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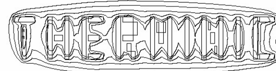

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TRANSIENT THERMAL SIMULATION OF A POWER HYBRID MODULE USING AN ANALYTICAL SOLUTION OF THE HEAT EQUATION

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ABSTRACT

This paper presents a method for the analysis of transient thermal states in electronic circuits using an analytical solution of the heat equation. The time dependent, three-dimensional temperature fields in multilayered structures can be computed analytically using the Green's functions. The proposed method is illustrated based on a practical example, where the results of thermal simulations of a real hybrid module are compared with infrared temperature measurements. Additionally, the discussion of simulation errors caused mainly by different non-linear phenomena is included.

1. INTRODUCTION

Owing to the latest technological advances it was possible to miniaturize electronic devices and drastically increase their operating frequency thus augmenting the dissipated power density and as a result circuit temperature. Because elevated operating temperatures can have serious impact on circuit performance or even can lead to its destruction, the analysis of thermal phenomena has become a very important issue in modern electronics. Consequently, the urgent need has arisen for reliable circuit thermal models and simulation tools capable of performing relatively fast but accurate thermal analyses of electronic circuits, which are required both in the process of circuit design and for the on-line circuit temperature monitoring purposes. One of such models was employed in this paper by the authors for thermal simulations of a power hybrid module.

Most commercial electro-thermal simulators employ various numerical methods for the solution of the heat equation resulting from the adopted circuit thermal model. In order to obtain exact results using numerical methods, it is necessary to apply a sufficiently dense discretisation mesh especially where the temperature gradient values are important. This, however, significantly increases both the memory demand and the simulation time. Thus, regarding costs and the time of an entire circuit design process, the analytical solutions yielding explicit formulas relating the dissipated power density to the temperature rise at any point of a circuit are much more desirable, but in most cases they are difficult to find.

Fortunately, there exists a large variety of circuits having simple geometrical shapes, which can be simulated with satisfactory accuracy employing thermal models, for which it is possible to find analytical solutions of the heat equation. Then, when the thermal model is not excessively simplified, the analytical solutions produce more accurate results than the numerical ones, which are inherently not exact and depend on the choice of the discretisation mesh. The main goal of this paper is to present a thermal model allowing analytical solutions of the heat equation suitable for fast circuit thermal simulation purposes.

The contents of the paper can be divided into two main parts: the theoretical and the experimental ones. The first part covers the theory concerning the methodology of solving the heat equation in non-homogeneous solids using the analytical Green's functions. This is followed by the description of the particular circuit thermal model used in the subsequent simulations and its solution. Next, in the experimental part of the paper, all the mathematical apparatus is employed for thermal simulations of transient processes occurring in a real hybrid power module. The transient thermal simulations are compared with infrared measurements providing some important conclusions and indications for future work.

2. CIRCUIT THERMAL MODELLING

The main purpose of this theoretical part is to introduce a methodology for solving analytically transient thermal problems in multilayered structures. This part of the paper will be divided into several subsections. First, the three-dimensional heat equation, modelling thermal processes occurring in solids, will be presented. Next, the Green's functions, on which the proposed solution method of the

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heat equation is based, will be introduced. Finally, the solution of the heat equation for the particular circuit thermal model will be found determining the appropriate Green's functions.

2.1. Heat equation in non-homogenous solids

For the heat conduction processes, obeying the Fourier's law of conduction stating that the heat flux is directly proportional to the temperature gradient, the elementary energy balance performed for a unitary solid volume leads to the following three-dimensional Fourier-Kirchhoff heat equation [1]-[4]:

$$\nabla\circ\left[\lambda(r)\,\nabla T\right]+g_v(r,t)=c_v(r)\frac{\partial T}{\partial t} \tag{1}$$

where:
λ - thermal conductivity [W/mK]; t – time [s];
r – position vector [m]; T – temperature [K];
c_v – specific heat per unit volume [J/m^{3}K];
g_v – generated heat per unit volume [W/m^3];

The solution of Equation 1 can be determined given material thermophysical properties as well as initial and boundary conditions. The initial conditions determine the temperature distribution in a structure at the starting point of the thermal analysis. The boundary conditions describe the heat exchange with the surrounding fluid at the outer structure surfaces. Generally, there are distinguished three fundamental types of boundary conditions: the Dirichlet condition (prescribed surface temperature), the Neumann condition (prescribed surface heat flux) and the Robin condition (convective heat exchange).

2.2. Green's functions

Green's functions (GFs) are versatile mathematical tools suitable for obtaining solutions of linear heat conduction problems. There exist two possible interpretations of GFs. Firstly, they can be regarded as a temperature response in a point x at time t caused by an instantaneous heat generation occurring in a point x' at time τ. Thus, in order to obtain the temperature response in time, it is enough to integrate a GF over the entire volume and time as far as heat is generated. For the heat diffusion equation, a GF describes also the temperature distribution at a point x in time t due to an initial temperature rise at a point x'. Then, the entire temperature field can be computed as an integral of a GF evaluated at the analysis initial time over the entire domain.

The GFs can be derived using different methods, such as the method of images, the Laplace transform method or the Fourier method of separation of variables. All these methods yield solutions in different but mathematically equivalent forms. From the computational point of view, the main difference between all these methods is the rate of series convergence. Generally, the first two methods are rapidly convergent for short times whereas the Fourier method produces series, which are better convergent for large times, and thus are more convenient for thermal simulations of electronic circuits.

Particular GFs depend on the structure geometry and the applied boundary conditions. Therefore, the same GF can be used then for solving different problems. For linear problems the overall temperature rise can be found as the sum of the individual temperature rises caused by the various influencing factors, such as the initial temperature distribution, the internal heat generation and some non-homogeneous boundary conditions [5]-[7].

Taking into account that one-dimensional GFs are geometry dependent solutions of the heat equation with homogeneous boundary conditions, they can be easily tabulated and used for the construction of other more sophisticated solutions. For instance, multi-dimensional GFs can be determined in many cases, especially in the rectangular co-ordinate systems, as the products of one-dimensional GFs. More information on the GFs and the particular methods of deriving them can be found in [5].

2.3. Green's function solution

The procedure of solving Equation 1 employing Green's functions begins with the solution of the complementary Equation 2, which is almost identical with the original one except for the fact that there is no internal heat generation and that the boundary conditions are of the same type but homogeneous ones [5].

$$\nabla\circ\left[\lambda(r)\,\nabla\Theta\right]=c_v(r)\frac{\partial \Theta}{\partial t} \tag{2}$$

According to the separation of variables method, the temperature Θ being the solution of the complementary equation, can be expressed as follows:

$$\Theta(r,t)=\sum_n A_n\;\varphi_n(r)\exp(-\gamma_n\;t) \tag{3}$$

where:
A – series coefficients; n – series index;
φ – eigenfunctions; γ – eigenvalues;

Substituting the solution form into Equation 2 leads to the eigenvalue problem, which can be solved following the so-called Galerkin procedure by the integration over the entire structure volume. Finally, the procedure yields the following formula to compute the eigenvalues γ_n [5]:

$$\gamma_n=\frac{-\int_v\varphi_n\,\nabla\circ\left(\lambda(r)\nabla\varphi_n\right)dV}{\int_v c_v(r)\varphi_n^2\;dV} \tag{4}$$

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Typically, pursuing the Fourier method, the next step would be to determine, applying the initial condition, the unknown series coefficients A_n and hence finding the complementary solution. However, in the considered case, the appropriate GFs have to be found before. This is done repeating the entire procedure with the substitution of the solution and the integration over the volume, however this time for Equation 1. Then, after algebraic manipulations, the unknown GFs can be obtained using Equation 5 [5]. The remaining part of the solution for the case considered here will be presented after the description of the circuit thermal model.

$$G(r,t/r',\tau)=c_v(r)* \sum_n \frac{\varphi_n(r)\,\varphi_n(r')}{\int_v c_v(r)\,\varphi_n^2(r)\,dV}\exp(-\gamma_n\,(t-\tau)) \quad (5)$$

2.4. Circuit thermal model

The particular hybrid circuit considered in this publication has rectangular shapes and can be approximated, as many other electronic circuits, by a multilayered parallelepiped. The model, owing to its simplicity, allows the application of the Green's function analytical approach yet provides satisfactory accuracy. The dissipated power is represented in the model by the heat flux penetrating into the structure through its top surface. Since most circuits are relatively thin in comparison to their area, the four lateral surfaces are assumed to be adiabatic ones and the heat is removed only at the remaining surfaces, according to the Newton's law, according to which the heat flux is proportional to the temperature difference between the circuit surface and surrounding fluid [8]-[9].

The entire set of equations describing the proposed thermal model and its boundary conditions is summarised in Equations 6. Additionally, the thermal model is shown in Figure 1. The imperfect contact between the structure layers is characterised by the thermal conductance g. The letter i and the numbers 1 and 2 are the layer indices.

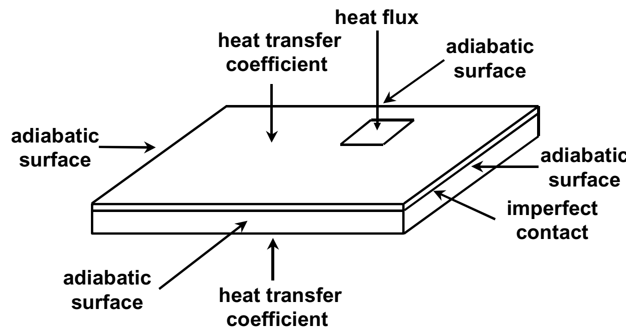

Figure 1: Circuit thermal model.

Governing equation: $$\nabla\circ[\lambda_i\,\nabla T_i]=c_{vi}\frac{\partial T_i}{\partial t} \quad (6)$$

Boundary conditions:

Top surface: $$-\lambda_1\frac{\partial T_1}{\partial n}=q-h_1\,(T_1-T_\infty) \quad (6a)$$

Lateral surfaces: $$\frac{\partial T_i}{\partial n}=0 \quad (6b)$$

Layer contact: $$\lambda_1\frac{\partial T_1}{\partial n}=g\,(T_1-T_2) \quad (6c)$$

Bottom surface: $$-\lambda_2\frac{\partial T_2}{\partial n}=h_2\,(T_2-T_\infty) \quad (6d)$$

where:

h – heat transfer coefficient [W/m^{2}K];
n – outward drawn normal to surface;
q – heat flux [W/m^2]; *T_∞ – ambient temperature [K];*

2.5. Model solution

For the above-proposed circuit thermal model consisting of two layers, its time dependent solution can be found in accordance with the earlier presented solution outline. First, the three-dimensional eigenfunctions ϕ_n, expressed by Equation 7, were determined computing the unknown values of coefficients C, Γ and β by the application of the specific boundary and contact conditions consistent with the thermal model.

$$\varphi_{m,n,k}(x,y,z)=C_{ik}\cos\left(\frac{m\,\pi}{a}x\right)*\cos\left(\frac{n\,\pi}{b}y\right)* \left(\sin\left(\frac{\beta_k}{d}z\right)+\Gamma_{ik}\cos\left(\frac{\beta_k}{d}z\right)\right) \quad (7)$$

where:

m, n, k – series indices;a, b, d – top layer dimensions [m];

In particular, the eigenvalues in the vertical direction β_k were found as the solutions of the transcendental equation:

$$\lambda_2\left(\left(g-h_1\,\Gamma_{1k}^2\right)tg\,\beta_k+\Gamma_{1k}\,(g+h_1)\right)\left(1-\Gamma_{2k}\,tg\,\beta_k\right) =g\lambda_1\left(1-\Gamma_{1k}\,tg\,\beta_k\right)\left(tg\,\beta_k+\Gamma_{2k}\right) \quad (8)$$

When the temperature rise does not result from the initial temperature distribution and there is no internal energy generation, the only non-homogenous boundary condition is the heat flux diffusing into the structure through its top surface. Then, the final formula to compute temperature rise in the circuit can be expressed in the terms of Green's function as given in Equation 9. More detailed description of the solution method can be found in [10]-[11].

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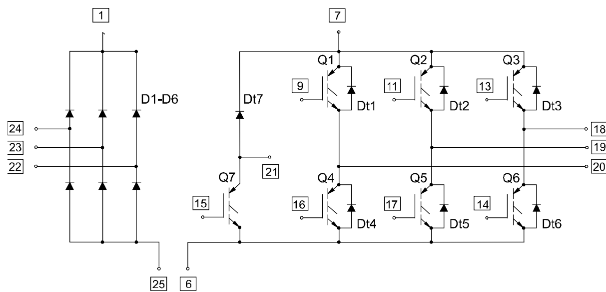

Figure 2: IGBT module electrical scheme.

$$c_v(r)T(r,t) = \int_{\tau=0}^{t} \int_S (q\ G)_{S'}\ dS'\ d\tau \qquad (9)$$

3. MEASUREMENT AND SIMULATION

The earlier presented mathematical considerations will be applied in this section for the analysis of transient thermal process in a hybrid power circuit. After a short description of the circuit, its thermal simulations based on the GF solution of the heat equation resulting from the thermal model will be presented in detail. The simulations will be validated with infrared temperature measurements.

3.1. Circuit description

The analyses presented in this paper concerned an IGBT power hybrid module manufactured in the insulated metal substrate technology [12]. The module is a commercial three-phase AC motor driver consisting of the three-phase input rectifier (diodes D1-D6), the three-phase output inverter (transistors T1-T6 with free-wheel diodes DT1-DT6) and the protection circuit (transistor T7 and diode DT7). All these devices, together with the molybdenum heat spreaders are attached to the common copper frame providing electrical contact. The entire circuit is mounted on a large 50mm x 75mm x 1.5 mm aluminium plate and insulated electrically from the plate by a thin raisin layer. The electrical scheme and the layout of the circuit are given in Figures 2 and 3 respectively.

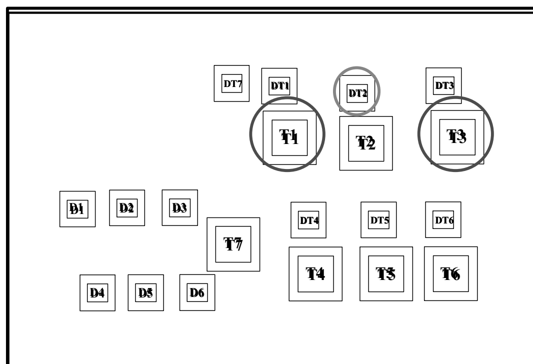

Figure 3: IGBT module layout.

3.2. Thermal model

In order to perform the thermal simulations, an adequate circuit thermal model was required. Originally, a full five-layer model was considered. This model had been solved numerically employing the finite difference method. The preliminary simulations showed that the temperature drop in the top layers (silicon dies, molybdenum heat spreaders and the copper frame) is not significant, thus a simplified model has been used involving only two layers; namely the aluminium base plate and the top layer representing the other layers. The model allows the direct application of the earlier described heat equation solution method based on Green's functions.

The cross-section of the thermal model is presented in Figure 4. The symbols λ, α and h denote the thermal conductivity, the thermal diffusivity and the heat transfer coefficient respectively. The thermal properties of the top layer correspond to the thermal properties of silicon and molybdenum, which are very much alike. The value of the heat transfer coefficient at the top surface represents the typical combined radiation and convection cooling at this temperature whereas the coefficient value at the bottom reflecting the presence of a large aluminum block as well as the contact conductance were set experimentally so that to minimize the relative error with respect to the infrared measurements. Finally, the thermal diffusivity, being the ratio of the thermal conductivity to the thermal capacity, was found from the transient temperature curve so that to obtain the correct temperature rise rate.

3.3. Infrared measurements

All bodies having temperatures higher than the absolute zero emit the thermal electromagnetic waves. According to the Stefan-Boltzmann law the total amount of radiated energy is proportional to the fourth power of temperature. Moreover, it can be calculated form the Planck formula that the spectrum density of the thermal radiation has its maximum in the range of infrared waves. Thus, using appropriate detectors of infrared radiation, it is possible to measure temperature by measuring thermal radiation.

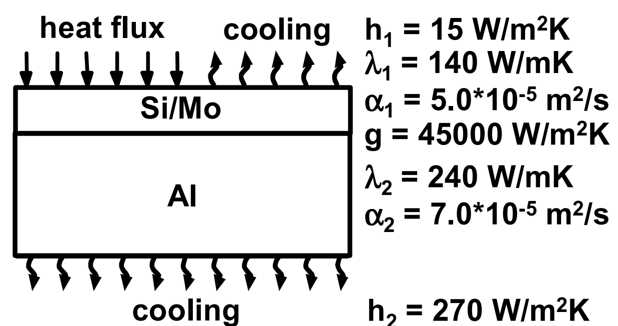

Figure 4: Circuit thermal model.

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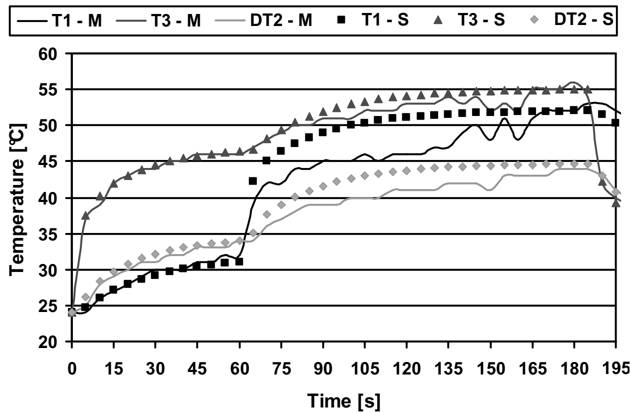

Figure 5: Measured and simulated temperature values.

For the infrared temperature measurements the circuit was placed horizontally on a massive block of aluminum. The transient temperature measurements were taken using the AGEMA Thermovision 900 infrared camera at the rate of 3 images per second. Before the measurements, the circuit was sprayed with black matt paint so as to assure uniform and known surface emissivity [13].

Initially, during the experiment, the power of 8.4 W was dissipated in the transistor T3. Then, after 63.5 s, the transistor T1 was also switched on and dissipating the same amount of power. Finally, after 187 s the transistor T3 was switched off and the power was dissipated only in the transistor T1.

The particular measured temperature curves obtained for the hot spots, i.e. transistors T1 and T3, and the diode DT2 placed between the transistors, encircled in Figure 3, are presented with solid lines in Figure 5. Additionally, an exemplary infrared picture obtained after 180 s is shown in Figure 6.

The presented results revealed that there were some instabilities observed just after switching the transistor T1 on, probably due to unexpected transients occurring in the power supply cables. Moreover, it should be also clearly said that the diode temperature could not be determined accurately from the infrared measurements because of the problems with establishing the exact device location in the registered infrared images.

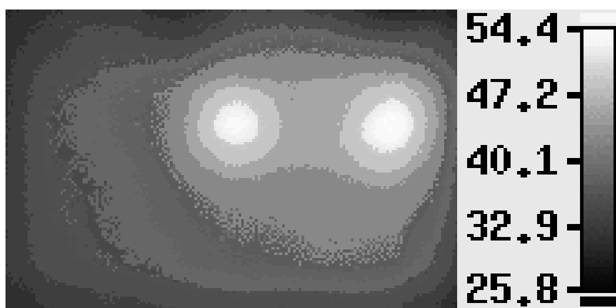

Figure 6: Exemplary infrared picture (after 180s).

3.4. Thermal simulation

First, during the simulations, the temperature values for different time instants were computed in the transistors, where the heat was generated, and at the location of the diode DT2. The simulation results, represented by various markers, are compared with the measurement in Figure 5. Moreover, the temperature map, shown in Figure 7, was computed using the analytical method exactly for the same time instant as the earlier presented infrared image. As can be seen, the simulated temperature values are in relatively good accordance with the infrared measurements. During transient states the temperature differences do not exceed 5 K, which is acceptable considering the limited accuracy of measurements and the relative simplicity of the circuit thermal model. The discrepancies between the simulation and the measurement are clearly much more important for the transistor T1. As already mentioned, this is caused the most probably by the fluctuations of the power dissipated in this transistor observed also in the measurement curves.

The other possible sources of simulation errors result from various non-linearities occurring in the real structure. Because in reality the material thermal properties, such as the conductivity or the diffusivity, depend quite strongly on temperature, the computation of the total temperature rise by the simple addition of temperature rises resulting from the power dissipation in the individual transistors might lead to important errors.

All the presented thermal simulations were performed in the Matlab environment on the computer equipped with the 1.1 GHz AMD Athlon processor and a 512 MB RAM memory. During all the simulations only 20 eigenvalues were considered, because the introduction of additional series components did not change the simulation results by more than 1 K. When the simulation time is concerned, for the transient simulations of 2 heat sources with the resolution of 5 seconds some 10 minutes were required, whereas it took almost 2 hours to compute the 4 000 point temperature map.

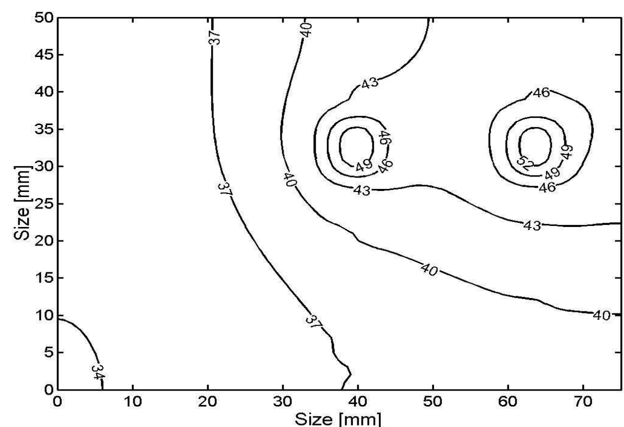

Figure 7: Simulated temperature distribution (after 180s).

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4. CONCLUSIONS

The authors presented in this paper an analytical approach to the thermal analysis of power electronic circuit. Owing to the relative simplicity of the proposed circuit thermal model, it was possible to apply for the solution of the heat equation resulting from the model an analytical method based on the Green's functions. The simulation results are in good agreement with infrared measurements, however the true accuracy of the methodology cannot be assessed because the measurements themselves bear some errors.

The proposed analytical solution is more elegant and has many obvious advantages, e.g. it renders possible the computation of the temperature values only in selected locations from the same mathematical formula without the necessity for solving the entire set of difference equation as it is in the case of various numerical methods. Thus, the presented approach might prove to be competitive with numerical methods, especially when the temperature has to be computed only in a limited number of points, e.g. for the determination of the hot spot temperature. However, numerical methods certainly are more advantageous, if the whole temperature distribution map has to be computed.

The hereby-presented methodology is very powerful and allows thermal analyses of structures having relatively complex shapes and various types of boundary conditions. However, the main difficulty of the method consists in the computation of the eigenvalues. Fortunately, the Galerkin integral procedure is very universal and, even if the exact solutions cannot be determined, quite accurate results can be obtained using linear combinations of non-orthogonal basis functions.

The simulations proved that the model could be used with constant model parameter values only in a limited range of temperatures because the heat transfer processes are strongly temperature-dependent phenomena. However, some of the non-linearities can considered in simulations in a relatively simple way by updating the thermal model parameters with each iteration, as shown in [14].

Moreover, it is worth mentioning that all the physical model parameter values are almost identical with the ones commonly encountered in the literature. The only values which had to be found directly from the measurements were the thermal contact resistance and the heat transfer coefficient value at the bottom of the structure. These two quantities are responsible mainly for the temperature map profile on the circuit top surface and the temperature rise over the ambient temperature.

5. ACKNOWLEDGEMENTS

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