\text{A})(R_1 + R_2) \end{cases}$$

The two unknowns here are R_2 and n . Since Q_3 and Q_4 are biased at a relatively low current, we assume

$$V_{BE3} = V_{BE4} \approx 700 \text{ mV} \Rightarrow \left(1 + \frac{R_2}{R_1}\right) (2V_T) \ln(mn) \approx 1.8 \text{ V}$$

From the second equation, $R_1 + R_2 \approx 36 \text{ k}\Omega$, $\Rightarrow R_2 = 35 \text{ k}\Omega$.

11.16 $g_m \propto T^{-3/4}$ Thus, $\frac{\partial R}{\partial T} = T^{-3/4}$

11.17 The current thru R_1 is PTAT

and $V_x = V_y = V_{BE1} / R_3$.

The current thru each PNPES

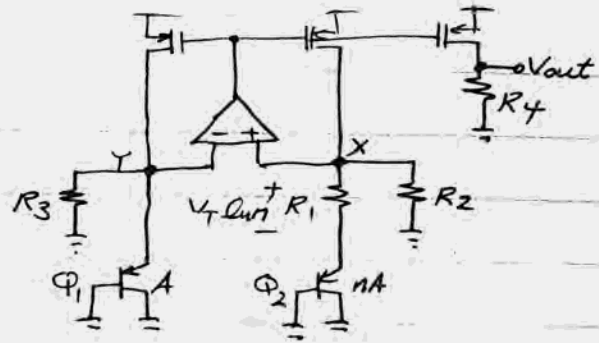
device is $\frac{V_T \ln n}{R_1} + \frac{V_{BE1}}{R_3}$

and hence

$$V_{out} = \frac{R_4}{R_3} \left(\frac{V_T \ln n}{R_1} + \frac{V_{BE1}}{R_3} \right)$$

$$= \frac{R_4}{R_3} V_{BE1} + \frac{R_4}{R_1} V_T \ln n.$$

Since V_{BE1} is multiplied by R_4/R_3 , the output voltage can be arbitrarily scaled.



11.18 $V_x = V_y - V_{os} \Rightarrow V_{REF} = V_{BE1} \frac{R_4}{R_3} + \frac{R_4}{R_1} V_T \ln n$

$$- \frac{R_4}{R_3} \left(1 + \frac{R_2}{R_1} \right) V_{os}$$

11.19 (a) When S_1 is on and S_2 is off, $V_{out} \approx V_T \ln \frac{I_1}{I_{S1}}$.

(b) when S_1 turns off and S_2 turns on,

$V_x = V_T \ln \frac{I_1 + I_2}{I_{S1}}$. This change is amplified

by $\frac{C_2 + 1}{C_1}$ and added to the original voltage across

C_2 : $V_{out} = \left(1 + \frac{C_2}{C_1} \right) \left(V_T \ln \frac{I_1 + I_2}{I_{S1}} - V_T \ln \frac{I_1}{I_{S1}} \right)$

$+ V_T \ln \frac{I_1}{I_{S1}}$

$\underbrace{\hspace{2cm}}_{V_{BE}}$

$= \left(1 + \frac{C_2}{C_1} \right) V_T \ln \left(1 + \frac{I_2}{I_1} \right) + V_{BE}$

$= \left(1 + \frac{C_2}{C_1} \right) V_T \ln m + V_{BE}$

11.16 $g_m \propto T^{-3/4}$ Thus, $\frac{\partial g_m}{\partial T} = -\frac{3}{4} \frac{g_m}{T}$.

11.17 The current thru R_1 is PTAT

and $V_x = V_y = V_{BE1} / R_3$.

The current thru each PNP

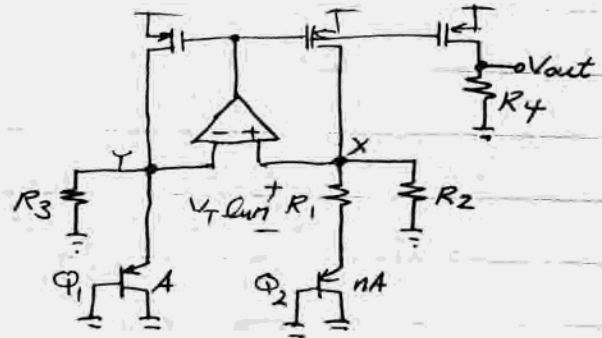
device is $\frac{V_T \ln n}{R_1} + \frac{V_{BE1}}{R_3}$

and hence

$$V_{out} = R_4 \left(\frac{V_T \ln n}{R_1} + \frac{V_{BE1}}{R_3} \right)$$

$$= \frac{R_4}{R_3} V_{BE1} + \frac{R_4}{R_1} V_T \ln n.$$

Since V_{BE1} is multiplied by R_4/R_3 , the output voltage can be arbitrarily scaled.



11.18 $V_x = V_y - V_{os} \Rightarrow V_{REF} = V_{BE1} \frac{R_4}{R_3} + \frac{R_4}{R_1} V_T \ln n$

$$- \frac{R_4}{R_3} \left(1 + \frac{R_2}{R_1} \right) V_{os}$$

11.19 (a) When S_1 is on and S_2 is off, $V_{out} \approx V_T \ln \frac{I_1}{I_{S1}}$.

(b) when S_1 turns off and S_2 turns on,

$V_x = V_T \ln \frac{I_1 + I_2}{I_{S1}}$. This change is amplified by $\frac{C_2 + 1}{C_1}$ and added to the original voltage across

C_2 : $V_{out} = \left(1 + \frac{C_2}{C_1} \right) \left(V_T \ln \frac{I_1 + I_2}{I_{S1}} - V_T \ln \frac{I_1}{I_{S1}} \right)$

$$+ \underbrace{V_T \ln \frac{I_1}{I_{S1}}}_{V_{BE}}$$

$$= \left(1 + \frac{C_2}{C_1} \right) V_T \ln \left(1 + \frac{I_2}{I_1} \right) + V_{BE}$$

$$= \left(1 + \frac{C_2}{C_1} \right) V_T \ln m + V_{BE}$$

cc) for zero TC: $(1 + \frac{C_2}{C_1}) \ln(1 + \frac{I_2}{I_1}) \approx 17.2$.

11.20 $V_{out} = (1 + \frac{C_2}{C_1}) V_T \ln(1 + \frac{I_2}{I_1}) + V_{BE}$

If $\frac{I_2}{I_1} = N + \epsilon \Rightarrow V_{out} = (1 + \frac{C_2}{C_1}) V_T \ln(1 + N + \epsilon) + V_{BE}$

$= (1 + \frac{C_2}{C_1}) V_T [\ln(1 + N) + \ln(1 + \frac{\epsilon}{1 + N})] + V_{BE}$

$\approx (1 + \frac{C_2}{C_1}) V_T [\ln N + \frac{\epsilon}{1 + N}] + V_{BE}$

The error is thus equal to $(1 + \frac{C_2}{C_1}) V_T \frac{\epsilon}{1 + N}$.

11.21 $R_1 = 1 \text{ k}\Omega, R_2 = 2 \text{ k}\Omega$

(a) $V_{out} = \frac{V_T \ln n}{R_1} \cdot R_2 + V_{BE3} \Rightarrow \ln n \approx \frac{17.2}{2} = 8.6$

$\Rightarrow n = 5432 (!)$

Alternatively, we can scale (W/L)₅ up by a factor

α such that: $V_{out} = \frac{V_T \ln n}{R_1} \alpha \cdot R_2 + V_{BE3}$.

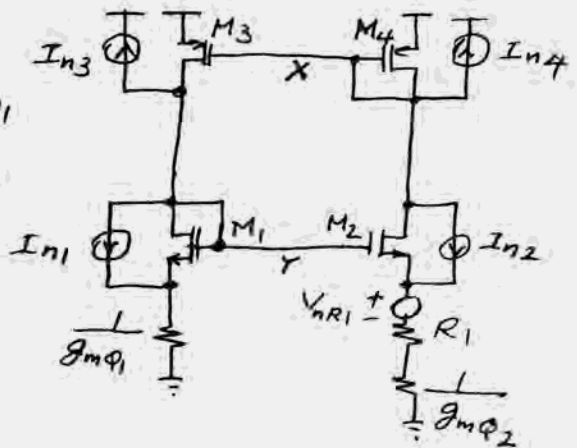
For example, for $\alpha = 4$, $n = 8.58$.

(b) V_x is given by:

$(-g_{m3} V_x - I_{n3} - I_{n1}) \frac{1}{g_{m1}} + (-g_{m3} V_x - I_{n3}) \frac{1}{g_{m4}}$
 (under I_{n3} and I_{n1} terms) I_{D3} current thru M_1
 (under I_{n3} term) current thru $\frac{1}{g_{m4}}$

Also:

$(-g_{m4} V_x - I_{n4})(R_1 + \frac{1}{g_{m2}}) + V_{nR1}$
 $+ (-g_{m4} V_x - I_{n4} - I_{n2}) \frac{1}{g_{m2}} = V_x$
 (under I_{n4} and I_{n2} terms) I_{D2}



Equating these, we have

$V_x = \frac{1}{g_{m4} R_1} \left[I_{n3} \left(\frac{1}{g_{m1}} + \frac{1}{g_{m4}} \right) + \frac{I_{n1}}{g_{m1}} + I_{n4} \left(R_1 + \frac{1}{g_{m2}} + \frac{1}{g_{m2}} \right) + \frac{I_{n2}}{g_{m2}} + V_{nR1} \right]$

This noise is amplified by $g_{m5} (R_2 + \frac{1}{g_{m3}})$ when it appears at the output.

$$\overline{V_{n,out,tot}}^2 = \frac{g_{m5}^2 (R_2 + \frac{1}{g_{m3}})^2}{(g_{m4} R_1)^2} \left[2 I_{n3}^2 \left(\frac{1}{g_{m1}} + \frac{1}{g_{m4}} \right)^2 + \frac{2 I_{n1}^2}{g_{m1}^2} + I_{n4}^2 R_1^2 + V_{nR1}^2 \right] \\ + I_{n5}^2 (R_2 + \frac{1}{g_{m3}})^2 + V_{nR2}^2$$

11.22 $f_{ck} = 50 \text{ MHz}$ power budget = 1 mW. $g_{m1} = \frac{1}{500 \Omega}$
 $g_{m1} = \frac{2}{R_S} \left(1 - \frac{1}{\sqrt{K}} \right)$ $R_S = \frac{1}{f_{ck} C_S}$ $I_{D1} = I_{D2} = \frac{0.5 \text{ mW}}{3 \text{ V}}$
 $= 167 \mu\text{A}$

$$I_{out} = \frac{2}{\mu_n C_{ox} (W/L)_N} \cdot \frac{1}{R^2} \left(1 - \frac{1}{\sqrt{K}} \right)^2 = \frac{2}{\mu_n C_{ox} (W/L)_N} \left(\frac{g_{m1}}{2} \right)^2$$

$$\Rightarrow \left(\frac{W}{L} \right)_N = 89.4$$

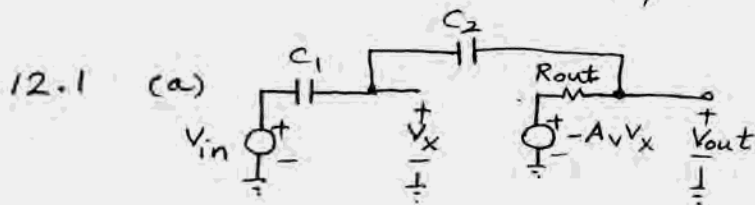
We assume $K = 4$. $\Rightarrow \frac{1}{500 \Omega} = \frac{2}{R_S} \left(1 - \frac{1}{2} \right) = \frac{1}{R_S}$

$$\Rightarrow R_S = 500 \Omega \Rightarrow C_S = 40 \text{ pF}$$

$\left(\frac{W}{L} \right)_2 = 4 \times 89.4$ For M_3 and M_4 , there is some freedom so long as the transistors remain saturated. For example

$$\left(\frac{W}{L} \right)_3 = \left(\frac{W}{L} \right)_4 = 50$$

Chapter 12



$$\frac{V_{out} - (-A_v V_x)}{R_{out}} = -\frac{V_{out} - V_x}{\frac{1}{C_2 s}} \Rightarrow$$

$$V_x = (V_{out} - V_{in}) \frac{C_2}{C_1 + C_2} + V_{in}$$

$$= \frac{C_2}{C_1 + C_2} V_{out} - \frac{C_1}{C_1 + C_2} V_{in}$$

$$\Rightarrow V_{out} \left[1 + \frac{A_v C_2}{C_1 + C_2} + \frac{C_2}{C_1 + C_2} R_{out} C_2 s \right] = V_{in} \left[A_v \frac{C_1}{C_1 + C_2} + \frac{C_1}{C_1 + C_2} R_{out} C_2 s \right]$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = A_v \frac{C_1}{C_2} \frac{1 + R_{out} C_2 s}{1 + \frac{C_1}{C_2} + A_v + R_{out} C_2 s}$$

(b)
$$A_v(s) = -\frac{R_F \parallel \frac{1}{C_2 s}}{\frac{1}{C_1 s}} = -\frac{R_F C_1 s}{R_F C_2 s + 1}$$

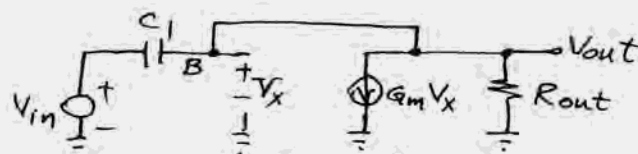
Nominal Gain = 4

$$\frac{R_F C_1 \omega}{\sqrt{1 + R_F^2 C_2^2 \omega^2}} = 3.96 \quad \omega = 2\pi (1 \text{ MHz})$$

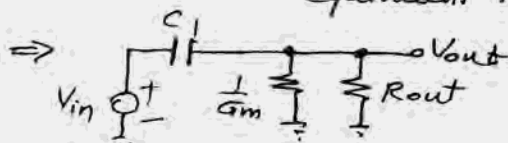
$$(3.96^2 R_F^2 C_2^2 - R_F^2 C_1^2) \omega^2 + 3.96^2 = 0 \Rightarrow R_F^2 = \frac{3.96^2}{\omega^2 (C_2^2 - 3.96^2 C_1^2)}$$

$$\Rightarrow R_F = 2.23 \text{ M}\Omega$$

12.2 (a)



equivalent to a resistor of $\frac{1}{g_m}$



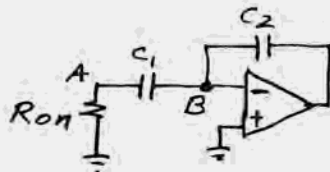
$$\frac{V_{out}}{V_{in}} = \frac{(\frac{1}{g_m} \parallel R_{out}) C_1 s}{(\frac{1}{g_m} \parallel R_{out}) C_1 s + 1}$$

$$\approx \frac{(1/g_m) C_1 s}{\frac{C_1}{g_m} s + 1}$$

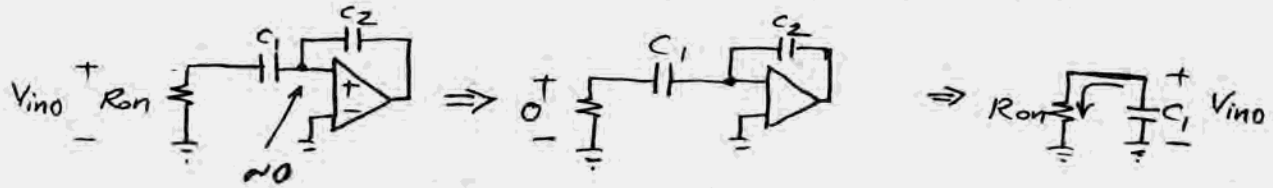
(b) $\omega = 2\pi (100 \text{ MHz}), C_1 = 1 \text{ pF}, \frac{1}{g_m} = 100 \Omega \Rightarrow \frac{C_1}{g_m} \omega = 0.0628$

$$\Rightarrow \frac{V_{out}}{V_{in}} \approx 0.0628, \text{ with a phase shift of nearly } 90^\circ.$$

12.3



Since node B is at virtual ground, $\tau \approx R_{on} C_1$.



\Rightarrow Total energy is that stored on $C_1 = \frac{1}{2} C_1 V_{in0}^2$

12.4 (a) $\left(\frac{W}{L}\right)_1 = \frac{20}{0.5}$, $C_H = 1 \text{ pF}$, $I_{D,sat} = 20.8 \text{ mA}$

$$\Rightarrow t_1 = 146 \text{ ps}$$

$$+1 \text{ mV} = \frac{2(2.3 \text{ V}) \exp\left[-(2.3 \text{ V}) \frac{\mu_n C_{ox} W/L (t-t_1)}{C_H}\right]}{1 + \exp\left[-(2.3 \text{ V}) \frac{\mu_n C_{ox} W/L (t-t_1)}{C_H}\right]}$$

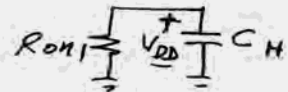
$$\Rightarrow \exp\left[-(2.3 \text{ V}) \frac{\mu_n C_{ox} W/L (t-t_1)}{C_H}\right] \approx \frac{+1 \text{ mV}}{2(2.3 \text{ V})}$$

$$\Rightarrow t - t_1 = 465 \text{ ps} \Rightarrow \text{total time} = 611 \text{ ps}$$

(b) $R_{on1} = 55 \Omega \Rightarrow \tau = 55 \text{ ps}$

$$V_{out} = V_{DD} \exp\left(-\frac{t}{\tau}\right) \Rightarrow t = 440 \text{ ps}$$

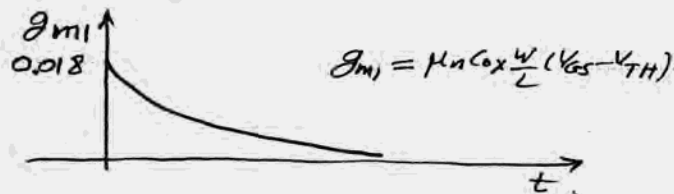
It underestimates the required time.



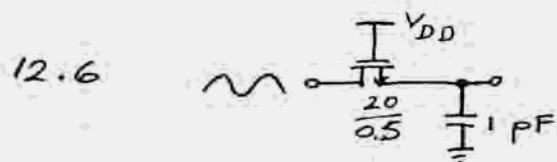
12.5 (a) $2.1 = 2.3 - \frac{1}{\frac{1}{2} \frac{\mu_n C_{ox}}{C_H} \frac{W}{L} t + \frac{1}{2.3}}$ ($\gamma=0$)

$$\Rightarrow \frac{1}{2} \frac{\mu_n C_{ox}}{C_H} \frac{W}{L} t + \frac{1}{2.3} = 5 \Rightarrow t \approx 1.16 \text{ ns}$$

(b)

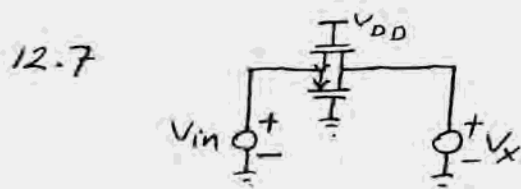


$$\Rightarrow g_{m1}(t=0) = 0.018 \text{ S}$$

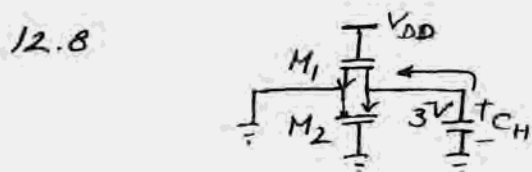


(a) $R_{on1} = 55 \Omega$
 $|\theta| = \tan^{-1}(RC\omega)$
 $= 1.98^\circ$

(b) $R_{on1} = 97.6 \Omega \Rightarrow |\theta| \approx 3.96^\circ$



V_x is a voltage-dependent voltage source that follows V_{in} with a, say, 20-mV difference. We can then monitor the current drawn by either source, invert it, and normalize it to 20 mV in a dc sweep that varied V_{in} across the range of interest.



$R_{on1} = 55 \Omega$ $R_{on2} = 64.3 \Omega$, $C_H = 1 \text{ pF}$
 $R_{on1} || R_{on2} = 29.6 \Omega \Rightarrow \tau = 29.6 \text{ ps}$
 $\Rightarrow +1 \text{ mV} = +3 \text{ V} \exp\left(-\frac{t}{\tau}\right)$

$\Rightarrow t \approx 237 \text{ ps}$

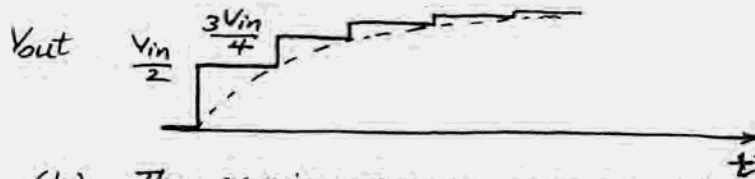
12.9 $V_{GS} - V_{TH} = 2.3 \text{ V} \Rightarrow V_{\text{error}} = \frac{W L \overset{\text{eff.}}{C_{ox}} (V_{GS} - V_{TH})}{C_H}$
 $= 60 \text{ mV}$

For clock feedthrough: $C_{ov} = (0.4 \times 10^{-11} \text{ F/m}) \times 20 \mu\text{m} = 0.08 \text{ fF}$

$V_{\text{error}} \approx \frac{C_{ov}}{C_H} V_{CK} = 0.24 \text{ mV}$

The overlap capacitance in Table 2.1 should actually be $0.4 \text{ e-}9$ for NMOS. Thus, the error due to clock feedthrough will be about 24 mV, somewhat less than that due to worst-case charge injection.

- 12.10 (a) C_1 together with M_1 and M_2 can be viewed as a resistor. Thus, C_2 charged to $2V$ with an envelope given by $1 - \exp\left(-\frac{t}{\tau}\right)$, where $\tau = \frac{1}{f_{ck} C_1} \cdot C_2$.



- (b) The maximum error occurs when $V_{GS} - V_{TH}$ is maximum.

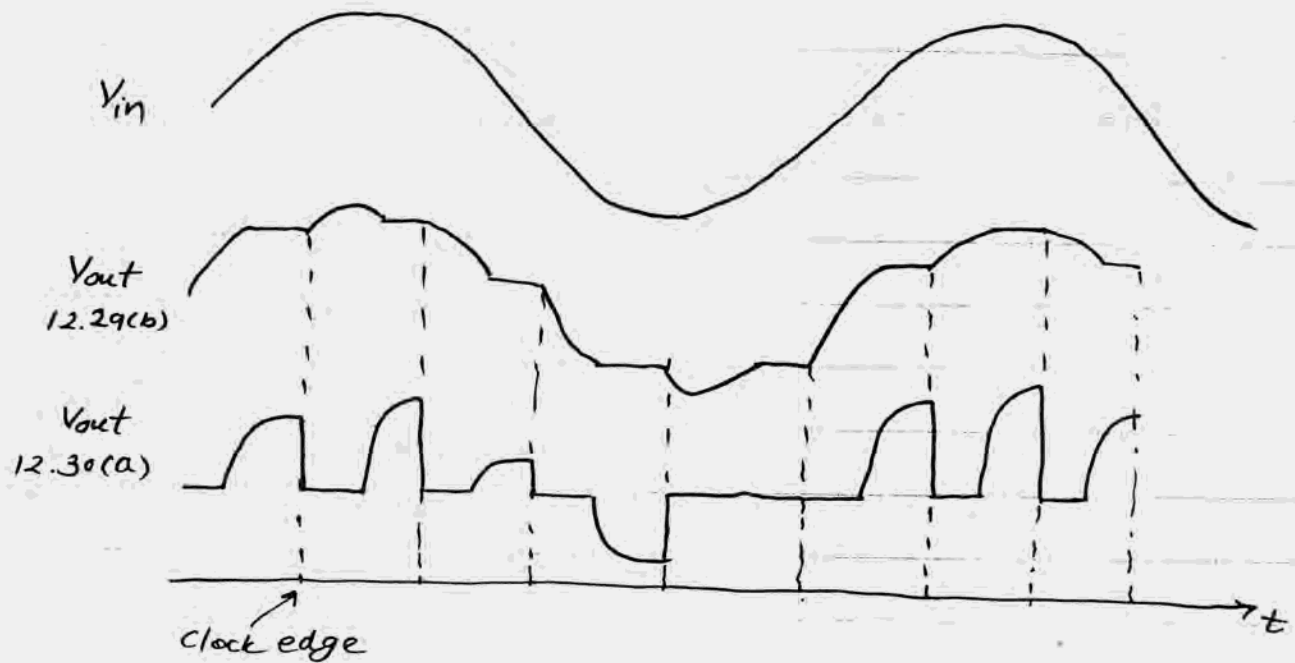
If all of M_1 channel charge is injected onto C_1 , then after V_{C_1} has reached V_{in} and M_1 turns off, V_{C_1} incurs an error equal to $(V_{GS} - V_{in} - V_{TH})WL C_{ox} / C_1$. When M_2 turns on, it absorbs some charge into its channel and when it turns off, it injects the charge back onto C_1 and C_2 . Thus, only the charge due to M_1 need be considered. This error is divided equally between C_1 and C_2 , yielding an overall output error of $\frac{WL C_{ox} (V_{GS} - V_{in} - V_{TH})}{2 C_1}$.

- (c) When M_1 turns off, a voltage equal to $\sqrt{\frac{kT}{C_1}}$ is stored across C_1 . When M_2 is on, this voltage is distributed between C_1 and C_2 . Moreover, M_2 itself produces thermal noise:

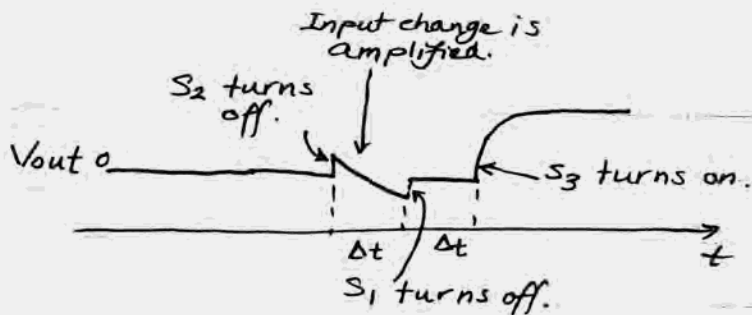
$$\Rightarrow V_{n,out} = \sqrt{\frac{kT}{2C_1}}$$

$$\Rightarrow V_{n,out,tot}^2 = \frac{1}{4} \frac{kT}{C_1} + \frac{kT}{2C_1} = \frac{3kT}{4C_1}$$

$$\Rightarrow V_{n,out,tot} = \sqrt{\frac{3kT}{4C_1}}$$



12.12



12.13

$$\text{Gain error} \approx (C_2 + C_1 + C_{in}) / (C_2 A_{v1}) = 0.01$$

$$\Rightarrow 1 + \frac{C_1}{C_2} + \frac{C_{in}}{C_2} = 10 \Rightarrow 9C_2 = C_1 + C_{in} = 2.2 \text{ pF}$$

$$\Rightarrow C_2 = \frac{2.2 \text{ pF}}{9}$$

$$\frac{C_L}{C_2} = 8.2 \rightarrow 8.0$$

12.14

$$G_m = 100^{-1} \text{ V}$$

$$\tau_{amp} = \frac{C_L C_{eq} + C_L C_2 + C_{eq} C_2}{G_m C_2}$$

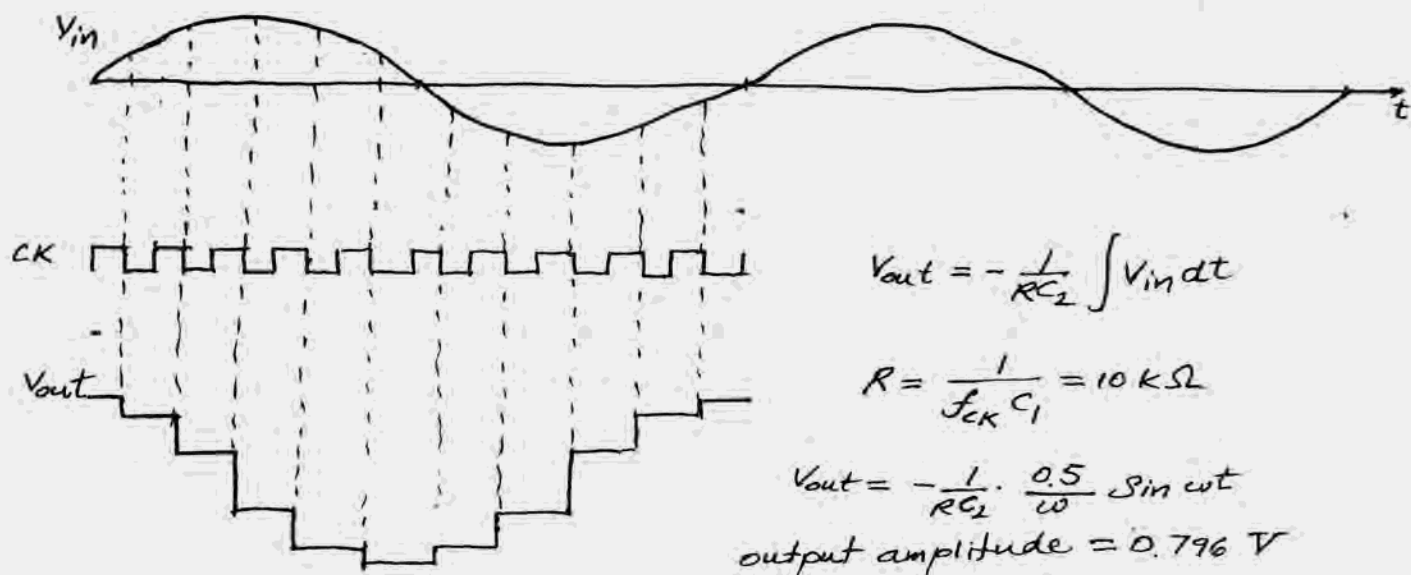
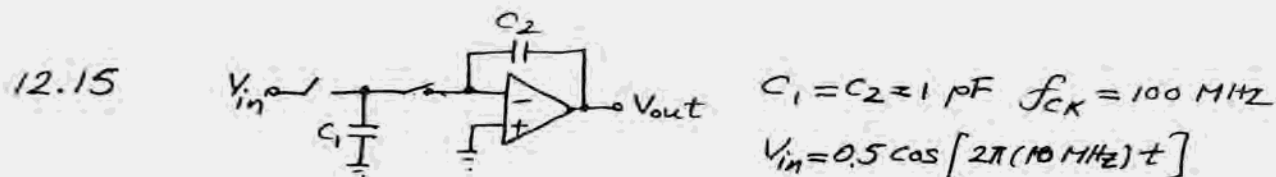
$$C_{eq} = C_1 + C_{in}$$

$$= \frac{C_{eq}}{G_m} \quad \text{because } C_L = 0$$

$$= 2 \text{ ns} \Rightarrow C_{eq} = 20 \text{ pF}$$

Since $C_{in} = 0.2 \text{ pF}$, $C_1 = 19.8 \text{ pF}$. Also, $9C_2 = 20 \text{ pF} \Rightarrow$

$$\frac{C_L}{C_2} = 44$$



12.16 (a) The minimum level = $1.5 \text{ V} - V_{TH1,2} \approx 0.8 \text{ V}$.

The maximum level places M_3 or M_4 at the edge of the triode region. $|V_{GS} - V_{TH}|_{3,4} = 0.421 \text{ V} \Rightarrow \text{max. level} = 2.58 \text{ V}$.
 $\Rightarrow \text{Max. Swing} = 1.78 \text{ V}$.

(b) $A_{v,open} \approx \beta_{m1,2} (r_{o1} || r_{o2}) = 27.3$

Gain Error = $\frac{C_1 + C_3}{C_3 A_v} = 18.3\%$

(c) $T_{amp} \approx \frac{C_1}{g_m} = 0.488 \text{ ns}$

12.17 (a) same.

(b) The gate-source cap is equal to $\frac{2}{3} W_{left} C_{ox} + W C_{ov}$
 $\approx 44 \text{ fF}$

(The overlap cap in Table 2.1 must actually be 0.4×10^{-9} , in which case $C_{in} \approx 64 \text{ fF}$.)

The gate-drain overlap capacitance ^(of $M_{1,2}$) changes the gain equation because it appears in parallel with the feedback capacitor:

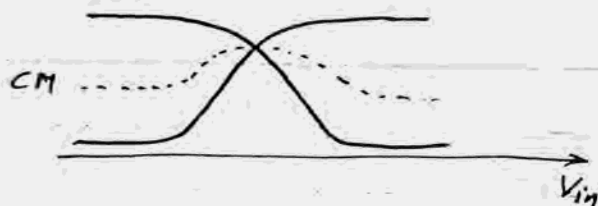
$$\begin{aligned} \frac{V_{out}}{V_{in}} &\approx -\frac{C_1}{C_1 + W C_{ov}} \left(1 - \frac{C_3 + W C_{ov} + C_1 + C_{in}}{C_3 + W C_{ov}} \cdot \frac{1}{A_v}\right) \\ &\approx -\frac{C_1}{C_3} \left(1 - \frac{W C_{ov}}{C_3}\right) \left(1 - \frac{C_3 + W C_{ov} + C_1 + C_{in}}{C_3 + W C_{ov}} \cdot \frac{1}{A_v}\right) \end{aligned}$$

Thus, the gain error rises to $\frac{W C_{ov}}{C_3} + \frac{C_3 + C_1 + W C_{ov} + C_{in}}{C_3 + W C_{ov}} \cdot \frac{1}{A_v}$.

Assuming $C_{ov} = 0.4 \text{ e-}9$, we obtain a gain error of 22.2%.

(c) Neglecting the drain junction caps at the output, we have $\tau_{amp} \approx \frac{C_1 + C_{in}}{g_m} \approx 0.503 \text{ ns}$.

12.18 Plotting the CM level, we see that it changes with the differential output.



This usually means that the CM feedback network, in particular the devices sensing the CM level, are quite nonlinear.

12.19 Since $I_{D5} = 1 \text{ mA}$, $(V_{GS} - V_{TH})_5 = 319 \text{ mV} \Rightarrow$ Minimum input level = $V_{GS1,2} + 319 \text{ mV} \approx 1.245 \text{ V}$ (neglecting body effect.)

Since $I_{D6} = 50 \mu\text{A}$, $(V_{GS} - V_{TH})_6 = 71.3 \text{ mV} \Rightarrow$

$V_{out, CM} = 71.3 \text{ mV} + V_{TH6} + V_{GS5} \approx 1.79$ (neglecting body effect.)

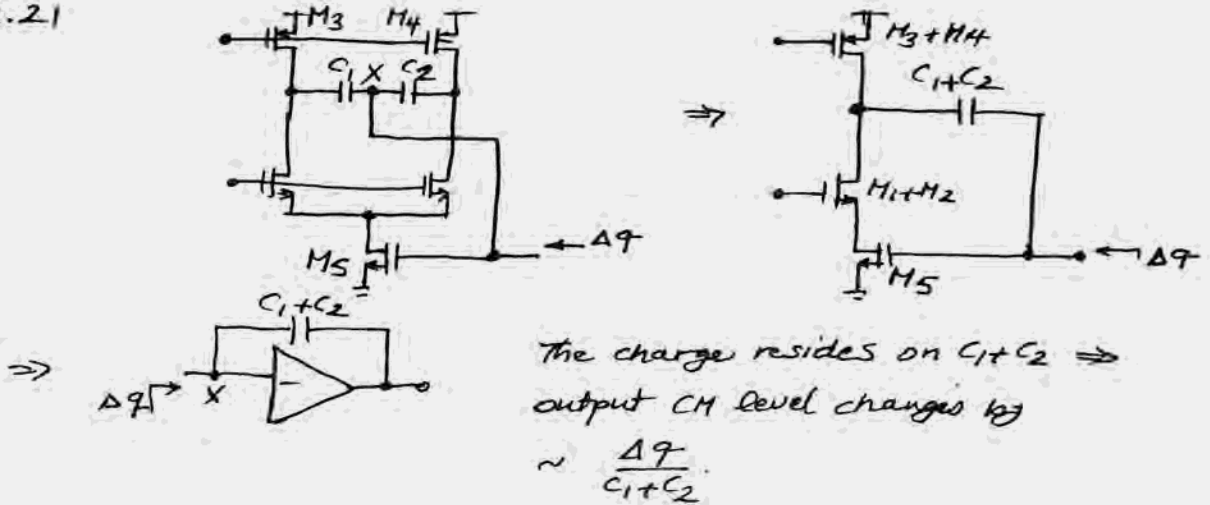
$\Rightarrow V_{in, max} = 1.79 + V_{TH1,2} \approx 2.49 \text{ V}$

12.20 $V_{in, min}$ remains the same.

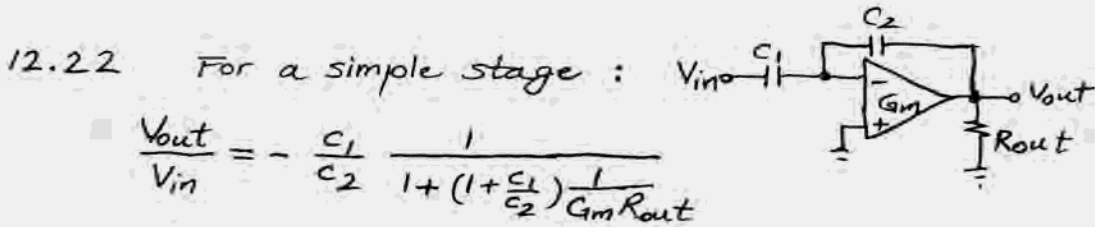
$$V_{in, max} = V_{GS6} + V_{GS5} + V_{TH1,2}$$

$$= 0.859 + 1.019 + 0.7 = 2.578 \text{ V (neglecting body effect)}$$

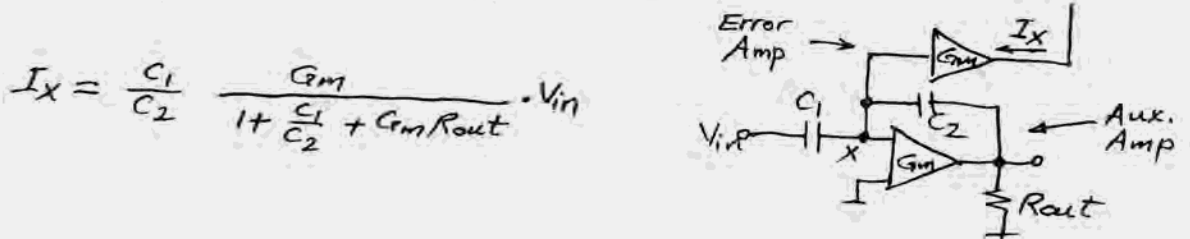
12.21



The voltage at node X changes by $\frac{\Delta Q}{C_1 + C_2} \cdot \frac{1}{A_v}$,
 where $A_v = g_{m5} [r_{o3+4} \parallel (\beta_{m1+2} r_{o1+2} r_{o5})]$.



Thus, the voltage at node X and hence the current drawn by the error amplifier can be easily calculated.



$$\left[-G_m(V_{in} - V_1) + \frac{G_m \frac{C_1}{C_2} V_{in}}{1 + \frac{C_1}{C_2} + G_m R_{out}} \right] \frac{R_{out}}{2} = V_{out}$$

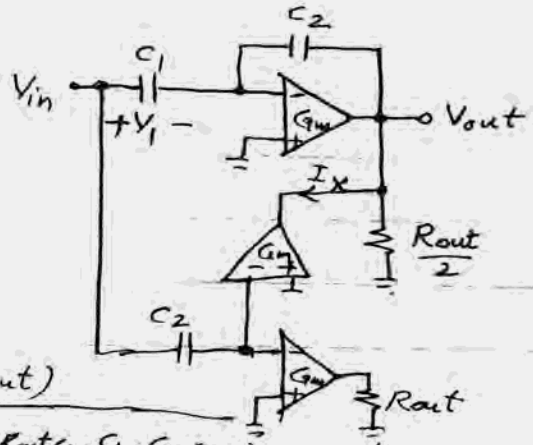
$$V_1 = \frac{V_{in} - V_{out}}{1 + \frac{C_1}{C_2}}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2} \frac{\frac{G_m R_{out}}{2} (2 + 2\frac{C_1}{C_2} + G_m R_{out})}{(1 + \frac{C_1}{C_2}) (1 + \frac{C_1}{C_2} + G_m R_{out}) + \frac{G_m R_{out}}{2} (1 + \frac{C_1}{C_2} + G_m R_{out})}$$

$$= -\frac{C_1}{C_2} \frac{1 + 2\frac{1 + \frac{C_1}{C_2}}{G_m R_{out}}}{1 + 3\frac{1 + \frac{C_1}{C_2}}{G_m R_{out}} + 2\left(\frac{1 + \frac{C_1}{C_2}}{G_m R_{out}}\right)^2}$$

$$= -\frac{C_1}{C_2} \frac{1 + 2X}{(1+X)(1+2X)} \quad X = \frac{1 + \frac{C_1}{C_2}}{G_m R_{out}}$$

$$= -\frac{C_1}{C_2} \frac{1}{1 + \frac{1 + \frac{C_1}{C_2}}{G_m R_{out}}}$$



Interestingly, the gain error is the same. But if the G_m stage in the error amplifier has a very high output impedance, then the load resistor of the main amplifier is R_{out} rather than $R_{out}/2$ and

$$\frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2} \frac{1 + 2\frac{1 + \frac{C_1}{C_2}}{G_m R_{out}}}{1 + 2\frac{1 + \frac{C_1}{C_2}}{G_m R_{out}} + \left(\frac{1 + \frac{C_1}{C_2}}{G_m R_{out}}\right)^2}$$

$$= -\frac{C_1}{C_2} \frac{1}{1 + \frac{(1 + \frac{C_1}{C_2})^2 / (G_m R_{out})^2}{1 + 2\frac{1 + \frac{C_1}{C_2}}{G_m R_{out}}}}$$

$$\approx -\frac{C_1}{C_2} \frac{1}{1 + \left(\frac{1 + \frac{C_1}{C_2}}{G_m R_{out}}\right)^2}, \text{ as if the open-loop gain of the amplifier is squared.}$$

Chapter 13

$$13.1 \quad y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) \quad x = [0 \ x_{\max}]$$

(a) Straight line passing through the end points:

$$y_1 = \frac{\alpha_1 x_{\max} + \alpha_2 x_{\max}^2}{x_{\max}} \cdot x = (\alpha_1 + \alpha_2 x_{\max}) x$$

$$y(t) - y_1 = -\alpha_2 x_{\max} \cdot x + \alpha_2 x^2$$

⇒ Error is maximum at $x = \frac{x_{\max}}{2}$ and equal to $-\frac{\alpha_2 x_{\max}^2}{4}$. This value is usually normalized to the maximum output level.

$$(b) \quad y(t) = \alpha_1 \frac{x_{\max} \cos \omega t}{2} + \alpha_1 \frac{x_{\max}}{2} + \frac{\alpha_2}{4} x_{\max}^2 \underbrace{\cos^2 \omega t}_{\frac{1 + \cos 2\omega t}{2}} \\ + \frac{\alpha_2}{4} 2 x_{\max}^2 \cos \omega t + \frac{\alpha_2}{4} x_{\max}^2$$

$$\Rightarrow \text{Fundamental: } \left(\frac{\alpha_1 x_{\max}}{2} + \frac{\alpha_2}{2} x_{\max}^2 \right) \cos \omega t$$

$$\text{Second Harmonic: } \frac{\alpha_2}{8} x_{\max}^2 \cos 2\omega t$$

$$\Rightarrow \text{THD} = \frac{\alpha_2^2 x_{\max}^4 / 64}{\left(\frac{\alpha_1 x_{\max}}{2} + \frac{\alpha_2}{2} x_{\max}^2 \right)^2} \\ = \frac{\alpha_2^2 x_{\max}^2}{16 (\alpha_1 + \alpha_2 x_{\max})^2}$$

$$13.2 \quad \text{For Fig. 13.6 (a): } \frac{A_{HD2}}{A_F} = \frac{V_m}{4(V_{GS} - V_{TH})} \quad V_{GS} - V_{TH} = 356 \text{ mV}$$

$$\Rightarrow \frac{A_{H2}}{A_F} = 7\% \quad (-23 \text{ dB})$$

$$\text{For Fig. 13.6 (b): } \frac{A_{HD3}}{A_F} \approx \frac{V_m^2}{32(V_{GS} - V_{TH})^2} \\ = 0.25\% \quad (-52 \text{ dB})$$

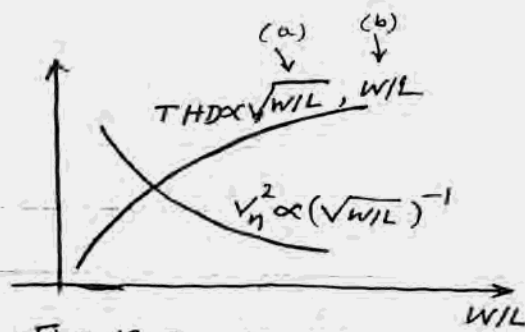
• If we double W/L , $V_{GS} - V_{TH}$ is divided by $\sqrt{2}$.

⇒ For (a), distortion goes up by $\sqrt{2}$ (3 dB) and for (b), by 2 (6 dB).

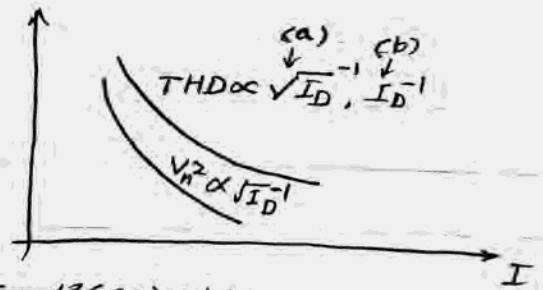
• If we double I , $V_{GS} - V_{TH}$ is multiplied by $\sqrt{2}$.

⇒ Distortion goes down by $\sqrt{2}$ and 2 for (a) & (b), respectively.

13.3



Figs. 13.6 (a), (b)



Figs. 13.6 (a), (b)

(Note that here THD is the ratio of voltages. If we take the ratio of powers, the relations must be squared.)

Increasing I and hence power dissipation decreases both the noise and the nonlinearity, whereas increasing W/L degrades the linearity while reducing the noise.

13.4 (1) As (W/L) is increased to increase the voltage gain, the linearity degrades (with a constant I).

(2) As I is increased to linearize the circuit, the load resistance must be decreased to maintain the same voltage headroom \Rightarrow gain \downarrow .

$$13.5 \quad \frac{b}{a} = \frac{\alpha_2}{2} V_m \frac{1}{\alpha_1} \frac{1}{(1 + \beta \alpha_1)^2}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{G50} + V_m \cos \omega t - V_{TH})^2 \quad V_{G50} - V_{TH} = \text{overdrive}$$

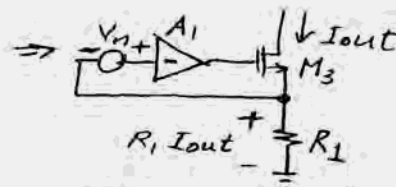
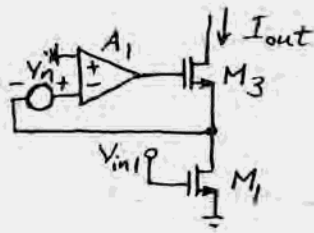
$$= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[V_m^2 \cos^2 \omega t + 2V_m \cos \omega t \cdot (V_{G50} - V_{TH}) + (V_{G50} - V_{TH})^2 \right] \quad \text{with } I_D = 1 \text{ mA}$$

$$\Rightarrow \left| \frac{\alpha_2}{\alpha_1} \right| = \frac{1}{2 \cdot (V_{G50} - V_{TH})} = 1.57 \text{ V}^{-1}$$

$$\left\{ \alpha_1 = \mu_n C_{ox} \frac{W}{L} (V_{G50} - V_{TH}) R_D = 6286 \mu\text{A/V} \times 2 \text{ k}\Omega = 12.57 \right.$$

$$\Rightarrow \frac{b}{a} = 6.36 \times 10^{-4}$$

13.6



$$R_1^{-1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 [2(V_{GS} - V_{TH}) - 2V_{DS}]$$

$$[(R_1 I_{out} + V_n)(-A_1) - R_1 I_{out}] g_{m3} = I_{out}$$

$$\Rightarrow I_{out} = \frac{-g_{m3} A_1}{1 + g_{m3} R_1 A_1 + g_{m3} R_1} V_n \approx \frac{-A_1}{R_1 (A_1 + 1)} V_n$$

$$\approx \frac{-1}{R_1} V_n$$

$$\Rightarrow |V_{n,in}| = \frac{\frac{1}{R_1} V_n}{g_{m1}}, \quad g_{m1} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (2V_{DS})$$

13.7 while increasing W/L raises the open-loop gain, it also makes the circuit more nonlinear (if I remains constant.)

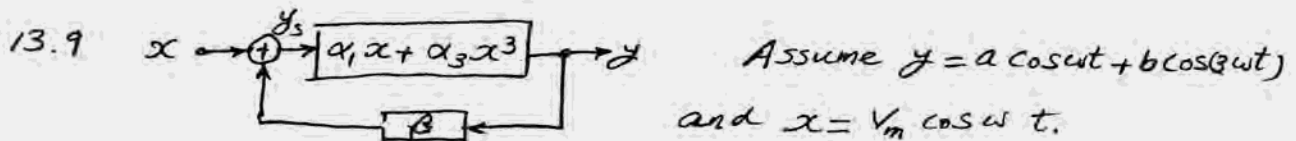
Since $\frac{W}{L}$ is multiplied by a factor of 4 $\Rightarrow \left|\frac{\alpha_2}{\alpha_1}\right| \uparrow$ by 2x,

and $\alpha_1 \uparrow$ by 2x \Rightarrow

$$\frac{b}{a} = 4.32 \times 10^{-4}$$

13.8

$$\beta \alpha_1 = 5.03 \Rightarrow \frac{b}{a} \approx \frac{\alpha_2}{\alpha_1} \cdot \frac{V_m}{2} \cdot \frac{1}{\beta^2 \alpha_1^2} = 3.1 \times 10^{-4}$$



$$y_s = V_m \cos \omega t - \beta (a \cos \omega t + b \cos 3\omega t)$$

$$\Rightarrow y(t) = \alpha_1 (V_m - \beta a) \cos \omega t - \alpha_1 \beta b \cos 3\omega t + \alpha_3 (V_m - \beta a)^3 \cos^3 \omega t - \alpha_3 \beta^3 b^3 \cos^3 3\omega t - 3\alpha_3 (V_m - \beta a)^2 \cos^2 \omega t \cdot \beta b \cos 3\omega t + 3\alpha_3 (V_m - \beta a) \cos \omega t \cdot \beta^2 b^2 \cos^2 3\omega t$$

Neglecting higher order terms: $a \approx \frac{\alpha_1}{1 + \beta \alpha_1} V_m$, $V_m - \beta a \approx \frac{a}{\alpha_1}$

$$b \approx -\alpha_1 \beta b + \frac{\alpha_3}{4} (V_m - \beta a)^3$$

$$\Rightarrow \frac{b}{a} \approx \frac{1}{4} \frac{\alpha_3}{\alpha_1} \frac{V_m^2}{(1 + \beta \alpha_1)^3}$$

13.10 $I_D = I_0 \exp \frac{V_{GS}}{\xi V_T}$ $V_{GS} = V_{GS0} + V_m \cos \omega t$

$$\Rightarrow I_D = (I_0 \exp \frac{V_{GS0}}{\xi V_T}) \left[1 + \frac{V_m \cos \omega t}{\xi V_T} + \frac{1}{2} \left(\frac{V_m \cos \omega t}{\xi V_T} \right)^2 + \dots \right]$$

If $V_m \ll \xi V_T$, only second harmonic is significant: $\frac{1}{4} \left(\frac{V_m}{\xi V_T} \right)^2 \cos 2\omega t$.

For the differential pair, $I_{D1} + I_{D2} = I_{SS}$, and

$$V_{in} - V_{GS1} + V_{GS2} = 0 \Rightarrow V_{in} = \xi V_T \ln \frac{I_{D1}}{I_0} - \xi V_T \ln \frac{I_{D2}}{I_0}$$

$$= \xi V_T \ln \frac{I_{D1}}{I_{D2}}$$

It follows that: $I_{D1} = \frac{I_{SS} \exp[V_{in}/(\xi V_T)]}{1 + \exp[V_{in}/(\xi V_T)]}$

$$I_{D2} = \frac{I_{SS}}{1 + \exp[V_{in}/(\xi V_T)]}$$

Thus, $I_{D1} - I_{D2} = I_{SS} \tanh \frac{V_{in}}{2\xi V_T}$ $\tanh \varepsilon \approx \varepsilon - \frac{\varepsilon^3}{3}$

$$\approx I_{SS} \left[\frac{V_{in}}{2\xi V_T} - \left(\frac{V_{in}}{2\xi V_T} \right)^3 \right]$$

If $V_{in} = V_{in0} + V_m \cos \omega t$, the third harmonic is given

by $-I_{SS} \frac{1}{(2\xi V_T)^3} V_m^3 \frac{1}{4} \cos 3\omega t$.

13.11 $I_D = \frac{1}{2} \frac{\mu_0 C_{ox}}{1 + \theta(V_{GS} - V_{TH})} \frac{W}{L} (V_{GS} - V_{TH})^2$

$$\approx \frac{1}{2} \mu_0 C_{ox} \frac{W}{L} [1 - \theta(V_{GS} - V_{TH})] (V_{GS} - V_{TH})^2$$

If $V_{GS} = V_{GS0} + V_m \cos \omega t$, then the third harmonic is

given by $\frac{1}{2} \mu_0 C_{ox} \frac{W}{L} (-\theta) \frac{V_m^3}{4} \cos 3\omega t$.

$$13.12 \quad (a) \quad \Delta V_{TH} = \frac{0.1 t_{ox}}{\sqrt{WL}} \quad t_{ox} = 90 \text{ \AA}$$

$$\Rightarrow W = 6.5 \text{ \mu m}$$

$$(b) \quad THD = \frac{V_m^2}{32(V_{GS} - V_{TH})^2} \quad I_D = 1 \text{ mA} \quad \frac{W}{L} = \frac{6.5}{0.5}$$

$$\Rightarrow V_{GS} - V_{TH} = 1.07 \text{ V}$$

$$\Rightarrow V_{m, \max} = 0.61 \text{ V}$$

$$13.13 \quad (a) \quad W = 6.5 \text{ \mu m} \times \left(\frac{5 \text{ mV}}{2 \text{ mV}}\right)^2 = 41 \text{ \mu m}$$

$$(b) \quad V_{GS} - V_{TH} = 1.07 \text{ V} \times \sqrt{\frac{6.5}{41}} = 0.426 \text{ V}$$

$$\Rightarrow V_{m, \max} = 0.61 \times \sqrt{\frac{6.5}{41}} = 0.243 \text{ V}$$

We see a trade-off between input offset and nonlinearity (if the channel length remains constant.)

13.14

$$\left| \frac{\Delta I_D}{I_D} \right| = \frac{2 \Delta V_{TH}}{V_{GS} - V_{TH}} = 0.02 \Rightarrow \Delta V_{TH} = 5 \text{ mV} = \frac{0.1 \times 90 \text{ \AA}}{\sqrt{WL}}$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right) (V_{GS} - V_{TH})^2 \Rightarrow \frac{W}{L} = 29.9$$

$$\Rightarrow \begin{cases} L = 0.033 \text{ \mu m} \\ W = 0.984 \end{cases} \quad \text{But if } L_{\min} \approx 0.5 \text{ \mu m} \Rightarrow \frac{W}{L} = \frac{15 \text{ \mu m}}{0.5 \text{ \mu m}}$$

13.15

$$I_D R_S + \sqrt{\frac{2 I_D}{\mu_n C_{ox} W L}} + V_{TH} = V_b$$

Take the total differential of both sides and substitute $g_m = \frac{2 I_D}{V_{GS} - V_{TH}}$. Then, the result is obtained.

$$13.16 \quad y_1(t) = \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t$$

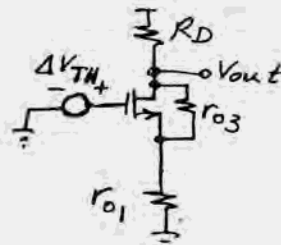
$$y_2(t) = \alpha_1 A \cos(\omega t + \theta) + \alpha_2 A^2 \cos^2(\omega t + \theta) + \alpha_3 A^3 \cos^3(\omega t + \theta)$$

The second harmonic arises from $\alpha_2 A^2 [\cos^2 \omega t - \cos^2(\omega t + \theta)]$

$$= \alpha_2 A^2 \frac{\cos(2\omega t) - \cos(2\omega t + 2\theta)}{2}$$

$$= \frac{\alpha_2 A^2}{4} \sin \theta \sin(2\omega t)$$

13.17 We calculate the output offset first. Viewing offset as noise, we have the following circuit:



$$\Rightarrow v_{out} = \Delta V_{TH} \frac{-g_{m3} r_{o3} R_D}{R_D + r_{o1} + r_{o3} + g_{m3} r_{o3} r_{o1}}$$

This must be divided by the voltage gain, which for moderate R_D is given by $g_{m1} R_D \Rightarrow |v_{os,in}| \approx \frac{g_{m3} r_{o3}}{g_{m1} R_D + g_{m3} r_{o3} r_{o1}}$

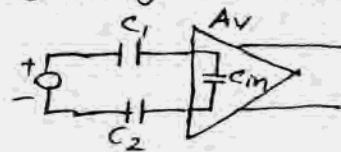
If $R_D \rightarrow \infty$, $|v_{out}| \rightarrow \Delta V_{TH} \cdot g_{m3} r_{o3}$. The voltage gain is obtained from Eq. (3.119) as $\sim g_{m1} r_{o1} g_{m3} r_{o3} \Rightarrow$

$|v_{os,in}| \approx \frac{\Delta V_{TH}}{g_{m1} r_{o1}}$. This is why we usually neglect the offset contributed by cascode devices.

13.18 With a finite input capacitance, the gain of the circuit is no longer A_v .

$$A_{v,tot} = \frac{C_1}{C_1 + 2C_{in}} \cdot A_v$$

$$\Rightarrow v_{os,in} = \frac{v_{os}}{\frac{C_1}{C_1 + 2C_{in}} \cdot A_v}$$



13.19 If W is doubled, the gain increases by approximately a factor of $\sqrt{2}$. Also, the input devices exhibit a smaller mismatch. For example, the threshold voltage mismatch decreases by a factor of $\sqrt{2}$. Thus, if the input devices dominate the offset, the overall input offset drops by a factor of 2.

13.20 To minimize the input offset, we maximize the overdrive of M_3 and M_4 . But this limits the high level of the output swings.

14.1

$$I_{D, \text{scaled}} = \frac{1}{2} \mu_n C_{ox} \frac{W/\alpha}{L/\alpha} \left(\frac{V_{GS}}{\alpha} - \frac{V_{TH}}{\alpha} \right)^2$$

$$= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \frac{1}{\alpha^2}$$

$$C_{ch, \text{scaled}} = \frac{W}{\alpha} \cdot \frac{L}{\alpha} C_{ox}$$

$$= \frac{1}{\alpha^2} WL C_{ox}$$

If the junction capacitances of SID are neglected, then,

$$T_d, \text{scaled} = \frac{C/\alpha^2}{I/\alpha^2} \cdot \frac{V_{DD}}{\alpha}$$

$$= \left(\frac{C}{I} V_{DD} \right) \frac{1}{\alpha}$$

Same as ideal scaling. But,

$$g_m, \text{scaled} = \mu C_{ox} \frac{W/\alpha}{L/\alpha} \frac{V_{GS} - V_{TH}}{\alpha}$$

$$= \mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) \cdot \frac{1}{\alpha}$$

14.2

$$W_d, \text{scaled} \approx \sqrt{\frac{2E_s \epsilon_i}{q} \left(\frac{1}{N_A} + \frac{1}{N_D} \right) \frac{V_R}{\alpha}}$$

SID sub.

$$\approx \frac{1}{\sqrt{\alpha}} \sqrt{\frac{2E_s \epsilon_i}{q} \frac{1}{N_A} V_R}$$

$$N_A \gg \alpha N_D$$

The depletion region capacitance per unit area therefore increases by $\sqrt{\alpha}$ rather than α . The series resistance increases.

DIBL arises primarily from the depletion region in the substrate rather than in the drain. Thus, DIBL remains relatively constant.

14.3 (a) Since $V_{n, \text{rms}} = \sqrt{\frac{kT}{q}}$, the capacitors must increase by a factor of 4.

(b) G_m should increase by a factor of 4.

(c) For square-law devices, W/L and bias current must increase by a factor of 4. \Rightarrow Power increases by a factor of 2.

(d) $SR = I/C \Rightarrow$ Bias current must increase by a factor of 4.

14.4

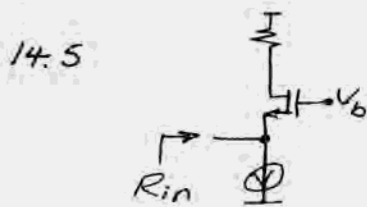
$$I_D = \mu_n C_d \frac{W}{L} V_T^2 \left(\exp \frac{V_{GS} - V_{TH}}{3V_T} \right) \left(1 - \exp \frac{-V_{DS}}{V_T} \right)$$

$$C_d = \sqrt{\epsilon_{Si} q N_{sub} / (4\phi_B)} \quad N_{sub} \rightarrow \alpha N_{sub}, \quad \phi_B \sim \text{constant.}$$

$$V_{GS} - V_{TH} \rightarrow \frac{V_{GS} - V_{TH}}{\alpha} \quad \beta = 1 + \frac{C_d}{C_{ox}} \rightarrow 1 + \frac{\sqrt{\alpha} C_d}{\alpha C_{ox}}$$

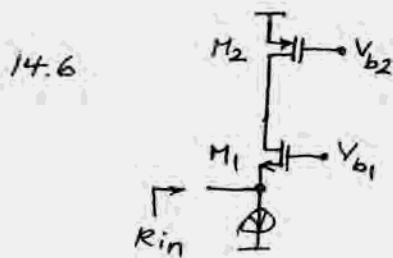
$$V_{DS} \rightarrow \frac{V_{DS}}{\alpha}$$

$$S_{scaled} = 2.3 V_T \left(1 + \frac{\sqrt{\alpha} C_d}{\alpha C_{ox}} \right) \Rightarrow S \downarrow, \text{ i.e., subthreshold behavior improves.}$$



$$R_{in} = \frac{1}{\sqrt{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}} = 50 \Omega$$

$$R_{in, scaled} = \frac{1}{\sqrt{\alpha \frac{\alpha}{\alpha} \frac{1}{\alpha}}} \times 50 \Omega = 50 \Omega$$



$$R_{in} = \frac{r_{o2} + r_{o1}}{1 + (g_{m1} + g_{mb1})r_{o1}} \approx \frac{r_{o2} + r_{o1}}{(g_{m1} + g_{mb1})r_{o1}}$$

$$R_{in, scaled} \approx \frac{r_{o2} + r_{o1}}{(g_{m1} + g_{mb1, scaled})r_{o1}}$$

$$g_{mb1, scaled} = \frac{\beta_{scaled}}{2\sqrt{2\phi_F + \frac{V_{sub}}{\alpha}}} g_{m1} \quad \beta_{scaled} = \frac{\sqrt{2q\epsilon_{Si} \alpha N_{sub}}}{\alpha C_{ox}}$$

If $\frac{V_{sub}}{\alpha} \gg 2\phi_F \Rightarrow g_{mb1, scaled} = g_{mb1}$.

14.7

$$\frac{g_m}{I_D} \Big|_{sat.} = \frac{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}}{I_D} = \sqrt{\frac{2\mu_n C_{ox} \frac{W}{L}}{I_D}}$$

$$\frac{g_m}{I_D} \Big|_{sub.} \approx \frac{\frac{I_D}{3V_T}}{I_D} = \frac{1}{3V_T}$$

The two are equal at $I_D = \frac{2\mu_n C_{ox} \frac{W}{L}}{(3V_T)^2}$.

14.4

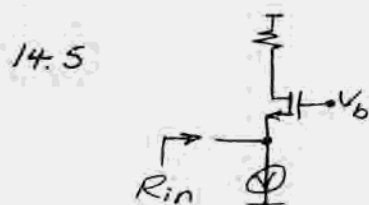
$$I_D = \mu_n C_{ox} \frac{W}{L} V_T^2 \left(\exp \frac{V_{GS} - V_{TH}}{3V_T} \right) \left(1 - \exp \frac{-V_{DS}}{V_T} \right)$$

$$C_{ox} = \sqrt{\epsilon_{Si} q N_{sub} / (4\phi_B)} \quad N_{sub} \rightarrow \alpha N_{sub}, \quad \phi_B \sim \text{constant.}$$

$$V_{GS} - V_{TH} \rightarrow \frac{V_{GS} - V_{TH}}{\alpha} \quad \beta = 1 + \frac{C_{ox}}{\alpha} \rightarrow 1 + \frac{\sqrt{\alpha} C_{ox}}{\alpha}$$

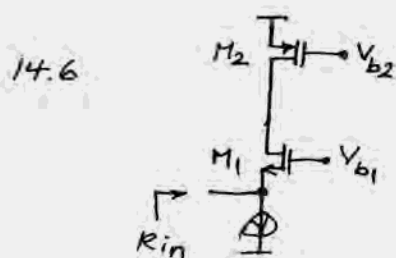
$$V_{DS} \rightarrow \frac{V_{DS}}{\alpha}$$

$$S_{scaled} = 2.3 V_T \left(1 + \frac{\sqrt{\alpha} C_{ox}}{\alpha} \right) \Rightarrow S \downarrow, \text{ i.e., subthreshold behavior improves.}$$



$$R_{in} = \frac{1}{\sqrt{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}} = 50 \Omega$$

$$R_{in, scaled} = \frac{1}{\sqrt{\alpha \frac{\alpha}{\alpha} \frac{1}{\alpha}}} \times 50 \Omega = 50 \Omega$$



$$R_{in} = \frac{r_{o2} + r_{o1}}{1 + (g_{m1} + g_{mb1})r_{o1}} \approx \frac{r_{o2} + r_{o1}}{(g_{m1} + g_{mb1})r_{o1}}$$

$$R_{in, scaled} \approx \frac{r_{o2} + r_{o1}}{(g_{m1} + g_{mb1, scaled})r_{o1}}$$

$$g_{mb1, scaled} = \frac{\beta_{scaled}}{2\sqrt{2\phi_F + \frac{V_{sub}}{\alpha}}} g_{m1} \quad \beta_{scaled} = \frac{\sqrt{2q\epsilon_{Si} \alpha N_{sub}}}{\alpha C_{ox}}$$

$$\text{If } \frac{V_{sub}}{\alpha} \gg 2\phi_F \Rightarrow g_{mb1, scaled} = g_{mb1}$$

14.7

$$\frac{g_m}{I_D} \Big|_{sat.} = \frac{\sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}}{I_D} = \sqrt{\frac{2\mu_n C_{ox} \frac{W}{L}}{I_D}}$$

$$\frac{g_m}{I_D} \Big|_{sub.} \approx \frac{\frac{I_D}{3V_T}}{I_D} = \frac{1}{3V_T}$$

The two are equal at $I_D = \frac{2\mu_n C_{ox} \frac{W}{L}}{(3V_T)^2}$.

14.8 Since $I = v \cdot Q$, if Q drops to zero, $v \rightarrow \infty$. But the velocity is limited to v_{sat} . Thus, at the pinch-off point, the charge density is not zero. Carriers reach their saturated velocity and shoot through the depletion region surrounding the drain.

$$14.9 \quad I_D = \frac{1}{2} \frac{\mu_0 C_{ox}}{1 + \theta(V_{GS} - V_{TH})} \frac{W}{L} (V_{GS} - V_{TH})^2$$

$$\frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2} \mu_0 C_{ox} \frac{W}{L} \left[\frac{-\theta(V_{GS} - V_{TH})^2}{[1 + \theta(V_{GS} - V_{TH})]^2} + \frac{2(V_{GS} - V_{TH})}{1 + \theta(V_{GS} - V_{TH})} \right]$$

$$= \mu_0 C_{ox} \frac{W}{L} \frac{V_{GS} - V_{TH}}{1 + \theta(V_{GS} - V_{TH})} \left[1 - \frac{\theta(V_{GS} - V_{TH})}{1 + \theta(V_{GS} - V_{TH})} \right]$$

$$= \mu_0 C_{ox} \frac{W}{L} \frac{V_{GS} - V_{TH}}{1 + \theta(V_{GS} - V_{TH})} \frac{1 + \frac{\theta}{2}(V_{GS} - V_{TH})}{1 + \theta(V_{GS} - V_{TH})}$$

$$= \frac{2I_D}{V_{GS} - V_{TH}} \cdot \frac{1 + \frac{\theta}{2}(V_{GS} - V_{TH})}{1 + \theta(V_{GS} - V_{TH})}$$

For small overdrives, $g_m \rightarrow \frac{2I_D}{V_{GS} - V_{TH}}$. For large overdrives,

$$g_m \rightarrow \frac{I_D}{V_{GS} - V_{TH}}$$

14.10 Using the results of Prob. 14.9 and replacing θ with

$\frac{\mu_0}{2v_{sat}L} + \theta$, we have:

$$g_m = \frac{I_D}{V_{GS} - V_{TH}} \frac{2 + \left(\frac{\mu_0}{2v_{sat}L} + \theta\right)(V_{GS} - V_{TH})}{1 + \left(\frac{\mu_0}{2v_{sat}L} + \theta\right)(V_{GS} - V_{TH})}$$

$$= \frac{I_D}{V_{GS} - V_{TH}} \left[1 + \frac{1}{1 + \left(\frac{\mu_0}{2v_{sat}L} + \theta\right)(V_{GS} - V_{TH})} \right]$$

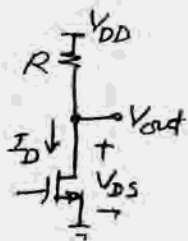
14.11

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \left(1 + \frac{\lambda}{1+k} V_{DS}\right)$$

$$r_o^{-1} = \frac{\partial I_D}{\partial V_{DS}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 \frac{\lambda V_{DS}}{(1+k V_{DS})^2}$$

$$\approx I_D \frac{\lambda V_{DS}}{(1+k V_{DS})^2}$$

$$\Rightarrow r_o = \frac{1}{\lambda \frac{I_D V_{DS}}{(1+k V_{DS})^2}}$$



$$I_D \approx \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 [1 + \lambda V_{DS} (1 - k V_{DS})] \quad \text{if } k V_{DS} \ll 1$$

$$= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS} - \lambda k V_{DS}^2)$$

We note that the voltage across $R_D = V_{DD} - I_D R_D$
 $= V_{DD} - \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS} - \lambda k V_{DS}^2) R_D$.
 Thus, even if V_{GS} changed by a very small value,
 the nonlinear dependence on V_{DS} results in nonlinearity
 in the voltage across R_D .

14.12

$$(a) \quad g_m = v_{sat} W C_{ox} \Rightarrow |A_V| \approx v_{sat} W C_{ox}$$

$$(b) \quad |A_V| \approx \frac{g_{m1}}{g_{m3}} = \frac{v_{sat} W_1 C_{ox}}{v_{sat} W_3 C_{ox}} = \frac{W_1}{W_3}$$

14.13

$$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \mu C_{ox} \frac{W}{L} \left(-\frac{2}{3} \gamma \left(\frac{-3/2}{\sqrt{V_{DS} - V_{BS} + 2\phi_F}} + \frac{-3/2}{\sqrt{-V_{BS} + 2\phi_F}} \right) \right)$$

$$= \mu C_{ox} \frac{W}{L} \gamma \frac{\sqrt{-V_{BS} + 2\phi_F} - \sqrt{V_{DS} - V_{BS} + 2\phi_F}}{\sqrt{V_{DS} - V_{BS} + 2\phi_F} \sqrt{-V_{BS} + 2\phi_F}}$$

$$14.14 \quad \frac{\partial E_g}{\partial T} = -7.02 \times 10^{-4} \frac{2T(T+1108) - T^2}{(T+1108)^2}$$

$$= -7.02 \times 10^{-4} \frac{T^2 + 2216T}{(T+1108)^2}$$

For example, at $T=300^\circ\text{K}$, $\frac{\partial E_g}{\partial T} = -0.267 \text{ meV/K}$

For bandgap references, Eq. (11.10) must be modified:

$$\frac{\partial I_s}{\partial T} = b(4+m)T^{3+m} \exp \frac{-E_g}{kT} + bT^{4+m} \left(\exp \frac{E_g}{kT} \right) \left(\frac{E_g}{kT^2} - \frac{1}{kT} \frac{\partial E_g}{\partial T} \right)$$

$$\Rightarrow \frac{V_T}{I_s} \cdot \frac{\partial I_s}{\partial T} = (4+m) \frac{V_T}{T} + \left(\frac{E_g}{kT^2} - \frac{1}{kT} \frac{\partial E_g}{\partial T} \right) V_T$$

$$\Rightarrow \frac{\partial V_{BE}}{\partial T} = \frac{V_{BE} - (4+m)V_T}{T} - \left(\frac{E_g}{kT^2} - \frac{1}{kT} \frac{\partial E_g}{\partial T} \right) V_T$$

Thus, the TC of V_{BE} is slightly more positive.

$$14.15 \quad (a) \quad |A_v| = \frac{g_{m1}}{g_{m2}} = \sqrt{\frac{\mu_n C_{ox} \left(\frac{W}{L}\right)_1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2}} \Rightarrow |A_v| \text{ is highest for fast N, slow P, etc.}$$

Input thermal noise voltage:

$$\overline{v_n^2} = 4kT \frac{2}{3g_{m1}} + 4kT \frac{2}{3} \frac{g_{m2}}{g_{m1}^2} \Rightarrow v_n \text{ is lowest for fast N, slow P, etc.}$$

(b) $|A_v| = g_{m1} (r_{o1} || r_{o2}) \Rightarrow |A_v|$ highest for fast N.
input noise: same as above.

$$14.16 \quad (a) \quad \text{If } V_{GS1} \text{ and } V_{GS2} \text{ are constant} \Rightarrow g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

$$\Rightarrow |A_v| = \frac{\mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})_1}{\mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{GS} - V_{TH})_2} \rightarrow |A_v| \text{ highest for fast N, slow P, etc.}$$

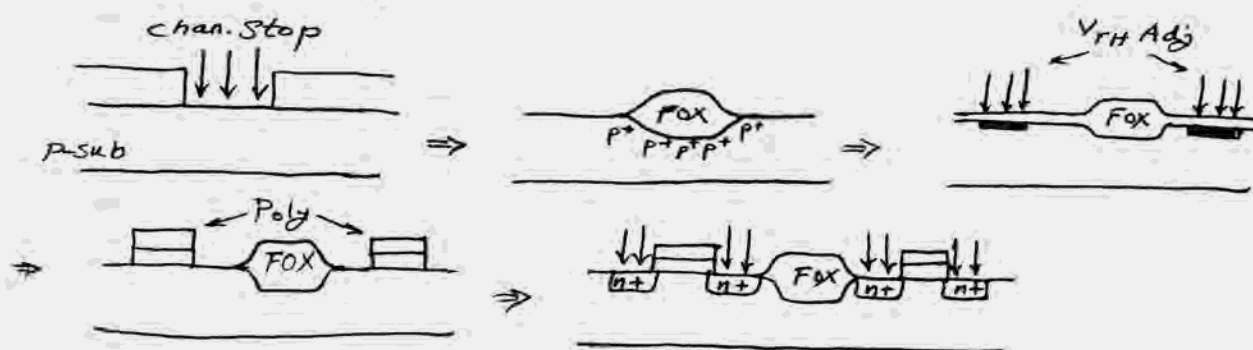
same result for thermal noise.

$$(b) \quad |A_v| = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})_1 \left[\frac{1}{\lambda_1 \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})_1^2} \parallel \frac{1}{\frac{\lambda_2}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{GS} - V_{TH})_2^2} \right]$$

$\Rightarrow |A_v|$ highest for fast N and slow P, etc.
Noise is as before.

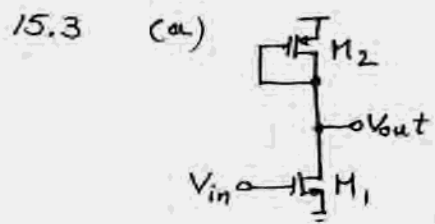
Chapter 15

15.1 Simplifying the flow shown in Fig. 15.8, we note that n-well is not necessary.



The back-end processing is similar to that shown in Figs. 15.10 and 15.11. Thus, the process requires one fewer mask.

15.2 Since the dopants are not concentrated near the surface, their effect is less than expected. For example, if the implant aims to increase the threshold of an NFET from zero to 0.5 V, the actual value will be less than 0.5 V.



For M_1 in saturation:

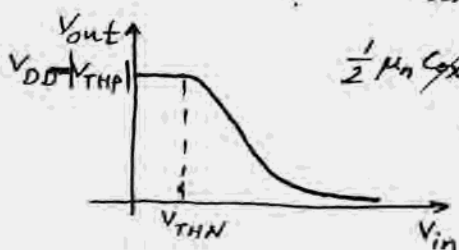
$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{in} - V_{THN})^2 = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{out} - |V_{THP}|)^2$$

\Rightarrow result independent of C_{ox} .

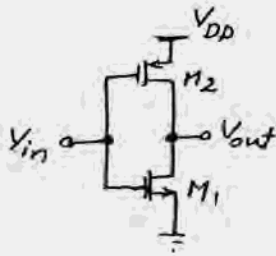
when M_1 enters the triode region:

$$\frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_1 [2(V_{in} - V_{THN})V_{out} - V_{out}^2] = \frac{1}{2} \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{out} - |V_{THP}|)^2$$

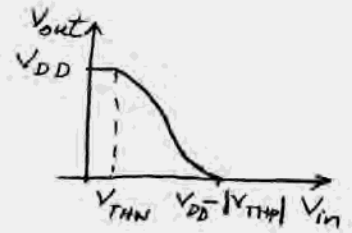
\Rightarrow same.



(b)

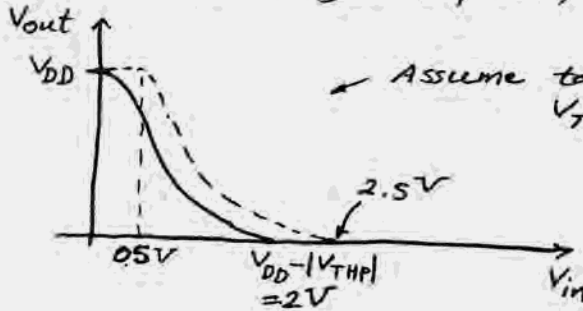


Using similar arguments, one can show that the result does not depend on C_{ox} .



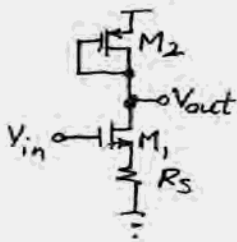
15.4 Without a threshold-adjust implant, $V_{THN} \approx 0$ and $V_{THP} \approx -1V$.

$V_{DD} = 3V$



Assume target values are: $V_{THN} = +0.5V$, $V_{THP} = -0.5V$

15.5



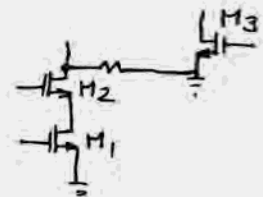
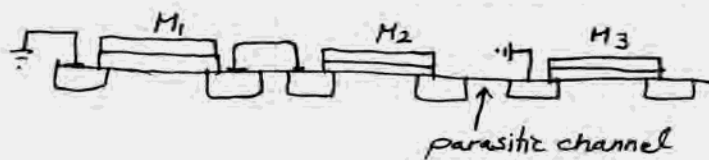
(a) Source of M_1 is spiked to the substrate, shorting R_S out.

(b) Drain of M_2 is spiked to its n-well.

15.6

(a) Channeling during SD implant leads to deep junctions, intensifying DIBL. But the effect is not significant as far as the output impedance is concerned. (Just slightly lower.)

(b) With no channel-stop implant, it is possible that an unrelated high-voltage line passing over the field oxide between the transistors creates a channel between them.:

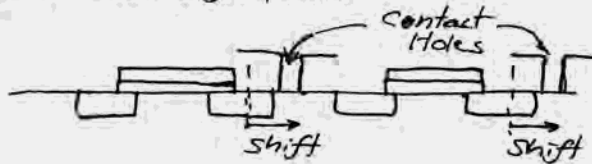


(c) Insufficient gate oxide growth typically does not degrade the output impedance.

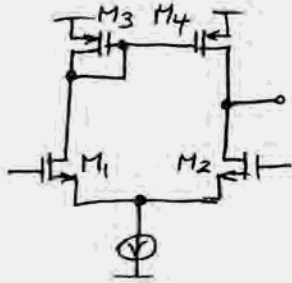
15.7



The zero output current is probably caused by a contact misalignment.



15.8

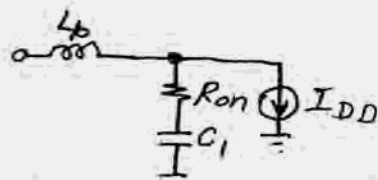


Long gate oxidation cycle is probably the reason: $A_v = g_{m1,2} (r_{o2} || r_{o4})$, $g_{m1,2}$ is lower than expected. The output resistance remains constant or decreases as $t_{ox} \uparrow$.

15.9

(b) If the bottom plate of C_1 is heavily doped, then the oxide grows faster in C_1 , leading to a smaller value for the capacitor. From Chapter 12, we note that if the input capacitance of the op amp is taken into account, then a lower value of C_1 yields a higher gain error.

15.10 (a) $R_{on} = \left[\mu_n C_{ox} \left(\frac{W}{L} \right)_1 (V_{GS} - V_{TH}) \right]^{-1} = 11 \Omega$



$$C_1 = 100 \times 0.34 \times 3.84 + 100 \times 2 \times 0.4 = 210.6 \text{ fF}$$

For critically-damped response: $R_{on} = 2 \sqrt{\frac{L_b}{C_1}} \Rightarrow L_b \leq 6.37 \text{ pH}$.

$$15.11 \quad g_m r \frac{N(N+1)}{2} = 0.01$$

$$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) = 1/(254 \Omega)$$

$$\Rightarrow r = 4.8 \text{ m}\Omega.$$

15.12 $t = 1 \mu\text{m}$, $h = 3 \mu\text{m}$ Parallel Plate $\propto \frac{W}{h}$, the remaining terms determine the fringe capacitance:

$$\frac{W}{3} = 0.77 + 1.06 \left(\frac{W}{3}\right)^{0.25} + 1.06 \left(\frac{1}{3}\right)^{0.5}$$

$$\Rightarrow W \approx 8.25 \mu\text{m}$$

If $h = 5 \mu\text{m}$, then:

$$\frac{W}{8} = 0.77 + 1.06 \left(\frac{W}{8}\right)^{0.25} + 1.06 \left(\frac{1}{8}\right)^{0.5}$$

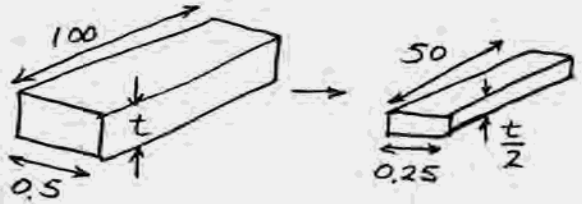
$$\Rightarrow W \approx 19.7 \mu\text{m}$$

$$16.1 \quad R_{\square, \text{poly}} = 30 \Omega/\square \quad R_{\square, \text{M1}} = 80 \text{ m}\Omega/\square$$

$$R_{\square} = \frac{\rho}{t} \quad \Rightarrow \quad \frac{\rho_{\text{poly}}}{\rho_{\text{M1}}} = \frac{R_{\square, \text{poly}} \times t_{\text{poly}}}{R_{\square, \text{M1}} \times t_{\text{M1}}} = \frac{30 \times 0.2}{0.08 \times 1.0} = 75$$

16.2

$$\frac{W}{L} = \frac{100}{0.5} \rightarrow \frac{50}{0.25}$$



The sheet resistivity increases by a factor of 2. Since the number of squares is constant, the total gate resistance also increases by a factor of 2.

16.3

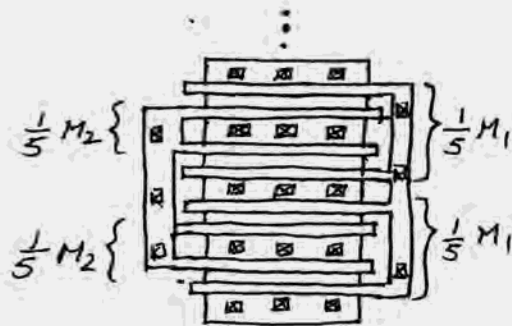
For a total gate resistance of 10Ω , suppose each device consists of N fingers each $\frac{100 \mu\text{m}}{N}$ wide. The total gate resistance is then equal to

$$R_G = \left(\frac{200}{N}\right) \cdot \frac{1}{N} \cdot (5 \Omega/\square)$$

$$= \frac{1000}{N^2} \Omega$$

$$\Rightarrow N = 10$$

From Fig. 16.13 (c), a possible solution is:

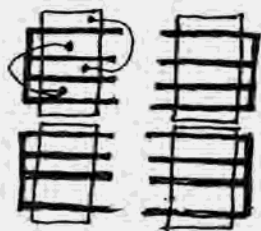


16.4.

- A_1 : a finite resistance may appear between the drains, degrading the voltage gain.
- A_2 : a large resistance may appear ^{in series} with the sources, introducing unwanted degeneration or, more importantly, input-referred offset.
- A_3 : Gate of NMOS current source on the bottom may be shorted to its source.
- A_4 : Part of contact hole may fall on FOX, increasing the contact resistance: source degeneration or offsets.
- A_5 : If the poly contact area is too close to the active area, the active area may be damaged during the etching of poly \Rightarrow offsets, even poor transistor operation.
- A_6 : Latch-up may occur.
- A_7 : Latch-up may occur.
- A_8 : A finite resistance may appear between the gates of the input transistors.

16.5.

In principle, only two layers of interconnect are sufficient for any routing. However, for reasonable symmetry, interconnect resistance, and area, approximately four layers are needed here.



16.6 In Fig. 6.22, temp. gradients introduce threshold and mobility mismatch between M_{REF} and each of $M_1 - M_n$. Thus, the output currents suffer from additional mismatches.

In Fig. 6.23, temp. gradients have much less effect because M_{REF1} and M_{REF2} are quite close to their mirrors.

16.7 $R = 500 \Omega \Rightarrow$ poly must be $\frac{500}{60}$ squares long and
n-well must be $\frac{500}{2000}$ square long.

Poly width = $3 \mu\text{m} \Rightarrow$ Poly length = $25 \mu\text{m}$

n-well length = $6 \mu\text{m} \Rightarrow$ n-well width = $24 \mu\text{m}$.

\Rightarrow Poly Cap = $3 \times 25 \times 100 \text{ aF}/\mu\text{m}^2 = 7.5 \text{ fF}$

n-well Cap = $6 \times 24 \times 1000 \text{ aF}/\mu\text{m}^2 = 144 \text{ fF}$.

Thus, the poly structure is preferable.

16.8 Assuming $C_1 = C_2 = C_3 = 40 \text{ aF}/\mu\text{m}^2$ and $C_4 = 60 \text{ aF}/\mu\text{m}^2$ in Fig. 16.34(d), we have (neglecting fringe cap.):

Fig. 16.34(a): $C_1 = 40 \text{ aF}/\mu\text{m}^2$, $C_p = 9 \text{ aF}/\mu\text{m}^2$

(b): $C_1 + C_2 = 80 \text{ aF}/\mu\text{m}^2$, $C_p = 15 \text{ aF}/\mu\text{m}^2$

(c): $C_1 + C_2 + C_3 = 120 \text{ aF}/\mu\text{m}^2$, $C_p = 30 \text{ aF}/\mu\text{m}^2$

(d): $C_1 + \dots + C_4 = 180 \text{ aF}/\mu\text{m}^2$, $C_p = 90 \text{ aF}/\mu\text{m}^2$

Thus, the lowest c_p/c occurs for (b).

16.9 Wire Propagation Delay $\approx \frac{R_{tot} C_{tot}}{2} = \frac{40 \times 37 \text{ fF}}{2}$
 $= 0.74 \text{ ps}$

Lumped Delay $\approx 500 \Omega \times 37 \text{ fF}$
 $= 18.5 \text{ ps}$

Thus, the propagation delay thru the wire is negligible.

$$16.10 \quad \text{Wire Delay} \approx \frac{20 \Omega \times 44 \text{ aF}}{2} \\ = 0.44 \text{ ps}$$

$$\text{Lumped Delay} \approx 22 \text{ ps}$$

$$16.11 \quad \text{Metal 1: } C_{\text{tot}} = (1000 \mu\text{m} \times 0.35 \mu\text{m} \times 30 \text{ aF}/\mu\text{m}^2) + 1000 \mu\text{m} \times 80 \text{ aF}/\mu\text{m} \\ = 90.5 \text{ fF}$$

$$\text{Metal 2: } C_{\text{tot}} = (1000 \mu\text{m} \times 0.45 \mu\text{m} \times 15 \text{ aF}/\mu\text{m}^2) + 1000 \mu\text{m} \times 50 \text{ aF}/\mu\text{m} \\ = 56.75 \text{ fF}$$

$$\text{Metal 3: } C_{\text{tot}} = (1000 \mu\text{m} \times 0.5 \mu\text{m} \times 9 \text{ aF}/\mu\text{m}^2) + 1000 \mu\text{m} \times 40 \text{ aF}/\mu\text{m} \\ = 44.5 \text{ fF}$$

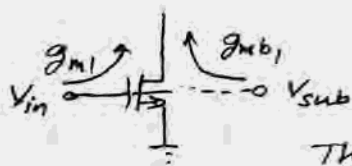
$$\text{Metal 4: } C_{\text{tot}} = (1000 \mu\text{m} \times 0.6 \mu\text{m} \times 7 \text{ aF}/\mu\text{m}^2) + 1000 \mu\text{m} \times 30 \text{ aF}/\mu\text{m} \\ = 34.2 \text{ fF}$$

Thus, metal 4 provided the smallest delay.

16.12 The results do not change because the capacitance of metal 4 is still largest.

$$16.13 \quad (W/L)_1 = 100/0.5, I_{D1} = 1 \text{ mA} \Rightarrow g_{m1} = \sqrt{2 \times 1 \text{ mA} \times \frac{100}{0.34} \times 134 \mu\text{A}/\text{V}^2} \\ = 8.88 \text{ mS}$$

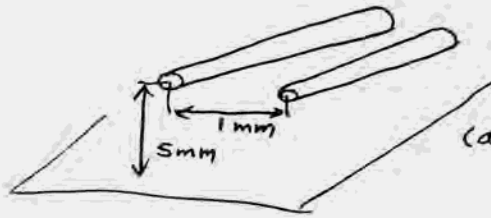
$$g_{mb1} = \frac{\gamma g_{m1}}{2 \sqrt{V_{SB} + |2\phi_F|}} = \frac{0.45}{2 \sqrt{0.9}} \times 8.88 \text{ mS} = 2.11 \text{ mS}$$



V_{sub} generates a drain current of $g_{mb1} V_{\text{sub}}$.

Thus, referred to the gate, the effect is equal to $\frac{g_{mb1}}{g_{m1}} = \frac{\gamma}{2 \sqrt{|2\phi_F|}} = 4.21^{-1} \Rightarrow$ input-referred noise = 11.9 mV_{pp}.

16.14.



$$(a) \quad L_m = 0.1 \ln \left[1 + \left(\frac{10}{1} \right)^2 \right] \times 4 \text{ mm} \\ = 1.85 \text{ nH.}$$

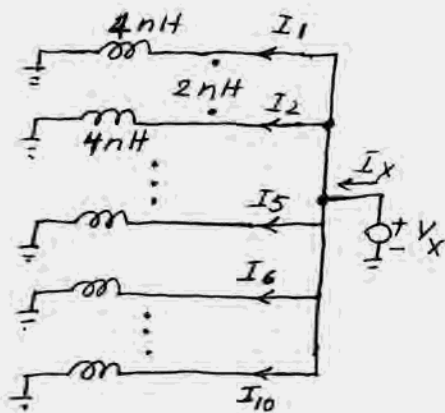
$$(b) \quad V = L_m \frac{dI}{dt} \\ = 1.85 \text{ nH} \times 2\pi \times 10^8 \times 1 \mu\text{A} \\ = 1.16 \text{ mVp.}$$

16.15 L_m must decrease by a factor of 4. \Rightarrow

$$0.1 \ln \left[1 + \left(\frac{2h}{d} \right)^2 \right] \times 4 \text{ mm} = \frac{1.85}{4}$$

$$\Rightarrow \frac{2h}{d} = 1.476 \Rightarrow d = 6.78 \text{ mm.}$$

16.16 (a)



By symmetry:

$$I_1 = I_{10}, I_2 = I_9, \dots, I_5 = I_6$$

we then construct equations for

$$I_1 - I_5$$

$$\begin{cases} (4 \text{ nH}) s I_1 + (2 \text{ nH}) s I_2 = V_x \\ (4 \text{ nH}) s I_2 + (2 \text{ nH}) s I_1 + (2 \text{ nH}) s I_3 = V_x \\ \vdots \end{cases}$$

$$\Rightarrow I_1 = \frac{5}{22} \frac{V_x}{s}, I_2 = \frac{1}{22} \frac{V_x}{s}, I_3 = \frac{4}{22} \frac{V_x}{s}, I_4 = \frac{2}{22} \frac{V_x}{s}, I_5 = \frac{3}{22} \frac{V_x}{s}$$

$$I_x = 2(I_1 + \dots + I_5) \Rightarrow L_{eq} = \frac{22}{30} \text{ nH for each of ground and } V_{DD} \text{ lines.}$$

$$16.17 \quad (a) \quad L_a = 0.2 \ln \frac{2h}{25 \mu\text{m}} \quad \text{nH} \quad C_a = 100^2 C_0$$

$$(b) \quad L_b = 0.2 \ln \frac{2h}{12.5 \mu\text{m}} \quad \text{nH} \quad C_b = 50^2 C_0$$

$$\frac{L_a C_a}{L_b C_b} = \frac{\ln \frac{2h}{25}}{\ln \frac{2h}{12.5}} \cdot \frac{4}{1} \quad \text{Is the first fraction greater or less than 1/4?}$$

$$\frac{\ln \frac{2h}{25}}{\ln \frac{2h}{12.5}} = \frac{1}{4} \Rightarrow h \approx 15.7 \mu\text{m}$$

Thus, for $h > 15.7 \mu\text{m}$ (which is quite realistic), case (b) is certainly preferable. For $h < 15.7 \mu\text{m}$, case (a) may be preferable.

Design of Analog CMOS Integrated Circuits

Behzad Razavi

Errata in Problem Sets

Chapter 2

- In Eq. (2.44), μ_n must be in the numerator.

Chapter 3

- Call the third problem 3.2'.
- In Problem 3.2, Fig. 3.68(d), change the gate voltage of M_2 to V_{b2} .
- In Problem 3.4, Fig. 3.71(a), change the gate voltage of M_1 to V_{b1} .
- In Fig. 3.72(e), V_{b1} must be changed to V_{in} .
- In Fig. 3.73(h), the output is at the source of M_2 .
- In Problem 3.10(c), the question must be phrased as: Which device enters the triode region first as V_{out} falls?
- In Problem 3.13, first sentence should read: ... with $W/L = 50/0.5$...
- In Problem 3.16(a), do not neglect channel-length modulation in the triode region.

Chapter 4

- In Problem 4.2, assume $I_{SS} = 1$ mA and change part (a) to: Determine the voltage gain.
- In Problem 4.6, assume $\lambda = 0$.
- In Problem 4.9, assume $\lambda = \gamma = 0$.
- In Problem 4.11, assume $I_{D5} = 20$ μ A.
- In Problem 4.13, change the figure number to 4.8(a).

Chapter 5

- In Problem 5.16(d), assume V_{TH} does not vary with temperature.

Chapter 6

- In Problem 6.4(b) and (d), assume $\lambda \neq 0$.

Chapter 7

- The second sentence of Problem 7.2 should read: Assume $(W/L)_1 = 50/0.5$, $I_{D1} = I_{D2} = 0.1$ mA ...

- In Problem 7.20, change I_{D1} and I_{D2} to 0.05 mA.

- In Problem 7.24, change the bias current to 0.1 mA.

Chapter 8

- In Problem 8.10, change the tolerable gain error to 5%.
- In Problem 8.15, Fig. 8.55(b), call label the top G_m block G_{m2} . The output is at the output nodes of G_{m2} .

Chapter 10

- In Problem 10.11, change I_{SS} to 0.25 mA and $(W/L)_{5,6}$ to 60/0.5.
- In Problem 10.12, add: Maximize $V_{GS14} = V_{GS15}$ while leaving at least 0.5 V across I_1 . Also, in part (b), change M_2 to M_1 .
- Problem 10.17 should read: ... between the gate and the drain of M_2 or M_3 .

- In Fig. 10.42, change the gate voltage of $M_{3,4}$ to V_{b1} .

- In Problem 10.19(c), change A_0 in the numerator to A .

Chapter 11

- In Problem 11.13, ... such that the circuit operates with $V_{DD} = 3$ V.
- In Problems 11.17 and 11.18, the top terminal of R_2 should be connected to the top terminal of R_1 .
- In Problem 11.22, assume $K = 4$.

Chapter 12

- In Problem 12.8, assume $C_H = 1$ pF.
- In Problem 12.12, assume all switches are NMOS devices.
- In Problem 12.14, assume $C_{in} = 0.2$ pF and calculate C_1 and C_2 .
- In Problem 12.16, the output is sensed at the drains of M_1 and M_2 .

Chapter 13

- In Problem 13.5, change the figure number to 13.6(a).